Title: MOND habitats within the solar system

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URL: http://pirsa.org/06020034

Abstract:

MOND habitats within the Solar System

João Magueijo

Perimeter Institute
Imperial College

Based on Bekenstein & Magueijo, **astro-ph/0602266**

The MOND/Dark Matter conflict

- Every gravitating body we see outside the Solar system refuses to follow the laws of General Relativity/ Newtonian gravity

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- Either gravity is fine, but there is an extra source we can't see: the dark matter.

- Or there is no dark matter, and the observations are telling us to modify gravity.

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- LeVerrier prediction of Neptune based on Uranus orbital anomalies (first example of dark matter).

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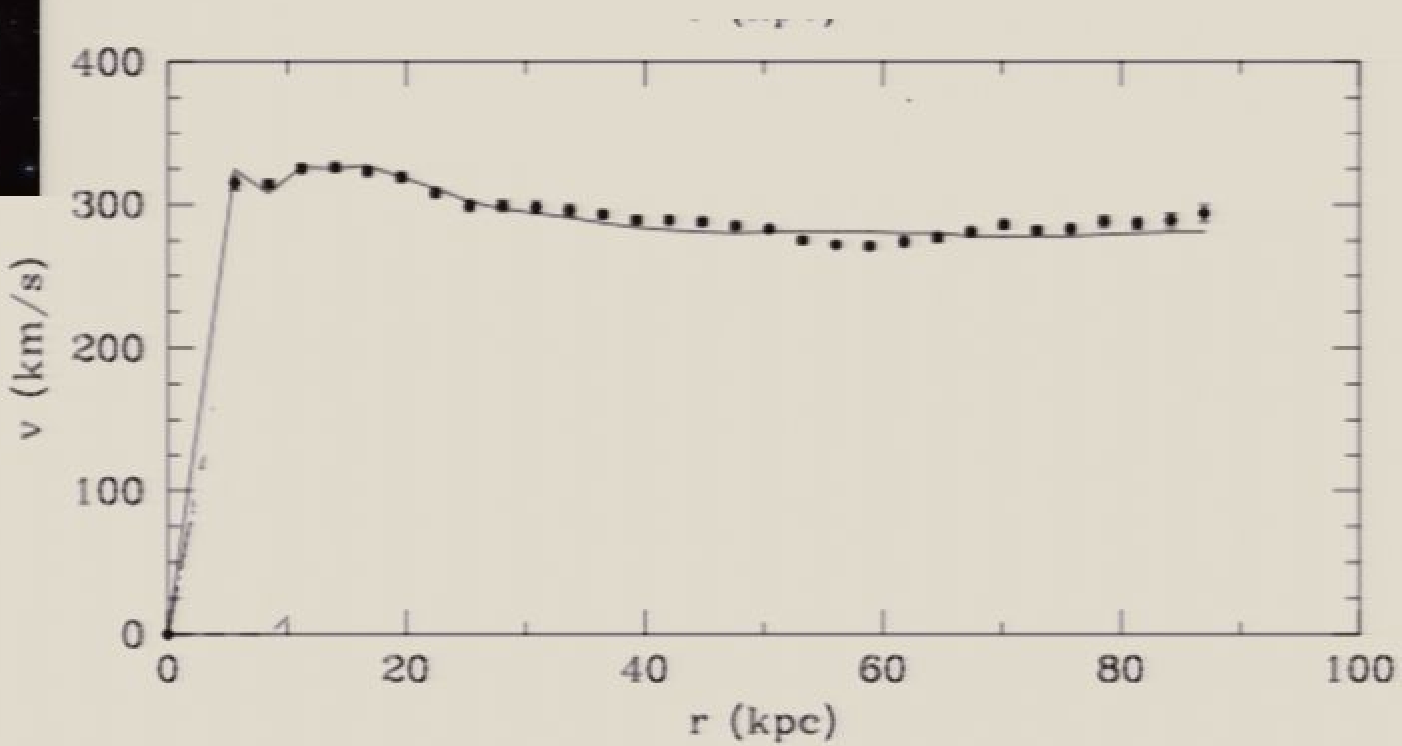
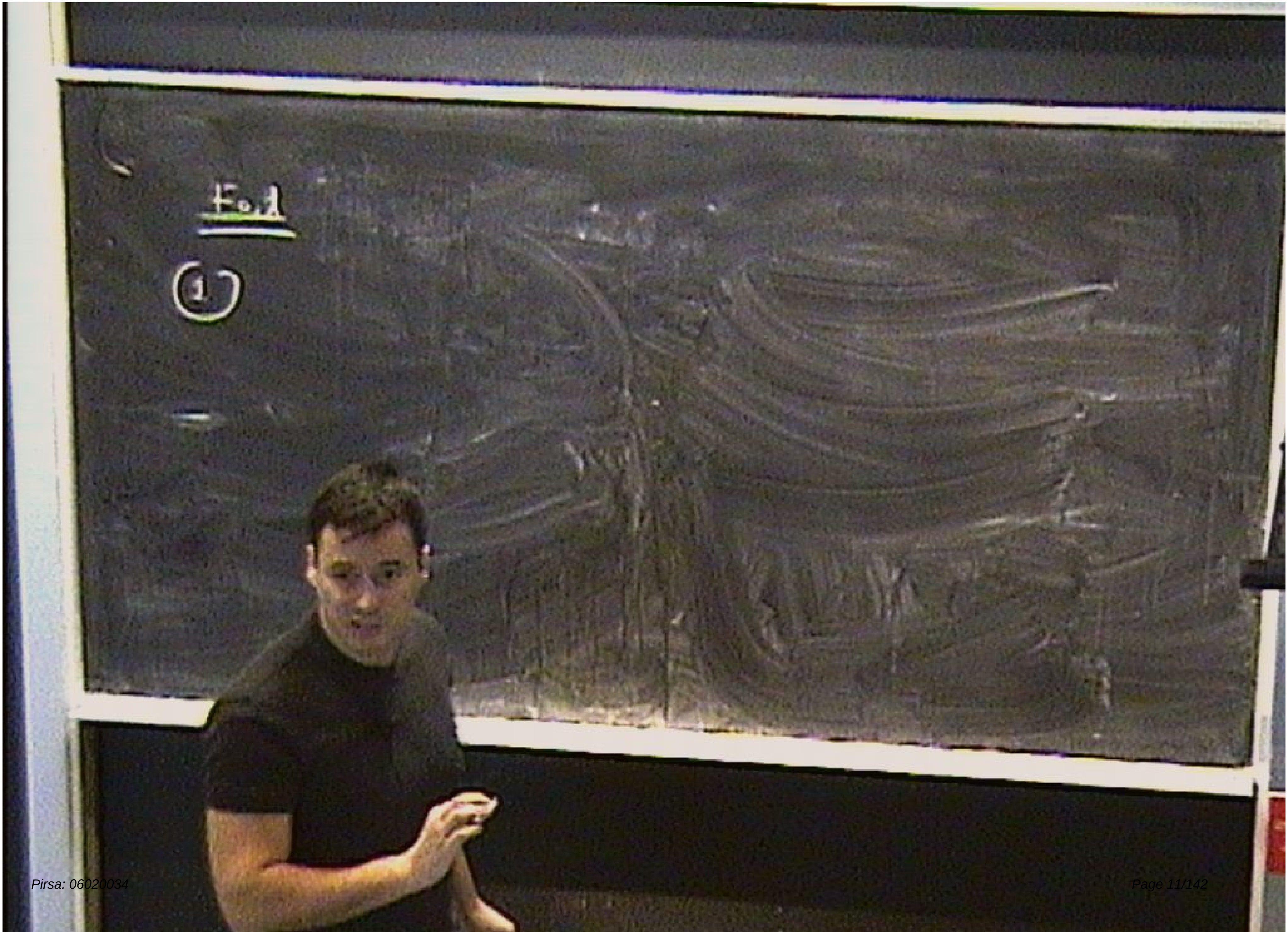
- LeVerrier prediction of Neptune based on Uranus orbital anomalies (first example of dark matter).
- He then tried to explain the anomalous precession of the perihelion of Mercury with "Vulcanus".

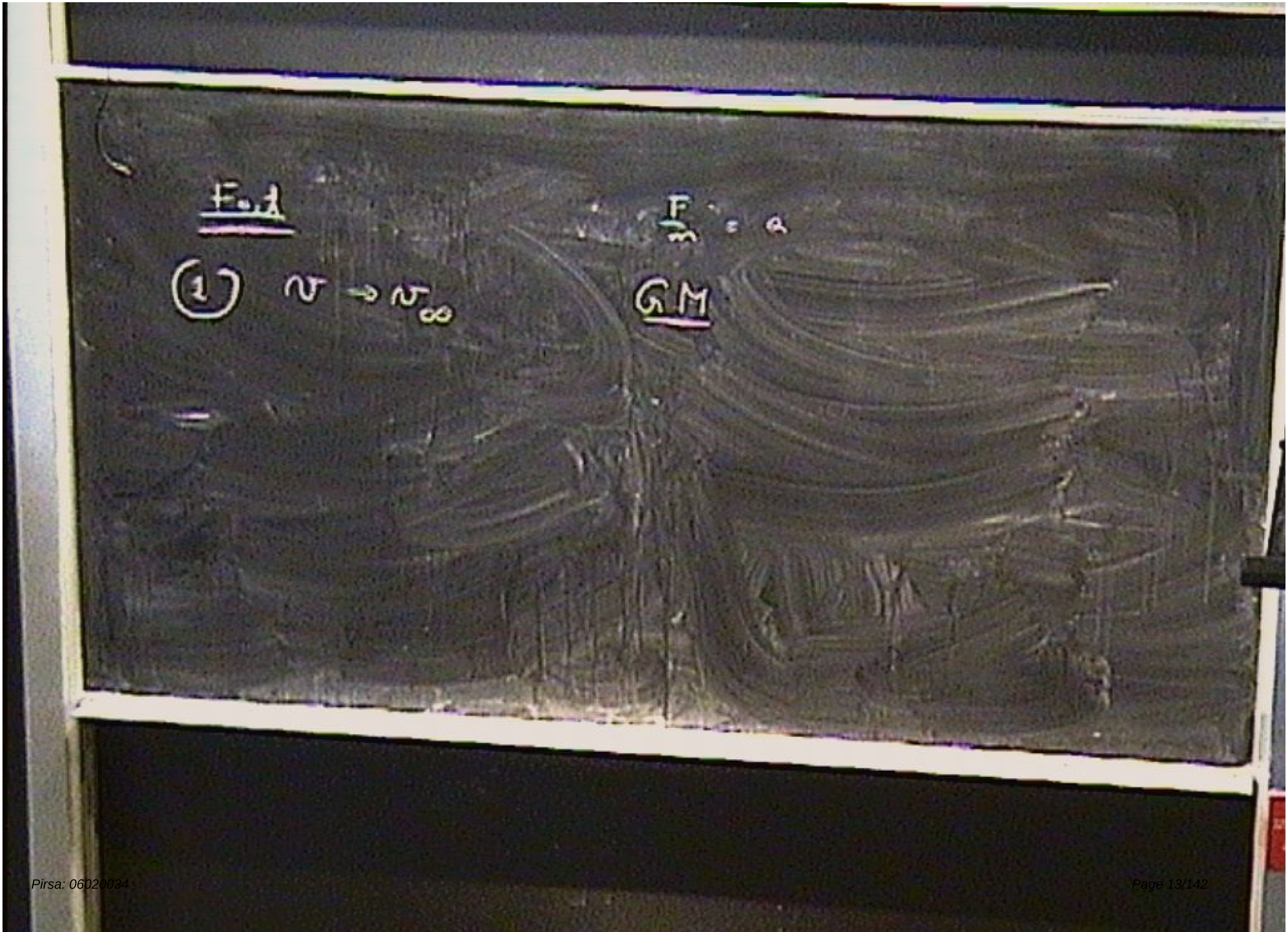
Galactic anomalies - normal surface brightness disks

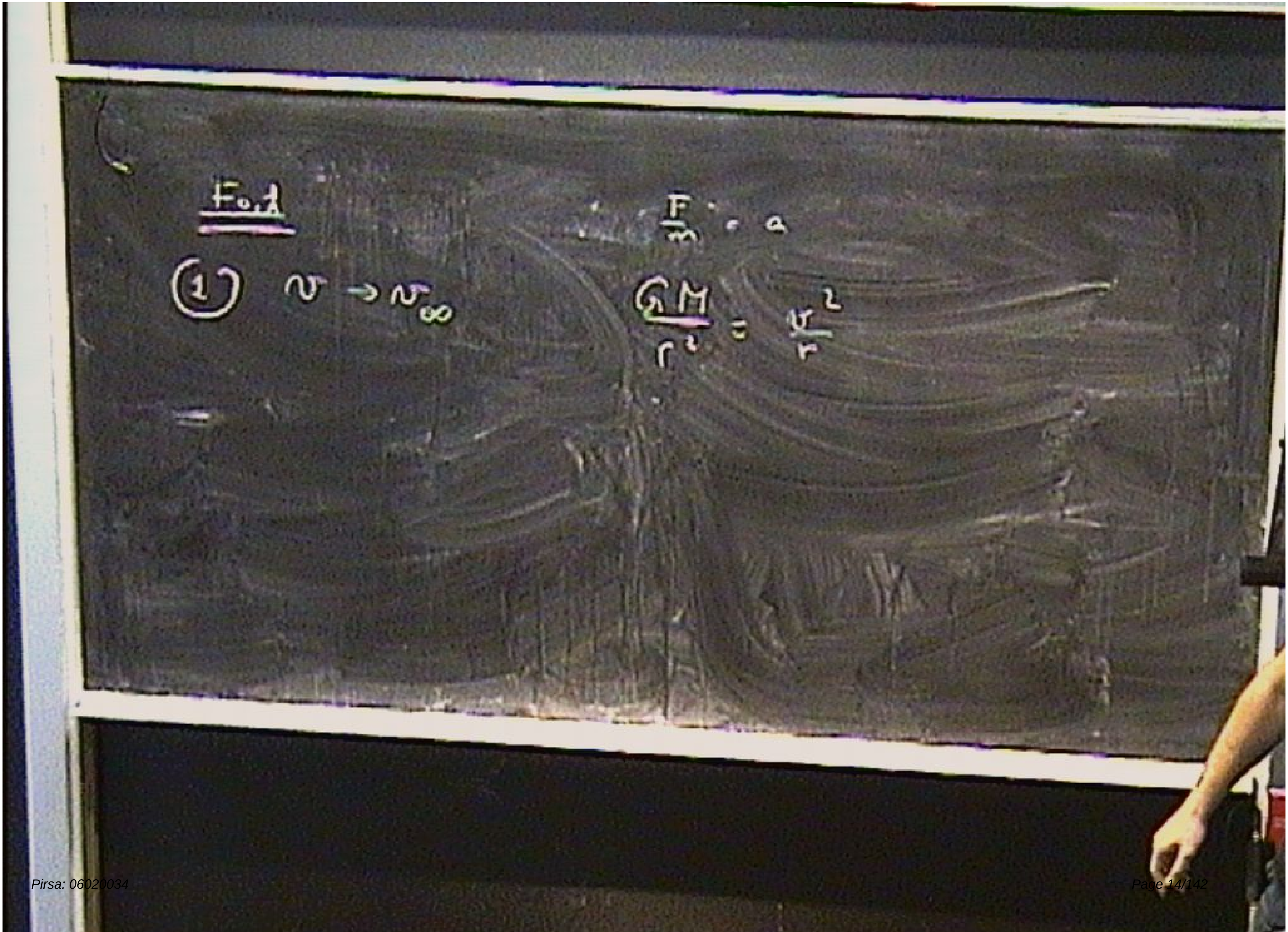




<u>Ford</u>

(1) $\upsilon \to \upsilon_\infty$


Fact
F/m = a
(1) v → v∞
GM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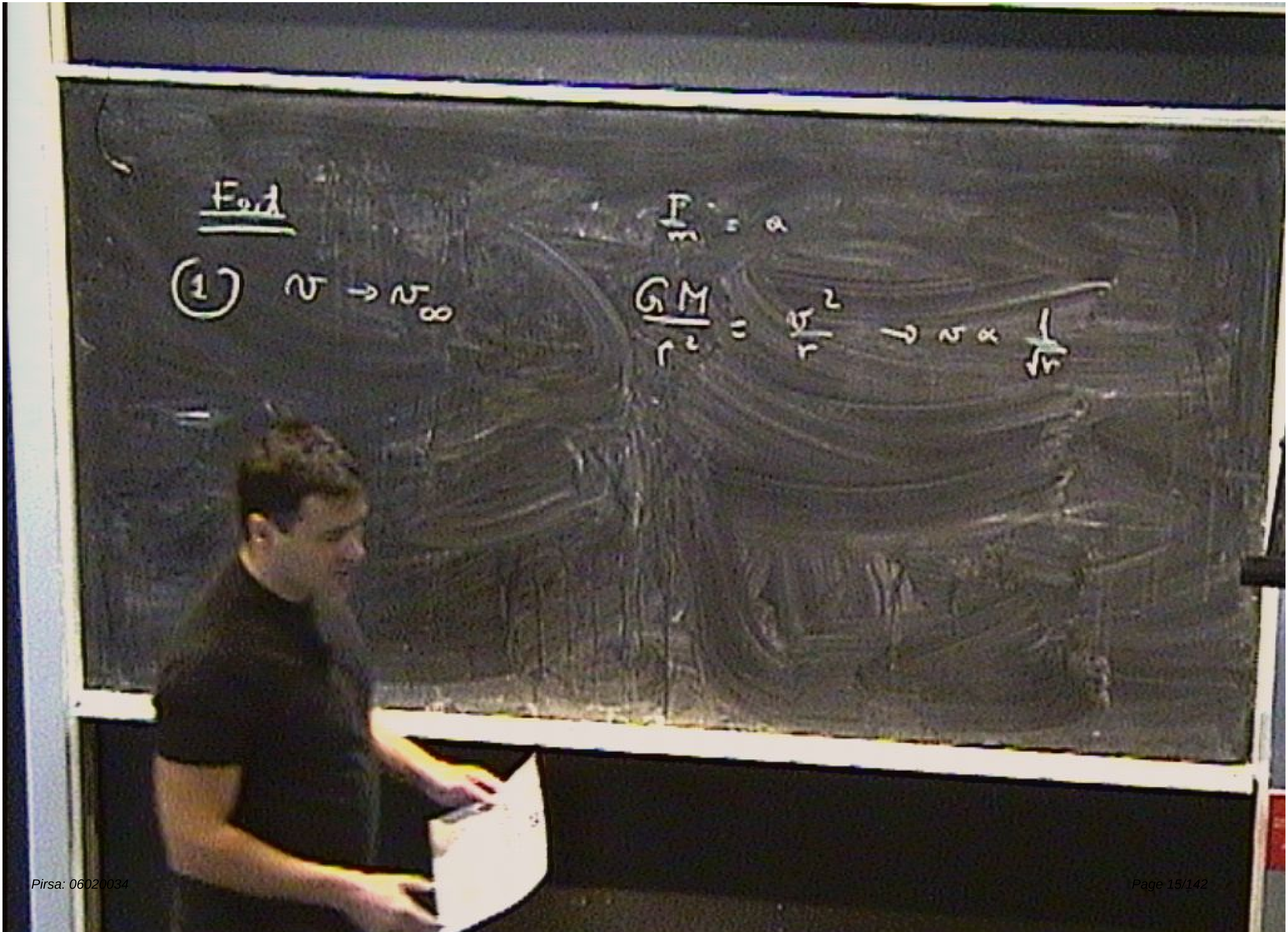
Fact
(1) $v \to v_\infty$
$\frac{F}{m} = a$
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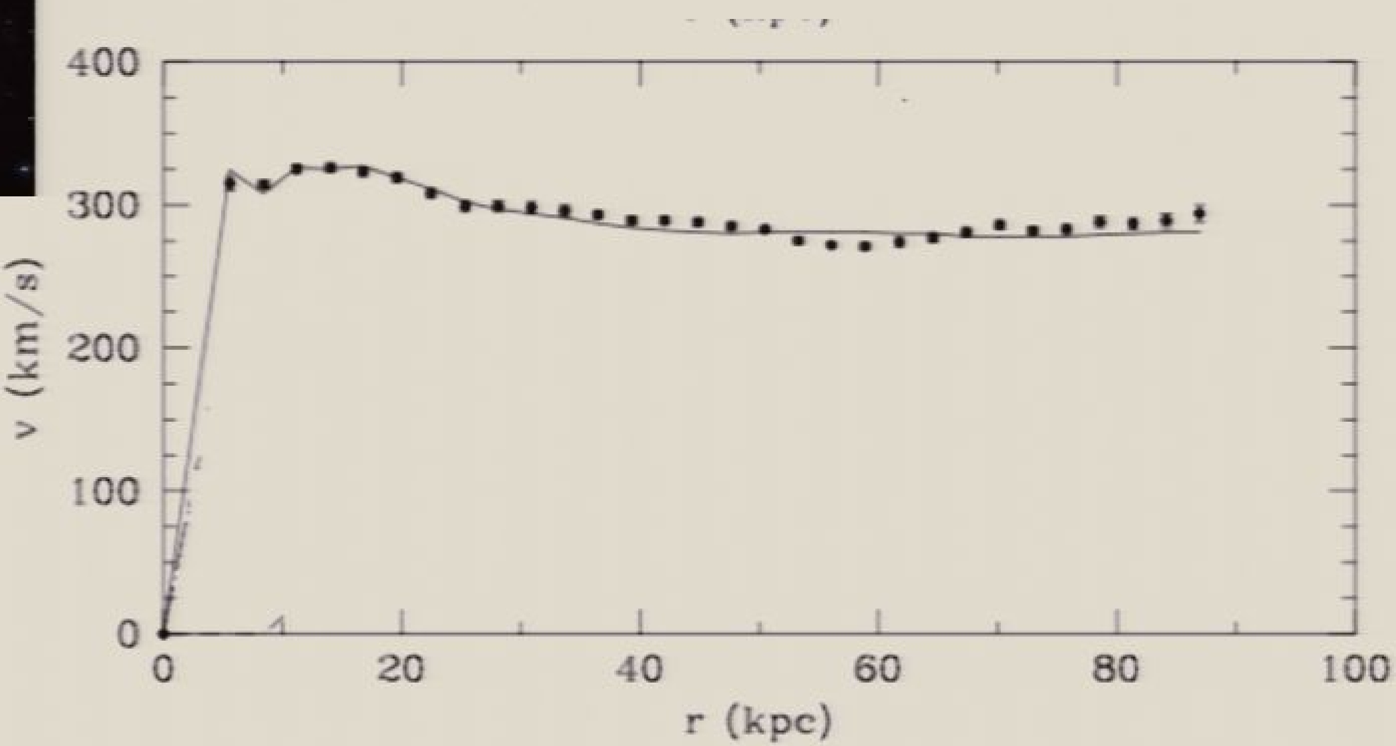
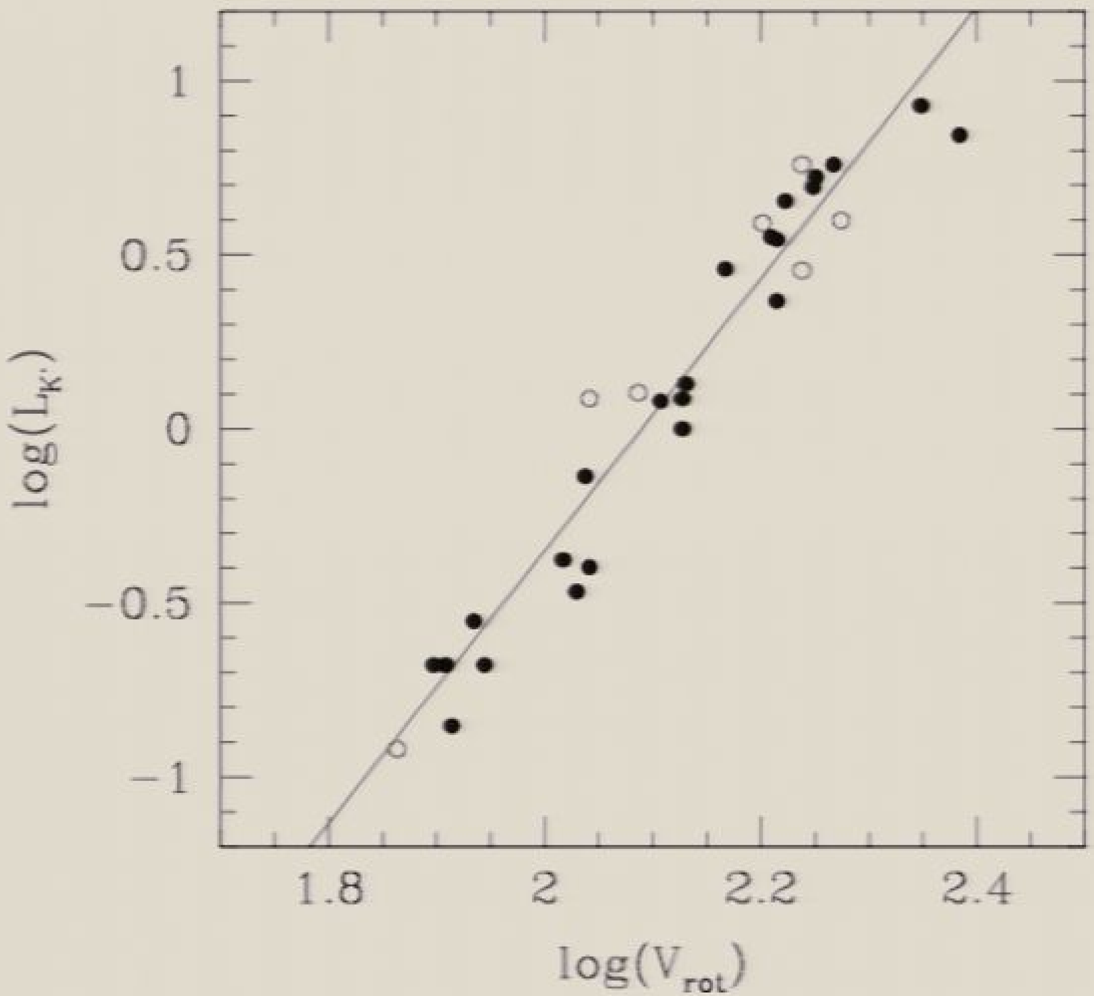
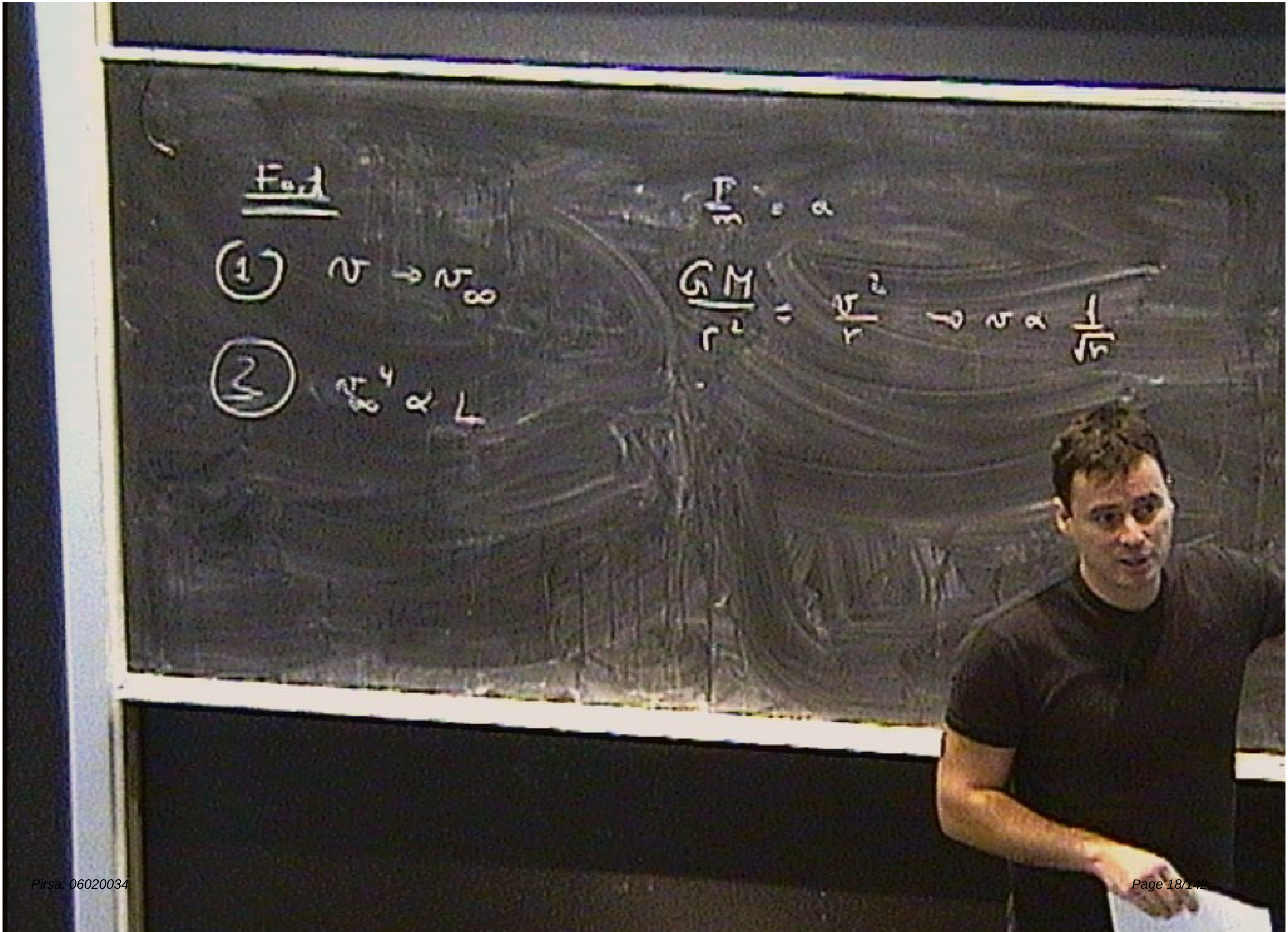


Tully-Fisher relation



$$L_{K'} \propto {V_{rot}}^4$$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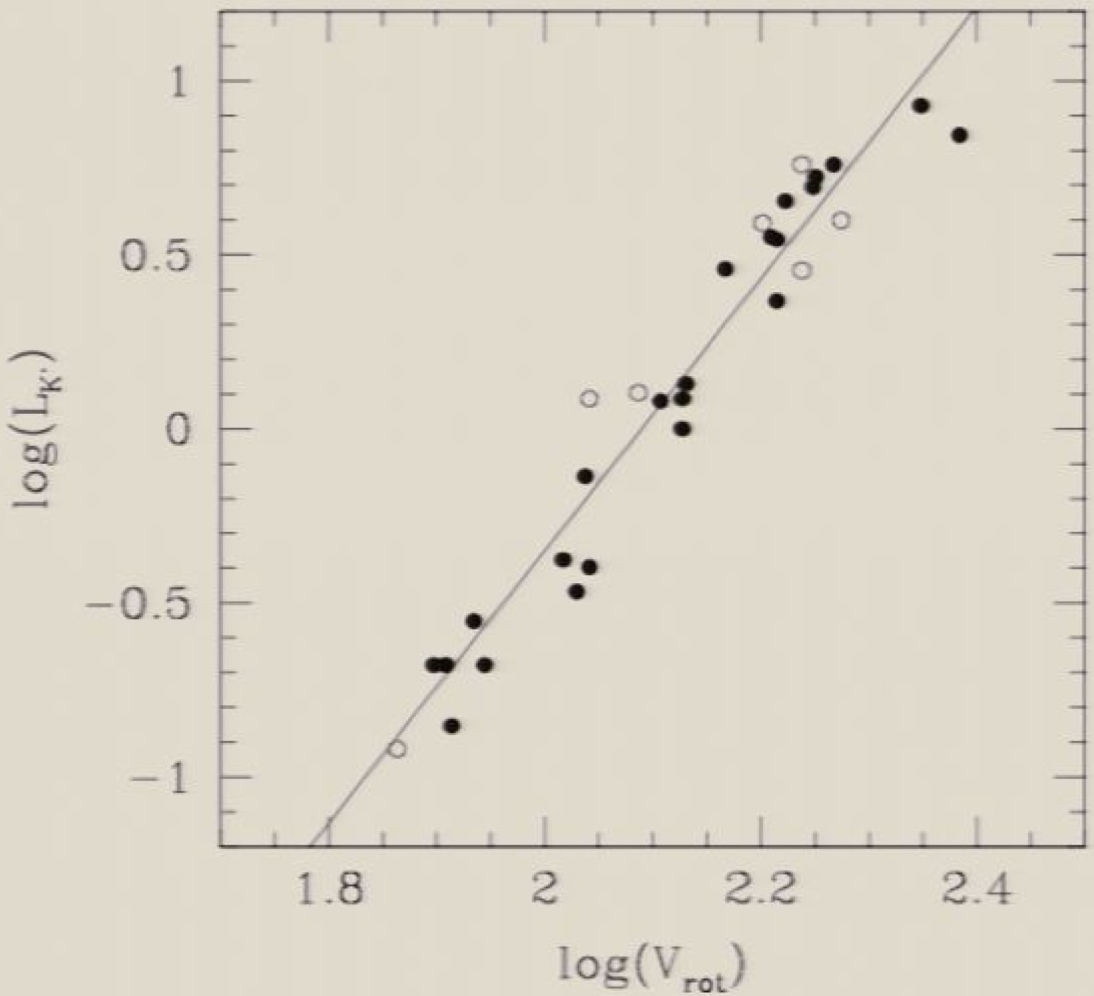

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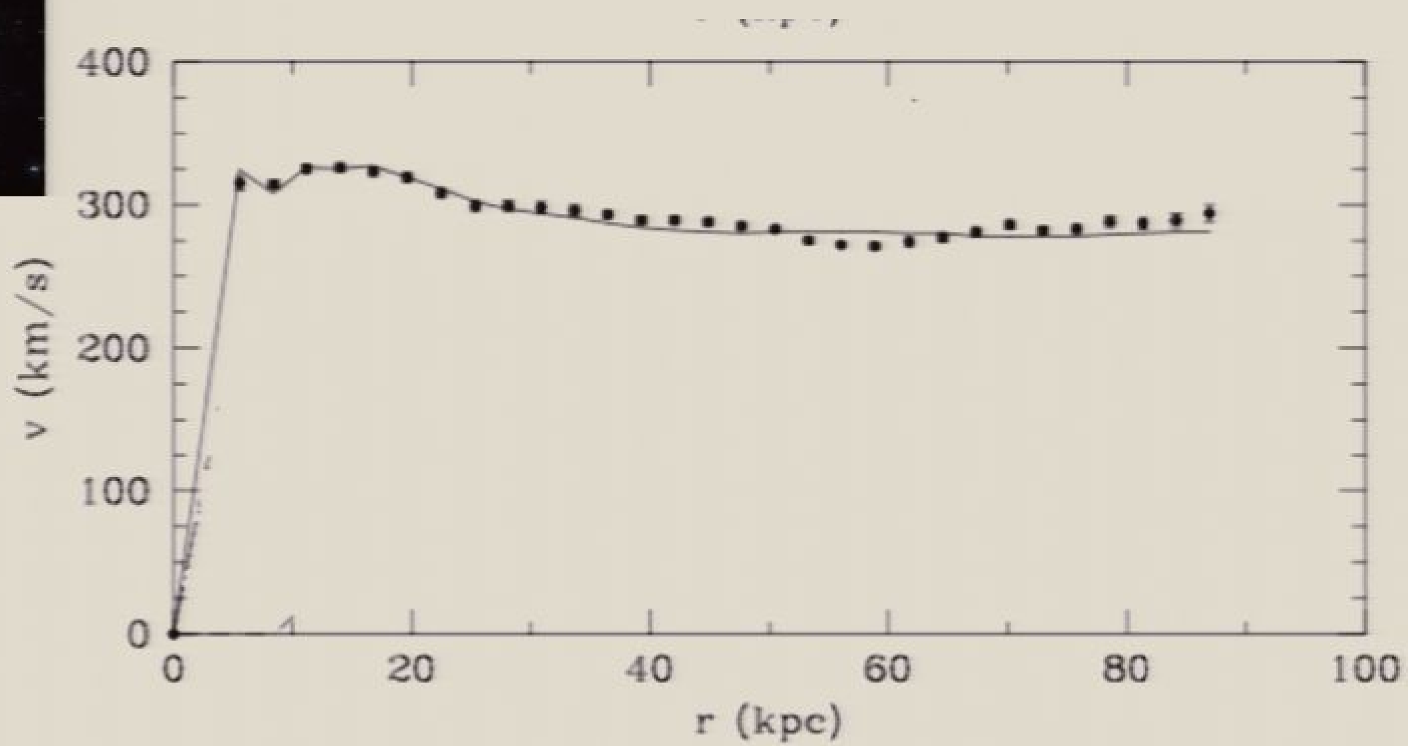
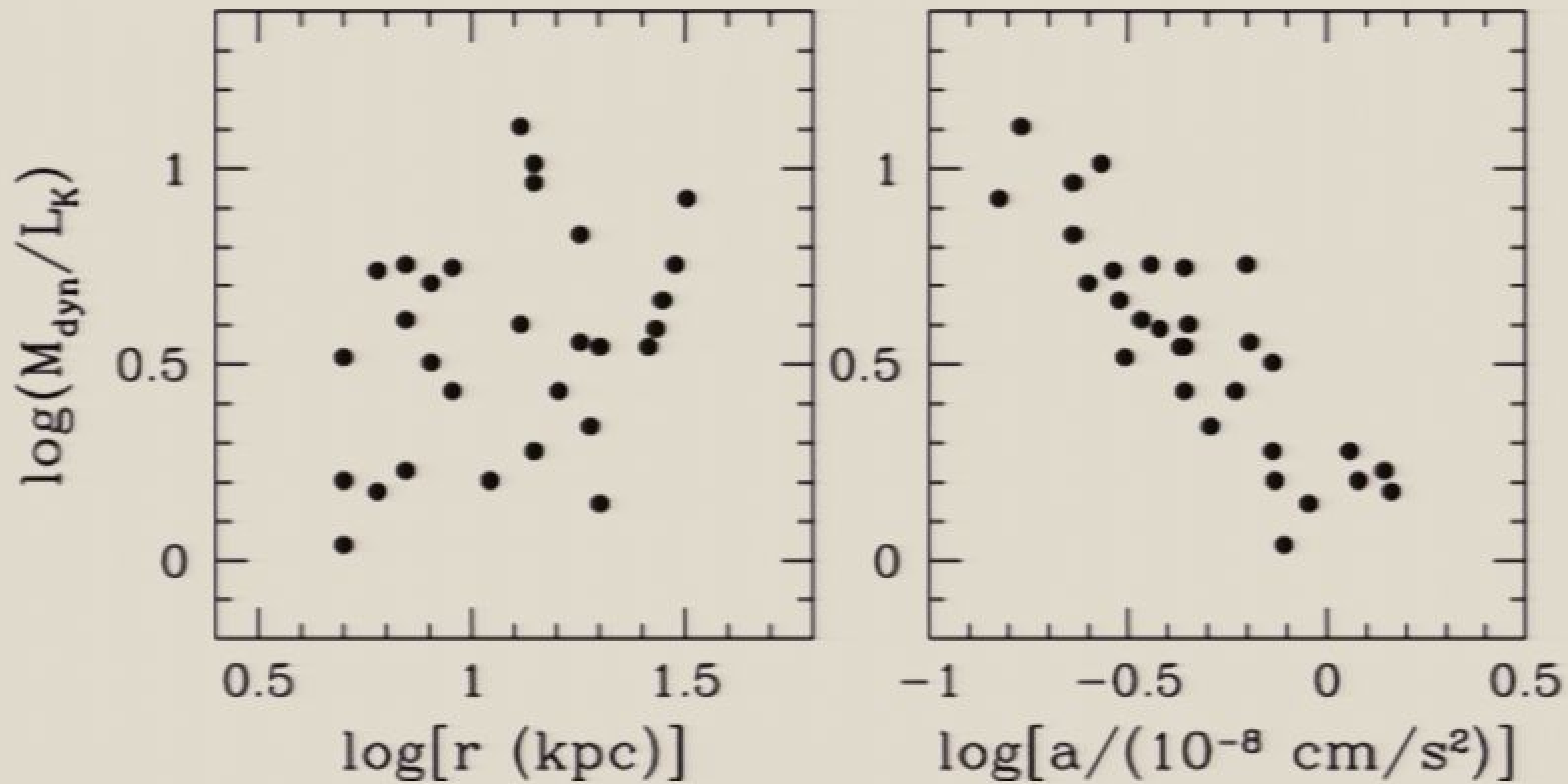
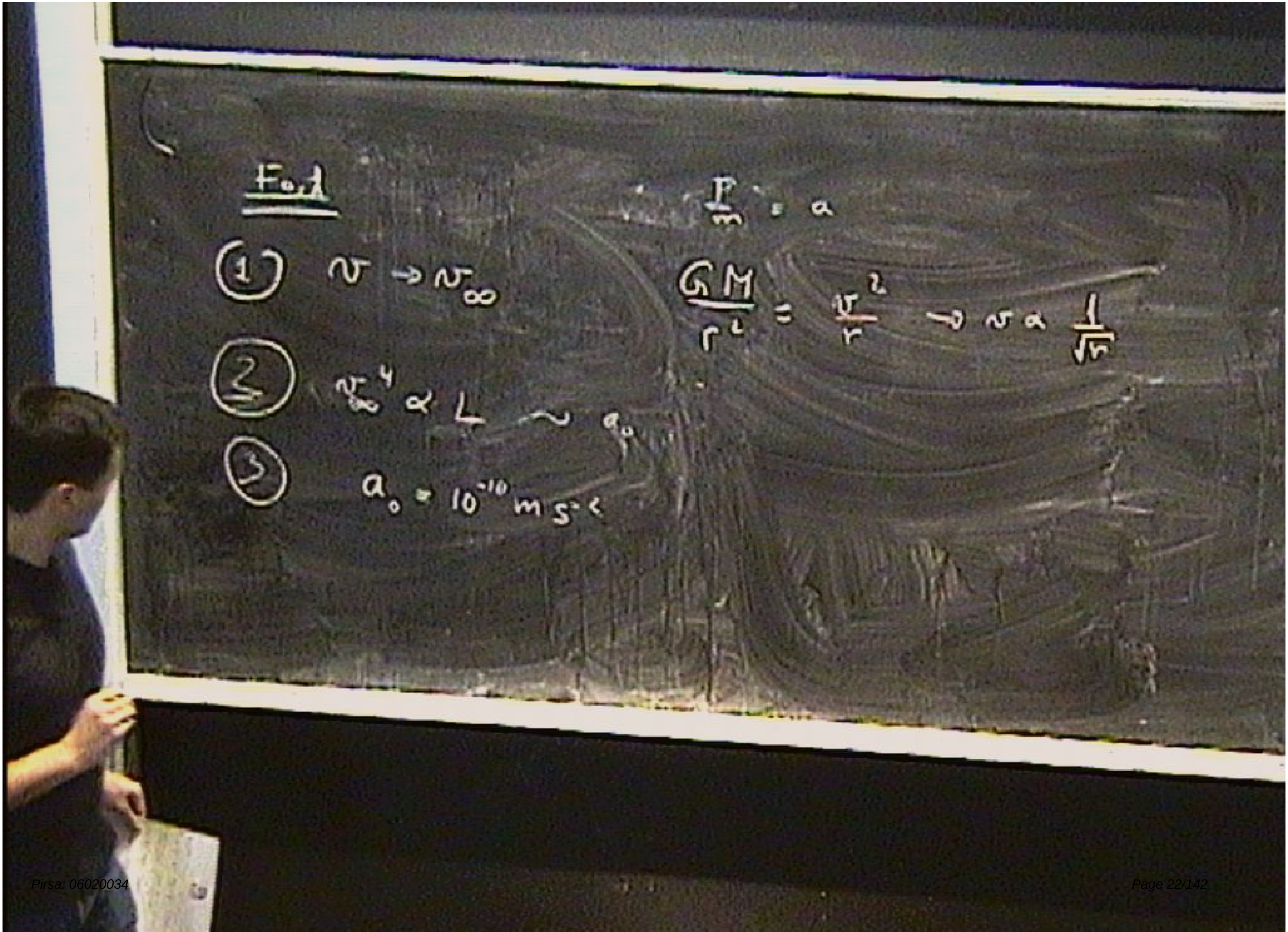


Acceleration, not length scale, is the key



Data: Tully et al. (1996); Verheijen and Sancisi, (2001)

Plots: Sanders and McGaugh (2003)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(3) a0 ≈ 10^-10 m s^-2
F/m = a
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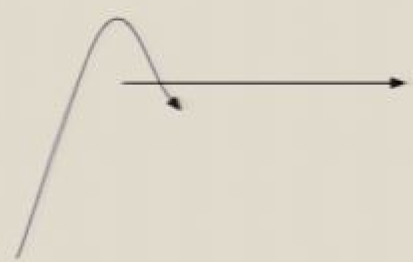
Dark matter paradigm

Each galaxy nested in a roundish extended dark halo

$$\rho \propto 1/r^2 \longrightarrow v = \text{const.}$$

Halo model parameters: $\sigma^2, \; R_c, \Upsilon = M/L$

Dark clouds:



Need to fine tune

$L_{K'} \propto \mathrm{V_{rot}}^4$

Where is the dark matter ?

$$\frac{F}{m} = a$$

$$\frac{GM}{r^2} = \frac{v^2}{r} \rightarrow v \propto \frac{1}{\sqrt{r}}$$

$\sim \; a_0$

s^{-2}

DM

$$M_{DM} \propto r \qquad \rho_{DM} \sim \frac{1}{r^2}$$

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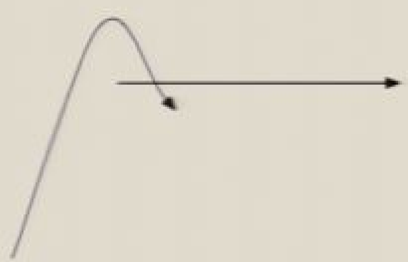

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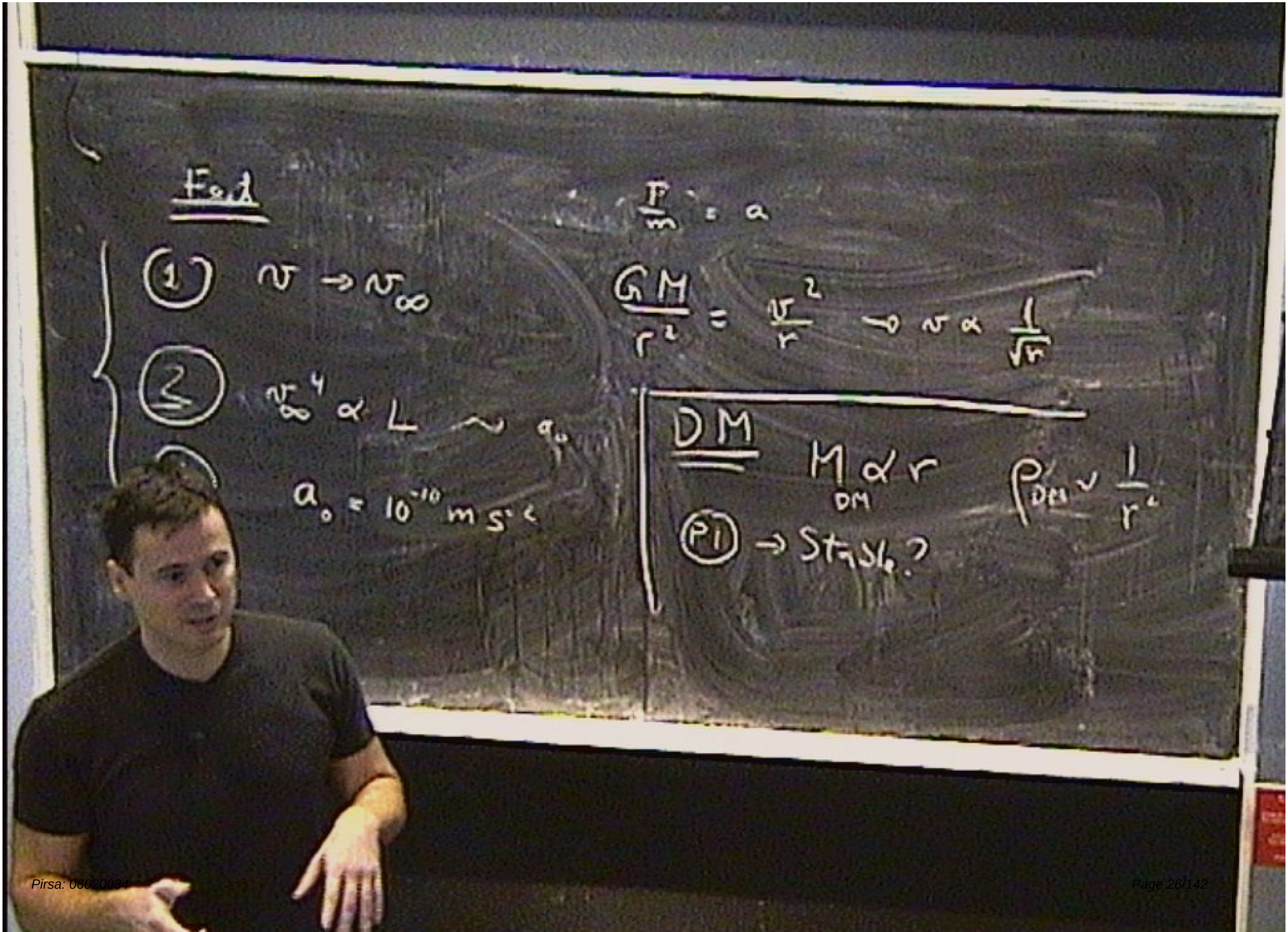

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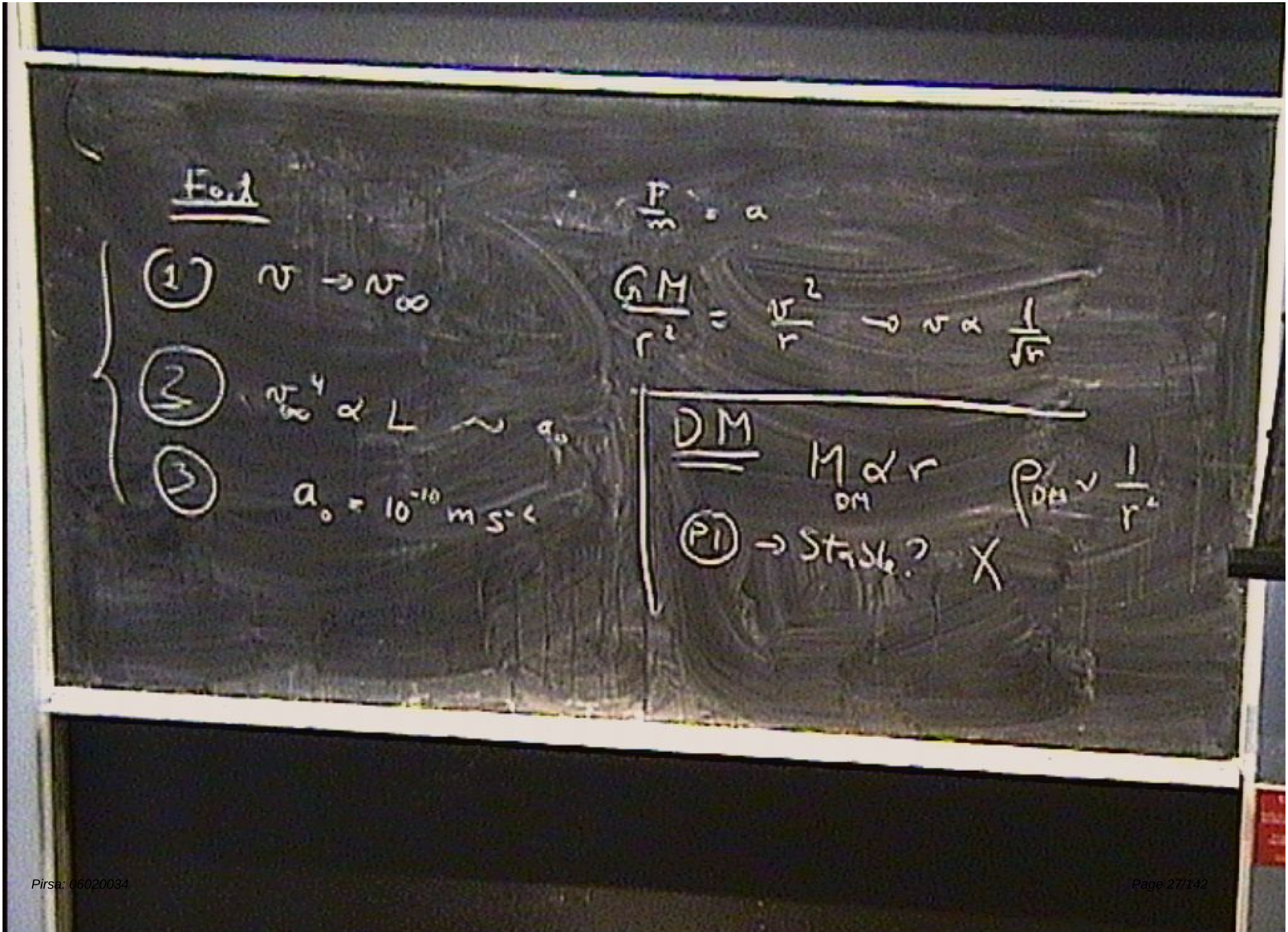
② v∞⁴ ∝ L ~ a₀
a₀ = 10⁻¹⁰ m s⁻²
DM
M_DM ∝ r
ρ_DM ~ 1/r²
P1 → Stable?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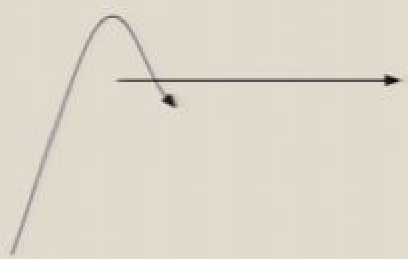

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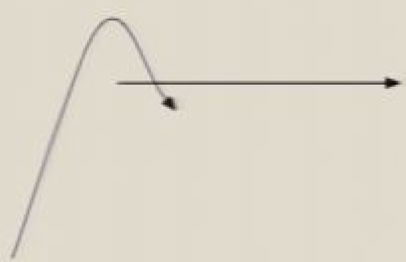

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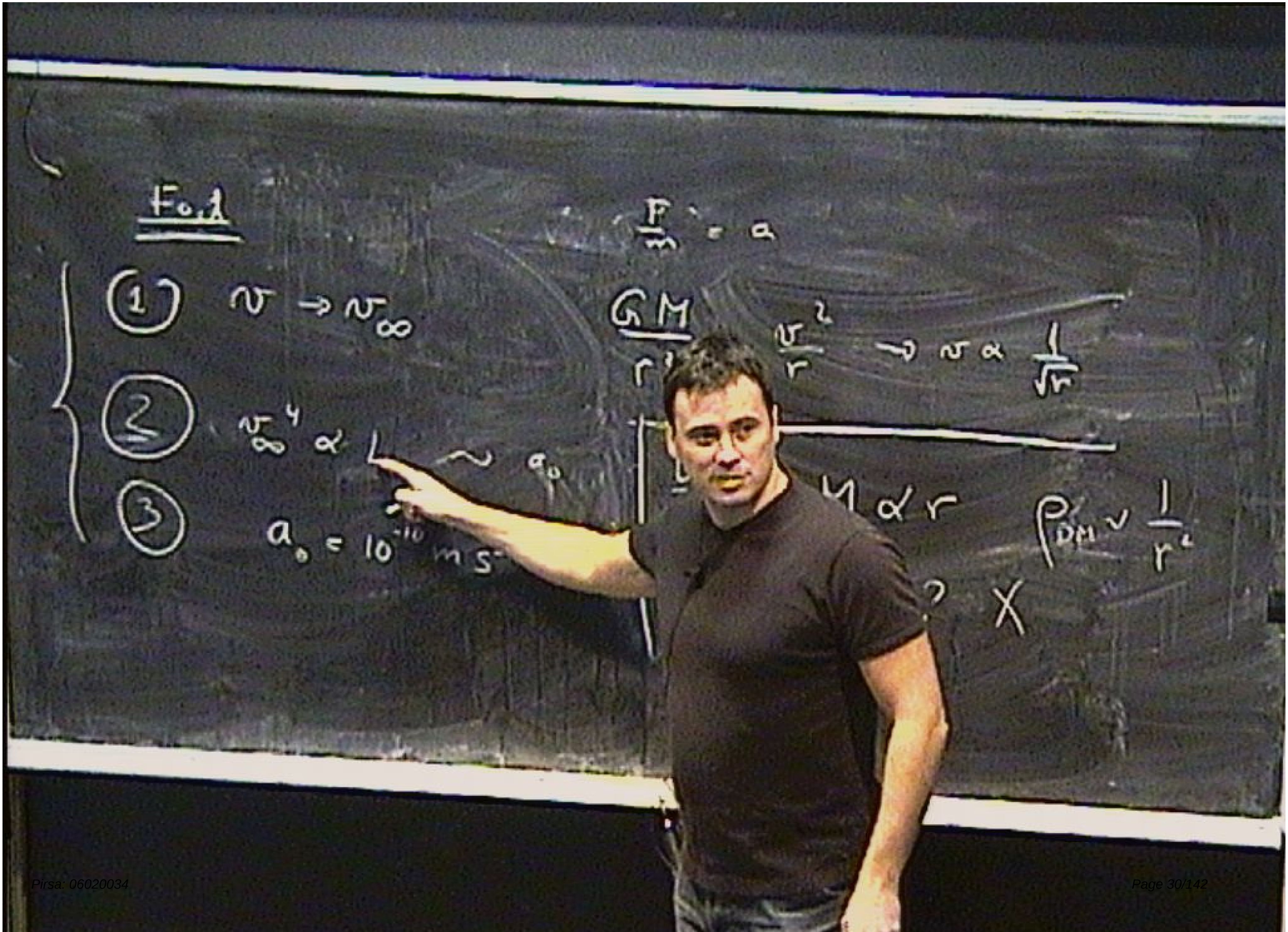
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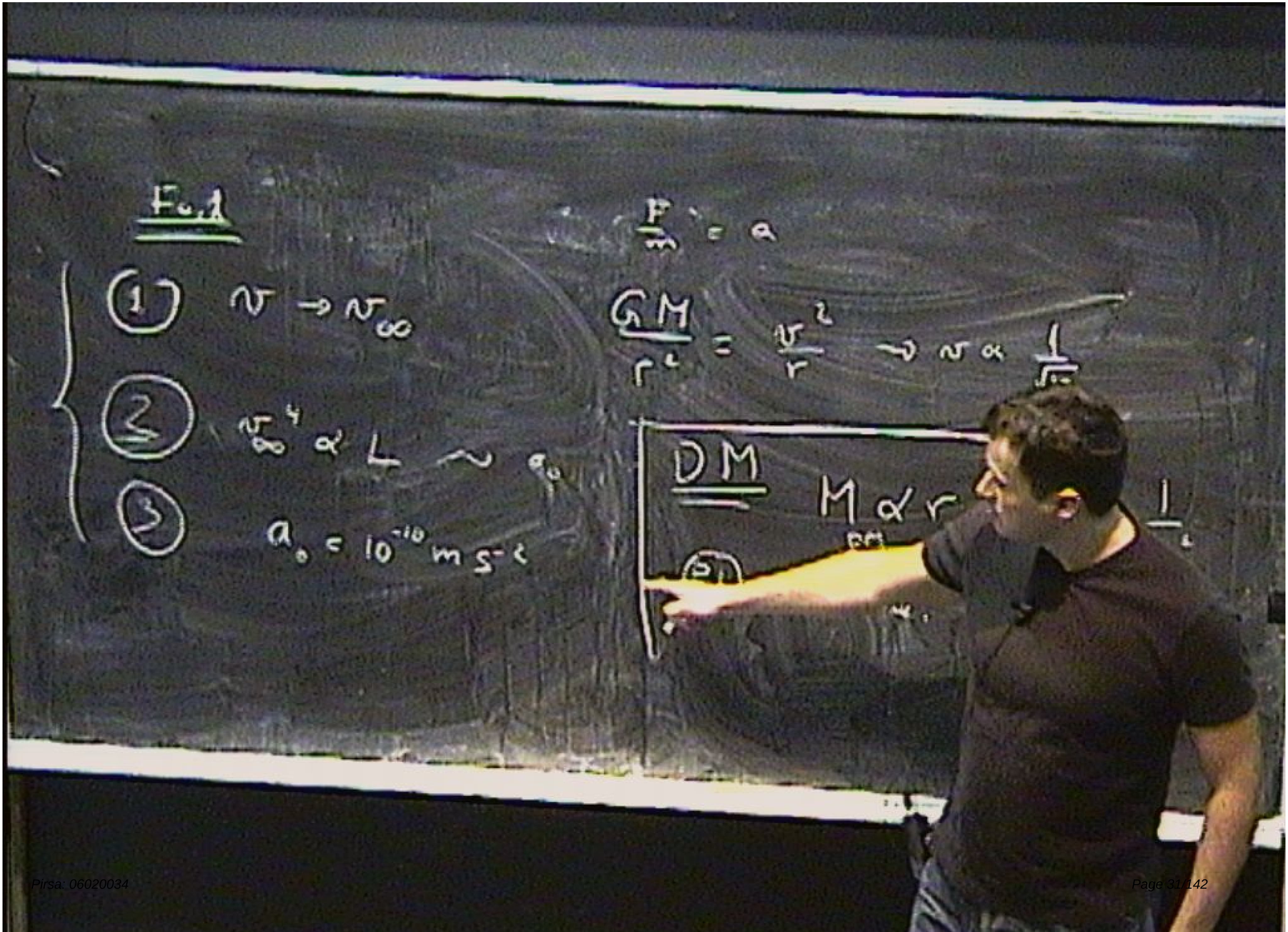
X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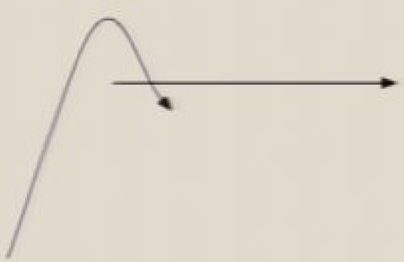


Where is the dark matter ?

The MOND paradigm

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_{\rm N}$$

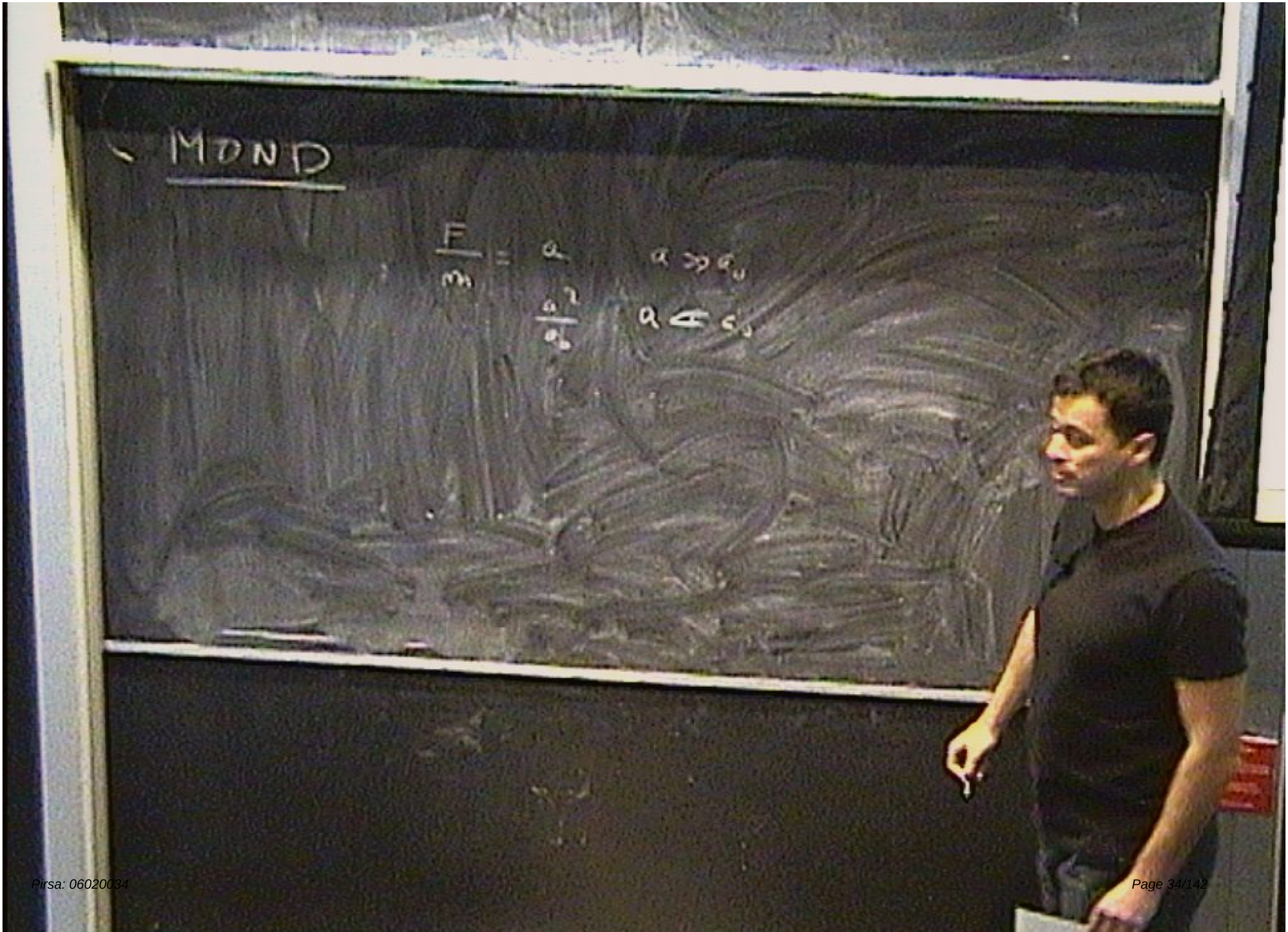
$$a_0 \approx 10^{-8}\,\mathrm{cm\ s}^{-2}$$

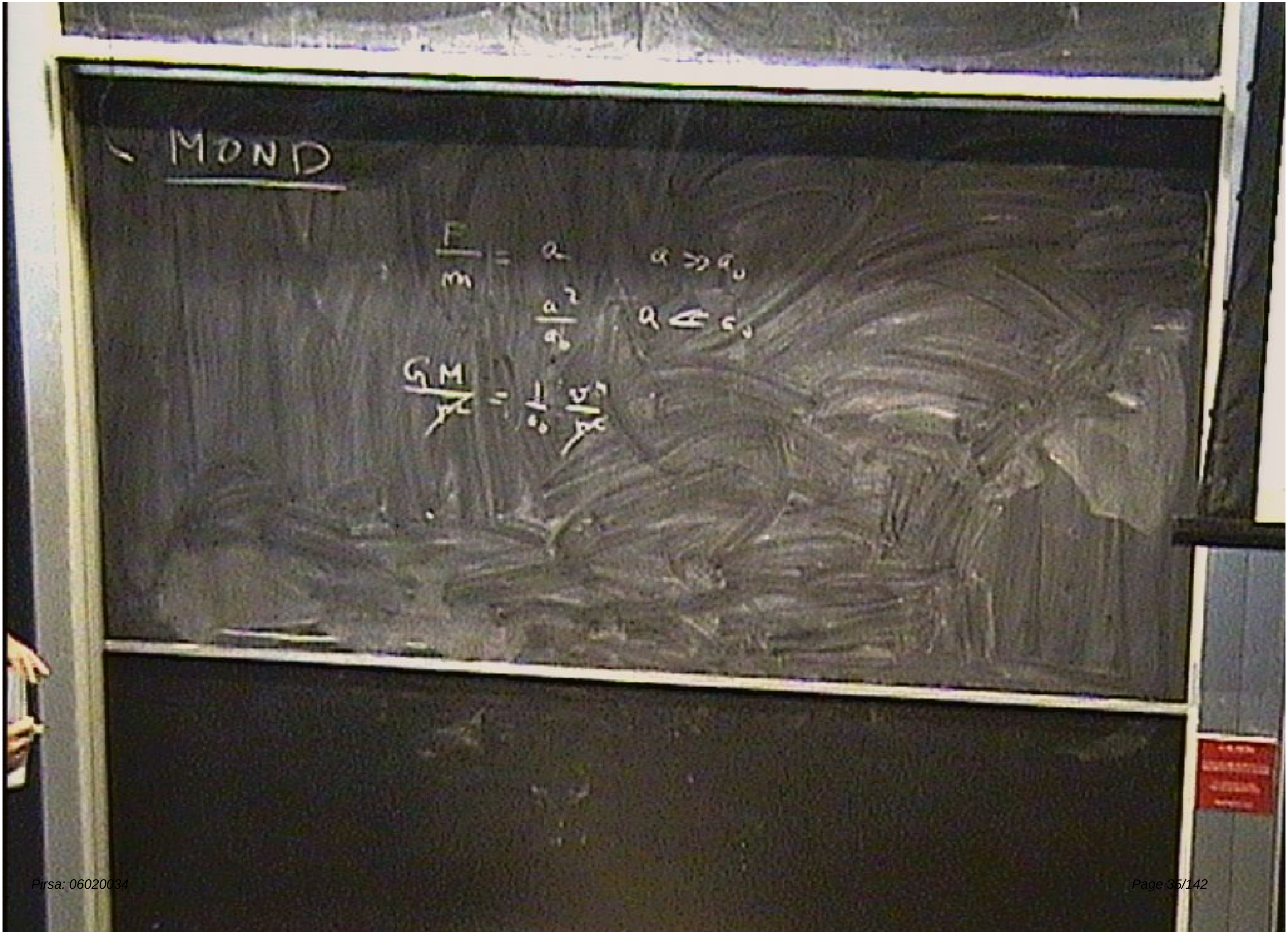
In extreme MOND limit

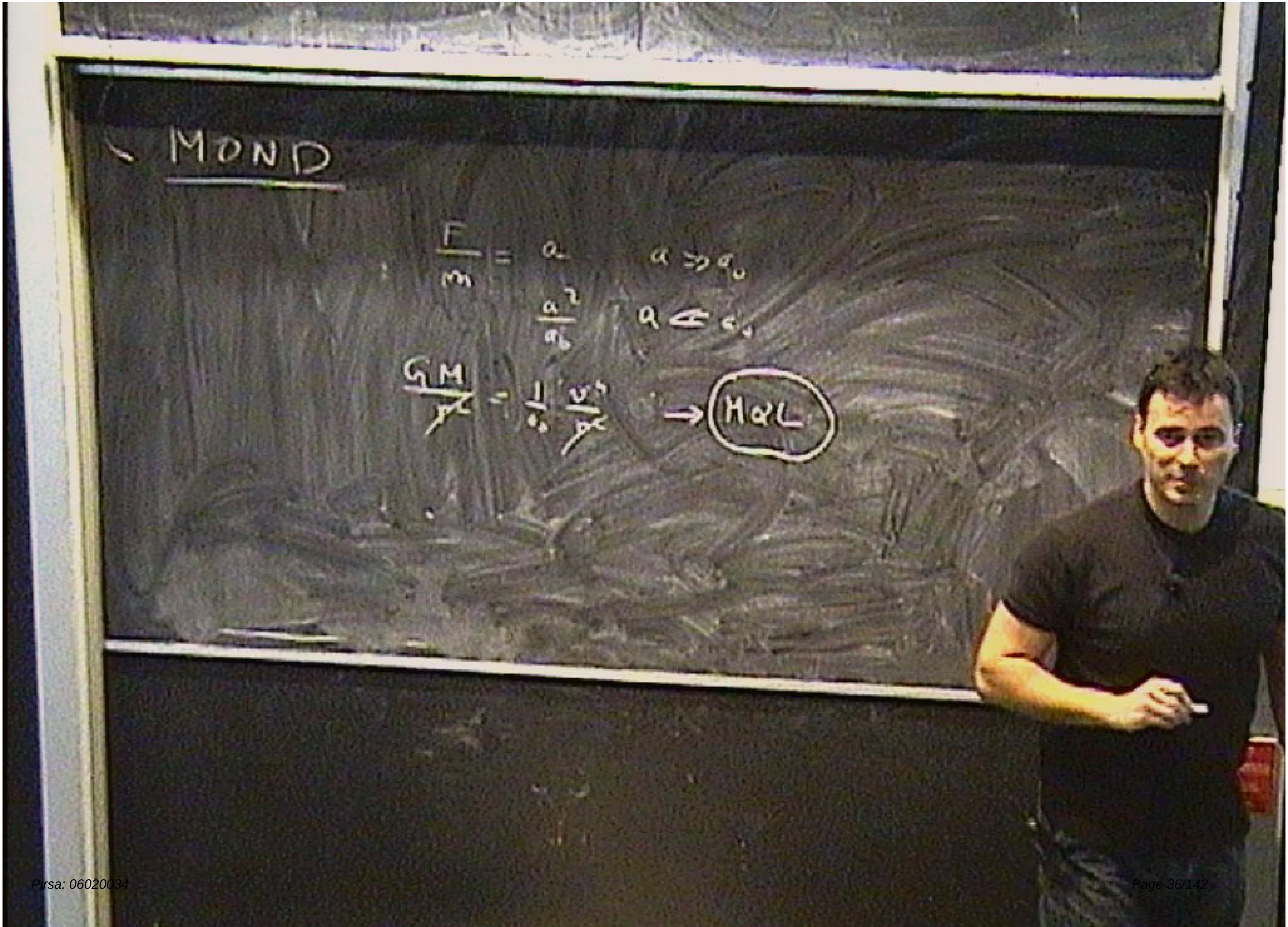
$$|\mathbf{a}|\mathbf{a}/a_0 = -\nabla\Phi_{\rm N}$$

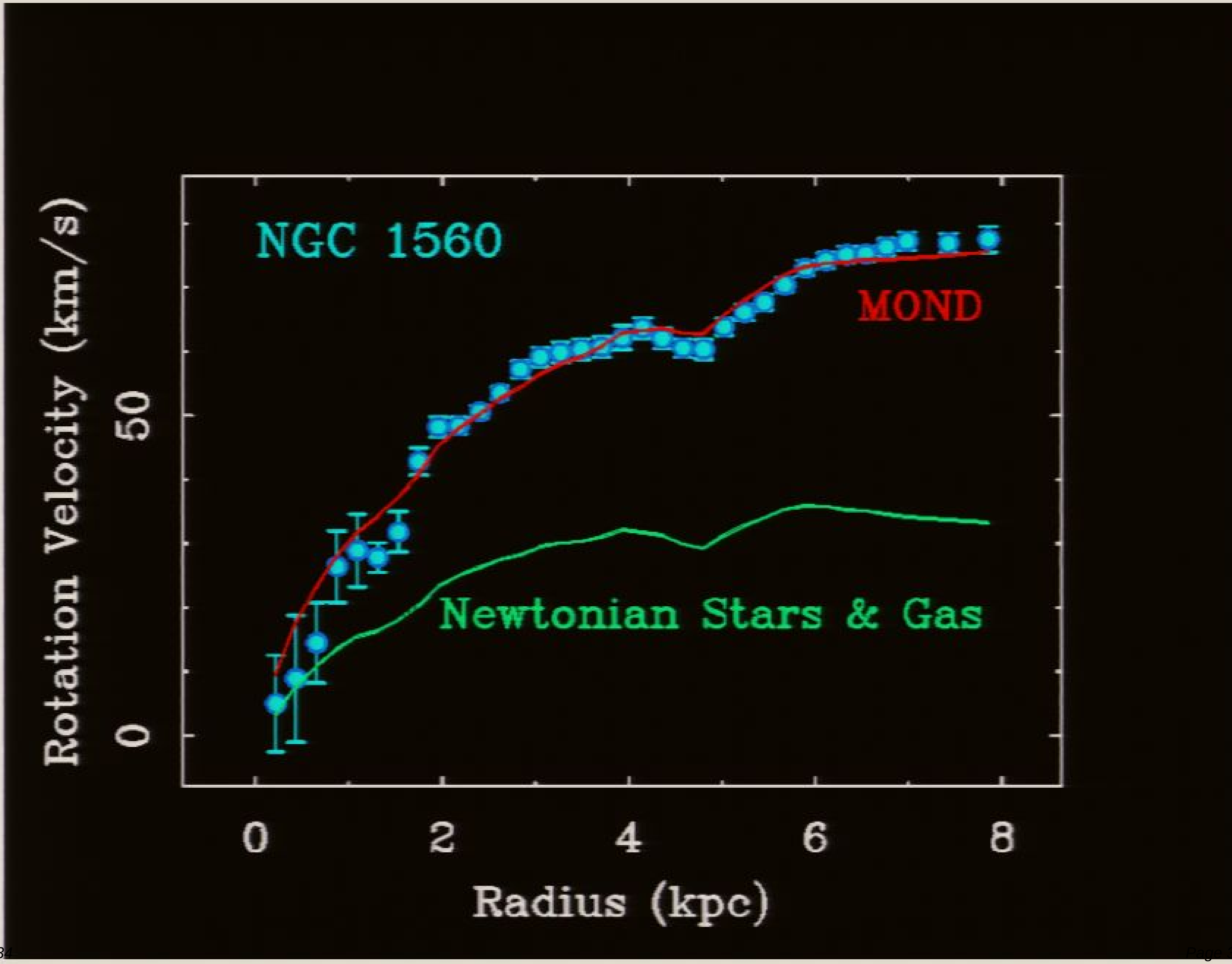
$$|\mathbf{a}| = v^2/r \qquad |\nabla\Phi_N| = GM/r^2$$

$$M = v^4/Ga_0 \qquad L = v^4/\Upsilon G a_0$$


MOND
F/m = a   a ≫ a0
a^2/a0   a ≪ a0


MOND
F/m = a   a >> a0
a²/a0   a << a0


MOND
$\frac{F}{m} = a \quad a \gg a_0$
$\frac{a^2}{a_0} \quad a \ll a_0$
$\frac{GM}{r^2} = \frac{1}{a_0}\frac{v^4}{r^2} \rightarrow M \propto L$


NGC 1560
MOND
Newtonian Stars & Gas
Rotation Velocity (km/s)
50
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2
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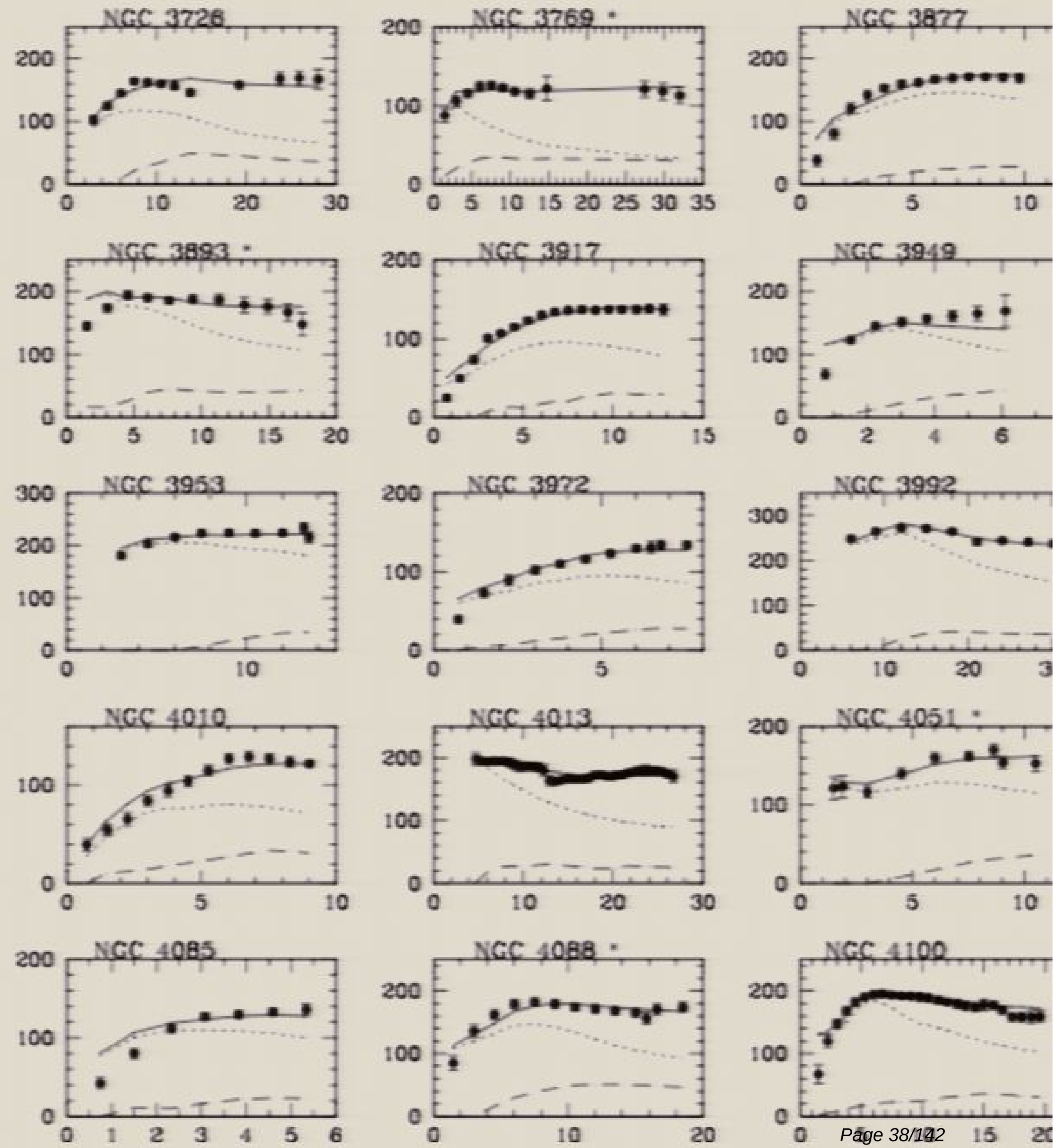
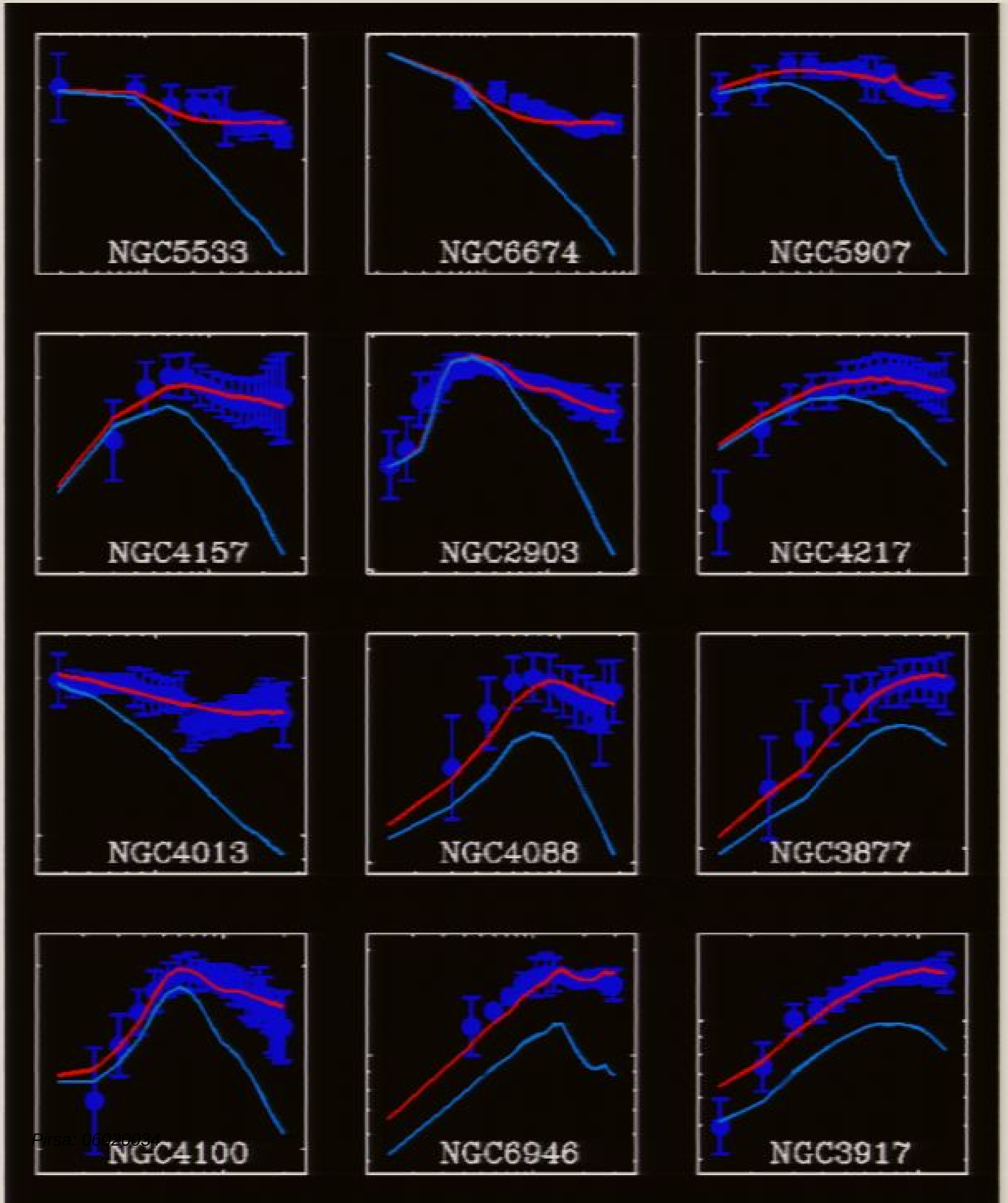
Radius (kpc)

MOND fits to disk galaxies



Data for the Ursa Major group: Verheijen (1997)

Fit: Sanders and Verheijen (1998)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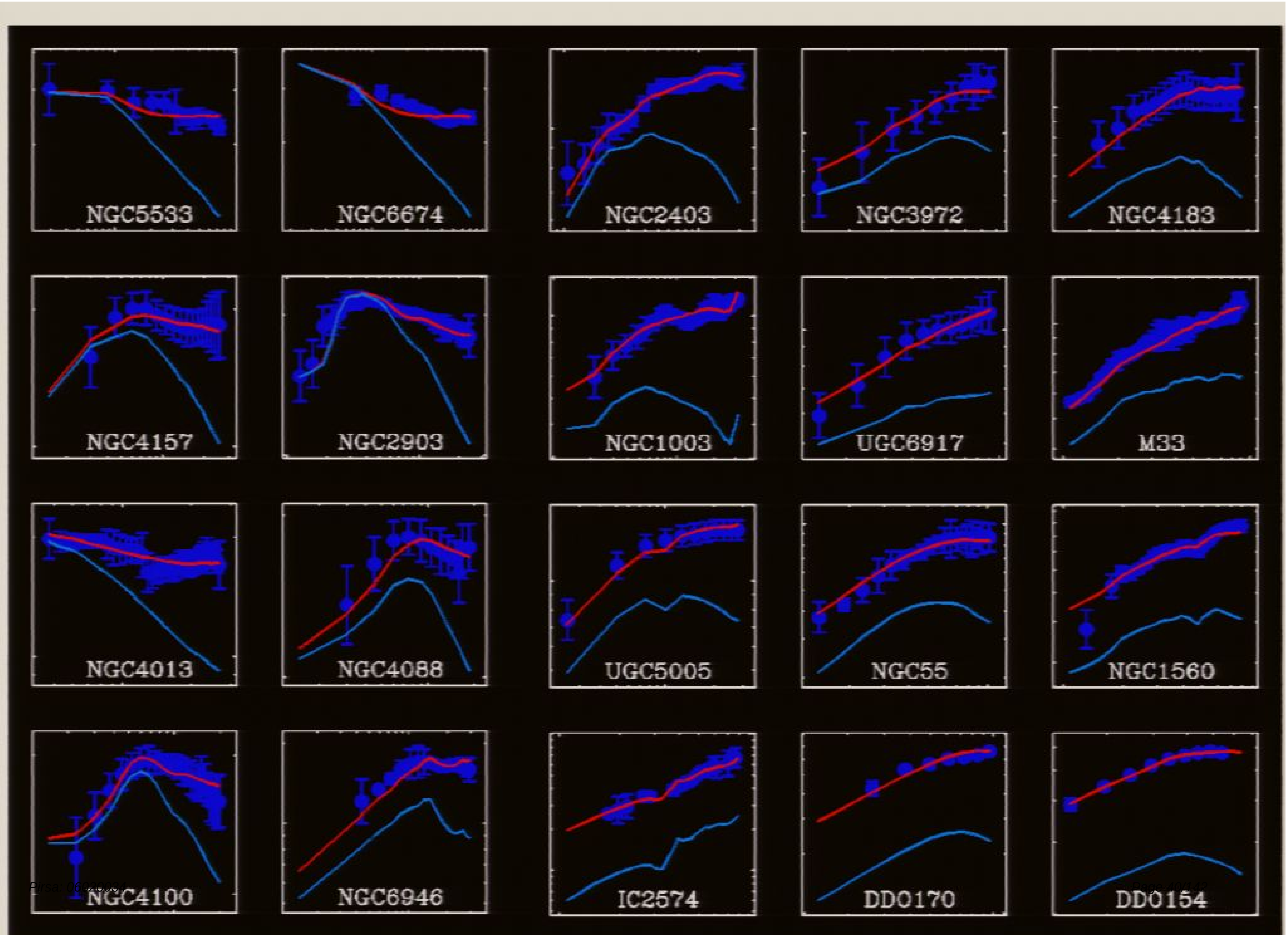
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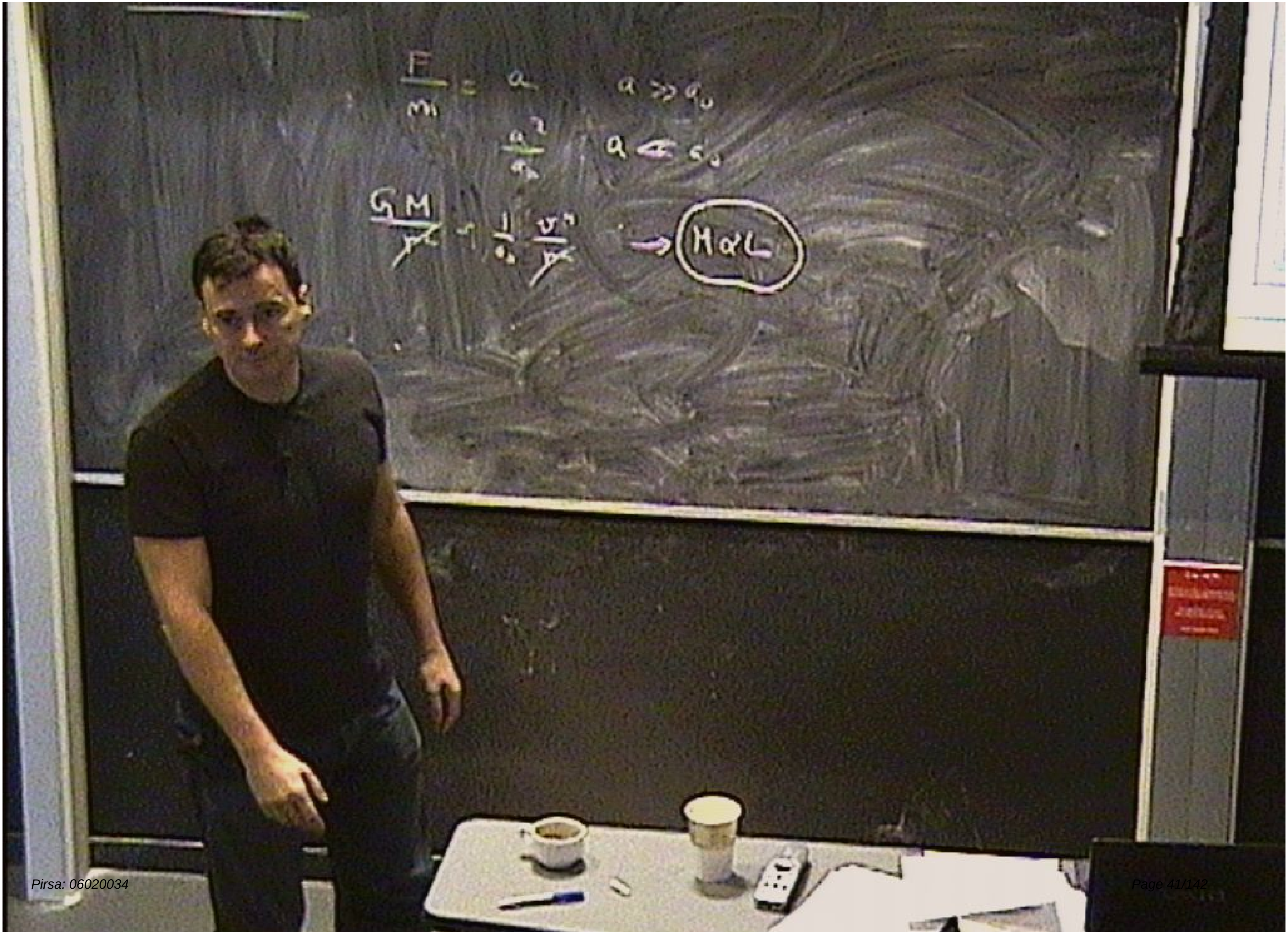
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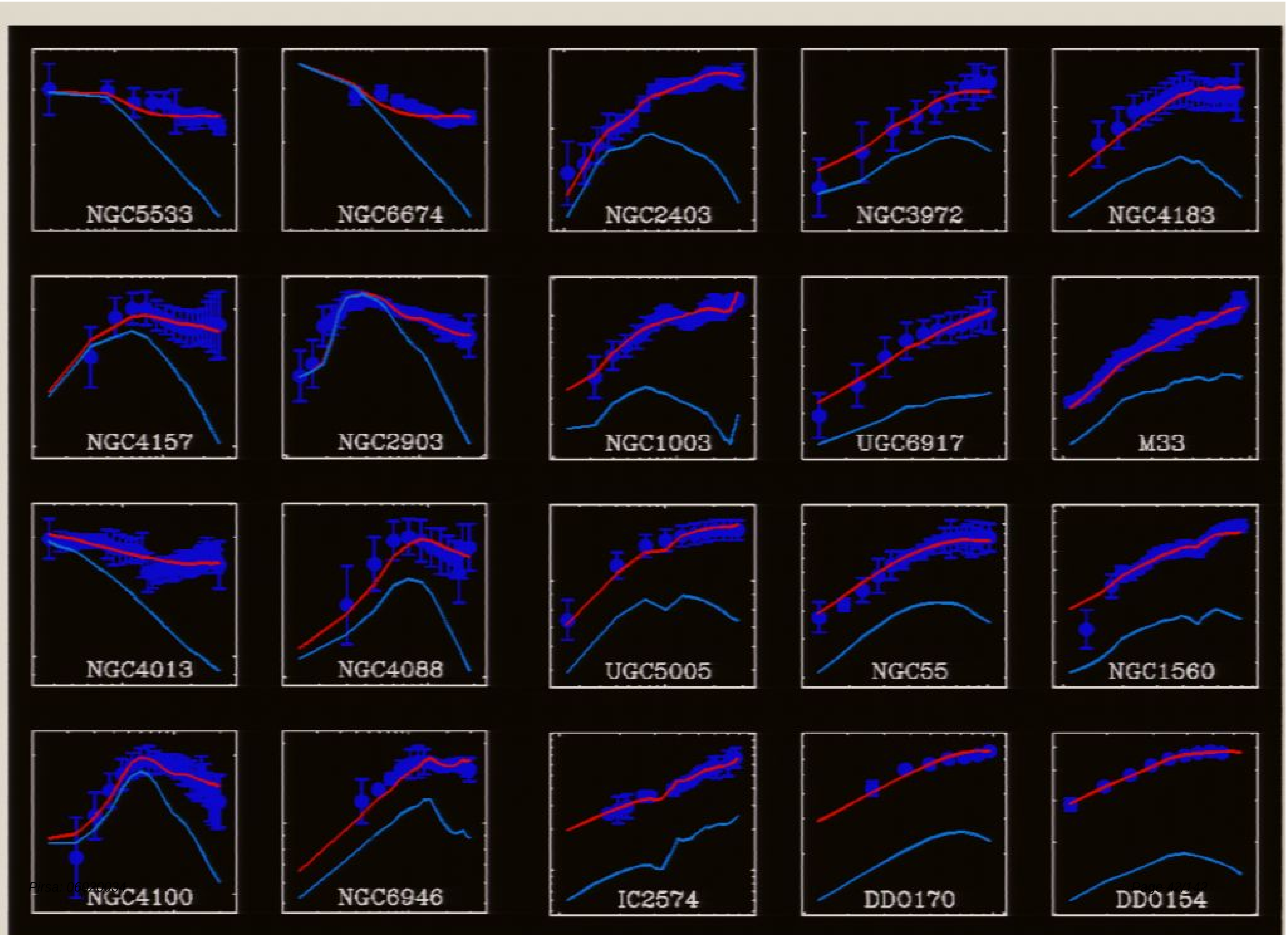
→ M ∝ L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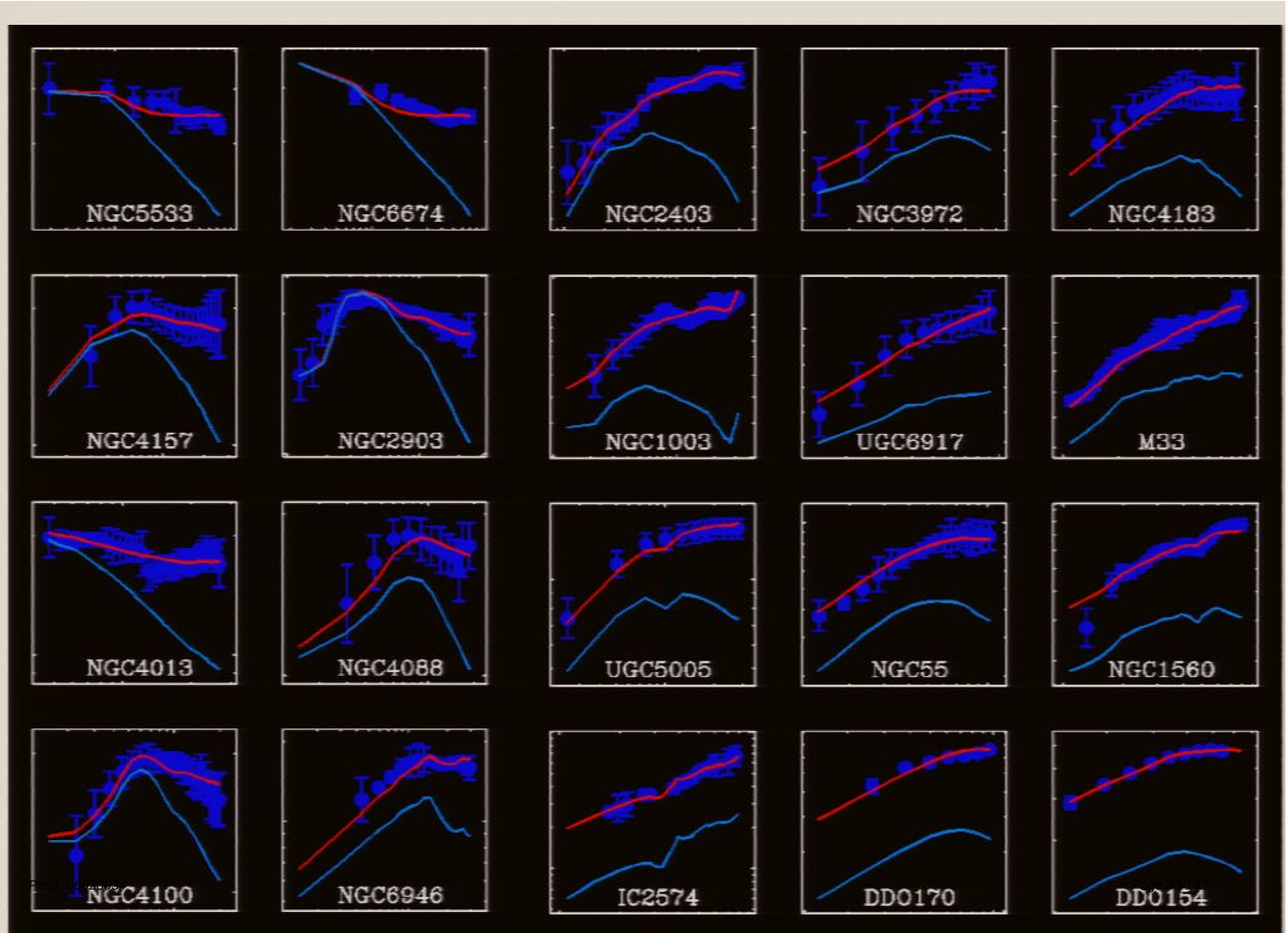

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Conceptual problems with MOND (Milgrom 1983)

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_{\rm N}$$

2

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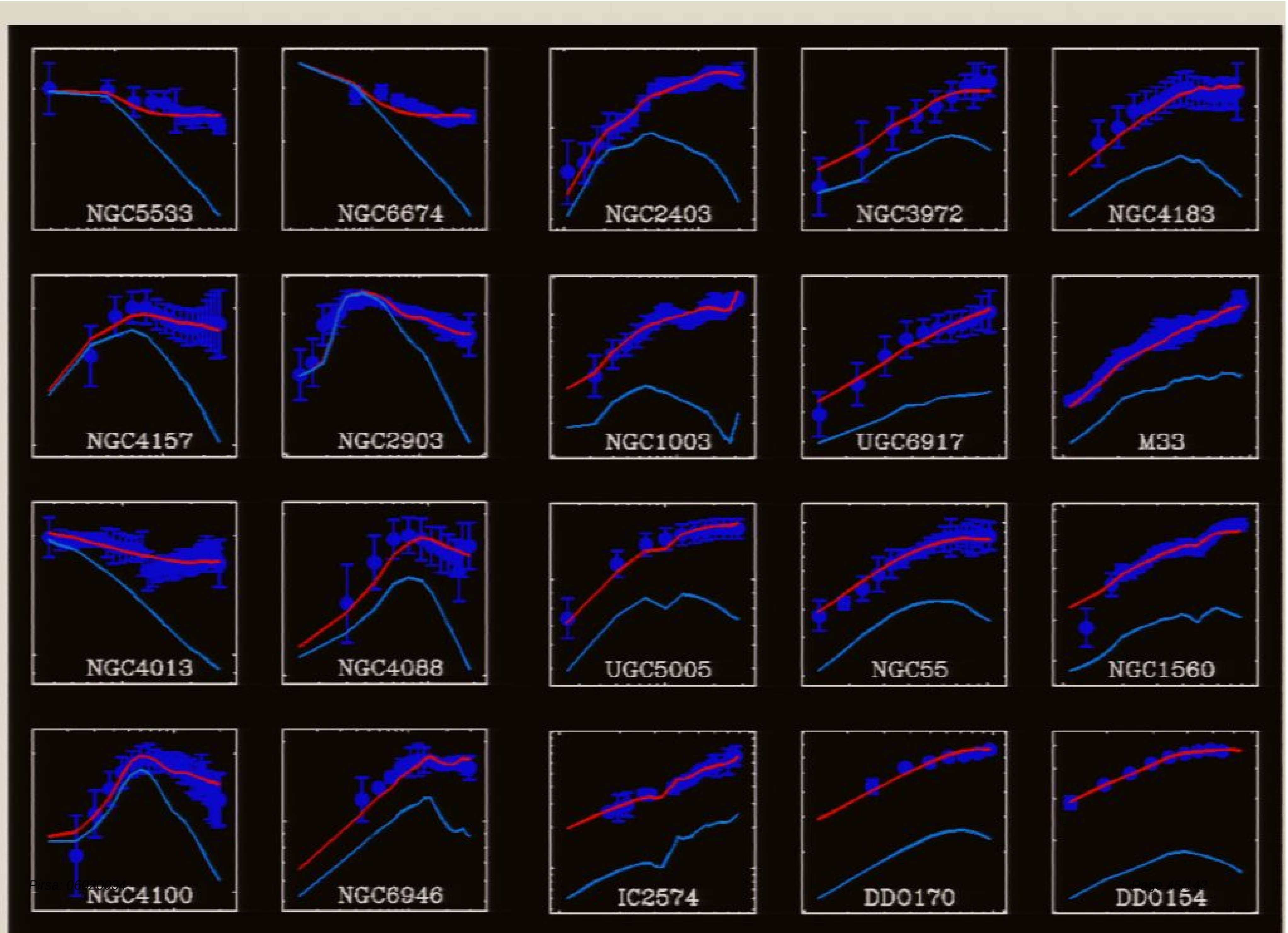
The "part - whole" paradox

M 81

M35 and NGC 2158

$\tilde{\mu} \approx 1$

"Environmental" effect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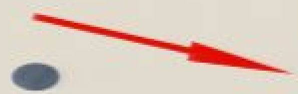

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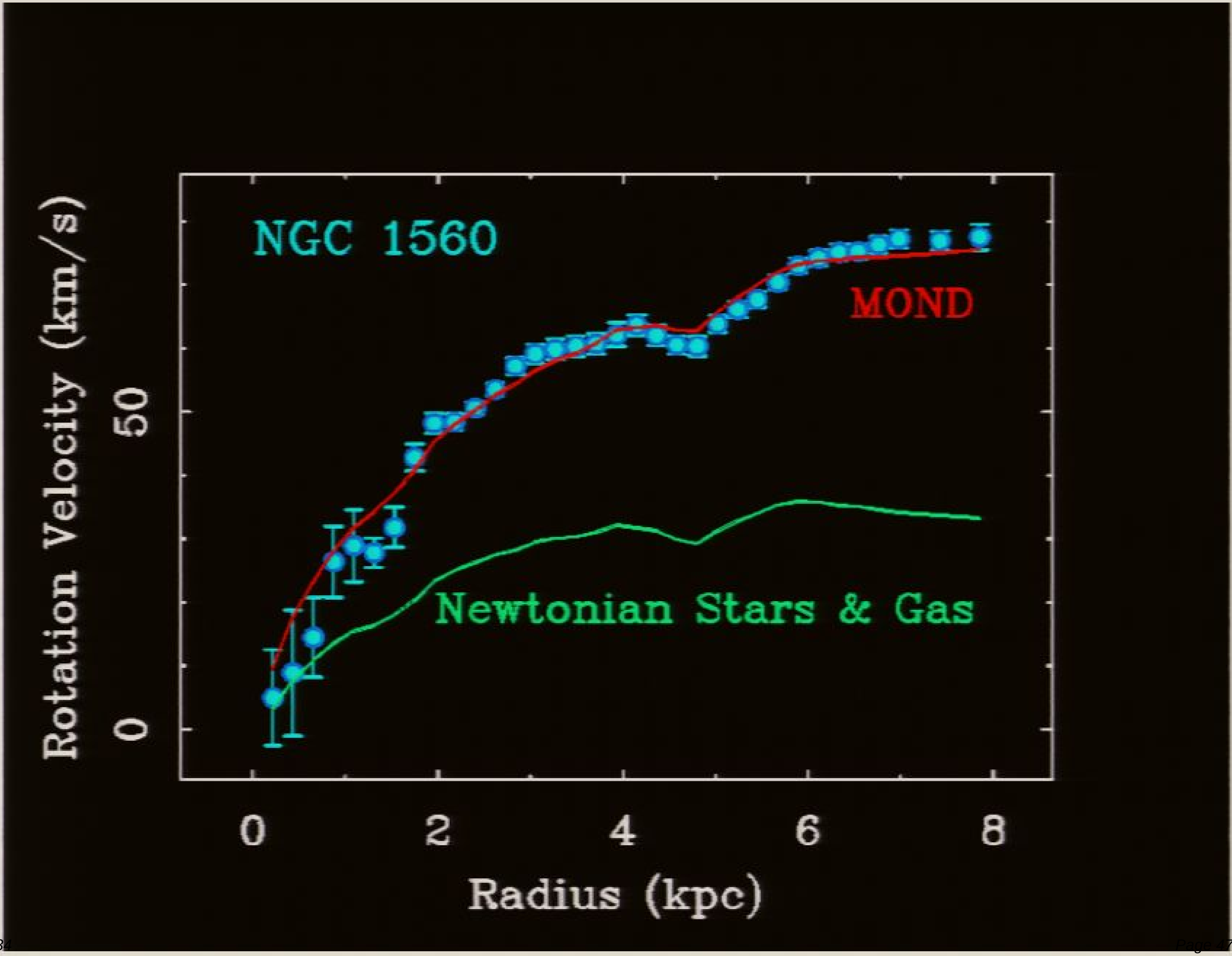
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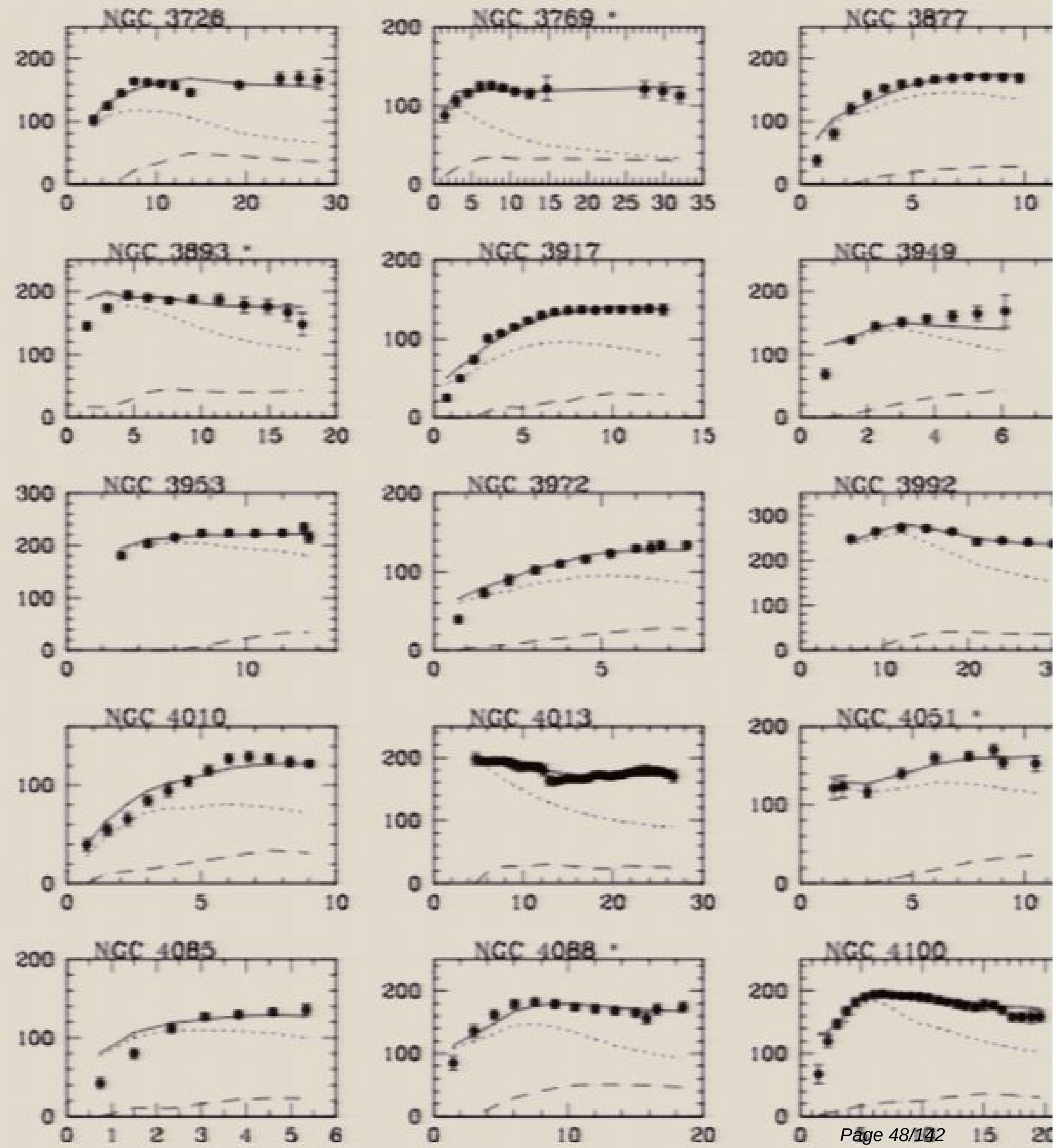

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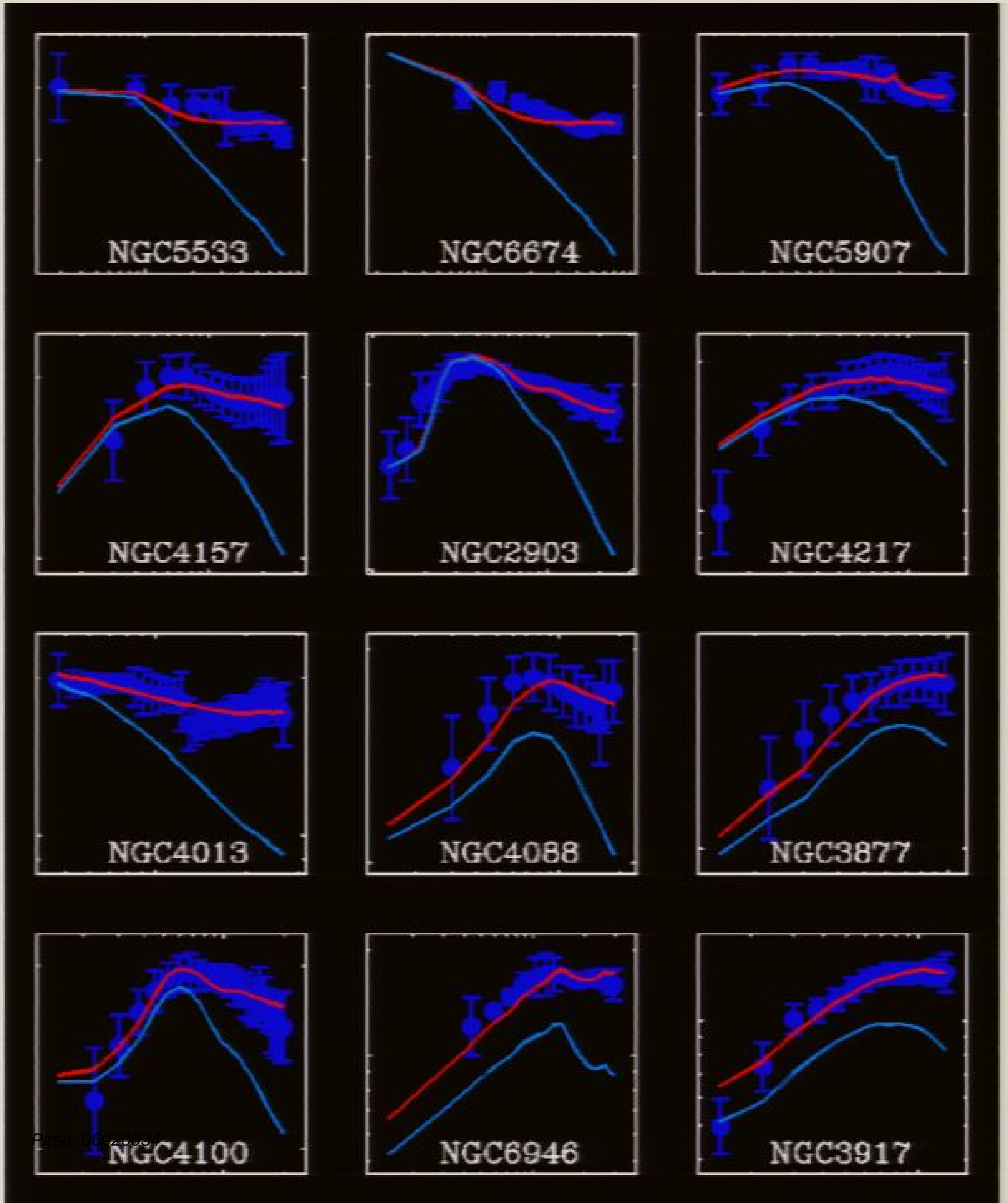

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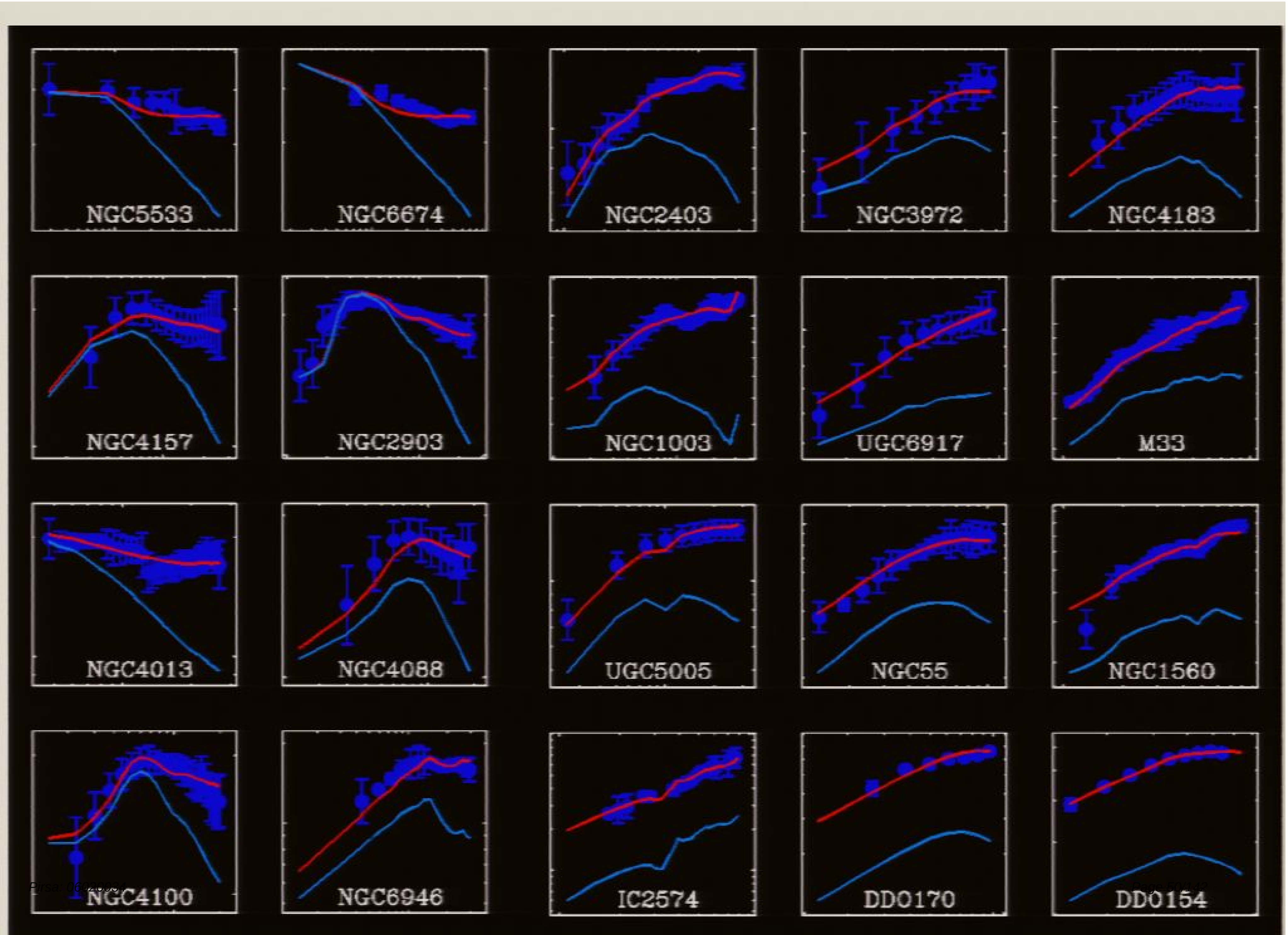
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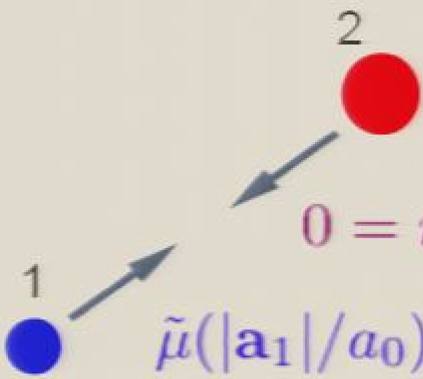

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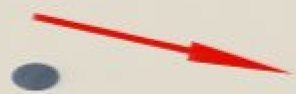

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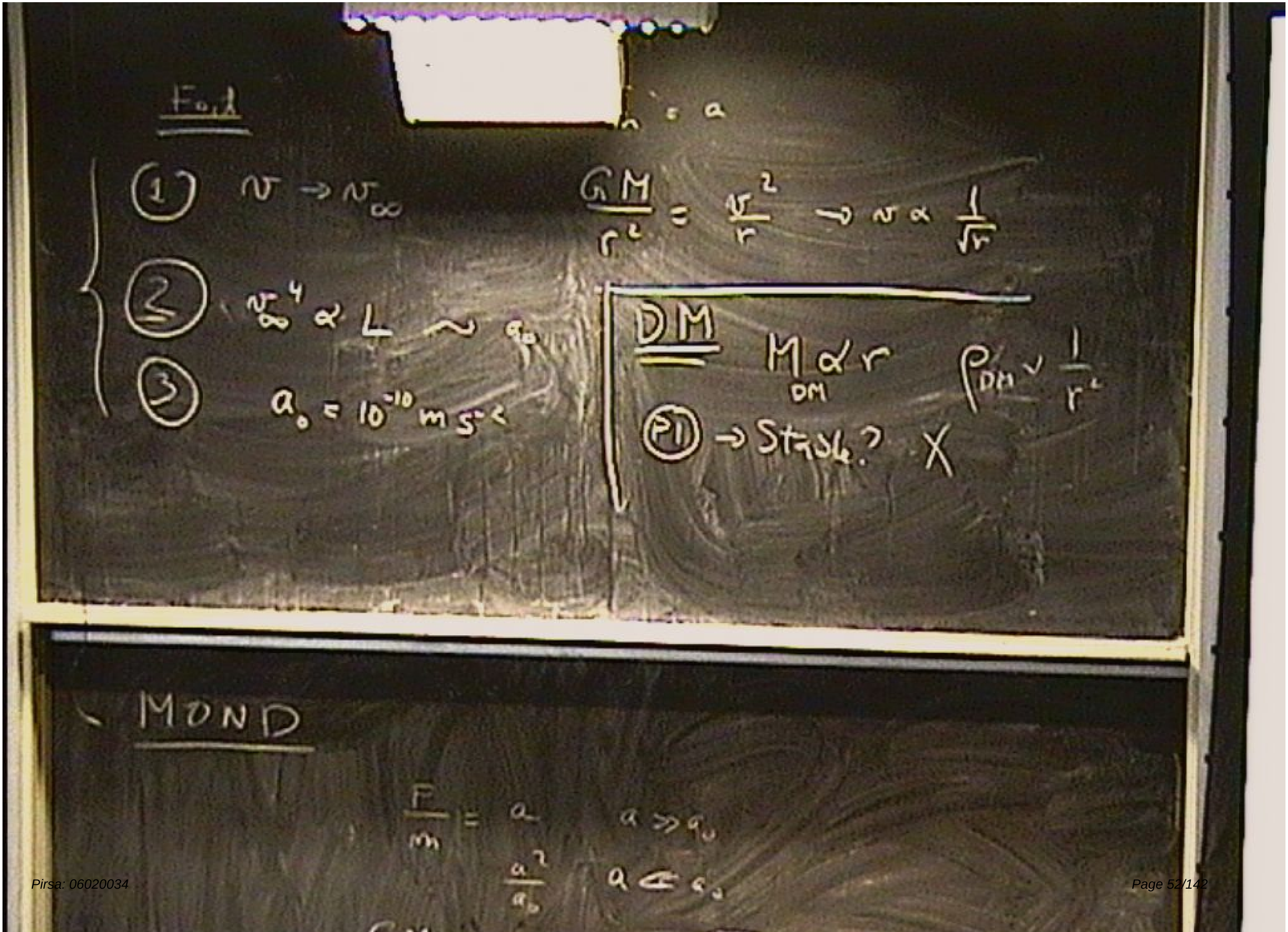
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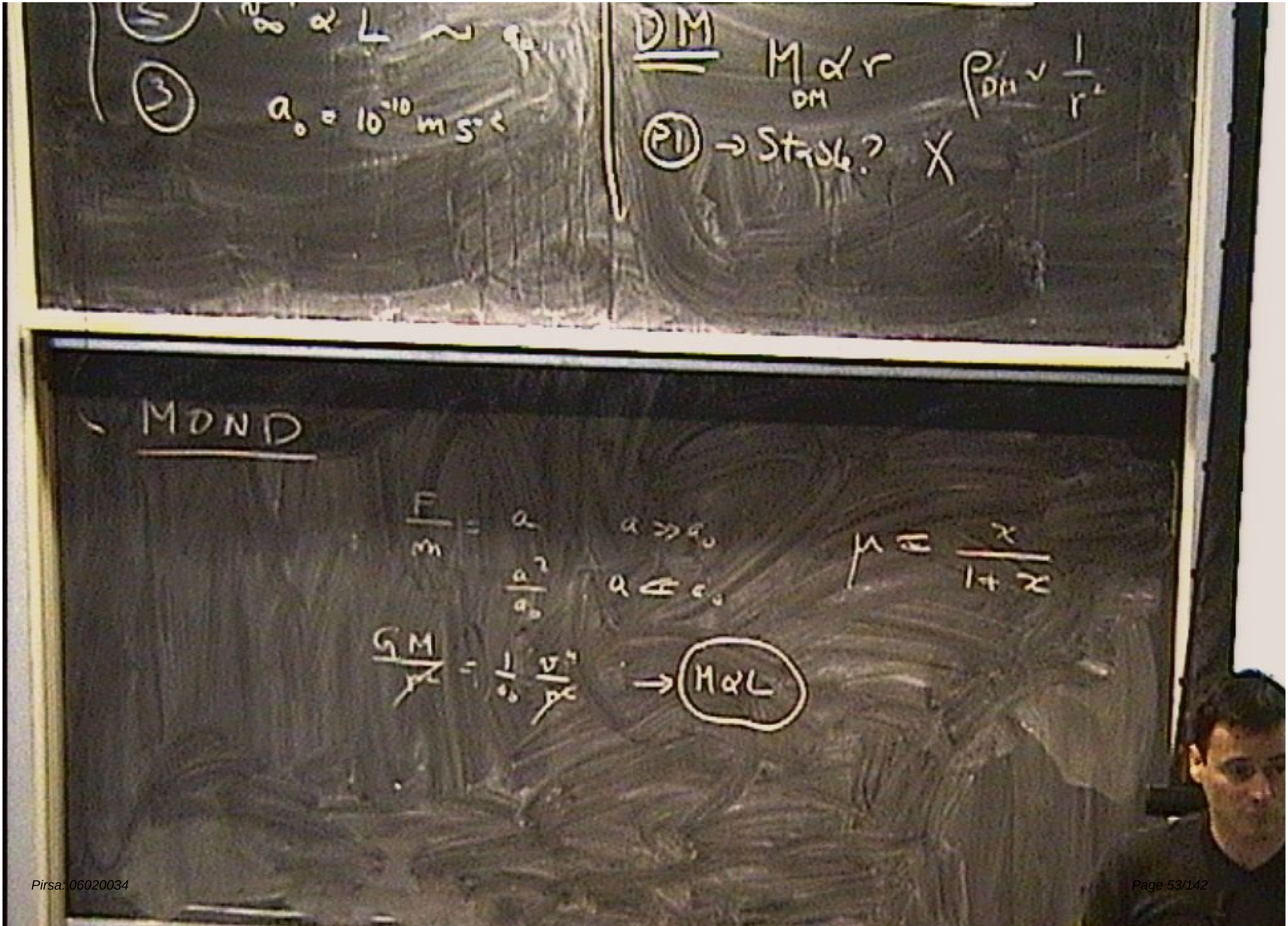

DM
MOND
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→ Strob.? X
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$$0 = m_1\tilde{\mu}(|\mathbf{a}_1|/a_0)\mathbf{a}_1 + m_2\tilde{\mu}(|\mathbf{a}_2|/a_0)\mathbf{a}_2 \neq m_1\mathbf{a}_1 + m_2\mathbf{a}_2$$

1

$$\tilde{\mu}(|\mathbf{a}_1|/a_0)\mathbf{a}_1 = (Gm_2/r^3)\mathbf{r}$$

The “part - whole” paradox

M 81

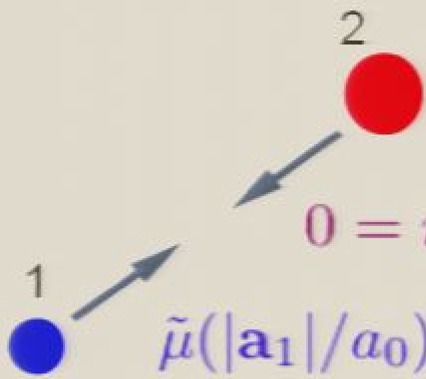
M35 and NGC 2158

$\tilde{\mu} \approx 1$

“Environmental” effect

Conceptual problems with MOND (Milgrom 1983)

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_{\mathrm{N}}$$



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The “part - whole” paradox

M 81

M35 and NGC 2158

$\tilde{\mu} \approx 1$

“Environmental” effect

Missing mass in clusters of galaxies

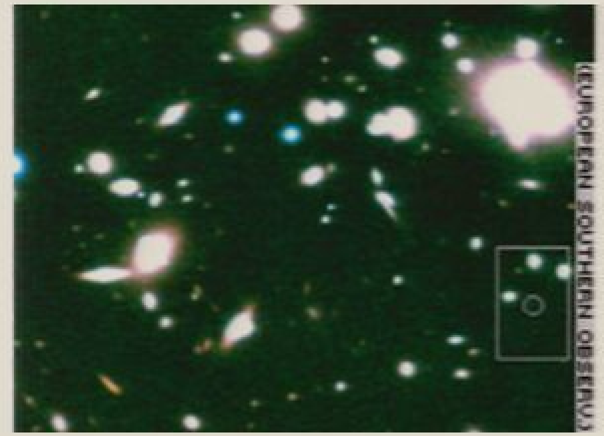
- From galaxy dynamics

  random motions define v

  $$M \approx Rv^2/G$$

  lots of missing mass;

  X-ray emitting gas explains part

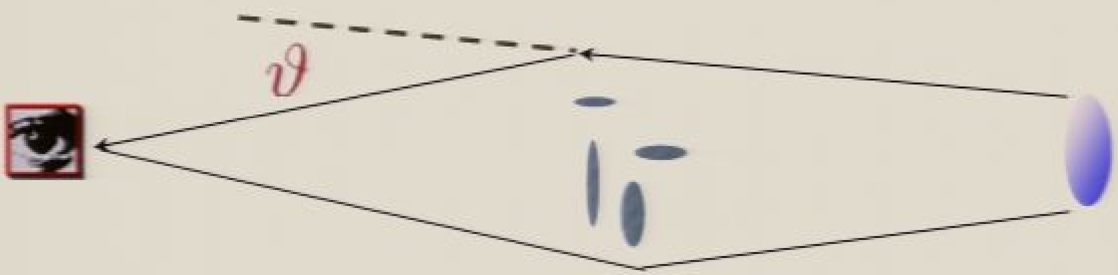


Abell 1835

- X-ray gas temperature distribution gives independent measure of mass

Abell 1689

- From gravitational lensing



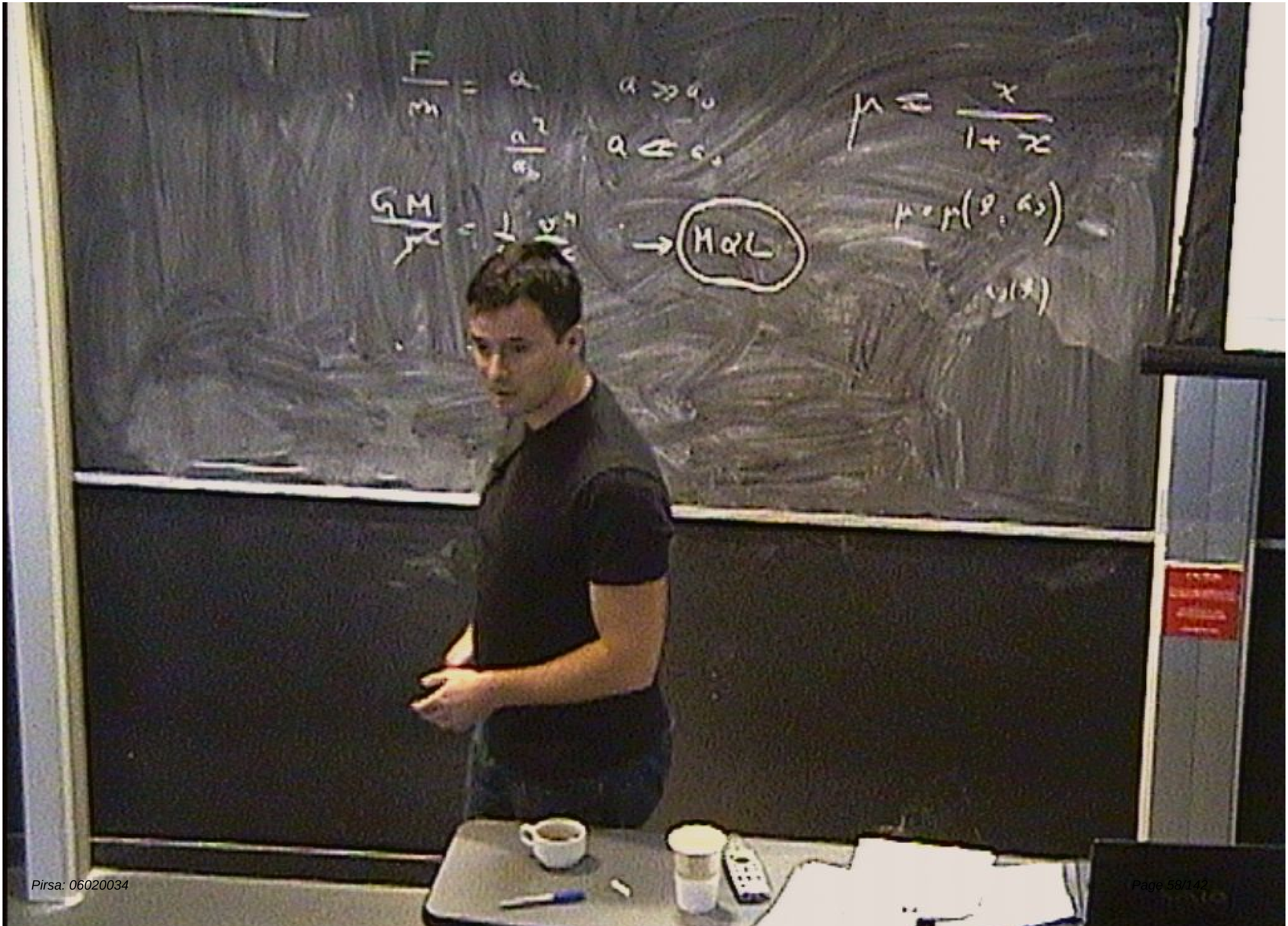
$$M \approx R\vartheta/4G$$

What to make of it ?

A. MOND is just an efficient way to parametrize dark matter

B. MOND reflects a departure of the effective theory of gravity from general relativity on macroscopic scales

- Construct such new gravity theory; ideally it should
  - have Newtonian nonrelativistic limit for large accelerations
  - have MOND nonrelativistic limit for small accelerations
  - reduce to GR for relativistic large acceleration situations
  - explain anomalous large lensing by clusters of galaxies
  - pass the post-Newtonian tests in the solar system
  - tell us how to do cosmology



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We need a direct detection!

- Dark matter searches: the game is over if a dark matter particle is detected!
- What is the equivalent "backyard" detection of MONDian behavior?

Tensor Vector Scalar theory

Bekenstein, Phys. Rev. D **70**, 083509 (2004); astro-ph/0412652

Dimensionless parameters K, k and scale of length ℓ

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} (-g)^{1/2} d^4x$$

$$S_v = -\frac{K}{32\pi G} \int \left[g^{\alpha\beta} g^{\mu\nu} \mathcal{U}_{[\alpha,\mu]} \mathcal{U}_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} \mathcal{U}_\mu \mathcal{U}_\nu + 1) \right] (-g)^{1/2} d^4x$$

$$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)(-\tilde{g})^{1/2} d^4x$$

$$\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + \mathcal{U}_\alpha \mathcal{U}_\beta) + e^{2\phi} \mathcal{U}_\alpha \mathcal{U}_\beta$$

$$S_s = -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \right] (-g)^{1/2} d^4x$$

$$\downarrow$$

$$h^{\alpha\beta} = g^{\alpha\beta} - \mathcal{U}^\alpha \mathcal{U}^\beta$$

AQUAL gravitational field theory (NR)

Bekenstein and Milgrom (1984)

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_\mathrm{N}$$

$$\mathcal{L} = -\frac{a_0^2}{8\pi G} f\left(\frac{|\nabla\Phi|^2}{a_0^2}\right) - \rho\Phi$$ (AQUAdratic Lagrangian)

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$$\tilde{\mu}(\sqrt{y}) \equiv df(y)/dy$$

Compare with $\nabla \cdot \nabla\Phi_N = 4\pi G\rho$

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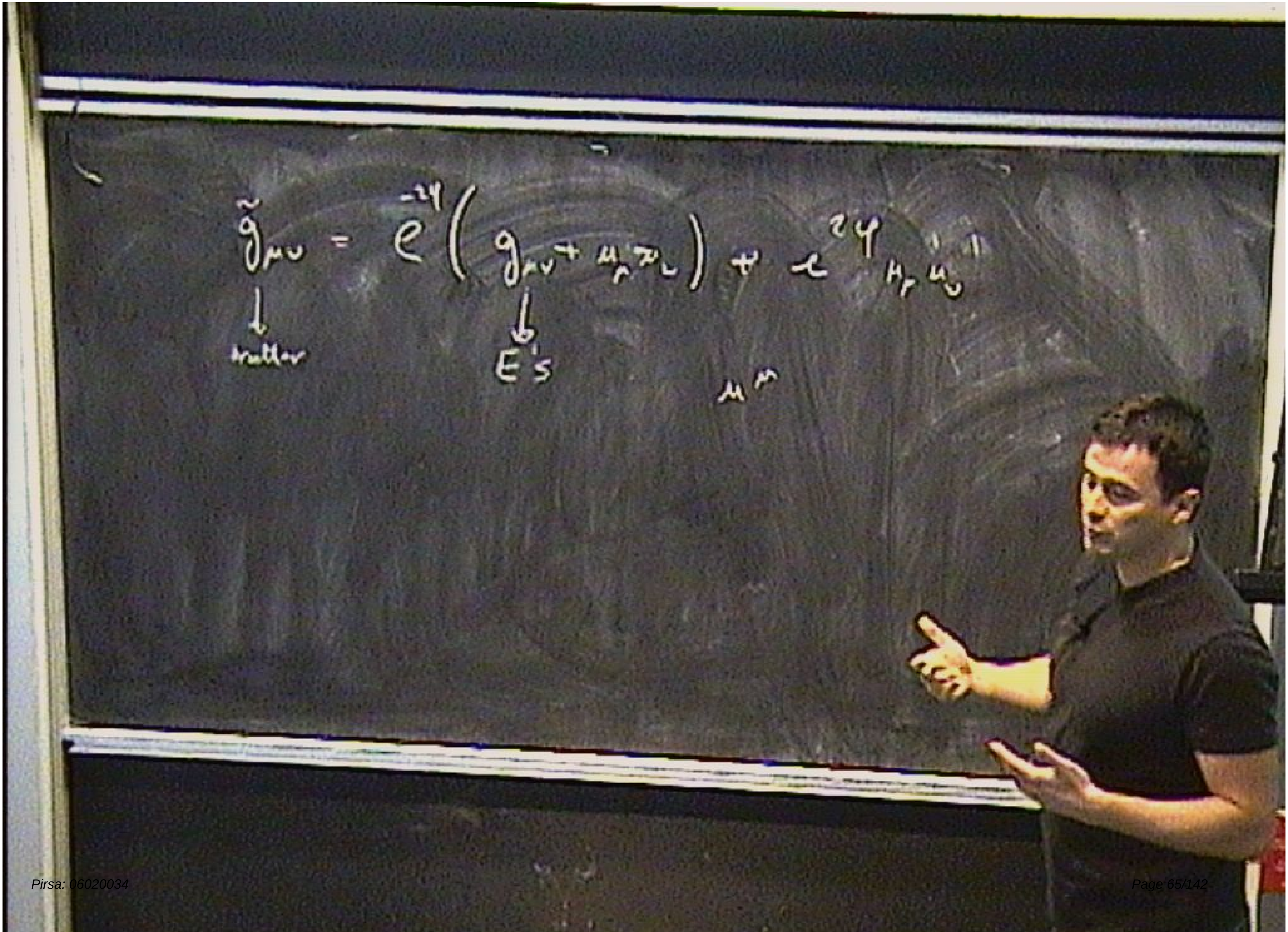
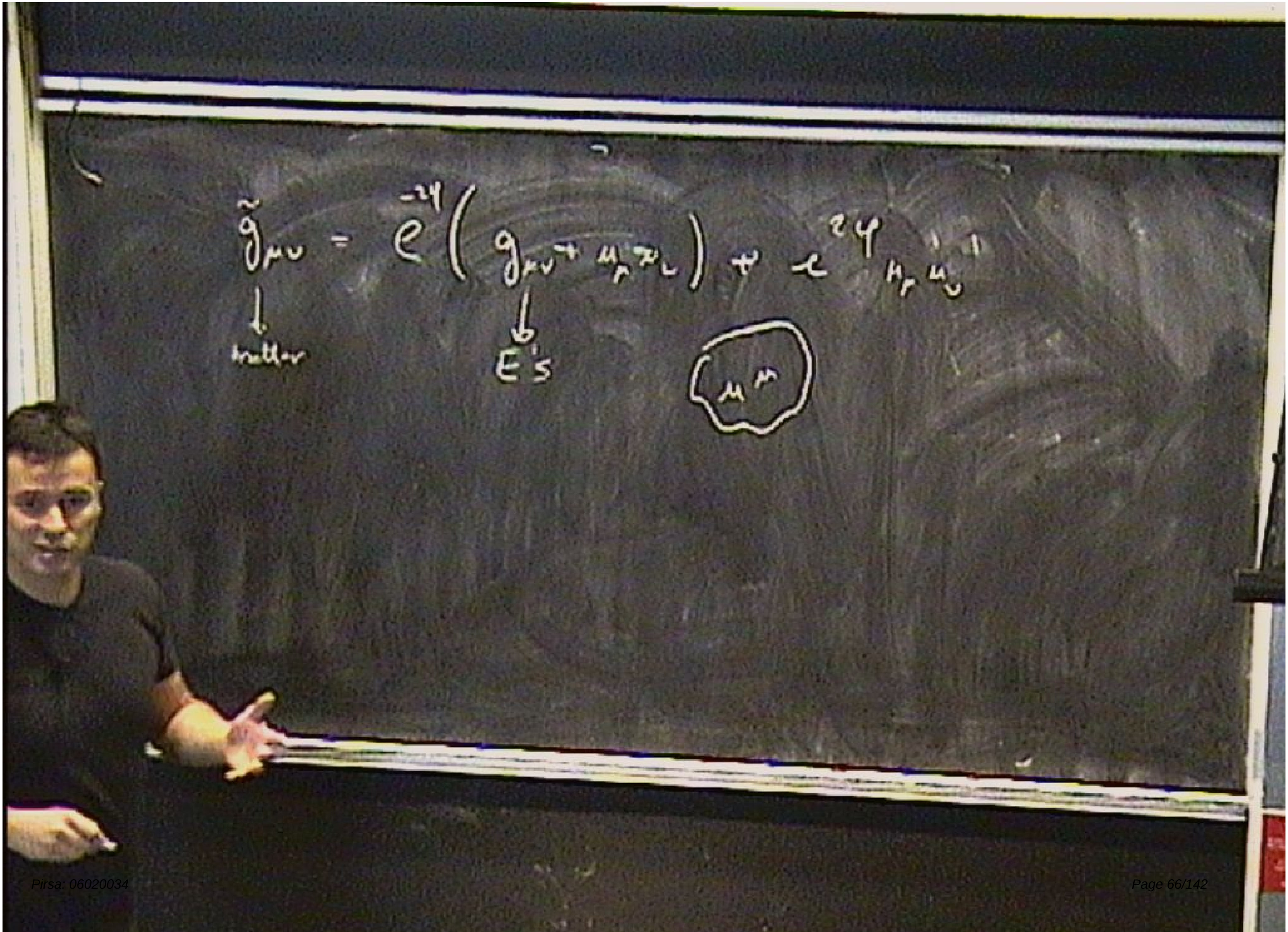
$$h^{\alpha\beta} = g^{\alpha\beta} - \mathcal{U}^\alpha \mathcal{U}^\beta$$

$$\tilde{g}_{\mu\nu} = e \quad g_{\mu\nu}$$

$\tilde{g}_{\mu\nu}$ ↓ matter

$g_{\mu\nu}$ ↓ E's


matter
E's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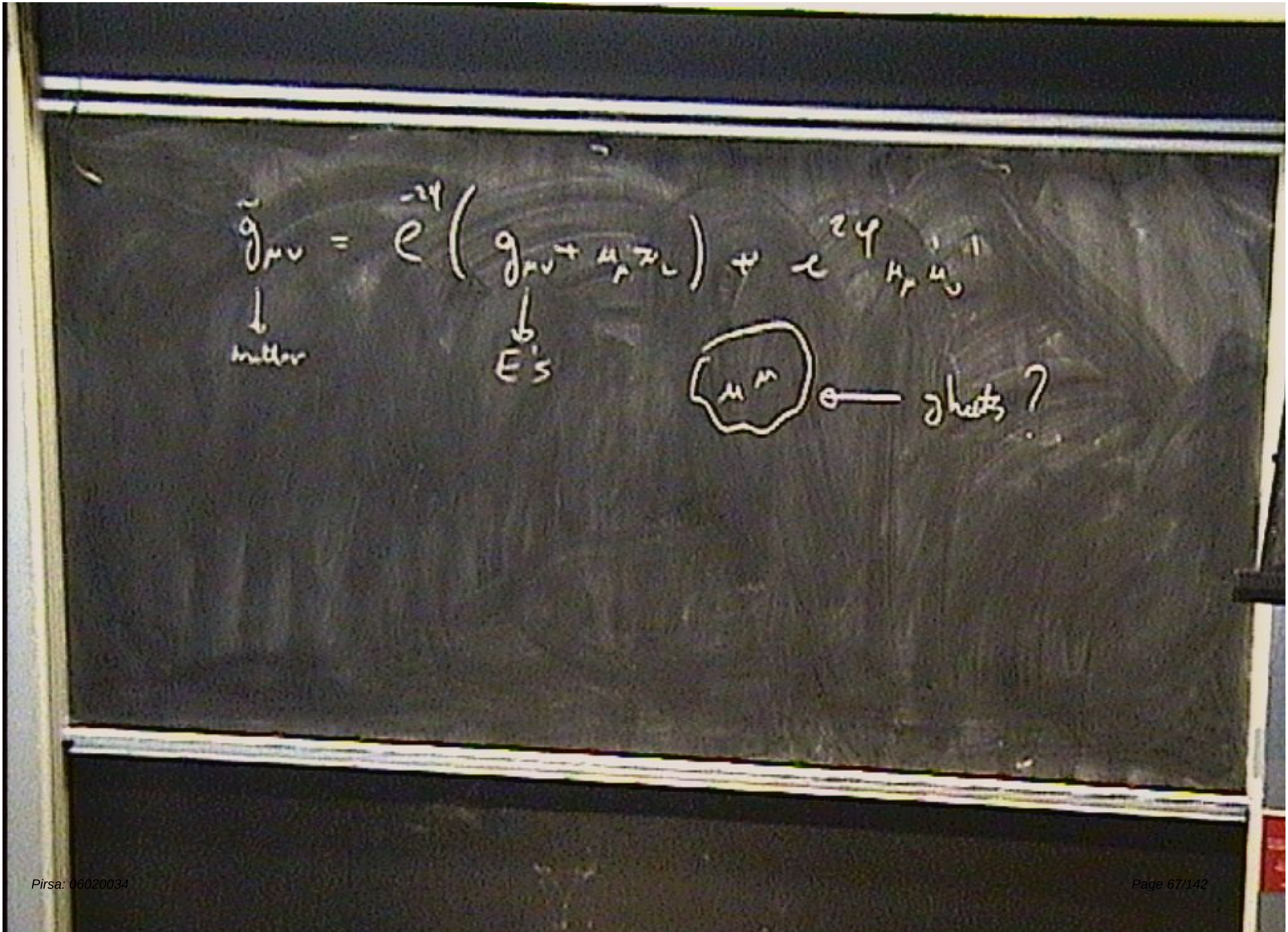
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matter
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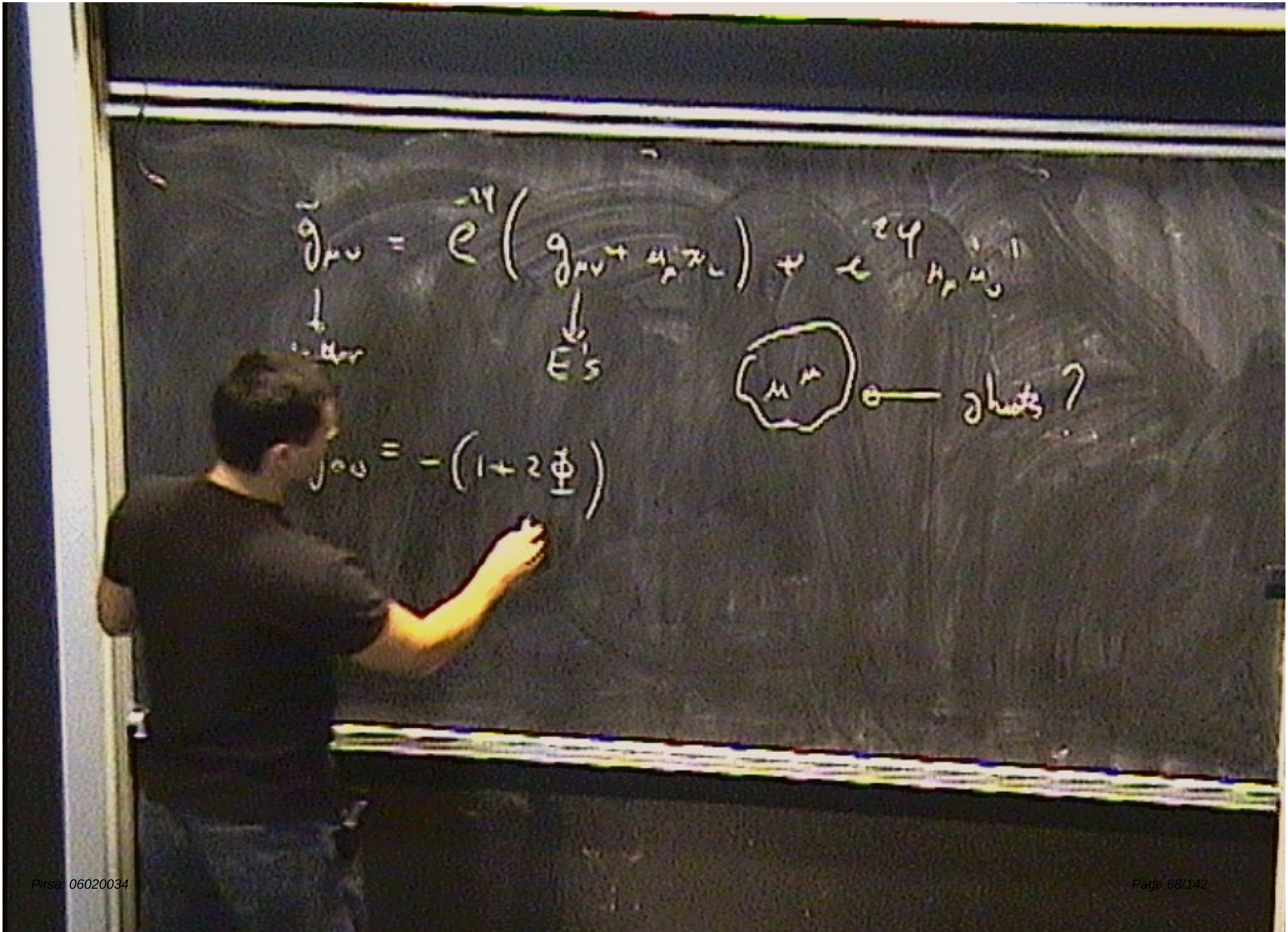
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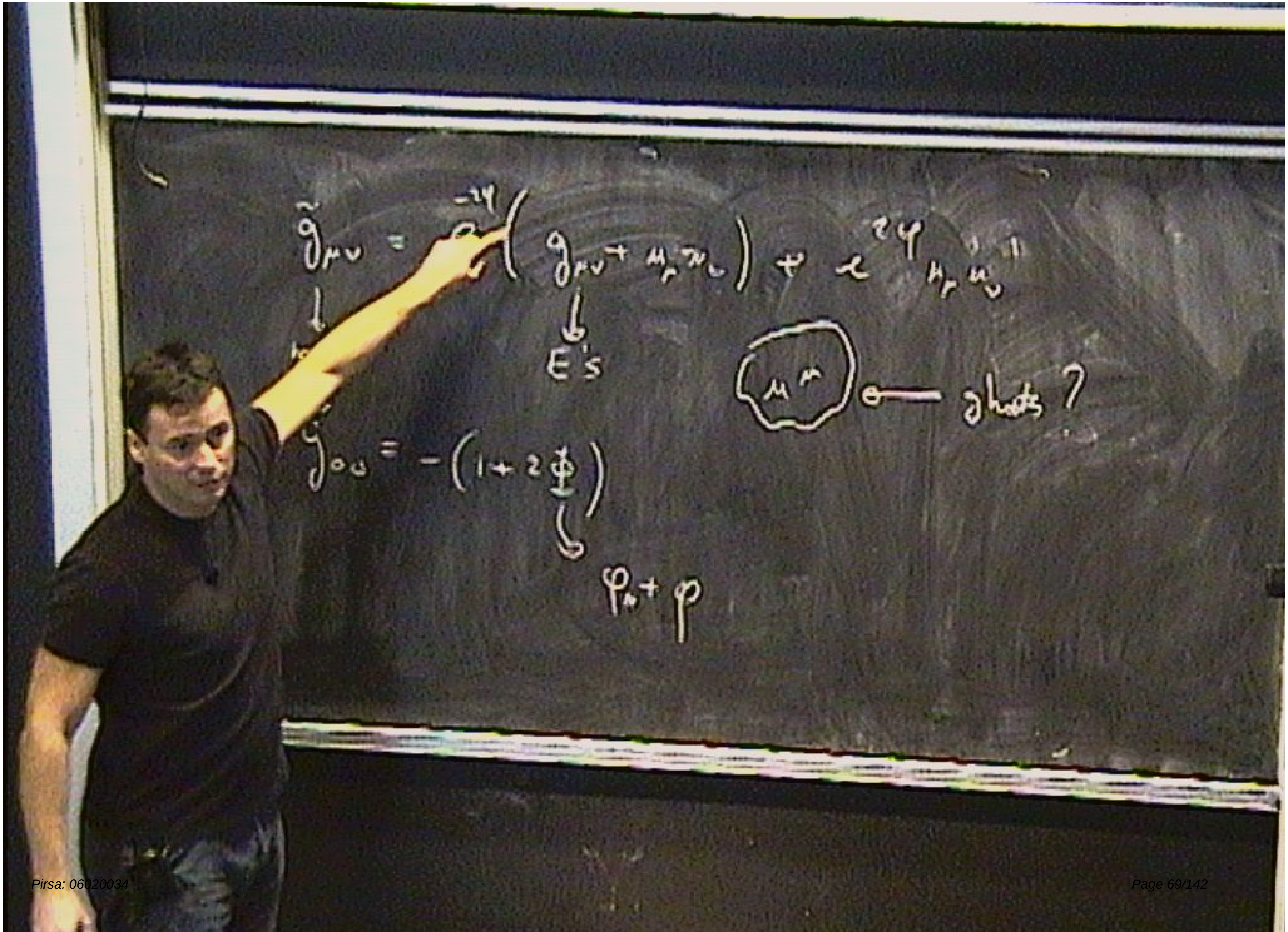

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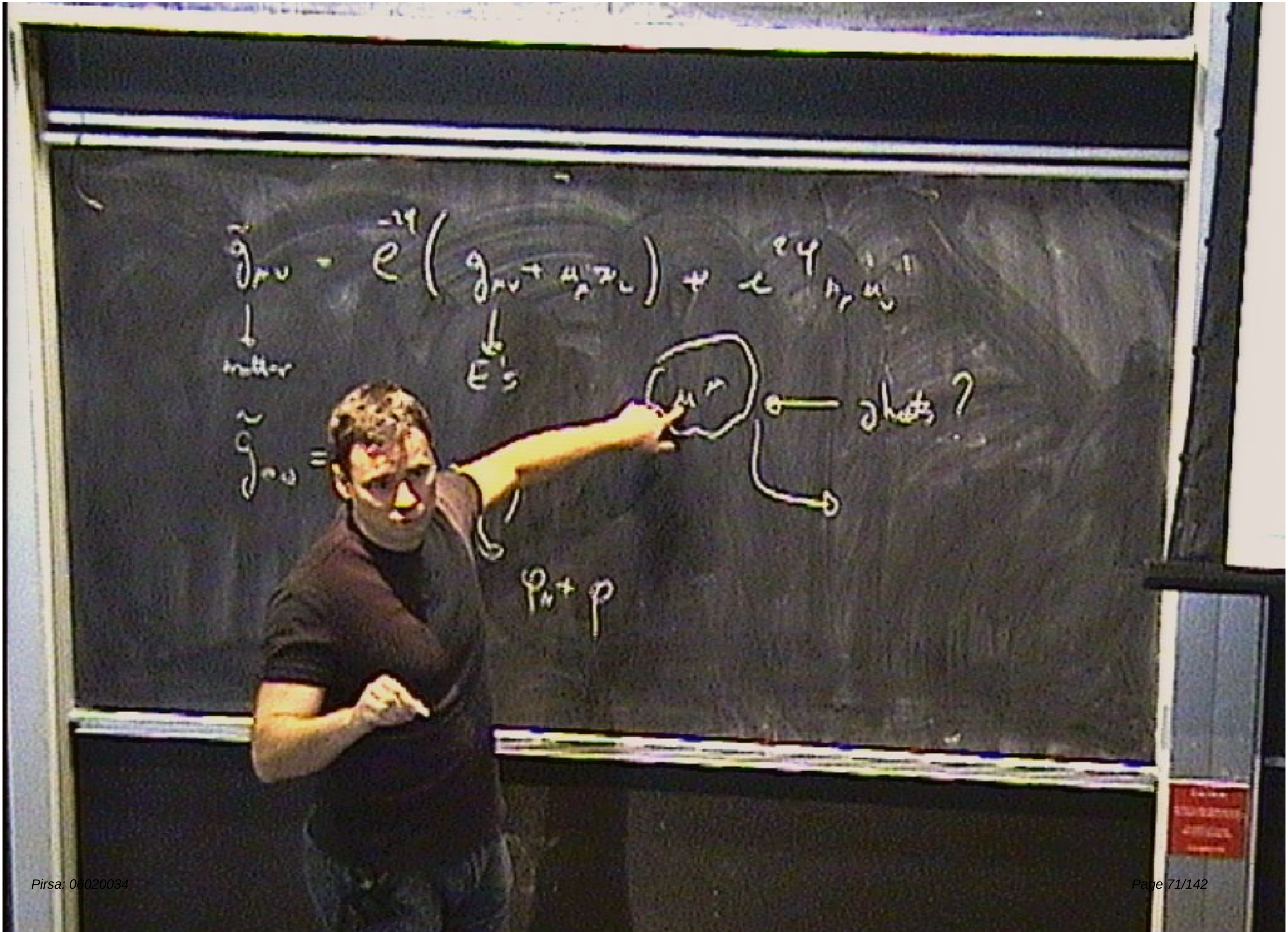
u^μ — ghosts ?

$$\tilde{g}_{00} = -(1 + 2\Phi)$$

Φ → $\varphi_N + \varphi$


matter
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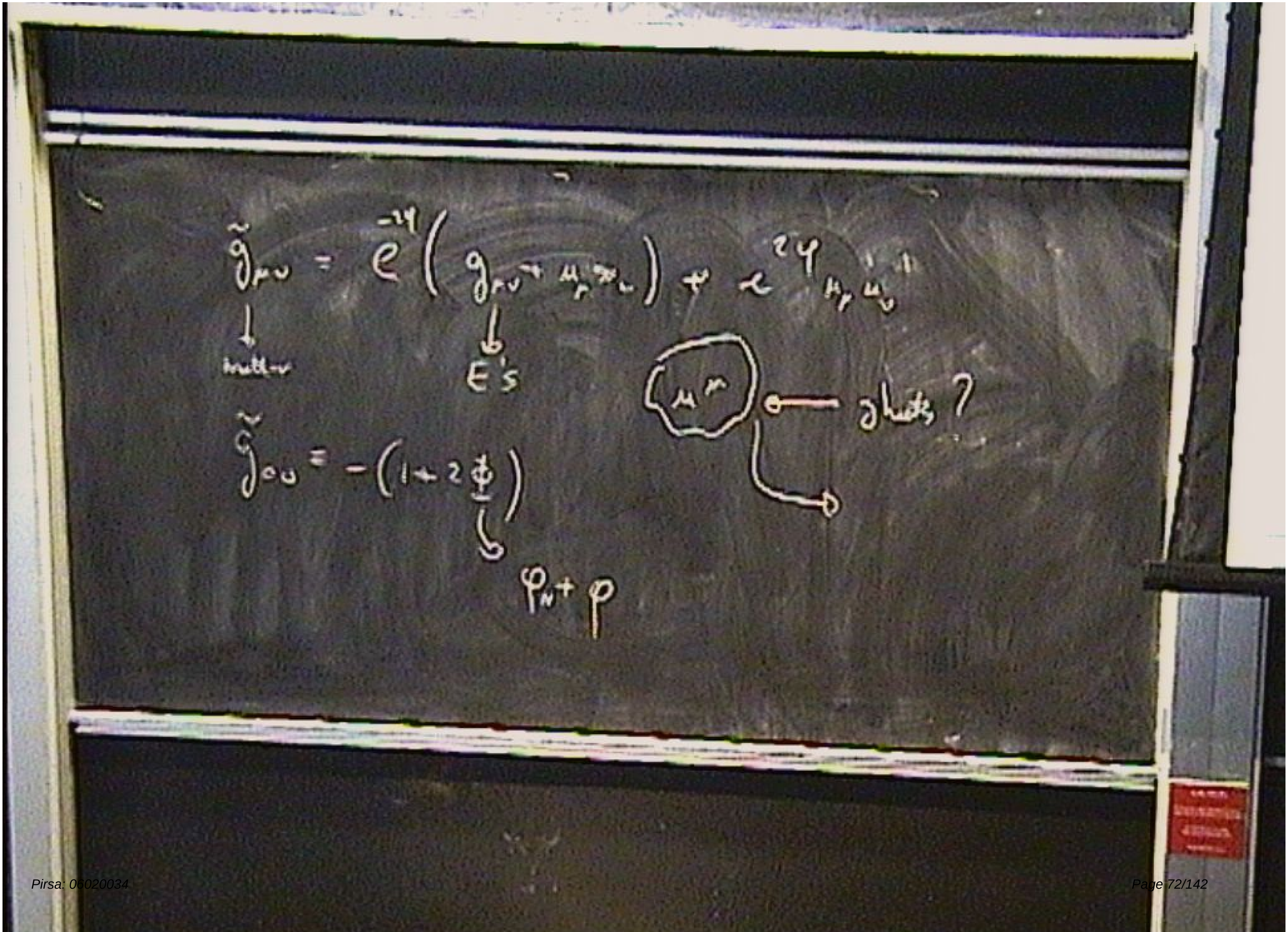
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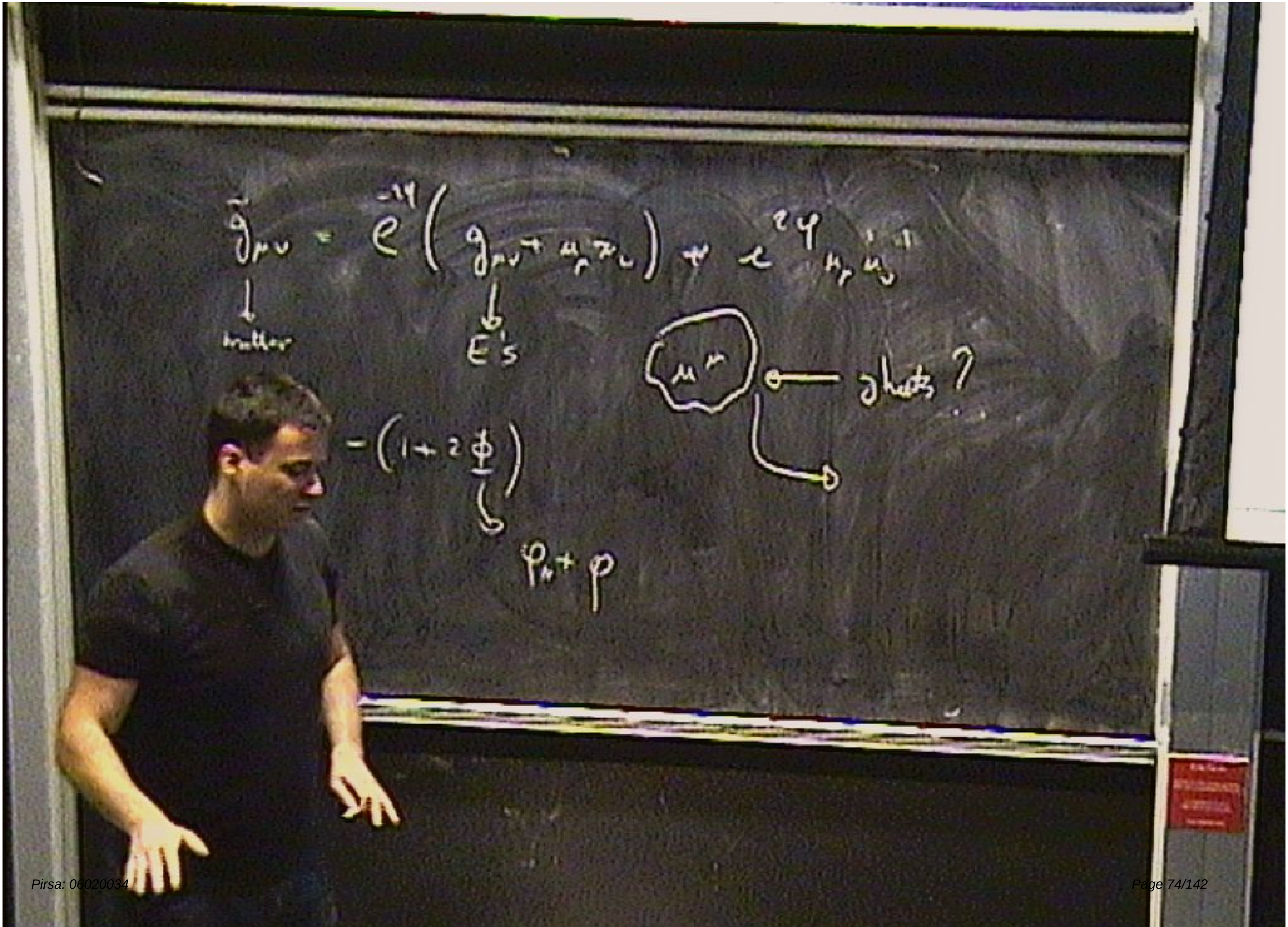
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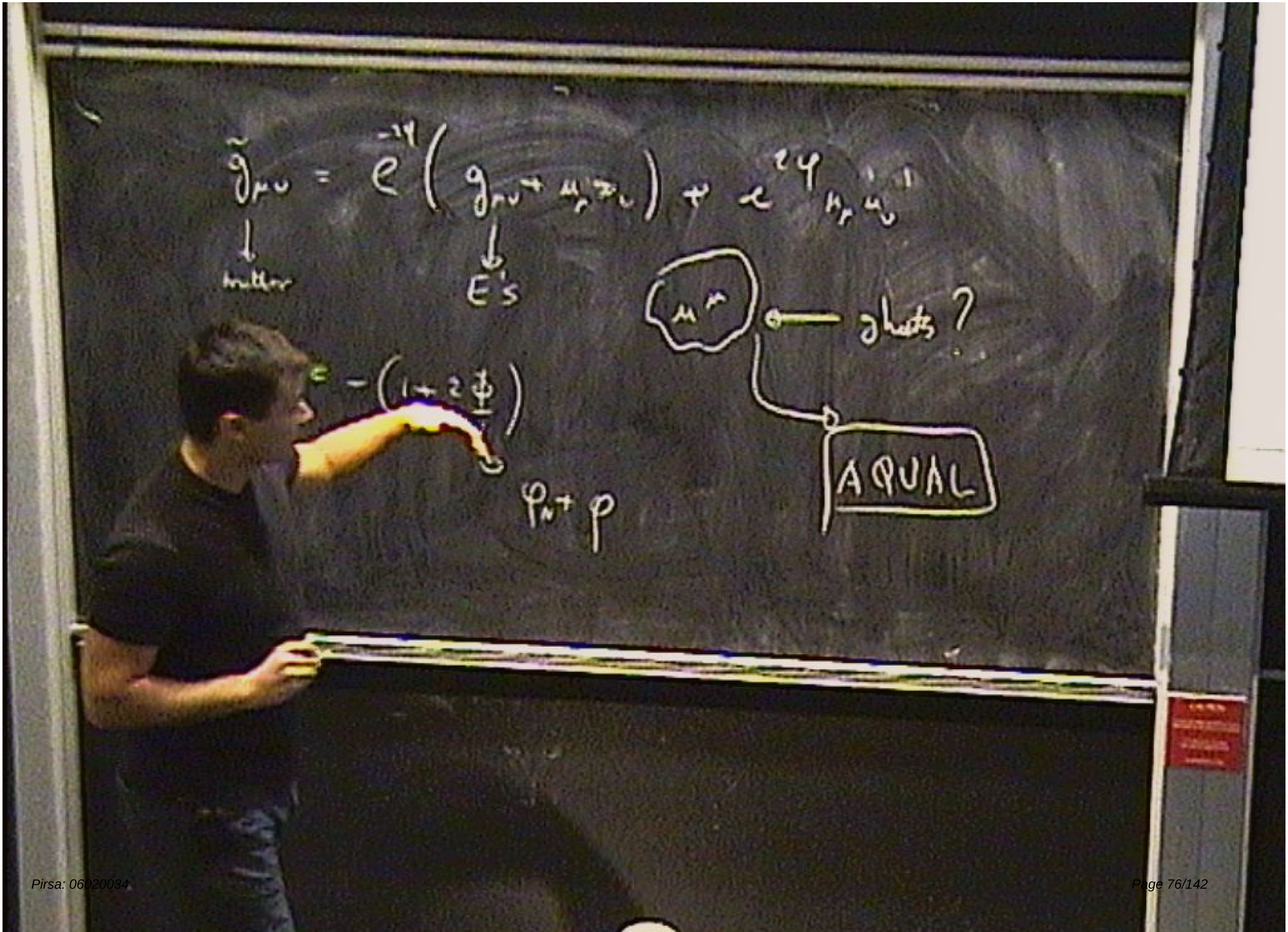
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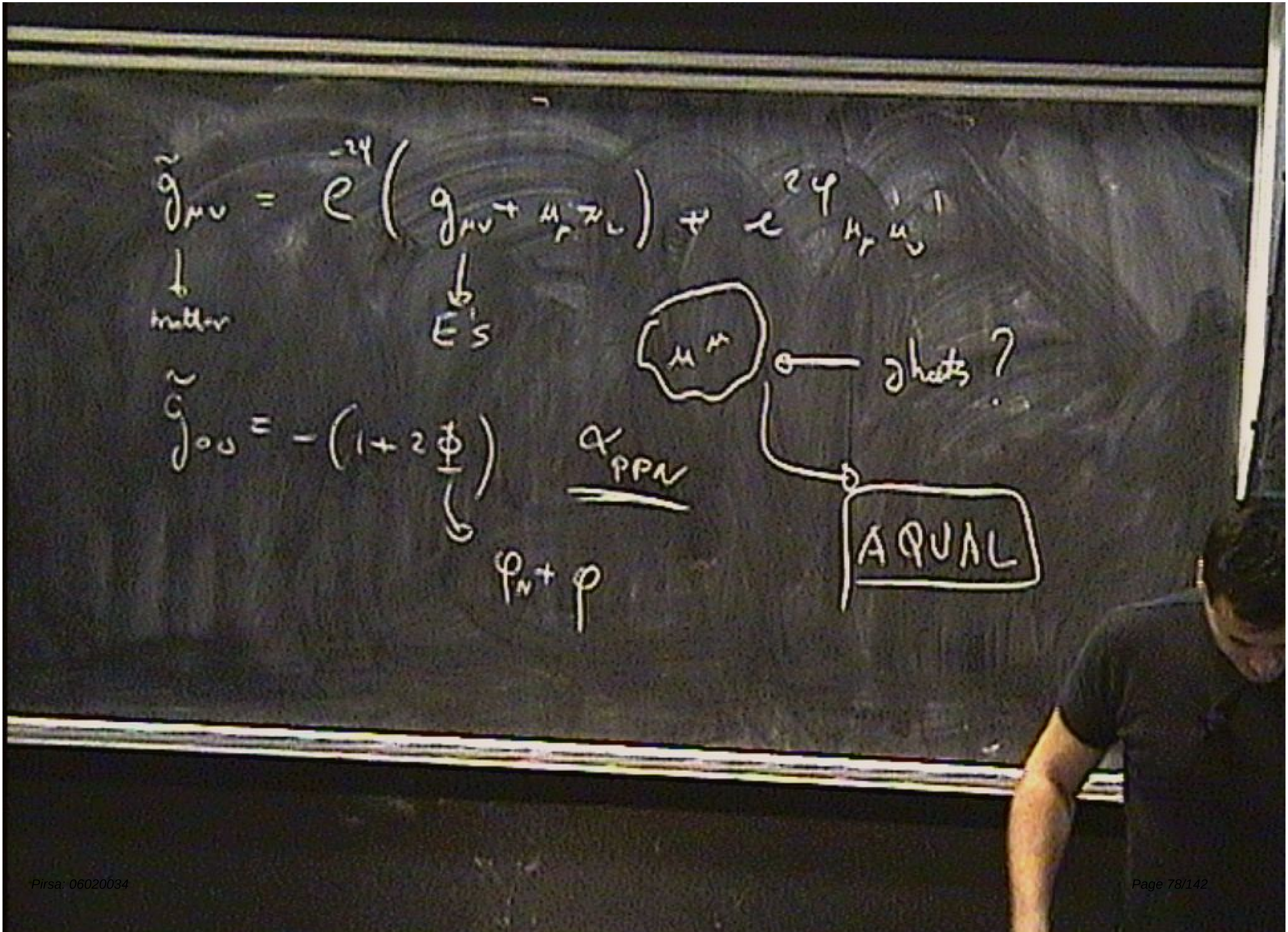

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matter
E's
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$\varphi_N + \varphi$
α_{PPN}
u^μ
ghosts ?
AQUAL

Summary: accomplishments of AQUAL

- ☑ Original MOND formula - an approximation
- ☑ Energy, momentum and angular momentum conserved
- ☑ Resolution of “part - whole” paradox
- ☑ Recovers the “environmental” effect
-

A backyard detection of MOND

- Solar system perturbations?
- Lagrange points?
- Saddles?

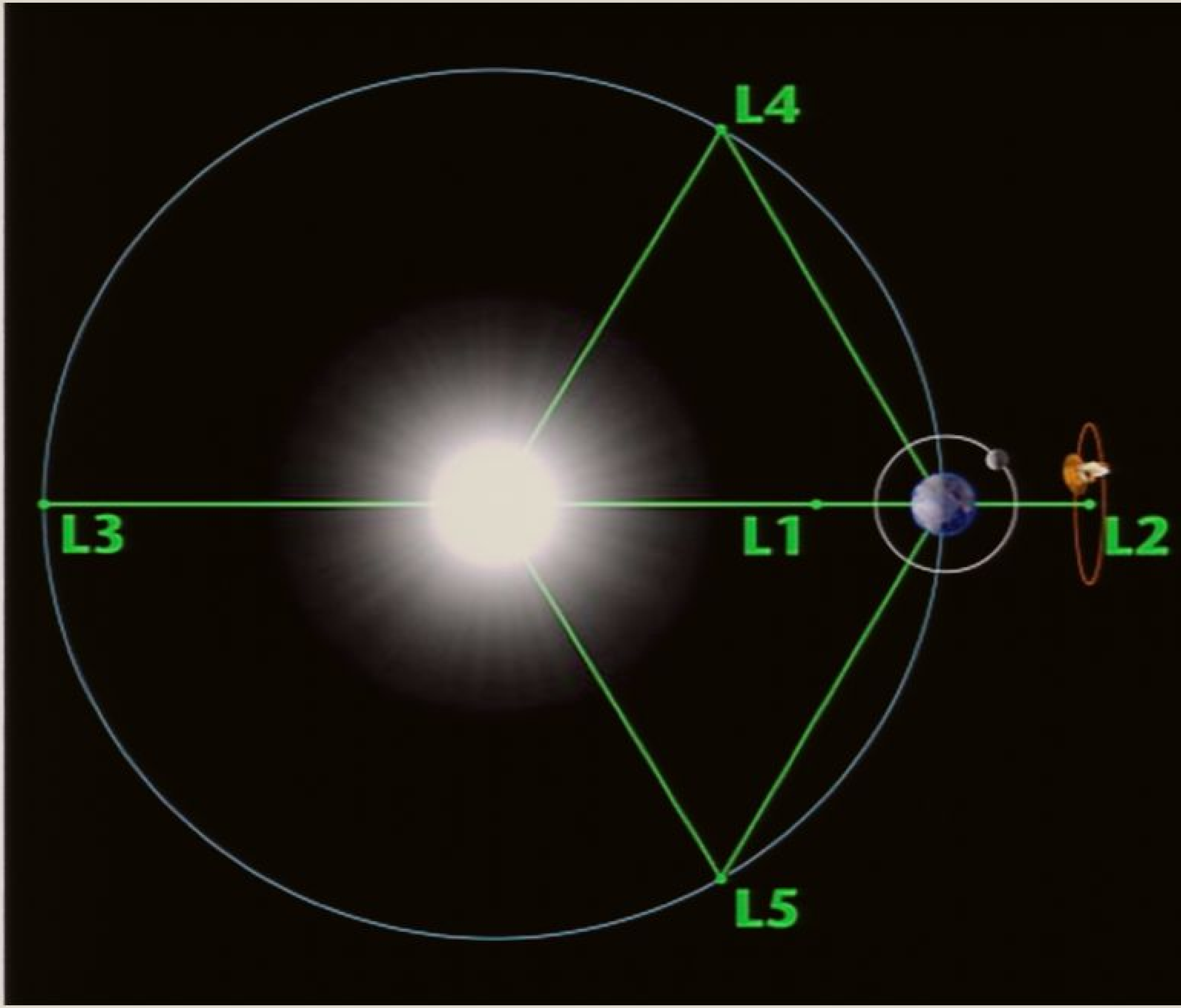
$$\underline{\nabla} \cdot \left(\mu \, \underline{\nabla} \phi \right) \doteq K G \rho$$

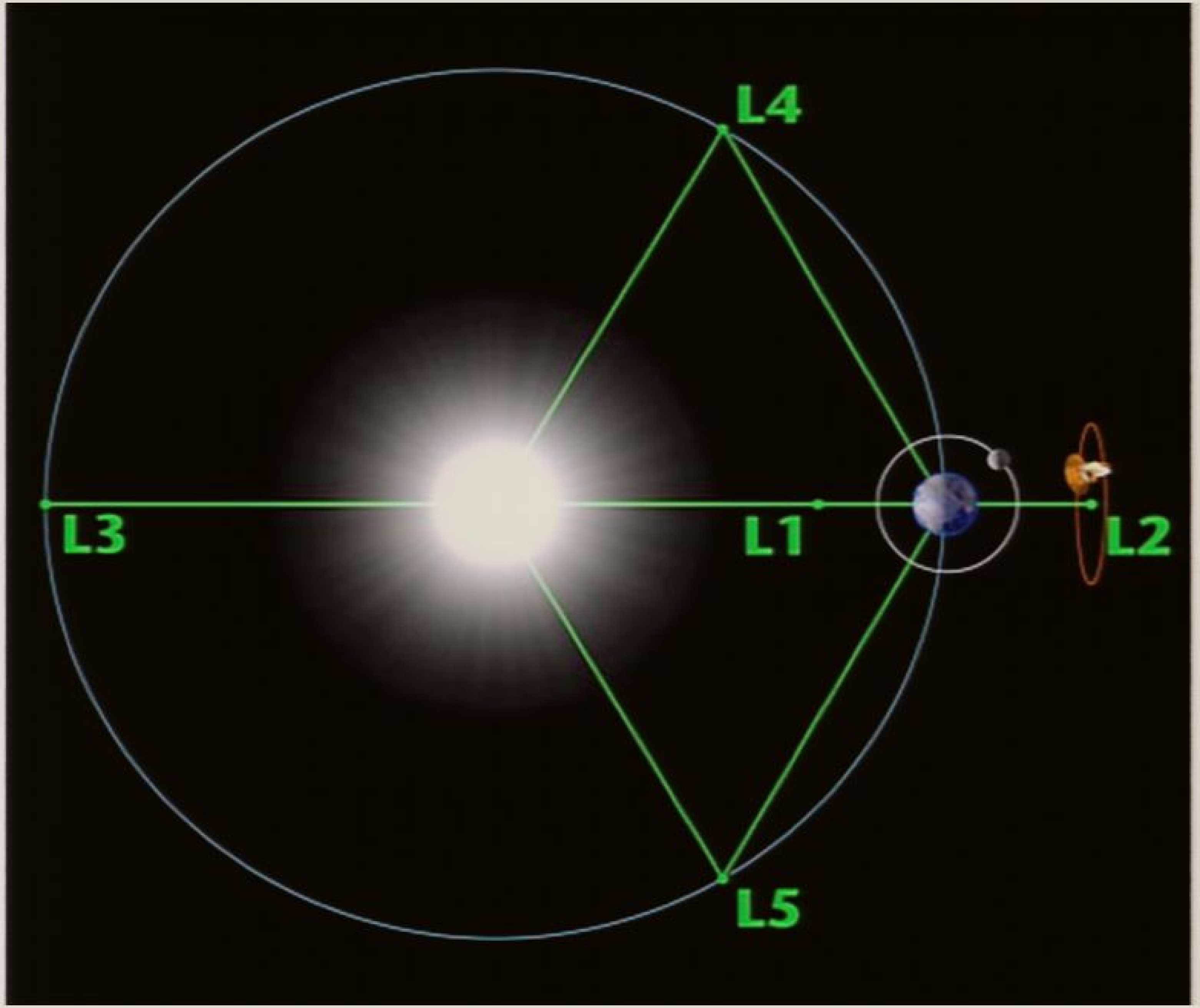
$$\underline{\nabla} \cdot \left(\mu \, \underline{\nabla} \varphi \right) \doteq K \, a \, \rho$$

$$\underline{\nabla} \varphi \ll a_0$$

$\downarrow$

$\underline{\underline{\text{tot}\,\varphi}}$


L4
L3
L1
L2
L5


L4
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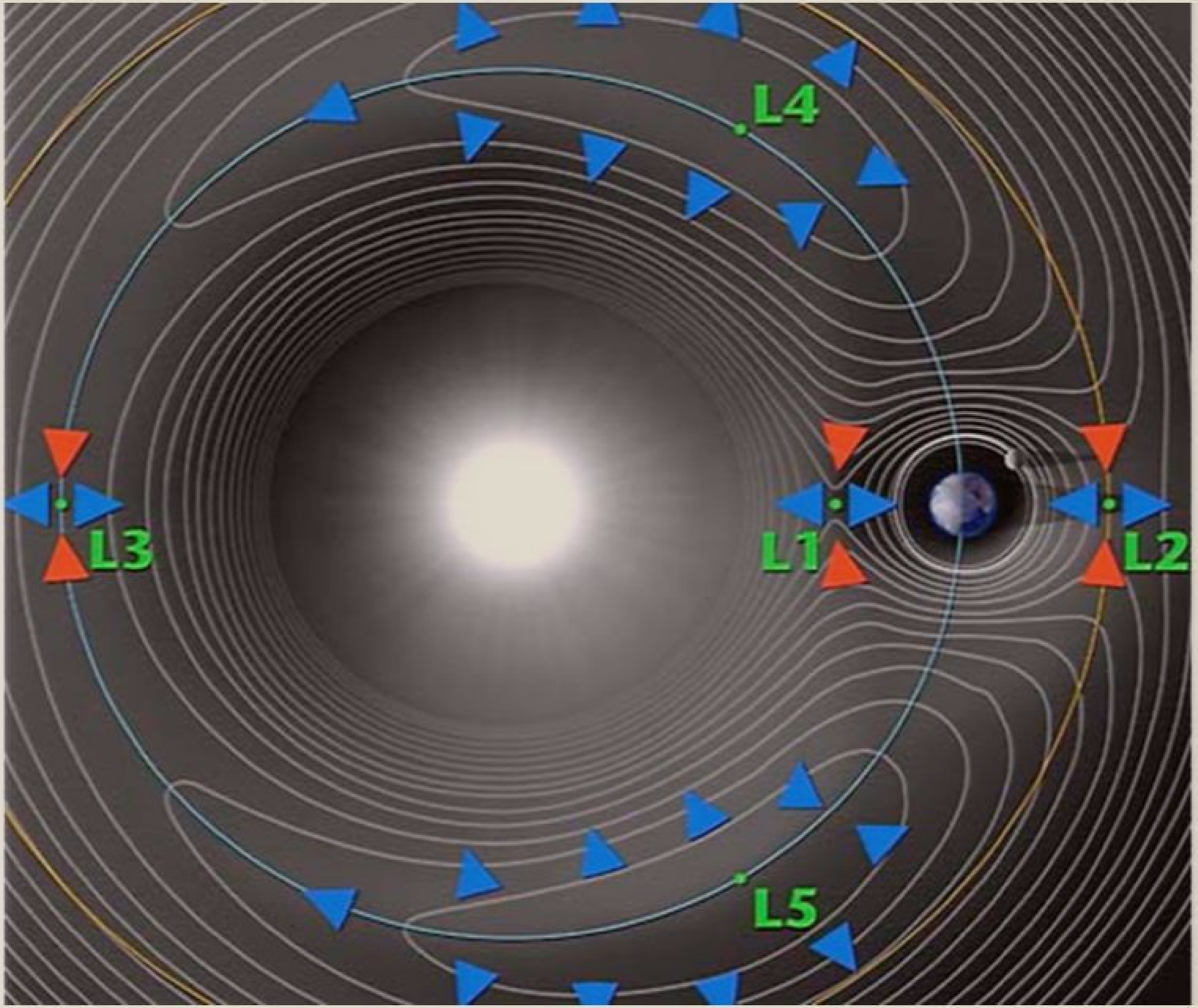

L4
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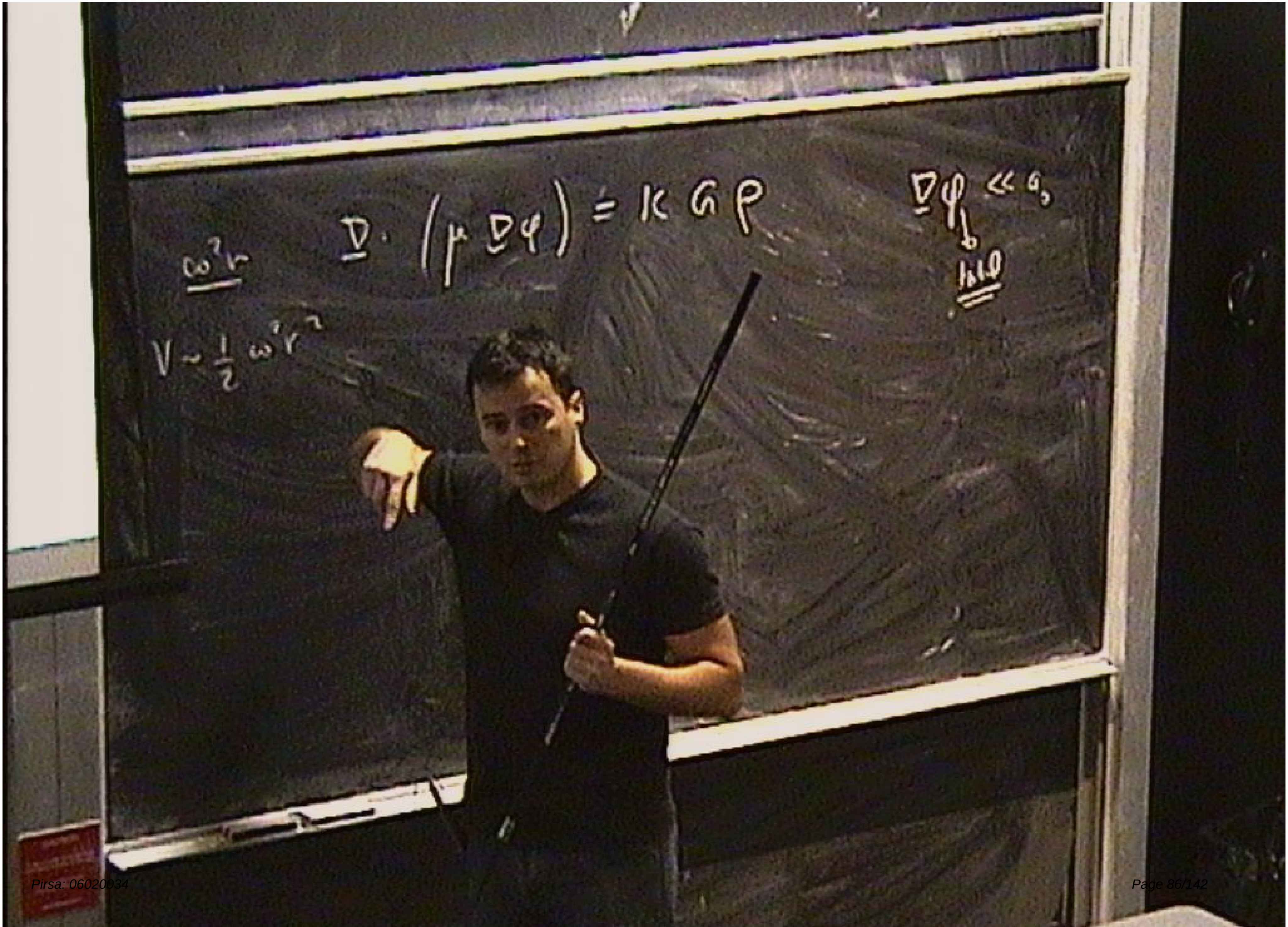
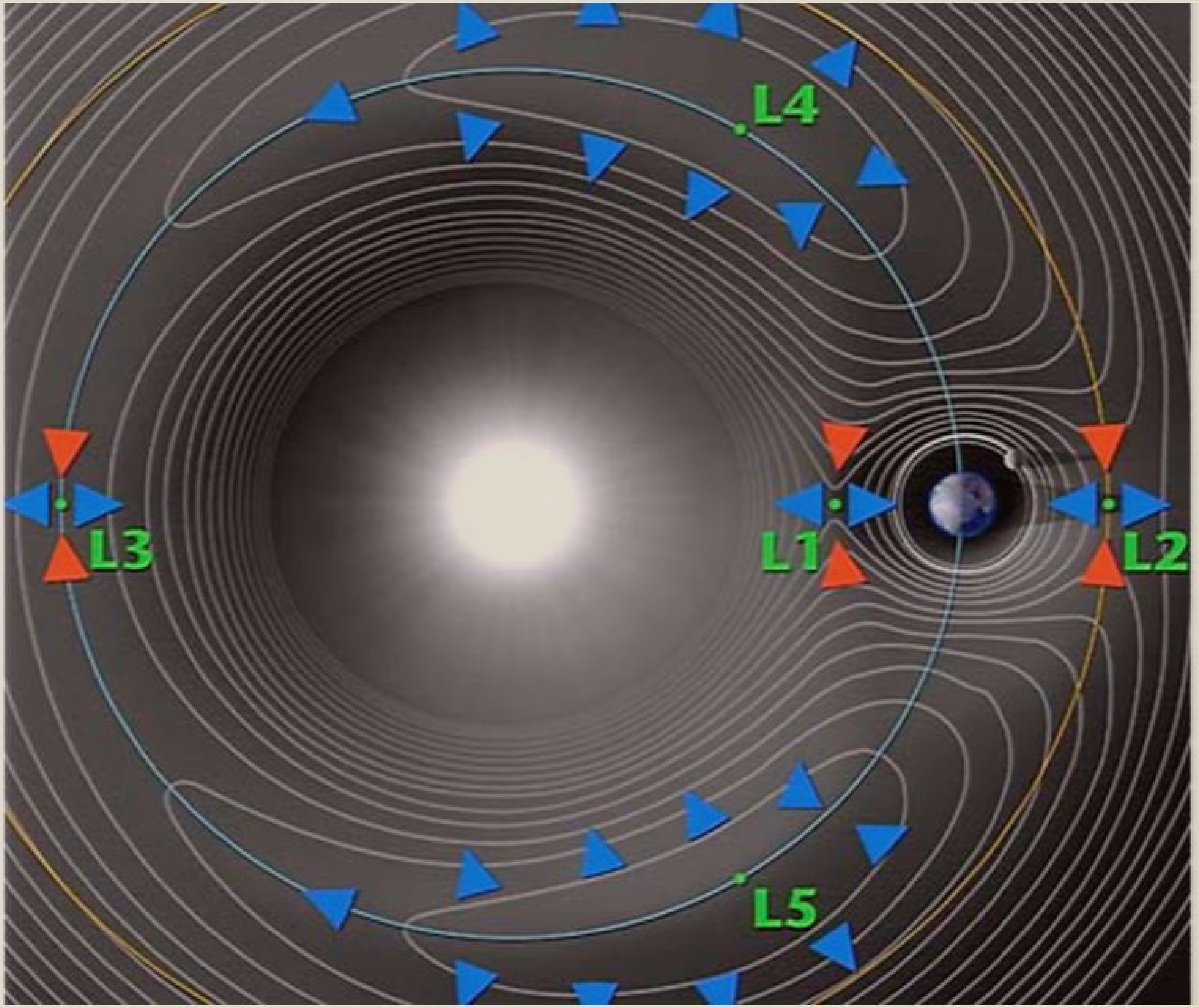

∇·(μ∇φ) = κGρ
ω²r
V ~ ½ ω²r²
∇φ ≪ a₀


L4
L3
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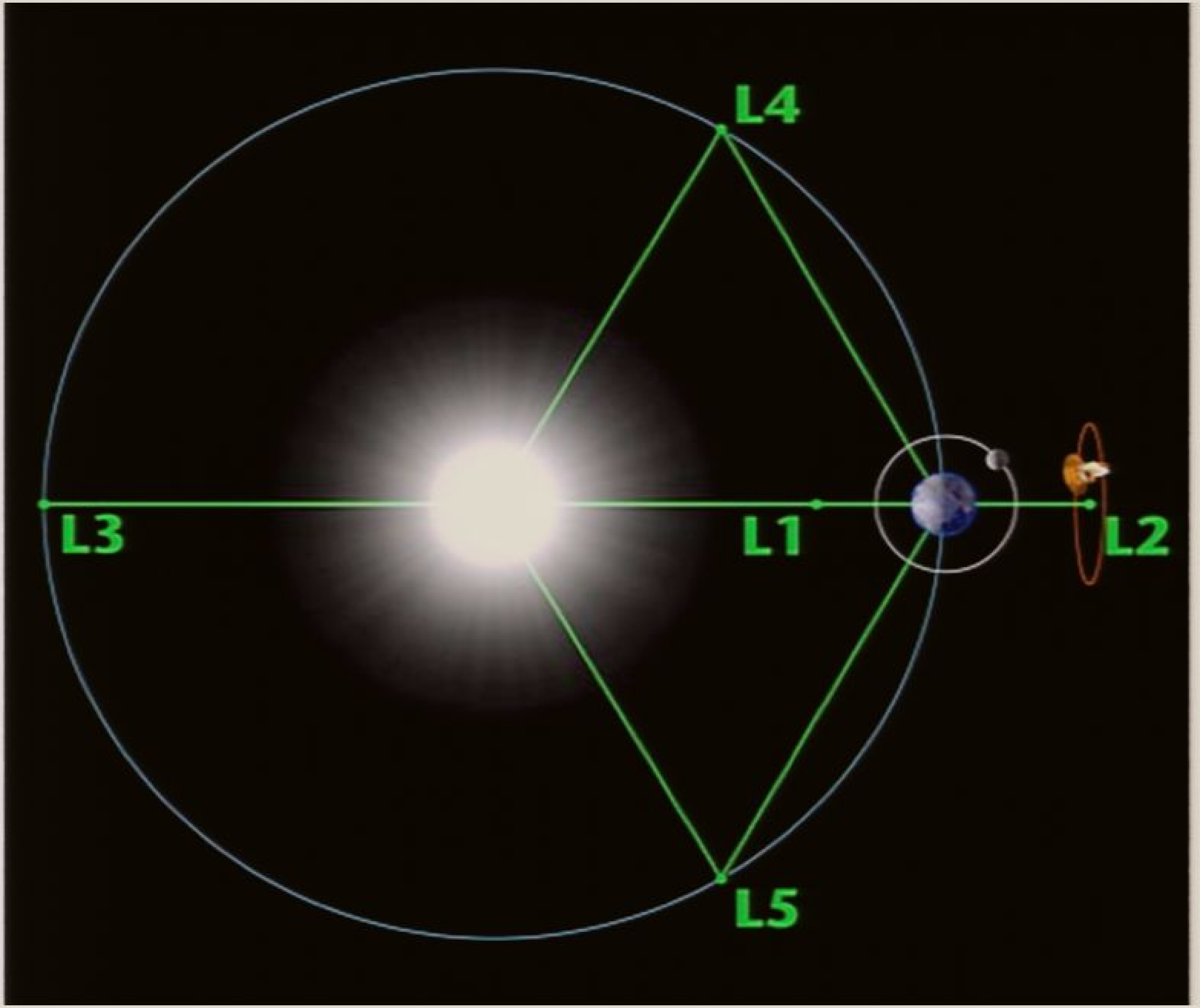
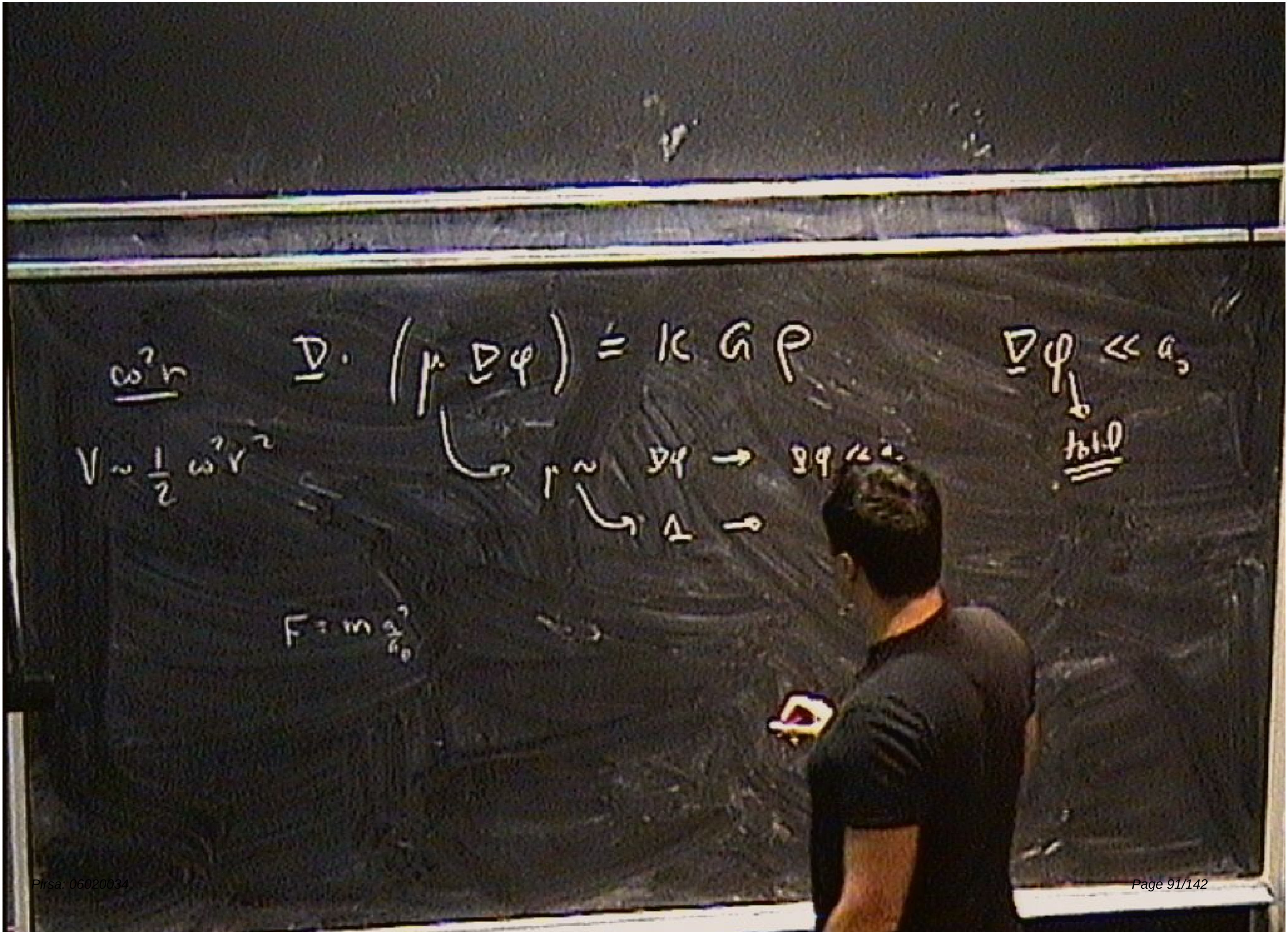
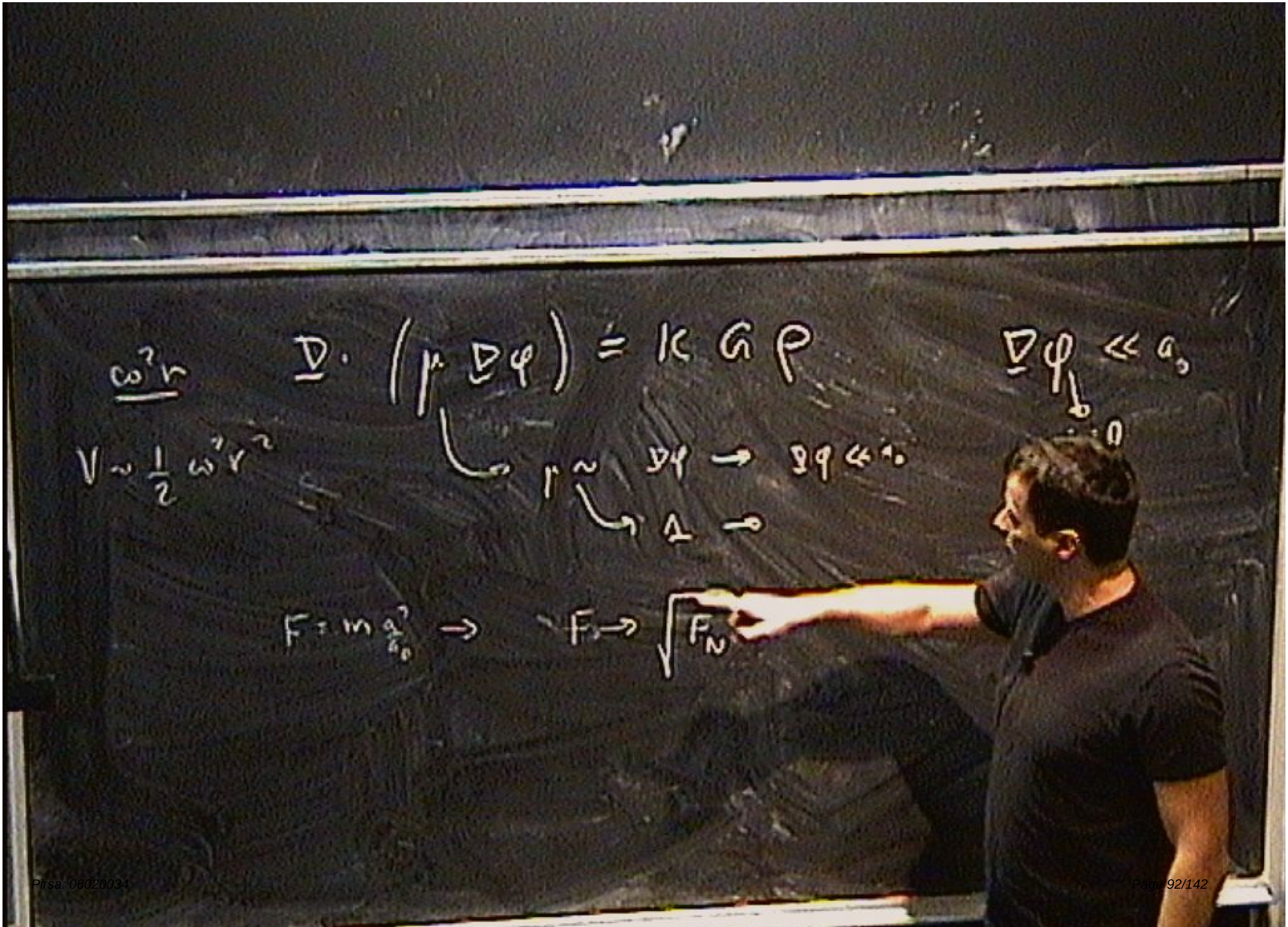
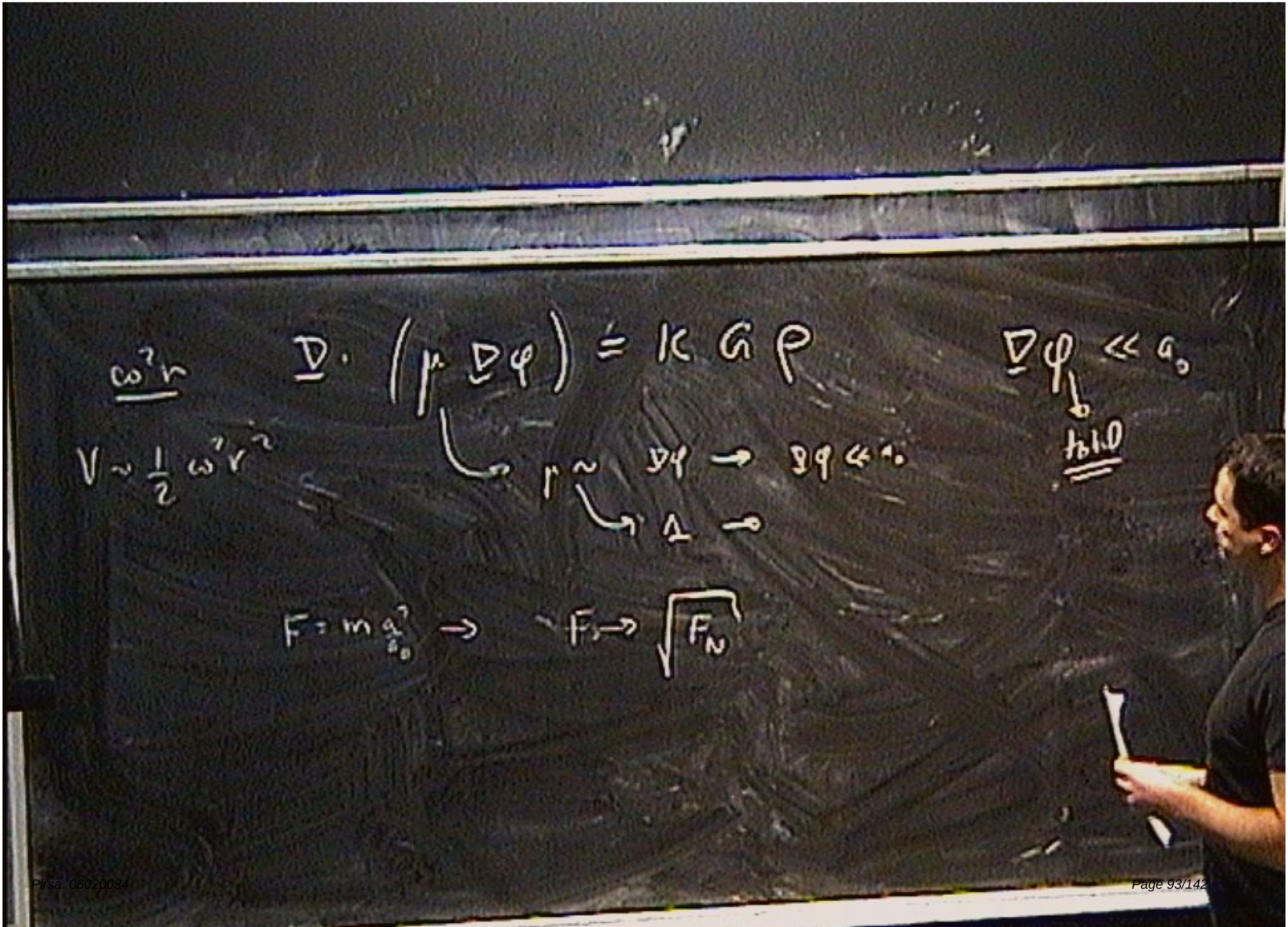

L4
L3
L1
L2
L5

$$u = -\frac{4\pi\mu}{k}\nabla\phi$$


ω²r
V ~ ½ ω²r²
∇·(μ ∇φ) = K G ρ
∇φ << a₀
F = m a²/a₀


ω²r
∇·(μ ∇φ) = KGρ
∇φ ≪ a₀
V ~ ½ ω²r²
μ ~ ∇φ → ∇φ ≪ a₀
1 →
F = m a²/a₀ →
F → √F_N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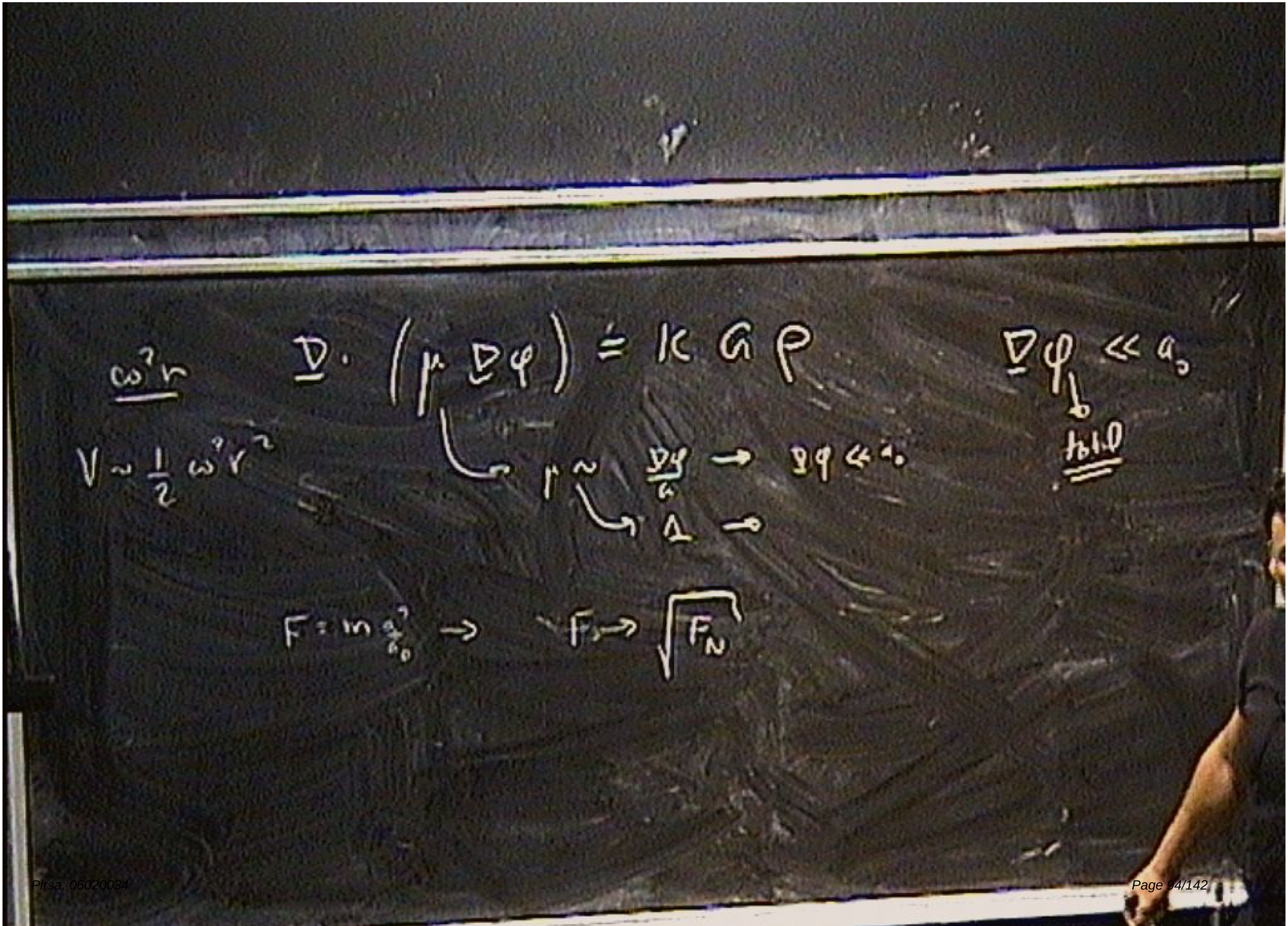

$\omega^2 r$
$\nabla\cdot(\mu\,\nabla\varphi) = K G \rho$
$\nabla\varphi \ll a_0$
$V \sim \frac{1}{2}\omega^2 r^2$
$\mu \sim \frac{\nabla\varphi}{a_0} \rightarrow \nabla\varphi \ll$
$F = m\frac{a^2}{a_0} \rightarrow$
$F \rightarrow \sqrt{F_N}$

$\underline{\omega^2 r}$

$V \sim \frac{1}{2}\omega^2 r^2$

$\nabla \cdot (\mu \nabla\varphi) = K G \rho$

$\mu \sim \frac{\nabla\varphi}{a_0} \rightarrow \nabla\varphi \ll a_0$

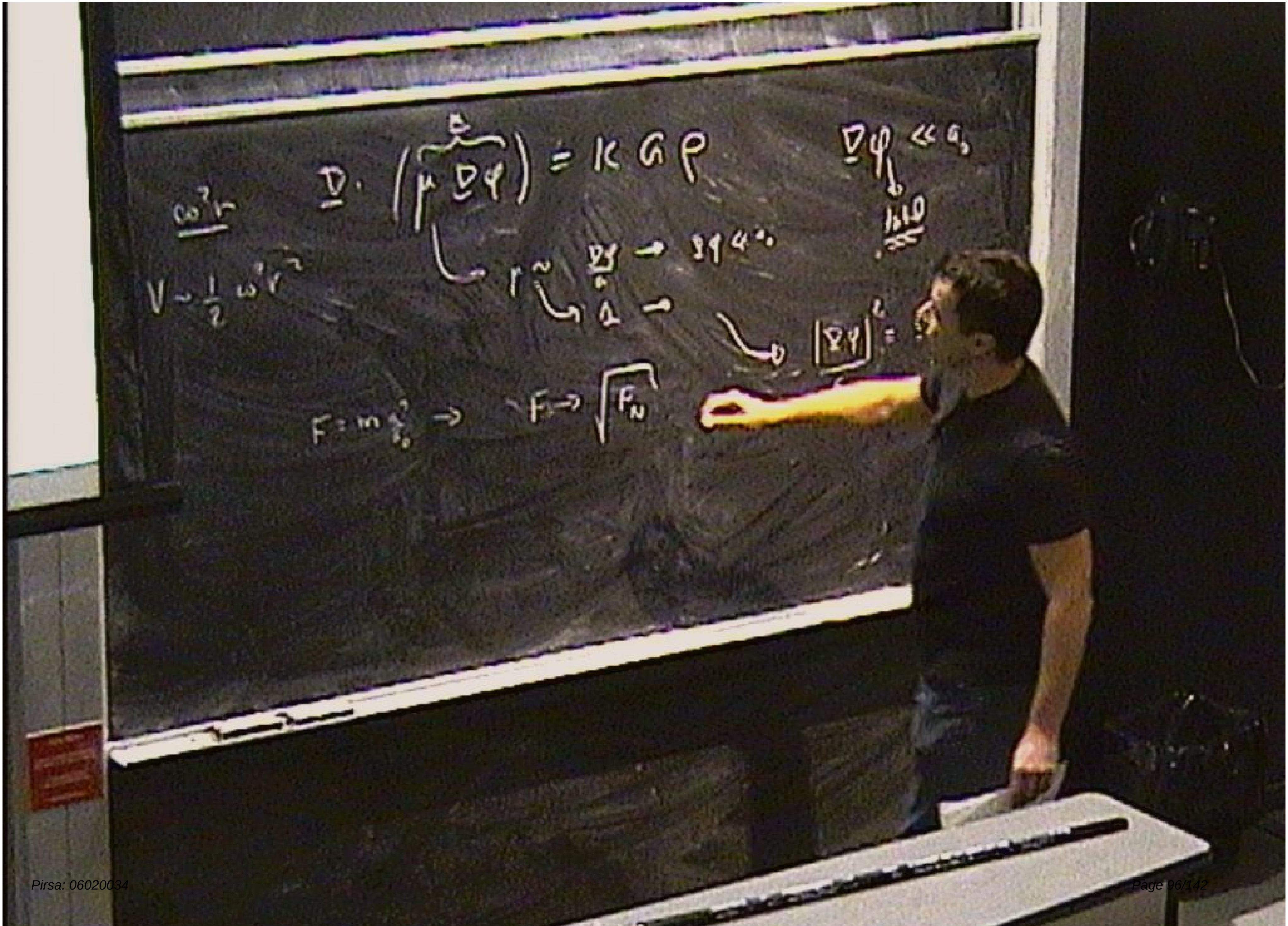
$\mu \sim 1 \rightarrow$

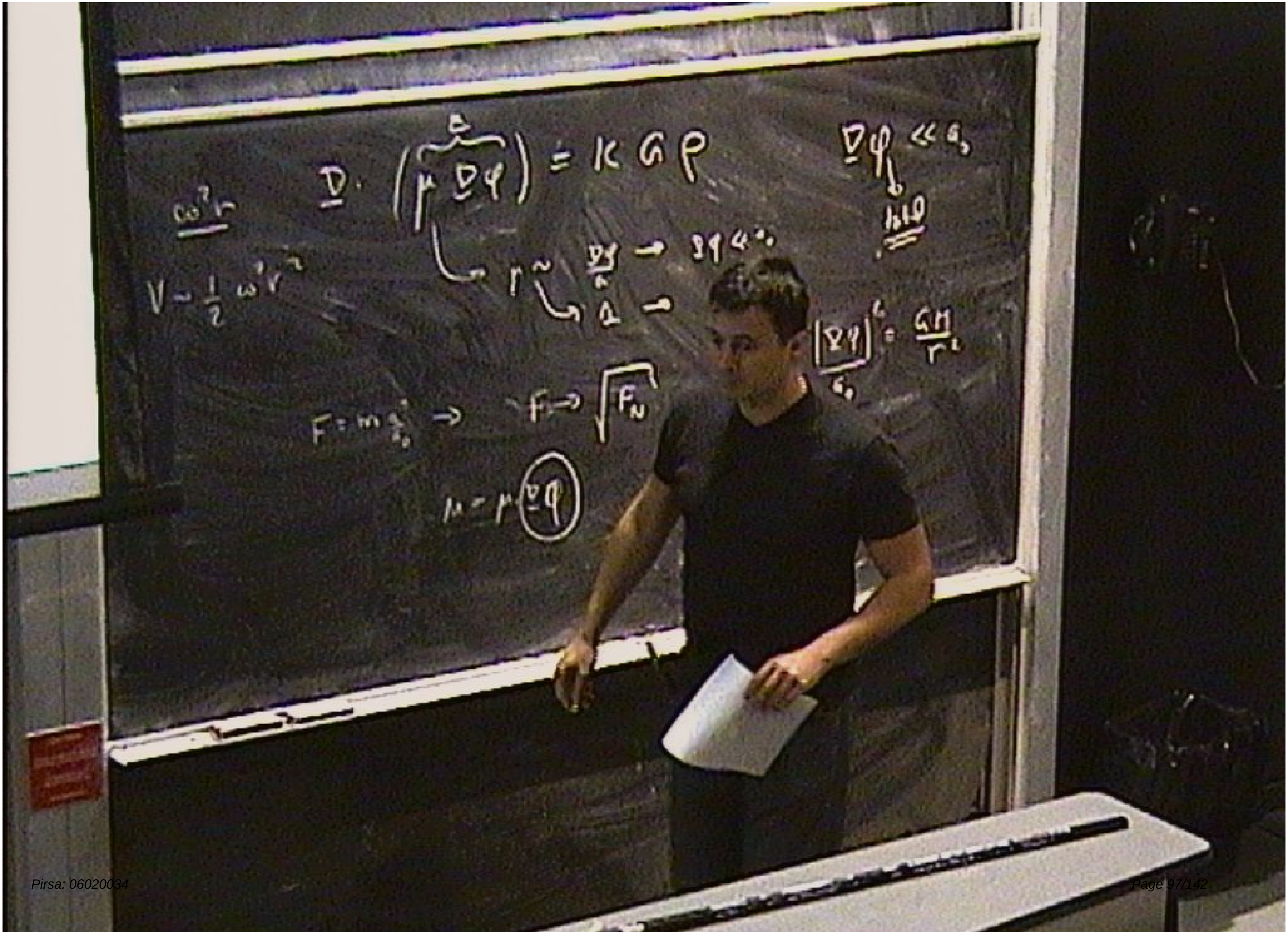
$\nabla\varphi \ll a_0$

MOND

$\frac{|\nabla\varphi|^2}{a_0} = \frac{GM}{r^2}$

$F = m\frac{a^2}{a_0} \rightarrow \quad F \rightarrow \sqrt{F_N}$

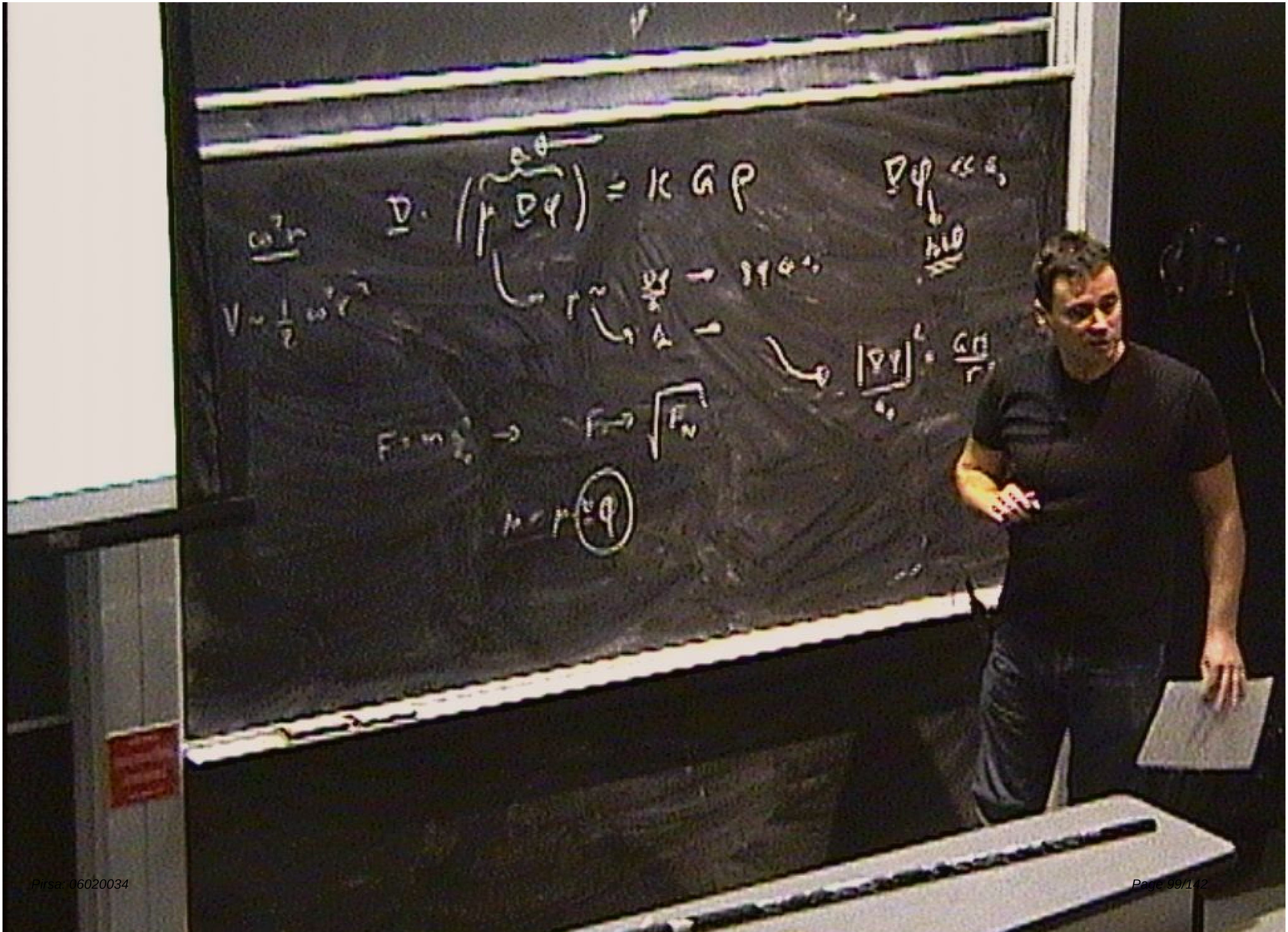


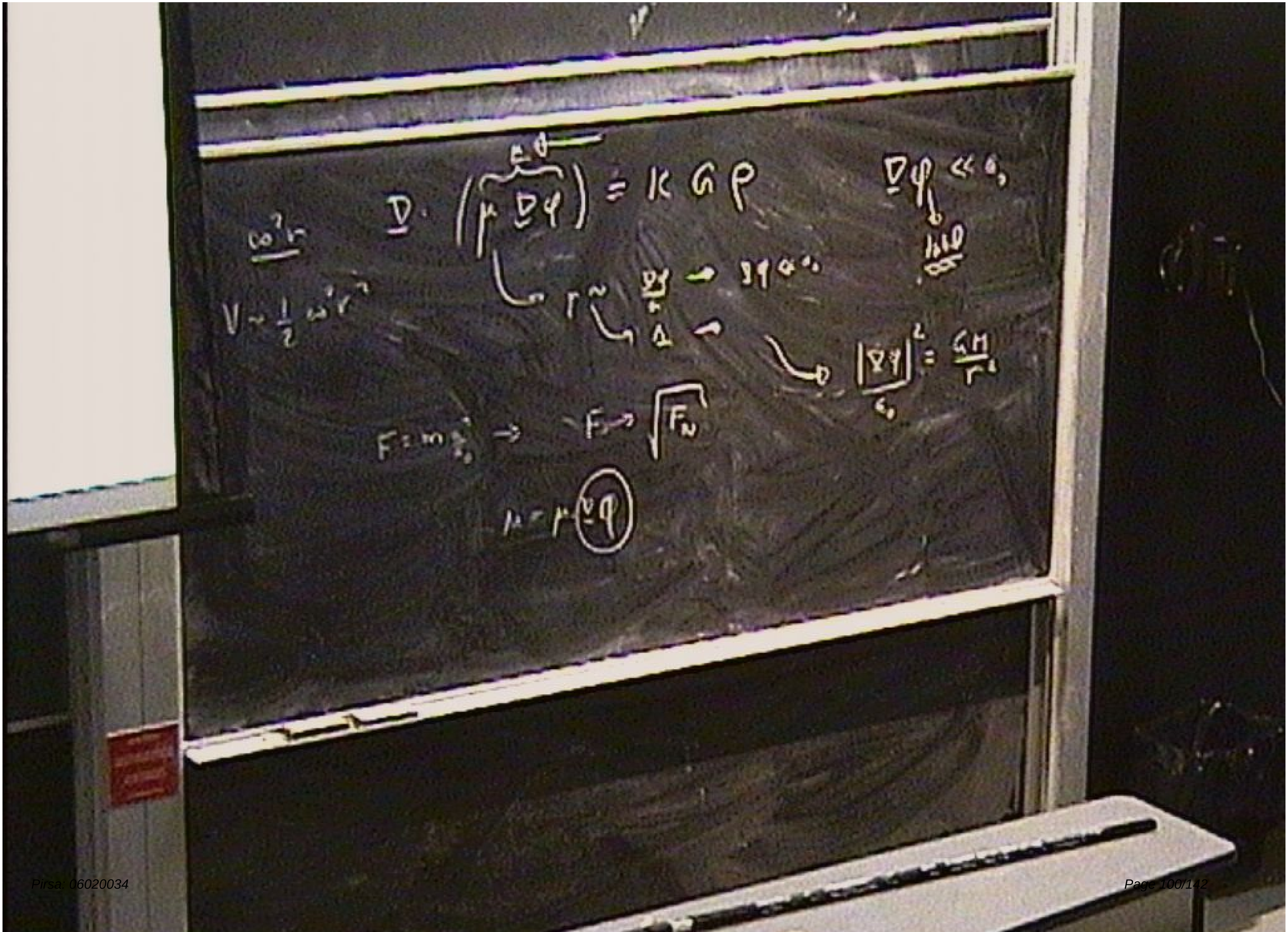


$$u = -\frac{4\pi\mu}{k}\nabla\phi$$

$$\nabla \cdot u = -4\pi G\rho$$

$$\nabla \wedge \frac{u}{\mu} = 0$$





$$\nabla \cdot u = -4\pi G\rho$$

$$u = -\frac{4\pi\mu}{k}\nabla\phi$$

$$\nabla \wedge \frac{u}{\mu} = 0$$

$$u = F^{(N)} + \nabla \wedge h$$

$$U = -\frac{k^2}{16\pi^2 a_0} u$$

$$\nabla \cdot U = 0$$

$$4(1+U^2)U^2 \nabla \wedge U + U \wedge \nabla U^2 = 0$$

Curl is zero under spherical symmetry

Curl is subdominant when U is larger than 1

$$U = -\frac{k^2}{16\pi^2 a_0} u$$

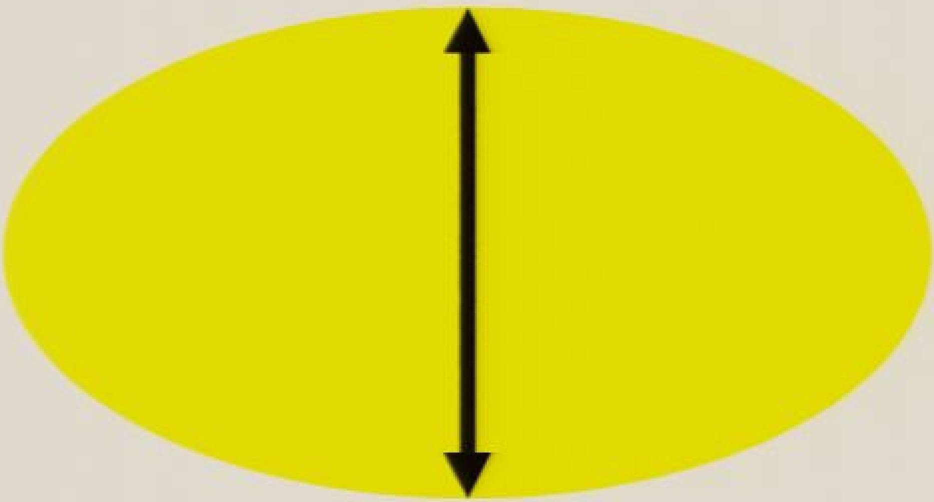
$$\nabla \cdot U = 0$$

$$4(1+U^2)U^2 \nabla \wedge U + U \wedge \nabla U^2 = 0$$

Curl is zero under spherical symmetry

Curl is subdominant when U is larger than 1

$$U^2 = 1$$

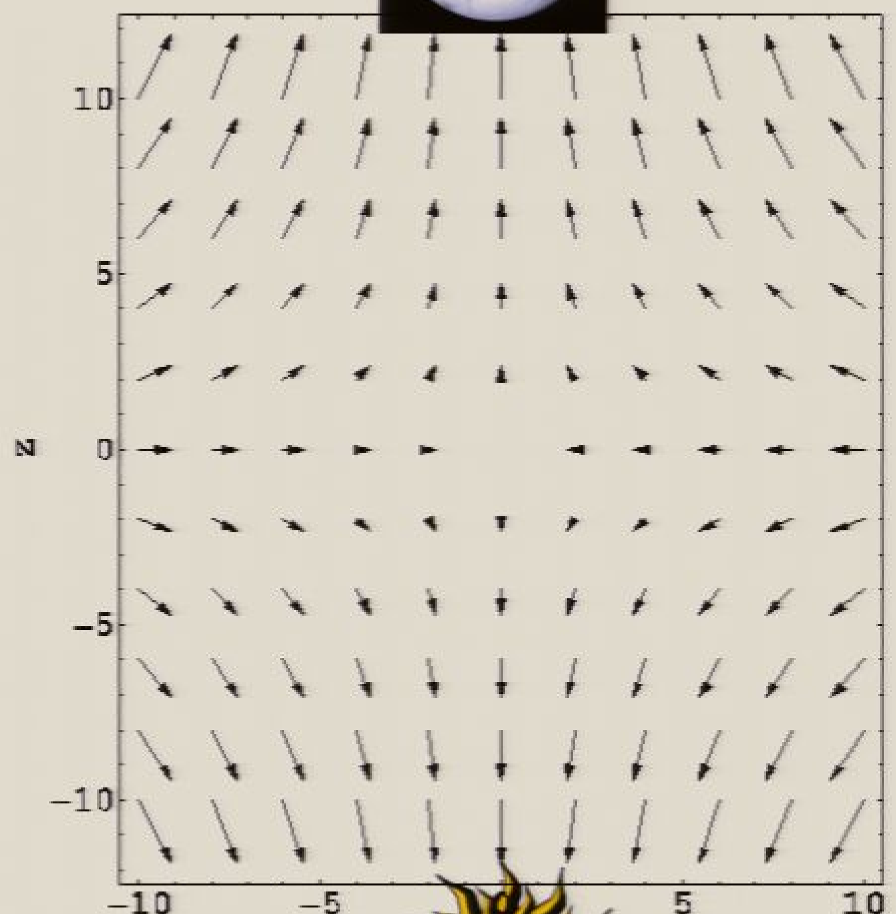


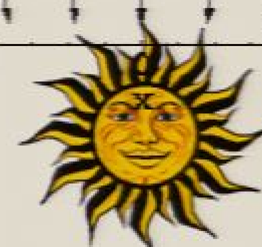
$$r_0 = \frac{16\pi^2 a_0}{k^2 A}$$

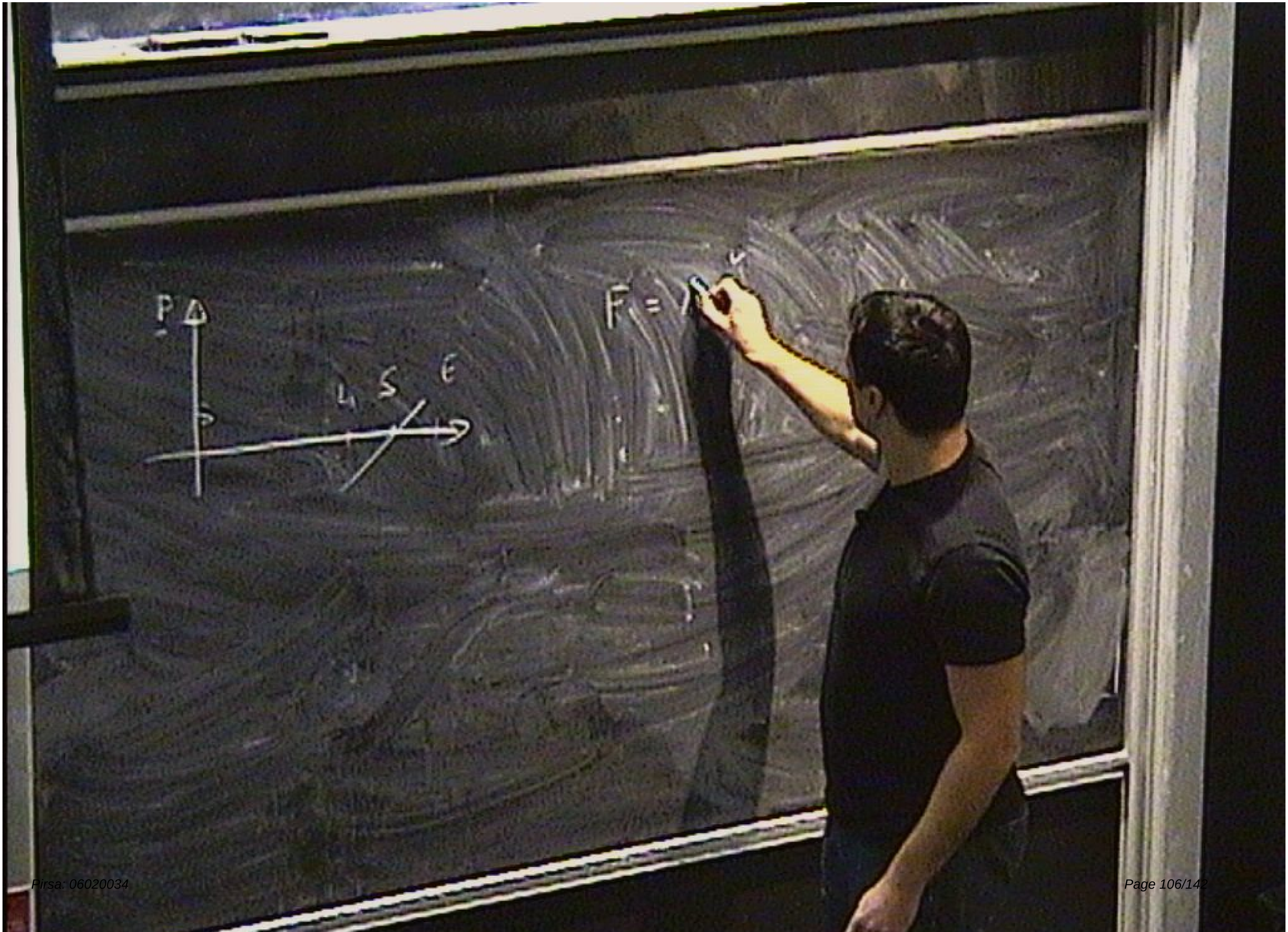
383 Km	Earth/Sun
9.6×10^5 Km	Jupiter /Sun
140Km	Earth/Moon

Near Newtonian field around saddle

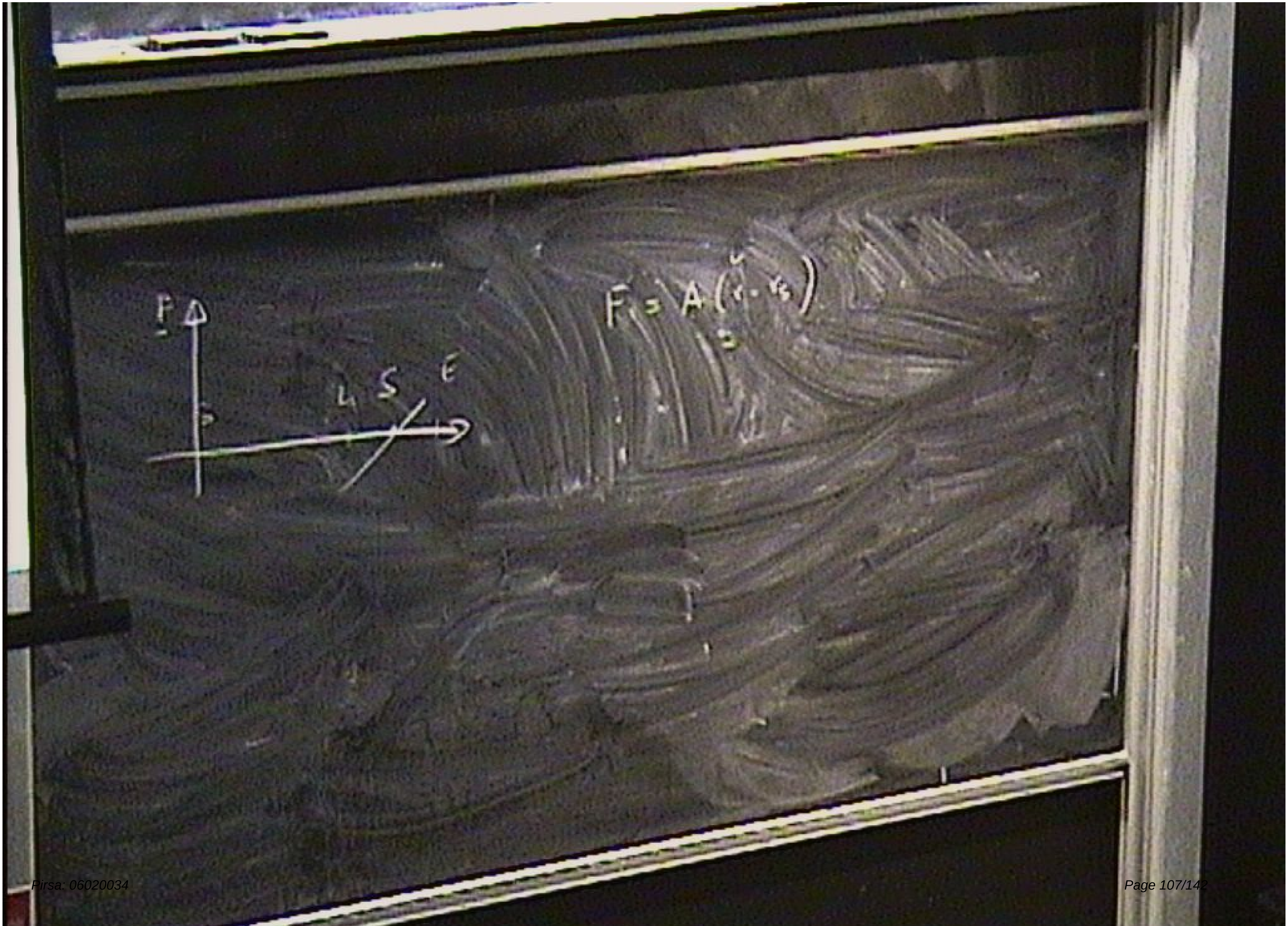
- Can separate curl neatly

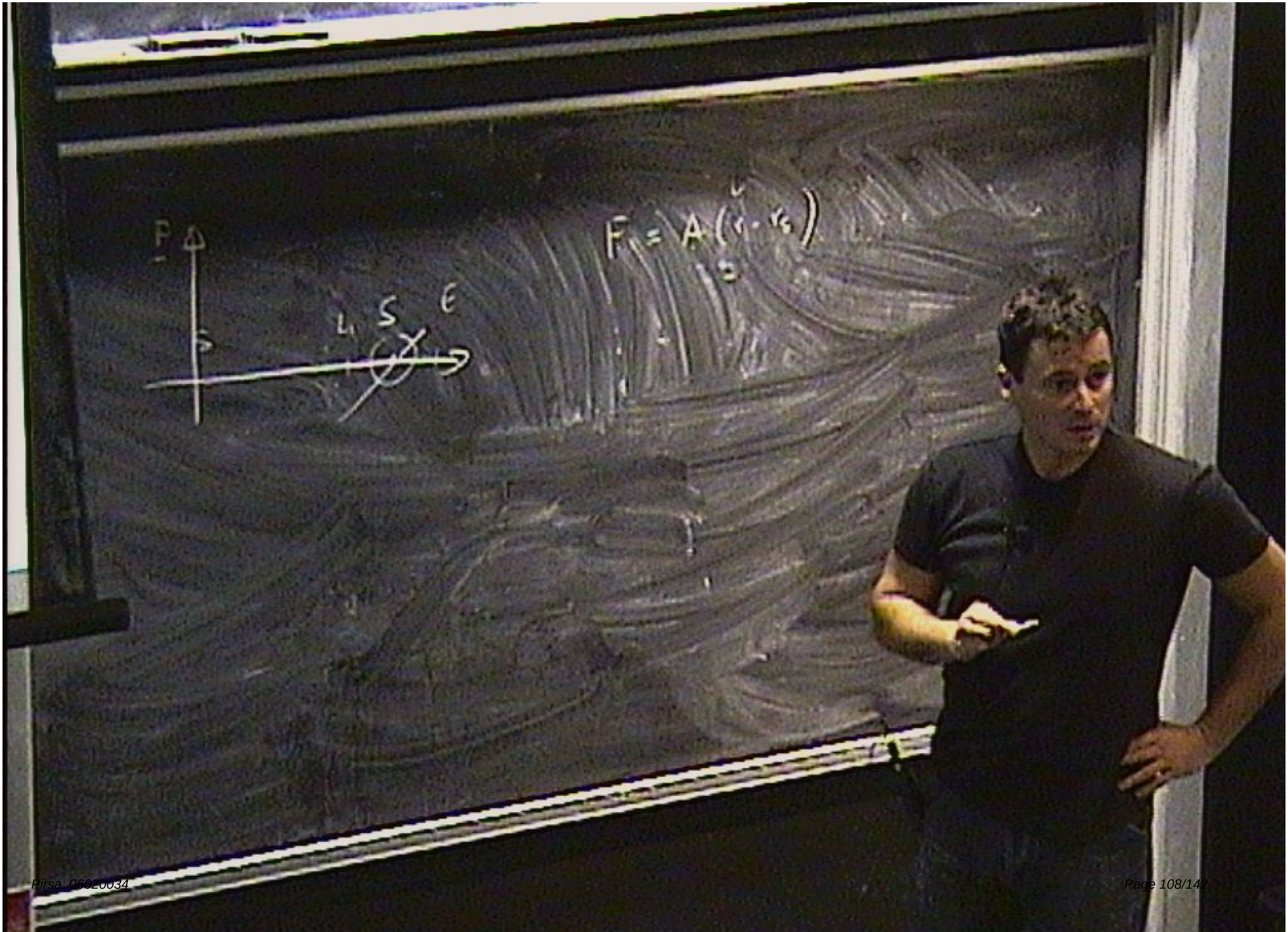


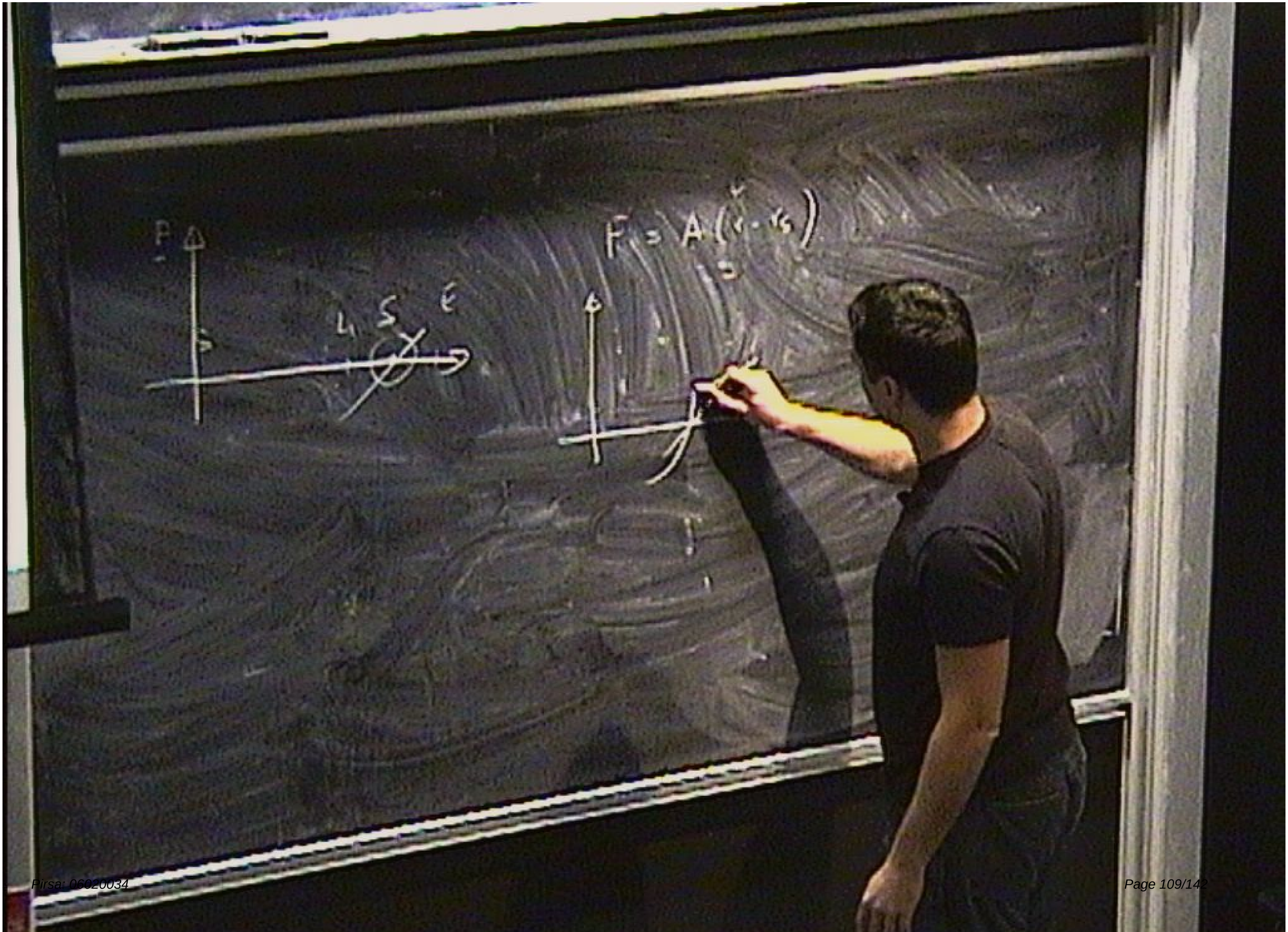


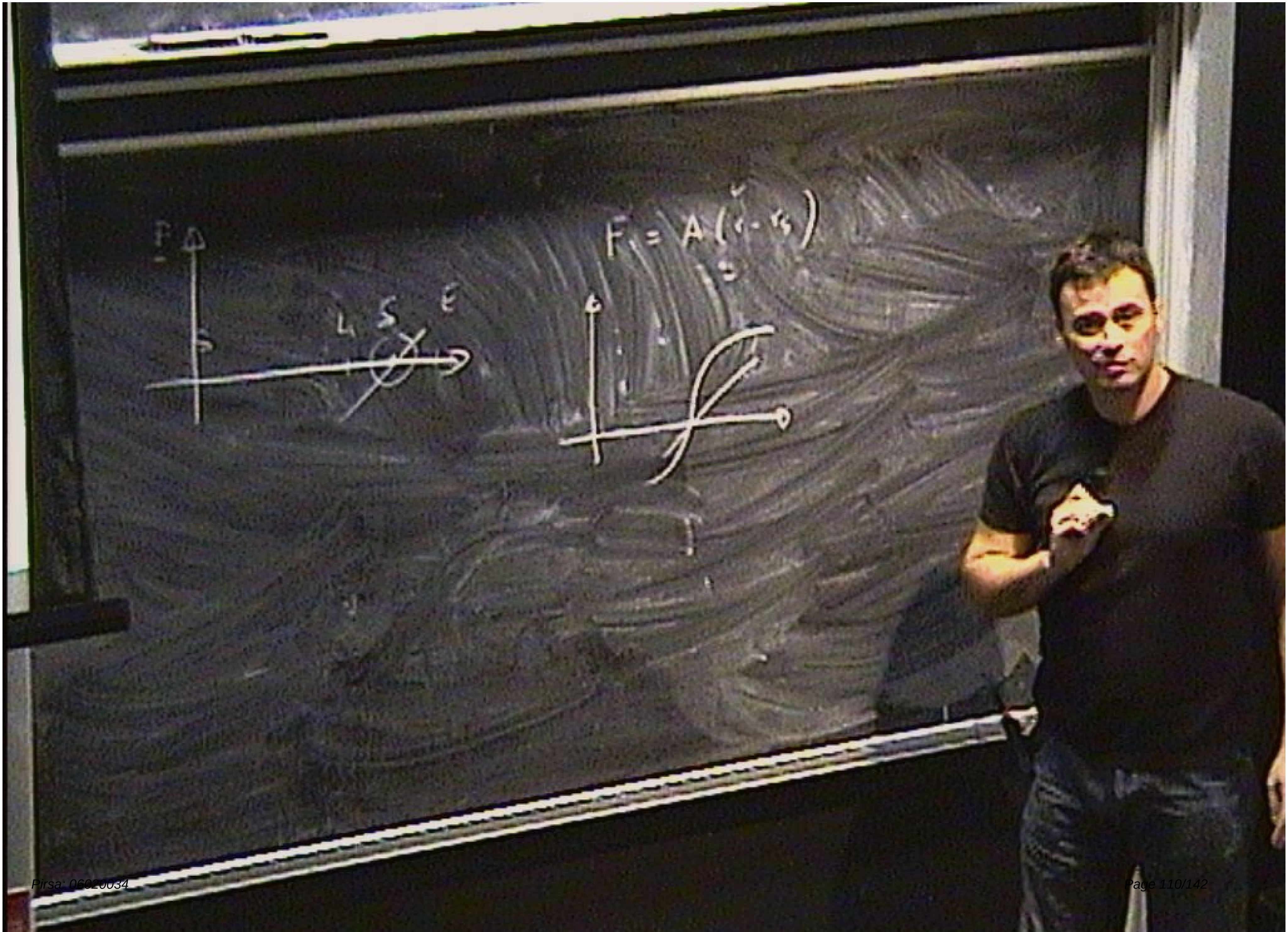


P
4, 5, 6
F =



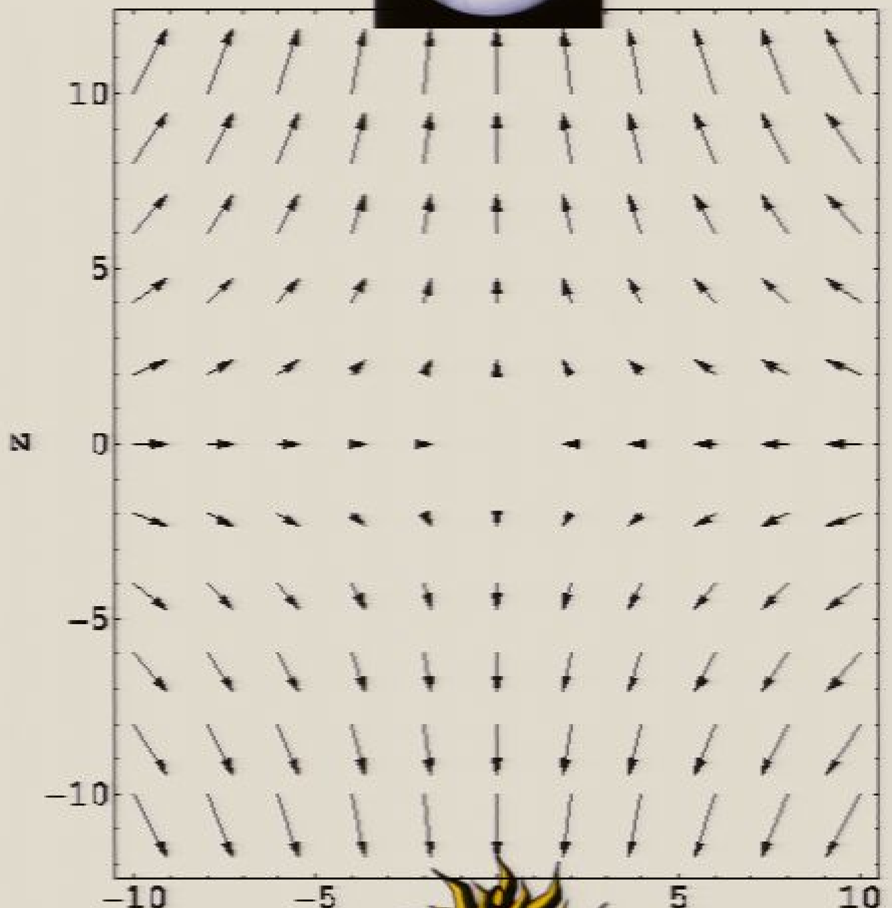


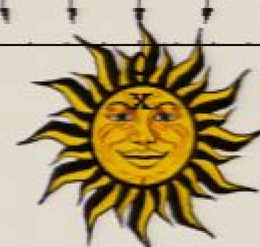




Near Newtonian field around saddle

- Can separate curl neatly





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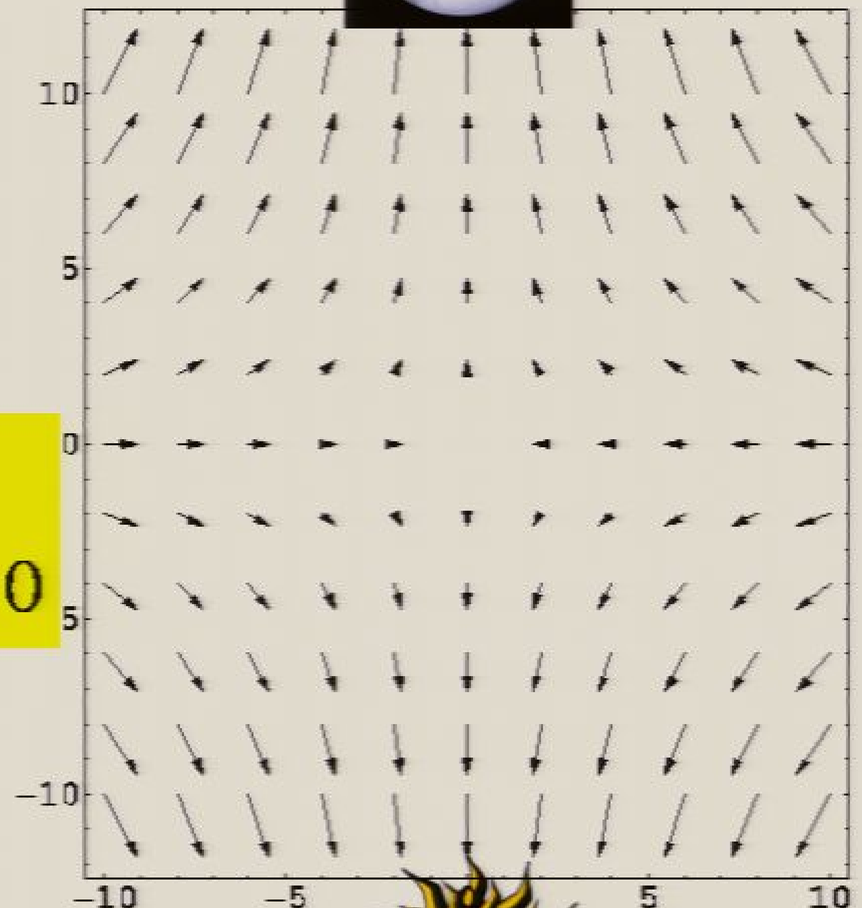
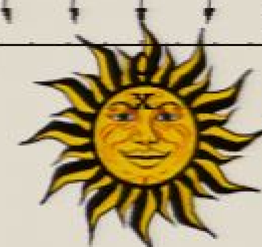


$$\nabla \cdot U = 0$$

$$4(1+U^2)U^2\nabla \wedge U + U \wedge \nabla U^2 = 0$$



Near Newtonian field around saddle

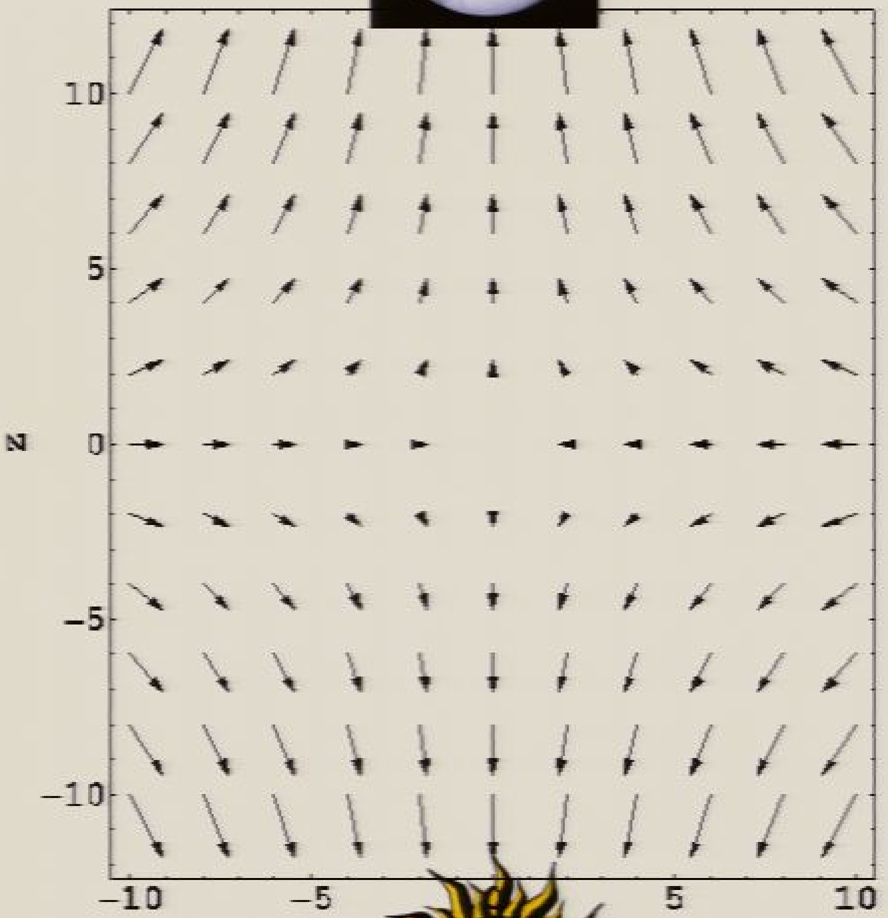
- Can separate curl neatly

$$U = U_0 + U_2$$

$$U_0 = \frac{r}{r_0} N(\psi)$$

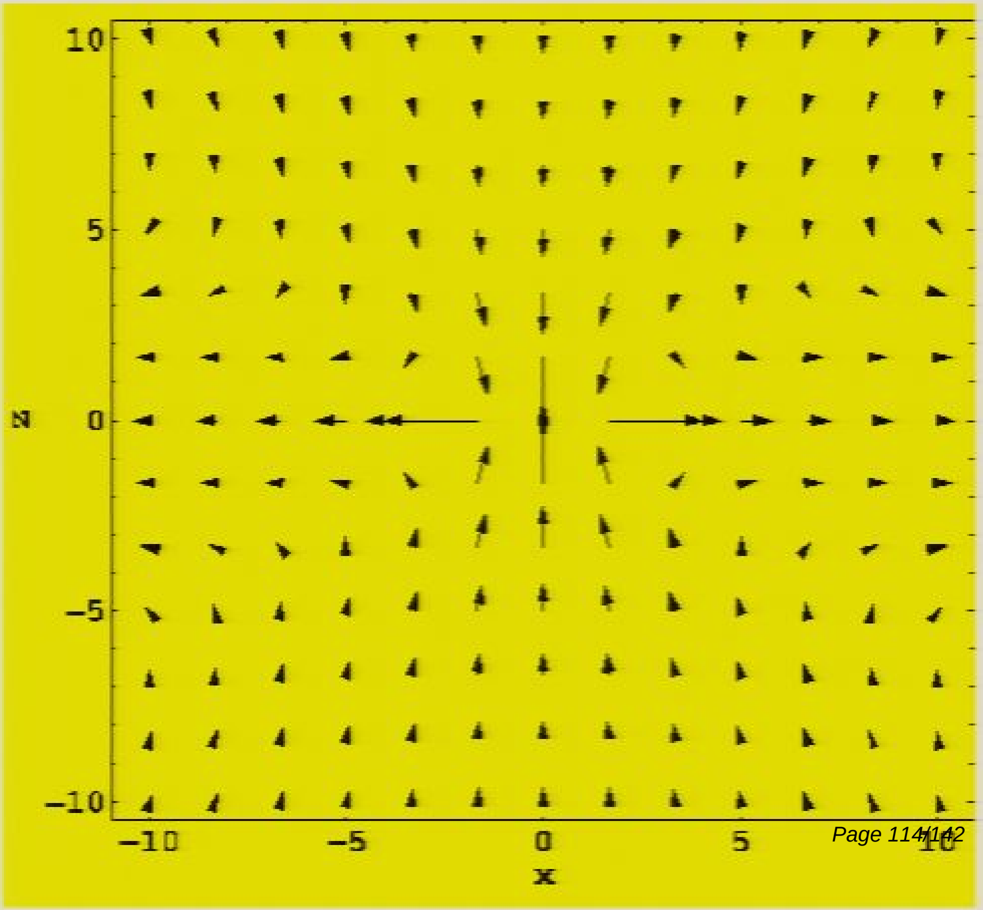
$$\nabla \cdot U_2 = 0$$

$$\nabla \wedge U_2 = -\frac{U_0 \wedge \nabla U_0^2}{4U_0^4}$$



The far-out region B field

- It can be found analytically
- It's not negligible



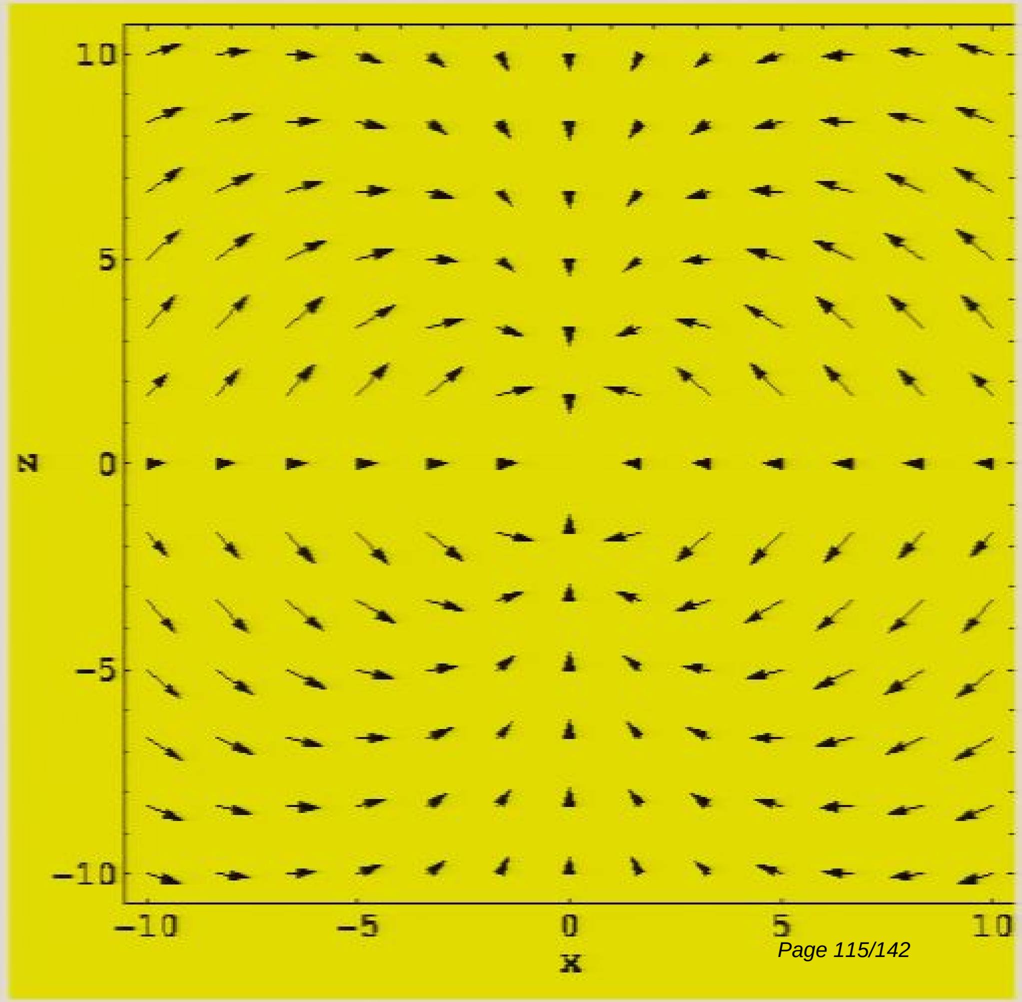
Angular profile of total correction

- Maximal fractional effect is at the border ellipsoid

- Its value is

$$\frac{\delta F}{F} \approx \frac{k}{4\pi} \approx 0.0025$$

- It then falls off as 1/r^2



The inner region

- Strong non-linearities do not allow separating divergence and curl terms. They're intertwined.
- There is a scale invariant solution (as expected)

$$U = \left(\frac{r}{r_0}\right)^{\beta}\left[F(\psi)e_r + G(\psi)e_\psi\right]$$

- Surprisingly

$$\beta \approx 1.528$$

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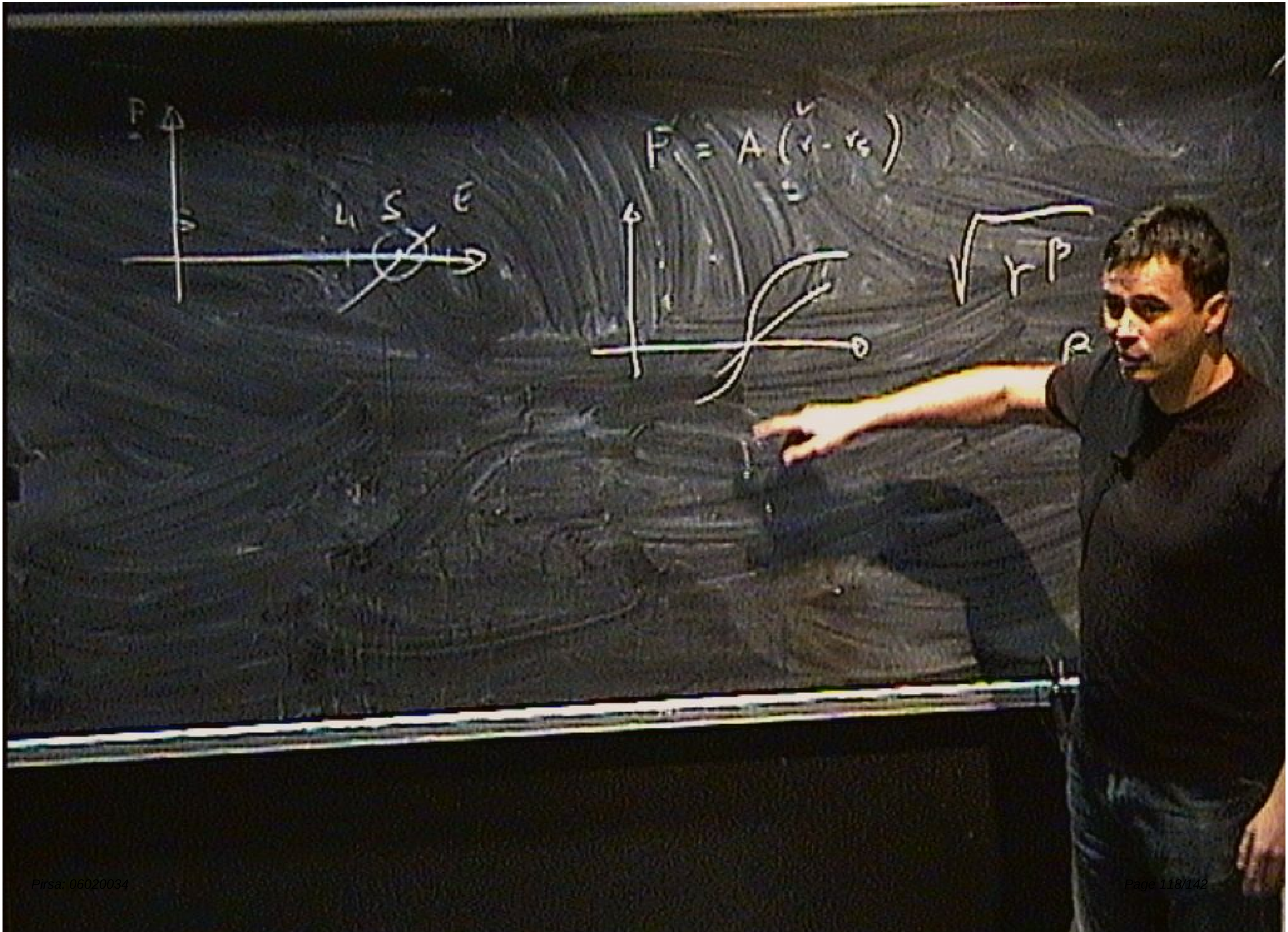
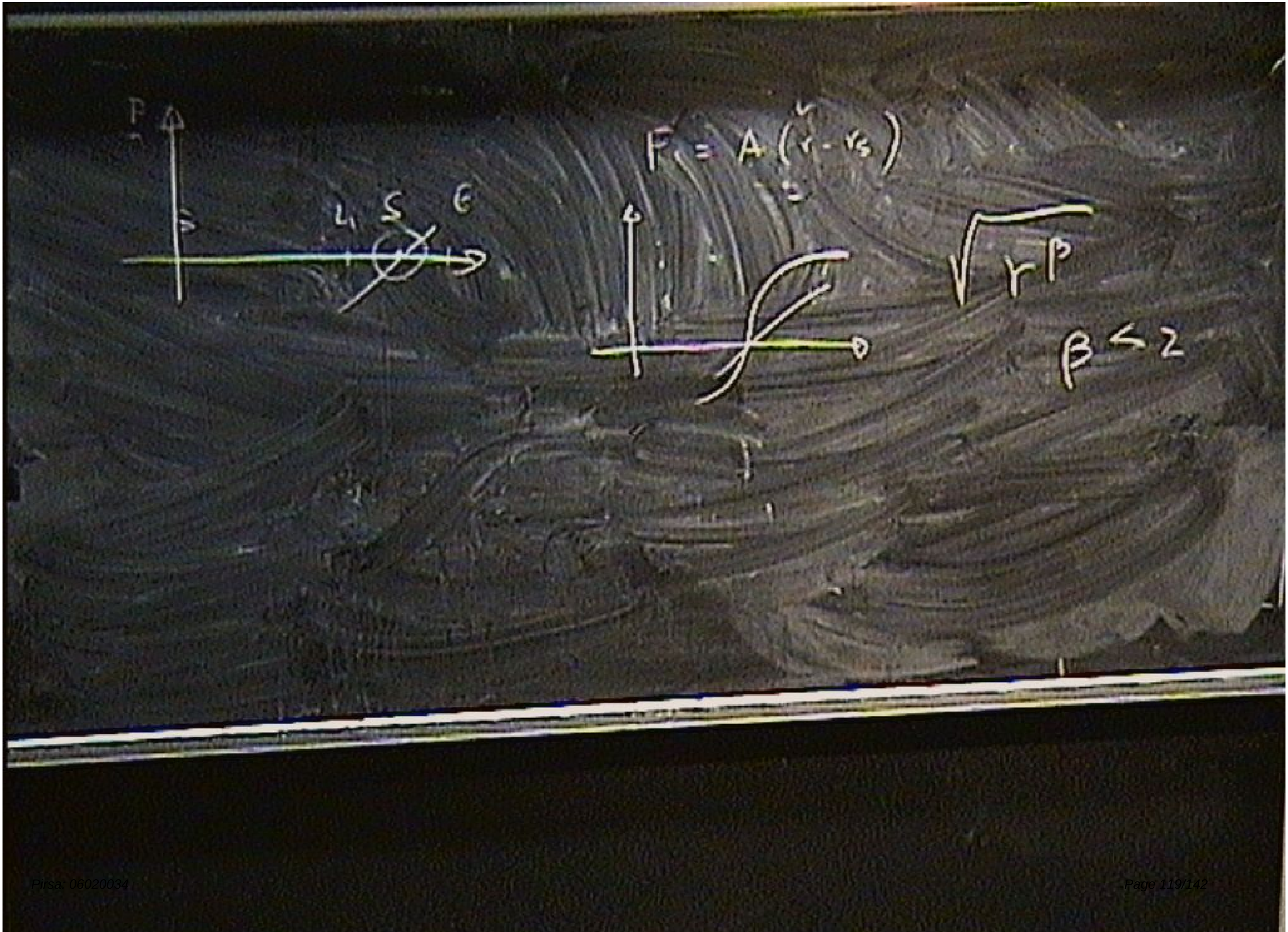
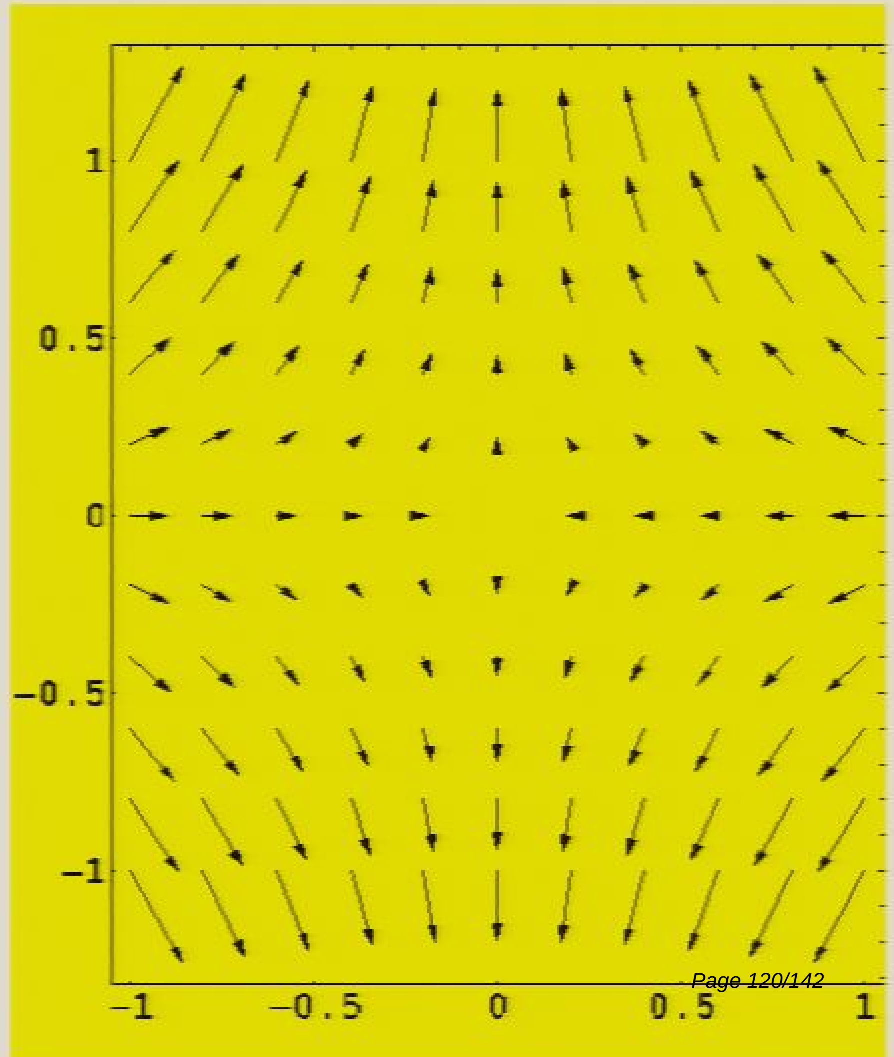

F
L S E
F = A(r - rc)
√rβ


F = A(r - rs)
√rβ
β < 2

The inner region profile

- The angular profile is approximately Newtonian
- The radial dependence is different: the tidal stress divergence is still there but it's much softer due to the curl term



The physical effects

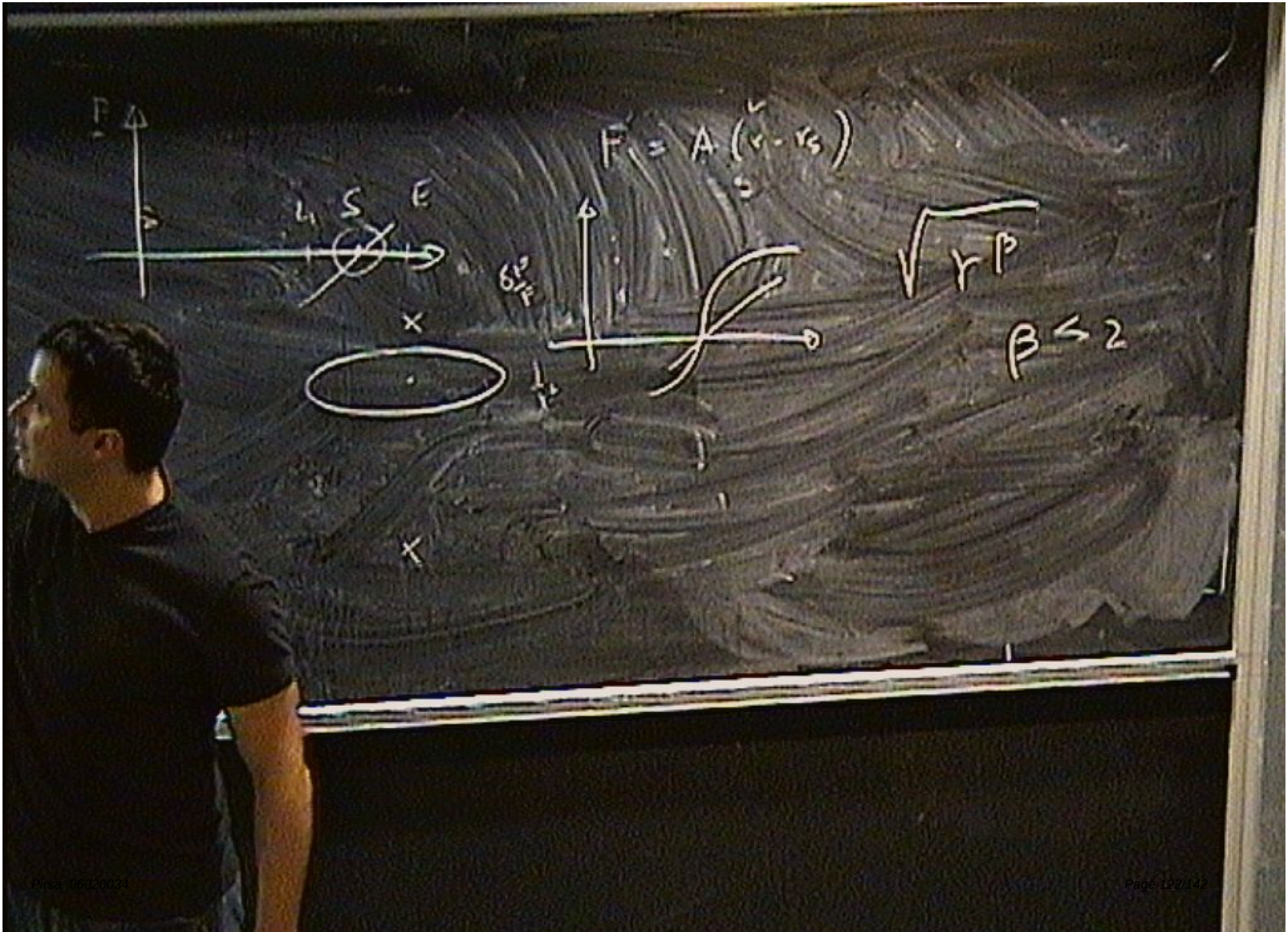
- The new physical force is

$$-\nabla\phi = \frac{4\pi a_0}{k}(1+U^2)^{1/4}\frac{U}{|U|^{1/2}} \approx \frac{4\pi a_0}{k}\frac{U}{|U|^{1/2}}$$

- The fractional corrections to Newtonian gravity continue to grow inside the border ellipsoid, but more softly than naively expected

$$\frac{\delta F}{F} \approx \frac{k}{4\pi}\left(\frac{r}{r_0}\right)^{-0.24}$$

- The divergence is still there because $\beta < 2$



L S E
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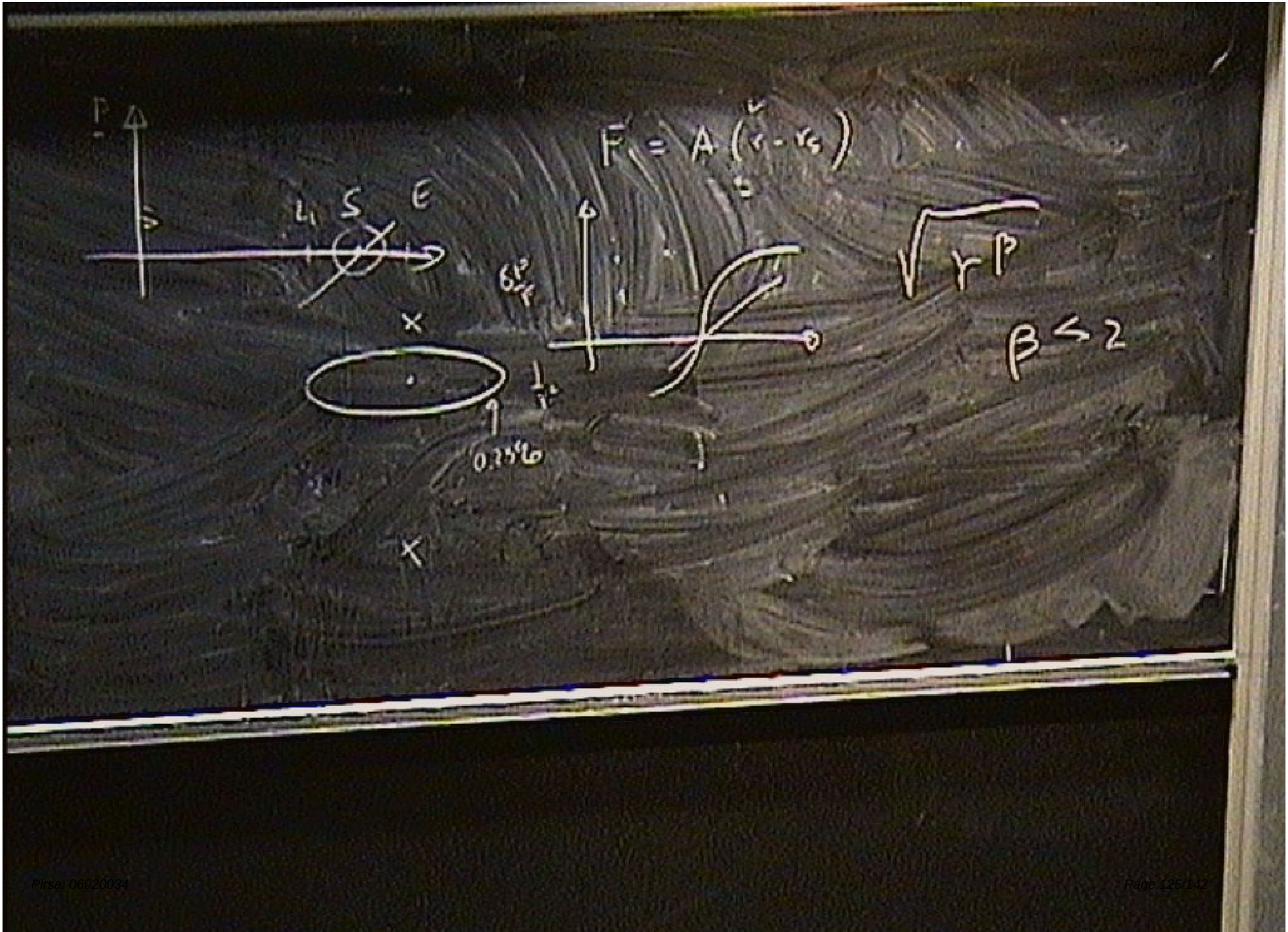
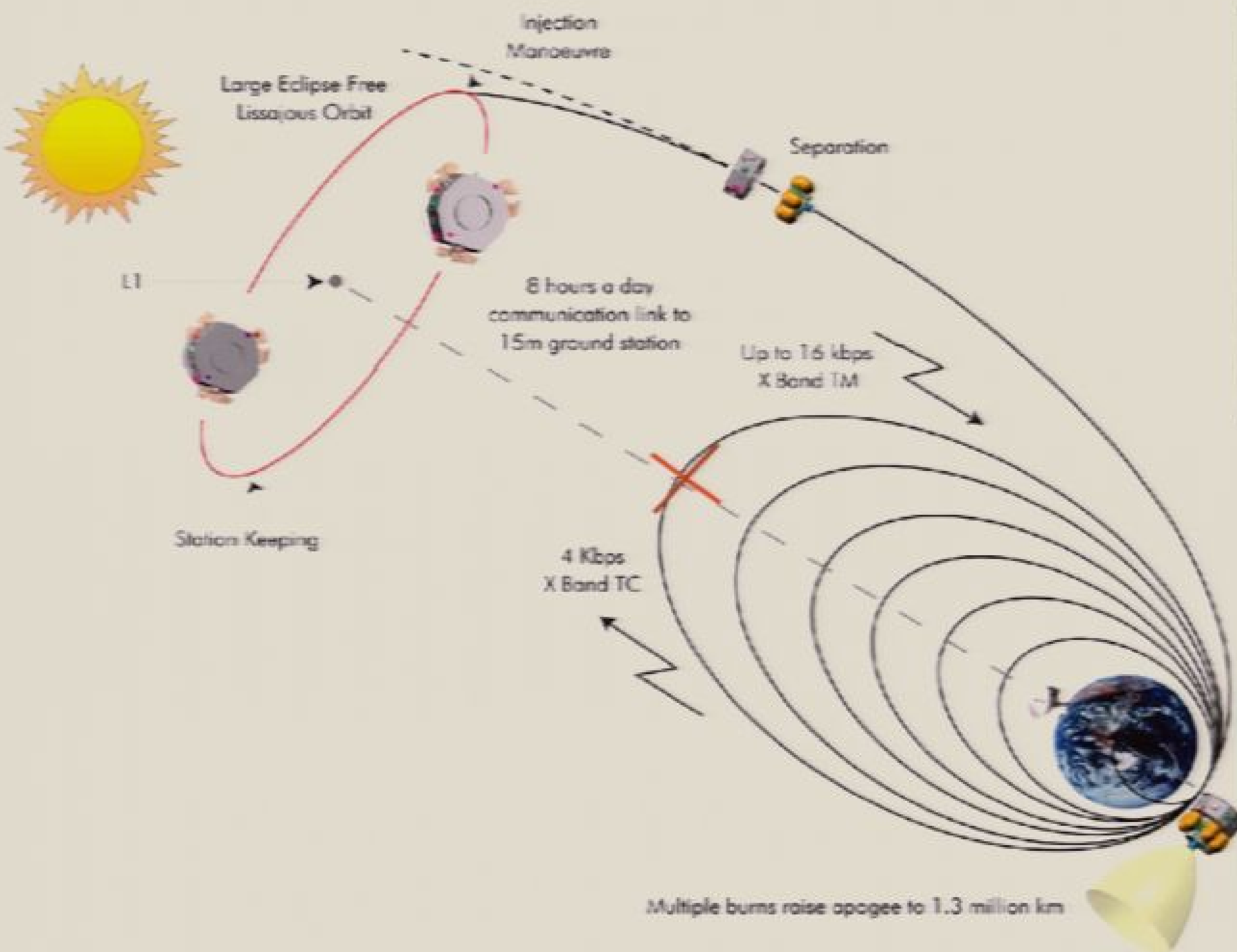
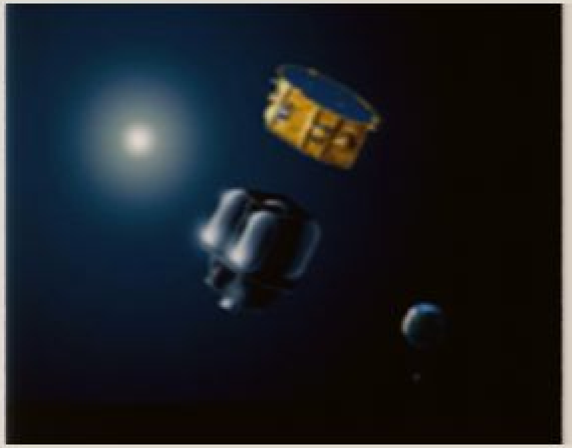

F = A (r - r_s)
β ≤ 2
0.35 G

LISA Pathfinder mission of ESA





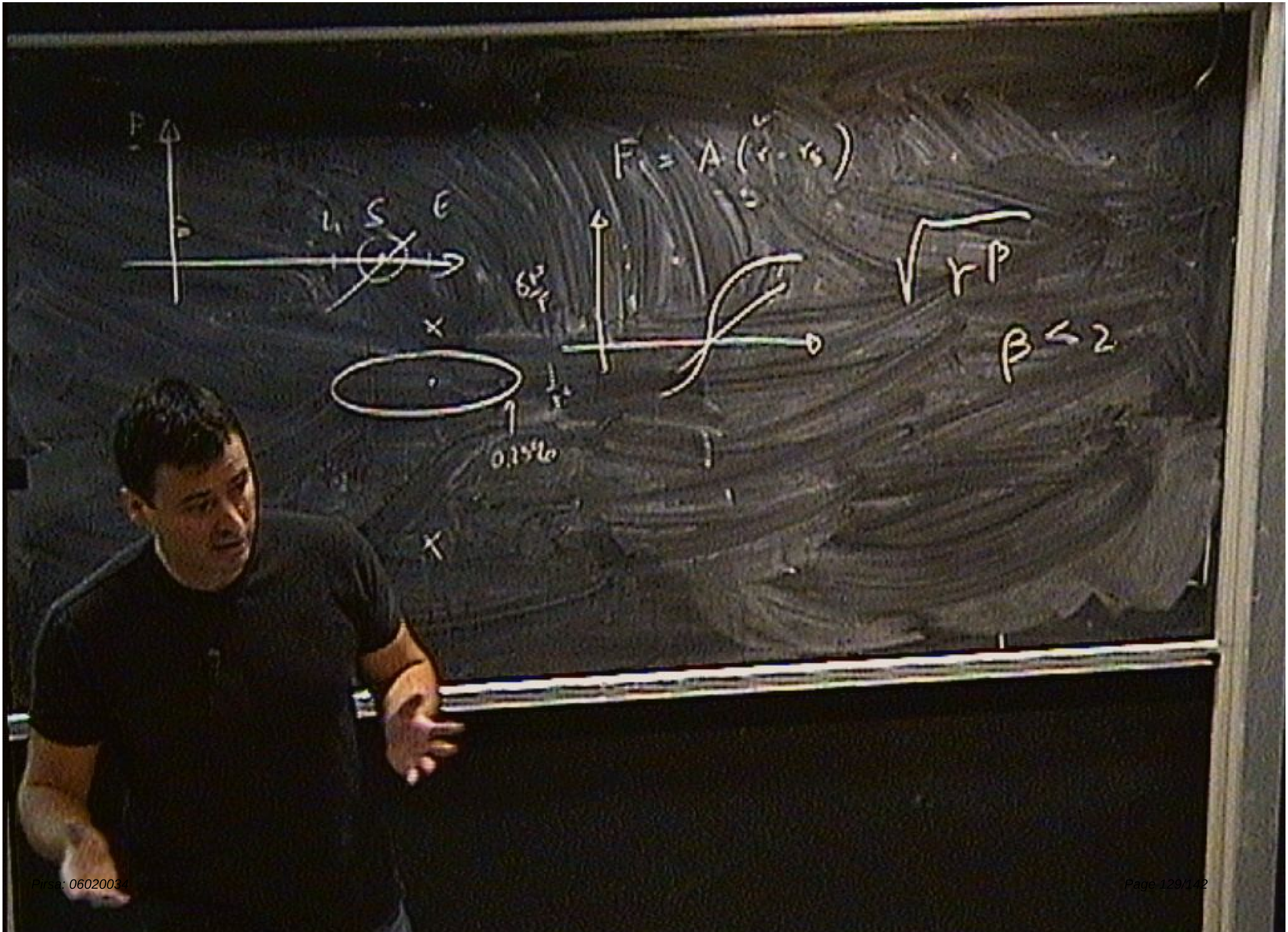
A target for LPF?

- No problems with radiation pressure well inside the Solar system
- Accelerometers have a sensitivity of

$$\frac{\Delta a}{\Delta r} \approx 10^{-15} s^{-2}$$

- Target region for Sun/Earth saddle has size

$$\Delta r \approx 3830 \,\mathrm{Km}$$


β < 2

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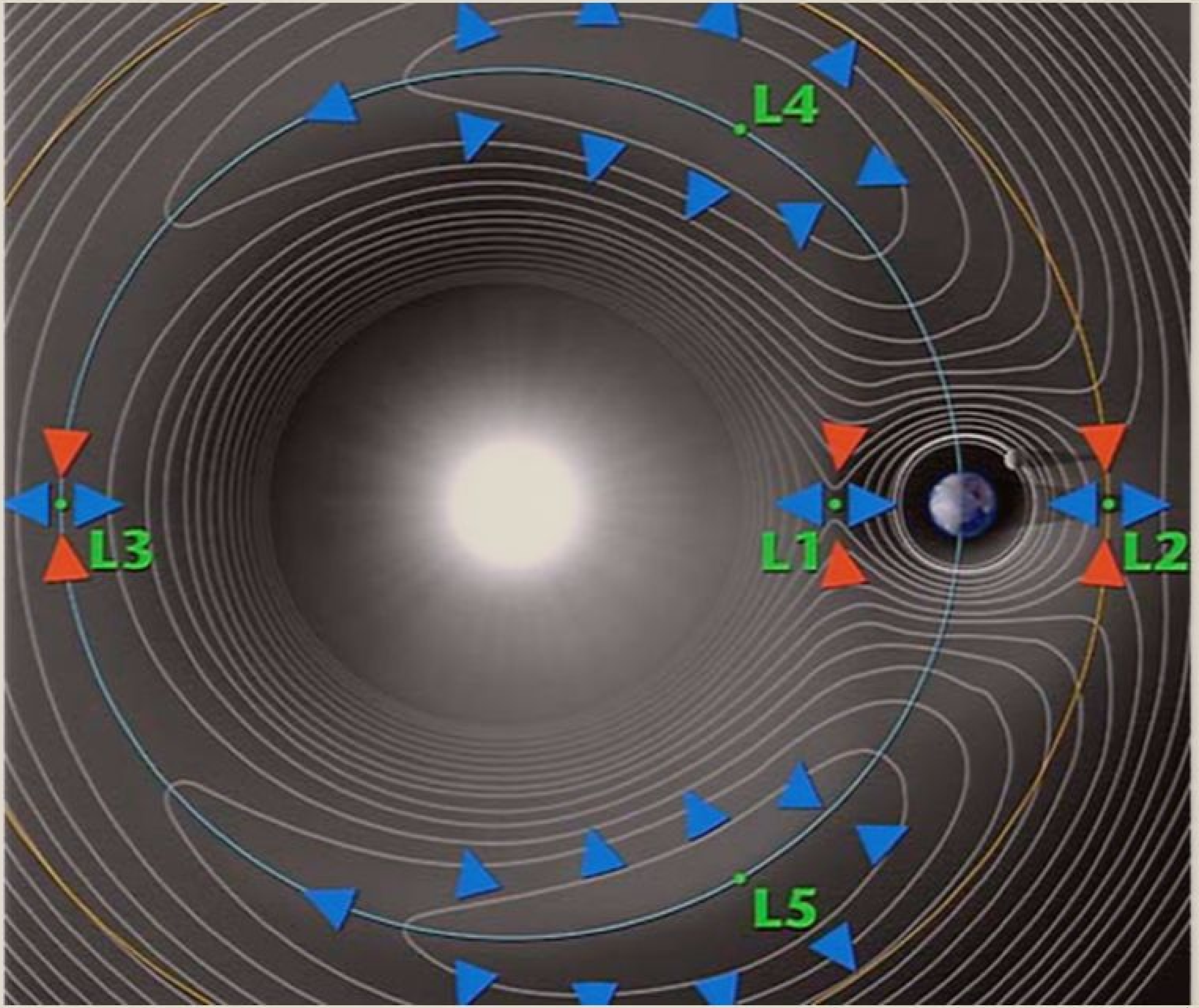
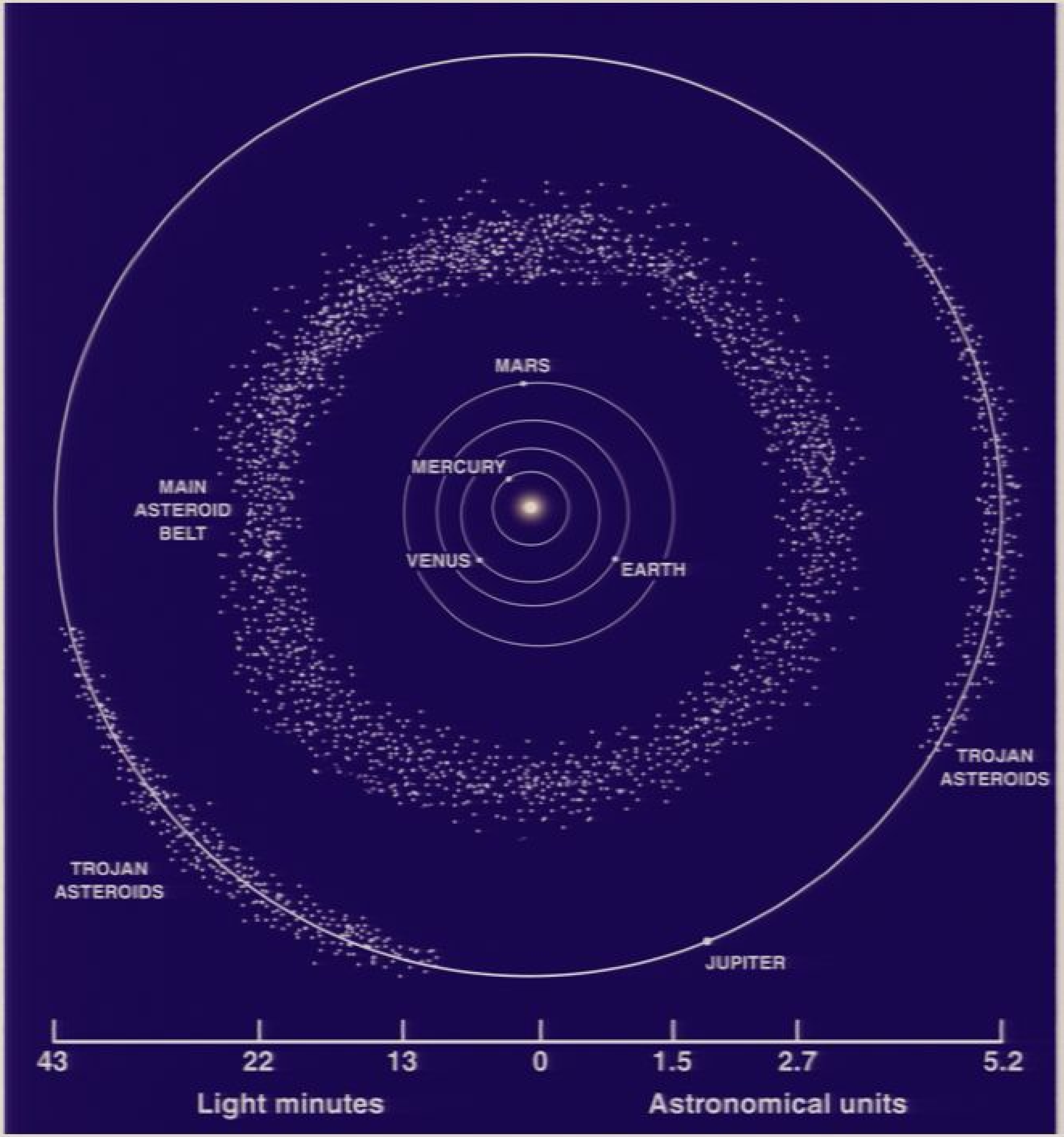

L4
L3
L1
L2
L5


MARS
MERCURY
MAIN
ASTEROID
BELT
VENUS
EARTH
TROJAN
ASTEROIDS
TROJAN
ASTEROIDS
JUPITER
43
22
13
0
1.5
2.7
5.2
Light minutes
Astronomical units

Conclusions

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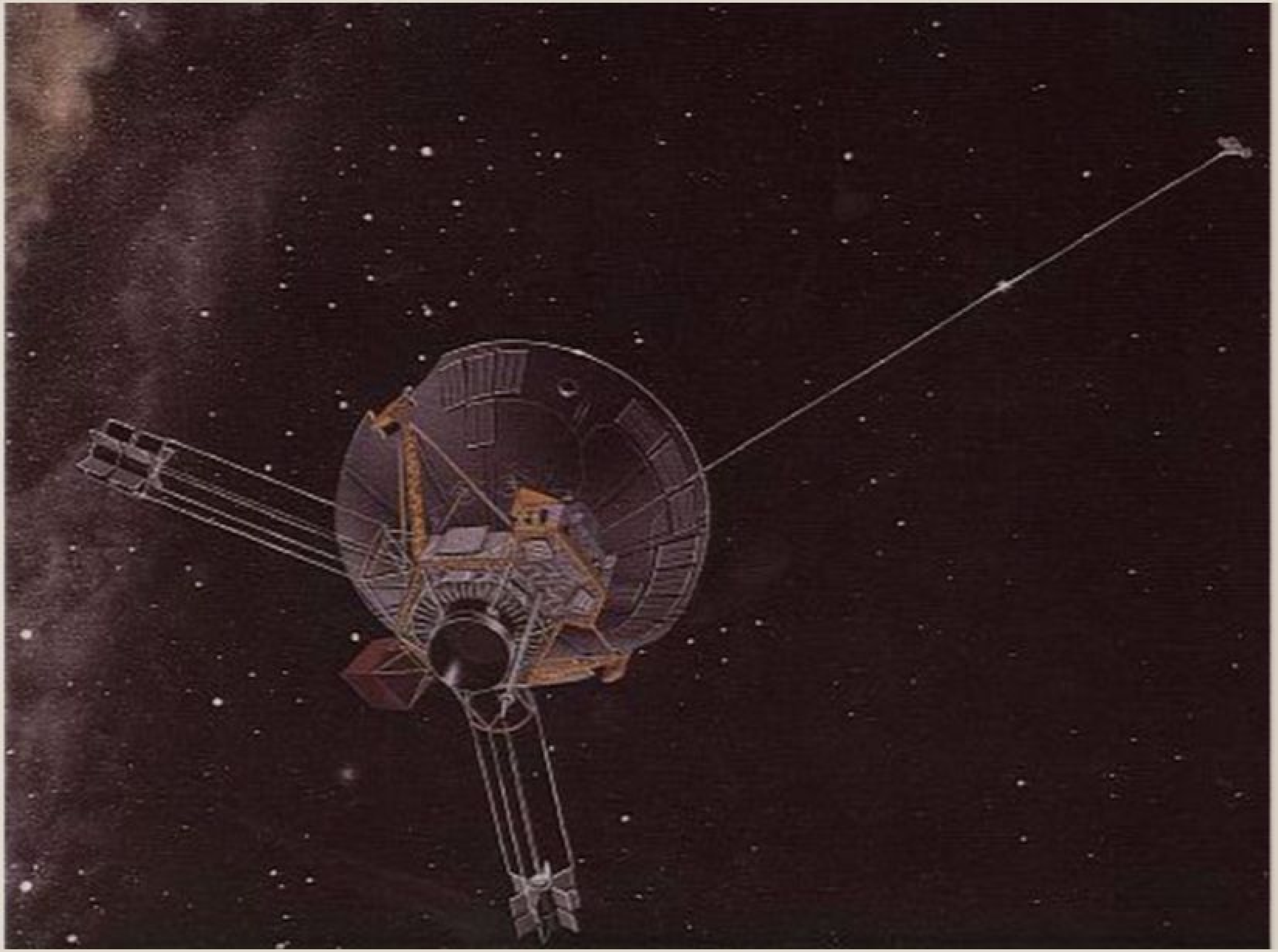
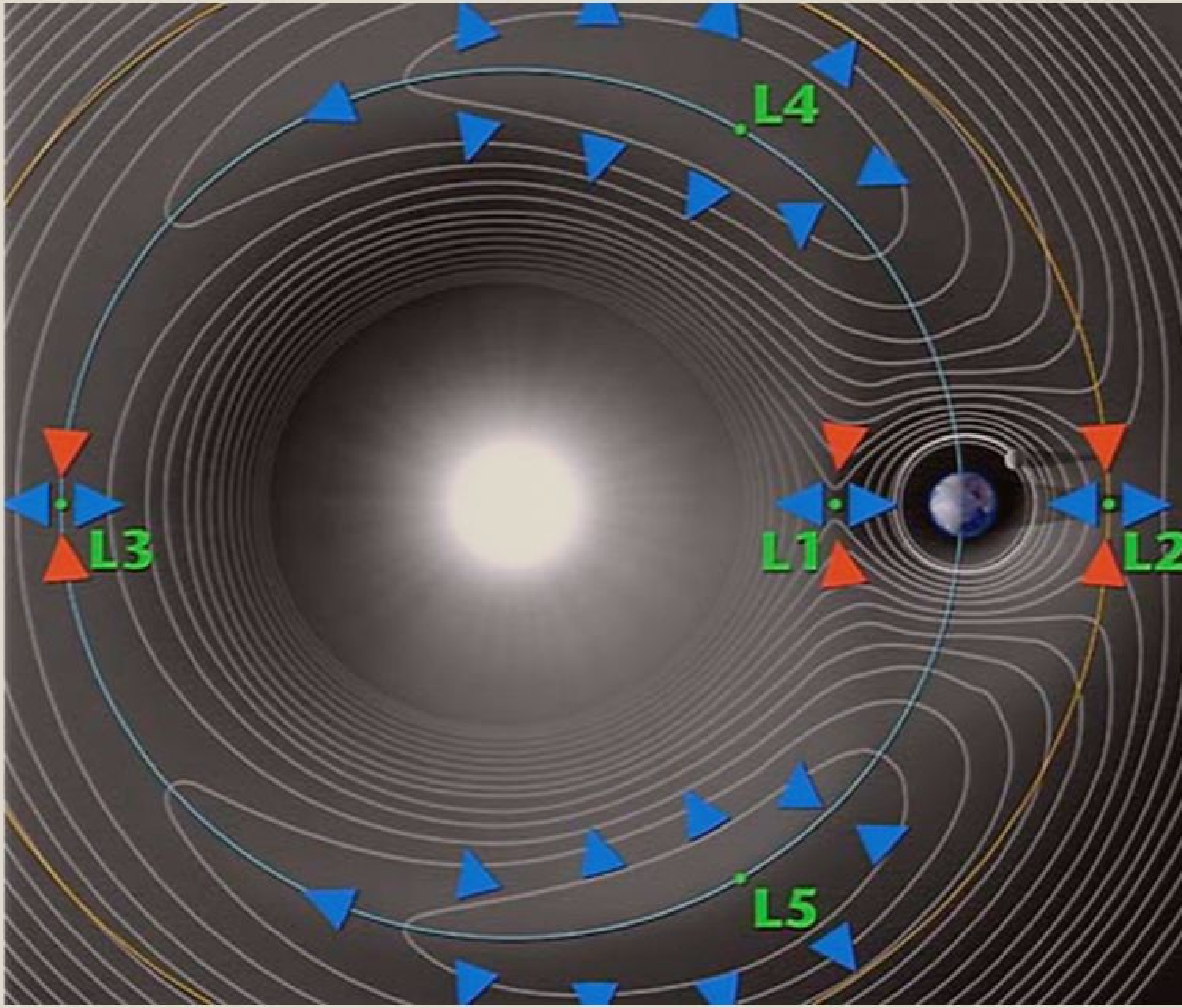
Conclusions

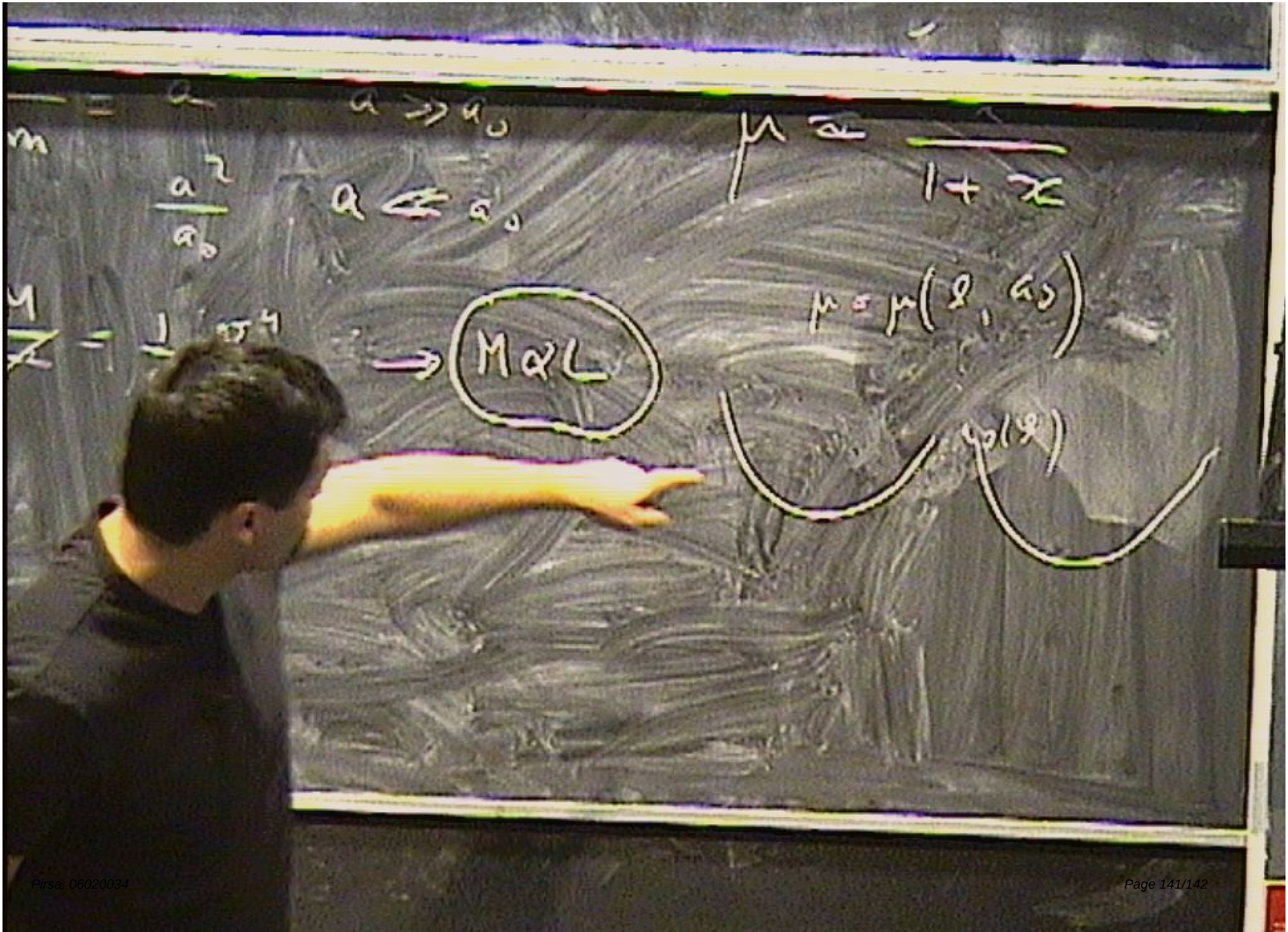
- When do you throw in the towel? What is proof beyond reasonable doubt?
- MOND should be a concern to everyone… if dark matter is wrong we could all be wide off the mark.
- Clearly none of current MOND theories is serious; but they are interesting if you're a masochist.

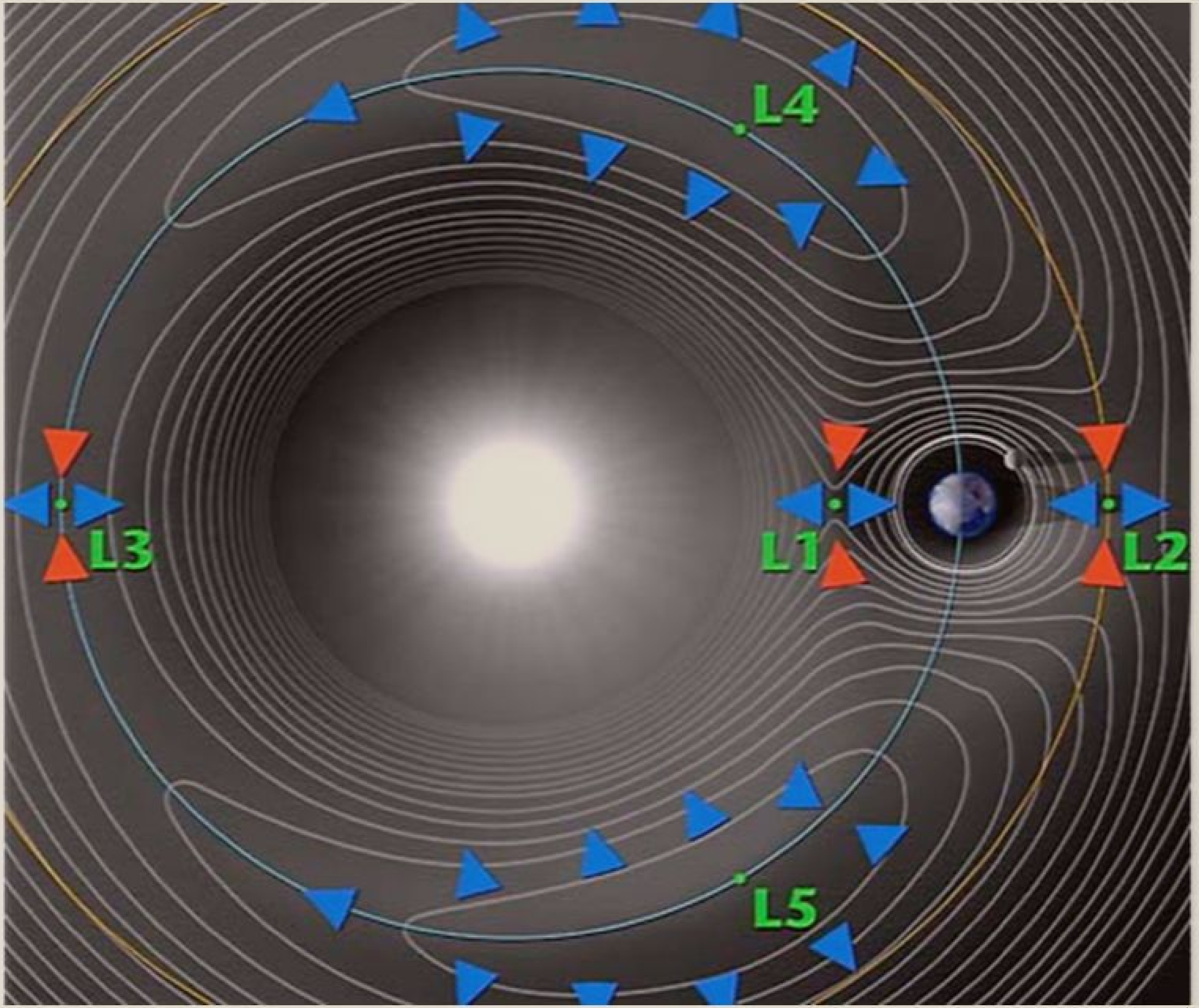


Warning: it's all a big soup!

- The Solar system is a many-body problem, and finding the exact location of the saddles involves taking all components into account.
- In general secondary bodies induce a quasi-elliptical precession to saddle.
- There are several excellent many-body codes capable of pinpointing exact saddle location.


L4
L3
L1
L2
L5


MaL


L4
L3
L1
L2
L5