Title: Recent developments in collapse models

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Abstract: Collapse models are one of the most promising attempts to overcome the measurement problem of quanum mechanics: they descibe, within one single framework, both the quantum properties of microscopic systems and the classical properties of macroscopic objects, and in particular they explain why measurements always have definite outcomes, distributed according to the Born probability rule. We will discuss some recent developments in this field: i) we will show how it is possible to formulate collapse models in such a way that the mean energy of physical system does non increse indefinitely, a typical feature of the models first proposed in the literature; ii) we will discuss recent experiments aiming at testing the validity of the superposition principle, thus of collapse models, at the mesoscopic level.

Recent Developments in Collapse Models

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OUTLINE

Introduction

- The measurement problem
- The Dynamical Reduction Program

Topic 1

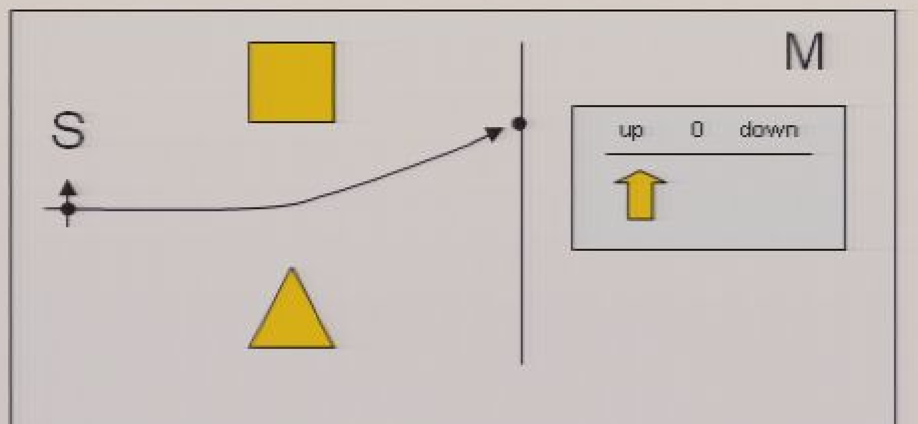
- The experiment of Marshall *et al.*
- Implications for Collapse Models

Topic 2

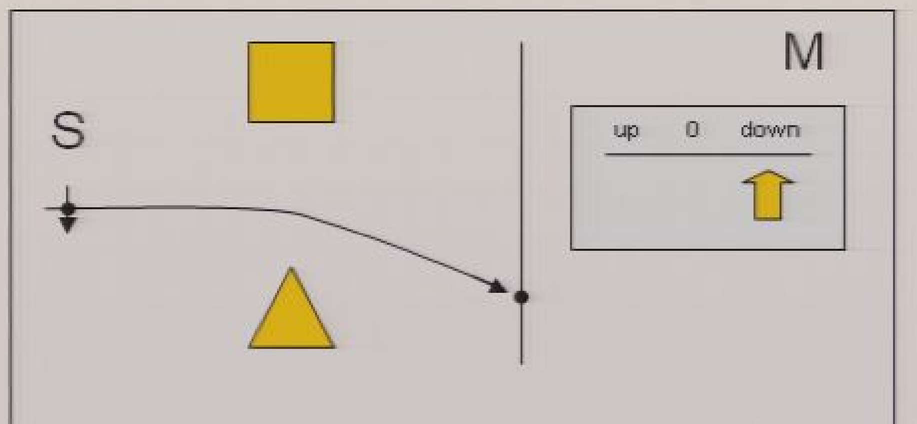
- The energy increase in isolated systems
- A new proposal
- Is it possible to restore the energy conservation principle?

THE VON NEUMANN MEASUREMENT SCHEME

J. von Neumann: "Mathematical Foundations of Q.M." (1932)



Initial state: $|\uparrow\rangle_S \otimes |0\rangle_M$ Final state: $|\uparrow\rangle_S \otimes |\mathbf{up}\rangle_M$



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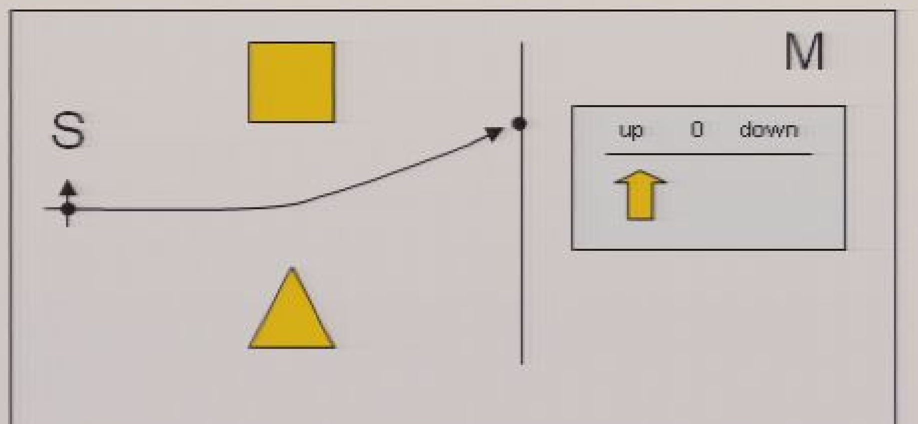

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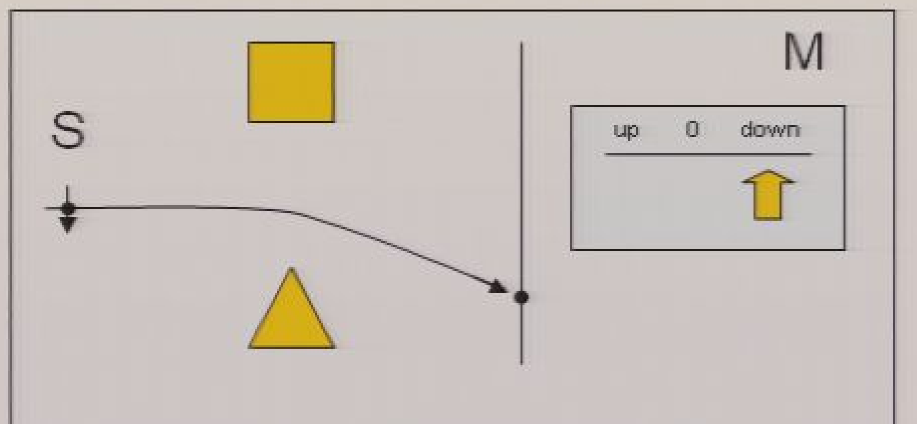

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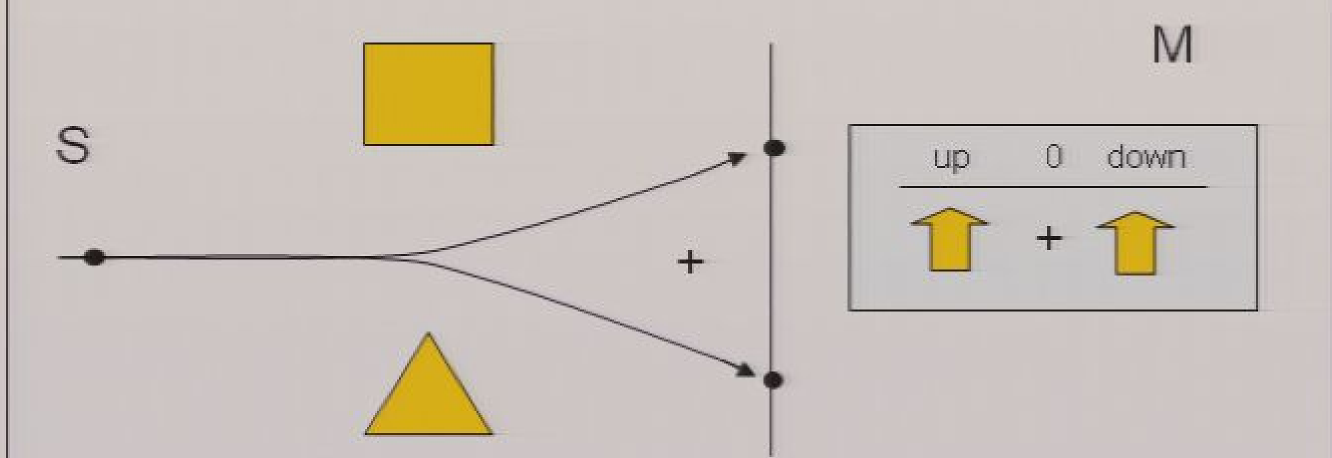
THE MEASUREMENT PROBLEM

From Linearity of Schrödinger Equation:



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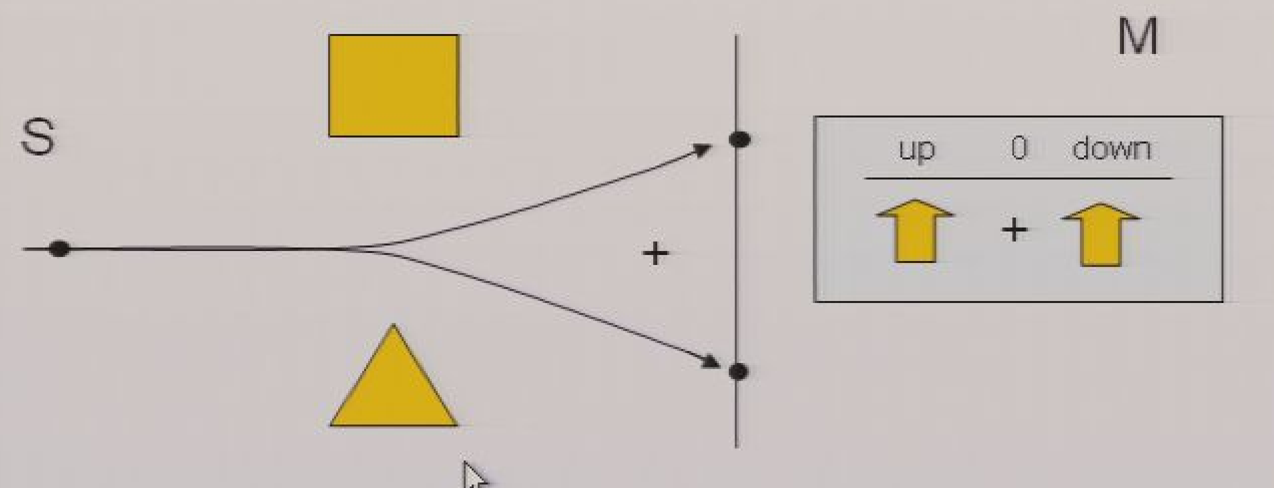

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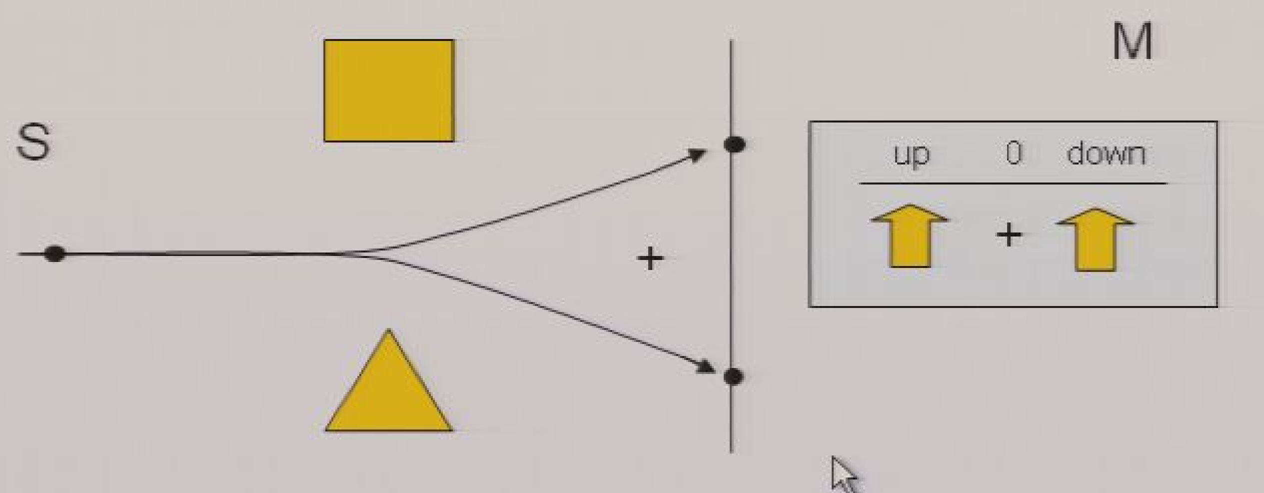

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STANDARD SOLUTION

POSTULATE OF WAVEPACKET REDUCTION

"At the end of a **measurement process**, the wavefunction is reduced into one of the possible outcomes …"

$$|\uparrow\rangle_S |\text{up}\rangle_M \quad \text{or} \quad |\downarrow\rangle_S |\text{down}\rangle_M$$

"… with a probability given by the square modulus of the coefficient associated to that term."

$$\frac{1}{\sqrt{2}}\left[|\uparrow\rangle_S |\text{up}\rangle_M + |\downarrow\rangle_S |\text{down}\rangle_M\right] \Rightarrow \begin{array}{ll} 50\% & |\uparrow\rangle_S |\text{up}\rangle_M \\ 50\% & |\downarrow\rangle_S |\text{down}\rangle_M \end{array}$$

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PROBLEMS WITH THE POSTULATE OF WAVEPACKET REDUCTION

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1 – TWO "FUNDAMENTALLY DIFFERENT" TYPES OF DYNAMICAL EVOLUTIONS:

A) SCHRÖDINGER EQUATION: LINEAR, DETERMINISTIC, REVERSIBLE.

B) WAVEPACKET REDUCTION: NON-LINEAR, STOCHASTIC, IRREVERSIBLE.

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→ MEASURE OR MACRO-OBJECTIFICATION PROBLEM

DYNAMICAL REDUCTION PROGRAM

IDEA: Joining the linear and deterministic Schrödinger evolution with wavepacket reduction process (non-linear and stochastic) in a unique universal dynamics in order to describe both quantum properties of microsystems and classical properties of macrosystems

Fundamental requests:

1. At the **microscopic level**, there must be no detectable difference with respect to standard quantum mechanics.
2. At the **macroscopic level**, one must recover classical mechanics.
3. In **measurement processes** on quantum systems (interaction between a micro and a macro system) one must get the correct outcomes, with the correct probabilities.

GRW MODEL

G.C. Ghirardi, A. Rimini, and T. Weber: Phys. Rev. D 34, 470 (1986)

1. Each particle of a system of n distinguishable particles experiences, with a mean rate λ_i, a sudden spontaneous localization process (hitting):

$$|\psi\rangle \xrightarrow{\text{localization}} \frac{L^i_{\mathbf{x}}|\psi\rangle}{\| L^i_{\mathbf{x}} |\psi\rangle \|} \qquad L^i_{\mathbf{x}} = \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\frac{\alpha}{2}(\mathbf{q}_i - \mathbf{x})^2}$$

2. The probability density for the occurrence of a localization at point $\mathbf{x}$ is assumed to be:

$$P_i(\mathbf{x}) = \| L^i_{\mathbf{x}} |\psi\rangle \|^2$$

3. In the time interval between two successive spontaneous processes the system evolves according to the usual Schrödinger equation

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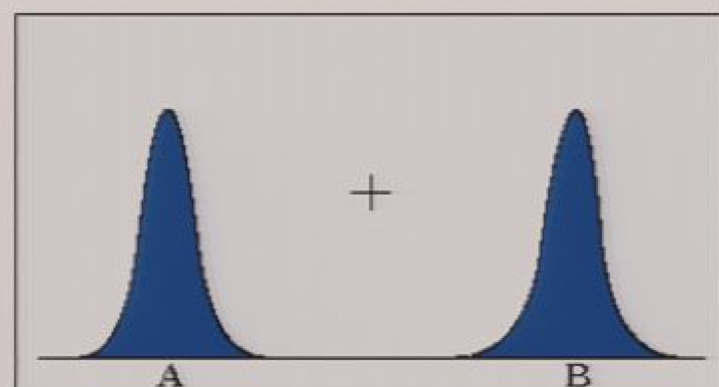
LOCALIZATION MECHANISM

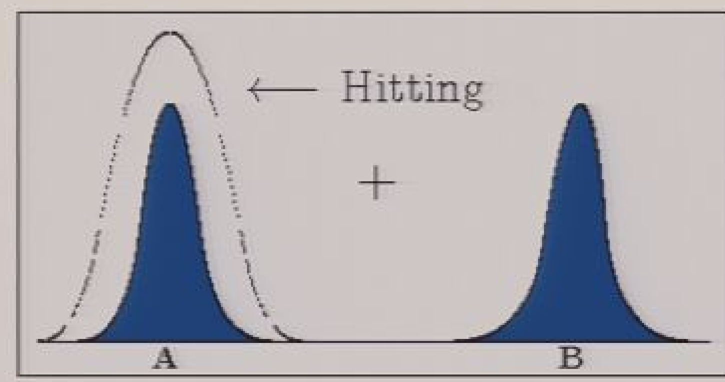
Let us consider the superposition of two Gaussian functions, one centered around position A and the other around position B:

$$\psi(z) = \frac{1}{N}\left[e^{-\frac{\gamma}{2}(z-A)^2} + e^{-\frac{\gamma}{2}(z-B)^2}\right]$$



Hitting around A

$$L_{\mathbf{A}} \quad = \quad \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\frac{\alpha}{2}(\mathbf{q}-\mathbf{A})^2} \qquad 1/\sqrt{\gamma} \ll 1/\sqrt{\alpha}$$



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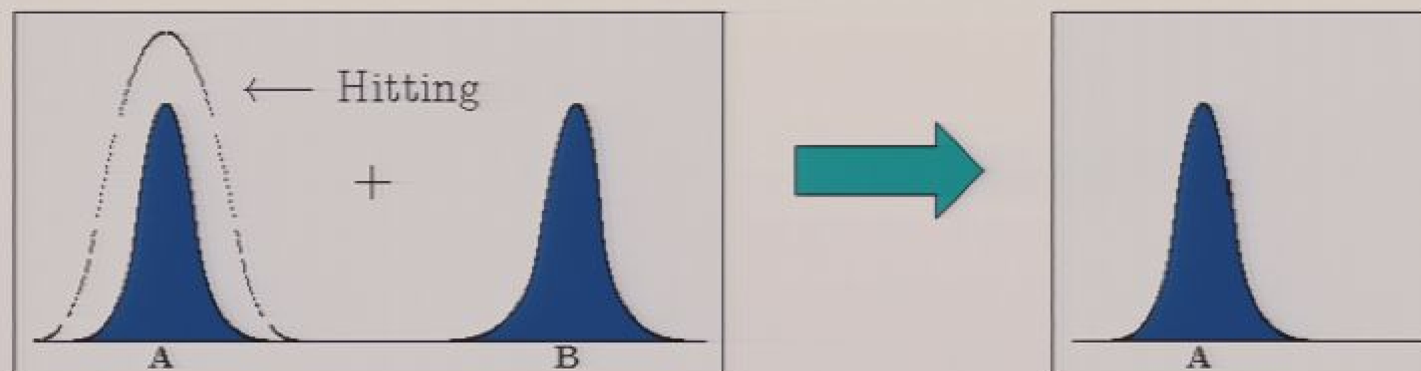

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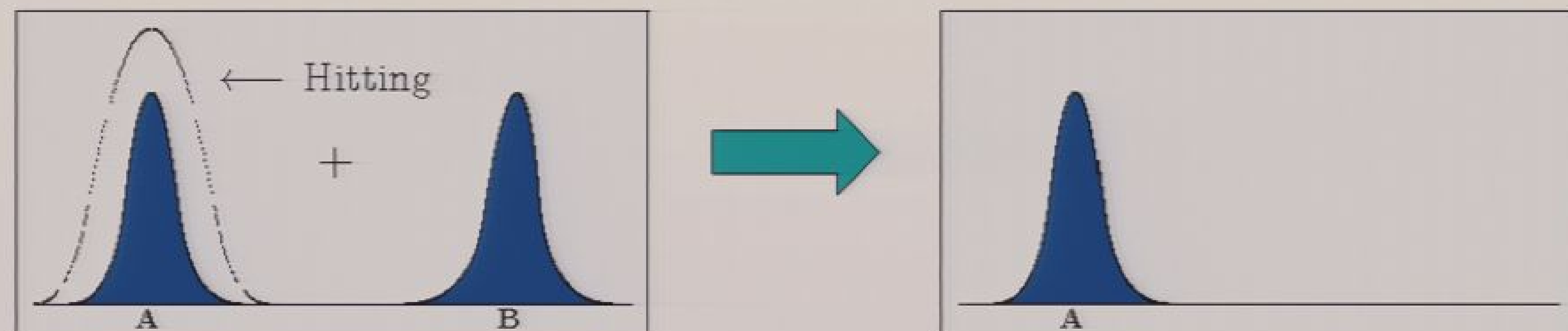

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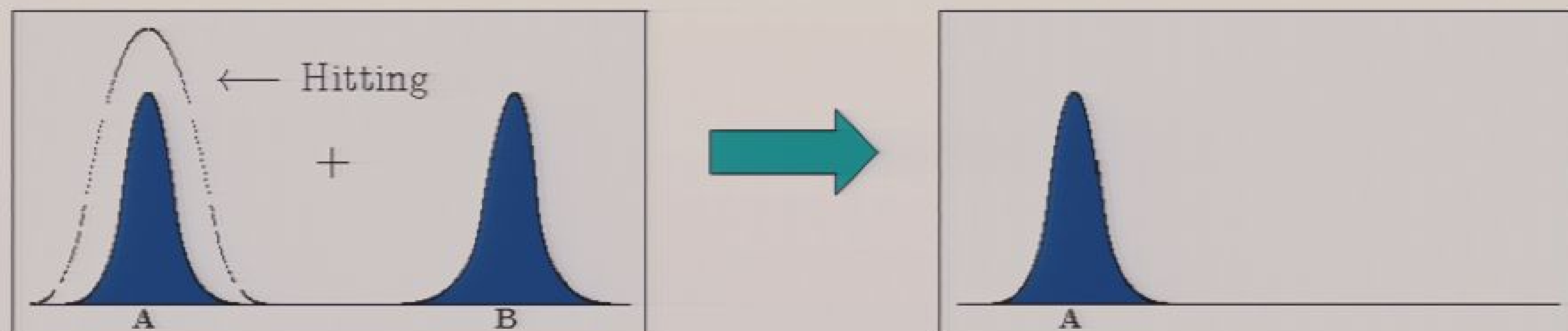
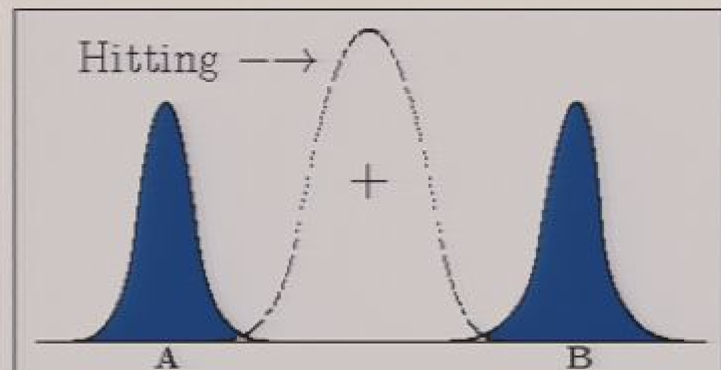


Probability of this event is $\quad \mathrm{P(z)} \;=\; \|L_A|\psi(z)\rangle\|^2 \;\approx\; \dfrac{1}{2}$

Hitting around the middle point

$$L_{\frac{\mathbf{A}+\mathbf{B}}{2}} = \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\frac{\alpha}{2}\left(\mathbf{q}-\frac{\mathbf{A}+\mathbf{B}}{2}\right)^2} \qquad 1/\sqrt{\gamma} \ll 1/\sqrt{\alpha}$$



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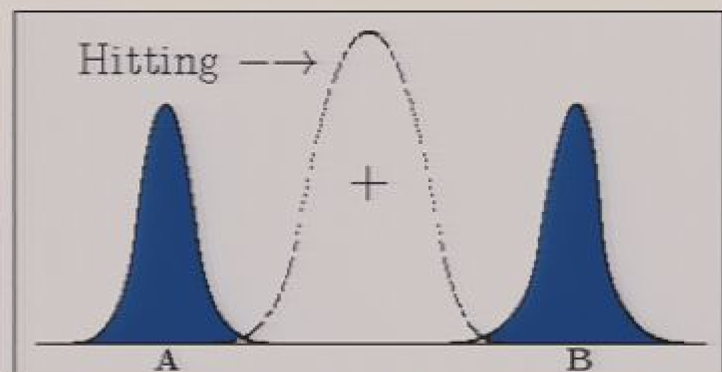

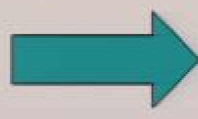

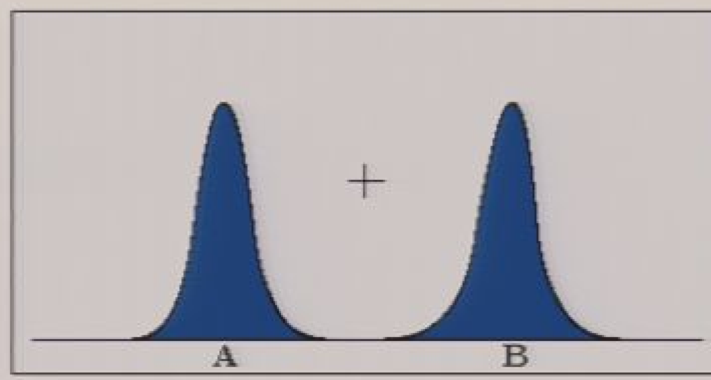

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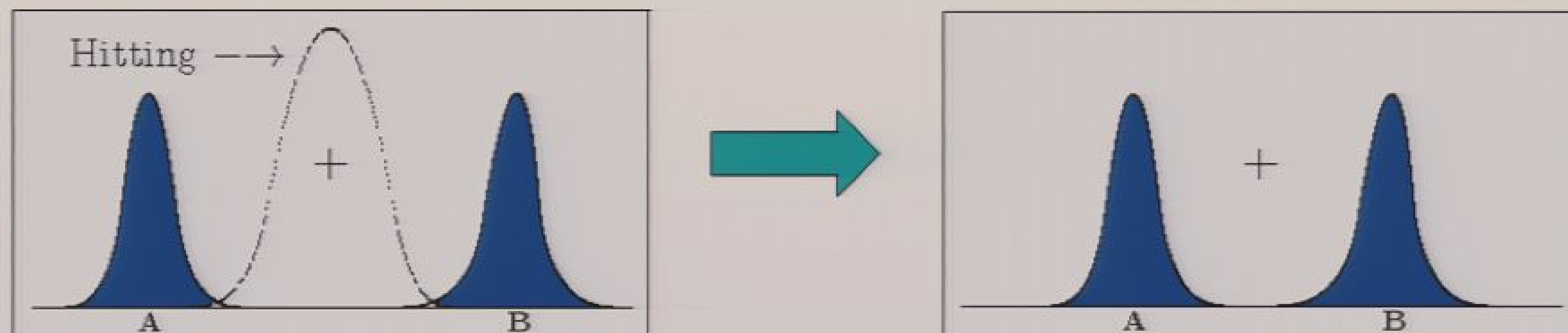

Probability of this event is $\mathrm{P(z)} = \left\| L_{\frac{A+B}{2}} |\psi(z)\rangle \right\|^2 \simeq 0$

MATHEMATICAL STRUCTURE OF COLLAPSE MODELS

$$d\psi_t = \left[-\frac{i}{\hbar} H \, dt + \sqrt{\eta} \, (A - r_t) \, dW_t - \frac{\eta}{2} \left(A^\dagger A - 2 r_t A + r_t^2 \right) dt \right] \psi_t$$

$$r_t = \frac{1}{2} \langle \psi_t | (A + A^\dagger) | \psi_t \rangle$$

H is related to the standard quantum Hamiltonian

A is the reduction operator on whose eigenmanifolds one wants to reduce the state vector (usually a function of the position operator q).

W_t is a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$

$$\mathrm{E}\,[\mathrm{W}_t] = 0 \qquad \mathbb{E}\,[W_t^2] = dt$$

The equation is nonlinear but preserves the norm of the state vector

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QMUPL MODEL

L. Diosi: Phys. Rev. A 40, 1165 (1989)

$$\mathrm{d}|\psi_t\rangle = \left[-\frac{i}{\hbar} H \,\mathrm{d}t + \sqrt{\eta}\,(q - \langle q\rangle_t)\,\mathrm{d}W_t - \frac{\eta}{2}\,(q - \langle q\rangle_t)^2 \mathrm{d}t \right] |\psi_t\rangle$$

H	Hamiltonian
q	Position Operator
$\langle q\rangle_t \equiv \langle\psi_t\|q\|\psi_t\rangle$	
$\eta \sim \mathbf{10^{13}}\,\mathrm{s}^{-1}\mathrm{m}^{-2}$	Strength of the collapse mechanism

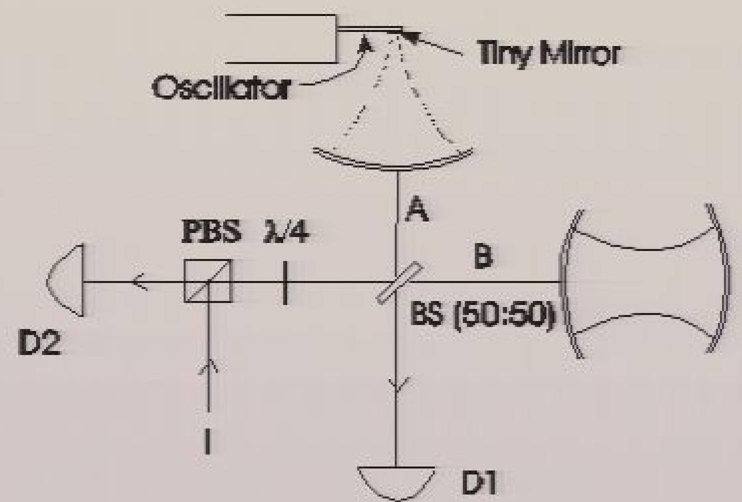
TOPIC 1

A PROPOSED EXPERIMENT TO TEST

THE VALIDITY OF COLLAPSE MODELS

Marshall *et al.* Experiment Proposal

W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester: Phys. Rev. Lett 91, 130401 (2003)



L is the Equilibrium cavity Length

M is the "Quntum" Tiny Mirror Mass

$$H = \hbar\omega_c(a_A^\dagger a_A + a_B^\dagger a_B) + \hbar\omega_m b^\dagger b - \hbar G a_A^\dagger a_A (b + b^\dagger)$$

$$G = \frac{\omega_c \sigma}{L} \qquad \sigma = \sqrt{\frac{\hbar}{2M\omega_m}}$$

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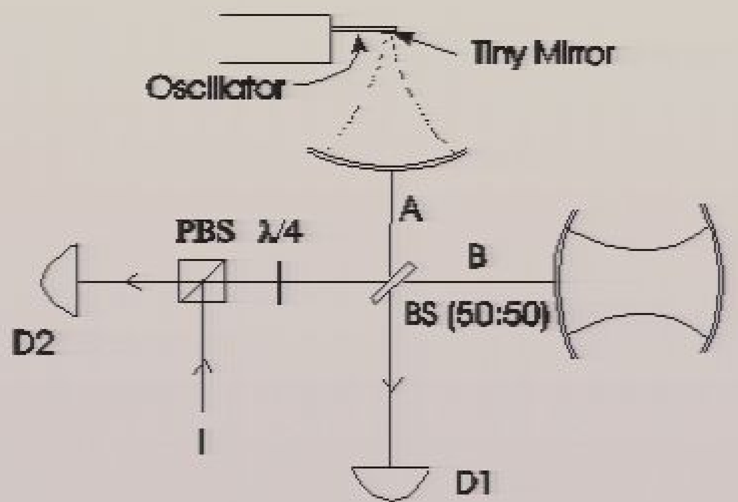

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EVOLUTION OF THE STATE

Initial State:

$$|\psi_0\rangle \;=\; \frac{1}{\sqrt{2}}\left[|0\rangle_A|1\rangle_B \;+\; |1\rangle_A|0\rangle_B\right]|0\rangle_m$$

State at time t:

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}e^{-i\omega_c t}\left[|0\rangle_A|1\rangle_B|0\rangle_m \;+ e^{i\kappa^2(\omega_m t - \sin\omega_m t)}\;|1\rangle_A|0\rangle_B|\alpha_t\rangle_m\right]$$

$|0\rangle_m$ Mirror at rest in its equilibrium position

$|\alpha_t\rangle_m$ Mirror oscillating between 0 and $4\kappa\sigma \sim 10^{-11}$cm $\Rightarrow \quad \kappa \geq \frac{1}{4}$

$$\kappa = \frac{G}{\omega_m}$$

Marshall *et al.* Experiment Proposal

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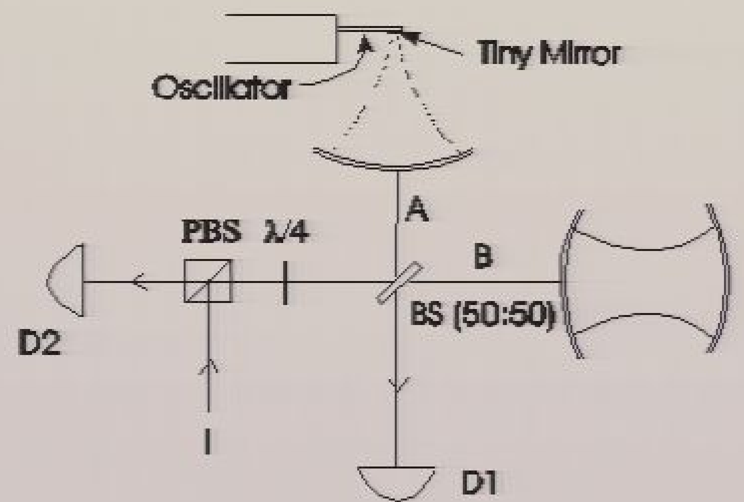

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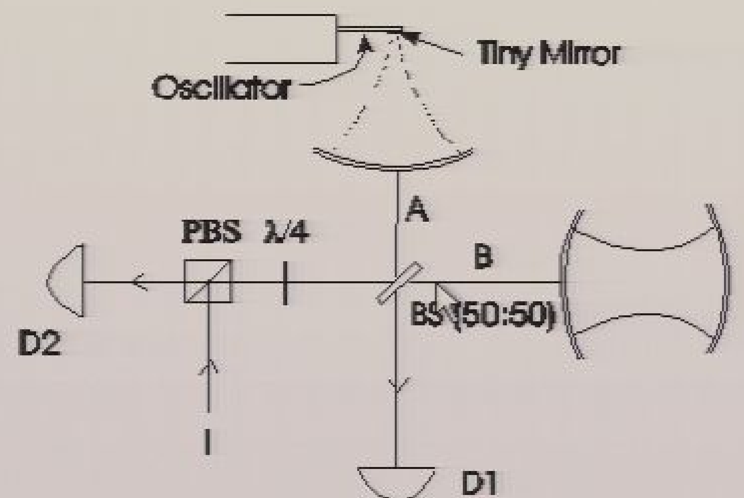

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$|0\rangle_m$ Mirror at rest in its equilibrium position

$|\alpha_t\rangle_m$ Mirror oscillating between 0 and $4\kappa\sigma \sim 10^{-11}$cm $\Rightarrow \kappa \geq \frac{1}{4}$

$$\kappa = \frac{G}{\omega_m}$$

MAXIMUM INTERFERENCE VISIBILITY ν

Full Density Matrix of Photon + Mirror: $\rho = |\psi_t\rangle\langle\psi_t|$

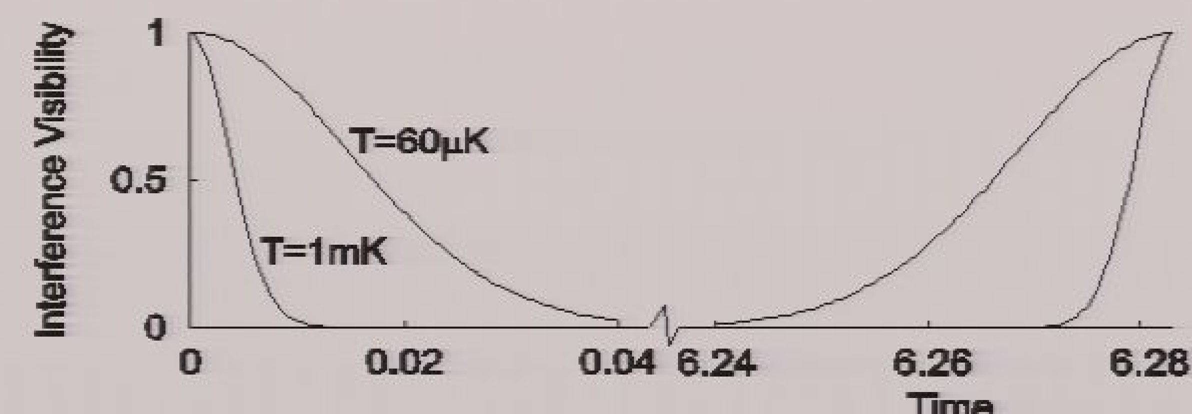
Reduced Density Matrix: $\rho_p = \mathrm{Tr}_m[\rho]$

Analytic Expression

$$\nu = 2 \cdot \left| {}_A\langle 1|_B\langle 0|\rho_p|1\rangle_B|0\rangle_A \right|$$

Standard Quantum Result

$$\nu(t) = e^{-\kappa^2(1-\cos\omega_m t)}$$



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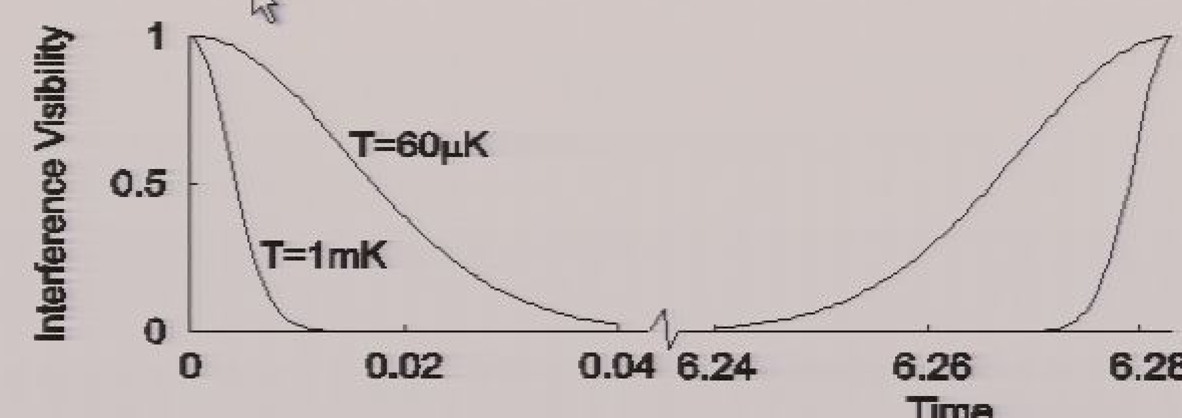

EVOLUTION OF THE STATE

Initial State:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right]|0\rangle_m$$

State at time t:

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}e^{-i\omega_c t}\left[|0\rangle_A|1\rangle_B|0\rangle_m + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B|\alpha_t\rangle_m\right]$$

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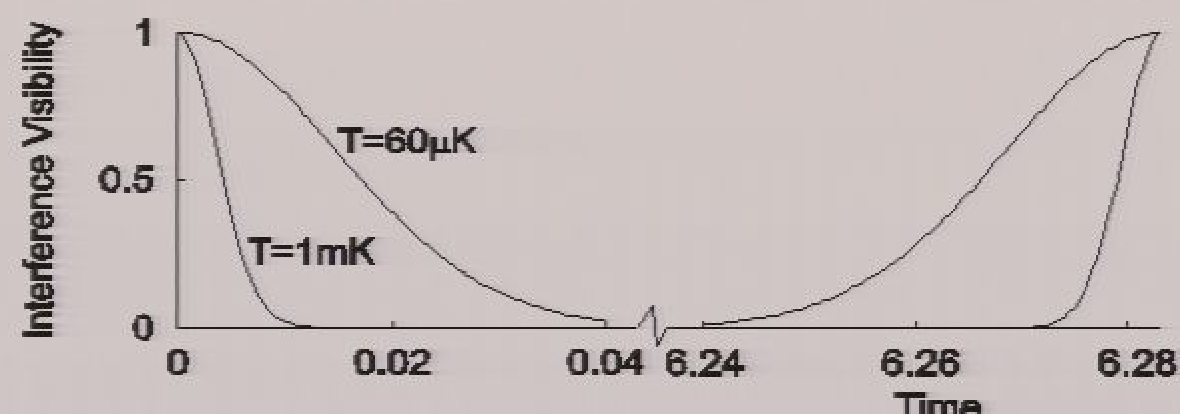

CHOICE OF COLLAPSE MODEL

We have chosen Diosi model for two main reasons:

1. This model allows to get an **exact** formula for visibility.
2. QMUPL corresponds to the leading term in the small-displacement Taylor expansion of **more significant models** like GRW or CSL.

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L. Diosi: Phys. Rev. A 40, 1165 (1989)

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DENSITY MATRIX EQUATION FOR QMUPL

From the stochastic differential equation:

$$\mathrm{d}|\psi_t\rangle = \left[-\frac{i}{\hbar} H\, \mathrm{d}t + \sqrt{\eta}\,(q - \langle q\rangle_t)\, \mathrm{d}W_t - \frac{\eta}{2}\,(q - \langle q\rangle_t)^2 \mathrm{d}t \right] |\psi_t\rangle$$

Using Itô calculus the evolution of density matrix $\hat{\rho} = |\psi_t\rangle\langle\psi_t|$ is

$$\mathrm{d}\hat{\rho} = -\frac{i}{\hbar}[H, \hat{\rho}]\mathrm{d}t - \sqrt{\eta}[\hat{\rho}, [\hat{\rho}, q]]\mathrm{d}W_t + \frac{1}{2}\eta[q, [q, \hat{\rho}]]\mathrm{d}t$$

Since to observe interference fringes requires passing to an ensemble of identically prepared photons, the relevant density matrix is the ensemble expectation $\rho = \mathbb{E}[\hat{\rho}]$, which obeys

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta[q, [q, \rho]] = -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta\sigma^2[b + b^\dagger, [b + b^\dagger, \rho]]$$

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Visibility can then rewrite as:

$$\nu(t) = 2 \cdot \left| \int_{-\infty}^{+\infty} \mathbb{E}\left[\phi_t^0(x)^* \, \phi_t^1(x)\right] dx \right|.$$

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The final step is to take the average with respect to the noise.

INTERFERENCE VISIBILITY FOR QMUPL

A. Bassi, E. Ippoliti, and S.L. Adler: Phys. Rev. Lett. 94, 030401 (2005)

$$\nu(t) = e^{-\kappa^2(1-\cos\omega_m t)} e^{-3\eta\kappa^2\sigma^2\left(t - \frac{4\sin\omega_m t}{3\omega_m} + \frac{\sin 2\omega_m t}{6\omega_m}\right)}$$

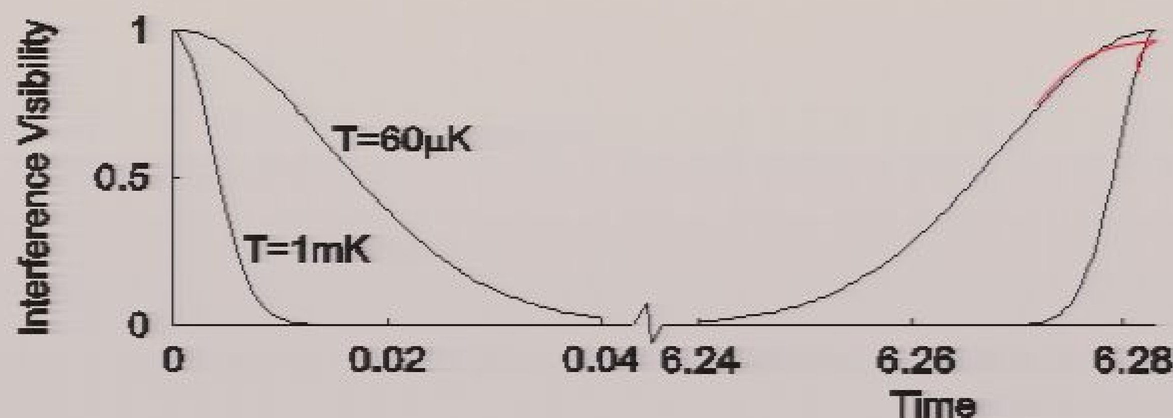
Maximum Interference Visibility is the product of two parts:

1. A "deterministic" part (the blue one) which does not depend on the noise and that coincides with the result derived from standard quantum evolution.

2. A "stochastic" part (the red one) which is the result of the noise terms and that corresponds to the correction to standard result.

EXPERIMENTAL MEASURE

$$\nu\left(\frac{2\pi}{\omega_m}\right) = 1 \cdot e^{-6\pi \frac{\eta \kappa^2 \sigma^2}{\omega_m}}$$



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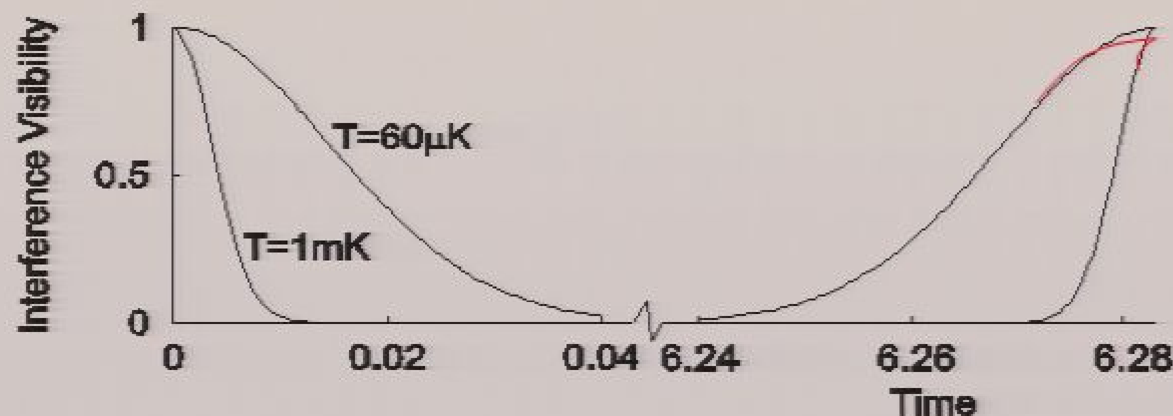

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QMUPL

$$\eta \sim \mathbf{10^{13}}\mathrm{s}^{-1}\mathrm{m}^{-2}$$

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$\alpha \sim 10^{10}\mathrm{cm}^{-2}$ $\lambda \sim 10^{-16}\mathrm{s}^{-1}$

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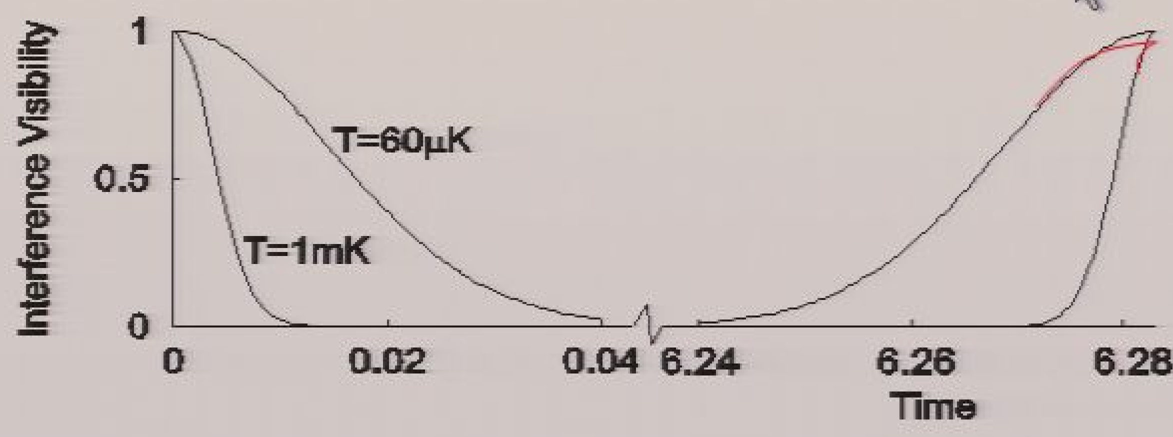

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DAMPING FACTOR OF THE VISIBILITY

The CSL model gives an extimate of η larger than that of QMUPL and GRW models by a factor of 10^8, so we will take this more conservative value for our analysis → $\eta \sim 0.6 \times 10^{21}\mathrm{s}^{-1}\mathrm{m}^{-2}$

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If dynamical reductions happen, the mirror maximum interference visibility after one oscillation period $2\pi/\omega_m = 2 \times 10^{-3}\mathrm{s}$ is damped by a factor $e^{-\Lambda}$ with:

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If the Marshall *et al* experiment were to observe maintenance of coherence to 0.2 percent accuracy, a realistic value in present-day laboratories, it would set only the weak bound

$$\gamma < 10^{-24}\mathrm{cm}^3\mathrm{s}^{-1}$$

on the CSL model stochasticity parameter. This would be considerably better than the bound set by fullerene diffraction experiments, the better one up to now:

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TOPIC 2

A SPACE-COLLAPSE MODEL WITHOUT

INFINITE ENERGY INCREASE

THE "PROBLEM" OF THE ENERGY

One of the characteristic features of the models that reduce in position is the violation of Principle of Energy Conservation for isolated systems. Such a violation is determined by the stochastic process responsible for the localization mechanism:

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For example in the GRW model:

$$\delta E / t \simeq 10^{-25}\,\mathrm{eV\,s^{-1}}$$ for a nucleon

$$\delta T / t \simeq 10^{-15}\,\mathrm{K\,y^{-1}}$$ for an ideal monoatomic gas

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- Any relativistic generalization of such models devised so far shows divergences for energy density originating from similar reasons

OUR PROPOSAL

A. Bassi, E. Ippoliti, and B. Vacchini: J. Phys. A 38, 8017 (2005)

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In the following we will focus our attention to the case of a free particle:

$$H_0 = \frac{p^2}{2m}$$

ENERGY MEAN VALUE

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whose solution is:

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$$H^0_{TOT} = H^0_{CM} + H^0_{REL}$$ ➡ It is easy to prove that the dynamics of the two types of degrees of freedom decouples

So we have for the dynamics of the center of mass:

$$d\psi_t(R) = \left[-\frac{i}{\hbar} H_{\text{CM}}\,dt + \sqrt{\eta_{\text{CM}}}\,(A_{\text{CM}} - r_{\text{CM},t})\,dW_t - \frac{\eta_{\text{CM}}}{2}\left(A^\dagger_{\text{CM}}A_{\text{CM}} - 2r_{\text{CM},t}A_{\text{CM}} + r^2_{\text{CM},t}\right)dt\right]\psi_t(R)$$

$$\begin{aligned} H_{\text{CM}} &= H^0_{\text{CM}} + \tfrac{\eta_{\text{CM}}\tau_{\text{CM}}}{2}\{Q, P\} \\ r_{\text{CM,t}} &= \tfrac{1}{2}\langle\psi_t|[A^\dagger_{\text{CM}} + A_{\text{CM}}]|\psi_t\rangle \\ A_{\text{CM}} &= Q + i\tfrac{\tau_{\text{CM}}}{\hbar}P \\ \text{W}_t &= \textstyle\sum_{n=1}^N \sqrt{\frac{m_n}{M}}\,W^n_t \end{aligned}$$

$$\begin{aligned} \eta_{\text{CM}} &= \left(\frac{M}{m_0}\right)\eta_0 \\ \tau_{\text{CM}} &= \left(\frac{m_0}{M}\right)\tau_0 \end{aligned}$$

that is the same equation for the case of a single particle except for the value of the mass which is M

MICROSCOPIC LEVEL

Microscopic systems can be observed only by resorting to suitable measurement procedures

All physical predictions have the form $\mathbb{E}_{\mathbb{P}}[\langle\psi_t|O|\psi_t\rangle] = \mathrm{Tr}[O\rho_t]$

The testable effects of the stochastic process on the wavefunction are similar to the effects induced by quantum environment on the particle, when both friction and diffusion are taken into account

With our choice of the parameters η e τ the measurable effects of stochastic process are of the same order of magnitude of those induced by the interaction of the system with particles and radiation of **intergalactic space**: such effects are very small and masked by most other source of decoherence, so that they cannot be tested by present-day technology

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THE EQUATION FOR a_t

$$a_t^{\mathrm{R}} = \frac{m\omega}{2\sqrt{2}\hbar} \frac{\sin\theta \sinh(\omega_1 t + \varphi_1) + \cos\theta \sin(\omega_2 t + \varphi_2)}{\cosh(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2)}$$

$$a_t^{\mathrm{I}} = \frac{-m\omega}{2\sqrt{2}\hbar} \left[\frac{\cos\theta \sinh(\omega_1 t + \varphi_1) - \sin\theta \sin(\omega_2 t + \varphi_2)}{\cosh(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2)} - \frac{2\sqrt{2}\eta\tau}{\omega} \right]$$

where we have introduced the following **mass independent** parameters:

$$\theta = \frac{1}{2}\tan^{-1}\left[\frac{\hbar}{2\eta_0\tau_0^2 m_0}\right] \simeq \frac{\pi}{4} \qquad \omega_1 = \sqrt{2}\,\omega\,\cos\theta$$

$$\omega = 2\sqrt[4]{4\eta_0^4\tau_0^4 + \frac{\eta_0^2\hbar^2}{m_0^2}} \simeq 10^{-5}\ \mathrm{sec}^{-1} \qquad \omega_2 = \sqrt{2}\,\omega\,\sin\theta$$

FUTURE DEVELOPMENTS

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So by taking into account the energy of both the quantum system and the stochastic medium one could restore perfect energy conservation not only on the average but also for single realizations of the stochastic process.

A REMARK

Whatever its nature can be, the stochastic medium **cannot be quantum** in the usual sense since its coupling to the particle is not a standard coupling between two quantum systems, i.e. the equation of evolution for the particle

$$d\psi_t = \left[-\frac{i}{\hbar} H \, dt + \sqrt{\eta} \, (A - r_t) \, dW_t - \frac{\eta}{2} \left(A^\dagger A - 2 r_t A + r_t^2 \right) dt \right] \psi_t$$

is not a standard Schrödinger equation with a stochastic potential.