

Title: New instanton effects in string theory

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Abstract:

New Instanton Effects in String Theory¹
C.B. E. Wilton, hep-th/0512054

New Instanton Effects in String Theory¹
C.B. E. Wilton, hep-th/0512089

$d=4$, $N=1$ SUSY

New Instanton Effects in String Theory¹
C.B. E. Witten, hep-th/0512039

$d=4$, $N=1$ SUSY, $\mathcal{H}_4$ on $\mathbb{R}^{3,1} \times X$

"New Instanton Effects in String Theory"
C.B. E. Witten, hep-th/0512039

$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

γ stable, hol gauge ball



X CY3

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$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{ad}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge ball} \\ \downarrow \\ X \text{ CY3 (Kähler)} \end{array} \right.$

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$d=4$, $N=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge bundle (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

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$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{ad}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \gamma \text{ stable hol gauge field (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

$\mathcal{M}_{\text{cl}} = \text{classical moduli space of } (\gamma, V)$

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$d=4$, $N=1$ SUSY, $\mathcal{H}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \gamma \text{ stable hol gauge field (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

$\mathcal{M}(c) = \text{classical/moduli space of } (\gamma, V)$

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$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \mathcal{V} \text{ stable, hol gauge bundle (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

$\mathcal{M}(c, \nu) = (\text{classical}) / \text{moduli space of } (x, \mathcal{V})$

$$\int_{\mathcal{M}} \mathcal{H} = \int d^4x \int d^3\sigma W(\vec{\Phi}^i)$$

(singlet $\chi \Phi$)

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$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{ad}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \mathcal{V} \text{ stable, hol gauge bundle (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

$\mathcal{M}(c, \nu) = \text{classical/moduli space of } (\gamma, \mathcal{V})$

$$\int \mathcal{L}_{\text{eff}} = \int d^4x \int_0^1 d\tau W(\tilde{\mathcal{Q}}^i)$$

(singlet χ_i)

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Worldsheet Instantons

$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge bundle (cpd, etc)} \\ \downarrow \\ X \text{ CY3 (Kähler)} \end{array} \right.$

$\mathcal{M}(c, \gamma) = \text{classical model}$

$$\delta \mathcal{L}_{\text{eff}} = \int d^4x \, d^3\sigma \, W$$



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Worldsheet Instantons



$d=4, N=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge bundle (cplx str)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx str)} \end{array} \right.$

$\mathcal{M}(c, \gamma) = (\text{classical}) \text{ moduli space of } (Y, \gamma)$

$$\int \mathcal{L}_{\text{eff}} = \int d^4x \int d^2\sigma W(\tilde{\Phi}^i)$$

(singlet χ_i)

"New Instanton Effects in String Theory"
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$d=4$, $N=1$ SUSY, $\mathcal{H}_{\text{ad}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \gamma \\ \downarrow \\ X \end{array} \right.$ stable hol gauge bdl (cplx str)

X CY3 (Kähler, cplx str)

$\mathcal{M} = \text{(classical) moduli space of } (\gamma, \psi)$

$$= \int_{\mathcal{M}} \int_{\mathcal{D}} W(\vec{\Phi})$$

(singlet χ_F)

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

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$d=4$, $N=1$ SUSY, Had on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge ball (cplx st)} \\ \downarrow \\ X \text{ CY3 (plasma)} \end{array} \right.$

$\mathcal{M}(c) = \text{(classical)}$

$\int \mathcal{L}_H = \int \text{over}$

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{D}$

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$d=4$, $N=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \mathcal{V} \text{ stable hol gauge bundle (cplx str)} \\ \downarrow \\ X \text{ CY3 (K3, 256)} \end{array} \right.$

$\mathcal{M}(c) = \{\text{classical vac}\}$

$$\delta \mathcal{L}_{\text{eff}} = \int d^4x \, d^3x$$

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{O}$ genus $g=0$

$\mathcal{O}C$ isolated in X , $N=0$ mod α'

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$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{eff}} \sim \mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} V \text{ stable hol gauge bundle (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

Small/moduli space of (X, V)

$$\mathcal{Z}_{\text{eff}} = \int \mathcal{D}V \mathcal{D}\Phi W(\Phi)$$

(symlet χ_{eff})

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{O}$ genus $g=0$

$\mathcal{O} C$ isolated in X , $N=0$ moduli

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$d=4, N=1$ SUSY. H_{eff} on $\mathbb{R}^{3,1} \times X$

$\begin{cases} Y & \text{stable, hol gauge bdl (color ss)} \\ \downarrow \\ X & \text{CY3 (Kahler)} \end{cases}$

$\mathcal{M}_{(c)} = \{ \text{classical vacua} \}$

$$\delta S_{\text{eff}} = \int_{\mathcal{M}_{(c)}} \delta \mathcal{L}$$

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\oplus$ genus $g=0$

$\oplus C$ isolated in X , $N = \dim \mathcal{M}_{(c)}$

$S_{\text{eff}} \subset$

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$d=4$, $\mathcal{N}=1$ SUSY, $H_{\text{eff}} \sim \mathbb{R}^{3,1} \times X$

$\begin{cases} Y & \text{stable, hol gauge ball (cplx st)} \\ \downarrow \\ X & \text{CY3 (K3)} \end{cases}$

$\mathcal{M}_{\text{eff}} = \text{classical}$

$\int \mathbb{Z}_H$

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\textcircled{0}$ genus $g=0$

$\textcircled{0} C$ isolated in X , $N_{\text{ol}} \text{ or } \text{moduli}$

$S_{\text{eff}} \subset$ has $g \geq 0$, or moves in family??

Eg, IIA in X

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$d=4, N=1$ SUSY. $\mathcal{H}$ on $\mathbb{R}^{3,1} \times X$



$\mathcal{M}_{(c)} = \{c\}$

$\delta S_{eff} =$

space of (γ, V)

(γ, V)

(γ, V)

(γ, V)

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{O}$ genus $g=0$

$\mathcal{O}C$ isolated in X , $N=0$ or $N=1$

$S_{eff} \subset$ has $g>0$, or moves in family??

$E_8, II A$ on X

$\delta S_{eff} = \int_{C \times \mathbb{R}^{3,1}} F_g(X) (W_{eff} W_{eff}^\dagger)$

New Instanton Effects in String Theory

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$d=4$, $N=1$ SUSY, H_{eff} on $\mathbb{R}^{3,1} \times X$

$\begin{cases} Y & \text{stable hol gauge bundle (cplx st)} \\ \downarrow \\ X & \text{CY3 (Kähler, cplx st)} \end{cases}$

$\mathcal{M}_{\text{cl}} = \text{classical moduli space of } (Y, \nabla)$

$$\int_{\mathcal{M}_{\text{cl}}} \int_{\mathcal{M}_{\text{cl}}} W(\nabla) \quad (\text{coupling } \lambda F)$$

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{O}$ genus $g=0$

$\mathcal{O}C$ isolated in X , $N.O. moduli$
 $\cdot S^1 \subset C$ has $g \geq 0$, or moduli in family??

Eg $\Pi A \rightarrow X$

$$\int_{\mathcal{M}_{\text{cl}}} \int_{\mathcal{M}_{\text{cl}}} F_g(X) (W_{\text{eff}} W^{-n})$$

Physical Gauge

Physical Gauge : (D1-Brane)

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$$C \hookrightarrow \mathbb{C}^2 \times X$$

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

$$P = (x^{\mu}, y^{\alpha}), \quad \mu, \alpha = 1, 2$$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\hat{m}}, y^{\hat{n}})$, $m, n = 1, 2$ $\theta^{\hat{a}} \otimes N$



Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\hat{\mu}}, y^{\hat{\alpha}}), \mu, \nu = 1, 2 \quad \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

$$\sigma: (x^{\hat{a}}, y^{\hat{a}}), \quad a=1,2 \quad \theta^2 \otimes N$$

$$\text{coordinates: } (L, \delta) \quad \lambda \quad S_-(\tau) \otimes V = V$$

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times$$

$$\theta = (x, y), \text{ with } \theta \in N$$

$$(L, \bar{\lambda}) \rightarrow S(\pi) \circ V: V$$

$$(f, \lambda) \Psi$$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\hat{\mu}}, y^{\hat{\alpha}}), \mu, \alpha = 1, 2, \dots, \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_-(TC) \otimes V = V_-$

$(\psi, \bar{\psi}) \quad \Psi \quad S_+(TC) \otimes S_+(\theta^2 \otimes N)$

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$d=4$, $N=1$ SUSY, $\mathcal{H}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stable, hol gauge bdl (cplx st)} \\ \downarrow \\ X \text{ CY3 (Kähler, cplx st)} \end{array} \right.$

$\mathcal{M}(c) = \text{classical moduli space of } (Y, V)$

$$\int_{\mathcal{M}(c)} d^4x \int_X W(\tilde{\varphi})$$

(coupling λF)

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{D}$ genus $g=0$

$\mathcal{D} \subset C$ isolated in X , $N \cdot \text{Obl. moduli}$
 - $S_{\mathcal{D}} \subset C$ has $g \geq 0$, or moves in family??

Eg IIA on X

$$\int_{\mathcal{M}(c)} d^4x \int_X F_g(X) (W_{\text{eff}} W^{\text{eff}})$$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(X^{\mu})_{\mu=0,1,2} \quad \theta^2 \otimes N$

Fermions: $(\psi^{\mu})_{\mu=0,1,2} \quad S_-(TC) \otimes V = V$

$\psi_+(TC) \otimes S_-(N)$

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\hat{\mu}}, y^{\hat{\nu}}), \mu, \nu = 1, 2 \quad \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_-(TC) \otimes V \equiv V_-$

$(\psi, \bar{\psi}) \quad \mathbb{F} \quad S_+(TC) \otimes S_+(\theta^2 \otimes N)$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(X^{\mu}, \dots, \theta^{\mu}, \dots)$

Fermions: $(L, \bar{\psi}) \quad \lambda \quad S_{-1/2}$

$(R, \psi) \quad \bar{\lambda} \quad S_{+1/2}$

Twisting: $\mathbb{Z} \rightarrow$

$$\theta^{\mu} \quad S_{-}(0) \otimes \sigma$$

$$\theta^{\mu} \quad S_{-}(0) \otimes \sigma^{\dagger}$$

$$\bar{\lambda} \quad S_{+}(0) \otimes \bar{N}$$

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\hat{\mu}}, y^{\hat{\mu}}), \mu, \nu = 1, 2 \quad \theta^{\hat{\mu}} \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_-(\tau_C) \otimes V = V$

$$(\psi, \bar{\psi}) \quad \underline{V} \quad S_+(\tau_C) \otimes S_+(\partial^{\hat{\mu}} N)$$

Twisting: $\Psi \mapsto$

$$\theta^{\hat{\mu}} \quad S_-(\partial^{\hat{\mu}}) \otimes \bar{\theta}$$

$$\bar{\theta}_{\hat{\mu}} \quad S_-(\partial^{\hat{\mu}}) \otimes \bar{\psi}^{\hat{\mu}}$$

$$\bar{\psi}_{\hat{\mu}} \quad S_+(\partial^{\hat{\mu}}) \otimes N$$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\mu}, y^{\alpha}), \mu=0,1,2 \quad \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_{-}(\tau) \otimes V = \bar{V}$

$(\psi, \bar{\psi}) \quad \Psi \quad S_{+}(\tau) \otimes S_{+}(\sigma^2 \otimes N)$

Twisting: $\Psi \Rightarrow$

$\theta^{\alpha} \quad S_{-}(\sigma^2) \otimes \sigma$

$\theta_{\beta}^{\dagger} \quad S_{-}(\sigma^2) \otimes \sigma^{\dagger}_{\beta}$

$\bar{\chi}_{\alpha}^{\dagger} \quad S_{+}(\sigma^2) \otimes \bar{N}$

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

$(\mathbb{R}, \mathbb{Z})$

Bosons: $(x^{\hat{\mu}}, y^{\hat{\mu}}), \mu, \nu = 1, 2 \quad \theta^2 \otimes N$

Fermions: $(L, \bar{L}) \quad \lambda \quad S_-(\tau c) \otimes V = \bar{V}$

$(R, \bar{R}) \quad \bar{\Psi} \quad S_+(\tau c) \otimes S_+(\partial^2 \otimes N)$

Twisting: $\mathbb{Z} \Rightarrow$

$$\theta^{\hat{\mu}} \quad S_-(\partial^2) \otimes \bar{\theta}^{\hat{\mu}}$$

$$\theta^{\hat{\mu}} \quad S_-(\partial^2) \otimes \bar{\theta}^{\hat{\mu}}$$

$$\mathbb{Z}_2 \quad S_-(\partial^2) \otimes \bar{\theta}^{\hat{\mu}}$$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(X^{\mu}, y^{\mu}), \lambda_1, \lambda_2, \lambda_3, \theta^{\alpha} \otimes N$

Fermions: $(\psi, \bar{\psi}) \lambda \quad S_{-}(\tau) \otimes V_{-}, V_{+}$

$(\psi, \bar{\psi}) \Psi \quad S_{+}(\tau) \otimes S_{+}(\sigma \otimes N)$

Twisting: $\Psi \rightarrow$

$\theta^{\alpha} \quad S_{-}(\theta) \otimes \bar{\sigma}$

$\bar{\sigma}_i \quad S_{-}(\theta) \otimes \bar{\sigma}_i$

$\bar{\chi}_a^{\alpha} \quad S_{+}(\theta) \otimes \bar{N}$

D-modules

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

(2,2)

Bosons: $(x^{\hat{\mu}}, y^{\hat{\mu}}), \mu, \nu = 1, 2 \quad \theta^2 \otimes N$

Fermions: $(L, \bar{\lambda}) \quad \lambda \quad S_-(\tau) \otimes V = \bar{V}$

$(R, \lambda) \quad \bar{\Psi} \quad S_+(\tau) \otimes S_+(\theta^2 \otimes N)$

Twisting: $\mathbb{Z} \Rightarrow$

$$\theta^{\hat{\mu}} \quad S_-(\theta^2) \otimes \bar{\theta}^{\hat{\mu}}$$

$$\theta^{\hat{\mu}} \quad S_-(\theta^2) \otimes \bar{\theta}^{\hat{\mu}}$$

$$\bar{\theta}^{\hat{\mu}} \quad S_+(\theta^2) \otimes N$$

D-modes
2 (adding $\Rightarrow d^2\theta$)

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\mu}, y^{\mu}), \mu=1,2,3 \quad \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_-(\tau_C) \otimes V; \bar{V}$

$(\psi, \bar{\psi}) \quad \Psi \quad S_+(\tau_C) \otimes S_-(\theta^2 \otimes N)$

Twisting: $\mathbb{F} \Rightarrow$

$\theta^2 \quad S_-(\theta^2) \otimes \bar{\theta}$

$\bar{\theta}^2 \quad S_-(\theta^2) \otimes \bar{\theta}^2$

$\bar{\theta}^2 \quad S_-(\theta^2) \otimes \bar{N} \quad \text{on } P = d m_e H^2(c, N)$

D-modes

2 (additive) $\Rightarrow d^2 \theta$

Cases:

Physical Gauge: (D1-Branes)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

(2, 2)

Bosons: $(x^{\hat{\mu}}, y^{\hat{\mu}}), \text{ massless } \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_-(\tau c) \otimes V = \bar{V}$

$(\psi, \bar{\psi}) \quad \Psi \quad S_+(\tau c) \otimes S_-(\theta^2 \otimes N)$

Twisting: $\Psi \Rightarrow$

$$\theta^2 \quad S_-(\theta^2) \otimes \bar{\theta}^2$$

$$\bar{\theta}^2 \quad S_-(\theta^2) \otimes \bar{\theta}^2$$

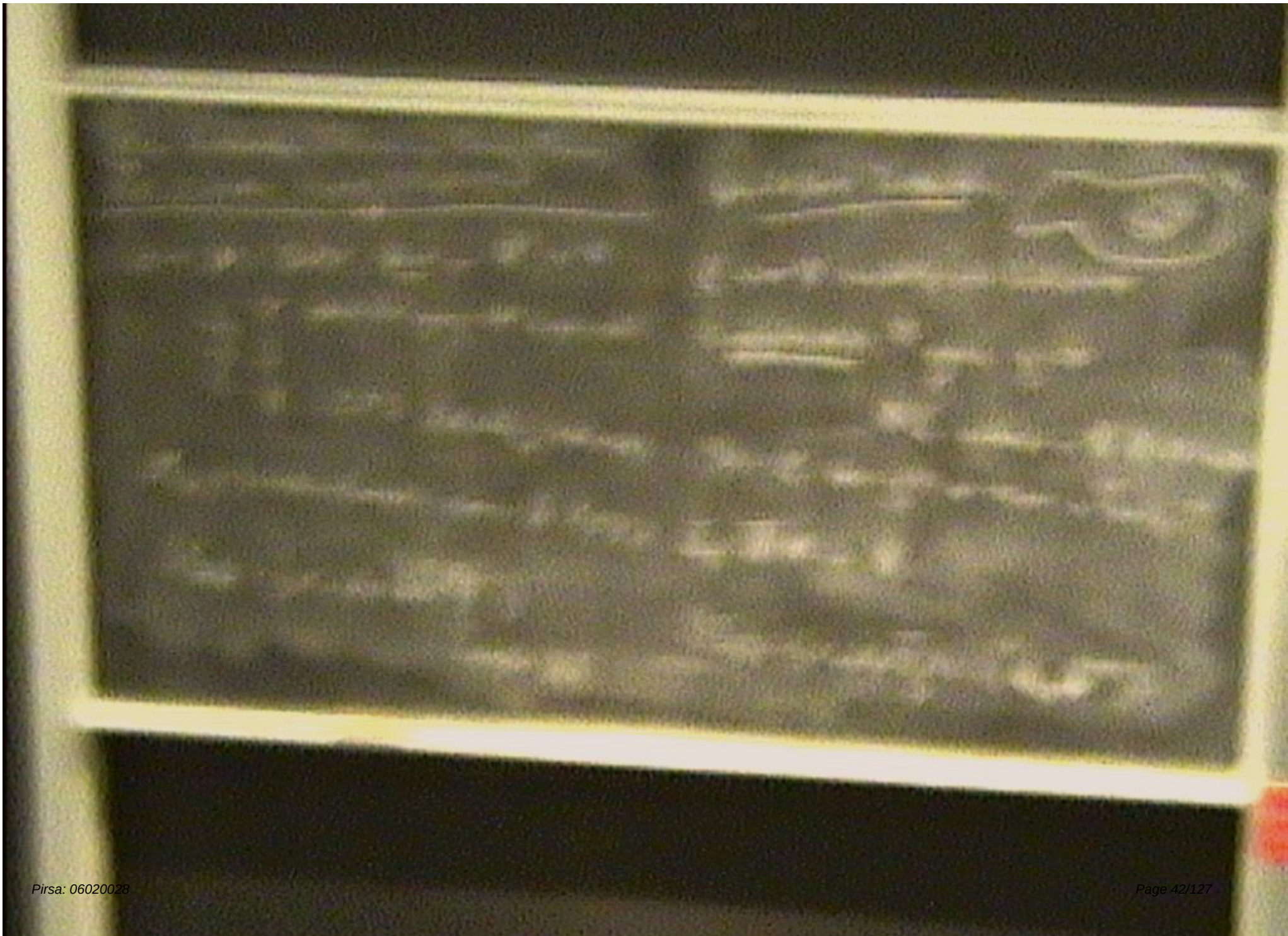
$$\bar{\theta}^2 \quad S_-(\theta^2) \otimes \bar{N} \quad 2\pi, \int d\mu_c \parallel^2(c, N)$$

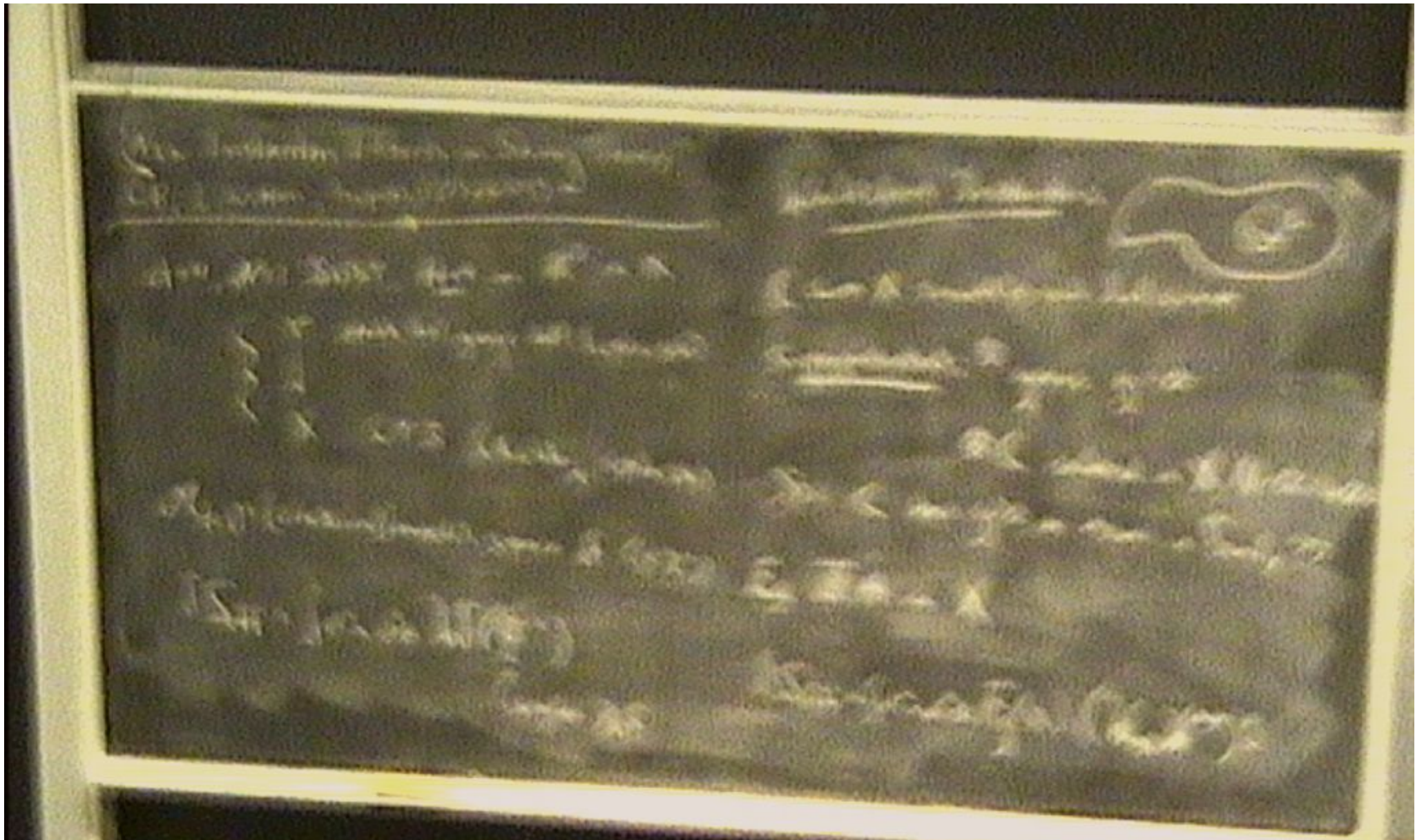
Cases: ① $g=0, p=0 \Rightarrow$ Supersymmetry

②

2-modes

2 (additive $\Rightarrow d^2\theta$)





Neu: Instanzen & Werte im Spring-Tagung
 CT, L. 11.11.11, 11.11.11

11.11.11, 11.11.11, 11.11.11, 11.11.11

Y 11.11.11, 11.11.11, 11.11.11, 11.11.11

X CT3 (11.11.11, 11.11.11)

11.11.11 (11.11.11, 11.11.11, 11.11.11)

11.11.11 (11.11.11, 11.11.11)

11.11.11

11.11.11, 11.11.11



11.11.11, 11.11.11, 11.11.11

11.11.11, 11.11.11, 11.11.11

11.11.11, 11.11.11, 11.11.11

11.11.11, 11.11.11, 11.11.11

11.11.11, 11.11.11

11.11.11, 11.11.11, 11.11.11

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$d=4$, $\mathcal{N}=1$ SUSY, $H_{\text{eff}} \sim \mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} Y \text{ stack of gauge branes (e.g. } SU(2)) \\ X \text{ CY3 (Kahler, cplx)} \end{array} \right.$

$\mathcal{M}_{\text{cl}} = \text{classical moduli space of } (Y, V)$

$\mathcal{S}_{\text{eff}} = \int_{\mathcal{M}_{\text{cl}}} W(\vec{q})$

(couple to X)

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Supersymmetric: 0 genus $g=0$

$\odot C$ isolated in X , N -orbits

$S^1 \subset C$ has $g=0$, or more in family??

E.g. $\Pi A \rightarrow X$

$\mathcal{S}_{\text{eff}} = \int_{\mathcal{M}_{\text{cl}}} F_g(X) (W_{\text{eff}} W^{-1})$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^{\mu}, y^{\mu}), \mu=1,2 \quad \theta^2 \otimes N$

Fermions: $(\psi, \bar{\psi}) \quad \lambda \quad S_{-}(\tau) \otimes V = \bar{V}$

$(\psi, \bar{\psi}) \quad \Psi \quad S_{+}(\tau) \otimes S_{+}(\partial \otimes N)$

Twisting: $\Psi \Rightarrow$

$$\theta^{\mu}$$

$$S_{-}(\partial) \otimes \bar{\theta}$$

D-modes
2 (addition $\Rightarrow d^2\theta$)

$$\theta^{\mu}$$

$$S_{-}(\partial) \otimes \bar{\theta}$$

$$2)$$

$$\bar{\theta}^{\mu}$$

$$S_{+}(\partial) \otimes \bar{N}$$

21, $p = \dim_{\mathbb{C}} H^1(C, N)$

Case: ① $g=0, p=0 \Rightarrow$ Supersymmetry

② $g=1, p=0 \Rightarrow (W, \omega)$

Physical Gauge: (D1-Brane)

$$C \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: (x^μ, y^ν) , where $\partial^\mu \in N$

Fermions: $(\psi, \bar{\psi}) \lambda \quad S_-(\tau) \circ V: V_-$

$$(\psi, \bar{\psi}) \quad \Psi \quad S_-(\tau) \circ S_-(\partial^2 W)$$

Twisting: $\Psi \mapsto$

$$\theta^x$$

$$S_-(\theta^x) \circ \bar{\theta}^x$$

2-indices
 $\mathbb{Z}$ (shift line $\Rightarrow d^2\theta$)

$$\theta^y$$

$$S_-(\theta^y) \circ \bar{\theta}^y$$

?)

$$\bar{\chi}_\mu^x$$

$$S_-(\theta^x) \circ \bar{N}$$

$2\eta, \int d\mu_c H^*(c, N)$

Cases: ① $g=0, p=0 \Rightarrow$ Supersymmetric

② $g=1, p=0 \Rightarrow (W_\mu W^\mu)$

③ $g=0, p \geq 1 \Rightarrow$ Multi-form F-terms

① Superpotential (ϕ, ψ, λ)

① Superpotential $(= \mathbb{C}^n, N = 0, \dots, n)$

$$SS = \int$$

① Superpotential $(\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi), M = M(\phi, \partial_\mu \phi))$

$$\mathcal{H} = \int d^3x d^0 W_c$$

① Superpotential $(\mathcal{L} = \mathcal{L}_m^2, N = O(1) \rightarrow O(n))$

$$\mathcal{S} = \int d^4x d\theta W_c$$

$$W_c = \exp\left(\frac{-A(\phi)}{2\pi\alpha'}\right)$$

① Superpotential: $(\mathbb{C}^n, \omega = \frac{i}{2} \partial \bar{\partial} \phi)$

$$SS' = \int dV \, d\theta \, W_C$$

$$W_C = \exp\left(-\frac{A(C)}{2\pi i} + i \int_C B\right)$$

① Spectral density $\rho(\omega)$ $\omega \rightarrow \omega + d\omega$

$$S = \int_{-\infty}^{\infty} W_E$$

$$W_E = \frac{1}{2} \cdot \frac{\partial \rho(\omega)}{\partial \omega}$$

① Superpotential $(-\frac{1}{2} \hbar^2, N=0, 1, 2, \dots)$

$$SS = \int d^4x d\theta W_C$$

$$W_C = \exp\left(\frac{-A(\phi)}{2\pi i} + \frac{1}{\epsilon} \int B\right) \cdot \frac{\mathcal{H}_N(\phi(\tilde{\sigma}_N))}{(\det \tilde{\sigma})^2}$$

① Superpotential $(-\epsilon \bar{m}^2, N=0, 1, 2, 4)$

$$SS = \int d^4x d\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{A(\phi)}{2\pi\alpha'}}_{\text{Kähler}} + i \underbrace{\int B}_{\text{up to 2d}}\right) \cdot \frac{\tilde{H}_A(\phi(\bar{\partial}_0))}{(\det \bar{\partial}_0)^2 (\det \bar{\partial}_{-1})}$$

John Jost, "Lectures on String Theory"
 1985, 2nd ed. (1985) / 0712.0019

John Jost, "Supergravity", $H_{\text{dR}}^2 = \mathbb{R}^2 = \mathbb{R}$

$\left\{ \begin{array}{l} \mathbb{R} \text{ gauge field group (11d)} \\ 1 \end{array} \right.$

$\left\{ \begin{array}{l} X \text{ CY3 (K3, 6 planes)} \end{array} \right.$

M_{dR}^2 (classical moduli space of (1,1))

$\text{Set} = \text{zero of } W(\varphi)$

large λ

Worldsheet Instantons



[Can X smoothly and holomorphically

Superpotential: $\mathbb{D}$ gives $g=0$

$\mathbb{D} \subset \text{moduli} = X, N$ -dim

Sp $\subset$ has $g=0$, = moduli space?

$E_8, II, A = X$

$\text{Set} = \text{zero of } F_2(x) (N, N-1)$

Physical Gauge: (D1-Brane)

$$\mathbb{C} \hookrightarrow \mathbb{C}^2 \times X$$

Bosons: $(x^\mu, y^a), \mu=0,1,2 \quad \theta^2 \otimes N$

Fermions: $(L, \bar{\psi}) \quad \lambda \quad S_-(\tau c) \otimes V = \bar{V}$

$(P, \psi) \quad \Psi \quad S_+(\tau c) \otimes S_-(\theta^2 \otimes N)$

Twisting: $\Phi \Rightarrow$

θ^μ

$S_-(\theta^2) \otimes \bar{\psi}$

0-modes

2 (additive $\Rightarrow d^2\theta$)

θ_3

$S_-(\theta^2) \otimes \bar{\psi}_c^i$

2)

$\bar{\chi}_2^{\bar{N}}$

$S_+(\theta^2) \otimes \bar{N}$

2P, $P = \dim_{\mathbb{C}} H^2(\mathbb{C}, N)$

Cases: ① $g=0, p=0 \Rightarrow$ Supersymmetric

② $g \geq 1, p=0 \Rightarrow (W, W^*)$

③ $g=0, p \geq 1 \Rightarrow$ "Multi-fermion F-terms"

moder
line $\Rightarrow d\theta$

dim $H^1(C, W)$

trivial

 $W \rightarrow 0$

even F-terms

① Superpotential $(-\dim^2, N \cdot \dim + \dim)$

$$SS = \int d^4x \int d\theta \mathcal{W}_C$$

$$\mathcal{W}_C = \exp\left(\underbrace{-\frac{ACE}{2\pi\alpha'}}_{\text{F-terms}} + \underbrace{i\beta}_{\text{F-terms}}\right) \cdot \underbrace{\frac{\text{Pfaff}(\tilde{\gamma}_2)}{(\det \tilde{\gamma}_0)^2 (\det \tilde{\gamma}_{n-1})^2}}_{\text{up to 16}}$$

② Higher Genus F-terms g^2, ℓ^2

$$\begin{array}{ccc} \text{III} & \theta_i^2 & \theta_i^2 \\ & \theta_i^2 & \theta_i^2 \\ & \theta_i^2 & \theta_i^2 \\ & \theta_i^2 & \theta_i^2 \end{array}$$



moduli

time $\Rightarrow d^4\theta$

$\dim_{\mathbb{C}} H^1(C, N)$

trivial

$W^{\pm} \rangle$

cases F-terms

① Superpotential $(-\frac{1}{4}m^2, \text{dimension})$

$$SS = \int d^4\theta \int d^3x W_C$$

$$W_C = \exp\left(\underbrace{-\frac{A(\phi)}{2\pi i}}_{\text{Feynman}}, \underbrace{\int d^3x}_{\text{space}}\right) \cdot \frac{\text{Pfaff}(\tilde{\Delta}_C)}{(\det \tilde{\Delta}_3)^2 (\det \tilde{\Delta}_{-1})^2}$$

② Higher genus F-terms g^2, τ^2

$$\text{II} \quad \left. \begin{array}{cc} \theta_i^+ & \theta_i^- \\ \theta_i^+ & \theta_i^- \\ \chi_i^+ & \chi_i^- \end{array} \right\} \begin{array}{c} 2 \\ 2 \\ 0 \end{array}$$

moder
line $\Rightarrow d^4\theta$

$\dim_{\mathbb{C}} W(c, w)$

trivial

$W^{\sim})$

even F-terms

① Superpotential $(-d^4\theta, \text{Majorana})$

$$SS = \int d^4x d^4\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{d^2 c}{2d^2 x}}_{\text{Feynman}} \cdot \underbrace{\left(\frac{d^2}{d^2 x}\right)}_{\text{operator}}\right) \cdot \frac{\tilde{H}_C(\tilde{\Sigma}_C)}{(\det \tilde{\Sigma}_C)^2 (\det \tilde{\Sigma}_C)^2} SS_{\Sigma_C} \int d^4x d^4\theta$$

② Higher terms F-terms $g^{\alpha\beta} \theta^{\alpha} \theta^{\beta}$

$$\begin{matrix} \theta^{\alpha} & \theta^{\beta} & \theta^{\gamma} & \theta^{\delta} \\ \theta^{\alpha} & \theta^{\beta} & \theta^{\gamma} & \theta^{\delta} \\ \theta^{\alpha} & \theta^{\beta} & \theta^{\gamma} & \theta^{\delta} \end{matrix} \left\{ \begin{matrix} \theta^{\alpha} \theta^{\beta} \theta^{\gamma} \theta^{\delta} \\ \theta^{\alpha} \theta^{\beta} \theta^{\gamma} \theta^{\delta} \end{matrix} \right.$$

under
 then $\Rightarrow d^2\theta$
 dim $H^2(C, \mathbb{R})$
 central
 $W^{\pm} \rightarrow$
 some F-terms

① Superpotential $(\phi, \psi, \lambda, \mu, \nu, \omega)$

$$SS = \int d^4x d^4\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{A(\phi)}{24\pi^2}}_{\text{Kahler}} + \underbrace{\int \tilde{\phi}}_{\text{super}}\right) \cdot \frac{\pi \text{SEC}(\tilde{\phi}_0)}{(\det \tilde{\phi}_0)^3 (\det \tilde{\phi}_0)^3} SS_{\text{gauge}} \int d^4x d^4\theta$$

② Higher genus F-terms g^2, g^4
 III $\theta_i^2 = \theta_i^2$
 $\theta_i^2 \gamma \theta_i^2 \gamma$
 $\gamma^2 0 \gamma^2 0$



moduli
line $\Rightarrow d\theta$

$\dim_{\mathbb{C}} H^1(C, \mathcal{N})$

central

$W^{\pm} \rangle$

curvature tensors

① Superpotential $(\phi, \psi, \lambda, \mu, \nu)$

$$SS = \int d^4x d\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{\lambda(\phi)}{g}}_{\text{Feynman}} + \underbrace{\frac{1}{c}}_{\text{coupling}}\right) \cdot \underbrace{\frac{\tilde{F}_A(\phi(\tilde{\lambda}_A))}{(\det \tilde{\lambda}_B)^3 (\det \tilde{\lambda}_C)^3}}_{\text{spin 16}} \delta S_{2A} \int d\theta_1 d\theta_2 d\theta_3 \left(\frac{1}{2} W_{\alpha\beta\gamma} \tilde{\lambda}^{\alpha\beta\gamma} \right)$$

② Higher genus functions g^2, τ^2

$$\begin{matrix} \text{II} & \theta_1^2 & \theta_2^2 & \theta_3^2 & \theta_4^2 \\ & \theta_1^2 & \theta_2^2 & \theta_3^2 & \theta_4^2 \\ & \theta_1^2 & \theta_2^2 & \theta_3^2 & \theta_4^2 \\ & \theta_1^2 & \theta_2^2 & \theta_3^2 & \theta_4^2 \end{matrix} \left\{ \begin{matrix} \theta_1^2 \theta_2^2 \theta_3^2 \theta_4^2 \\ \theta_1^2 \theta_2^2 \theta_3^2 \theta_4^2 \\ \theta_1^2 \theta_2^2 \theta_3^2 \theta_4^2 \\ \theta_1^2 \theta_2^2 \theta_3^2 \theta_4^2 \end{matrix} \right\}$$



moduli
 $\text{Area} \Rightarrow d^4\theta$
 $\dim_{\mathbb{C}} W(\mathbb{C}, W)$
 local
 $W^{\pm} \otimes$
 inner F-terms

① Superpotential $(-\infty^4, N=1, D=4)$

$$SS = \int d^4x d^4\theta W_c$$

$$W_c = \exp\left(\underbrace{-\frac{A(\phi)}{2\pi\alpha'}}_{\text{F-term}} \cdot \underbrace{\beta}_{\text{F-term}}\right) = \frac{\mathcal{H}_c(\tilde{\phi}_c)}{(\det \tilde{\gamma}_3)^3 (\det \tilde{\gamma}_4)^3}$$

② Higher terms $\int d^4x d^4\theta$

$$\text{II} \quad \left. \begin{matrix} \theta_i^2 & \theta_i^2 & \theta_i^2 & \theta_i^2 \\ \theta_i^2 & \theta_i^2 & \theta_i^2 & \theta_i^2 \\ \theta_i^2 & \theta_i^2 & \theta_i^2 & \theta_i^2 \end{matrix} \right\} \theta_i^2$$

$$SS_{\text{H.T.}} = \int d^4x d^4\theta \left(\sum_i \frac{1}{2} \tilde{\gamma}_i^{\mu\nu} \tilde{\gamma}_i^{\mu\nu} \right) \exp(-S)$$



C.B. L.W. memo, WCP-40/25-14-1971-4

S. 10. 10. 10. 10. 10. 10. 10. 10. 10. 10.

X CTB (C. trachomatis)

M. j. (C. 2000) (mod. 5000) 5000

Sur. (no. 15)



Alfred Russel Wallace

(1) A number of the

Summary 1-10

⑤ 1944 - XN-100

511-X

San-San-Fan (1627)

① Superpotential: $(-d\mathcal{M}^2, N=0, 1, 2, 0, 1, 2)$

$$\mathcal{S} = \int d^4x d^4\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{A(C)}{2\pi i}}_{\text{Kahler}} + \underbrace{\int B}_C\right) \cdot \underbrace{\frac{\mathcal{P}_{\text{Higgs}}(\tilde{\mathcal{D}}_V)}{(\det \tilde{\mathcal{D}}_D)^2 (\det \tilde{\mathcal{D}}_{\text{aux}})^4}}_{\text{up to st}}$$

② Higher Genus f. terms g.c. p. 10

$$\text{III} \quad \left. \begin{array}{cc} \theta_L^* & \theta_R^* \\ \theta_L^* & \theta_R^* \\ \bar{\chi}_L^* & \bar{\chi}_R^* \end{array} \right\} \begin{array}{l} 2 \\ 2 \\ 0 \end{array} \quad \left. \begin{array}{l} p^* = \theta_L^* d\tau + \theta_R^* d\bar{\tau} \\ \theta_L^* d\tau \\ \theta_R^* d\bar{\tau} \end{array} \right\}$$

$$\mathcal{S}_{\text{Higgs}} = \int d^4x d^4\theta d^4p \left(\int_C W_{\text{Higgs}} p^* p^* \right) \exp(-S_U)$$

$\mathcal{S}_{\text{Higgs}}$

① Superpotential $(C = 4\pi^2, N = 0, 1, 2, 3, 4)$

$$\delta S = \int d^4x d^4\theta W_C$$

$$W_C = \exp\left(\underbrace{\frac{-A(C)}{2\pi i}}_{\text{Kähler}} + i \int B\right) \cdot \underbrace{\frac{\text{Pf}(\tilde{\mathcal{D}}_V)}{(\det \tilde{\mathcal{D}}_0)^2 (\det \tilde{\mathcal{D}}_{\infty})^2}}_{\text{up to st}}$$

② Higher Genus F. terms g^2, p^2

$$\text{III} \quad \left. \begin{array}{cc} \theta_1^2 & \theta_2^2 \\ \theta_1^2 & \theta_2^2 \\ \bar{\chi}_1^2 & \bar{\chi}_2^2 \end{array} \right\} \begin{array}{l} p^2 = \theta_1^2 \theta_2^2 \\ \theta_1^2 \theta_2^2 \\ 0 \end{array}$$

$$\delta S_{\text{Higgs}} = \int d^4x d^4\theta d^4p \left(\int \tilde{W}_{\text{Higgs}} \tilde{p}^2 \tilde{p}^2 \right) \exp(-S_V)$$

$$\delta S_{\text{Higgs}} = \int d^4x d^4\theta d^4p \tilde{W}_{\text{Higgs}}$$

① Superpotential: $(-8\pi^2, N=0, 1, 2, 3, 4, \dots)$

$$\mathcal{S} = \int d^4x d^3\theta W_C$$

$$W_C = \exp\left(\underbrace{\frac{-A(c)}{2\pi i}}_{\text{Kähler}} + \underbrace{\int \beta}_{\text{up to 1st}}\right) = \frac{\text{Pfaff}(\tilde{\gamma}_\nu)}{(\det \tilde{\gamma}_\nu)^2 (\det \tilde{\gamma}_{\alpha\beta})^4}$$

② Higher Genus F-terms g^2, p^2

$$\text{III} \quad \left. \begin{array}{cc} \theta_i^2 & \theta_j^2 \\ \theta_i^2 & \theta_j^2 \\ \theta_i^2 & \theta_j^2 \end{array} \right\} \begin{array}{l} p^2 = \theta_i^2 d^2 \\ \theta_j^2 d^2 \end{array}$$

$$\mathcal{S}_{\text{Hef}} = \int d^4x d^3\theta d^3\theta_f \left(\int_C W_{\text{Hef}} g^2 p^2 \right) \exp(-S_1)$$

$$= \int d^4x d^3\theta d^3\theta_f \left(\text{Tr}(W_{\text{Hef}} g^2) \right)$$

① Superpotential $(\mathcal{N} = 0, 1, 2, 3, 4)$

$$\mathcal{S} = \int d^4x d\theta W_c$$

$$W_c = \exp\left(\underbrace{-\frac{A(c)}{2nd}}_{\text{Kähler}} + \underbrace{\beta}_{\text{up to 16}}\right) \cdot \frac{\mathcal{F}_A(c(\bar{\partial}_0))}{(\det \bar{\partial}_0)^2 (\det \bar{\partial}_{\infty})^2}$$

② Higher Genus F-terms $g^{\text{ol}}, p^{\text{ol}}$

$$\text{IIA} \quad \left. \begin{array}{cc} \theta_L^* & \theta_R^* \\ \theta_L^* & \theta_R^* \\ \bar{\chi}_L^* & \bar{\chi}_R^* \end{array} \right\} \begin{array}{l} p^* \theta_i^* d\tau_i \\ \theta_i^* d\tau_i \end{array}$$

$$\mathcal{S}_{\text{IIA}} = \int d^4x d\theta d^3p \left[\int_c W_{\text{IIA}} p^* p^* \right] \exp(-S_1)$$

$$\mathcal{S}_{\text{Heter}} = \int d^4x d\theta d^3\theta_i \left[\int_c \text{Tr}(W_{\text{Heter}} \theta_i^*) \right]$$

New Instanton Effects in String Theory CB, E. Witten, hep-th/0512059

$d=4$, $\mathcal{N}=1$ SUSY, $\mathcal{H}_{\text{eff}}$ on $\mathbb{R}^{3,1} \times X$

$\left\{ \begin{array}{l} \gamma \text{ stable, hol gauge hol (cplx stab)} \\ \downarrow \\ X \text{ CY3 (Kahler, cplx stab)} \end{array} \right.$

$\mathcal{M}_{\text{eff}} = (\text{classical}) \text{ moduli space of } (\gamma, V)$

$$\int \mathcal{L}_{\text{eff}} = \int d^4x \int d^3\sigma W(\tilde{\varphi}^i)$$

(single λF)

Worldsheet Instantons



$C \hookrightarrow X$ smoothly emb. hol. curve

Superpotential: $\mathcal{O}$ genus $g=0$

$\mathcal{O}C$ isolated in X , $N=0$ moduli
 $S_{\text{eff}} \subset$ has $g>0$, or moves in family??

Eg IIA on X

$$\int \mathcal{L}_{\text{eff}} = \int d^4x \int d^3\sigma F_g(X) (W_{\text{eff}} W^{\text{eff}})$$

① Superpotential $(-8\pi^2, N=0, 1, 2, 0, 1)$

$$\mathcal{S} = \int d^4x d\theta W_C$$

$$W_C = \exp\left(\underbrace{-\frac{A(\phi)}{2\pi i}}_{\text{Kähler}}, \underbrace{\int B}_C\right) \cdot \frac{\tilde{F}_A((\tilde{\nabla}_\mu))}{(\det \tilde{\nabla}_0)^2 (\det \tilde{\nabla}_m)^2}$$

up to 1/4

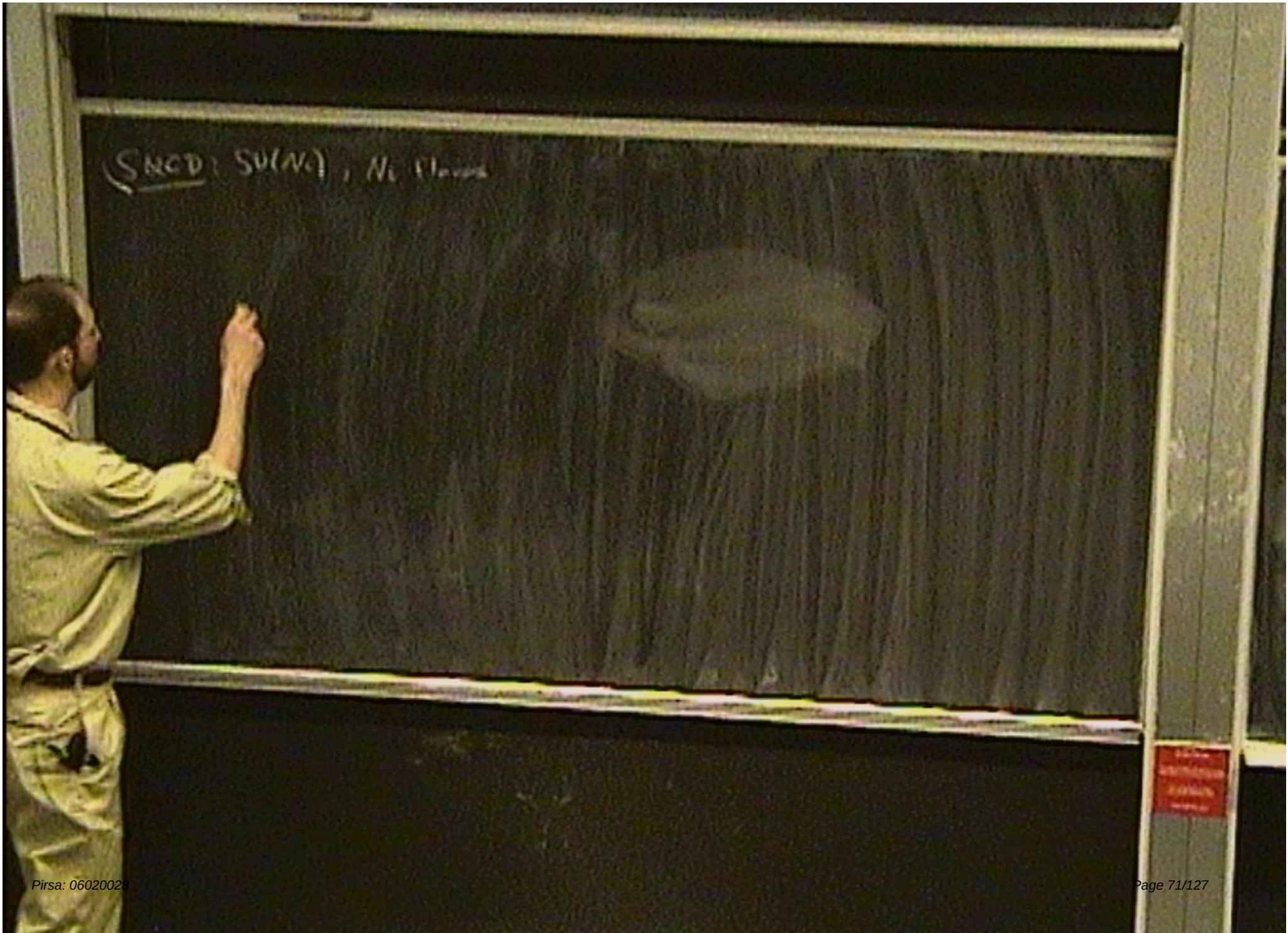
② Higher Genus F-terms $g^{2,1}, p^{1,0}$

$$\text{IIA} \quad \left. \begin{array}{cc} \theta_1^* & \theta_2^* \\ \theta_1 & \theta_2 \end{array} \right\} \begin{array}{l} p^* = \theta_1^* d\theta_1 + \theta_2^* d\theta_2 \\ \theta_1^* d\theta_1 \\ \theta_2^* d\theta_2 \end{array}$$

$$\left. \begin{array}{cc} \chi_1^* & \chi_2^* \\ \chi_1 & \chi_2 \end{array} \right\} \begin{array}{l} 0 \\ 0 \end{array}$$

$$\mathcal{S}_{\text{IIA}} = \int d^4x d\theta d\theta^* \left(\int_C W_{\text{IIA}} p^* p^* \right) \exp(-S_L)$$

$$\mathcal{S}_{\text{HII}} = \int d^4x d\theta d\theta^* \left(\int_C \text{Tr}(W_{\text{HII}} \theta_i^*) \right)$$



SNCD SV(N) , Ni Flavours

SUBD SUBD, M. (1990)

$p = N_1 - N_2 = 1$

$p = 0$

S&CD: $SU(N_c)$, N_c Flavours

$$p = N_c - N_L = 1$$

$$p = 0$$

S&CD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f + 1$$

$$p=0 \quad W_{\text{Ans}} \neq 0, \quad \mu \rightarrow$$

SACD: SULNA, Ne Flows

$p = N_1 - N_{1-1}$

Wads $\neq 0$, M_{1-1}

SACD: $SU(N)$, N_c flavors

$$p = N_c - N_f + 1$$

$$p=0 \quad W_{\text{ads}} \neq 0, \text{ ~~U(1)~~$$

$$p=1$$

S&CD: $SU(N_c)$, N_c Flavours

$$p = N_c = N_f = 1$$

$$p=0 \quad W_{\text{AdS}} \neq 0, \quad \mathcal{U}_{\text{AdS}}$$

$$p=1 \quad \text{Cplx str deformation of } \mathcal{M}_{\text{AdS}}$$

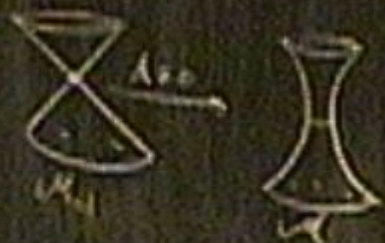


S&CD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f + 1$$

$$p=0 \quad W_{\text{AdS}} \neq 0, \mathcal{M}_{\text{AdS}}$$

$$p=1 \quad \text{Cplx str deformation of } \mathcal{M}_{\text{AdS}}$$

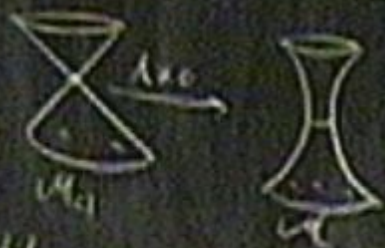


SQCD: $SU(N_c)$, N_f Flavors

$$p = N_c - N_f = 1$$

$p=0$ $W_{\text{dof}} \neq 0$, M_{11}

$p=1$ Cplx str deformation of M_{11}



$p>1$ Higher Fermions

General Pbs on Multi-Fermion F-terms

SNGD: $SU(N)$, N flavors

$$p: N_1 = N_2 = 1$$

$$p=0 \quad W_{AB} \neq 0, \quad \mathcal{M}_{11}$$

$$p=1$$

str. determination of $\mathcal{M}_{11}$



General PLS in Multi-Fermion Systems

• global Syst

• effective action: $N=1$ atom

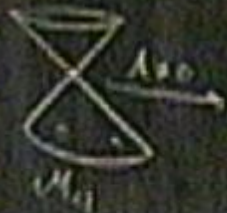
$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_d$$

SQCD: $SU(N_c)$, N_f flavors

$$p = N_c - N_f + 1$$

$p=0$ Wads $\neq D$, $\mathcal{M}_{d-1}$

$p=1$ Cplx str J $\mathcal{M}_{d-1}$



$p=1$ Hyper Red

General PLS on Multi-Fermion Systems

- global Syst
- effective action: $N=1$ atom

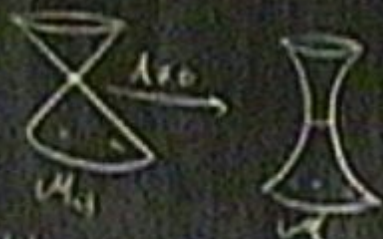
$$\Phi: \mathbb{R}^d \rightarrow \mathcal{M}_d, \mathcal{S}_i(d) \circ \mathcal{K}(\pm, \pm)$$

SQCD: $SU(N_c)$, N_f flavors

$$p = N_c - N_f + 1$$

$p = 0$ $W_{\text{ads}} \neq 0$, $\mathcal{M}_{\text{cl}}$

$p = 1$ Cplx str deformation of $\mathcal{M}_{\text{cl}}$



$p > 1$ Higher F-terms

General Phys in Multi-Fermion Systems

• global sym

• effective action: $N+1$ atoms

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_0, S: (d\Phi)^2 K(\Phi, \Phi)$$

Deformation

Deformation: (intrinsic)

$\bar{\gamma}_3 \rightarrow$

Deformation: (intrinsic)

$$\tilde{\gamma}_3 \mapsto \tilde{\gamma}_3 + \omega \dot{\gamma}_3 \partial_c, [\omega] \in H_2^c(M_1, \mathbb{Z})$$

Deformation: (intrinsic)

$$\delta_j \mapsto \delta_j + \omega_j \partial_i, [\omega] \in H^1_2(M, T^*M)$$

$$\Rightarrow d\phi' \mapsto d\phi' - \omega_j d\tau^j$$

$$\int g_{ic} d\phi' d\tau^i \mapsto \int g_{ic} (d\phi' - \omega_j d\tau^j) d\tau^i$$

Deformation: (intrinsic)

$$\bar{\partial}_j \mapsto \bar{\partial}_j + \omega_j^i \partial_i, [\omega] \in H_2^c(M, \mathbb{C})$$

$$\Rightarrow d\phi' \mapsto d\phi' - \omega_j^i d\tau^j$$

$$\Rightarrow g_{i\bar{j}} d\phi^i d\bar{\phi}^{\bar{j}} \mapsto g_{i\bar{j}} (d\phi^i - \omega_j^i d\tau^j) d\bar{\phi}^{\bar{j}}$$

(1.1)

Deformation: (intrinsic)

$$\bar{\partial}_Z \rightarrow \bar{\partial}_Z + \omega \bar{\partial}_Z, [\omega] \in H^1_Z(M, T^*M)$$

$$\Rightarrow d\phi' \rightarrow d\phi' - \omega \bar{\partial} \phi'$$

$$\Rightarrow g_{i\bar{j}} d\phi^i d\bar{\phi}^{\bar{j}} \rightarrow (d\phi^i - \omega \bar{\partial} \phi^i) d\bar{\phi}^{\bar{j}}$$

(1,1) (0,2)

$$\omega = \bar{\partial} \gamma = \frac{1}{2} (g_{i\bar{j}} \omega \bar{\partial} \phi^i + g_{i\bar{j}} \bar{\partial} \phi^i)$$

$$= \text{sym. sect. of } \bar{\partial}^{\perp}_{M_1} \otimes \bar{\partial}^{\perp}_{M_2}$$

(1,0)

Deformation: (inverse)

$$\tilde{\gamma}_3 \rightarrow \tilde{\gamma}_3 + \omega_3^c \partial_c, [\omega] \in H_2^c(M, \mathbb{R})$$

$$\Rightarrow d\phi^i \rightarrow d\phi^i + \omega_3^c dx^c$$

$$\Rightarrow g_{i,c} d\phi^i dx^c \rightarrow g_{i,c} (d\phi^i + \omega_3^c dx^c) dx^c$$

(1,1) (0,2)

$$\omega = \tilde{\gamma} = \frac{1}{2} (g_{i,c} \omega_3^c + g_{i,c} \omega_3^c)$$

$$= \text{sym} \text{ part of } \tilde{\gamma}_{i,c}^+ \otimes \tilde{\gamma}_{i,c}^+$$

(1,1)

SQCD: $SU(N_c)$, N_f flavors

$$p = N_c - N_f = 1$$

$$p=0 \quad W_{\text{ads}} \neq 0, \mathcal{H}_{\text{eff}}$$

$p=1$ Cplx str deformation of $\mathcal{M}$



$p>1$ Higher F-terms

General Phys in Multi-Fermion Systems

• global sym

• effective action $N=1$ atom

$$\Phi: \mathbb{R}^4 \rightarrow \mathcal{M}_U, S^1(\text{d}x \otimes K(\Phi, \bar{\Phi}))$$

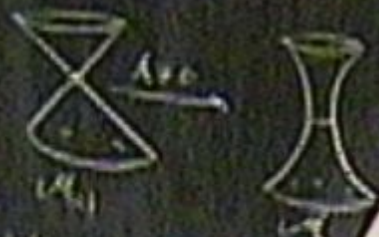
$$S = \int \text{d}x \otimes \text{d}y \otimes \bar{D}_\alpha \bar{\Phi}^c D^\alpha \Phi^c$$

SQCD: $SU(N_c)$, N_f flavors

$$p = N_c - N_f = 1$$

$p = 0$ $W_{\text{ADS}} \neq 0$, $\mathcal{M}_{\text{NS}} \rightarrow$

$p = 1$ Cplx str deformation of $\mathcal{M}_{\text{NS}}$



$p > 1$ Higher Fermions

General PHS on Multi-Fermion F-terms

• global QFT

• effective action: $N = 1$ atom

$$\mathbb{R}^n \rightarrow \mathcal{M}_U, S^1(\mathbb{R}^n, \mathcal{K}(\mathbb{E}, \mathbb{E}^*))$$

$$\int d^4x \, \text{tr} \, \bar{D}_\alpha \mathbb{E}^c D^\alpha \mathbb{E}^T$$

$$= \int d^4x \, \text{tr} \, \bar{\psi}^i \psi^j \dots$$

SNCD: $SU(N)$, N_c flavors

$$p = N_c - N_f + 1$$

$p=0$ Wads $\neq 0$, M_{11}

$p=1$ Cplx str dde M_{11}



$p>1$ Hyper

General PLS on Minkowski-Fermion F-terms

• global syst

• effective action: $N=1$ atom

$$\Phi: \mathbb{R}^n \rightarrow M_n, S^1 \text{ (diffeo } K(\mathbb{R}^n, \mathbb{R}^n))$$

$$S_1 S^1 = \int d^4x d^2\theta \omega_{ij} \bar{D}_\alpha \bar{\chi}^c D^\alpha \chi^j$$

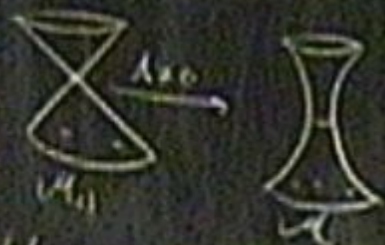
$$= \int d^4x \omega_{ij} d\bar{\chi}^i d\chi^j \dots$$

SQCD: $SU(N_c)$, N_c Flavours

$$p = N_c - N_f + 1$$

$p=0$ $W_{\text{dynam}} \neq 0, \mathcal{M}_{\text{eff}}$

$p=1$ Cplx str deformation of $\mathcal{M}_{\text{eff}}$



$p>1$ Higher Fermions

General Phys via Multi-Fermion F-theory

• global SUSY

• effective action $N=1$ terms

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_d, S^1(d\mu \otimes K(\underline{\mathbb{Z}}, \underline{\mathbb{Z}}^c))$$

$$S[S] = \int d^4x \, \omega_{\text{eff}} \, \bar{D}_\mu \underline{\mathbb{Z}}^c \, \bar{D}^\mu \underline{\mathbb{Z}}^c$$

$$= \int d^4x \, \omega_{\text{eff}} \, d\bar{\mathbb{Z}}^c \, d\mathbb{Z}^c \dots$$

Deformation: (intrinsic)

$$\gamma_j \mapsto \gamma_j + \omega_j^\dagger \partial_i, [\omega] \in H_2^1(M, \mathbb{R})$$

$$\Rightarrow d\varphi^i \mapsto d\varphi^i - \omega_j^\dagger d\varphi^j$$

$$\Rightarrow g_{i\bar{j}} d\varphi^i d\bar{\varphi}^{\bar{j}} \mapsto g_{i\bar{j}} (d\varphi^i - \omega_j^\dagger d\varphi^j) d\bar{\varphi}^{\bar{j}}$$

(1,1) (0,2)

$$\omega = \bar{\omega} = \frac{1}{2} (g_{i\bar{j}} \omega_j^\dagger + g_{j\bar{i}} \omega_i^\dagger)$$

$$= \text{sym part of } \bar{\partial}_i^\dagger \otimes \bar{\partial}_{\bar{j}}^\dagger$$

(1,1)

SACD: $SU(N)$, N_c flavors

$p = N_c - N_f + 1$

$p > 0$ $W_{\text{ADS}} \neq 0$, $\mathcal{M}_{p,1}$

$p = 1$ Cplx str deformation of $\mathcal{M}_{p,1}$



for F-strings

General Phys in Multi-Fermion Systems

• global SUSY

• effective action $N=1$ atom

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_n, S^1(d\tau \otimes K(\Phi, \bar{\Phi}))$$

$$S, S' = \int d^4x d\tau \omega_{ij} \bar{D}_\alpha \bar{\Phi}^i D^\alpha \Phi^j$$

$$= \int d^4x \omega_{ij} d\tau^i d\tau^j \dots$$

S&CD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f + 1$$

$p=0$ $W_{\text{ads}} \neq 0, \mathcal{M}_{\text{eff}}$

$p=1$ Cplx str dtd $\mathcal{M}_{\text{eff}}$



$p=1$ Higgs

General PLS on Auto-Fermion Fixations

• global SUSY

• effective action: $N=1$ atoms

$$\Phi: \mathbb{R}^4 \rightarrow \mathcal{M}_d, S: (d^2 \Phi) \in K(\pm, \pm)$$

$$S[S] = \int d^4x d\theta d\bar{\theta} \mathcal{L}(\Phi, \bar{\Phi}, D_\mu \Phi, \bar{D}_\mu \bar{\Phi})$$

$$= \int d^4x \mathcal{L}(\Phi, \bar{\Phi}, d\Phi^i, d\bar{\Phi}^{\bar{i}}) \dots$$

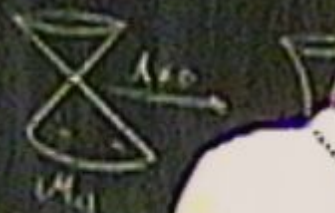
Locally $= \int d^4x d\theta d\bar{\theta} S[\Phi]$

SQCD: $SU(N_c)$, N_c Flavors

$$p = N_c - N_f + 1$$

$$p = 0 \quad W_{\text{ads}} \neq 0, \mathcal{M}_{1,1}$$

$p = 1$ Cplx str adjoint



$p > 1$ Higher

General Phys on Multi-Fermion F-theory

- global syst
- effective action: $N=1$ atom

$$\Phi: \mathbb{R}^4 \rightarrow \mathcal{M}_d, S^1(d^3 d^0 K(\Phi, \bar{\Phi}^c))$$

$$S[S] = \int d^4 x d^0 \omega_{ij} \bar{D}_0 \bar{\Phi}^c D^0 \Phi^i$$

$$= \int d^4 x \omega_{ij} d\Phi^i d\bar{\Phi}^j \dots$$

$$\text{Locally} = \int d^4 x d^0 S[K] = \int d^4 x d^0 (D_0 \Phi^i S[K]) (D_0 \bar{\Phi}^j D_0 \bar{\Phi}^j)$$

SACD: $SU(N)$, N_c flavors

$p: N_c - N_c + 1$

$p=0$ $W_{adj} \neq 0, \mathcal{M}_{11}$

$p=1$ Cplx str deformation of $\mathcal{M}_{11}$



$p=1$ Hyper-Fuchsian

General Phys in Matrix Form in 1-dim

• global SUSY

• effective action $N=1$ atom

$$\Phi: \mathbb{R}^4 \rightarrow \mathcal{M}_d, S_1^i(d^2x) \circ K(\Phi, \Xi^c)$$

$$S_1^i = \int d^2x d^2y \bar{D}_\alpha \Xi^c \bar{D}^\alpha \Xi^c$$

$$= \int d^2x d^2y d\bar{z}^c d\bar{z}^c \dots$$

$$\text{Locally} = \int d^2x d^2y SK = \int d^2x d^2y (D_\alpha SK) (D^\alpha SK) (D_\alpha \Xi^c D^\alpha \Xi^c)$$

$$\text{Globally} = \dots$$

Deformation: (intrinsic)

$$\bar{\partial}_j \rightarrow \bar{\partial}_j + \omega_j^i \partial_i, [\omega] \in H_2^c(M, \mathbb{C})$$

$$\Rightarrow d\varphi^i \rightarrow d\varphi^i - \omega_j^i d\varphi^j$$

$$\Rightarrow g_{i\bar{j}} d\varphi^i d\bar{\varphi}^j \rightarrow g_{i\bar{j}} d\varphi^i d\bar{\varphi}^j$$

(1,1)

$$\omega = \bar{\omega} = \frac{1}{2}(g_{i\bar{j}} \omega_j^i)$$

= sym sect. of $(T^* \otimes T)$

Generalization: $p \geq 0$

$$\omega_{i_1 \dots i_p j_1 \dots j_p} = \text{sym sect. of } \bigotimes_{k=1}^p T^* \otimes \bigotimes_{l=1}^p T$$

$$SS = \int_M d^d x \omega_{i_1 j_1 \dots i_p j_p} (\partial_{x^1} \varphi^{i_1} \partial_{x^2} \varphi^{j_1} \dots \partial_{x^p} \varphi^{i_p} \partial_{x^p} \varphi^{j_p})$$

Deformation: (intrinsic)

$$\gamma_j \rightarrow \gamma_j + \omega_j^i \partial_i, [\omega] \in H^2(M, T^*M)$$

$$\Rightarrow d\phi^i \rightarrow d\phi^i - \omega_j^i d\phi^j$$

$$\Rightarrow g_{ic} d\phi^i d\phi^c \rightarrow g_{ic} (d\phi^i - \omega_j^i d\phi^j) d\phi^c$$

(1,1) (1,1)

$$\omega = \bar{\omega} = \frac{1}{2} (g_{ic} \omega_j^i + g_{ij} \omega_c^i)$$

= sym sect of $\bar{\omega}$

Generalization: $p \geq 0$

$$\omega_{i_1 \dots i_p j_1 \dots j_p} = \text{sym sect. of } \sum_{u,v} T_{u,v}^i \bar{T}_{u,v}^j$$

$$\mathcal{SS} = \{ \mu, d\phi^i, \omega_{i_1 \dots i_p j_1 \dots j_p} (T_{u,v}^i \bar{T}_{u,v}^j), \dots \}$$

(0,0) (p,p)

$$\text{curvature} = \omega_{i_1 \dots i_p j_1 \dots j_p}$$

Deformation: (intrinsic)

$$\bar{\partial}_\gamma \rightarrow \bar{\partial}_\gamma + \omega_\gamma^\dagger \partial_\gamma, \quad (\omega_\gamma) \in H^1_2(M, T^*M)$$

$$\Rightarrow d\phi' \rightarrow d\phi' - \omega_\gamma^\dagger d\phi'$$

$$\Rightarrow g_{i\bar{j}} d\phi^i d\bar{\phi}^{\bar{j}} \rightarrow g_{i\bar{j}} (d\phi^i - \omega_\gamma^\dagger d\phi^i) d\bar{\phi}^{\bar{j}} \quad \begin{matrix} (1,1) \\ (0,2) \end{matrix}$$

$$\omega_\gamma = \frac{1}{2} (g_{i\bar{j}} \omega_\gamma^\dagger + g_{j\bar{i}} \omega_\gamma)$$

$$= \text{sym sect of } \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma$$

Generalization: $p \geq 0$

$$\omega_\gamma, \dots, \omega_{p+1} \in \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma = \text{sym sect of } \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma$$

$$CS = \int_M d\phi \omega_\gamma, \dots, \omega_{p+1} \in \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma$$

$$\text{Reality: } \omega_\gamma, \dots, \omega_{p+1} \in \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma \text{ sect } \bar{\partial}_\gamma^\dagger \otimes \bar{\partial}_\gamma$$

Deformation: (intrinsic)

$$\bar{\partial}_\gamma \rightarrow \bar{\partial}_\gamma + \omega \bar{\partial}_\gamma, [\omega] \in H^1_2(M, T^*M)$$

$$\Rightarrow d\bar{\partial}^i \rightarrow d\bar{\partial}^i - \omega \bar{\partial}^i$$

$$\Rightarrow g_{i\bar{j}} d\bar{\partial}^i d\bar{\partial}^{\bar{j}} \xrightarrow{(1,1)} g_{i\bar{j}} (d\bar{\partial}^i - \omega \bar{\partial}^i) d\bar{\partial}^{\bar{j}} \xrightarrow{(0,2)}$$

$$\begin{aligned} \omega = \bar{\partial} &= \frac{1}{2} (g_{i\bar{j}} \omega \bar{\partial}^i + g_{j\bar{i}} \omega \bar{\partial}^j) \\ &= \text{sym part of } \bar{\partial}^i_{\bar{\partial}^j} \otimes \bar{\partial}^{\bar{j}}_{\bar{\partial}^i} \\ &\quad (comp) \end{aligned}$$

Generalization: $p \neq 0$

$$\omega_0, \dots, \omega_{p-1} \bar{\partial}^i = \text{sym part of } \bar{\partial}^i_{\bar{\partial}^j} \otimes \bar{\partial}^{\bar{j}}_{\bar{\partial}^i}$$

$$SS = \{ \mu, d\bar{\partial}^i \omega_0, \dots, \omega_{p-1} \bar{\partial}^i \} \dots$$

Chirality: $\omega_i, \bar{\partial}^i \bar{\partial}^j \bar{\partial}^{\bar{k}} \dots$ and $\bar{\partial}^i_{\bar{\partial}^j} \otimes \bar{\partial}^{\bar{j}}_{\bar{\partial}^i}$
 $\bar{\partial} \omega = 0$

Calculus:

Deformation: (intrinsic)

$$\delta g \rightarrow \delta g + \omega_j^i \partial_i, [\omega] \in H^1_{\mathbb{R}}(M, T^*M)$$

$$\Rightarrow d\phi^i \rightarrow d\phi^i - \omega_j^i dx^j$$

$$\Rightarrow g_{ic} d\phi^i d\phi^c \rightarrow g_{ic} (d\phi^i - \omega_j^i dx^j) d\phi^c$$

(1,1) (0,2)

$$\omega = \bar{\omega} = \frac{1}{2} (g_{ic} \omega_j^i + g_{ij} \bar{\omega}_c^i) dx^j dx^c$$

$$= \text{sym sect. of } \bar{\omega}_{\mu\nu}^T$$

(1,1)

Generalization: p2 0

$$\omega_{\mu_1 \dots \mu_p} = \text{sym sect. of } \bar{\omega}_{\mu_1 \dots \mu_p}^T$$

$$\delta S = \int d^4x \sqrt{-g} \omega_{\mu_1 \dots \mu_p} \bar{\omega}^{\mu_1 \dots \mu_p} = \int d^4x \sqrt{-g} \omega_{\mu_1 \dots \mu_p} \bar{\omega}^{\mu_1 \dots \mu_p}$$

a.

$$\text{curvature} = \omega_{\mu\nu}^i \partial_\rho - \omega_{\nu\rho}^i \partial_\mu = 0$$

$$\text{curvature: } \partial_\mu \omega_\nu - \partial_\nu \omega_\mu = [\bar{\omega}_\mu, \bar{\omega}_\nu]$$

Deformation: (infinitesimal)

$$\bar{\partial}_Z \rightarrow \bar{\partial}_Z + \omega_Z^i \partial_i, [\omega] \in H_Z^1(M_1, T_{M_1})$$

$$\Rightarrow d\phi^i \rightarrow d\phi^i - \omega_Z^i \partial \bar{\partial} \bar{Z}$$

$$\rightarrow \text{give } d\phi^i \text{ give } (d\phi^i - \omega_Z^i \partial \bar{\partial} \bar{Z}) \text{ on } (0,2)$$

$$\text{ext of } \bar{\partial}_Z^i \otimes \bar{\partial}_Z^i$$

Generalization: $p \geq 0$

$$\omega_{Z, \dots, p} \bar{\partial}_Z^{p+1} = \text{sym sect. of } \bar{\partial}_Z^T \otimes \bar{\partial}_Z^p$$

$$\mathcal{SS} = \int \mu \wedge d\bar{\omega}_{Z, \dots, p} \bar{\partial}_Z^{p+1} \frac{(\bar{\partial}_Z^i \bar{\partial}_Z^j \bar{\partial}_Z^k) \dots}{(\bar{\partial}_Z^i \bar{\partial}_Z^j \bar{\partial}_Z^k)}$$

Chirality: $\omega_{Z, \dots, p} \bar{\partial}_Z^{p+1}$ just $\bar{\partial}_Z^T \otimes \bar{\partial}_Z^p$
 $\bar{\partial}_Z = 0$

Cohomology: $\mathcal{O}_Z \cup \mathcal{O}_Z = \{\bar{\partial}_Z, \dots\}$

Deformation: (intrinsic)

$$\bar{\partial}_j \rightarrow \bar{\partial}_j + \omega_j^i \partial_i, [\omega] \in H_2^1(M, T^*M)$$

$$\Rightarrow d\varphi^i \rightarrow d\varphi^i - \omega_j^i d\bar{\varphi}^j$$

$$\xrightarrow{(1,1)} g_{i\bar{j}} d\varphi^i d\bar{\varphi}^j \xrightarrow{(0,2)} g_{i\bar{j}} (d\varphi^i - \omega_j^i d\bar{\varphi}^j) d\bar{\varphi}^j$$

$$\frac{1}{2} (g_{i\bar{j}} \omega_j^i + g_{j\bar{i}} \bar{\omega}_i^j)$$

$$\text{you need } d\bar{\partial}_i^T \otimes \bar{\partial}_i^T$$

Generalization: $p \geq 0$

$$\omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_p} = \text{sym. sect. of } \bar{\partial}_{i_1}^T \otimes \dots \otimes \bar{\partial}_{i_p}^T$$

$$SS = \int_M d^4x \underbrace{\omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_p} (D_{i_1}^T \bar{\partial}_{\bar{j}_1}^T) \dots (D_{i_p}^T \bar{\partial}_{\bar{j}_p}^T)}_{d_p}$$

Chirality: $\omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_p} \text{ sect } \bar{\partial}_{i_1}^T \otimes \dots \otimes \bar{\partial}_{i_p}^T$
 $\bar{\partial} \omega = 0$

Calculus: $\partial \omega = \partial \omega - [\bar{\partial} \omega, -]$

SQCD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f = 1$$

$$p = 0 \quad W_{\text{ads}} \neq 0, \mathcal{M}_{\text{cl}}$$

$p = 1$ Cplx str deformation of $\mathcal{M}_{\text{cl}}$



$p > 1$ Higher F-terms

General Physics in Multi-Fermion F-theory

• global SUSY

• effective action: $N = 1$ terms

$$\Phi: \mathbb{R}^4 \rightarrow \mathcal{M}_4, S = \int d^4x \sqrt{-g} K(\Phi, \bar{\Phi})$$

$$S, \bar{S} = \int d^4x \sqrt{-g} \bar{D}_\mu \bar{\Phi}^\dagger D^\mu \Phi$$

$$= \int d^4x \sqrt{-g} \bar{\Phi}^\dagger D^\mu \Phi D_\mu \Phi \dots$$

$$\text{Locally} = \int d^4x \sqrt{-g} S K = \int d^4x \sqrt{-g} (D_\mu \Phi)^\dagger (D^\mu \Phi) S K$$

$$\text{Globally} \checkmark \quad \dots$$

3) Multi-Fermion F-terms: $C =$

3) Multi-Fermion F-terms: $C = \mathcal{O}^N$, per

$$\theta^2 \quad 2$$

$$\theta_3^2 \quad 0$$

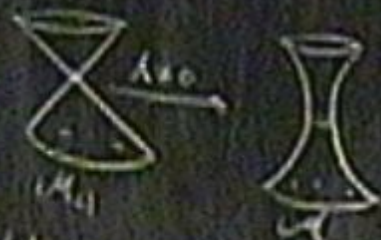
$$\gamma_{\alpha}^{\beta} \quad 2p$$

SNCD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f + 1$$

$$p = 0 \quad W_{\text{ads}} \neq 0, \mathcal{M}_1$$

$$p = 1 \quad \text{Cplx str deformation of } \mathcal{M}_1$$



$$p > 1 \quad \text{Higher Fermions}$$

General Physics on Multi-Fermion Fermions

• global SUSY

• effective action $N = 1$ atom

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_1, S = \int d^4x d\theta K(\Phi, \bar{\Phi})$$

$$S, \bar{S} = \int d^4x d\theta \omega_{ij} \bar{D}_\alpha \bar{\Phi}^i D^\alpha \Phi^j$$

$$= \int d^4x d\theta \omega_{ij} d\bar{\theta}^i d\theta^j \dots$$

$$\text{Locally} = \int d^4x d\theta SK = \int d^4x d\theta (D_\alpha D^\alpha SK) (D_\beta \bar{\Phi}^i D^\beta \bar{\Phi}^j)$$

3) Multi-Fermion F-fermion: C, CP, P, T

$$\theta^{\pm} \quad 2$$

$$\theta_0^{\pm} \quad 0$$

$$\chi_{\pm}^{\pm} \quad 2p$$

SA

moduli

3) Multi-Fermion F-terms: $C = \mathbb{CP}^1$, $p=1$
 $\delta A = H^1_S(X, \text{End } V)$

$$\theta^+ \quad 2$$

$$\theta^- \quad 0$$

$$\chi^\pm \quad 2p$$

moduli

3) Multi-Fermion F-terms: $C-CP^1$, p.21

$$SA = H^1_S(X, F_{ind} V) \text{ (b)}_1$$

$$\text{moduli: } ST = H^1_S(X, \Omega^1_X)$$

$$\Theta^2 = 2$$

$$\Theta^3 = 0$$

$$\chi^2 = 2p$$

3) Multifermion F-term: $C-CP^1$, p. 1

$$\Theta^4 \quad 2$$

$$\Theta^2_2 \quad 0$$

$$\chi^{\pi}_2 \quad 2p$$

$$\delta A \in H^3_2(X, \text{End } V) \text{ w/}$$

moduli: $\delta T \in H^3_2(X, \Omega^2_x) \text{ Kähler}$

$$\delta U \in H^2_2(X, TX) \text{ cpl. sh.}$$

3) Multi-Fermion F-term: $C = \mathbb{CP}^1$, pol

$$\theta^2 = 2$$

$$\theta_0^2 = 0$$

$$\chi_n^2 = 2p$$

$$I_{int} = \int d^2z \bar{\partial} \chi_n^2$$

$$\text{moduli: } \begin{cases} \delta A = H_5^4(X, \text{End } V) \text{ wll} \\ \delta T = H_5^2(X, \Omega_{X, \mathbb{C}}^2) \text{ Kähler} \\ \delta U = \dots TX) \text{ gauge} \end{cases}$$

3) Multi-Fermion Fermion: $C = \mathbb{CP}^1$, $p=1$

$$\theta^+ \quad 2$$

$$\theta^- \quad 0$$

$$\chi^\pm \quad 2p$$

$$\text{moduli: } \begin{cases} SA = H^2_2(X, \text{End } V)_{\text{local}} \\ ST = H^2_2(X, \Omega^2_{X/C})_{\text{Kubert}} \\ SU = H^2_2(X, \mathbb{C})_{\text{cpt. sc.}} \end{cases}$$

$$I_{\text{int}} = \int_C d^2z \bar{\partial}_{\bar{z}} \left(\bar{\chi}^\pm \bar{D}^\mu \bar{\chi}^\pm \right)$$

3) Multi-Fermion F-term: $C = \mathbb{C}P^1$, $p=1$

$$\Theta^x \quad 2$$

$$\Theta^y \quad 0$$

$$\bar{\chi}_x \quad 2p$$

$$\text{moduli: } \begin{cases} \delta A \cdot H_5^1(X, \text{End}(V)) \text{ Lie} \\ \delta T \cdot H_5^2(X, \Omega_X^2) \text{ Klv} \\ \delta U \cdot H_5^3(X, T) \text{ su} \end{cases}$$

$$I_{\text{int}} = \int_C d^2z \bar{\mathcal{D}}_{\bar{z}} \left(\bar{\chi}_x \bar{\mathcal{D}}^{\bar{z}} \bar{\chi}^x \right) / \text{Tr}$$

3) Multifermion F-term: $C = \mathbb{CP}^1$, $p \geq 1$

$$\Theta^x = 2$$

$$\Theta^y = 0$$

$$\chi^x = 2p$$

$$\text{moduli: } \begin{cases} SA = H^2_5(X, F_{\text{nd}} V) \text{ ball} \\ ST = H^3_5(X, \Omega^2_{\mathbb{C}}) \text{ K3 surface} \\ SU = H^1_5(X, TX) \text{ elliptic curve} \end{cases}$$

$$I_{\text{int}} = \int_C d^4x \sqrt{g} \left(\chi^x \bar{D}^\mu \bar{\chi}^\mu \right) / \text{Tr}(SA^2, j^2)$$

3) Multi-Fermion F-term: $C = \text{eff. pot.}$

$$\Theta^c \quad 2$$

$$\Theta^c_3 \quad 0$$

$$\chi^c_a \quad 2p$$

$$\text{moduli: } \begin{cases} SA = H^1_2(X, F_{\text{dV}}) \text{ Lefschetz} \\ ST = H^1_2(X, \Omega^c_X) \text{ Kähler} \\ SU = H^1_2(X, TX) \text{ complex} \end{cases}$$

$$I_{\text{int}} = \int d^4x \int_{\text{moduli}} (\chi^c_a \bar{\chi}^c_b \bar{\chi}^c_c) \text{Tr}(SA_{\text{eff}})$$

3) Multi-Fermion Fermion: $C = \mathbb{CP}^1$, $p \geq 1$

$$\Theta^+ = 2$$

$$\Theta^+ = 0$$

$$\chi^+ = 2p$$

$$\text{moduli: } \begin{cases} SA = H^2_2(X, F_{nd}(V)) \text{ Lefschetz} \\ ST = H^2_2(X, \Omega^2_X) \text{ Kähler} \\ SU = H^2_2(X, TX) \text{ complex} \end{cases}$$

$$I_{int} = \int_C d^2z \bar{D}_\mu \bar{D}_\nu \bar{X}^\mu \bar{D}^\nu \bar{X}^\mu \text{Tr}(S \bar{A}_{\mu\nu}^{\epsilon})$$

$$+ \int d^2z \left[\text{---} \text{---} \right] (S \bar{U}_{\mu\nu}^{\epsilon}) \bar{X}^\mu \bar{X}^\nu$$

$$+ \int d^2z \left[\text{---} \text{---} \right] (S \bar{T}_{\mu\nu}^{\epsilon}) (\bar{X}^\mu \bar{X}^\nu) \cdot (\bar{X}^\mu \bar{X}^\nu)$$

3) Multi-Fermion F-term: $G = \mathbb{CP}^1$, $p=1$

$$\Theta^+ = 2$$

$$\Theta_-^+ = 0$$

$$\chi_-^+ = 2p$$

$$\text{mult.} \left\{ \begin{array}{l} SA = H_5^1(X, \text{End } V) \text{ Weil} \\ ST = H_5^1(X, \Omega_{X/C}^2) \text{ Kulkarni} \\ SU = H_5^1(X, TX) \text{ Cpl. SU} \end{array} \right.$$

$$I_{int} = \int_C d^2z \bar{D}_+ \pi_F (\bar{\chi}_-^+ \bar{D}_+ \bar{\chi}^+) / \text{Tr}(S \bar{A}_{\alpha i}^{\dot{\alpha}}) +$$

$$+ \int d^2z \left[\text{---} \right] (S \bar{D}_{\alpha i}^+ \lambda_2 \gamma^m) +$$

$$+ \int d^2z \left[\text{---} \right] (S \bar{T}_{\alpha i}^{\dot{\alpha}} \partial_i \bar{\chi}^+)$$

3) Multi-Fermion F-term: $C = \mathbb{CP}^1$, $p=1$

$$\Theta^x = 2$$

$$\Theta^{\bar{x}} = 0$$

$$\bar{\chi}_x = 2p$$

$$\text{moduli: } \begin{cases} SA = H^1_2(X, \text{End } V) \text{ ball} \\ ST = H^2_2(X, \Omega^2_x) \text{ Kähler} \\ SU = H^2_2(X, TX) \text{ complex} \end{cases}$$

$$I_{\text{int}} = \int_C d^2z \bar{\mathcal{D}}_{\bar{x}} \mathcal{D}_x (\bar{\chi}_x \bar{\mathcal{D}}^x \bar{\chi}^{\bar{x}}) (\text{Tr}(S \bar{A}_{x\bar{x}}^x) + \dots)$$

$$+ \int d^2z \left[\text{---} \right] (SU_{x\bar{x}}^x \partial_{\bar{x}} y^m) +$$

$$+ \int d^2z \left[\text{---} \right] (S \bar{T}_{x\bar{x}}^x \partial_{\bar{x}} \bar{\chi}^{\bar{x}}) = (2 + 1/2)$$

3) Multi-Fermion F-ferm: $C=CP^1$, P^1

$$\theta^x = 2$$

$$\theta^z = 0$$

$$\chi^x = 2p$$

$$\text{moduli: } \begin{cases} SA = H^1_2(X, End(V)) \text{ hol} \\ ST = H^1_2(X, \Omega^2_X) \text{ Kähler} \\ SU = H^1_2(X, TX) \text{ spinor} \end{cases}$$

(P^1)

$$\omega \sim \int_{C \times C} S \bar{A} \delta \bar{A} \langle j, j-2 \rangle'$$

$$I_{int} = \int_C d^2x \sqrt{g} \left(\bar{\chi}^x \not{D}^x \chi^x \right) \left(\text{Tr}(S \bar{A}_{\alpha i} j^x) \right) +$$

$$+ \int_C d^2x \left[\text{---} \right] (S \bar{U}_{\alpha i} \lambda^x y^x) +$$

$$+ \int_C d^2x \left[\text{---} \right] (S \bar{T}_{\alpha i} \partial_i \bar{\chi}^x) \cdot (\bar{\chi}^x + c.c.)$$

3) Multi-Fermion F-term: $C = \mathbb{CP}^1$, $p=1$

$\Theta^x = 2$
 $\Theta^y = 0$
 $\chi^x = 2p$

moduli: $\begin{cases} SA \cdot H^1_2(X, \text{End } V) \text{ full} \\ ST \cdot H^1_2(X, \Omega^2_X) \text{ Kähler} \\ SU \cdot H^1_2(X, TX) \text{ complex} \end{cases}$

$\omega \sim \int_{C \times C} SA \delta \bar{A} \langle j, j \rangle$
 $\int_{C \times C} ST \delta \bar{U} \langle j, j \rangle$

$$I_{int} = \int_C d^2x \int_{C \times C} (\bar{\chi}^x \bar{D}^x \bar{\chi}^x) \text{Tr}(SA_{(1)}^x j_2)$$

$$+ \int_C d^2x \left[\text{---} \right] (SU_{(1)}^x j_2 \gamma^m)$$

$$+ \int_C d^2x \left[\text{---} \right] (ST_{(1)}^x \partial_i \bar{\chi}^x) \cdot (j_2 \gamma^m)$$

3) Multi-Fermion F-term: $C \in \mathbb{H}^3, p=1$

$$\theta^x = 2$$

$$\theta^y = 0$$

$$\chi^x = 2p$$

$$\text{moduli} \left\{ \begin{array}{l} SA \in H^4_2(X, \text{End } V) \text{ bulk} \\ ST \in H^3_2(X, \Omega^2_{\mathbb{C}}) \text{ Kähler} \\ SU \in H^2_2(X, TX) \text{ spinors} \end{array} \right. \quad \omega \sim \int_{\text{cell}} SA \delta \bar{A} \langle j, j \rangle + \int_{\text{cell}} ST \delta \bar{U} \langle \lambda, \lambda \rangle$$

$$I_{\text{int}} = \int_C d^4x \, \bar{\psi} \gamma^\mu \psi (\bar{\chi}^x \bar{D}^\mu \chi^x) (\text{Tr}(S \bar{A}_{\mu}^x j_\mu)) + \int_C d^3x \, \text{---} \text{---} \text{---} (S \bar{U}_{\mu}^x j_\mu \gamma^\mu) + \int_C d^2x \, \text{---} \text{---} \text{---} (S \bar{T}_{\mu}^x j_\mu \gamma^\mu) \cdot (\bar{\chi}^x \chi^x)$$

3) Multi-Fermion F-term: $C = \mathbb{C}P^1$, $p=1$ (F.1)

$$\Theta^+ \quad 2$$

$$\Theta^- \quad 0$$

$$\chi_\alpha \quad 2p$$

$$\text{moduli: } \begin{cases} SA = H^1_2(X, \text{End } V) \text{ w.r.t. } \omega \sim \int_{C \times C} SA \delta \bar{A} \langle j, j \rangle' \\ ST = H^1_2(X, \Omega^2_{X/C}) \text{ Kähler} \\ SU = H^1_2(X, TX) \text{ spinor} \end{cases}$$

$$\omega \sim \int_{C \times C} SA \delta \bar{A} \langle j, j \rangle' + \int_{C \times C} ST \delta \bar{U} \langle \lambda, \lambda \rangle'$$

$$I_{\text{int}} = \int_C d^2z \int_{\Sigma_{g,p}} (\bar{\chi}_\alpha^\dagger \bar{D}^\mu \chi^\alpha) (\text{Tr}(SA_{\alpha\beta}^\dagger j_\mu)) + \delta S_{1,2} = \int_C d^2z \int_{\Sigma_{g,p}} \mathcal{U}_C$$

$$= \int_C d^2z \left[\text{---} \right] (SD_{\alpha\beta}^\dagger \lambda_2 \gamma^\mu) +$$

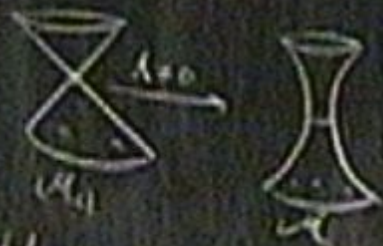
$$+ \int_C d^2z \left[\text{---} \right] (ST_{\alpha\beta}^\dagger \partial_i \bar{\gamma}^\alpha) \cdot (\partial_i \gamma^\beta)$$

SNCD: $SU(N)$, N_c flavors

$$p = N_c - N_f + 1$$

$p=0$ $W_{\text{ads}} \neq 0, \mathcal{M}_{c,1}$

$p=1$ Cplx str deformation of $\mathcal{M}_{c,1}$



$p>1$ Higher Fermions

General PLS on Multi-Fermion Systems

• global syst

• effective action: $N+1$ atoms

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_U, S_c(d\Phi, \mathbb{E}^c, \mathbb{E}^c)$$

$$S[S] = \int d^4x d\tau w_{\mathbb{E}^c} \bar{D}_\mu \mathbb{E}^c D^\mu \mathbb{E}^c$$

$$= \int d^4x w_{\mathbb{E}^c} \mathbb{E}^c \bar{\mathbb{E}}^c$$

Locally $= \int d^4x d\tau S[\mathbb{E}^c] = \int d^4x d\tau$

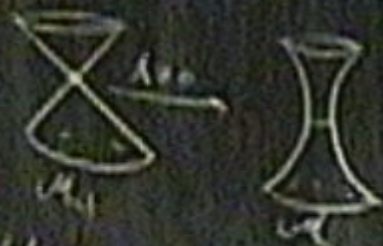
Globally $\checkmark$ — — —

S&CD: $SU(N_c)$, N_c flavors

$$p = N_c - N_f + 1$$

$p=0$ $W_{\text{ads}} \neq 0$, $\mathcal{M}_{c,1}$

$p=1$ Cplx. str. deformation of $\mathcal{M}_{c,1}$



$p=1$ Hyper-Fuchsian

General Phys. in Multi-Fermion Fock spaces

- global syst
- effective action $N=1$ atom

$$\Phi: \mathbb{R}^n \rightarrow \mathcal{M}_n, S^1 \text{ (and } \mathbb{K}(\mathbb{R}^n, \mathbb{R}^n))$$

$$S^1 = \int d\tau d\sigma \omega_{ij} \bar{D}_\tau \bar{X}^i D^\sigma X^j$$

$$= \int d\tau d\sigma \omega_{ij} d\tau^i d\sigma^j \dots$$

$$\text{Locally} = \int d\tau d\sigma S\mathbb{K} = \int d\tau d\sigma (D_\tau D_\sigma \delta)$$

$$\text{Globally} \quad \gamma = \dots$$