Title: Spin chains and strings in Ypq and Lpq|r manifolds

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Abstract:

Spin chains and strings in $Y^{p,q}$ and $L^{p,q|r}$ manifolds

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Based on hep-th/0505046
and hep-th/0505206
w/ S. Benvenuti (Scuola Normale & INFN, Pisa, Italy)

Summary

Introduction:

N=4 AdS/CFT: Strings ↔ Long ops.

- Point-like strings ↔ BPS ops.
- String excitations ↔ insertions
- Long strings ↔ semiclassical states

N=1 AdS/CFT

- $AdS_5 \times X_5$; X_5 is Sasaki-Einstein

- X_5 : $T^{1,1}$ SU(2) x SU(2) x U(1)
  $Y^{p,q}$ SU(2) x U(1) x U(1)
  $L^{p,q\,|\,r}$ U(1) x U(1) x U(1)

$Y^{p,q}$

- Massless geodesics and long operators:
  matching of R and flavor charges.

- Extended strings (Qualitative)
  Eff. action appears in f.t. and is similar but not equal to string side.

$L^{p,q|r}$

- Derivation of dual field theory

- Massless geodesics and long operators:
  matching of R and flavor charges.

N=4 SYM $\Leftrightarrow$ II B on $AdS_5 \times S^5$

Strings? Long operators $\leftrightarrow$ Strings

(BMN, GKP) an. dimension $\leftrightarrow$ Energy

Frolov-Tseytlin: Long strings

Minahan-Zarembo: Spin chains

Bethe Ansatz (BFST)

$X=\phi_1+i\,\phi_2$; $Y=\phi_3+i\,\phi_4$; $O=Tr(XY\ldots Y)$

Long chains ($J\rightarrow\infty$) are classical

$$S_{eff} = J\left[\int \cos\theta\,\dot{\phi} - \frac{\lambda}{J^2}\int\left(\partial_\sigma \vec{n}\right)^2\right]$$

$$n = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

S_{eff}: Action of spin chain and fast strings.

(M.K.; Ryzhov, Tseytlin, M.K.; ...)

Strings as states of N=4 SYM on $R \times S^3$

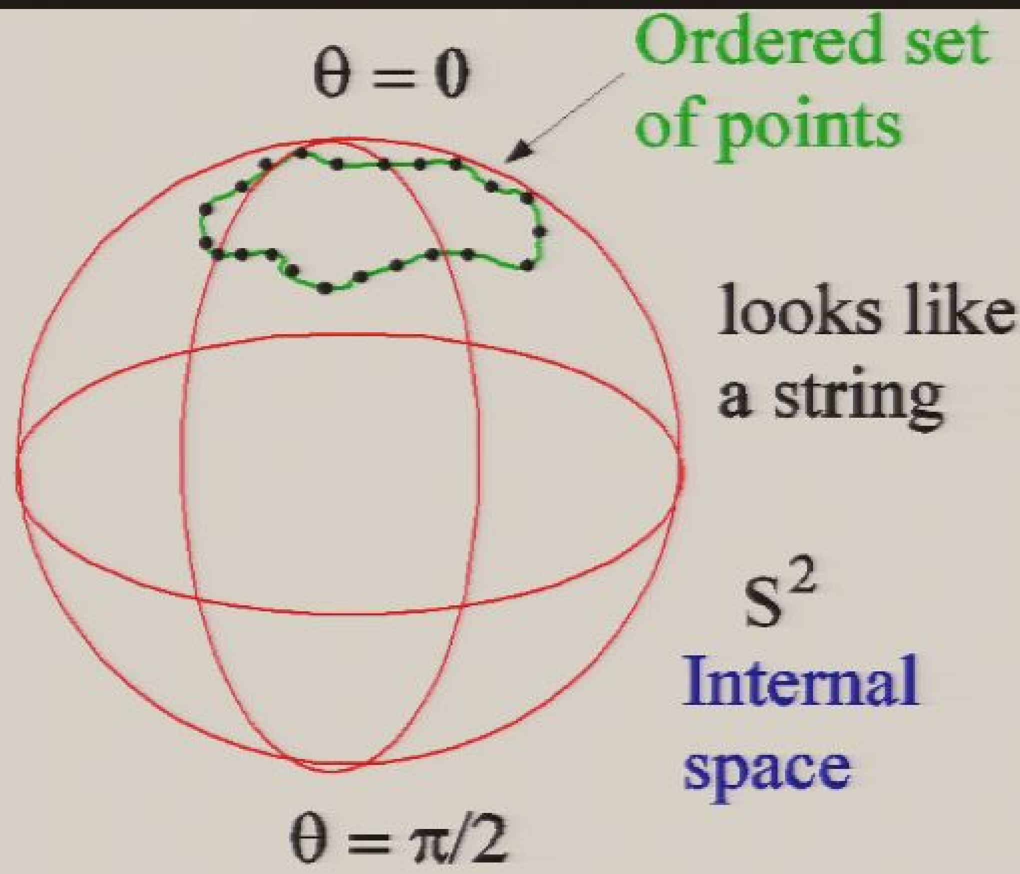
The operator $O = \sum c_i \, [\, Tr(XY \ldots Y) \,]_i$ maps to a state of the field th. on S^3

State: delocalized and has a large number of particles (X and Y).

Q.M. : $|\psi\rangle = \cos(\theta/2) \exp(i\phi/2) \, |X\rangle + \sin(\theta/2) \exp(-i\phi/2) \, |Y\rangle$

We can use $v_i = |\psi(\theta_i, \phi_i)\rangle$ to construct: $O = Tr\,(v_1 v_2 v_3 \ldots v_n)$

(Coherent state)



Strings are useful to describe states of a large number of particles (in the large N limit)

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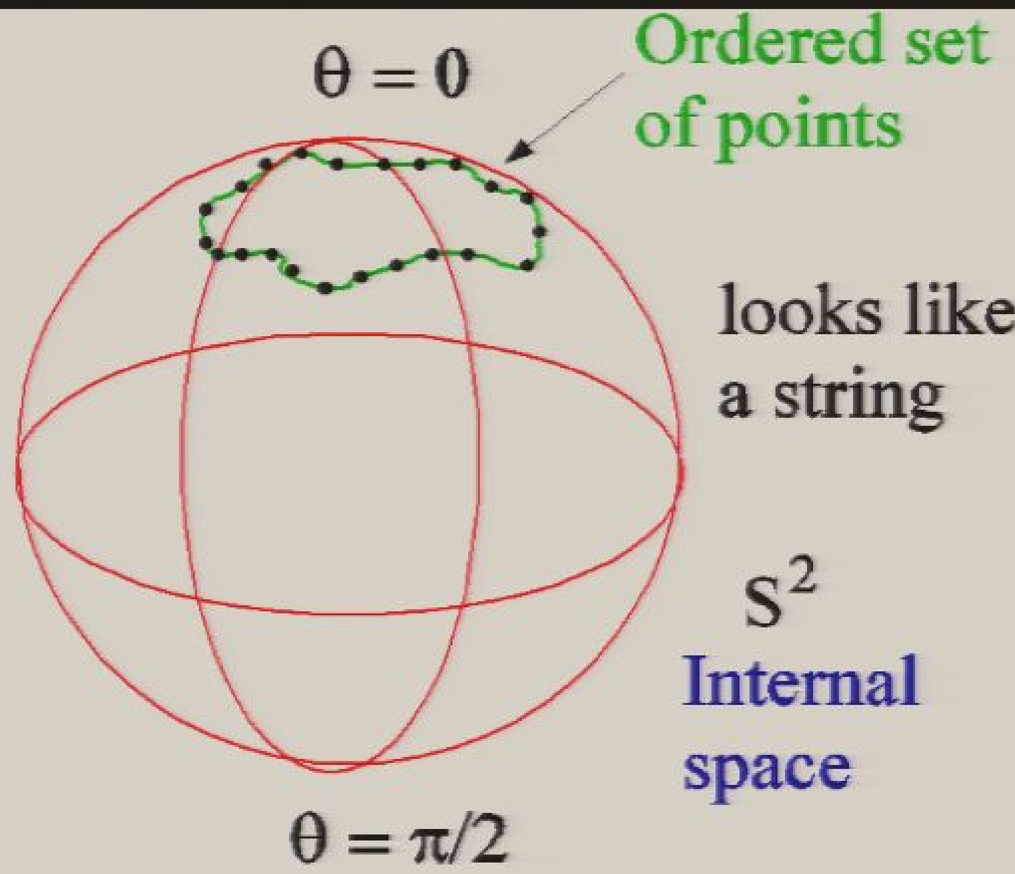

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N=1 $AdS_5 \times X_5$: X_5 : Sasaki-Einstein

$$ds^2 = dx^2_{[4]} + \underline{dr^2 + r^2 \; dX_5^2}$$

⇓

CY cone

Put D3 branes at $r = 0$ and take near horizon limit:

$$ds^2 = \quad r^2 dx^2_{[4]} + \frac{dr^2}{r^2} \quad + dX_5^2 \;, \quad AdS_5 \times X_5$$

$T^{1,1}$ (conifold) Klebanov-Witten

$Y^{p,q}$ **Gauntlett, Martelli, Sparks, Waldram**

Benvenuti, Franco, Hanany, Martelli, Sparks

$L^{p,q|r}$ **Cvetic, Lu, Page, Pope; Benvenuti, M.K.;**

Franco, Hanany, Martelli, Sparks, Vegh, Wecht;

Butti, Forcella, Zaffaroni

$T^{1,1}$

$$ds^2 = \frac{1}{9}\left[d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right]^2 + \frac{1}{6}\sum_{i=1}^{2}\left[d\theta_i^2 + \sin\theta_i d\phi_i^2\right]$$

$Y^{p,q}$ (Gauntlett, Martelli, Sparks, Waldram)

$$ds^2 = w(y)\left[d\alpha + f(y)(d\psi - \cos\theta)\right]^2 + \frac{1-y}{6}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$+ \frac{dy^2}{6p(y)} + \frac{q(y)}{9}\left(d\psi - \cos\theta\, d\phi\right)^2$$

If $\alpha \to \beta = 6\alpha + \psi$; only p(y) appears.

$$p(y) = \frac{a - 3y^2 + 2y^3}{3(1-y)}$$

; $y_1 < y < y_2$

$$y_{1,2} = \left(2p \mp 3q - \sqrt{4p^2 - 3q^2}\right)/(4p)$$

Point-like strings (massless geodesics)

$$S=\frac{\sqrt{\lambda}}{2}\int d\tau\left(-\partial_\tau t\,\partial_\tau t+g_{ab}\partial_\tau x^a\partial_\tau x^b\right)$$

Const.: $-\partial_\tau t\,\partial_\tau t+g_{ab}\partial_\tau x^a\partial_\tau x^b=0$

$t=\kappa\,\tau\,;\quad P_t=\Delta=\sqrt{\lambda}\;\;\kappa$

$$p_a=\frac{\delta L}{\delta(\partial_\tau x^a)}\;;\qquad H=g^{ab}p_a p_b$$

$$\Delta^2=\left(\frac{3}{2}Q_R\right)^2+\frac{1}{6p(y)}\left(P_\alpha+3yQ_R\right)^2+6p(y)P_y^2+\frac{6}{1-y}\left(J^2-P_\psi^2\right)$$

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BPS geodesics

$$\Delta = (3/2)\,Q_R \quad \Rightarrow \quad P_y = 0\;,\; J = P_\psi\,,\;\; y_0 = -\frac{P_\alpha}{3\,Q_R}$$

Therefore, y is the ratio between U(1) and R-charges.

Also: $J = \frac{1}{2}(1 - y_0)\,Q_R$ (from $J = P_\psi$)

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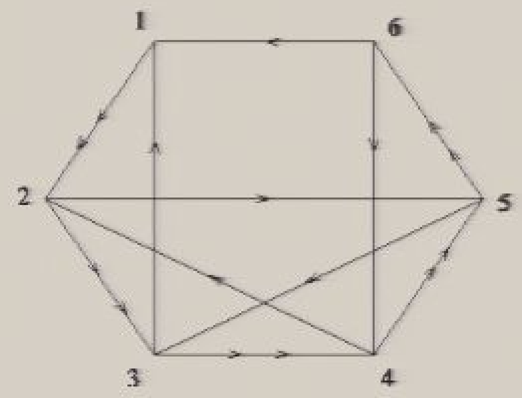
Dual gauge theories

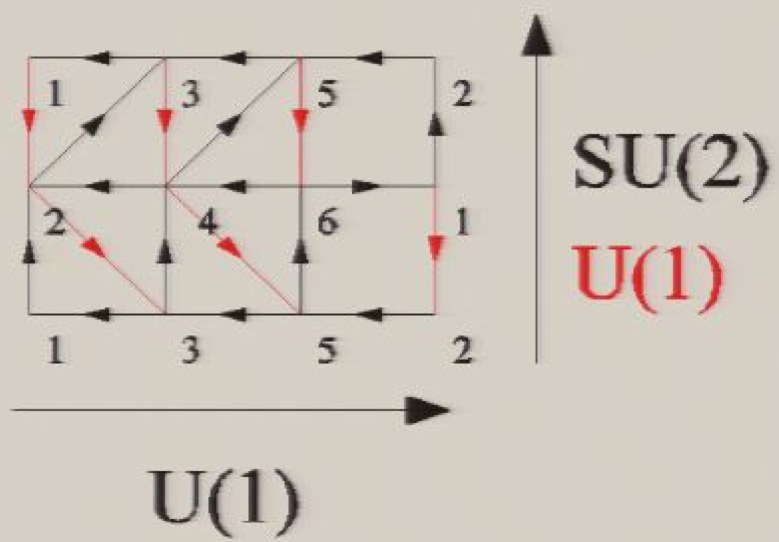
(Benvenuti, Franco, Hanany, Martelli, Sparks)

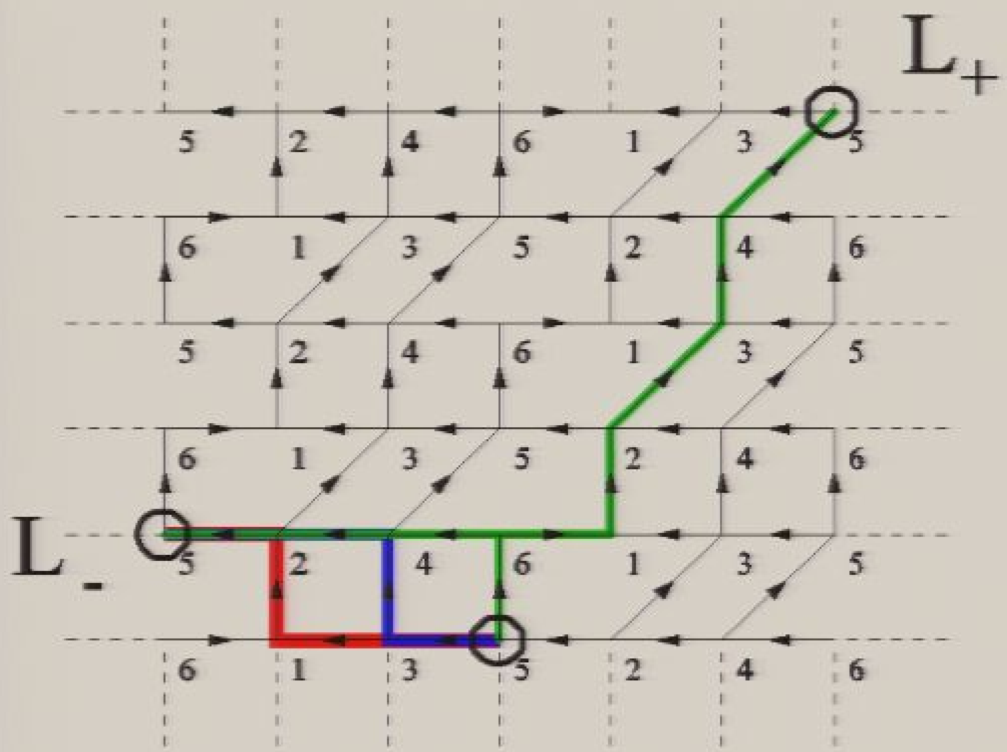
Example: $Y^{3,2}$



Periodic rep.:

(Hanany, Kennaway, Franco, Vegh, Wecht)





Operators of maximum and minimum slopes can be identified with the geodesics at $y=y_1$ and $y=y_2$

R-charges and flavor charges match.

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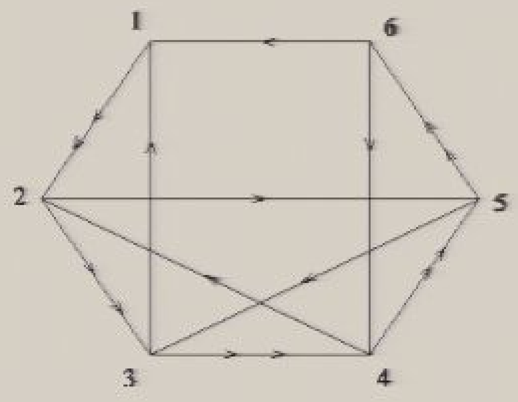

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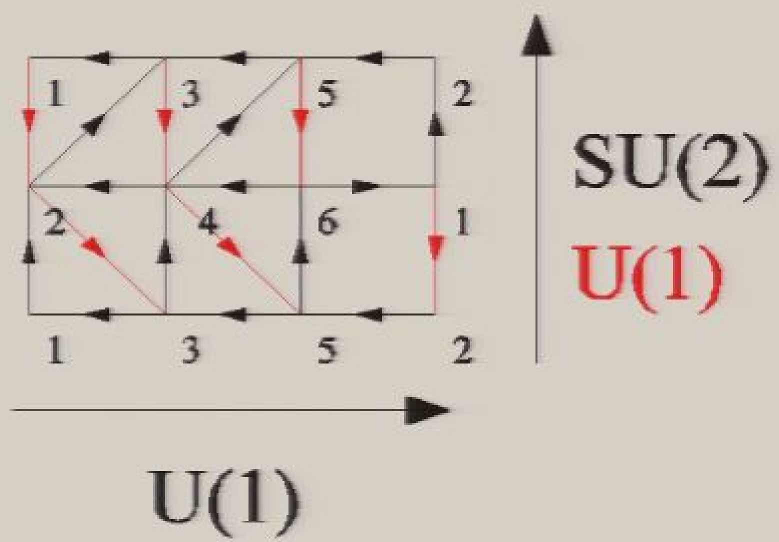

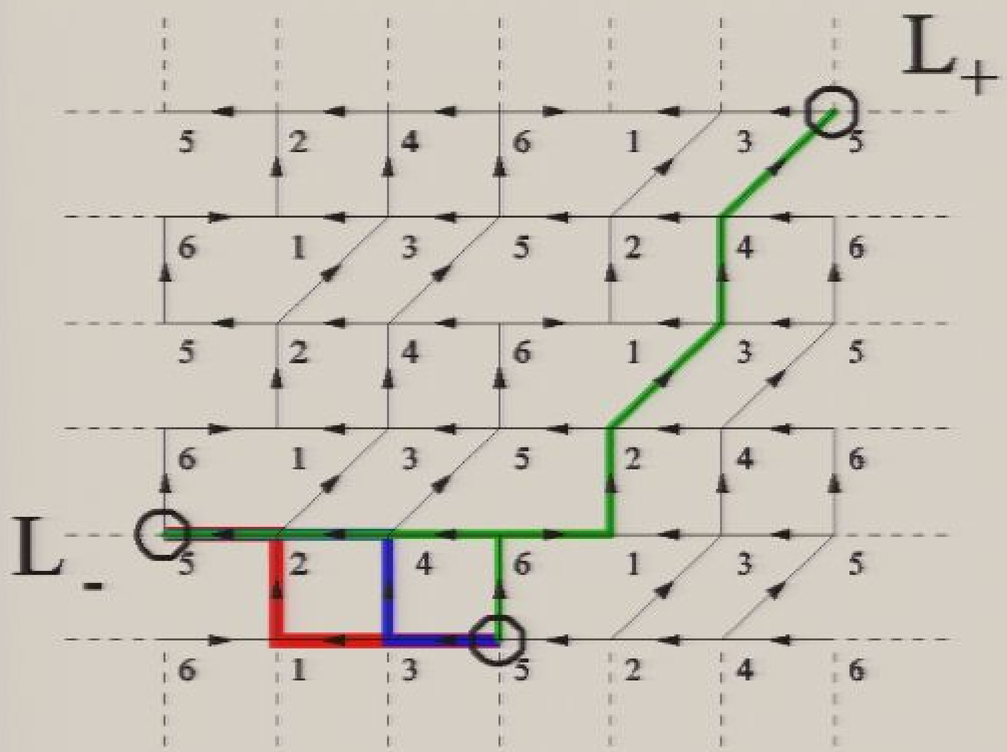

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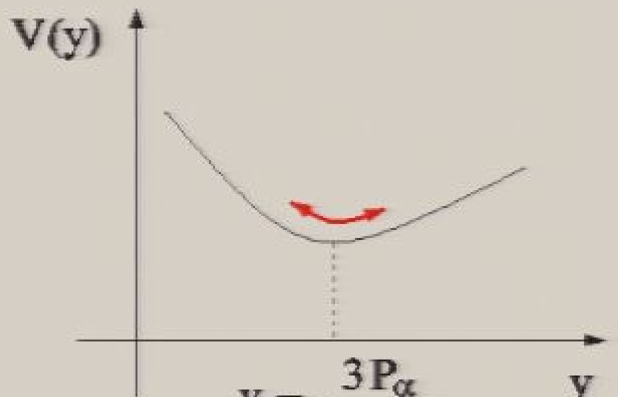
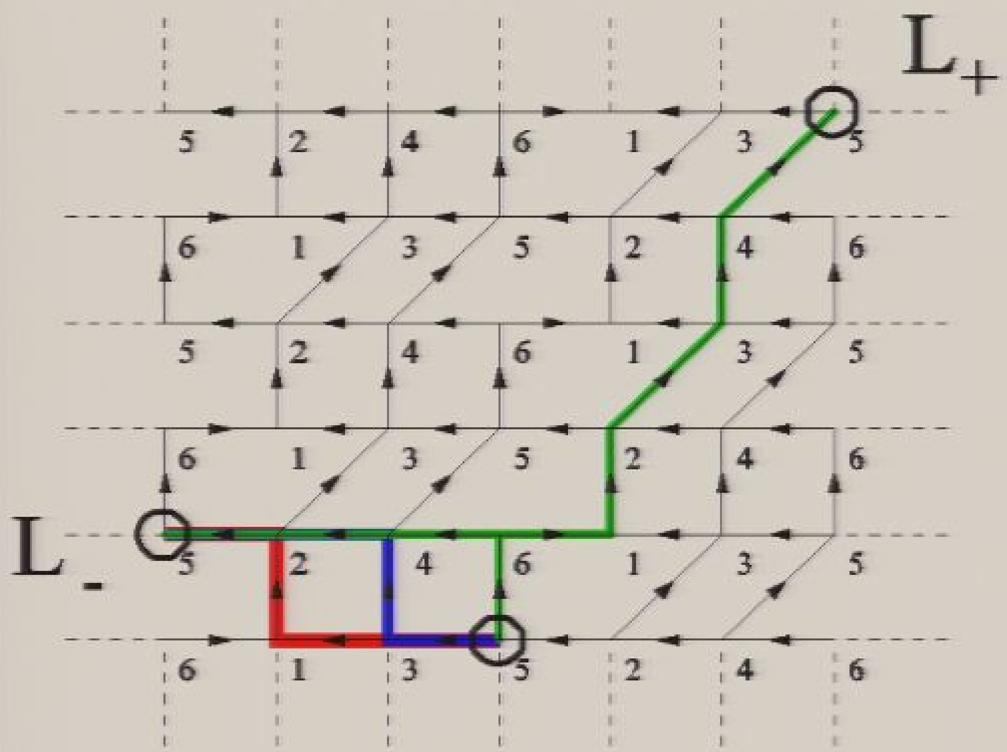


$$y_0 = -\frac{3P_\alpha}{Q_R}$$

Quantize: $\delta\Delta=2n$, $\delta Q_R=0$, $\delta P_\alpha=0$, $\delta J=0$

Also ($J>P_\psi$): $\delta\Delta=2n$, $\delta Q_R=0$, $\delta P_\alpha=0$, $\delta J=n$

Agree with quantum numbers of U(1) and SU(2) currents
$\Rightarrow$ we identify these non-BPS geodesics with insertions of the currents. (Also Sonnenschein et al.)



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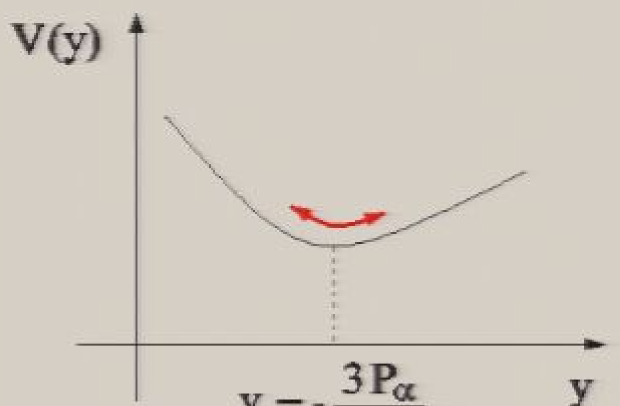

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Extended Strings

We consider closed extended strings such that each point moves approximately along a BPS geodesic.

Effective action for such strings:

$$ds^2 = -dt^2 + \frac{1}{6} g_{ij} dx^i dx^j + \frac{1}{9}(d\psi + A_j dx^j)^2$$

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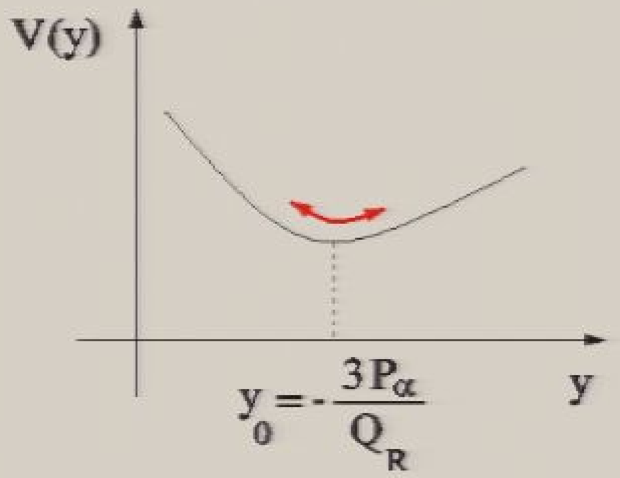

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and take the limit: $\partial_\tau X \to 0$, $\kappa \to \infty$, $\kappa\,\partial_\tau X$ fixed.

We get a reduced action:

$$S = \frac{1}{3}\int \kappa\left(\partial_\tau \psi_1 + A_i\,\partial_\tau x^i\right) - \frac{1}{12} g_{ij}\partial_\sigma x^i \partial_\sigma x^j$$

In our case:

$$S = \frac{Q_R}{4\pi}\left\{\int\left(\partial_t \psi_1 - y\,\partial_t \beta - (1-y)\cos\theta\,\partial_t \phi\right)\right.$$

$$\left. - \frac{4\pi^2}{9}\frac{\lambda}{Q_R^2}\left[(1-y)(\partial_\sigma n)^2 + \frac{(\partial_\sigma y)^2}{p(y)} + p(y)\left(\partial_\sigma \beta - \cos\theta\,\partial_\sigma \phi\right)^2\right]\right\}$$

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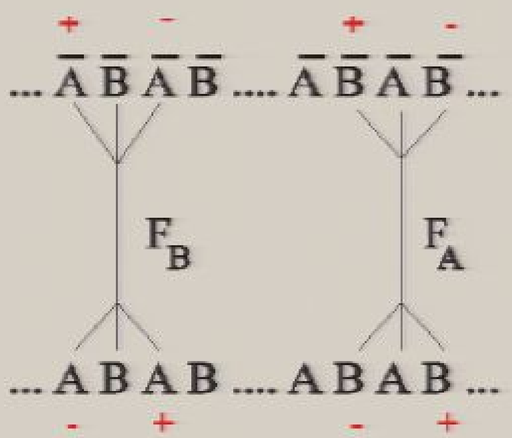
Example of $T^{1,1}$ (Angelova, Pando-Zayas, M.K. ; Strassler et al.)

$$S_{\text{eff}} = J\left\{\int \cos\theta_1\, \dot{\phi}_1 + \cos\theta_2\, \dot{\phi}_2 - \frac{\lambda}{J^2}\int \left[\left(\partial_\sigma \vec{n}_1\right) + \left(\partial_\sigma \vec{n}_2\right)\right]\right\}$$

Field theory:

Tr(ABABAB…ABAB) ; SU(2)xSU(2)



$W = \varepsilon_{ab}\,\varepsilon_{cd}\ \mathrm{Tr}\,(A^a B^c A^b B^d)$

+ , - : doublet of SU(2)

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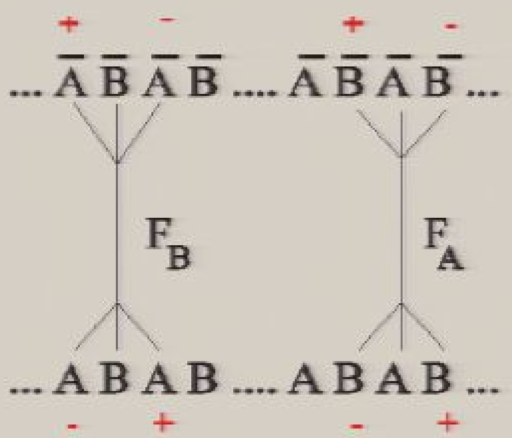

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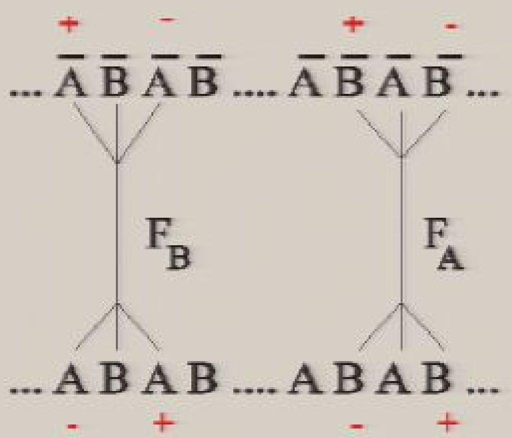

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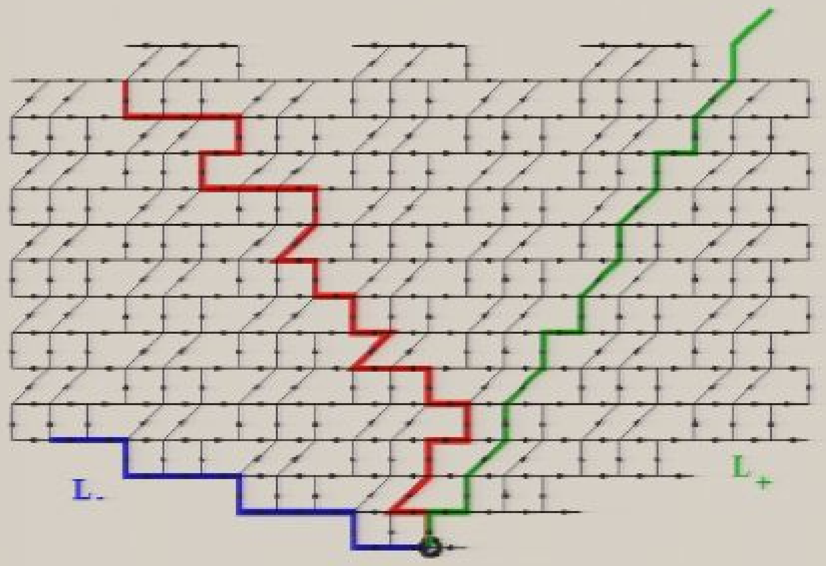
For $Y^{p,q}$



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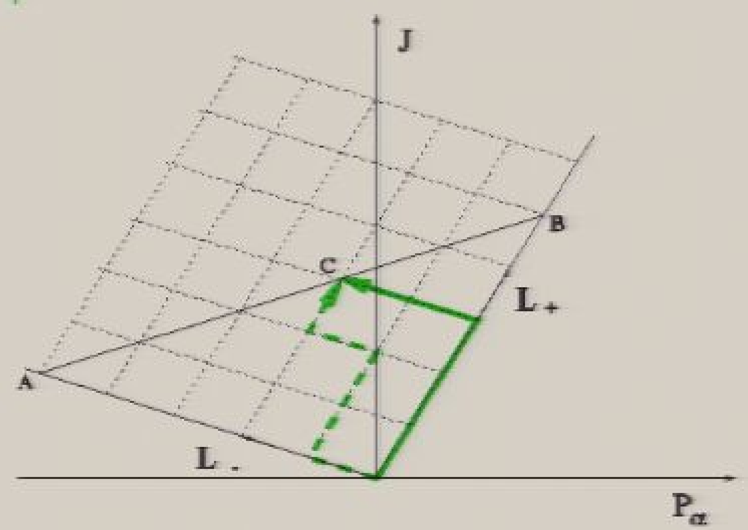
β: conjugate of y

$+$ spin $\rightarrow (\theta, \phi)$

Effective model:

(max and min slopes)



Example of $T^{1,1}$ (Angelova, Pando-Zayas, M.K. ; Strassler et al.)

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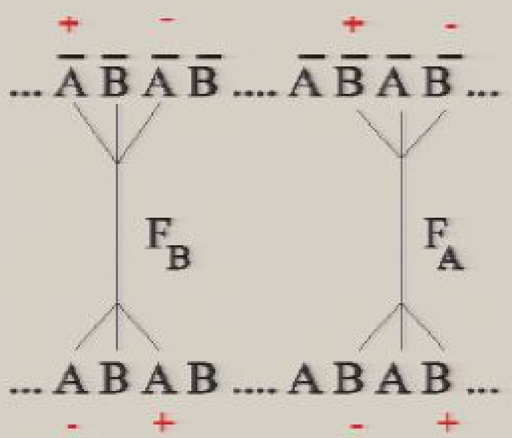
Tr(ABABAB…ABAB) ; SU(2)xSU(2)



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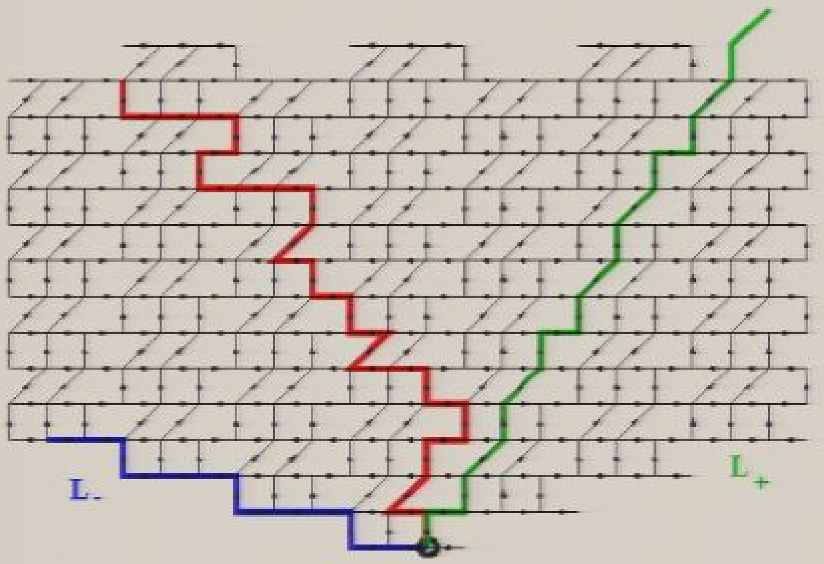

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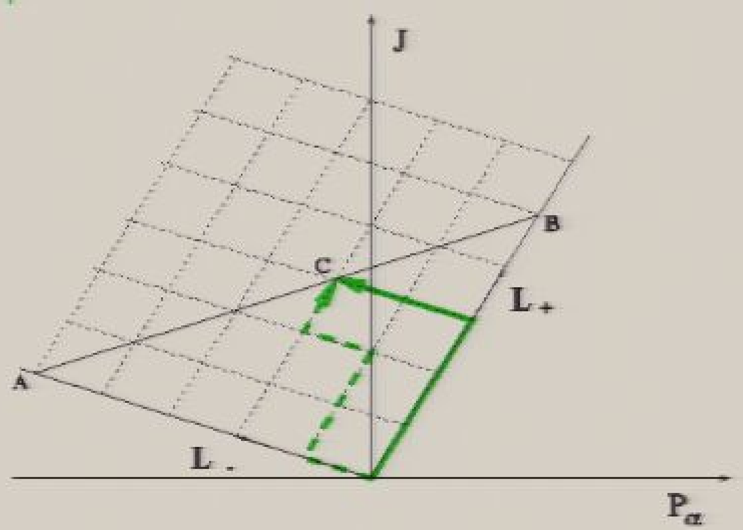

Hamiltonian $H = h_{eff.} \sum (1 - P_{i\,i+1})$

Coherent States: $$O = \rho_{1i} \exp(iP_\alpha^{(1)}\alpha_i)\, U(\theta_i, \phi_i, \psi_i)\, L_1 + \rho_{2i} \exp(iP_\alpha^{(2)}\alpha_i)\, U(\theta_i, \phi_i, \psi_i)\, L_2$$

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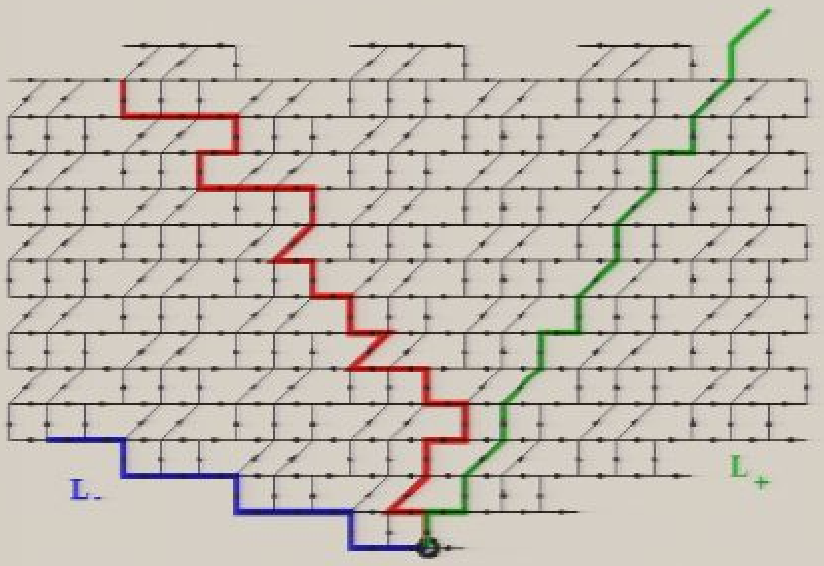

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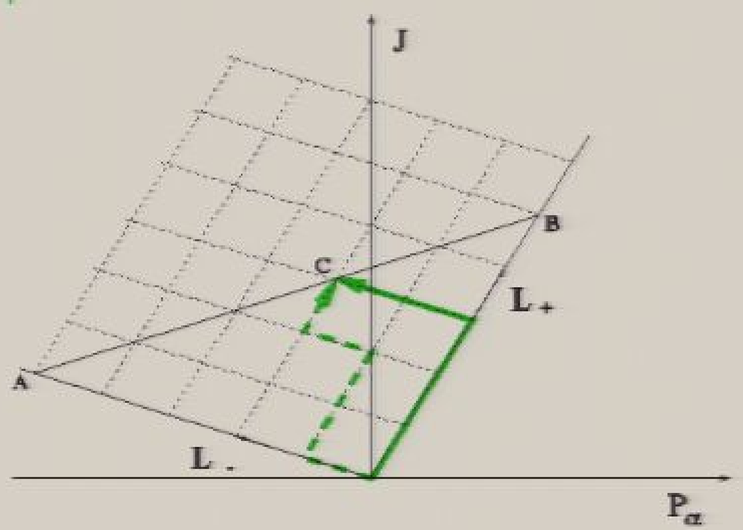

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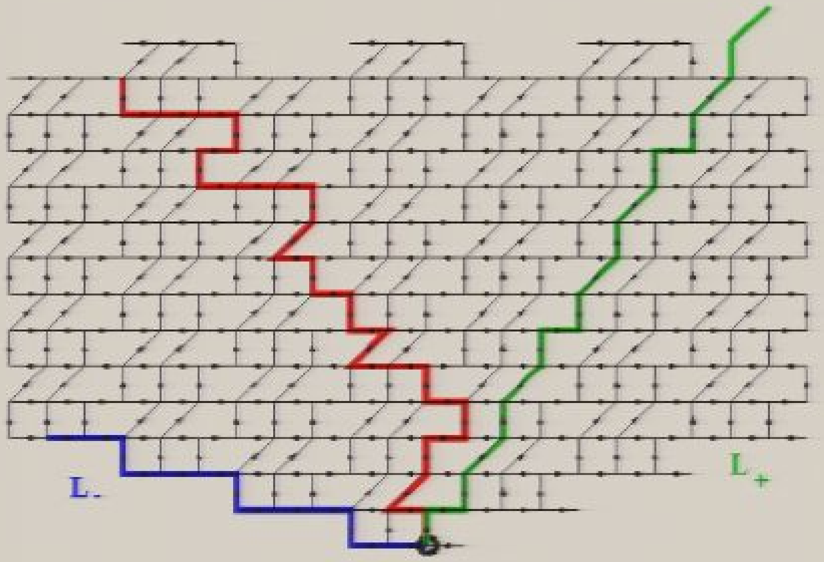

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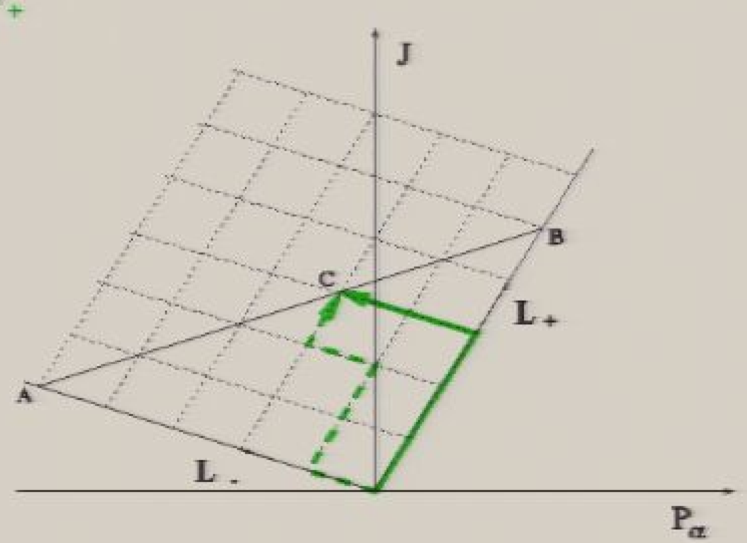

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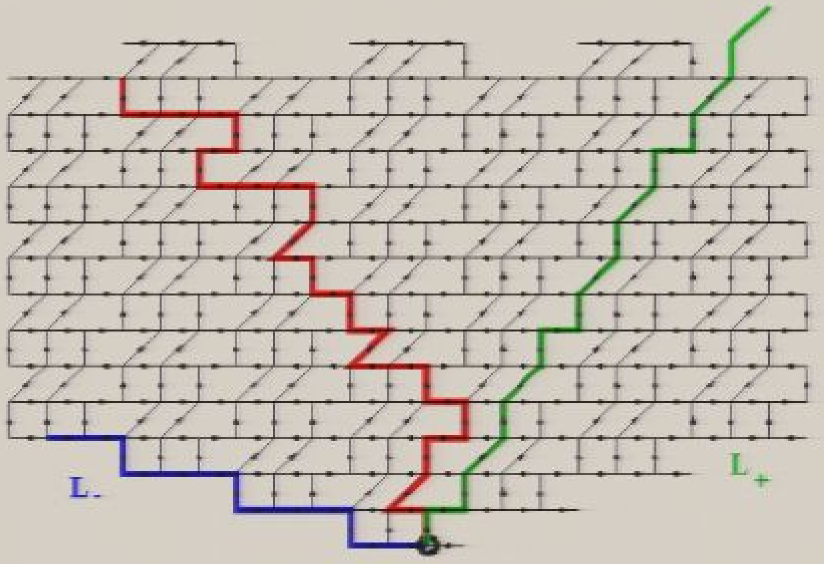

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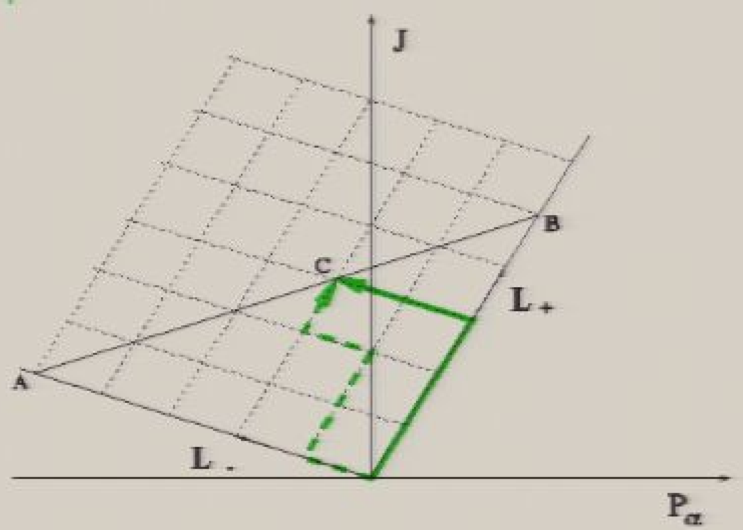

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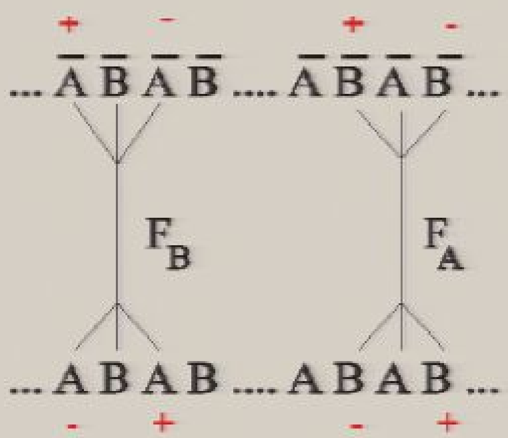

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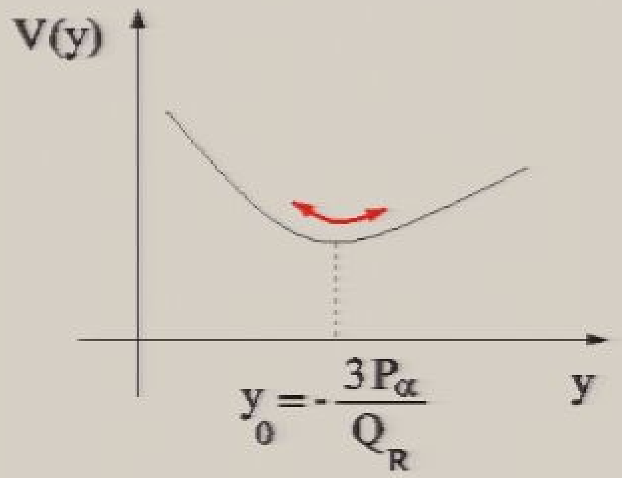
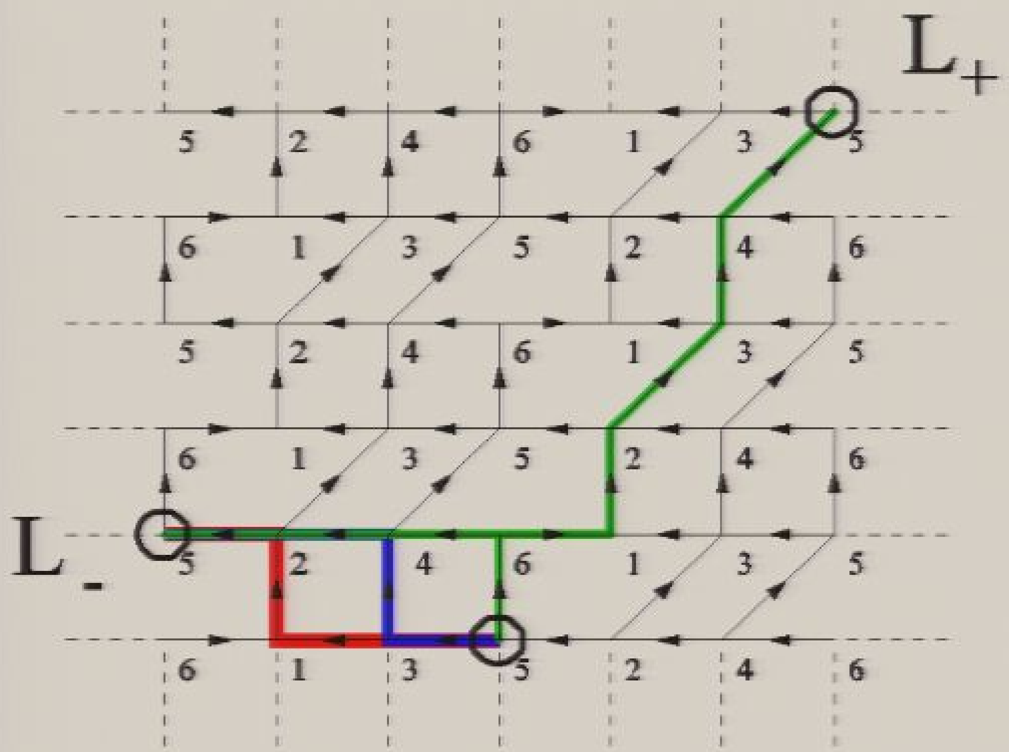
Small Fluctuations (BMN, GKP, Gomis-Ooguri, Klebanov et al.)



Quantize: $\delta\Delta=2n,\ \delta Q_R=0,\ \delta P_\alpha=0,\ \delta J=0$

Also ($J>P_\psi$): $\delta\Delta=2n,\ \delta Q_R=0,\ \delta P_\alpha=0,\ \delta J=n$

Agree with quantum numbers of U(1) and SU(2) currents
$\Rightarrow$ we identify these non-BPS geodesics with insertions
of the currents. (Also Sonnenschein et al.)

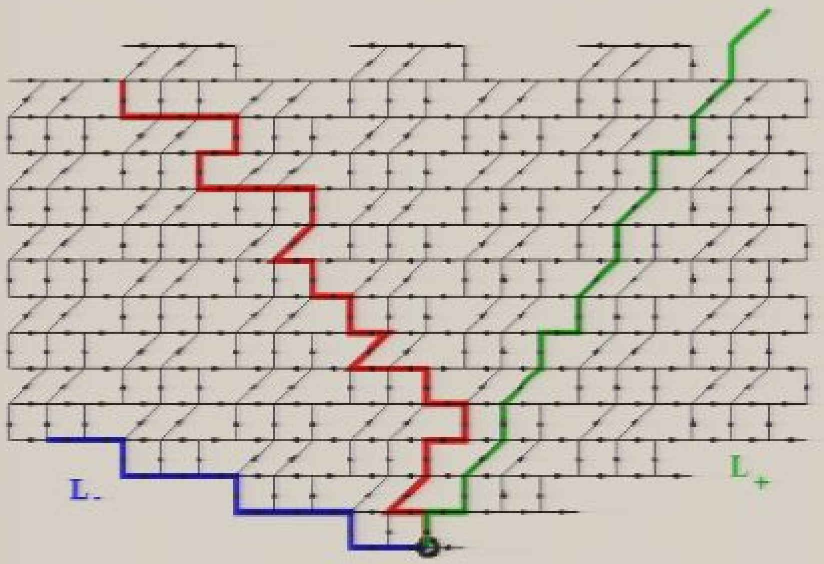


Operators of maximum and minimum slopes can be identified with the geodesics at $y=y_1$ and $y=y_2$

R-charges and flavor charges match.

(Also: Berenstein, Herzog, Ouyang, Pinansky)

For $Y^{p,q}$



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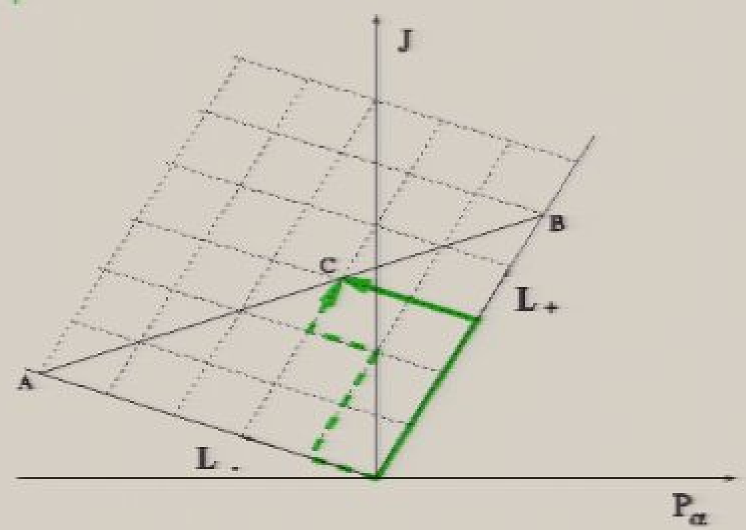

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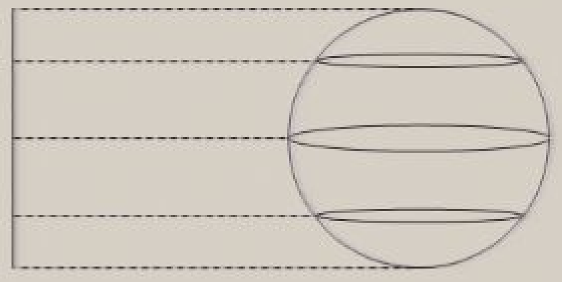
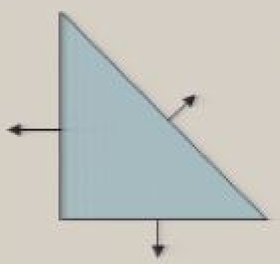
Toric Varieties

S^2



$(z_1, z_2) \equiv w\,(z_1, z_2)$

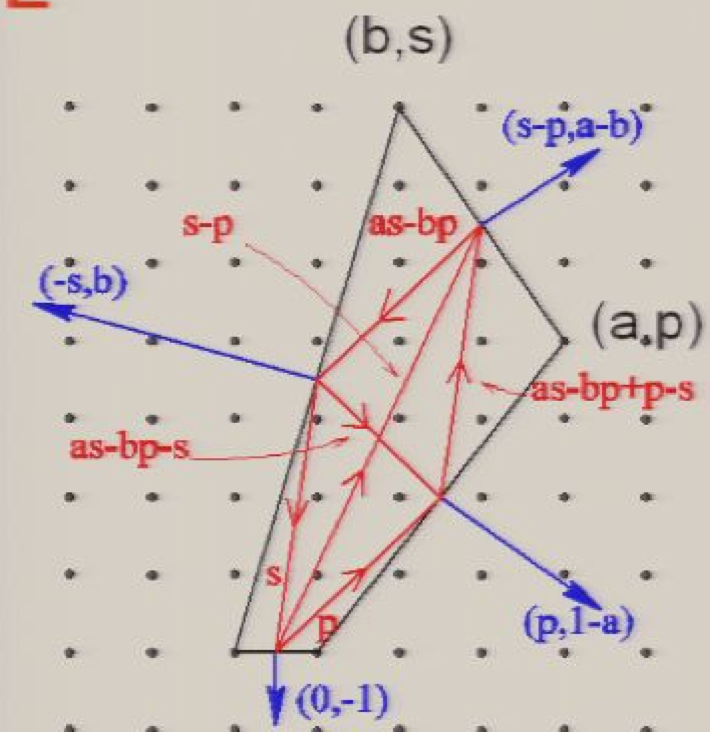


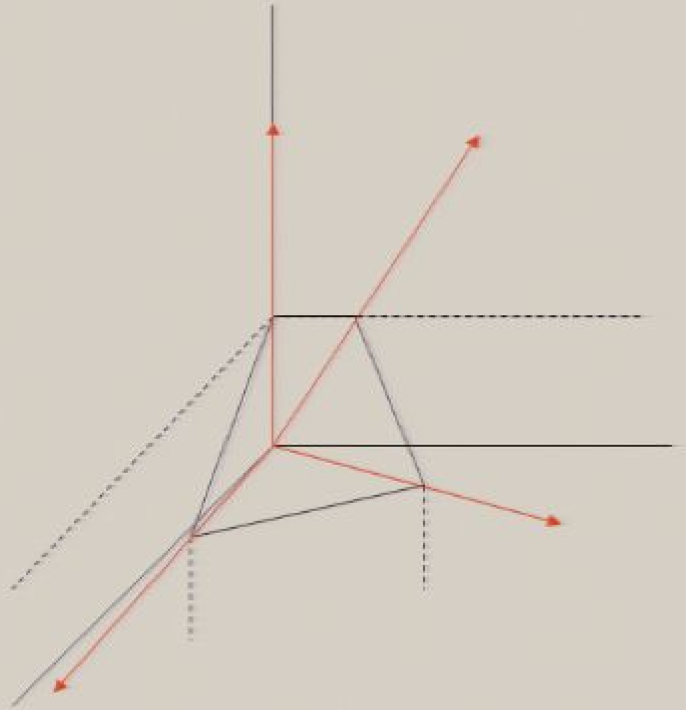
$(C^3)^* / C$

$(z_1, z_2, z_3) \equiv w\,(z_1, z_2, z_3)$

$(z_1 e^{i\alpha}, z_2 e^{i\beta}, z_3)$

$L^{pq|r}$





$$r = p + q - s \qquad q = a\,s - p\,b$$

(Also Butti, Zaffaroni, generic polygon)

$L^{p,q|r}$

Metric: **(Cvetic, Lu, Page, Pope)**

$$ds^2 = \left(\frac{1}{3}\mathrm{d}\psi_R + \mathrm{A}\right)^2 + \frac{1}{4}\mathrm{d}s^2_{[4]}$$

$$A = \frac{\alpha - 3\alpha y - 3x + 3xy}{6\alpha}\mathrm{d}\phi + \frac{\beta + 3\beta y - 3x - 3xy}{6\alpha}\mathrm{d}\psi$$

$$\mathrm{d}s^2_{[4]} = \frac{\rho^2}{f(x)}\mathrm{d}x^2 + \frac{f(x)}{\rho^2}\left(\frac{1-y}{\alpha}\mathrm{d}\phi + \frac{1+y}{\beta}\mathrm{d}\psi\right)^2 + \frac{\rho^2}{g(y)}\mathrm{d}y^2 + \frac{g(y)}{\rho^2}\left(\frac{\alpha - x}{\alpha}\mathrm{d}\phi - \frac{\beta - x}{\beta}\mathrm{d}\psi\right)^2$$

$$\rho^2 = \alpha + \beta - y(\alpha - \beta) - 2x$$

$$\psi_R = 3\xi + \phi + \psi$$

$x_1 < x < x_2$

$-1 < y < 1$

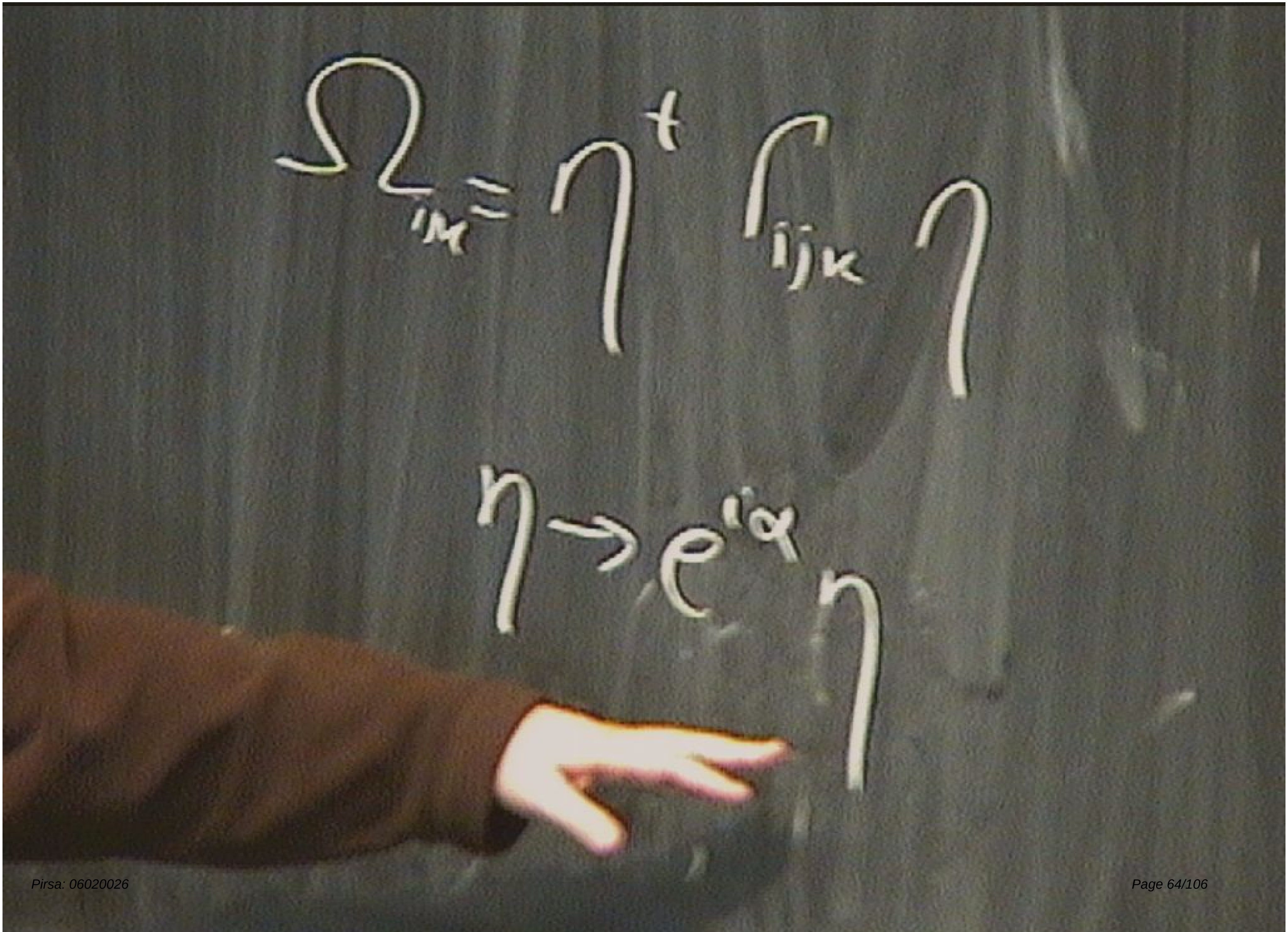
To write the metric in this way we needed to identify the angle conjugated to the R-charge.

This can be done by finding the covariantly constant holomorphic three form in the Calabi-Yau.

$$\Omega_{[3]} = \sqrt{f(x)g(y)}\; e^{i\psi_R}\; r^3\; \eta_1 \wedge \eta_2 \wedge \eta_3$$

$$\eta_1 = \frac{x-\beta}{f(x)}\,dx - \frac{1+y}{g(y)}\,dy + \frac{2i}{\alpha}d\phi \qquad \eta_3 = \frac{dr}{r} + i\,(d\xi + \sigma)$$

$$\eta_2 = \frac{x-\alpha}{f(x)}\,dx + \frac{1-y}{g(y)}\,dy + \frac{2i}{\beta}d\psi \qquad \sigma = \frac{(\alpha-x)(1-y)}{2\alpha}d\phi + \frac{(\beta-x)(1+y)}{2\beta}d\psi$$


$\Omega_{IM} = \eta^{t} \Gamma_{IJK} \eta$
$\eta \rightarrow e^{i\alpha} \eta$

$$\Omega_{iM} = \eta^t \Gamma_{ijk} \eta$$

$$\eta \to e^{i\alpha} \eta$$

$L^{p,q\,|\,r}$

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Massless geodesics

$$H = \frac{9}{2}P_{\psi_R}^2 + \frac{1}{2b_2}P_y^2 + \frac{1}{2b_1}P_x^2 + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)$$

$$\sigma_1 = -\frac{-d_2 P_\phi - d_1 P_\psi + 3(a_1 d_2 + a_2 d_1) P_{\psi_R}}{c_1 d_2 + c_2 d_1}$$

$$\sigma_2 = -\frac{-c_2 P_\phi + c_1 P_\psi + 3(a_1 c_2 - a_2 c_1) P_{\psi_R}}{c_1 d_2 + c_2 d_1}$$

$$a_1 = \frac{(\alpha - x)(1-y)}{2\alpha} - \frac{1}{3}, \quad a_2 = \frac{(\beta - x)(1+y)}{2\beta} - \frac{1}{3}, \quad b_1 = \frac{\rho^2}{4f(x)}, \quad b_2 = \frac{\rho^2}{4g(y)}$$

$$c_1 = \frac{\sqrt{f(x)}}{2\rho}\frac{(1-y)}{\alpha}, \quad c_2 = \frac{\sqrt{f(x)}}{2\rho}\frac{(1+y)}{\beta}, \quad d_1 = \frac{\sqrt{g(y)}}{2\rho}\frac{(\alpha - x)}{\alpha}, \quad d_2 = \frac{\sqrt{g(y)}}{2\rho}\frac{(\beta - x)}{\beta}$$

Massless geodesics

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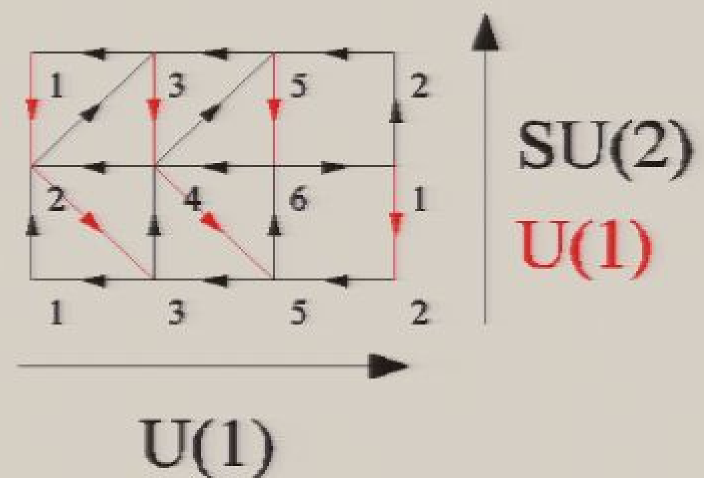
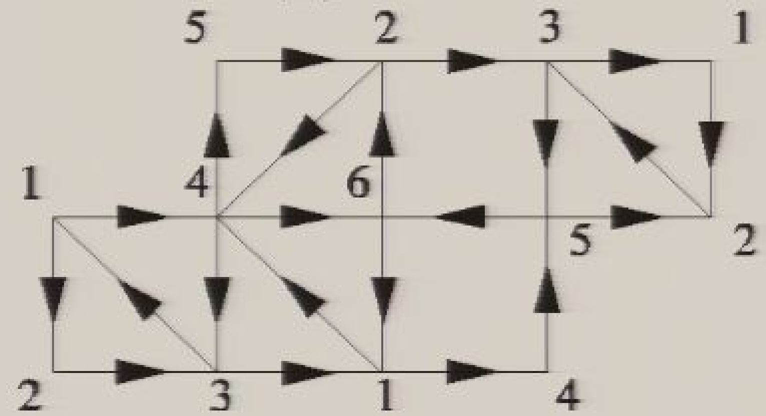
Example: $Y^{3,2} = L^{1,5\,|\,3}$



$L^{1,5\,|\,4}$



Massless geodesics

$$H = \frac{9}{2}P_{\psi_R}^2 + \frac{1}{2b_2}P_y^2 + \frac{1}{2b_1}P_x^2 + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)$$

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Metric: **(Cvetic, Lu, Page, Pope)**

$$ds^2 = \left(\frac{1}{3}\mathrm{d}\psi_R + A\right)^2 + \frac{1}{4}\mathrm{d}s^2_{[4]}$$

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$x_1 < x < x_2$

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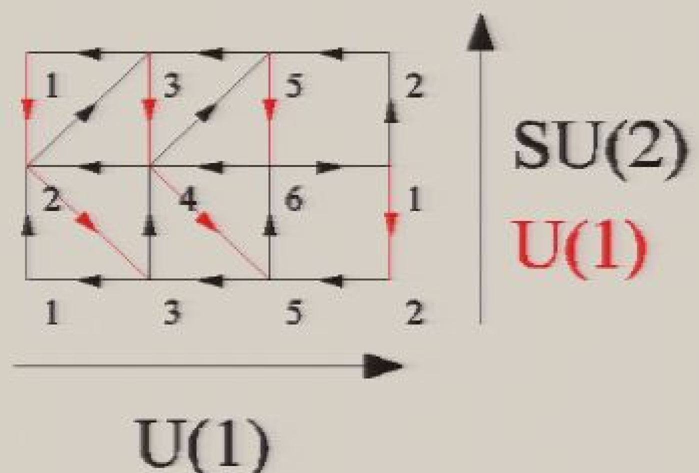
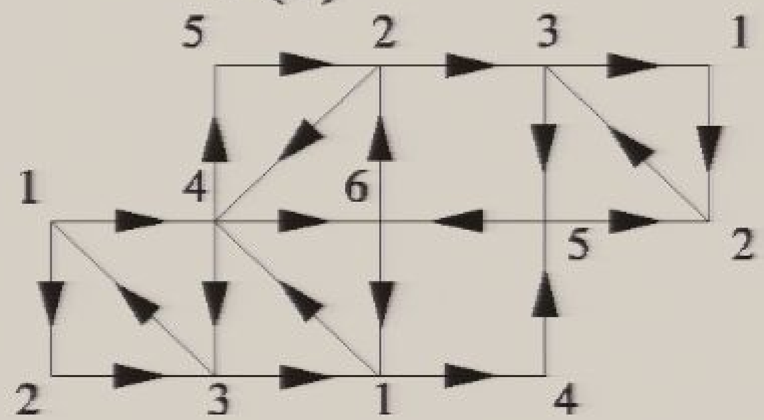
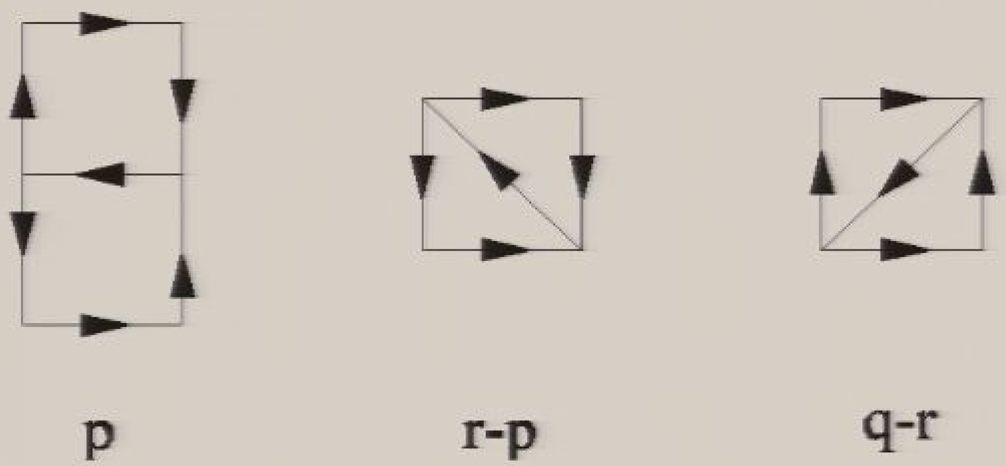


$L^{1,5\,|\,4}$



In general



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Using a-max one can compute the R-charges and the central charge. Everything matches. The parameters can be mapped from one description to the other.

$$a = \frac{\pi^3}{4\mathrm{V}} = \frac{1}{4}\frac{8pqrs}{(p+q)^3}\frac{1}{W}$$

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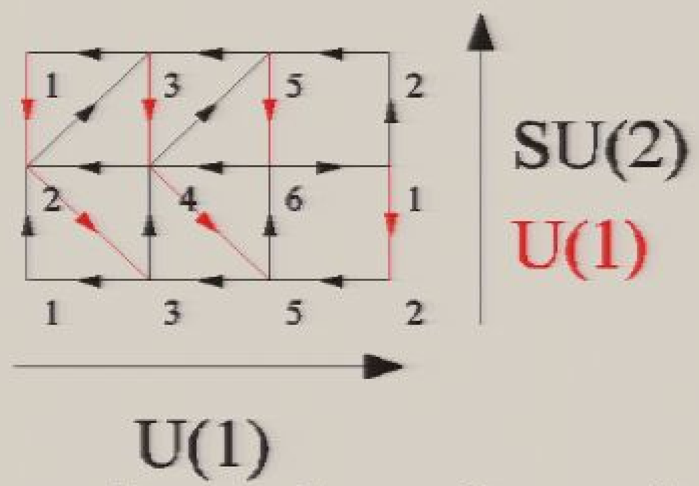

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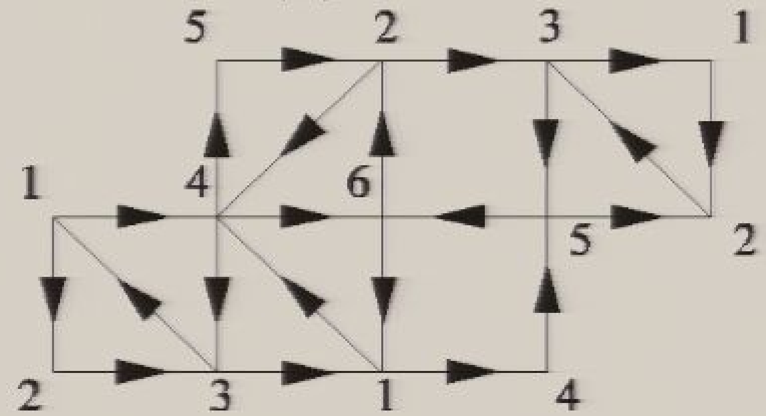

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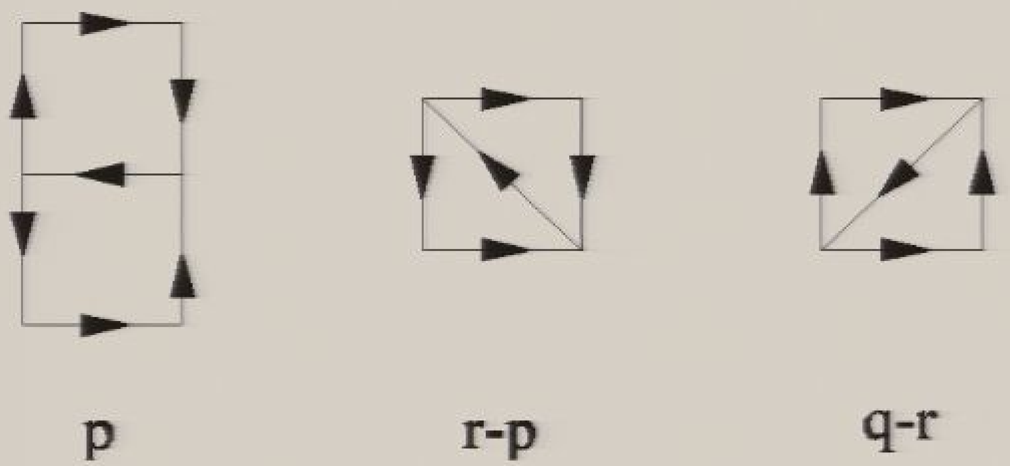

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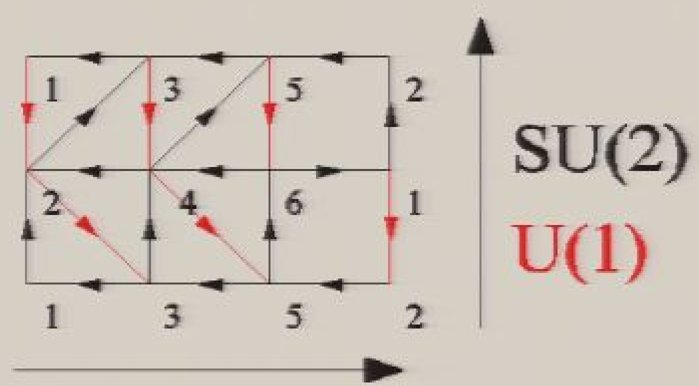

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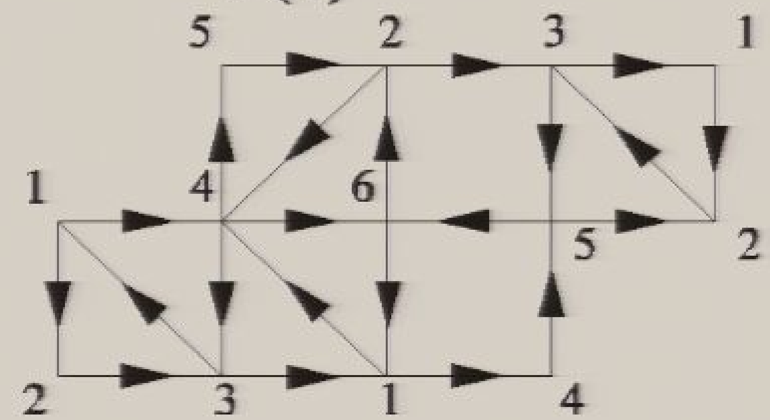

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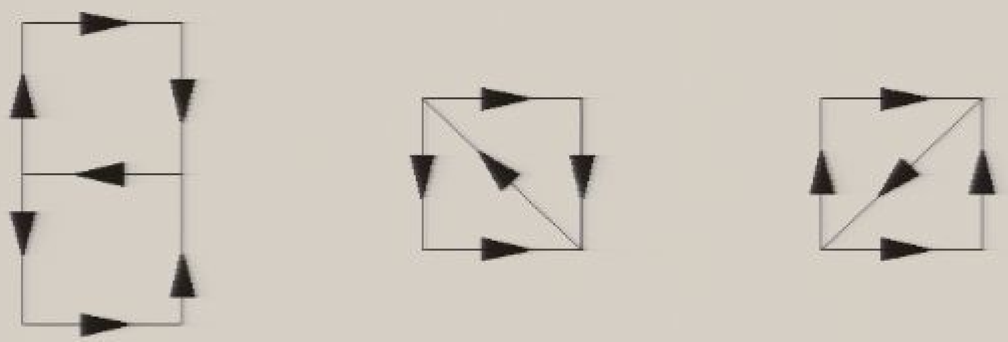

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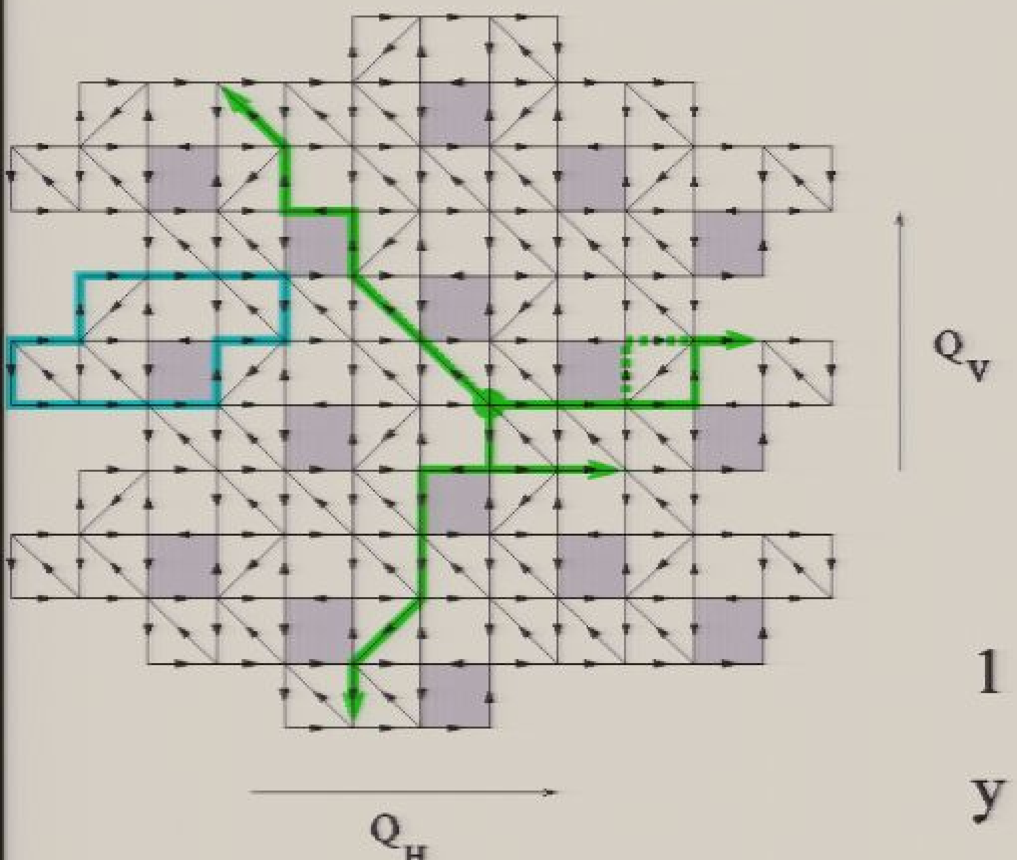

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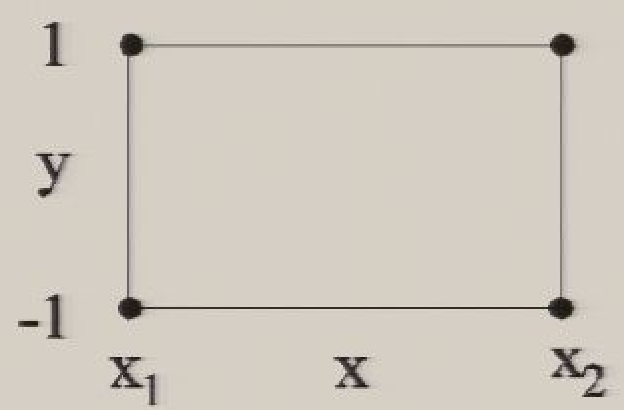

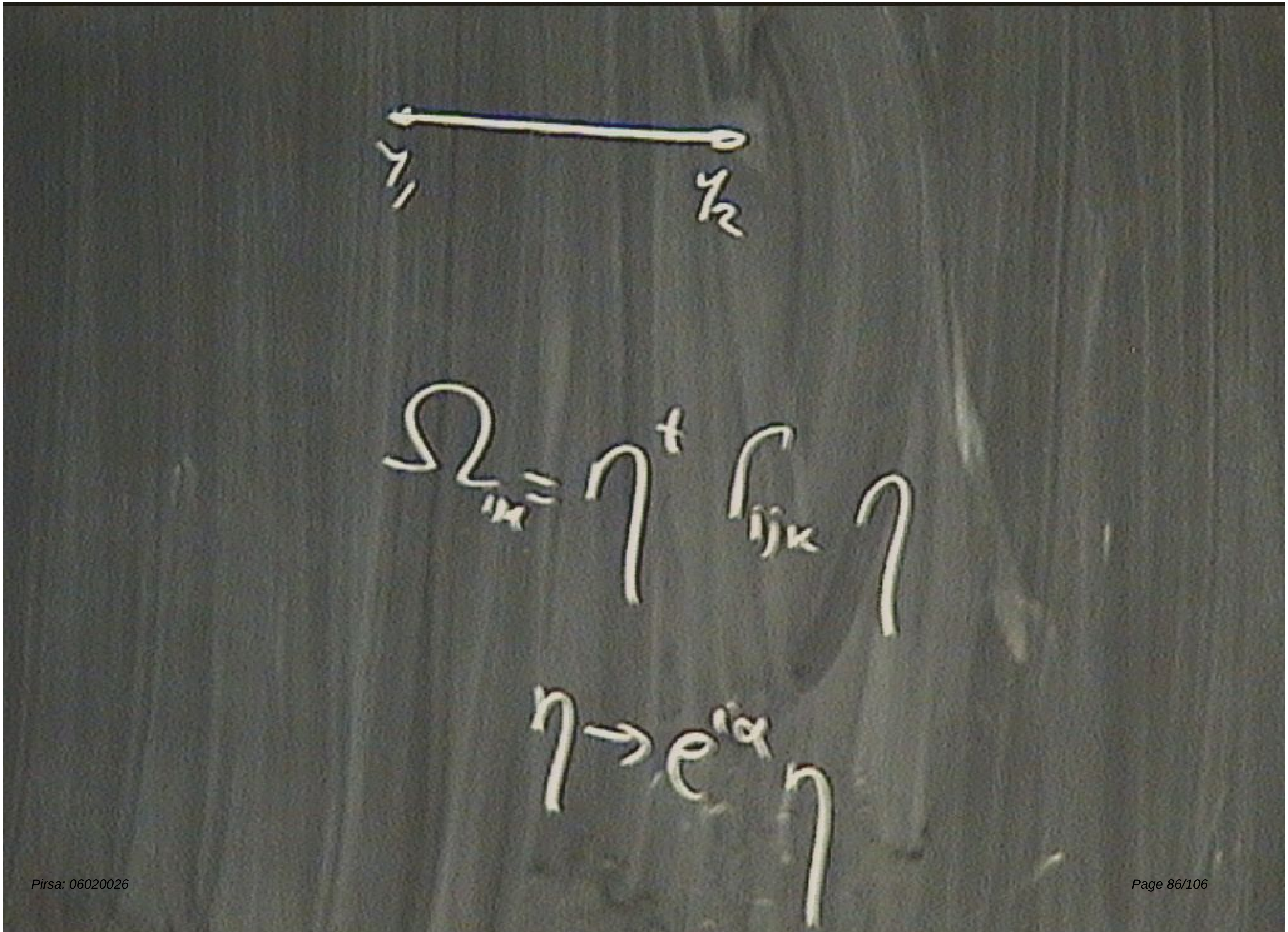
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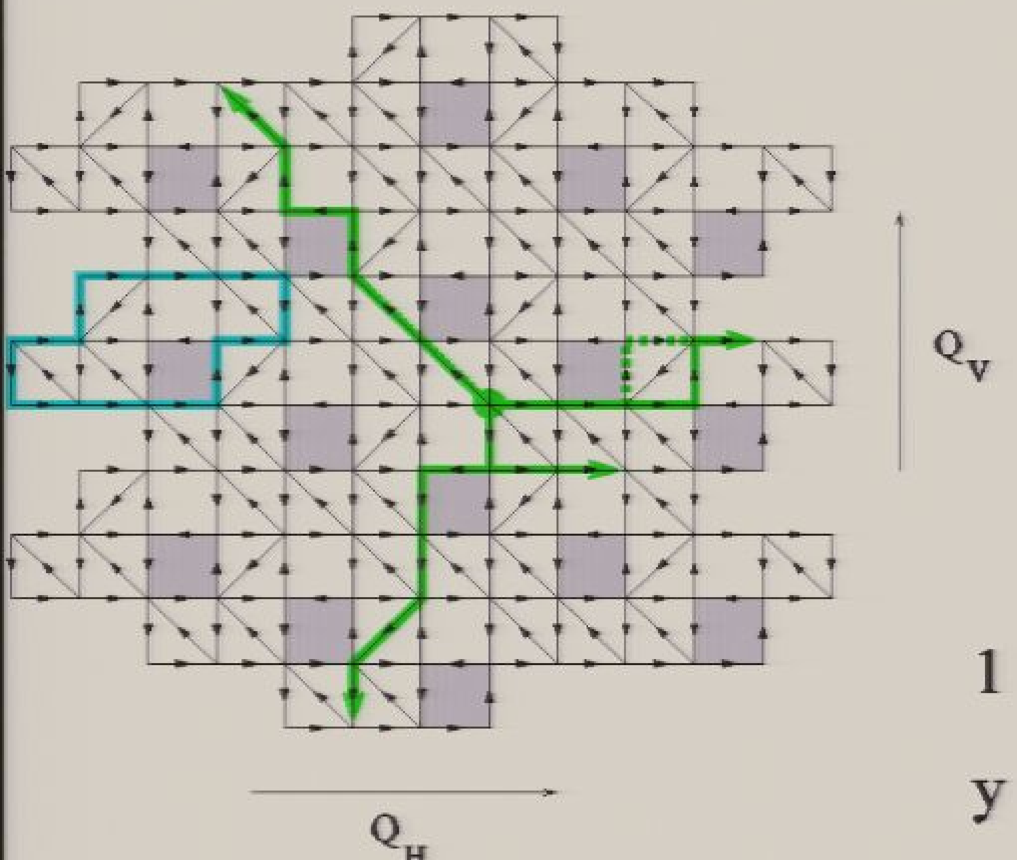

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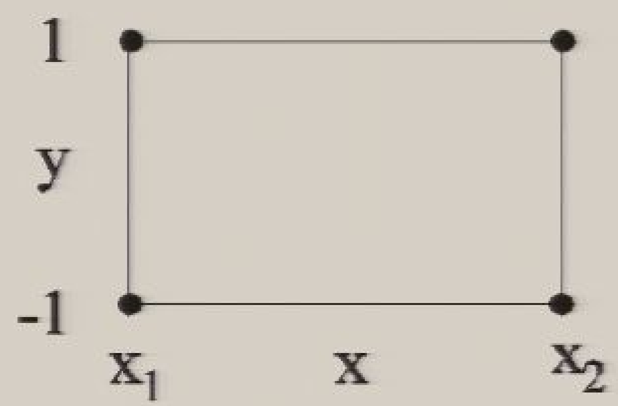

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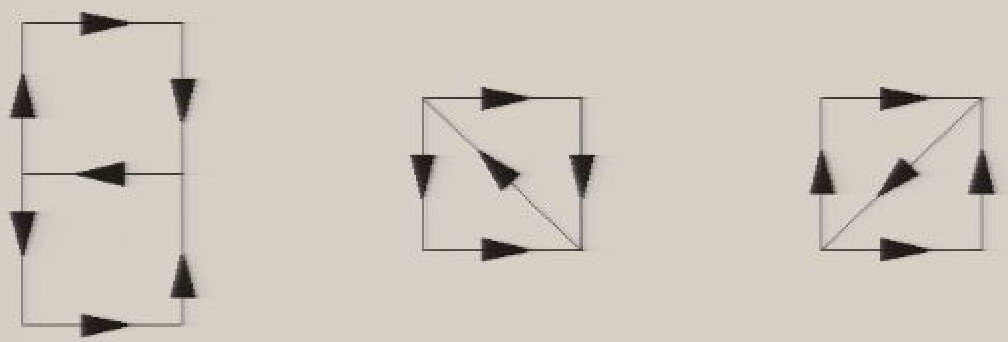

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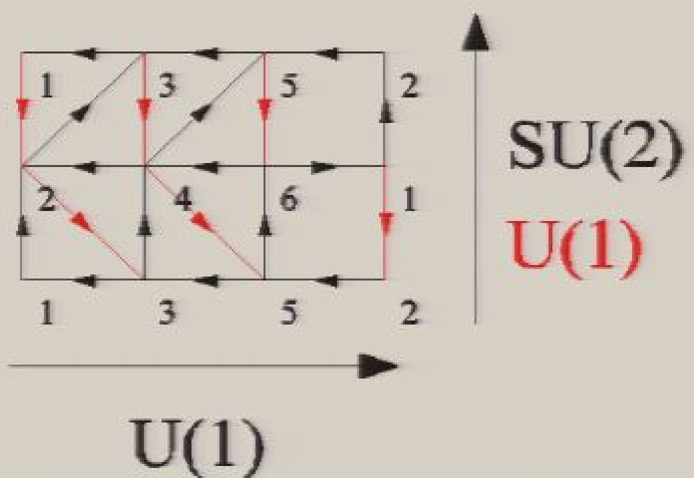

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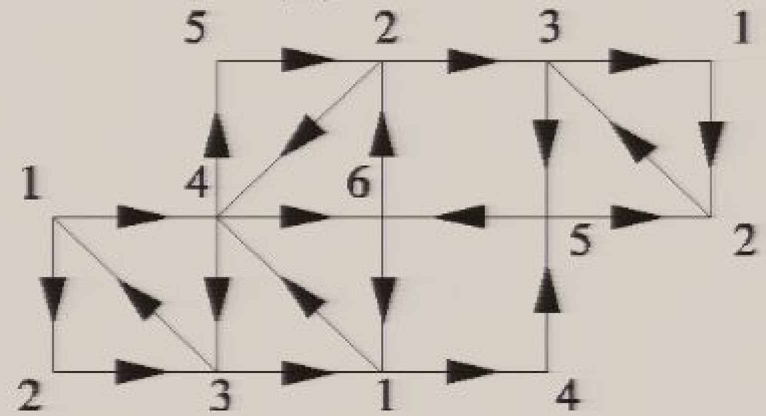

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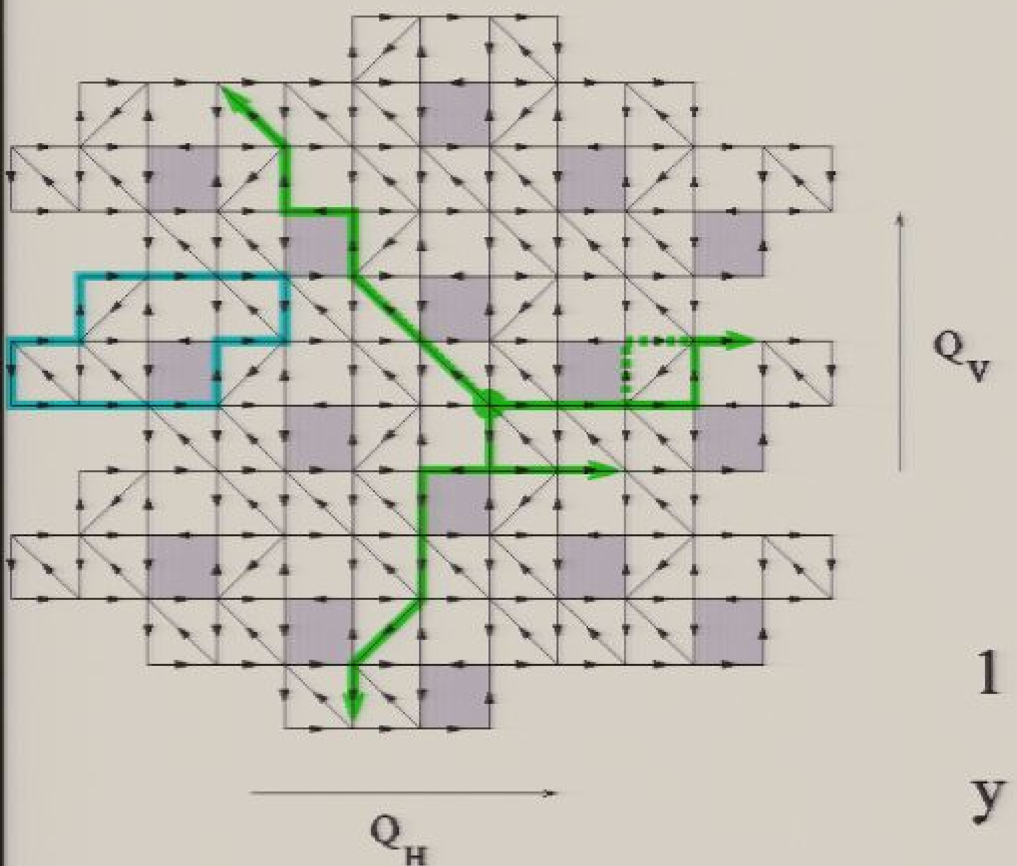

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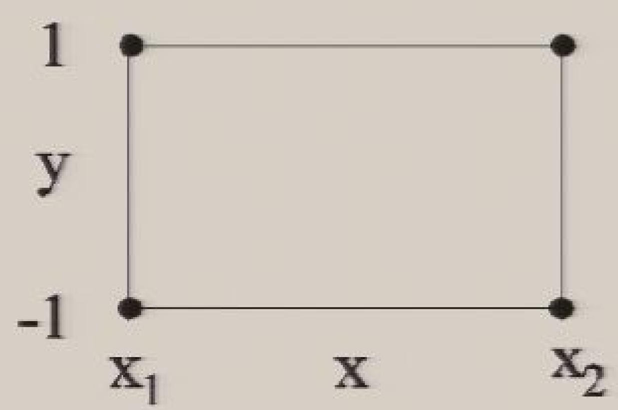

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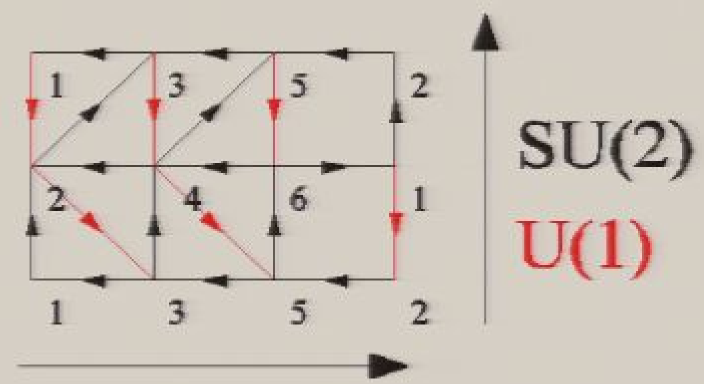

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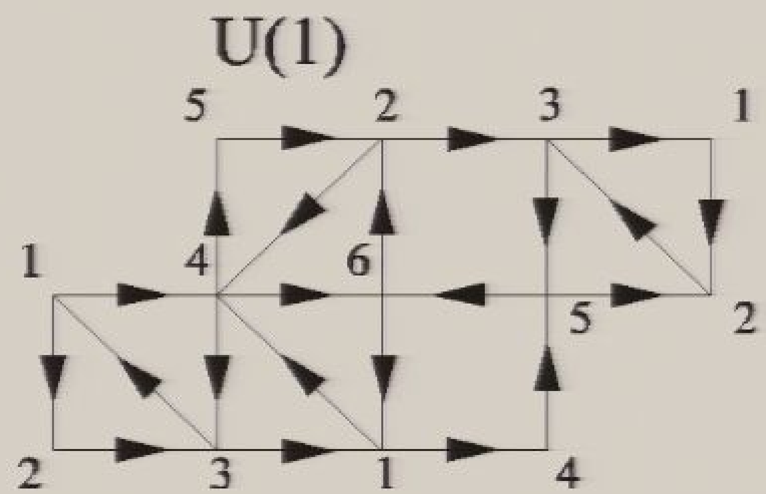

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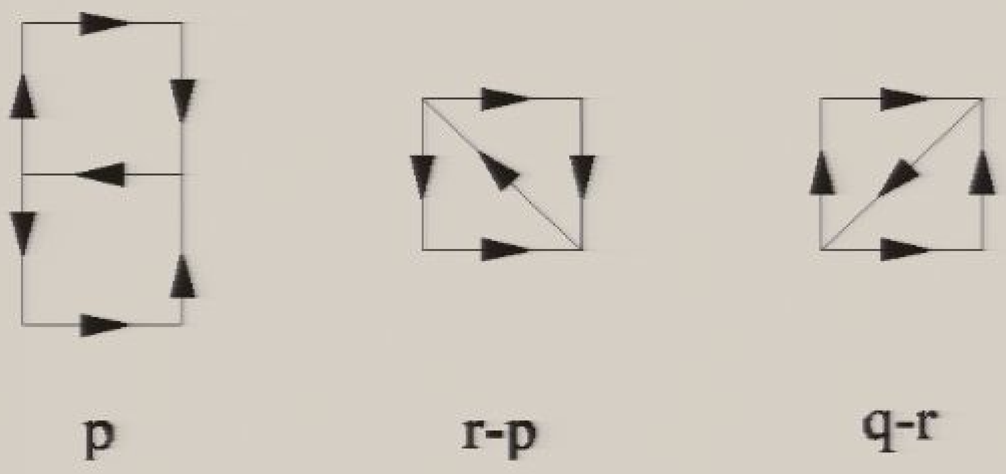

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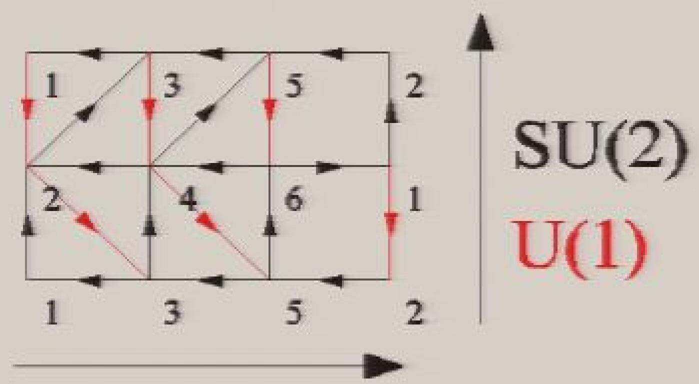

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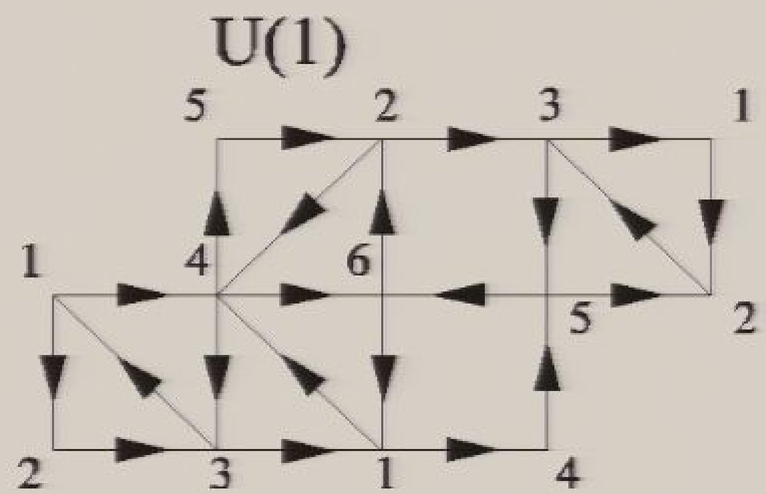

Conclusions

$Y^{p,q}$

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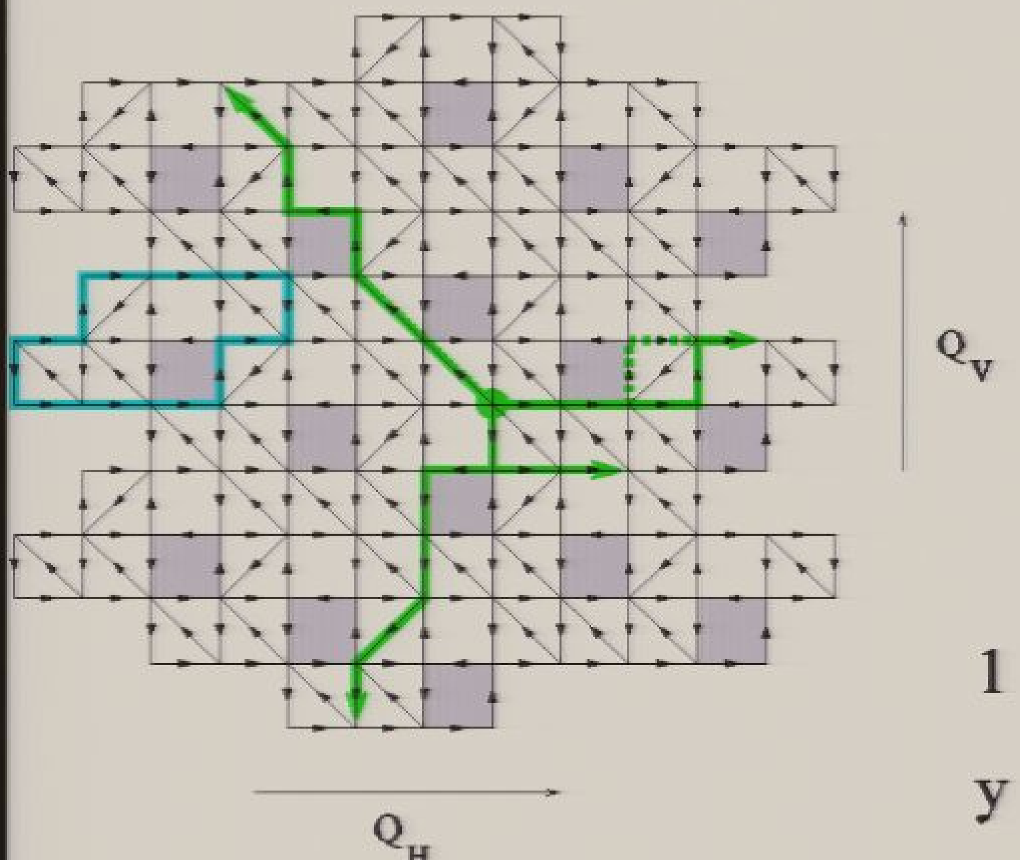

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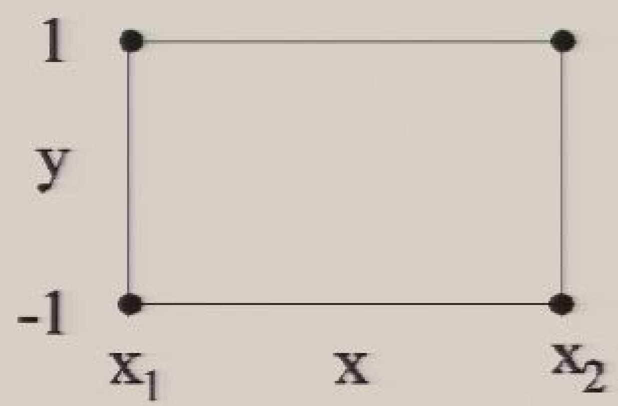

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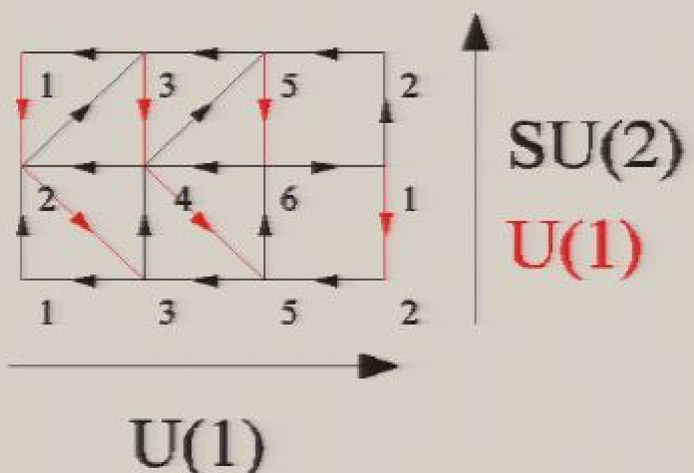

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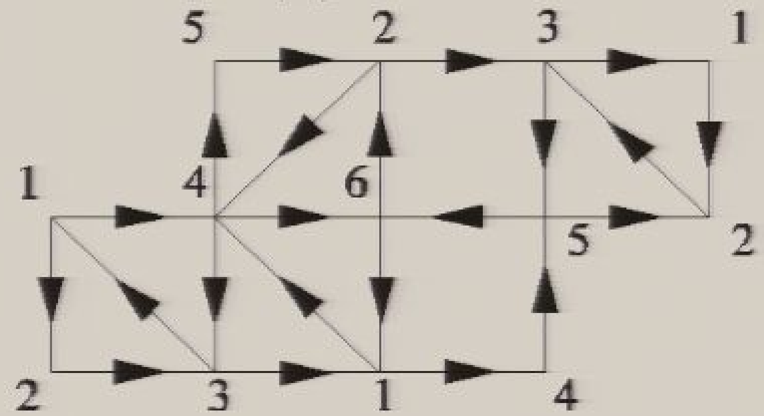

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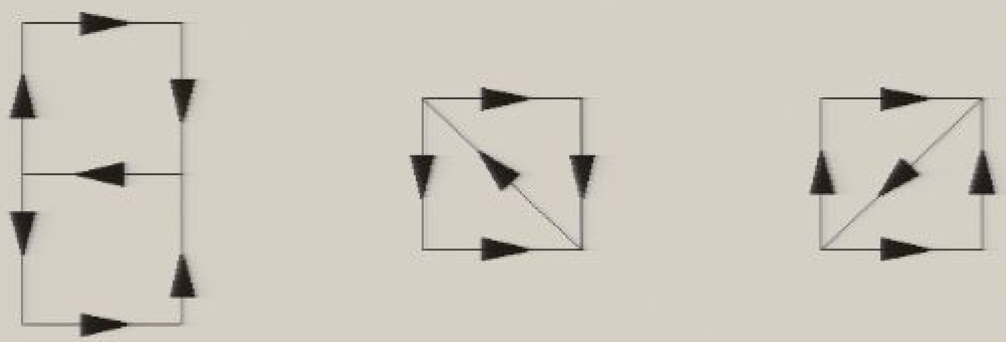

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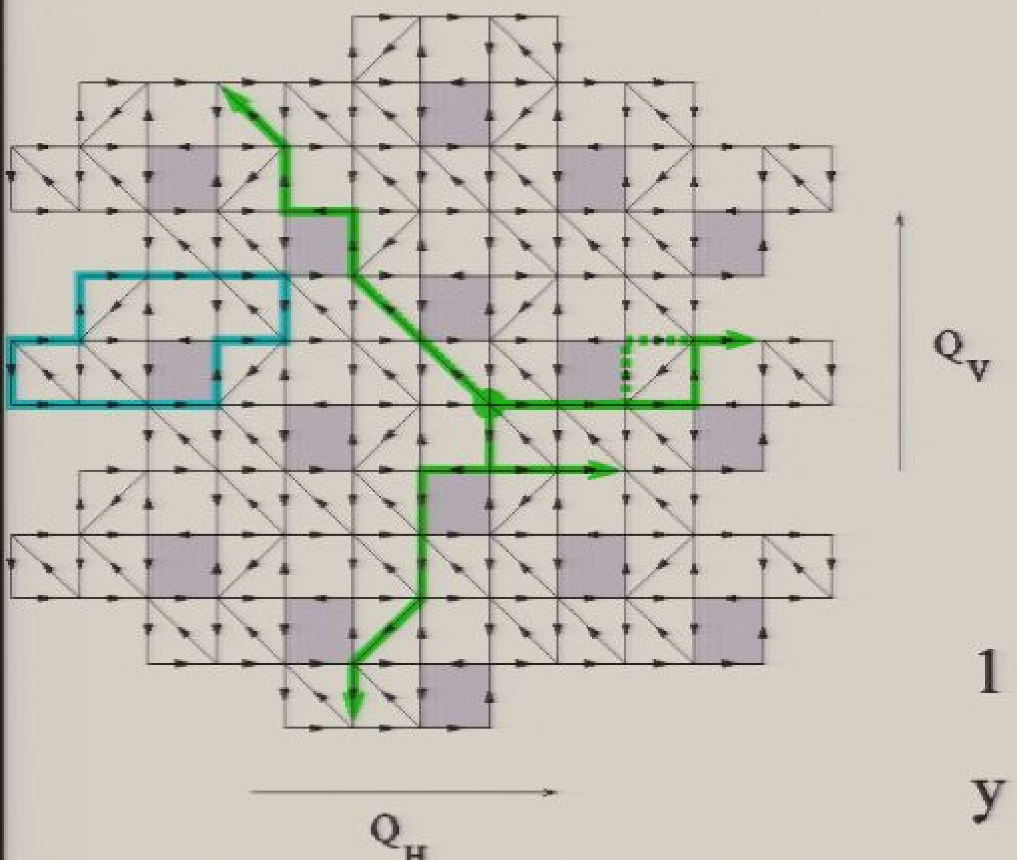

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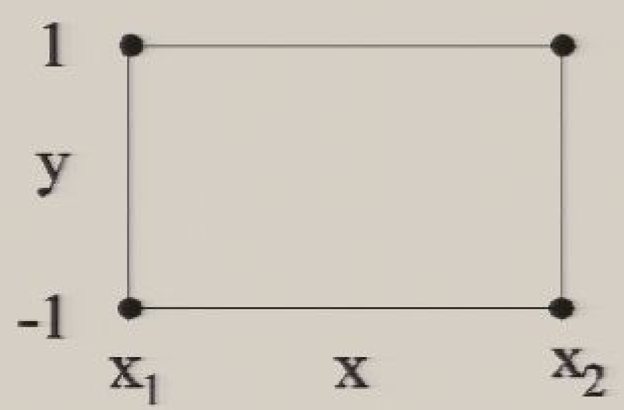

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End of slide show, click to exit.

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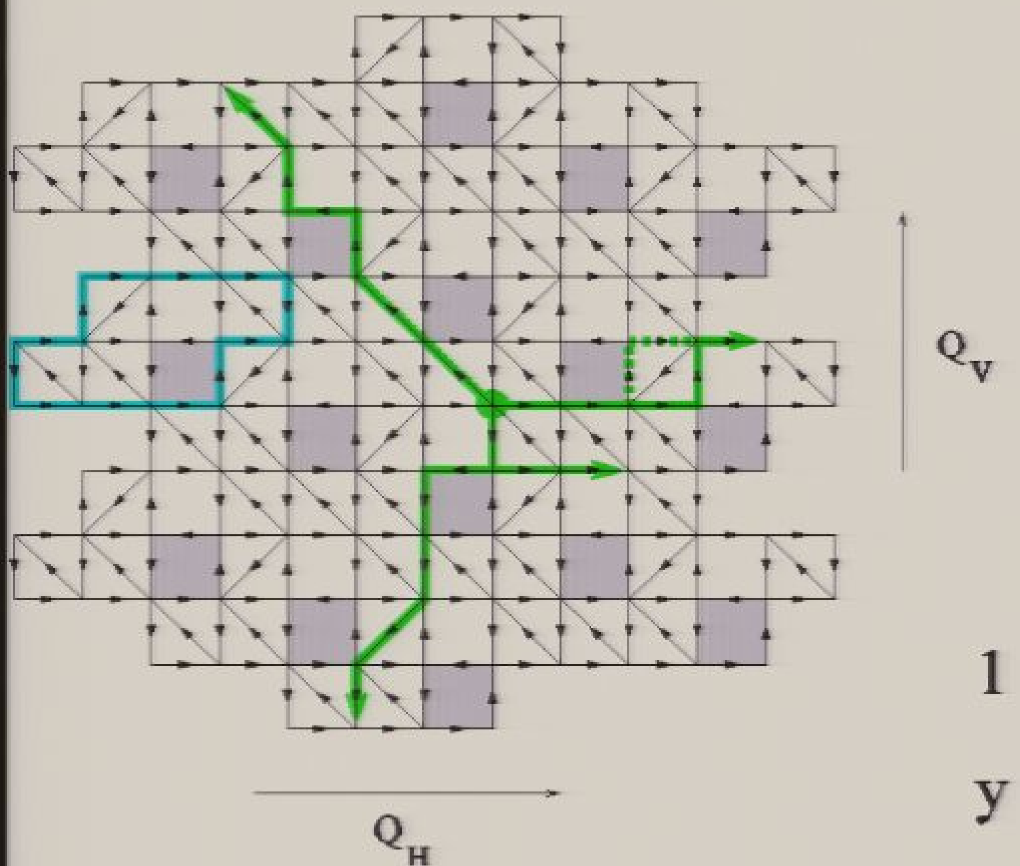

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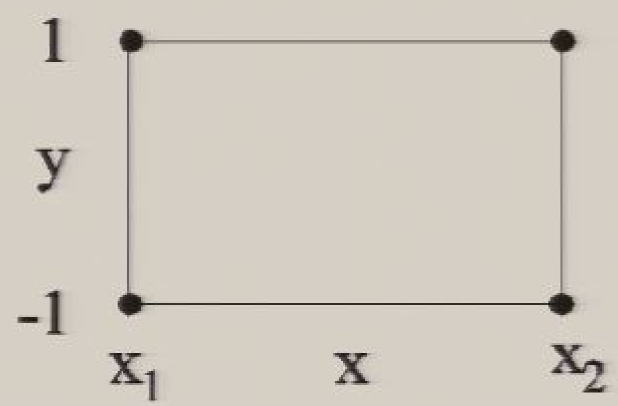

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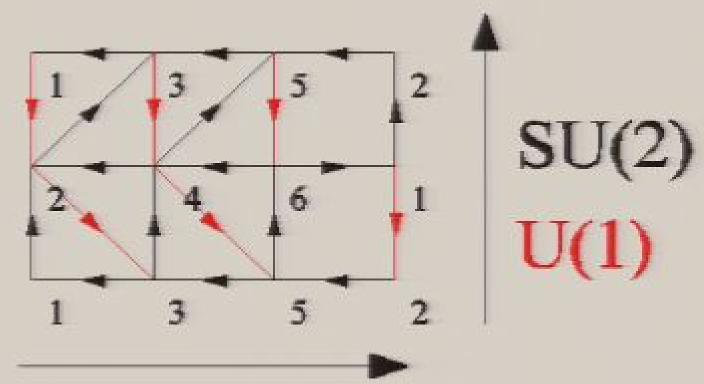
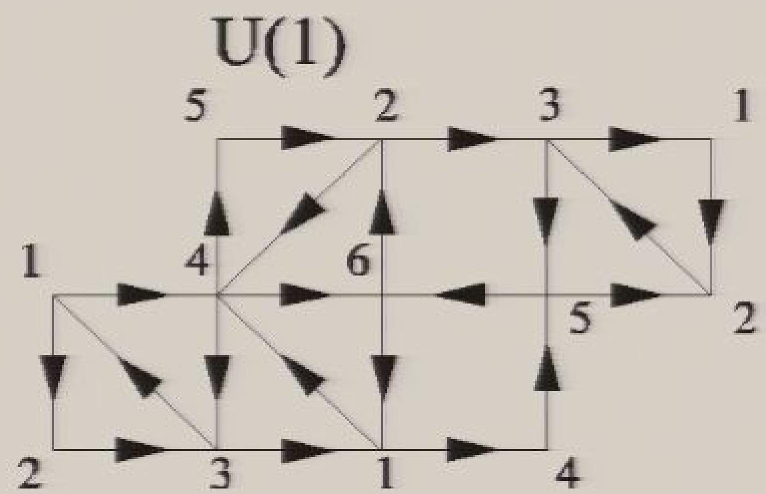
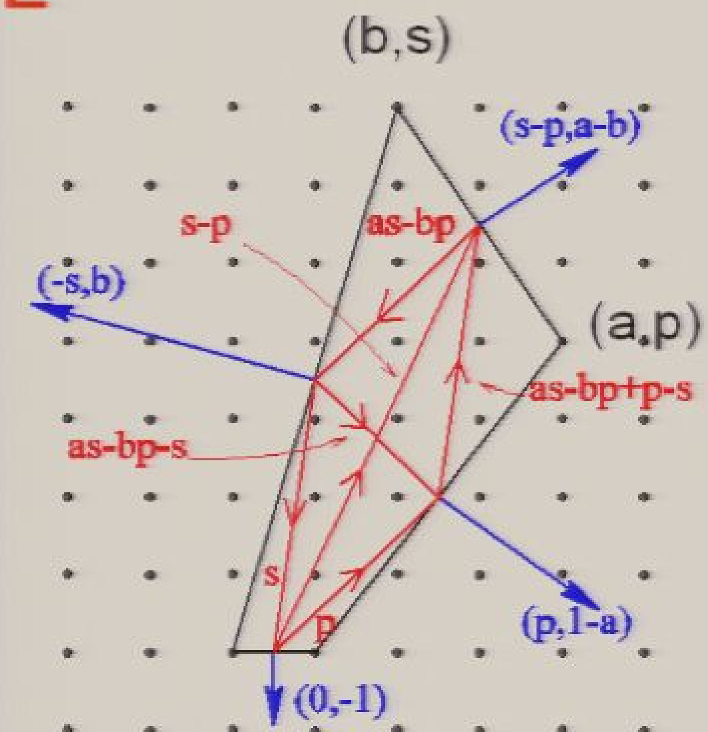
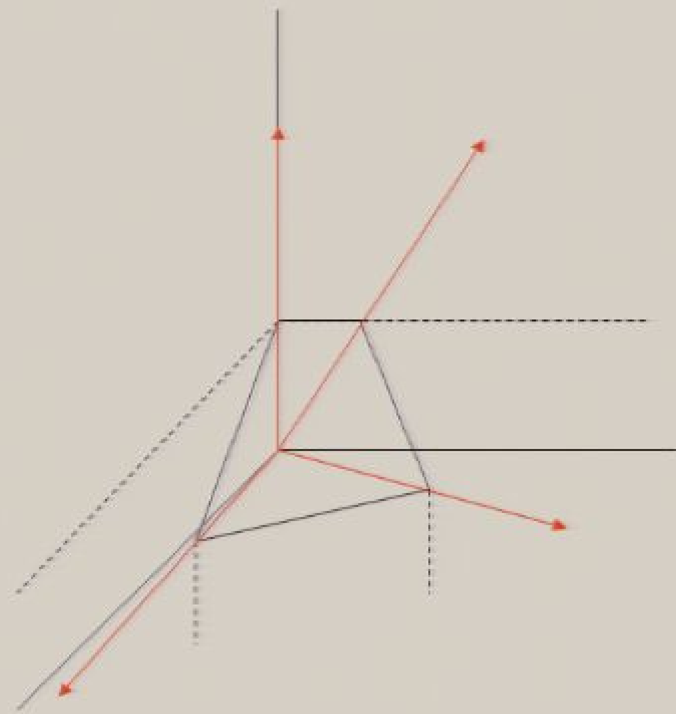


$L^{1,5\,|\,4}$



$L^{pq|r}$

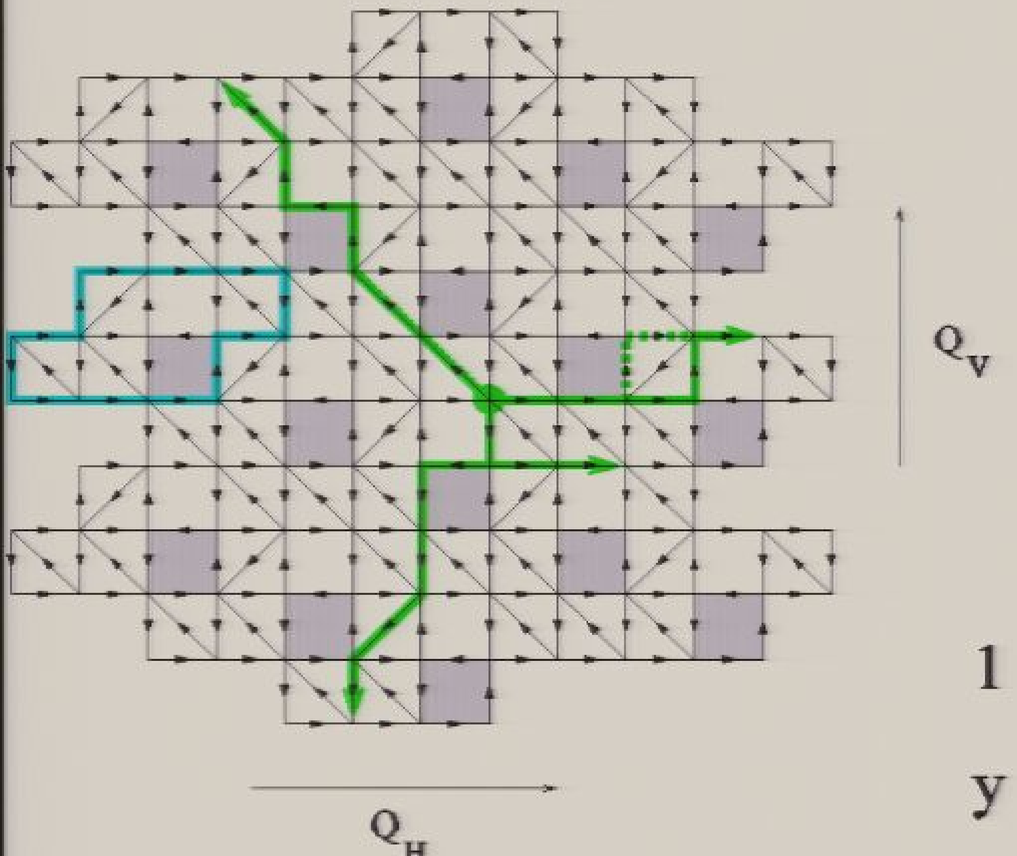




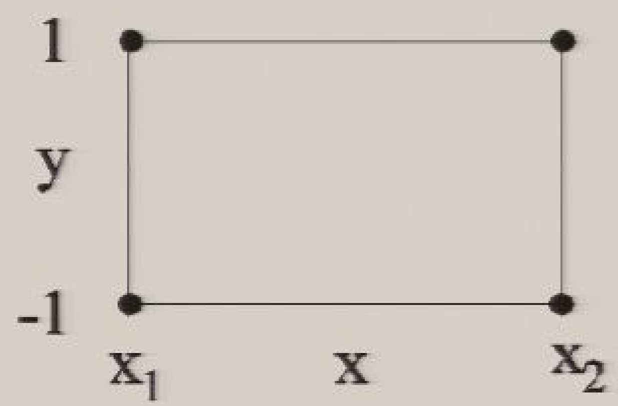
$r = p + q - s \qquad q = a\,s - p\,b$

(Also Butti, Zaffaroni, generic polygon)

Long chiral primaries:



can be matched to geodesics at the "corners":



Conclusions

$Y^{p,q}$

- Massless BPS geod. $\leftrightarrow$ Chiral primaries.
- Non-BPS geodesics $\leftrightarrow$ Current insertions.
- Extended strings $\leftrightarrow$ Long operators.
- Effective action emerges in f.t. Similar but not equal to bulk eff. action.

$L^{p,q|r}$

- Found dual gauge theories.
- Matched BPS geodesics with long chiral primaries.
- Strings? Three dimensional quiver?