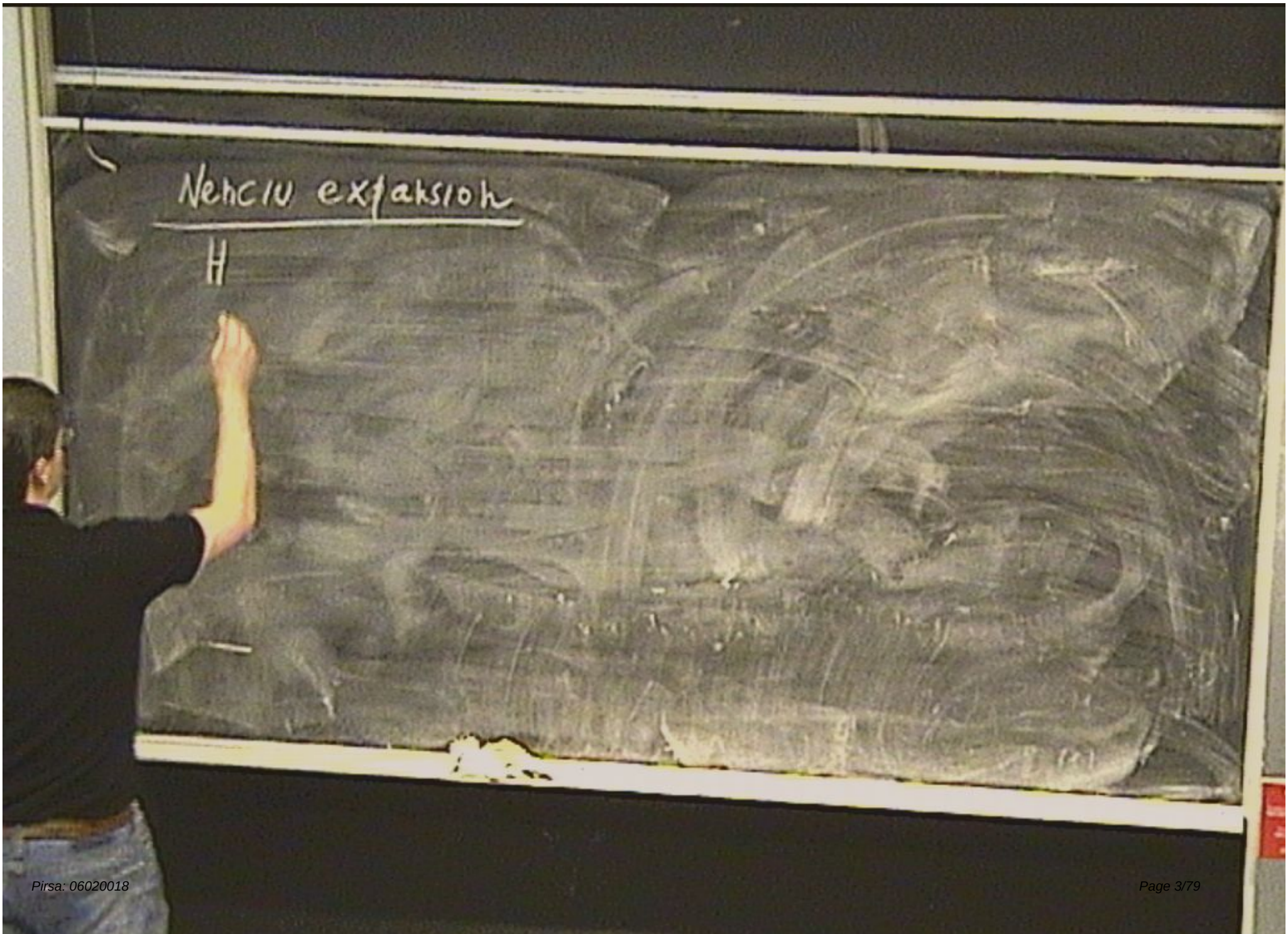
Title: Deviations from adiabatic approximation

Date: Feb 09, 2006 04:00 PM

URL: http://pirsa.org/06020018

Abstract:

Newton expansion


Newton expansion
H

Nenciu expansion

Heisenberg picture

Neumann expansion

Heisenberg picture

IVP : $\begin{cases} i\dot{\rho}_\tau = \tau[\end{cases}$

Nenciu expansion

Heisenberg picture

IVP : $\left\{ i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \right.$

Nehciu expansion

Heisenberg picture

IVP :
$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) \end{cases}$$

Nelson expansion

Heisenberg picture

IVP :
$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

Nenciu expansion

Heisenberg picture

IVP:
$$\begin{cases} \dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

ρ_0 - spectral projection of $H(0)$

Nenciu expansion

Heisenberg picture

IVP :
$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

ρ_0 - spectral projection of $H(0)$

H_0

Nehciu expansion

Heisenberg picture

IVP :
$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

ρ_0 = spectral projection of $H(0)$

Nehciu expansion

Heisenberg picture

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ρ_0 - spectral projection of $H(0)$

Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

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$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

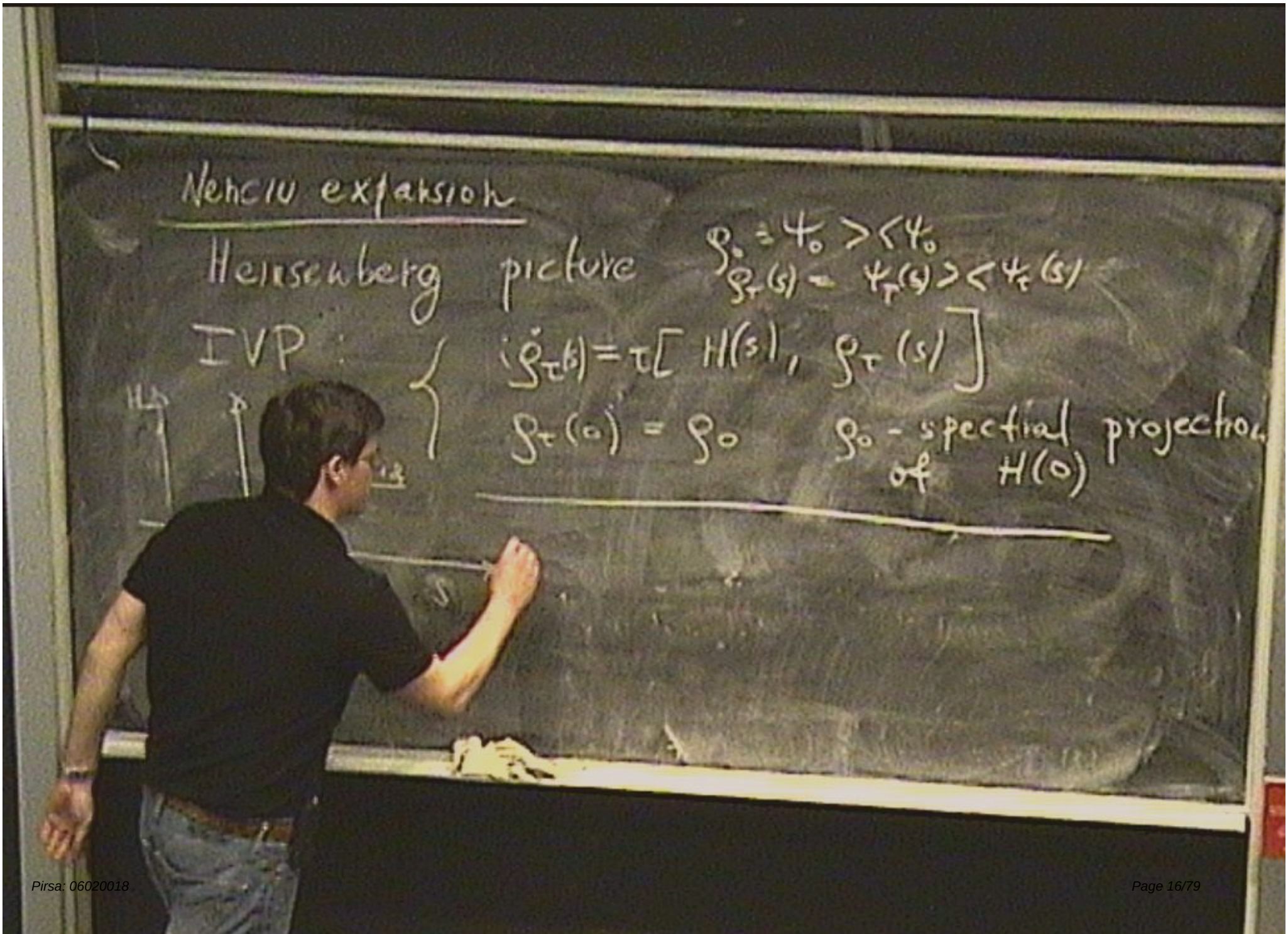
ρ_0 - spectral projection of $H(0)$

Newiu expansion

Heisenberg picture $\rho_0 = \psi_0 > < \psi_0$

$\rho_T(s) = \psi_T(s) > < \psi_T(s)$

IVP : $\begin{cases} i\dot{\rho}_T(s) = T[\, H(s), \rho_T(s) \,] \\ \rho_T(0) = \rho_0 \end{cases}$ ρ_0 - spectral projection of $H(0)$


Nenciu expansion
Heisenberg picture
$\rho_0 = \psi_0 > < \psi_0$
$\rho_\tau(s) = \psi_\tau(s) > < \psi_\tau(s)$
IVP :
$i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)]$
$\rho_\tau(0) = \rho_0$
ρ_0 - spectral projection of $H(0)$

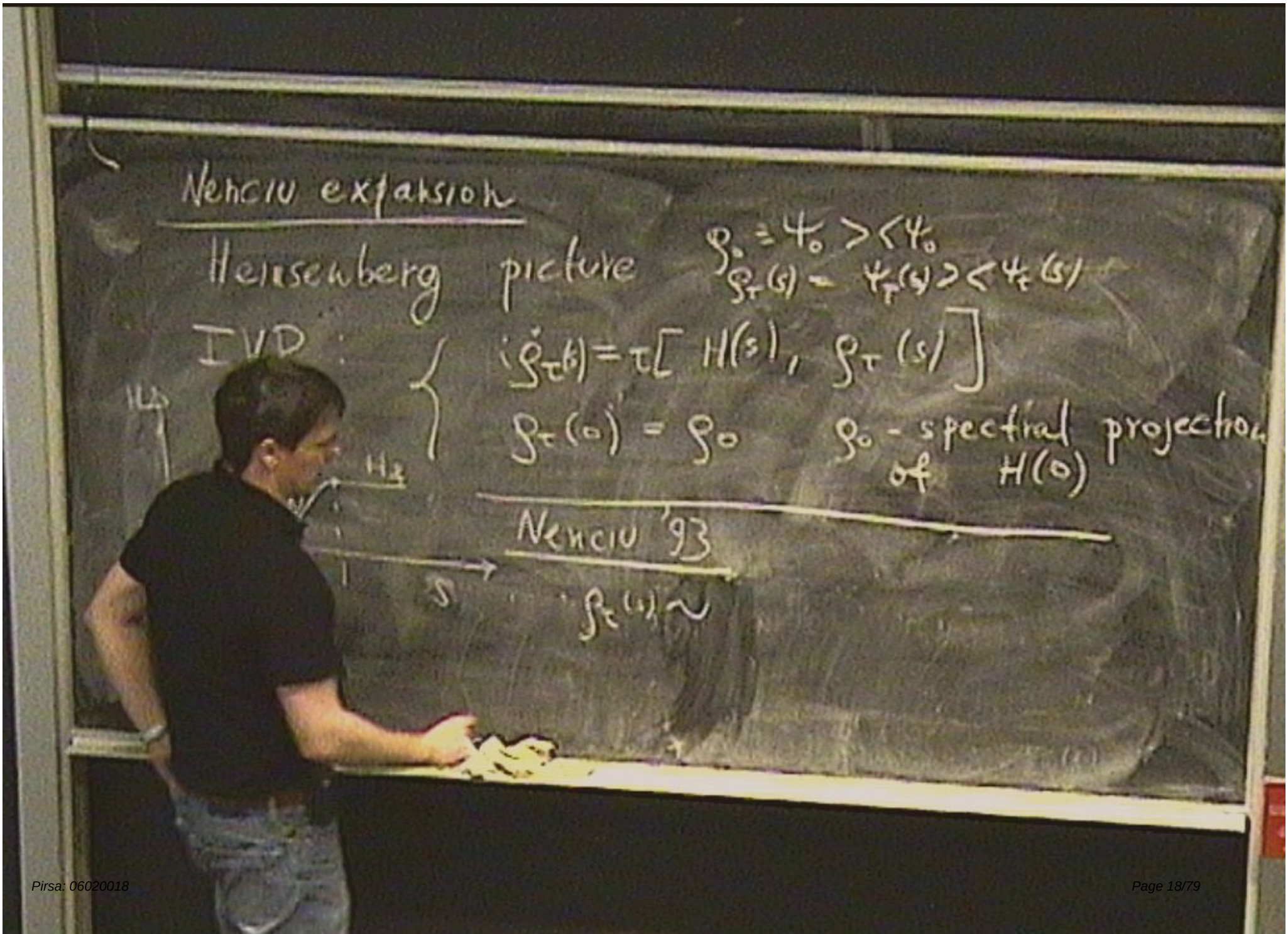
Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$

IVP : $\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$ ρ_0 - spectral projection of $H(0)$

Nenciu '93


Nenciu expansion
Heisenberg picture
$\rho_0 = \psi_0 > < \psi_0$
$\rho_\tau(s) = \psi_\tau(s) > < \psi_\tau(s)$
IVP :
$i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)]$
$\rho_\tau(0) = \rho_0$
ρ_0 - spectral projection of $H(0)$
Nenciu '93
$\rho_\tau(s) \sim$
H_1
H_2
s

Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

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ρ_0 - spectral projection of $H(0)$

Nenciu '93

$$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \ldots$$

Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

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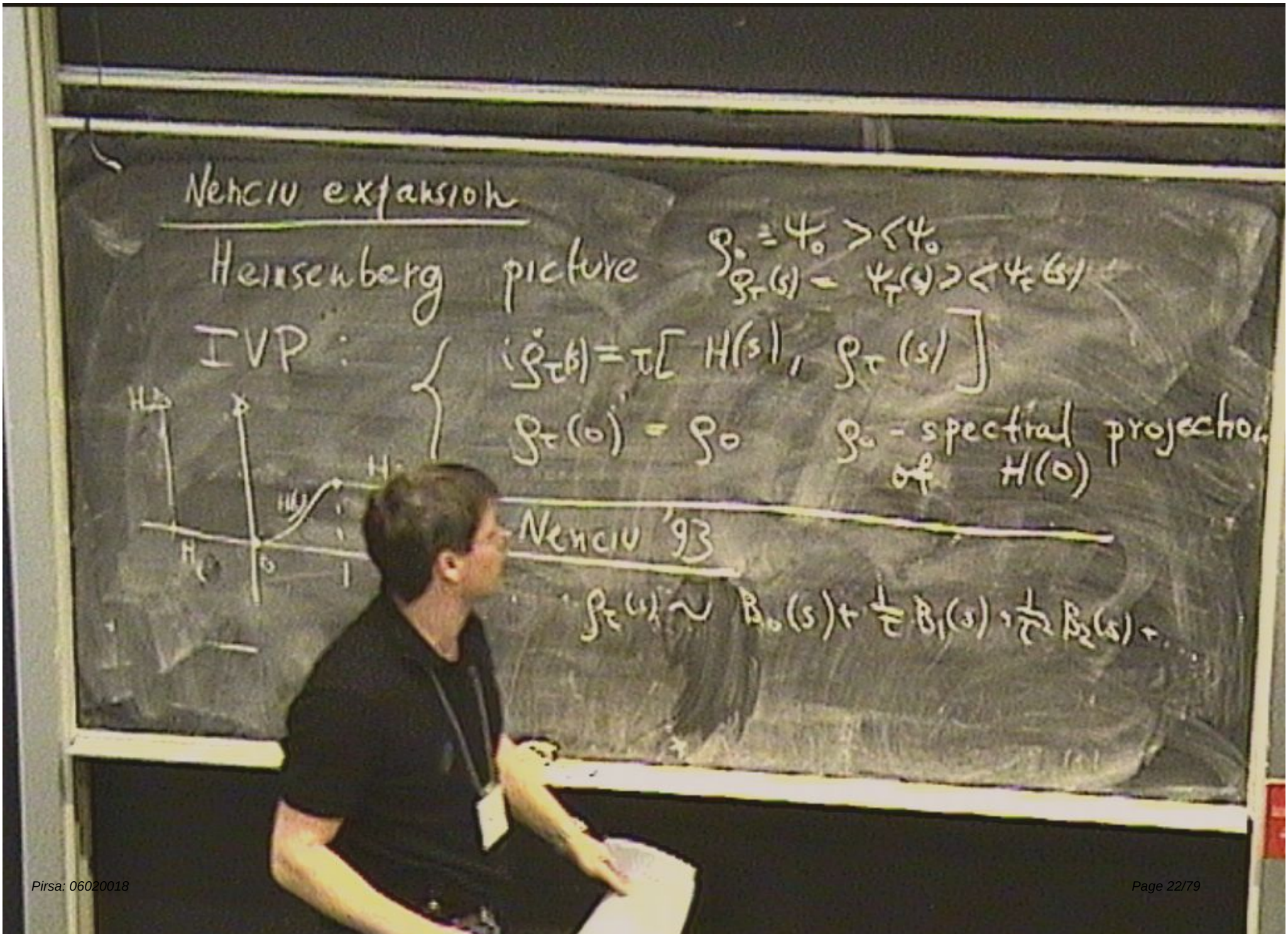
$$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \dots$$

$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$


Nenciu expansion
Heisenberg picture
$\rho_0 = \psi_0 > < \psi_0$
$\rho_\tau(s) = \psi_\tau(s) > < \psi_\tau(s)$
IVP :
$i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)]$
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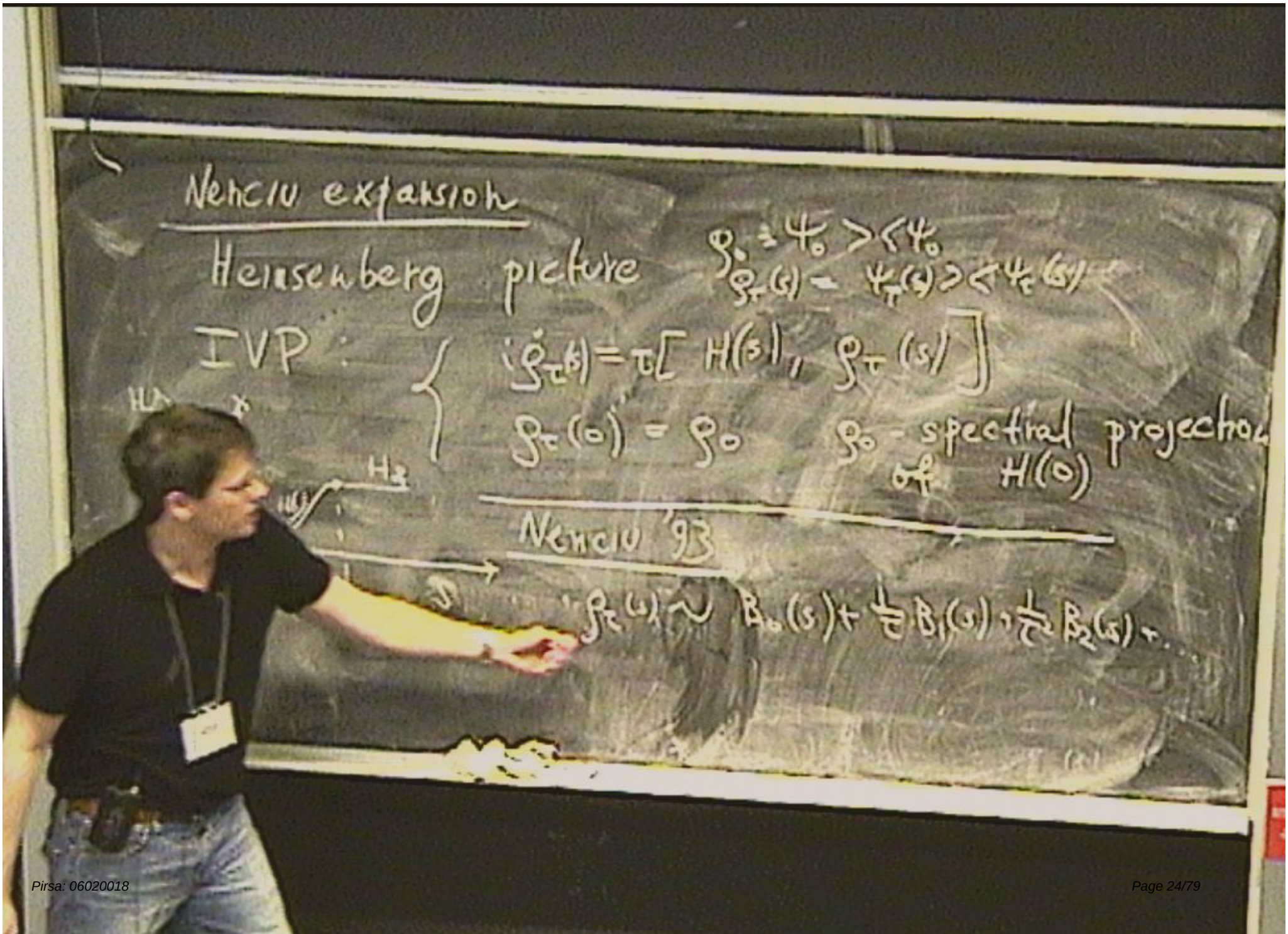
ρ_0 - spectral projection of $H(0)$
Nenciu '93
$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \ldots$

$$i\dot{\mathcal{S}}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

$$(*) \quad i\,\dot{B}_j(s) = \left[H(s), B_{j+1}(s)\right]$$


Nenciu expansion
Heisenberg picture
$\rho_0 = \psi_0 \rangle\langle \psi_0$
$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$
IVP :
$\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)]$
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$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \ldots$

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Solution of hier. is not unique!

$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

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Solution of hier. is not unique!

$\rho_\tau(s)$ is a projection if ρ_0 is a projection!

Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$

IVP:
$$\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

ρ_0 - spectral projection of $H(0)$

Nenciu '93

$$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \dots$$

$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

(*) $i\dot{B}_j(s) = [H(s), B_{j+1}(s)]$

Solution of hier. is not unique!

$\rho_\tau(s)$ is a projection if ρ_0 is a projection!

$\rho_\tau^2(s)$

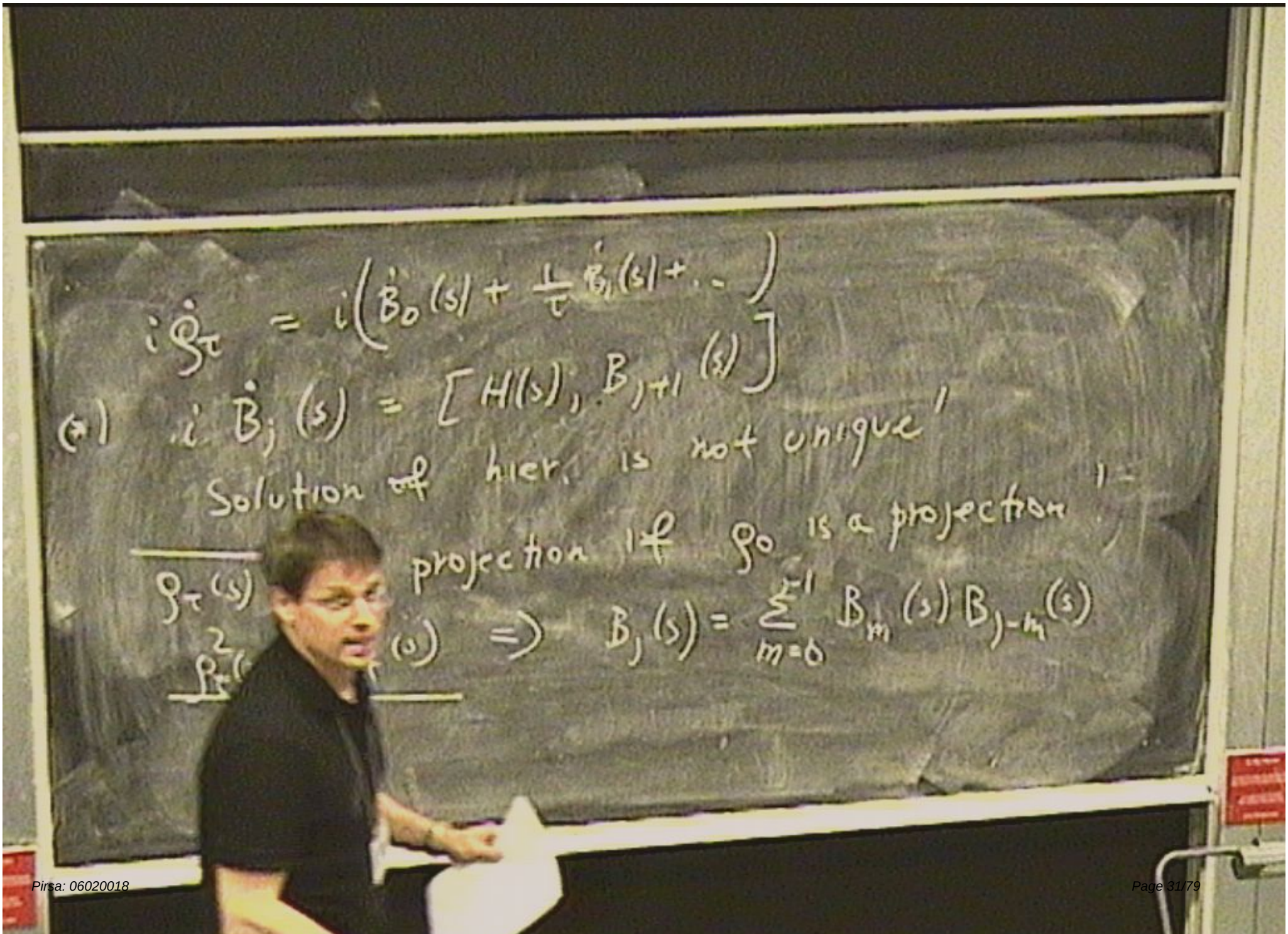
$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

(*) $i\dot{B}_j(s) = [H(s), B_{j+1}(s)]$

Solution of hier. is not unique!

$\rho_\tau(s)$ is a projection if ρ_0 is a projection!

$\rho_\tau^2(s) = \rho_\tau(s) \Rightarrow$


$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$
(*) $i\dot{B}_j(s) = [H(s), B_{j+1}(s)]$
Solution of hier. is not unique!
$\rho_\tau(s)$ projection if ρ_0 is a projection!
$\rho_\tau^2(s) = \rho_\tau(s) \Rightarrow B_j(s) = \sum_{m=0}^{j-1} B_m(s) B_{j-m}(s)$

$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

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Heisenberg picture $\rho_0 = |\psi_0\rangle\langle\psi_0|$

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IVP : $\begin{cases} \dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$ ρ_0 - spectral projection of $H(0)$

Nenciu '93

$$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \dots$$

$$i\dot{\rho}_\tau = i\left(\dot{B}_0(s) + \frac{1}{\tau}\dot{B}_1(s) + \dots\right)$$

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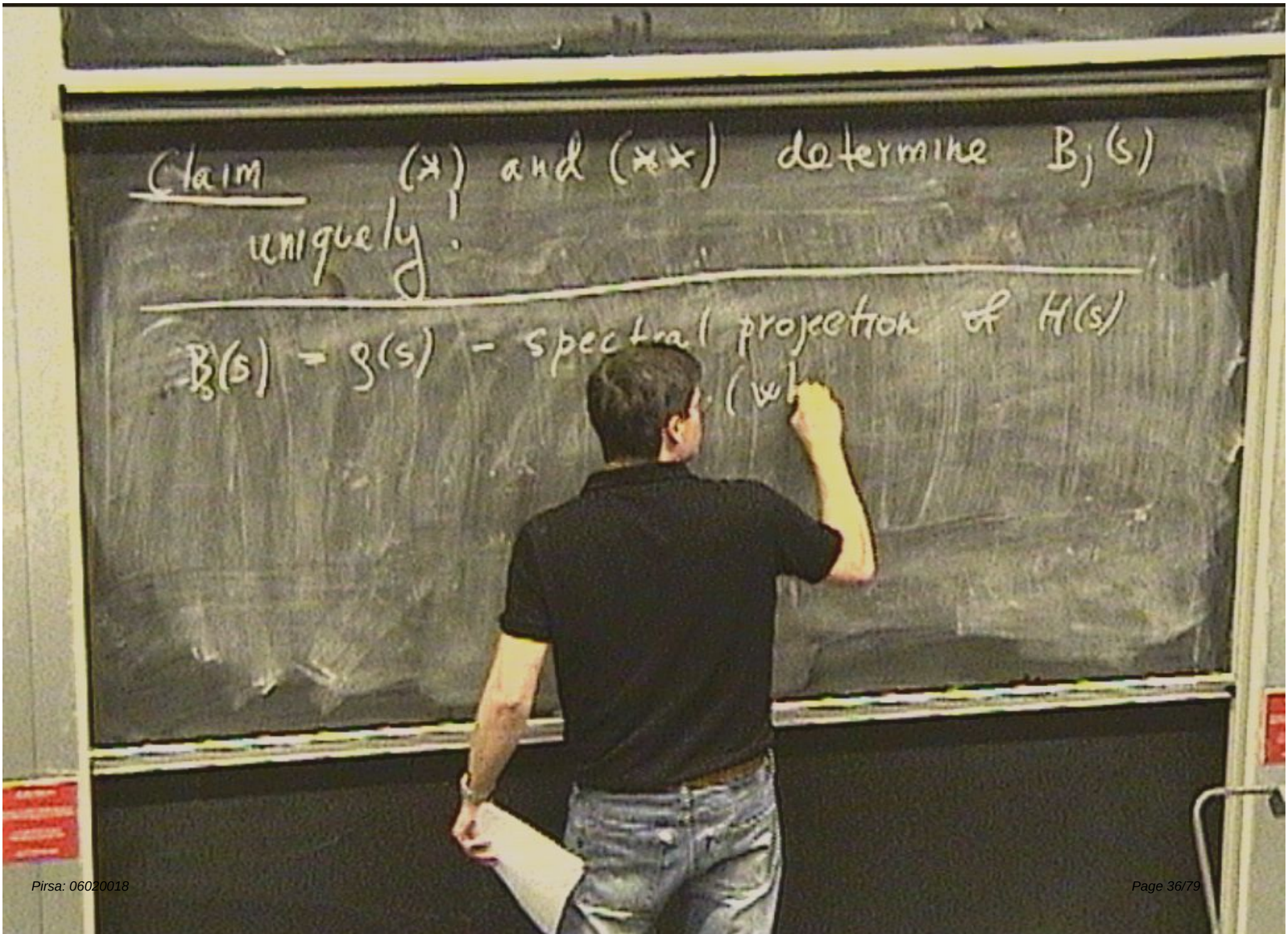
Solution of hier. is not unique!

$\rho_\tau(s)$ is a projection if ρ_0 is a projection!

$$\rho_\tau^2(s) = \rho_\tau(s) \Rightarrow \boxed{B_j(s) = \sum_{m=0}^{j-1} B_m(s) B_{j-m}(s)} \quad (**)$$

<u>Claim</u> (*) and (**) determine $B_j(s)$ uniquely!


Claim (*) and (**) determine $B_j(s)$
uniquely!
$B_j(s) = \rho(s)$ - spectral projection of $H(s)$
(w

<u>Claim</u> (*) and (**) determine $B_j(s)$ uniquely!

$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0)

<u>Claim</u> (*) and (**) determine $B_j(s)$ uniquely!

$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) \left[\rho(s), \dot{B}_{j-1}(s)\right] R_z(s)\, dz$$

$B_0(s) = \rho(s)$ – spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi i} \oint_\Gamma R_z(s) \left[\rho(s), \dot{B}_{j-1}(s)\right] R_z(s)\, dz$$

$$+ S_j(s) - \rho(s) S_j(s) \rho(s),$$ where

$B_0(s) = \rho(s)$ – spectral projection of $H(s)$ (which correspond to ρ_0)

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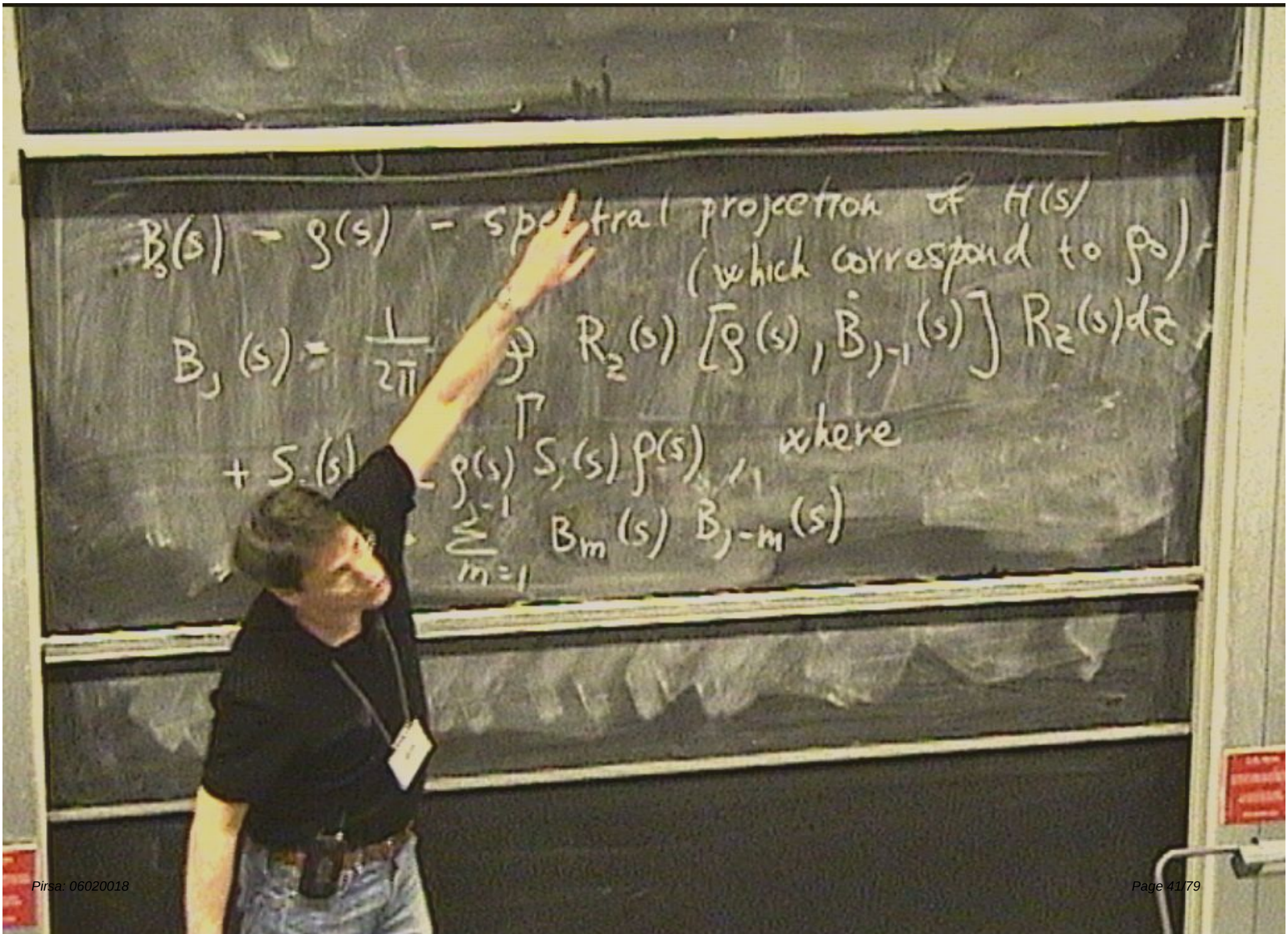
$$+ S_j(s) - 2\rho(s) S_j(s) \rho(s),$$ where

$S_j(s) =$


$B_0(s) = \rho(s)$ - spectral projection of $H(s)$
(which correspond to ρ_0)
$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) [\rho(s), \dot{B}_{j-1}(s)] R_z(s) dz$
$+ S_j(s) - \rho(s) S_j(s) \rho(s)$, where
$\sum_{m=1}^{j-1} B_m(s) \dot{B}_{j-m}(s)$

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$$S_j(s) = \sum_{m=1}^{j-1} B_m(s) \dot{B}_{j-m}(s)$$

$$i\dot{\rho}_\tau = i\left(B_0(s) + \frac{\tau}{2} B_1(s) + \dots\right)$$

(c) $i\dot{B}_j(s) = [H(s), B_{j+1}(s)]$

Solution of hier. is not unique!

$\rho_\tau(s)$ is a projection iff ρ_0 is a projection!

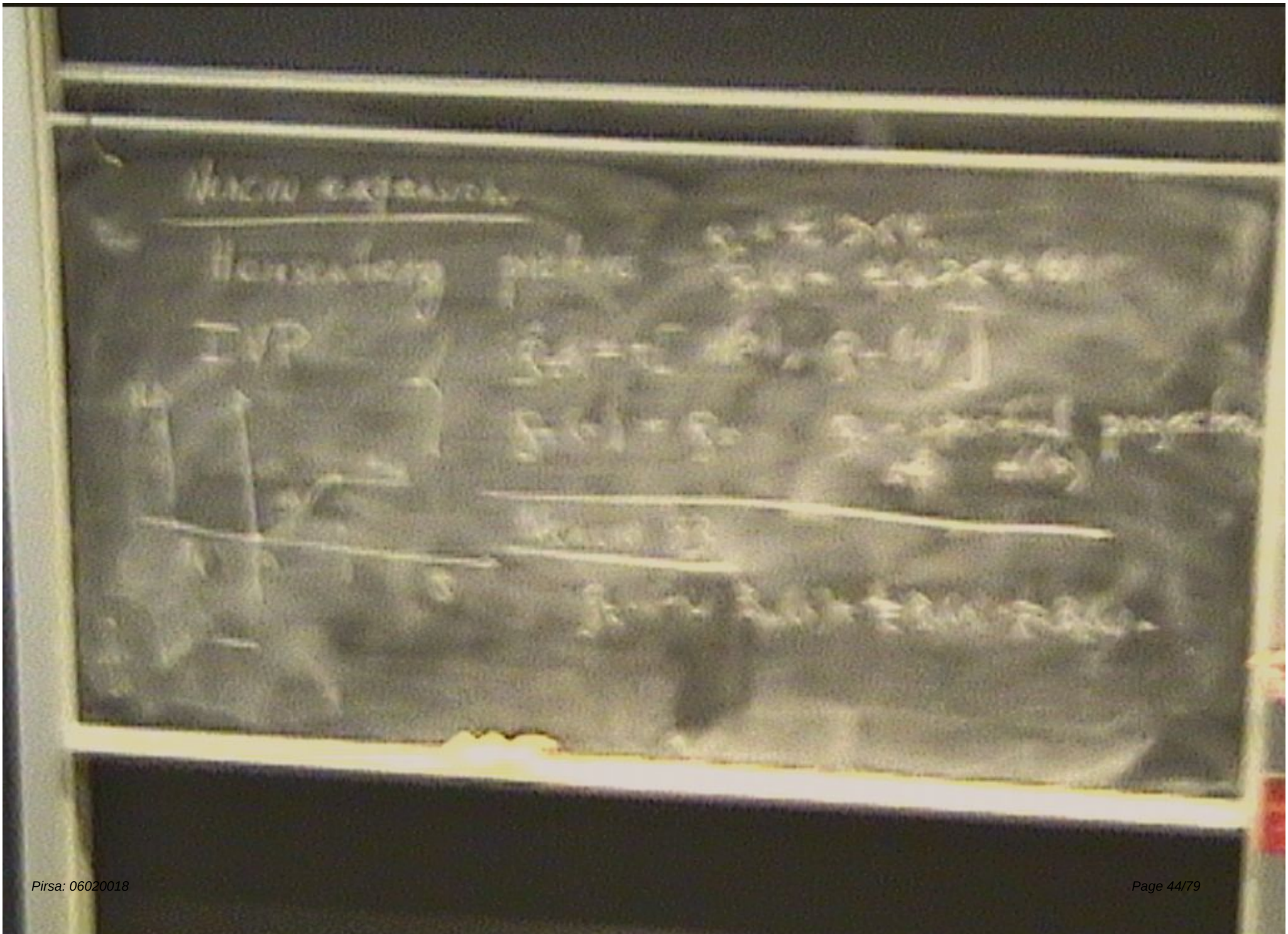
$$\rho_\tau^2(s) = \rho_\tau(s) \Rightarrow \boxed{\dot{B}_j(s) = \sum_{m=0}^{j-1} B_m(s) B_{j-m}(s)} \quad (**)$$

Claim (*) and (**) determine $B_j(s)$ uniquely!

$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) [\rho(s), \dot{B}_{j-1}(s)] R_z(s) dz + S_j(s) - 2\rho(s) S_j(s) \rho(s),$$ where

$$S_j(s) = \sum_{m=1}^{j-1} B_m(s) \dot{B}_{j-m}(s)$$



Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

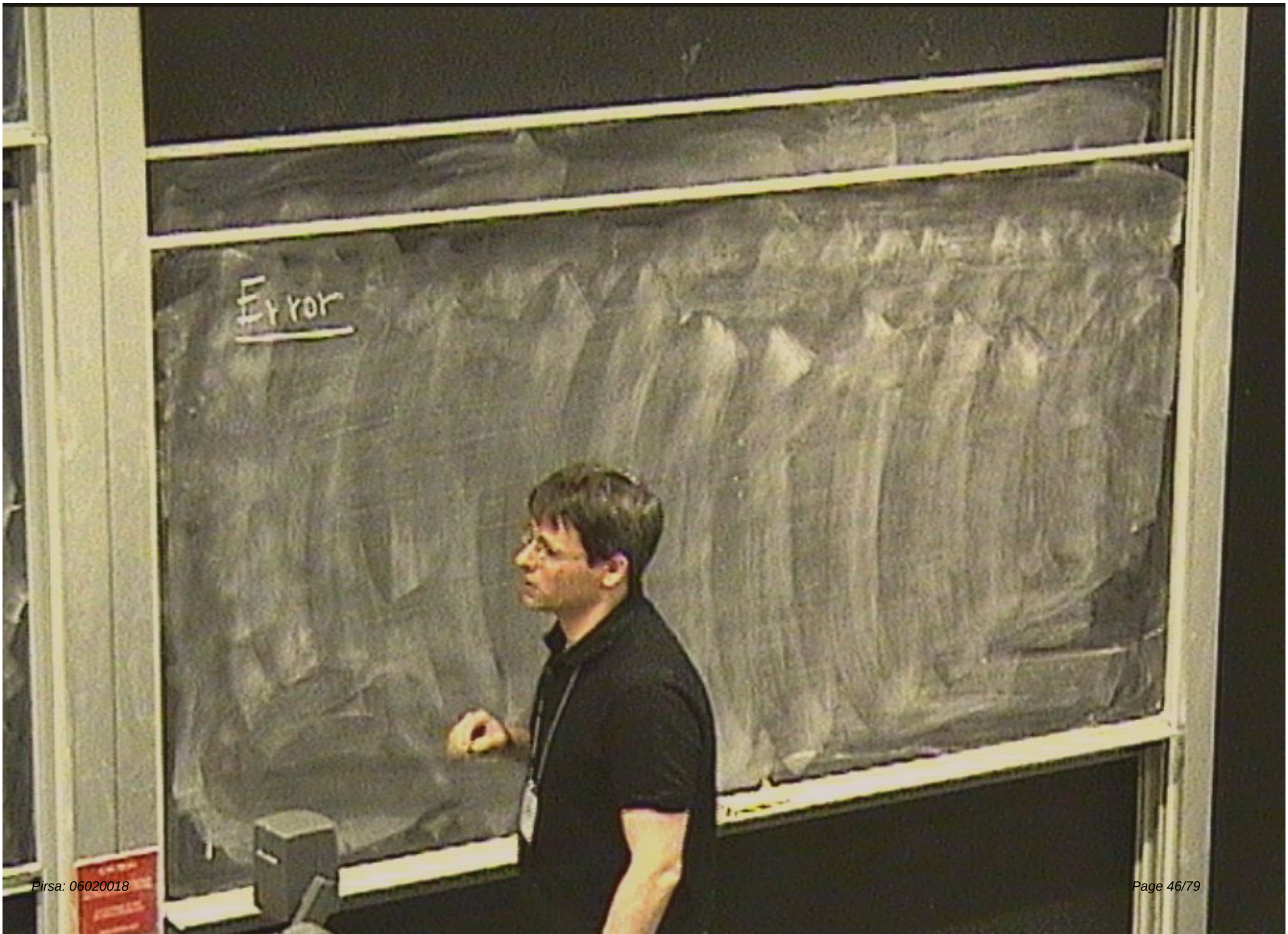
$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$

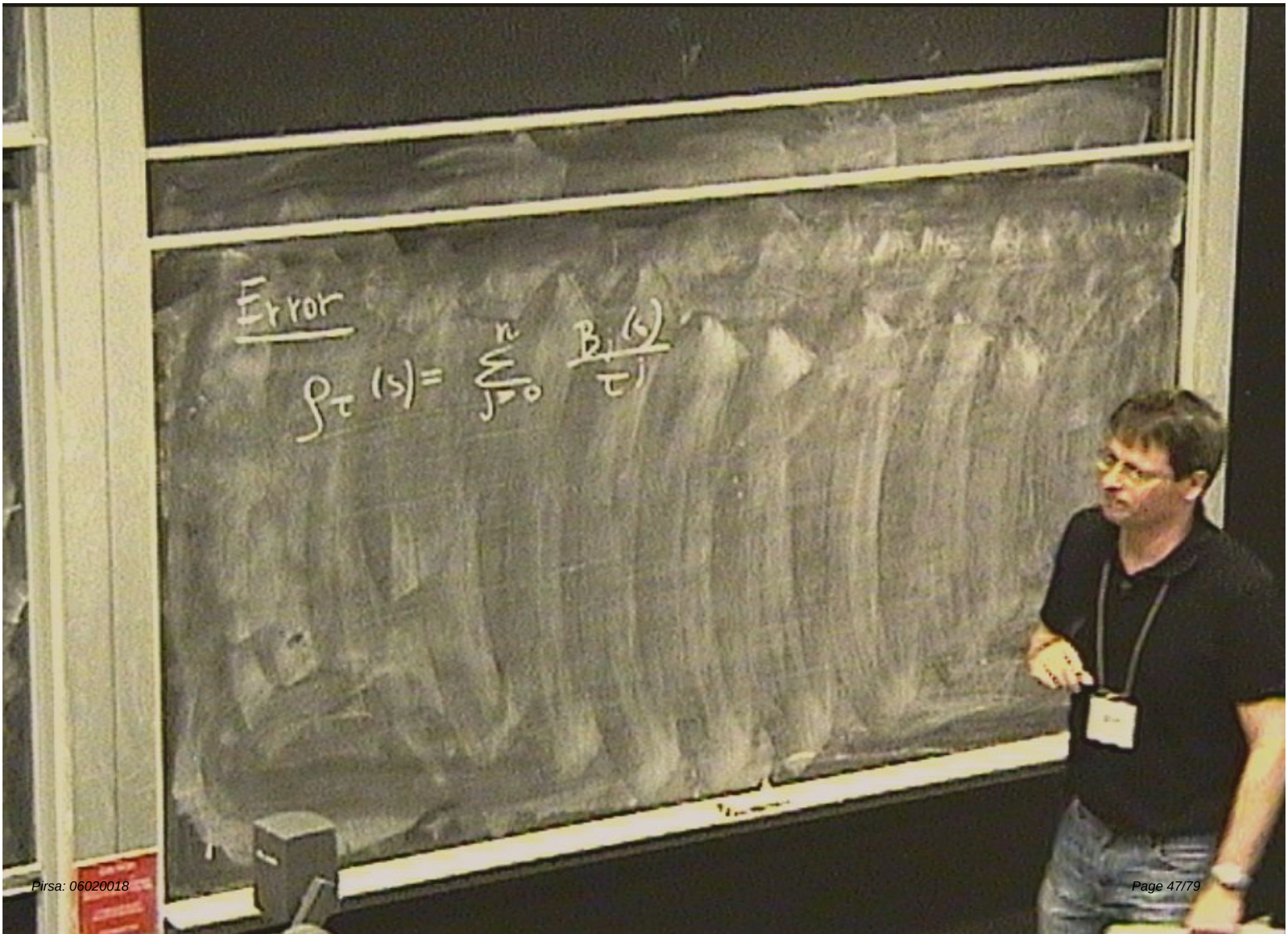
IVP
$$\begin{cases} \dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$$

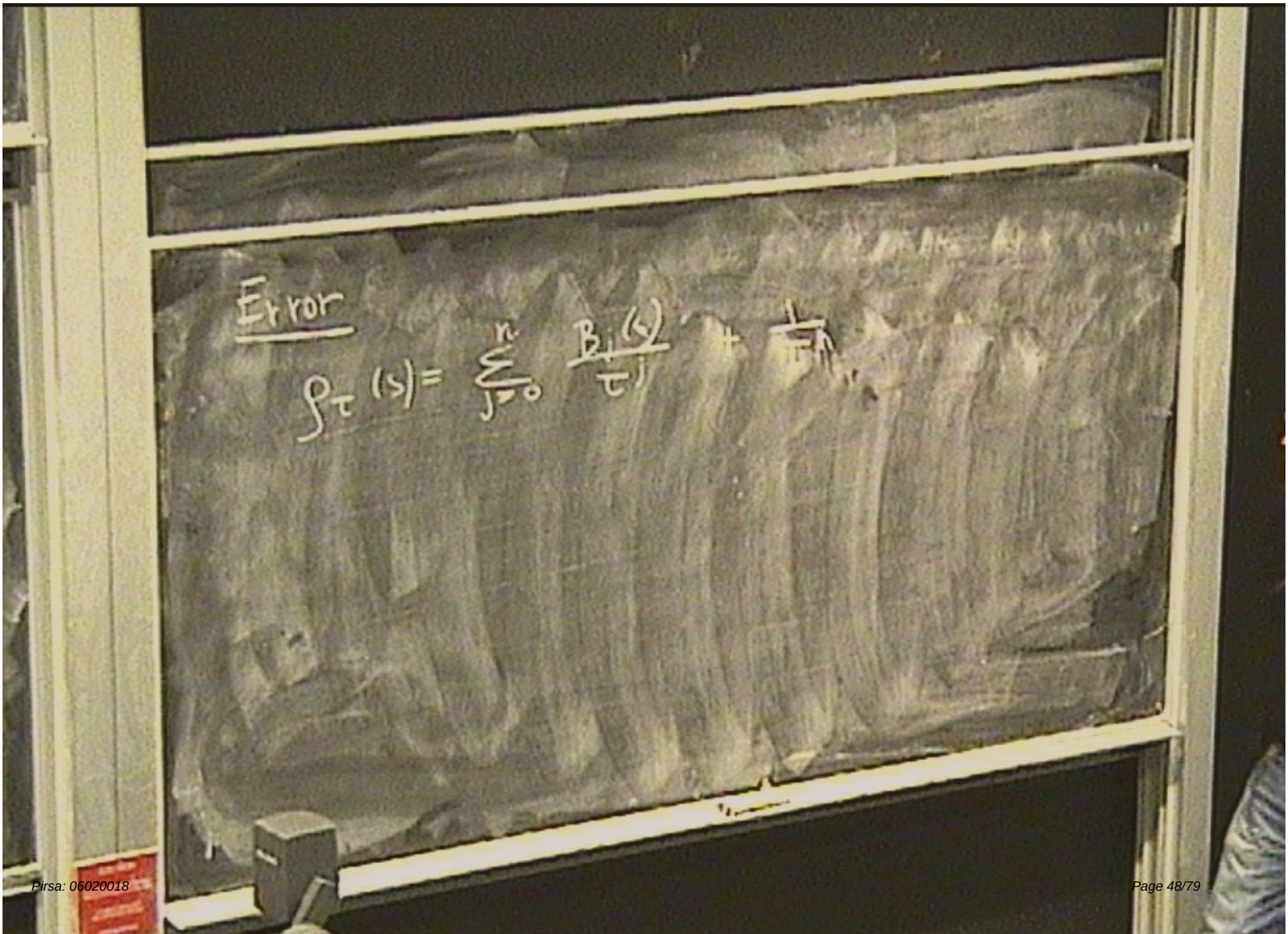
ρ_0 - spectral projection of $H(0)$

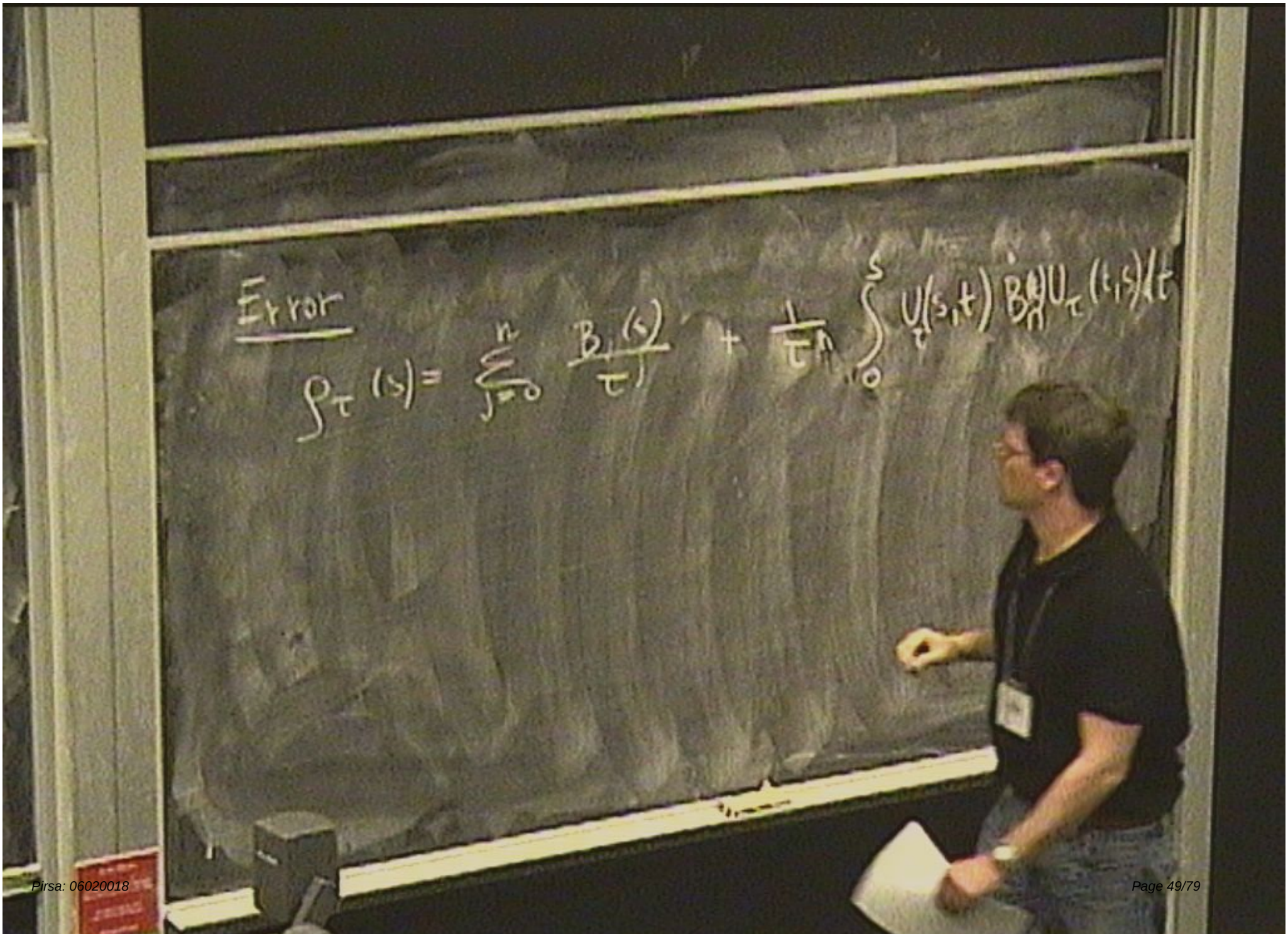
Nenciu '93

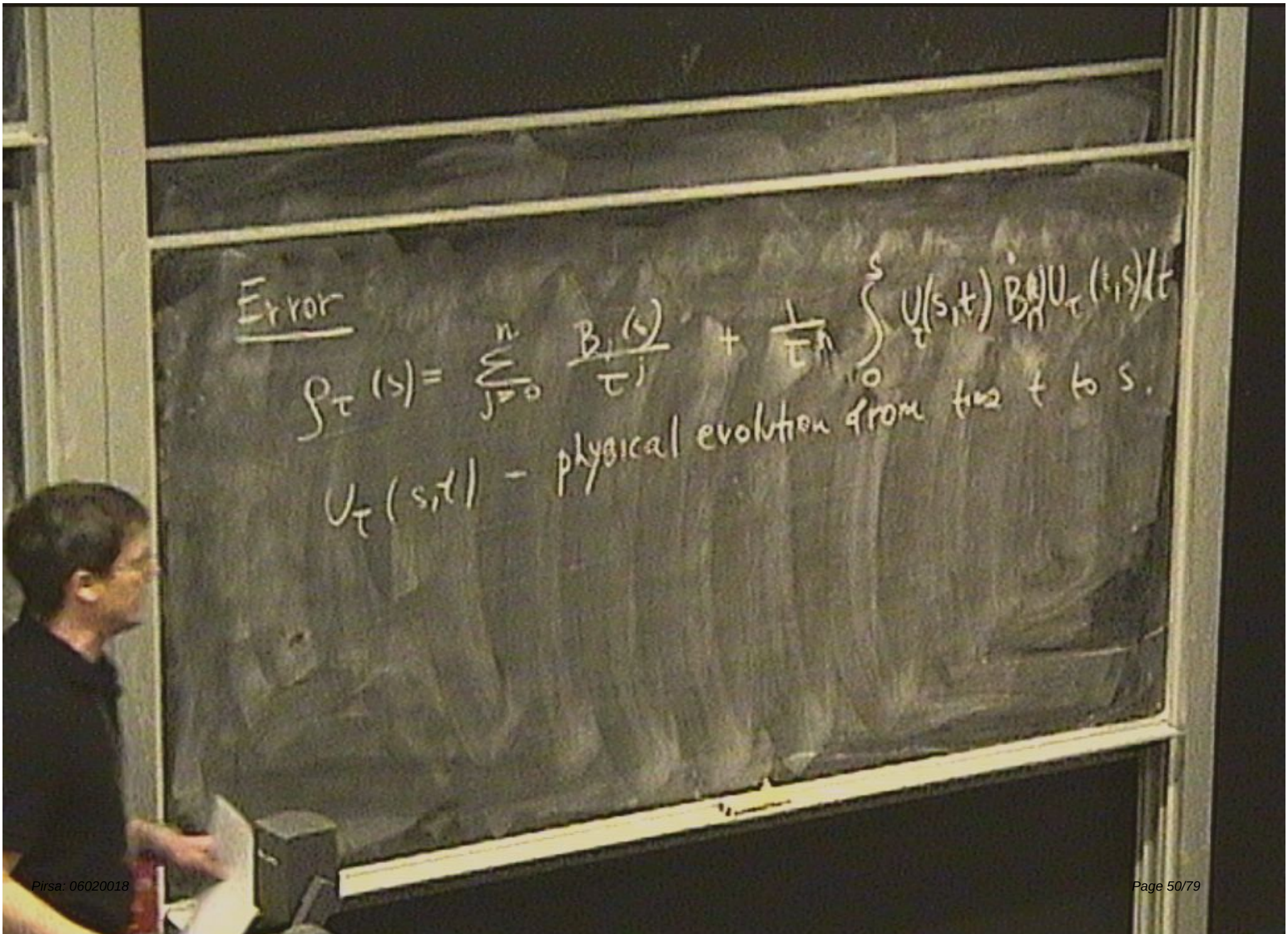
$$\rho_\tau(s) \sim P_0(s) + \frac{1}{\tau} P_1(s) + \frac{1}{\tau^2} P_2(s) + \ldots$$


Error


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j}$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n}$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_\tau(s,t)\, B_n(t)\, U_\tau(t,s)\, dt$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$U_\tau(s,t)$ - physical evolution from time t to s.

Error

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{i\hbar}\int_0^s U_\tau(s,t)\, B_n(t)\, U_\tau(t,s)\, dt$$

$U_\tau(s,t)$ - physical evolution from time t to s.

<u>Error</u>

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_\tau(s,t)\, B_n(t)\, U_\tau(t,s)\, dt$$

$U_\tau(s,t)$ – physical evolution from time t to s.

$$H(s) = \begin{cases} H_f & s \geq 1 \\ H_i & \end{cases}$$

Nenciu expansion

Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$

IVP : $\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$ ρ_0 - spectral projection of $H(0)$

Nenciu '93

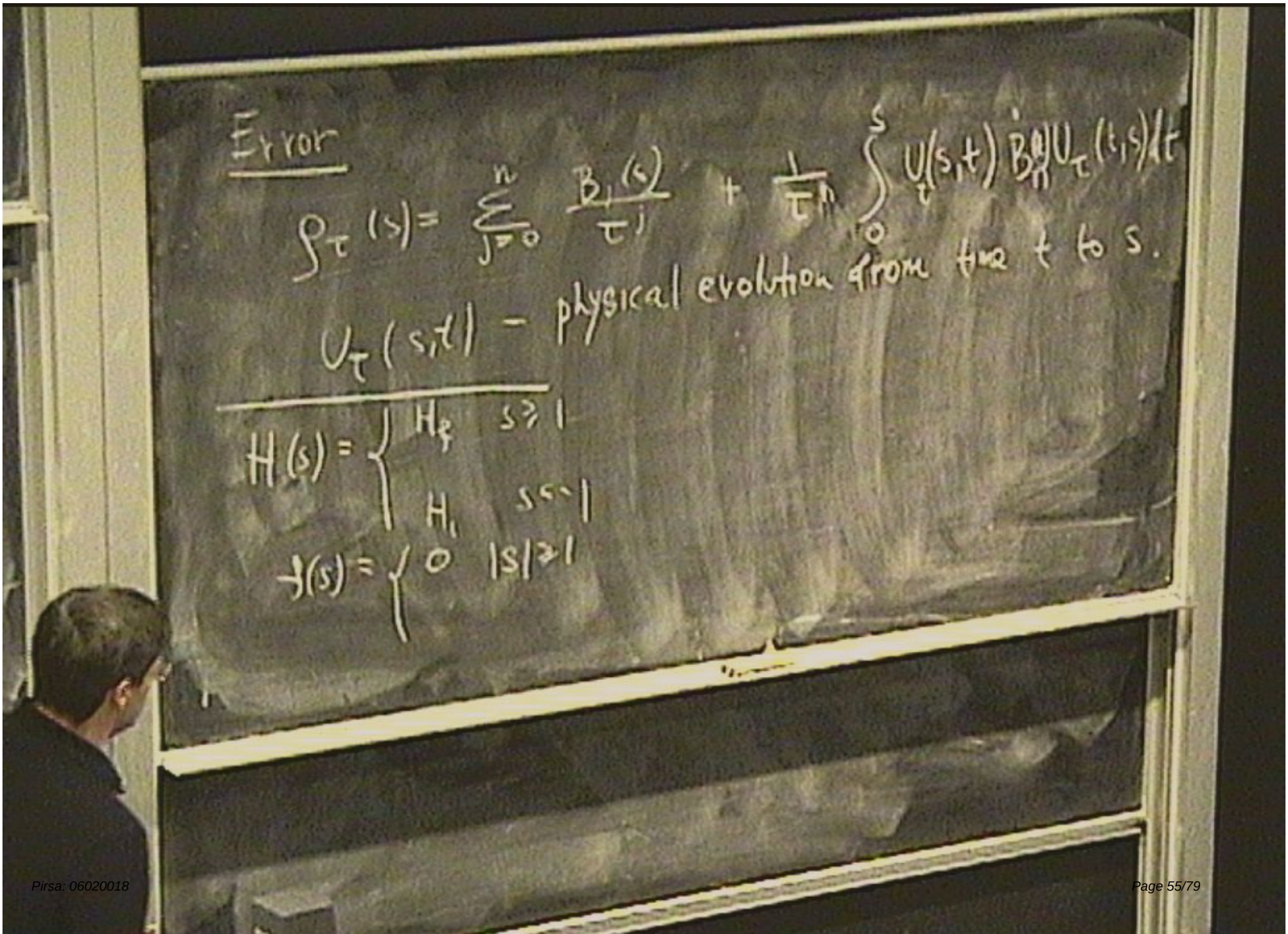
$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \dots$

Error

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n}\int_0^s U_\tau(s,t)\, B_n(t)\, U_\tau(t,s)\, dt$$

$U_\tau(s,t)$ – physical evolution from time t to s.

$$H(s) = \begin{cases} H_f & s \geq 1 \\ H_i & s \leq 1 \end{cases}$$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_\tau(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$
$U_\tau(s,t)$ – physical evolution from time t to s.
$H(s) = \begin{cases} H_f & s \geq 1 \\ H_i & s \leq 1 \end{cases}$
$\dot{H}(s) = \begin{cases} 0 & |s| \geq 1 \end{cases}$

Error

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n}\int_0^s U_\tau(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$$

$U_\tau(s,t)$ – physical evolution from time t to s.

$$H(s) = \begin{cases} H_f & s \geq 1 \\ H_i & s \leq -1 \end{cases}$$

$$\mathcal{J}(s) = \begin{cases} 0 & |s| \geq 1 \\ e^{-\frac{1}{1-s^2}} & |s| < 1 \end{cases}$$

Error

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n}\int_0^s U_t(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$$

$U_\tau(s,t)$ – physical evolution from time t to s.

$$H(s) = \begin{cases} H_f & s \geq 1 \\ H_i & s \leq -1 \end{cases}$$

$$f(s) = \begin{cases} 0 & |s| \geq 1 \\ e^{-\frac{1}{1-s^2}} & |s| < 1 \end{cases}$$

$$|f^{(n)}(s)| \leq C^n (n!)^2$$

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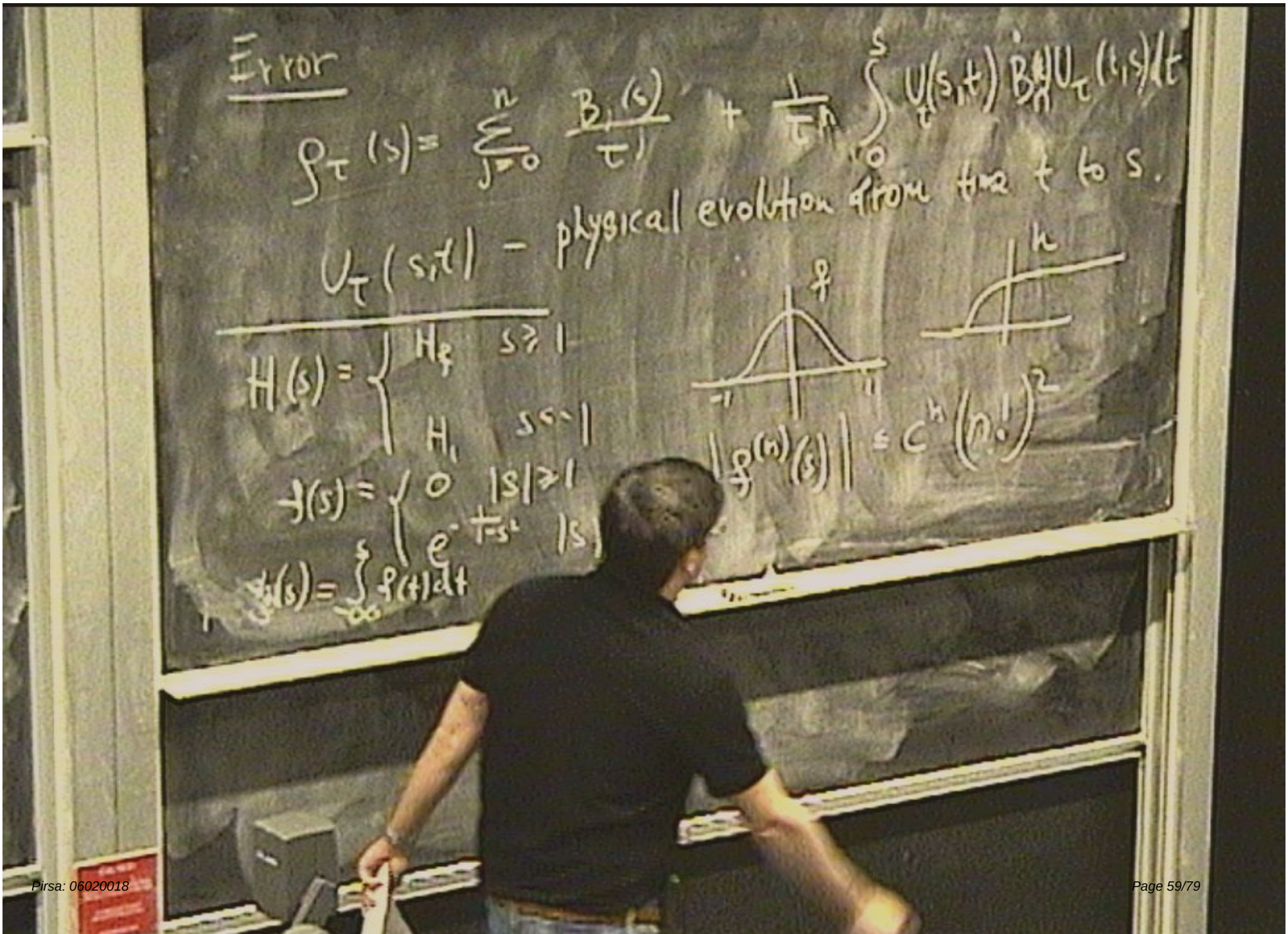
$$\left|\rho^{(n)}(s)\right| \leq C^n (n!)^2$$

$$\rho(s) = \int_{-\infty}^{s} f(t)\, dt$$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$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_t(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$
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$H(s) = \begin{cases} H_f & s \geq 1 \\ H_1 & s \leq -1 \end{cases}$
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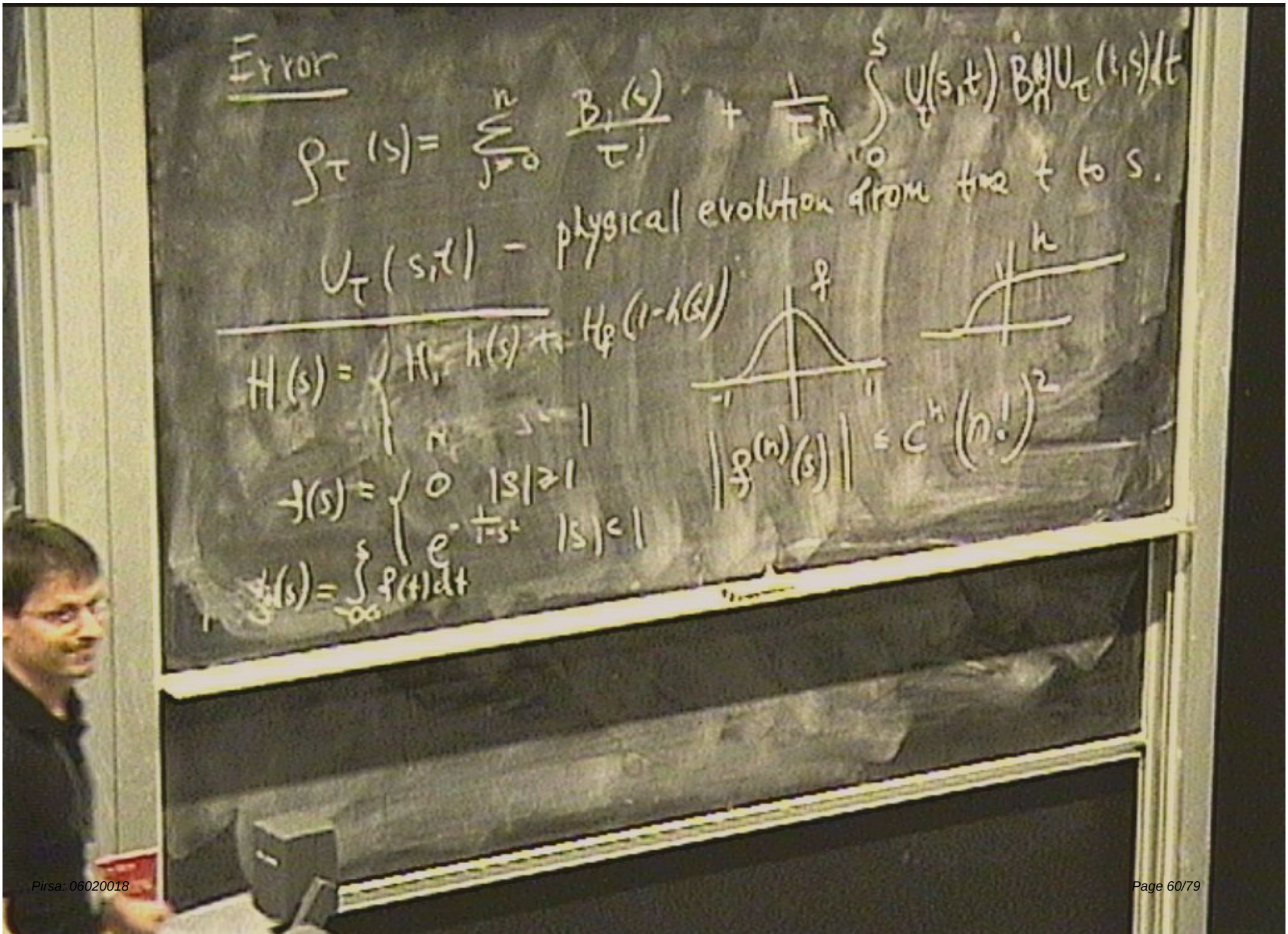
$\hat{f}(s) = \int_{-\infty}^{s} f(t)\, dt$
$|f^{(n)}(s)| \leq C^n (n!)^2$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_\tau(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$
$U_\tau(s,t)$ - physical evolution from time t to s.
$H(s) = \begin{cases} H_1 - h(s) + H_f(1-h(s)) \\ H_f \quad s \le 1 \end{cases}$
$f(s) = \begin{cases} 0 & |s| \ge 1 \\ e^{-\frac{1}{1-s^2}} & |s| < 1 \end{cases}$
$\mathcal{F}(s) = \int_{-\infty}^{s} f(t)\, dt$
f
-1
1
h
$\|f^{(n)}(s)\| \le C^n (n!)^2$

Error

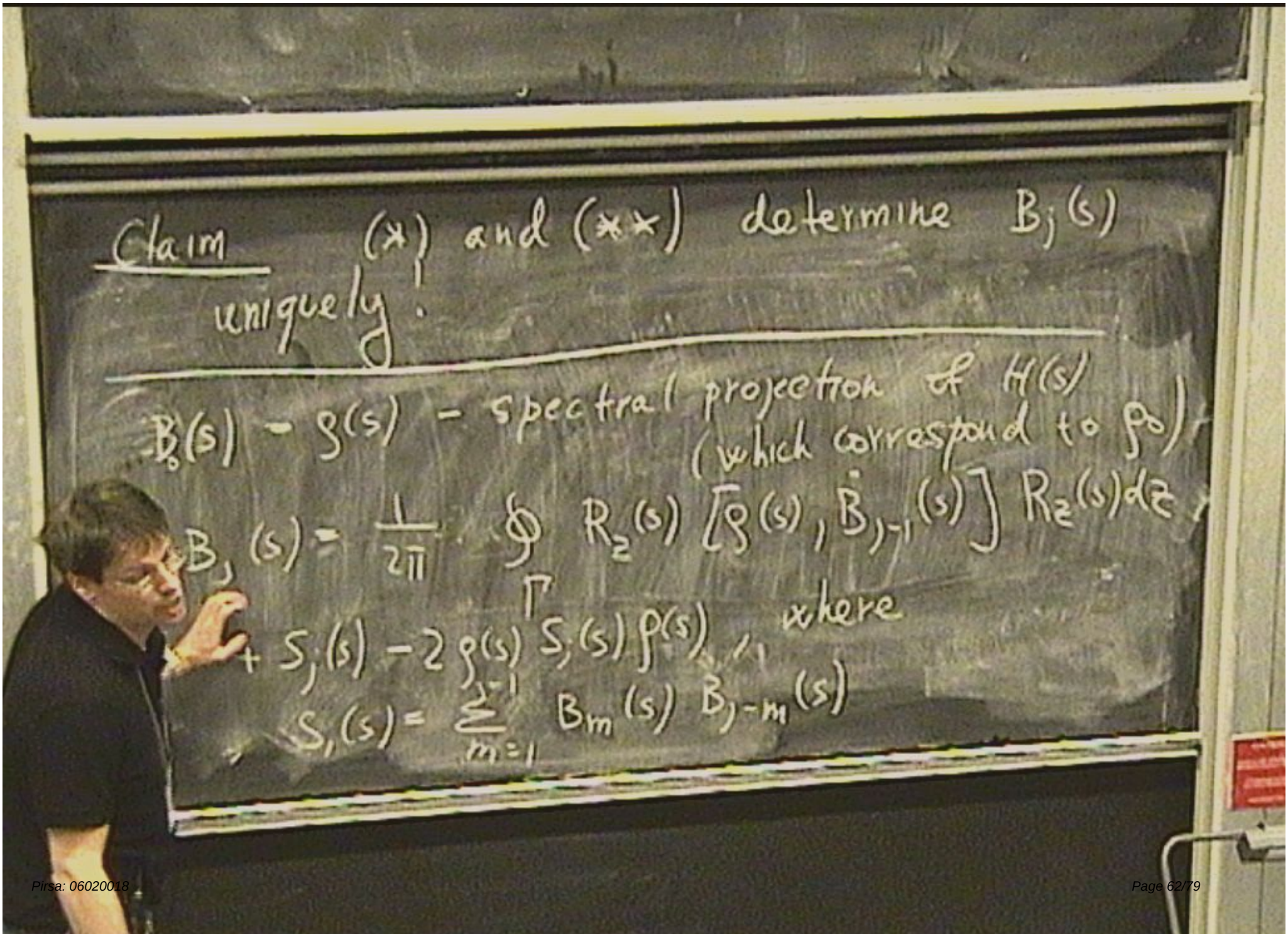
$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n}\int_0^s U_\tau(s,t)\,\dot{B}_n(t)\,U_\tau(t,s)\,dt$$

$U_\tau(s,t)$ – physical evolution from time t to s.

$$H(s) = \begin{cases} H_1 = h(s) + H_f(1-h(s)) \\ \|H^{(n)}(s)\| \le C_1^n (n!)^2 \end{cases}$$

$$f(s) = \begin{cases} 0 & |s| \ge 1 \\ e^{-\frac{1}{1-s^2}} & |s| < 1 \end{cases} \qquad \|f^{(n)}(s)\| \le C^n (n!)^2$$

$$g(s) = \int_{-\infty}^{s} f(t)\,dt$$


Claim (*) and (**) determine $B_j(s)$ uniquely!
$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0),
$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) [\rho(s), \dot{B}_{j-1}(s)] R_z(s) dz$,
$+ S_j(s) - 2\rho(s) S_j(s) \rho(s)$, where
$S_j(s) = \sum_{m=1}^{j-1} B_m(s) B_{j-m}(s)$

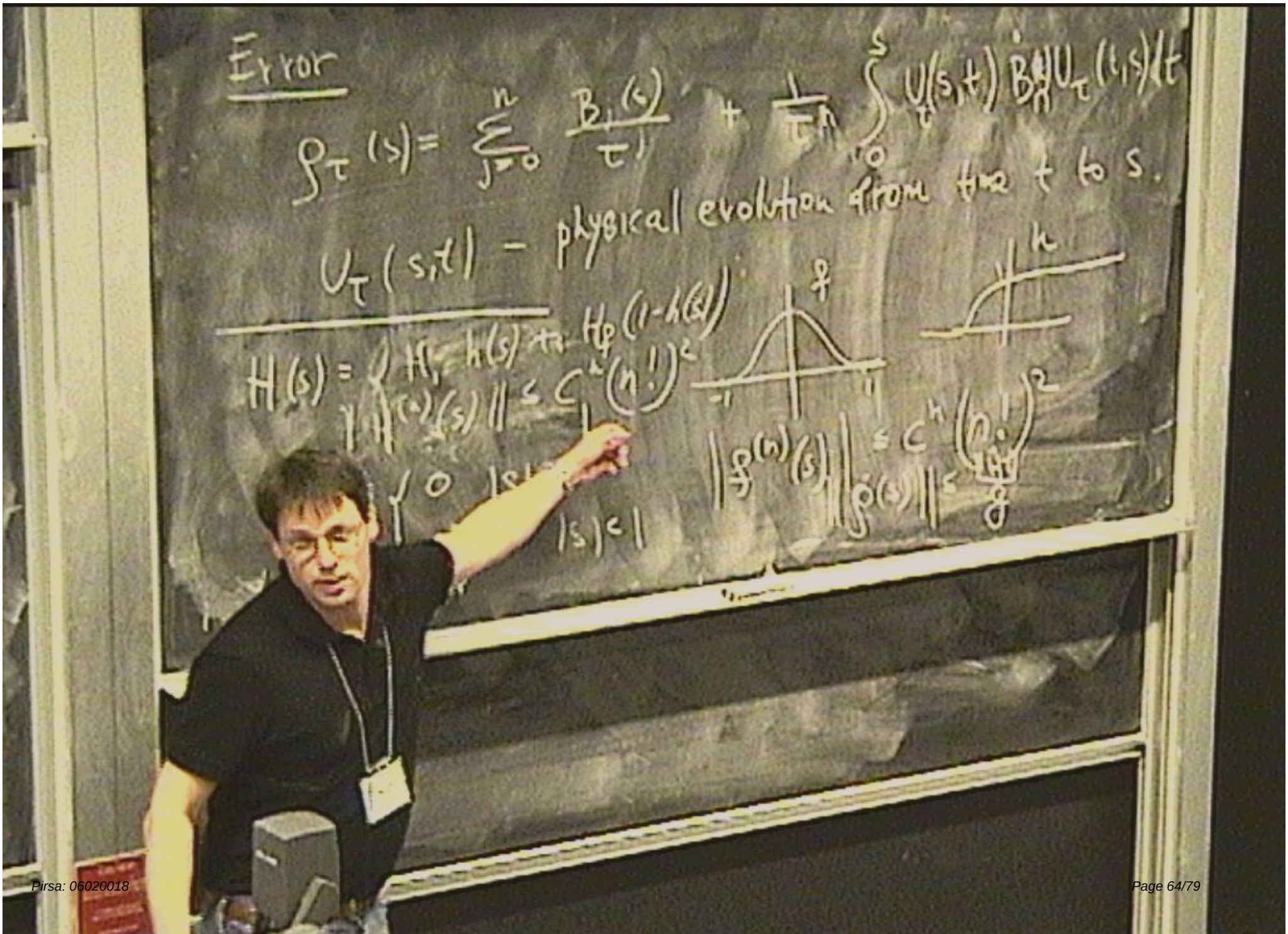
<u>Claim</u> (*) and (**) determine $B_j(s)$ uniquely!

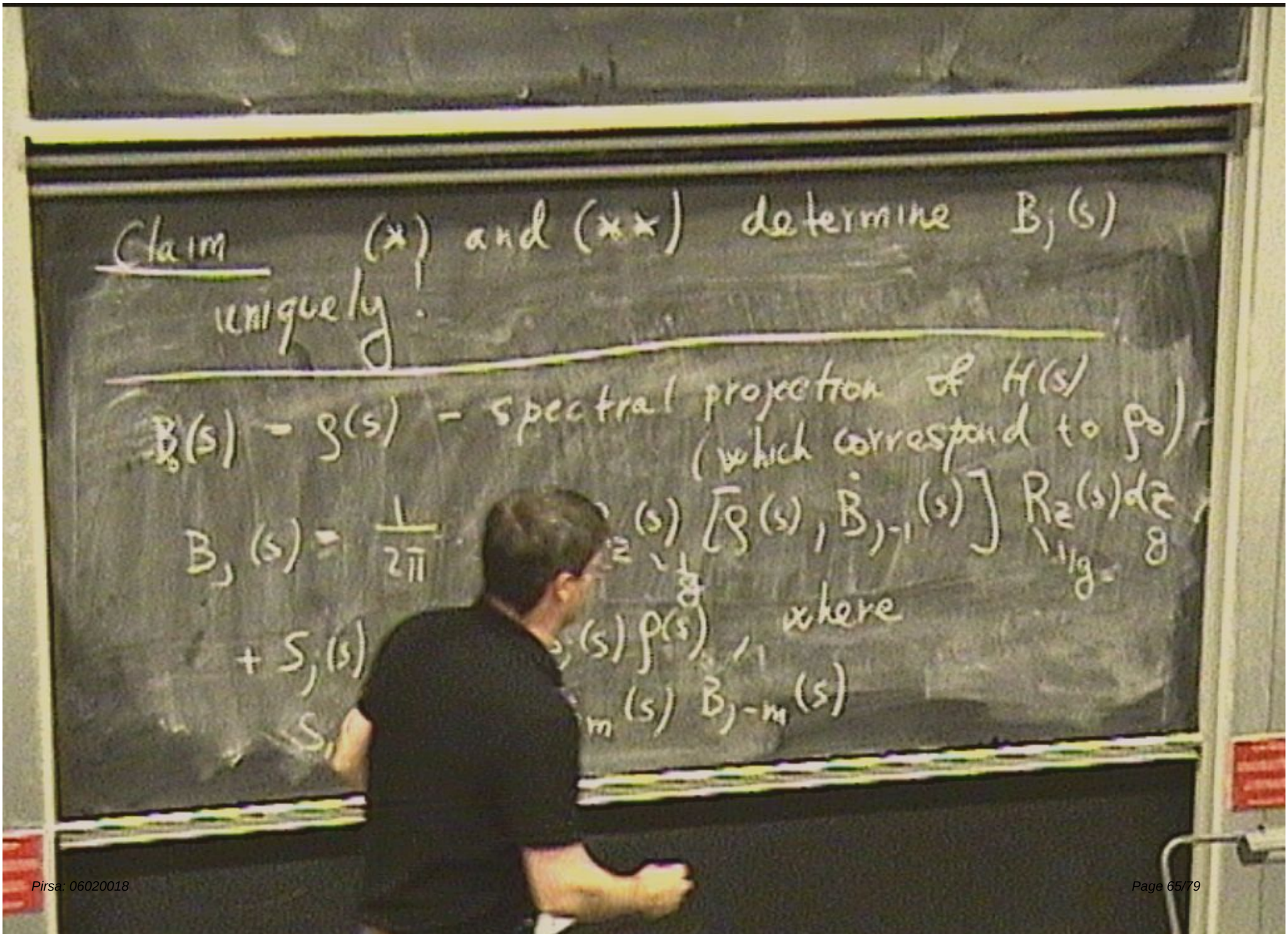
$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) \left[\rho(s), \dot{B}_{j-1}(s)\right] R_z(s)\, dz$$

$$+ S_j(s) - 2\rho(s) S_j(s) \rho(s), \quad \text{where}$$

$$S_j(s) = \sum_{m=1}^{j-1} B_m(s) B_{j-m}(s)$$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_t(s,t)\, \dot{B}_n(t)\, U_\tau(t,s)\, dt$
$U_\tau(s,t)$ – physical evolution from time t to s.
$H(s) = \{ H_1 = h(s) + H_0(1-h(s))$
$\|H^{(n)}(s)\| \le C^n (n!)^2$
$|s| < 1$
$\|\rho^{(n)}(s)\| \le C^n (n!)^2$
$\|\rho(s)\| \le$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uniquely!
$B_0(s) = \rho(s)$ - spectral projection of $H(s)$
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$B_j(s) = \frac{1}{2\pi}$ $(s)\,[\rho(s), B_{j-1}(s)]\,R_z(s)\,dz$
$+ S_j(s)$ $(s)\,\rho(s)$, where
S_j $m(s)\,B_{j-m}(s)$

$B_0(s) = \rho(s)$ - spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) \left[\rho(s), B_{j-1}(s)\right] R_z(s)\, dz$$

$R_z(s) \sim \frac{1}{g}$, $R_z(s) \sim 1/g$

$$+ S_j(s) - 2\rho(s) S_j(s) \rho(s)$$

where finally $B_j(s) \sim \frac{1}{g^j}$

$$S_j(s) = \sum_{m=1}^{j-1} B_m(s) B_{j-m}(s)$$

Nenciu expansion

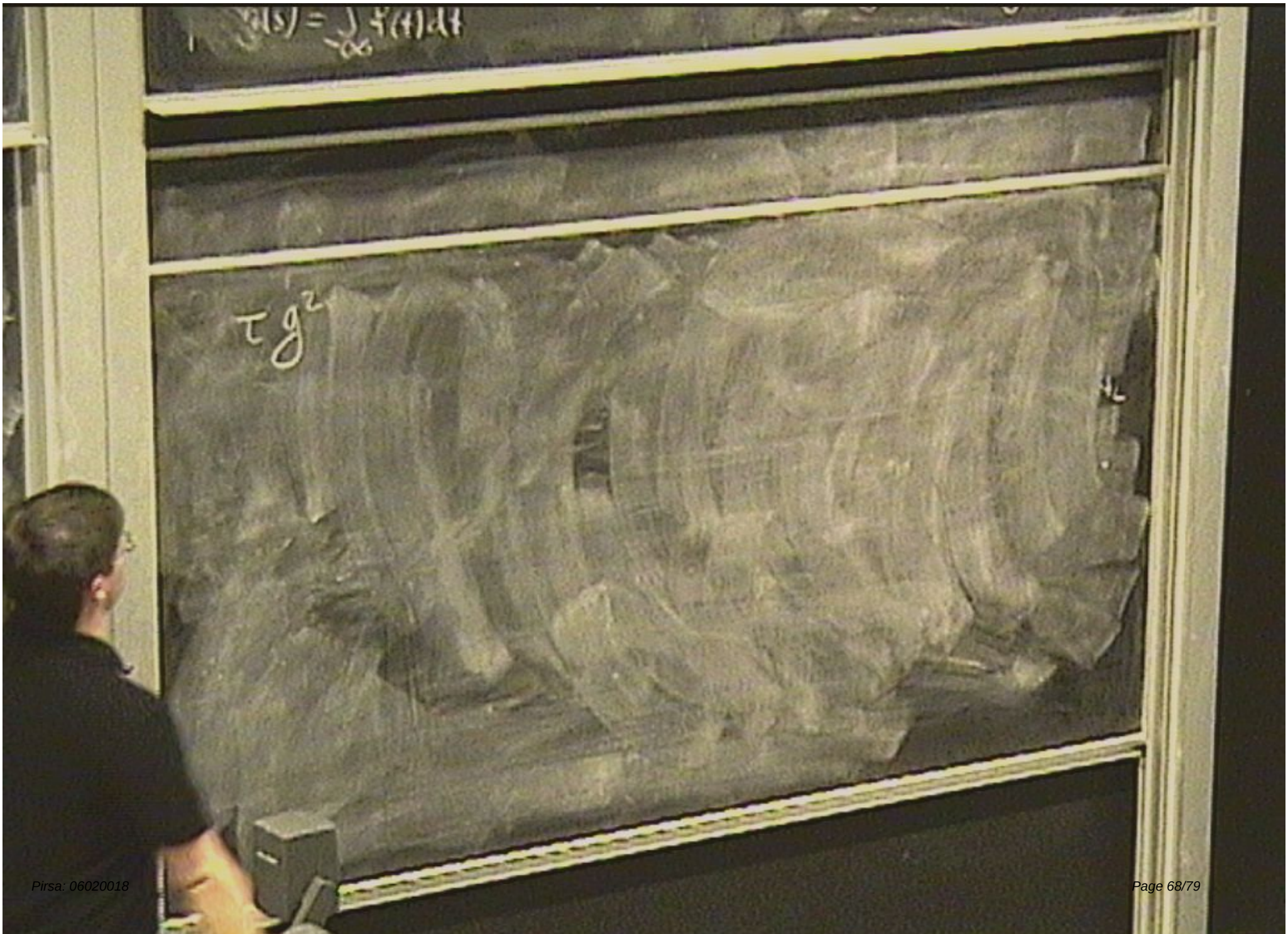
Heisenberg picture $\rho_0 = \psi_0 \rangle\langle \psi_0$

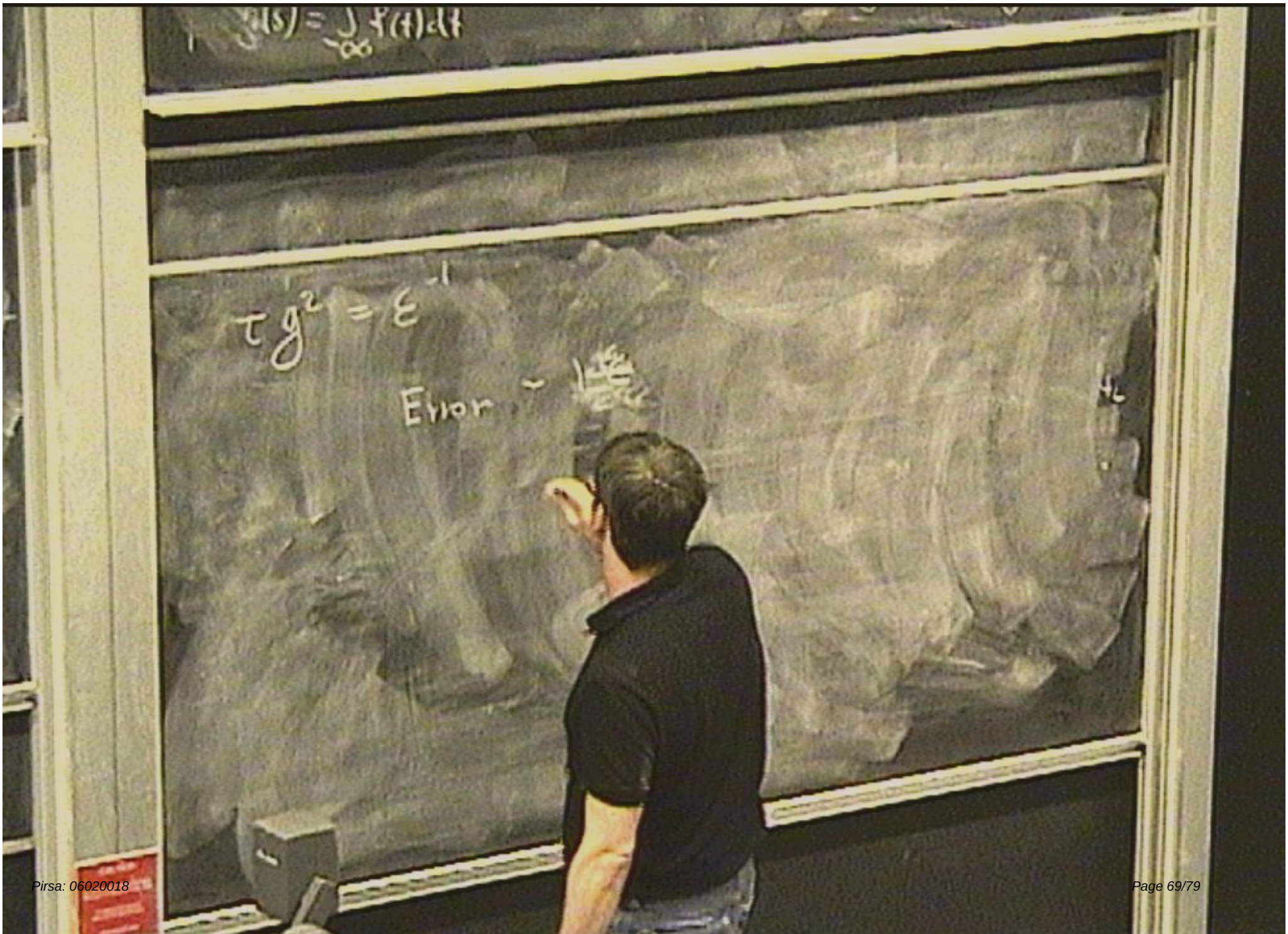
$\rho_\tau(s) = \psi_\tau(s) \rangle\langle \psi_\tau(s)$

IVP: $\begin{cases} i\dot{\rho}_\tau(s) = \tau[H(s), \rho_\tau(s)] \\ \rho_\tau(0) = \rho_0 \end{cases}$ ρ_0 - spectral projection of $H(0)$

Nenciu '93

$$\rho_\tau(s) \sim B_0(s) + \frac{1}{\tau} B_1(s) + \frac{1}{\tau^2} B_2(s) + \dots$$


τg^2


$\tau g^2 = \varepsilon^{-1}$
Error ~

$$\tau g^2 = \varepsilon^{-1}$$

$$\text{Error} \sim \varepsilon + \varepsilon^2 + \varepsilon^3 + \varepsilon^4 + \cdots$$

$B_0(s) = \rho(s)$ – spectral projection of $H(s)$ (which correspond to ρ_0)

$$B_j(s) = \frac{1}{2\pi} \oint_\Gamma R_z(s) \left[\rho(s), \dot{B}_{j-1}(s)\right] R_z(s)\, dz$$

$R_z(s) \sim \frac{1}{g}$, $R_z(s) \sim 1/g$

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$$S_j(s) = \sum_{m=1}^{j-1} B_m(s) \dot{B}_{j-m}(s)$$

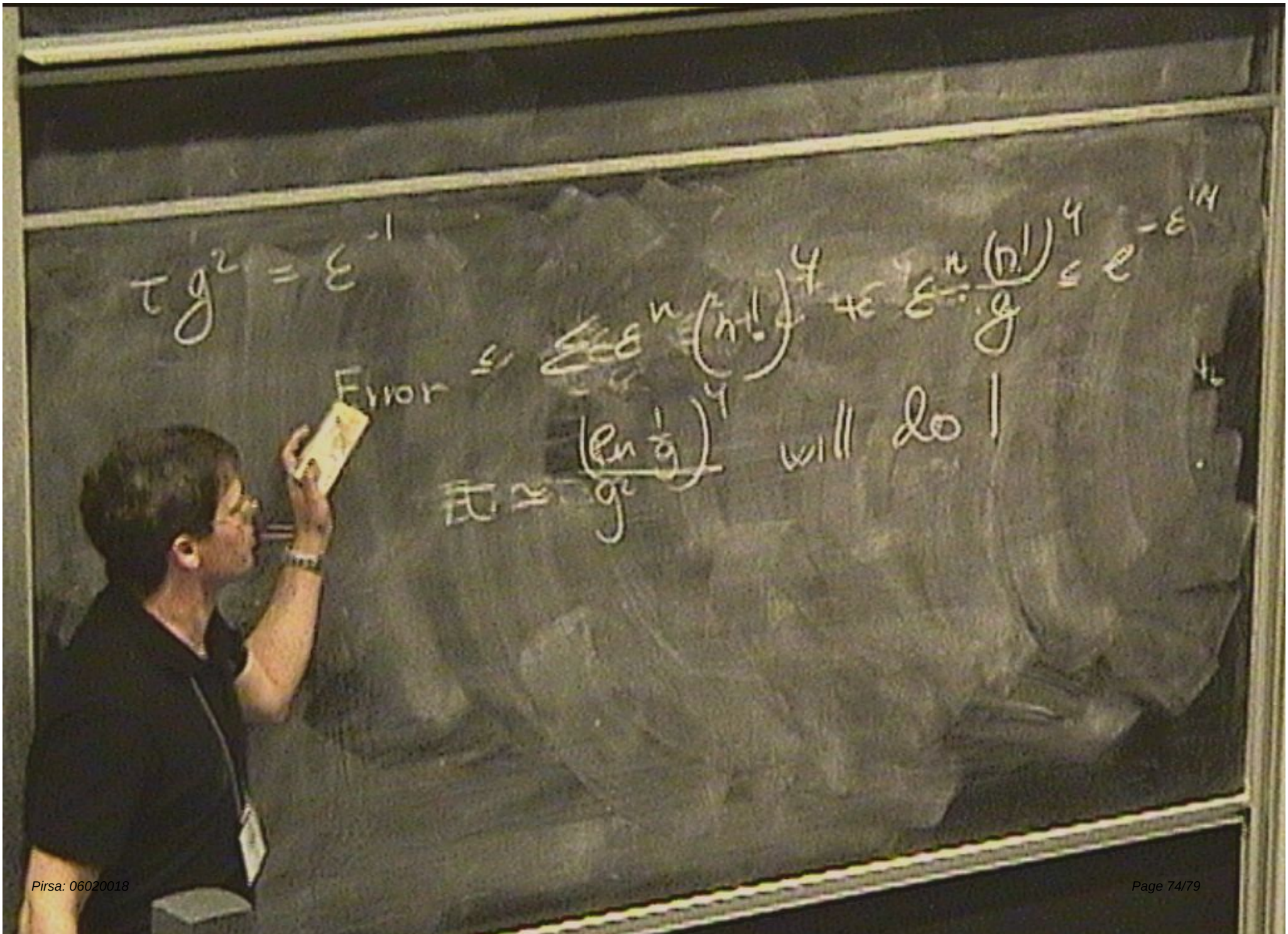
$$\mathcal{F}(s) = \int_{-\infty}^{\infty} f(t)\,dt$$

$$\tau g^2 = \varepsilon^{-1}$$

$$\text{Error} \lesssim \sum_n \varepsilon^n (n!)^4 + \varepsilon^{-1} + \cdots$$

$$\tau g^2 = \varepsilon^{-1}$$

$$\text{Error} \lesssim \sum_{n} g^n (n!)^4 \approx \sum_{n} \frac{1}{g}$$

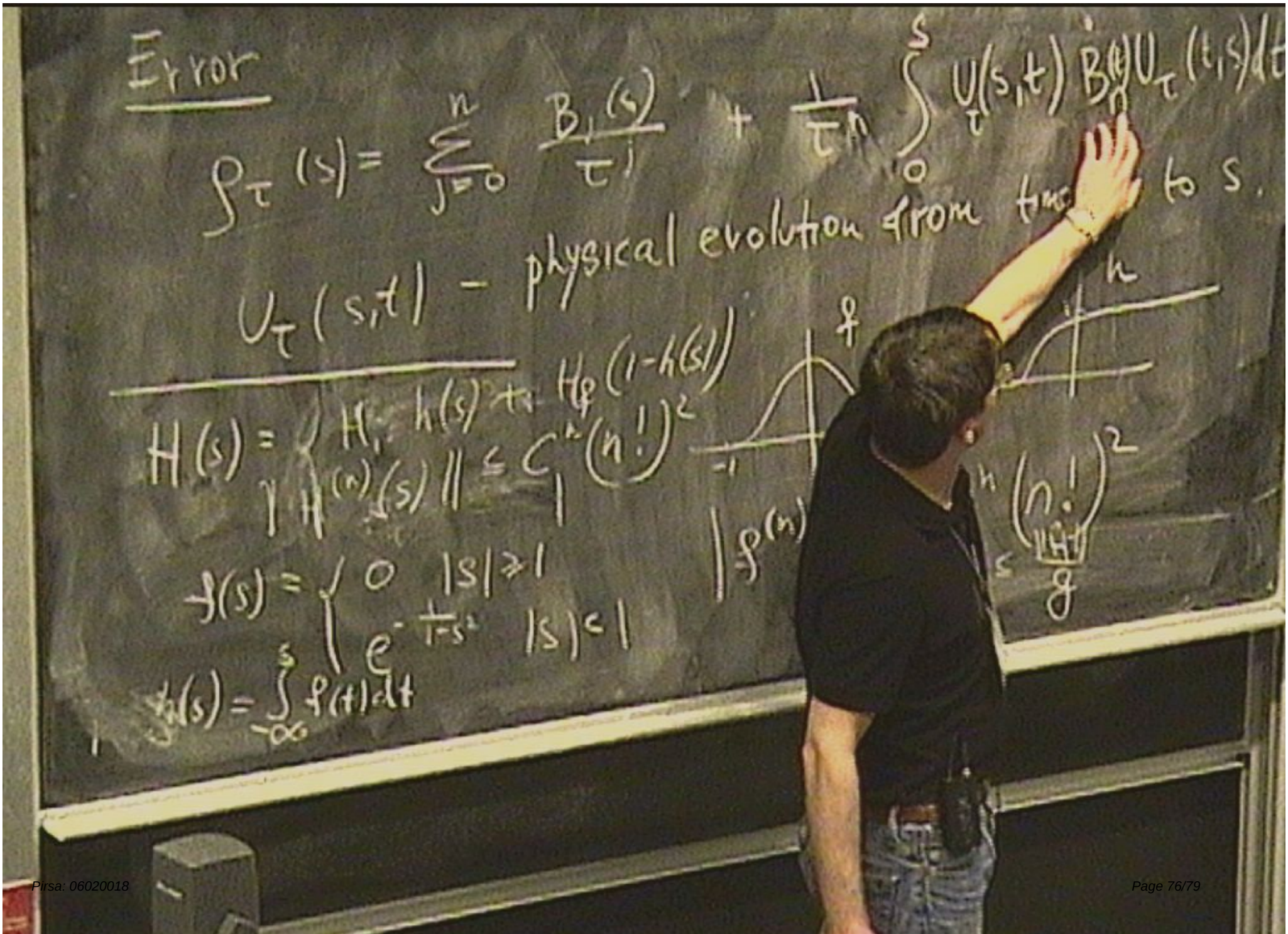

$\tau g^2 = \varepsilon^{-1}$
Error $\leq \varepsilon^n (n!)^4 \tau^4 \varepsilon^{-4} \frac{n}{g} (n!)^4 \leq e^{-\varepsilon^{1/4}}$
$\left(\frac{\ln \frac{1}{g}}{g^2}\right)^4$ will do!
$\tau \simeq \frac{1}{g^2}$

$$\tau g^2 = \varepsilon^{-1}$$

$$\text{Error} \lesssim \varepsilon^n (n!)^4 \lesssim \varepsilon^n \frac{(n!)^4}{g} \lesssim e^{-\varepsilon^{1/4}}$$

$$\| \quad 0 \quad 1$$

$$\Rightarrow \quad \tau \simeq \frac{\left(\ln \frac{1}{g^2}\right)^4}{g^2}$$


Error
$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^s U_\tau(s,t) B_n(t) U_\tau(t,s) dt$
$U_\tau(s,t)$ - physical evolution from time to s.
$H(s) = \begin{cases} H_1 \cdot h(s) + H_2(1-h(s)) \\ \| H^{(n)}(s) \| \le C_1^n (n!)^2 \end{cases}$
$f(s) = \begin{cases} 0 & |s| \ge 1 \\ e^{-\frac{1}{1-s^2}} & |s| < 1 \end{cases}$
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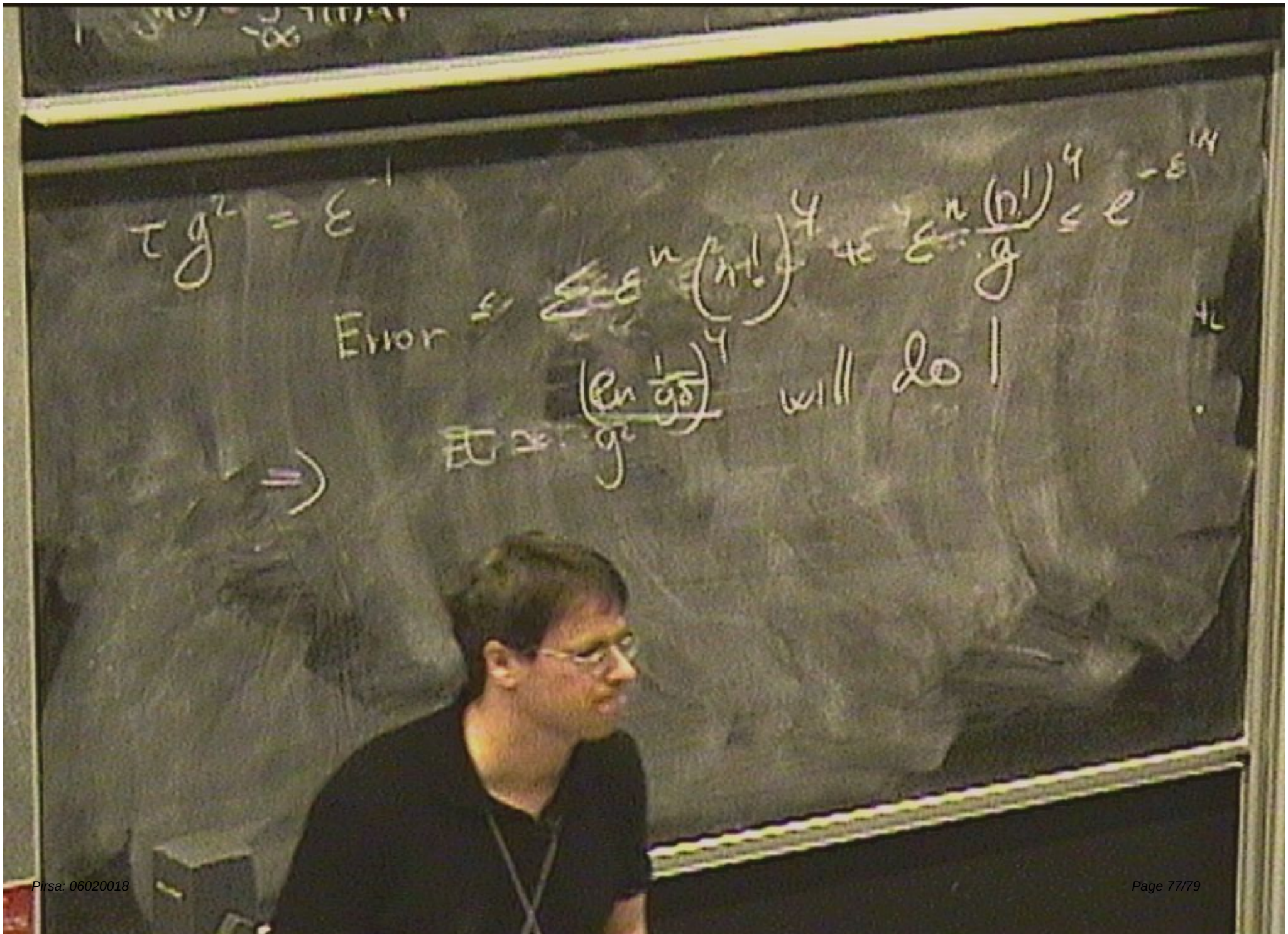
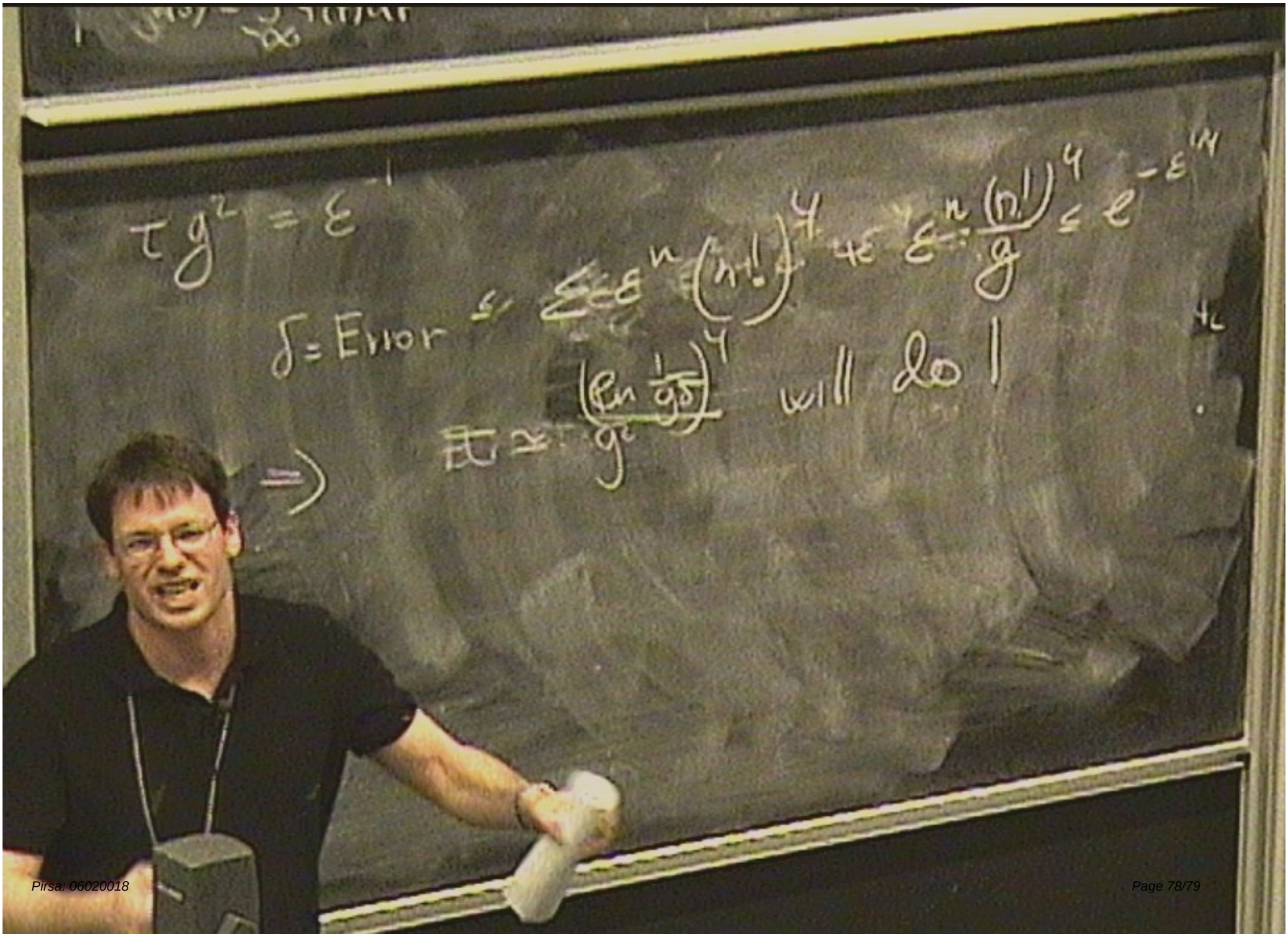
f
h
-1
$|\rho^{(n)}$
$\le \frac{\|H\|}{g}$


$\tau g^2 = \varepsilon^{-1}$
Error $\leq \varepsilon^n (n!)^4 \leq \varepsilon^n \frac{(n!)^4}{g} \leq e^{-\varepsilon^{1/4}}$
$\left(\frac{\ln \frac{1}{g\delta}}{g^2}\right)^4$ will do!
$\Rightarrow$


$\tau g^2 = \varepsilon^{-1}$
$\delta = \text{Error} \le \varepsilon^n (n!)^4 \le \varepsilon^n \frac{(n!)^4}{g} \le e^{-\varepsilon^{1/4}}$
$\Rightarrow \quad \tau \approx \frac{(\ln \frac{1}{g\delta})^4}{g^2}$ will do!

$$\rho_\tau(s) = \sum_{j=0}^{n} \frac{B_j(s)}{\tau^j} + \frac{1}{\tau^n} \int_0^{\tau} U_t(s,t)\, B_n(t)\, U_\tau(t,s)\, dt$$

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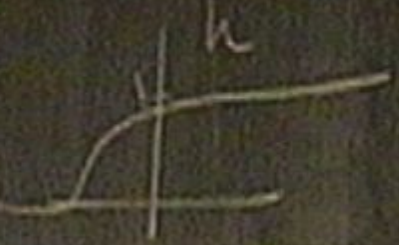

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$$h(s) = \int_{-\infty}^{s} f(t)\, dt$$

$$|\rho^{(n)}(s)| \le C^n (n!)^2$$

$$\|\rho(s)\| \le \frac{\|H\|}{g}$$

$$\tau g^2 = \varepsilon^{-1}$$

$$\le \varepsilon^n (n!)^4 + \varepsilon^4 \varepsilon^n \frac{(n!)^4}{g} \le e^{-\varepsilon^{1/4}}$$