Title: Entanglement loss properties and classical simulability of quantum many-body systems

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Abstract: In this talk I will expose different results concerning the properties of quantum many-body systems: on the one hand, I will introduce the concept of fine-grained entanglement loss together with its relation with majorization relations along parameter flows and Renormalization Group flows. The machinery of Conformal Field Theory will allow us to derive very general analytical properties, and some examples -like the XY quantum spin chain- will also be considered. On the other hand, I will describe results concerning the classical simulability of quantum many-body systems by means of Matrix Product States. In particular, I will present an approximated classical simulation of a quantum algorithm by adiabatic evolution solving hard instances of an NP-Complete problem up to 100 qubits.

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Entanglement loss properties and classical simulability of quantum many-body systems

Román Orús

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Perimeter Institute, 6 of February 2006

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The working scenario…

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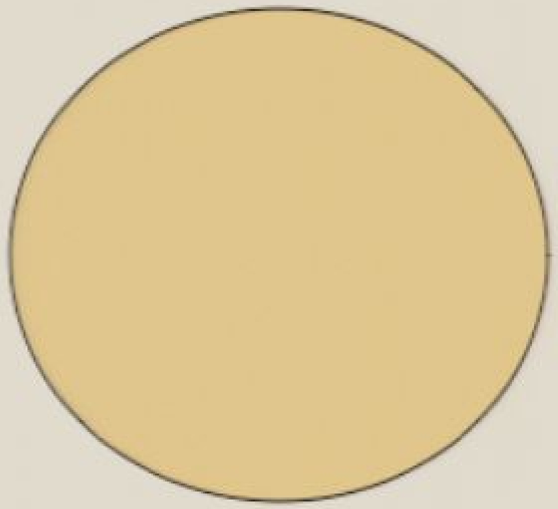
Quantum information and computation



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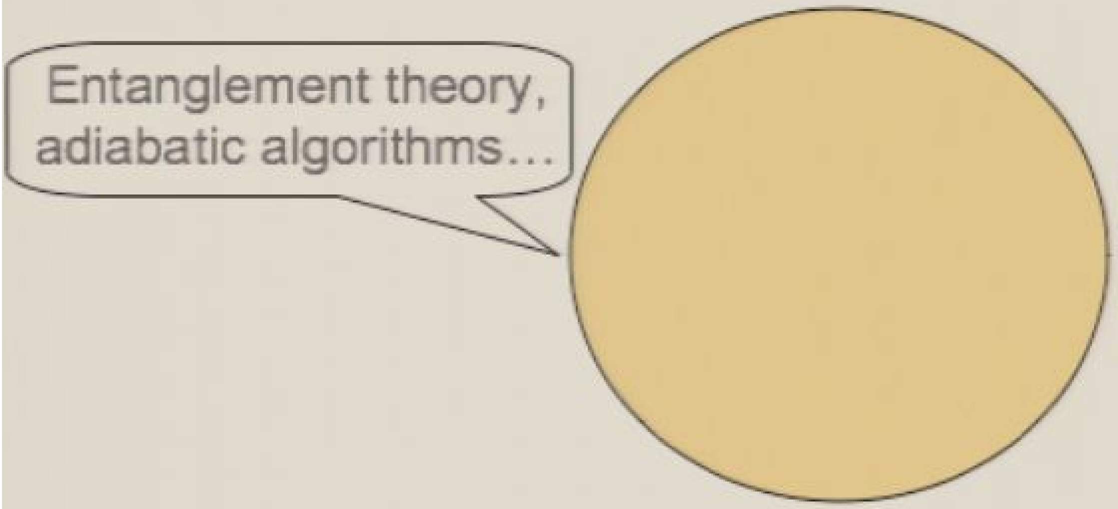
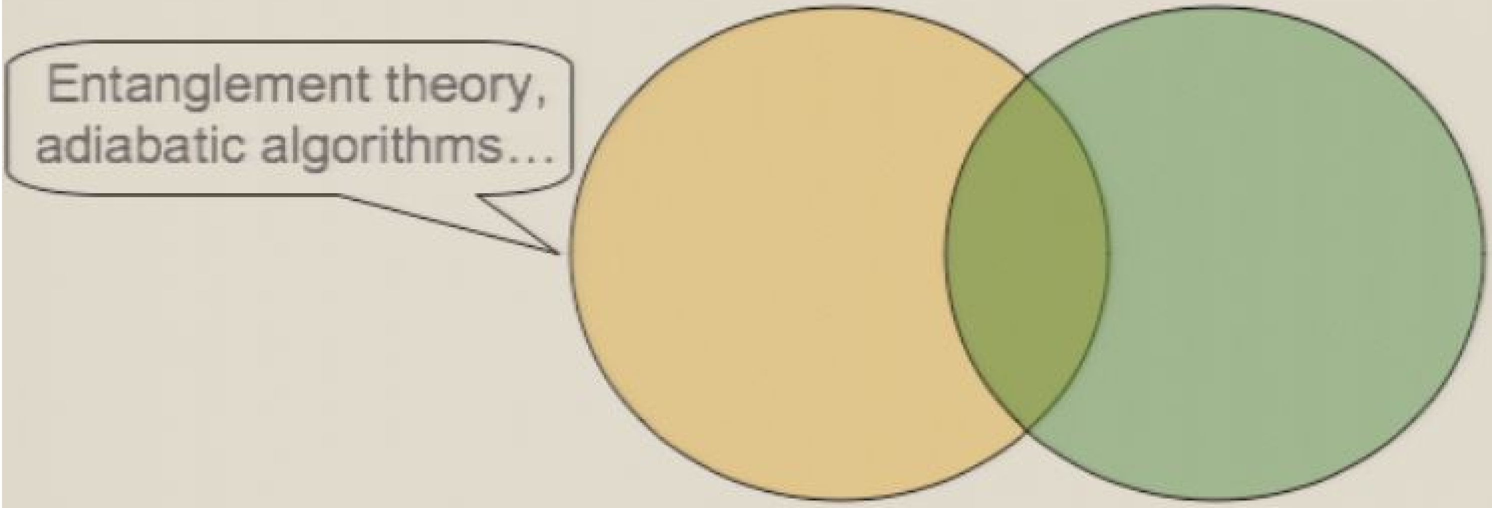


The working scenario…

Quantum information and computation

Condensed matter physics



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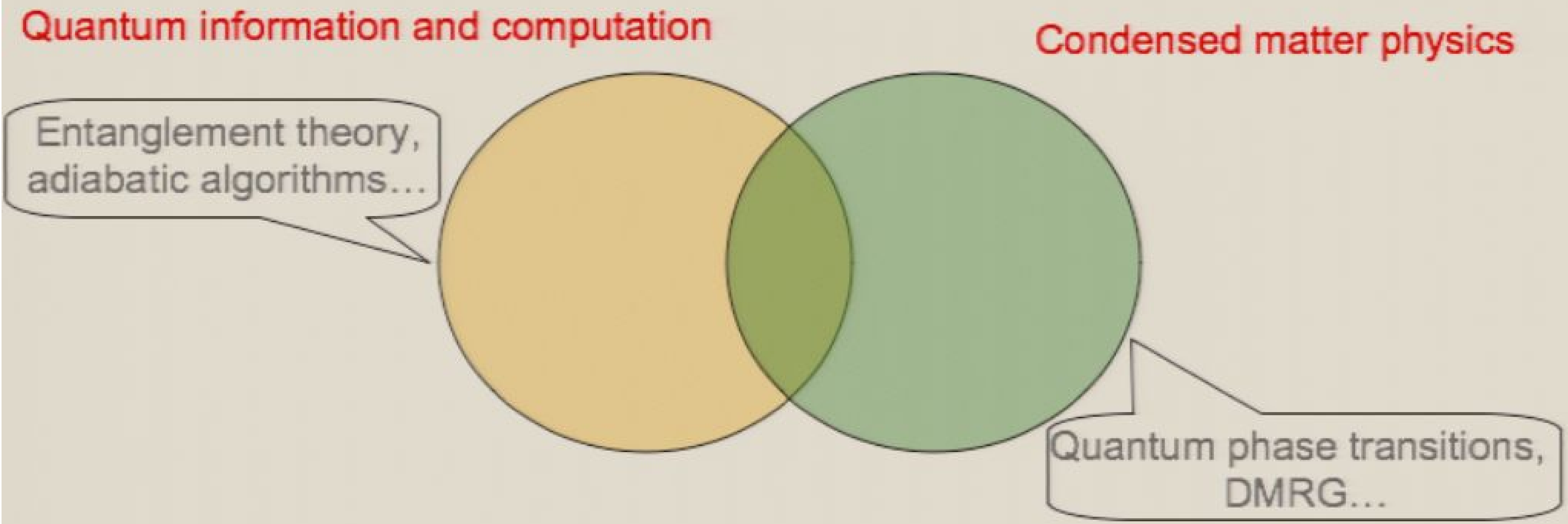

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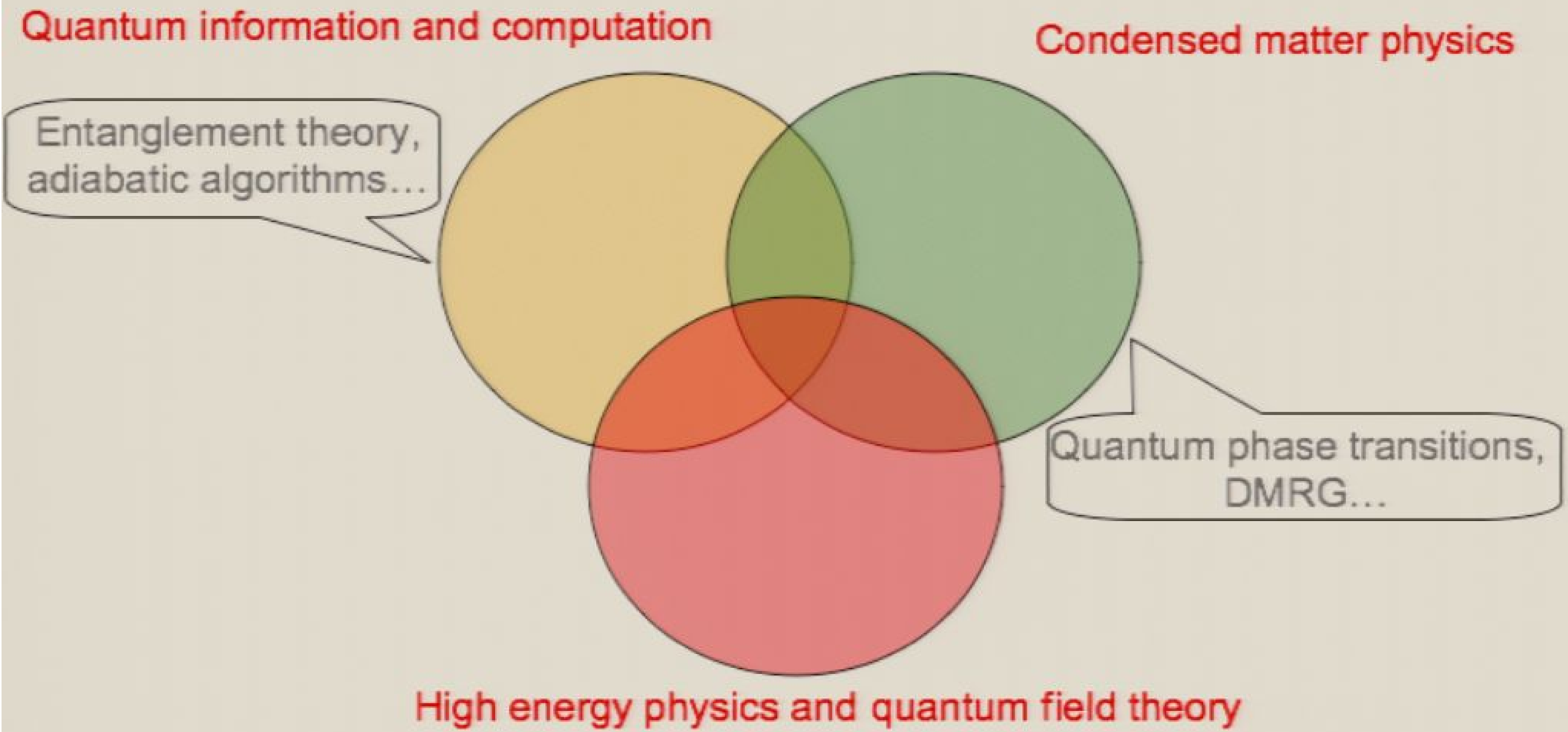

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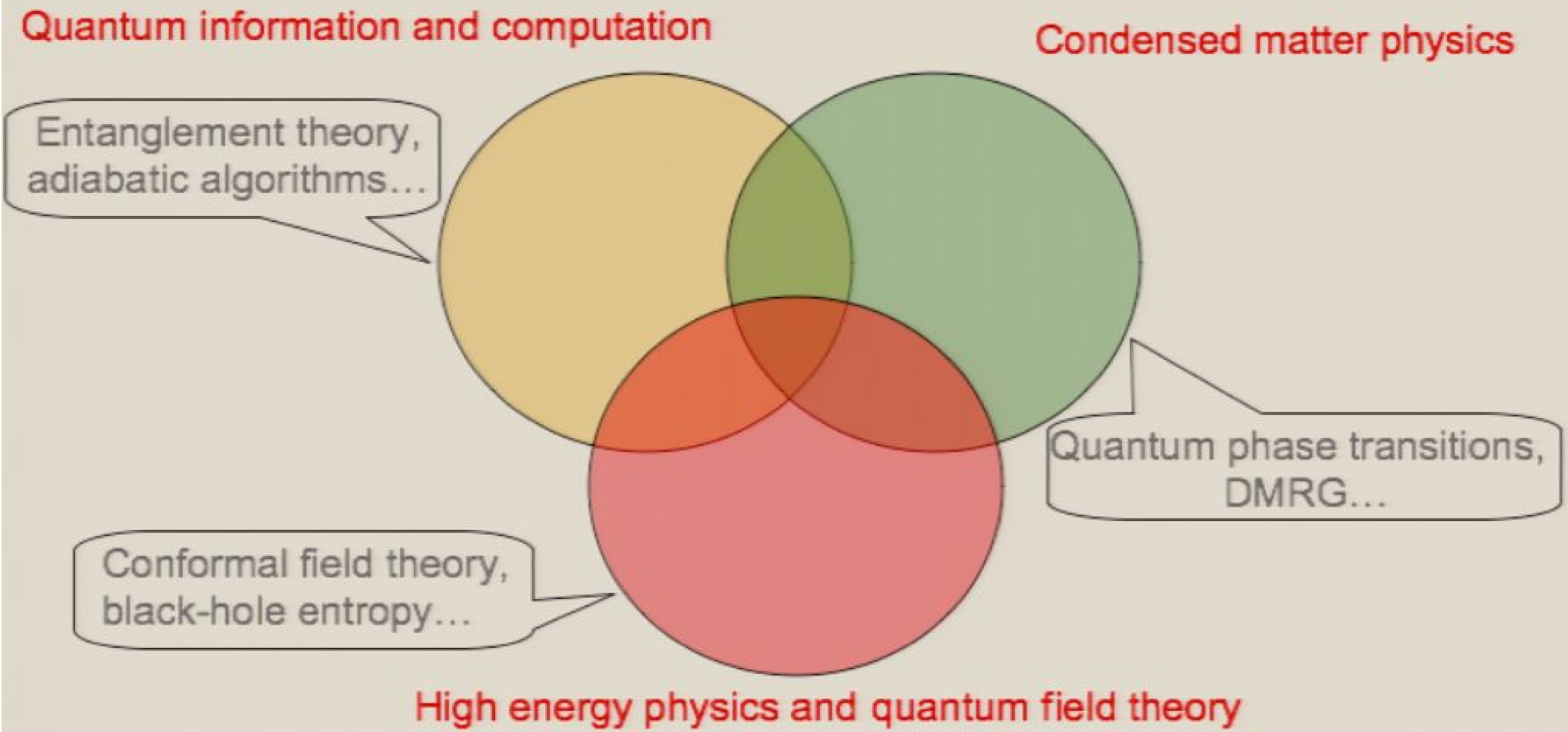

My work at Barcelona

- Robustness of weakly-entangled states

  PRA 70, 050101 (2004)
- Majorization in quantum algorithms

  QIP 1 (4), 283-302 (2002); EPJD 29, 119-132 (2004)
- Classical computational complexity of quantum problems

  QIC 5 (6), 449-455 (2005)
- Entanglement in quantum algorithms and quantum phase transitions

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- Majorization and entanglement loss along parameter flows and RG-flows

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  quant-ph/0503174 (to appear in PRA), and more to (hopefully) come…

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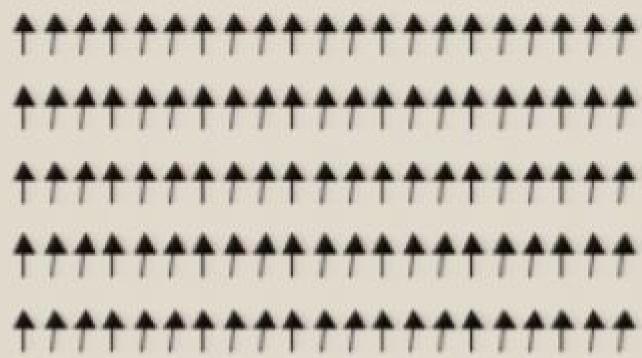

Majorization theory

$\vec{x},\vec{y} \in R^d$ probability vectors $\left(\sum_{i=1}^{d} x_i = \sum_{i=1}^{d} y_i = 1\right)$

- **Definition:** $\vec{x}$ is majorized by $\vec{y}$ $(\vec{x} \prec \vec{y})$ if and only if
  - $\vec{x} = \sum_j p_j P_j \vec{y}$ (P_j: permutation matrices; p_j: probabilities)
  - $\sum_{i=1}^{k} x_i \le \sum_{i=1}^{k} y_i \quad k = 1,2,..d-1$ (components sorted into decreasing order)
  - $\vec{x} = D\vec{y}$ (D: doubly stochastic matrix)

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Majorization imposes a very strong sense of order in probability distributions.

Global, monotonous and fine-grained entanglement loss

[Latorre et *al*., 2004]

$\rho = \rho_A = tr_B\left(|\Omega\rangle\langle\Omega|\right)$ $\quad |\Omega\rangle$ ground state (vacuum) of a physical system that depends on a parameter t

Transformation: $\begin{cases} t_1 \rightarrow t_2 \\ \rho(t_1) \rightarrow \rho(t_2) \end{cases}$ $\quad t_2 > t_1$

- Global entanglement loss: $S(\rho(t_1)) > S(\rho(t_2))$
- Monotonous entanglement loss: $\dfrac{dS}{dt} < 0$
- Fine-grained entanglement loss: $\vec{\rho}(t) \prec \vec{\rho}(t + \delta t)$

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Conformal symmetry in a nutshell

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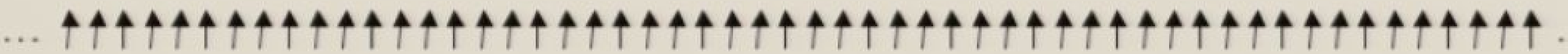

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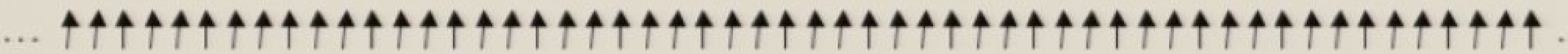

- In the continuum limit, when these theories are described in terms of a quantum field theory, one sees that the symmetry group is even larger: **conformal group.** The theory is then a **conformal field theory (CFT).**

Conformal transformation: $g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \Omega(x) g_{\mu\nu}(x)$

metric

space-time coordinate

conformal factor

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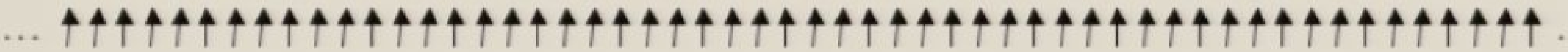

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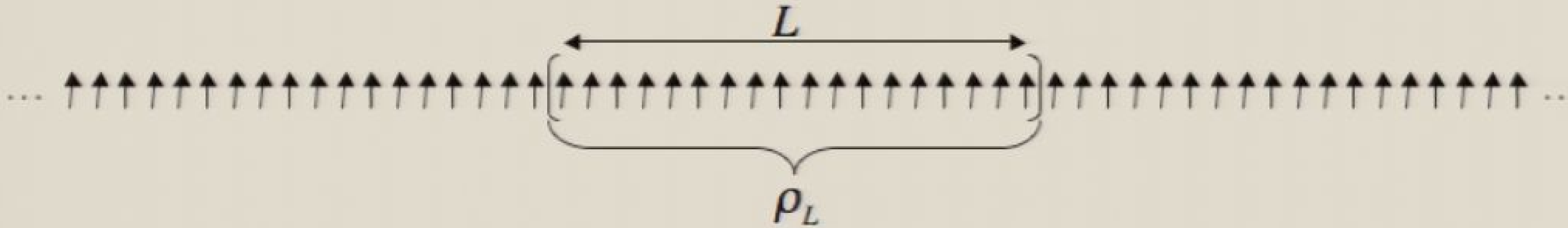
metric — space-time coordinate — conformal factor

- **Conformal symmetry completely determines all the physical properties of critical theories in (1+1) dimensions (e.g. some critical spin chains).**

space — time (usually fixed)

Majorization with L in CFT with (1+1) dimensions



(see also [Zhou et *al*., 2005])

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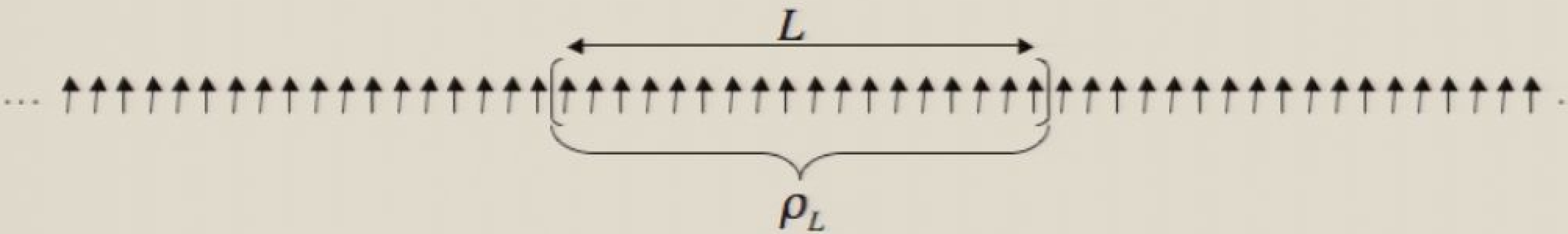


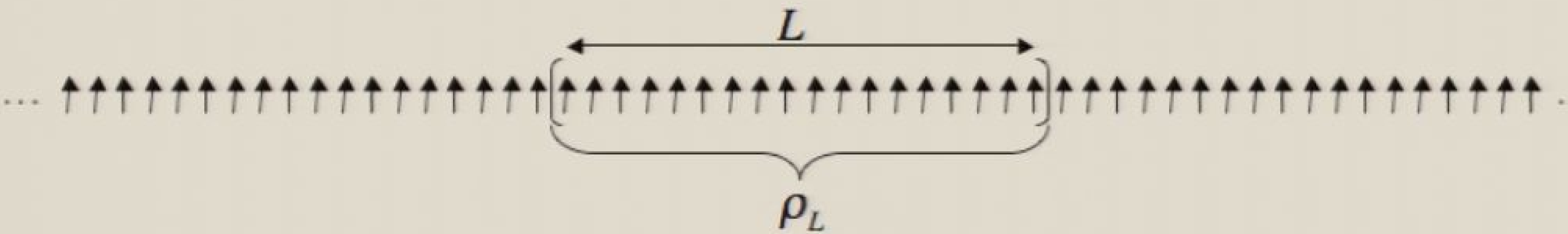
- **Theorem:** $\rho_L \prec \rho_{L'}$ If $L \geq L'$ for all possible (1+1)-dimensional CFT.

$$\rho_L = \frac{1}{Z(q)} q^{-\frac{c}{12}} q^{\left(L_0 + \bar{L}_0\right)} \qquad \log(q) = -\frac{2\pi^2}{\log(L/\varepsilon)}$$

- c: central charge
- $Z(q)$: partition function
- L_0, $\bar{L}_0$: 0th Virasoro operators
- ε: short-distance cut-off (=1 for spin chains)

(see also [Zhou et *al*., 2005])

Majorization with L in CFT with (1+1) dimensions



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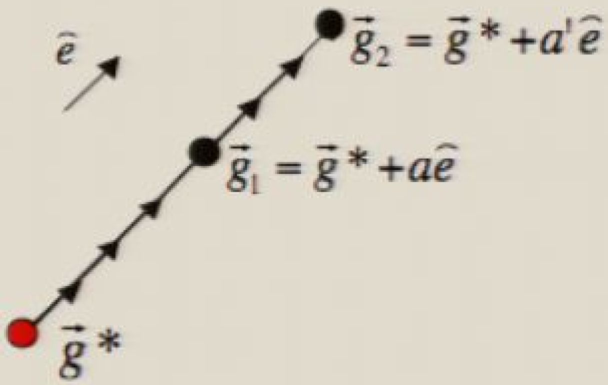
short-distance cut-off
(=1 for spin chains)

- **Example:** XX quantum spin chain $H = -\sum_{l=1}^{\infty}\left(\sigma_l^x \sigma_{l+1}^x + \sigma_l^y \sigma_{l+1}^y\right)$ [Peschel, 2004]

(see also [Zhou et *al.*, 2005])

Majorization along parameter flows

$\vec{g} = (g_1, g_2, \ldots)$ vector of parameters



$\vec{g}^*$ conformal point

$\hat{e}$ unity vector

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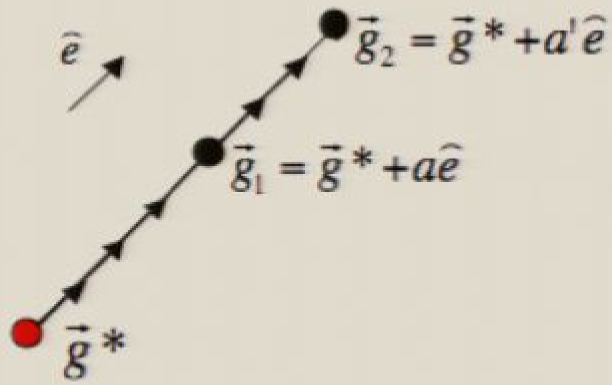

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- **Theorem:** if the reduced density matrices at $\vec{g} = \vec{g}^* + a\hat{e}, a > 0$ preserve the same structure than at $\vec{g}^*$, with a monotonically decreasing $q(\vec{g})$ along the flow in a, then $\rho(\vec{g}_1) \prec \rho(\vec{g}_2)$.

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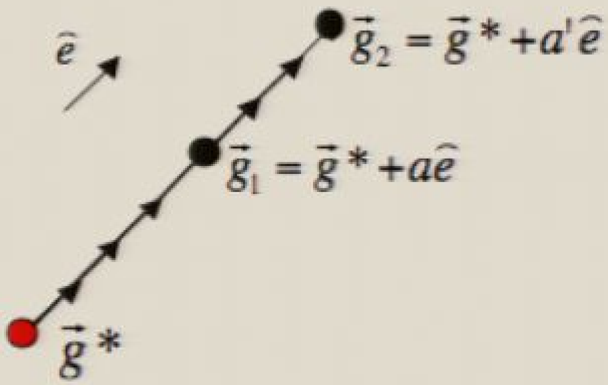

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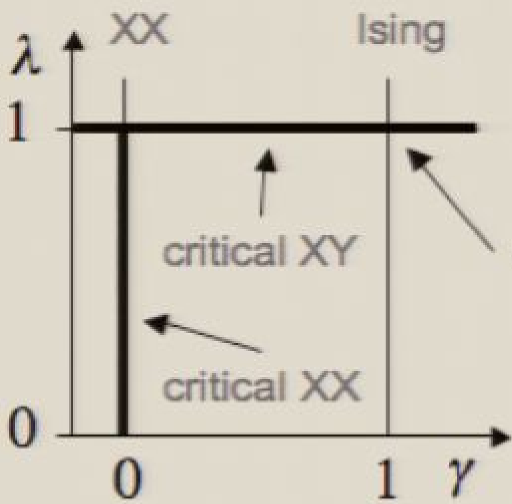
- If a coincides with the scale of an RG-flow, then we have proven continuous majorization along an RG-flow of some integrable theories (by construction).
- If the theory has only one parameter, motonicity of the parameter along RG is trivial, but majorization of the reduced vacuum is not.

Example: XY quantum spin chain

$$H = -\sum_{l=1}^{\infty}\left((1+\gamma)\sigma_l^x\sigma_{l+1}^x + (1-\gamma)\sigma_l^y\sigma_{l+1}^y + 2\lambda\sigma_l^z\right)$$



$\rho_{(\lambda,\gamma)}$ half-infinite chain

$$\rho_{(\lambda,\gamma)} = \frac{1}{Z(\lambda,\gamma)}\exp\left(-\sum_{k=0}^{\infty} n_k \varepsilon_k(\lambda,\gamma)\right)$$ [Peschel, 2004]

$n_k = 0,1$ fermionic ocupation numbers

$\varepsilon_k(\lambda,\gamma)$ monotonic function of the two parameters

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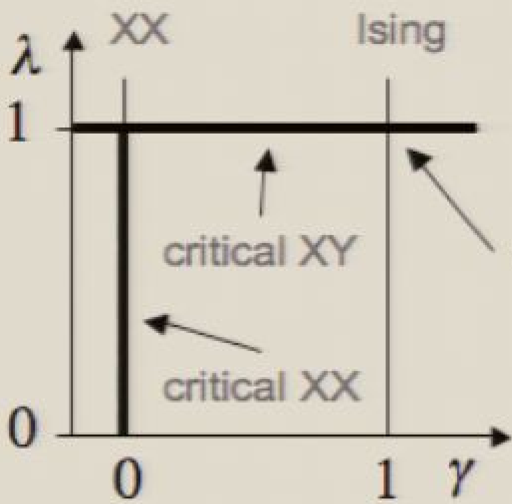

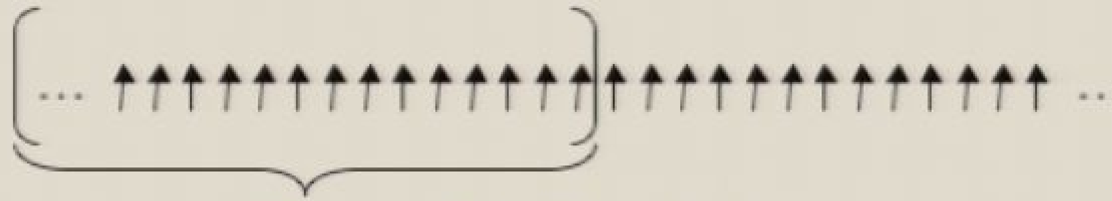

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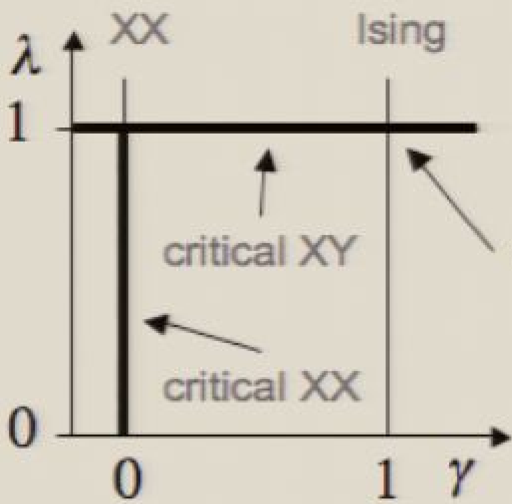

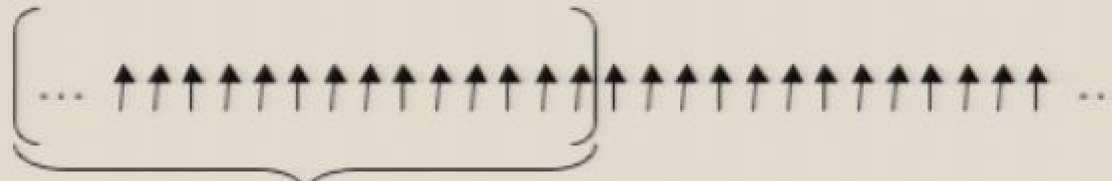

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- Majorization relations hold along the flows in λ and γ. For $\gamma = 1$, the flow in λ along the Ising line coincides with an RG-flow.
- Majorization may hint irreversibility of RG-flows in **higher dimensions** also. [Riera, Latorre, *in prepration*]

Other interesting results with CFT

- Single-copy entanglement scales assimptotically as half the entanglement entropy for conformal field theories in (1+1) dimensions.
  [Orús et *al*., 2005]

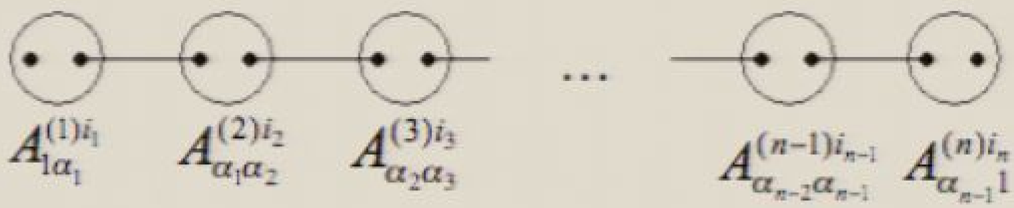
$$E_1(\rho_L) = \frac{c}{6}\log L - \frac{c}{6}\frac{\pi^2}{\log L} + O(1/L)$$

$$S(\rho_L) = \frac{c}{3}\log L + O(1/L)$$

- Entanglement entropy in the Lipkin-Meshkov-Glick model behaves *as if* the model, though defined on a simplex, were governed by an underlying conformal symmetry in (1+1) dimensions.
  [Latorre et *al*., 2004]

$$S(\rho_L) \approx \frac{c_{eff}}{3}\log L$$

Classical simulation of quantum many-body systems



A "bit" of motivation…

State of an n-qubit quantum system: $|\psi\rangle = \sum_{\{i\}} c_{i_1 i_2 \ldots i_n} |i_1, i_2, \ldots, i_n\rangle$

$c_{i_1 i_2 \ldots i_n}$ ⟹ 2^n coefficients

⇓

Exact classical simulations of general Hamiltonians
only possible up to ~24 spins (qubits) with exact methods
(motivation to build a quantum computer [Feynmann, 1982]).

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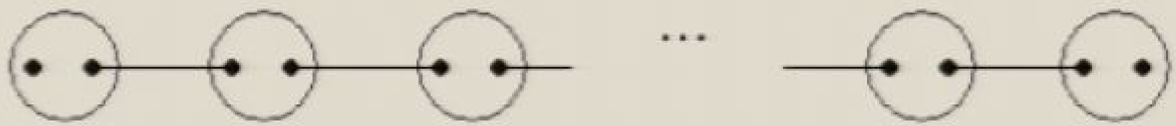
$\Downarrow$

Exact classical simulations of general Hamiltonians only possible up to ~24 spins (qubits) with exact methods (motivation to build a quantum computer [Feynmann, 1982]).

We have efficiently simulated *approximated* quantum computations up to 100 qubits with matrix product states.

Matrix product states (MPS)

[Afflek et *al*., 1987]



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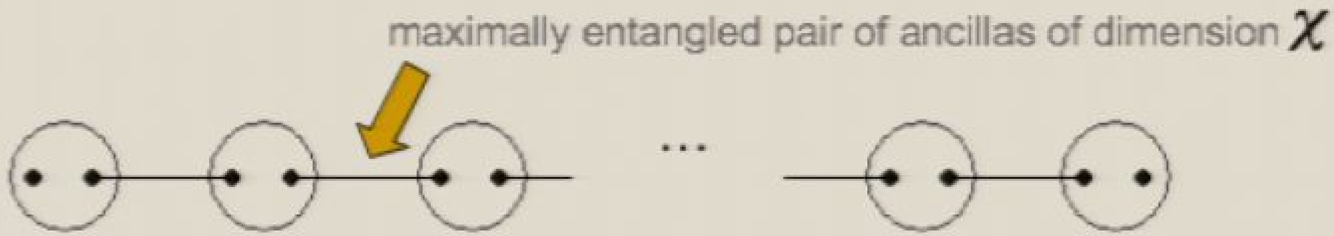

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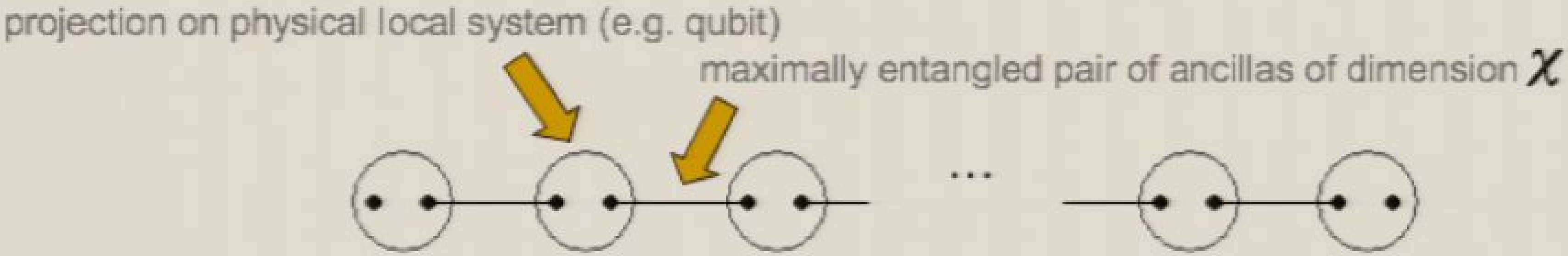

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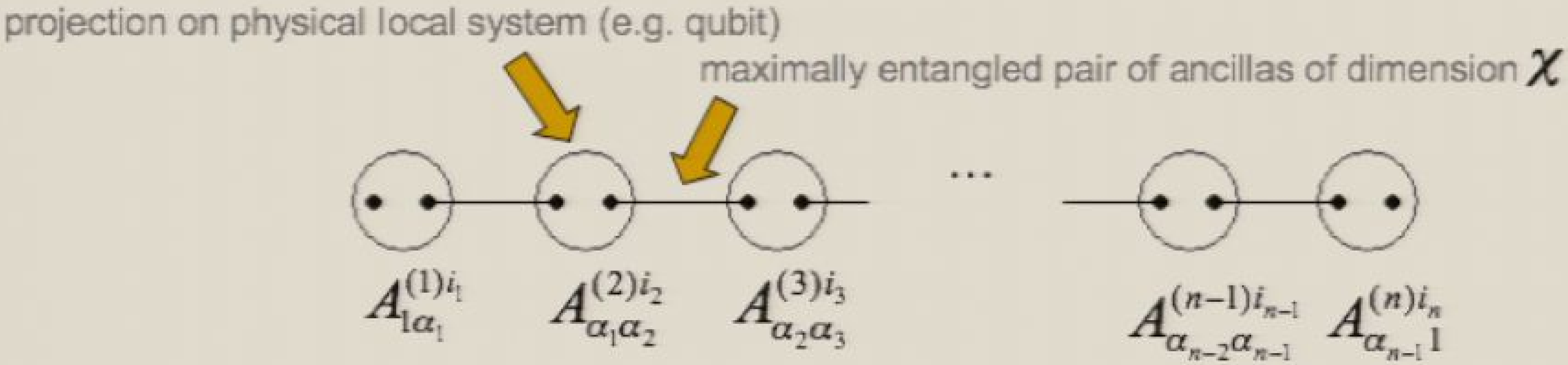
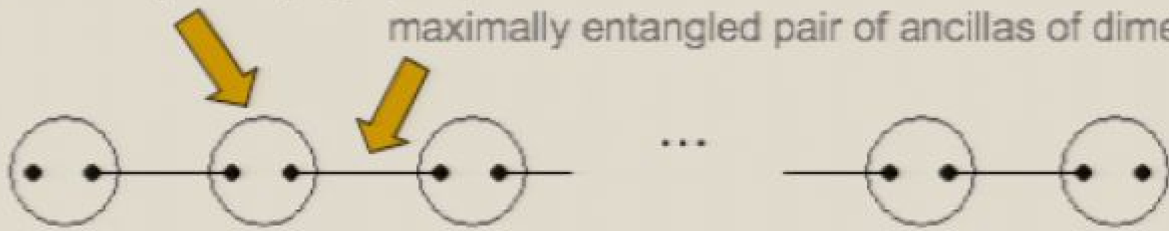


Matrix product states (MPS)

[Afflek et *al*., 1987]

projection on physical local system (e.g. qubit)

maximally entangled pair of ancillas of dimension χ



$A^{(1)i_1}_{1\alpha_1}$ $A^{(2)i_2}_{\alpha_1\alpha_2}$ $A^{(3)i_3}_{\alpha_2\alpha_3}$ $A^{(n-1)i_{n-1}}_{\alpha_{n-2}\alpha_{n-1}}$ $A^{(n)i_n}_{\alpha_{n-1}1}$

$$|\psi\rangle = \sum_{\{i\}}\sum_{\{\alpha\}} A^{(1)i_1}_{1\alpha_1} A^{(2)i_2}_{\alpha_1\alpha_2} \ldots A^{(n)i_n}_{\alpha_{n-1}1} |i_1, i_2, \ldots, i_n\rangle \qquad i_a = 1,2,\ldots d \quad \alpha_a = 1,\ldots,\chi$$

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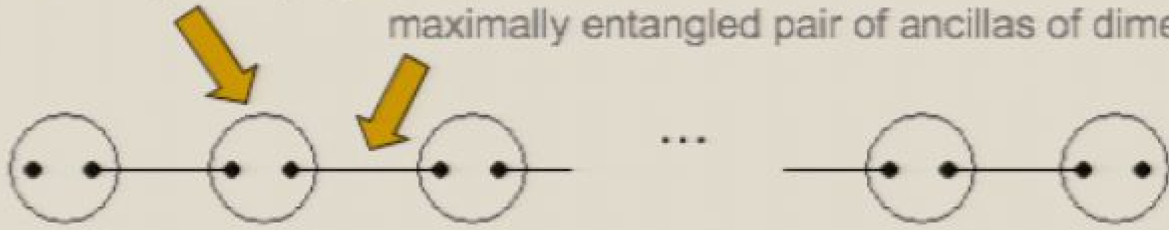

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Schmidt coefficients [Vidal, 2003]

(measure of entanglement; $\chi \approx 2^{S(\rho)}$)

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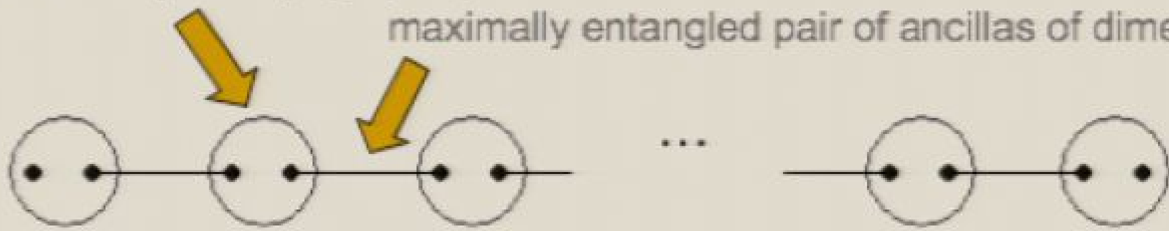

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(measure of entanglement; $\chi \approx 2^{S(\rho)}$)

Generalizations
- To more than 1 dimension $\longrightarrow$ PEPS [Verstraete, Cirac, 2004]
- With disentangling unitaries $\longrightarrow$ MERA [Vidal, 2005]

Why are MPS "good"?

- The state is represented with $nd\chi^2$ parameters, instead of d^n.
- Any quantum state can be represented as an MPS, with large enough χ.
- Physical observables (e.g. correlators) can be computed in $O(poly(\chi))$ time.

- They allow to compute low-energy properties of Hamiltonians (DMRG, euclidean time evolution…).
- They allow to compute the dynamics of quantum states.

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Works great for quantum many-body systems in 1 spatial dimension, because of the logarithmic scaling of the entanglement entropy.

[Vidal et *al.*, 2002]

Computing ground states with MPS

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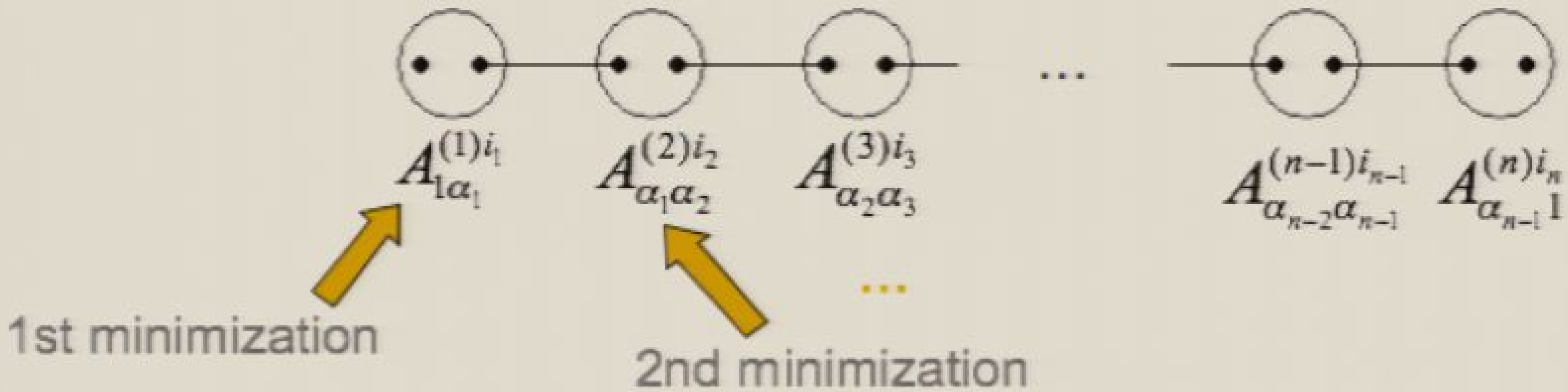

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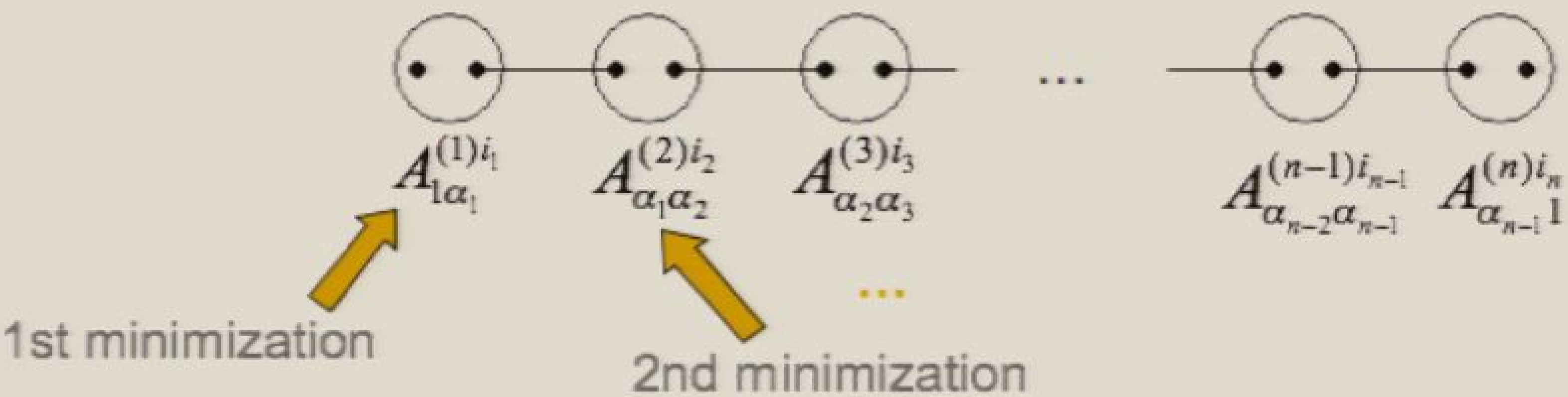


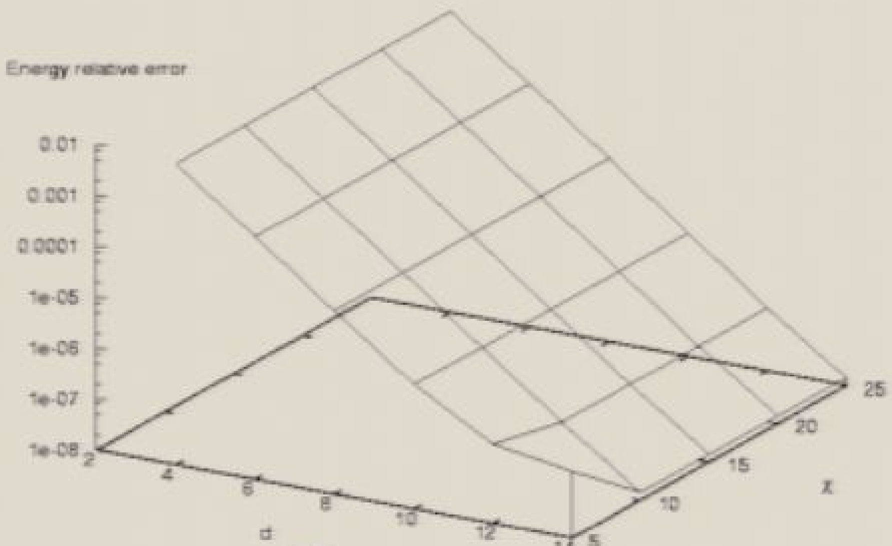
This procedure is equivalent to the DMRG algorithm. [White, 1992]

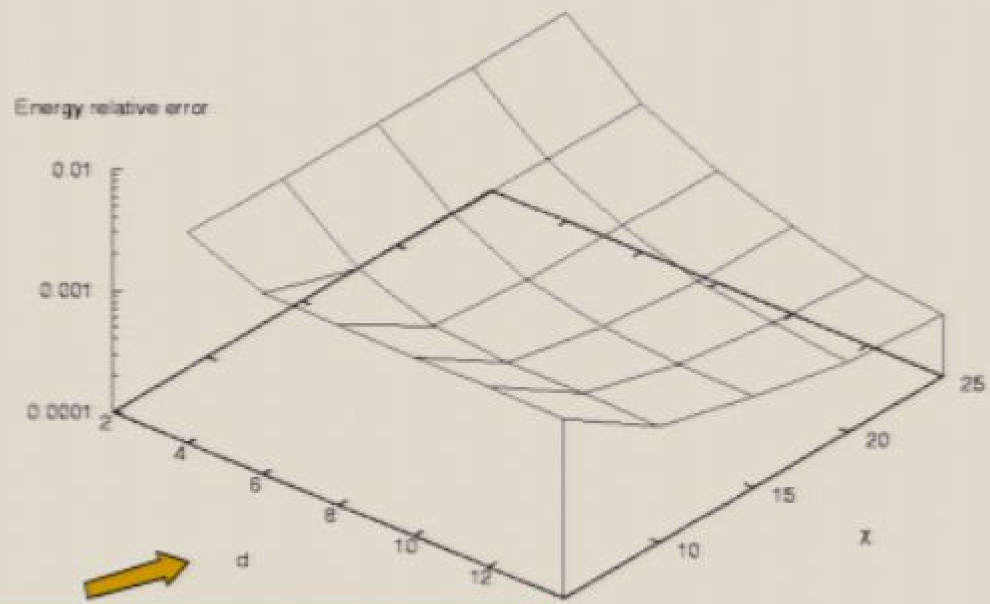
Example: harmonic chain (ϕ^2 theory)

[Iblisdir, Orús, Latorre, *in preparation*]

$$H=\frac{1}{2}\sum_{i=1}^{N}p_i^2+\frac{1}{2}\sum_{i=1}^{N-1}(x_i-x_{i+1})^2+\frac{\mu}{2}\sum_{i=1}^{N}x_i^2\approx\int\left(\frac{1}{2}\dot{\phi}^2+\frac{1}{2}(\nabla\phi)^2+\frac{\mu}{2}\phi^2\right)dx$$

Off-critical $(\mu=1)$ $(N=30)$ Critical $(\mu\cong 0)$





Truncation in the local dimension of the Hilbert space (Fock basis of free oscillators).

Computing dynamics with MPS

[Vidal, 2003]

- Local gate on system a: $A'^{(a)i'_a}_{\alpha\beta} = U^{(a)}_{i_a i'_a} A^{(a)i_a}_{\alpha\beta}$

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diagonalization of $\rho^{ij}_{\alpha\gamma} = \left|\lambda^{(a-1)}_{\beta}\right|^2 \Theta^{ki}_{\beta\alpha} \Theta^{*kj}_{\beta\gamma}$ (rank $d\chi$)

$$\Theta^{i'_a i'_{a+1}}_{\alpha\gamma} = U^{(a,a+1)}_{i'_a i'_{a+1}, i_a i_{a+1}} A^{(a)i_a}_{\alpha\beta} A^{(a+1)i_{a+1}}_{\beta\gamma} = A'^{(a)i'_a}_{\alpha\beta} A'^{(a+1)i'_{a+1}}_{\beta\gamma}$$

$1,\ldots,\chi$

$1,\ldots,d\chi$

creation of entanglement

(exponential growth! Some truncation is needed…)

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- Non-local gate on non-adjacent systems: **SWAP** systems until they are together.

Truncation schemes

- Local procedure [Vidal, 2003]: retain only a fixed number χ of terms that carry most of the entanglement in the decomposition.

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Entanglement-inspired truncation method

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- Non-local procedure [Verstraete, Cirac, 2004]:

compute *all* the matrices $\tilde{A}^{(a)i_a}_{\alpha\beta}$ with indices up to a fixed χ such that

$\left\| |\tilde{\psi}\rangle - |\psi'\rangle \right\|^2$ is minimum.

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Precission-inspired truncation method

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Precission-inspired truncation method

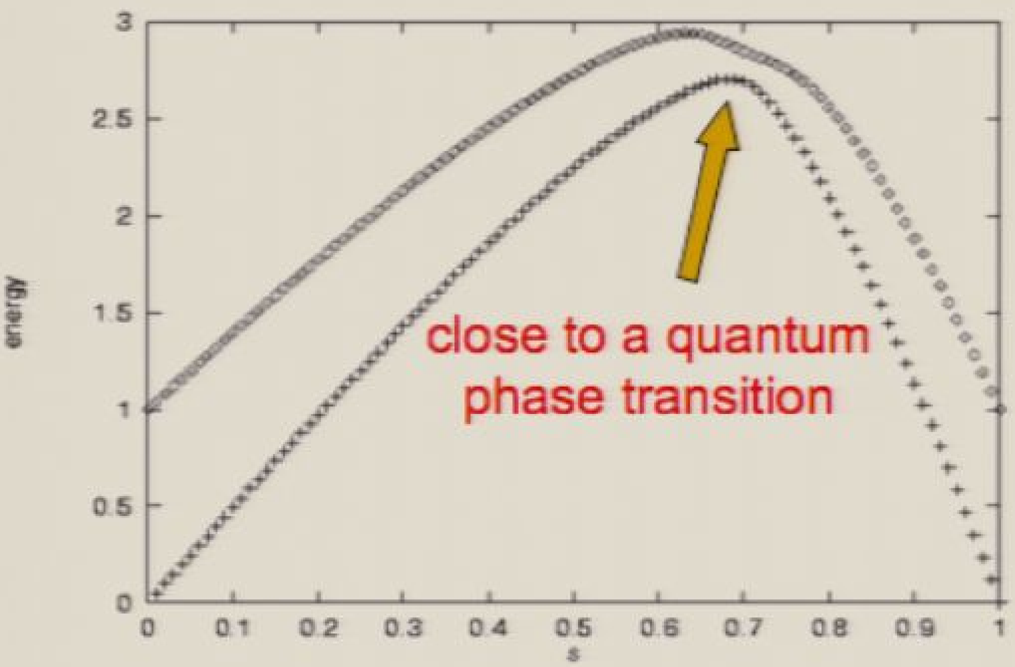
We applied the first local procedure in our time-evolution simulations.

Adiabatic quantum algorithms

[Farhi et *al.*, 2000]

H_p codifies the solution to a problem in its ground state

H_0 with an easy-to-prepare ground state



At $t = 0$ prepare the ground state of H_0 and evolve adiabatically with the time-dependent Hamiltonian

$$H(t/T) = (1 - t/T)H_0 + (t/T)H_p$$

Adiabatic theorem $\Rightarrow$ $T > \frac{\Delta}{g_{\min}^2}$

$$\Delta = \max\left|\frac{dH_{10}(t/T)}{dt}\right| \qquad s = t/T$$

$$g_{\min} = \min(E_1(t) - E_0(t))$$

$$\left|\langle\psi(T)|E_0(T)\rangle\right|^2 \approx 1$$

The Exact Cover NP-Complete problem

Instance: set of m clauses (constraints) over n bits

$$\text{Clause for bits } i,j,k: \quad \begin{array}{c} i,j,k \\ 100 \\ 010 \\ 001 \end{array} \Bigg\} \text{ Satisfies the clause}$$

Problem: is there an assignment of the n bits that satisfies *all* the clauses?

Hard classical instances ⟶ $\frac{m}{n} \approx 0.8$

Adiabatic algorithm solving Exact Cover

$$H_0 = \sum_{i=1}^{n} \frac{d_i}{2}\left(1-\sigma_i^x\right) \qquad H_p = \sum_{c(i,j,k)}^{m} \left(z_i + z_j + z_k - 1\right)^2$$

d_i = # clauses for i $\qquad z_i = \frac{1}{2}\left(1-\sigma_i^z\right)$

$$H(s) = (1-s)H_0 + sH_p$$

Highly non-local interacting spin Hamiltonian

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Average numerical scalings for ~20 qubits:

- $T \approx n^2$ [Farhi et *al.*, 2001]
- $\chi_{\max} \approx 2^{0.1n}$ [Orús, Latorre, 2004]

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Small! (but exponential).
Possibility of a good MPS simulation for "many" qubits?

Discretization in gates

Time evolution ⟹ $U_{T,0} = T\left(e^{-i\int_0^T H(t/T)dt}\right) \approx U_{T,T-\Delta} \ldots U_{2\Delta,\Delta} U_{\Delta,0}$

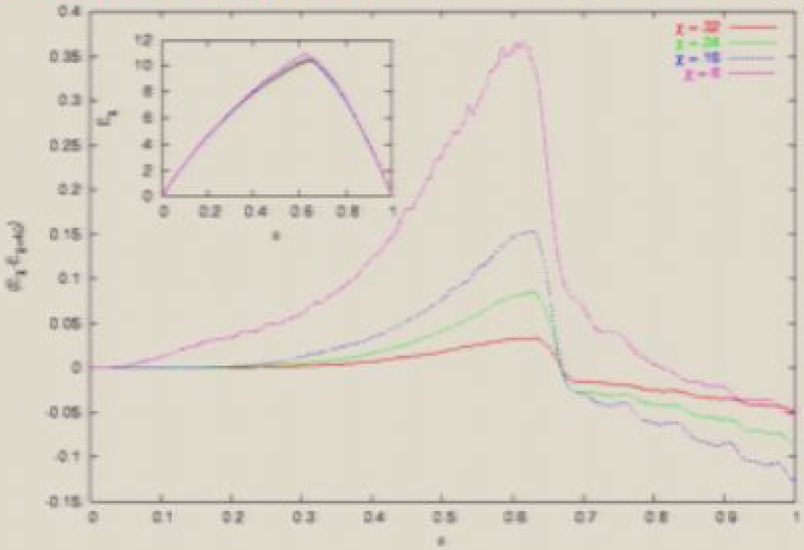
$$U_{(l+1)\Delta,l\Delta} = e^{-i\Delta H(s)} \approx \left(e^{-i\frac{\delta}{2}(1-s)H_0} e^{-i\delta s H_p} e^{-i\frac{\delta}{2}(1-s)H_0}\right)^{\frac{\Delta}{\delta}} \qquad s = \frac{l\Delta}{T}$$

2nd order Trotter expansion

$$\begin{cases} e^{-i\frac{\delta}{2}(1-s)H_0} = \prod_{i=1}^{n} e^{-i\frac{\delta}{4}(1-s)d_i(1-\sigma_i^x)} \\ e^{-i\delta s H_p} = \prod_{c(i,j,k)} e^{-i\delta s(z_i^2 - 2z_i)} e^{-i\delta s(z_j^2 - 2z_j)} e^{-i\delta s(z_k^2 - 2z_k)} e^{-i\delta s} \\ \qquad e^{-i2\delta s z_i z_j} e^{-i2\delta s z_i z_k} e^{-i2\delta s z_j z_k} \end{cases}$$

Simulation results (1)

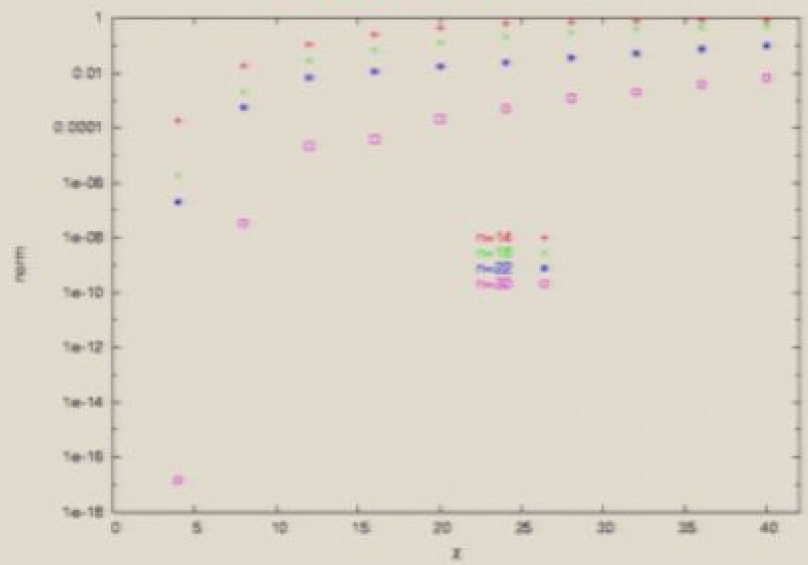
Expected energy for $n = 30, m = 24, T = 100$



⬅ Good accuracy in "small" computation time (as compared to the exact calculation)

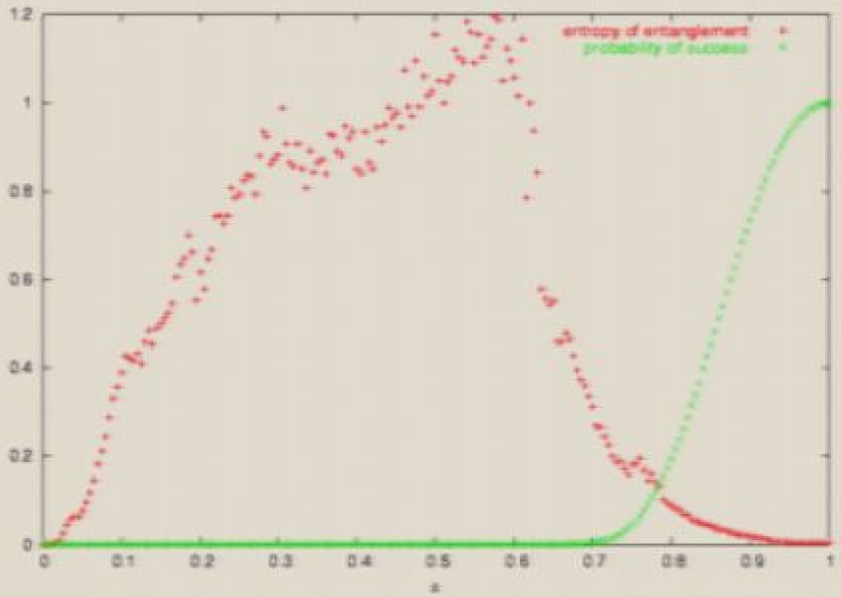
Loss of norm

Increasing precission with increasing χ ➡



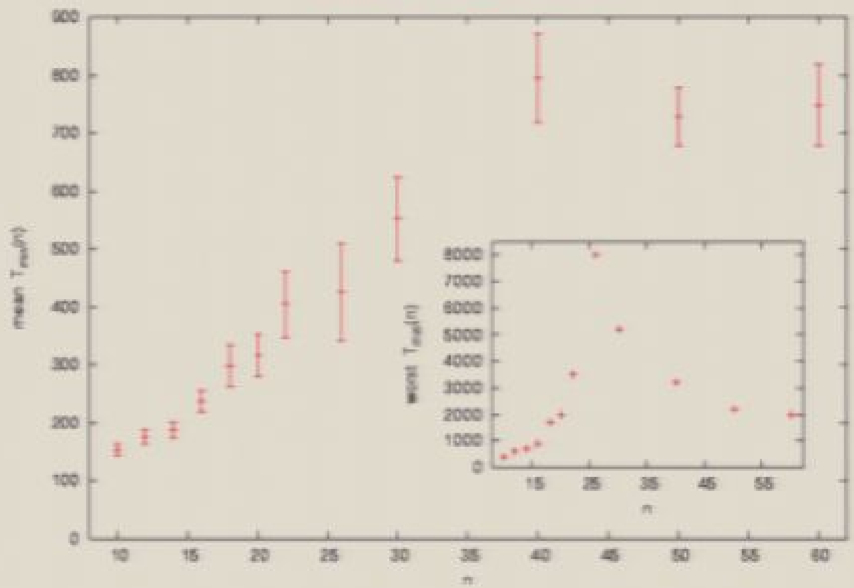
Simulation results (2)

100-qubit intance with $n = 100, m = 84, T = 2000, \chi = 14 << 2^{50}$



Finds the correct solution among $2^{100} \approx 10^{30}$ possibilities!

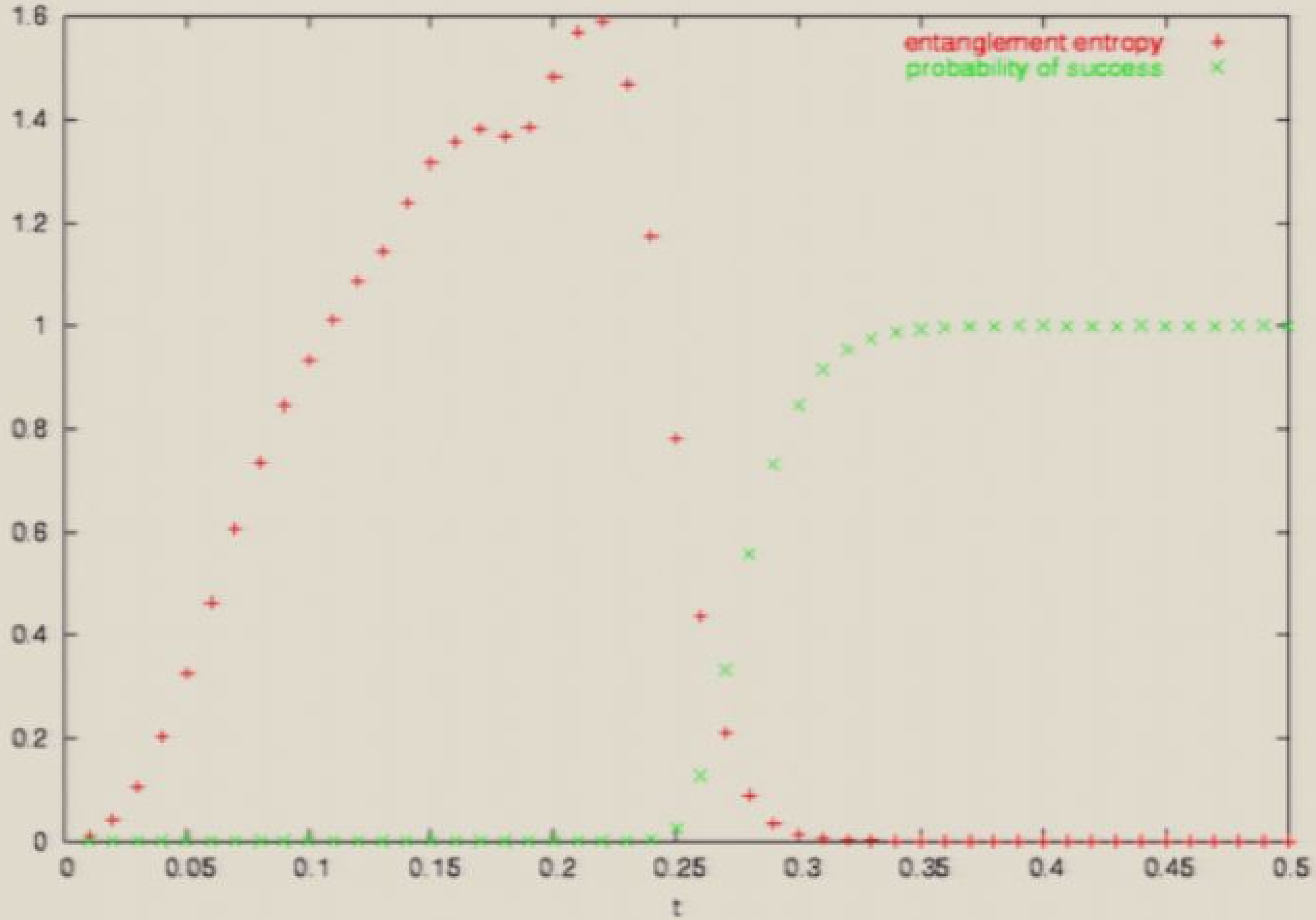
Time statistics



Average appears to grow very slowly with n

Euclidean time evolution: $e^{-H_p t}$

$n = 26, \chi = 6$



Unphysical (not unitary)

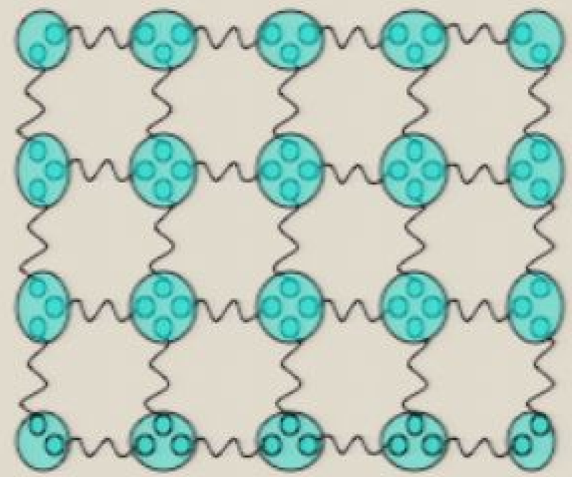
Finds the correct solution much faster than the adiabatic simulations

Conclusions

- Majorization along L and parameter/RG-flows can be proven for some integrable theories in (1+1) dimensions under appropiate assumptions using CFT. This proves some conjectures previously raised for quantum spin chains, and hints a way to prove irreversibility of RG-flows in more dimensions.

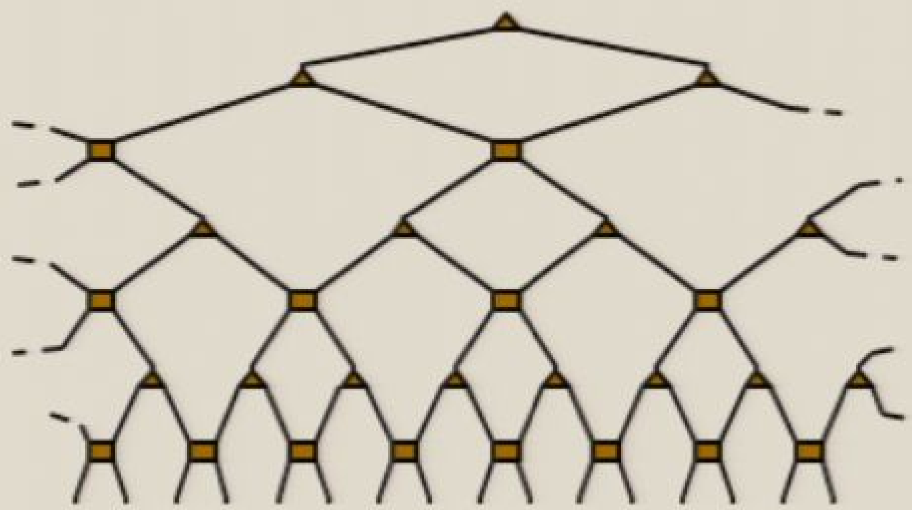
- MPS are a useful computational tool for calculating ground states and dynamical properties of many interesting physical systems. On top, MPS-simulations of quantum computations can handle a relatively large number of qubits with reasonable approximation schemes.

Future work



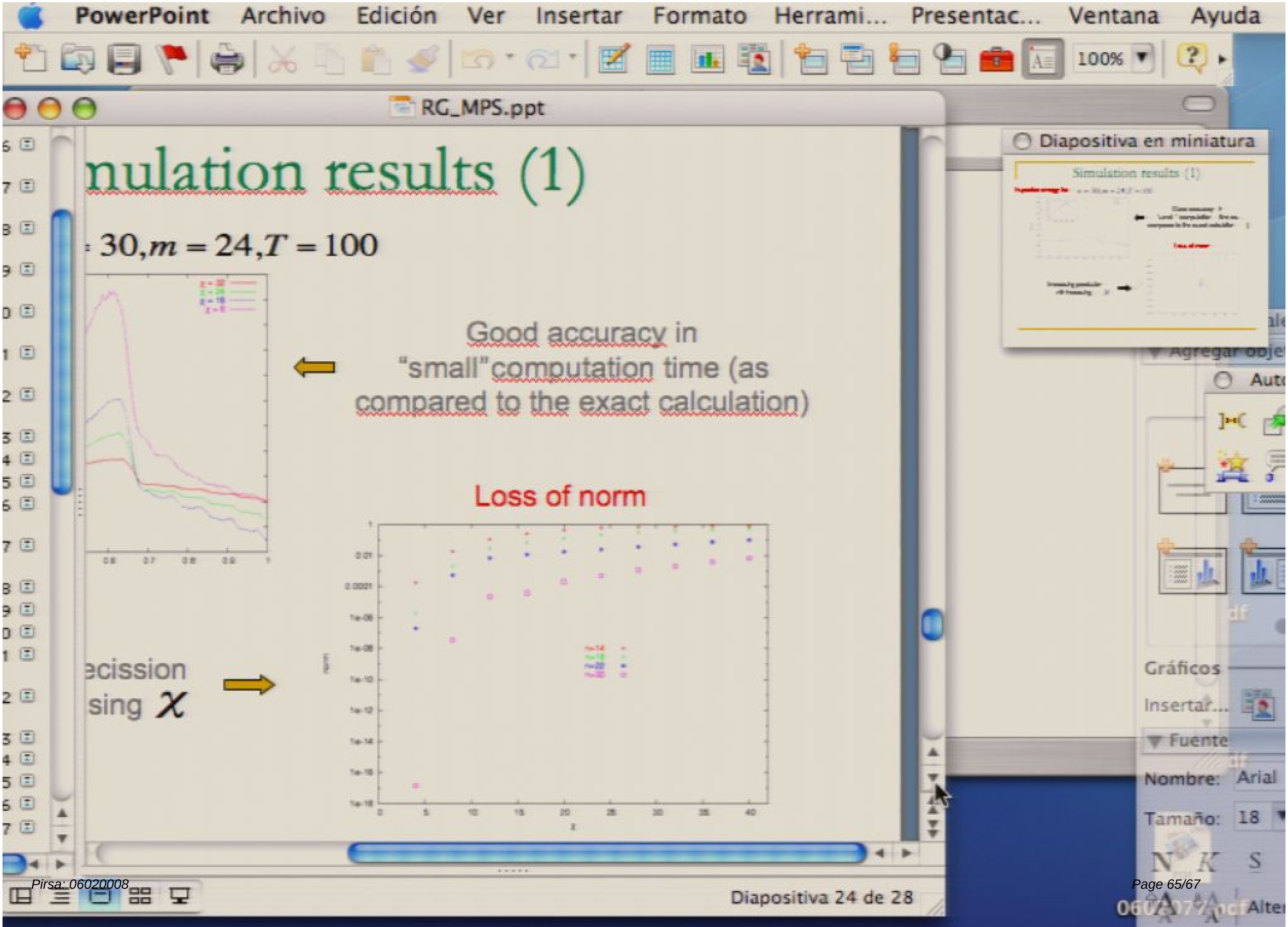
- **PEPS** in more than 1 dimension.

  [Verstraete, Cirac, 2004]



- **MERA** (disentangling unitaries) in 1 and more dimensions.

  [Vidal, 2005]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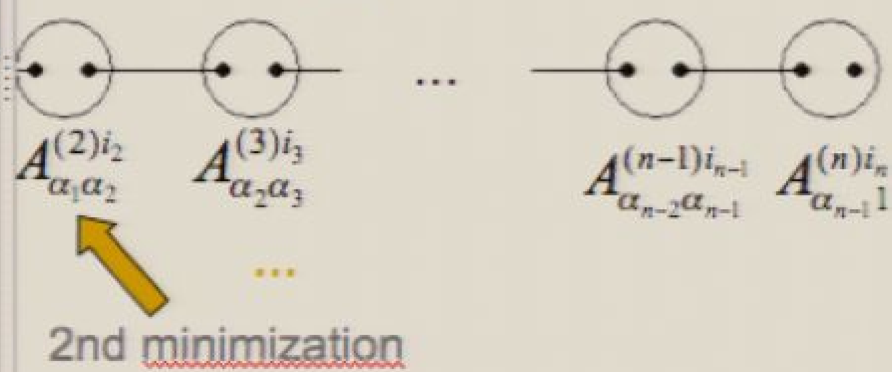

...equivalent to the DMRG algorithm. [White, 1992]

Diapositiva en miniatura

Computing ground states with MPS

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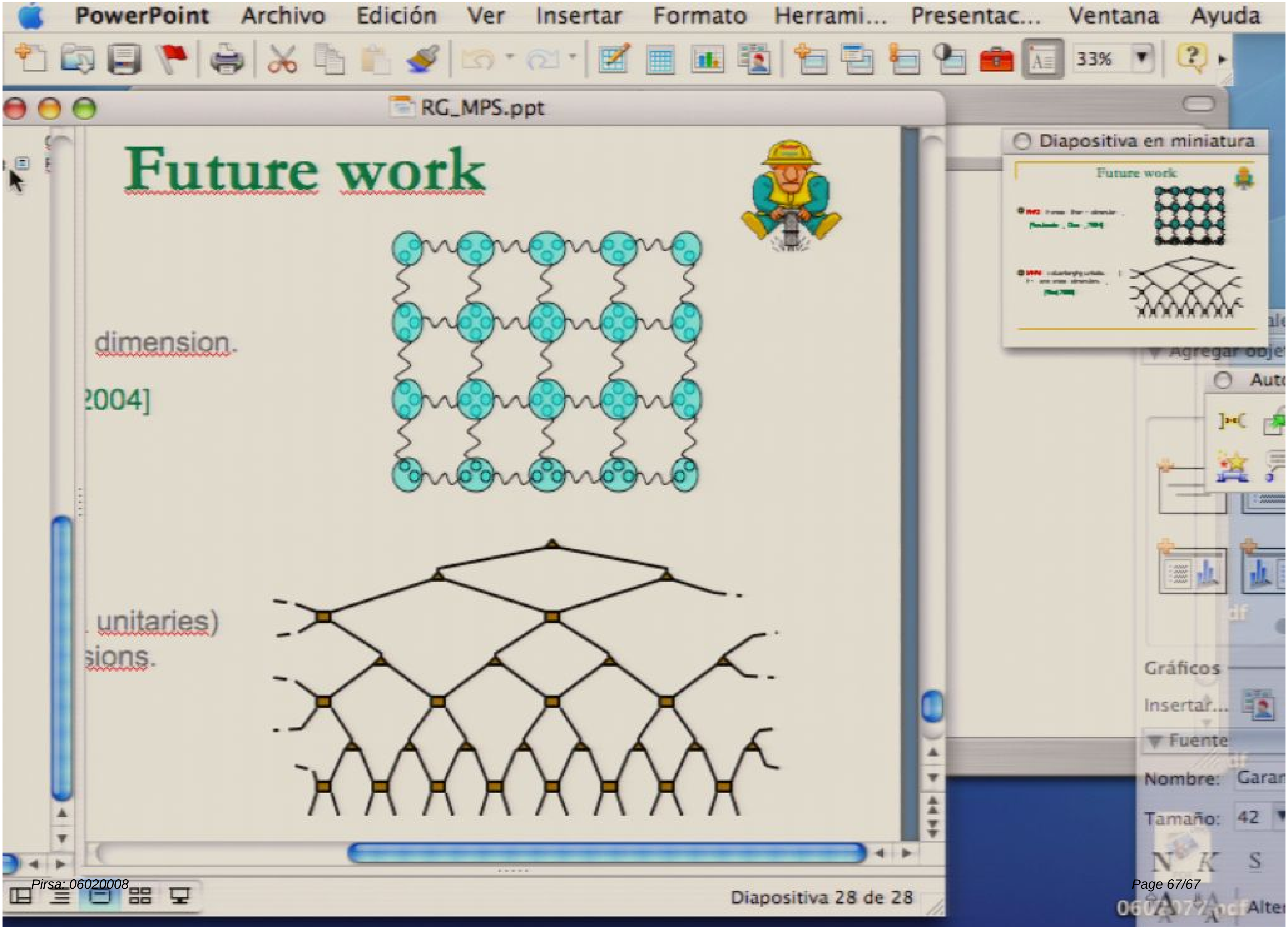
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