Title: Introduction to quantum gravity - Part 6

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URL: http://pirsa.org/06020004

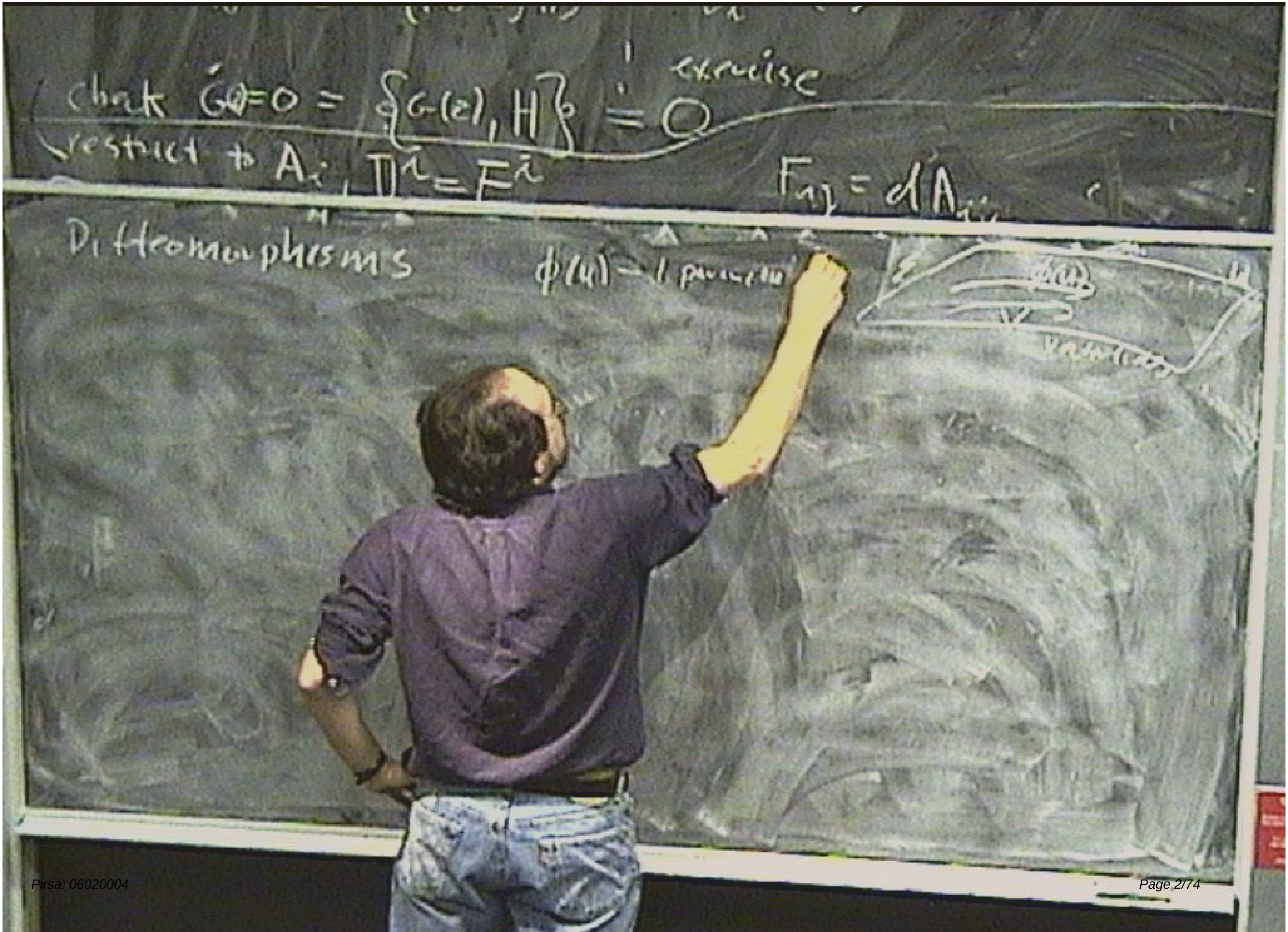
Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravityy with a positive cosmological constant, Lee Smolin, hep-th/0209079
-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

-undergraduate quantum mechanics
-basics of classical gauge field theories
-basic general relativity
-hamiltonian and lagrangian mechanics
-basics of lie algebras


exercise
restrict to
Diffeomorphisms

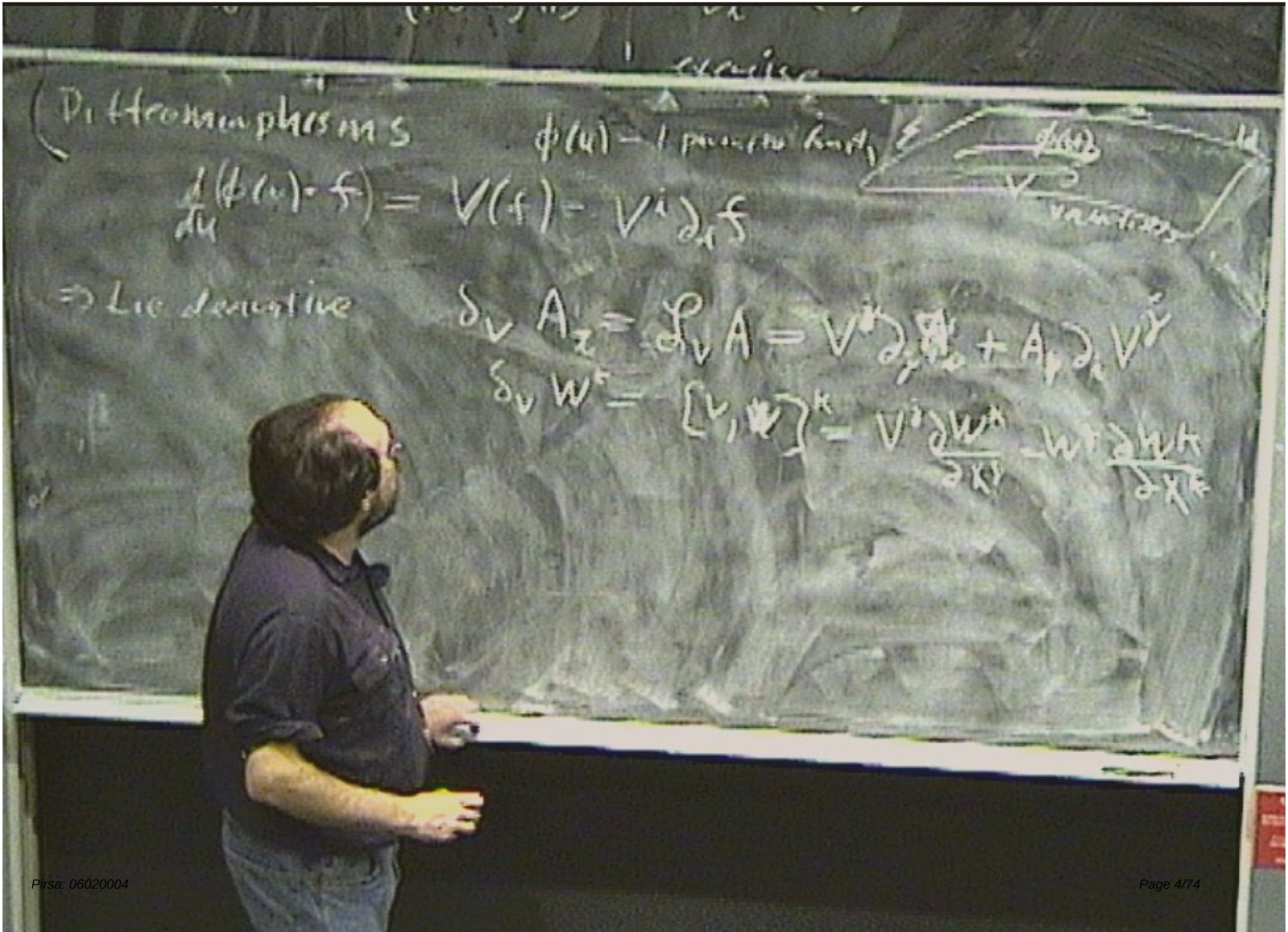
check $\dot{G}(z) = 0 = \{G(z), H\} \doteq 0$ exercise

restrict to A_i, $\Pi^i = F^i$ $\qquad F_{ij} = dA_{ij}$

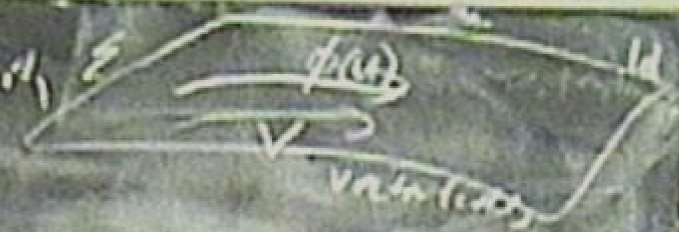
Diffeomorphisms $\qquad \phi(u)$ — 1 parameter family

$$\frac{d}{du}(\phi(u)\cdot f) = V(f) = V^i \partial_i f$$

$\Rightarrow$ Lie derivative $\qquad \delta_V A_i$


Diffeomorphisms
⇒ Lie derivative
vanishes

Diffeomorphisms $\phi(u)$ — 1 parameter family



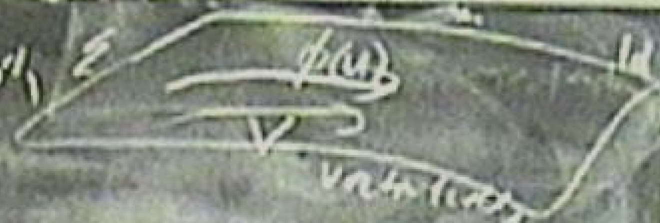
$$\frac{d}{du}(\phi(u)\cdot f) = V(f) = V^i \partial_i f$$

$\Rightarrow$ Lie derivative

$$\delta_V A_i = \mathcal{L}_V A = V^j \partial_j A_i + A_j \partial_i V^j$$

$$\delta_V W^k = [V, W]^k = V^j \frac{\partial W^k}{\partial x^j} - W^j \frac{\partial V^k}{\partial x^j}$$

Diffeomorphisms $\quad \phi(u)$ – 1 parameter family



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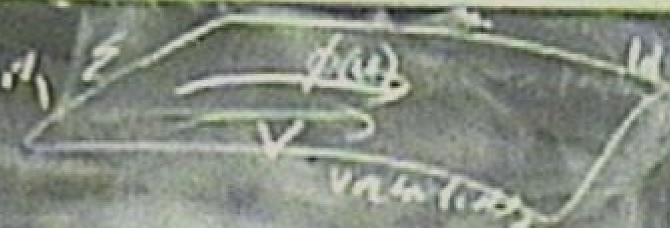
Diffeomorphisms $\quad \phi(u)$ — 1 parameter family



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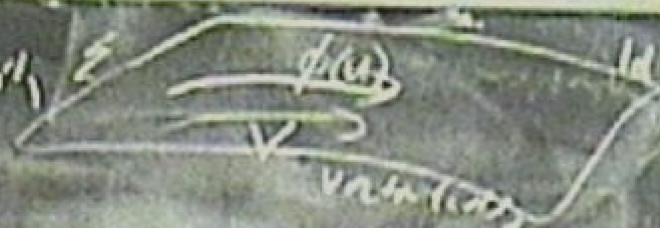
$$\delta_\lambda A_i = \partial_i \lambda$$

$\lambda =$

Diffeomorphisms $\quad \phi(u)$ — 1 parameter family



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$\Rightarrow$ Lie derivative

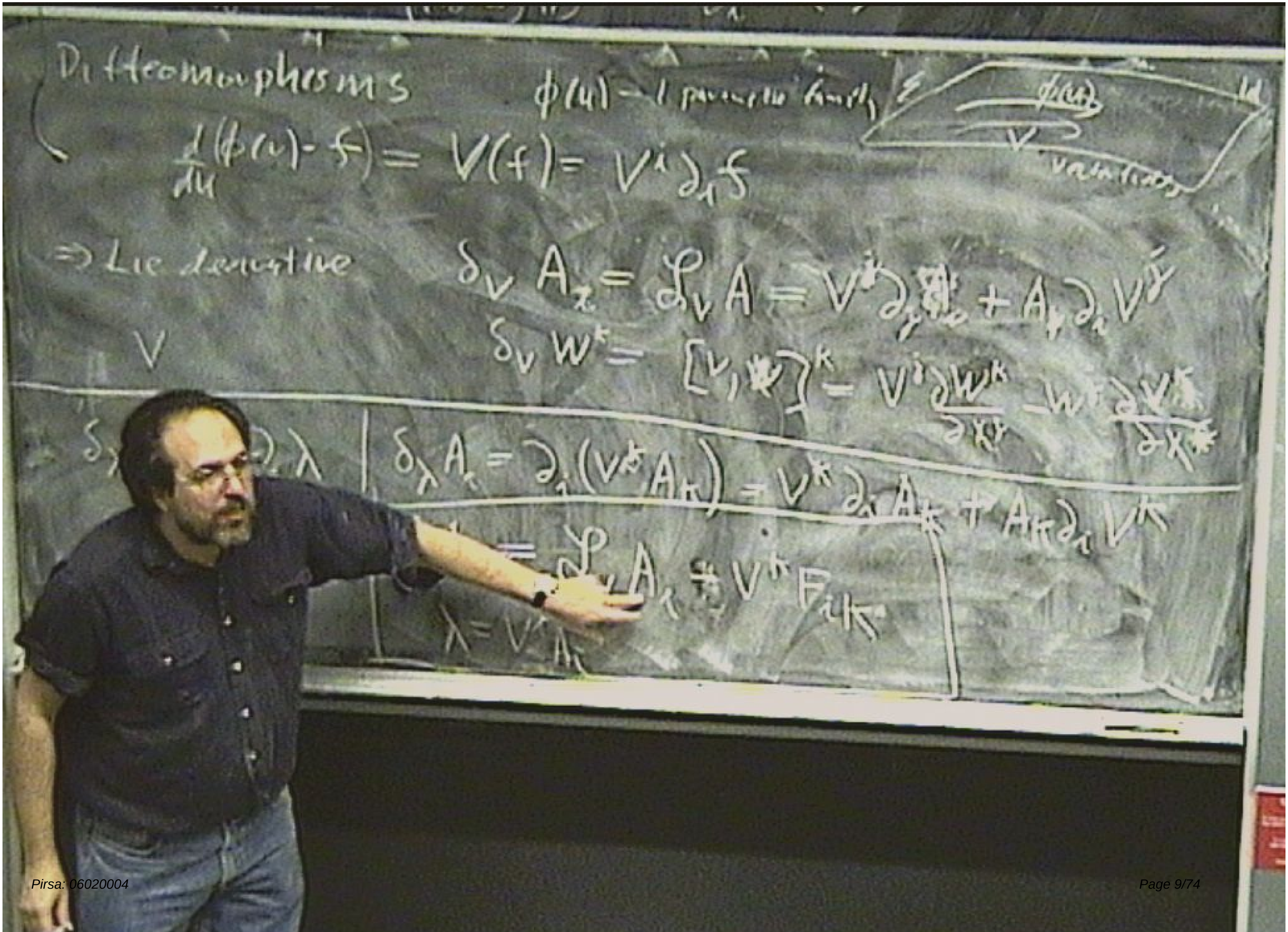
$$\delta_V A_i = \mathcal{L}_V A = V^j \partial_j A_i + A_j \partial_i V^j$$

V

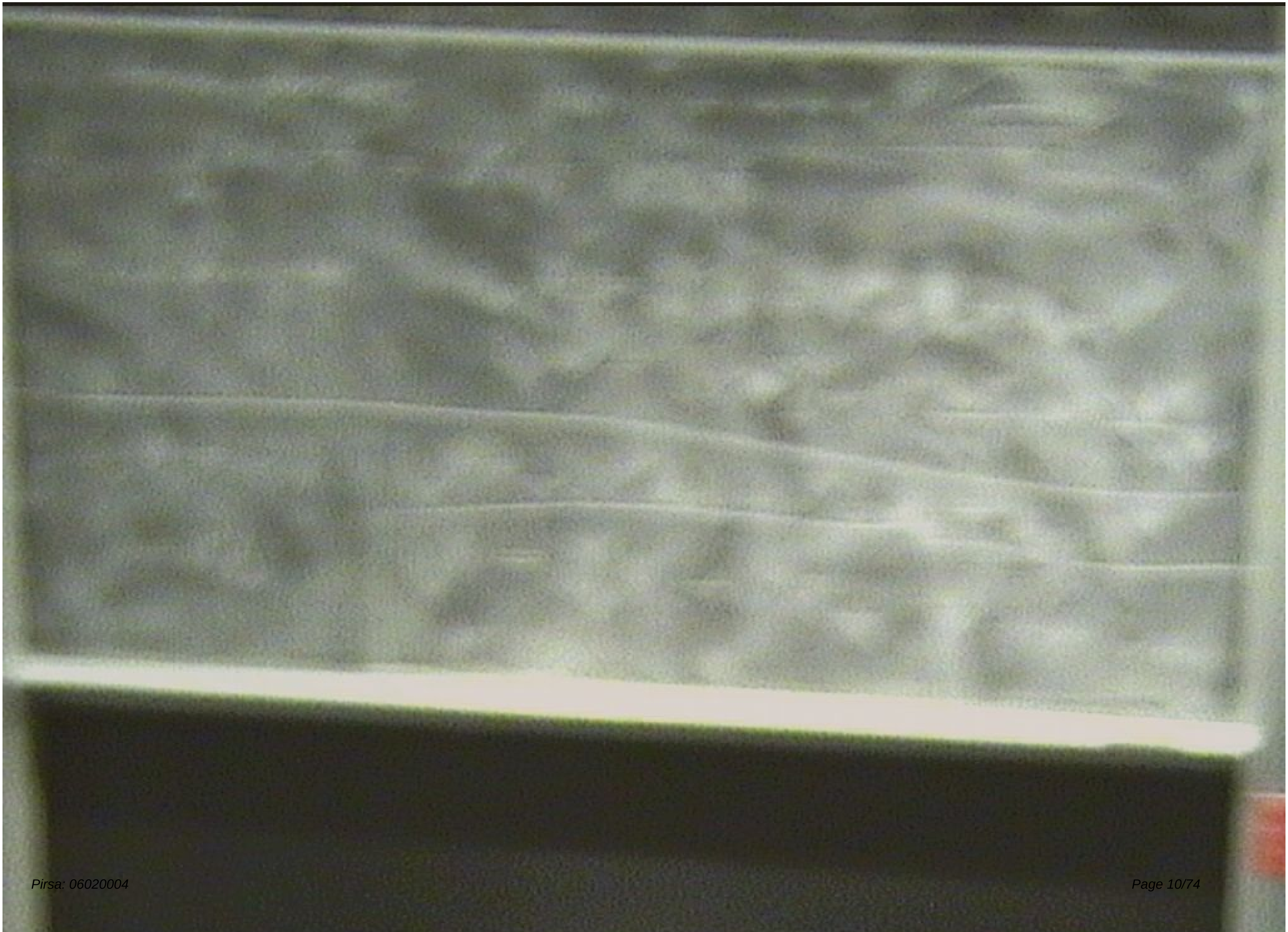
$$\delta_V W^k = [V, W]^k = V^i \frac{\partial W^k}{\partial x^i} - W^i \frac{\partial V^k}{\partial x^i}$$

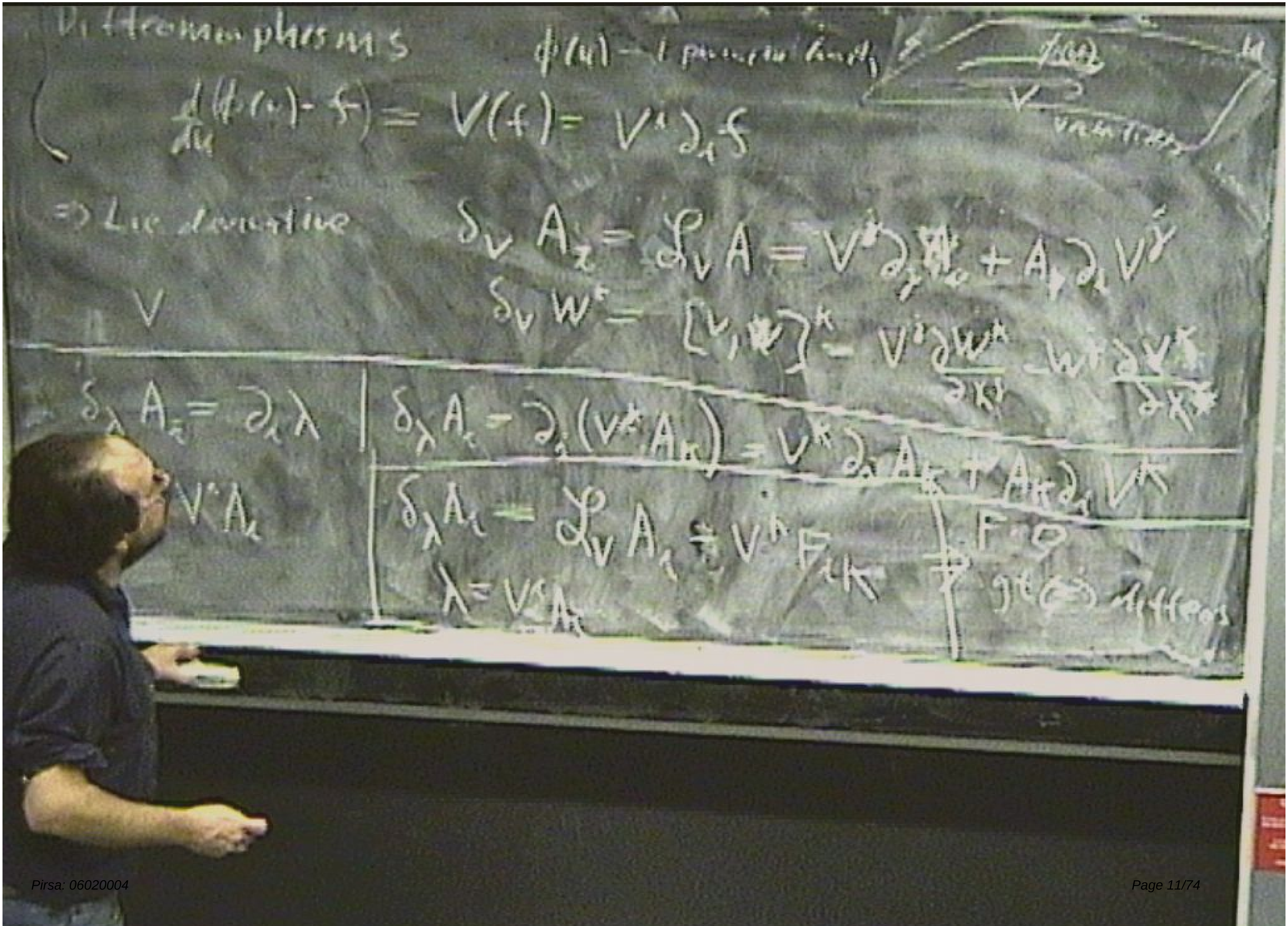
$$\delta_\lambda A_i = \partial_i \lambda \quad \Big| \quad \delta_\lambda A_i = \partial_i (V^k A_k)$$

$$\lambda = V^i A_i$$



Diffeomorphisms
φ(u) – 1 parameter family
d/du (φ(u)·f) = V(f) = V^i ∂_i f
⇒ Lie derivative
V
δ_V W^k = [V, W]^k = V^i ∂W^k/∂x^i − W^i ∂V^k/∂x^i
δ_λ A_i = ∂_i (V^k A_k) = V^k ∂_i A_k + A_k ∂_i V^k
λ = V^k A_k





Diffeomorphisms
⇒ Lie derivative
$\delta_V A_i = \mathcal{L}_V A = V^j \partial_j A_i + A_j \partial_i V^j$
$\delta_\lambda A_i = \partial_i \lambda$

$\lambda = V^i A_i$

$\delta_\lambda A_i = \mathcal{L}_V A_i + V^k F_{ik}$

$\lambda = V^k A_k$

$F = 0 \Rightarrow gt \Leftrightarrow$ diffeos

$S = \int$ density $= \int$ 3-form $= \int A \wedge dA = \int \varepsilon^{abc} A_a \partial_b A_c$

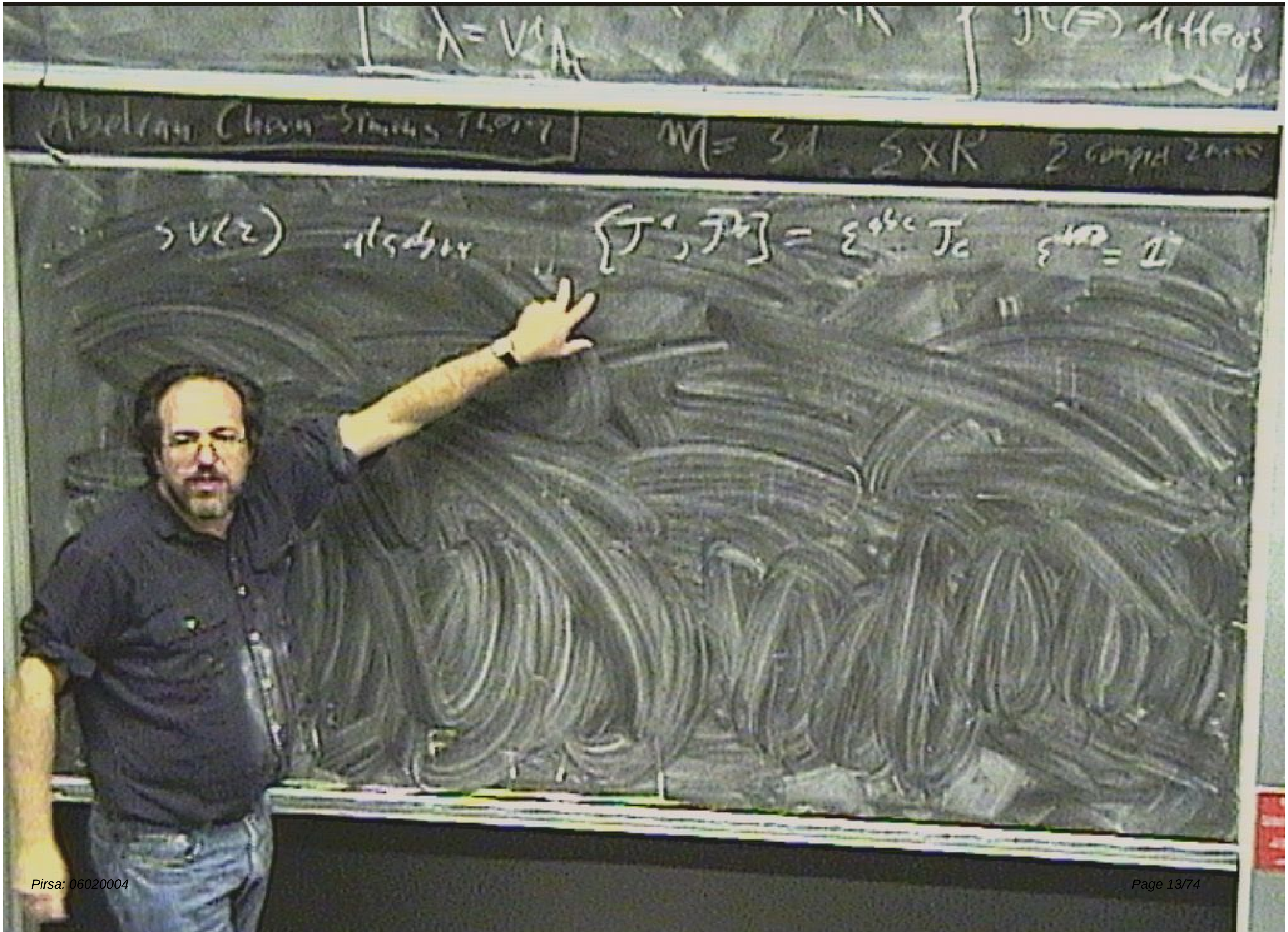
$a = 0, \; i = 1,2$

$S = \int \varepsilon^{ij} [A_0 2 \partial_i A_j + A_j \partial_0 A_i + \cancel{A_i \partial_j A_0}]$

$\pi^i = \frac{\delta S}{\delta \dot{A}_i(z)} = \varepsilon^{ij} A_j(z) \Rightarrow \boxed{\{ A_1(x), A_2(y) \} = \delta^2(x,y)\, \varepsilon_{12}}$

$\pi^0 = 0$

$\{A_i(x), A_j(y)\} = \varepsilon_{ij}\, \delta^2(x,y)$



Abelian Chern-Simons Theory
M = 3d Σ × R
[J^a, J^b] = ε^abc J_c

$\lambda = V^a A$ ... $\mathcal{G}(\Sigma)$ diffeos

Abelian Chern-Simons Theory

$\mathcal{M} = 3d \quad \Sigma \times \mathbb{R}' \quad \Sigma$ compact 2-mfd

$SU(2)$ algebra $\quad [J^a, J^b] = \epsilon^{abc} J_c \quad \epsilon^{123} = 1$

2-dim rep $\quad J^a \Rightarrow \sigma^a \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\lambda = V^a A_a$ $\quad$ R $\quad$ $\mathcal{G}(\Sigma)$ diffeo's

$\boxed{\text{Abelian Chern-Simons Theory}}$ $\quad$ $M = 3d \quad \Sigma \times R^1 \quad \Sigma$ compact 2 mfd

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A_a $\qquad$ $A = \sum_a c_a \sigma^a \quad c_a \in \mathbb{C}$

$\lambda = V^{-1} A$ … K … $\mathcal{G}(\Sigma)$ diffeos

Abelian Chern-Simons Theory

$M = 3d$, $\Sigma \times \mathbb{R}$ $\quad$ Σ compact 2 mfld

$SU(2)$ algebra $\quad [J^a, J^b] = i\varepsilon^{abc} J_c \quad \varepsilon^{123} = 1$

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A_a $\Phi = \sum_a c_a \sigma^a$, $c_a \in \mathbb{C}$

$\lambda = V^a A_a$ $\mathcal{G} \Leftrightarrow$ diffeos

Abelian Chern-Simons Theory $M = 3d$ $\Sigma \times R'$ Σ compact 2 mfd

$su(2)$ algebra $[T^a, T^b] = i\epsilon^{abc} T_c$ $\epsilon^{123} = 1$

2-dim rep $T^a \Rightarrow \sigma^a$ $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$A_a \in \mathcal{A}$ vector field V^a $\Phi = \sum_a c_a \sigma^a$ $c_a \in \mathbb{C}$

$V^a A_a \in$ function in $\mathcal{A}$

$\lambda = V^a A_a$ $gt (=)$ diffeos

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$A_a \in \mathcal{a}$ vector field V^a $\mathcal{a} = \sum_a c_a \sigma^a$

$V^a A_a \in$ function in $\mathcal{a}$ $\delta A_a = \partial_a$

$f \in \mathcal{a}$ $f = f^I \sigma^I$

$A_a = A_a^I \sigma^I$

$\lambda = V^a A_a$ ... $\exists$ $\mathcal{G}(\Rightarrow)$ diffeos

$SU(2)$ algebra $[J^a, J^b] = i\varepsilon^{abc} J_c$ $\quad \varepsilon^{123} = 1$

2-dim rep $J^a \Rightarrow \sigma^a$ $\quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$A_a \in \mathcal{a}$ vector field V^a $\quad \alpha = \sum_a c_a \sigma^a$, $c_a \in \mathbb{C}$

$V^a A_a \in$ function in $\mathcal{a}$ $\qquad \delta A_a = \partial_a f + [A_a, f]$

$f \in \mathcal{a}$ $\quad f = f_I \sigma^I$

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$\lambda = V^a A_a$ gt (=) diffeos

$SU(2)$ algebra $[T^a, T^b] = i\epsilon^{abc} T_c$ $\epsilon^{123} = 1$

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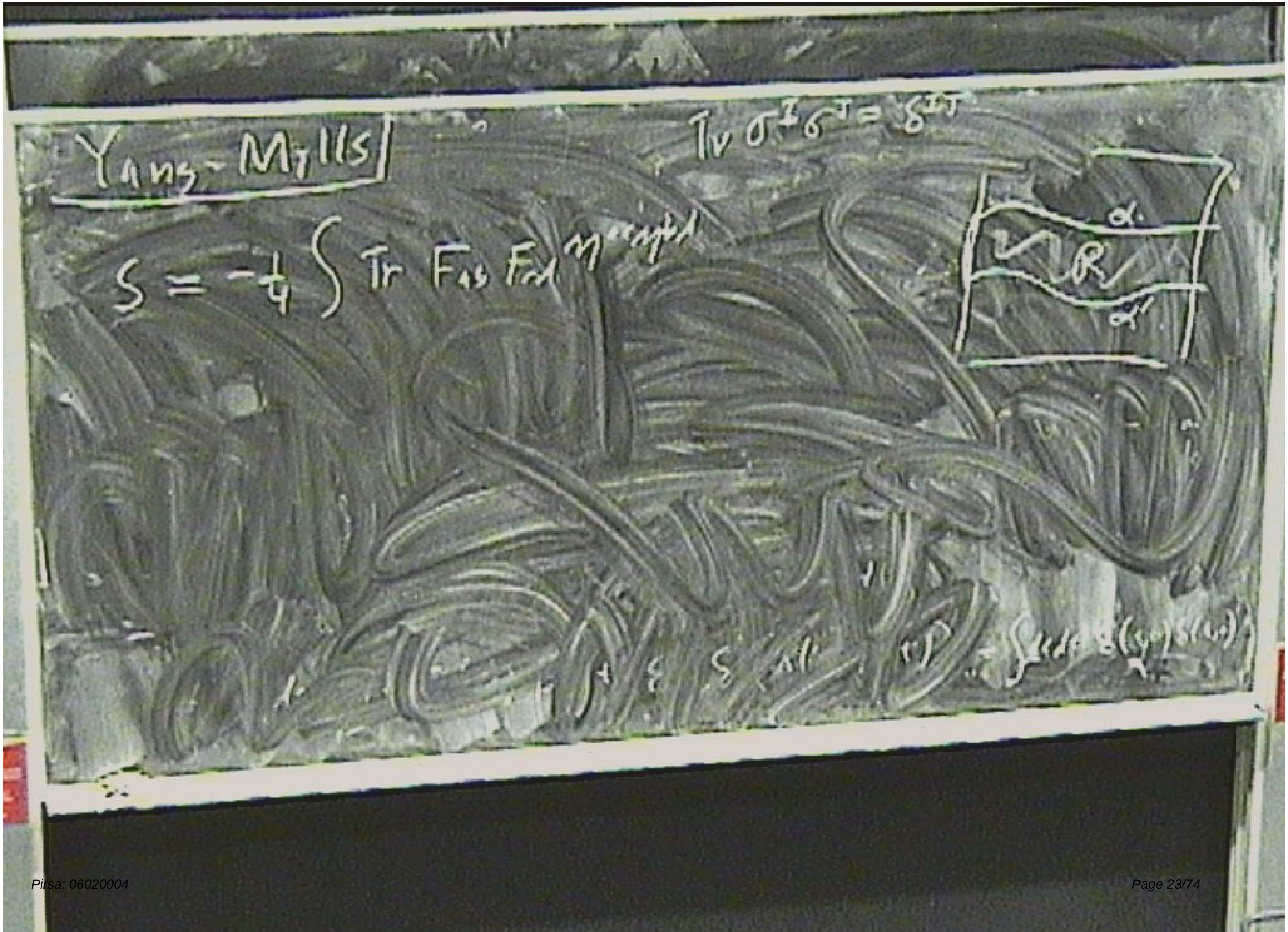
$$F_{ab} = dA_{ab} + [A_a, A_b]$$

SU(2)
Non-Abelian
gauge fields

$$\delta_f F_{ab} = [f, F_{ab}]$$


Yang-Mills
$\mathrm{Tr}\,\sigma^I\sigma^J = \delta^{IJ}$
$S = -\frac{1}{4}\int \mathrm{Tr}\, F_{ab} F^{ab}$ Manifold
R
α
α'

$\lambda = V^a A_a$ $\quad$ $\mathfrak{g}(\Leftarrow)$ diffeos

$SU(2)$ algebra $\quad [J^a, J^b] = i\varepsilon^{abc} J_c \quad \varepsilon^{123} = 1$

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$V^a A_a \in$ function in $\mathcal{a}$

$f \in \mathcal{a} \quad f = f^I \sigma^I$

$\delta A_a = \partial_a f + [A_a, f]$

$A_a = A_a^I \sigma^I$

$F_{ab} = dA_{ab} + [A_a, A_b]$ $\quad$ $SU(2)$ Non-abelian gauge fields

$\delta_f F_{ab} = [f, F_{ab}] = F_{ab}^I \sigma^I$

Yang-Mills

$$\mathrm{Tr}\, \sigma^I \sigma^J = \delta^{IJ}$$

$$S = -\frac{1}{4}\int \mathrm{Tr}\, F_{ab} F_{cd}\, \eta^{ac}\eta^{bd}$$

$$= -\frac{1}{4}\int \sum_I F^I_{ab} F^I_{cd}\, \eta^{ac}\eta^{bd}$$

Yang-Mills

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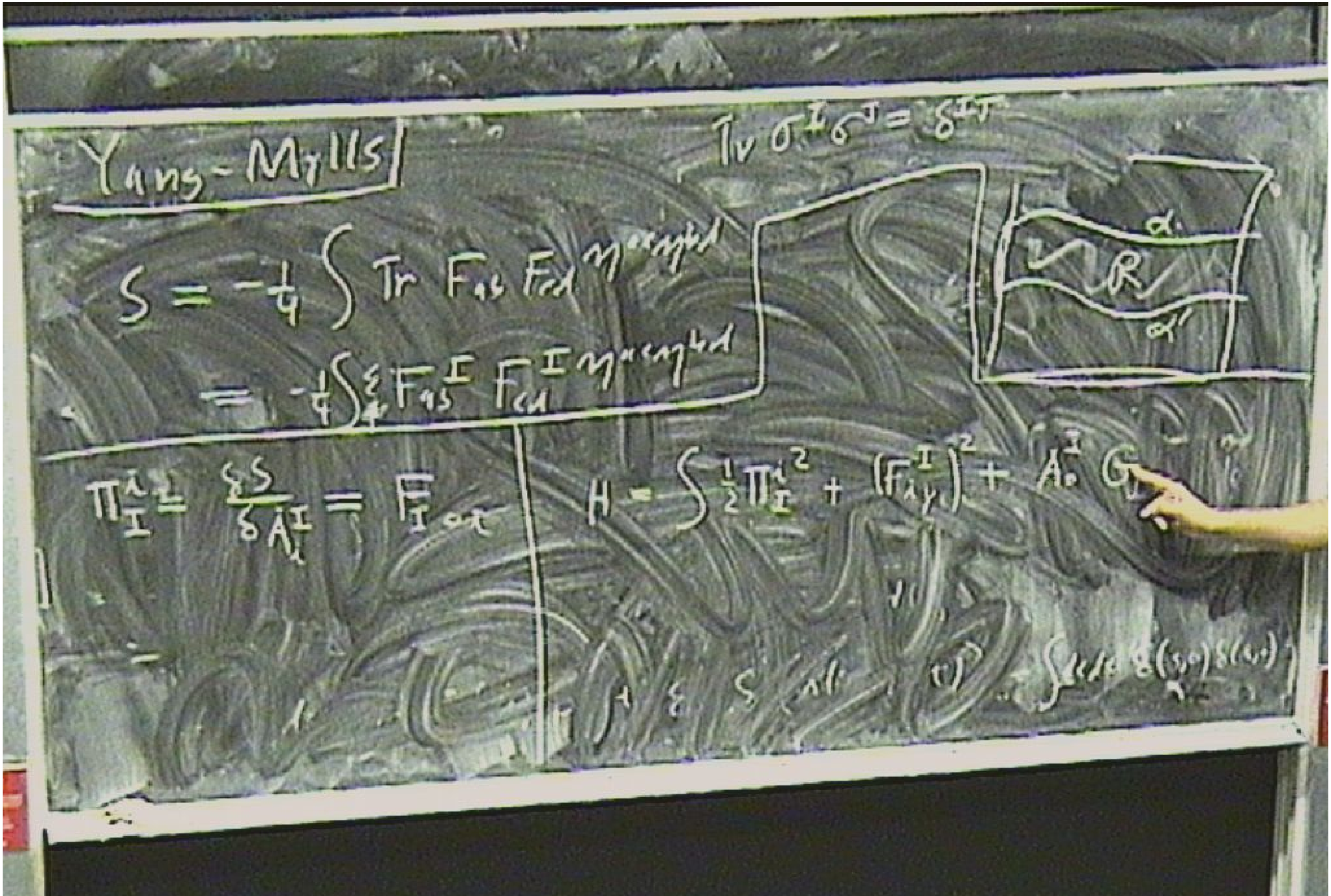
$$= -\frac{1}{4}\int \sum_I F^I_{ab} F^I_{cd}\, \eta^{ac}\eta^{bd}$$

$$\Pi^a_I = \frac{\delta S}{\delta A^I_a} = F_{0a}$$

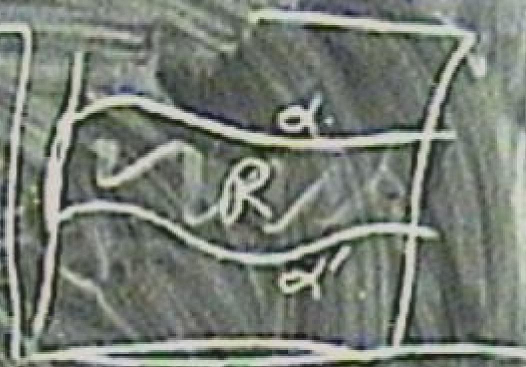

Yang-Mills
$\mathrm{Tr}\,\sigma^I \sigma^J = \delta^{IJ}$
$S = -\frac{1}{4}\int \mathrm{Tr}\, F_{ab} F_{cd} \eta^{ac}\eta^{bd}$
$= -\frac{1}{4}\int \sum_I F_{ab}^I F_{cd}^I \eta^{ac}\eta^{bd}$
α
R
α'
$\Pi_I^i = \frac{\delta S}{\delta A_i^I} = F_{I\,0i}$
$H = \int \frac{1}{2}{\Pi_I^i}^2 + (F_{ij}^I)^2 + A_0^I G_I$

Yang-Mills

$$\mathrm{Tr}\,\sigma^I\sigma^J=\delta^{IJ}$$

$$S=-\frac{1}{4}\int \mathrm{Tr}\,F_{ab}F_{cd}\,\eta^{ac}\eta^{bd}$$

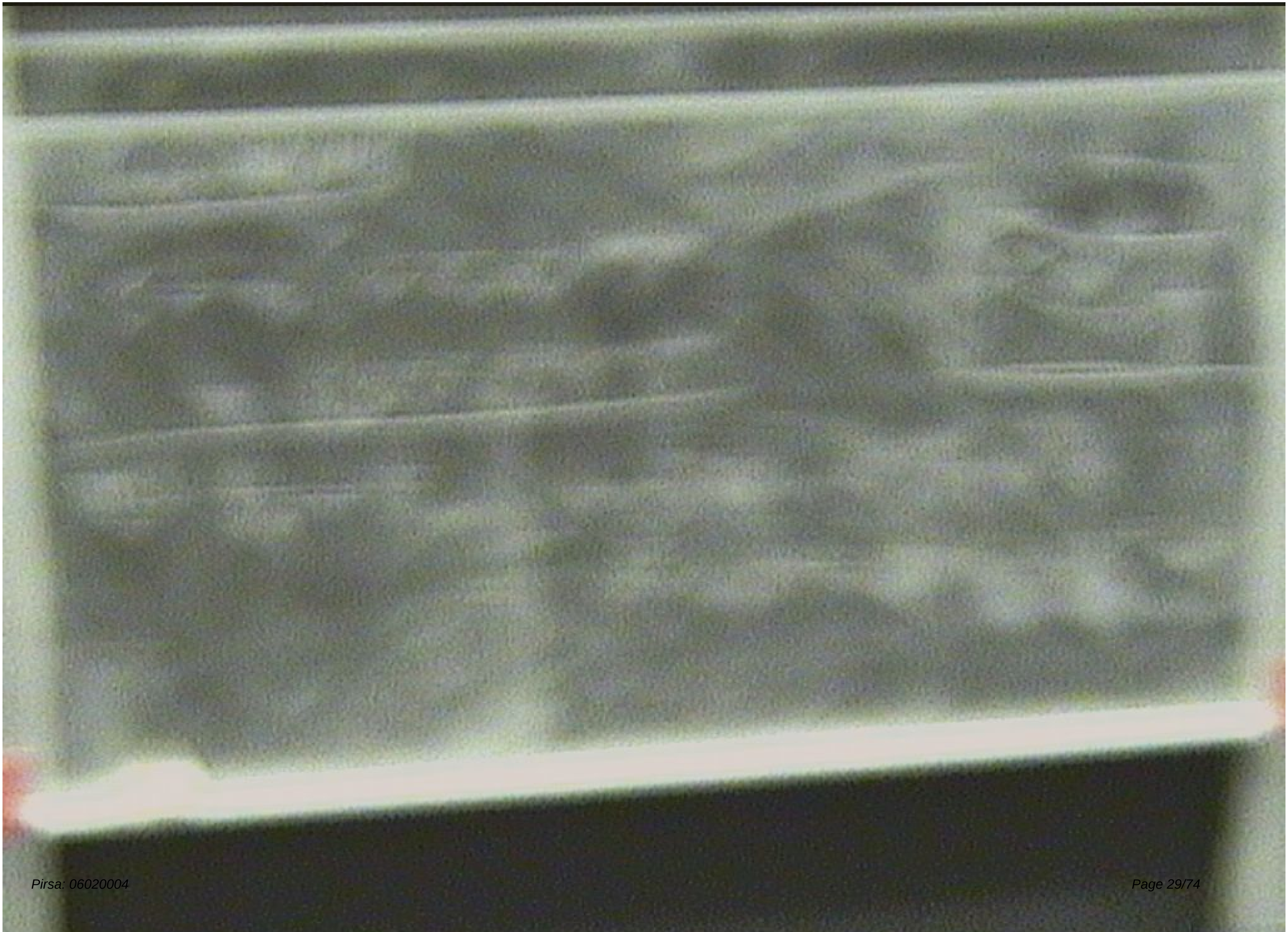
$$=-\frac{1}{4}\int \frac{\varepsilon}{4}\,F^I_{ab}\,F^I_{cd}\,\eta^{ac}\eta^{bd}$$

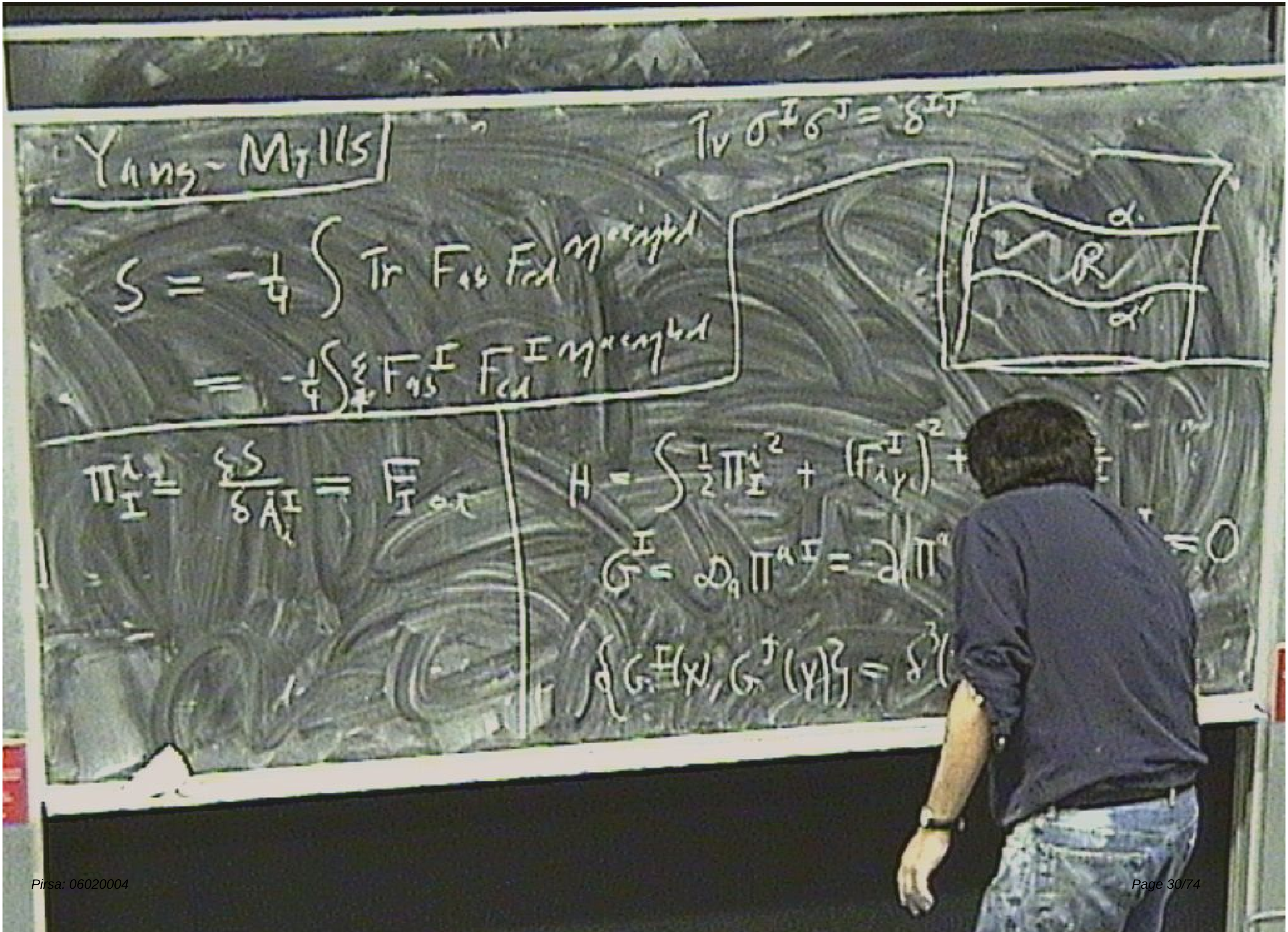


$$\Pi^\lambda_I=\frac{\delta S}{\delta A^I_\lambda}=F_{I\,0\lambda}$$

$$H=\int \frac{1}{2}\Pi^{i\,2}_I+\left(F^I_{xy}\right)^2+A^I_0\,G_I$$

$$G=$$




Yang-Mills
$\mathrm{Tr}\,\sigma^I\sigma^J = \delta^{IJ}$
$S = -\frac{1}{4}\int \mathrm{Tr}\, F_{ab} F_{cd}\, \eta^{ac}\eta^{bd}$
$= -\frac{1}{4}\int \frac{\epsilon}{4} F_{ab}^I F_{cd}^I\, \eta^{ac}\eta^{bd}$
$\pi^a_I = \frac{\delta S}{\delta \dot{A}^I_a} = F_{I\,0a}$
$H = \int \frac{1}{2}\pi^{a\,2}_I + (F^I_{xy})^2 +$
$G^I = \mathcal{D}_a \pi^{aI} = \partial_a \pi^a$
$= 0$
$\{G^I(x), G^J(y)\} = \delta^3($
R
α
α'

Yang-Mills

$\mathrm{Tr}\,\sigma^I \sigma^J = \delta^{IJ}$

$$S = -\frac{1}{4}\int \mathrm{Tr}\, F_{ab} F_{cd}\, \eta^{ac}\eta^{bd}$$

$$= -\frac{1}{4}\int \frac{\delta}{4} F^I_{ab} F^I_{cd}\, \eta^{ac}\eta^{bd}$$

$$\Pi^a_I = \frac{\delta S}{\delta A^I_a} = F_{I\,oa}$$

$$H = \int \frac{1}{2}\Pi^{a\,2}_I + (F^I_{xy})^2 + A_0^I G_I$$

$$G^I = D_a \Pi^{aI} = \partial_a \Pi^{aI} + [A_a, \Pi^a]^I = 0$$

$$\boxed{\{G^I(x), G^J(y)\} = \delta^3(x,y)\,\epsilon^{IJK} G_K(x)}$$

3-manifold $A_\mu \in su(2)$

3-manifold $A_a \in su(2)$, $e_a \in su(2)$

3-manifold $A_a \in su(2)$, $e_a \in su(2)$ $e_a = e_a^I \sigma_I$

3-manifold
No metric
$M = \Sigma_{\text{compact}} \times R$

$A_a \in su(2)$, $e_a \in su(2)$ $e_a = e_a^I \tau_I$

3-manifold

No metric

$\mathcal{M} = \Sigma_{compact} \times R$

$A_a \in su(2)$, $e_a \in su(2)$ $e_a = e_a^I \tau_I$

$$S = \int e \wedge F$$

3-manifold

No metric

$\mathcal{M} = \Sigma_{compact} \times R$

$A_a \in su(2)$, $e_a \in su(2)$ $e_a = e_a^I \tau_I$

$$S = \int_{\mathcal{M}} e^I \wedge F^I$$

$$= \mathrm{Tr} \int_{\mathcal{M}} e \wedge F$$

3-manifold
No metric
$M = \Sigma_{\text{compact}} \times R$

$A_a \in su(2)$, $e_a \in su(2)$ $e_a = e_a^I \sigma_I$

$$S = \int_M e^I \wedge F^I(A)$$

$$= \mathrm{Tr} \int_M e \wedge F(A)$$

$$S = \int \epsilon^{abc} e_a^I \left(\partial_b A_c^I + \right.$$

Non-local observables

$$\{P^1(x), P^2(y)\} = \{\Pi^1 - A_2, \Pi^2 + A_1\} = -\{A_1, \Pi^1\} + \{\Pi^2, A_2\} = 2$$

$$H = \int \lambda\, \epsilon^{ij} \partial_i A_j = -\int A_j\, \epsilon^{ij} (\partial_i \lambda)$$

$$\dot{P}_1 = -\{\Pi^1 - A_2(x), \int d^2x\, A_k\, \epsilon^{ik} \partial_i \lambda\} = \partial_1 \lambda$$

$$F_{ij}^I = \partial_i A_j^I + \epsilon^I{}_{JK} A_i^J A_j^K$$

No local observables

$$\{P^1(x), P^2(y)\} = \{\Pi^1 - A_2, \Pi^2 + A_1\} = -\{A_1, \Pi^1\} + \{\Pi^2, A_2\} = 2$$

$$H = \int \lambda\, \varepsilon^{ij} \partial_i A_j = -\int A_j\, \varepsilon^{ij}(\partial_i \lambda)$$

$$\dot{P}_1' = -\left\{\Pi^1 - A_2(x), \int d^2x\, A_j\, \varepsilon^{ij} \partial_i \lambda\right) = \partial_1 \lambda$$

$$F_{ij}^{\ I} = 2\partial_{[i} A_{j]}^{\ I} + \varepsilon^I{}_{JK} A_i^J A_j^K$$

$= 2$

3-manifold $\quad A_a \in su(2)$, $e_a \in su(2)$ $\quad e_a = e_a^I \sigma_I$

No metric

$M = \Sigma_{\text{compact}} \times R$

$$S = \int_M e^I \wedge F^I(A)\,,$$

$$\text{Tr} \int_M e \wedge F(A)\,,$$

$$S = \int \varepsilon^{abc} e_a^I \left[\partial_b A_c^I + \varepsilon^I{}_{JK} A_b^J A_c^K \right]$$

3-manifold $\quad A_a \in su(2)\ ,\ e_a \in su(2) \quad e_a = e_a^I \sigma_I$

No metric

$\mathcal{M} = \Sigma_{compact} \times R \qquad S = \int_{\mathcal{M}} e^I \wedge F^I(A)$

$$S = \int \epsilon^{abc} e_a^I \left[\partial_b A_c^I + \epsilon^I{}_{JK} A_b^J A_c^K\right] = \int \epsilon^{ijk} e_i^I F_{jk}^I$$

3-manifold

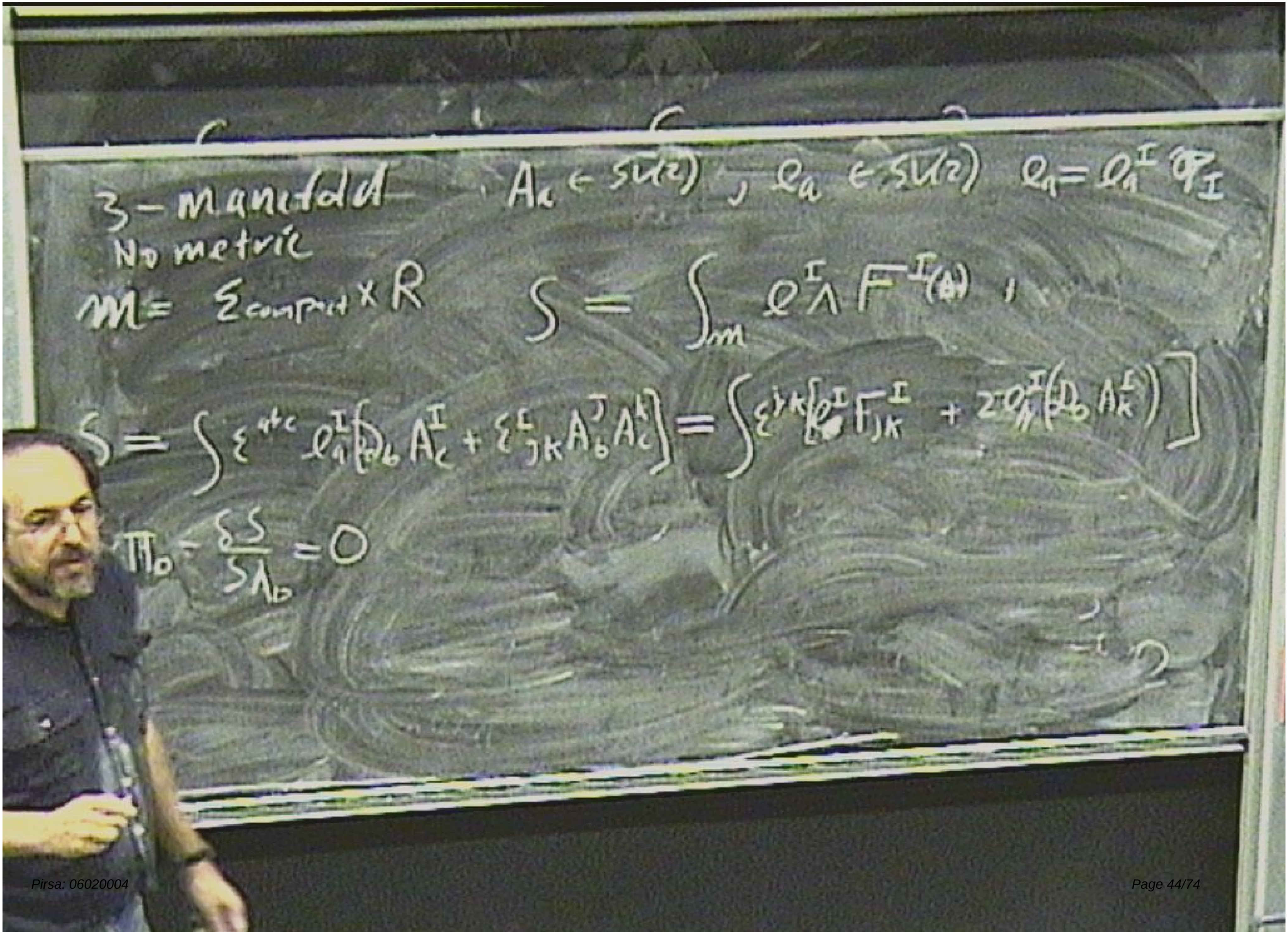
No metric

$\mathcal{M} = \Sigma_{\text{compact}} \times R$

$A_a \in SU(2)$, $e_a \in SU(2)$, $e_a = e_a^I \tau_I$

$$S = \int_{\mathcal{M}} e^I \wedge F^I(A),$$

$$S = \int \epsilon^{abc} e_a^I \left[\partial_b A_c^I + \epsilon^I{}_{JK} A_b^J A_c^K\right] = \int \epsilon^{jk} e_0^I F_{jk}^I + 2 e_k^I (\partial_0 A_k^I)$$


3-manifold
No metric
M = Σ compact × R
A_a ∈ su(2) , e_a ∈ su(2) e_a = e_a^I τ_I
S = ∫_M e^I ∧ F^I(A)
S = ∫ ε^abc e_a^I [∂_b A_c^I + ε^I_JK A_b^J A_c^K] = ∫ ε^ijk [e_i^I F_jk^I + 2 e_i^I (∂_b A_k^I)]
Π_b = δS/δA_b = 0

3-manifold

No metric

$M = \Sigma_{compact} \times R$

$A_a \in su(2)$, $e_a \in su(2)$ $\quad e_a = e_a^I \sigma_I$

$$S = \int_M e^I \wedge F^I(A)$$

$$S = \int \epsilon^{abc} e_a^I [\partial_b A_c^I + \epsilon^I_{JK} A_b^J A_c^K] = \int \epsilon^{ijk} [e_i^I F_{jk}^I + 2 e_0^I (\partial_b A_k^I)]$$

$$\Pi_0 = \frac{\delta S}{\delta \dot{A}_0} = 0 \qquad \Pi^i = \delta$$

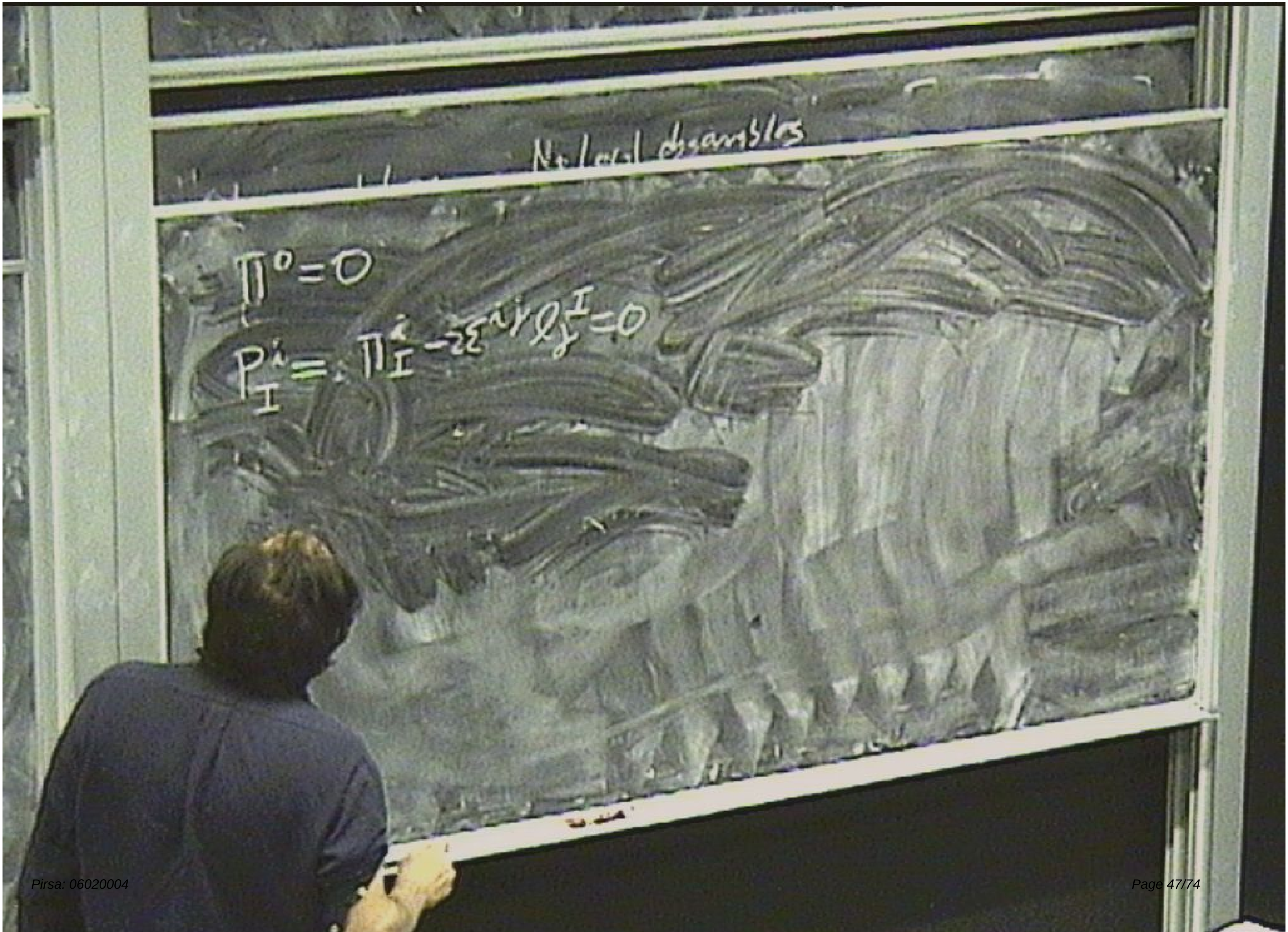
3-manifold $\quad A_a \in su(2)\,,\ e_a \in su(2) \quad e_a = e_a^I \sigma_I$

No metric

$M = \Sigma_{\text{compact}} \times R \qquad S = \int_M e^I \wedge F^I(A)\,, \quad \partial_0 A_k$

$$S = \int \epsilon^{abc} e_a^I [\partial_b A_c^I + \epsilon^I{}_{JK} A_b^J A_c^K] = \int \epsilon^{ijk} [e_0^I F_{jk}^I + 2 e_k^I (\partial_0 A_k^I)]$$

$$\Pi_0 = \frac{\delta S}{\delta A_0} = 0 \qquad \Pi_I^k = \frac{\delta S}{\delta A_k^I} = 2\epsilon^{ij} e_j^I$$


No local observables
$\Pi^0 = 0$
$P^i_I = \Pi^i_I - 2\epsilon^{ij} e^I_j = 0$

N_t local ensembles

$$\Pi^0 = 0$$

$$P^i_I = \Pi^i_I - i\epsilon^{ij} e^I_j = 0$$

2-dim rep $\quad J^a \Rightarrow \sigma^a$

$A_a \in \mathcal{A}$ vector field V

$V^a A_a \in$ function in $\mathcal{A}$ ...

$f \in \mathcal{A} \quad f = f^I, \sigma^I$

$A_a = A_a^I \sigma^I \qquad F_{ab} =$

$\delta e_a = -[f, e_a]_F \qquad \delta_f F_{ab} =$

3-manifold $\quad A_a \in su(2)$, $e_a \in su(2)$ $\quad e_a = e_a^I \tau_I$

No metric

$M = \Sigma_{compact} \times R$ $\qquad S = \int_M e^I \wedge F^I(A)$ $\qquad \partial_0 A_k$

$$S = \int \epsilon^{abc} e_a^I [\partial_b A_c^I + \epsilon^I_{JK} A_b^J A_c^K] = \int \epsilon^{ijk} [e_i^I F_{jk}^I + 2 e_0^I (\partial_b A_k^I)]$$

$\Pi_0 = \frac{\delta S}{\delta A_0} = 0$ $\qquad \Pi_I^a = \frac{\delta S}{\delta A_a^I} = 2\epsilon^{ij} e_j^I$ $\quad \| \quad P_I^a = \frac{\delta S}{\delta e_a^I} = 0$

No local observables

$$\Pi^0 = 0$$

$$P^i_I = \Pi^i_I - 2\epsilon^{ij} e^I_j = 0$$

$$S_I = 0$$

No metric

$\mathcal{M} = \Sigma_{\text{compact}} \times R \qquad S = \int_{\mathcal{M}} e^I \wedge F^I(A) \, , \qquad \partial_0 A_k$

$$S = \int \epsilon^{abc} e^I_a [\partial_b A^I_c + \epsilon^I_{JK} A^J_b A^K_c] = \int \epsilon^{ijk} [e^I_0 F^I_{jk} + 2 e^I_i (\partial_0 A^I_k)]$$

$$\Pi_0 = \frac{\delta S}{\delta A_0} = 0 \qquad \Pi^i_I = \frac{\delta S}{\delta \dot{A}^I_i} = 2 \epsilon^{ijk} e^I_j \quad \Big\| \quad P^a_I = \frac{\delta S}{\delta \dot{e}^I_a} = 0$$

$$H = \int [\Pi^i_I \dot{A}^I_i + \Pi^0_I \dot{A}^I_0 + P \dot{e}$$

No metric

$M = \Sigma_{compact} \times R$ $\qquad S = \int_M e^I \wedge F^I(A)$ $\qquad \partial_b A_K$

$$S = \int \epsilon^{abc} e^I_a [\partial_b A^I_c + \epsilon^I_{JK} A^J_b A^K_c] = \int \epsilon^{ijk}[e^I_i F^I_{jk} + 2 e^I_0 (D_b A^I_k)]$$

$$\Pi_0 = \frac{\delta S}{\delta A_0} = 0 \qquad \Pi^i_I = \frac{\delta S}{\delta A^I_i} = 2\epsilon^{ij} e^I_j \quad \Big\| \quad \rho^i_I = \frac{\delta S}{\delta e^I_i} = 0$$

$$H = \int [\Pi^i_I \dot{A}^I_i + \Pi^0_I \dot{A}^I_0 + \rho \dot{e} - \mathcal{L}] = \int d^3x$$

No metric

$\mathcal{M} = \Sigma_{\text{compact}} \times R \qquad S = \int_{\mathcal{M}} e^I \wedge F^I(A)$ $\quad \partial_0 A_k$

$$S = \int \epsilon^{abc} e_a^I \left[\partial_b A_c^I + \epsilon^I{}_{JK} A_b^J A_c^K\right] = \int \epsilon^{ijk} \left[e_0^I F_{jk}^I + 2 e_i^I (\partial_0 A_k^I)\right]$$

$$\Pi_0 = \frac{\delta S}{\delta \dot A_0} = 0 \qquad \Pi_I^i = \frac{\delta S}{\delta \dot A_i^I} = 2\epsilon^{ijk} e_j^I \qquad \Big\| \qquad P_I^a = \frac{\delta S}{\delta \dot e_a^I} = 0$$

$$H = \int \left[\Pi_I^i \dot A_i^I + \Pi_I^0 \dot A_0^I + P \dot e - \mathcal{L}\right] = \int d^3x \left[e_0^I F^I + A_0^I G^I\right.$$

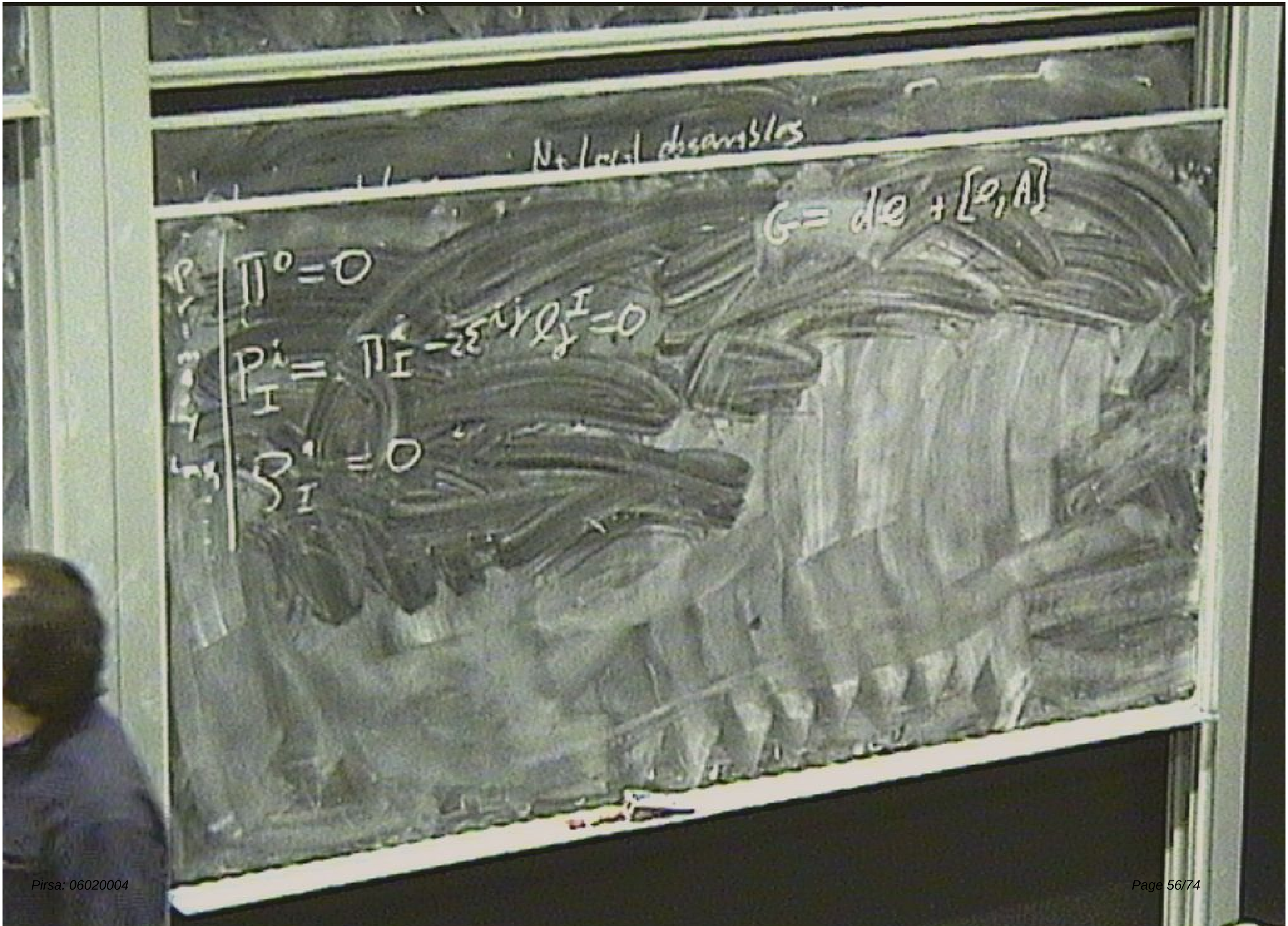
No metric

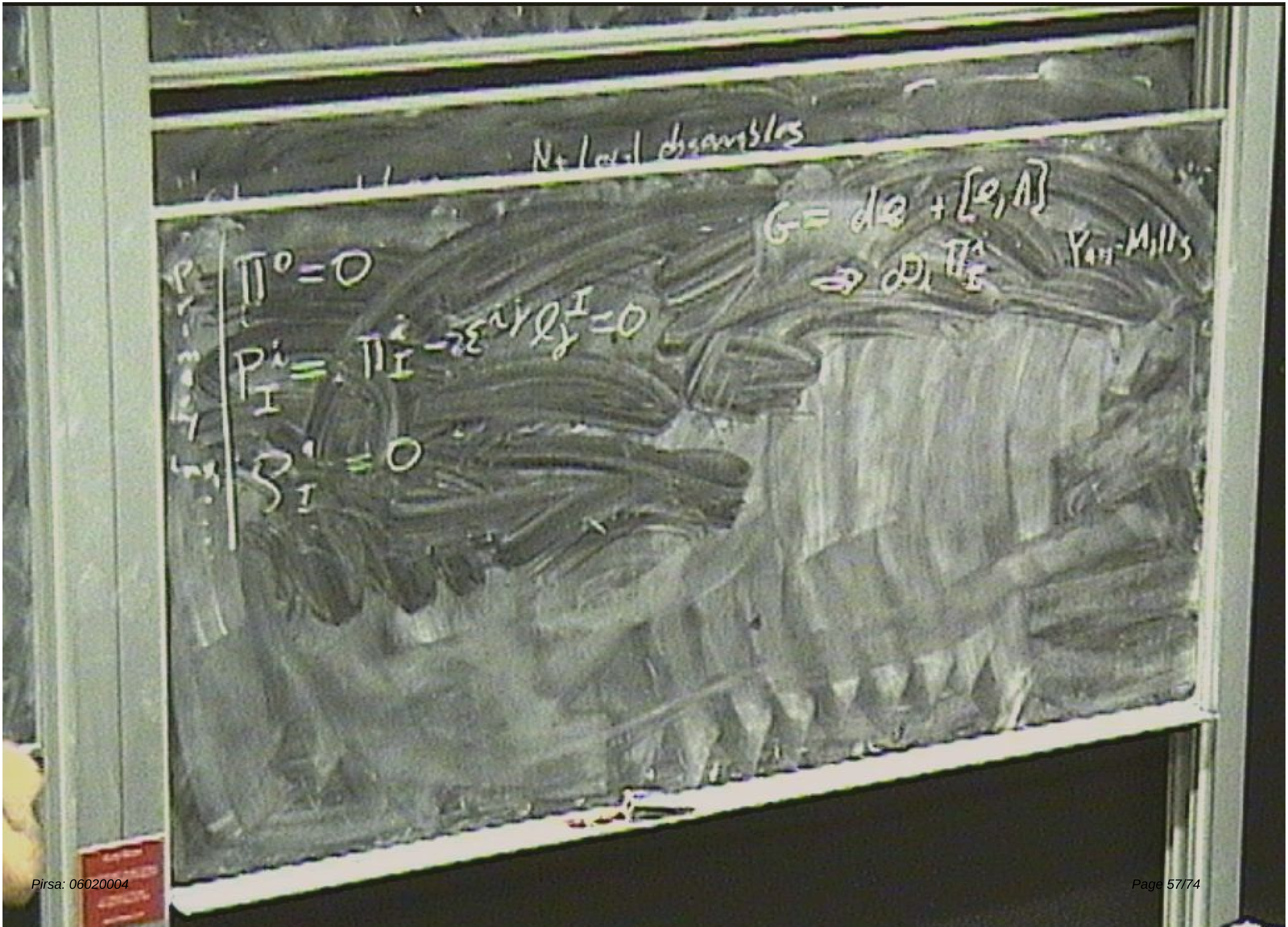
$$\mathcal{M} = \Sigma_{\text{compact}} \times R \qquad S = \int_{\mathcal{M}} e^I \wedge F^I(A) \;, \;\cdots\; \partial_b A_k$$

$$S = \int \epsilon^{abc} e^I_a \left[\partial_b A^I_c + \epsilon^I_{JK} A^J_b A^K_c\right] = \int \epsilon^{ijk} \left[e^I_0 F^I_{jk} + 2 e^I_j \left(\partial_0 A^I_k\right)\right]$$

$$\Pi_0 = \frac{\delta S}{\delta A_0} = 0 \qquad \Pi^i_I = \frac{\delta S}{\delta \dot{A}^I_i} = 2\epsilon^{ij} e^I_j \;\Big\|\; P^a_I = \frac{\delta S}{\delta \dot{e}^I_a} = 0$$

$$H = \int \left[\Pi^i_I \dot{A}^I_i + \Pi^0_I \dot{A}^I_0 + P\dot{e} - \mathcal{L}\right] = \int d^3x \left[e^I_0 F^I + A^I_0 G^I\right] \rightarrow 0$$


G = de + [e, A]
Π° = 0
P_I^i = Π_I^i − 2ε^{ij} e_j^I = 0
S_I^i = 0


Not local observables
$\Pi^0 = 0$
$P^i_I = \Pi^i_I - \varepsilon^{ij} e^I_j = 0$
$S_I = 0$
$G = de + [e, A]$
$\Rightarrow \partial_i \Pi^i_I$
Yang-Mills

No local dsgmsles

$$\Pi^0 = 0$$

$$P^i_I = \Pi^i_I - 2\epsilon^{ij} e_j^I = 0$$

$$S^i_I = 0$$

$$\hat{G} = de + [e, \Lambda]$$

Yang-Mills

$$\Rightarrow \mathcal{D}_i \Pi^i_I$$

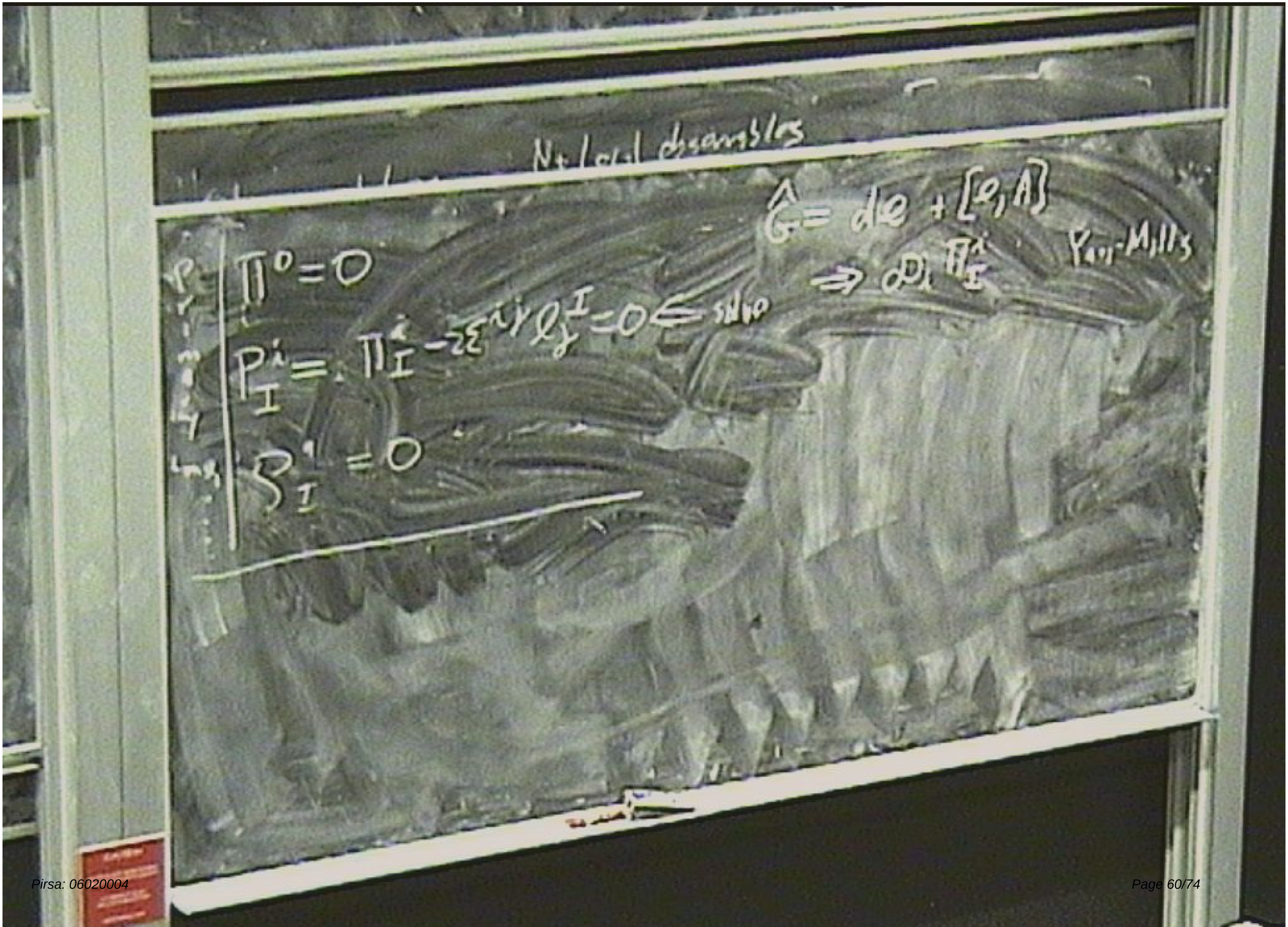
No metric

$M = \Sigma_{compact} \times R$ $\qquad S = \int_M e^I \wedge F^I(A) \qquad \partial_0 A_k$

$$S = \int \epsilon^{abc}\, e^I_a \left[\partial_b A^I_c + \epsilon^I{}_{JK} A^J_b A^K_c\right] = \int \epsilon^{ijk}\left[e^I_0 F^I_{jk} + 2 e^I_i \left(\partial_0 A^I_k\right)\right]$$

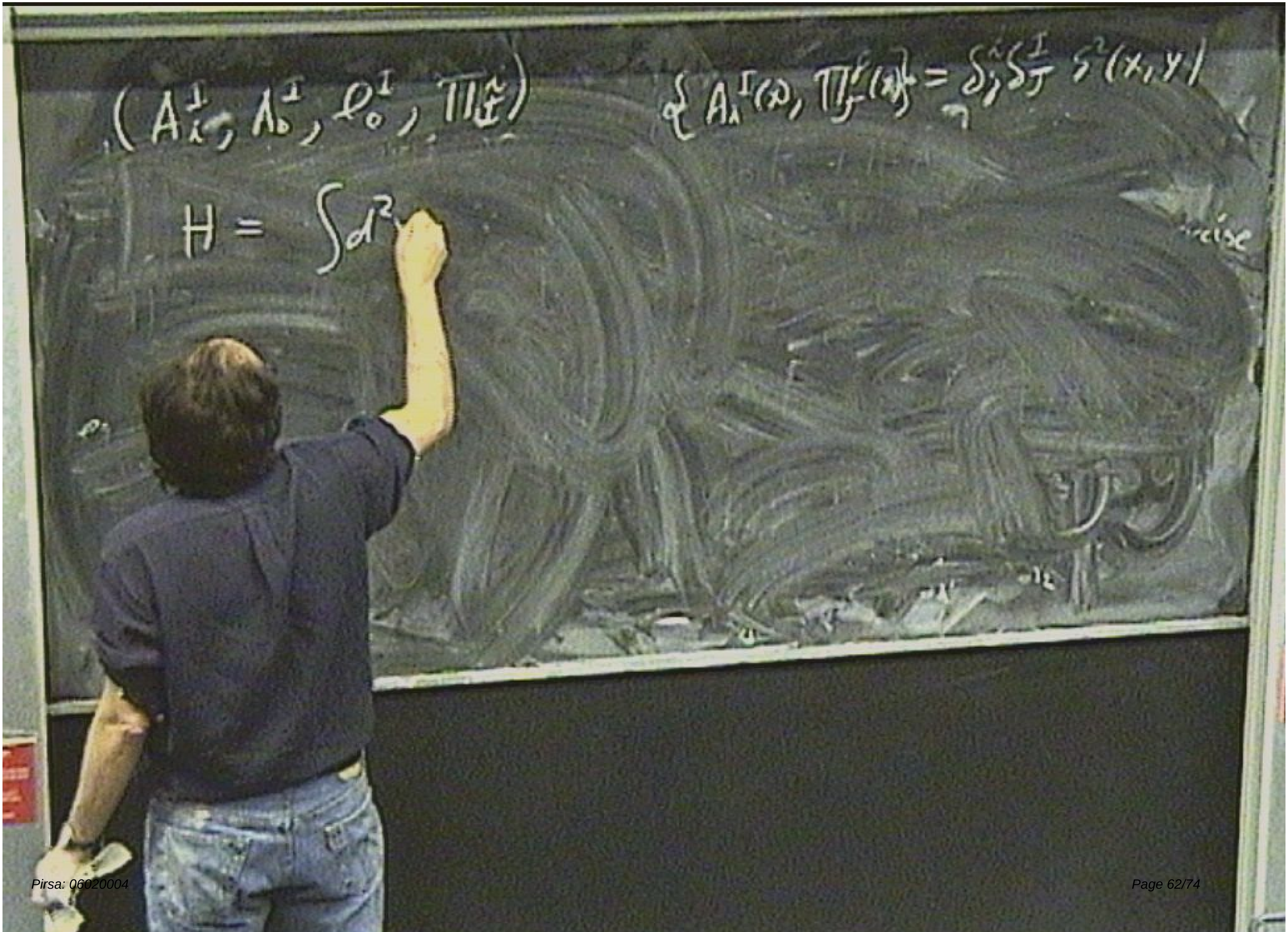
$$\Pi_0 = \frac{\delta S}{\delta A_0} = 0 \qquad \Pi^i_I = \frac{\delta S}{\delta A^I_i} = 2\epsilon^{ijk} e^I_j \quad \Big\| \quad P^a_I = \frac{\delta S}{\delta e^I_a} = 0$$

$$H = \int \left[\Pi^i_I \dot{A}^I_i + \Pi^0_I \dot{A}^I_0 + P\dot{e} - \mathcal{L}\right] = \int d^3x \left[e^I_0 F^I + A^I_0 \hat{G}^I + \Pi^i_I \dot{A}^I_i + P e\right]$$



Yang-Mills
$\Pi^0 = 0$
$P_I^i = \Pi_I^i - 2\epsilon^{ij} e_j^I = 0$
$S_I = 0$

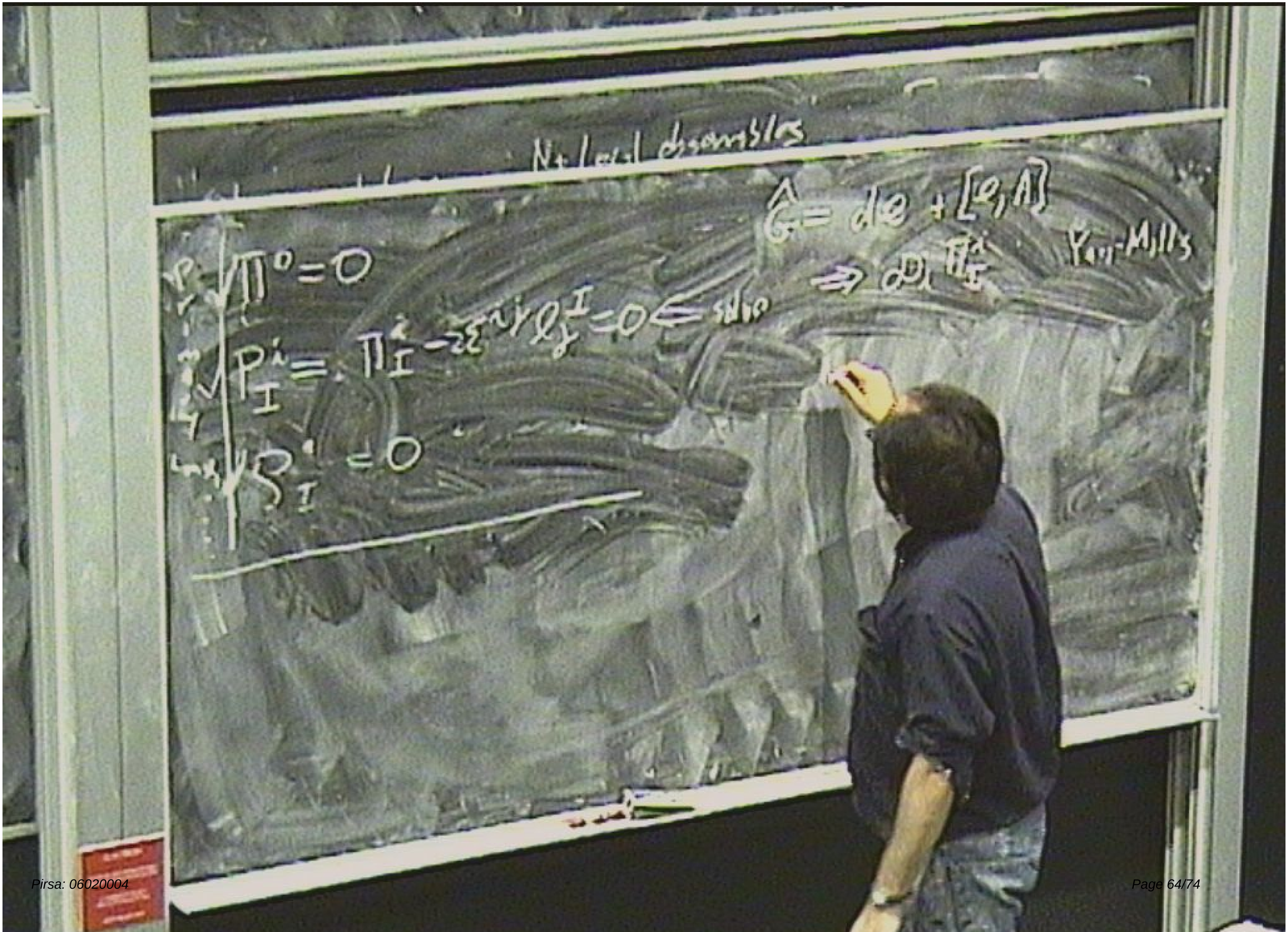
$(A^I_i, \Lambda^I_0, e^I_0, \Pi^i_I)$

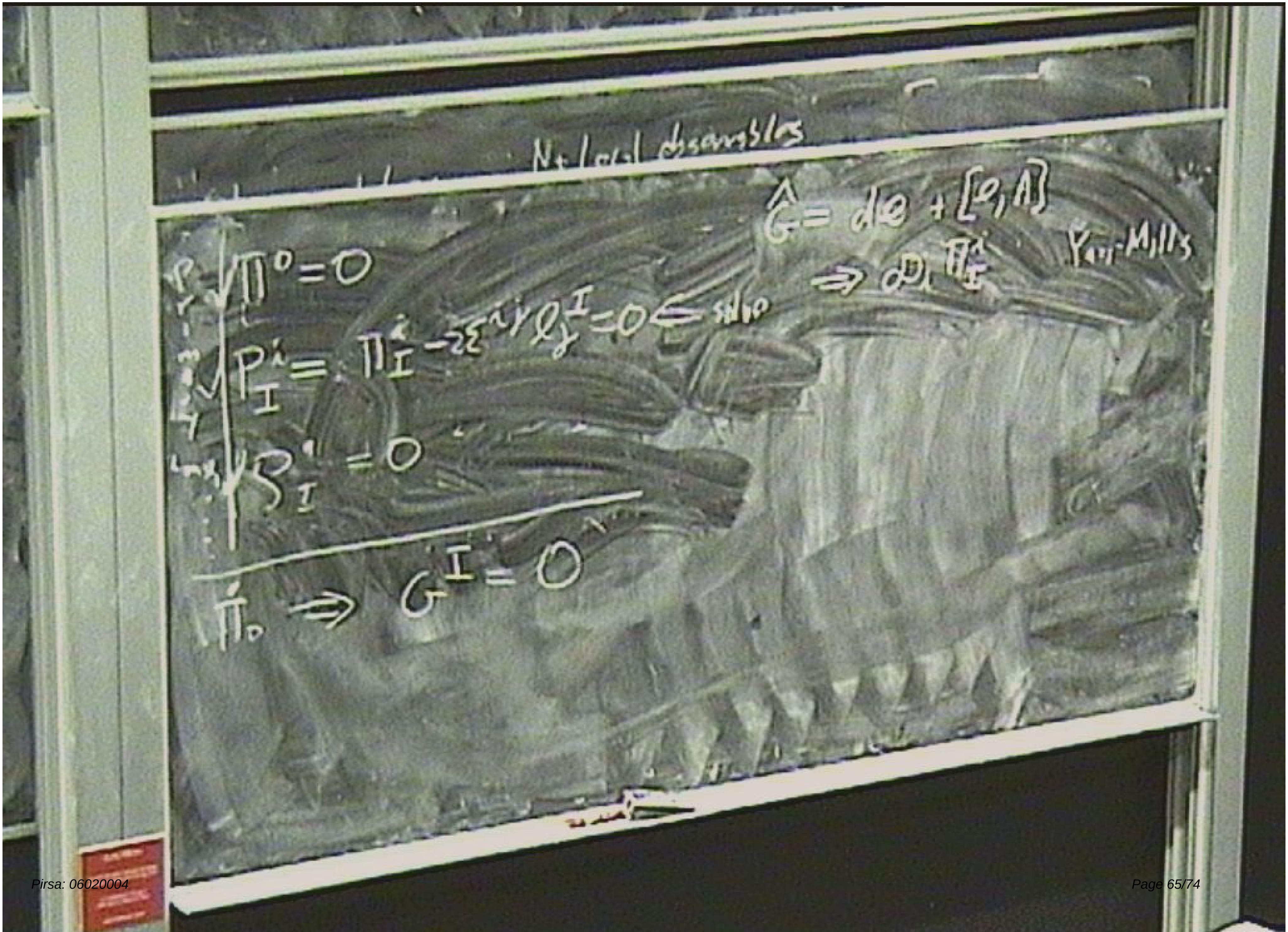

$H = \int d^2$

$$(A^I_i, A^I_b, e^I_0, \Pi^i_I)$$

$$\{A^I_i(x), \Pi^j_J(y)\} = \delta^j_i \delta^I_J \delta^2(x,y)$$

$$H = \int_\Sigma d^2x \left[e_0^I F^I + A_b^I G^I \right]$$


Π^0 = 0



No local observables
$\hat{G} = de + [e,\Lambda]$
Yang-Mills
$\Rightarrow \mathcal{D}_i \Pi^i_I$
$\Pi^0 = 0$
$\Pi^i_I - 2\epsilon^{ij} e_j^I = 0 \Leftarrow$
$P^i_I = \Pi^i_I$
$S^0_I = 0$
$\dot{\Pi}_0 \Rightarrow G^I = 0$

Not local observables

$\hat{G} = de + [e, A]$ Yang-Mills

$\Rightarrow \partial_i \Pi^i_I$

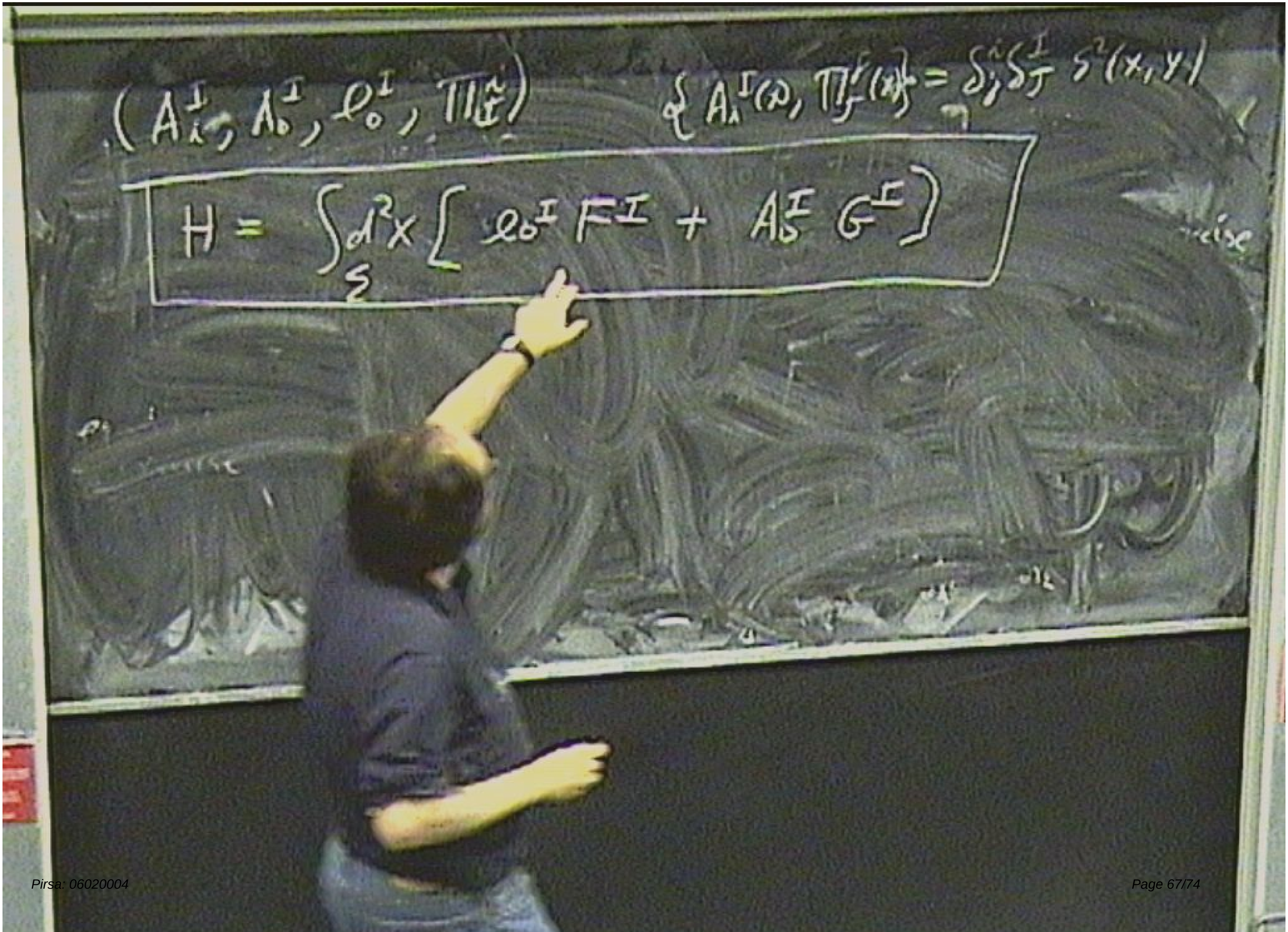
$\Pi^0 = 0$

$P^i_I = \Pi^i_I - 2\epsilon^{ij} e^I_j = 0 \Leftarrow$ SNO

$S^i_I = 0$

$\Pi_0 \Rightarrow G^I = 0$

$\xi \Rightarrow F^I = 0$


$(A_i^I, A_0^I, e_0^I, \Pi_I^i)$
$\{A_i^I(x), \Pi_J^j(y)\} = \delta_i^j \delta_J^I \delta^2(x,y)$
$H = \int_\Sigma d^2x \, [\, e_0^I F^I + A_0^I G^I \,]$

$$(A_i^I,\ A_0^I,\ e_0^I,\ \Pi_I^i) \qquad \{A_i^I(x), \Pi_J^j(y)\} = \delta_i^j \delta_J^I\, \delta^2(x,y)$$

$$\boxed{H = \int_\Sigma d^2x \left[e_0^I F^I + A_0^I G^I \right]}$$

$(A_i^I, A_0^I, e_0^I, \Pi_I^i)$ $\qquad \{A_i^I(x), \Pi_J^j(y)\} = \delta_i^j \delta_J^I \delta^2(x,y)$

$$\boxed{H = \int_\Sigma d^2x \left[e_0^I F^I + A_0^I G^I \right]}$$

$$\dot{G} = \{G, H\}$$

$$\dot{F} = \{F, H\}$$

Not local disamsles

$$\hat{G} = d\varrho + [\varrho, A]$$

Yang-Mills

$$\Rightarrow \mathcal{D}_i \Pi^i_I$$

$$\Pi^0 = 0$$

$$P^i_I = \Pi^i_I - 2\epsilon^{ij} e^I_j = 0 \Leftarrow \text{sdyn}$$

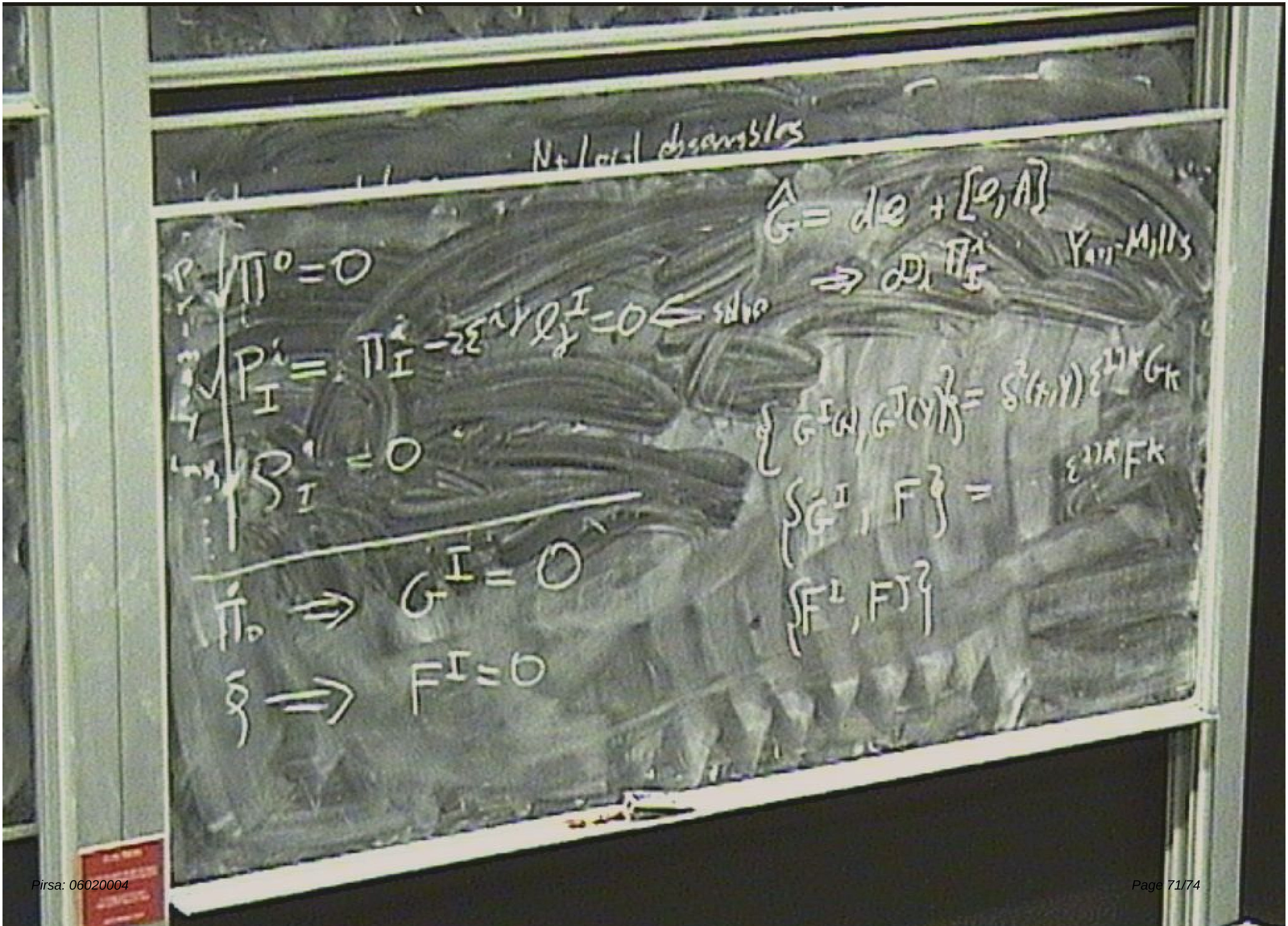
$$\{G^I(x), G^J(y)\} = \delta^2(x,y)\epsilon^{IJK} G_K$$

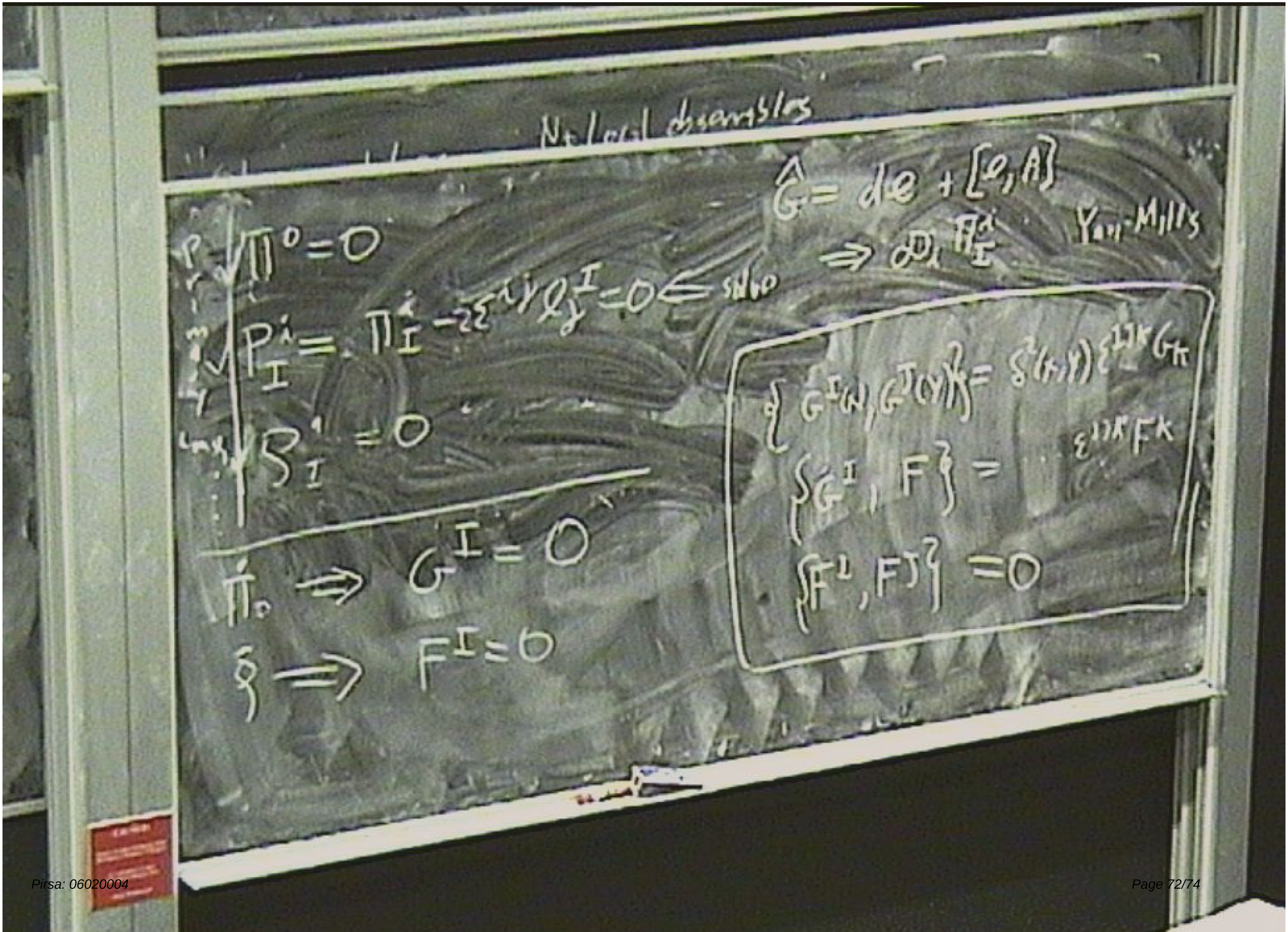
$$S^i_I = 0$$

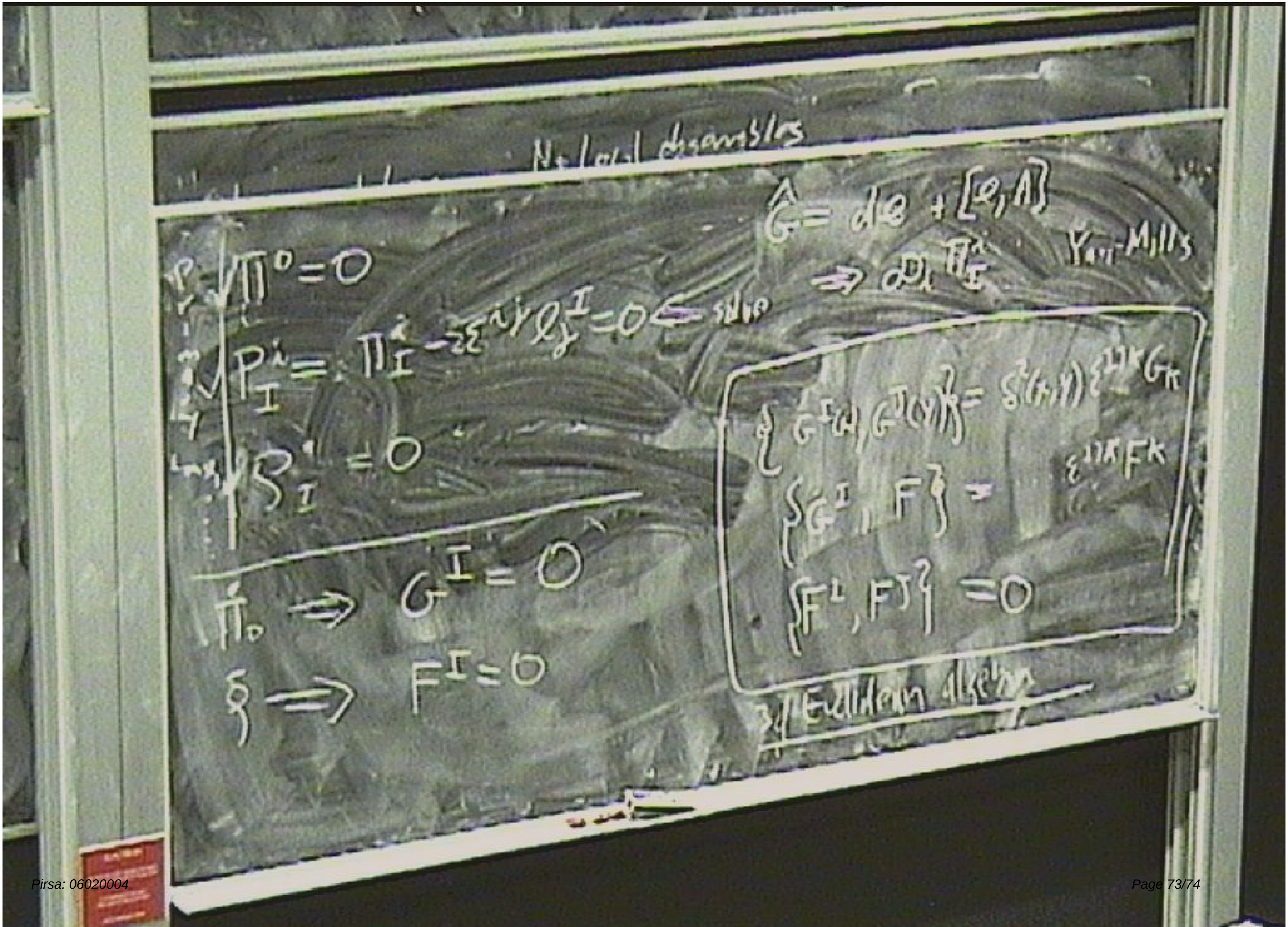
$$\{G^I, F\} = \epsilon^{IJK} F^K$$

$$\Pi_0 \Rightarrow G^I = 0$$

$$\xi \Rightarrow F^I = 0$$


$\hat{G} = de + [e, A]$
Yang-Mills
$\Pi^0 = 0$
$P_I^i = \Pi_I^i - 2\epsilon^{ij} e_j^I = 0$
$S_I = 0$
$G^I = 0$
$F^I = 0$
$\{G^I(x), G^J(y)\} = \delta^2(x,y)\epsilon^{IJK} G_K$
$\{G^I, F\} = \epsilon^{IJK} F_K$
$\{F^I, F^J\}$


$G^I = 0$
$F^I = 0$
$\{F^I, F^J\} = 0$


Yang-Mills
$\Pi^0 = 0$
$P^i_I = \Pi^i_I - \epsilon^{ij} e^I_j = 0$
$S_I = 0$
$\Pi_0 \Rightarrow G^I = 0$
$S \Rightarrow F^I = 0$
$\{F^I, F^J\} = 0$
Euclidean algebra

$(A^I_i, A^I_0, e^I_0, \Pi^i_I)$

$\{A^I_i(x), \Pi^j_J(y)\} = \delta^j_i \delta^I_J \delta^2(x,y)$

$$\boxed{H = \int_\Sigma d^2x \left[e_0^I F^I + A_0^I G^I \right]}$$

GR in 3 spacetime dimensions
in Euclidean signature

$\dot{G} = \{G, H\}$

$\dot{F} = \{F, H\}$