Title: Inflationary cosmology as a probe of primordial quantum mechanics

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Abstract: It is shown that inflationary cosmology may be used to test the statistical predictions of quantum theory at very short distances. Hidden-variables theories, such as the pilot-wave theory of de Broglie and Bohm, allow the existence of vacuum states with non-standard field fluctuations (Â“quantum non-equilibriumÂ”). It is shown that such non-equilibrium vacua lead to statistical anomalies, such as a breaking of scale invariance for the primordial power spectrum. The results depend only weakly on the details of the de Broglie-Bohm dynamics. Recent observations of the cosmic microwave background are used to set limits on violations of quantum theory in the early universe.

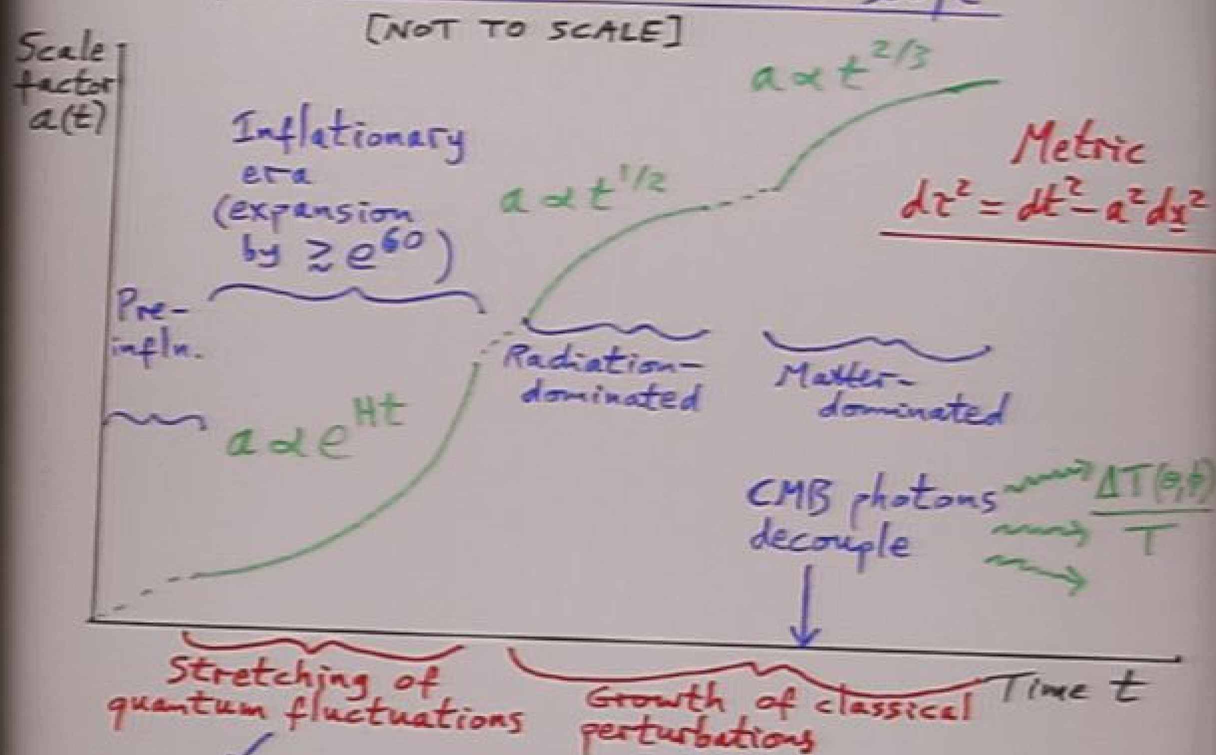
1)

Inflationary Cosmology as a Probe of Primordial Quantum Mechanics

Antony Valentini
(Perimeter Institute)

- Background and motivation (quick review)
- Cosmic Microwave Background (CMB) and inflationary cosmology
- How CMB observations can constrain early violations of quantum theory

2/ Inflation as a Cosmic Microscope



$\lambda_{phys} = \frac{a(t)}{a_0}\lambda \propto e^{Ht}$

Mode "exit" when $\lambda_{phys} \gtrsim H^{-1}$ (= const.) at time $t_{exit} = t_{exit}(k)$. } Pertn. "freezes"

Smaller k (larger λ) exit earlier ..... (stretched most)

$\lambda_{phys} \propto a \propto \begin{cases} t^{1/2} \\ t^{2/3} \end{cases}$

grow slower than $H^{-1} \equiv \frac{a}{\dot{a}} \propto t$

Mode "re-entry" when $\lambda_{phys} \lesssim H^{-1}$ and perturbations start to grow

(→ inhomogeneous CMB, galaxies, ... etc)

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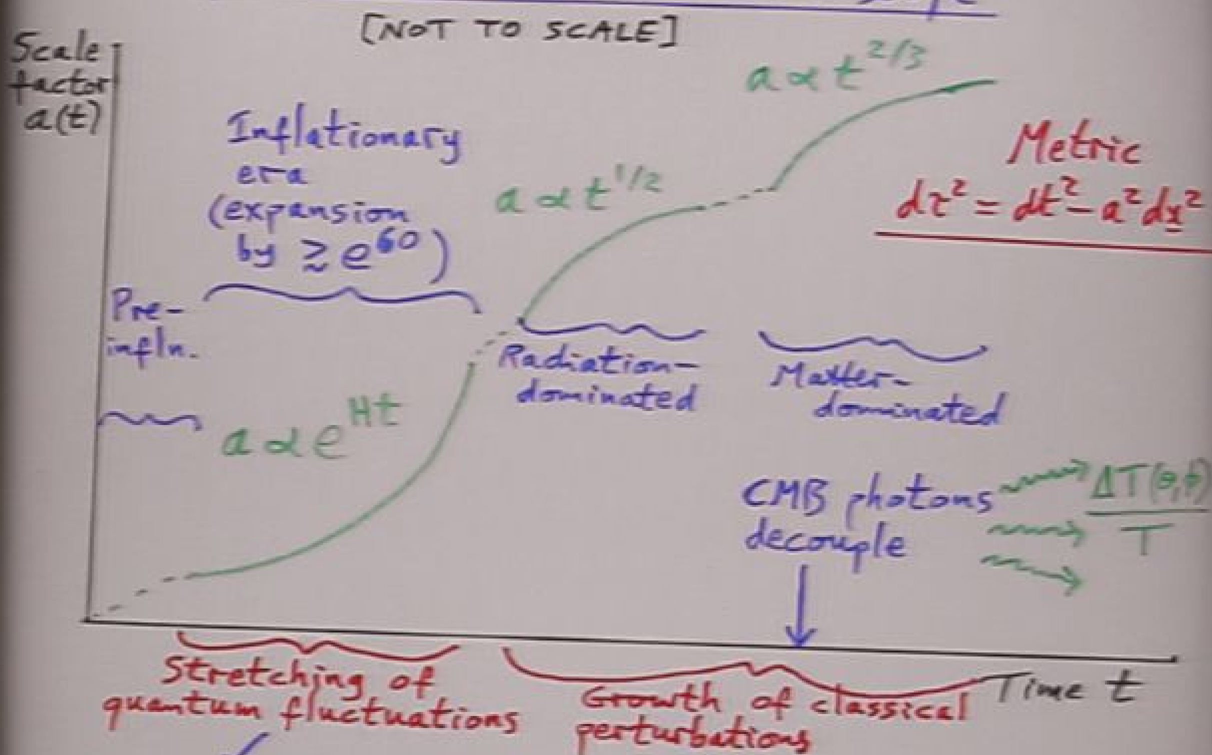

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3/

If inflation occurred, measurements of CMB (and large-scale structure generally) can probe physics at very early times and at very short distances (possibly even $\lesssim l_{Planck}$):

- Modified dispersion relations:

Martin and Brandenberger 2001
Niemeyer 2001
Niemeyer and Parentani 2001 } "Ad hoc"

Kowalski-Glikman 2001 } Quantum gravity/ Deformed special relativity

Hofmann and Winkler 2004 } Quantum cosmology

- UV cutoff from deformed uncertainty relations:

Kempf 2001, Kempf & Niemeyer 2001
Easther et al. 2001 } quantum gravity? String theory?

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- Short-distance non-commutative geometry:

Lizzi, Mangano, Miele and Peloso 2002

- Different choices of quantum vacuum:

Danielsson 2002

Greene, Schalm and van der Schaar 2005 (review)

- Excited (non-vacuum) states:

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- Generally agreed (in an inflationary context) that primordial perturbations have a quantum origin.

- Yet, not a single paper considers effects on CMB from modifications of or corrections to .....

QUANTUM THEORY

(Hilbert state space, unitary evolution, Born rule probabilities)

- Why not?

No good scientific reason ....

Quantum theory might [illegible]

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Quantum theory might not be "final".

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Frequently-heard arguments for "finality" of QT:

- "Universal framework" (electrons, fields, atoms...)
- "Simple, elegant axioms" (von Neumann, Hardy, ...)
- "Basis for new technologies" (lasers, semiconductors, cryptography, computing, ...)
- "Agrees with experiment (so far)"

Cf. Newtonian mechanics in 18th and 19th centuries:

- "Universal framework" (dust, rocks, fluids, planets...)
- "Simple, elegant axioms" (3 laws, Principia 1687)
- "Basis for new technologies" (hydraulics, mechanical engineering, gyrocompass, ...)
- "Agrees with experiment (so far)"

Yet, Newtonian mechanics is approximate + emergent:

Quantum field theory $\xrightarrow{\text{(Low-energy limit)}}$ Non-relativistic quantum mechanics

$\xrightarrow{\text{(Classical limit, decoherence)}}$ Newton's 3 axioms/laws

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$\longrightarrow$ (classical limit, decoherence) Newton's 3 axioms/laws

Scientific approach:

- test theories experimentally, to determine domain of validity
- all scientific theories are subject to possible modification in hitherto untested regimes

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Helpful to have a "foil" against which to test quantum theory, that is:

a model giving quantum theory only in some limit

Many possibilities:

- nonlinear evolution
- collapse models
- nonequi[illegible] hidden variables ....

(Here, look [illegible] tion.)

7/

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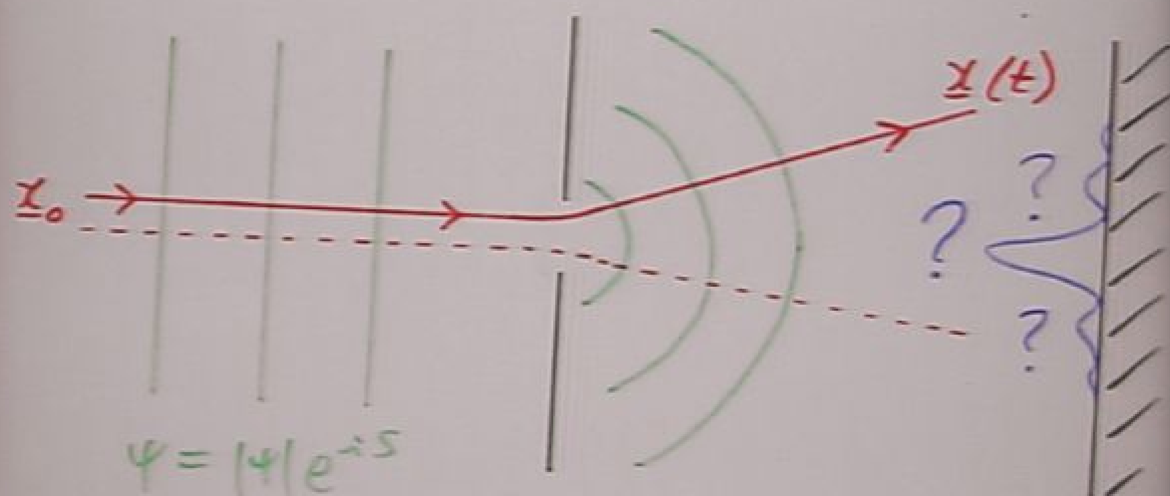
8) Nonequilibrium hidden variables: the case of pilot-wave dynamics (AV 1991)

Single low-energy particle:



$$m \frac{d\underline{x}}{dt} = \vec{\nabla} S \qquad \left(S = S(\underline{x}, t) \right)$$

(locally)

"Equilibrium" input $\rho(\underline{x}, 0) = |\psi(\underline{x}, 0)|^2$

$\Longrightarrow$ quantum output $\rho(\underline{x}, t) = |\psi(\underline{x}, t)|^2$ at the backstop

"Nonequilibrium" input $\rho(\underline{x}, 0) \neq |\psi(\underline{x}, 0)|^2$

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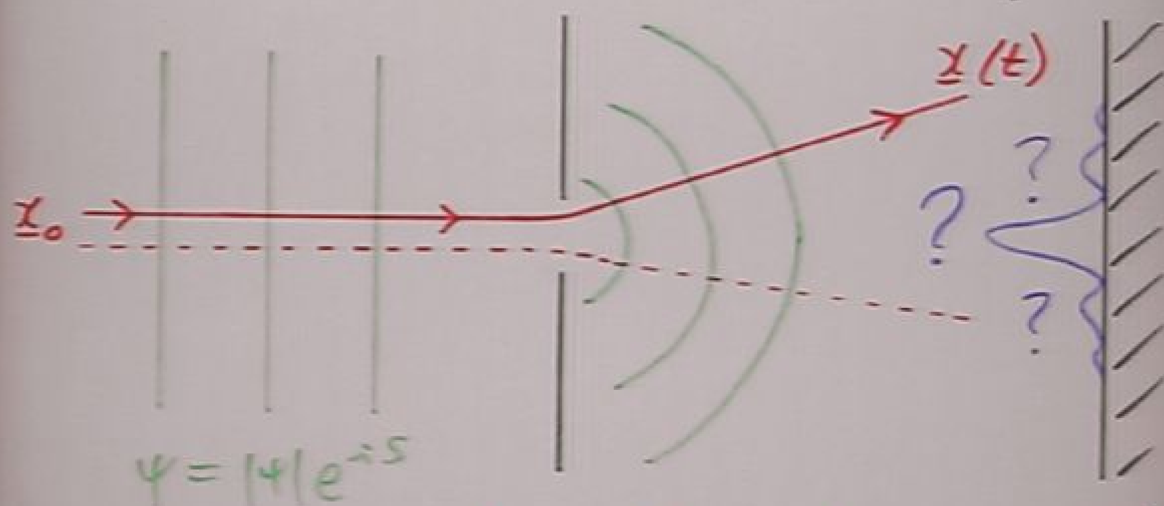

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9)

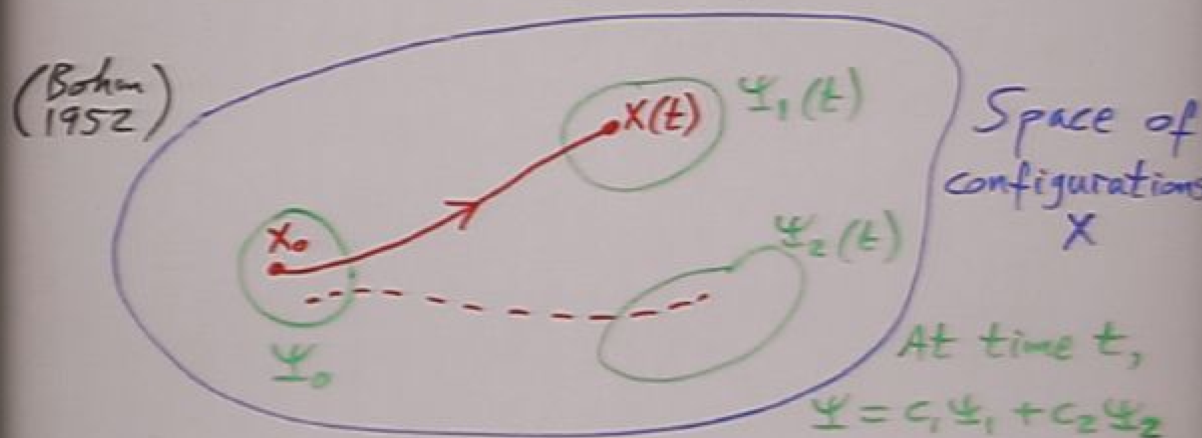
n particles (including apparatus, pointers, etc):

$\Psi(\underline{x}_1, \ldots, \underline{x}_n, t)$ $\qquad$ $m_i \dfrac{d\underline{x}_i}{dt} = \underline{\nabla}_i S$

(deBroglie 1927)

$$X(0),\ \Psi(X,0) \longrightarrow X(t),\ \Psi(X,t)$$

deterministic evolution
(for each run of an experiment)



– Each outcome determined by initial conditions X_0, Ψ_0

– Quantum probabilities for ensembles $\Longleftrightarrow P_0(X) = |\Psi_0(X)|^2$

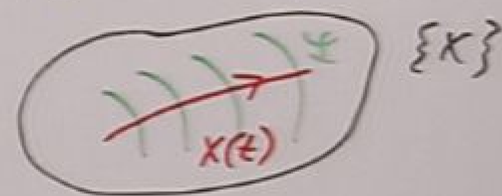
(– Field theory: $X(t) \rightarrow \phi(\underline{x},t)$, $\Psi(X,t) \rightarrow \Psi[\phi(\underline{x}),t]$)

NB. Ψ is a physical field on configuration space

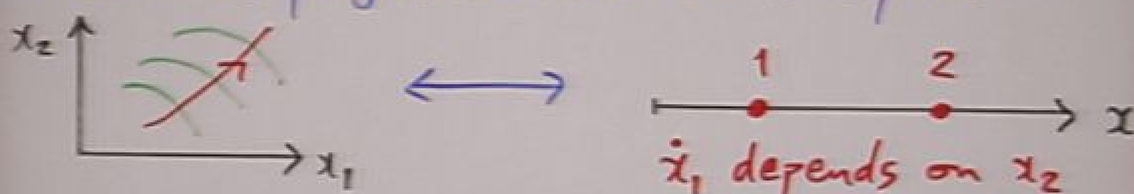
10)

Radically new form of dynamics:

— Grounded in configuration space, where Ψ propagates



— Nonlocal projection down to 3-space



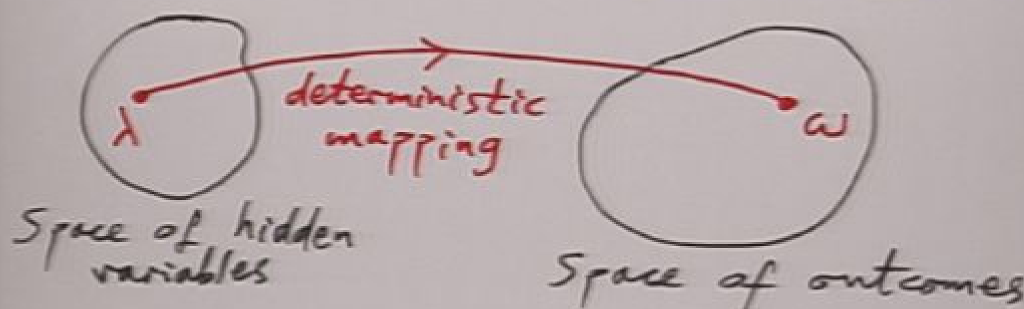
— First-order in configuration (not phase) space, law of motion has form $\frac{dX}{dt} = \cdots\cdots$.

— "Aristotelian" dynamics

$\Longleftrightarrow$ "Aristotelian" kinematics (AV 1997)

Natural state of rest,
natural preferred slicing of spacetime.

(ii) Can consider nonequilibrium in *any* deterministic hidden-variables theory:



Distribution $\rho_{QT}(\lambda) \longleftrightarrow$ Born rule for $P(\omega)$

Arbitrary $\rho(\lambda) \neq \rho_{QT}(\lambda) \longleftrightarrow$ violations of Born rule

– Recent models (based on classical Hamiltonian dynamics):

- Adler 2004: $\lambda =$ (matrix field elements)
  thermal equilibrium $\Longleftrightarrow$ quantum theory (details obscure)

- Markopoulou + Smolin 2004: $\lambda =$ (discrete graph)
  thermal equilibrium $\longrightarrow$ Nelson's (1966) stochastic mechanics: density ρ and current velocity $\underline{v}$; assume/derive $\bar{\nabla} \times \underline{v} = 0, \Rightarrow \underline{v} = \bar{\nabla} S \longrightarrow \psi \equiv \sqrt{\rho}\, e^{iS}$; but recover Schrödinger equation only for exceptional (nodeless) wave functions (cf. Wallstrom 1994 objection to Nelson's theory)

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- For purposes of this talk, suffices that there exists at least one model of quantum nonequilibrium, to serve as a "foil".
  No real need to "motivate" possible existence of nonequilibrium.

  Even so, let us provide some [m]otivation .....

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13/ Relaxation $P \longrightarrow |\Psi|^2$:

- Analogous to classical thermal relaxation, but with H-function $H = \int dx\, P \ln (P/|\Psi|^2)$ (AV 1991, 1992)
- Accessible systems emerge from long and violent astrophysical history $\Rightarrow$ we see quantum equilibrium

Nonlocality:

- Nonlocal signals $\Longleftrightarrow P \neq |\Psi|^2$ (or $\rho \neq \rho_{QT}$) (AV 1991, 2002)
- Explains "conspiracy" of "hidden nonlocality": we happen to live in a state of quantum equilibrium (to good approxn.)
- Our physics = physics of equilibrium state (cf. SSB)

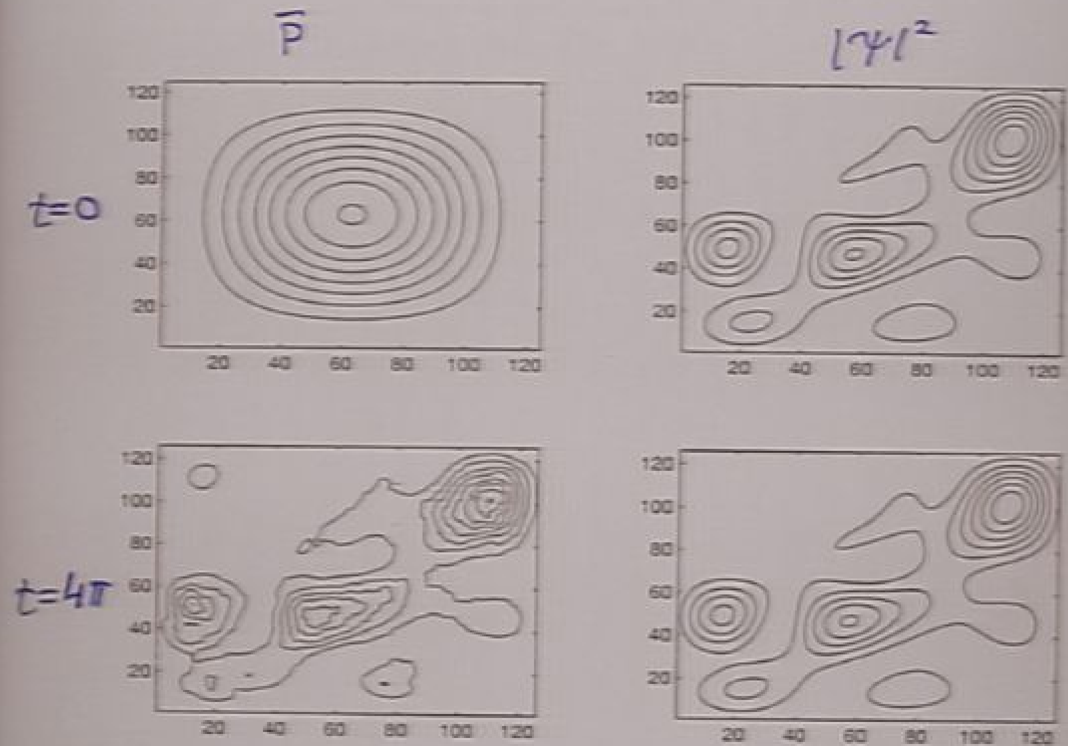
Nonequilibrium Physics

- Expect it will exist somewhere, sometime ("If something can happen, it probably will")
- Natural place and time: the early universe (quantum nonequilibrium at the big bang)
- Also motivated by cosmological horizon problem (solved by nonequilibrium nonlocality? — AV 1992)

13'

(AV and H. Westman, Proc. Roy. Soc. A 2005)

Relaxation in 2D:



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Relaxation in field theory: (Minkowski spacetime)

Free, massless $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\bar{\nabla}\phi)^2$, evolution of $\phi(\underline{x},t)$ determined by $\Psi[\phi,t]$:

$$i\frac{\partial\Psi}{\partial t} = \frac{1}{2}\int d^3\underline{x}\left(-\frac{\delta^2}{\delta\phi^2} + (\bar{\nabla}\phi)^2\right)\Psi, \quad \frac{\partial\phi(\underline{x},t)}{\partial t} = \frac{\delta S[\phi,t]}{\delta\phi(\underline{x})}$$

Fourier components $\phi_{\underline{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\underline{k}1} + i q_{\underline{k}2})$

Look at one decoupled mode $\underline{k}$: $\Psi = \psi(q_{\underline{k}1}, q_{\underline{k}2}, t).\chi$

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2}{\partial q_{\underline{k}1}^2} + \frac{\partial^2}{\partial q_{\underline{k}2}^2}\right)\psi + \frac{1}{2}k^2\left(q_{\underline{k}1}^2 + q_{\underline{k}2}^2\right)\psi$$

$$\dot{q}_{\underline{k}1} = \frac{\partial S}{\partial q_{\underline{k}1}}, \quad \dot{q}_{\underline{k}2} = \frac{\partial S}{\partial q_{\underline{k}2}}$$

State $|\Psi\rangle \sim |1_{\underline{k}}\rangle + |2_{\underline{k}}\rangle + |3_{\underline{k}}\rangle + \ldots$

$\Rightarrow$ rapid relaxation

(just as for 2D harmonic oscillator)

(14) Field theory on an expanding background:

Flat (for simplicity), $d\tau^2 = dt^2 - a^2 d\underline{x}^2$ $(a = a(t))$

Massless ϕ, $\mathcal{L} = \frac{1}{2} a^3 \dot{\phi}^2 - \frac{1}{2} a (\bar{\nabla}\phi)^2$, $\pi = a^3 \dot{\phi}$

$$\mathcal{H} = \frac{1}{2} \frac{\pi^2}{a^3} + \frac{1}{2} a (\bar{\nabla}\phi)^2$$

Schrödinger equation for wave functional $\Psi = \Psi[\phi, t]$:

$$i \frac{\partial \Psi}{\partial t} = \int d^3\underline{x} \left(-\frac{1}{2a^3} \frac{\delta^2}{\delta\phi^2} + \frac{1}{2} a (\bar{\nabla}\phi)^2 \right) \Psi$$

$\Rightarrow$ continuity equation for $|\Psi|^2$:

$$\frac{\partial |\Psi|^2}{\partial t} + \int d^3\underline{x} \frac{\delta}{\delta\phi} \left(|\Psi|^2 \frac{1}{a^3} \frac{\delta S}{\delta\phi} \right) = 0$$

$\Rightarrow$ de Broglie velocity

$$\frac{\partial \phi}{\partial t} = \frac{1}{a^3} \frac{\delta S}{\delta \phi}$$

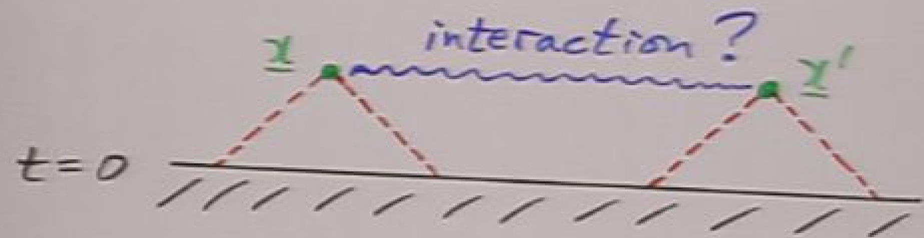
$\Rightarrow$ arbitrary distribution $P[\phi, t]$ (generally $\neq |\Psi|^2$) evolves according to

$$\frac{\partial P}{\partial t} + \int d^3\underline{x} \frac{\delta}{\delta\phi} \left(P \frac{1}{a^3} \frac{\delta S}{\delta\phi} \right) = 0$$

15) Standard Friedmann cosmology:

(conformally equivalent to section of Minkowski)

Early homogeneity over causally-disconnected domains:



Generic (entangled) Ψ, field velocity at $\mathbf{x}$

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = \frac{1}{a^3}\frac{\delta S[\phi,t]}{\delta \phi(\mathbf{x})}$$, depends on $\phi(\mathbf{x}')$.

- Initial nonequilibrium solves "horizon problem" in sense of "natural" initial conditions (AV 1992, 1996), but homogenisation not yet studied (cf. other approaches).
- At least some inflationary models require homogeneity as an initial condition, in order for inflation to begin (Vachaspati + Trodden 2000).
- Here: motivation, ... evolution of very early (pre-inflationary?) nonequilibrium .... some modes in nonequilibrium as enter inflation ...

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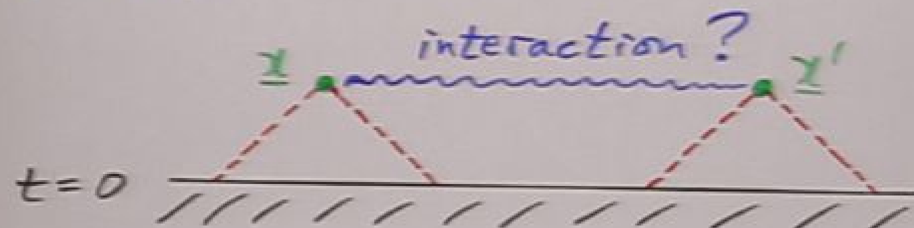

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16) Pre-inflationary / Early Friedmann phase:

- E.g., again, decoupled mode, $\Psi = \psi(q_{\underline{k}1}, q_{\underline{k}2}, t)\chi$

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2a^3}\left(\frac{\partial^2}{\partial q_{\underline{k}1}^2} + \frac{\partial^2}{\partial q_{\underline{k}2}^2}\right)\psi + \frac{1}{2}ak^2\left(q_{\underline{k}1}^2 + q_{\underline{k}2}^2\right)\psi$$

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Effect of $a(t)$? Relaxation suppressed as $a \to 0$?

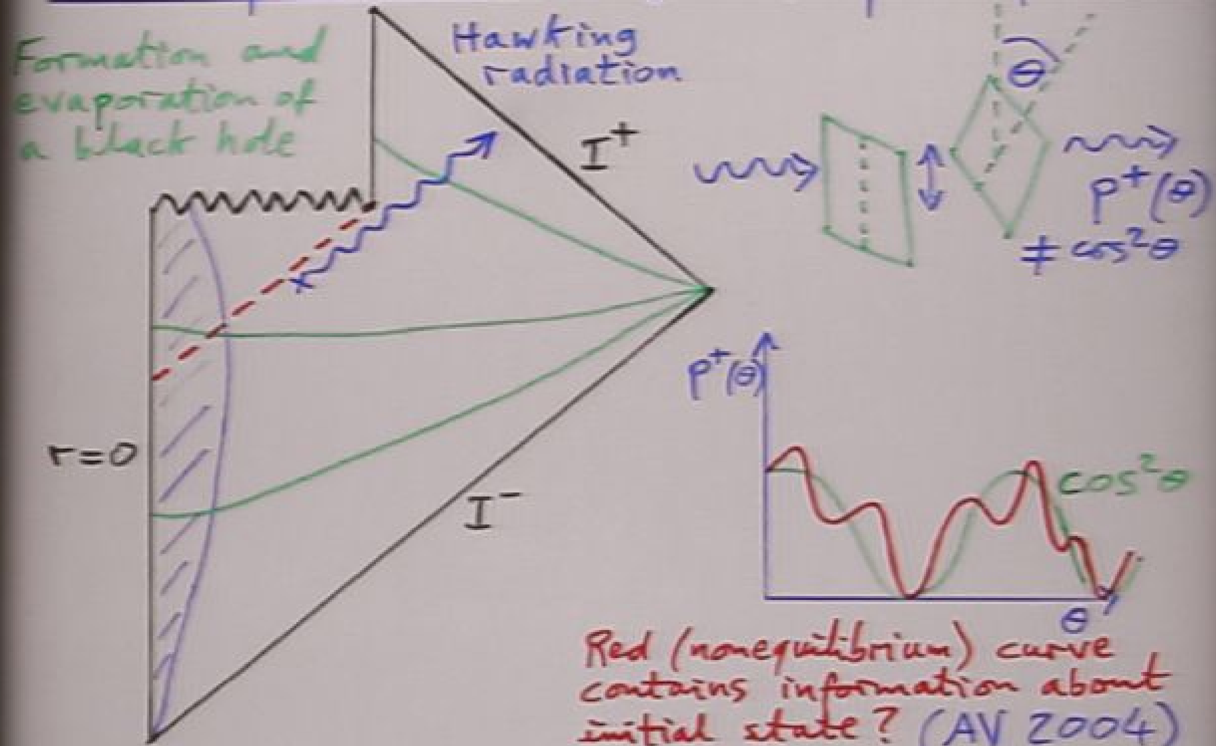
- Two competing effects:
  1. Relaxation (known for $a(t) = \text{const.}$)
  2. Spatial expansion, transfers nonequilibrium to larger spatial scales (cf. late-time deSitter)

$$\lambda_{phys}(t) = a(t)\lambda$$

- Here: "phenomenology", assume that some modes are in quantum nonequilibrium at beginning of inflationary phase.

Use CMB data to ...

17/ Another poss. motivation: gravity upsets equilibrium?



Red (nonequilibrium) curve contains information about initial state? (AV 2004)

- Microscopic (virtual?) black holes
  $\longrightarrow$ microscopic nonequilibrium (at $\sim l_P$?)
- "Phenomenology": generates nonequilibrium modes during inflationary phase (assume here). Use CMB data to set bounds.

General point is:

During inflation, some field modes may be in quantum nonequilibrium.

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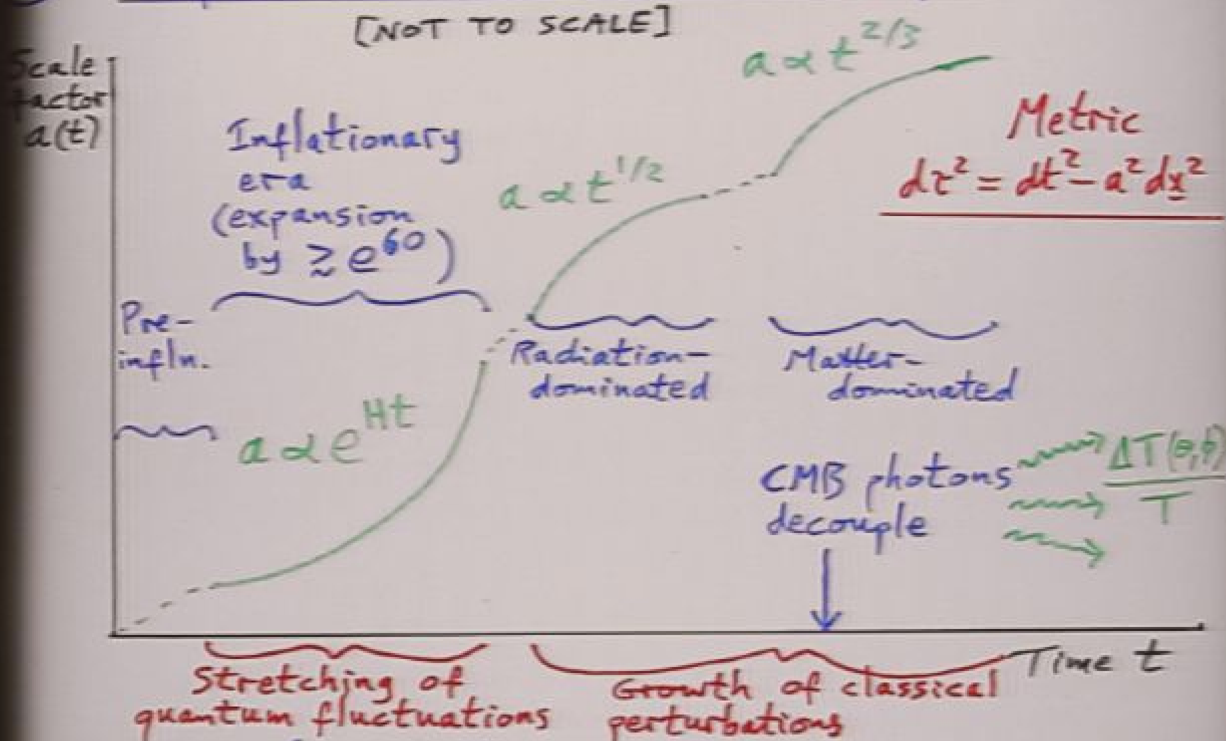

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Mode "re-entry" when

$$\lambda_{phys} \lesssim H^{-1}$$

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(→ inhomogeneous CMB, galaxies ..., etc.)

19/

CMB Observations

- One temperature function $T(\theta,\phi)$ on the sky.
  Harmonic decomposition

$$\Delta T(\theta,\phi)/T = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{+\ell} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

One set of coefficients, $\{a_{\ell m}\}$.

- Assume $T(\theta,\phi)$ is one realisation of a stochastic process. "Ensemble of skies", $P[T(\theta,\phi)]$.
  Statistical rotational invariance:

$$P[T(\theta-\delta\theta,\phi-\delta\phi)] = P[T(\theta,\phi)] \Rightarrow \underline{p_{\ell m}(a_{\ell m}) \text{ indep. of } m}$$

- Each ℓ: $(2\ell+1)$-realisations of one random variable.
  For large enough ℓ,

$$\langle |a_{\ell m}|^2 \rangle_{ens.} \approx \frac{1}{2\ell+1} \sum_{m=-\ell}^{+\ell} |a_{\ell m}|^2 \equiv C_\ell$$

(Low ℓ, "cosmic variance": C_ℓ inaccurate estimate of $\langle |a_{\ell m}|^2 \rangle_{ens.}$)

- Usually plot $\frac{\ell(\ell+1)}{2\pi} C_\ell$ versus ℓ
  (angular power spectrum)

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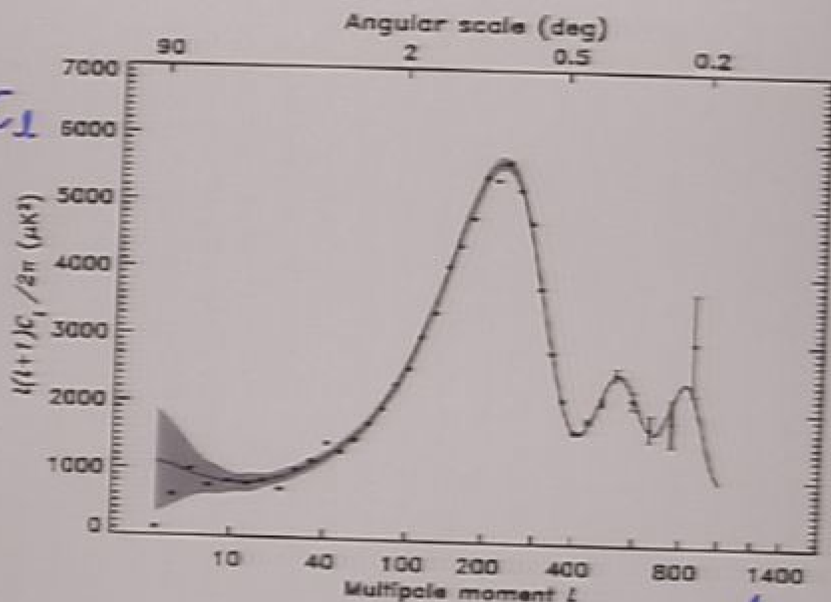
First year WMAP data for the angular power spectrum

(Hinshaw et al. 2003)

$\frac{l(l+1)}{2\pi} C_l$



l

Fig. [illegible].— The final angular power spectrum, $l(l+1)C_l/2\pi$, obtained from the 28 cross-power spectra, as described in §5. The data are plotted with 1σ measurement errors only which reflect the combined uncertainty due to noise, beam, calibration, and source subtraction uncertainties. The solid line shows the best-fit ΛCDM model from Spergel et al. (2003). The grey band around the model is the 1σ uncertainty due to cosmic variance on the cut sky. For this plot, both the model and the error band have been binned with the same boundaries as the data, but they have been plotted as a splined curve to guide the eye. On the scale of this plot the unbinned model curve would be virtually indistinguishable from the binned curve except in the vicinity of the third peak.

(Grey band: 1σ uncertainty from cosmic variance)

Solid line: best-fit cosmological model,
— Spergel et al. 2003

21/ Growth of (Classical) Primordial Perturbations

- Linear regime (scales >> bound systems), well-established theory for ΔT in terms of primordial perturbations.
- "Primordial era": long period between $t_{exit}(k)$ and $t_{enter}(k)$ ($k << Ha$, $\lambda_{phys} >> H^{-1}$; simple evolution)
- Only one independent dof for primordial perturbation (ignoring gravity waves),

$$\mathcal{R}_{\underline{k}} = \frac{1}{4}\left(\frac{a}{k}\right)^2 {}^{(3)}R_{\underline{k}} \quad \text{(time independent during primordial era)}$$

- In terms of $\mathcal{R}_{\underline{k}}$, (Lyth + Riotto, Phys. Rep. 1999)

$$a_{\ell m} = \frac{i^\ell}{2\pi^2}\int d^3\underline{k}\, \underbrace{\mathcal{I}(k,\ell)}_{\text{encodes astrophysical evolution}} \mathcal{R}_{\underline{k}} Y_{\ell m}(\hat{\underline{k}})$$

- Prob.($\mathcal{R}_{\underline{k}}$) $\longrightarrow$ Prob.($a_{\ell m}$)
- Main sources of anisotropies $\Delta T(\theta, \phi)$:
  - velocity of matter non-uniform over last scattering surface
  - local gravitational potential " " " " " (Sachs-Wolfe effect, dominates at large angles)
  - energy density of radiation " " " " " (dominates at small angles)
- If Prob.($\mathcal{R}_{\underline{k}}$) is translationally invariant .....

(22)

$$P[\mathcal{R}(\underline{x}-\underline{d})] = P[\mathcal{R}(\underline{x})] \Longrightarrow \langle \mathcal{R}_{\underline{k}} \mathcal{R}^*_{\underline{k}'}\rangle = \delta_{\underline{k}\underline{k}'}\langle|\mathcal{R}_{\underline{k}}|^2\rangle$$

$$\Longrightarrow \langle |a_{\ell m}|^2\rangle = (---)\int d^3\underline{k}\,(----)\langle|\mathcal{R}_{\underline{k}}|^2\rangle$$

and h... expression for angular power spectrum

$$C_\ell = \ldots \int^\infty \frac{dk}{k}\, I^2(k,\ell).\, P_{\mathcal{R}}(k)$$

...al power spectrum

$$\ldots \frac{4\pi k^3}{V}\langle|\mathcal{R}_{\underline{k}}|^2\rangle$$

... $P_{\mathcal{R}}(k) \approx$ const. (scale free)

...t ($\ell \lesssim 20$), Sachs-Wolfe dominates,

... $_\ell(2k/H_0)$, and if $P_{\mathcal{R}}(k) =$ const.

$$\ldots \propto \int_0^\infty \frac{dk}{k}\, j_\ell^2(k) = \frac{1}{2\ell(\ell+1)}$$

have

$$\ell(\ell+1)C_\ell = \text{const.} \quad (\text{low } \ell)$$

as approximately observed.

(or, anomalous low power?)

(22)

$$P[\mathcal{R}(\underline{x}-\underline{d})] = P[\mathcal{R}(\underline{x})] \implies \langle \mathcal{R}_{\underline{k}} \mathcal{R}^*_{\underline{k}'} \rangle = \delta_{\underline{k}\underline{k}'} \langle |\mathcal{R}_{\underline{k}}|^2 \rangle$$

$$\implies \langle |a_{\ell m}|^2 \rangle = (---) \int d^3\underline{k} \, (----) \langle |\mathcal{R}_{\underline{k}}|^2 \rangle$$

and have expression for angular power spectrum

$$C_\ell = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \, I^2(k,\ell) \, . \, P_{\mathcal{R}}(k)$$

in terms of primordial power spectrum

$$P_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V} \langle |\mathcal{R}_{\underline{k}}|^2 \rangle$$

- Data for $C_\ell \implies P_{\mathcal{R}}(k) \simeq$ const. (scale free)
- E.g. low-ℓ limit ($\ell \lesssim 20$), Sachs-Wolfe dominates,

$I^2(k,\ell) \propto j_\ell^2(2k/H_0)$, and if $P_{\mathcal{R}}(k) =$ const.

then $C_\ell \propto \int_0^\infty \frac{dk}{k} j_\ell^2(k) = \frac{1}{2\ell(\ell+1)}$

and have

$$\ell(\ell+1) C_\ell = \text{const.} \quad (\text{low } \ell)$$

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- E.g. low-ℓ limit ($\ell \lesssim 20$), Sachs-Wolfe dominates,

$\mathcal{I}^2(k,\ell) \propto j_\ell^2(2k/H_0)$, and if $P_{\mathcal{R}}(k) = \text{const.}$

then $C_\ell \propto \int_0^\infty \frac{dk}{k} j_\ell^2(k) = \frac{1}{2\ell(\ell+1)}$

and have

$$\ell(\ell+1)C_\ell = \text{const.} \quad (\text{low } \ell)$$

as approximately observed.

(or, anomalous low power?)

and have expression for angular power spectrum

$$C_\ell = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}\, T^2(k,\ell).\, P_{\mathcal{R}}(k)$$

in terms of primordial power spectrum

$$P_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V}\langle|\mathcal{R}_k|^2\rangle$$

- Data for $C_\ell \Longrightarrow P_{\mathcal{R}}(k) \simeq$ const. (scale free)
- E.g. low-ℓ limit ($\ell \lesssim 20$), Sachs-Wolfe dominates,

$T^2(k,\ell) \propto j_\ell^2(2k/H_0)$, and if $P_{\mathcal{R}}(k) =$ const.

then $C_\ell \propto \int_0^\infty \frac{dk}{k} j_\ell^2(k) = \frac{1}{2\ell(\ell+1)}$

and have

$$\ell(\ell+1)C_\ell = \text{const.} \quad (\text{low } \ell)$$

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and have expression for angular power spectrum

$$C_\ell = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}\, \mathcal{I}^2(k,\ell)\,.\, P_{\mathcal{R}}(k)$$

in terms of primordial power spectrum

$$P_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V}\langle |\mathcal{R}_k|^2\rangle$$

- Data for $C_\ell \Longrightarrow P_{\mathcal{R}}(k) \simeq$ const. (scale free)
- E.g. low-ℓ limit ($\ell \lesssim 20$), Sachs-Wolfe dominates,

$\mathcal{I}^2(k,\ell) \propto j_\ell^2(2k/H_0)$, and if $P_{\mathcal{R}}(k) = $ const.

then $C_\ell \propto \int_0^\infty \frac{dk}{k} j_\ell^2(k) = \frac{1}{2\ell(\ell+1)}$

and have

$$\ell(\ell+1)C_\ell = \text{const.} \quad (\text{low } \ell)$$

as approximately observed.
(or, anomalous low power?)

23)

Inflationary Slow Roll

- Approx.ly homogeneous inflaton field $\phi_0(t) + \phi(\underline{x}, t)$
- Energy density $\rho \approx \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) \approx \text{const.} \equiv V_0$
- Friedmann equation $\longrightarrow$ approx. deSitter

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \Rightarrow a \propto e^{Ht}, \quad H = \sqrt{\frac{8\pi G}{3}}\, V_0^{1/2}$$

- Time evolution of ϕ_0, slow-roll approximation

$$\underbrace{\ddot{\phi}_0}_{\text{neglect}} + 3\frac{\dot{a}}{a}\dot{\phi}_0 + \frac{dV}{d\phi_0} = 0$$

- Flatness of V:

$\varepsilon << 1$, $|\eta| << 1$ where

$$\varepsilon \equiv \frac{1}{16\pi G}\left(\frac{1}{V}\frac{dV}{d\phi_0}\right)^2$$

$$\eta \equiv \frac{1}{8\pi G}\frac{1}{V}\frac{d^2V}{d\phi_0^2}$$

- Quantum fluctuations during slow roll $\longrightarrow$ primordial perturbations
- Many models, refinements, unknowns (e.g. form of V)
- First approximation: eternal deSitter expansion ($\longrightarrow$ scale-free primordial power spectrum)

Quantum Theory of Primordial Perturbations

- Slow-roll limit ($\epsilon \to 0$), and flatness of V, $\Rightarrow \phi$ evolves like a free, massless field
- Assume vacuum state for inflaton perturbation $\delta\hat{\phi}$
- A few Hubble times (e-foldings) after $t_{exit}(k)$, vacuum fluctuations $\longrightarrow$ "classical" perturbation
- Corresponding curvature pert.

$$\mathcal{R}_k = -\left[\frac{H}{\dot{\phi}}\delta\phi_k\right]_{t=t_*(k)}$$

where $t_*(k)$ is a time a few e-folds after $t_{exit}(k)$

- Calculate spectrum of $\mathcal{R}_k$ at $t=t_*$ ("primordial spectrum")

$$\mathcal{P}_{\mathcal{R}}(k) = \left[\frac{H^2}{\dot{\phi}^2}\mathcal{P}_{\delta\phi}\right]_{t_*(k)} \propto \left[\frac{H^4}{\dot{\phi}^2}\right]_{t_*(k)}$$

- First approximation $\mathcal{P}_{\mathcal{R}}(k)$ independent of k (scale free)
- Slow roll corrections: H and $\dot{\phi}$ slowly changing $\Rightarrow$ small ... k

24) Quantum Theory of Primordial Perturbations

(e.g. Liddle & Lyth 2000)

- Slow-roll limit ($\ddot{H} \to 0$), and flatness of V, $\Rightarrow$ ϕ evolves like a free, massless field
- Assume vacuum state for inflaton pertns. $\hat{\phi}$
- A few Hubble times/e-foldings after $t_{exit}(k)$, vacuum fluctuations $\longrightarrow$ "classical" pertns.
- Corresponding curvature pertn.

$$\mathcal{R}_{\underline{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\underline{k}}\right]_{t=t_*(k)}$$

where $t_*(k)$ is a time a few e-folds after $t_{exit}(k)$

- Predicted spectrum of $\mathcal{R}_{\underline{k}}$ at $t=t_*$ ("primordial spectrum")

$$P_{\mathcal{R}}^{QT}(k) = \left[\frac{H^2}{\dot{\phi}_0^2}P_\phi\right]_{t_*(k)} \propto \left[\frac{H^4}{\dot{\phi}_0^2}\right]_{t_*(k)}$$

— First approximation, $P_{\mathcal{R}}(k)$ independent of k (scale free)

— Slow roll corrections, H and $\dot{\phi}_0$ slowly changing $\Rightarrow$ small dependence on k

- Here, consider only the first approximation. Study effect of quantum nonequilibrium at this level.

$\Rightarrow$ ϕ evolves like a free, massless field

- Assume vacuum state for inflaton pertns. $\hat{\phi}$
- A few Hubble times/e-foldings after $t_{exit}(k)$, vacuum fluctuations $\longrightarrow$ "classical" pertns.
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— First approximation, $P_{\mathcal{R}}(k)$ independent of k (scale free)

— Slow roll corrections, H and $\dot{\phi}_0$ slowly changing $\Rightarrow$ small dependence on k

- Here, consider only the first approximation. Study effect of quantum nonequilibrium at this level.

25/ Quantum Vacuum Fluctuations in deSitter Space

- Standard field operator expansion

$$\hat{\phi}(\underline{x},t) = \sum_{\underline{k}} \left[\frac{(k/a + i H)}{k\sqrt{2Vk}} \hat{a}_{\underline{k}} e^{-i(\underline{k}\cdot\underline{x} + k/Ha)} + \text{H.c.} \right]$$

Bunch–Davies vacuum, $\hat{a}_{\underline{k}} |0\rangle = 0 \quad \forall \underline{k}$

- $$\langle 0 | \hat{\phi}(\underline{x},t) \hat{\phi}(\underline{x}',t) | 0 \rangle = \sum_{\underline{k}} \frac{(k/a)^2 + H^2}{2Vk^3} e^{-i\underline{k}\cdot(\underline{x}-\underline{x}')}$$

$$\Longrightarrow \quad \langle |\phi_{\underline{k}}|^2 \rangle_{QT} = \frac{V}{2(2\pi)^3} \cdot \frac{H^2}{k^3} \left(1 + \frac{k^2}{H^2 a^2} \right)$$

(decreasing width $\longrightarrow$ const.)

- Power spectrum

$$P_\phi^{QT}(k) \equiv \frac{4\pi k^3}{V} \langle |\phi_{\underline{k}}|^2 \rangle_{QT} = \frac{k^2}{4\pi^2 a^2} + \frac{H^2}{4\pi^2}$$

- Long wavelength limit, $k/a \ll H$ $\left(\lambda_{phys} \gg H^{-1} \right)$ $\updownarrow$ (mode exits Hubble radius)

$$P_\phi^{QT}(k) = \left(\frac{H}{2\pi} \right)^2$$

(or, $\times 2$ if set $k = Ha$)

- Generates scale-free spectrum for primordial perturbations $\mathcal{R}_{\underline{k}} \propto \phi_{\underline{k}}$ outside Hubble radius

5) Quantum Vacuum Fluctuations in deSitter Space

- Standard field operator expansion

$$\hat{\phi}(x,t) = \sum_{\underline{k}} \left[\frac{(k/a + iH)}{k\sqrt{2Vk}} \hat{a}_{\underline{k}} e^{i(\underline{k}\cdot x + k/Ha)} + \text{H.c.} \right]$$

Bunch–Davies vacuum, $\hat{a}_{\underline{k}}|0\rangle = 0 \quad \forall \underline{k}$

- $$\langle 0|\hat{\phi}(x,t)\hat{\phi}(x',t)|0\rangle = \sum_{\underline{k}} \frac{(k/a)^2 + H^2}{2Vk^3} e^{i\underline{k}\cdot(x-x')}$$

$$\Longrightarrow \quad \langle|\phi_{\underline{k}}|^2\rangle_{QT} = \frac{V}{2(2\pi)^3}\cdot\frac{H^2}{k^3}\left(1 + \frac{k^2}{H^2a^2}\right)$$

(decreasing width $\longrightarrow$ const.)

- Power spectrum

$$P_\phi^{QT}(k) \equiv \frac{4\pi k^3}{V}\langle|\phi_{\underline{k}}|^2\rangle_{QT} = \frac{k^2}{4\pi^2a^2} + \frac{H^2}{4\pi^2}$$

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- Generates scale-free spectrum for primordial perturbations $\mathcal{R}_{\underline{k}} \propto \phi_{\underline{k}}$ outside Hubble radius

26/ Pilot-Wave Field Theory on de Sitter Space

- $\frac{\partial \phi}{\partial t} = \frac{1}{a^3}\frac{\delta S}{\delta \phi}$, $\; i\frac{\partial \Psi}{\partial t} = \int d^3x \left(-\frac{1}{2a^3}\frac{\delta^2}{\delta \phi^2} + \frac{1}{2}a(\bar{\nabla}\phi)^2 \right)\Psi$

- Rewrite in Fourier space, $\phi_k = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{k1} + i q_{k2})$
have

$$\frac{dq_{kr}}{dt} = \frac{1}{a^3}\frac{\partial S}{\partial q_{kr}} \qquad (r = 1, 2)$$

$$i\frac{\partial \Psi}{\partial t} = \sum_{kr}\left(-\frac{1}{2a^3}\frac{\partial^2}{\partial q_{kr}^2} + \frac{a}{2}k^2 q_{kr}^2 \right)\Psi$$

- For product $\Psi[q_{kr}, t] = \prod_{kr} \psi_{kr}(q_{kr}, t)$ (e.g. B-D vacuum)
wave function for a single mode satisfies

$$i\frac{\partial \psi_{kr}}{\partial t} = \left(-\frac{1}{2a^3}\frac{\partial^2}{\partial q_{kr}^2} + \frac{1}{2}ak^2 q_{kr}^2 \right)\psi_{kr}$$

and writing $\psi_{kr} = |\psi_{kr}| e^{iS_{kr}}$ $\left(S = \sum_{kr} S_{kr} \right)$
have
$$\frac{dq_{kr}}{dt} = \frac{1}{a^3}\frac{\partial S_{kr}}{\partial q_{kr}}$$

- Initial product distribution $P[q_{kr}, t_i] = \prod_{kr} \rho_{kr}(q_{kr}, t_i)$
evolves according to

$$\frac{\partial \rho_{kr}}{\partial t} + \frac{\partial}{\partial q_{kr}}\left(\rho_{kr}\frac{1}{a^3}\frac{\partial S_{kr}}{\partial q_{kr}} \right) = 0$$

26/ Pilot-Wave Field Theory on deSitter Space

- $\dfrac{\partial\phi}{\partial t} = \dfrac{1}{a^3}\dfrac{\delta S}{\delta\phi}$, $\quad i\dfrac{\partial\Psi}{\partial t} = \displaystyle\int d^3x \left(-\frac{1}{2a^3}\frac{\delta^2}{\delta\phi^2} + \frac{1}{2}a(\nabla\phi)^2\right)\Psi$

- Rewrite in Fourier space, $\phi_{\mathbf{k}} = \dfrac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + iq_{\mathbf{k}2})$, have

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3}\frac{\partial S}{\partial q_{\mathbf{k}r}} \qquad (r = 1, 2)$$

$$i\frac{\partial\Psi}{\partial t} = \sum_{\mathbf{k}r}\left(-\frac{1}{2a^3}\frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{a}{2}k^2 q_{\mathbf{k}r}^2\right)\Psi$$

- For product $\Psi[q_{\mathbf{k}r}, t] = \prod_{\mathbf{k}r}\psi_{\mathbf{k}r}(q_{\mathbf{k}r}, t)$ (e.g. B–D vacuum)

wave function for a single mode satisfies

$$i\frac{\partial\psi_{\mathbf{k}r}}{\partial t} = \left(-\frac{1}{2a^3}\frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{1}{2}ak^2 q_{\mathbf{k}r}^2\right)\psi_{\mathbf{k}r}$$

and writing $\psi_{\mathbf{k}r} = |\psi_{\mathbf{k}r}|e^{is_{\mathbf{k}r}}$ $\left(S = \sum_{\mathbf{k}r} s_{\mathbf{k}r}\right)$

have
$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}}$$

- Initial product distribution $P[q_{\mathbf{k}r}, t_i] = \prod_{\mathbf{k}r}\rho_{\mathbf{k}r}(q_{\mathbf{k}r}, t_i)$ evolves according to

$$\frac{\partial\rho_{\mathbf{k}r}}{\partial t} + \frac{\partial}{\partial q_{\mathbf{k}r}}\left(\rho_{\mathbf{k}r}\frac{1}{a^3}\frac{\partial s_{\mathbf{k}r}}{\partial q_{\mathbf{k}r}}\right) = 0$$

27/ Time Evolution of Nonequilibrium Vacua

- Bunch-Davies wave function $\Psi = \Psi(q,t)$ for mode $\underline{k}r$ (dropping indices $\underline{k}r$), $\Psi = |\Psi| e^{iS}$

$$|\Psi|^2 = \frac{1}{\sqrt{2\pi\Delta^2}} e^{-q^2/2\Delta^2}, \quad \Delta^2 = \frac{H^2}{2k^3}\left(1 + \frac{k^2}{H^2 a^2}\right)$$

(contracting Gaussian)

$$S = -\frac{a k^2 q^2}{2H(1 + k^2/H^2 a^2)} + (\text{function of } t)$$

- Quantum vacuum: $q_{\underline{k}r}$ are indep. random variables, each with Gaussian distribution of zero mean. Width decreases with time, asymptotic value $H/\sqrt{2}\,k^{3/2}$
- de Broglie velocity $\frac{dq}{dt} = \frac{1}{a^3}\frac{\partial S}{\partial q} = -\frac{k^2 H q}{k^2 + H^2 a^2}$

Solve with conformal time $\eta \in (-\infty, 0)$ $\left[d\eta = \frac{dt}{a}, \quad a \propto e^{Ht}, \quad \eta = -\frac{1}{Ha} \right]$
$(d\tau^2 = a^2(d\eta^2 - d\underline{x}^2))$

$$\Delta^2 = \frac{H^2}{2k^3}(1 + k^2\eta^2)$$

$$\frac{dq}{d\eta} = \frac{k^2 q \eta}{1 + k^2\eta^2} \Longrightarrow q(\eta) = q(0)\sqrt{1 + k^2\eta^2}$$

- Arbitrary distribution $\rho(q,\eta)$ $\left(\neq |\Psi(q,\eta)|^2\right)$

$$\frac{\partial \rho}{\partial \eta} + \frac{\partial}{\partial q}\left(\rho \frac{dq}{d\eta}\right) = 0 \Longrightarrow \rho(q,\eta) = \frac{1}{\sqrt{1 + k^2\eta^2}}\,\rho\left(\frac{q}{\sqrt{1 + k^2\eta^2}}, 0\right)$$

(solution for any $\rho(q,0)$)

27) Time Evolution of Nonequilibrium Vacua

- Bunch-Davies wave function $\Psi = \Psi(q,t)$ for mode $\underline{k}r$ (dropping indices $\underline{k}r$), $\Psi = |\Psi| e^{iS}$

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Solve with conformal time $\eta \in (-\infty, 0)$ $\left[d\eta = \frac{dt}{a}, \; a \propto e^{Ht}, \; \eta = -\frac{1}{Ha} \right]$

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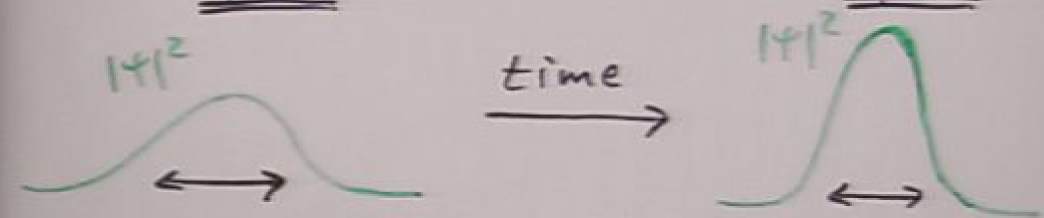
- Arbitrary distribution $\rho(q,\eta)$ $(\neq |\Psi(q,\eta)|^2)$

$$\frac{\partial \rho}{\partial \eta} + \frac{\partial}{\partial q}\left(\rho \frac{dq}{d\eta}\right) = 0 \implies \rho(q,\eta) = \frac{1}{\sqrt{1+k^2\eta^2}}\, \rho\left(\frac{q}{\sqrt{1+k^2\eta^2}}, 0\right)$$

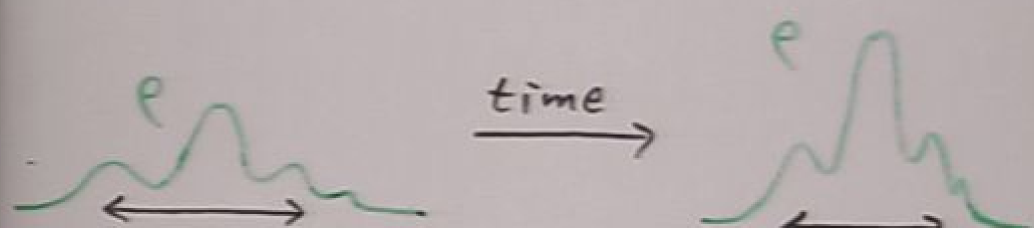
(solution for any $\rho(q,0)$)

28/ Uniform contraction of both $|\psi|^2$ and ρ:

$\underline{\eta<0}$ $\qquad\qquad$ $\underline{\eta=0}$



$$\Delta(\eta)=\Delta(0)\sqrt{1+k^2\eta^2} \qquad \Delta(0)=H/\sqrt{2}k^{3/2}$$



$$D(\eta)=D(0)\sqrt{1+k^2\eta^2} \qquad D(0)$$

Result:

(any convenient fiducial time)

$$\frac{D_{\mathbf{k}r}(t)}{\Delta_{\mathbf{k}r}(t)}=(\text{const. in time})\equiv\sqrt{\xi(k)}$$

$$\Longrightarrow \langle|\phi_{\mathbf{k}}|^2\rangle=\langle|\phi_{\mathbf{k}}|^2\rangle_{QT}\cdot\xi(k)$$

$$\Longrightarrow \underline{P_\phi(k)=P_\phi^{QT}(k)\cdot\xi(k)=\left(\frac{H}{2\pi}\right)^2\cdot\xi(k)}$$

(BROKEN SCALE INVARIANCE) $(k/a<<H)$

29/

<u>Note</u>: Weakly dependent on details of de Broglie–Bohm dynamics:

In one dimension, local conservation of $|\psi|^2$

$$\frac{\partial|\psi|^2}{\partial t} + \frac{\partial(|\psi|^2 v)}{\partial q} = 0$$

uniquely fixes the velocity field v as

$$v(q,t) = \frac{1}{|\psi(q,t)|^2}\int_q^{\infty} dq' \, \frac{\partial|\psi(q',t)|^2}{\partial t}$$

Therefore, obtain identical evolution in any hidden variables theory with

(1) Continuously evolving field beable $\phi(x,t)$

(2) Separable dynamics for product $\Psi = \prod_{\underline{k}r} \psi_{\underline{k}r}$ (velocity of $q_{\underline{k}r}$ indep. of other $q_{\underline{k}r}$s)

In any deterministic theory of this form:

Quantum nonequilibrium (if it exists) will not relax during inflationary phase.

It is preserved, and transferred to cosmological scales.

30)

In any deterministic theory of this form:

Quantum nonequilibrium (if it exists) will not relax during inflationary phase.

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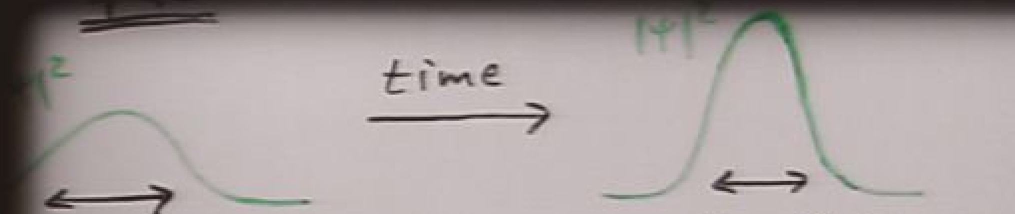
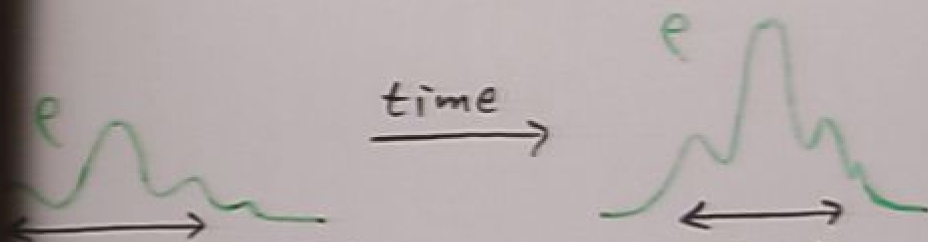


$) = \Delta(0)\sqrt{1+k^2\eta^2}$ $\qquad$ $\Delta(0) = H/\sqrt{2}\,k^{3/2}$



$) = D(0)\sqrt{1+k^2\eta^2}$ $\qquad$ $D(0)$

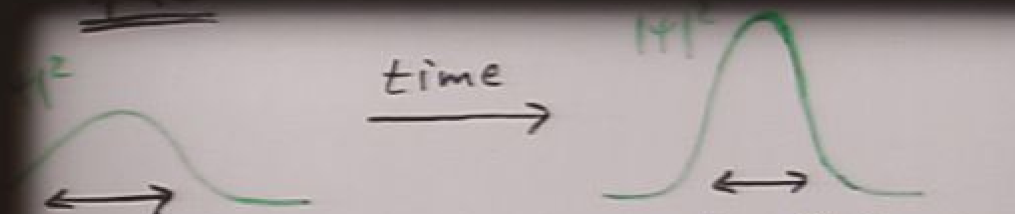
ult: (any convenient fiducial time)

$$\frac{D_{kr}(t)}{\Delta_{kr}(t)} = (\text{const. in time}) \equiv \sqrt{\xi(k)}$$

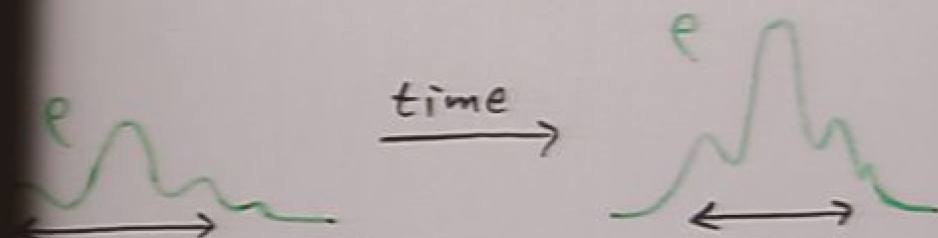
$\Rightarrow \quad \langle|\phi_k|^2\rangle = \langle|\phi_k|^2\rangle_{QT}\cdot\xi(k)$

$$\Rightarrow \quad P_\phi(k) = P_\phi^{QT}(k)\cdot\xi(k) = \left(\frac{H}{2\pi}\right)^2\cdot\xi(k)$$

KEN … RIANCE) $(k/a \ll H)$



$$\ldots) = \Delta(0)\sqrt{1+k^2\eta^2} \qquad \Delta(0) = H/\sqrt{2}\,k^{3/2}$$



$$\ldots) = D(0)\sqrt{1+k^2\eta^2} \qquad D(0)$$

…ult: (any convenient fiducial time)

$$\frac{D_{\underline{k}r}(t)}{\Delta_{\underline{k}r}(t)} = (\text{const. in time}) \equiv \sqrt{\xi(k)}$$

$$\Rightarrow \langle|\phi_{\underline{k}}|^2\rangle = \langle|\phi_{\underline{k}}|^2\rangle_{QT}\cdot\xi(k)$$

$$\Rightarrow P_\phi(k) = P_\phi^{QT}(k)\cdot\xi(k) = \left(\frac{H}{2\pi}\right)^2\cdot\xi(k)$$

…KEN SCALE INVARIANCE) $(k/a \ll H)$

3.1) Bounds on Violation of Quantum Theory in the Early Universe

- Use data to constrain "nonequilibrium function" $\xi(k)$
- $P_\phi(k) = \left(\frac{H}{2\pi}\right)^2 \xi(k) \implies P_{\mathcal{R}}(k) = P_{\mathcal{R}}^{QT}(k) \cdot \xi(k)$

where
$$P_{\mathcal{R}}^{QT}(k) = \frac{1}{4\pi^2}\left[\frac{H^4}{\dot{\phi}_0^2}\right]_{t_*(k)}$$

- Active field of research: k-dependence of $P_{\mathcal{R}}^{QT}(k)$

Common parameterisation, spectral index $n(k)$, $n(k) - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k}$ and running of spectral index, $n'(k) \equiv \frac{dn}{d \ln k}$ (e.g. slow-roll inflation predicts $n - 1 = -6\varepsilon + 2\eta$)

- Can regard observed bounds on $n(k)$, $n'(k)$ as bounds on early quantum nonequilibrium

(assume no "conspiratorial" cancellation by other effects)

- Recent limits: e.g. Bennett et al. 2003 (first year WMAP)

$$n = 0.93 \pm 0.03 \quad (\text{at } k = 0.05\ \text{Mpc}^{-1})$$
$$n' = -0.031^{+0.016}_{-0.018}$$

angular power spectrum

(Hinshaw et al. 2003)

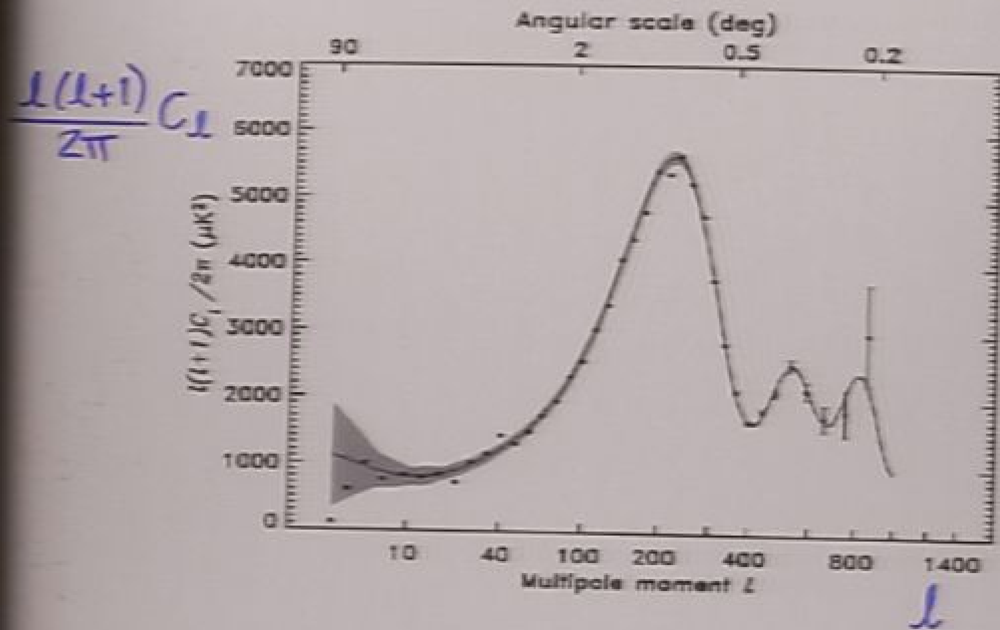


Fig. 8.— The final angular power spectrum, $l(l+1)C_l/2\pi$, obtained from the 28 cross-power spectra, as described in §5. The data are plotted with 1σ measurement errors only which reflect the combined uncertainty due to noise, beam, calibration, and source subtraction uncertainties. The solid line shows the best-fit ΛCDM model from Spergel et al. (2003). The grey band around the model is the 1σ uncertainty due to cosmic variance on the cut sky. For this plot, both the model and the error band have been binned with the same boundaries as the data, but they have been plotted as a splined curve to guide the eye. On the scale of this plot the unbinned model curve would be virtually indistinguishable from the binned curve except in the vicinity of the third peak.

(Grey band: 1σ uncertainty from cosmic variance)

Solid line: best-fit cosmological model,

— Spergel et al. 2003

3) Anomalous Low Power at Small ℓ ?

- Some indication of low power at small ℓ
- At small ℓ (Sachs–Wolfe domination)

$$C_\ell = \frac{H_0^4}{2\pi}\int_0^\infty \frac{dk}{k}\cdot j_\ell^2(2k/H_0)\cdot P_{\mathcal{R}}(k)$$

- For $P_{\mathcal{R}}(k) = \text{const.}$, have $\ell(\ell+1)C_\ell = \text{const.}$
- Taking $P_{\mathcal{R}}(k) = P_{\mathcal{R}}^{QT}(k)\cdot\xi(k)$, and assuming $P_{\mathcal{R}}^{QT}(k) = \text{const.}$, we have

$$\frac{\ell(\ell+1)C_\ell}{\ell(\ell+1)C_\ell^{QT}} = 2\ell(\ell+1)\cdot\int_0^\infty \frac{dk}{k}\cdot j_\ell^2(2k/H_0)\cdot\xi(k)$$

- If the low power anomaly is real, could be due to $\xi(k) < 1$ (e.g. at dominant scale $k \sim \ell H_0$)
- On this assumption, can gain information about the form of quantum nonequilibrium in the early universe

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Non-Random Primordial Phases

- Phases contain a lot of information (e.g. Coles 2005)

- Trajectories $q_{\mathbf{k}r}(\eta) = q_{\mathbf{k}r}(0)\sqrt{1+k^2\eta^2}$

$$\Longrightarrow \quad \theta_{\mathbf{k}} = \tan^{-1}\left(\frac{q_{\mathbf{k}2}}{q_{\mathbf{k}1}}\right) = \text{const. in time}$$

Phase $\theta_{\mathbf{k}}$ of each mode is static

- Quantum equilibrium: phases are random on $(0, 2\pi)$, $\rho_{\mathbf{k}}^{QT}(\theta_{\mathbf{k}}) = \frac{1}{2\pi}$

- Nonequilibrium, non-random phases, with

$$\rho_{\mathbf{k}}(\theta_{\mathbf{k}}) = \text{const. in time}$$

- Explore effect on phases of a_{lm} s

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Other Effects

- Correlated nonequilibrium:
  prob. can be correlated across modes,
  even if $|\Psi|^2$ is not

- Non-Gaussianity:

Quantum prob. $[\phi] = \prod_{k\sigma} \underbrace{|\psi_{k\sigma}|^2}_{\text{Gaussian}}$

Can be badly broken in nonequilibrium

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35/
Violation of Scalar–Tensor Consistency Relation

- Primordial gravitational waves ("tensor perturbations"), small effect on temp. anisotropy $\frac{\Delta T}{T}$
- Tensor contribution to polarisation anisotropy might be measureable
- Slow-roll inflation predicts

$$\frac{P_{tensor}(k)}{P_{scalar}(k)} = 16\varepsilon \quad \left(\varepsilon = \frac{1}{16\pi G}\left(\frac{1}{V}\frac{dV}{d\phi_0}\right)^2, \; V, V' \text{ evaluated at } t_{exit}(k), \text{ or } k = aH \right)$$

- Hidden variables perspective: no reason why fluctuations should be related for physically distinct degrees of freedom (cf. distinct boxes of classical gas)
- Not every anomaly is compatible with nonequilibrium:

  anomalous $P_{tensor}(k)$ and/or $P_{scalar}(k)$, but with consistency $P_t/P_s = 16\varepsilon$ preserved, would be strong evidence against nonequilibrium

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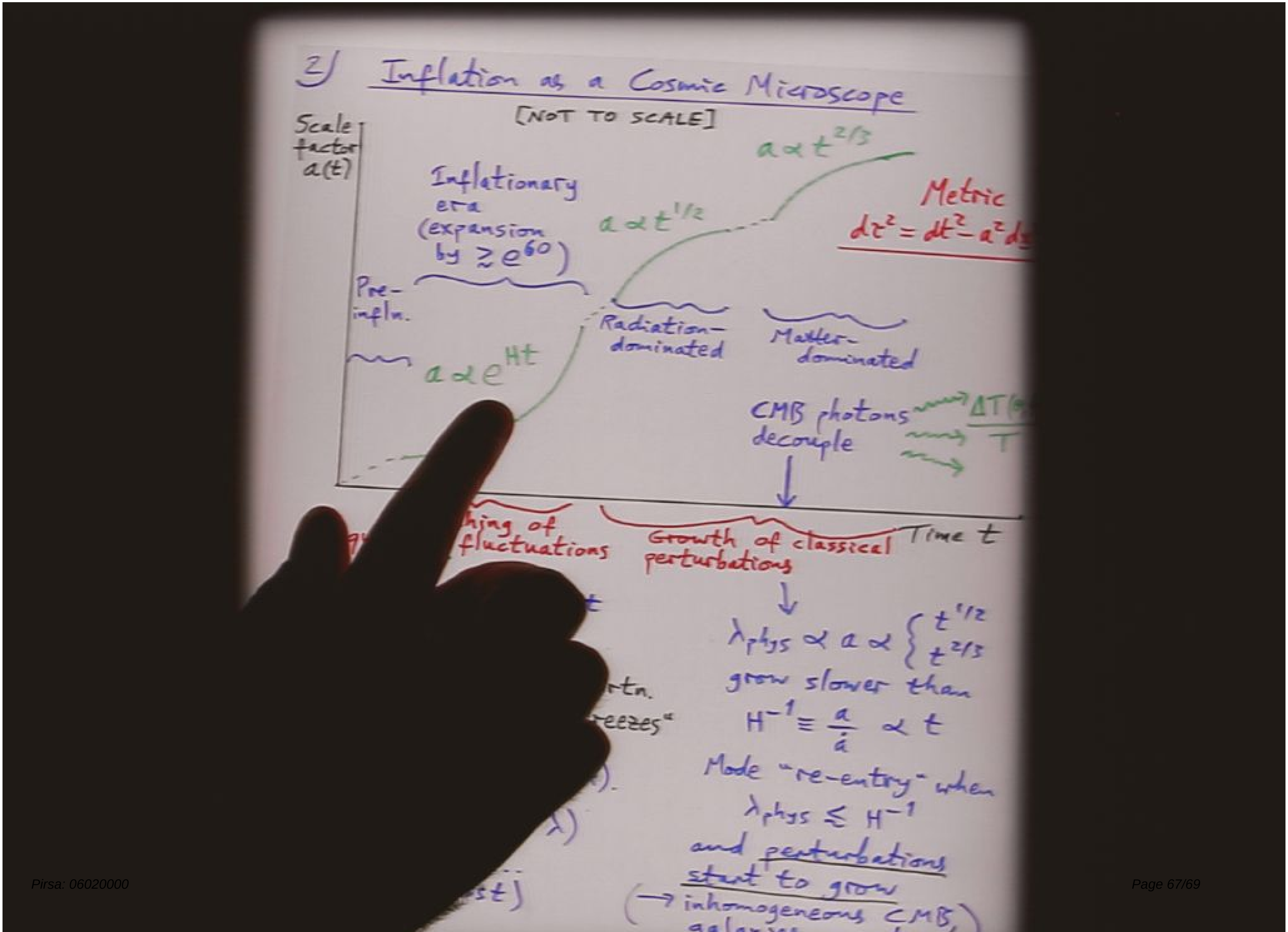
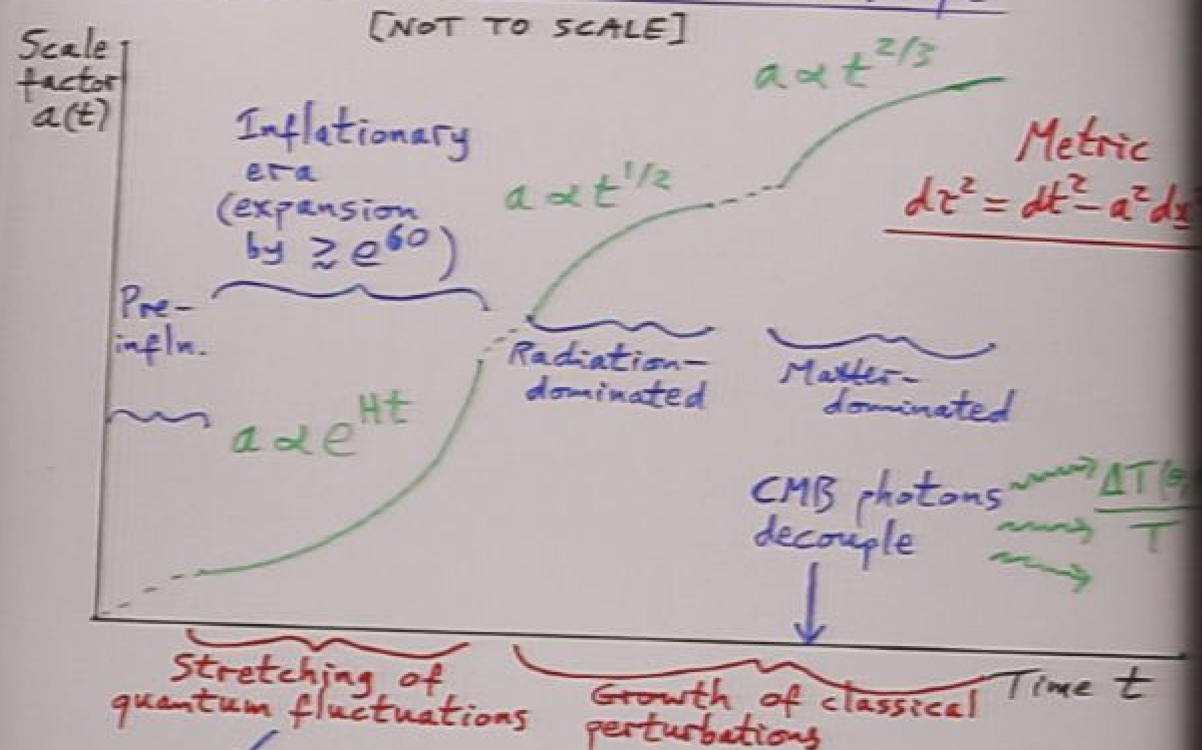

2/ Inflation as a Cosmic Microscope
[NOT TO SCALE]
Scale factor a(t)
a ∝ t^2/3
Inflationary era (expansion by ≳ e^60)
a ∝ t^1/2
Metric
dτ² = dt² − a²d
Pre-infln.
Radiation-dominated
Matter-dominated
a ∝ e^Ht
CMB photons decouple
ΔT
T
Time t
fluctuations
Growth of classical perturbations
λphys ∝ a ∝ { t^1/2, t^2/3
grow slower than
H^−1 ≡ a/ȧ ∝ t
Mode "re-entry" when
λphys ≲ H^−1
and perturbations start to grow
(→ inhomogeneous CMB,

2) Inflation as a Cosmic Microscope



$$\lambda_{phys} = \frac{a(t)}{a_0}\lambda \propto e^{Ht}$$

Mode "exit" when $\lambda_{phys} \gtrsim H^{-1}$ (= const.) } Pertn. "freezes"

at time $t_{exit} = t_{exit}(k)$.

Smaller k (larger λ) exit earlier ..... (stretched most)

$$\lambda_{phys} \propto a \propto \begin{cases} t^{1/2} \\ t^{2/3} \end{cases}$$

grow slower than

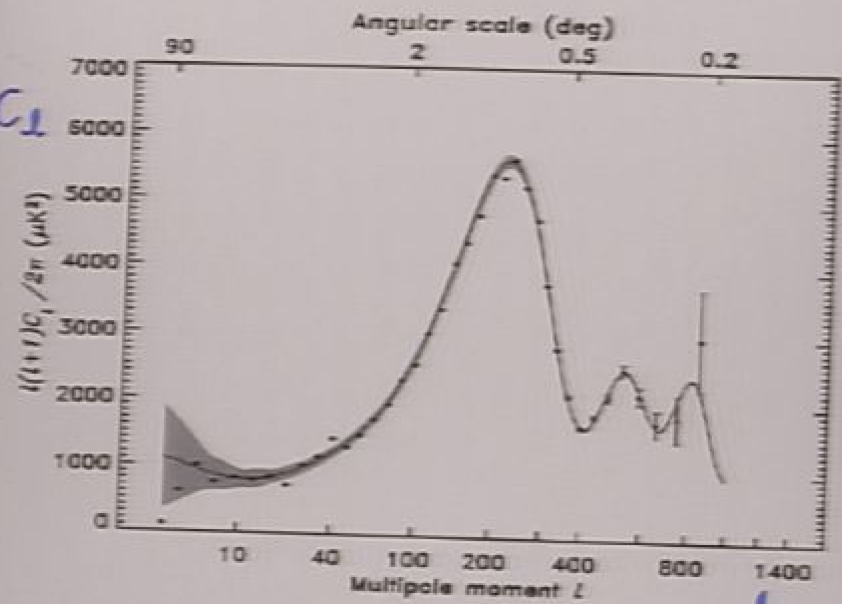
$$H^{-1} \equiv \frac{a}{\dot{a}} \propto t$$

Mode "re-entry" when $\lambda_{phys} \lesssim H^{-1}$

and perturbations start to grow

(→ inhomogeneous CMB, galaxies, ... etc)

First year WMAP data for the angular power spectrum (Hinshaw et al. 2003)



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