Title: Confining the electroweak model to a brane

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Abstract: From the Quantum Field Theory point of view, matter and gauge fields are generally expected to be localised around branes (topological defects) occurring in extra dimensions. I will discuss a simple scenario where, by starting with a five dimensional SU(3) gauge theory, we end up with several 4-D parallel braneworlds with localised 'chiral' fermions and gauge fields to them. I will show that it is possible to reproduce the electroweak model confined to a single brane, allowing a simple and geometrical approach to the hierarchy problem. Some nice results of this construction are: Gauge and Higgs fields are unified at the 5-D level; and new particles are predicted: a left-handed neutrino (with zero-hypercharge) and a massive vector field coupling together the new neutrino to other leptons.

Confining the Electroweak Model to a Brane

(hep-th/0505170)

Gonzalo A. Palma
DAMTP, Cambridge

Perimeter, February 2006

Confining the Electroweak Model to a Brane

or

Gauge-Higgs Unification on the Brane

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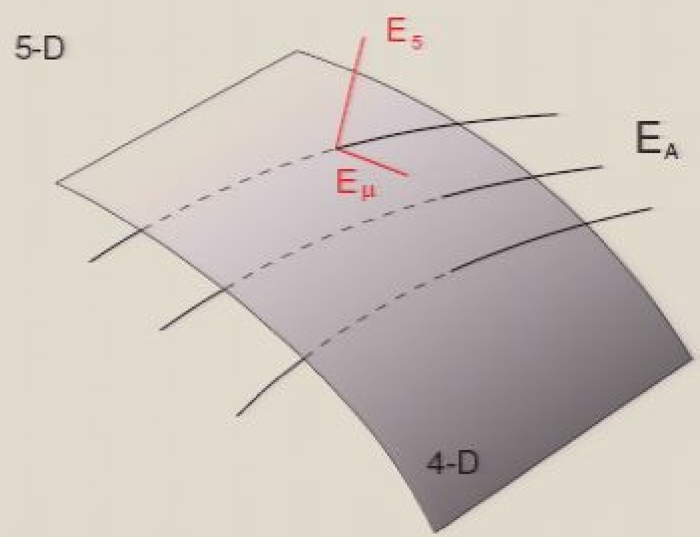
Motivation 1 Gauge-Higgs Unification:

Gauge and Higgs fields could be components of the same field in higher dimensions.



The Higgs scalar is the transverse component of the extra dimensional Gauge field.

N.S. Manton, Nucl.Phys. B **158**, 136 (1979)

Toy model: Consider a 5-D system consisting of a spin-1/2 fermion Ψ and a real scalar Φ:

$$\mathcal{L}^{(5)} = -\bar{\Psi}\left[\gamma^A \partial_A + m + y\,\Phi\right]\Psi - \frac{1}{2}(\partial_A \Phi)^2 - \frac{\sigma}{4}\left[\Phi^2 - v^2\right]^2$$

$$A = 1, \ldots, 5$$
$$z = x^5 \quad \mu = 1, \ldots, 4, \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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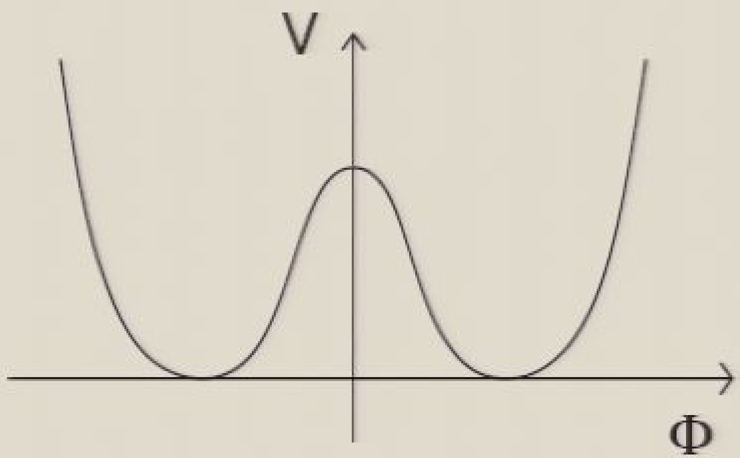
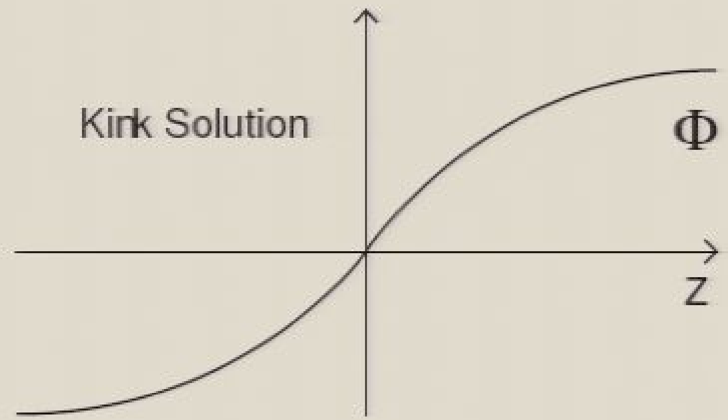
The system admits a kink solution of the form: $\Phi(z) = v \tanh(kz)$ with $k = v\sqrt{\sigma/2}$.





So we want to solve: $\left[\gamma^\mu \partial_\mu + \gamma^5 \partial_z + y\,\Phi(z)\right]\Psi = 0$

To solve this equation notice that the translational invariance along z is broken:

$$\gamma^5 \Psi_L = +\Psi_L \qquad \text{and} \qquad \gamma^5 \Psi_R = -\Psi_R$$

The solution is:

$$\Psi_{L,R} = A \exp\left\{\mp y \int_0^z \Phi(z)\, dz\right\} \psi_{L,R}(x)$$

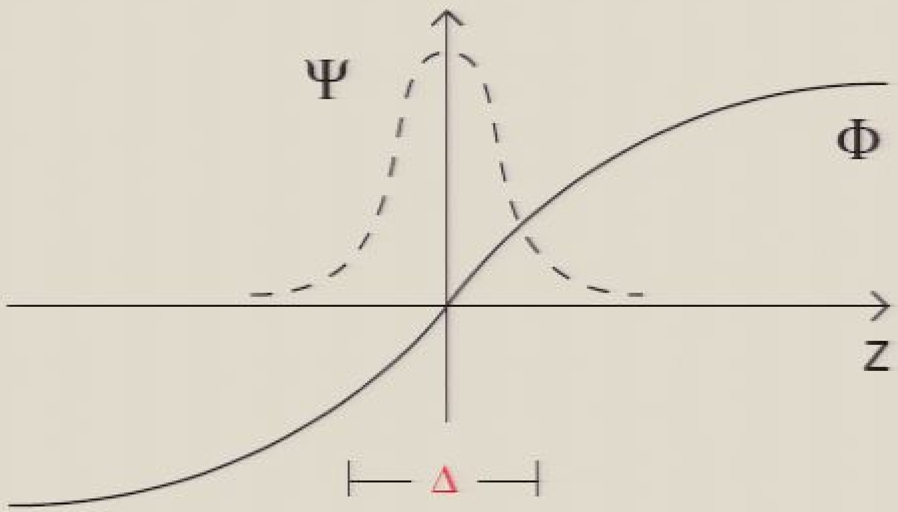
where A is such that:

$$\int dz\, |\Psi|^2 = |\psi(x)|^2.$$

If $y > 0$ then the left handed fermion is normalizable.

If $y < 0$ then the right handed fermion is normalizable.

For example, consider the case $y > 0$:



Δ depends on parameters of the theory (y, σ and v). We can compute the 4-D Lagrangian:

$$\mathcal{L}^{(4)} = -\bar{\psi}_L(\gamma^\mu \partial_\mu)\psi_L$$

In the limit $\Delta \to 0$, we obtain:

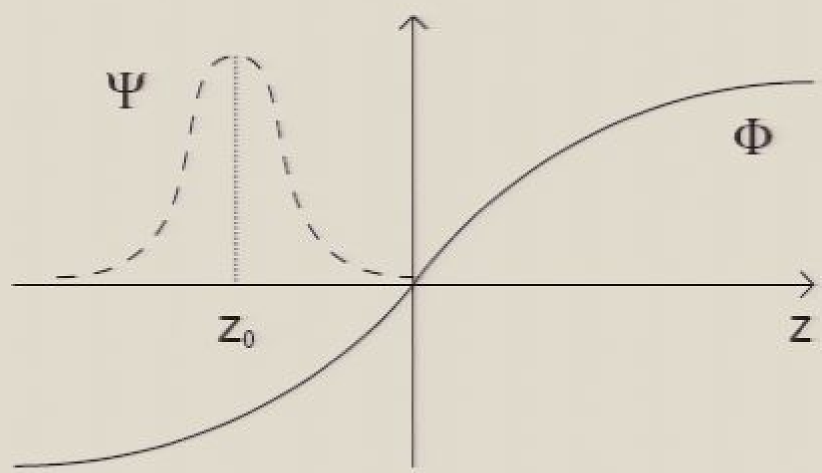
$$\mathcal{L}^{(5)} = \delta(z)\mathcal{L}^{(4)}$$

We could also add a mass m:

$$\left[\gamma^\mu \partial_\mu + \gamma^5 \partial_z + m + y\,\Phi(z)\right]\Psi = 0$$

The solution is:

$$\Psi_{L,R} = A \exp\left\{\mp \int_0^z [m + y\Phi(z)]\, dz\right\} \psi_{L,R}(x)$$



Where $z_0 = -m\Delta^2$. Again the theory is massless:

$$\mathcal{L}^{(4)} = -\bar{\psi}_L(\gamma^\mu \partial_\mu)\psi_L$$

A few questions:

1. What about gauge fields?
2. Can we reproduce the correct chiral structure for the standard model?
3. If so, any specific predictions?
4. Hierarchy problem?

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Rest of the talk:

1. More on domain wall fermions.
2. Quasi-localization of gauge fields.
3. "Electroweak Brane" construction.

Localizing $SU(3)$ fermions:

Consider a 5-D space: $M = \mathbb{R}^4 \times S^1$ with $z = x^5 \in [0, L]$ and the Lagrangian:

$$\mathcal{L}_\Psi^{(5)} = -\bar{\Psi}[\gamma^A D_A + Y(\Phi)]\Psi,$$
$$D_A \Psi = (\partial_A - i E_A^\alpha T_\alpha)\Psi.$$

E_A^α: $SU(3)$ bulk gauge field ($\alpha = 1, \ldots, 8$)
T_α: $SU(3)$ generators
$\Phi = \Phi^\alpha T_\alpha$: Adjoint repr. scalar field

$SU(3)$ charges are: $Q = (T_3, T_8)$.

Localizing $SU(3)$ fermions:

The gauge part of the theory has the action:

$$\mathcal{L}_{\mathrm{G}}^{(5)} = -\frac{1}{4g^2} F^{\alpha}_{AB} F_{\alpha}^{AB} + iE^{\alpha}_{A} \bar{\Psi} T_{\alpha} \Psi$$

$$F^{\alpha}_{AB} = \partial_A E^{\alpha}_B - \partial_B E^{\alpha}_A + C^{\alpha}_{\beta\gamma} E^{\beta}_A E^{\gamma}_B$$

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$$\begin{array}{rcllr} W^a_\mu & = & E^a_\mu & \text{with} & a = 1,2,3, \\ V^i_\mu & = & E^i_\mu & \text{with} & i = 4,5,6,7, \\ \phi^i & = & E^i_5 & \text{with} & i = 4,5,6,7, \\ B_\mu & = & E^8_\mu . & & \end{array}$$

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The potential for Φ is:

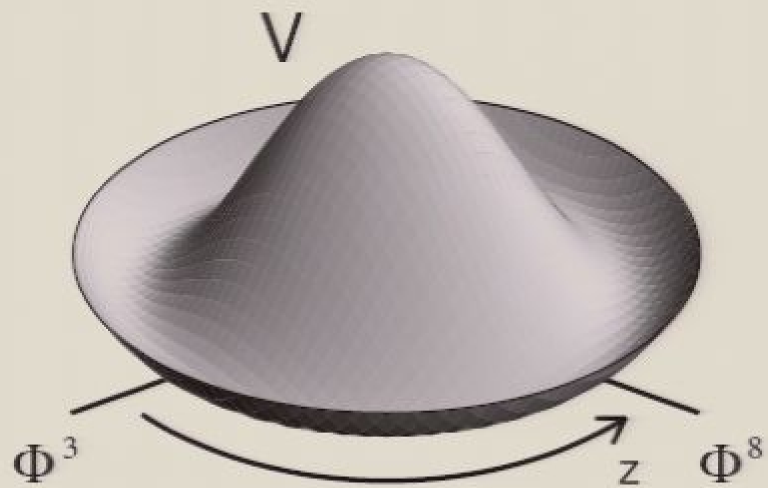
$$V(\Phi) = \frac{\sigma}{4}\left[\Phi^\alpha \Phi_\alpha - v^2\right]^2.$$

Only Φ^3 and Φ^8 can acquire non-zero v.e.v's

There is a single-winding solution:

$$\langle \Phi(z) \rangle = \Phi_0 \left[\cos(kz) T_3 + \sin(kz) T_8 \right]$$

where $k = 2\pi/L$ and $\Phi_0^2 = v^2 + k^2/\sigma$.



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$$\Psi_{L,R} = A \exp\left\{ \mp \int_0^z Y(\Phi)\, dz \right\} \psi_{L,R}(x),$$

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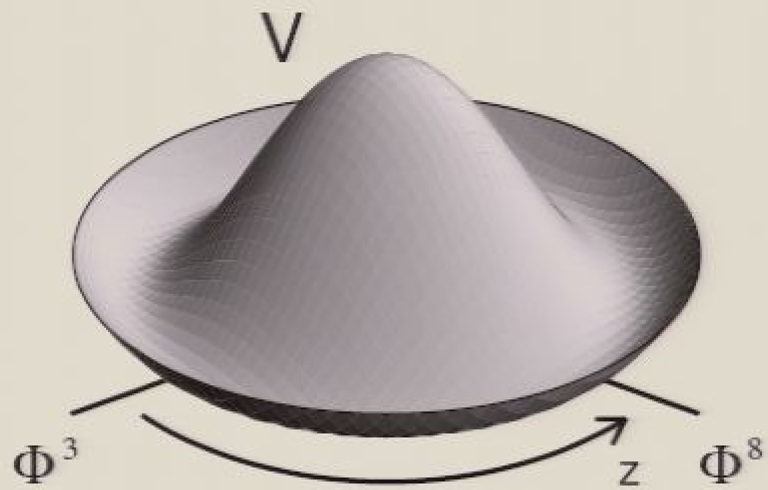

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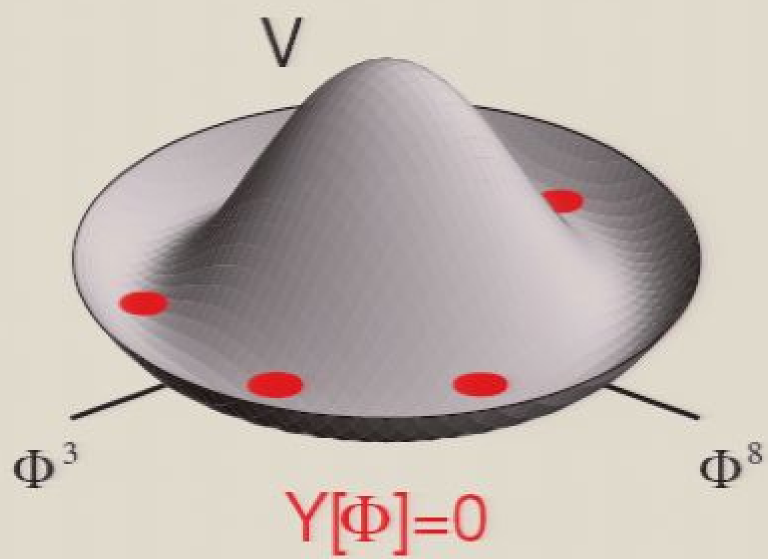

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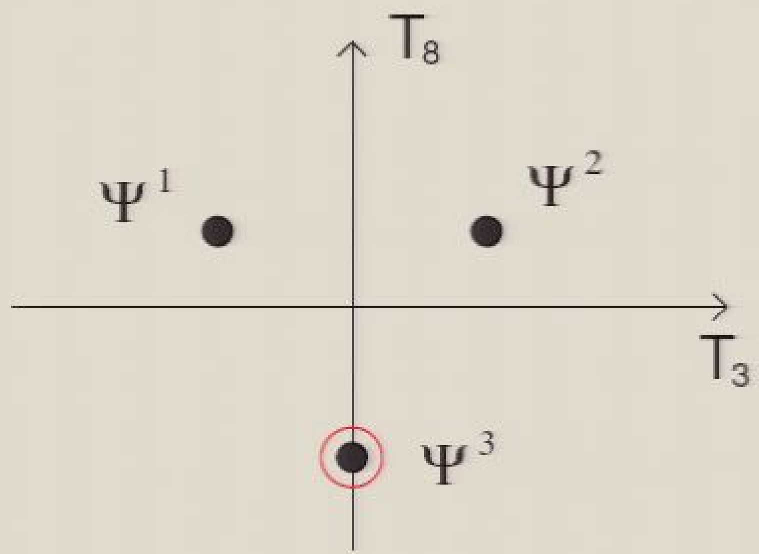
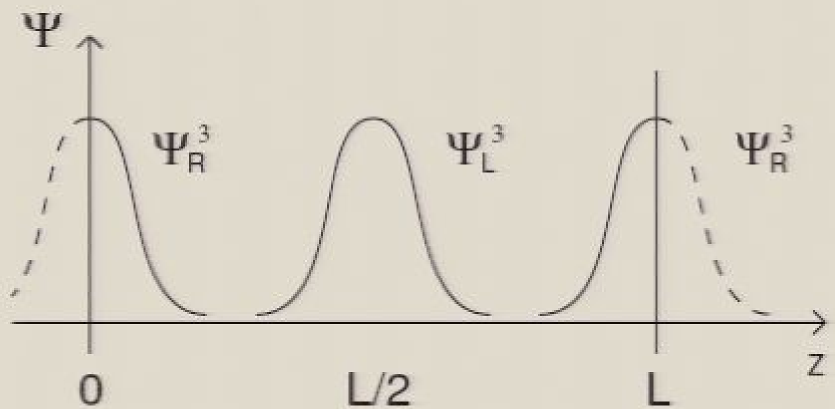
Example: Consider the fundamental representation of $SU(3)$, the 3, and assume the simple case: $Y(\Phi) = y\Phi = y\Phi^{\alpha} T_{\alpha}$.



$$Y(z)\Psi^3 = -y\frac{\sqrt{3}}{3}\Phi_0 \sin(kz)\Psi^3$$

The confining scale is $\Delta = 1/\sqrt{|y\Phi_0 k|}$.

Then if $y > 0$:



The representation is decomposed along the extra dimension!

The theory at $z = 0$ when $\Delta \to 0$ is:

$$\mathcal{L}_{\text{eff}} = -\delta(z)\bar{\psi}_R^3\gamma^\mu\left[\partial_\mu + i\frac{\sqrt{3}}{3}E_\mu^8\right]\psi_R^3.$$

$$J_8^\mu = -i\frac{\sqrt{3}}{3}\bar{\psi}_R^3\gamma^\mu\psi_R^3,$$

Constructing the Electroweak Brane:

Consider $\Phi = \Phi^\alpha T_\alpha$ and $\Theta = \Theta^\alpha T_\alpha$ with the following potentials:

$$V \propto \left[\Phi^\alpha \Phi_\alpha - v^2\right]^2, \quad U \propto \left[\Theta^\alpha \Theta_\alpha - u^2\right]^2.$$

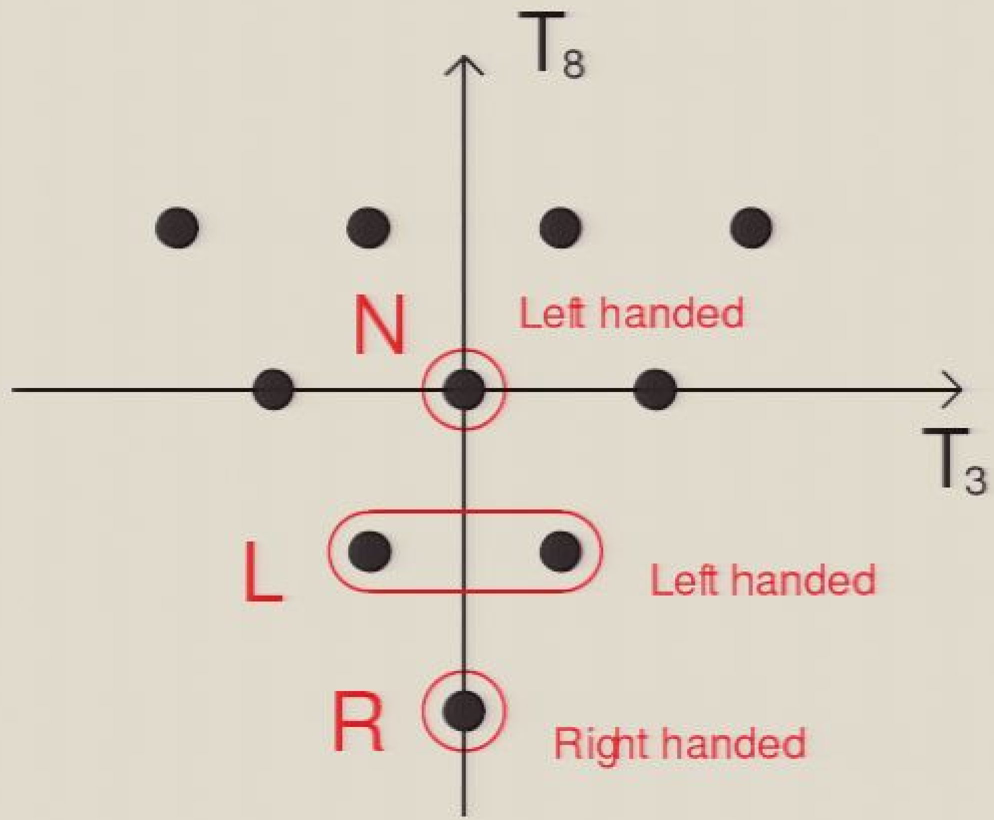
And consider a Y coupling having the form:

$$Y = -y \left(\frac{1}{2}\{\Phi, \Theta\} - \frac{1}{4}\Theta^\alpha \Phi_\alpha + p\frac{\sqrt{3}}{2}|\Theta|\, \Phi \right)$$

$p = 1$ if Y couples to the $\mathbf{10}$
$p = -1/3$ if Y couples to the $\bar{\mathbf{6}}$

Now assume that $\langle \Theta \rangle = u T_8$, and that Φ has the same solution as before

Example: Consider the **10** (p=1) and the case $y\Phi_0 u > 0$. The following states confine to $z = 0$.



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This is just the electroweak model for leptons with gauge couplings $g_1 = \lambda_H$ and $g_2 = \sqrt{3}\lambda_G$, with Higgs field ϕ^i, and new fields V^i_μ and N.

Localization of gauge fields:

We have a domain-wall at $z = 0$ s.t:

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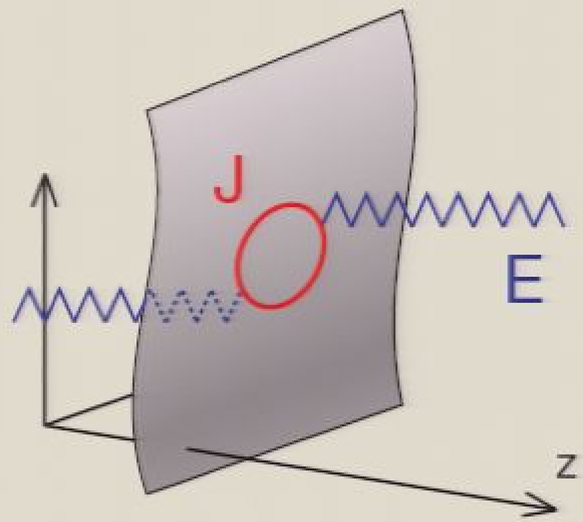

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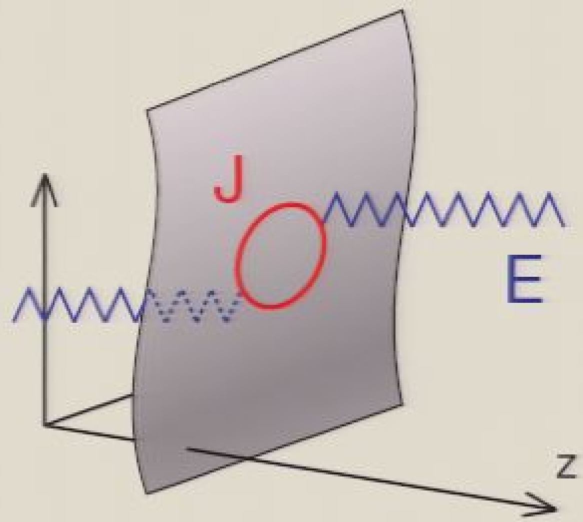

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Then the current term is:

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Where $\mathcal{L}_{\mathsf{G}}^{(4)}$ is the induced Lagrangian:

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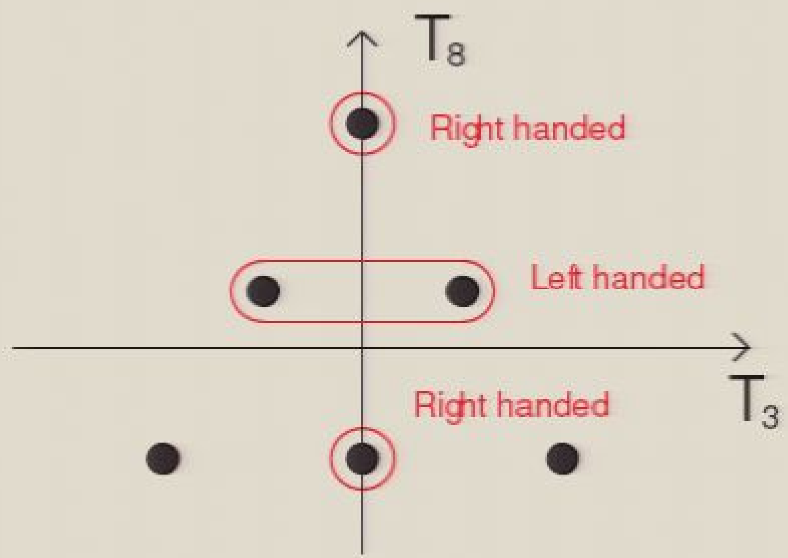

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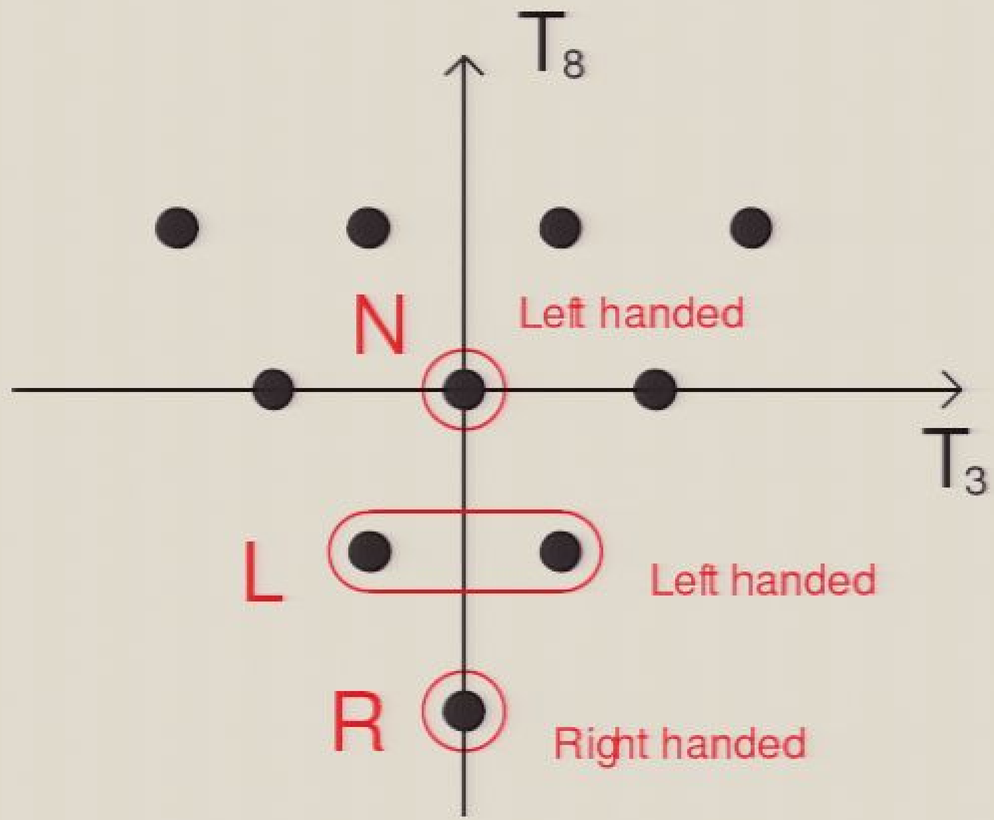

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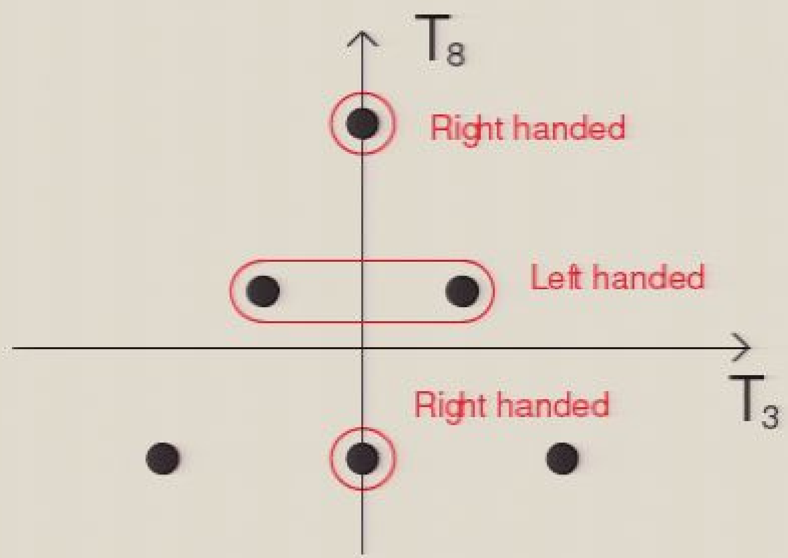
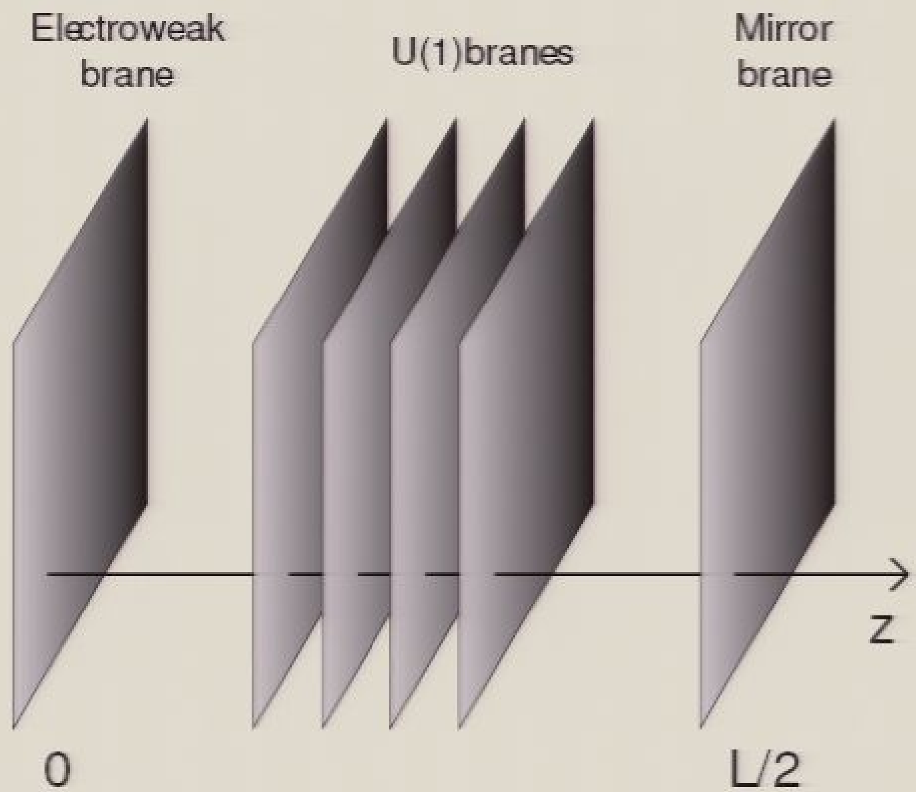


Other branes:



The Hierarchy problem: Observe that if ϕ^i is the Higgs, then:

$$M_{\text{electron}} \sim M_{\text{quarks}} \sim M_{\text{W}} \sim M_{\text{V}}$$

This is just the hierarchy problem for this model! To solve it, we modify Y:

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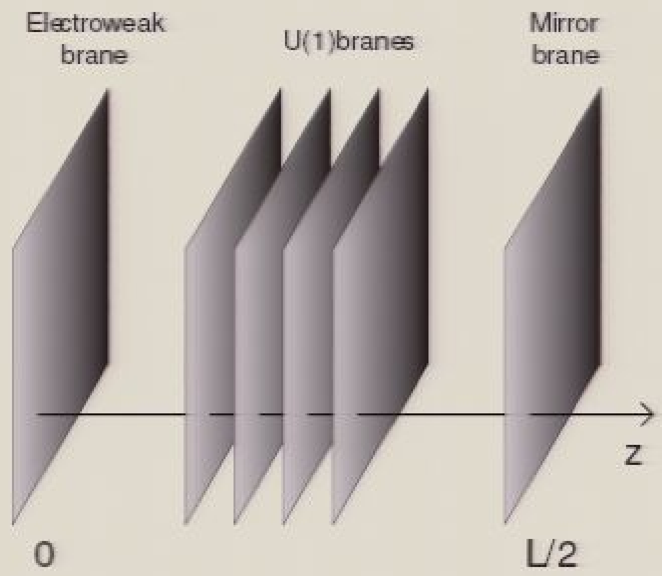

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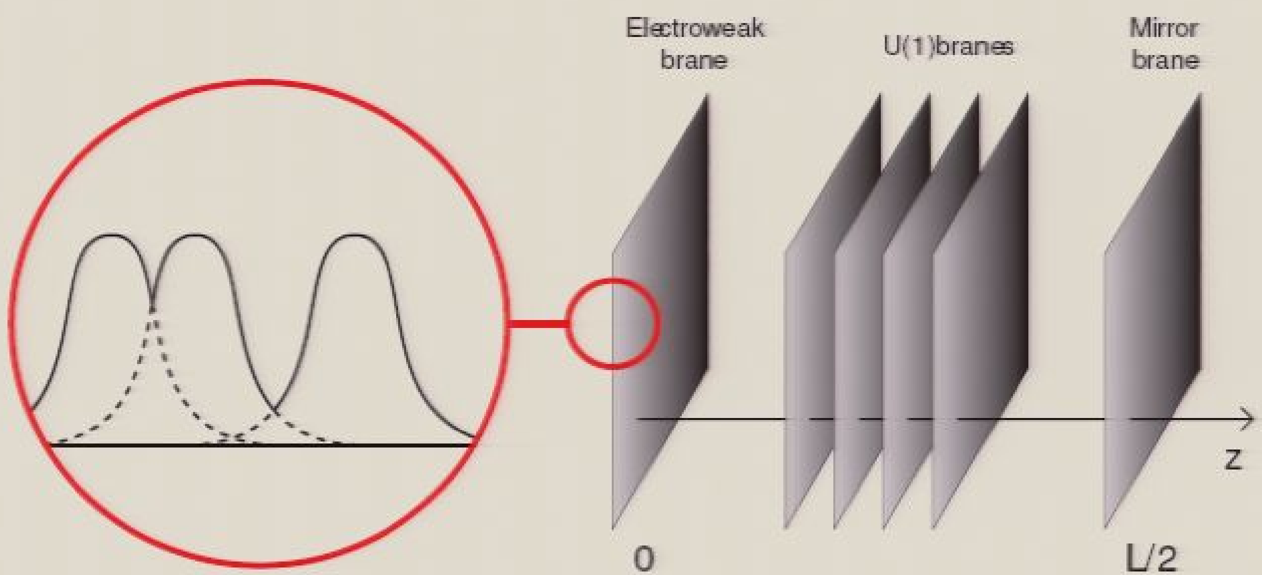

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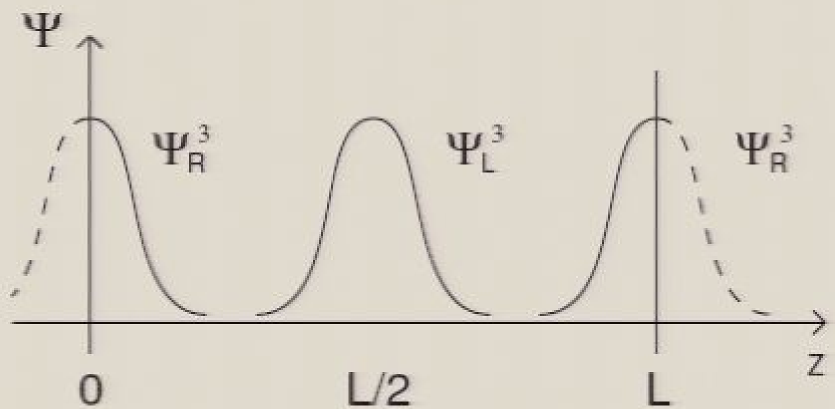
Looking forward: Compute the Higgs v.e.v. ϕ_0. Include mixing of generations. Phenomenology of N and V fields.

Then if $y > 0$:



The representation is decomposed along the extra dimension!

The theory at $z = 0$ when $\Delta \to 0$ is:

$$\mathcal{L}_{\text{eff}} = -\delta(z)\bar{\psi}_R^3\gamma^\mu\left[\partial_\mu + i\frac{\sqrt{3}}{3}E_\mu^8\right]\psi_R^3.$$

$$J_8^\mu = -i\frac{\sqrt{3}}{3}\bar{\psi}_R^3\gamma^\mu\psi_R^3,$$

Constructing the Electroweak Brane:

Consider $\Phi = \Phi^\alpha T_\alpha$ and $\Theta = \Theta^\alpha T_\alpha$ with the following potentials:

$$V \propto \left[\Phi^\alpha \Phi_\alpha - v^2\right]^2, \quad U \propto \left[\Theta^\alpha \Theta_\alpha - u^2\right]^2.$$

And consider a Y coupling having the form:

$$Y = -y\left(\frac{1}{2}\{\Phi, \Theta\} - \frac{1}{4}\Theta^\alpha \Phi_\alpha + p\frac{\sqrt{3}}{2}|\Theta|\,\Phi\right)$$

$p = 1$ if Y couples to the $\mathbf{10}$
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Now assume that $\langle\Theta\rangle = uT_8$, and that Φ has the same solution as before

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