

Title: Introduction to quantum gravity - Part 4

Date: Jan 26, 2006 08:10 PM

URL: <http://pirsa.org/06010018>

Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

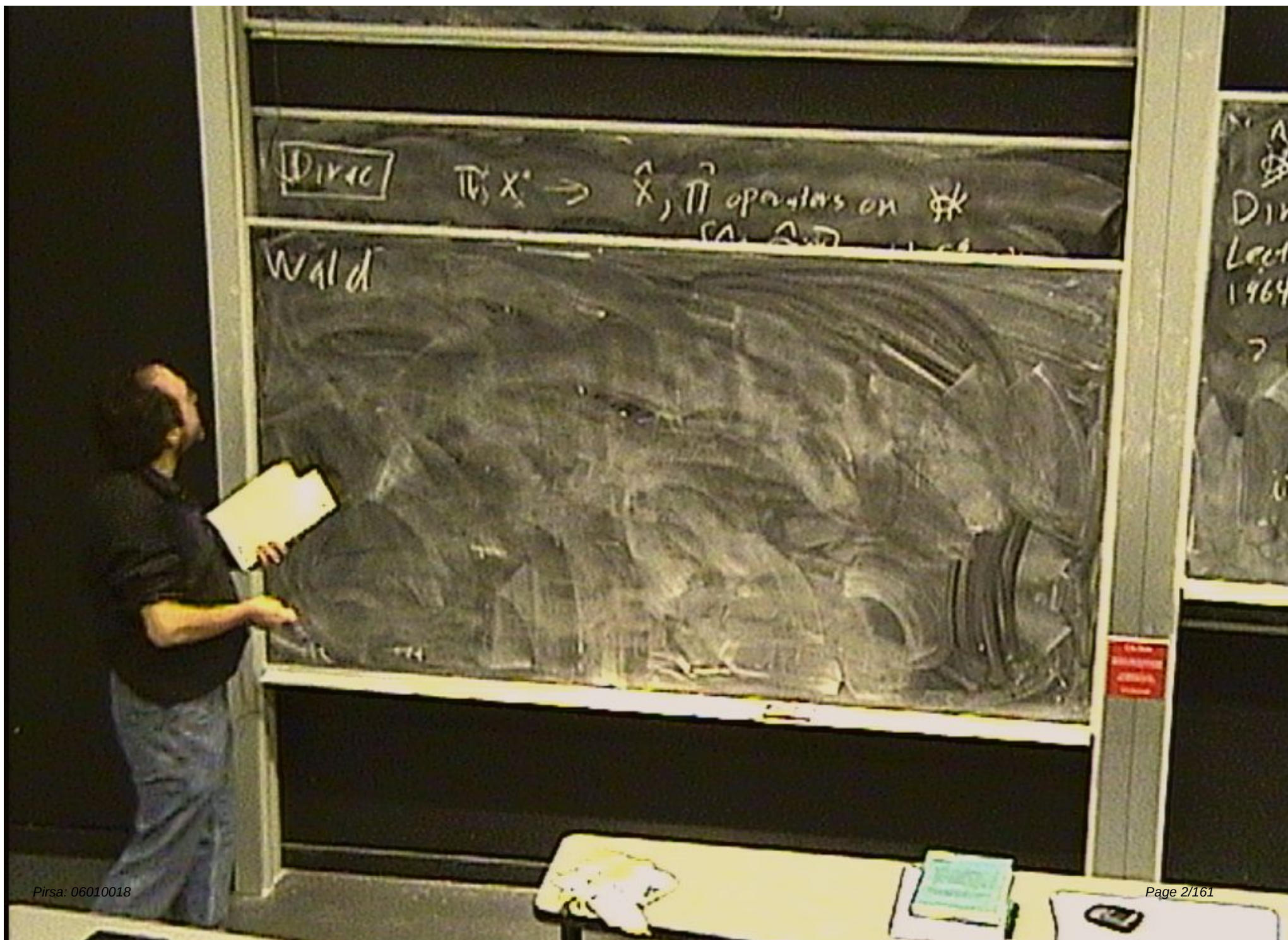
Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras



Dirac

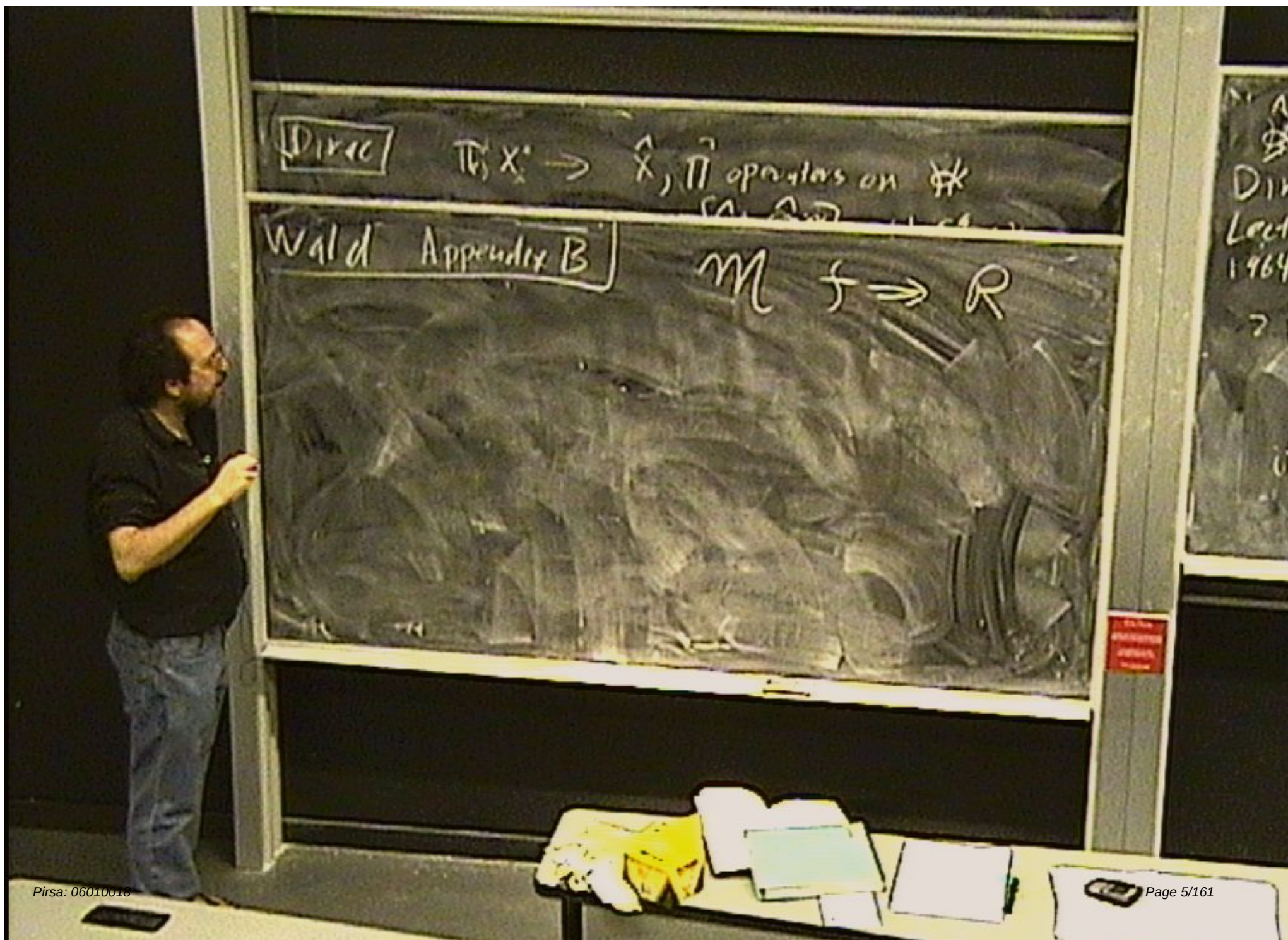
$\pi: X \rightarrow \hat{X}$, π operates on $\mathfrak{H}$

Wald Appendix B

Dirac
Lect
1964

Dirac $\Pi, X \rightarrow \hat{X}, \hat{\Pi}$ operators on $\mathcal{H}$
SA, SW, ...
Wald Appendix B





Dirac

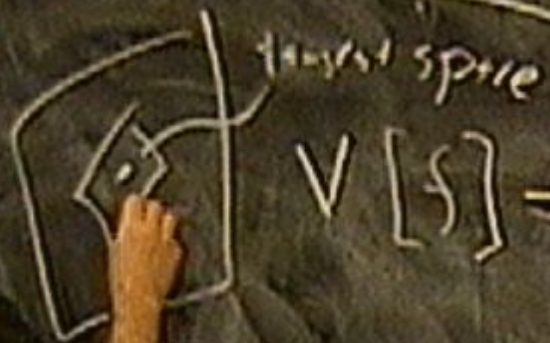
$\Pi, X \rightarrow \hat{X}, \hat{\Pi}$ operators on $\mathcal{H}$

Wald Appendix B

$M \rightarrow R$

$\Pi, X \rightarrow \hat{X}, \Pi$ operators on $\mathcal{H}$

Wald Appendix B



$$V[f] = V^a \partial_a f$$

$$\mathcal{M} \xrightarrow{f} \mathcal{R}$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$\Pi, X \rightarrow \hat{X}, \Pi$ operators on $\mathcal{H}$
 Wald Appendix B



$$V[f] = V^a \partial_a f$$

$$M \rightarrow R$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$\Pi, X \rightarrow \hat{X}, \Pi$ operators on Φ

Wald Appendix B



tangent space

$$V[f] = V^a \partial_a f$$

vector fields

$$M \rightarrow R$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

Wald Appendix B



tangent space

$$V[f] = V^a \partial_a f$$

vector fields

$$\omega[V] = f = \omega_a V^a$$

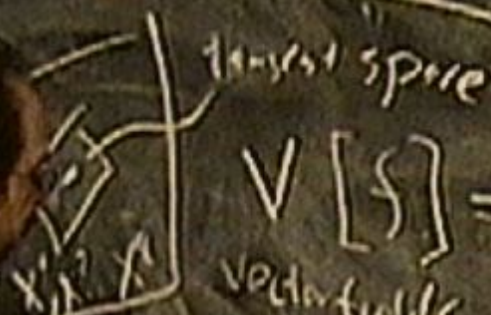
$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$$M \rightarrow R$$

$\mathbb{R}, \Pi$ operators on $\mathbb{R}$

$\Pi, X \rightarrow \hat{X}, \Pi$ operators on Φ
 Wald Appendix B

$$\mathcal{M} \xrightarrow{f} \mathcal{R}$$



$$V[f] = V^a \partial_a f$$

vector fields

$$\omega[V] = f = \omega_a V^a$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$\Pi, X_i \rightarrow \hat{X}, \Pi$ operators on $\mathcal{H}$

Wald Appendix B

$$\mathcal{M} \xrightarrow{f} \mathbb{R}$$



tangent space

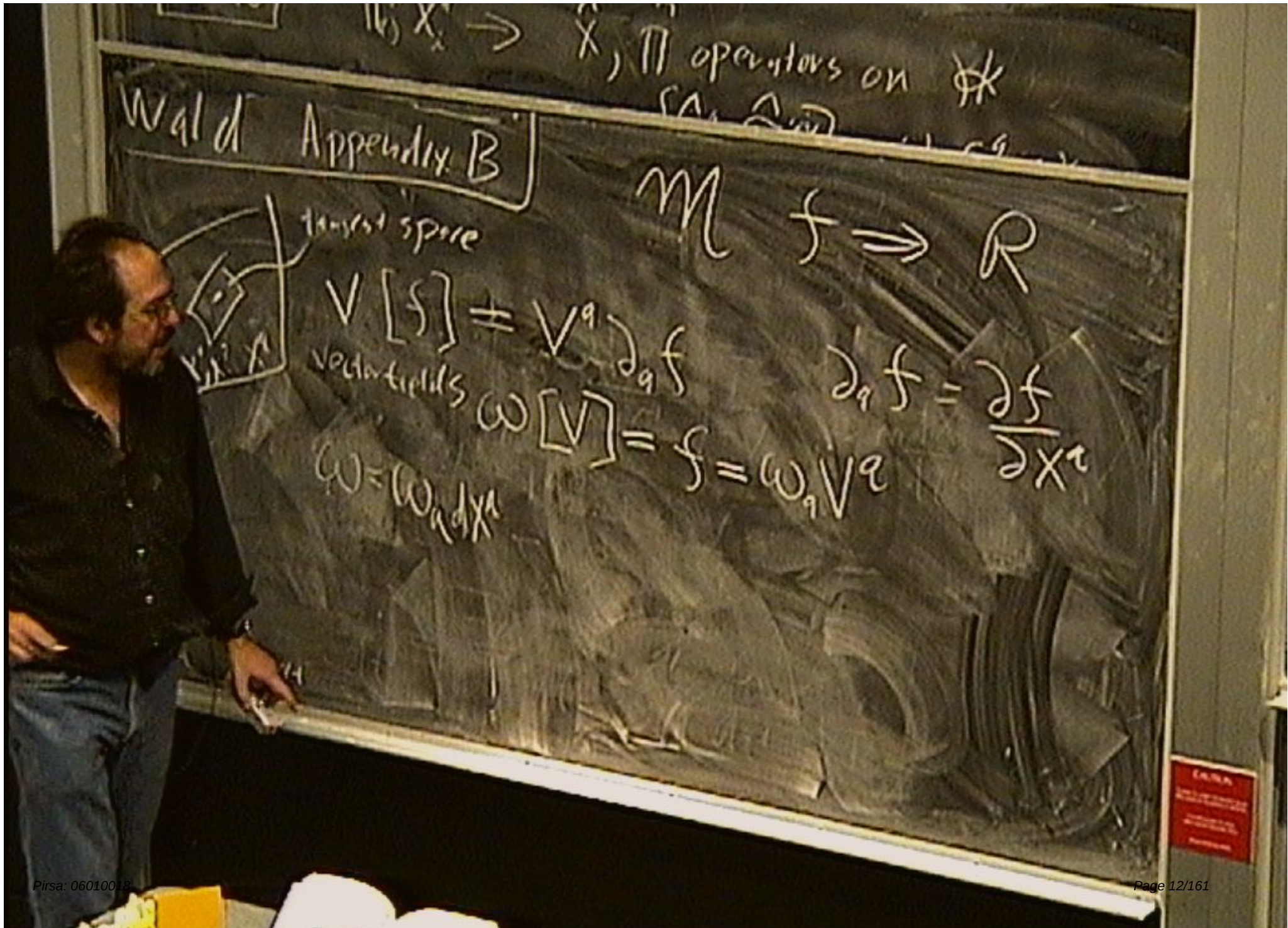
$$V[f] = V^a \partial_a f$$

vector fields

$$\omega[V] = f = \omega_a V^a$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$$\omega = \omega_a dx^a$$



$\mathbb{R}^n \rightarrow \hat{X}, \Pi$ operators on Φ

Wald Appendix B

$$\mathcal{M} \rightarrow \mathbb{R}$$



tangent space

$$V[f] = V^a \partial_a f$$

vector fields

$$\omega[V] = f = \omega_a V^a$$

$$\omega = \omega_a dx^a$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$X_i \rightarrow \hat{X}, \Pi$ operators on Φ

Wald Appendix B

$$\mathcal{M} \xrightarrow{f} \mathbb{R}$$



$$V[f] = V^a \partial_a f$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$$\omega[V] = f = \omega_a V^a$$

$$\omega = \omega_a dx^a \quad \langle dx^a, dx^b \rangle = \delta^{ab}$$

16) $X_i \rightarrow \hat{X}, \Pi$ operators on $\mathcal{H}$

Wald Appendix B

$$\mathcal{M} \xrightarrow{f} \mathcal{R}$$



tangent space

$$V[f] = V^a \partial_a f$$

vector fields

$$\omega[V] = f = \omega_a V^a$$

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

dx^a basis of T_p

$$\omega = \omega_a dx^a$$

$$\langle dx^a, dx^b \rangle = \delta^a_b$$

1-forms

Wald Appendix B

$$M \xrightarrow{f} R$$



tangent space

$$V[f] = V^a \partial_a f$$

vector fields

$$\partial_a f = \frac{\partial f}{\partial x^a}$$

$$\omega[V] = f = \omega_a V^a$$

dx^a basis of T_x

$$\omega = \omega_a dx^a$$

$$\langle dx^a, \partial_b \rangle = \delta^a_b$$

1-forms

$$f[V, W, X] \rightarrow R = \beta_{abc} V^a W^b X^c$$

f d a S

$$\dot{\gamma} \Rightarrow \underline{\partial_a \mathcal{L}} dx^a = d\mathcal{L}$$

$$\dot{f} \Rightarrow \underline{\partial_a S} dx^a = d\dot{f}$$

derivatives of function
are not defined

$$\dot{f} \Rightarrow \underline{\partial_a S} dx^a = d\dot{f}$$

derivatives of function
are not defined

$$v^i v^a \partial_a v^i$$

$$\dot{\gamma} \Rightarrow \underline{\partial_a S} dx^a = d\hat{S}$$

derivatives of function
are not defined

$$v \cdot v^a \partial_a \gamma^b$$

$$\dot{f} \Rightarrow \underline{\partial_a S} dx^a = d\dot{f}$$

derivatives of function
are not defined

$$V \quad V^a \quad \partial_b V^a$$

$$\dot{f} \Rightarrow \underline{\partial_a S} dx^a = d\dot{f}$$

derivatives of function
are well defined

$V^f \quad V^g$

$\partial_b V^H$

Not well defined

$$\dot{f} \Rightarrow \underline{\partial_a S} dx^a = d\dot{f}$$

derivatives of function
are well defined

$$V^i, V^a$$

$$\partial_b V^H$$

Not well defined

$$\frac{\partial \dot{f}}{\partial X^H} = \frac{\partial X^H}{\partial X^H} = 1$$

$$f \Rightarrow \underline{\partial_a f} dx^a = df$$

derivatives of function
are not defined

$$V \quad V^a \quad \partial_b V^a$$

Not well defined

$$x^a \rightarrow x'^a$$

$$\frac{\partial}{\partial x^a} = \frac{\partial x'^b}{\partial x^a} \frac{\partial}{\partial x'^b}$$

$$dx^a \rightarrow dx'^a \frac{\partial x'^a}{\partial x^a}$$

$$f \Rightarrow \underline{\partial_a S} dx^a = d\tilde{f}$$

derivatives of function
are well defined

$$V \neq V^a$$

$$\partial_b V^a$$

Not well defined

$$x^a \rightarrow x'^a$$

$$\frac{\partial}{\partial x^a} = \frac{\partial x'^b}{\partial x^a} \frac{\partial}{\partial x'^b}$$

$$dx^a \rightarrow dx'^a \frac{\partial x^a}{\partial x'^a}$$

$$V^a \rightarrow V'^a = \frac{\partial x^a}{\partial x'^b} V^b$$

$$\vec{f} \Rightarrow \underline{\partial_a S} dx^a = d\vec{f}$$

derivatives of function
are not defined

$$V \quad V^a$$

$$\partial_b V^a$$

Not well defined

y

$$\frac{\partial}{\partial x^a} = \frac{\partial x^b}{\partial x^a} \frac{\partial}{\partial x^b}$$

$$dx^a \rightarrow dx^{a'} \frac{\partial x^a}{\partial x^{a'}}$$

$$\frac{\partial x^a}{\partial x^{a'}} V^a$$

$$\frac{\partial x^a}{\partial x^{a'}} \frac{\partial}{\partial x^{a'}} \left(\frac{\partial x^a}{\partial x^{a'}} V^a \right)$$

$$\int \Rightarrow \underline{\partial_a S} dx^a = dS$$

derivatives of function
are not defined

$$V \text{ f. } V^a$$

$$\partial_b V^a$$

Not well defined

$$x^a \rightarrow x'^a$$

$$\frac{\partial}{\partial x^a} = \frac{\partial x'^b}{\partial x^a} \frac{\partial}{\partial x'^b}$$

$$dx^a \rightarrow dx'^a \frac{\partial x^a}{\partial x'^b}$$

$$V^a \rightarrow V'^a = \frac{\partial x'^a}{\partial x^b} V^b$$

$$\partial_b V^a \Rightarrow \frac{\partial}{\partial x^b} \left(\frac{\partial x'^a}{\partial x^c} V^c \right) = \frac{\partial x'^a}{\partial x^b} \frac{\partial}{\partial x^c} \left(\frac{\partial x^c}{\partial x'^d} V^d \right)$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A}$$

$E=0$ is the

Consider $\nabla_a \omega_b = \partial_a \omega_b - F_{ab}^c \omega_c$

Gauge fixed:

$$\boxed{F = \nabla \wedge A} \quad E=0 \text{ is the}$$

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - F_{ab}^c \omega_c$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative
depends on Γ_{ab}^c

$$\nabla_a g_{bc} = 0$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A = E}$$

$E=0$ is the

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative
depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_{a'} \omega_{b'} = \partial_{a'} \left(\frac{\partial x^b}{\partial x^{b'}} \right) \omega_b$$

$$\nabla_a g_{bc} = 0$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Covariant derivative $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative
depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_{a'} \omega_{b'} = \left(\partial_{a'} \frac{\partial x^b}{\partial x^{b'}} \right) \omega_b$$

$$\nabla_a g_{bc} = 0$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on Γ_{ab}^c

$$\partial_a \omega_b \rightarrow \partial_{[a} \omega_{b]} = \left(\delta_{a1} \frac{\partial x^b}{\partial x^{b1}} \right) \omega_b$$

$$\nabla_a g_{bc} = 0$$

$$\frac{\partial^2 x^b}{\partial x^{a1} \partial x^{b1}} \omega_b$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Consider $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_{[a} \omega_{b]} = \left(\delta_{a'}^a \frac{\partial x^{b'}}{\partial x^b} \right) \omega_{b'}$$

$$\nabla_a g_{bc} = 0$$

$$\frac{\partial^2 x^b}{\partial x^{a'} \partial x^{b'}} \omega_b + \dots$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A} \quad E=0 \text{ is the}$$

is the

Covariant derivative $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_{a'} \omega_{b'} = \left(\delta_{a'}^a \frac{\partial x^b}{\partial x^{b'}} \right) \omega_b$$

$$\nabla_a g_{bc} = 0$$

$$\frac{\partial^2 x^b}{\partial x^{a'} \partial x^{b'}} \omega_b + \dots$$

2 forms

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Consider $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on g_{ab}

$$\nabla_a g_{bc} = 0$$

$$\partial_a \omega_b \rightarrow \partial_{a'} \omega_b = \left(\delta_{a'}^a + \frac{\partial x^b}{\partial x^{a'}} \right) \omega_b$$

$$= \frac{\partial^2 x^b}{\partial x^{a'} \partial x^a} \omega_b + \dots$$

2 forms

$$\oint_{\gamma} dx^a \omega_a dx^b$$

$$A_{[ab]} = \frac{1}{2} (A_{ab} - A_{ba})$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E = 0} \quad E=0 \text{ is the}$$

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_a \omega_b = \left(\delta_a^c \frac{\partial x^b}{\partial x^c} \right) \omega_b$$

$$\nabla_a g_{bc} = 0$$

2 forms $\oint_{[a,b]} dx^a \omega_b dx^b$

$$= \frac{\partial^2 x^b}{\partial x^a \partial x^a} \omega_b + \dots$$

$$A_{[a,b]} = \frac{1}{2} (A_{ab} - A_{ba})$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Covariant der $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on Γ_{ab}^c

$$\nabla_a g_{bc} = 0$$

$$\partial_a \omega_b \rightarrow \partial_{[a} \omega_{b]} = \left(\delta_a^c \frac{\partial x^b}{\partial x^c} \right) \omega_b = \frac{\partial^2 x^b}{\partial x^a \partial x^c} \omega_b + \dots$$

2 forms $\oint_{[a,b]} dx^a \omega_b dx^b$

ω, ϑ 1 forms $\frac{1}{2} [\omega \otimes \vartheta - \vartheta \otimes \omega] = \omega \wedge \vartheta$

$$A_{[a,b]} = \frac{1}{2} (A_a b_b - A_b a_a)$$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Consider $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on Γ

$$\nabla_a g_{bc} = 0$$

$$\partial_a \omega_b \rightarrow \partial_{[a} \omega_{b]} = \left(\delta_{ab} \frac{\partial x^b}{\partial x^a} \right) \omega_b = \frac{\partial^2 x^b}{\partial x^a \partial x^a} \omega_b + \dots$$

2 forms $\oint \omega = \int dx^a \dots dx^b$

$$\omega, \text{ 1-forms } \oint \omega = \int \partial \omega = \omega \wedge \partial$$

$$A_{[5]} = \frac{1}{2} (A_{55} - A_{44})$$

$\omega_a dx^a$ $g_a dx^a$

Gauge fixed:

$$\boxed{F = \nabla \wedge A - E}$$

$E=0$ is the

Consider $\nabla_a \omega_b = \partial_a \omega_b - \Gamma_{ab}^c \omega_c$

Cov derivative depends on g_{ab}

$$\partial_a \omega_b \rightarrow \partial_{[a} \omega_{b]} = \left(\partial_a \frac{\partial x^b}{\partial x^a} \right) \omega_b =$$

$$\nabla_a g_{bc} = 0$$

$$\frac{\partial^2 x^b}{\partial x^a \partial x^a} \omega_b + \dots$$

2 forms $\oint_{[a,b]} dx^a \wedge dx^b$

$\omega, \varnothing$ 1 forms $\frac{1}{2} [\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing$

$$A_{[ab]} = \frac{1}{2} (A_{ab} - A_{ba})$$

" " $\omega_a dx^a \wedge \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$

$$\partial_a \omega_b \rightarrow \partial_a \omega_b = \left(\partial_a, \frac{\partial x^b}{\partial x^a} \right) \omega_b =$$

depend on ∂_a

$$\nabla_a g_{bc} = 0$$

$$\frac{\partial^2 x^b}{\partial x^a \partial x^c} \omega_b + \dots$$

2 forms $\phi_{[ab]} dx^a \wedge dx^b$

$\omega, \varnothing$ 1 forms $\frac{1}{2} [\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing$

$$A_{[ab]} = \frac{1}{2} (A_{ab} - A_{ba})$$

$\omega_a dx^a \quad \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$

$$d\omega =$$

$$\sum \pi^2 + V = 0$$

depend on ∂g

$$\nabla_a g_{bc} = 0$$

$$= \frac{\partial^2 x^b}{\partial x^a \partial x^c} \omega_b + \dots$$

$$A_{[ab]} = \frac{1}{2}(A_{ab} - A_{ba})$$

$$\partial_a \omega_b \rightarrow \partial_a \omega_b = \left(\partial_a, \frac{\partial x^b}{\partial x^a} \right) \omega_b$$

2 terms $\oint_{[ab]} dx^a \omega_b$

$\omega, \varnothing$ 1 forms $\frac{1}{2}[\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing$

" " $\omega_a dx^a \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$

$$d\omega = \partial_a \omega_b dx^a \wedge dx^b$$

$$H = \frac{1}{2m} \sum \pi^2 + V = 0$$

$$\omega, \varphi \text{ forms } \frac{1}{2} [\omega \otimes \varphi - \varphi \otimes \omega] = \omega \wedge \varphi \quad \text{Axiom: } \frac{1}{2} (A_{43} - A_{34})$$

$$\omega_a dx^a \quad \varphi_b dx^b \quad \omega \wedge \varphi = \frac{1}{2} \omega_a \varphi_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega = \frac{1}{2} \omega_a \omega_b dx^a \wedge dx^b}$$

P-form

$$\omega = \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

Construct:

$$\frac{1}{2m} \sum \pi_a^2 = \frac{m \dot{x}_a^2 \left(\frac{-V}{T} \right)}{2m} = -V$$

$$\boxed{\mathcal{H} = \frac{1}{2m} \sum \pi^2 + V = 0}$$

$$\omega, \vartheta \text{ 1-forms } \frac{1}{2} [\omega \otimes \vartheta - \vartheta \otimes \omega] = \omega \wedge \vartheta \quad A_{\mu\nu} = \frac{1}{2} (A_{\mu\nu} - A_{\nu\mu})$$

$$\omega_a dx^a \quad \vartheta_b dx^b \quad \omega \wedge \vartheta = \frac{1}{2} \omega_a \vartheta_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

P form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$\omega, \varphi \text{ 1 forms } \{ \omega \otimes \varphi - \varphi \otimes \omega \} = \omega \wedge \varphi \quad \Lambda^2 \mathbb{R}^2 = \frac{1}{2} (\Lambda_{12} - \Lambda_{21})$$

$$\omega_a dx^a \quad \varphi_b dx^b \quad \omega \wedge \varphi = \frac{1}{2} \omega_a \varphi_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

P-form $\omega = \frac{1}{p} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$

p+1 form $d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$

$$\omega, \varnothing \text{ 1 times } \frac{1}{2} [\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing$$

$$\omega_a dx^a \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

P form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

p+1 form

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$\omega, \varnothing \text{ 1 forms } \frac{1}{2} [\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing \quad A_{[1,2]} = \frac{1}{2} (A_{12} - A_{21})$$

$$\omega_a dx^a \quad \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

P form $\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$

p+1 form $d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$

$$d^2 = 0 \quad d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p} = 0$$

$$\omega, \varnothing \text{ 1-forms } \frac{1}{2} [\omega \otimes \varnothing - \varnothing \otimes \omega] = \omega \wedge \varnothing \quad A_{[5]} = \frac{1}{2} (A_{51} - A_{15})$$

$$\omega_a dx^a \quad \varnothing_b dx^b \quad \omega \wedge \varnothing = \frac{1}{2} \omega_a \varnothing_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

P-form $\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$

p+1 form $d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$

$$d^2 = 0 \quad d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p} = 0$$

vector potential in 4d spacetime 1-form $A_a dx^a$

$$\omega, \text{ 1-forms } \frac{1}{2} [\omega \otimes \varrho - \varrho \otimes \omega] = \omega \wedge \varrho \quad A_{[ab]} = \frac{1}{2} (A_{ab} - A_{ba})$$

$$\omega_a dx^a \quad \varrho_b dx^b \quad \omega \wedge \varrho = \frac{1}{2} \omega_a \varrho_b (dx^a \wedge dx^b)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

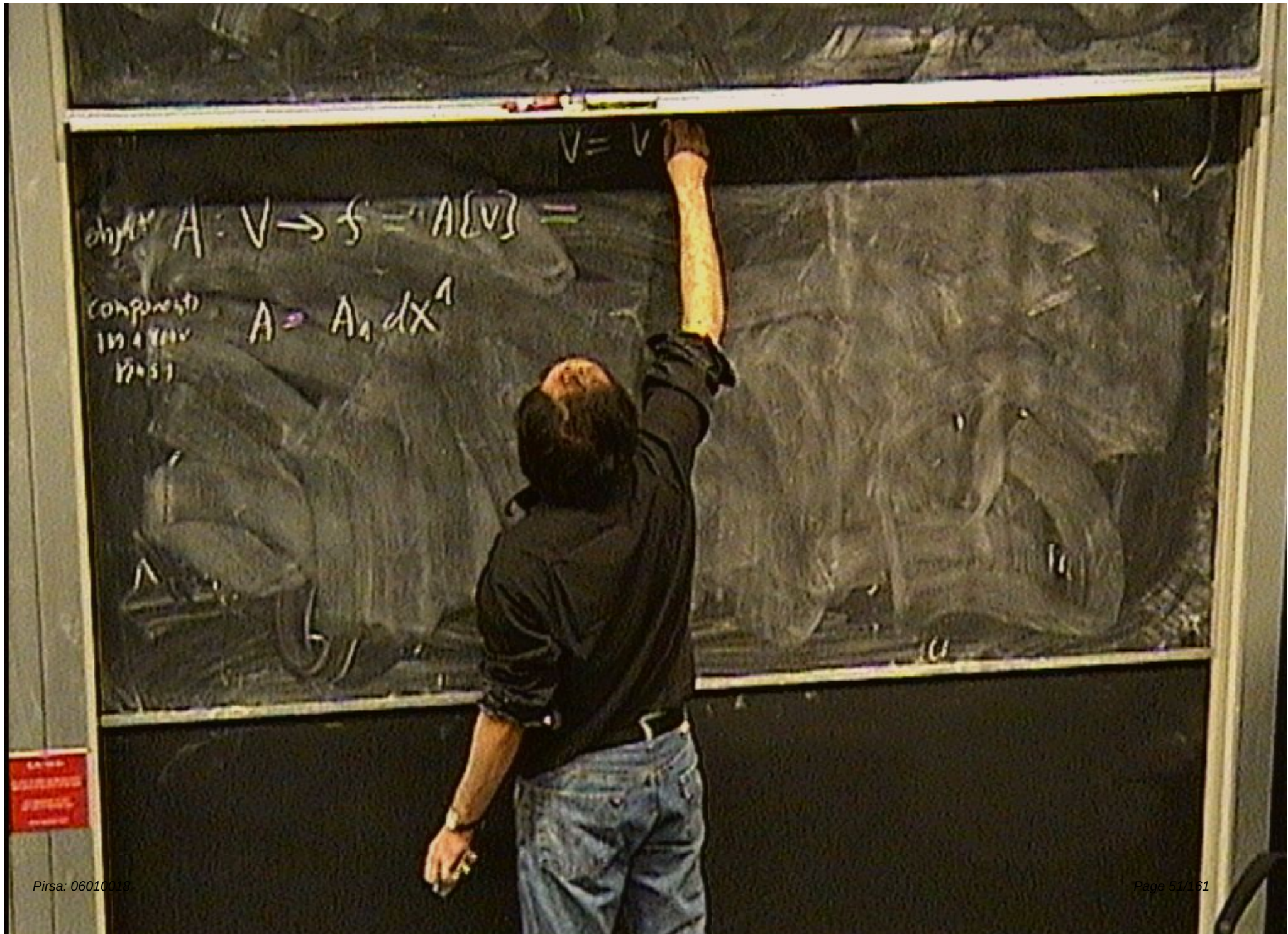
P-form $\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$

p+1 form $d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$

$$d^2 = 0 \quad d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p} = 0$$

Vector potential in 4-d spacetime 1-form $A_a dx^a$

$$\text{def } A: V \rightarrow \mathcal{F} = A[V]$$



$$V = V^{\mu} \frac{\partial}{\partial x^{\mu}}$$

defn $A: V \rightarrow \mathfrak{g} = A[V] = A_{\mu} V^{\mu}$

components
in a basis
basis $A = A_{\mu} dx^{\mu}$

$$V = V^i \frac{\partial}{\partial x^i}$$

defn $A: V \rightarrow \mathfrak{g} = A[V] = A_i V^i$

Component
in $\mathfrak{g}$
basis

$$A = A_i dx^i$$

$$V = V^i \frac{\partial}{\partial x^i}$$

chgt $A: V \rightarrow \mathfrak{g} = A[V] = A_i V^i$

components
in the
basis

$$A = A_i dx^i$$

(coordinate
notation)

$$f \Rightarrow \underline{\partial_a S} dx^a = dS$$

derivatives of function
are not defined

$$V f \quad V^a \quad \partial_b V^a$$

Not well defined

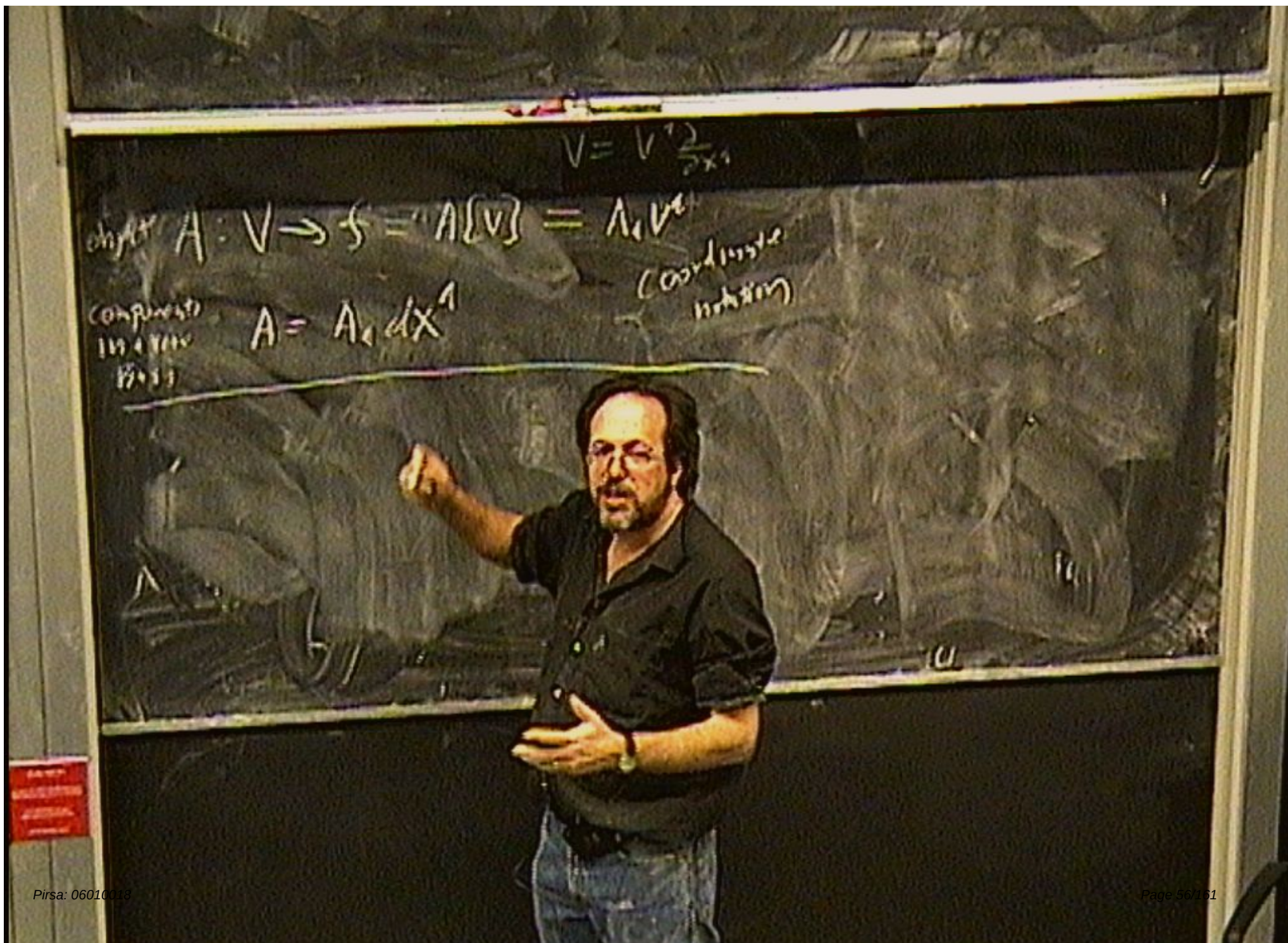
x^a

$$\frac{\partial}{\partial x^a} = \frac{\partial x^b}{\partial x^a} \frac{\partial}{\partial x^b}$$

$$dx^a \rightarrow dx^{a'} \frac{\partial x^a}{\partial x^{a'}}$$

$$= \frac{\partial x^{a'}}{\partial x^a} V^a$$

$$= \frac{\partial x^{a'}}{\partial x^b} \frac{\partial}{\partial x^{a'}} \left(\frac{\partial x^b}{\partial x^{a'}} V^a \right)$$



$$V = V^a \frac{\partial}{\partial x^a}$$

defn $A: V \rightarrow \mathfrak{g} = A[V] = A_a V^a$

components
in a basis
(1.1)

$$A = A_a dx^a$$

(coordinate
notation)

Wald $A_a V^a$

$$V = V^a \frac{\partial}{\partial x^a}$$

defn $A: V \rightarrow \mathfrak{g} = \Lambda[V] = A_a V^a$

components
in a given
basis

$$A = A_a dx^a$$

(coordinate
notation)

Wald $A_a \quad V^a \quad \Lambda(V) \quad A_a V^a$

$$V = V^i \frac{\partial}{\partial x^i}$$

dyad $A: V \rightarrow \mathfrak{f} = A[V] = A_i v^i$

Component
notation

$$A = A_i dx^i$$

(Coordinate
notation)

Wald

A_a

V^b

$A(v)$

$A_i v^i$

Abstract Index
notation

$$V = V^i \frac{\partial}{\partial x^i}$$

defn $A: V \rightarrow \mathfrak{g} = A[V] = A_i V^i$

components
in a given
basis

$$A = A_i dx^i$$

(coordinate
notation)

Wald

$$A_a V^b = V^a \left(\frac{\partial A_b}{\partial x^a} \right)$$

Abstract Index
form

$$V = V^a \frac{\partial}{\partial x^a}$$

define $A: V \rightarrow \mathfrak{g} = A[V] = A_a V^a$

components
in $\mathfrak{g}$
basis

$$A = A_a dx^a$$

(coordinate
notation)

Wald

$$A_a V^b = V^a \left(\frac{\partial A_b}{\partial x^a} \right)$$

A_a Index
has

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

P-form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

p+1 terms

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

vector potential in 4 dimensions

$$1\text{-form } A_a$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

p-form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

p+1 form

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

vector potential in 4d spacetime

1-form

$$A_a$$

$$F_{ab} = (dA)_{ab}$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

P-form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

p+1 form

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

Vector potential in 4 dimensions

1-form

A_a

$$F_{ab} = (dA)_{ab}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & B_3 & 0 & B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

p-form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p} \quad q =$$

p+1 form

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p} = 0$$

vector potential in 4d spacetime

1-form A_a

$$F_{ab} =$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$E^i = F_{0i}$$

$$B^i = \frac{1}{2} \epsilon^{ijk} F_{jk}$$

$$d\omega = \partial_a \omega_b dx^a \wedge dx^b$$

p-form

$$\omega = \frac{1}{p!} \omega_{a_1 \dots a_p} dx^{a_1} \wedge \dots \wedge dx^{a_p} \quad a = (1, 0)$$

p+1 form

$$d\omega = \partial_b \omega_{a_1 \dots a_p} dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p} \quad (1, 2, 3)$$

$$d^2 = 0$$

$$d^2 \omega = \partial_c \partial_b \omega_{a_1 \dots a_p} dx^c \wedge dx^b \wedge dx^{a_1} \wedge \dots \wedge dx^{a_p}$$

vector potential in 4d spacetime

1-form

A_a

$$F_{ab} = (dA)_{ab}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$E^i = F_{0i}$$

$$B^i = \frac{1}{2} \epsilon^{ijk} F_{jk}$$

$$V = V' \gamma_{v}$$

gauge invariance $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \phi$

$$V = V' \gamma_{v_1}$$

give invariant

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \xi$$

$$A \rightarrow A' = A + d\xi$$

$$V = V' \frac{2}{5}$$

give invariance

$$A_4 \rightarrow A'_4 = A_4 + \partial_4 \xi$$

$$A \rightarrow A' = A + \partial \xi$$

$$F \rightarrow F' = \partial A' - \partial A + \partial^2 \xi^0$$

$$V = V' \frac{2}{5}$$

give invariant

$$A_n \rightarrow A'_n = A_n + 2\delta$$

$$A \rightarrow A' = A + \delta$$

$$F \rightarrow F' = A' - A + \delta^2 = F$$

$$V = V' \frac{1}{\gamma}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

$$F \rightarrow F' = dA' - dA + d^2\chi = F$$

theoretically gauge invariant

$$V = V' \gamma_{\mu\nu}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

theoretic Eqs gauge invariant

$$F \rightarrow F' = dA' - dA + d\chi^2 = F$$

$$\Rightarrow dF = 0$$

$$V = V' + \vec{v}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

theoretic Eqs gauge invariant

$$F \rightarrow F' = dA' - dA + \cancel{d\chi} = F$$

$$\Rightarrow dF = 0 \quad \vec{E} + \nabla \times \vec{B} = 0$$

$$V = V' \gamma_{\beta\beta}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

$$F \rightarrow F' = dA' - dA + d\chi^2 = F$$

the electric field is gauge invariant

$$\Rightarrow dF = 0 \quad \nabla \times B = 0$$

$$V = V' \frac{1}{\gamma_{v_1}}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

theoretic Eqs gauge invariant

$$F \rightarrow F' = dA' - dA + d^2\chi = F$$

$$\Rightarrow dF = 0 \quad \nabla \times B = 0$$

Impose a metric

$$V = V' \frac{2}{\sqrt{3}}$$

gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

theoretic Eqs gauge invariant

$$F \rightarrow F' = dA' - dA + d\chi^2 = F$$

$$\Rightarrow dF = 0 \quad \nabla \cdot B = 0$$

Impose a metric



$$V = V' \frac{\partial x^\mu}{\partial x'^\mu}$$

gauge invariance $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \phi$

$$A \rightarrow A' = A + d\phi$$

$$F \rightarrow F' = dA' - dA + d^2\phi = F$$

theoretically gauge invariant

$$\Rightarrow dF = 0 \quad \nabla \cdot B = 0$$

Impose a metric $S = \int d^4x$

$$V = V' \frac{\partial x}{\partial x'}$$

gauge invariance $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$

$$A \rightarrow A' = A + d\chi$$

therefore E is gauge invariant

$$F \rightarrow F' = dA' = dA + d^2\chi = F$$

$$\Rightarrow dF = 0 \quad \nabla \cdot B = 0$$

Impose a metric

$$\eta_{ab} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$S = -\frac{1}{4} \int d^4x F_{ab} F^{ab} \eta^{ab}$$

$$V = V' \frac{\partial x^\mu}{\partial x'^\mu}$$

Gauge invariance

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$$

$$A \rightarrow A' = A + d\chi$$

theoretic Eqs gauge invariant

$$F \rightarrow F' = dA' - dA + d^2\chi = F$$

$$\Rightarrow dF = 0 \quad \nabla \cdot B = 0$$

Impose a metric

$$\eta_{ab} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$S = -\frac{1}{4} \int d^4x \quad F_{ab} F_{cd} \eta^{ac} \eta^{bd}$$

$$\int \Rightarrow \int \partial_a S dx^a = \int dS$$

derivatives of function
are not defined

free scalar field $\phi(x^a) = \phi(x_i, t)$ scalar

$$\dot{f} \Rightarrow \partial_a \dot{f} dx^a = d\dot{f}$$

derivatives of function
are well defined

free scalar field $\phi(x^a) = \phi(x^i, t)$ scalar

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\delta \Rightarrow \delta S = \int d^4x \delta \mathcal{L}$$

derivatives of function
are not defined

free scalar field $\phi(x) = \phi(x, t)$ scalar

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\dot{\gamma} \Rightarrow \partial_a S dx^a = d\dot{\gamma}$$

derivatives of function
are not defined

free scalar field $\phi(x^a) = \phi(x^a, t)$ scalar $[C=1]$

$$\square \phi = 0 \quad \square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\delta \Rightarrow \underline{\partial_a S} dx^a = d\hat{S}$$

derivatives of function
are not defined

free scalar field $\phi(x^a) = \phi(x^i, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

free scalar field $\phi(x^\mu) = \phi(x^\mu; t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \, \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\delta S = 0 \text{ eq's of motion}$$

Free scalar field $\phi(x) = \phi(x, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\delta S = 0 \text{ eq's of motion}$$

$$\delta S = \int d^4x \eta^{\mu\nu} \left(\delta \partial_\mu \phi \right) \partial_\nu \phi +$$

Free scalar field $\phi(x^\mu) = \phi(x^\mu, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \, \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\delta S = \frac{1}{2} \int d^4x \, \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)]$$

$$\delta S = 0 \quad \text{eq's of motion}$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

free scalar field $\phi(x) = \phi(x^\mu)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\begin{aligned} \delta S &= \frac{1}{2} \int d^4x \eta^{\mu\nu} ((\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)) \\ &= \int d^4x \eta^{\mu\nu} \partial_\nu (\delta \phi) \partial_\mu \phi \end{aligned}$$

$$\delta S = 0 \quad \text{eq's of motion}$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

free scalar field $\phi(x) = \phi(x, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\begin{aligned} \delta S &= \frac{1}{2} \int d^4x \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)] \\ &= \int d^4x \eta^{\mu\nu} \partial_\mu (\delta \phi) \partial_\nu \phi \end{aligned}$$

$$\delta S = 0 \text{ eq's of motion}$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

free scalar field $\phi(x) = \phi(x, t)$ scalar $\boxed{C=1}$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$



$$\delta S = 0 \quad \text{eq's of motion}$$

$$\begin{aligned} \delta S &= \frac{1}{2} \int d^4x \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)] \\ &= \int d^4x \eta^{\mu\nu} \partial_\mu (\delta \phi) \partial_\nu \phi \end{aligned}$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

$$\text{No boundary terms}$$

free scalar field $\phi(x) = \phi(x^\mu)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$



$$\delta S = 0 \quad \text{eq's of motion}$$

$$\begin{aligned} \delta S &= \frac{1}{2} \int d^4x \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)] \\ &= \int d^4x \eta^{\mu\nu} \partial_\nu (\delta \phi) \partial_\mu \phi \end{aligned}$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

No boundary
at $\phi|_b = 0$ s.k.o

free scalar field $\phi(x) = \phi(x^i, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = \frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\delta S = 0 \quad \text{eq's of motion}$$

$$\delta S = \frac{1}{2} \int d^4x \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)]$$

$$\delta \partial_\mu \phi = \partial_\mu \delta \phi$$

$$= \int d^4x \eta^{\mu\nu} \partial_\mu (\delta \phi) \partial_\nu \phi = - \int d^4x \delta \phi \eta^{\mu\nu} \partial_\mu \partial_\nu \phi$$

no boundary

or $\phi|_B = 0$ s.t.o

free scalar field $\phi(x) = \phi(x^i, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{ab} \partial_a \partial_b = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{ab} \partial_a \phi \partial_b \phi$$

$$\delta S = 0 \quad \text{eq's of motion}$$

$$\delta S = \frac{1}{2} \int d^4x \eta^{ab} [(\delta \partial_a \phi) \partial_b \phi + \partial_a \phi (\delta \partial_b \phi)]$$

$$\delta \partial_a \phi = \partial_a \delta \phi$$

$$= \int d^4x \eta^{ab} \partial_a (\delta \phi) \partial_b \phi = - \int d^4x \delta \phi (\eta^{ab} \partial_a \partial_b \phi)$$

no boundary
 $\phi|_b = 0$ s.t.o

free scalar field $\phi(x) = \phi(x^i, t)$ scalar $[C=1]$
 $\square \phi = 0$ $\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = \frac{\partial^2}{\partial t^2} + \nabla^2$

$$S = \frac{1}{2} \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad \delta S = 0 \text{ eq's of motion}$$

$$\delta S = \frac{1}{2} \int d^4x \eta^{\mu\nu} [(\delta \partial_\mu \phi) \partial_\nu \phi + \partial_\mu \phi (\delta \partial_\nu \phi)]$$

$$= \int d^4x \eta^{\mu\nu} \partial_\nu (\delta \phi) \partial_\mu \phi = - \int d^4x \delta \phi (\eta^{\mu\nu} \partial_\mu \partial_\nu \phi) = 0 \Rightarrow \square \phi = 0$$

No boundary
 $\phi|_0 = 0$ & $\phi|_1 = 0$

free scalar field $\phi(x) = \phi(x^i, t)$ scalar $[C=1]$

$$\square \phi = 0$$

$$\square = \eta^{ab} \partial_a \partial_b = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$S = \frac{1}{2} \int d^4x \eta^{ab} \partial_a \phi \partial_b \phi$$

$$\delta S = 0 \quad \text{eqs of motion}$$

$$\delta S = \frac{1}{2} \int d^4x \eta^{ab} [(\delta \partial_a \phi) \partial_b \phi + \partial_a \phi (\delta \partial_b \phi)]$$

$$\delta \partial_a \phi = \partial_a \delta \phi$$

$$= \int d^4x \eta^{ab} \partial_a (\delta \phi) \partial_b \phi$$

$$= - \int d^4x \delta \phi (\eta^{ab} \partial_a \partial_b \phi) = 0 \Rightarrow \square \phi = 0$$

no boundary

or $\phi|_b = 0$ s.t.o

$$S = \frac{1}{2} \int d^3x dt [-(\partial_0 \phi)^2 + (\partial_i \phi)^2]$$

$$S = \frac{1}{2} \int d^3x dt [-(\partial_0 \phi)^2 + (\partial_i \phi)^2]$$

$$\Pi(\phi) = \frac{\delta S}{\delta \dot{\phi}}$$

$$S = \frac{1}{2} \int d^3x dt [-(\partial_0 \phi)^2 + (\partial_i \phi)^2]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}}$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_x \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}}$$

$$S = -\frac{1}{2} \int d^3x dt [-(\partial_0 \phi)^2 + (\partial_i \phi)^2]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi} \quad H = \int d^3x$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x$$

$$S = \frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \dot{\phi}^2 + (\partial_i \phi)^2 \right]$$

$$S = \frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x, y)$$

$$S = \int d^3x dt \left[-\mathcal{L}_0(\phi) + \mathcal{L}_1(\phi) \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$= \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + (\partial_t \phi)^2 \right]$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x, y)$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x, t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + (\partial_i \phi)^2 \right]$$

$$\dot{\phi}(x) = \{ \phi(x), H \}$$

$$\{ \phi(x), \Pi(y) \} = \delta^3(x, y)$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \dot{\phi}^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x, y)$$

$$\int d^3x \left[\frac{1}{2} \Pi^2(x) + (\partial_i \phi(x))^2 \right]$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \dot{\phi}^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \phi(z)\}_\tau = 0$$

$$\{\phi(x), \Pi(y)\}_\tau = \delta^3(x, y)$$

$$\dot{\phi}(x) = \{\phi(x), H\} = \left\{ \phi(x), \int d^3z \left[\frac{1}{2} \Pi^2(z) + (\partial_i \phi(z))^2 \right] \right\}_\tau$$

$$S = -\frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$= \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \phi(y)\} = 0$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x-y)$$

$$\{H\} = \left\{ \phi(x), \int d^3z \left[\frac{1}{2} \Pi^2(z) + (\partial_i \phi(z))^2 \right] \right\}$$

$$\{A, B\} = B \{A, C\} + \{A, B\} C$$

$$S = \frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \dot{\phi}^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \phi(y)\} = 0$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x, y)$$

$$\begin{aligned} \dot{\phi}(x) = \{\phi(x), H\} &= \{\phi(x), \int d^3z \left[\frac{1}{2} \Pi^2(z) + (\partial_i \phi(z))^2 \right]\} \\ &= \int d^3z \left\{ \phi(x), \Pi(z) \Pi(z) + \Pi(z) \phi(z) \right\} \end{aligned}$$

$$\{A, B\} = B \{A, 1\} + \{A, B\}$$

$$S = \frac{1}{2} \int d^3x dt \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x [\Pi \dot{\phi} - \mathcal{L}]$$

$$H = \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \dot{\phi}^2 + (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \phi(z)\} = 0$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x, y)$$

$$\begin{aligned} \dot{\phi}(x) = \{\phi(x), H\} &= \{\phi(x), \int d^3z \left[\frac{1}{2} \Pi^2(z) + (\partial_i \phi(z))^2 \right]\} \\ &= \int d^3z \left\{ \phi(x), \Pi(z) \Pi(z) + \Pi(z) \phi(z) \right\} \end{aligned}$$

$$\{A, B\} = B \{A, C\} + \{A, B\} C$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a dx^b}$$

$$H_1 = S \rightarrow \text{mod } I$$

$$\hat{\phi}(x) = \left(\sqrt{\frac{2}{\pi}} \phi(x) \right)$$

$$f=0$$

$$= \tilde{S}(x, y)$$

$$f = B(A, z)$$

$$+ A(z)$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

$$H=1 \Rightarrow \text{mass } 1$$

$$\phi(x) = \int_{\mathbb{R}^3} f(\omega) d\omega$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

$$H_1(S) \rightarrow \text{case 1}$$

$$\dot{\phi}(x) = \int d^3z \delta(x,z) \pi(z)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

$$H=1 \rightarrow \text{case 1}$$

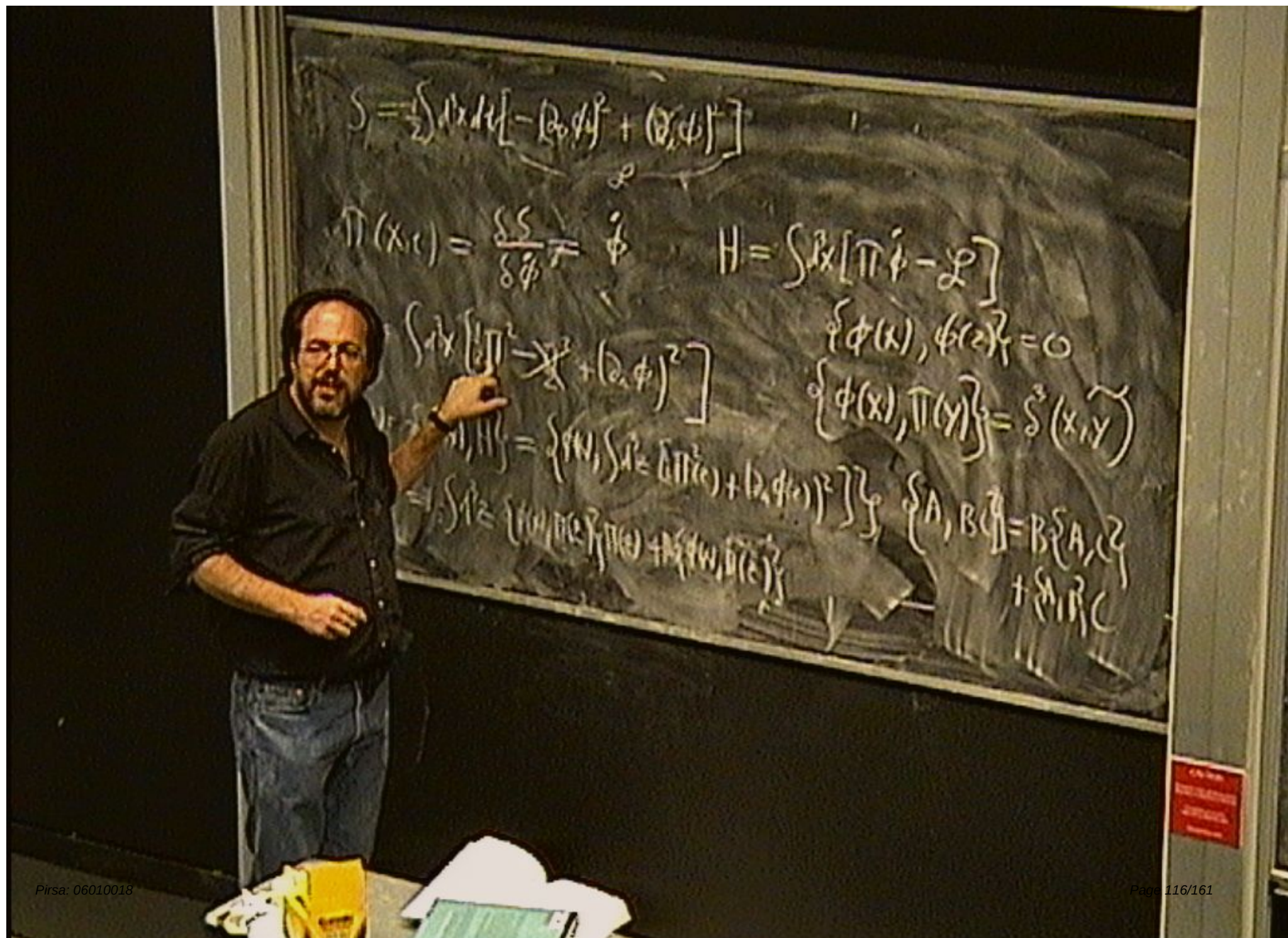
$$\dot{\phi}(x) = \int d^3z \delta(x,z) \pi(z) = \pi(x)$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

Het: $S \rightarrow \text{case 1}$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \Pi(z) = \Pi(x)$$

$$\dot{\Pi}(x) = \{ \Pi(x), H \}$$



$$S = \frac{1}{2} \int d^4x \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\Pi(x,t) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi}$$

$$H = \int d^3x \left[\Pi \dot{\phi} - \mathcal{L} \right]$$

$$\int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + \frac{1}{2} (\partial_i \phi)^2 \right]$$

$$\{\phi(x), \phi(y)\} = 0$$

$$\{\phi(x), \Pi(y)\} = \delta^3(x-y)$$

$$\{H, H\} = \left\{ \int d^3x \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + \frac{1}{2} (\partial_i \phi)^2 \right], \int d^3y \left[\frac{1}{2} \Pi^2 - \frac{1}{2} \nabla^2 \phi^2 + \frac{1}{2} (\partial_i \phi)^2 \right] \right\}$$

$$\{A, B\} = B \{A, C\} + \{A, B\} C$$

$H_1 \rightarrow \text{base 1}$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\dot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \{ \pi(x), \partial_i \phi \}$$

$H_1 = S \rightarrow \text{case 1}$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\ddot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^i} \right\} \frac{\partial \phi}{\partial z^i}$$

$H_1 = S \rightarrow \text{base } 1$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\ddot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\frac{1}{2} \int d^3x \left[-(\partial_0 \phi)^2 + (\partial_i \phi)^2 \right]$$

$$\phi(x) = \frac{\delta S}{\delta \dot{\phi}} = \dot{\phi} \quad H = \int d^3x [\pi \dot{\phi} - \mathcal{L}]$$

$$\int d^3x [\pi^2 - \mathcal{H} + (\partial_i \phi)^2]$$

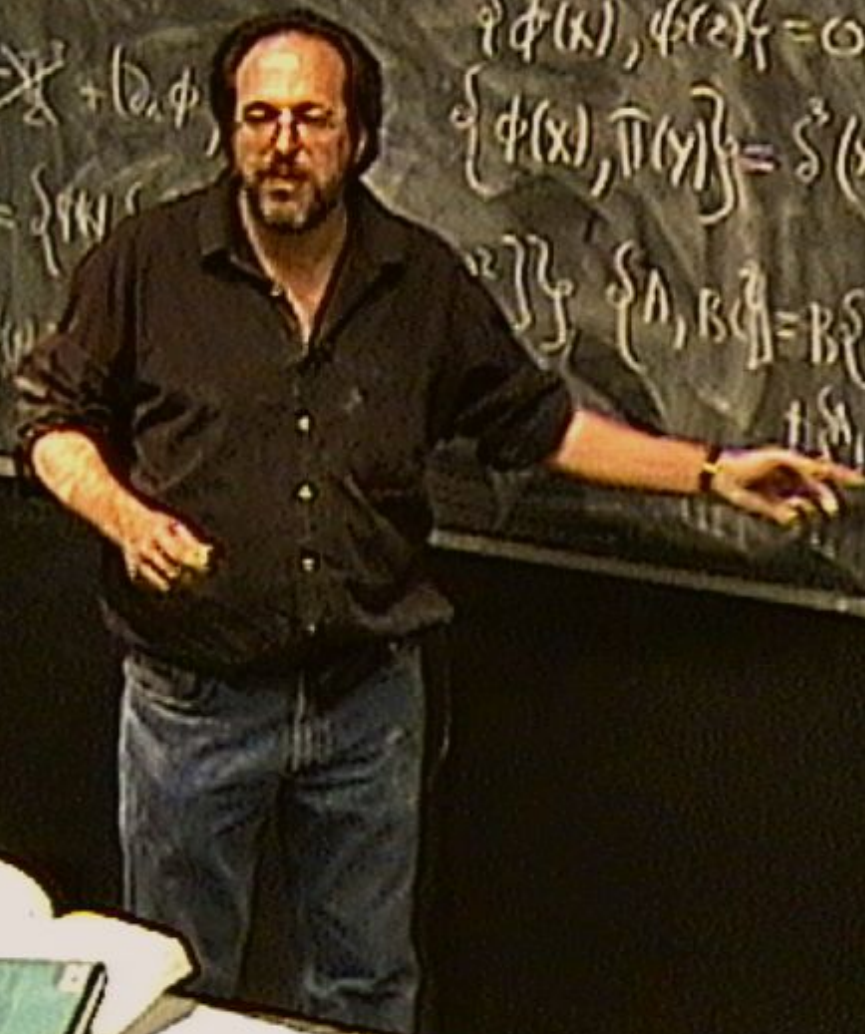
$$\{\phi(x), H\} = \dot{\phi}(x)$$

$$\Rightarrow \int d^3x \dot{\phi}(x)$$

$$\{\phi(x), \phi(y)\} = 0$$

$$\{\phi(x), \pi(y)\} = \delta^3(x, y)$$

$$\{A, B\} = B \{A, C\} + \{A, B\} C$$



$$H = \int d^3x$$

$$\dot{\phi}(x) = \int d^3x$$

$$\dot{\pi}(x) = \int d^3x [\pi(x),$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

$$H: S \rightarrow \text{base } V$$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\dot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

Het $S \rightarrow \text{base } 1$

$$\phi(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\dot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\left\{ \pi(x), \frac{\partial \phi(z)}{\partial z^a} \right\} = \frac{\partial}{\partial z^a} \{ \pi(x), \phi(z) \}$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

$$H = S \longrightarrow \text{base } 1$$

$$\phi(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\pi(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\left\{ \pi(x), \frac{\partial \phi(z)}{\partial z^a} \right\} = \frac{1}{2} \left\{ \pi(x), \phi(z) \right\}$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

Let $S \rightarrow \text{mass } 1$

$$\phi(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\left\{ \frac{\partial \phi(z)}{\partial z^a}, \right\} = \frac{\partial}{\partial z^a} \{ \pi(x), \phi(z) \}$$

$$= \int d^3x \frac{\partial}{\partial z^a} \delta^3(x, z)$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

$$H_1 = S \rightarrow \text{case 1}$$

$$\dot{\phi}(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\ddot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \{ \pi(x), \frac{\partial \phi}{\partial z^a} \} \frac{\partial \phi}{\partial z^a}$$

$$\{ \pi(x), \frac{\partial \phi(z)}{\partial z^a} \} = \frac{\partial}{\partial z^a} \{ \pi(x), \phi(z) \}$$

$$\ddot{\pi}(x) = - \int d^3z \frac{\partial}{\partial z^a} \left(\delta^3(x, z) \right) \frac{\partial \phi}{\partial z^a}$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

Het = S $\Rightarrow$ base 1

$$\phi(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\dot{\pi}(x) = \{ \pi(x), H \} = \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\left\{ \pi(x), \frac{\partial \phi(z)}{\partial z^a} \right\} = \frac{\partial}{\partial z^a} \left\{ \pi(x), \phi(z) \right\}$$

$$\frac{\partial \phi}{\partial z^a} \Big|_{\text{boundary}} = 0$$

$$\dot{\pi}(x) = \int d^3z \frac{\partial}{\partial z^a} \left(\delta^3(x, z) \right) \frac{\partial \phi}{\partial z^a} = \int d^3z \delta^3(x, z) \nabla^2 \phi(z)$$

$$d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b$$

Hint: $S \rightarrow \text{use 1}$

$$\phi(x) = \int d^3z \delta^3(x, z) \pi(z) = \pi(x)$$

$$\dot{\pi}(x) = \{ \pi(x), H \} = \frac{1}{2} \int d^3x \left\{ \pi(x), \frac{\partial \phi}{\partial z^a} \right\} \frac{\partial \phi}{\partial z^a}$$

$$\left\{ \pi(x), \frac{\partial \phi(z)}{\partial z^a} \right\} = \frac{\partial}{\partial z^a} \left\{ \pi(x), \phi(z) \right\}$$

$$\frac{\partial \phi}{\partial z^a} \Big|_{\text{boundary}} = 0$$

$$\begin{aligned} \dot{\pi}(x) &= - \int d^3z \frac{\partial}{\partial z^a} \left(\delta^3(x, z) \right) \frac{\partial \phi}{\partial z^a} = \int d^3z \delta^3(x, z) \nabla^2 \phi(z) \\ &= \nabla^2 \phi(x) \end{aligned}$$

$$\boxed{d\omega \equiv \partial_a \omega_b dx^a \wedge dx^b}$$

$$\dot{\Pi}(x) = \left\{ \Pi(x), H \right\} = \int d^3x \left[\Pi(x) \frac{\partial \mathcal{L}}{\partial z^a} \frac{\partial z^a}{\partial x^0} \right]$$

$$\left\{ \Pi(x) \frac{\partial \phi(z)}{\partial z^a} \right\} = \frac{\partial}{\partial z^a} \left\{ \Pi(x), \phi(z) \right\}$$

$$\left. \frac{\partial \phi}{\partial z^a} \right|_{\partial V} = 0$$

$$\begin{aligned} \dot{\Pi}(x) &= - \int d^3x \frac{\partial}{\partial z^a} \left(\delta^3(x, z) \right) \frac{\partial \phi}{\partial z^a} = \int d^3x \delta^3(x, z) \nabla^2 \phi(z) \\ &= \nabla^2 \phi(z) \end{aligned}$$

$$\dot{\phi}(x) = \dot{\Pi} = \nabla^2 \phi \Rightarrow \square \phi = 0$$

3-manifold Σ



3-manifold Σ compact

3-manifold Σ compact

d dim $\int$ w-forms

3-manifold Σ compact

$d \dim \int \omega$

$\int d^3 x dx^3$

3-manifold Σ compact

$d \dim \int \omega$

$\int dx^1 dx^2 dx^3$

$\omega_{ijk} dx^i dx^j dx^k$

3-manifold Σ compact

$$d \dim \int \omega_{\text{volume}} = \int dx^1 dx^2 dx^3$$

$$\omega_{ijk} dx^i \wedge dx^j \wedge dx^k = \omega_{123} dx^1 \wedge dx^2 \wedge dx^3$$

3-manifold Σ compact

$X \rightarrow X'$

$$\text{dim} \int \omega_{123} = \int dx^1 dx^2 dx^3 \Rightarrow \int$$

$$\omega_{ijk} dx^i dx^j dx^k = \omega_{123} dx^1 dx^2 dx^3$$

$$\int \omega = \int \omega_{123} dx^1 dx^2 dx^3 \text{ is coord. invariant}$$

3-manifold Σ compact

$X \rightarrow X'$

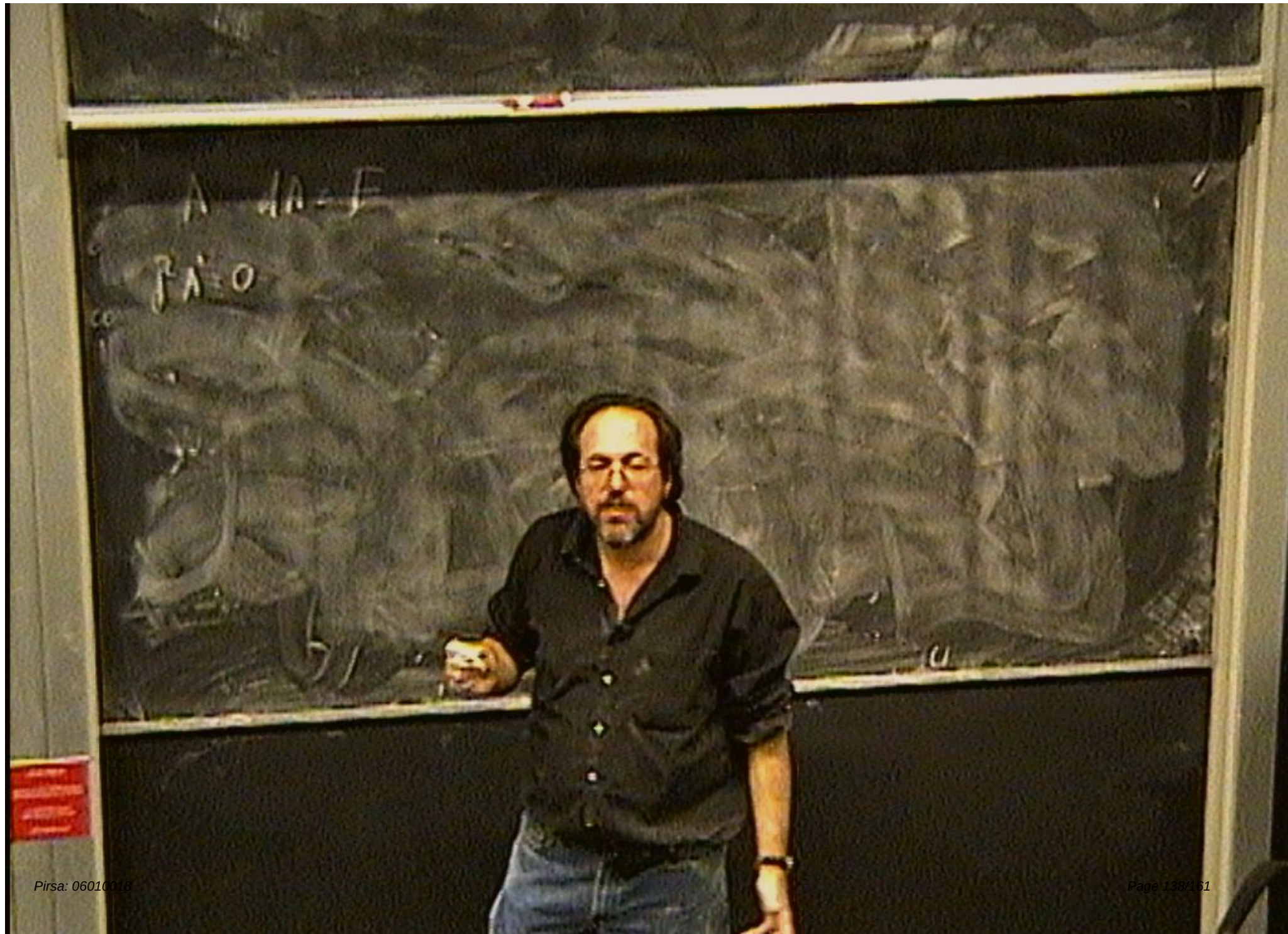
$$\dim \int \omega_{\text{volume}} = \int dx^1 dx^2 dx^3 \Rightarrow \int$$

$$\omega_{ijk} dx^i dx^j dx^k = \omega_{123} dx^1 dx^2 dx^3$$

check $\int \omega = \int \omega_{123} dx^1 dx^2 dx^3$ is const. invariant

$$A \cdot A = E$$





$$\Lambda \quad d\Lambda = F \quad \omega = \Lambda \wedge F = \Lambda \wedge d\Lambda$$

$$\beta \Lambda' = 0$$

W.



$$A \wedge A = F$$

$$\omega = A \wedge F = A \wedge dA$$

$$F A^* = 0$$

$$\Rightarrow A \wedge dA$$



$$A \, dA = F \quad \omega = A \wedge F = A \wedge dA$$

$$\partial A = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + dA \wedge A$$



$$A \wedge A = F$$

$$\omega = A \wedge F = A \wedge dA$$

$$dA = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + \underbrace{dA \wedge A}_{=0}$$

$$d(A \wedge B) = dA \wedge B + (-1)^{|A|} A \wedge dB$$

30.



$$A \quad dA = F$$

$$\omega = A \wedge F = A \wedge dA$$

$$dA = 0$$

$$\Rightarrow A \wedge dA = d(A \wedge A) \Rightarrow A \wedge dA + \underbrace{dA \wedge A}_{=0}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

30.



$$A \quad dA = E$$

$$\omega = A \wedge F = A \wedge dA$$

$$dA = 0$$

$$\Rightarrow A \wedge dA = d(A \wedge A) \Rightarrow A \wedge dA + \underbrace{dA \wedge A}$$

$$d(A \wedge B) = dA \wedge B + A \wedge dB$$

yu.

$$A \wedge A = F \quad \omega = A \wedge F = A \wedge dA$$

$$dA = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + \frac{d(A \wedge A)}{d(S \wedge A)}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(A \wedge A)$$

$$A \wedge A = F \quad \omega = A \wedge F = A \wedge dA$$

$$F \wedge A = 0$$

$$\Rightarrow A \wedge dA \rightarrow A \wedge dA + \underbrace{d(A \wedge A)}_{d(S \wedge A)}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(S \wedge A)$$

$$A \wedge A = F \quad \omega = A \wedge F = A \wedge dA$$

$$F \wedge A = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + \underline{d(A \wedge A)}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int d(\text{something}) = 0 \quad d(\int A \wedge A)$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(A \wedge A)$$

$$A \wedge A = F \quad \omega = A \wedge F = A \wedge dA$$

$$F A = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + \underbrace{d(A \wedge A)}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int d(\text{something}) = 0 \quad d(S \wedge A)$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(A \wedge A)$$

$$S^{\omega} = \int A \wedge dA$$

$$A \quad dA = F \quad \omega = A \wedge F = A \wedge dA$$

$$F A' = 0$$

$$\Rightarrow A \wedge dA \wedge dA \Rightarrow A \wedge dA + \underline{d(A \wedge A)}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int d(\text{something}) = 0 \quad d(S \wedge A)$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(A \wedge A)$$

$S' = \int A \wedge dA$	$\delta S = \int \delta A \wedge dA + A \wedge \delta A$
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$$A \wedge dA = F \quad \omega = A \wedge F = A \wedge dA$$

$$\oint A = 0$$

$$\Rightarrow A \wedge dA \Rightarrow A \wedge dA + \underbrace{dA \wedge A}$$

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB$$

$$\int d(\text{something}) = 0 \quad d(S \wedge A)$$

$$\int \omega = \int A \wedge dA \rightarrow \int A \wedge dA + \int d(A \wedge A)$$

$$S^{\text{cl}} = \int A \wedge dA$$

$$\delta S = \int \delta A \wedge dA + A \wedge d\delta A = 2 \int \delta A \wedge dA$$

$$\Rightarrow dA = 0 \Rightarrow F = 0$$

$$\Sigma = T^3$$



$$\begin{aligned} & \{ \phi, \pi(\phi) \} + \{ \phi, \pi(\phi) \} \\ & \{ A, B \} = B \wedge A, \wedge \\ & + \{ A, B \} \end{aligned}$$

$$\Sigma = T^3$$

$$\int_A A = A \cdot \text{vol}$$



$$\{ \{ \omega, \eta \} \} + \{ \{ \omega, \eta \} \} \} \} \} \{ A, B \} = B \{ A, C \} + A \{ B, C \}$$

$$\zeta = T^3$$

$$\int_{\mathcal{A}} A = A(B)$$

under 21

$$A\{0\} \Rightarrow \sum_1 A + dS \Rightarrow \sum_2 A + \left(\sum_1 dS \right) = 0$$



$$= \{ (w_1, w_2) \in W(A) \times W(B) : (w_1, w_2) \in W(A \cup B) \} = W(A) \cup W(B)$$

$$\Sigma = T^3$$

$$\int_{\Sigma} A = A(E)$$

under SL

$$A(E) \Rightarrow \int_{\Sigma} A + dS \Rightarrow \int_{\Sigma} A + \left(\int_{\Sigma} dS \right) = 0$$



$$\Sigma = T^3$$

$$\int_{\Sigma} A = A(\Sigma)$$



under gl.

$$A(\Sigma) \Rightarrow \int_{\Sigma} A + dS \Rightarrow \int_{\Sigma} A + \left(\int_{\Sigma} dS \right) = 0$$

$$\int_{\Sigma} A = \int A(n(x))$$



$$\Sigma = T^3$$



$$\int_{\alpha} A = A[\alpha]$$

under g_1

$$A[\alpha]$$

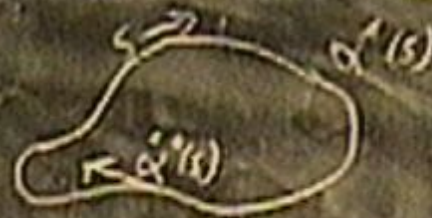
$$\int_{\alpha} A + dS \Rightarrow \int_{\alpha} A + \left(\int_{\alpha} dS \right) = 0$$



$$\int_{\alpha} A = \int_{\alpha} A(\gamma(s)) \alpha'(s) ds$$

$$\Sigma = T^3$$

$$\int_{\Sigma} A = A[\Sigma]$$



under gl.

$$A[\Sigma] \Rightarrow \int_{\Sigma} A + dS \Rightarrow \int_{\Sigma} A + \left(\int_{\Sigma} dS \right) = 0$$

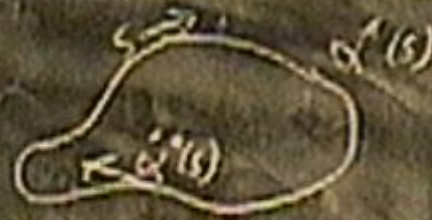
$$\int_{\Sigma} A = \int_{\Sigma} A_{\mu\nu} \alpha^{\mu} \alpha^{\nu} dS$$

$$\int_{\Sigma} dS = \int_{\Sigma} \frac{1}{2} \epsilon^{\mu\nu} \alpha^{\mu} \alpha^{\nu} dS =$$

$$\Sigma = T^3$$



$$\int_{\Sigma} A = A[\Sigma]$$



under gl.

$$A[\Sigma] \Rightarrow \int_{\Sigma} A + dS \Rightarrow \int_{\Sigma} A + \left(\int_{\Sigma} dS \right) = 0$$

$$\int_{\Sigma} A = \int_{\Sigma} A_{\mu\nu} \alpha^{\mu} \alpha^{\nu} ds$$

$$\int_{\Sigma} dS = \int_{\Sigma} \frac{\partial}{\partial x^i} [x^i(s)] \frac{dx^i}{ds} ds = \int_{\Sigma} ds \frac{dx^i}{ds}$$

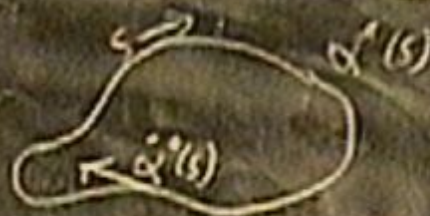
$$\Sigma = T^3$$



give
inv

$$\boxed{\int_{\alpha} A = A[\alpha]}$$

under gl.



$$A[\alpha] \Rightarrow \int_{\alpha} A + d\int_{\alpha} A \Rightarrow \int_{\alpha} A + \left(\int_{\alpha} d\int_{\alpha} A \right) = 0$$

$$\int_{\alpha} A = \int A(\alpha(s)) \alpha'(s) ds$$

$$\int_{\alpha} d\int_{\alpha} A = \int \frac{\partial A}{\partial x^i}(\alpha(s)) \frac{dx^i}{ds} ds = \int ds \frac{d\int_{\alpha} A}{ds} = 0$$

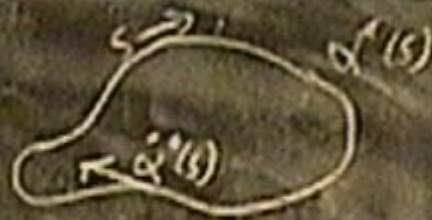
$$\Sigma = T^3$$



give
inv

$$\boxed{\int_{\alpha} A = A[\Sigma]}$$

under GL



$$A[\Sigma] \Rightarrow \int_{\alpha} A + dS \Rightarrow \int_{\Sigma} A + \left(\int_{\partial \Sigma} dS \right) = 0$$

$$\int_{\alpha} A = \int A_i(x) \alpha^i(s) ds$$

$$\int_{\partial \Sigma} dS = \int \frac{\partial}{\partial x^i} [A_i(s)] \frac{dx^i(s)}{ds} ds = \int ds \frac{dS}{ds} [A_i(s)] = 0$$

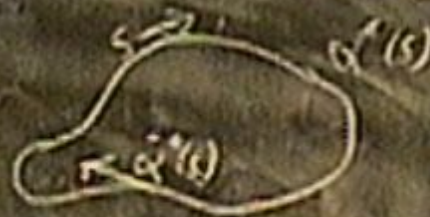
$$Z = T^3$$



giving
by

$$\boxed{\int_{\mathcal{V}} A = A[\mathcal{V}]}$$

under $\mathcal{V}$



$$A[\mathcal{V}] \Rightarrow \int_{\mathcal{V}} A + d\mathcal{V} \Rightarrow \int_{\mathcal{V}} A + \left(\int_{\partial\mathcal{V}} d\mathcal{V} \right) = 0$$

$$\int_{\mathcal{V}} A = \int_{\mathcal{V}} A(\vec{r}(s)) \frac{d\mathcal{V}}{ds} ds$$

$$\int_{\mathcal{V}} d\mathcal{V} = \int_{\mathcal{V}} \frac{d\mathcal{V}}{ds} \frac{ds}{ds} ds = \int_{\mathcal{V}} \frac{d\mathcal{V}}{ds} ds = 0$$