Title: Spacetime regions as "quantum subsystems": glimmers of a pre-geometric perspective

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Abstract: Space-time measurements and gravitational experiments are made by the mutual relations between objects, fields, particles etc... Any operationally meaningful assertion about spacetime is therefore intrinsic to the degrees of freedom of the matter (i.e. non-gravitational) fields and concepts such as ``locality" and ``proximity" should, at least in principle, be definible entirely within the dynamics of the matter fields. We propose to consider the regions of space just as general ``subsystems". By writing the Hilbert space of the matter fields as a generic tensor product of subsystems we analyse the evolution of a state vector on an information theoretical basis and discuss general principles to recover a posteriori the usual space-time relations. We apply such principles to generic interacting second quantized models with a finite number of fermionic degrees of freedom. Finally, we discuss the possible role of gravity in this framework.

Perimeter Institute, 19/1/2006

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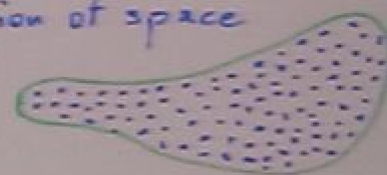
Space-time Regions as "Quantum Subsystems": Glimmers of a pre-geometric Perspective

hep-th/0506124

hep-th/0511285

Invitation

QFT with a UV cut-off: the number of degrees of freedom is finite and proportional to the volume of region of space



ALTHOUGH (when also gravity effects are taken into account...)

— Holographic principle ('t Hooft '93, Susskind '94)

— Some solutions to the B.H. information loss paradox...
(Susskind, Thorlacius, Uglum '93 ; Horowitz, Maldacena 2004)

— Strong gravitational back reaction (Giddings, Lippert 2004)

— dS Thermodynamics (Banks 2000, Witten 2001 etc...)

— ... spacetime is dynamical! (Einstein '16)

Traditionally →

These are jobs for Quantum Gravity

It should →

- be pre-geometric
- reproduce spacetime continuum and G.R. in some appropriate coarse-grained / low-energy limit
- ... ?

... ALTHOUGH:

Any operationally meaningfull assertion about space-time is in fact about the degrees of freedom of the matter- (non-gravitational) fields !!

The program

- No "new physics"
- Try to find a pre-geometric version of some simple quantum field theory (ideally: S.M., M.S.S.M, ...)
- Ask it about space-time

Example:

The "discovery" of Lorentz transformation in Special Relativity ...

1905

Which are the correct coordinate transformations between inertial observers?

a minimal, "bottom-up" approach:

"I don't know... ask physics!!"

① Principle of relativity
② Constancy of the velocity of light

i.e. Use physics that you already know and find the procedure by which the observers assign a set of coordinates to the physical events

"Like every other electrodynamics, the theory to be developed is based on the kinematics of the rigid body, since assertions of each and any theory concern the relations between rigid bodies (coordinate systems), clocks, and electromagnetic processes. Insufficient regard for this circumstance is at the root of the difficulties with which the electrodynamics of moving bodies must presently grapple"

"If, for example, I say that –the train arrives here at 7 o'clock, that means, more or less, – the pointing of the small hand of my clock to 7 and the arrival of the train are simultaneous events"

Assumption 0: The dynamics of the matter fields is described by quantum theories

what's going on while an observer assigns a "position" to an object ?

$\Downarrow$

The problem of "quantum measurement"

(GOSH !!)

A working hypothesis (Everett III 1957, 1983):

Assumption 1: • Every physical process is described by a unitary evolution of a state-vector in a Hilbert space • A measurement is a physical process during which the degree of correlation/information between the "measuring" and the "measured" increases

Example

A spin measurement

Traditional view:

$\mathcal{H}_S$ ← non unitary dynamics

$$\alpha|+\rangle + \beta|-\rangle \longrightarrow \begin{cases} |+\rangle & \text{prob. } |\alpha|^2 \\ |-\rangle & \text{"} \ |\beta|^2 \end{cases}$$

Everett's view:

$\mathcal{H}_S \otimes \mathcal{H}_M$ ← unitary evolution

$$(\alpha|+\rangle + \beta|-\rangle) \otimes |\text{init}\rangle \longrightarrow \alpha|+\rangle \otimes |U\rangle + \beta|-\rangle \otimes |D\rangle$$

A type of correlation: quantum entanglement

$$\mathcal{H}_{universe} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes \ldots$$

a separable state: $|+\rangle_A \otimes |+\rangle_B$

an entangled state: $|+\rangle_A \otimes |+\rangle_B + |-\rangle_A \otimes |-\rangle_B$

The entanglement between A and B depends only on:

- $|\psi\rangle_{Universe}$
- What you decided to call "A" and "B"!

(i.e. the tensor product structure (T.P.S.) that has been choosen for the system-Universe)

Call $I(A,B)$ our "bona fide" measure of entanglement between A and B.

If $|\psi\rangle_{universe} = |\xi\rangle_{AB} \otimes |\text{all the rest}\rangle$, then

$$I(A,B) = S(B) = S(A) \equiv -tr_A(\rho_A \log \rho_A)$$

Von Neumann entropy

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von Neumann entropy

The most elementary type of space-time relation between subsystems:

Space-time coincidence

(or "having been in touch")

① Choose a TPS for the system-Universe

$$\mathcal{H}_{universe} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \ldots$$

② Work out the dynamics: $|\psi, t\rangle_{universe}$

③ Calculate the entanglement

A SUFFICIENT CONDITION:

If before t_1 $S(A; t<t_1) = S(B; t<t_1) = 0$

and at $t_2 > t_1$ $S(A; t_2) = S(B, t_2) \gg 1$, with

$S(AB; t<t_2) = 0$, then A and B have been coincident

Vincent's information worldline

between, say, t_1 and t_2:

01110010001010111011111001010101110000100

Vincent's information worldline

between, say, t_1 and t_2:

0111001000101010111011111001010101110000100

"I met A"

- **Contiguity**: (def./assumption) two systems that were in a pure state and started exchanging information with each other "have been contiguous". Ex: $I(V, A; t)$ ↑

Vincent's information worldline

between, say, t_1 and t_2:

$\underbrace{0111001}_{\text{"I met A"}}000101011101111100101011100000100$

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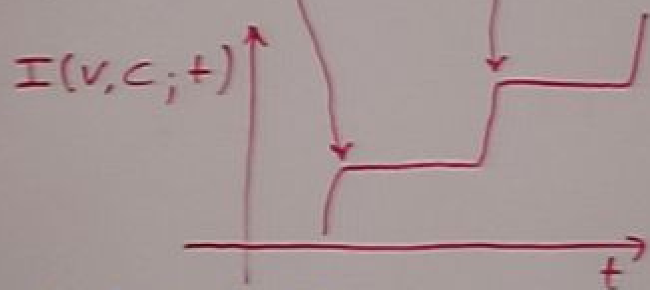
Vincent's information worldline

between, say, t_1 and t_2:

0111001000101010111011110010101110000100

"I met A"   "it 7 p.m."   "it's 8 p.m."

- **Vincent's clock**: a system C that regularly sends pulses of information to Vincent:



A good clock should be:

- recognizable
- predictable

} No particular relation with the "external", unobservable time t!

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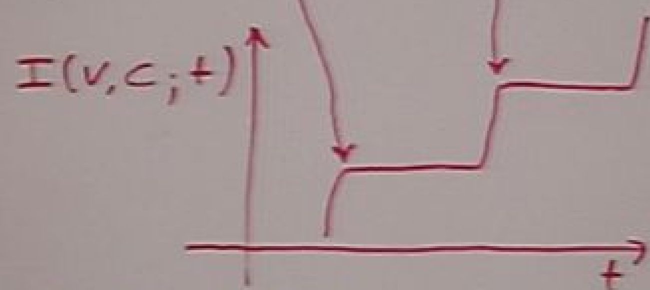

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Giving a Hilbert space a T.P.S.

(it's like choosing an element of a group...)

Ex: $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$

$$
\begin{array}{lcl}
|1\rangle & & |+\rangle \otimes |+\rangle \\
|2\rangle & \Longrightarrow & |+\rangle \otimes |-\rangle \\
|3\rangle & & |-\rangle \otimes |+\rangle \\
|4\rangle & \sim U(4)/U(2)\otimes U(2) & |-\rangle \otimes |-\rangle
\end{array}
$$

For a D-dim. Hilbert space the group has dimensions $\sim D^2$

Conjecture:

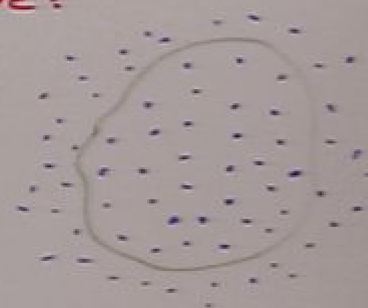
The T.P.S. that singles out the "localized" subsystems is the one that minimizes the tendency to entanglement

(... and all we can say about space-time can be extracted by the coincidence relations between "localized" subsystems ...)

Back to spatial regions

Traditional QFT picture:

From the beginning:



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \otimes \cdots \mathcal{H}_N$$

Picking up a "subregion" amounts to choose a partition of these spaces. For a given volume M you choose M factors.

... but we don't want to be said what spacetime is "a priory" $\longrightarrow$

$$\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_{N-M}$$

There is an infinite number of way of partitioning the Hilbert space!

A "quantum field" toy model

$$H = \underbrace{\sum_j^N \lambda_j c_j^+ c_j}_{H_0,\ \text{"free"}} + \underbrace{\sum_{jklm}^N \gamma_{jklm} c_j^+ c_k^+ c_l c_m}_{H_I,\ \text{"interacting"}}$$

N fermionic operators:

$$\{c_j^+, c_k\} = \delta_{jk} \qquad \{c_j, c_k\} = 0$$

A finite-dimensional Hilbert space! $\underline{\dim(\mathcal{H}) = 2^N}$

$$\mathcal{H} = \underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2}_{N \text{ times}}$$

← $\mathcal{H}$ has a natural Tensor product-structure:

N subsystems!

a basis: $c_N^{+\alpha_N} c_{N-1}^{+\alpha_{N-1}} \ldots c_1^{+\alpha_1} |0\rangle$

$\sim |\alpha_N\rangle \otimes |\alpha_{N-1}\rangle \otimes \ldots \otimes |\alpha_1\rangle$

Fixed "number of particles" sub spaces

$[H, N] = 0$

Fock structure → $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_N$

We consider only Fock structure-preserving T.P.S.

⇒ Bogoliubov transformations ⇒ $\tilde{c}_j^+ = \alpha_{jk}(t) c_k^+$

Results : (hep-th/0505124)

Free fields ($H_I = 0$):

A suitable time-dependent T.P.S. can always "reabsorb" the effects of the evolution: No coincidence relations between localized particles $\boxed{\text{NO SPACE-TIME}}$!

Interacting fields ($H_I \neq 0$)

The minimization problem is not trivial.

The case of a one-dimensional Heisenberg spin chain:

$$H = \sum_i \lambda_i c_i^\dagger c_i + \sum_{jklm} c_j^\dagger c_k^\dagger c_l c_m \gamma_{jklm}$$

$$\gamma_{jklm} = \delta(j+k-l-m)\left[\cos\frac{2\pi(j-l)}{N} - \cos\frac{2\pi(k-l)}{N}\right.$$

In the two-particles subspace entanglement tendency is minimized by "position"!

Conclusions:

- A scheme to interpret the unitary evolution of a (matter fields-) state vector $|\psi; t\rangle$ as "space-time relations" between "parties"
- Free fields $\Rightarrow$ No space-time
- One-dim. Heisenberg spin chain $\Rightarrow$ OK
- More generally: The "minimal" T.P.S.

$\Rightarrow$ the class of "localized systems"

$\Rightarrow$ the emerging space-time depends on the initial state

i.e. on the matter-content!

(GRAVITY?)

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