

Title: Introduction to quantum gravity

Date: Jan 18, 2006 06:30 PM

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras

I The basic structure of $\mathcal{LQG}$...

$$LGR \Rightarrow QR$$

$$\mathbb{C}G R \Leftarrow \mathbb{Q}G$$



I The basic structure of LQG+...

II Intro GR as a gauge theory

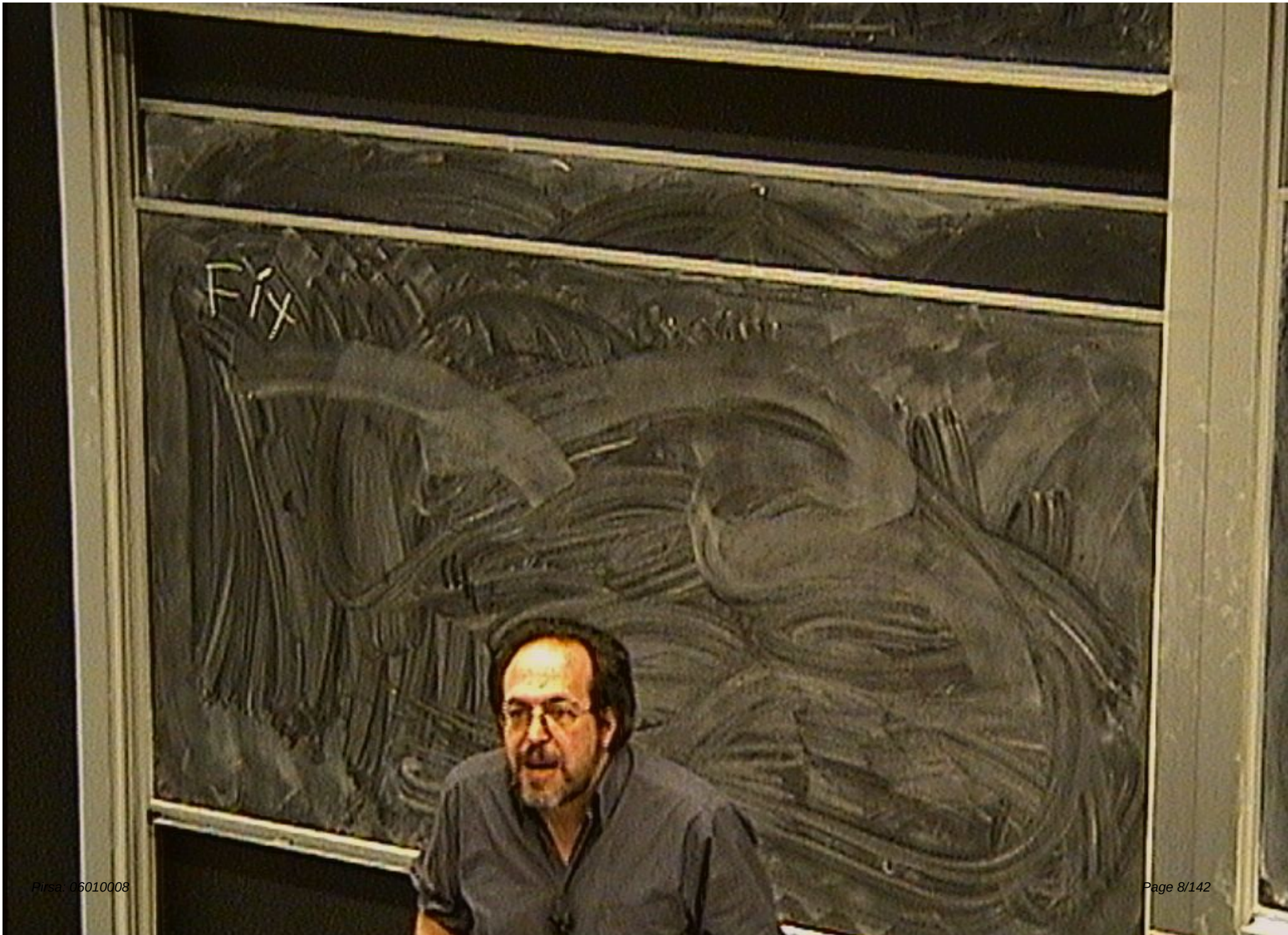
I The basic structure of LQG+...

II Intro GR as a gauge theory
hep-th/0709079

I The basic structure of LQG+...
hep-th/0408048

II Intro GR as a gauge theory
hep-th/0201079
Sections 2.3 - 6

tuesday



Fix Spatial Topology S^3

Fix Spatial Topology $S^3 = \Sigma$



Fix Spatial Topology $S^3 = \Sigma$



Fix Spatial Topology

Graph



$$S^3 = \Sigma$$



Fix Spatial Topology

All Graphs $\mathcal{P} =$



$$S^3 = \Sigma$$

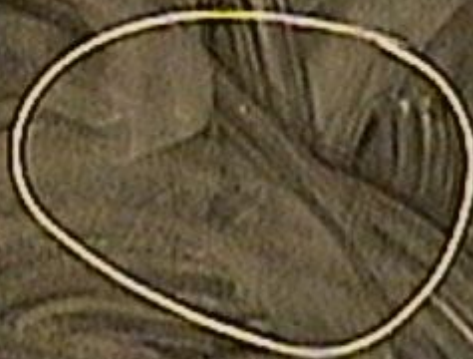


Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\Gamma =$



$G = \{ \text{all graphs} \}$

Finite # of vertices, links



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $P =$

$G =$



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

Finite of vertices, links



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\mathcal{P} =$



$\mathcal{G} = \{ \text{all graphs} \}$

Finite # of vertices, links

$\{G\} =$

$\{ \text{all embeddings of } G \text{ into } \Sigma \text{ upto topology} \}$



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\mathcal{P} =$

$$G = \{$$

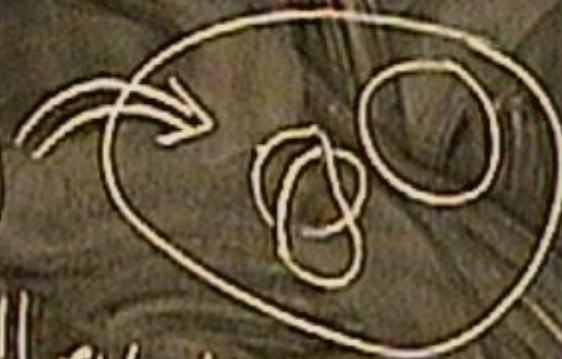
plus

vertices, links



$\{G\} =$

$\{ \text{all embeddings of } G \text{ into } \Sigma \text{ up to topology} \}$
countable set



Fix Spatial Topology

$$S^3 = \Sigma$$

All Graphs $\mathcal{P} =$

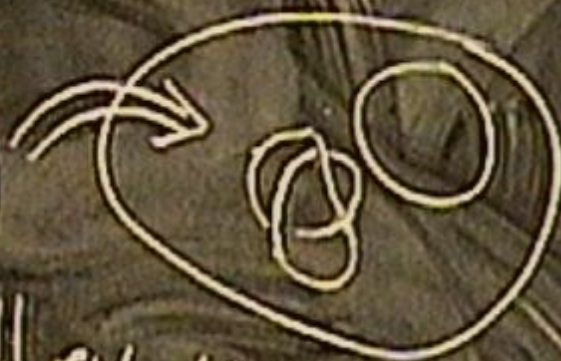


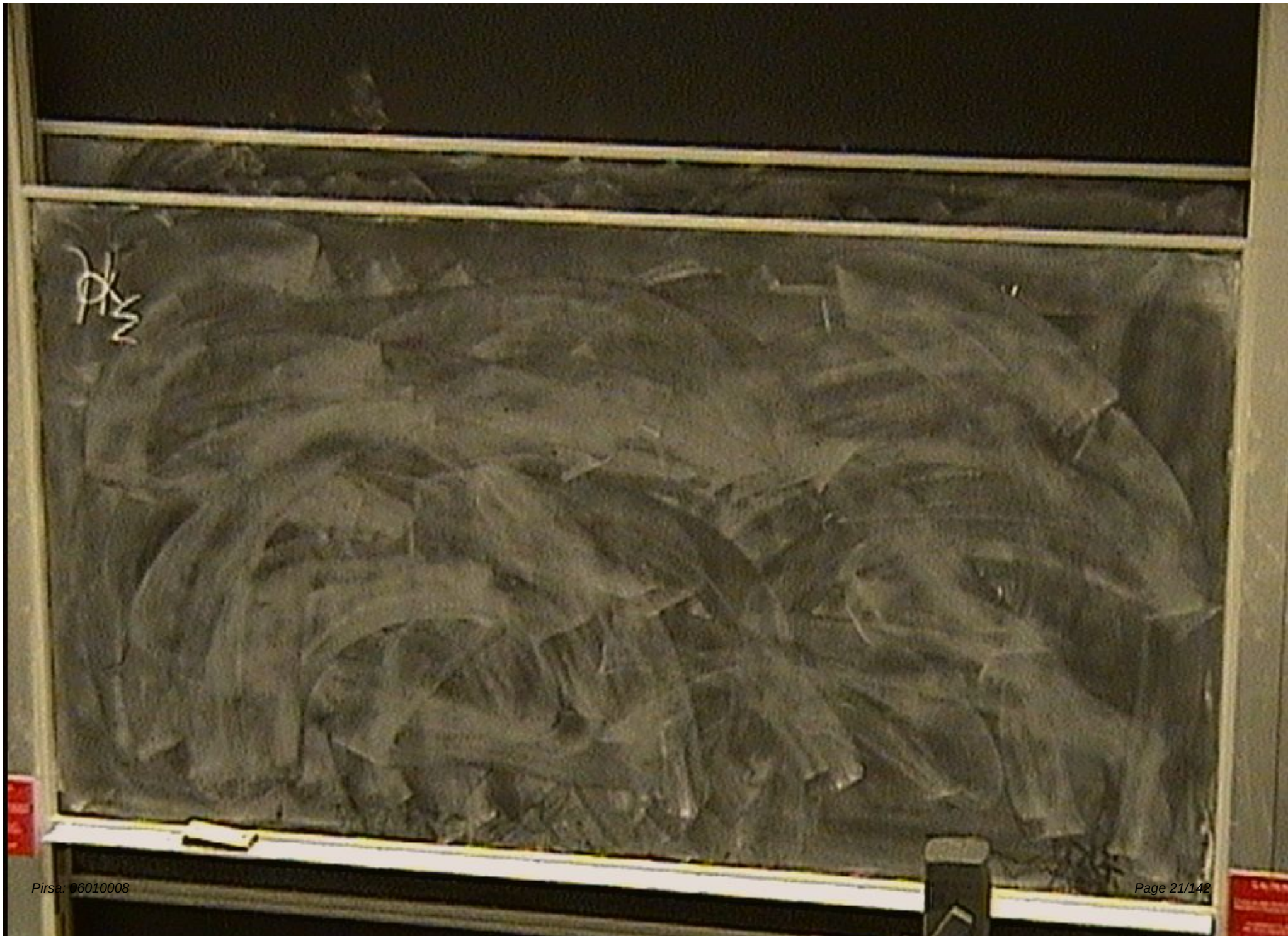
$\mathcal{G} = \{\text{all graphs}\}$

Finite n. of nodes, links

$\{G\}$

$= \{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$
 (countable set)





$\Phi_{\Sigma} = \text{orthonormal basis}$ |

Topology $S^3 = \Sigma$



$\{G\} = \left\{ \begin{array}{l} \text{all embeddings of } G \\ \text{into } \Sigma \text{ up to topology} \\ \text{conjugate set} \end{array} \right\}$

$\mathcal{H}_\Sigma = \text{orthonormal basis}$

Topology $S^3 = \Sigma$



$\{G\} = \{ \text{all embeddings of } G \text{ into } \Sigma \text{ upto topology} \}$
 (containing set)

$\mathcal{H}_\Sigma = \text{orthonormal basis}$

ϕ_k = orthonormal basis $|A\rangle$

$$|\psi\rangle = \sum_k c_k \phi_k$$

$\{ | \hat{A} \rangle \} = \text{orthonormal basis } | \hat{A} \rangle$

$$| \psi \rangle = \sum_{\hat{A}} a_{\hat{A}} | \hat{A} \rangle \quad \sum_{\hat{A}} | a_{\hat{A}} |^2 = 1$$

$\{ | \hat{A} \rangle \} = \text{orthonormal basis } | \hat{A} \rangle$

$$| \psi \rangle = \sum_{\hat{A}} a_{\hat{A}} | \hat{A} \rangle \quad \sum_{\hat{A}} | a_{\hat{A}} |^2 = 1$$

$$\langle \hat{A} | \hat{A}' \rangle = \delta_{\hat{A} \hat{A}'}$$

$\mathcal{H}_S$ = orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\hat{n}} a_{\hat{n}} |\hat{n}\rangle \quad \sum_{\hat{n}} |a_{\hat{n}}|^2 = 1$$

$$\langle \hat{n} | \hat{n}' \rangle = \delta_{\hat{n} \hat{n}'}$$

$\{ \hat{\psi} \} = \text{orthonormal basis } | \hat{\psi} \rangle$ separable

$$| \psi \rangle = \sum_{\hat{\psi}} a_{\hat{\psi}} | \hat{\psi} \rangle \quad \sum_{\hat{\psi}} | a_{\hat{\psi}} |^2 = 1$$

$$\langle \hat{\psi} | \hat{\psi}' \rangle = \delta_{\hat{\psi} \hat{\psi}'}$$



$\mathcal{H}_S$ = orthonormal basis $|\hat{n}\rangle$ separable

$$|\psi\rangle = \sum_{\hat{n}} a_{\hat{n}} |\hat{n}\rangle \quad \sum_{\hat{n}} |a_{\hat{n}}|^2 = 1$$

$$\langle \hat{n} | \hat{n}' \rangle = \delta_{\hat{n} \hat{n}'}$$



Volume operator in $\mathcal{H}_S$

$\mathcal{H}_\Sigma$ = orthonormal basis $|\hat{r}\rangle$ separable

$$|\psi\rangle = \sum_{\hat{r}} a_{\hat{r}} |\hat{r}\rangle \quad \sum_{\hat{r}} |a_{\hat{r}}|^2 = 1$$

$$\langle \hat{r} | \hat{r}' \rangle = \delta_{\hat{r} \hat{r}'}$$



Volume operator in $\mathcal{H}_\Sigma$

$$\hat{V} |\hat{r}\rangle$$

$\mathcal{H}_\Sigma$ = orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\hat{n}} a_{\hat{n}} |\hat{n}\rangle \quad \sum_{\hat{n}} |a_{\hat{n}}|^2 = 1$$

$$\langle \hat{n} | \hat{n}' \rangle = \delta_{\hat{n} \hat{n}'}$$



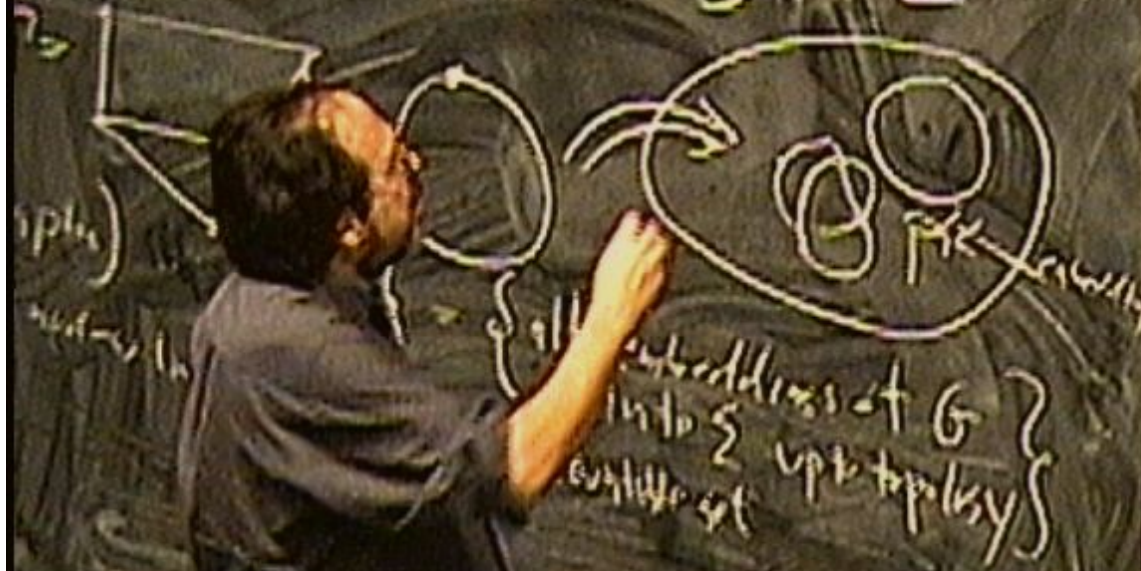
Volume operator in $\mathcal{H}_\Sigma$

$$\hat{V} |\hat{n}\rangle = |\hat{n}\rangle V_{\hat{n}}$$

$V_{\hat{n}} = \int_{\hat{n}} d^3x$ number of 4-vertex valent nodes

Spatial Topology

$$S^3 = \Sigma$$



{ all embeddings of G into Σ up to topology }
equivalence set

$$\mathcal{H}_\Sigma = \text{overlaid on } \mathcal{H}_\Sigma$$

$$|\psi\rangle = \sum_{\vec{p}} a_{\vec{p}}$$

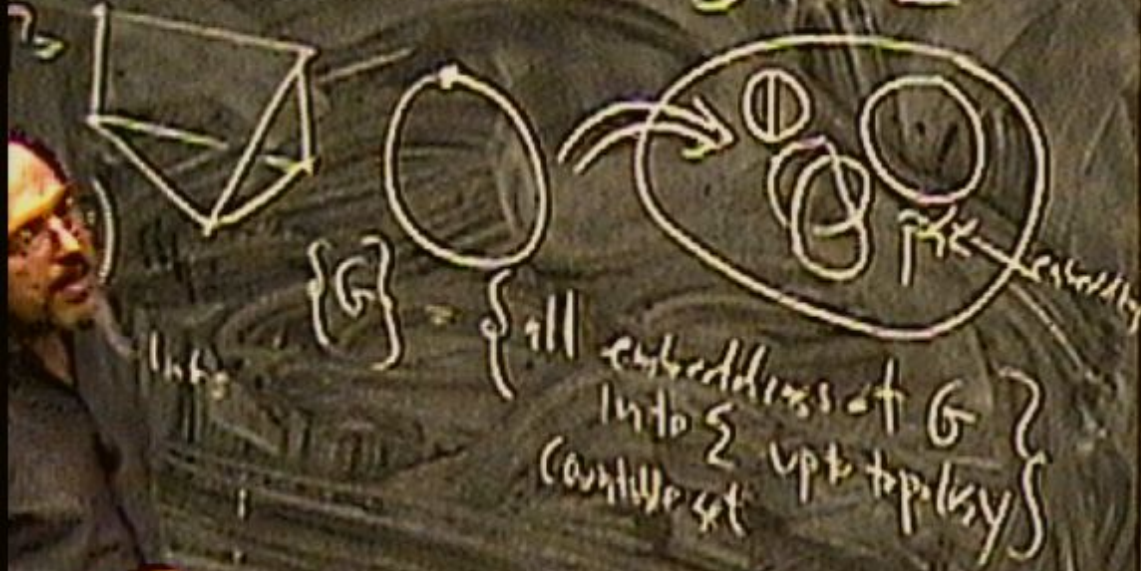
$$\langle \vec{p} | \vec{p}' \rangle = \delta_{\vec{p}, \vec{p}'}$$



Volume element in $\mathcal{H}_\Sigma$

Spatial Topology

$$S^3 = \Sigma$$



$$\mathcal{H}_\Sigma = \text{orthonormal basis}$$

$$|\psi\rangle = \sum_{\vec{p}} a_{\vec{p}}$$

$$\langle \vec{p} | \vec{p}' \rangle = \delta_{\vec{p}, \vec{p}'}$$



Volume operator in $\mathcal{H}_\Sigma$

$\mathcal{H}_\Sigma$ = orthonormal basis $|\vec{r}\rangle$ separable

$$|\psi\rangle = \sum_{\vec{r}} a_{\vec{r}} |\vec{r}\rangle \quad \sum_{\vec{r}} |a_{\vec{r}}|^2 = 1$$

$$\langle \vec{r} | \vec{r}' \rangle = \delta_{\vec{r} \vec{r}'}$$



Volume operator in $\mathcal{H}_\Sigma$

$$\hat{V} |\vec{r}\rangle = |\vec{r}\rangle V_{\vec{r}}$$

$V_{\vec{r}} = \ell_P^3$ number of 4-neighborly nearest nodes

$\{| \hat{n} \rangle\} = \text{orthonormal basis}$ $| \hat{n} \rangle$ separable

$$| \psi \rangle = \sum_{\hat{n}} a_{\hat{n}} | \hat{n} \rangle \quad \sum_{\hat{n}} | a_{\hat{n}} |^2 = 1$$

$$\langle \hat{n} | \hat{n}' \rangle = \delta_{\hat{n} \hat{n}'}$$



Volume operator in $\mathcal{H}_{\Sigma}$

$$\hat{V} | \hat{n} \rangle = | \hat{n} \rangle$$

$$V_{\hat{n}} = \int_{\hat{n}} d^3x \quad \text{number of}$$

glut holes

$\mathcal{H}_\Sigma$ = orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\hat{n}} a_{\hat{n}} |\hat{n}\rangle \quad \sum_{\hat{n}} |a_{\hat{n}}|^2 = 1$$

$$\langle \hat{n} | \hat{n}' \rangle = \delta_{\hat{n} \hat{n}'}$$



Volume operator in $\mathcal{H}_\Sigma$

$$\hat{V} |\hat{n}\rangle = |\hat{n}\rangle V_{\hat{n}}$$

$V_{\hat{n}} = l_{Pl}^3$ number of 4 or higher valent nodes

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$$

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$$

Area of boundary

All Graphs $\Gamma =$

$G = \{\text{all graphs}\}$

Finite # of nodes & links

If Σ has boundary



$\{G\} =$

$\{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$
 Countable set



All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

Finite of vertices, links

If Σ has boundary



$\{\mathcal{G}\} = \{\text{all embeddings of } \mathcal{G} \text{ into } \Sigma \text{ up to topology}\}$
 Countable set



Graphs can have edges end on boundary

All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

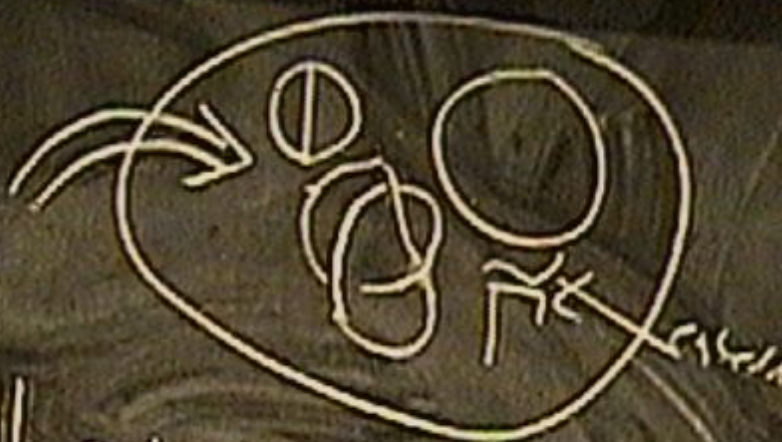
Finite of vertices, links

If Σ has boundary



$\{\mathcal{G}\} =$

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 Countable set



Graphs can have edges end on boundary

All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

Finite # of vertices, edges

If Σ has boundary



$\{G\} = \{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$
 Countable set

Graphs can have edges end on boundary

$V_{\text{in}} = \sum p_i^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle_{\text{AF}}$

$\partial D^3 = S^2$

$V_{\hat{P}} = l_{Pl}^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_{\hat{P}}$

$q_{\hat{P}} = l_{Pl}^2$ # of punctures

All Graphs $\mathbb{P} =$

$G = \{\text{all graphs}\}$

Finite set of nodes, links

if Σ has boundary



$\{G\} = \{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$
 countable set

Graphs can have edges end on boundary

$$V_{\hat{P}} = \ell_{Pl}^3 \text{ number of 4 or 4-ish valent nodes}$$

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$$

$$\text{Area of boundary } \hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{P}\rangle q_{\hat{P}}$$

$$q_{\hat{P}} = \ell_{Pl}^2 \# \text{ of punctures}$$

$$V_P = l_P^2 \text{ number of 4 or higher valent nodes}$$

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = \Sigma^2$$

$$\{\hat{A}, \hat{V}\} = 0$$

$$\text{Area of boundary } \hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle a_P$$

$$\hat{P} = l_P^2 \# \text{ of punctures}$$

$$V_P = l_P^2 \text{ number of } \psi \text{ or } \psi^\dagger \text{ at } P$$

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$$

$$\{A, \hat{V}\} = 0$$

$$\text{Area of boundary } \hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$$

$$q_P = l_P^2 \# \text{ of punctures}$$

$V_{\hat{P}} = \ell_{Pl}^3$ number of 4 or higher valent nodes

$\Sigma =$ boundary

D^3

$\partial \Sigma = S^2$

$\{\hat{A}, \hat{V}\} = 0$

Area of boundary

$$\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_{\hat{P}}$$

History

$q_{\hat{P}} = \ell_{Pl}^2$ # of punctures

$V_P = \ell_P^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{\hat{A}, \hat{V}\} = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$

$q_P = \ell_P^2$ # of punctures

History

Local dynamics

All Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

Finite # of vertices, links

if Σ has boundary



$\{\mathcal{G}\} = \{\text{all embeddings of } \mathcal{G} \text{ into } \Sigma \text{ up to topology}\}$
 Countable set

Graphs can have edges end on boundary

$V_P = \ell_P^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{\hat{A}, \hat{V}\} = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$

$q_P = \ell_P^2$ # of punctures

History

Local dynamics

4-valent nodes

$V_{\hat{P}} = l_{Pl}^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$[\hat{A}, \hat{V}] = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_{\hat{P}}$

$$q_{\hat{P}} = l_{Pl}^2 \# \text{ of punctures}$$

History

Local dynamics
means local
moves

$V_P = l_P^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$[\hat{A}, \hat{V}] = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$

$q_P = l_P^2$ # of punctures

3-valent nodes

History

Local dynamics

means local moves

$V_P = l_P^3$ number of 4-valent valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{\hat{A}, \hat{V}\} = 0$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle_{\text{AF}}$

$Q_P = l_P^2$ # of punctures

3-valent nodes

History

Local dynamics
means local
moves



$V_P = l_P^3$ number of 4-valent vertex nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{A, V\} = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle_{\text{AF}}$

$q_P = l_P^2$ # of punctures

3-valent nodes

History

Local dynamics
means local
moves



$V_P = l_P^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{A, V\} = 0$

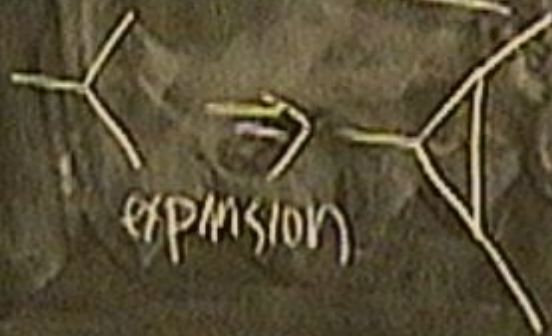
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$

$q_P = l_P^2$ # of punctures

3-valent nodes

History

Local dynamics
means local
moves



$$V_P = l_P^3 \text{ number of 4 or higher valent nodes}$$

$$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$$

$$\{\hat{A}, \hat{V}\} = 0$$

$$\text{Area of boundary } \hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_P$$

$$q_P = l_P^2 \text{ \# of punctures}$$

History

3-valent

Local dynamics
means local
moves



$V_P = l_P^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{\hat{A}, \hat{V}\} = 0$$

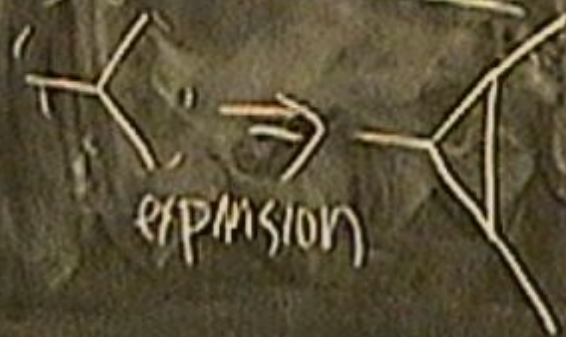
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_P$

$q_P = l_P^2$ # of punctures

History

3-valent nodes

Local dynamics
means local
moves



4-valent nodes



4-valent nodes



$V_P = l_P^3$ number of 4-valent vertex nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{\hat{A}, \hat{V}\} = 0$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle_{\text{AF}}$

$Q_P = l_P^2$ # of punctures

History

3-valent nodes

Local dynamics
means local
moves



All Graphs $\mathcal{P} =$

$G = \{\text{all graphs}\}$

Finite set of vertices, edges

if Σ has boundary



boundary



$\{G\}$

$= \{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$
(countable set)



Graphs can have edges end on boundary

$V_P = \ell_P^2$ number of 4-valent valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$[\hat{A}, \hat{V}] = 0$$

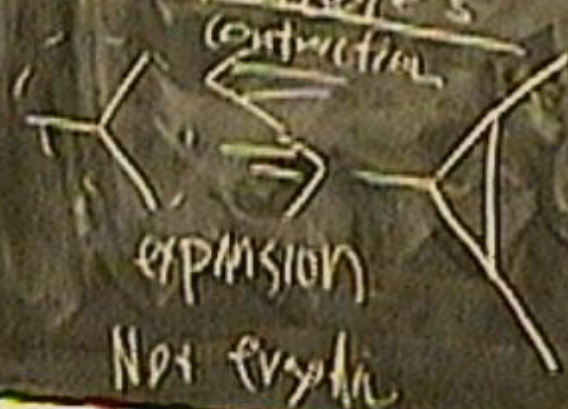
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle a_P$

$q_P = \ell_P^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



4-valent nodes



Markov Equivalence:

if you can go from any input graph to any output graph in a finite # of moves

4-valent nodes



Mixing Ergodic:

if you can go from any input graph to any output graph in a finite # of moves

4-valent nodes



Markov Equivalence:

if you can go from any input graph to any output graph in a finite # of moves

$V_{\hat{P}} = \ell_{Pl}^2$ number of 4 valent valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{\hat{A}, \hat{V}\} = 0$

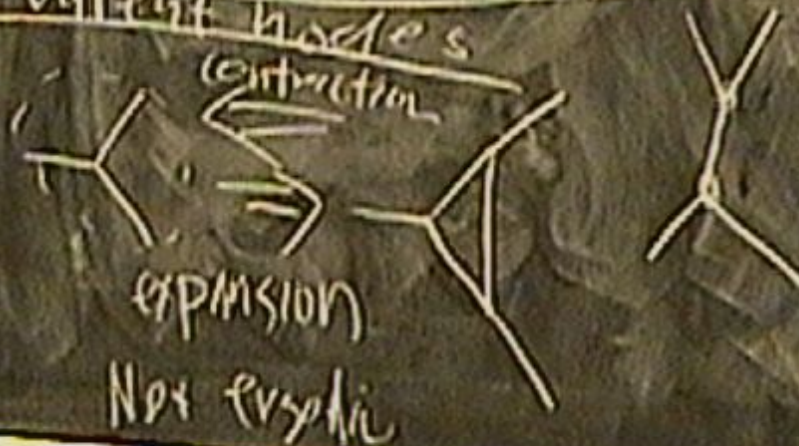
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_{\hat{P}}$

$q_{\hat{P}} = \ell_{Pl}^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



$V_P = \ell_P^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{\hat{A}, \hat{V}\} = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_P$

$q_P = \ell_P^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



$V_{\hat{P}} = \ell_{Pl}^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2$

$$\{\hat{A}, \hat{V}\} = 0$$

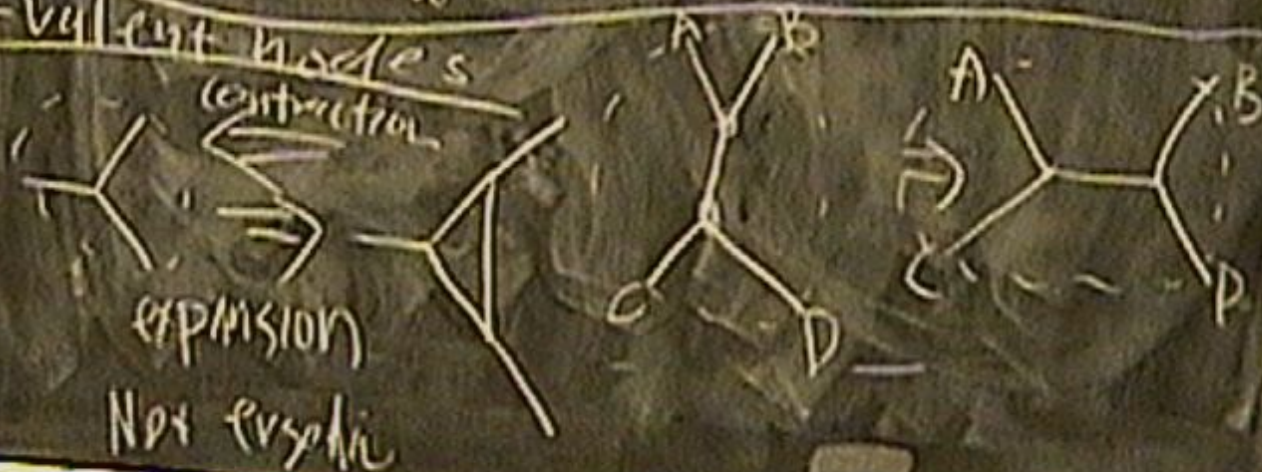
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_{\hat{P}}$

$q_{\hat{P}} = \ell_{Pl}^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



$V_{\hat{P}} = l_{Pl}^2$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{\hat{A}, \hat{V}\} = 0$

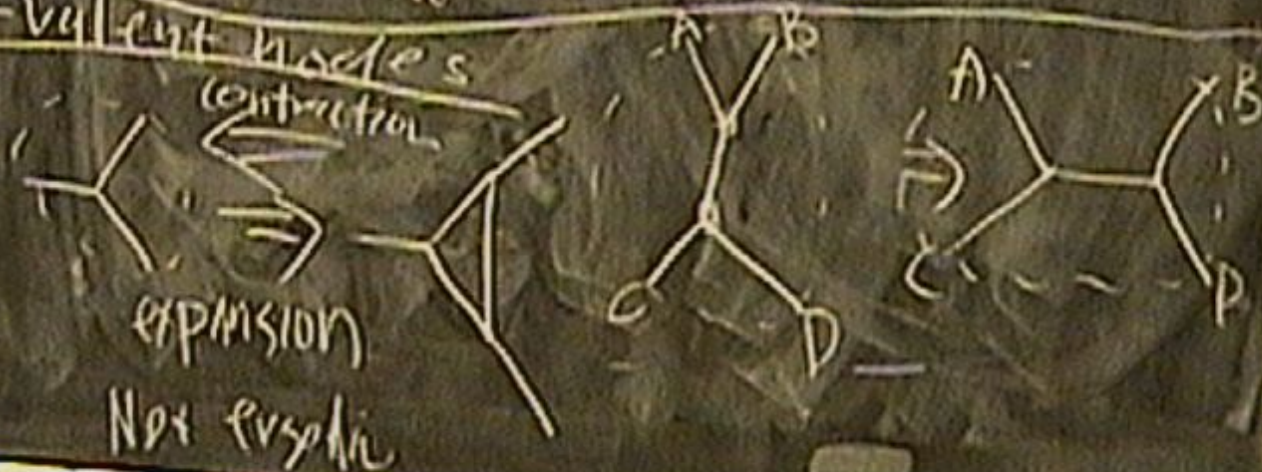
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_{\hat{P}}$

$q_{\hat{P}} = l_{Pl}^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



Not physical

$V_F = \ell_P^3$ number of 4 or higher valent nodes

$\Sigma = \text{boundary } D^3 \quad \partial \Sigma = S^2 \quad \{\hat{A}, \hat{V}\} = 0$

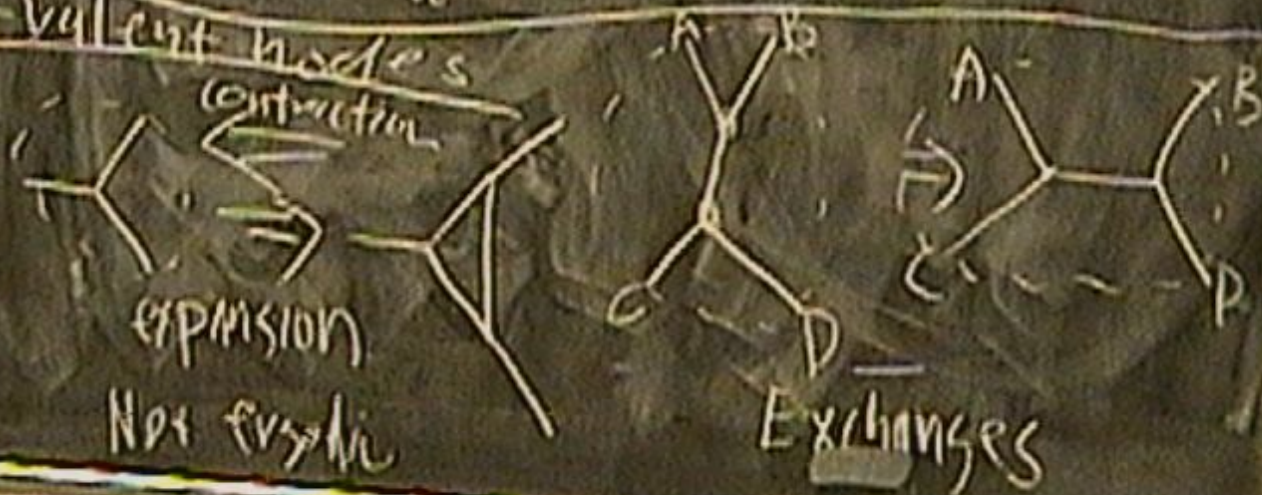
Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\hat{\Gamma}\rangle q_F$

$q_F = \ell_P^2$ # of punctures

History

Local dynamics
means local
moves

3-valent nodes



4-valent nodes



Markov Ergodic: both edges & configurations
if you can go from any input graph to any output graph in a finite # of moves

history (3-valent

$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_n$

$\downarrow$
each move is

$\Sigma = \text{boundary}$

D^3

$$\partial \Sigma = S^2$$

$$\{\hat{A}, \hat{V}\} = 0$$

Area of boundary $\hat{A}[\partial \Sigma] |\Gamma\rangle = |\Gamma\rangle q_F$

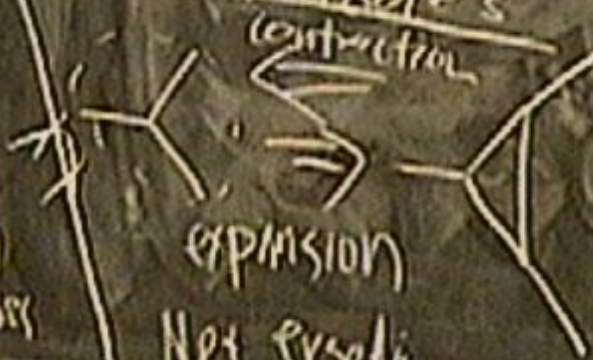
$$q_F = \ell_{Pl}^2 \# \text{ of punctures}$$

History

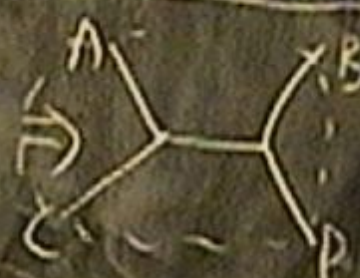
Local dynamics
means local
moves

All
local
moves

3-valent nodes



expansion
Not explicit



Exchanges

history (3-valent

$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_m$

$\downarrow$
each move is local move

4. Valent nodes

Markov Ergodic: both discrete & finite configurations

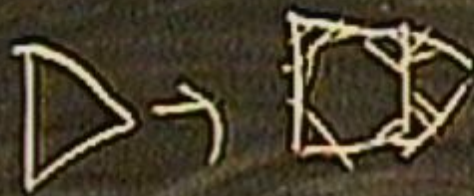
if you can go from any input graph to any output graph in a finite # of moves



4-valent nodes

Markov Equidic: both exchanges & splits & contractions

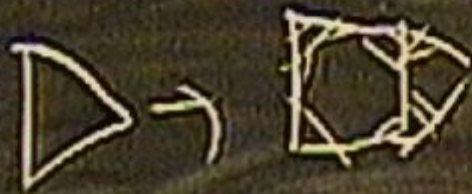
if you can go from any input graph to any output graph in a finite # of moves



4. Valent nodes

Morse-Eugodilic: both degrees & genus & configuration

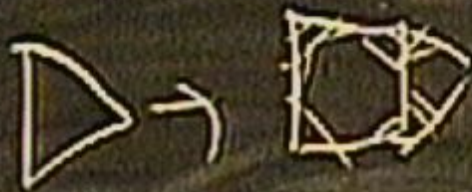
if you can go from any input graph to any output graph in a finite # of moves



4-valent nodes

Markov Ergodic: both exchanges & deletion & contraction

if you can go from any input graph to any output graph in a finite # of moves



history (3-valent) $\mathcal{F}$ = foam.

$$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_m$$

$\downarrow$
each move is local move

history (3-valent) $\mathcal{H}$

"form"

$$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_n$$

$\downarrow$
each move is local move

history (3-valent) $\mathcal{H}$

"found"

$$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_{\text{final}}$$

$\downarrow$
each move is local move

Dynamics $Q_{\text{top}}, Q_{\text{ent}}, Q_{\text{exchange}}$

history (3-valent) $\mathcal{F}$

"found"

$$\hat{\pi}_0 \rightarrow \hat{\pi}_1 \rightarrow \dots \hat{\pi}_m$$

$\downarrow$
each move is local move

Dynamics $q_{\text{top}}, q_{\text{center}}, q_{\text{exchange}}$

history has an
amplitude

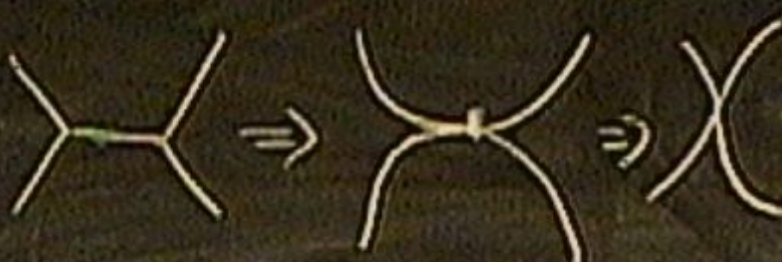
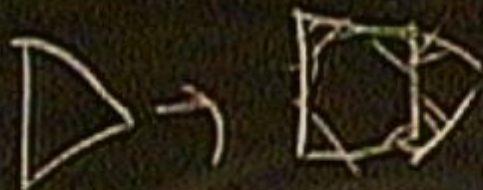
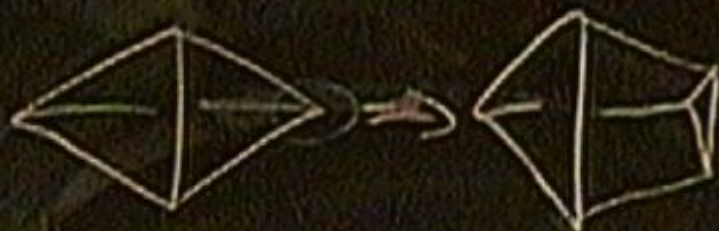
$$a(\mathcal{F}) = \prod_{\text{move}} a(\text{move})$$

4-Valent nodes

Ergodic:

both degrees & cycles & connectivity
all nodes?

is you can go from any input graph to any output graph in a finite # of moves



history (3-valent) $\mathbb{Z}$

"foam"

$\hat{\Gamma}_0 \rightarrow \hat{\Gamma}_1 \rightarrow \dots \hat{\Gamma}_n$

4-Valent Nodes



is you can go from any input graph to any output graph in a finite number of moves



history (3-valent) γ

$$\hat{\pi}_0 \Rightarrow \hat{\pi}_1 \Rightarrow \dots \hat{\pi}_{\text{final}}$$

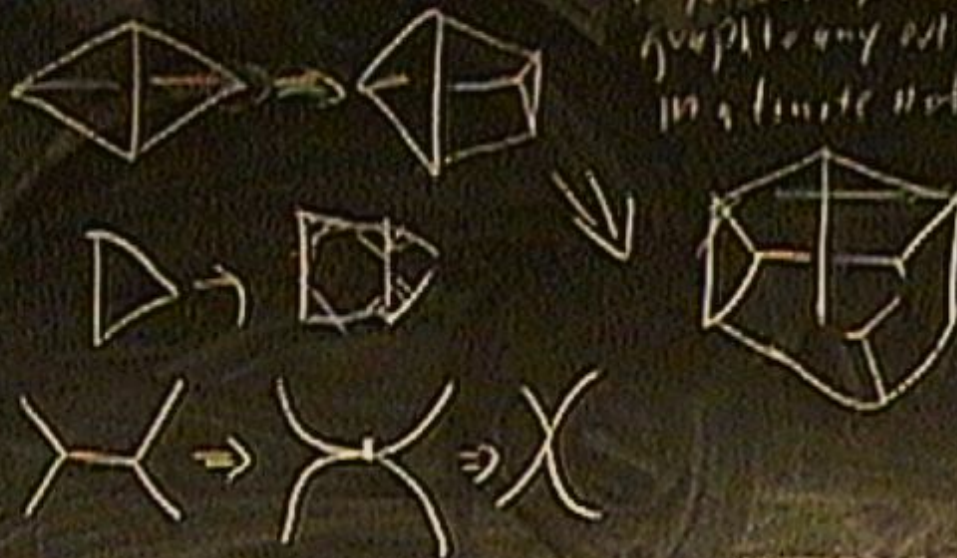
each move is local move

Dynamics $Q_{\text{top}}, Q_{\text{int}}, Q_{\text{edge}}$

history has an amplitude

$$A(\gamma) = \prod_{\text{move}} A(\text{move})$$

4. Valent nodes



if you can go from any input graph to any output graph in a finite number of moves

history (3-valent) γ

$\hat{p}_0 \Rightarrow \hat{p}_1 \Rightarrow \dots \hat{p}_{\text{final}}$
 $\downarrow$
 each move is local move

Dynamics $a_{\text{up}}, a_{\text{down}}, a_{\text{rotate}}$

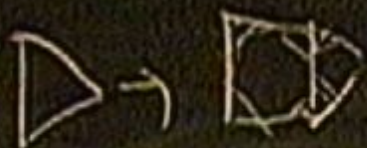
history has an amplitude $a(\gamma) = \prod_{\text{move}} a(\text{move})$

"found"

Evolution amplitude
 $\hat{p} \rightarrow \hat{p}'$
 $= \sum_{\gamma: \hat{p} \rightarrow \hat{p}'} a(\gamma)$



4-Valent Nodes



if you can go from any input graph to any output graph in a finite number of moves



history (3-valent) γ

$$\hat{P}_0 \Rightarrow \hat{P}_1 \Rightarrow \dots \hat{P}_{\text{final}}$$

each move is local move

Dynamics $q_{\text{diag}}, q_{\text{int}}, q_{\text{ext}} \dots$

history has an amplitude $a(\gamma) = \prod_{\text{move}} a(\text{move})$

"found"

Evolution amplitude

$$\hat{P}_0 \rightarrow \hat{P}_{\text{final}}$$

$$= \sum_{\gamma} a(\gamma)$$

$$\hat{P}_0 \rightarrow \hat{P}_{\text{final}}$$

Consider two moves P, Q

3m

Consider two moves P, Q

$P > Q$ if P acts on a subgraph altered by Q

4-valent nodes

Markov Equivalence: both exchanges of edges & contraction of 4 nodes

if you can go from any input graph to any output graph in a finite # of moves



4-valent nodes

Move no Edge: both ends of edge & configuration of nodes

if you can go from any input graph to any output graph in a finite # of moves



Consider two moves P, Q

$P > Q$

P

if P acts on a subgraph altered by Q

Consider two moves P, Q

$P > Q$ if P acts on a subgraph altered by Q
 $P \sim Q$ if not "causal" "causal"

4-valent nodes

Maximal Evolving: both exchanges & edges & configurations
is it needed?

if you can go from any input graph to any output graph in a finite # of moves



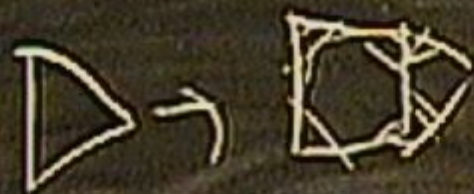
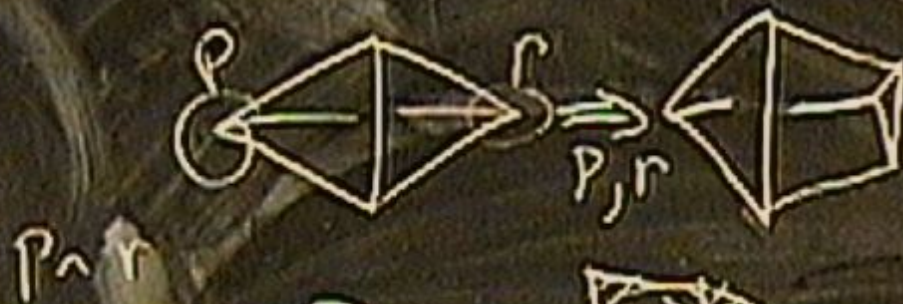
$P \sim 4$



4-valent nodes

Maximal Evagodic: both edges & edges & configurations
is it evagodic?

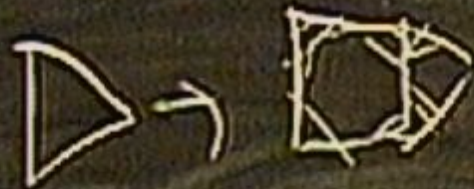
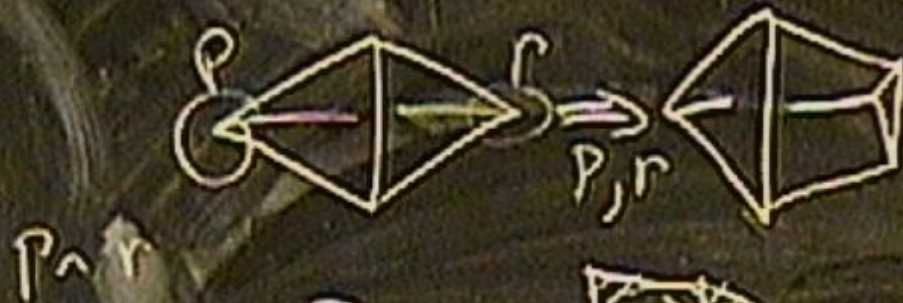
if you can go from any input graph to any output graph in a finite # of moves



4-valent nodes

Markov Equivalence: both discharge & split & contraction

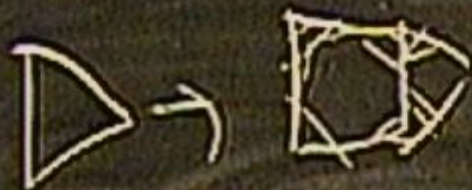
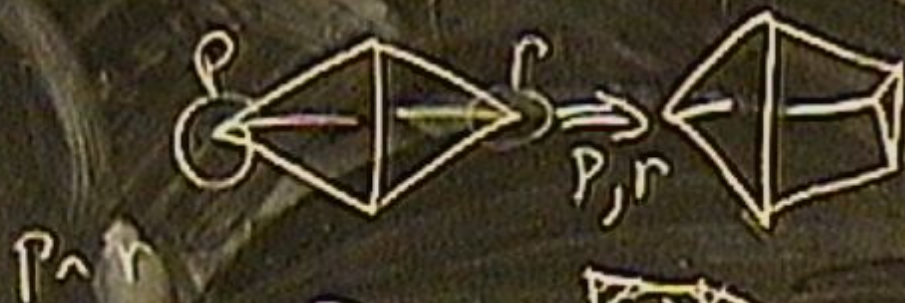
is you can go from any input graph to any output graph in a finite # of moves



4-Valent nodes

Move or Evagodic: both discharges & edges & configurations are needed

if you can go from any input graph to any output graph in a finite # of moves



q
 $\uparrow$
 P
 $\downarrow$
 r



Consider two moves P, Q

$P > Q$ if P acts on a subgraph altered by Q
 $P \sim Q$ if not "causal"

moves are a Partially ordered set

Consider two moves P, Q

$P > Q$ if P acts on a subgraph altered by Q
 $P \sim Q$ if not "causal"

moves are a Partially ordered set "Causal set"

history (3-valent) $\mathcal{F}$

$$\hat{\Gamma}_0 \rightarrow \hat{\Gamma}_1 \rightarrow \dots \hat{\Gamma}_{\text{final}}$$

$\downarrow$
each move is local move

Dynamics $a_{\text{occ}}, a_{\text{ent}}, a_{\text{exchange}}$

history has an amplitude

$$a(\mathcal{F}) = \prod_{\text{moves}} a(\text{move})$$

TOUWI

Evolution amplitude

$$\hat{\Gamma} \rightarrow \hat{\Gamma}' = E(\hat{\Gamma}, \hat{\Gamma}')$$

$$= \sum_{\mathcal{F}} a(\mathcal{F})$$

$$\hat{\Gamma} \rightarrow \hat{\Gamma}'$$

$$E(\hat{\Gamma}, \hat{\Gamma}', t)$$

history (3-valent) $\mathcal{F}$

$$\hat{P}_0 \rightarrow \hat{P}_1 \rightarrow \dots \hat{P}_{\text{final}}$$

$\downarrow$
each move is local move

Dynamics $a_{\text{loc}}, a_{\text{ent}}, a_{\text{exchange}}$

history has an amplitude

$$a(\mathcal{F}) = \prod_{\text{moves}} a(\text{move})$$

TOURNAMENT

Evolution amplitude

$$\hat{P} \rightarrow \hat{P}' = E(\hat{P}, \hat{P}')$$

$$= \sum_{\mathcal{F}} a(\mathcal{F})$$

$$\hat{P} \rightarrow \hat{P}'$$

$$E(\hat{P}, \hat{P}', t) = \sum_{\substack{\mathcal{F} \text{ (moves)} \\ \hat{P} \rightarrow \hat{P}'}}$$

X Spatial Topology

$$S^3 = \Sigma$$

Graphs $\mathcal{P} =$

$\mathcal{G} = \{\text{all graphs}\}$

Finite set of nodes and edges

if Σ has boundary



$\{\mathcal{G}\} = \{\text{all embeddings of } \mathcal{G} \text{ into } \Sigma \text{ up to topology}\}$
 countable set



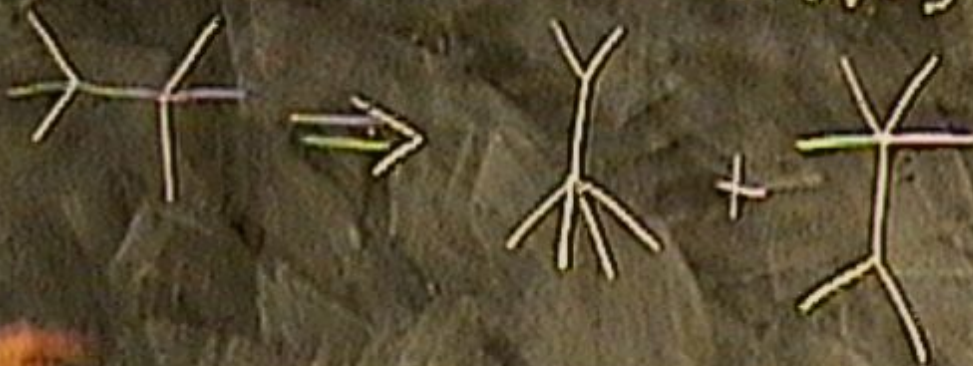
Graphs can have edges end on boundary

move nodes... move moves

more nodes... more moves



more nodes... more moves



move nodes... move moves



move nodes... move moves



spins

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

spins
addition

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

$$r_j = \sqrt{2j+1}$$

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

$$j \otimes k$$

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

$$j \otimes k = \sum_{l \in |j+k|, |j-k|} l$$

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

$$j \otimes k = \sum_{\substack{l \in |j+k| \\ l \in |j-k|}} l$$

commutative

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

$$j \otimes k = \sum_l$$

$$l \in \{j+k, j-k, |j-k|\}$$

commutative

is

$$j \otimes k \otimes l \ni 0 \text{ rep}$$

spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$\Gamma_j \oplus \Gamma_k = \bigoplus_l \Gamma_l$$

$$j \oplus k = \bigvee_{2j+1}^{2k+1}$$

complex
vector space

multiplication

$$\Gamma_j \otimes \Gamma_k = \sum_{l \in \{j+k, |j-k|\}} \Gamma_l$$

commutative

Is

$$\Gamma_j \otimes \Gamma_k \otimes \Gamma_l \ni 0 \text{ rep}$$

Yes - unique

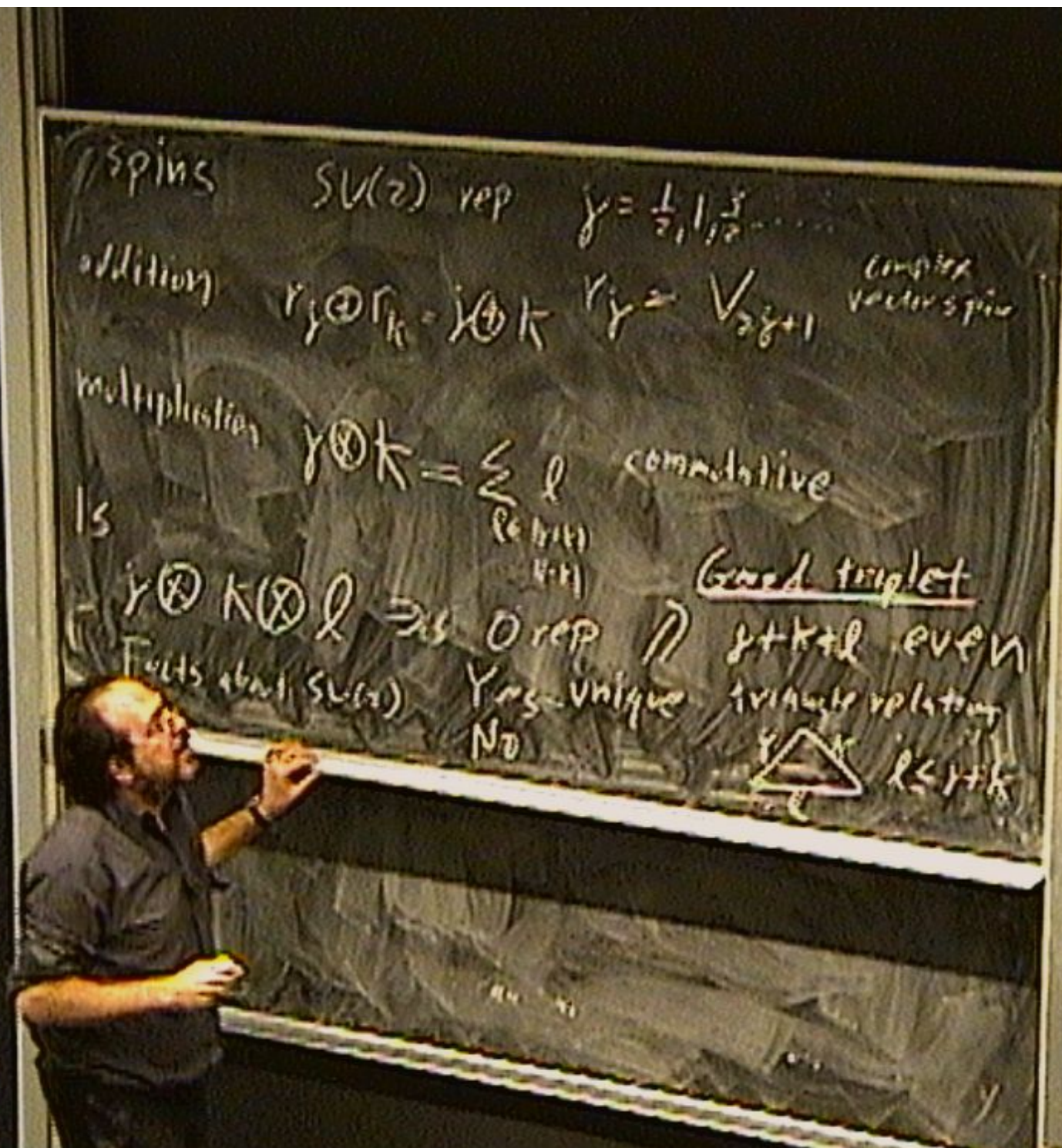
$j+k+l$ even

Facts about $SU(2)$

No

triangle relation

$$\begin{array}{c} j \\ \swarrow \quad \searrow \\ k \quad l \\ \swarrow \quad \searrow \\ j+k \end{array}$$



spins

$SU(2)$ rep

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

addition

$$r_j \oplus r_k = j \oplus k$$

$$r_j = V_{2j+1}$$

complex
vector space

multiplication

$$r_j \otimes r_k = \sum_l l$$

commutative

Is

$$r_j \otimes r_k \otimes r_l \ni 0$$

rep

Good triplet

$j+k+l$ even

Facts about $SU(2)$

Yas. unique

triangle relation

No

$$j \leq k+l$$

addition

$$r_j \oplus r_k = r_{j+k} \quad r_j = \sqrt{2j+1}$$

complex
vector space

multiplication

$$r_j \otimes r_k = \sum_{l=|j-k|}^{j+k} l \quad \text{commutative}$$

Is

$$r_j \otimes r_k \otimes r_l \ni 0 \text{ rep} \Rightarrow j+k+l \text{ even}$$

Facts about $su(2)$

Yes - unique
No

Good triplet

triangle relation



spin-network (Penrose (2))



addition $\gamma \oplus \gamma_k = \gamma \oplus k$ $\gamma \gamma = V_{2g+1}$

complex
vector space

multiplication $\gamma \otimes k = \sum_l$ commutative

is

$l \in \{k+1, \dots, u+1\}$

Good triplet

$\gamma \otimes k \otimes l \ni 0 \text{ rep } \Rightarrow \gamma+k+l \text{ even}$

Facts about $SU(2)$ Yes-unique triangle relation

No



spin-network (Penrose 67)

graph Γ where edges $| \gamma \rangle$ $\gamma \gamma_k$ all good



addition

$$\gamma_j \oplus \gamma_k = \gamma_{j+k} \quad \gamma_j = V_{2j+1}$$

complex
vector space

multiplication

$$\gamma \otimes k = \sum_{\substack{l \in \mathbb{N} \\ l+k=j}} l \quad \text{commutative}$$

Is

$$\gamma \otimes k \otimes l \ni 0 \text{ rep } \Rightarrow j+k+l \text{ even}$$

Facts about $su(2)$

Yes - unique
No

Good triplet

triangle relation



spin-network (Penrose 67)

(3 valent)

graph Γ where edges $\vdash \gamma$

$\gamma \vdash \gamma_k$ all good

Spinnet



$\mathcal{H}_\Sigma^{SU(2), \text{2-valent}}$

= orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\vec{n}} a_{\vec{n}} |\vec{n}\rangle \quad \sum_{\vec{n}} |a_{\vec{n}}|^2 = 1$$

$$\langle \vec{n} | \vec{n}' \rangle = \delta_{\vec{n} \vec{n}'}$$



Volume operator in $\mathcal{H}_\Sigma$

$$\hat{V} |\vec{n}\rangle = |\vec{n}\rangle V_{\vec{n}}$$

$V_{\vec{n}} = \ell_P^3$ number of 4-valent vertex nodes

Facts about $SU(2)$

Yes-unique
No

triangle relation



$$l \leq r+k$$

spin-network (Penrose '62)

(3-valent)

graph Γ where edges $|-\gamma$



all good

Spinnet embedding $\hat{\Gamma}$



Facts about $SU(2)$

Yes-unique
No

triangle relation



$$l \leq m + n$$

spin-network (Penrose 67) (3-valent)

graph Γ where edges $|-\gamma$ $\gamma \vee \kappa$ all good

Spinnet embedding $\tilde{\Gamma}$



$$\text{Inv}(\gamma \otimes \kappa \otimes l \otimes m)$$

$$\gamma \otimes \kappa \otimes l \otimes m \rightarrow 0$$

Facts about $SU(2)$

Yes - unique
No

triangle relation

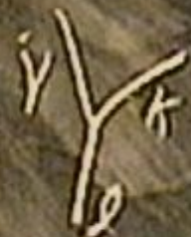


$$l \leq l+k$$

spin-network (Penrose '67)

(3-valent)

graph Γ where edges $|-\gamma$



all good

Spinnet embedding $\hat{\Gamma}$



$$\text{Inv}(\gamma \otimes k \otimes l \otimes m) \Rightarrow$$

$$\gamma \otimes k \otimes l \otimes m \rightarrow 0$$

Facts about $SU(2)$

Yes-unique
No

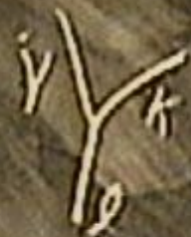
triangle relation



$$l \leq l+k$$

spin-network (Penrose 62) (3-valent)

graph Γ where edges $|-\gamma$



all good

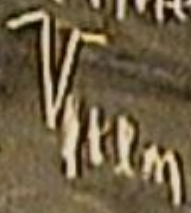
Spinnet embedding $\hat{\Gamma}$



$\text{Inv}(\gamma \otimes k \otimes l \otimes m)$

$\gamma \otimes k \otimes l \otimes m \rightarrow 0$

defines



graph 17

Spinnet embed


$$\text{Inv}(\tilde{Y} \otimes K(X))$$

YOKO LOM

$\mathcal{H}_{\Sigma}^{SU(2), \text{spin}}$ = orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\vec{n}} a_{\vec{n}} |\vec{n}\rangle \quad \sum_{\vec{n}} |a_{\vec{n}}|^2 = 1$$

$$\langle \vec{n} | \vec{n}' \rangle = \delta_{\vec{n} \vec{n}'}$$



A general spinnet graph Γ

$$|-i \quad \begin{array}{c} \nearrow \\ \times \\ \searrow \end{array} \Rightarrow |\psi\rangle \in \mathcal{H}_{\text{spin}}$$

$\mathcal{H}_S^{SU(2), \text{spin}}$ = orthonormal basis $|A\rangle$ separable

$$|\psi\rangle = \sum_{\vec{n}} a_{\vec{n}} |\vec{n}\rangle \quad \sum_{\vec{n}} |a_{\vec{n}}|^2 = 1$$

$$\langle \vec{n} | \vec{n}' \rangle = \delta_{\vec{n} \vec{n}'}$$



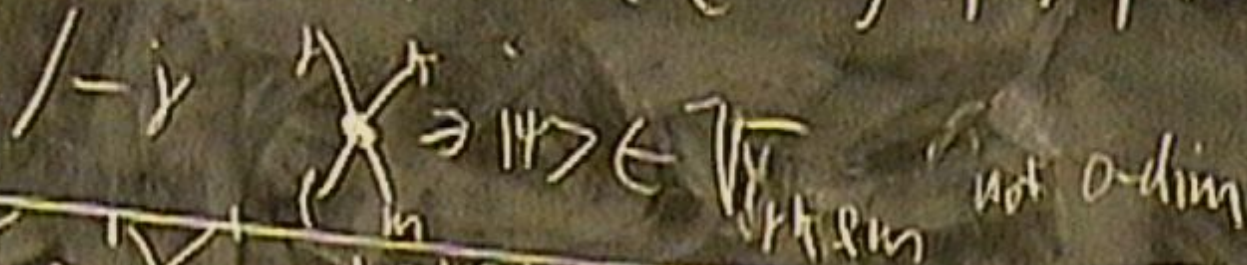
A general spinnet graph Γ

$1-y$  $\Rightarrow |\psi\rangle \in \mathcal{H}_{\Gamma}$ not 0-dim

$$\langle \hat{P} | \hat{P}' \rangle = \delta_{\hat{P} \hat{P}'}$$



A general SPINNET graph Γ

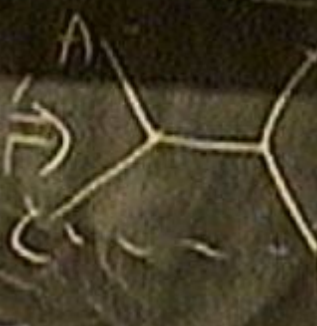
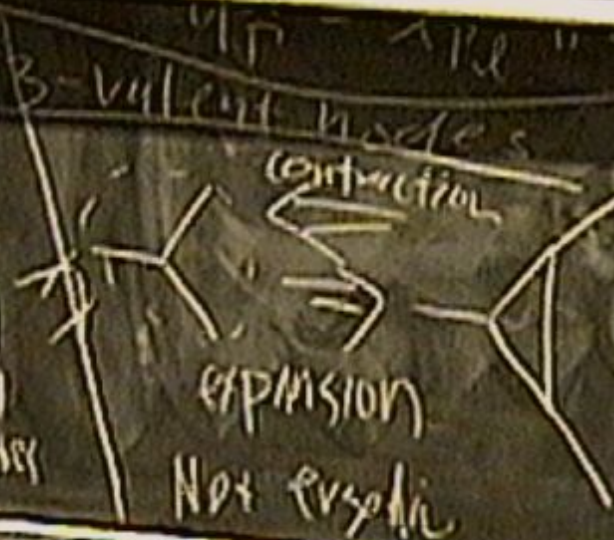


$$10010101 \rightarrow 0$$

History

Local dynamics
means local
moves

All
local
moves



Exchanges

$$\langle \hat{P} | \hat{P}' \rangle = \delta_{\hat{P} \hat{P}'}$$

A general SPINNET graph Γ

$1-y$  $\Rightarrow 14 \in \mathbb{N}$ not 0-dim

$X \Rightarrow$ 

$1010101 \rightarrow 0$
 $101 \rightarrow 101$

History

Local dynamics
 means local
 moves

All
 local
 moves



contraction

expansion

Not explicit



Exchanges

$\sum_{i=1}^n (2) \nu_i$

$\# \sum_{\text{class}}^{su(2)}$

morally

$\#$ quantum GK in $3+1$ in Σ

$\# \sum^{SU(2)}$

morally $\#$
topological embeddings
by piecewise
diffeomorphic
embeddings

quantum GK in $3+1$ in Σ

$\# \Sigma^{SU(2)}$

morally $\#$
topological embeddings
by piecewise
diffeomorphic
embeddings

quantum GR in 3+1 in Σ

LOST theorem

Facts about $SL(2)$

Yes-unique
No

triangle relation



$$l \leq r+k$$

spin-network (Penrose 62) (3-valent)

graph Γ where edges $\vdash \gamma$ $\gamma \vee \gamma_k$ all good

Spinnet embedding $\hat{\Gamma}$



$Inv(\gamma \otimes k)$
 $\gamma \otimes k \otimes l$



$\Rightarrow$ defines vector space

$\# \sum^{SU(2)}$

morally $\#$
topological embeddings
by piecewise
diffeomorphic
embeddings

quantum GK in $3+1$ in Σ

LOST chrom

history (3-valent) $\mathcal{F}$

$$\hat{P}_0 \rightarrow \hat{P}_1 \rightarrow \dots \hat{P}_{\text{final}}$$

each move is local move

Dynamics $a^{(\text{move})}_{\text{loc}}, a_{\text{entr}}, a_{\text{exchange}}$

history has an amplitude

$$a(\mathcal{F}) = \prod_{\text{moves}} a(\text{move})$$

"form"

Evolution amplitude

$$\hat{P} \rightarrow \hat{P}' = E(\hat{P}, \hat{P}')$$

$$= \sum_{\mathcal{F}} \sum_{\text{local moves}} a(\mathcal{F})$$

$$|\hat{P} \rightarrow \hat{P}'|$$

$$E(\hat{P}, \hat{P}', t) = \sum_{\substack{\mathcal{F} \text{ t moves} \\ \hat{P} \rightarrow \hat{P}'}}$$

Facts about $SU(2)$

Yes-unique
No

triangle relation



$l \leq k$

spin-network (Penrose 62) (3-valent)

graph Γ where edges $|-\gamma$ $\gamma \vee \pi$ all good

Spinnet embedding $\tilde{\Gamma}$



$\text{Inv}(\gamma \otimes k \otimes l \otimes m)$

$\gamma \otimes k \otimes l \otimes m \rightarrow 0$

defines vector space

history (3-valent) $\mathcal{F}$

$$\hat{P}_0 \rightarrow \hat{P}_1 \rightarrow \dots \hat{P}_n$$

$\downarrow$
each move is local move

Dynamics $a^{(move)}$
 $a_{loc}, a_{int}, a_{exchange}$

history has an
amplitude

$$a(\mathcal{F}) = \prod_{\text{moves}} a(\text{move})$$

"form"

Evolution amplitude

$$\hat{P} \rightarrow \hat{P}' = E(\hat{P}, \hat{P}')$$

$$= \sum_{\mathcal{F}} \sum_{\text{local moves}} a(\mathcal{F})$$

$$\hat{P} \rightarrow \hat{P}'$$

$$E(\hat{P}, \hat{P}', t) = \sum_{\substack{\mathcal{F} \text{ times} \\ \hat{P} \rightarrow \hat{P}'}}$$