Title: Generating scale-invaraint power spectrum in string gas cosmology

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Abstract: We study the generation of cosmological perturbations during the Hagedorn phase of string gas cosmology. Using tools of string thermodynamics we provide indications that it may be possible to obtain a nearly scale-invariant spectrum of cosmological fluctuations on scales which are of cosmological interest today. In our cosmological scenario, the early Hagedorn phase of string gas cosmology goes over smoothly into the radiation-dominated phase of standard cosmology, without having a period of cosmological inflation. Furthermore, we find that string thermodynamics implies that the fluctuations are Gaussian, and that the spectrum of tensor perturbations will exhibit a scale-invariant spectrum as well. We contrast the predictions of string gas cosmology in the Hagedorn phase with that of scalar field driven inflation, and comment on the possibility of observationally distinguishing between the two scenarios in future experiments.

String Gas Cosmology : A Scenario with Scale-Invariant Scalar Power Spectrum

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Wednesday, January 18, 2006

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- Our goal and our vision.
- Review of Brandenberger-Vafa, *a.k.a*, string gas cosmology.

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- Open strings[?]

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- String cosmology and dilaton.
- Statistical mechanics of closed strings.
- Calculating the power spectrum.
- Open strings[?]
- Conclusion.

Our Goal

Here we study the generation of cosmological fluctuations during the early Hagedorn phase of string gas cosmology using the tools of string statistical mechanics. Since this early phase is quasi-static, the Hubble radius (1/H(t)) is very large (infinite in the limit of the exactly static case). The approximation of thermodynamic equilibrium is justified on scales smaller than the Hubble radius. In this phase, a gas of closed strings induces a scale-invariant spectrum of scalar metric fluctuations on all scales smaller than the Hubble radius. Provided that the expansion of space is sufficiently slow, these scales will include all scales which are currently being probed by cosmological observations:

$$\langle(\delta M)^2\rangle = \langle E^2\rangle - \langle E\rangle^2 = T^2 C_V .$$

Our Vision

- Provided that the spectrum in these fluctuations is not distorted at the time of the transition from the Hagedorn phase to the usual phase of radiation-domination of standard cosmology (which is unlikely), it follows that string gas cosmology will lead -without invoking a period of inflation - to a scale-invariant spectrum of adiabatic curvature fluctuations.

Brandenberger-Vafa String Cosmology

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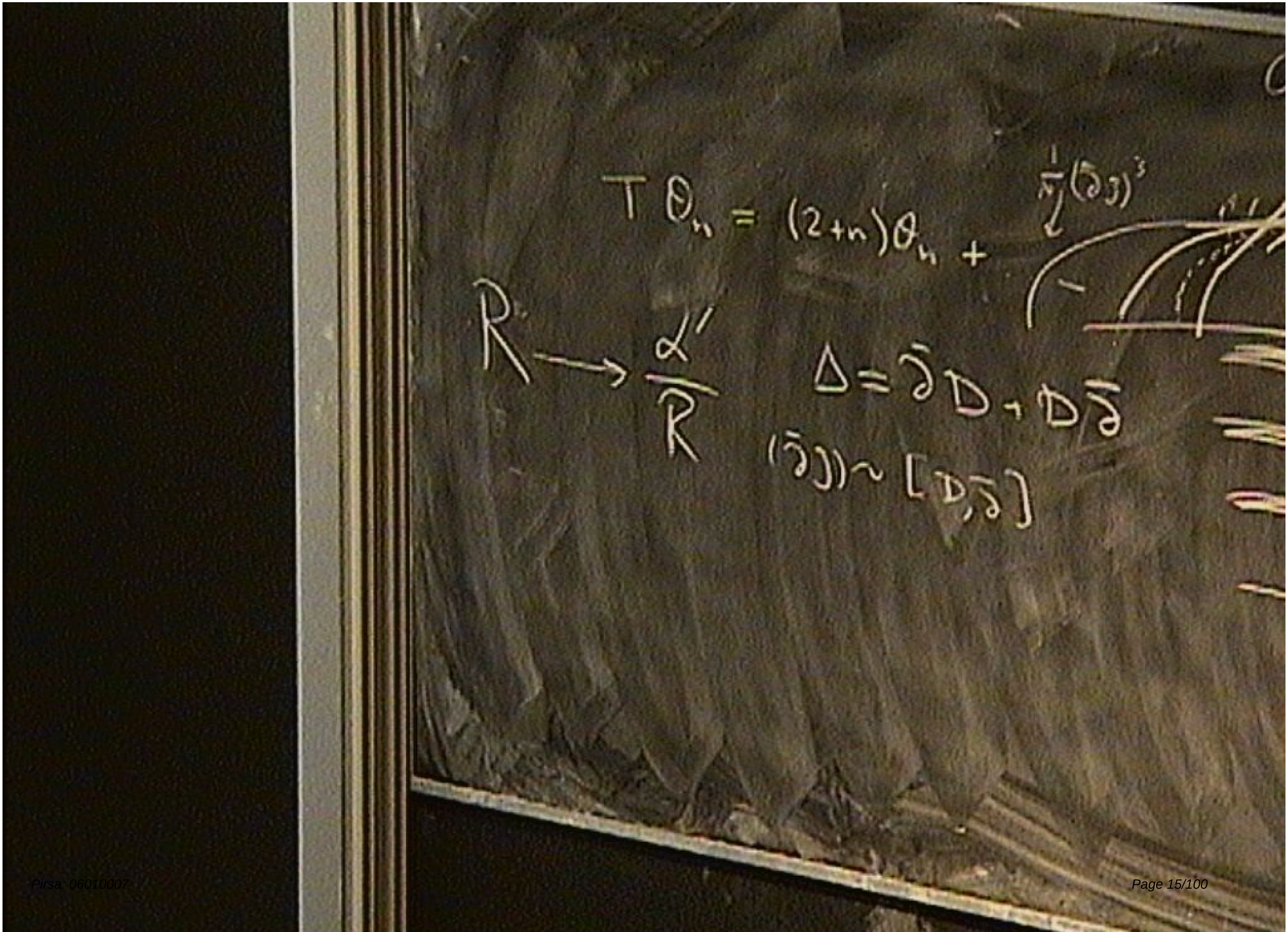
$T\theta_n = (2+n)\theta_n + \frac{1}{n}(\bar{\partial}J)^3$
$R \to \frac{\alpha'}{R}$
$\Delta = \bar{\partial}D + D\bar{\partial}$
$(\bar{\partial}J) \sim [D, \bar{\partial}]$

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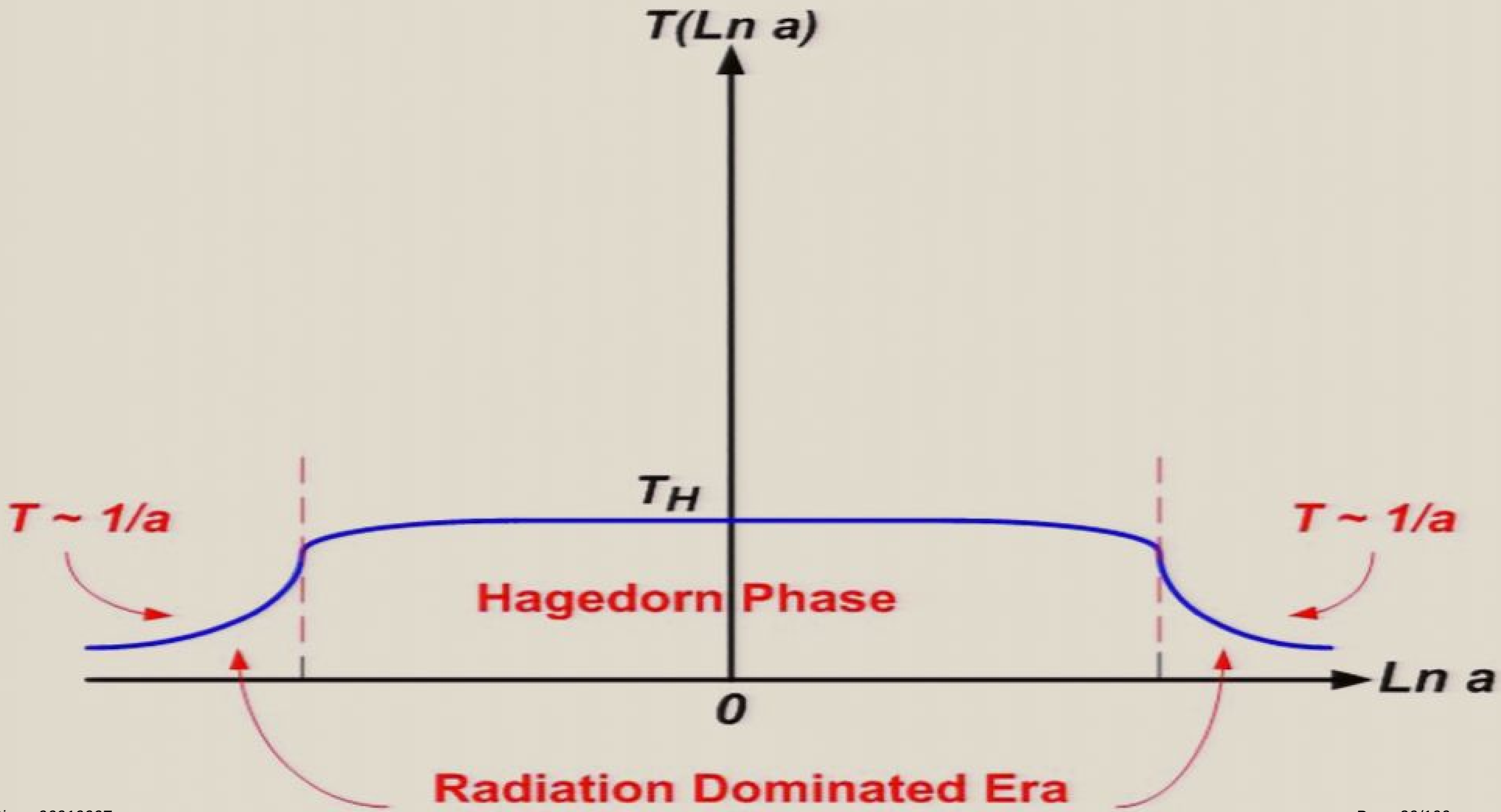
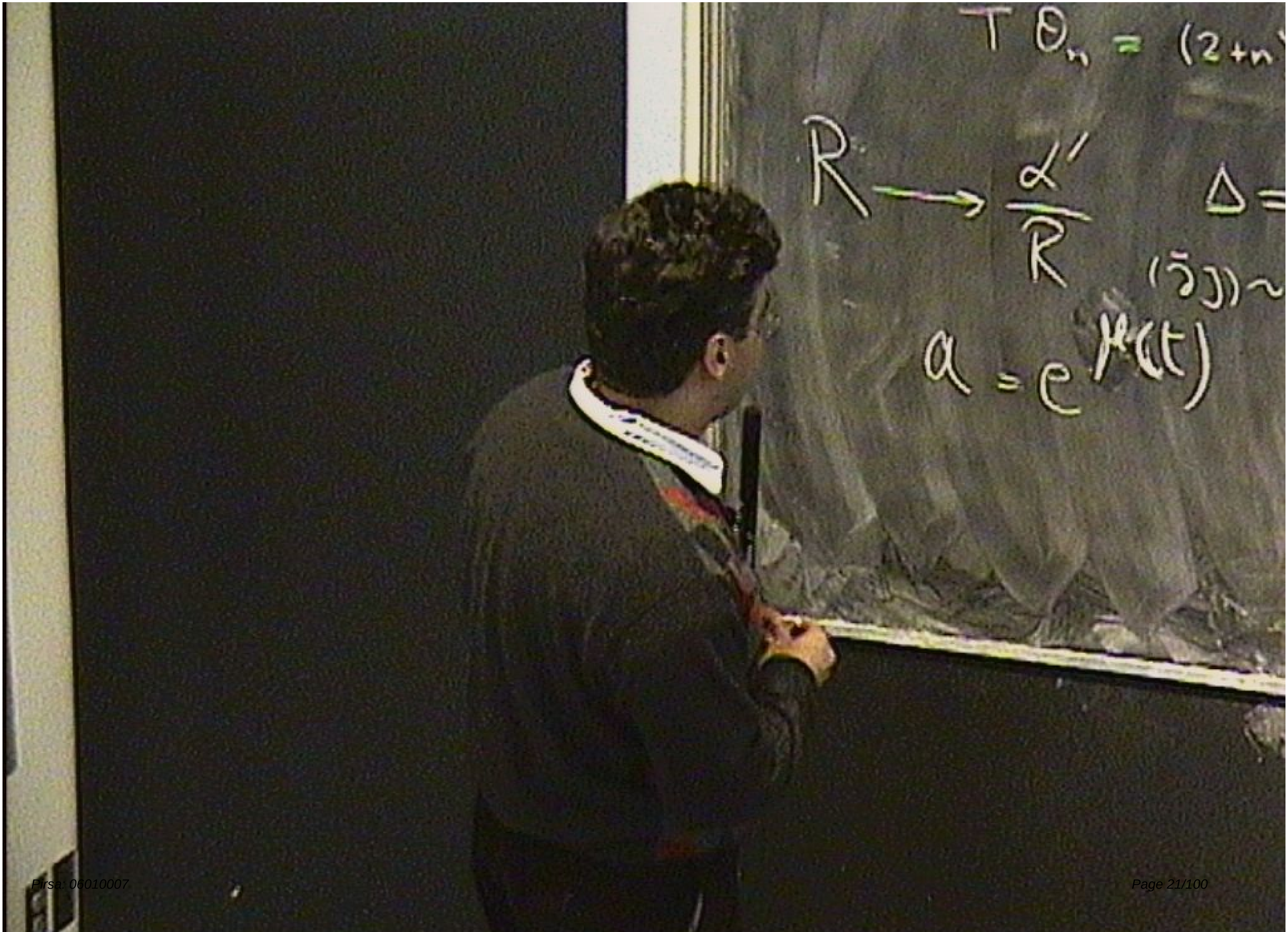
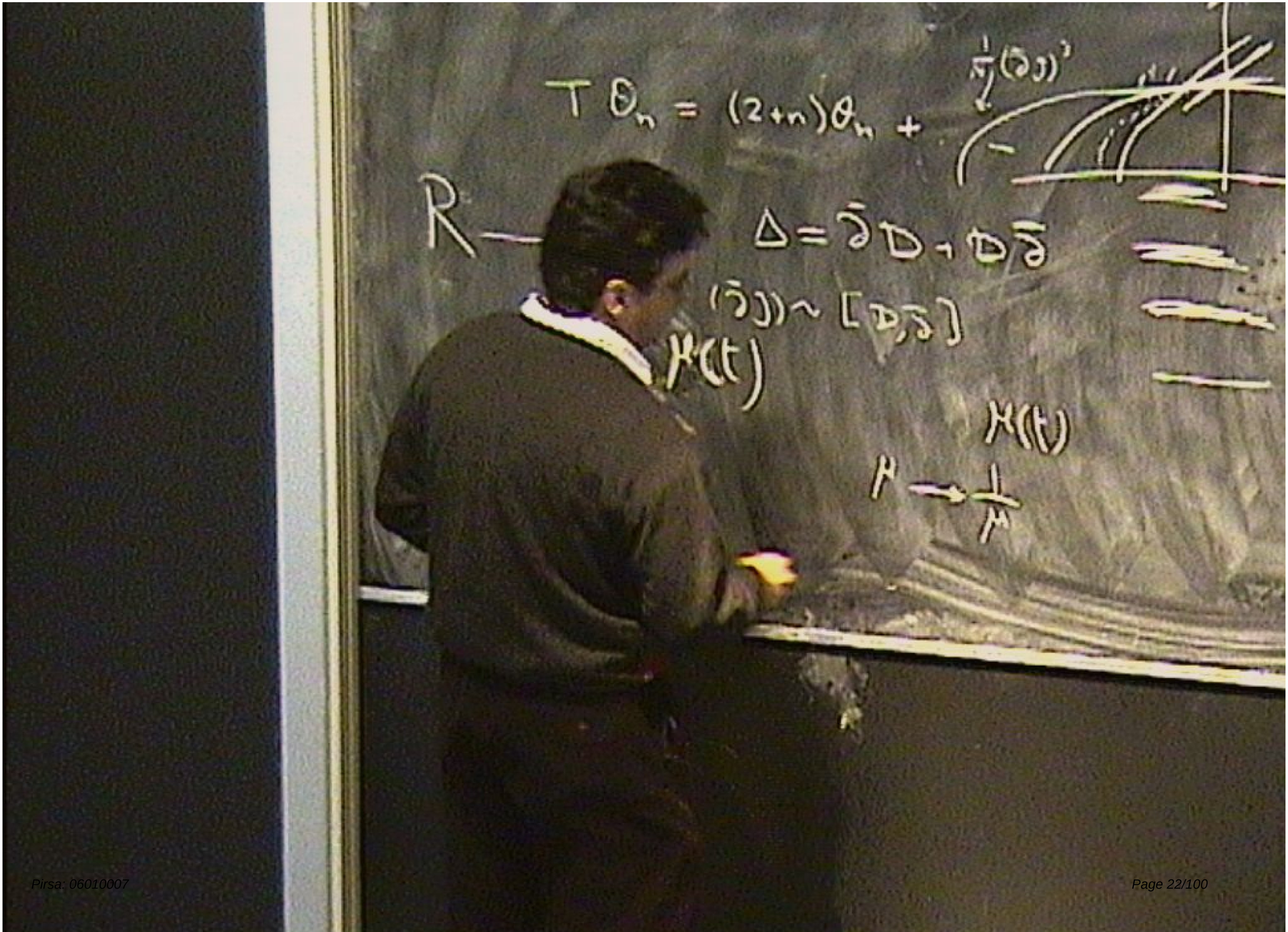
- More on thermodynamic side rather than dynamical side.

Behavior of Temperature




TΘn = (2+n
R → α'/R
Δ =
a = e^μ(t)



T θn = (2+n)θn +
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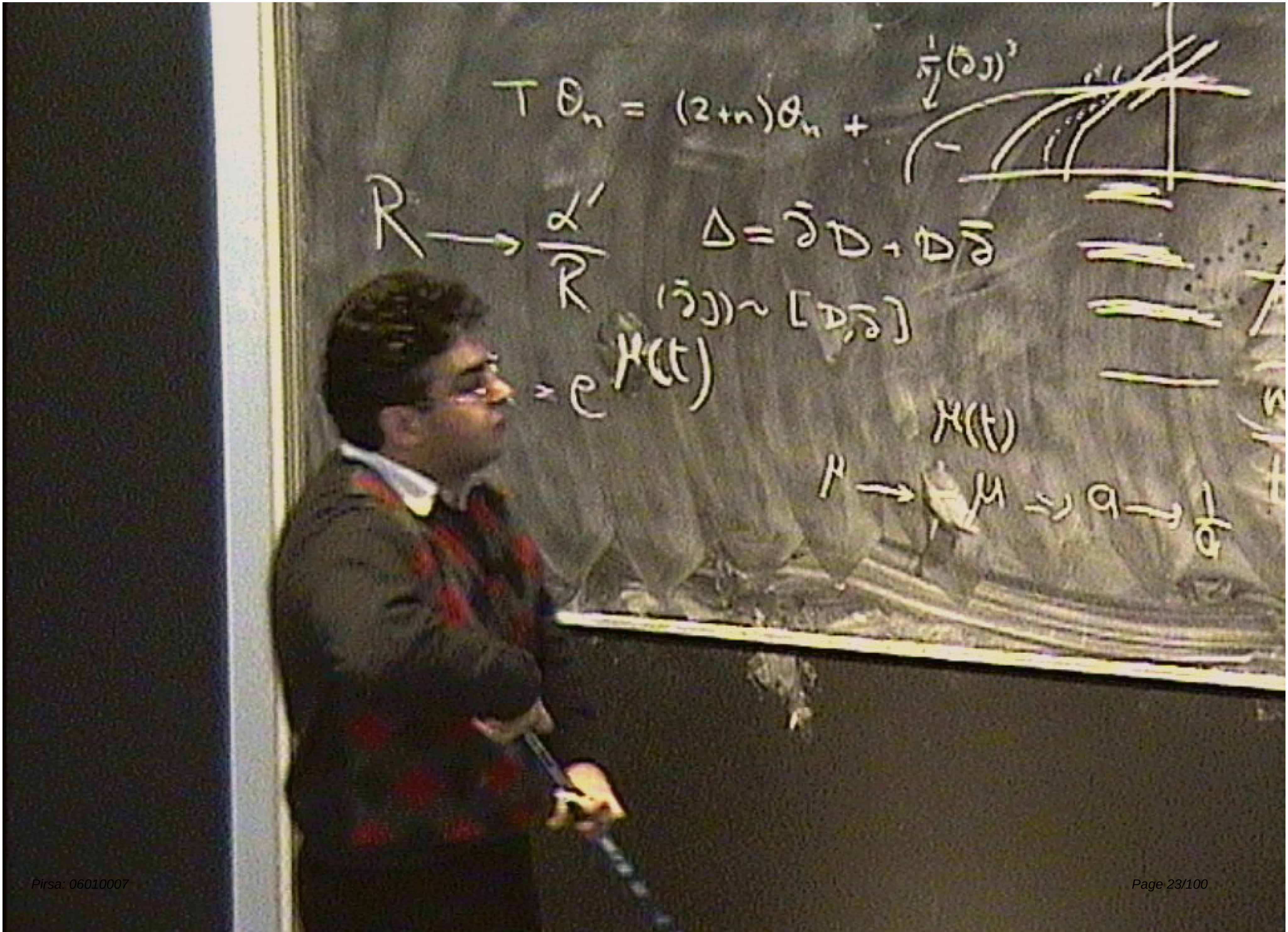
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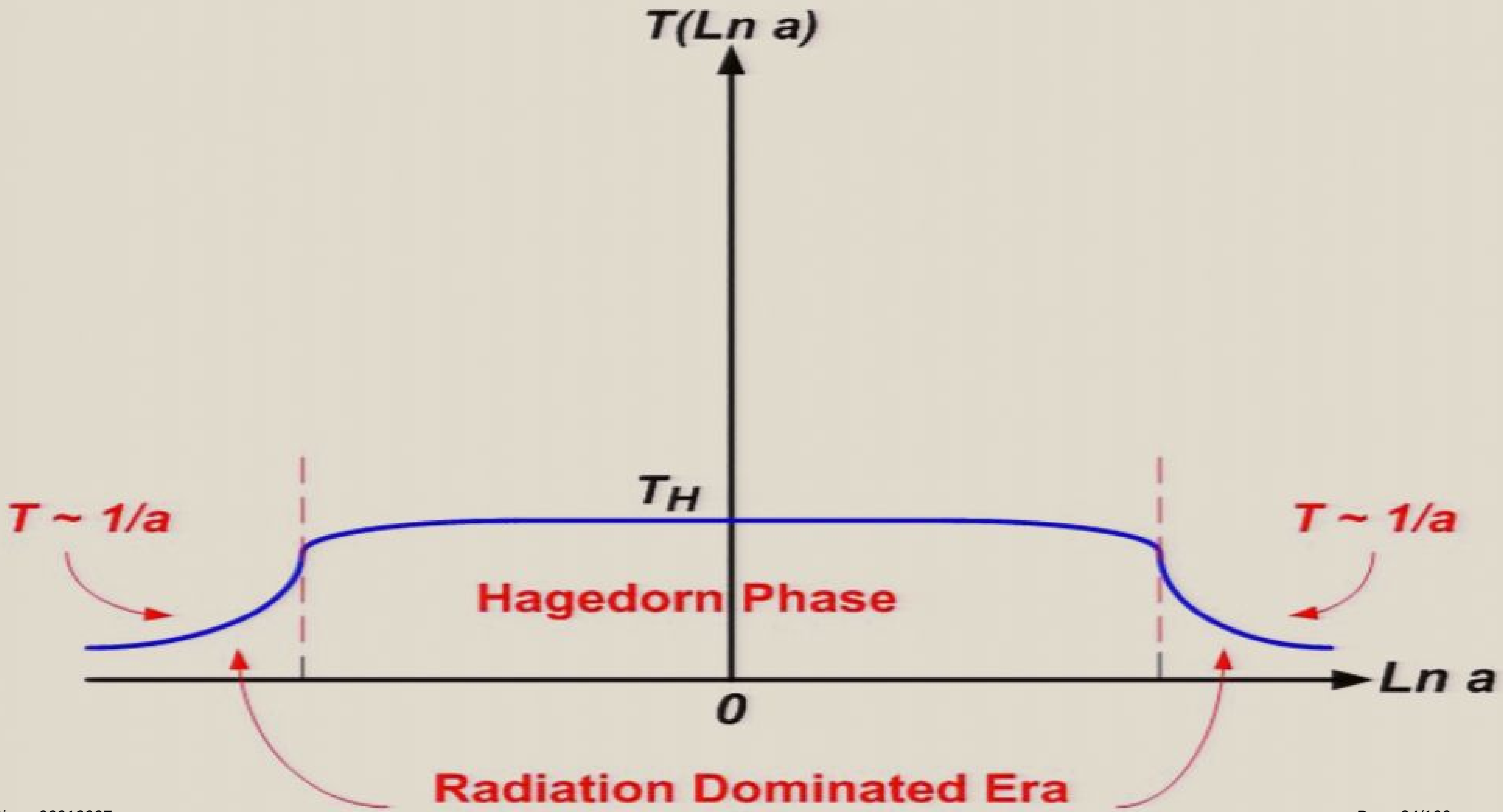

Dilatonic String Cosmology

$$\mathcal{A} = \int \sqrt{-{}^{(10)}g}\, d^{10}x\, e^{-2\phi} \left[{}^{(10)}R + 4(\nabla\phi)^2 + \dots \right] ,$$

$$ds^2 = -dt^2 + \sum_i^{(d)} a^2 dx_i^2 + \sum_i^{(9-d)} a_s^2 dx'^2_i ,$$

Dilatonic String Cosmology: Field Equations

$$-(d)\dot{\mu}^2 - (9-d)\dot{\nu}^2 + \dot{\varphi}^2 = e^{\varphi} E\,,$$

$$\ddot{\mu} - \dot{\varphi}\dot{\mu} = \frac{1}{2} e^{\varphi} P_d\,,$$

$$\dot{\nu} - \dot{\varphi}\dot{\nu} = \frac{1}{2} e^{\varphi} P_{9-d}\,,$$

$$\ddot{\varphi} - (d)\dot{\mu}^2 - (9-d)\dot{\nu}^2 = \frac{1}{2} e^{\varphi} E\,.$$

$$\varphi \equiv 2\phi - (d)\mu - (9-d)\nu\,,$$

$$a(t) = e^{\mu}(t)\,, \qquad a_s(t) = e^{\nu}(t)\,,$$

$$V = (2\pi\sqrt{\alpha'})^9 a^{(d)} a_s^{(9-d)} \equiv (2\pi\sqrt{\alpha'})^9 e^{(d)\mu} e^{(9-d)\nu}\,.$$

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Equations in Terms of Original Dilaton

$$-(d)\dot{\mu}^2-(9-d)\dot{\nu}^2+[\dot{\phi}-(d)\dot{\mu}-(9-d)\dot{\nu}]^2=e^{\phi}\rho\,,$$

$$\ddot{\mu}-[\dot{\phi}-(d)\dot{\mu}-(9-d)\dot{\nu}]\dot{\mu}=\frac{1}{2}e^{\phi}p_d\,,$$

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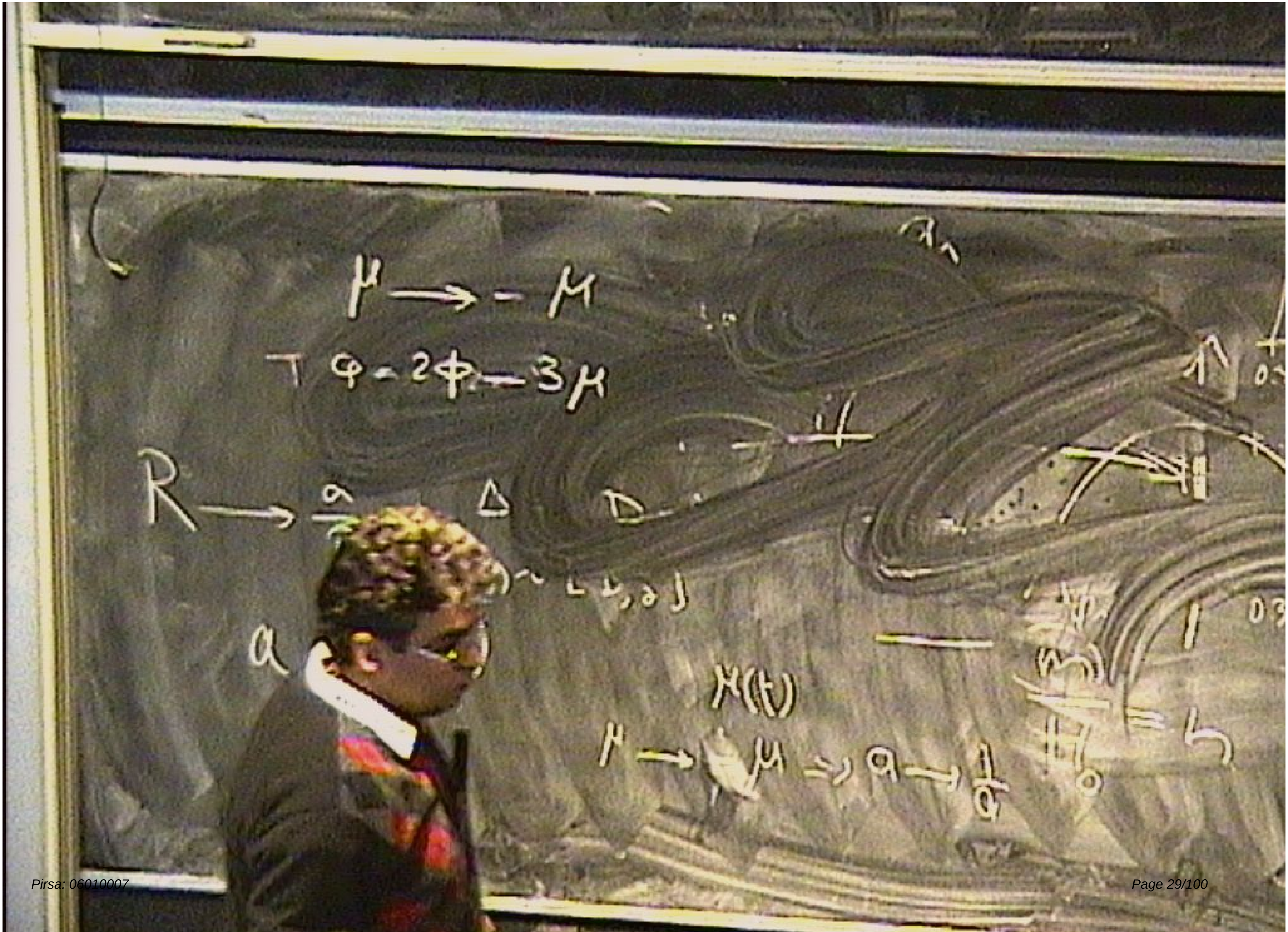

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$$\phi(t) = \ln\left[\frac{16\pi^2/E}{t(t+c)}\right]$$

$$\mu(t) = A + B\ln\left(\frac{t}{t+c}\right)$$

at late times

$$e^{\phi} \sim \frac{constant}{t^2} \quad a(t) \sim constant$$

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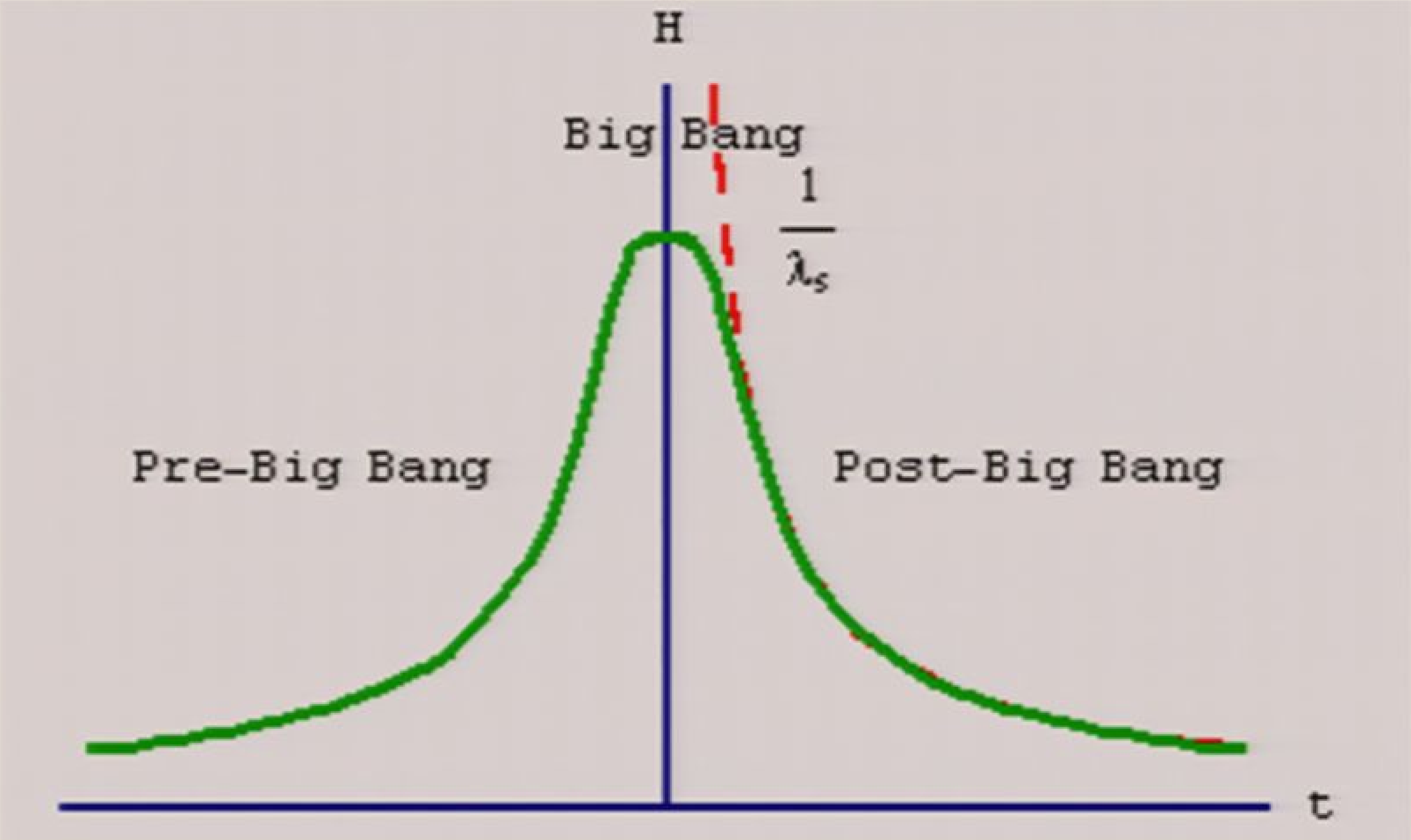
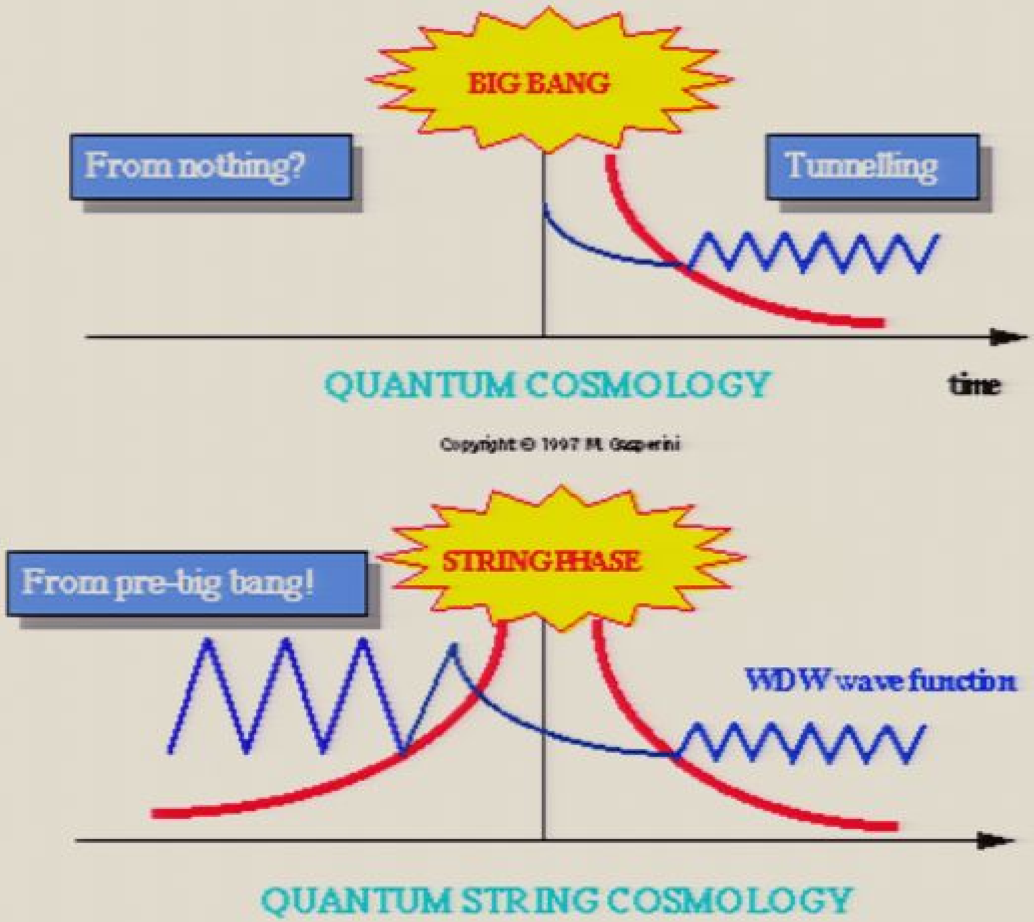
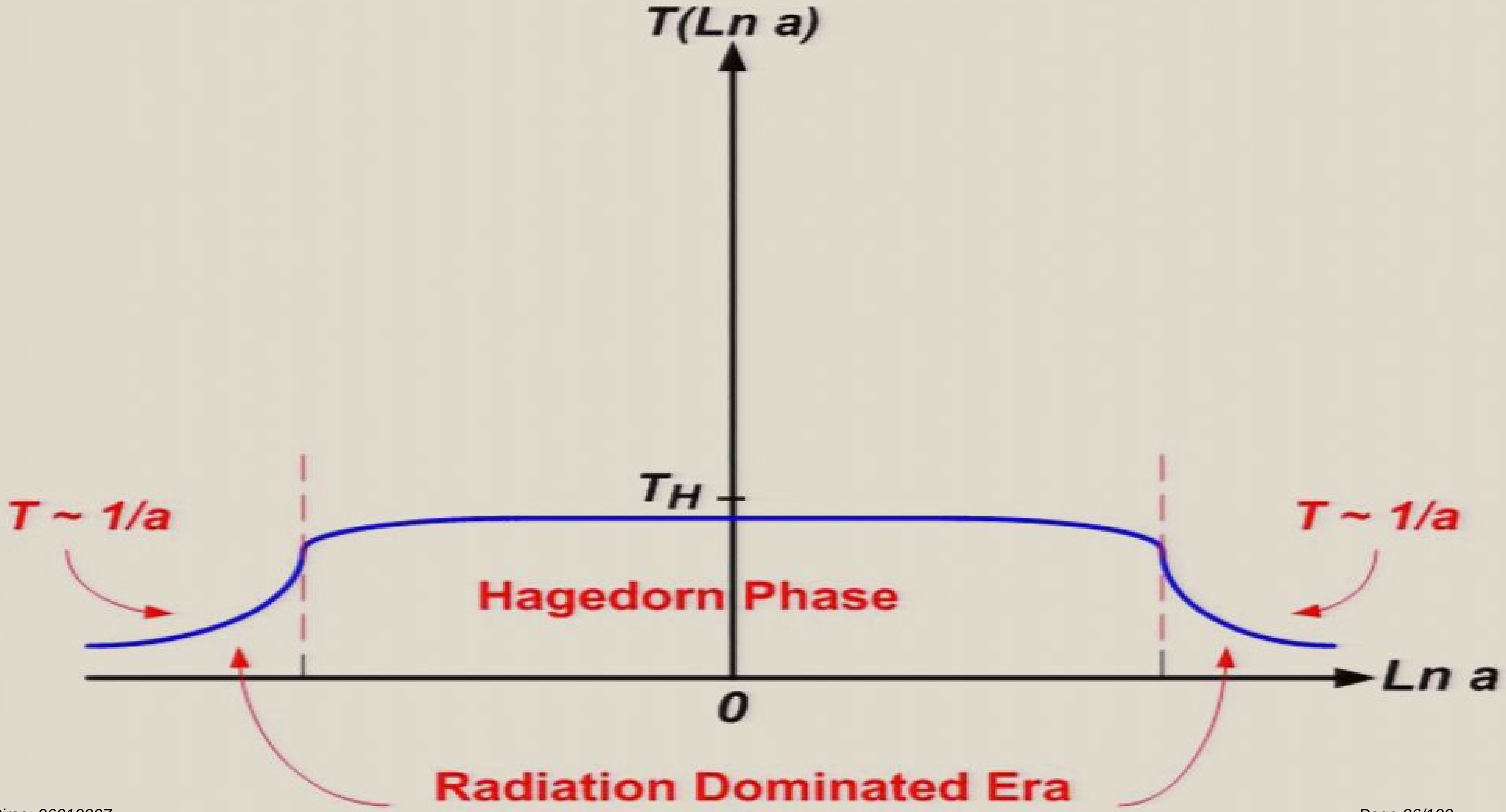
Classical String Cosmology



Quantum String Cosmology



Behavior of Temperature



Microcanonical Description

$$\Omega(E) \equiv \sum_i \delta(E - E_i)\,,$$

$$S(E) \equiv \ln \Omega(E)\,.$$

$$\frac{1}{T(E)} = \beta(E) \equiv \left(\frac{\partial S}{\partial E}\right)_V = \left(\frac{\partial \ln \Omega}{\partial E}\right)_V,$$

$$P \equiv T\left(\frac{\partial S}{\partial V}\right)_E = T\left(\frac{\partial \ln \Omega}{\partial V}\right)_E,$$

$$C_V \equiv \left(\frac{\partial E}{\partial T}\right)_V = -\beta^2\left(\frac{\partial E}{\partial \beta}\right)_V = -\left[T^2\left(\frac{\partial^2 S}{\partial E^2}\right)_V\right]^{-1}.$$

Canonical Description

$$Z(\beta) = \sum_i e^{-\beta E_i}\,,$$

$$Z(\beta) = \int_0^\infty dE e^{-\beta E}\Omega(E)\,,$$

$$\Omega(E) = \int_{L-i\infty}^{L+i\infty} \frac{d\beta}{2\pi i} e^{\beta E} Z(\beta)\,,$$

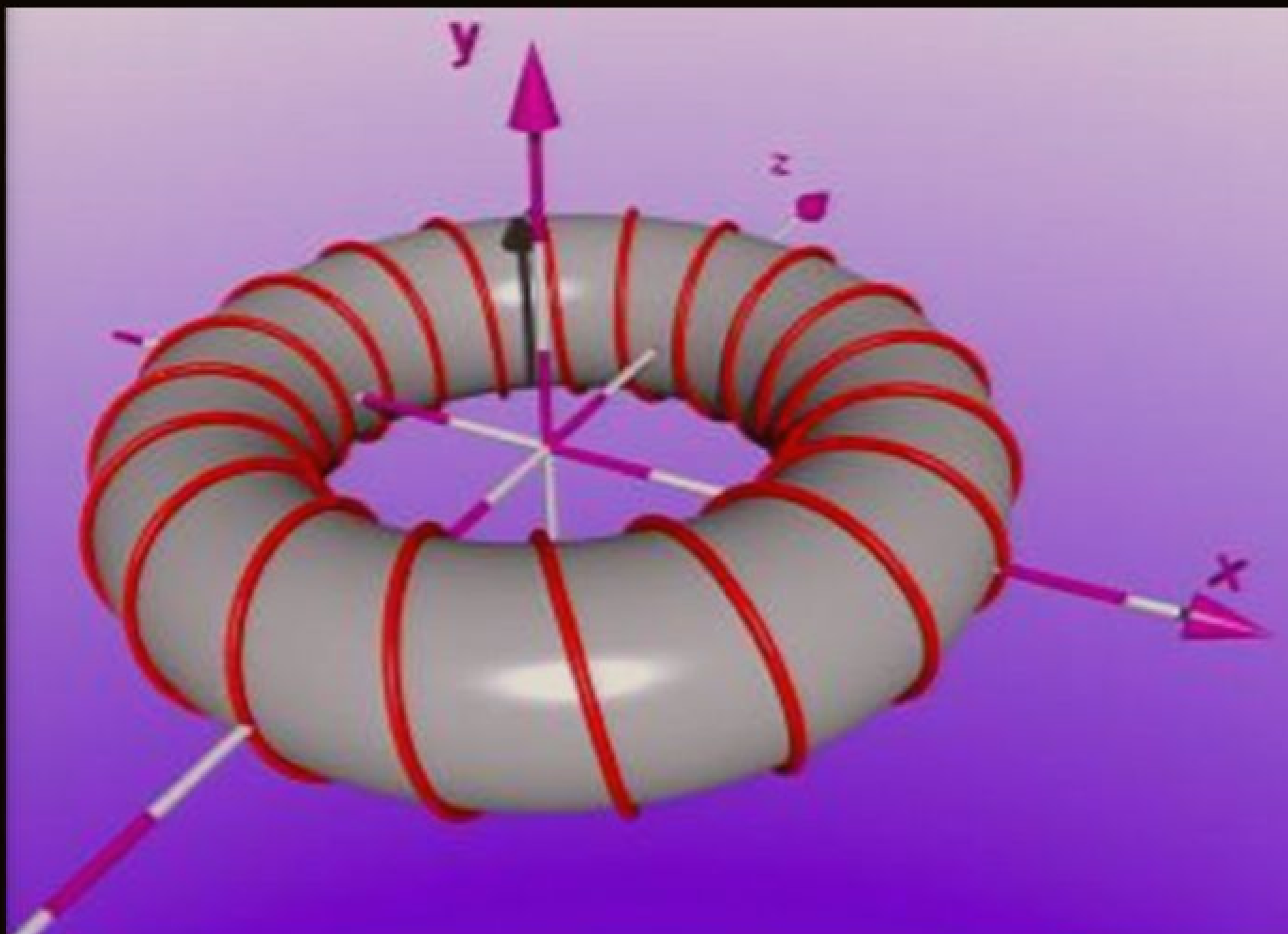
$$\frac{\Delta E}{\langle E\rangle} \equiv \sqrt{\frac{\langle E^2\rangle}{\langle E\rangle^2} - 1} \propto \frac{1}{\sqrt{\langle E\rangle}}\,.$$

$$\langle E\rangle \equiv -Z^{-1}(\partial Z/\partial\beta) = -(Z'/Z)$$

$$\langle E^2\rangle \equiv Z^{-1}(\partial^2 Z/\partial\beta^2) = (Z''/Z)$$

$$C_V = \beta^2\left(\langle E^2\rangle - \langle E\rangle^2\right) = \beta^2\left(\frac{Z''}{Z} - \frac{Z'^2}{Z^2}\right) = \beta^2\frac{\partial}{\partial\beta}\left(\frac{Z'}{Z}\right) = -\beta^2\frac{\partial\langle E\rangle}{\partial\beta}\,.$$

Statistical Mechanics of the Ideal String Gas



Density of States for the Ideal String Gas

When the string coupling is sufficiently small, $g_s << 1$, and the local spacetime geometry is close to flat $\mathbf{R}^{d+1}$ over the length scale of the finite size box of volume $V = R^d$, there are two distinct regimes that characterizes the statistical mechanics of string thermodynamics: the massless modes with field theoretic entropy :

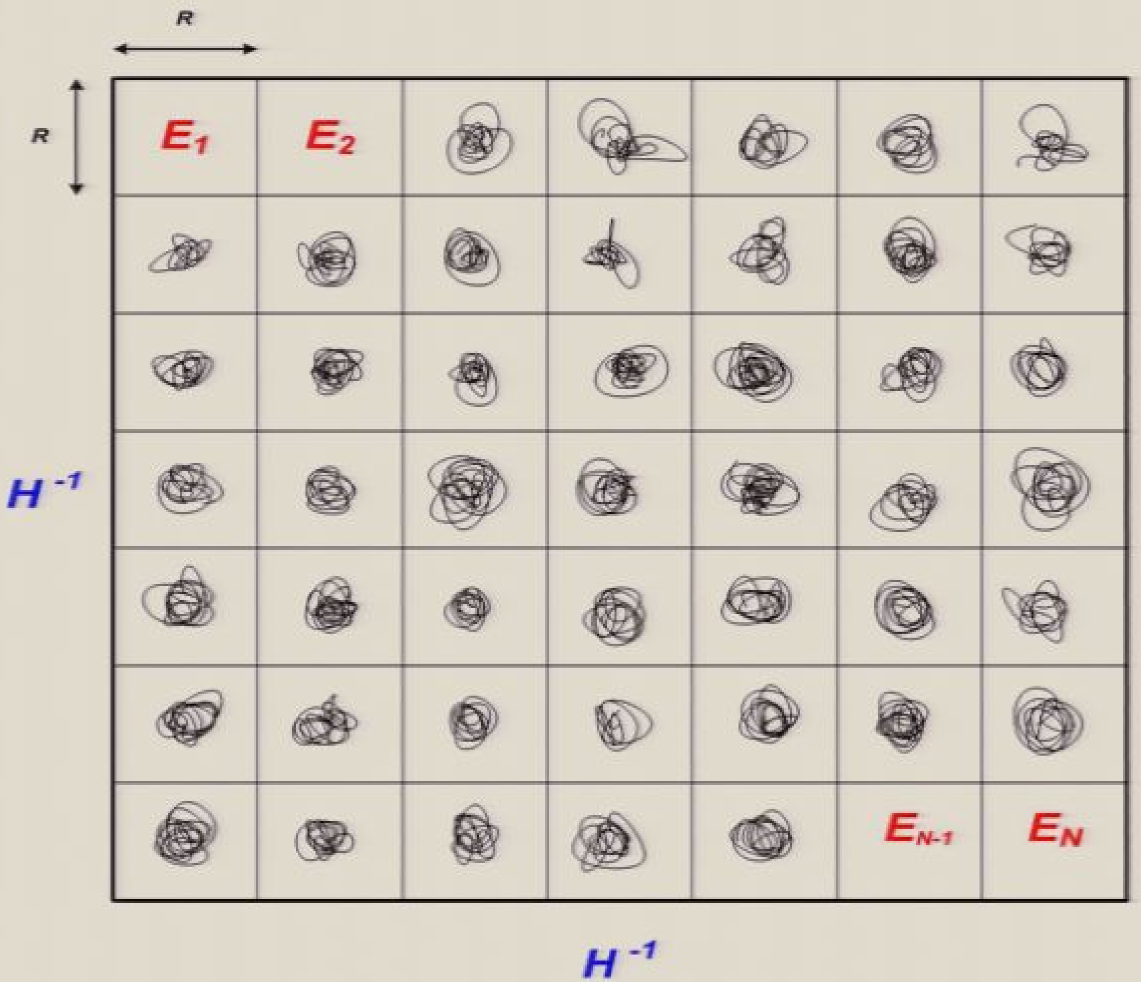
$$S \propto E^{d/(d+1)}$$

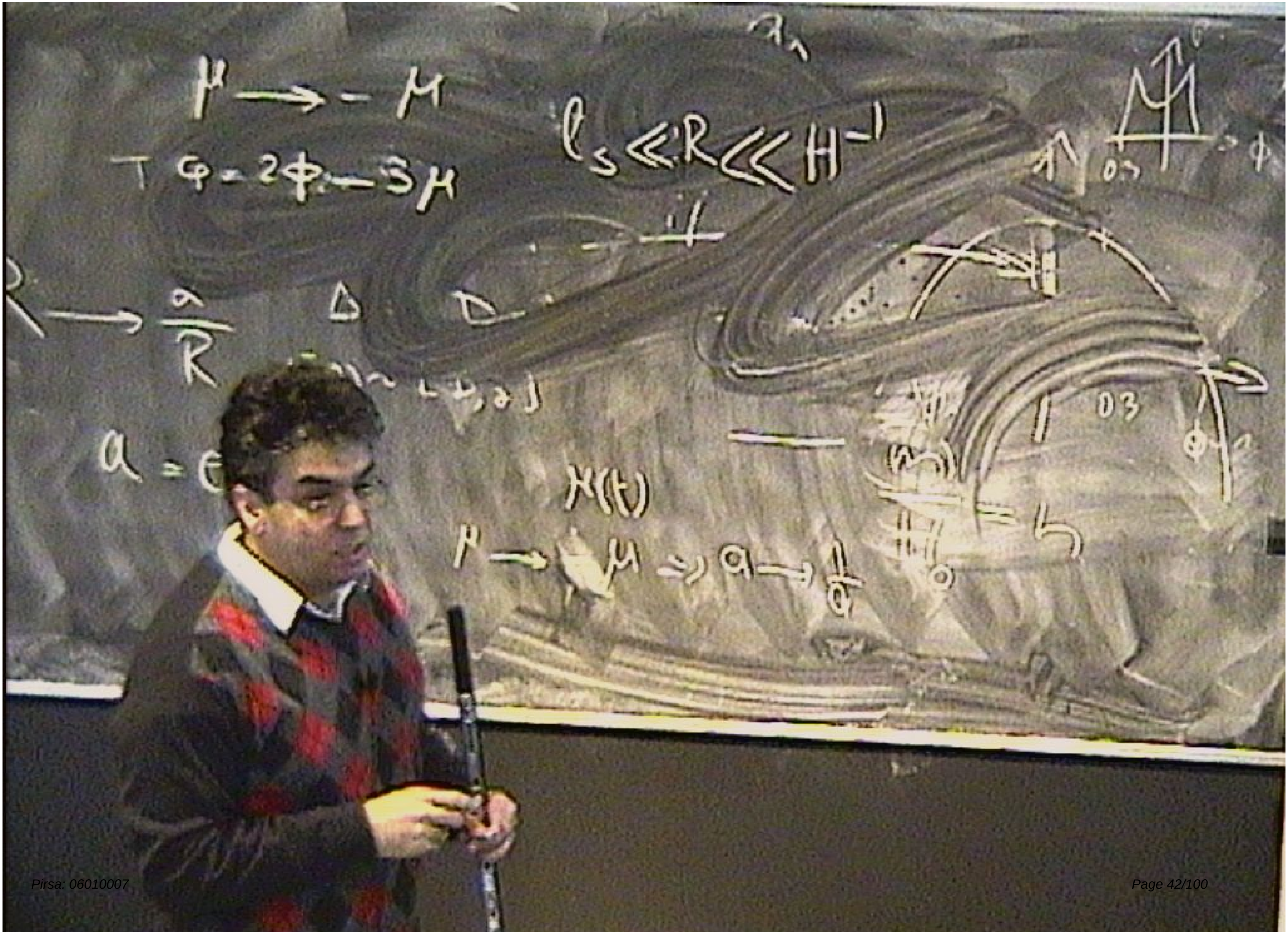
and the highly excited strings with

$$S \propto E$$

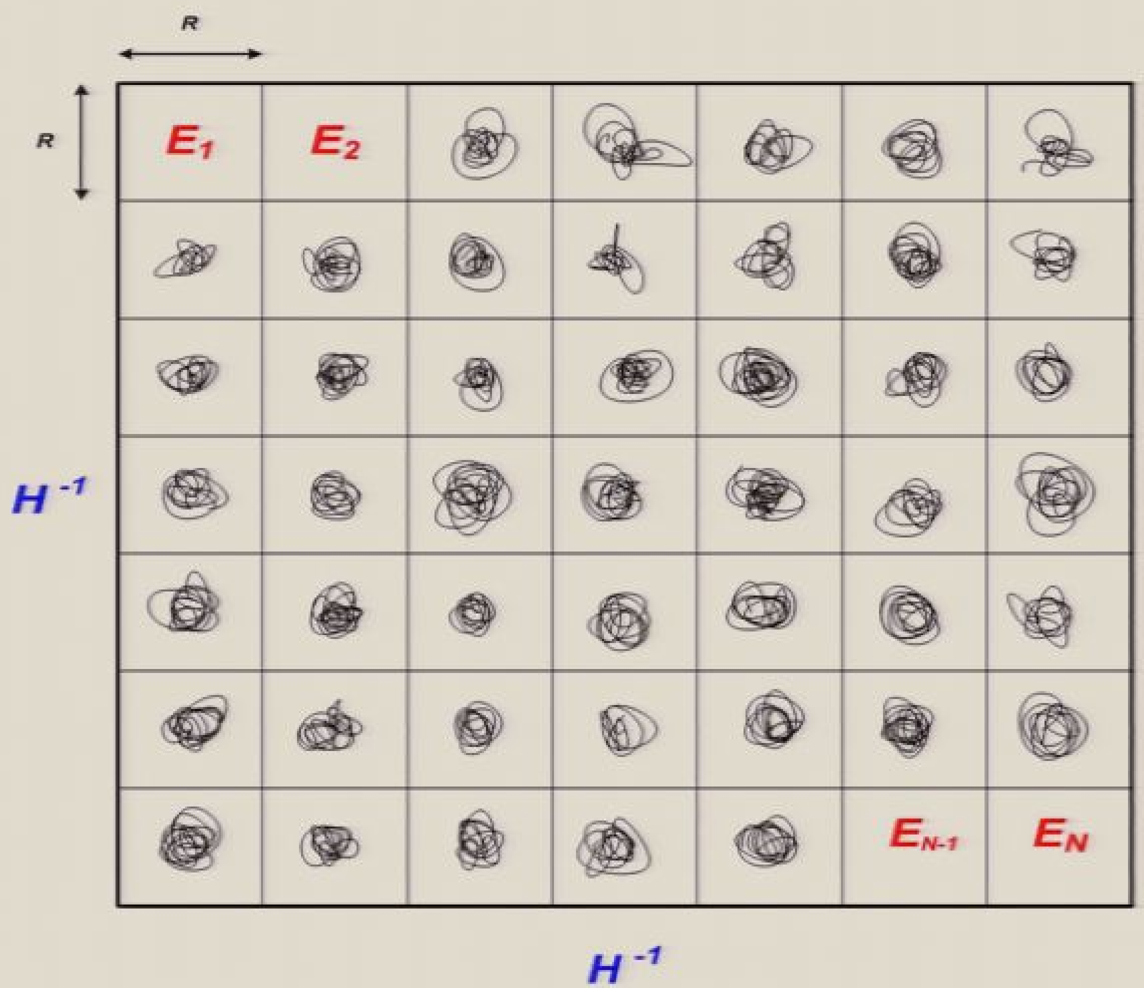
One simple realization of this setup for a string background is a spatial toroidal compactification with d dimensions of size L and 9 - d dimensions of string scale size. Small string coupling ensures us that we can measure energies with respect to the flat time coordinate.

Universe in Hagedorn Phase

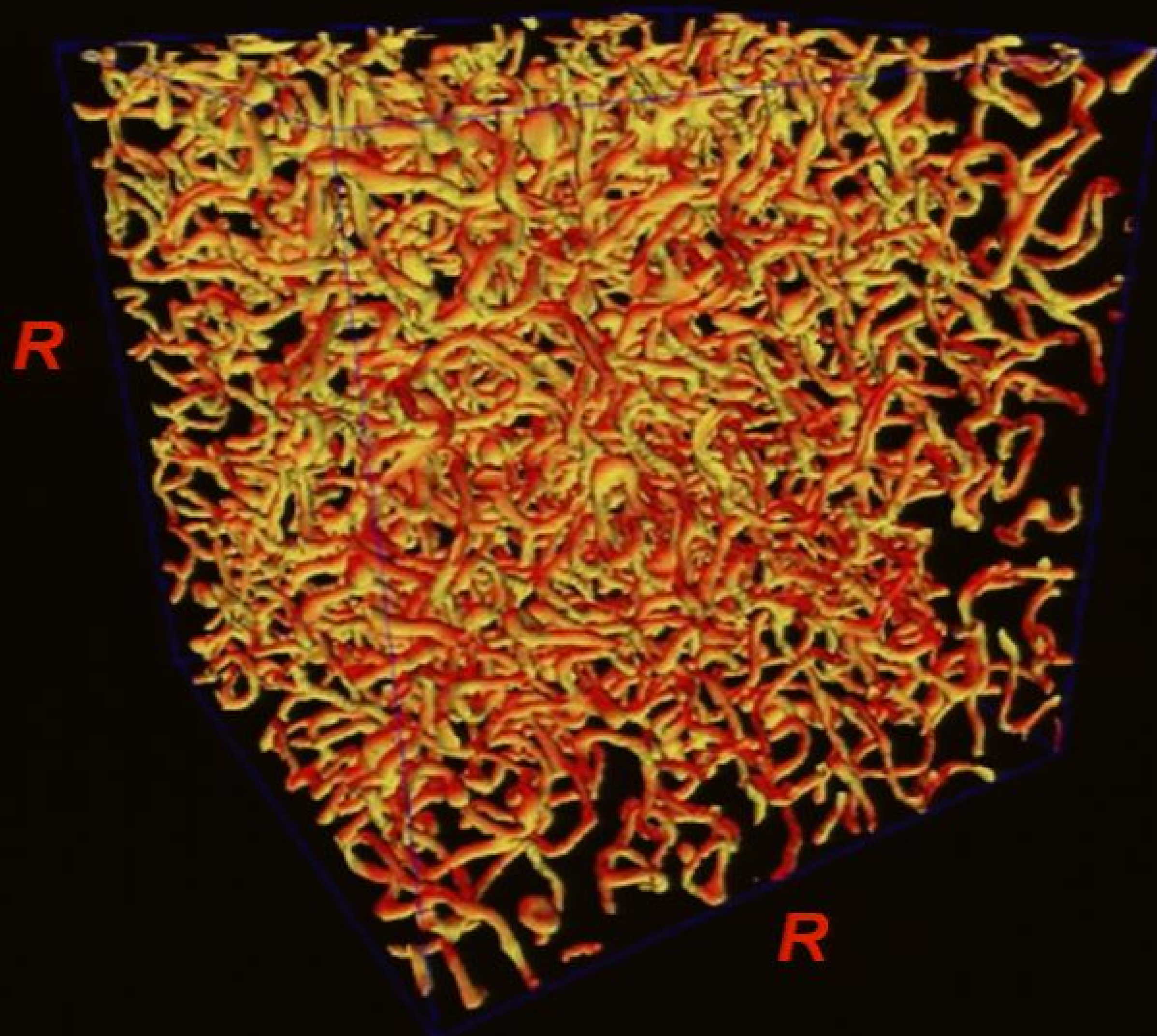



$\mu \to -\mu$
$l_s \ll R \ll H^{-1}$
$a = e$
$\mu(t)$

Universe in Hagedorn Phase



Stringy Block Universe



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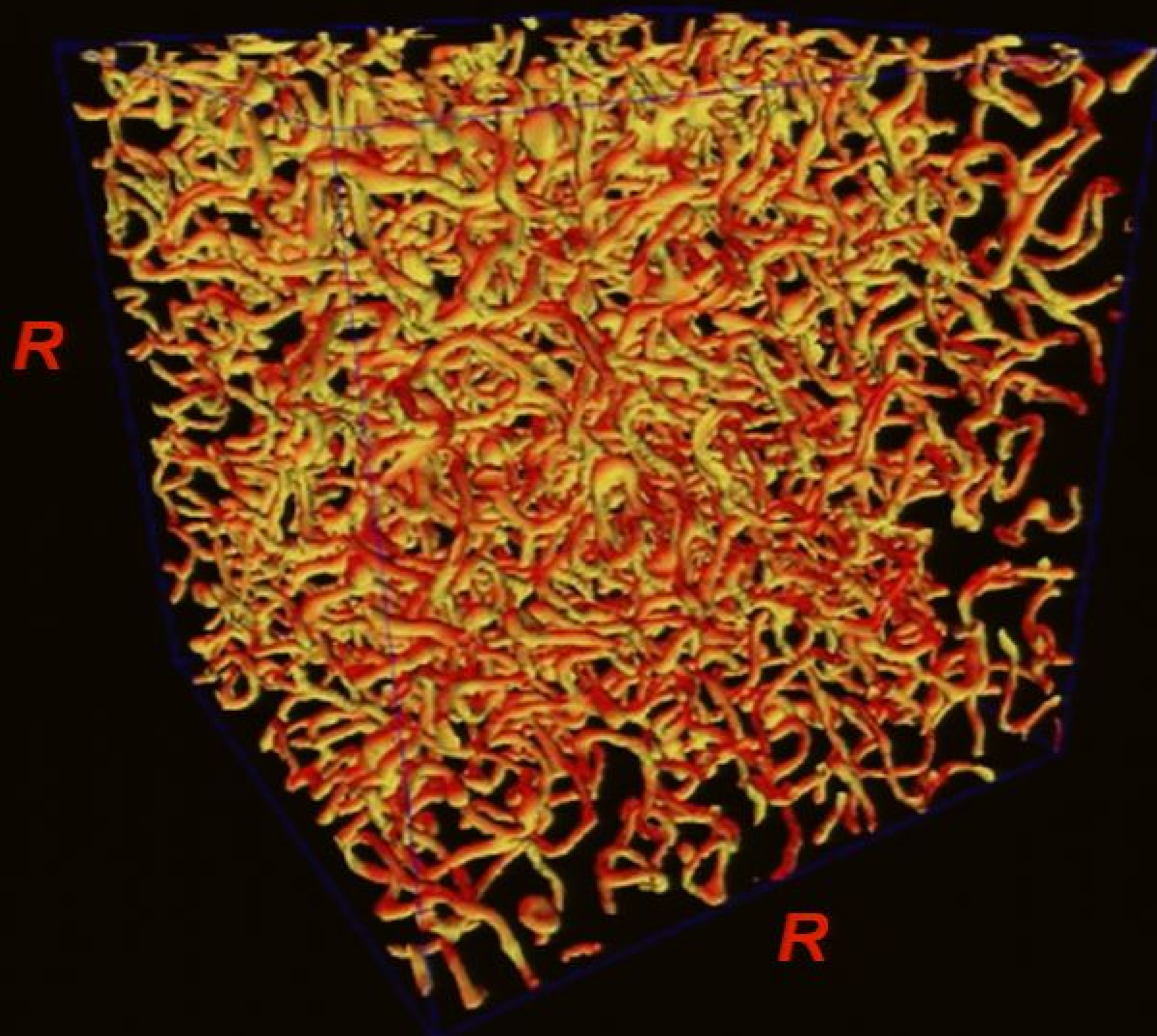

Density of Single State

- For a highly excited closed string represented as a random walk in a target space, the energy ε of the string is proportional to the length of the random walk. Thus the number of the strings with a fixed starting point grows as $e^{(\beta_H \varepsilon)}$, with $T_H = 1/\beta_H$ is the Hagedorn temperature. This explains the bulk of the entropy of highly energetic strings.

$$\omega_{closed}(\varepsilon) \sim V.\frac{1}{\varepsilon}.\frac{e^{\beta_H \varepsilon}}{V_{\mathrm{walk}}(\varepsilon)}.$$

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- Closed strings correspond to random walks that must close on themselves. This overcounts by a factor of roughly the volume of the walk, denoted $V_{walk}(\varepsilon)$. The global translation of the random walk in volume $V = R^d$ and (1/ε) due the fact that any point in the string can be considered as a new starting point are other factors that contribute to the number of closed string. Therefore, the final result is

$$\omega_{closed}(\varepsilon) \sim V.\frac{1}{\varepsilon}.\frac{e^{\beta_H \varepsilon}}{V_{\text{walk}}(\varepsilon)}.$$

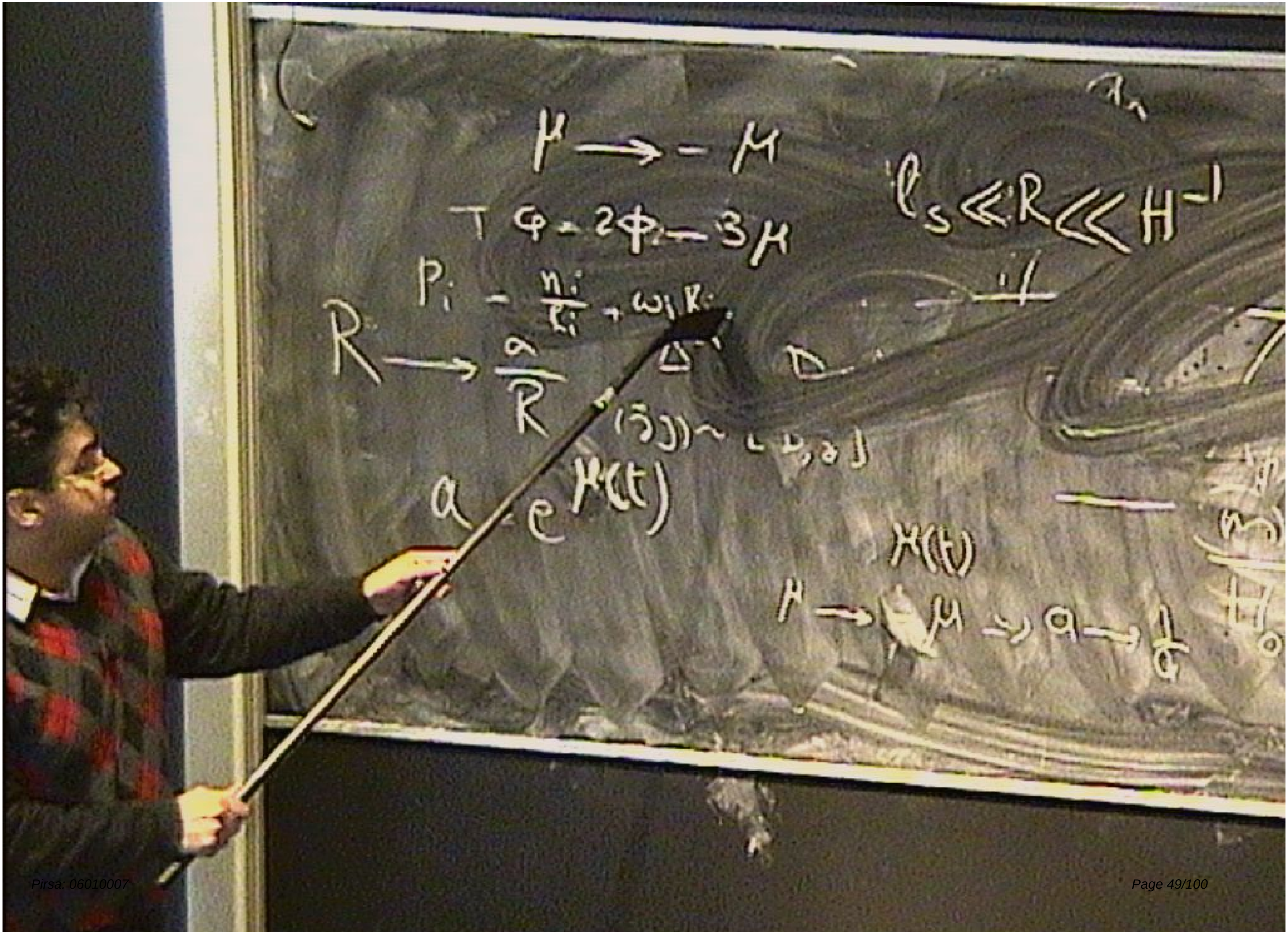
Density of Single State: Two limiting cases

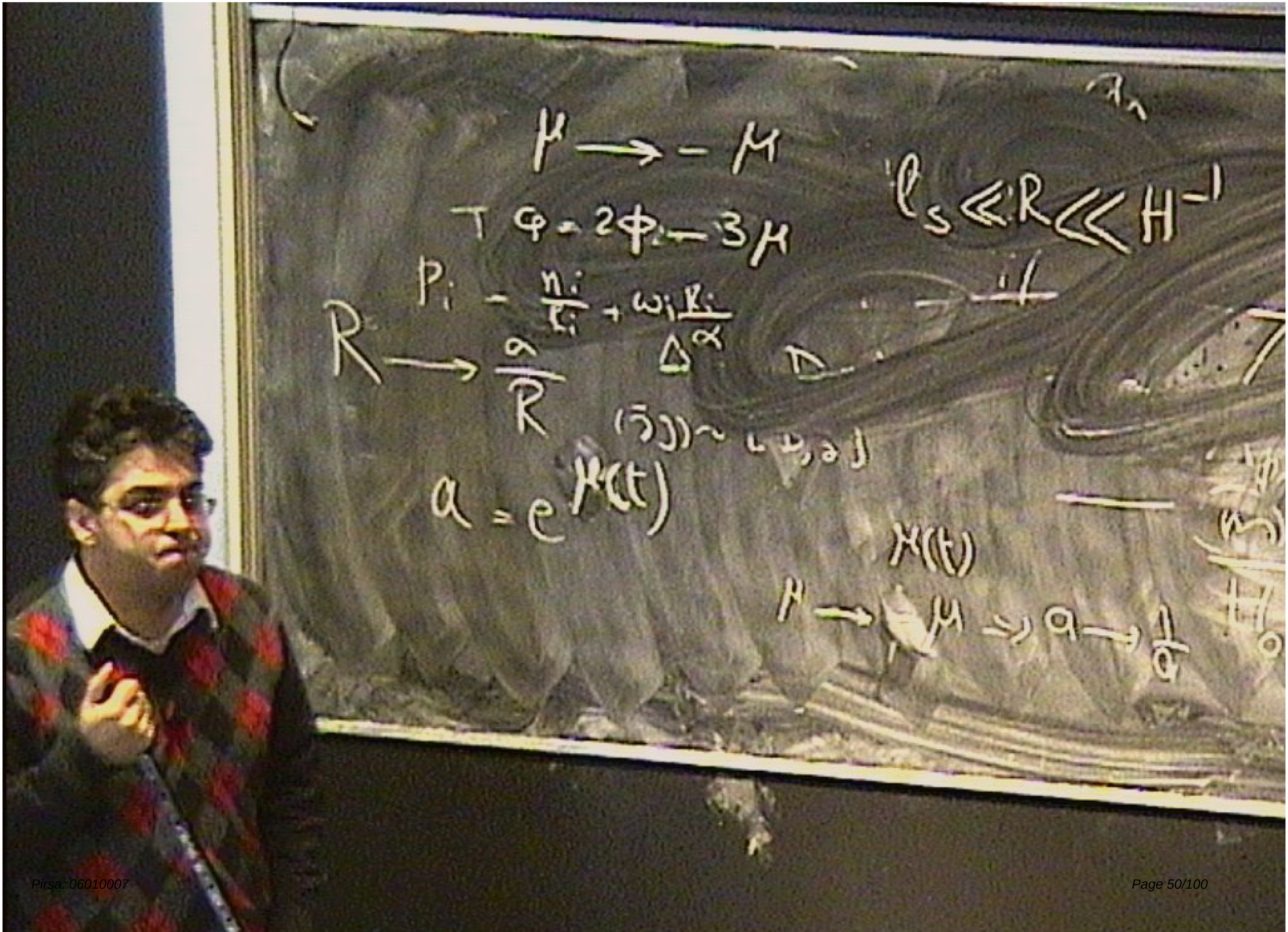
1. Volume of the random walk is well-contained in *d* spatial dimensions (i.e., $R >> (\varepsilon^{1/2})$) which corresponds to a string in *d* non-compact dimensions. In this case the $V(\varepsilon) \sim \varepsilon^{d/2}$, the density of states per unit volume is

$$\omega_{closed}(\varepsilon)/V \sim \frac{e^{\beta_H \varepsilon}}{\varepsilon^{d/2+1}}, \text{for } R \gg \sqrt{\varepsilon}.$$

2. Volume of the random walk is space-filling ($R << (\varepsilon^{1/2})$) and saturates at order V which corresponds to d compact dimensions that contains the highly excited string states. Hence,

$$\omega_{closed}(\varepsilon) = \frac{e^{\beta_H \varepsilon}}{\varepsilon}, \text{for } R \ll \sqrt{\varepsilon}.$$





μ → − μ
ℓs ≪ R ≪ H⁻¹
R → a/R
a = e^μ(t)
μ(t)

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Total Density

The total density of states, $\Omega(E)$, can be obtained. The partition function $Z(\beta)$ can be evaluated explicitly in the one loop approximation. Near the Hagedorn temperature, we can assume Maxwell-Boltzman statistics and thus treat the system quasiclassically Then we can write $Z(\beta) = \exp[z(\beta)]$, where $z(\beta)$ is the single-string partition function (free thermal energy)

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The singular part of the partition function at finite volume for closed strings is given by a set of poles of even multiplicity $g_\imath = 2k_\imath$

$$Z_{\imath,closed}^{\text{singular}} \sim \left(\frac{\beta_\imath}{\beta - \beta_\imath}\right)^{k_\imath} ,$$

with $k_\imath = k_0 = 1$ for the leading Hagedorn singularity $\beta_\imath = \beta_H$.

The regular part of the free energy to leading energy is

$$z_\imath^{\text{regular}} \sim a_\imath V_{D-1} - \rho_\imath V_{D-1}(\beta - \beta_\imath) + \mathcal{O}(V(\beta - \beta_\imath)^2) ,$$

where $a_\imath$ and $\rho_\imath$ have dimensions of number and energy density on the world-volume, respectively.

Whenever the specific heat is positive (and large), there is a correspondence between the canonical and microcanonical ensembles and thus the saddle point approximation is applicable. A necessary condition for this is that $\gamma < 1$, ensuring the canonical internal energy $E(\beta) \sim \partial_\beta z(\beta)$ diverges at the Hagedorn singularity that is these systems are unable to reach the Hagedorn temperature since their require an infinite amount of energy to do so. For these systems the Hagedorn temperature is limiting, and this is true for the closed strings the Hagedorn temperature is non-limiting for any model in which $D_c > 4$. In other words, stable canonical (i.e., no phase transition) can be achieved for the closed strings in low dimensional thermodynamical limits $d < 2$.

The case of closed strings has been studied in great details by Jain *et. al* [1992]. The leading singularity at very high and finite volume is always a simple pole of the partition function at the Hagedorn singularity,

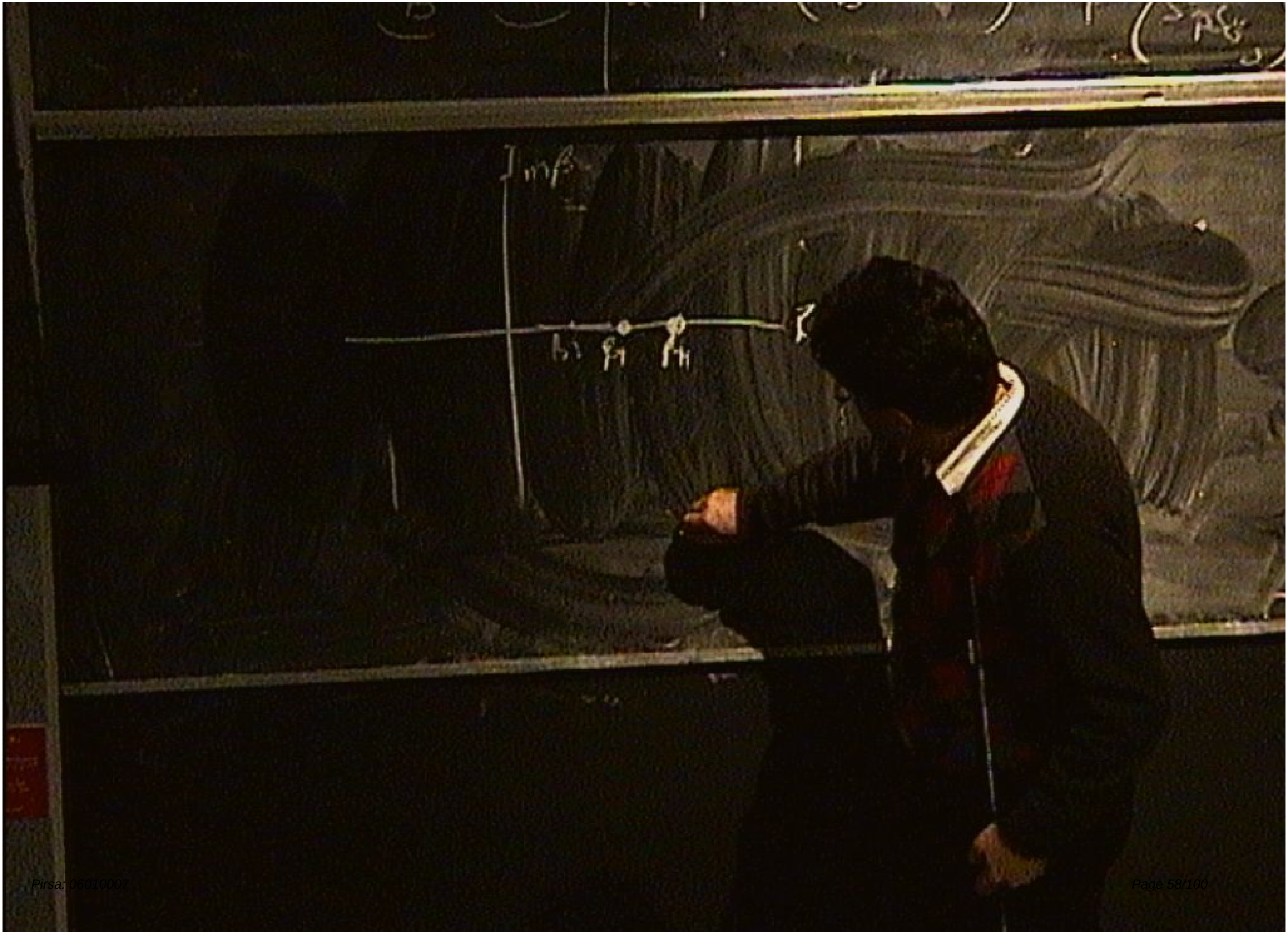
$$Z_{closed}(\beta) = (\beta - \beta_H)^{-1}.Z_{reg}(\beta)\,.$$

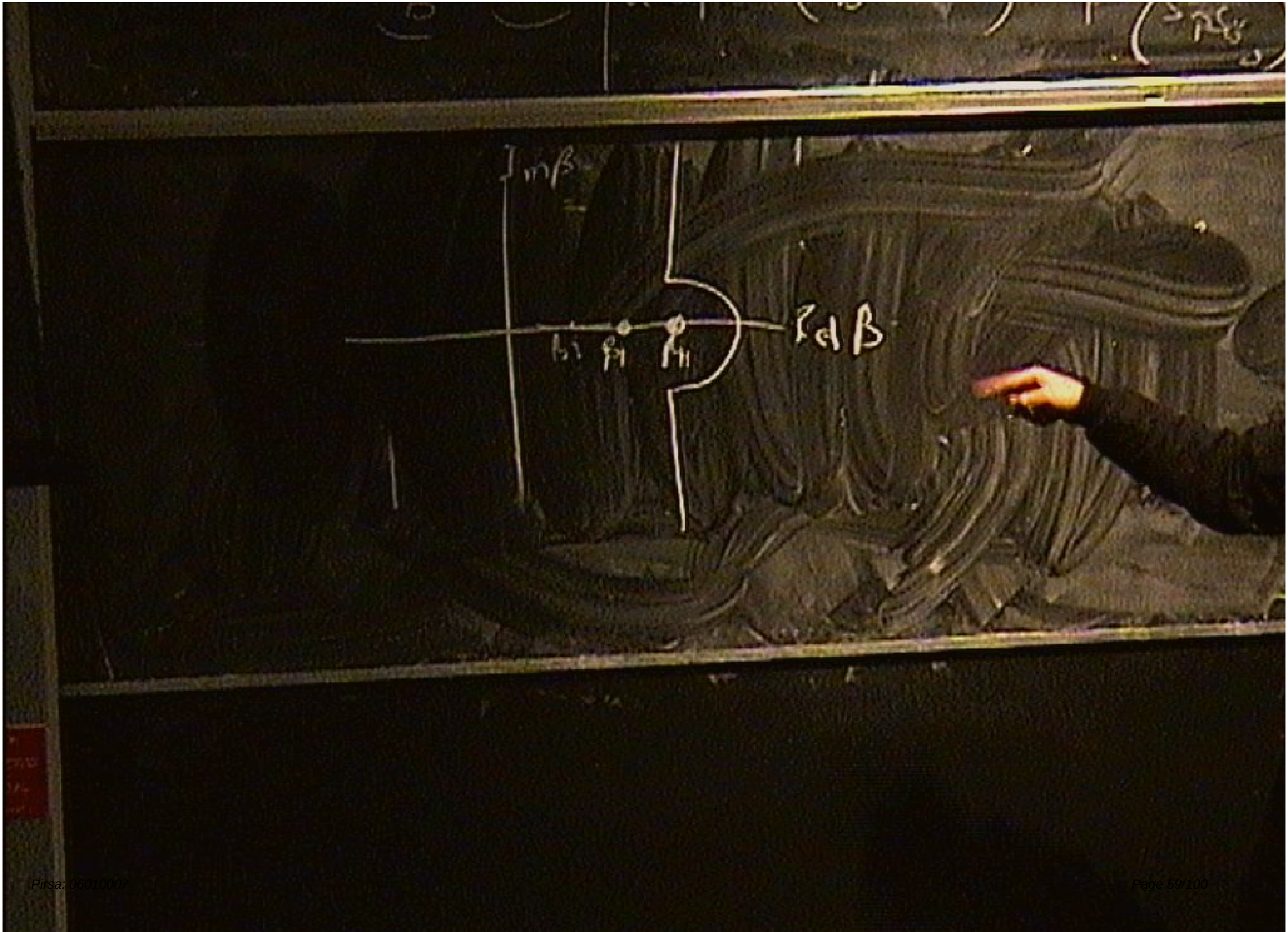
The next to leading singularity of the partition function β_1 is a pole of order $k_1 = 2D - 2$, located at

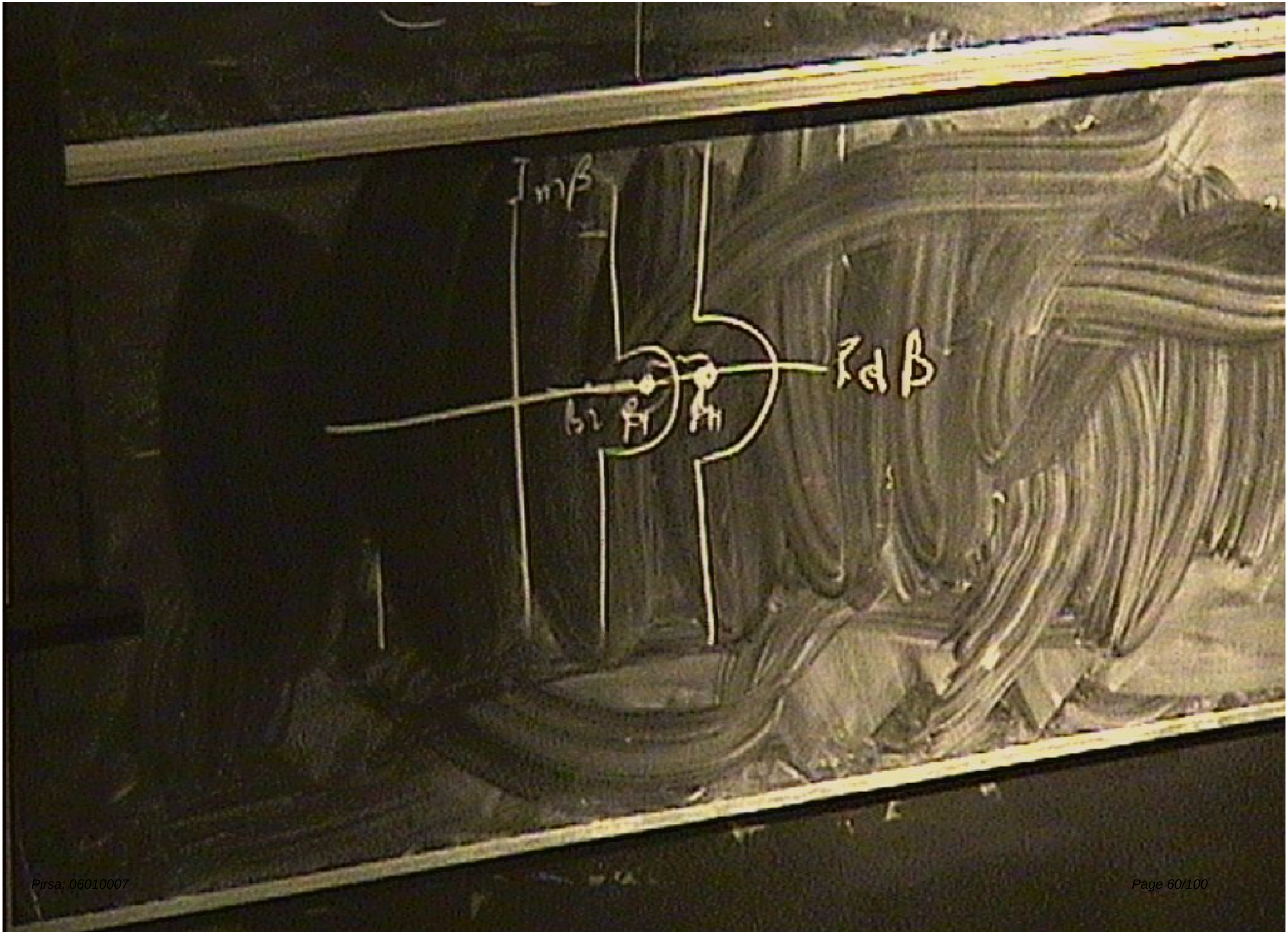
$$\beta_H - \beta_1 \sim \frac{\alpha'^{3/2}}{R^2} ,$$

with $\alpha' \sim \mathcal{O}(1)$ in string units. The total density of the states is

$$\Omega_{closed} \approx \beta_H e^{\beta_H E + a_H V_{D-1}} \left[1 \underbrace{- \frac{(\beta_H E)^{2D-3}}{(2D-3)!} e^{-(E - \rho_H V_{D-1})/R^2}}_{\delta\Omega_{(1)}} \right] ,$$




Im β
Re β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Entropy and Temperature

- The entropy of the universe in the Hagedorn phase is then

$$S(E,R) \simeq \beta_H E + a_H V + \ln\left(1 + \delta\Omega_{(1)}\right),$$

and therefore the temperature $T \equiv [(\partial S/\partial\langle E\rangle)_V]^{-1}$ will be

$$T(E,R) \simeq \left(\beta_H + \frac{\partial\delta\Omega_{(1)}/\partial E}{1+\delta\Omega_{(1)}}\right)^{-1} \simeq T_H\left(1 + \frac{\beta_H - \beta_1}{\beta_H}\delta\Omega_{(1)}\right).$$

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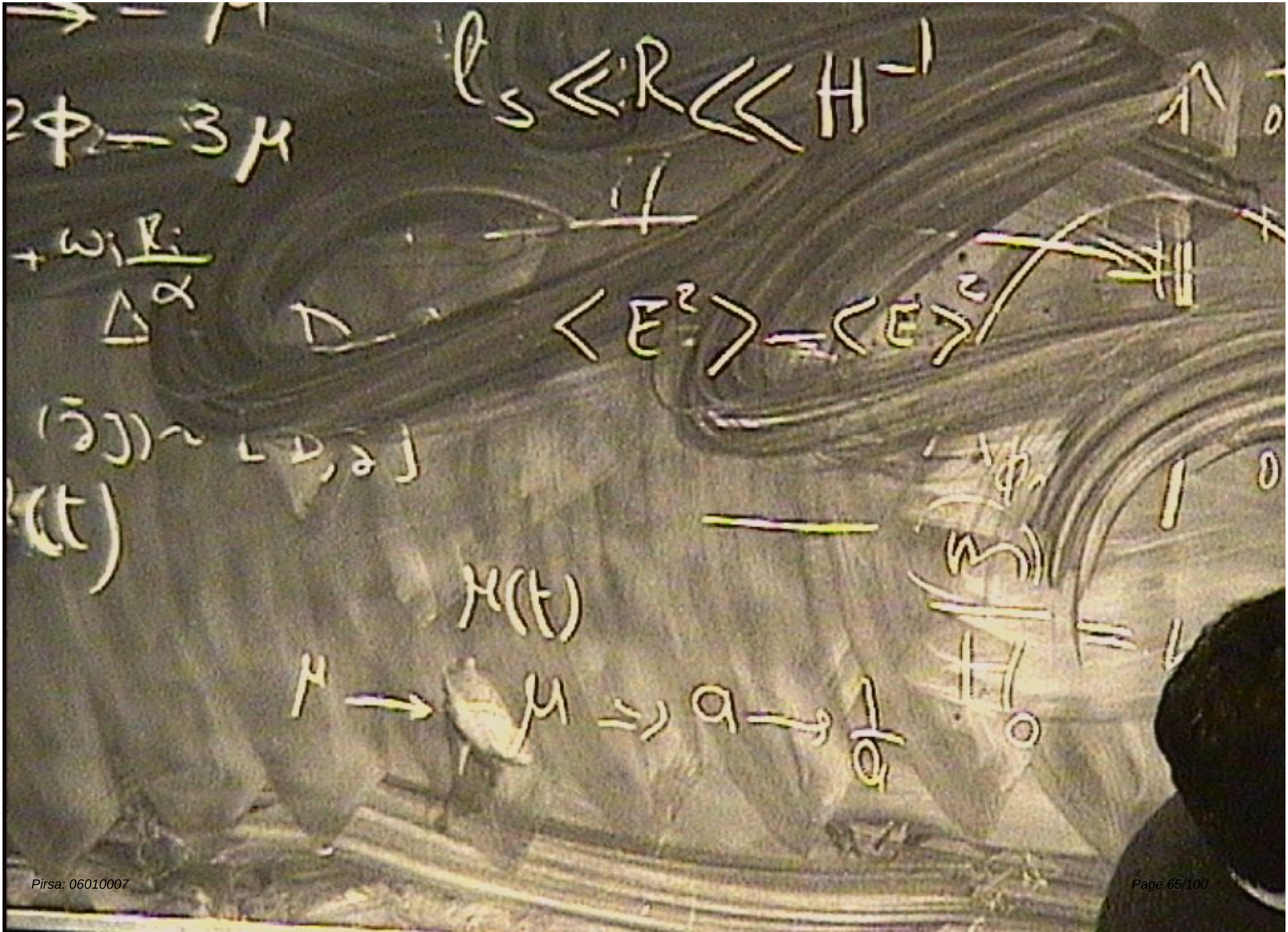
Correlation Function

$$C^{\mu}{}_{\nu}{}^{\sigma}{}_{\lambda} = \langle \delta T^{\mu}{}_{\nu} \delta T^{\sigma}{}_{\lambda} \rangle = \langle T^{\mu}{}_{\nu} T^{\sigma}{}_{\lambda} \rangle - \langle T^{\mu}{}_{\nu} \rangle \langle T^{\sigma}{}_{\lambda} \rangle$$
$$= 2\frac{G^{\mu\beta}}{\sqrt{-G}}\frac{\partial}{\partial G^{\beta\nu}}\left(\frac{G^{\sigma\delta}}{\sqrt{-G}}\frac{\partial \ln Z}{\partial G^{\delta\lambda}}\right)$$
$$+2\frac{G^{\sigma\beta}}{\sqrt{-G}}\frac{\partial}{\partial G^{\beta\lambda}}\left(\frac{G^{\alpha\delta}}{\sqrt{-G}}\frac{\partial \ln Z}{\partial G^{\delta\nu}}\right),$$

with $\delta T^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \langle T^{\mu}{}_{\nu} \rangle$ and

$$\langle T^{\mu}{}_{\nu} \rangle = 2\frac{G^{\mu\alpha}}{\sqrt{-G}}\frac{\partial \ln Z}{\partial G^{\alpha\nu}},$$


$l_s \ll R \ll H^{-1}$
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Energy Fluctuation

Now if we divide the universe inside the Hubble radius, H^{-1}, to small blocks of size $\ell_s \ll R \ll H^{-1}$, where R is almost independent of time during the Hagedorn phase. The partition function $Z = \exp(-\beta F)$, where $F = F(\beta\sqrt{-G_{00}}, R)$ is the string free energy with $\beta\sqrt{-G_{00}} = T^{-1}\sqrt{-G_{00}}$. Therefore $C^0{}_0{}^0{}_0$, becomes

$$\begin{aligned} C^0{}_0{}^0{}_0 &= \langle \delta\rho^2 \rangle = \langle \rho^2 \rangle - \langle \rho \rangle^2 \\ &= -\frac{1}{R^6}\frac{\partial}{\partial\beta}\left(F + \beta\frac{\partial F}{\partial\beta}\right) = -\frac{1}{R^6}\frac{\partial \langle E \rangle}{\partial\beta}, \\ &= \frac{T^2}{R^6} C_V \end{aligned}$$

Specific Heat

The specific heat C_V can be obtained from the entropy of the system,

$$C_V \equiv -\left[T^2\left(\frac{\partial S(E,R)}{\partial E}\right)_V\right]^{-1}$$

$$= \frac{R^2}{\alpha'^{3/2}T}\frac{1}{1-T/T_H},$$

and thus

$$C^0{}_0{}^0{}_0 = \langle\delta\rho^2\rangle = \langle\rho^2\rangle - \langle\rho\rangle^2$$

$$= \frac{T}{\alpha'^{3/2}R^4}\frac{1}{1-T/T_H}$$

Metric Fluctuation

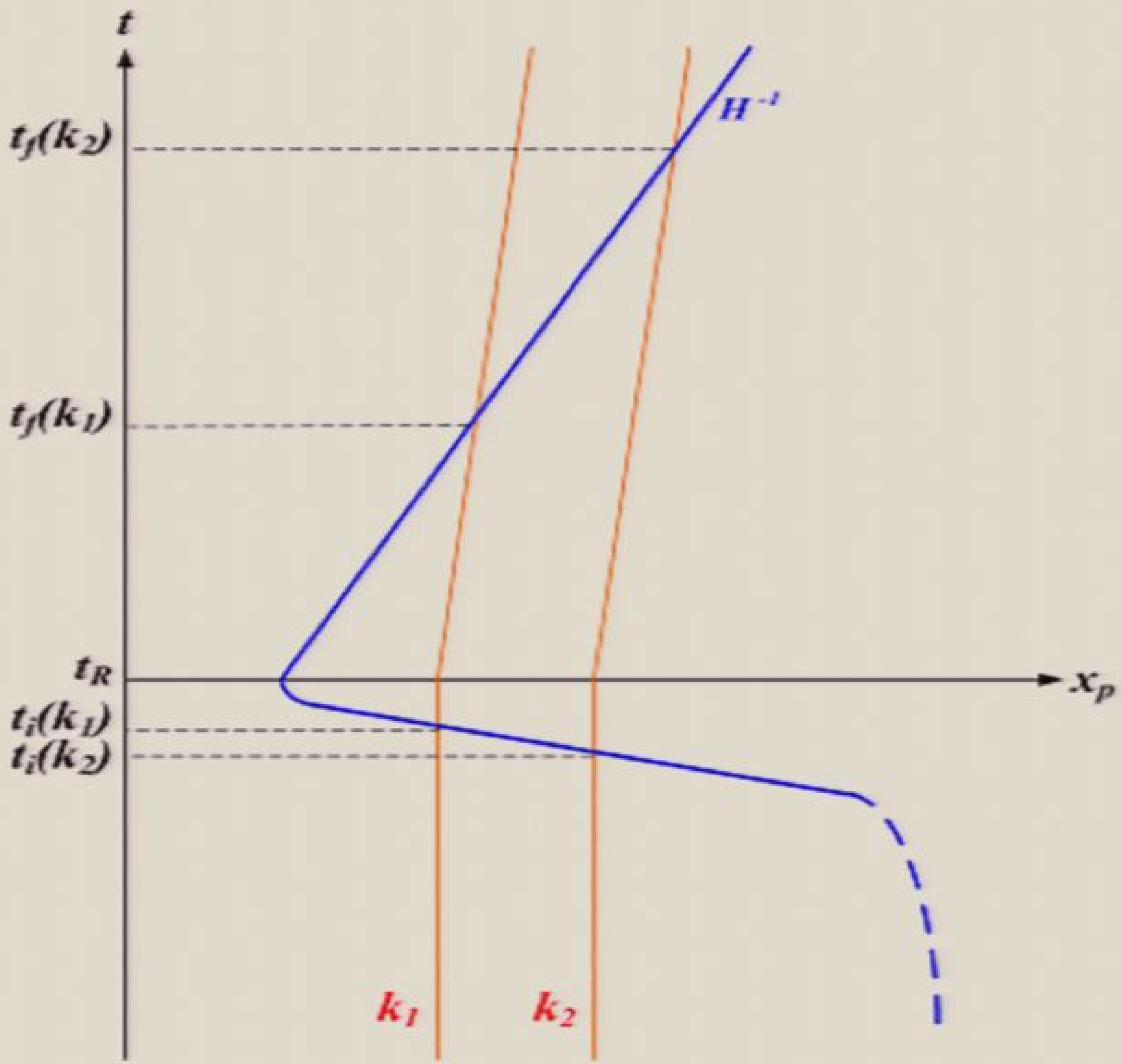
- On scales larger than the Hubble radius, gravity dominates the dynamics and metric fluctuations play the leading role. We will calculate here the spectrum of scalar metric fluctuations, fluctuation modes which couple to the matter sources. In the absence of anisotropic stress, there is only one physical degree of freedom, namely the relativistic generalization of the Newtonian gravitational potential. In longitudinal gauge, the metric then takes the form

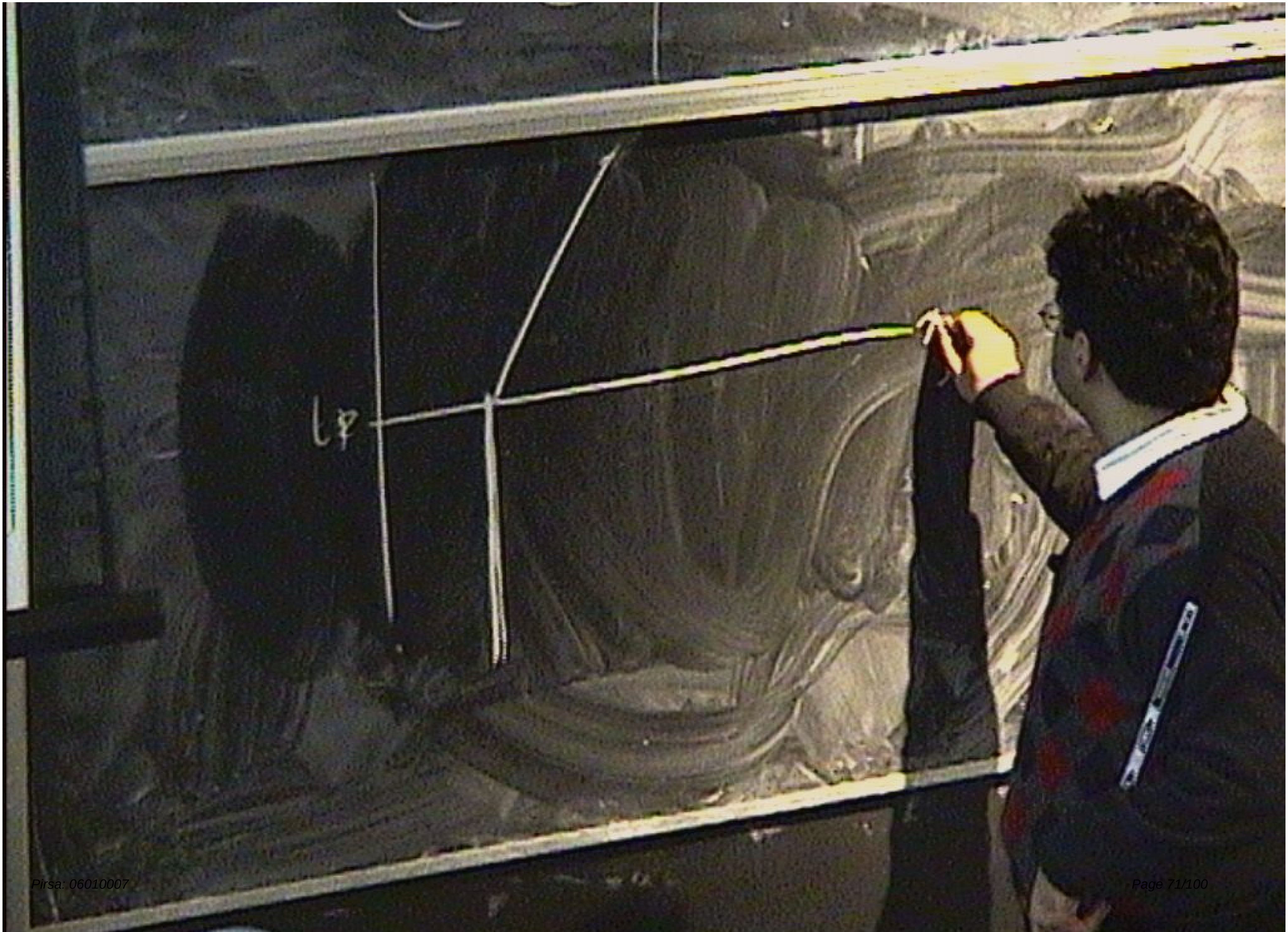
$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Phi)d\mathbf{x}^2,$$

- On scales smaller than the Hubble radius, the gravitational potential Φ is determined by the matter fluctuations via the Einstein constraint equation (the relativistic generalization of the Poisson equation of Newtonian gravitational perturbation theory)

$$\nabla^2\Phi = 4\pi G\delta\rho.$$

Spacetime diagram of the fluctuation modes

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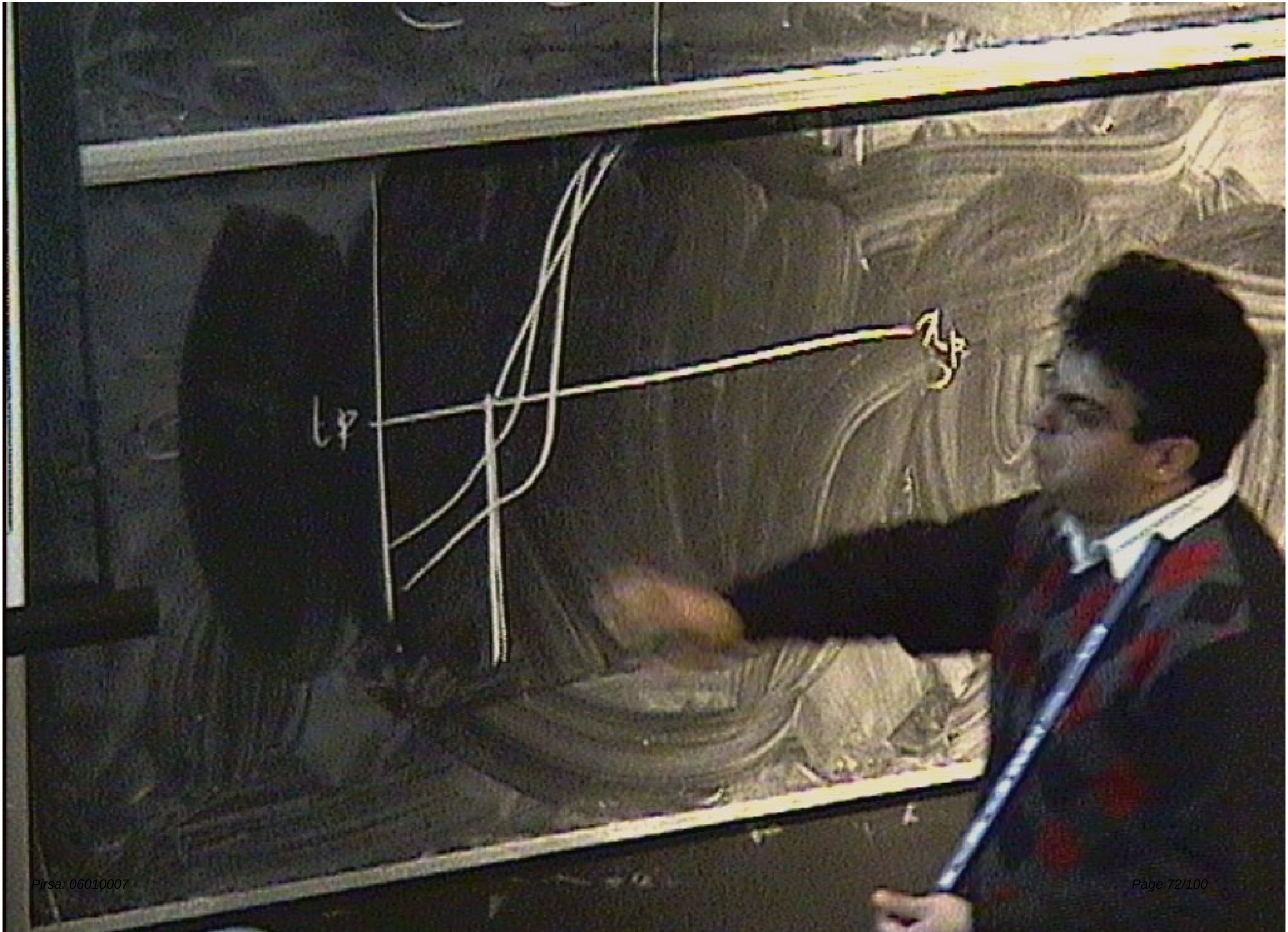

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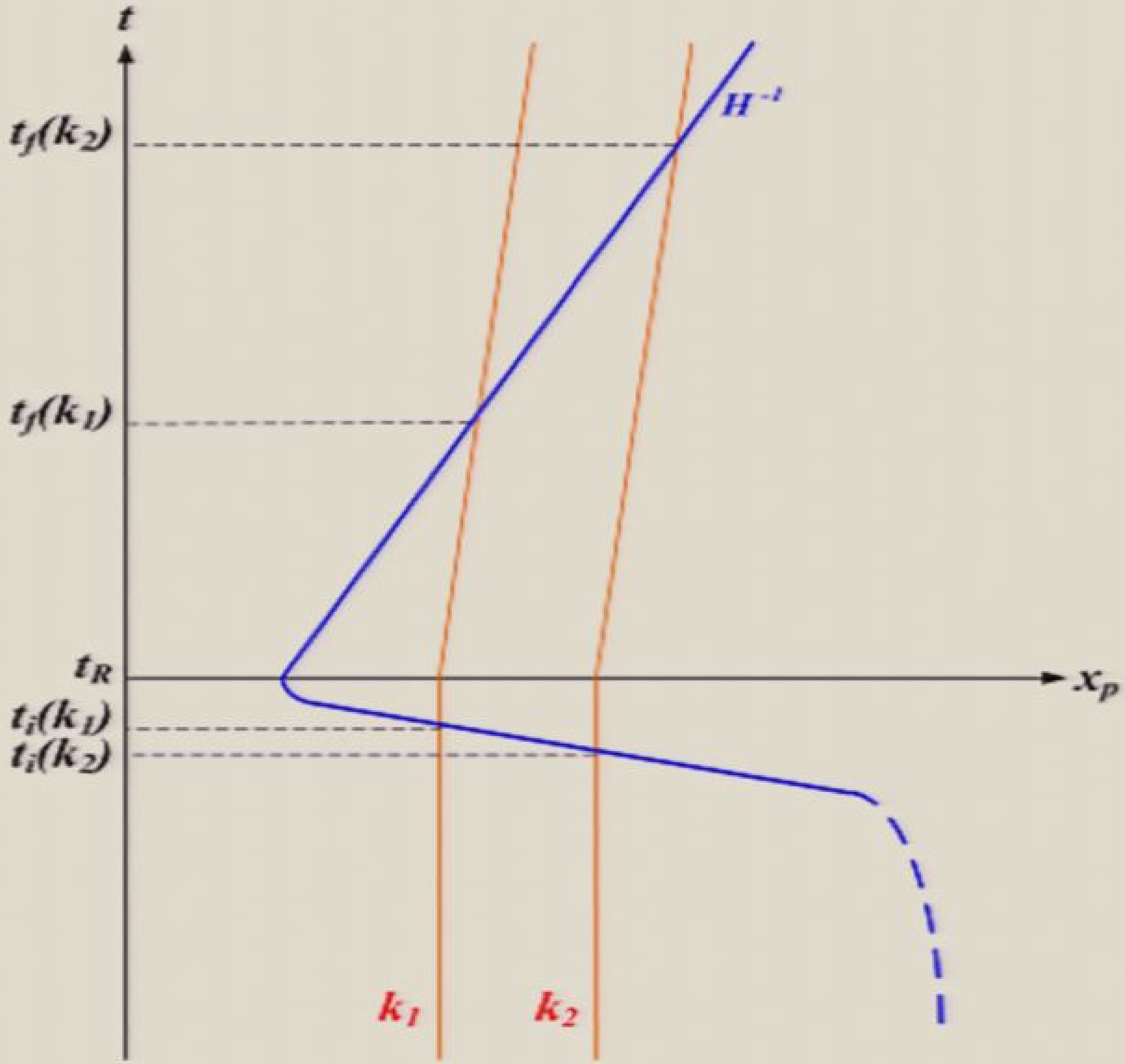

Power Spectrum

$$P_\Phi(k) \equiv k^3|\Phi(k)|^2 \sim k^{n-1}$$
$$= 16\pi^2 G^2 k^{-4}\langle(\delta\rho)^2\rangle$$
$$= 16\pi^2 G^2 \alpha'^{-3/2}\frac{T}{1-T/T_H}$$

Note that the amplitude is suppressed by the ratio $(\ell_{Pl}/\ell_s)^4$, In order to obtain the observed amplitude of fluctuations, a hierarchy of lengths of the order of 10^3 is required. This is consistent with our initial assumption that the string coupling constant should be really small since $(\ell_{Pl}/\ell_s) = g_s \sim 10^{-3} \ll 1$.

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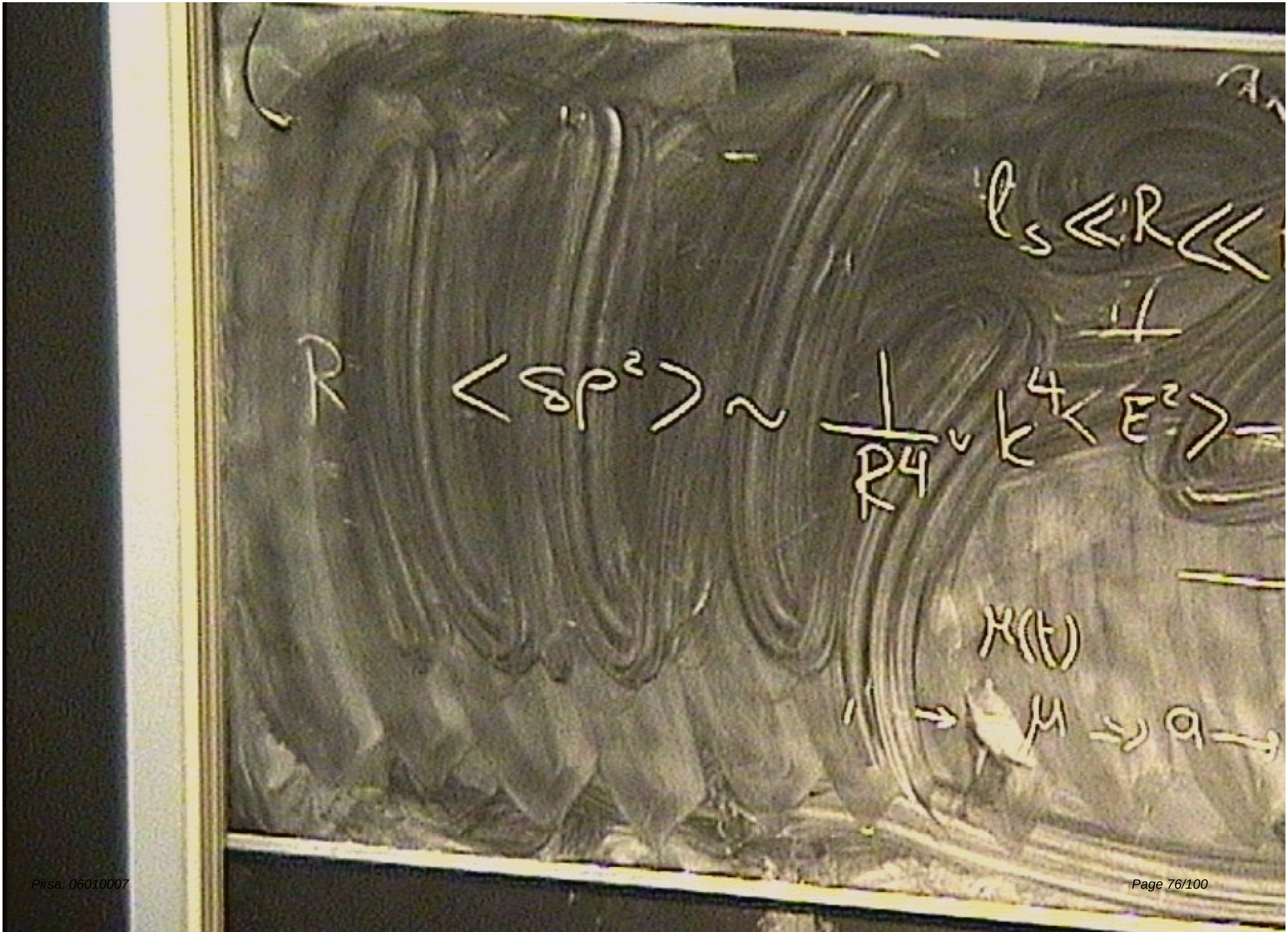
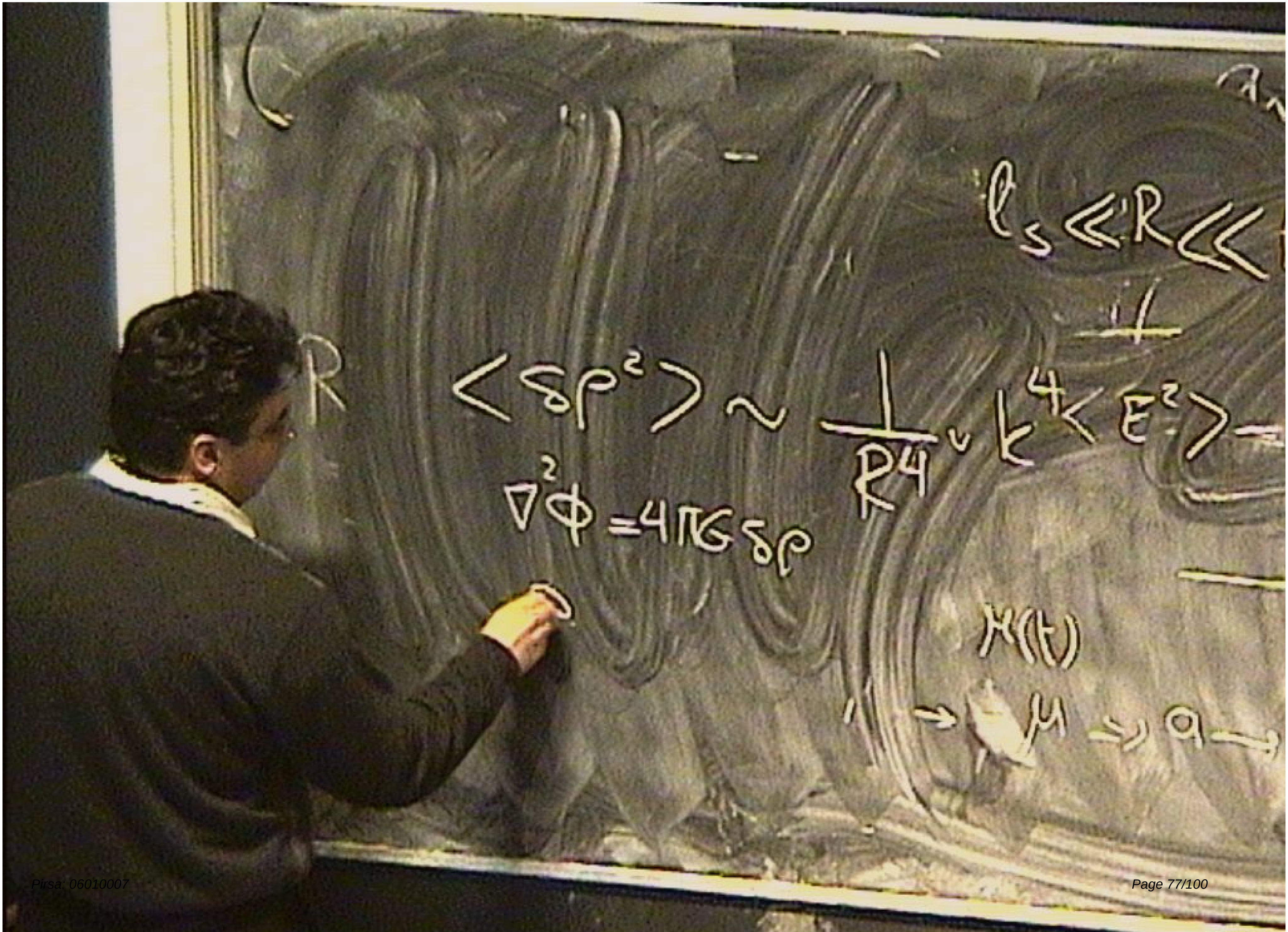
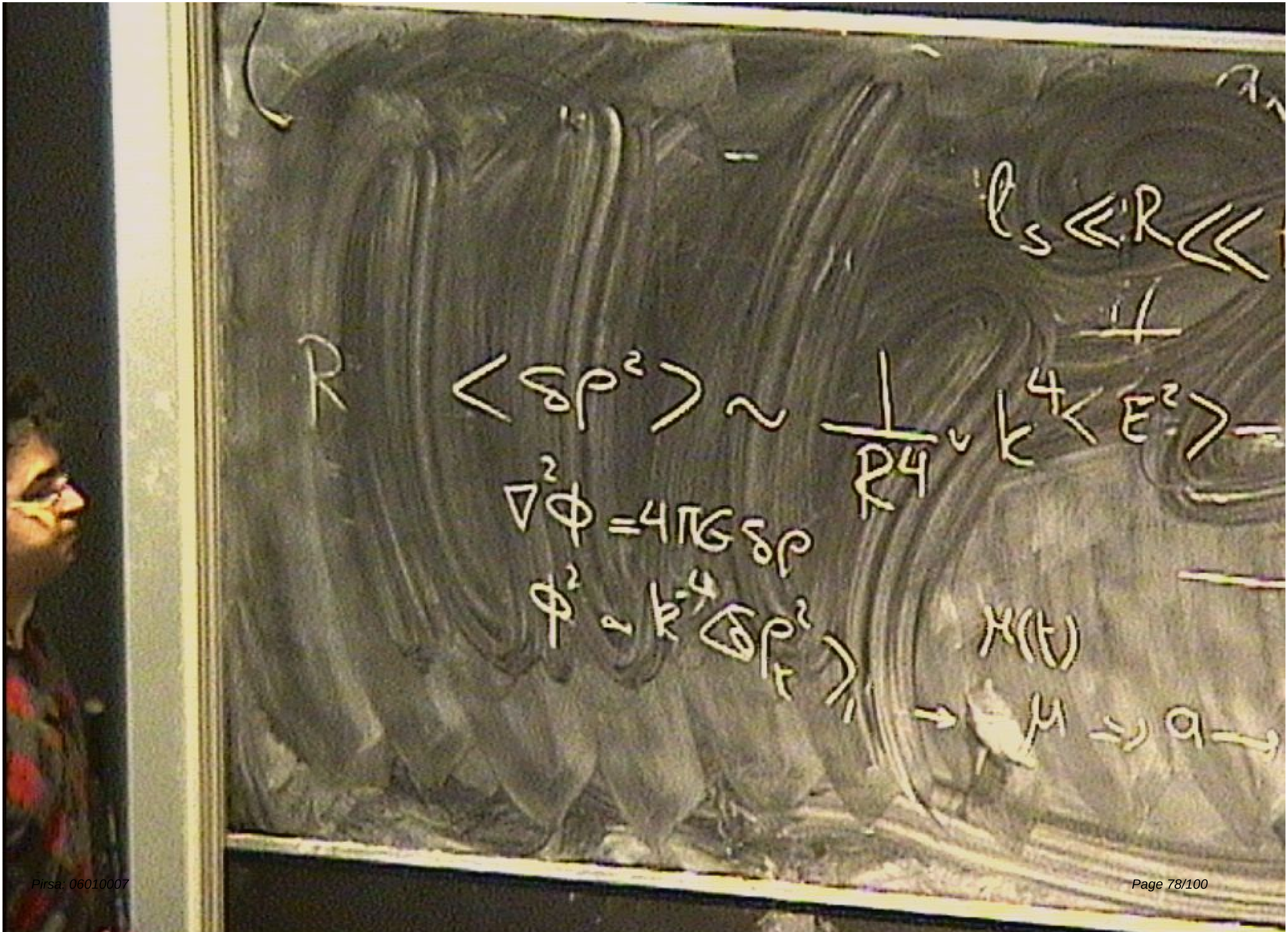

ls ≪ R ≪
R
<δρ²> ~ 1/R4 k4 <E²>
x(t)
μ → a →



ℓs ≪ R ≪
⟨δρ²⟩ ~ 1/R⁴ k⁴⟨E²⟩
∇²Φ = 4πGδρ
M(t)
M → a →


$\ell_s \ll R \ll$
R
$\langle \delta\rho^2 \rangle \sim \frac{1}{R^4} k^4 \langle \epsilon^2 \rangle$
$\nabla^2 \phi = 4\pi G \delta\rho$
$\phi^2 \sim k^{-4} \langle \delta\rho_t^2 \rangle$
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$\mu \rightarrow a \rightarrow$

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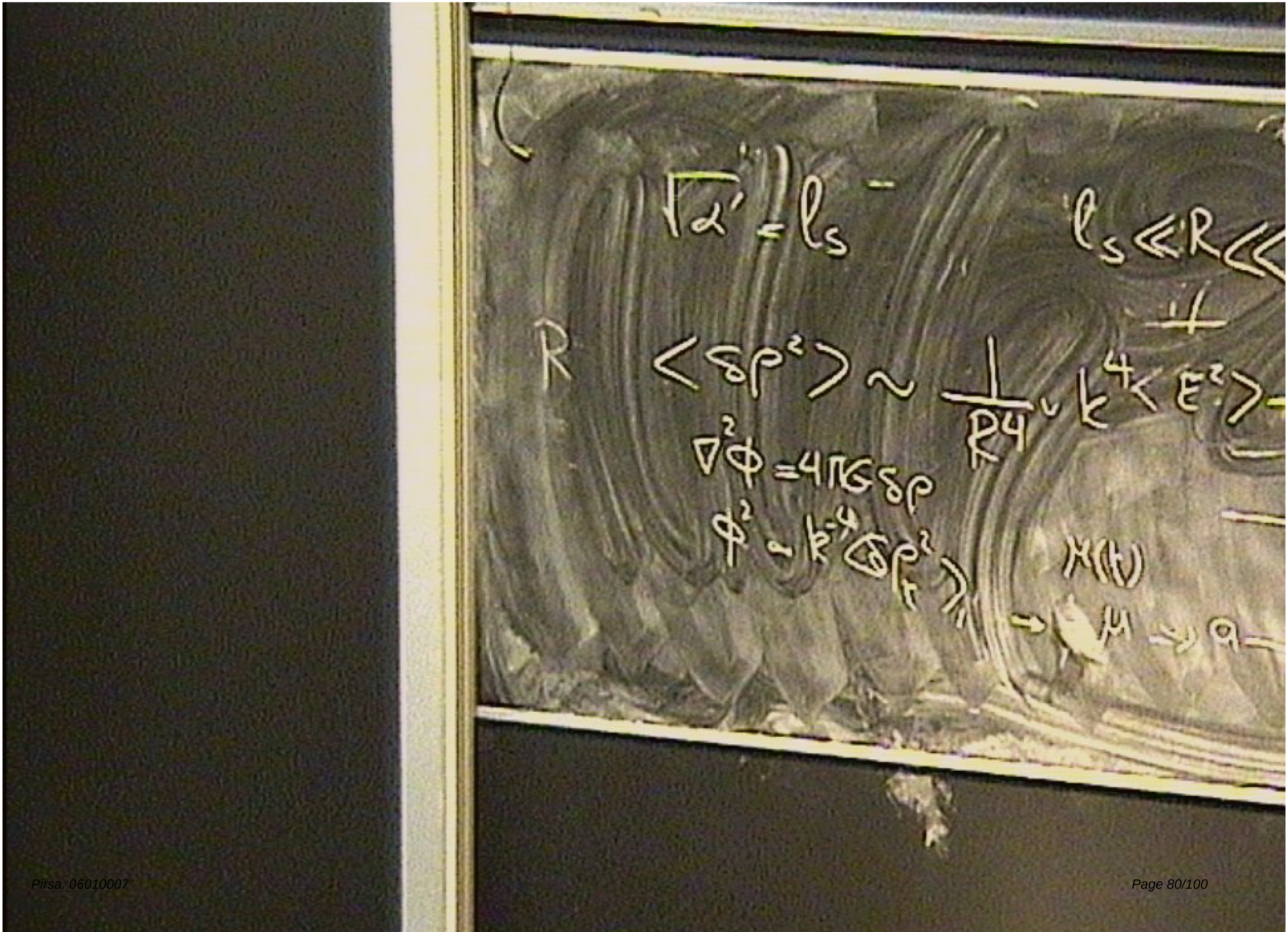
$\sqrt{\alpha'} = l_s$
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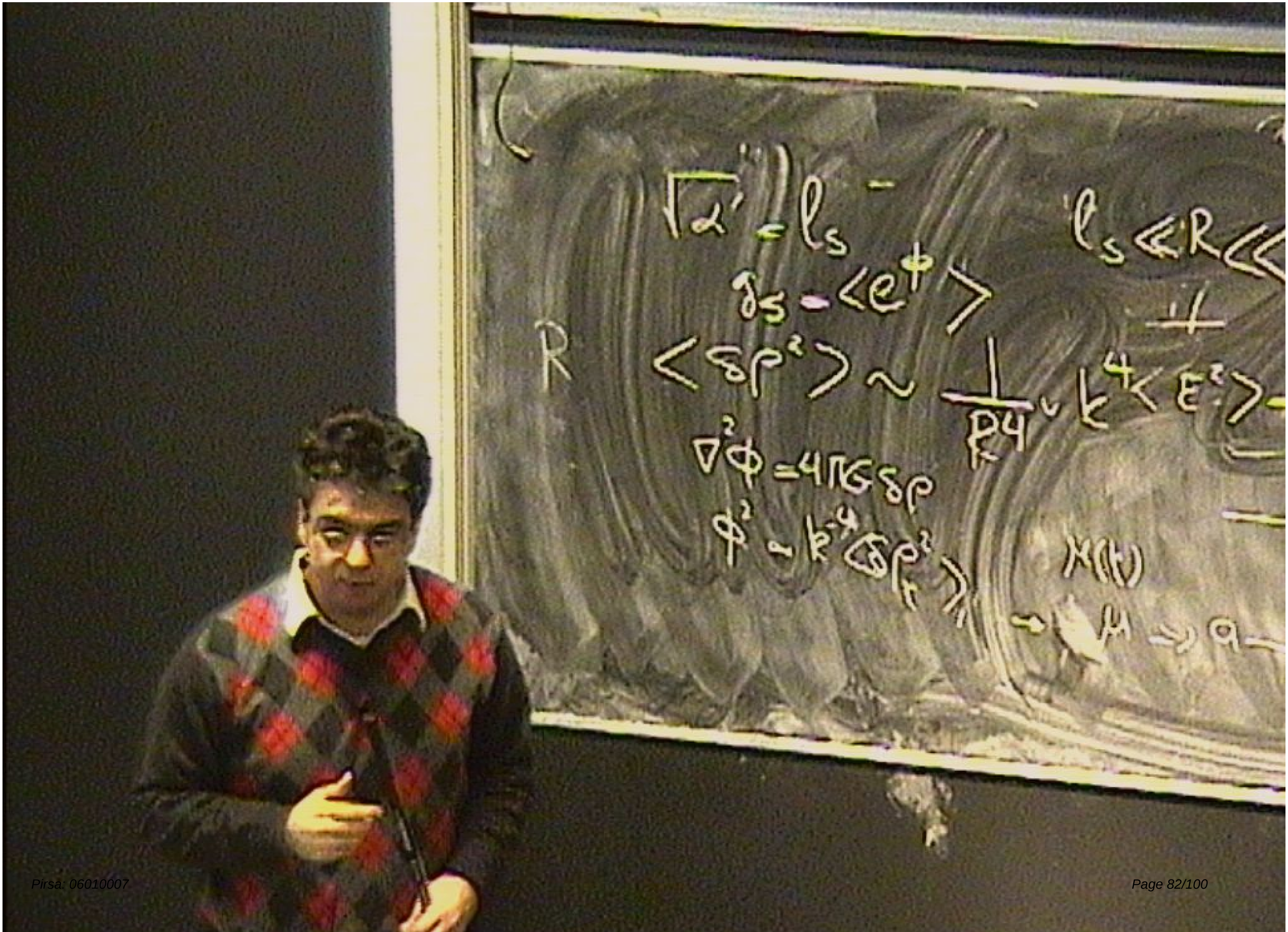

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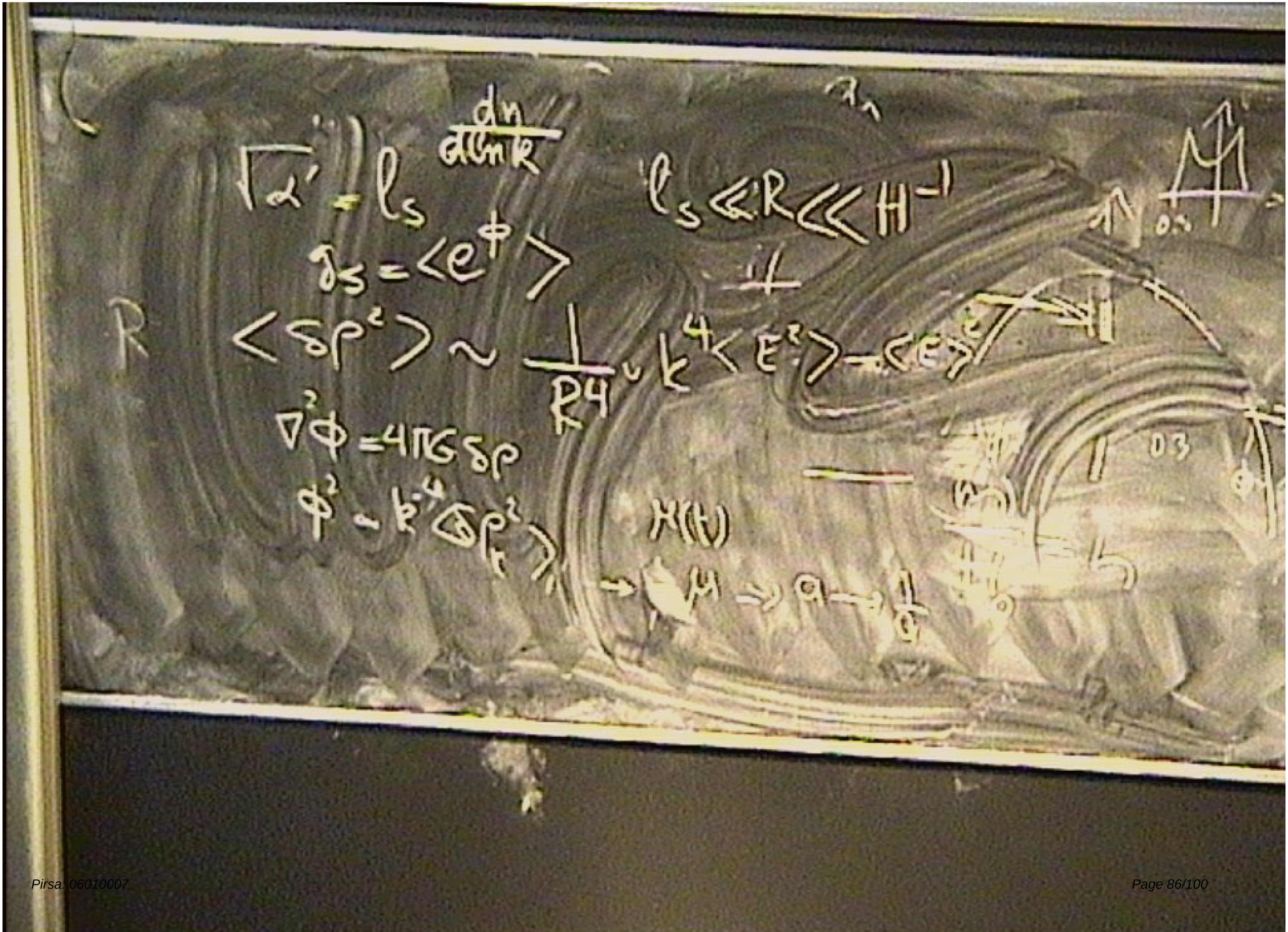
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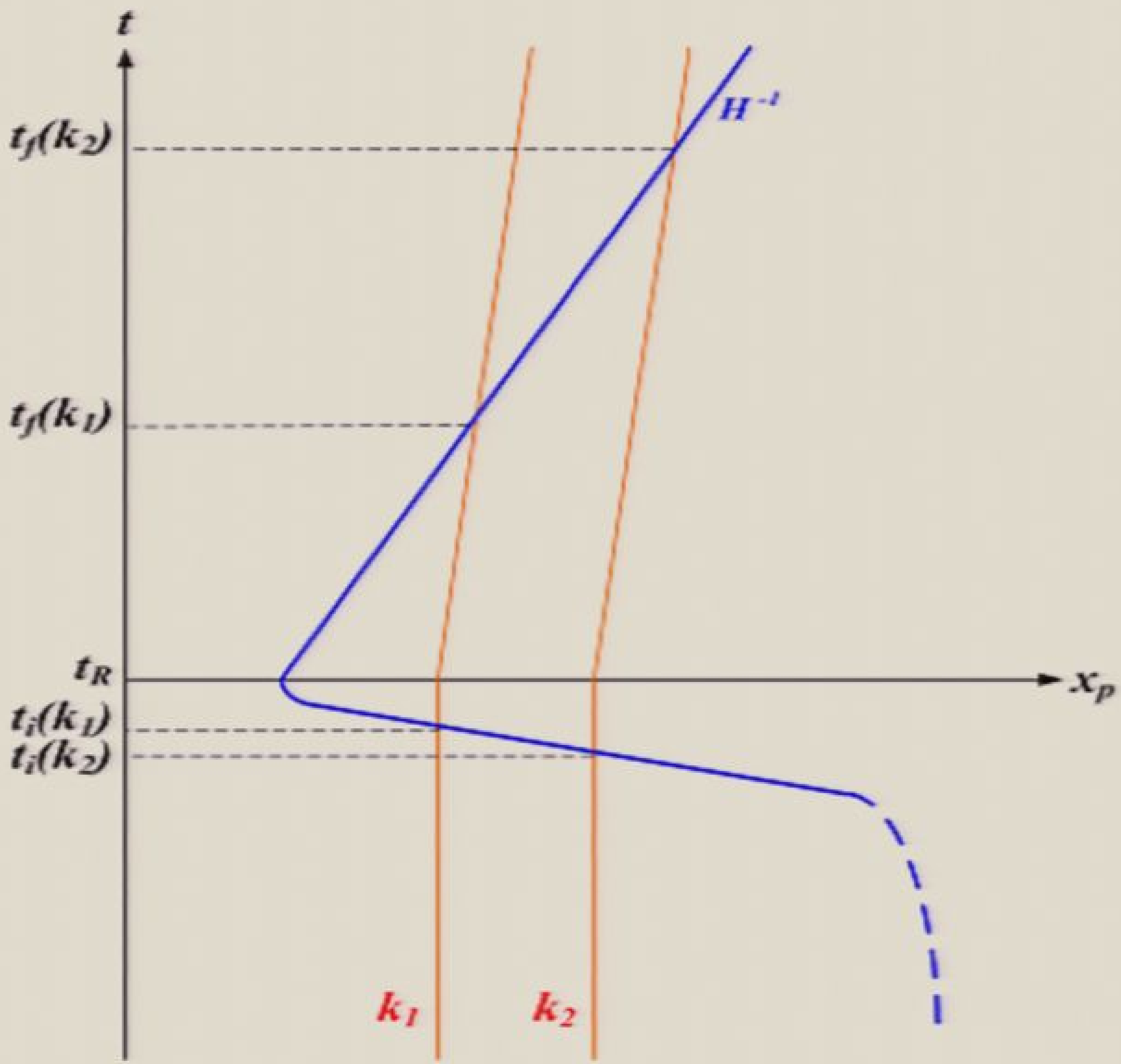

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Conclusion I

- Here, we have studied the generation and evolution of cosmological fluctuations in a model of string gas cosmology in which an early quasi-static Hagedorn phase is followed by the radiation-dominated phase of standard cosmology, without an intervening period of inflation.

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- In order to compute the spectrum of metric fluctuations at late times, we have applied the usual general relativistic theory of cosmological perturbations. Whereas this is clearly justified for times $t > t_R$, its use at earlier times is doubtful.

Conclusion II

- We have also assumed that the metric perturbation variable Φ does not change on super-Hubble scales during the transition between the Hagedorn phase and the radiation-dominated phase of standard cosmology. These assumptions are well justified in the context of the usual relativistic perturbation theory. However, the fact that the Hagedorn phase is described by a dilaton gravity background and not by a purely general relativistic background may lead to some modifications.

Conclusion II

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- Although our cosmological scenario provides a new mechanism for generating a scale-invariant spectrum of cosmological perturbations, it does not solve all of the problems which inflation solves. In particular, it does not solve the flatness problem. Without assuming that the three large spatial dimensions are much larger than the string scale, we do not obtain a universe which is sufficiently large today.

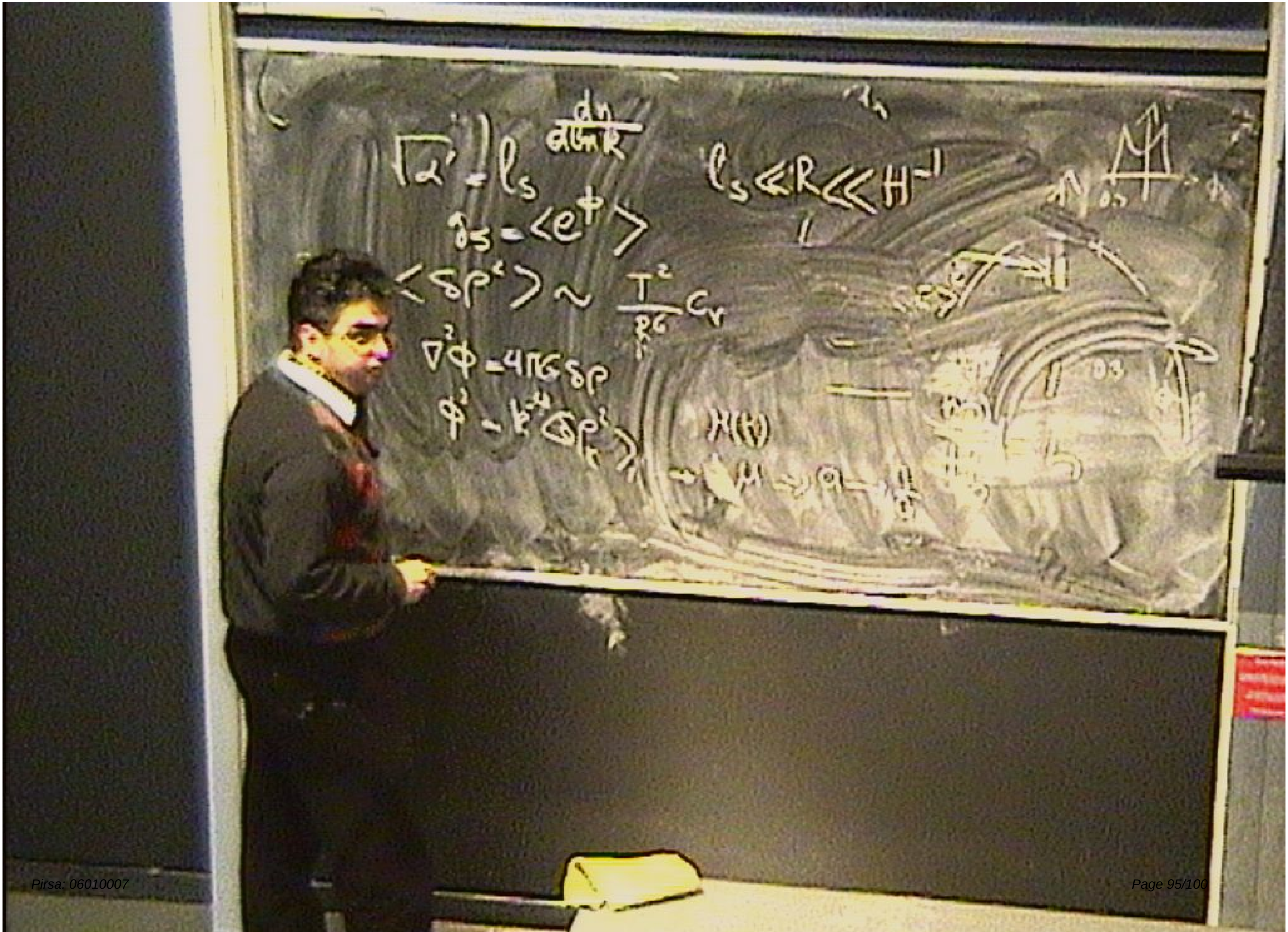
Future Hopes

Our scenario may well be testable observationally. Taking into account the fact that the temperature T evaluated at the time $t_i(k)$ when the scale k exits the Hubble radius depends slightly on k, the formula leads to a calculable deviation of the spectrum from exact scale invariance. Since $T(t_i(k))$ is decreased in gas k increases, a slightly red spectrum is predicted. Since the equation of state does not change by orders of magnitude during the transition between the initial phase and the radiation-dominated phase as it does in inflationary cosmology, the spectrum of tensor modes is not expected to be suppressed compared to that of scalar modes. Hence, a large ratio of tensor to scalar fluctuations might be a specific prediction of our model. This issue deserves further attention.

Dilatonic String Cosmology

$$\mathcal{A} = \int \sqrt{-{}^{(10)}g}\, d^{10}x\, e^{-2\phi} \left[{}^{(10)}R + 4(\nabla\phi)^2 + \ldots \right] ,$$

$$ds^2 = -dt^2 + \sum_i^{(d)} a^2 dx_i^2 + \sum_i^{(9-d)} a_s^2 dx'^2_i ,$$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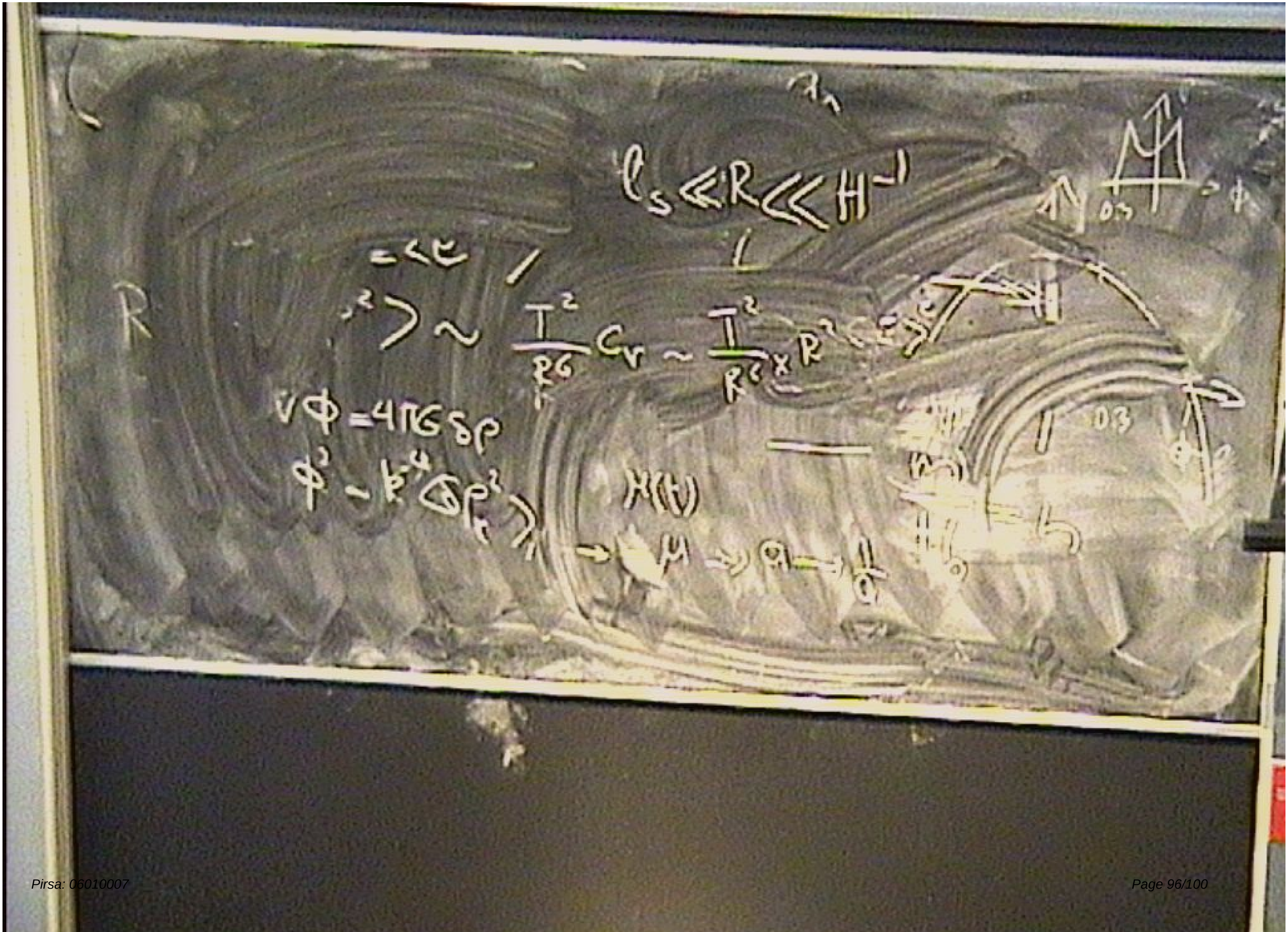
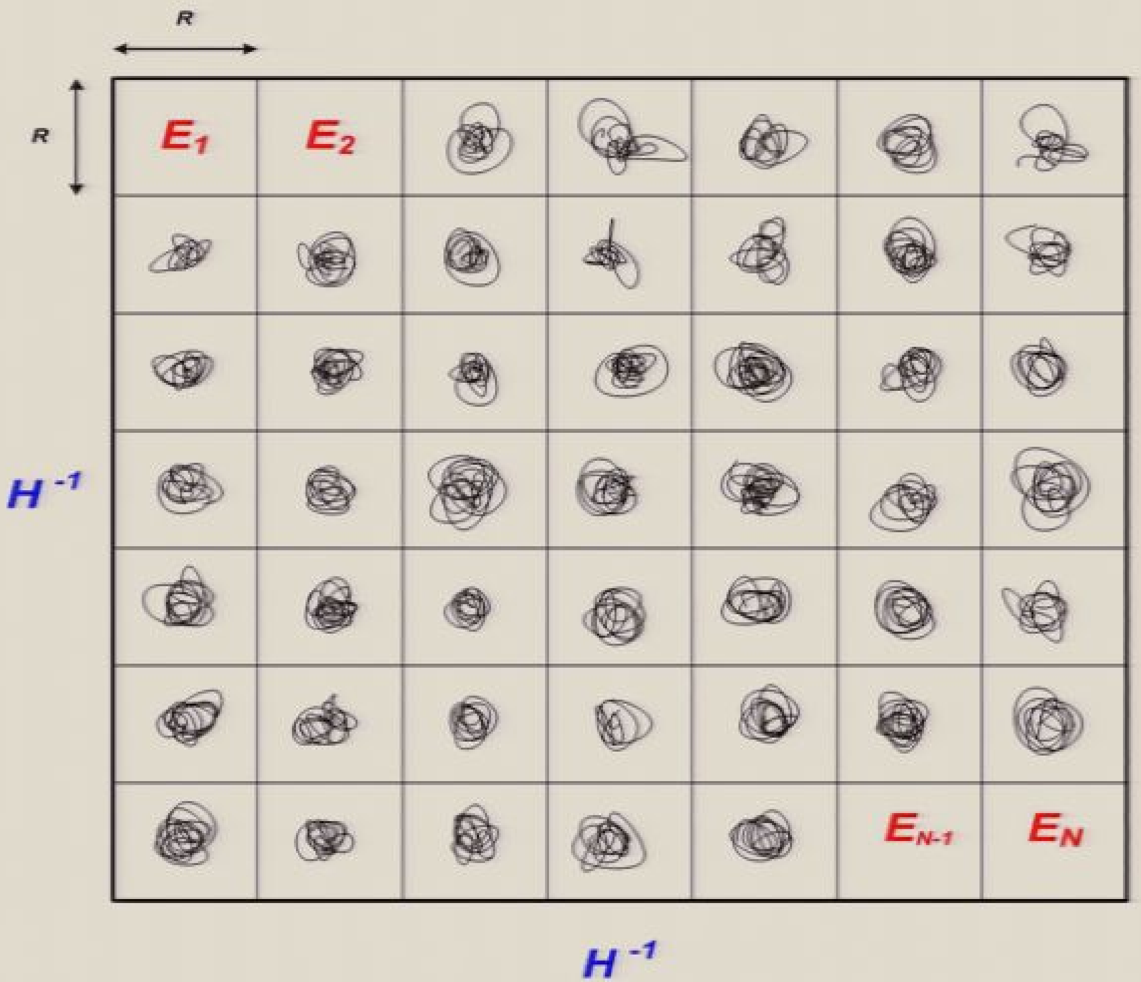

ls ≪ R ≪ H^-1
∇Φ = 4πG δρ
H(t)
μ → a → 1/a
D3

Universe in Hagedorn Phase



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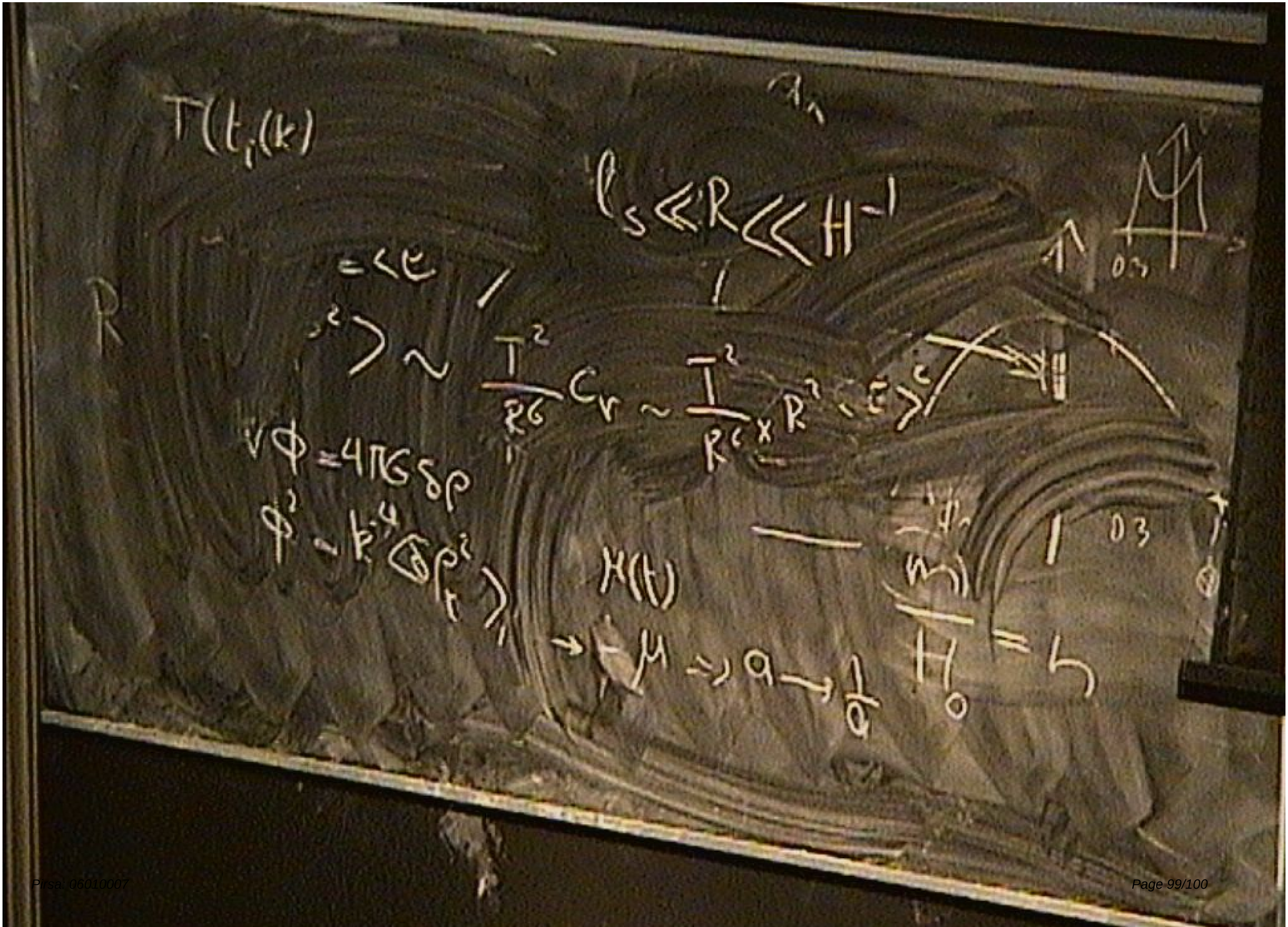
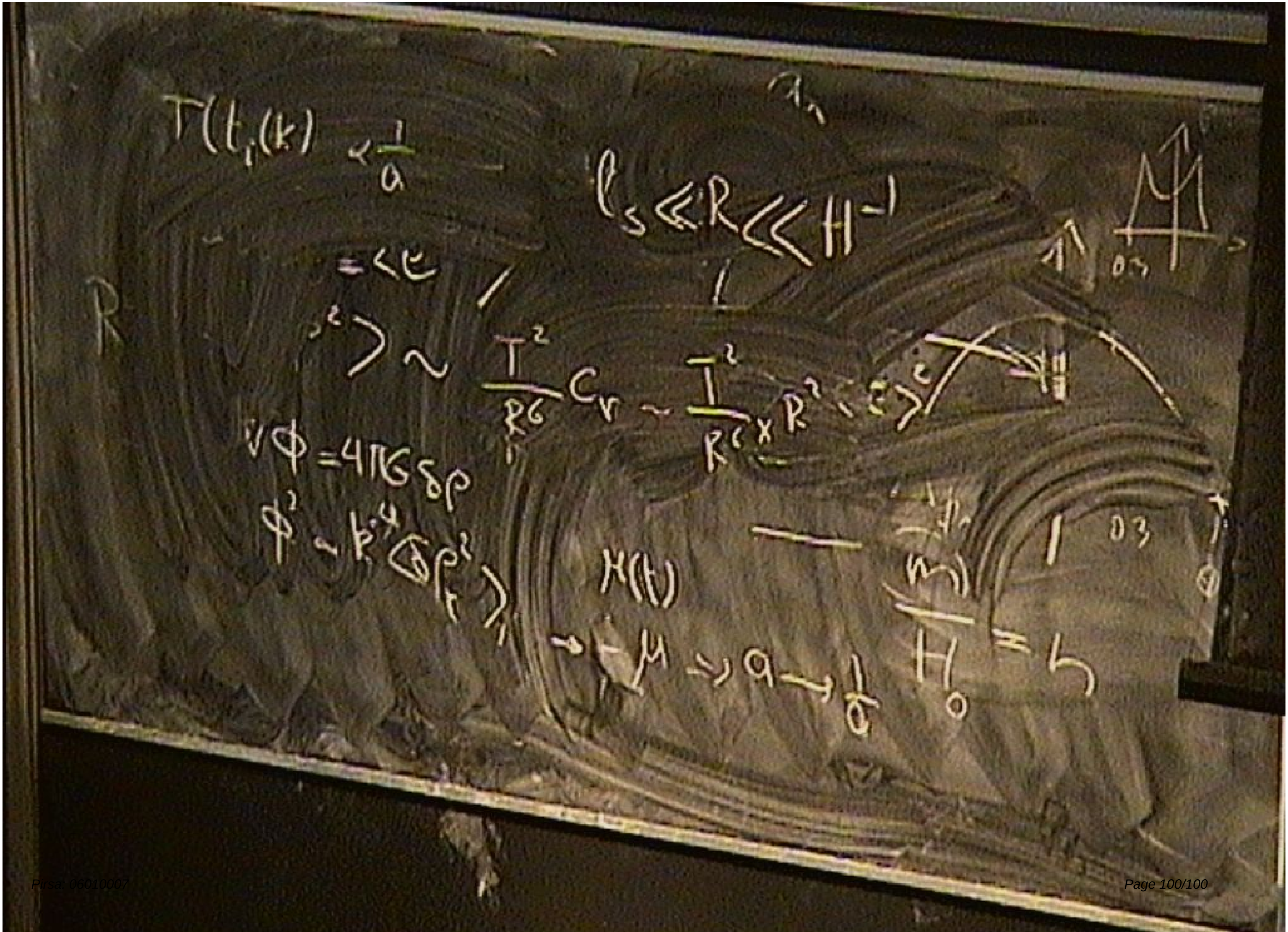

T(t_i(k))
l_s ≪ R ≪ H^-1
∇φ = 4πG δρ
H(t)
= h


T(t_i(k)) < 1/a
l_s << R << H^-1
=4πG δρ
H(t)