Title: The universality of highly damped quasinormal modes in generic single horizon black holes

Date: Jan 12, 2006 11:00 AM

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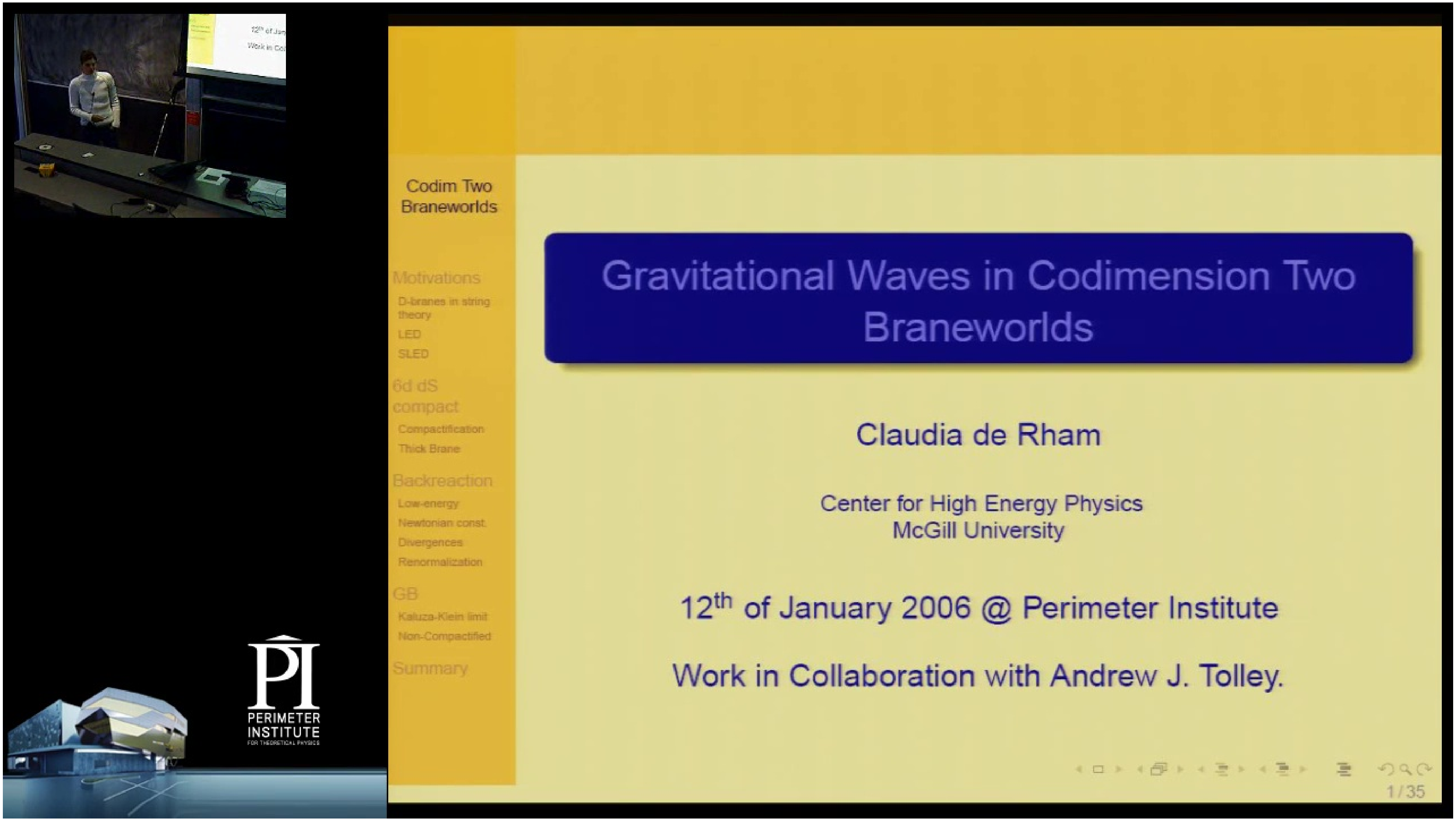
Abstract: We calculate analytically the highly damped quasinormal mode spectra of generic single-horizon black holes using the rigorous WKB techniques of Andersson and Howls. We thereby provide a firm foundation for previous analysis, and point out some of their possible limitations. The numerical coefficient in the real part of the highly damped frequency is generically determined by the behavior of coupling of the perturbation to the gravitational field near the origin, as expressed in tortoise coordinates. This fact makes it difficult to understand how the (in)famous ln(3) could be related to the quantum gravitational microstates near the horizon.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Compactification
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Backreaction
Low-energy
Newtonian const.
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Summary
Gravitational Waves in Codimension Two Braneworlds
Claudia de Rham
Center for High Energy Physics
McGill University
12th of January 2006 @ Perimeter Institute
Work in Collaboration with Andrew J. Tolley.
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1. Motivations
   - D-branes in string theory
   - Large Extra Dimensions
   - Supersymmetric Large Extra Dimensions
2. 6d de Sitter compactifications
   - Warped flux compactification
   - Thick Brane
3. Backreaction of Codimension two branes
   - Low-energy Expansion
   - Effective Newtonian constant
   - Divergences
   - Renormalization and EFT
4. Gauss-Bonnet terms
   - Kaluza-Klein limit
   - Uncompactified geometry

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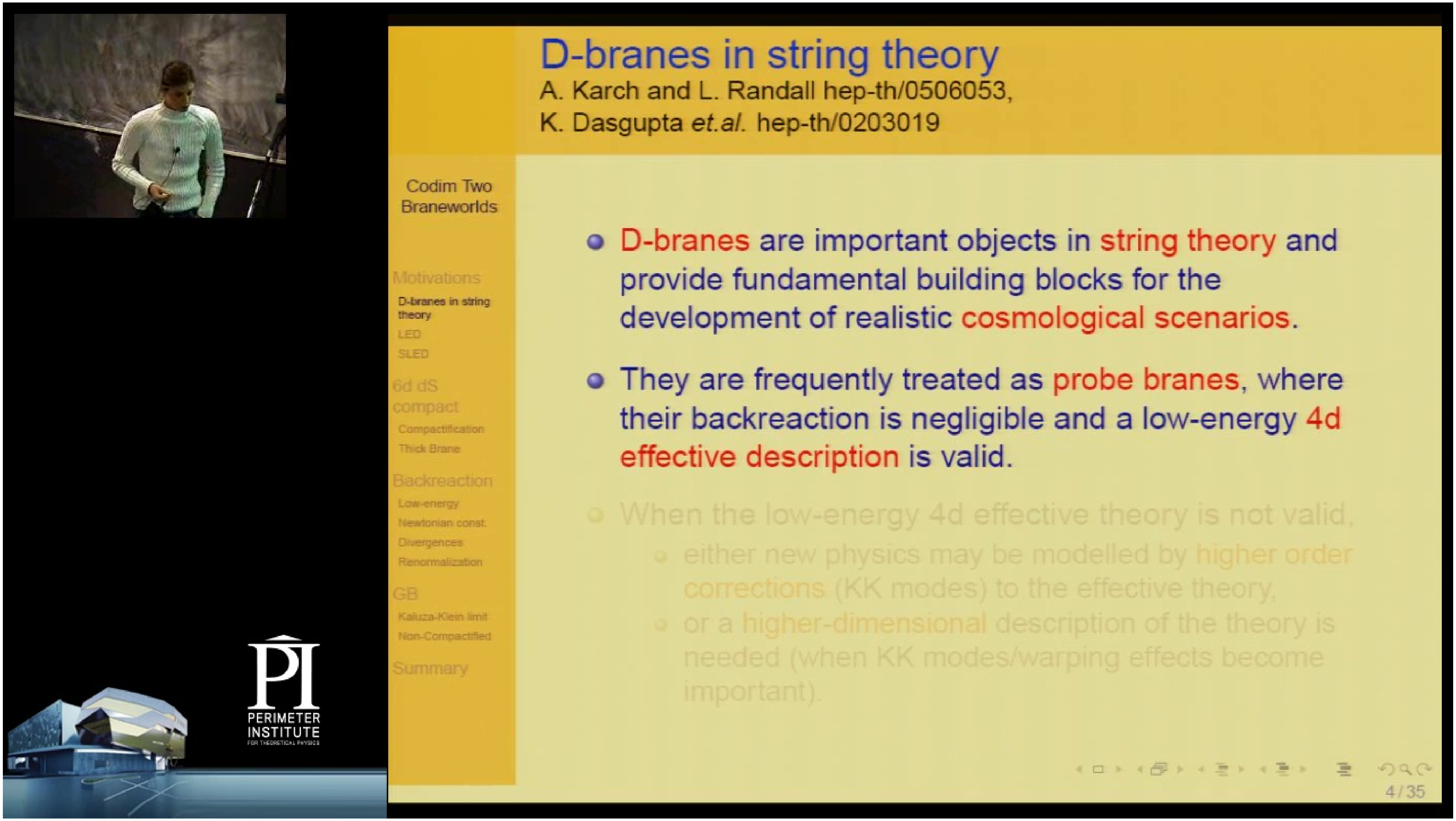
D-branes in string theory
A. Karch and L. Randall hep-th/0506053,
K. Dasgupta et.al. hep-th/0203019
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D-branes are important objects in string theory and provide fundamental building blocks for the development of realistic cosmological scenarios.
They are frequently treated as probe branes, where their backreaction is negligible and a low-energy 4d effective description is valid.
When the low-energy 4d effective theory is not valid,
either new physics may be modelled by higher order corrections (KK modes) to the effective theory,
or a higher-dimensional description of the theory is needed (when KK modes/warping effects become important).
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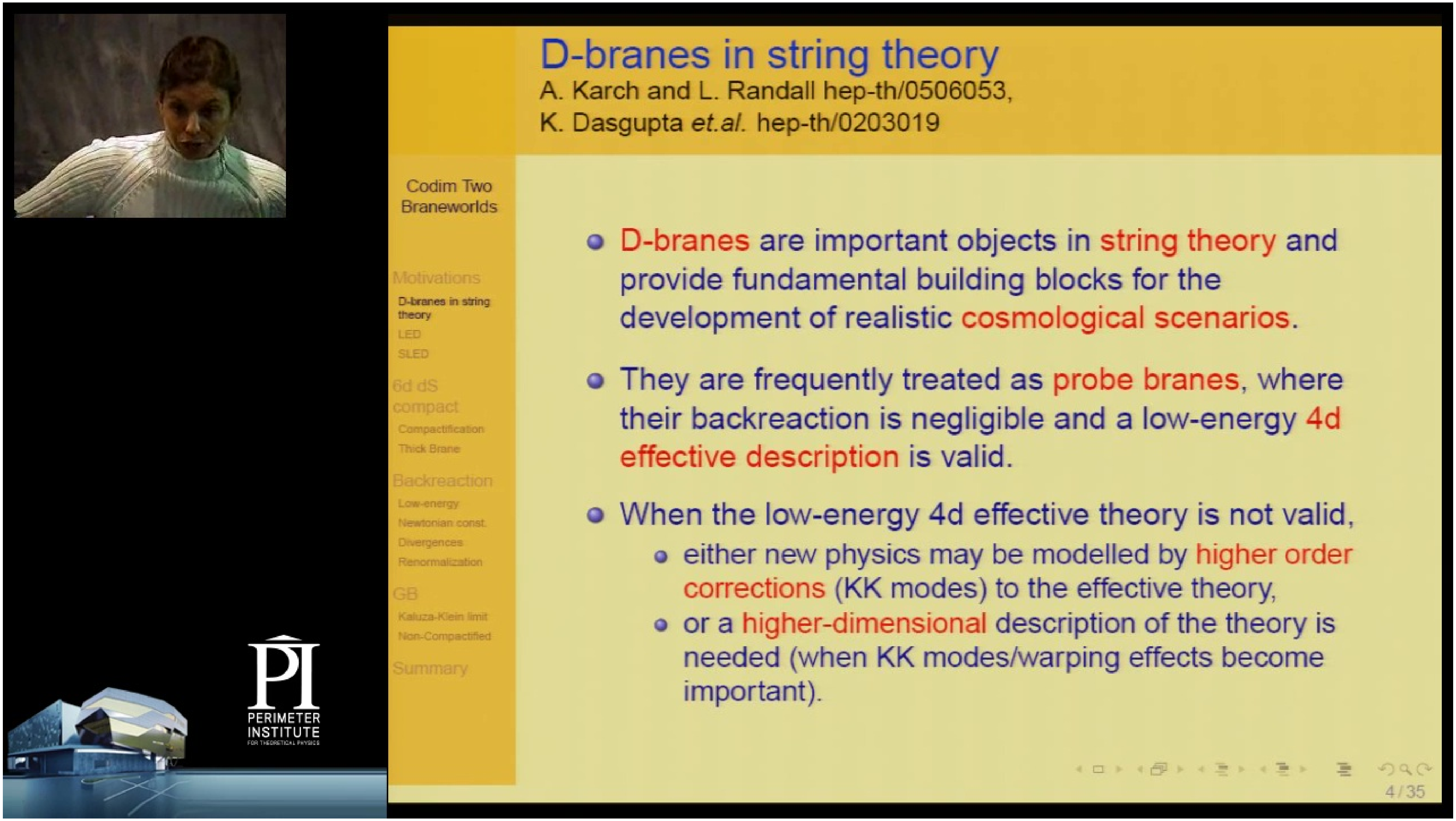
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Large extra dimensions

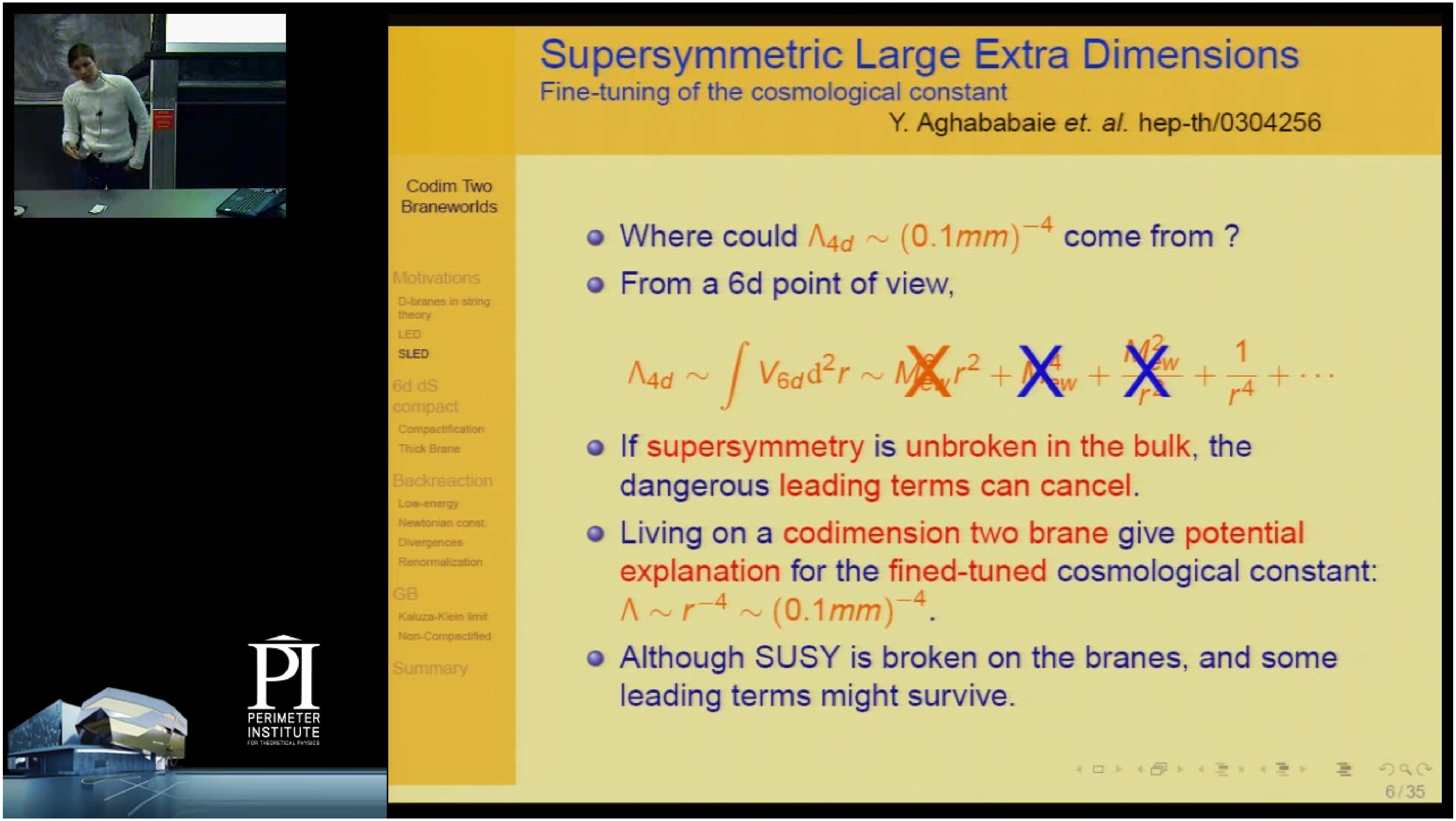
N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali hep-ph/9803315

- In the ADD model, the presence of large extra dimensions might solve the Hierarchy problem.
- If two extra dimensions are of the order of the mm, the fundamental Planck mass M_{6d} could be of the same order of magnitude as the electroweak scale:

$$M_{\rm Pl}^2 = V_d \, M_{d+4}^{d+2}$$

$$\text{if } d = 2, \text{ and } V_d \approx (0.1\text{mm})^2 \Rightarrow M_{d+4} \approx \text{TeV} \sim M_{ew}$$

- Gravity has not *yet* been tested for scales below the mm $\Rightarrow$ there is room for having modified gravity in the UV.
- The current cosmological constant could come from the same order of magnitude $\Lambda \approx \left(3 \times 10^{-12}\text{GeV}\right)^4 \sim (0.1mm)^{-4}$.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Supersymmetric Large Extra Dimensions
Fine-tuning of the cosmological constant
Y. Aghababaie *et. al.* hep-th/0304256
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Summary
Where could $\Lambda_{4d} \sim (0.1mm)^{-4}$ come from ?
From a 6d point of view,
$$\Lambda_{4d} \sim \int V_{6d} \mathrm{d}^2 r \sim M_{ew}^6 r^2 + M_{ew}^4 + \frac{M_{ew}^2}{r^2} + \frac{1}{r^4} + \cdots$$
If supersymmetry is unbroken in the bulk, the dangerous leading terms can cancel.
Living on a codimension two brane give potential explanation for the fined-tuned cosmological constant: $\Lambda \sim r^{-4} \sim (0.1mm)^{-4}$.
Although SUSY is broken on the branes, and some leading terms might survive.
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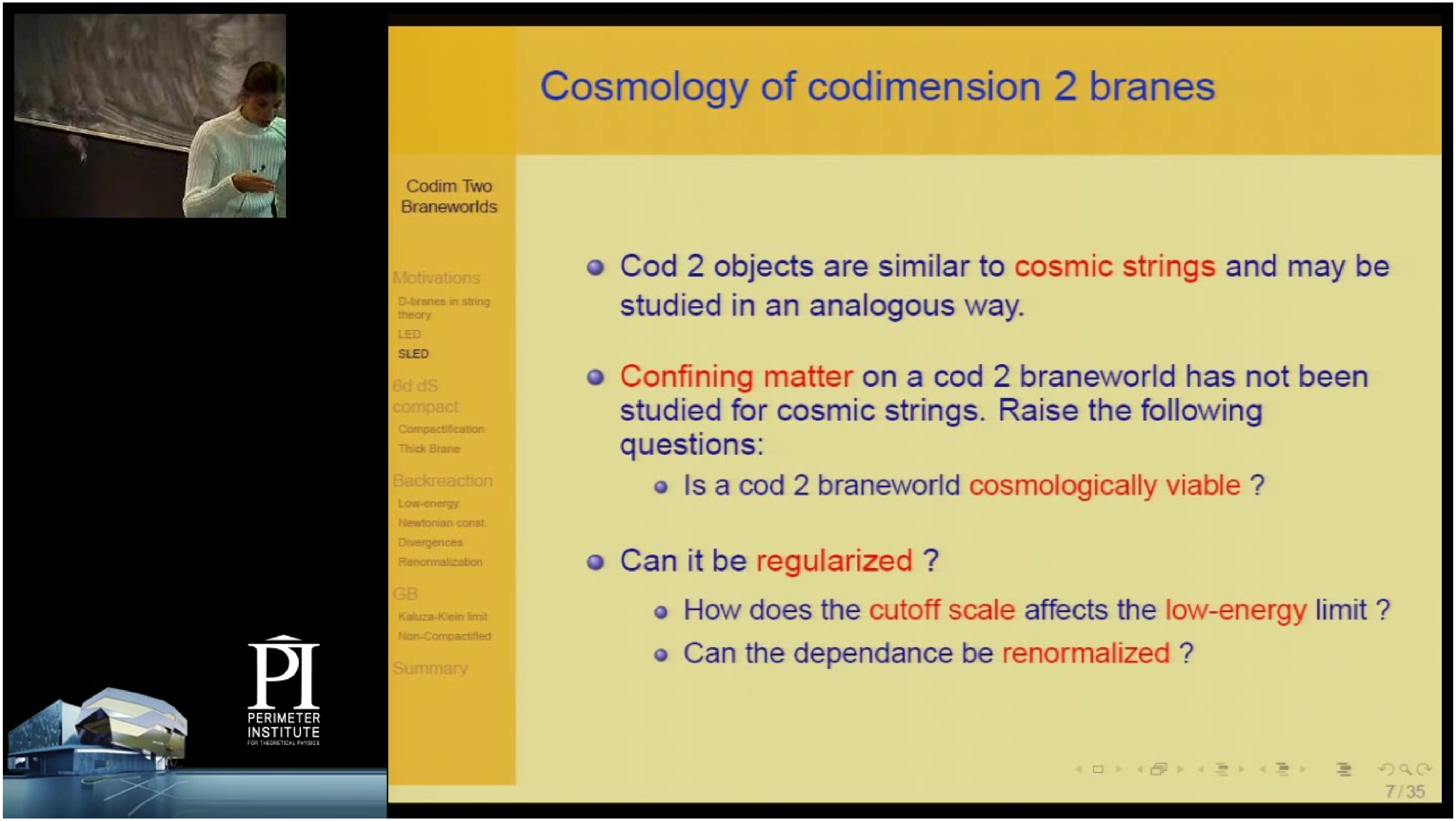
Cosmology of codimension 2 branes
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Summary
Cod 2 objects are similar to cosmic strings and may be studied in an analogous way.
Confining matter on a cod 2 braneworld has not been studied for cosmic strings. Raise the following questions:
Is a cod 2 braneworld cosmologically viable ?
Can it be regularized ?
How does the cutoff scale affects the low-energy limit ?
Can the dependance be renormalized ?
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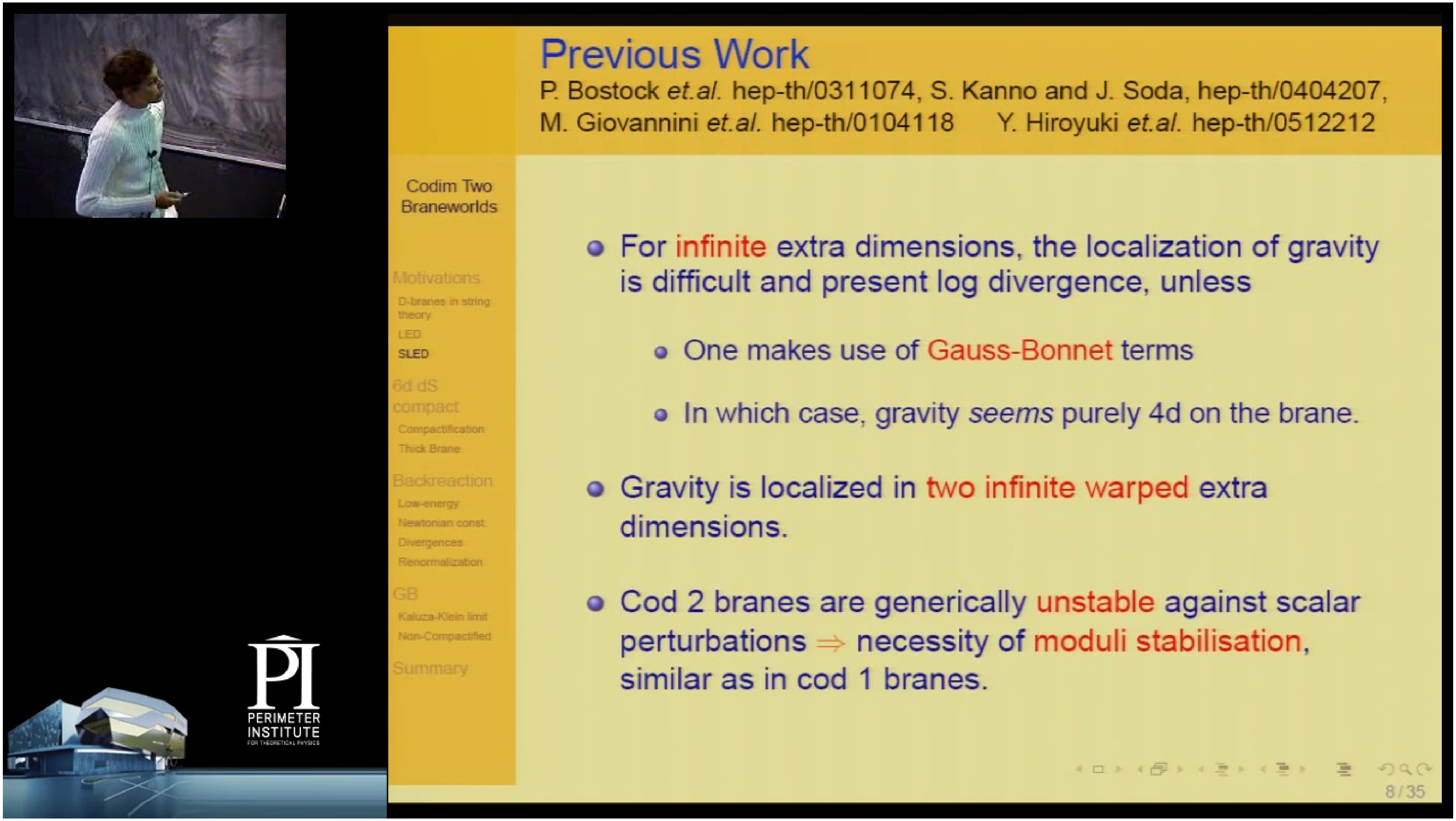
Previous Work
P. Bostock et.al. hep-th/0311074, S. Kanno and J. Soda, hep-th/0404207,
M. Giovannini et.al. hep-th/0104118 Y. Hiroyuki et.al. hep-th/0512212
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Summary
For infinite extra dimensions, the localization of gravity is difficult and present log divergence, unless
One makes use of Gauss-Bonnet terms
In which case, gravity seems purely 4d on the brane.
Gravity is localized in two infinite warped extra dimensions.
Cod 2 branes are generically unstable against scalar perturbations ⇒ necessity of moduli stabilisation, similar as in cod 1 branes.
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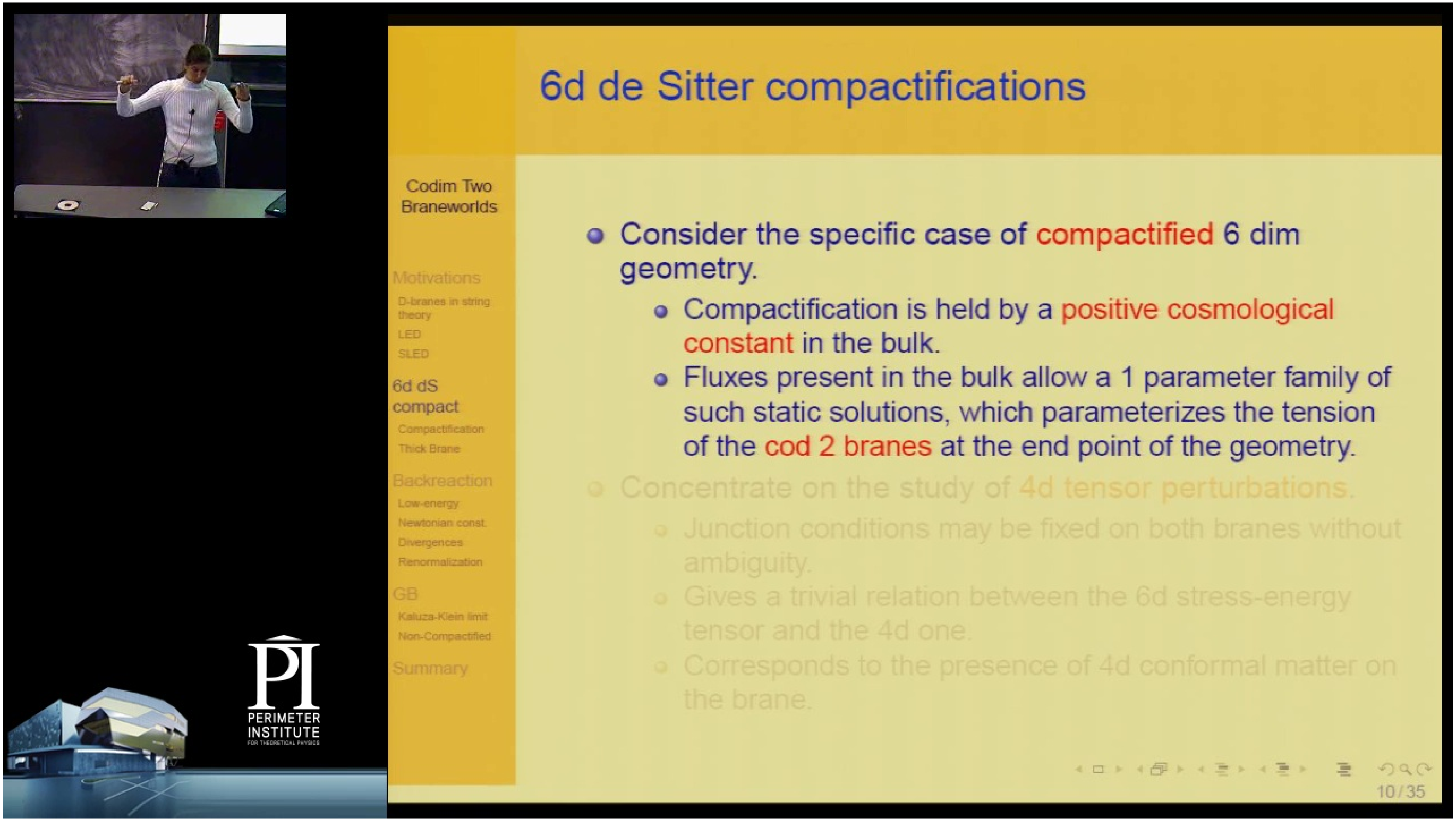
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Summary
Consider the specific case of compactified 6 dim geometry.
Compactification is held by a positive cosmological constant in the bulk.
Fluxes present in the bulk allow a 1 parameter family of such static solutions, which parameterizes the tension of the cod 2 branes at the end point of the geometry.
Concentrate on the study of 4d tensor perturbations.
Junction conditions may be fixed on both branes without ambiguity.
Gives a trivial relation between the 6d stress-energy tensor and the 4d one.
Corresponds to the presence of 4d conformal matter on the brane.
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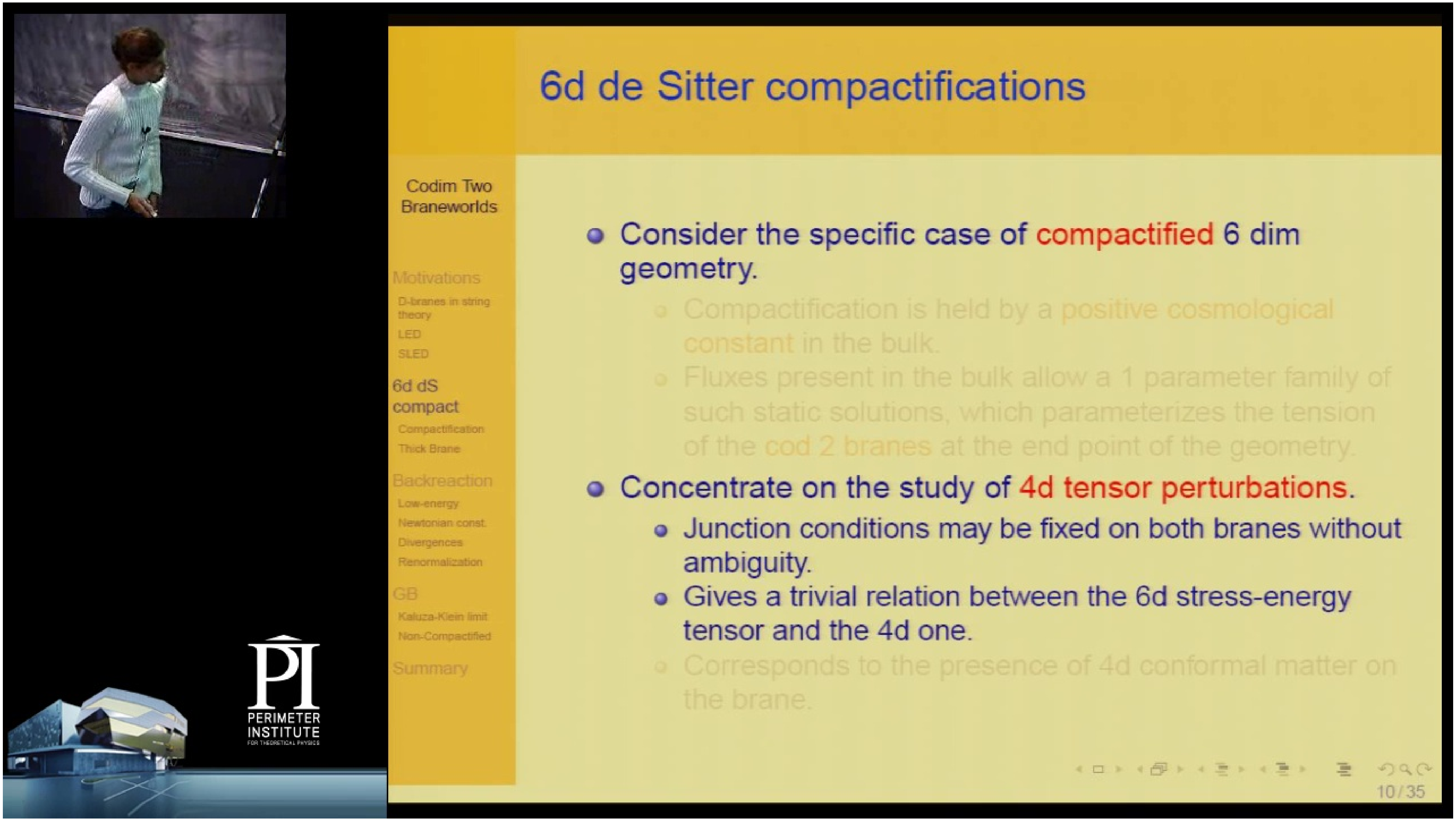
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6d de Sitter compactifications

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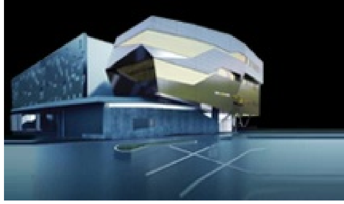


Warped flux compactification

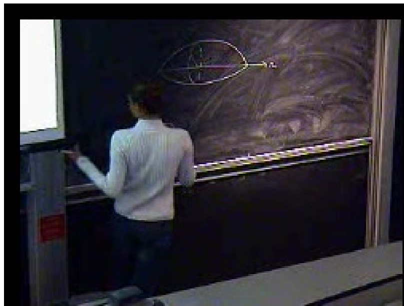
The model

S. Mukohyama *et.al.* hep-th/0506050

- As a toy model, we consider the model of flux compactification:

$$S = \int d^6x \sqrt{-g} \frac{1}{2\kappa} \left({}^{(6)}R - 2\Lambda - \frac{1}{2} F_{AB} F^{AB} \right),$$

- With Λ the 6d cosmological constant.
- And some bulk form field $F_{AB} = \partial_A A_B - \partial_B A_A$.



Warped flux compactification

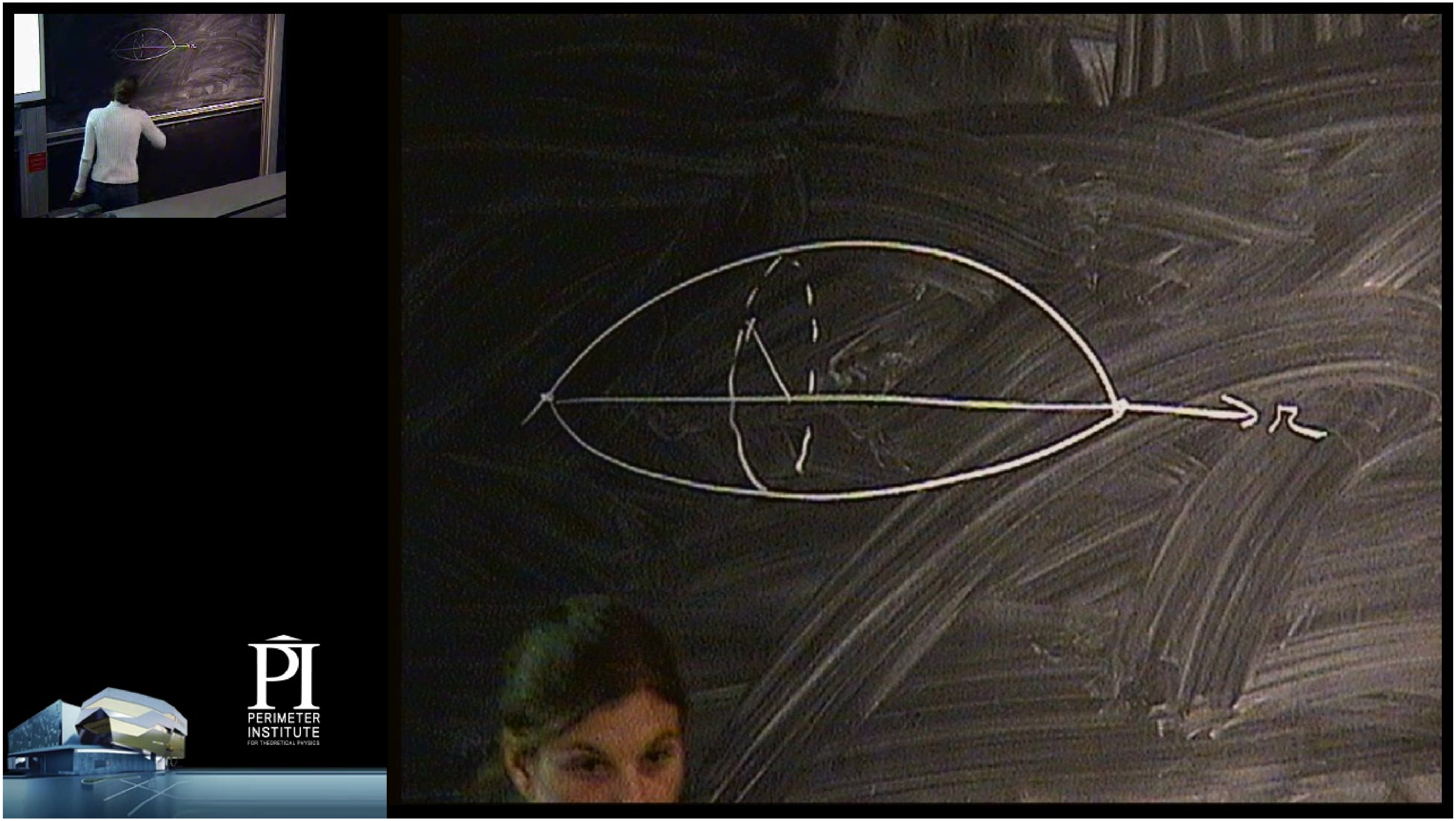
The solution

S. Mukohyama *et.al.* hep-th/0506050

- A solution of this theory has a metric of the form

$$
\begin{aligned}
\mathrm{d}s^2 &= f^{-1}(r)\mathrm{d}r^2 + L^2 f(r)\mathrm{d}\varphi^2 + H^2 r^2 q_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\
q_{\mu\nu} &= (H\tau)^{-2}\eta_{\mu\nu}, \\
f(r) &= 1 - \frac{\Lambda}{10}r^2 - \frac{\mu}{r^3} - \frac{b^2}{12r^6} \\
A_M \mathrm{d}x^M &= \frac{b}{3r^3} L\mathrm{d}\varphi.
\end{aligned}
$$

- Where the angular variable $\varphi \in [0, 2\pi]$ and the proper size of the φ direction is $2\pi\sqrt{f}L$.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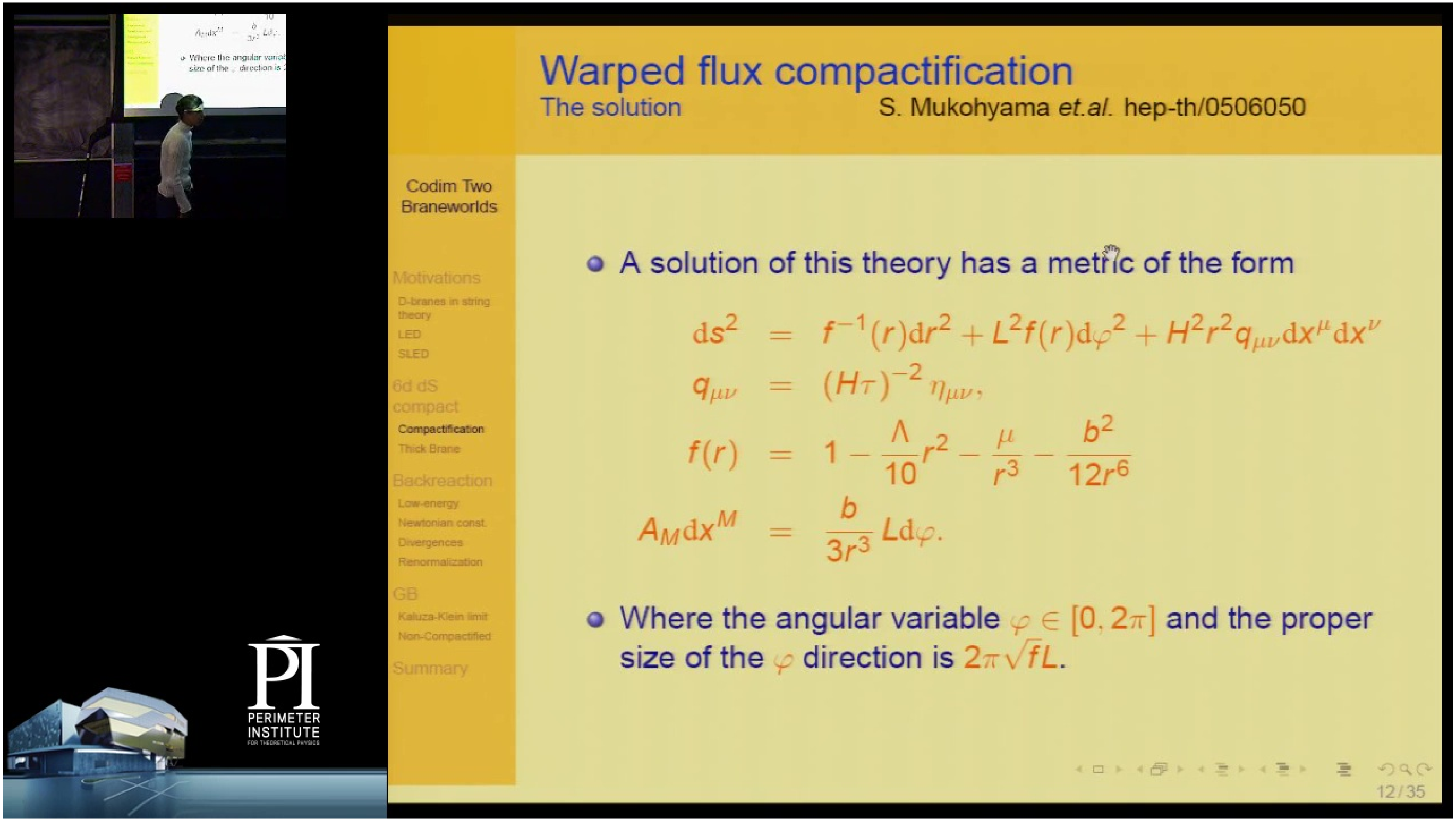
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$ds^2 = f^{-1}(r)dr^2 + L^2 f(r) d\varphi^2 + H^2 r^2 q_{\mu\nu} dx^\mu dx^\nu$
$q_{\mu\nu} = (H\tau)^{-2} \eta_{\mu\nu},$
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$A_M dx^M = \frac{b}{3r^3} L d\varphi.$
Where the angular variable $\varphi \in [0, 2\pi]$ and the proper size of the φ direction is $2\pi\sqrt{f}L$.
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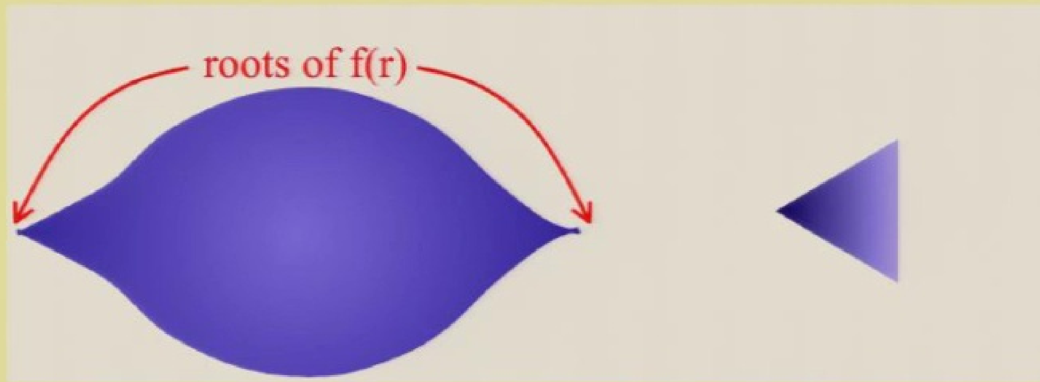
Warped flux compactification

Thick Brane

I. Navarro and J. Santagio hep-th/0411250

- The warping factor f has two real roots at $r_+ > r_- > 0$ where the branes are located.



- Close to the brane, the metric is flat Minkowski with a deficit angle $2\pi\left(1 \pm \frac{1}{2} f'(r_\pm)\right)$.

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Warped flux compactification

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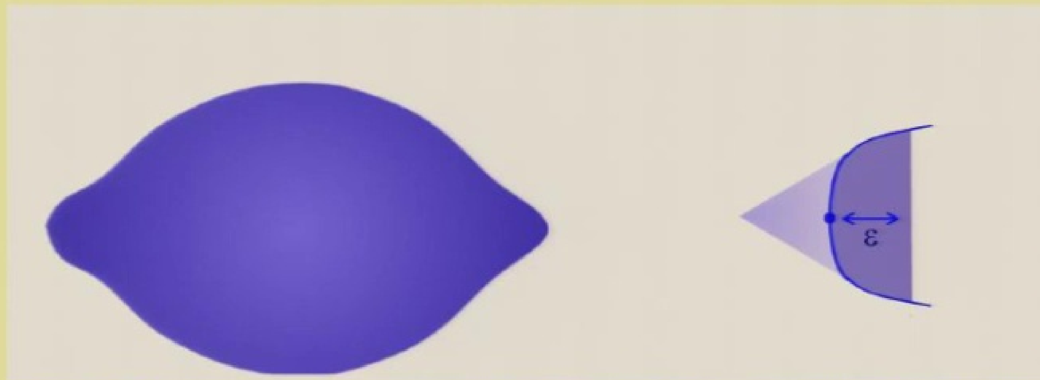

- To regulate the branes, one may replace them with a smooth stress energy source, or a thick brane, such as a cod two topological defect arising from Abelian-Higgs model.
- The conical deficits at the poles are replaced with smooth geometries.

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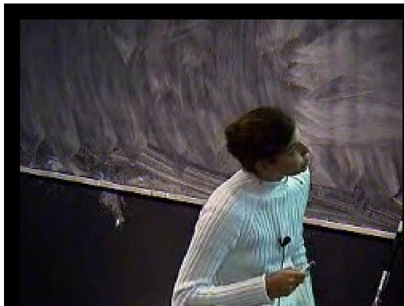
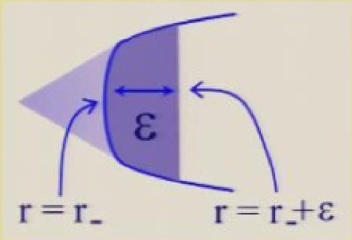


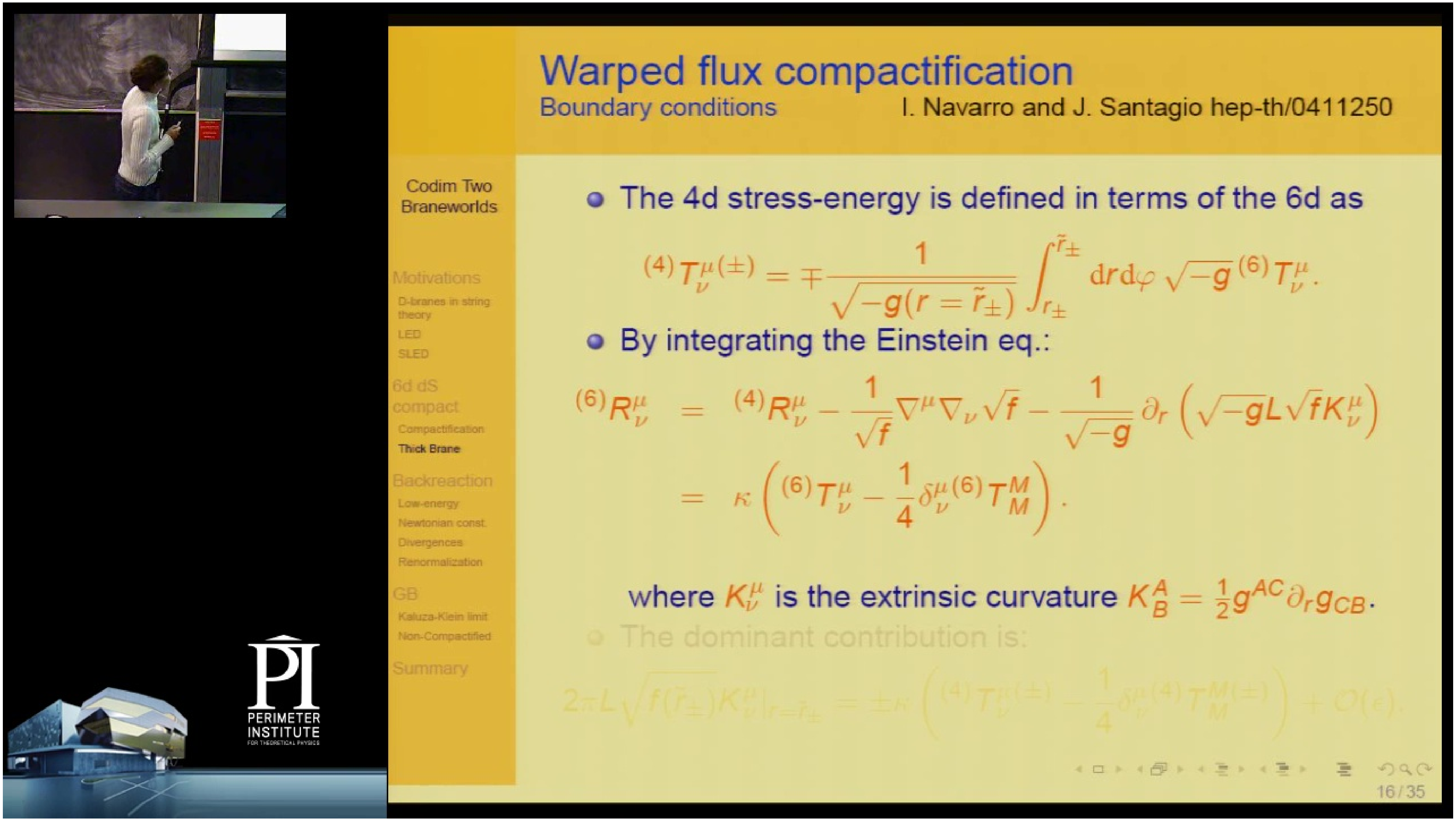
Warped flux compactification

Smooth Geometry

I. Navarro and J. Santagio hep-th/0411250



- One may integrate the Einstein equations over the brane thickness → infer the effective matching rule for the extrinsic curvature on the surface $r = \tilde{r}_\pm = r_\pm \mp \epsilon$ in terms of the integrated brane stress energies.
- This matching rules play the rôle of the Israël junction conditions for cod 1 branes.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Boundary conditions
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The 4d stress-energy is defined in terms of the 6d as
$$^{(4)}T^{\mu(\pm)}_{\nu} = \mp \frac{1}{\sqrt{-g(r=\tilde{r}_\pm)}} \int_{r_\pm}^{\tilde{r}_\pm} \mathrm{d}r\mathrm{d}\varphi \sqrt{-g}\, ^{(6)}T^{\mu}_{\nu}.$$
By integrating the Einstein eq.:
$$^{(6)}R^{\mu}_{\nu} = {}^{(4)}R^{\mu}_{\nu} - \frac{1}{\sqrt{f}}\nabla^\mu\nabla_\nu\sqrt{f} - \frac{1}{\sqrt{-g}}\partial_r\left(\sqrt{-g}L\sqrt{f}K^\mu_\nu\right) = \kappa\left(^{(6)}T^{\mu}_{\nu} - \frac{1}{4}\delta^{\mu}_{\nu}\, ^{(6)}T^{M}_{M}\right).$$
where K^μ_ν is the extrinsic curvature $K^A_B = \frac{1}{2}g^{AC}\partial_r g_{CB}$.
The dominant contribution is:
$$2\pi L\sqrt{f(\tilde{r}_\pm)}K^\mu_\nu|_{r=\tilde{r}_\pm} = \pm\kappa\left(^{(4)}T^{\mu(\pm)}_{\nu} - \frac{1}{4}\delta^\mu_\nu\, ^{(4)}T^{M(\pm)}_{M}\right) + \mathcal{O}(\epsilon).$$
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Warped flux compactification

Boundary conditions

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- The 4d stress-energy is defined in terms of the 6d as

$$ {}^{(4)}T^{\mu(\pm)}_{\nu} = \mp \frac{1}{\sqrt{-g(r=\tilde{r}_\pm)}} \int_{r_\pm}^{\tilde{r}_\pm} \mathrm{d}r\mathrm{d}\varphi \, \sqrt{-g}\, {}^{(6)}T^{\mu}_{\nu}. $$

- By integrating the Einstein eq.:

$$ \frac{1}{\sqrt{-g}} \int_{r_\pm}^{\tilde{r}_\pm} \mathrm{d}r\mathrm{d}\varphi \left[\sqrt{-g} \left(R^{\mu}_{\nu} - \frac{1}{\sqrt{f}} \nabla^{\mu}\nabla_{\nu}\sqrt{f} \right) - \partial_r \left(\sqrt{-g} L \sqrt{f} K^{\mu}_{\nu} \right) \right] $$

$$ = \frac{\kappa}{\sqrt{-g}} \int_{r_\pm}^{\tilde{r}_\pm} \mathrm{d}r\mathrm{d}\varphi \left({}^{(6)}T^{\mu}_{\nu} - \frac{1}{4} \delta^{\mu}_{\nu} {}^{(6)}T^{M}_{M} \right). $$

where K^{μ}_{ν} is the extrinsic curvature $K^{A}_{B} = \frac{1}{2} g^{AC} \partial_r g_{CB}$.

- The dominant contribution is:

$$ 2\pi L \sqrt{f(\tilde{r}_\pm)} K^{\mu}_{\nu}|_{r=\tilde{r}_\pm} = \pm\kappa \left({}^{(4)}T^{\mu(\pm)}_{\nu} - \frac{1}{4} \delta^{\mu}_{\nu} {}^{(4)}T^{M(\pm)}_{M} \right) + \mathcal{O}(\epsilon). $$

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Backreaction of Codimension two branes

cdr + A. J. Tolley hep-th/0511138

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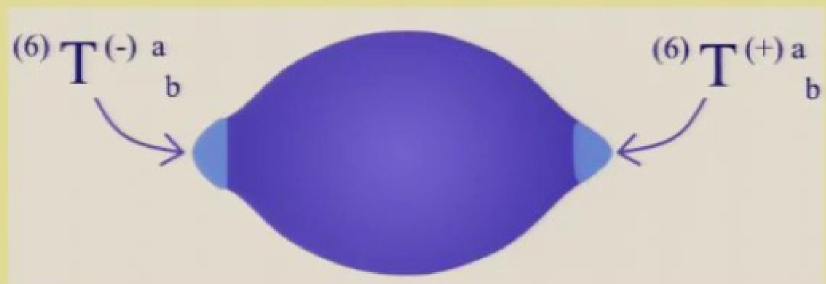
- Consider 4d tensor perturbations

$$
\begin{aligned}
q_{\mu\nu} &= (H\tau)^{-2}\eta_{\mu\nu} + h_{\mu\nu} \\
h^{\mu}_{\mu} &= 0 \qquad h^{\mu}_{\nu;\mu} = 0.
\end{aligned}
$$



- Sourced by a traceless stress-energy on each brane $^{(6)}T^{(\pm)a}{}_{b}$.

Backreaction of Codimension two branes

- Giving rise to the Einstein equation:

$$\delta^{(6)}R^{\mu}_{\nu} = \frac{1}{H^2 r^2}\underbrace{\delta^{(4)}R^{\mu}_{\nu}\left[q_{\mu\nu}\right]}_{-\Box h^{\mu}_{\nu}} - \frac{2}{r} f\, h^{\mu}_{\nu,r} - \frac{1}{2}\left(f\, h^{\mu}_{\nu,r}\right)_{,r} = 0,$$

- and the boundary conditions on each brane:

$$\partial_r h^{\mu}_{\nu}(\tilde{r}_{\pm}) = \pm \frac{\kappa}{\pi L H^2 \tilde{r}_{\pm}^2 f(\tilde{r}_{\pm})}\, {}^{(4)}T^{(\pm)\mu}_{\ \ \nu}.$$

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Backreaction of Codimension two branes

Low-energy expansion

- At low-energies, we may consider $\Box h \ll \partial_r^2 h$.
- And hence express the solution as an expansion in $\Box$:

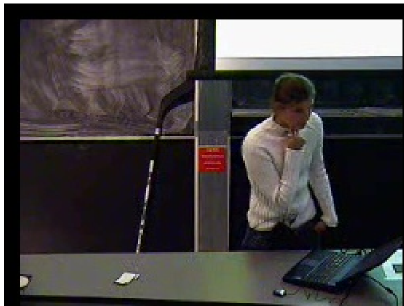
$$h(r, x^\mu) = \sum_{n\geq 0} \left(\frac{\Box}{H^2}\right)^{n-1} h_n(r, x^\mu),$$

- At low-energy, the zero mode, dominates:

$$h^\mu_\nu(\tilde{r}_\pm, x^\mu) = \frac{-2\tilde{\kappa}_\pm}{\Box}\left({}^{(4)}T^{\mu(\pm)}_\nu + \frac{\tilde{r}^2_\mp}{\tilde{r}^2_\pm}\,{}^{(4)}T^{\mu(\mp)}_\nu\right)$$

$$\tilde{\kappa}_\pm = \frac{3}{2\pi L H^2\left(\tilde{r}^3_+ - \tilde{r}^3_-\right)}\,\kappa.$$

- It is *finite* in the thin brane limit $\epsilon \to 0$.



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Backreaction of Codimension two branes

cdr + A. J. Tolley hep-th/0511138

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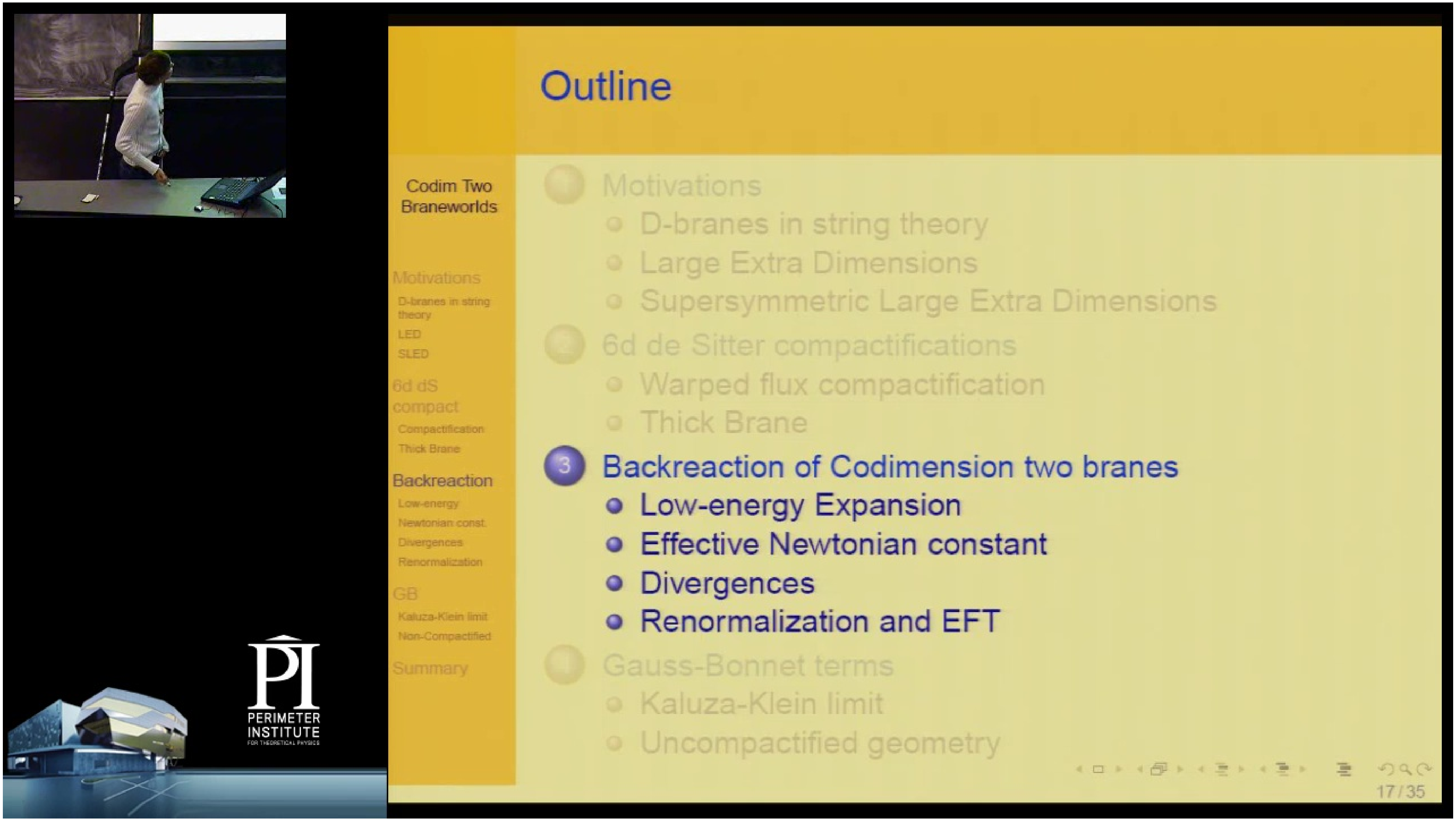
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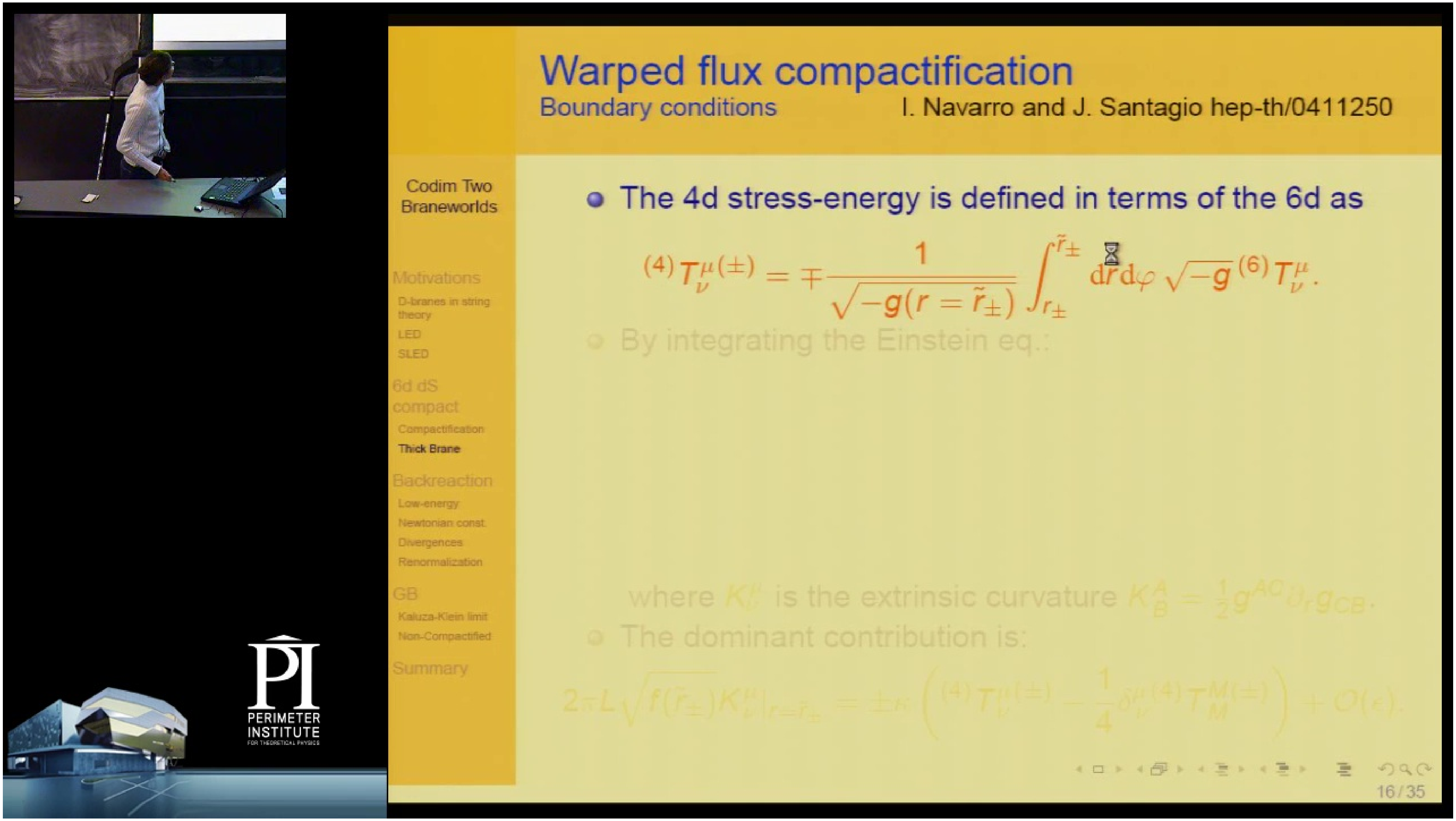
Warped flux compactification
Boundary conditions
I. Navarro and J. Santagio hep-th/0411250
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Summary
The 4d stress-energy is defined in terms of the 6d as
$^{(4)}T^{\mu\,(\pm)}_{\nu} = \mp \frac{1}{\sqrt{-g(r=\tilde{r}_\pm)}} \int_{r_\pm}^{\tilde{r}_\pm} \mathrm{d}r \mathrm{d}\varphi \sqrt{-g}\, {}^{(6)}T^{\mu}_{\nu}.$
By integrating the Einstein eq.:
The dominant contribution is:
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Warped flux compactification

The solution — S. Mukohyama *et.al.* hep-th/0506050

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- A solution of this theory has a metric of the form

$$
\begin{aligned}
\mathrm{d}s^2 &= f^{-1}(r)\mathrm{d}r^2 + L^2 f(r)\mathrm{d}\varphi^2 + H^2 r^2 q_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\
q_{\mu\nu} &= (H\tau)^{-2}\eta_{\mu\nu}, \\
f(r) &= 1 - \frac{\Lambda}{10}r^2 - \frac{\mu}{r^3} - \frac{b^2}{12r^6} \\
A_M \mathrm{d}x^M &= \frac{b}{3r^3} L\mathrm{d}\varphi.
\end{aligned}
$$

- Where the angular variable $\varphi \in [0, 2\pi]$ and the proper size of the φ direction is $2\pi\sqrt{f}L$.

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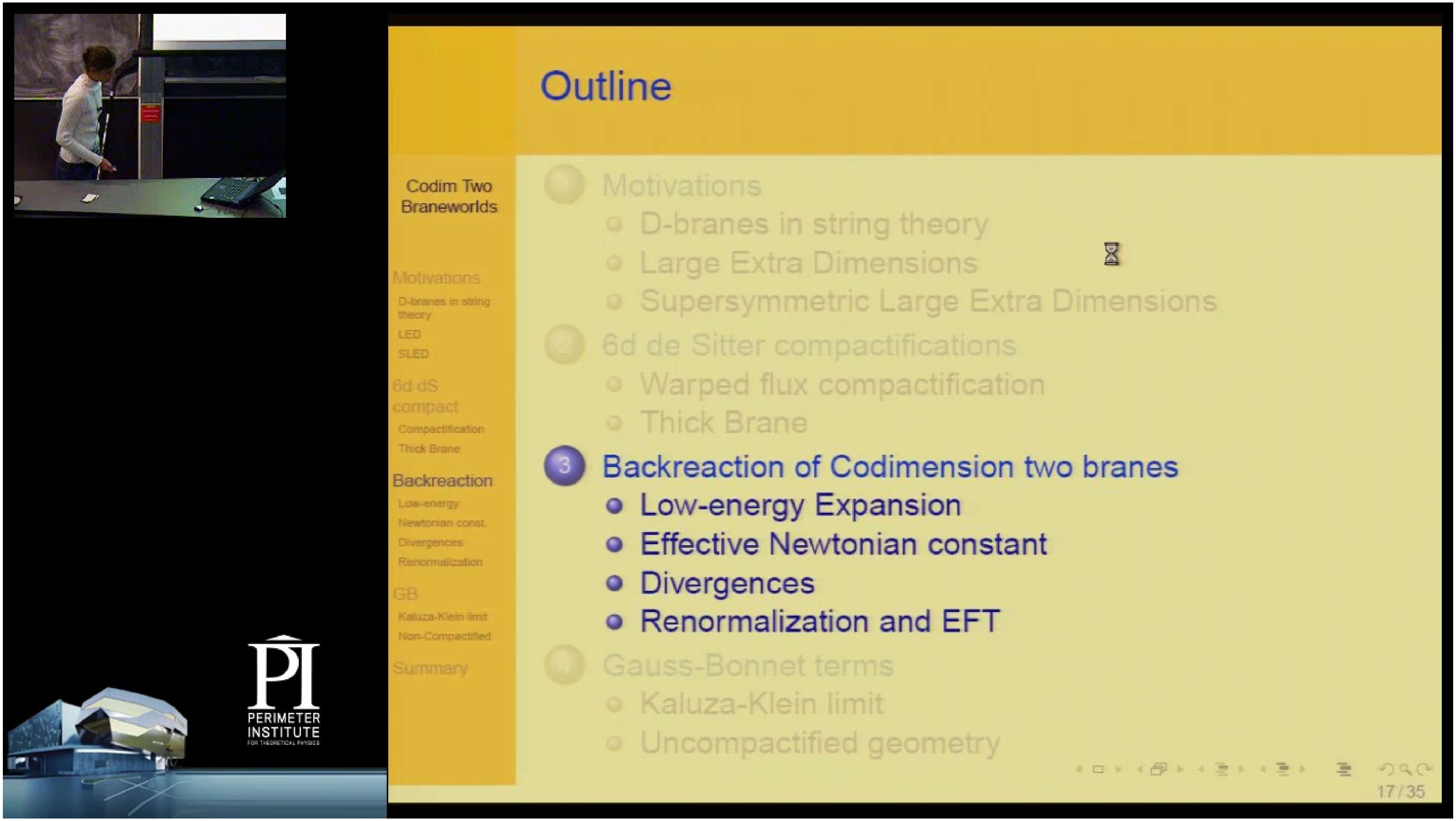
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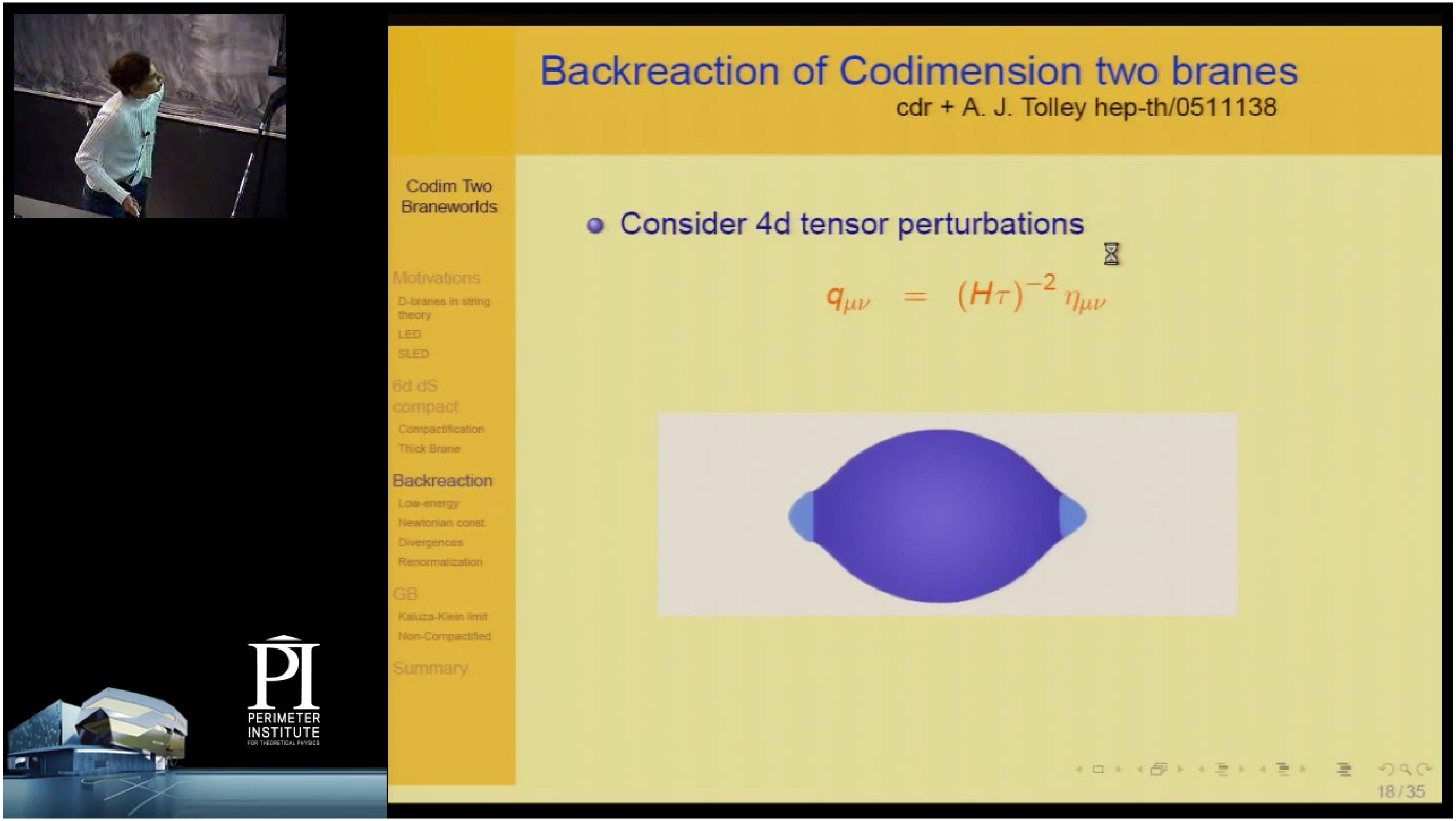
Backreaction of Codimension two branes
cdr + A. J. Tolley hep-th/0511138
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Summary
Consider 4d tensor perturbations
$q_{\mu\nu} = (H\tau)^{-2} \eta_{\mu\nu}$
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Backreaction of Codimension two branes

Low-energy expansion

Codim Two Braneworlds

- At low-energies, we may consider $\Box h \ll \partial_r^2 h$.
- And hence express the solution as an expansion in $\Box$:

$$h(r, x^\mu) = \sum_{n \geq 0} \left(\frac{\Box}{H^2} \right)^{n-1} h_n(r, x^\mu),$$

- At low-energy, the zero mode, dominates:

$$h^\mu_\nu(\tilde{r}_\pm, x^\mu) = \frac{-2\tilde{\kappa}_\pm}{\Box} \left({}^{(4)}T^{\mu(\pm)}_\nu + \frac{\tilde{r}^2_\mp}{\tilde{r}^2_\pm} {}^{(4)}T^{\mu(\mp)}_\nu \right)$$

$$\tilde{\kappa}_\pm = \frac{3}{2\pi L H^2 \left(\tilde{r}^3_+ - \tilde{r}^3_-\right)} \kappa.$$

- It is *finite* in the thin brane limit $\epsilon \to 0$.

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Backreaction of Codimension two branes

Effective Newtonian constant

- $\tilde{\kappa}_{\pm}$ is precisely what is expected using the "naive" argument

$$\frac{1}{2\kappa}\int \mathrm{d}^6x\sqrt{-g}\,^{(6)}R = \frac{1}{2\kappa}\int \mathrm{d}^4x\,\mathrm{d}r\,\mathrm{d}\varphi\sqrt{-g}\,g^{\mu\nu}\,^{(4)}R_{\mu\nu} + \dots$$
$$= \underbrace{\frac{2}{3}\pi L H^2\left(r_+^3 - r_-^3\right)\frac{1}{2\kappa}}_{1/2\tilde{\kappa}}\int \mathrm{d}^4x\sqrt{-q}\,q^{\mu\nu}\,^{(4)}R_{\mu\nu} + \dots .$$

- We indeed have

$$\tilde{\kappa}_{\pm} = \tilde{\kappa} = \frac{3}{2\pi L H^2\left(r_+^3 - r_-^3\right)}\,\kappa,$$

- which is precisely the result used in ADD scenario.

Backreaction of Codimension two branes

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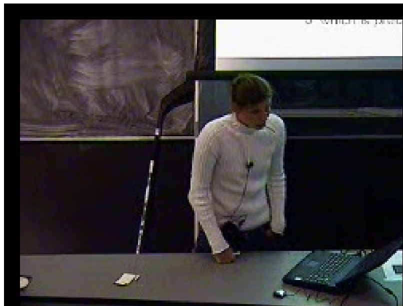

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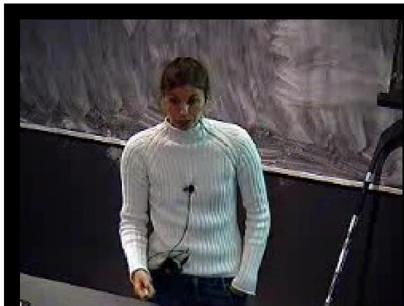
Backreaction of Codimension two branes

Logarithmic divergence

- Working at next order in the expansion,

$$h^{(\pm)} = \frac{-2\tilde{\kappa}_\pm}{\Box}\left({}^{(4)}T^{(\pm)} + \frac{r_\mp^2}{r_\pm^2}\, {}^{(4)}T^{(\mp)} \right) + \frac{\kappa \log \epsilon}{\pi L H^2 r_\pm^2 \, |f'(r_\pm)|}\, {}^{(4)}T^{(\pm)} + \mathcal{O}(\epsilon^0)$$

- On each brane, the dominant contribution is logarithmically divergent.



Backreaction of Codimension two branes

Logarithmic divergence

- The logarithmic divergence is a signature of the fact that the general solution are anisotropic Kasner-like. The presence of anisotropic stress-energy generates a solution of this kind.
- The divergence is expected to get worse at higher orders in perturbations.
- But the divergence does not get worse to next order in the derivative expansion which is well-defined.
- The log divergence is fundamentally different from the divergence in the Green function which is generic to cod 2 branes. Our log divergence is unlikely to disappear for higher codimensions.

Backreaction of Codimension two branes

Logarithmic divergence

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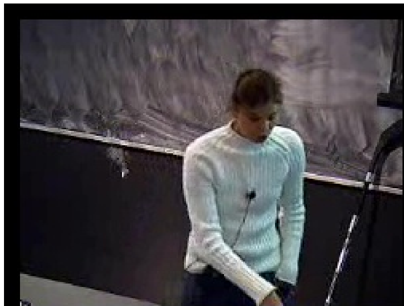

Backreaction of Codimension two branes

Logarithmic divergence

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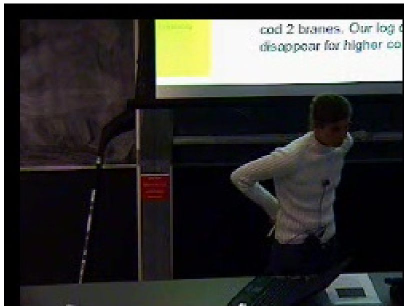

Backreaction of Codimension two branes

Logarithmic divergence

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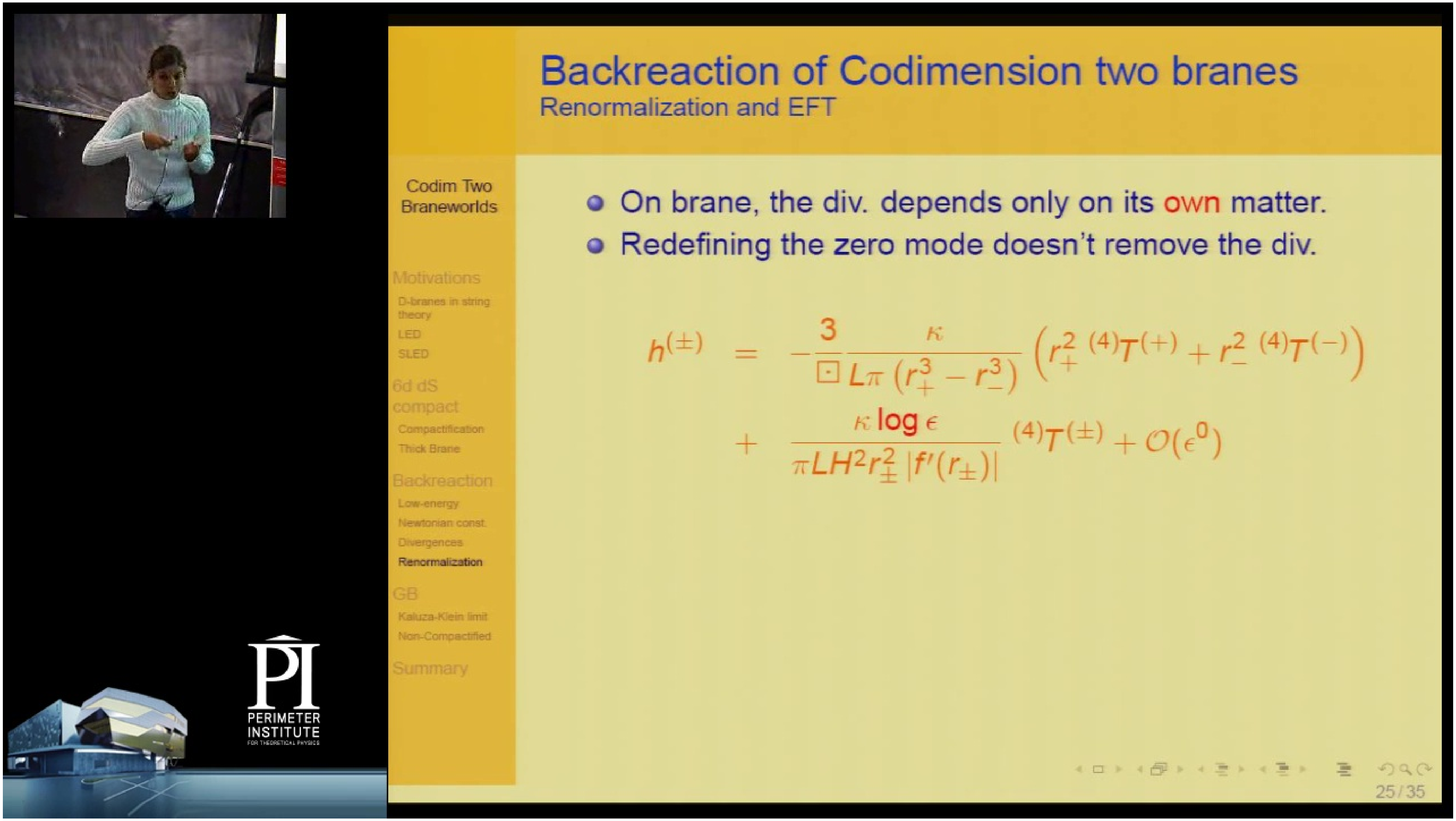
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On brane, the div. depends only on its own matter.
Redefining the zero mode doesn't remove the div.
$$h^{(\pm)} = -\frac{3}{\Box}\frac{\kappa}{L\pi\left(r_+^3 - r_-^3\right)}\left(r_+^2\,{}^{(4)}T^{(+)} + r_-^2\,{}^{(4)}T^{(-)}\right) + \frac{\kappa \log \epsilon}{\pi L H^2 r_\pm^2\,|f'(r_\pm)|}\,{}^{(4)}T^{(\pm)} + \mathcal{O}(\epsilon^0)$$
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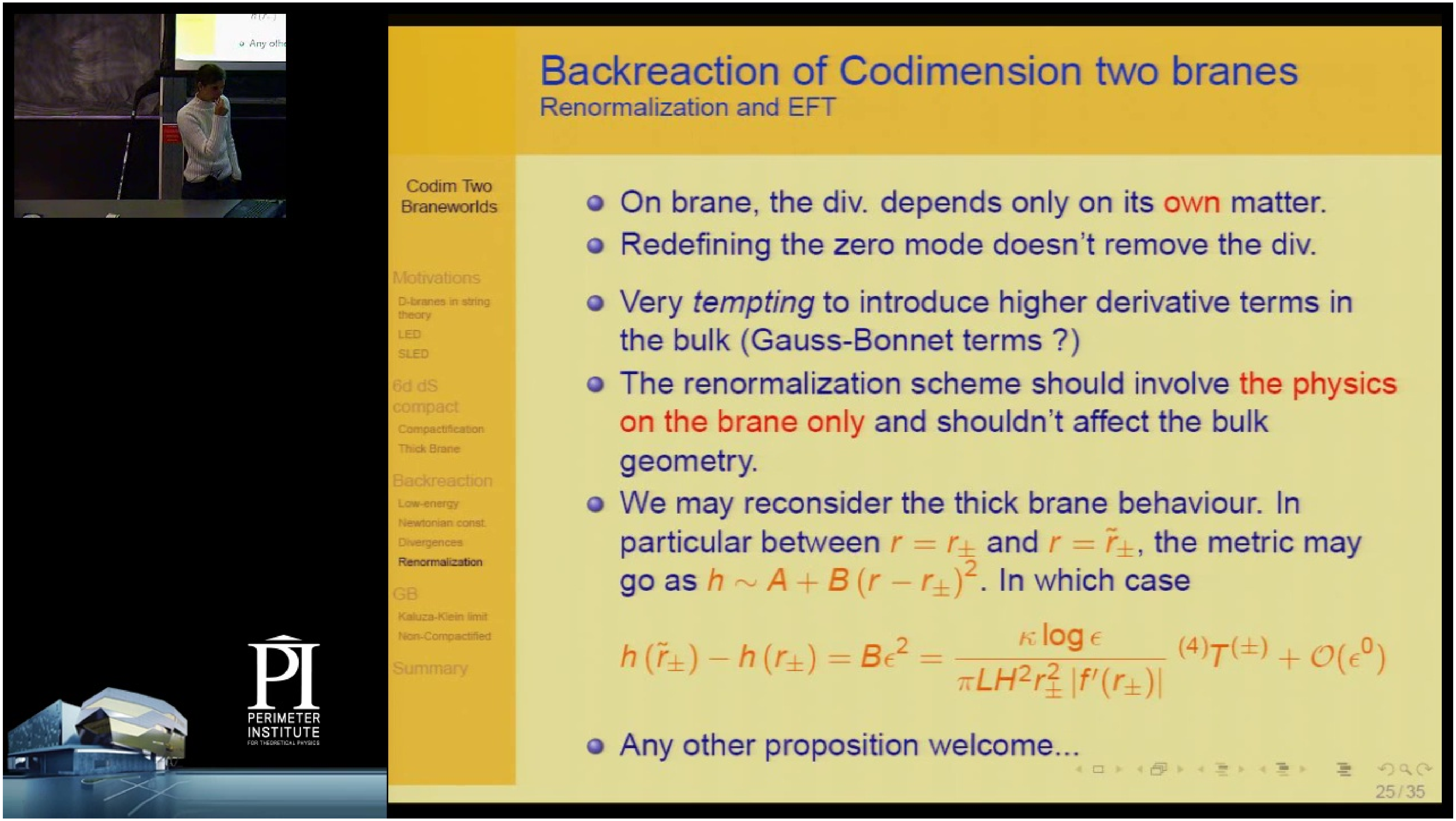
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On brane, the div. depends only on its own matter.
Redefining the zero mode doesn't remove the div.
Very tempting to introduce higher derivative terms in the bulk (Gauss-Bonnet terms ?)
The renormalization scheme should involve the physics on the brane only and shouldn't affect the bulk geometry.
We may reconsider the thick brane behaviour. In particular between $r = r_\pm$ and $r = \tilde{r}_\pm$, the metric may go as $h \sim A + B\,(r - r_\pm)^2$. In which case
$h\,(\tilde{r}_\pm) - h\,(r_\pm) = B\epsilon^2 = \frac{\kappa \log \epsilon}{\pi L H^2 r_\pm^2 \, |f'(r_\pm)|} \, {}^{(4)}T^{(\pm)} + \mathcal{O}(\epsilon^0)$
Any other proposition welcome...
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Gauss-Bonnet terms

Modification of boundary conditions

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- A convincing approach has been introduced in the literature.
- Introducing GB terms can change the form of the boundary conditions without affecting the bulk structure.
- Such terms are expected from string theory and derive from the action

$$S_{\mathrm{GB}} = \frac{\alpha}{2\sqrt{\kappa}} \int \mathrm{d}^6 x \sqrt{-g}\, \mathcal{R}^{\mathrm{GB}\,A}{}_{A}$$

$$\mathcal{R}^{\mathrm{GB}}_{AB} = R\, R_{AB} - 2R_{AC}\, R^{C}{}_{B} - 2\, R^{CD}\, R_{ACBD} + R_{A}{}^{DEF}\, R_{BDEF}\,.$$

Kaluza-Klein limit

Codim Two Braneworlds

- The general solution has a well-known Kaluza-Klein limit, then the solution describes *Minkowski* $\times S^2$:

$$ds^2 = ds_M^2 + \tilde{f}(\rho)^{-1} d\rho^2 + \ell^2 \tilde{f}(\rho) d\varphi^2,$$

with $\tilde{f}(\rho) = 1 - \rho^2/R_c^2$. Cod 2 branes at $\rho = \pm R_c$.

- The modes satisfy $\partial_\rho \left(\tilde{f} \partial_\rho h \right) = -\Box h$. The solution is of the form:

$$h = C_1 \mathcal{P}_m(\rho/R_c) + C_2 \mathcal{Q}_m(\rho/R_c),$$

with $m = -1/2 + \sqrt{1/4 + R_c^2 \Box}$.

- h diverges logarithmically at $\rho = \pm R_c$.

Gauss-Bonnet terms

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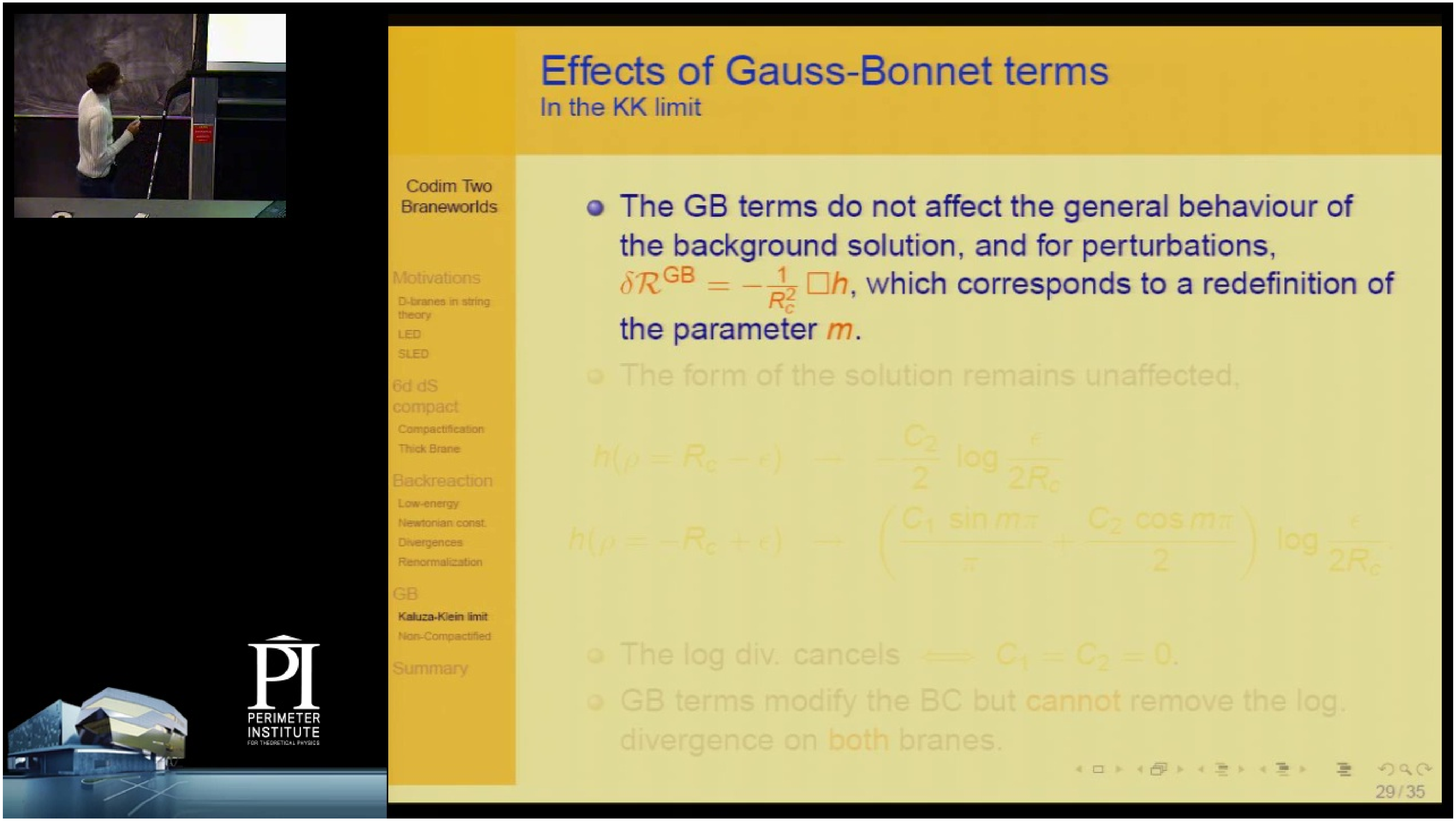
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Summary
The GB terms do not affect the general behaviour of the background solution, and for perturbations, $\delta\mathcal{R}^{\mathrm{GB}} = -\frac{1}{R_c^2}\Box h$, which corresponds to a redefinition of the parameter m.
The form of the solution remains unaffected.
$h(\rho = R_c - \epsilon) \to -\frac{C_2}{2}\log\frac{\epsilon}{2R_c}$
$h(\rho = -R_c + \epsilon) \to \left(\frac{C_1 \sin m\pi}{\pi} + \frac{C_2 \cos m\pi}{2}\right)\log\frac{\epsilon}{2R_c}$
The log div. cancels $\Longleftrightarrow$ $C_1 = C_2 = 0$.
GB terms modify the BC but cannot remove the log. divergence on both branes.
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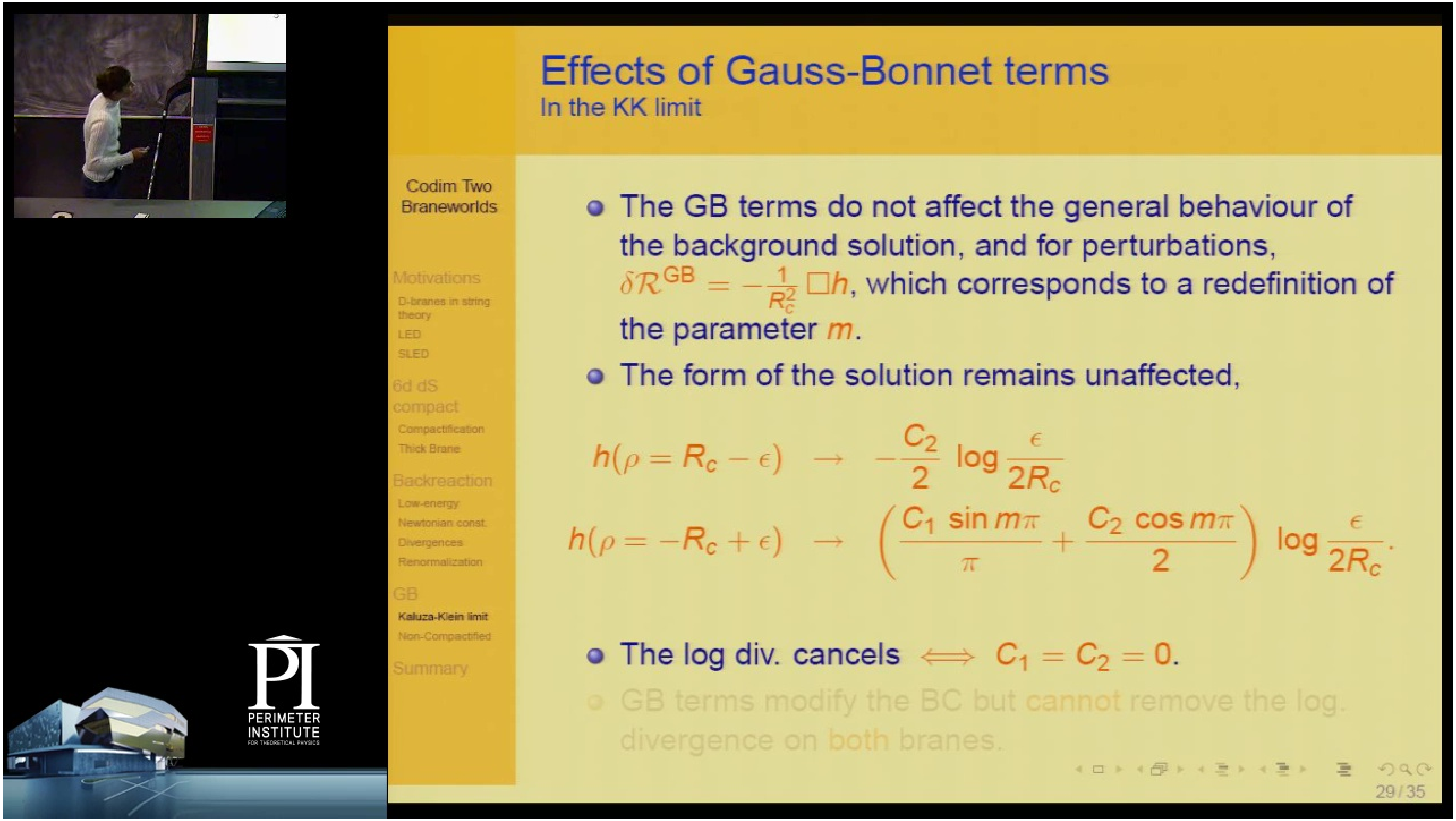
$$h = C_1 \mathcal{P}_m(\rho/R_c) + C_2 \mathcal{Q}_m(\rho/R_c),$$

  with $m = -1/2 + \sqrt{1/4 + R_c^2\,\Box}$.

- h diverges logarithmically at $\rho = \pm R_c$.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The log div. cancels $\iff C_1 = C_2 = 0$.
GB terms modify the BC but cannot remove the log. divergence on both branes.
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Effects of Gauss-Bonnet terms

In the KK limit

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Effects of Gauss-Bonnet terms

For an uncompactified geometry P. Bostock *et.al.* hep-th/0311074

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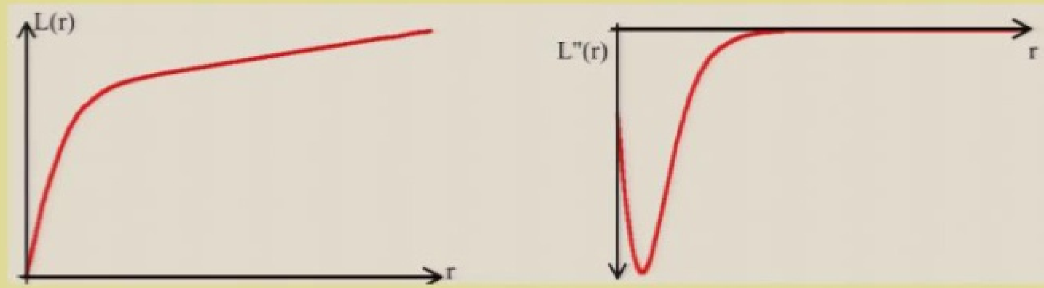
Summary

- Considering the cosmic string solution of the Abelian-Higgs model solution in flat spacetime:

$$ds^2 = M(r)^2 \eta_{\mu\nu} dx^\mu dx^\nu + L(r)^2 d\varphi^2 + dr^2$$



- $L'(\epsilon) = (1-\sigma)$, but $L'(0) = 1 \Rightarrow L''(r) = -\sigma\delta(r)$ in the thin brane limit:

$$\lim_{\epsilon \to 0} \int_0^\epsilon dr\, L''(r) = \lim_{\epsilon \to 0} \left(L'(\epsilon) - L'(0)\right) = -\sigma.$$

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Effects of Gauss-Bonnet terms

For an uncompactified geometry

Codim Two Braneworlds

- At the perturbed level, the Einstein Eq. is:

$$\delta R^{\mu}_{\nu} + \alpha\sqrt{\kappa}\mathcal{R}^{\mathrm{GB}\mu}{}_{\nu} = -\frac{1}{2M^2}\Box h^{\mu}_{\nu} - \frac{1}{2L}\partial_r\left[L\partial_r h^{\mu}_{\nu}\right] + \frac{2\alpha\sqrt{\kappa}\sigma}{LM^2}\Box h^{\mu}_{\nu}\delta(r)$$
$$= \frac{\kappa}{2\pi LM^2}{}^{(4)}T^{\mu}_{\nu}\,\delta(r).$$

- GB terms seem to localize gravity in two large uncompactified extra-dimensions:

$$h^{\mu}_{\nu} = \frac{\kappa}{4\pi\alpha\sqrt{\kappa}\sigma}\frac{1}{\Box}{}^{(4)}T^{\mu}_{\nu}.$$

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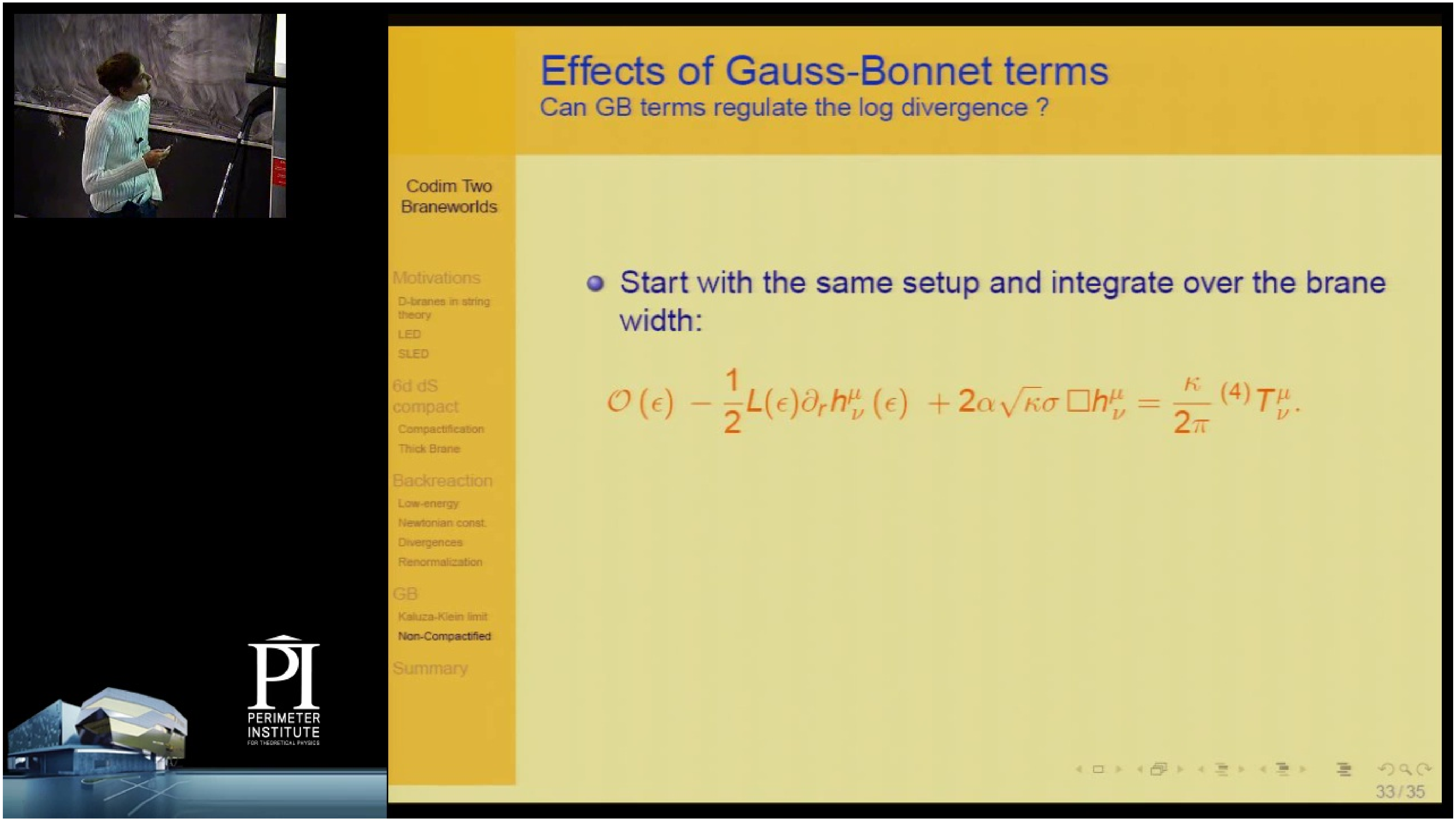
Effects of Gauss-Bonnet terms

Can GB terms regulate the log divergence ?

- Start with the same setup and integrate over the brane width:

$$-\int \mathrm{d}r\,\frac{L}{2M^2}\Box h^{\mu}_{\nu} - \frac{1}{2}\int \mathrm{d}r\,\partial_r\left[L\partial_r h^{\mu}_{\nu}\right] - \int \mathrm{d}r\,\frac{2\alpha\sqrt{\kappa}}{M^2}L''\,\Box h^{\mu}_{\nu} = \int \mathrm{d}r\,\frac{\kappa}{2\pi M^2}\,{}^{(4)}T^{\mu}_{\nu}\,\delta(r).$$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Start with the same setup and integrate over the brane width:
$\mathcal{O}(\epsilon) - \frac{1}{2}L(\epsilon)\partial_r h^\mu_\nu(\epsilon) + 2\alpha\sqrt{\kappa}\sigma\,\Box h^\mu_\nu = \frac{\kappa}{2\pi}\,{}^{(4)}T^\mu_\nu.$
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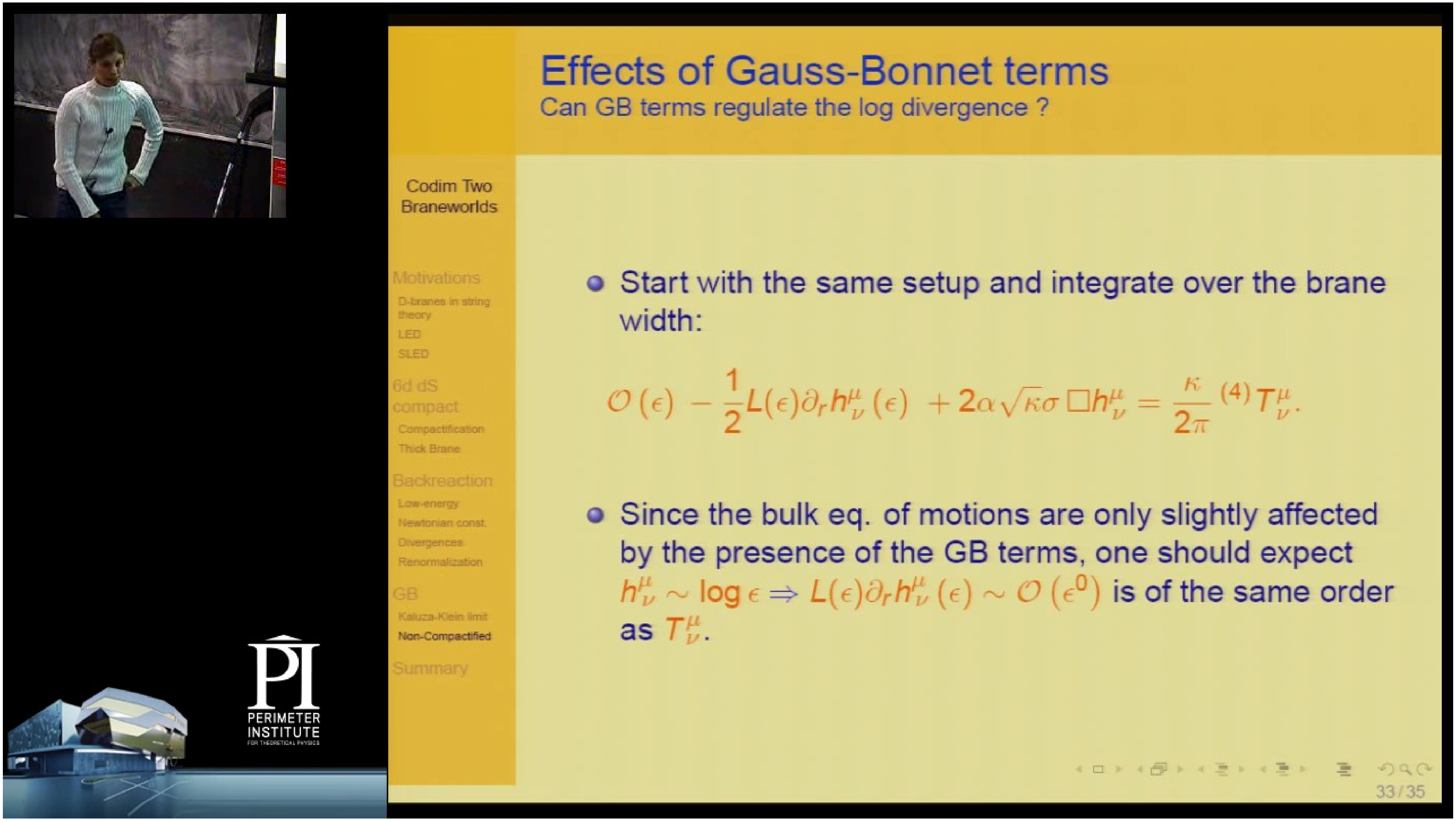
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Can GB terms regulate the log divergence ?
Codim Two Braneworlds
Motivations
D-branes in string theory
LED
SLED
6d dS compact
Compactification
Thick Brane
Backreaction
Low-energy
Newtonian const.
Divergences
Renormalization
GB
Kaluza-Klein limit
Non-Compactified
Summary
Start with the same setup and integrate over the brane width:
$\mathcal{O}(\epsilon) - \frac{1}{2} L(\epsilon)\partial_r h^\mu_\nu(\epsilon) + 2\alpha\sqrt{\kappa}\sigma \Box h^\mu_\nu = \frac{\kappa}{2\pi} {}^{(4)}T^\mu_\nu.$
Since the bulk eq. of motions are only slightly affected by the presence of the GB terms, one should expect $h^\mu_\nu \sim \log\epsilon \Rightarrow L(\epsilon)\partial_r h^\mu_\nu(\epsilon) \sim \mathcal{O}(\epsilon^0)$ is of the same order as T^μ_ν.
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General case

Can GB terms regulate the log divergence ?

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- The GB terms are genuinely second order in derivative and do not add new degrees of freedom to the theory. Their contribution cannot simultaneously remove the log div. on both branes in a compactified geometry.
- For an uncompactified geometry, the boundary conditions to be imposed at infinity are a priori unclear and the contribution of the GB could remove the log div. on the one brane if no conditions were imposed at infinity. This represents an unphysical choice.
- A more physical scenario would consider warped infinite extra dimensions for which perturbations are required to remain finite at infinity.

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Summary

- We looked at regularized cod 2 branes and studied their backreaction.
- Gravity is localised on cod 2 branes when the size of the extra dim is finite, and the effective Gravitational constant is related to the fundamental one times the proper size of the extra dimension.
- On the branes, the first KK correction to the low-energy zero mode has a log divergence in the cutoff scale.
- This div. remains when GB terms are considered.
- This result seems valid for a cod 2 brane embedded in infinite extra-dimension.
- This div. may not be trivially absorbed in a redefined zero mode, by adding some local counterterms. The renormalization scheme has to be understood in more details.

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