Title: Phenomenological quantum gravity: Pieces of the puzzle

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Abstract: The phenomenology of quantum gravity can be examined even though the underlying theory is not yet fully understood. Effective extensions of the standard model allow us to study specific features, such as the existence of extra dimensions or a minimal length scale. I will talk about some applications of this approach which can be used to make predictions for particle- and astrophysics, and fill in some blanks in the puzzle of quantum gravity. A central point of this investigations is the physics of black holes. I will comment on possible ways to proceed and on the missing pieces I find most important to look for.

The Puzzle of Quantum Gravity:

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- Theory

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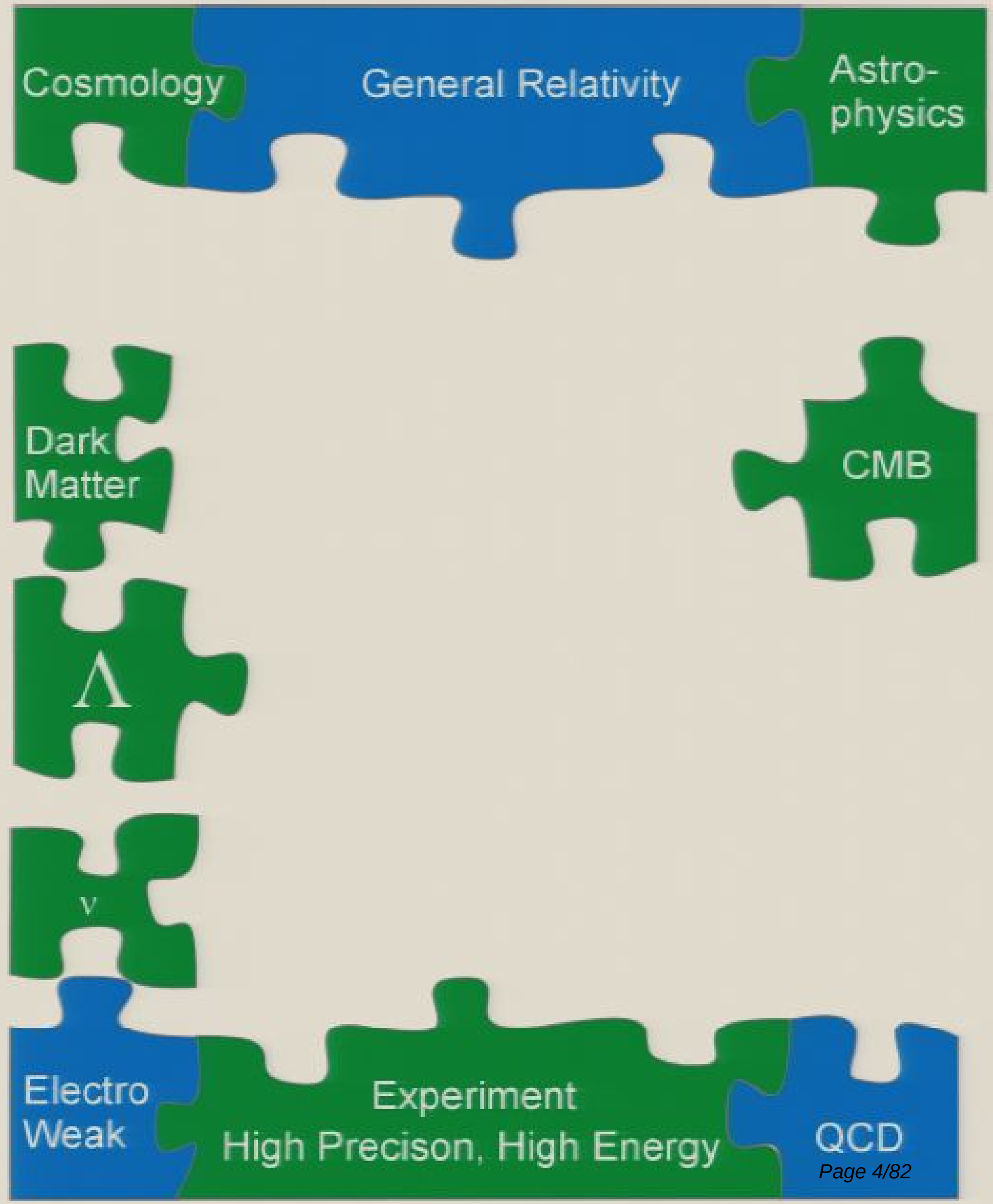
- Experiment



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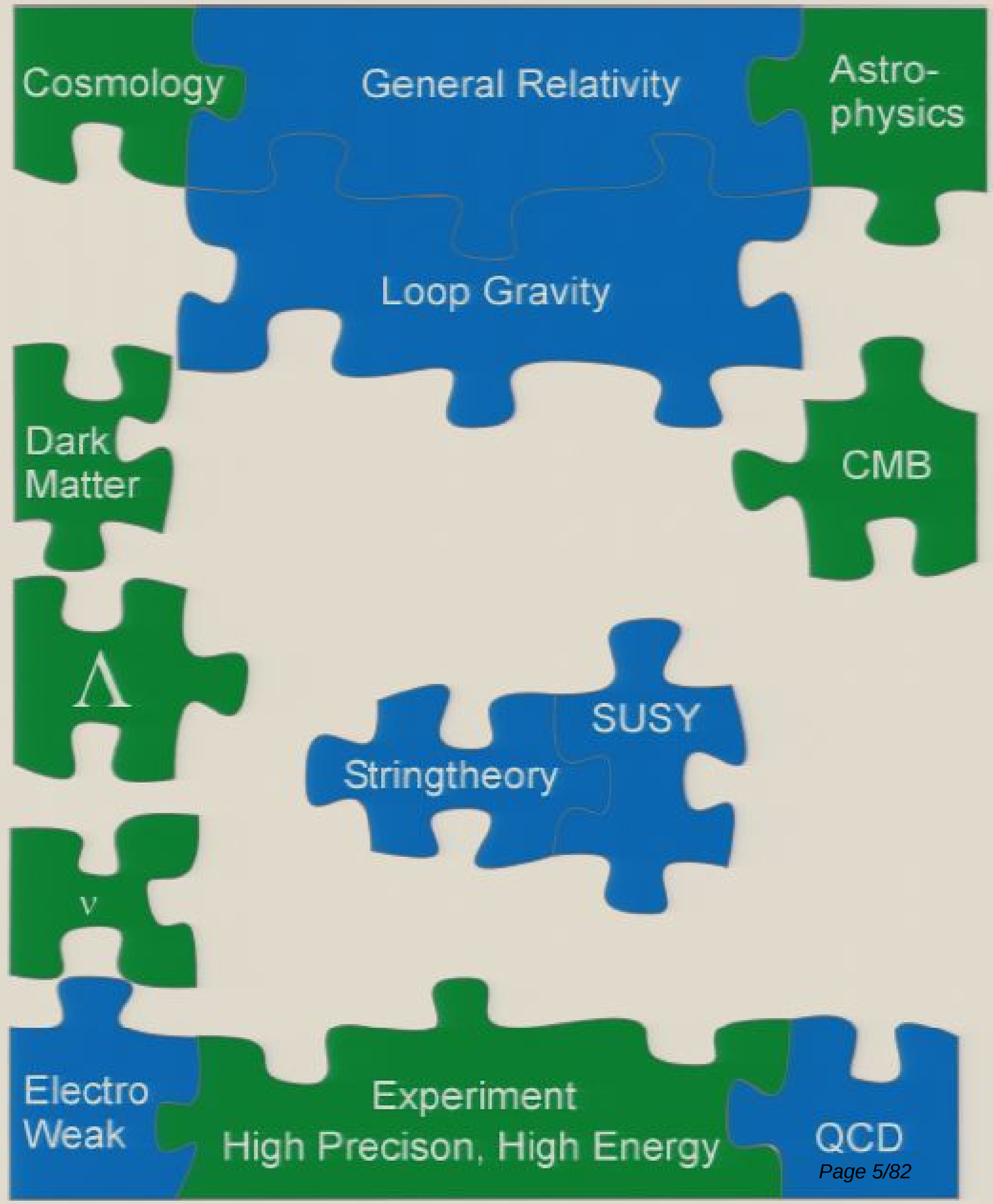

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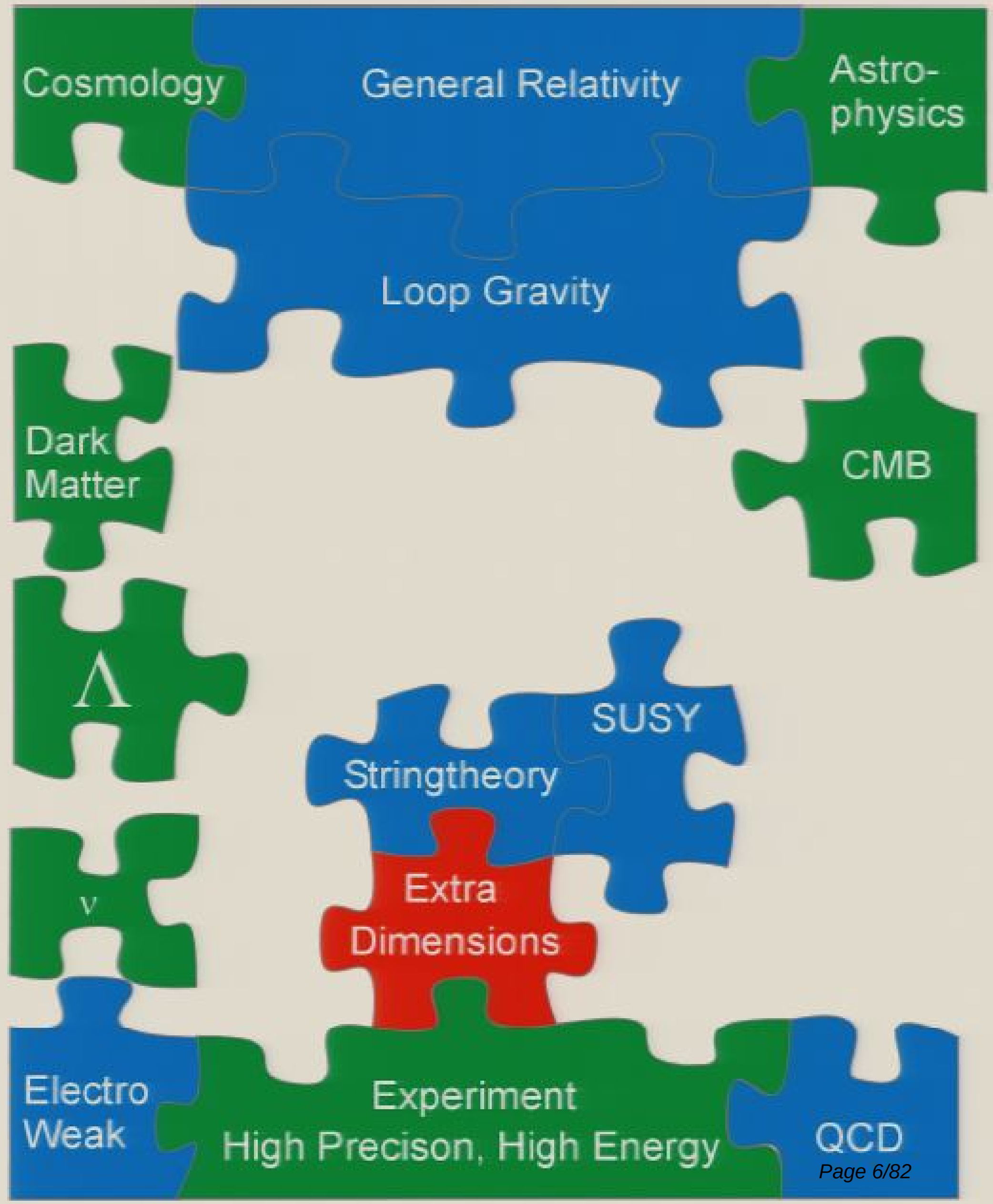
Non-orthodox approaches:

- Extra Dimensions



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- Extra Dimensions
- Minimal Length Scale
- Modified Special Relativity
- Black Hole Physics
- Holographic Principle
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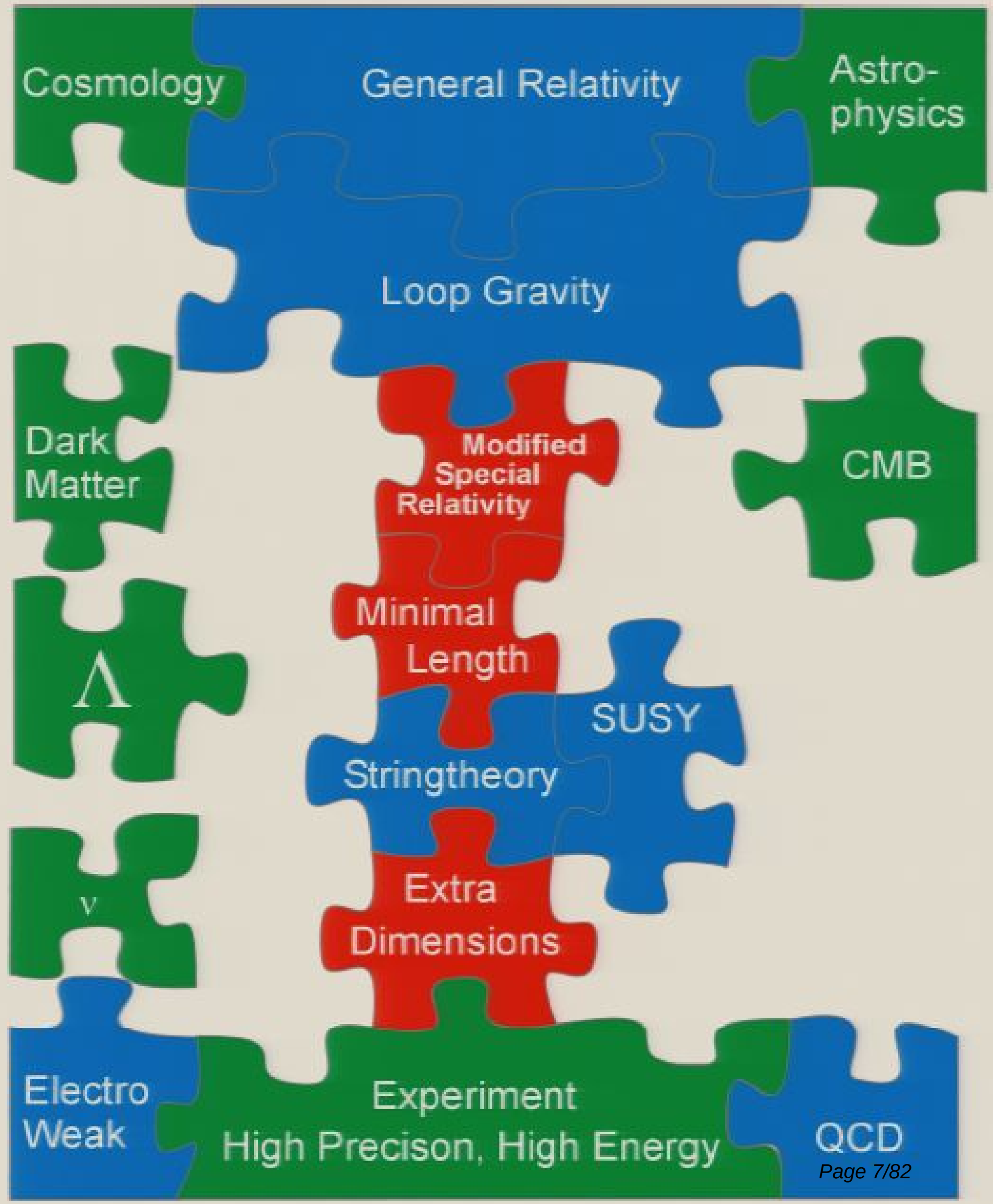

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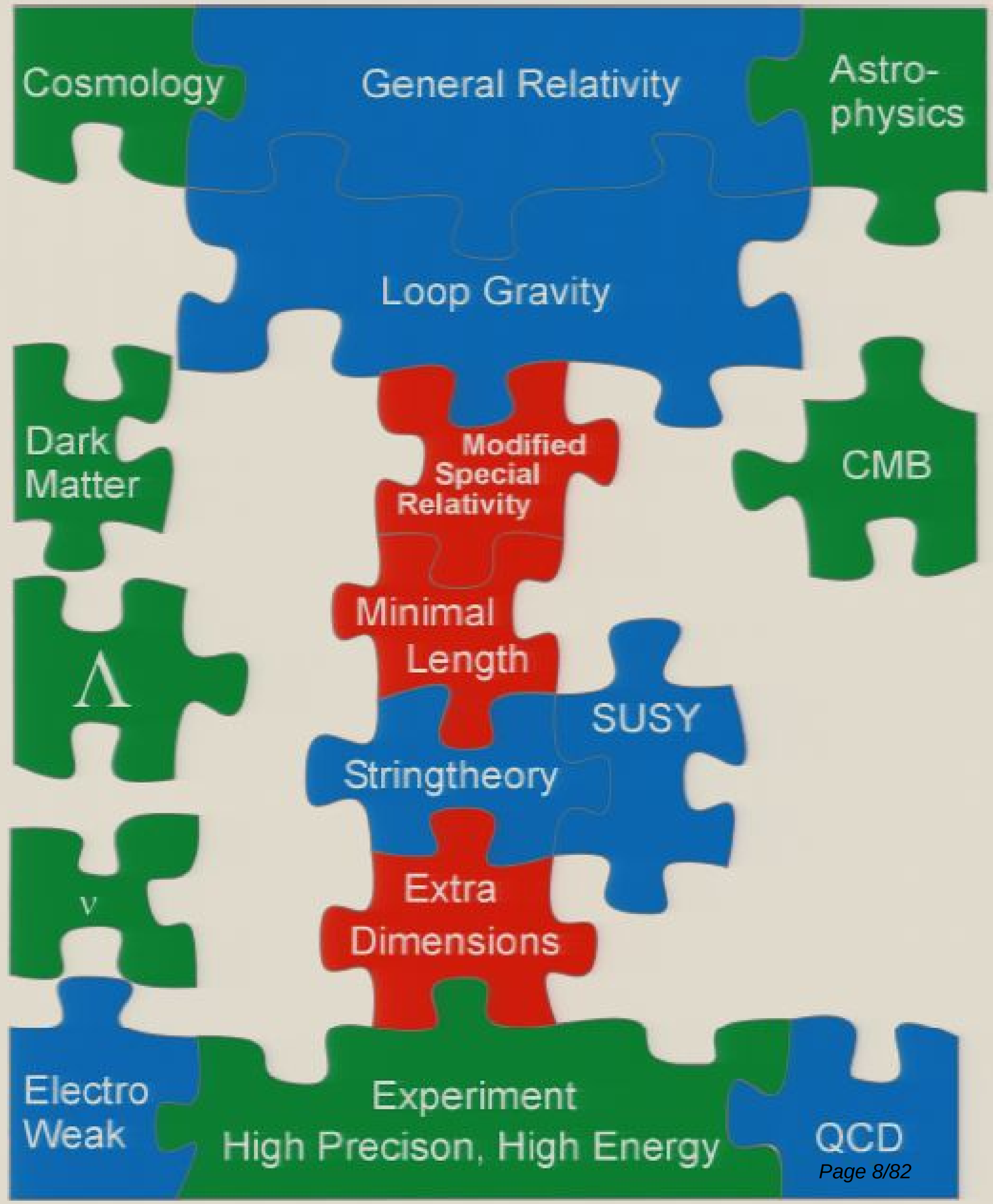

Models with Extra Dimensions

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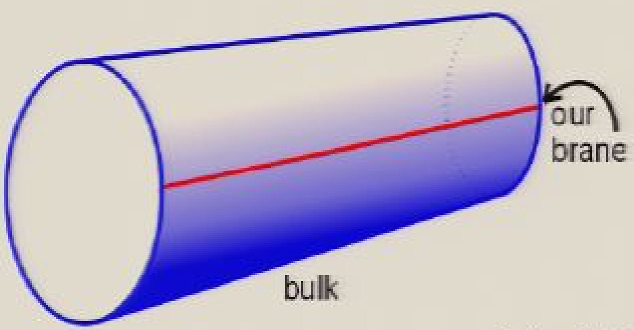
  - + Solves Hierarchy-problem, $m_p^2 = R^d M_f^{d+2}$



Arkani-Hamed, Dimopoulos and Dvali, Phys. Lett. B **429**, 263 (1998)
Antoniadis, Arkani-Hamed, Dimopoulos and Dvali, Phys. Lett. B **436**, 257 (1998)
Arkani-Hamed, Dimopoulos and Dvali, Phys. Rev. D **59**, 086004 (1999)

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  - + AdS-CFT Correspondence
  - + Allows non-compact extra dimension (volume stays finite)

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Randall and Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)
Randall and Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)

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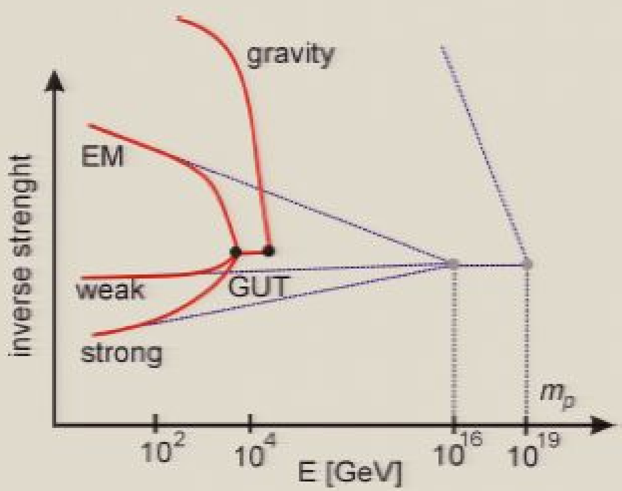
- UXD, TeV-scale dimensions
  - + Accelerated unification of coupling constants



Appelquist, Cheng and Dobrescu,Phys. Rev. D **64**, 035002 (2001)
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I. Antoniadis, Phys. Lett. B **246**, 377 (1990)
Dienes, Dudas and Gherghetta, Nucl. Phys. B **537**, 47 (1999)

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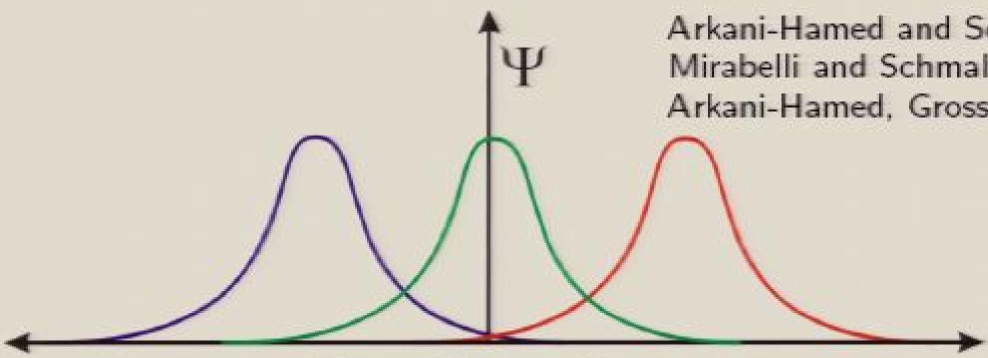
  - + Quick fix for several problems



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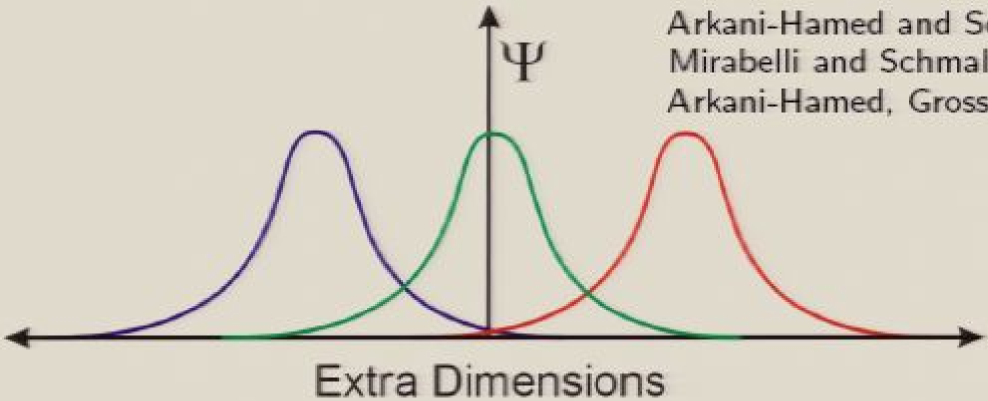

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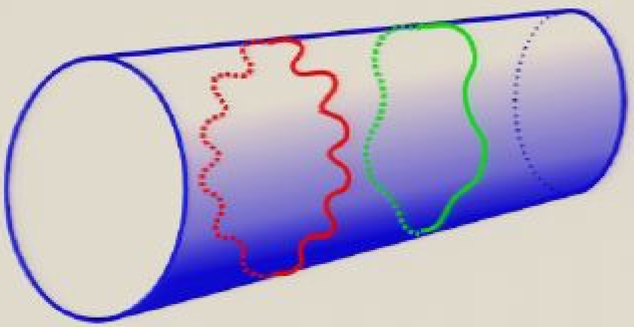
Observables of Extra Dimensions

- Excitation of KK-modes

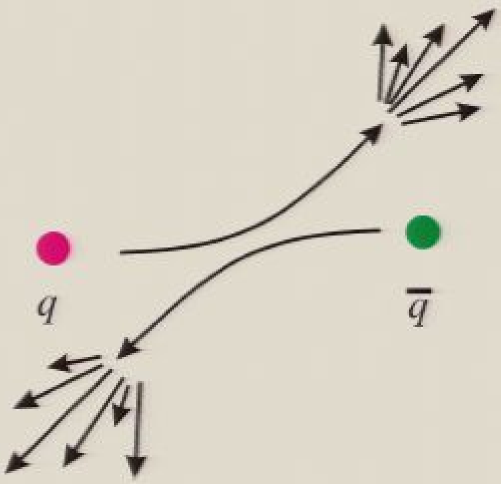
Fourier-expansion

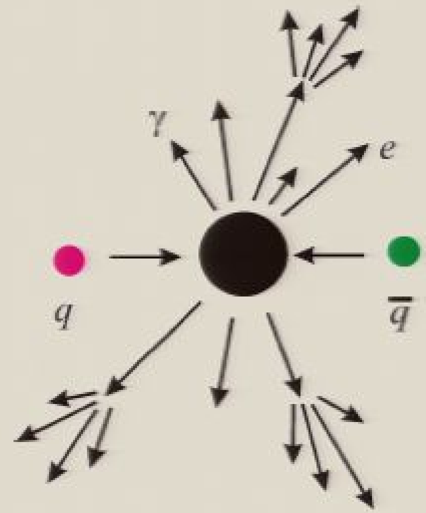


$$\psi(x,y) = \sum_{n=-\infty}^{+\infty} \psi^{(n)}(x) \exp(iny/R)$$

Observables of Extra Dimensions

- Excitation of KK-modes
- Black hole production and evaporation





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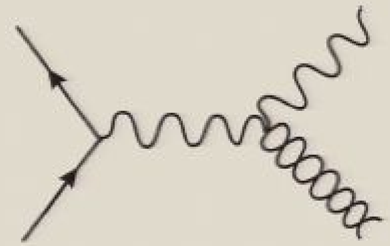
- Real graviton production



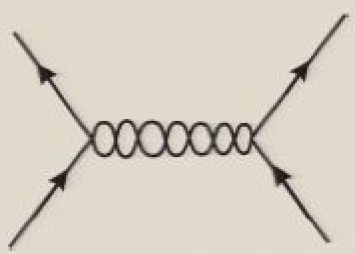
* $g_{AB} = \eta_{AB} + \Psi_{AB}$
* Decompose: spin-2 $h_{\mu\nu}$, vector $V_{\mu i}$, scalar ϕ_{ij} (trace $\phi^i{}_i = \phi$)
* Coupling $\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_M$
* Energy momentum tensor on brane $T_{AB} = \eta^{\mu}{}_{A}\eta^{\nu}{}_{B} T_{\mu\nu}(x)\delta(y)$

⟶ Yields coupling terms: $\mathcal{L}_{int} = -\frac{1}{2} T\phi - T^{\mu\nu} h_{\mu\nu}$

Observables of Extra Dimensions

- Excitation of KK-modes
- Black hole production and evaporation
- Real graviton production
- Modified cross-sections from virtual particle exchange

Summation over tower diverges:



$$\sum_n \int d^4p \frac{1}{[p^2 + (n/R)^2]^2} \to \infty$$

The Minimal Length Scale

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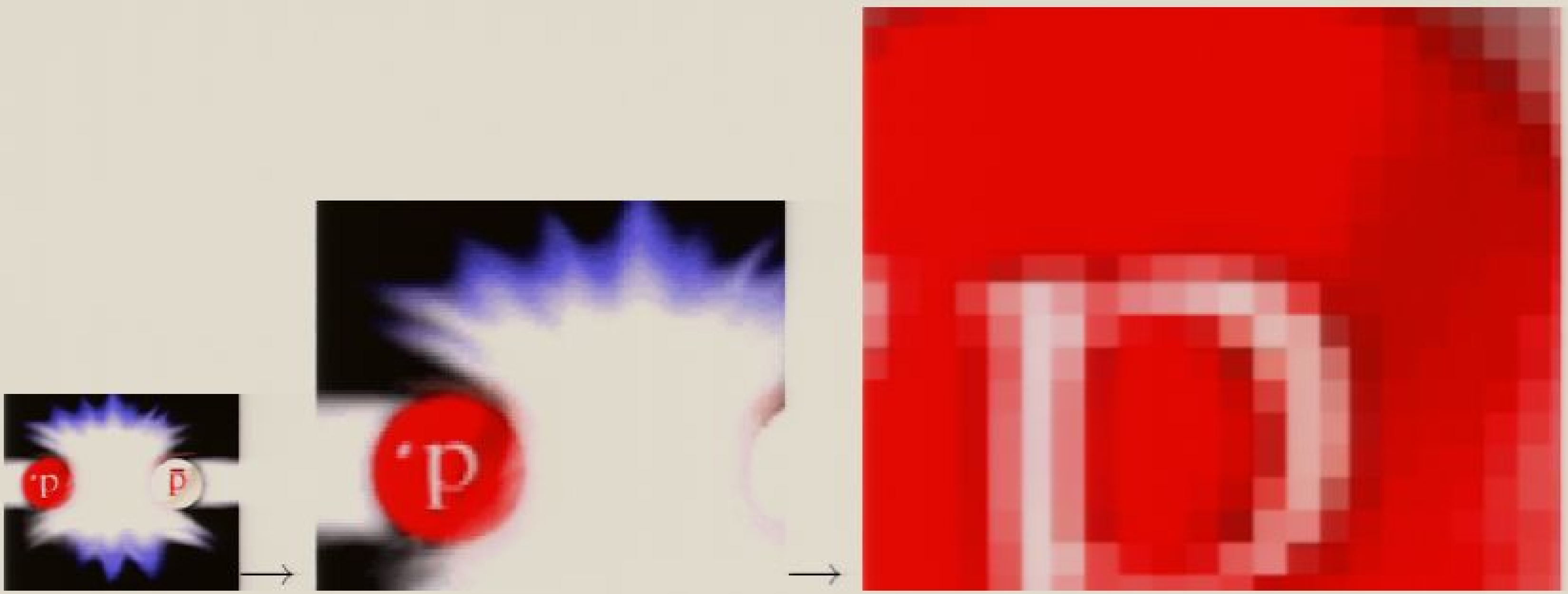

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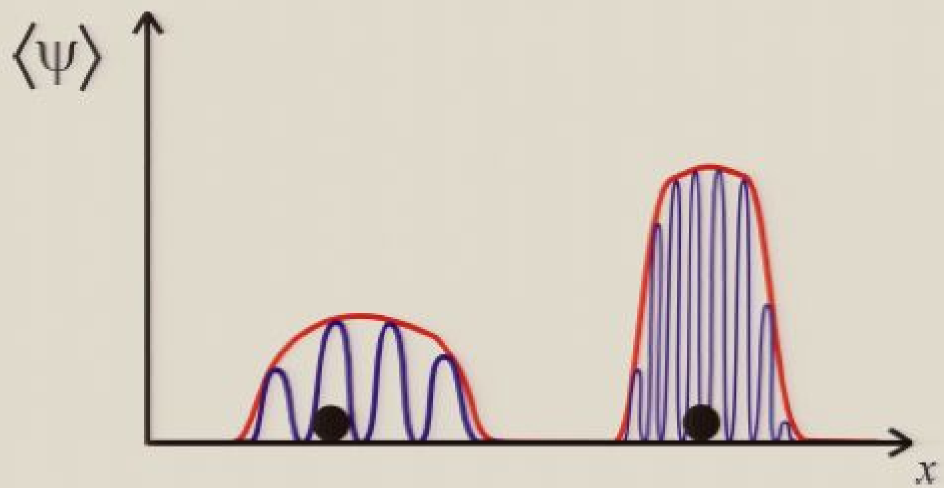
- Minimal length scales acts as UV cutoff

An Effective Model for the Minimal Length

- For large momenta, p, Compton-wavelength λ can not get arbitrarily small $\lambda > L_{\mathrm{f}} = 1/M_{\mathrm{f}}$



SH *et al*, Phys. Lett. B598 (2004) 92-98

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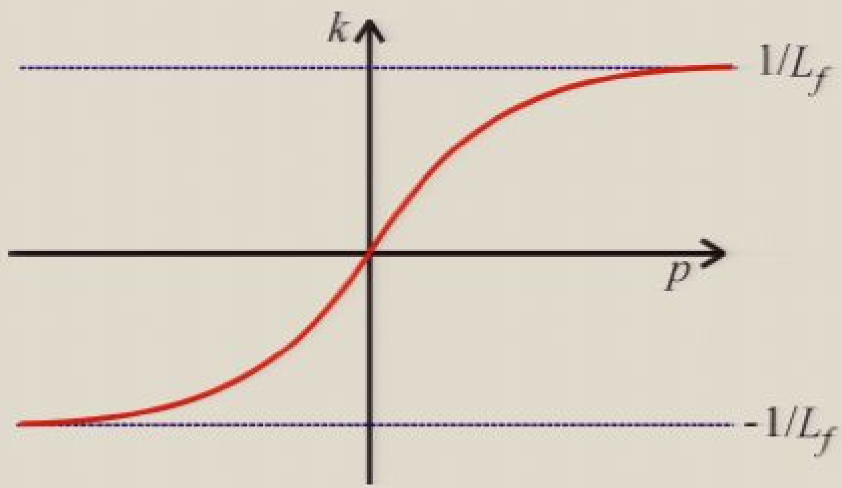
- Modify wave-vector k and commutation relations $k = k(p) = \hbar p + a_1 p^3 + a_2 p^5 ... \Rightarrow [p_i, x_j] = i\partial p_i / \partial k_j$



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$$\Delta x \Delta p \geq \frac{1}{2} \hbar \left(1 + b_1 \frac{\Delta p^2}{M_{\rm f}^2} \right) \quad ,$$

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- And a modified dispersion relation

$$\omega^2 - k^2 - \mu^2 = \Pi(k,\omega)$$

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Quantizing with a Minimal Length

- The Klein-Gordon equation

$$E^2 - p^2 = m^2 \quad \Rightarrow \quad \eta^{\mu\nu} p_\nu(k) p_\mu(k) \psi = m^2 \psi$$

Explicitely in NLO M_f

$$-\hbar^2 \eta^{\mu\nu} \left(\partial_\nu - \frac{L_f^2}{3} \partial_\nu^3 \right) \left(\partial_\mu - \frac{L_f^2}{3} \partial_\mu^3 \right) \psi = m^2 \psi$$

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$$\left[\hat{a}^\dagger(p), \hat{a}(p')\right]_\pm \quad = \quad \delta(\vec{p} - \vec{p}')\left|\frac{\partial p}{\partial k}\right|$$

- Lagrangian for free fermions $\mathcal{L}_f = \mathrm{i}\bar{\Psi}(\not{p}(k) - m)\Psi$.
- Coupling of the gauge field via $\partial_\nu \to D_\nu := \partial_\nu - \mathrm{i}eA_\nu$ yields the interaction $\mathcal{L} = \mathcal{L}_f - e\bar{\Psi}\gamma_\nu\Psi A^\nu + \mathcal{O}(e/M_f^2)$

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Transformation: $\mathbf{p} \to \mathbf{p}' = \Lambda \mathbf{p}$, $\mathbf{k} \to \mathbf{k}' = \widetilde{\Lambda}(\mathbf{k})$

SH, hep-th/0510245

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- Use to build self-consistent QFT with minimal length ...
- Multi-particle kinematics?

SH, hep-th/0510245

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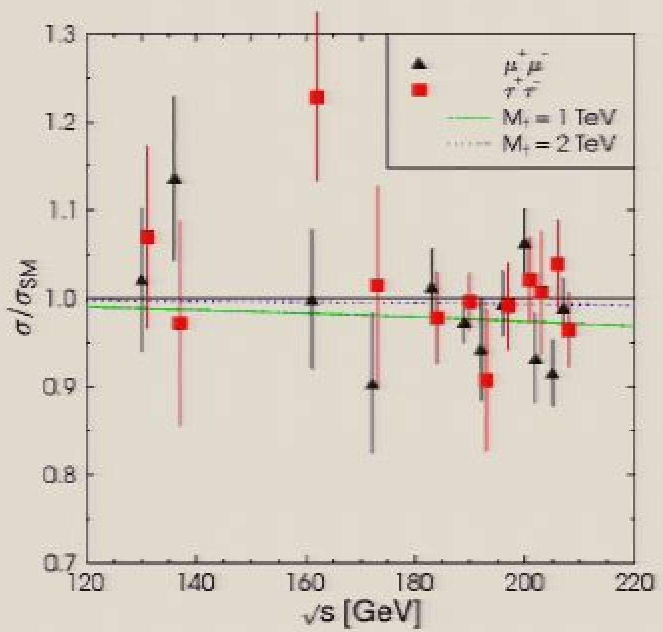

SH, M. Bleicher, S. Hofmann, S. Scherer, J. Ruppert and H. Stoecker, Phys. Lett. B598 (2004) 92-98

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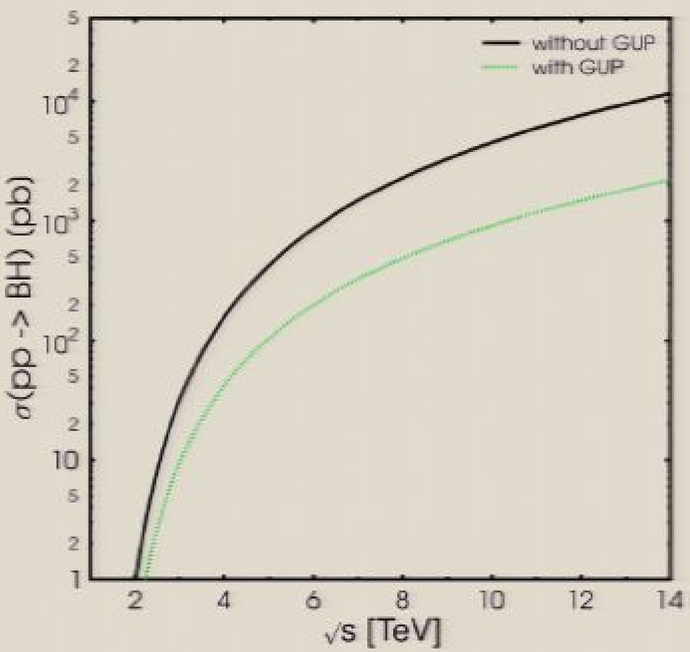

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Amelino-Camelia, Phys.Rev. D64 (2001) 036005
Magueijo and Smolin, Phys. Rev. Lett. **88** (2002) 190403
Judes and Visser, Phys. Rev. **D 68** (2003) 045001

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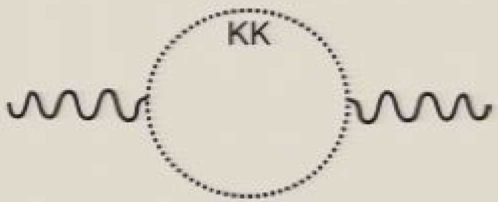

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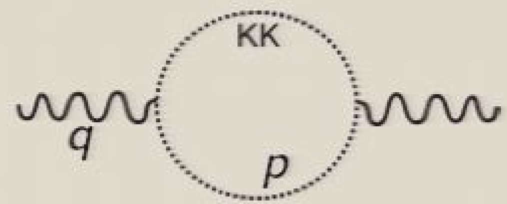
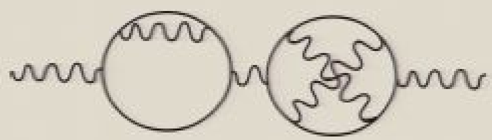
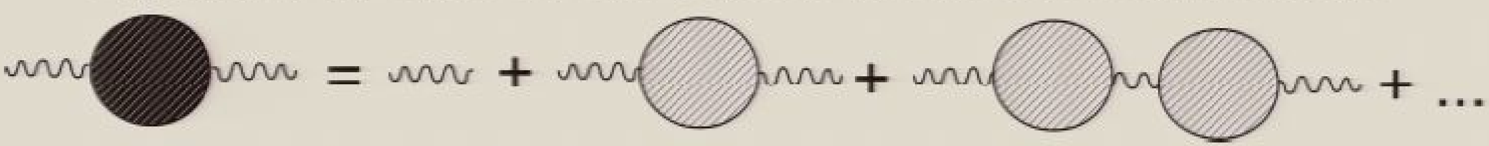


* Looking close, the propagator exhibits complex structures

$D^{\mu\nu}(q) = -g^{\mu\nu}/q^2$  $\rightarrow \widetilde{D}^{\mu\nu}$

* Expansion in series of one-particle irreducible contributions



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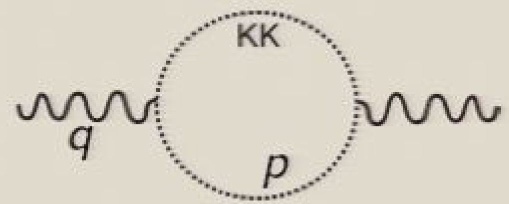

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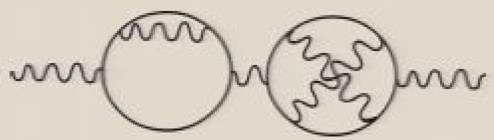
$D^{\mu\nu}(q) = -g^{\mu\nu}/q^2$  $\rightarrow \widetilde{D}^{\mu\nu}$

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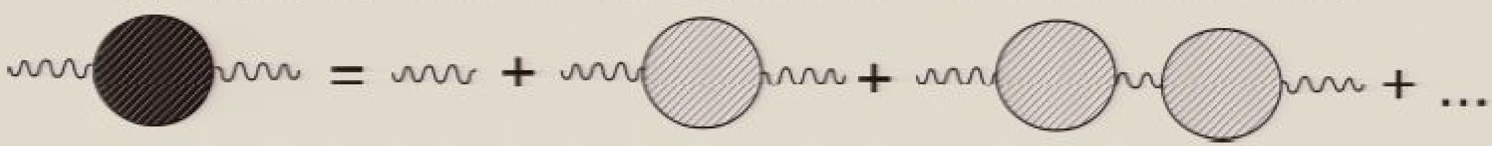

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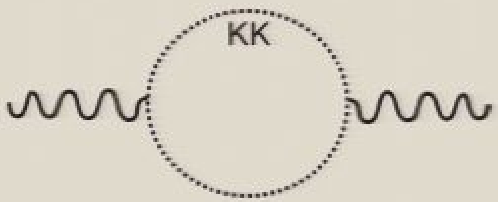

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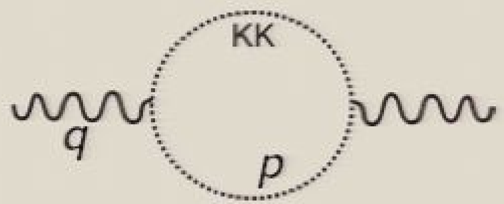

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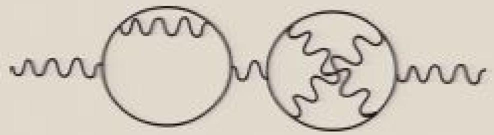
$D^{\mu\nu}(q) = -g^{\mu\nu}/q^2$  $\rightarrow \widetilde{D}^{\mu\nu}$

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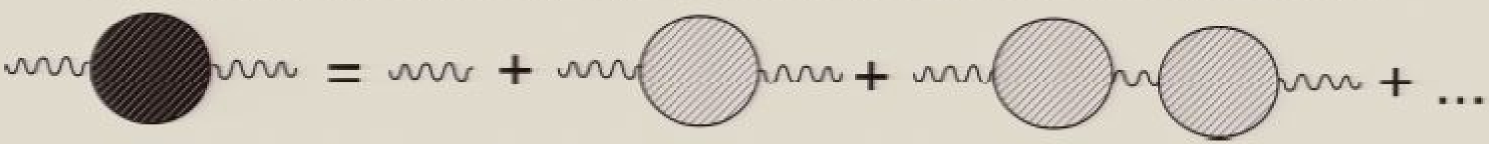

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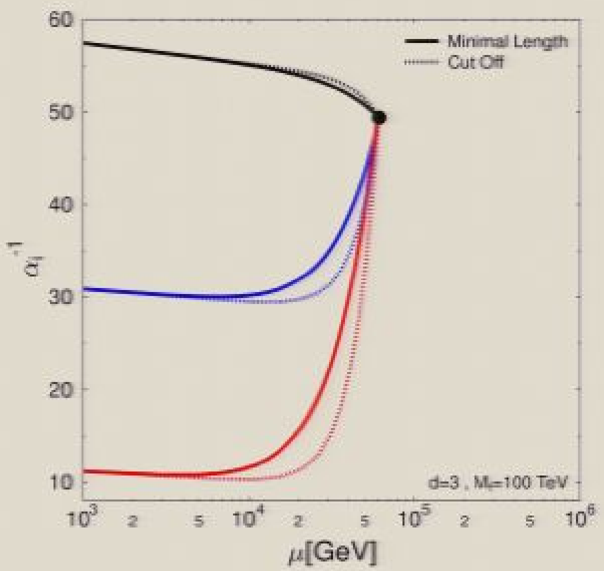

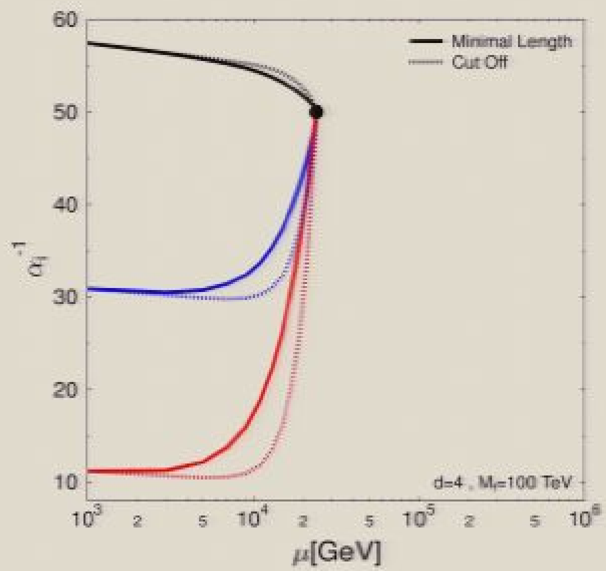

SH, Phys. Rev. **D70** (2004) 105003

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- Constrain model from top-down!
- Non-commutative position space - curved momentum space?

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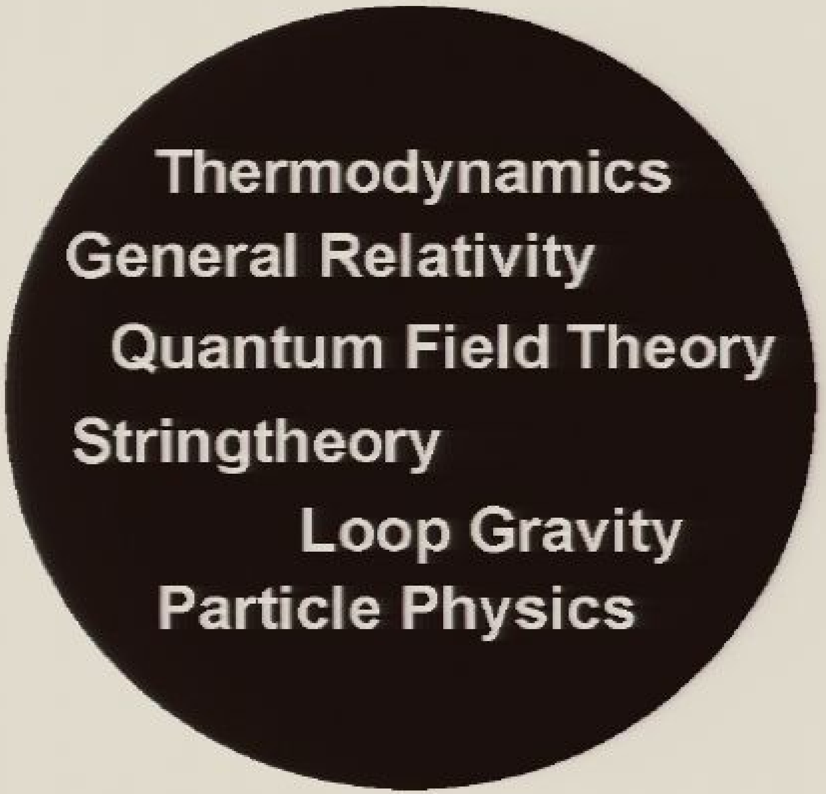

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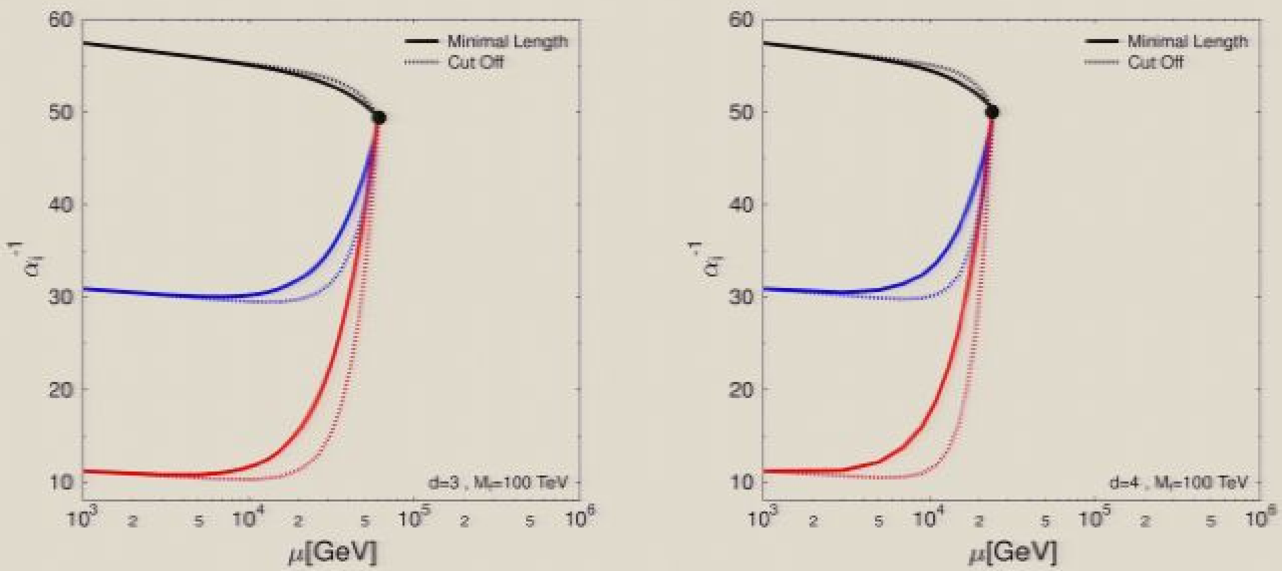

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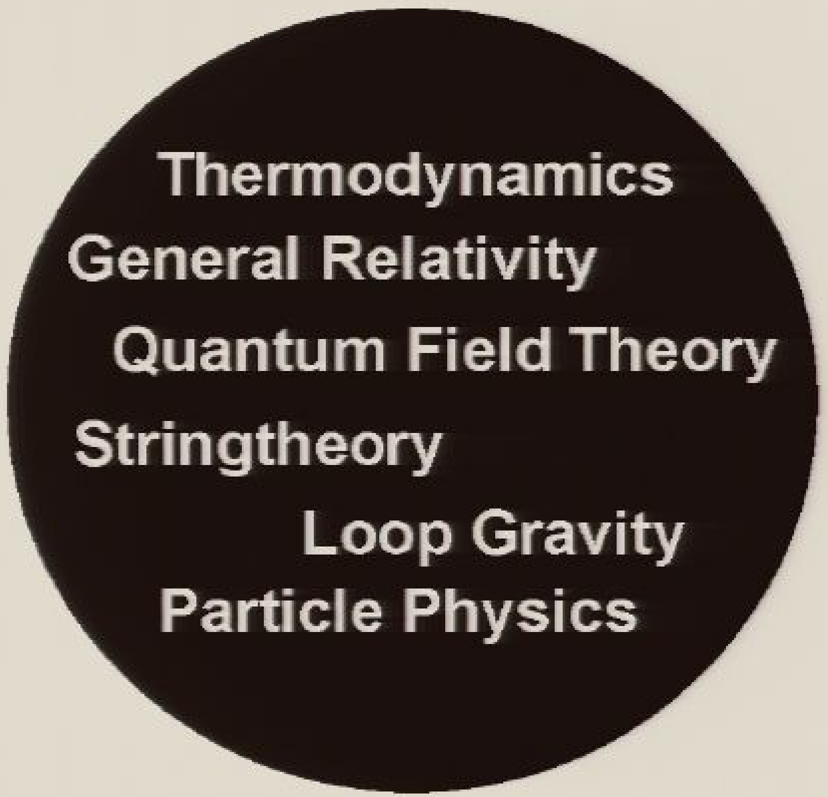
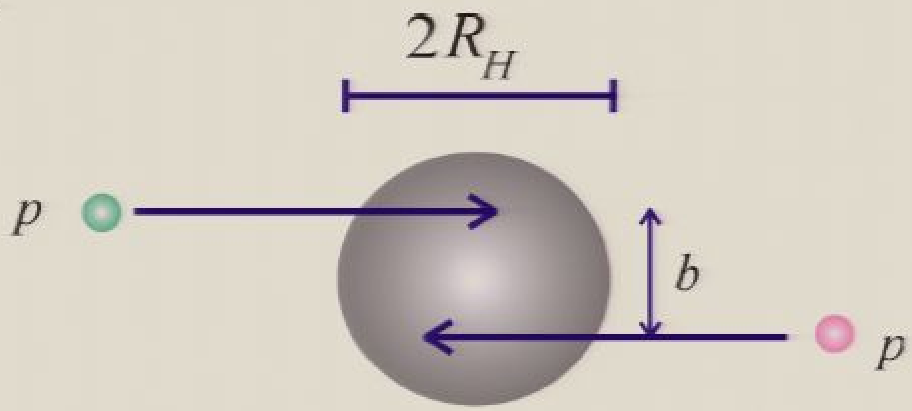


Black Holes in Particle Collisions

With large extra dimensions, black holes could become observable at the LHC.



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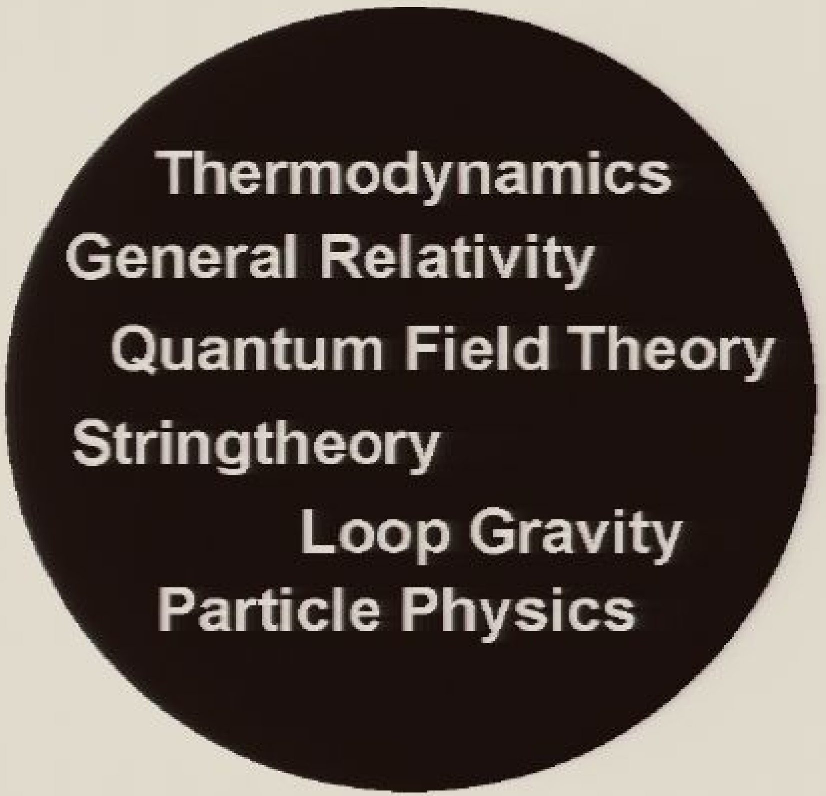

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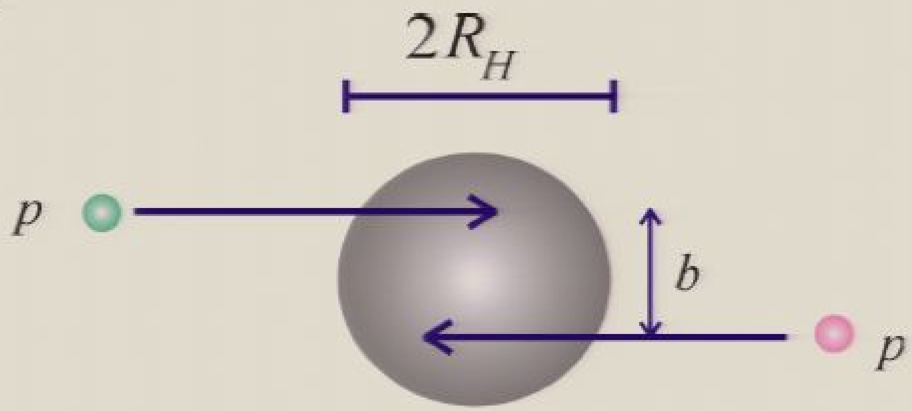

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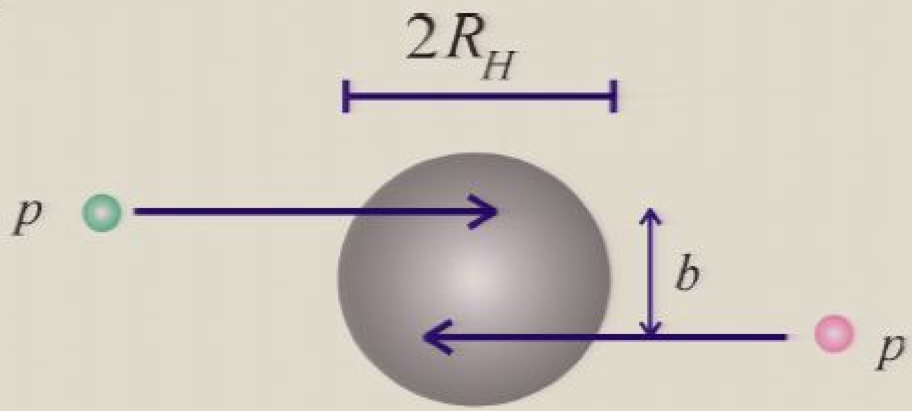

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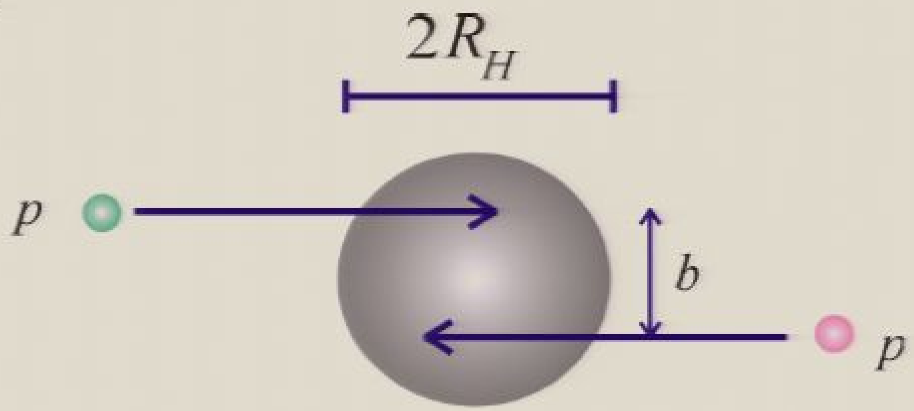
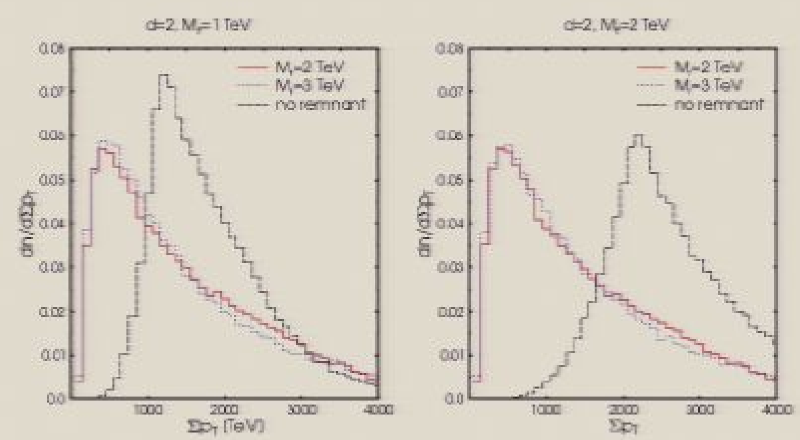


- Increased interest in examination of horizon formation and evaporation/decay
- Clear signature at LHC
- Virtual black holes
- Colored black holes ? (non-abelian)



Black Holes and the Minimal Length

A simple argument:

- Usual uncertainty for $\Delta E < m_{\rm p}$: $\Delta E \sim 1/\Delta x$
- General Relativity says: Schwarzschild-radius $R_H \sim \Delta E/m_p^2$
- So, a quantum particle with Compton wavelength $\lambda = 1/E$ receives an additional position uncertainty $\Delta x \sim E/m_p^2$

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- Minimal length simulates non-local interaction (locality bound)
- Reduces degrees of freedom $\rightarrow$ entropy bound?
- Holographic principle?
- Non-locality spoils unitarity $\rightarrow$ information loss problem?

Phenomenological Quantum Gravity is...

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... the attempt to connect pieces of the puzzle by focussing on specific questions.

Missing Pieces

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Homework:

- What is quantization?

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Missing Pieces

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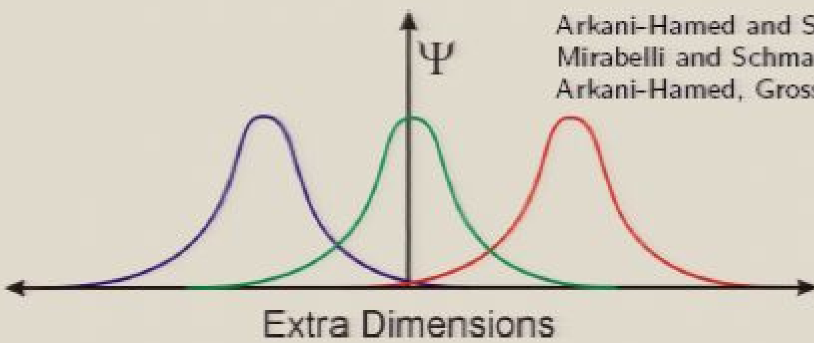
- What is matter?
- Why is our space-time 3+1 dimensional?


MERRY CHRISTMAS

Models with Extra Dimensions

- ADD-model: large extra dimensions $R \gg 1/M_f$
  - + Solves Hierarchy-problem, $m_p^2 = R^d M_f^{d+2}$
- RS-model (I and II), extra dimension is curved
  - + AdS-CFT Correspondence
  - + Allows non-compact extra dimension (volume stays finite)
- UXD, TeV-scale dimensions
  - + Accelerated unification of coupling constants
- Split fermion scenario: wave functions delocalized
  - + Quick fix for several problems



Arkani-Hamed and Schmaltz, Phys. Rev. D **61**, 033005 (2000)
Mirabelli and Schmaltz, Phys. Rev. D **61**, 113011 (2000)
Arkani-Hamed, Grossman and Schmaltz, Phys. Rev. D **61**, 115004 (2000).