

Title: Non-equilibrium physics of the very early universe

Date: Dec 08, 2005 11:00 AM

URL: <http://pirsa.org/05120010>

Abstract:

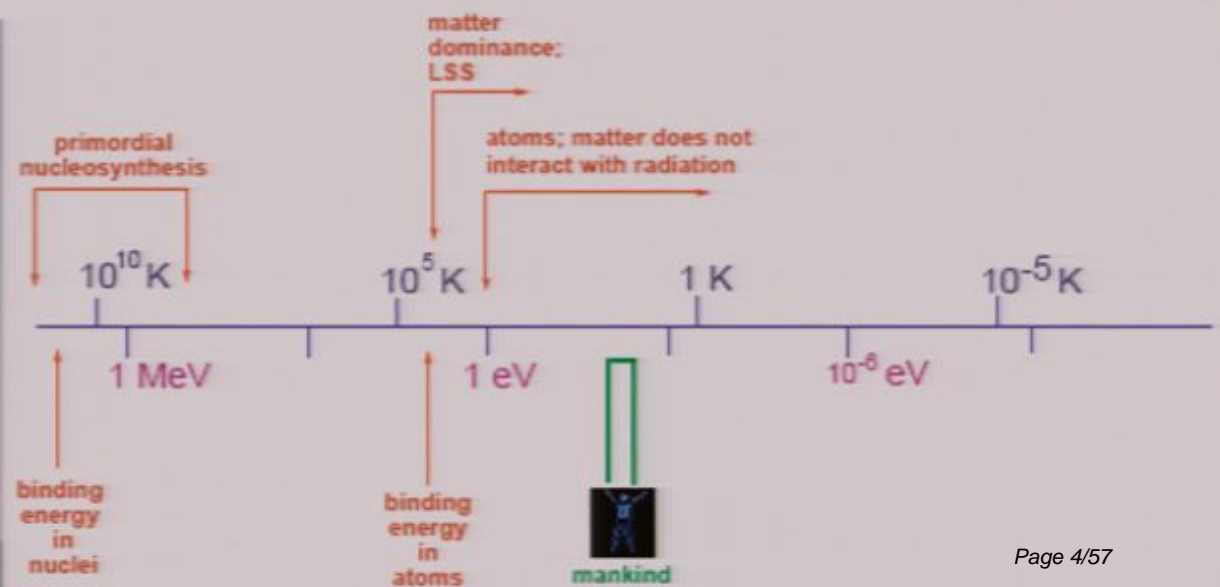
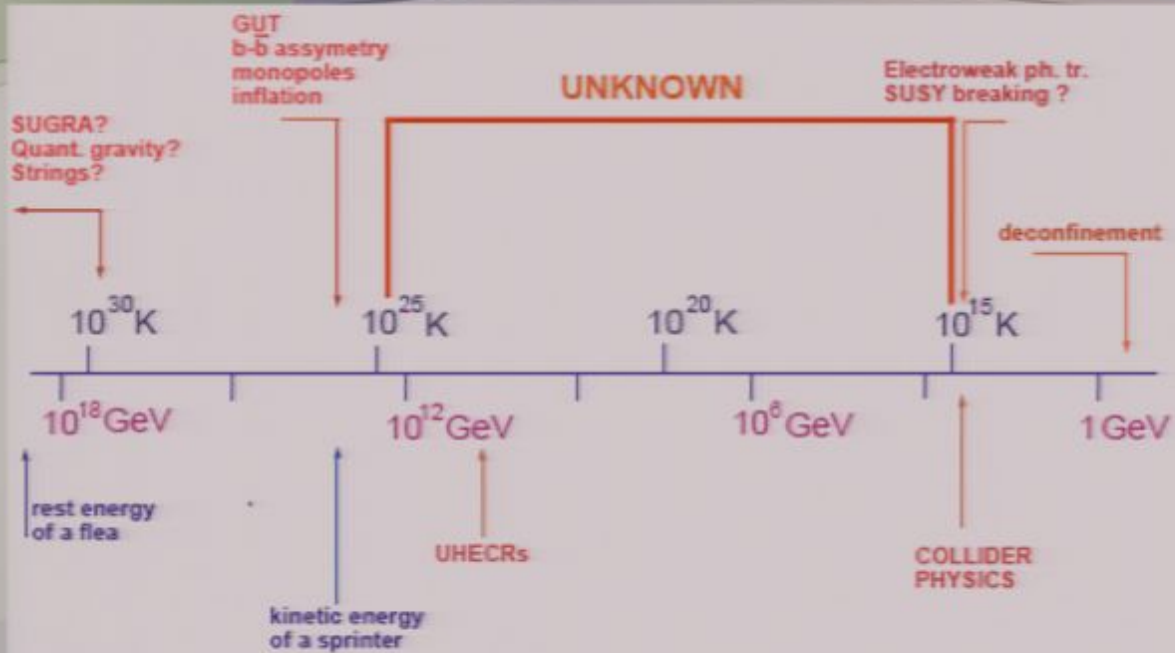
Non-Equilibrium Physics of the Very Early Universe

Dmitry Podolsky
CITA, University of Toronto

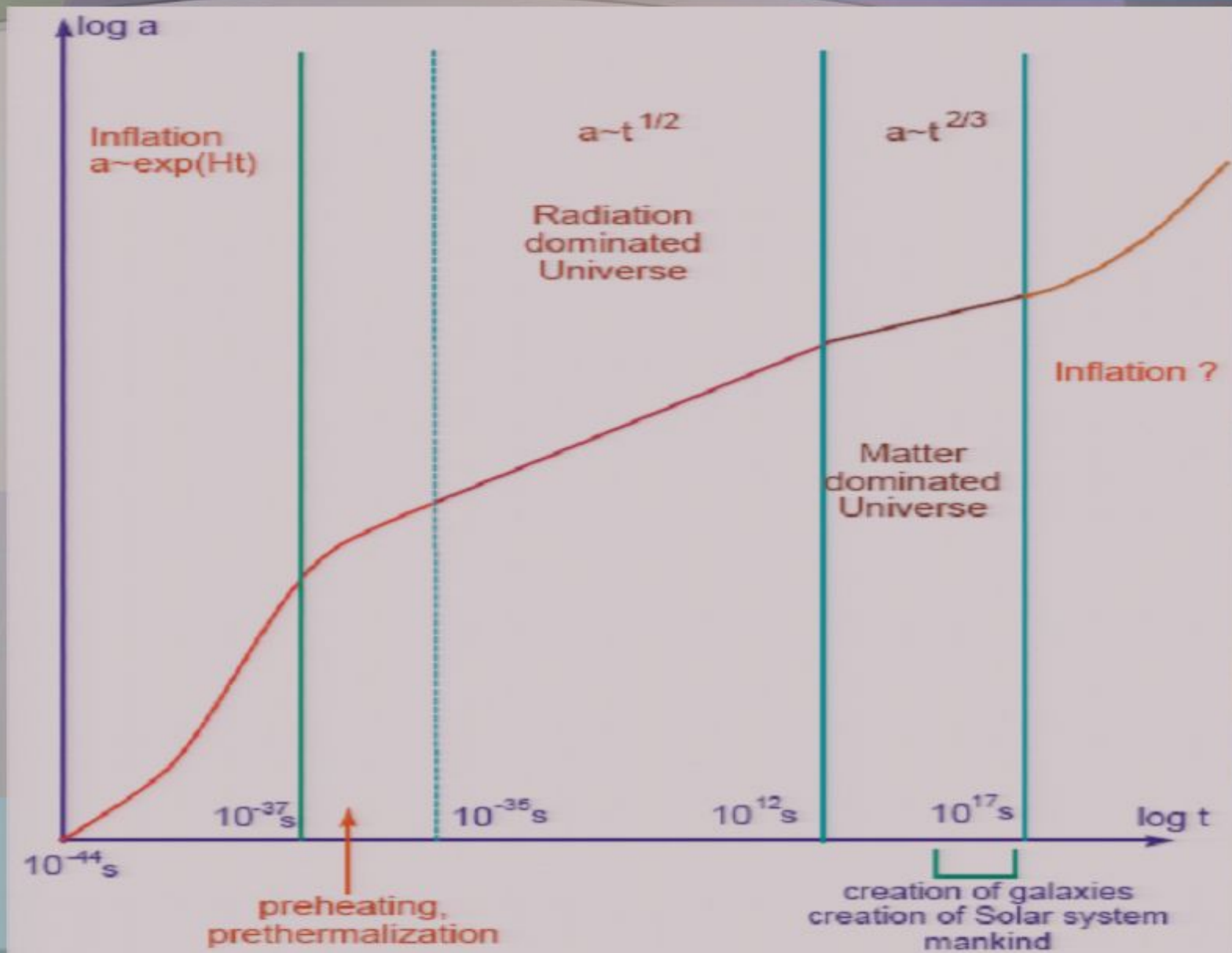
Outline

- **The very early Universe: HEP and cosmology**
- **Regimes of preheating: perturbative and non-perturbative**
- **Observable effects of preheating**
- **Prethermalization**

HEP point of view



Cosmologist's point of view



Decay of inflaton through gravitational interaction

Suppose that the inflaton field is completely decoupled from the matter sector and can decay only gravitationally:

$$\sqrt{-g}\mathcal{L}_{\text{int}} = \frac{1}{2}\sqrt{-g}g^{ik}\partial_i\phi\partial_k\phi - \frac{1}{2}\sqrt{-g}m^2\phi^2$$

No interaction term linear w.r.t. the inflaton field, therefore inflatons can only annihilate into gravitons, no decay

$$\langle\sigma v\rangle \sim \frac{m^2}{M_P^2} \frac{1}{M_P^2}, \text{ we have a freeze-out of annihilation processes in the expanding Universe, and the abundance of massive inflatons}$$

(m is taken from Cobe normalization)

$$\Omega h^2 \sim 10^{17}$$

The bottom line: massive inflatons overclose the Universe, there are also too many gravitons.

Therefore, the inflaton should interact with SM fields at the level of the Lagrangian.

Perturbative preheating

Model:
$$\mathcal{L} = -\frac{1}{16\pi G}R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\phi^2 - h_\chi\phi\chi^2 - h_\psi\phi\psi\bar{\psi}$$

- Single particle decay of inflatons into SM particles (both fermions and bosons)
- Products of the inflaton field's decay interact with each other (the same characteristic time scale), so all the information about pre-reheating Universe will be lost (except the reheating temperature T_R)
- Characteristic time scale is relatively long: $\Gamma(\phi \rightarrow \chi\chi) = \frac{h_\chi^2}{8\pi m}$ $\Gamma(\phi \rightarrow \psi\psi) = \frac{h_\psi^2 m}{8\pi}$

Numerically, if $h_\psi \sim 10^{-5}$, reheating takes about $t_{\text{reh}} \sim 10^{-26}$ sec

However, if a non-perturbative channel of preheating exists (in fact, in the theory under consideration it does), **preheating takes much shorter**.

Preheating: non-perturbative scenarios

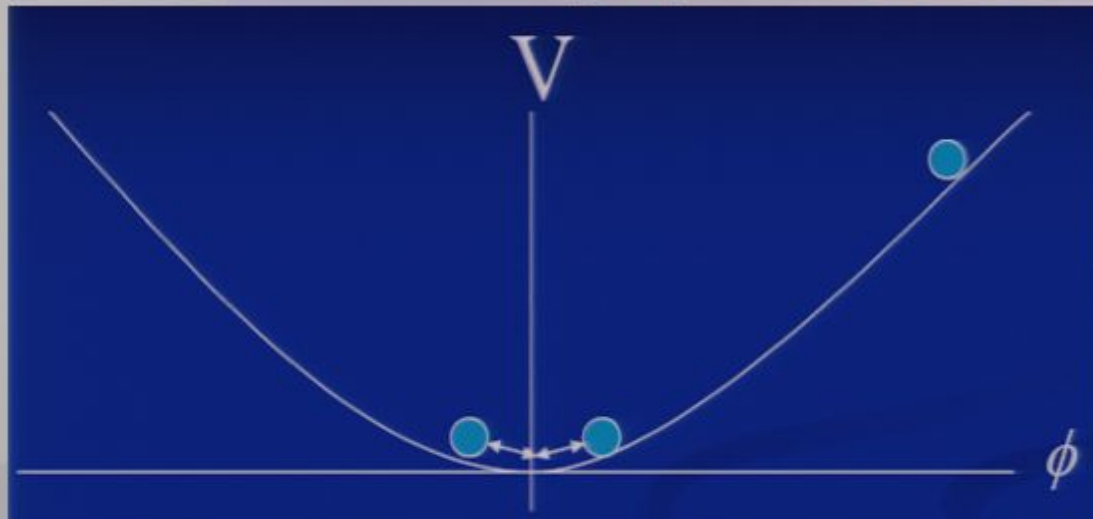
- **Fermions:** Pauli blocking, therefore, one cannot create fermions very effectively (non-perturbatively?)
- **Gauge bosons:** the inflaton cannot carry a charge; the charge conservation will block its decay (only annihilation is possible, not decay)

Decay of inflaton into quanta of other scalar fields

- Very effective decay is possible under certain conditions: parametric resonance, tachyonic instability etc.
- Even if only one such non-perturbative channel exists, it will completely dominate the dynamics of preheating
- Due to rescattering effects the numbers of quanta of other fields will rapidly increase as well

Parametric resonance 1

Model:
$$L = \underbrace{-\frac{1}{16\pi G}R}_{\text{gravity}} + \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{inflaton}} + \underbrace{\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}g^2\phi^2\chi^2}_{\text{matter}}$$

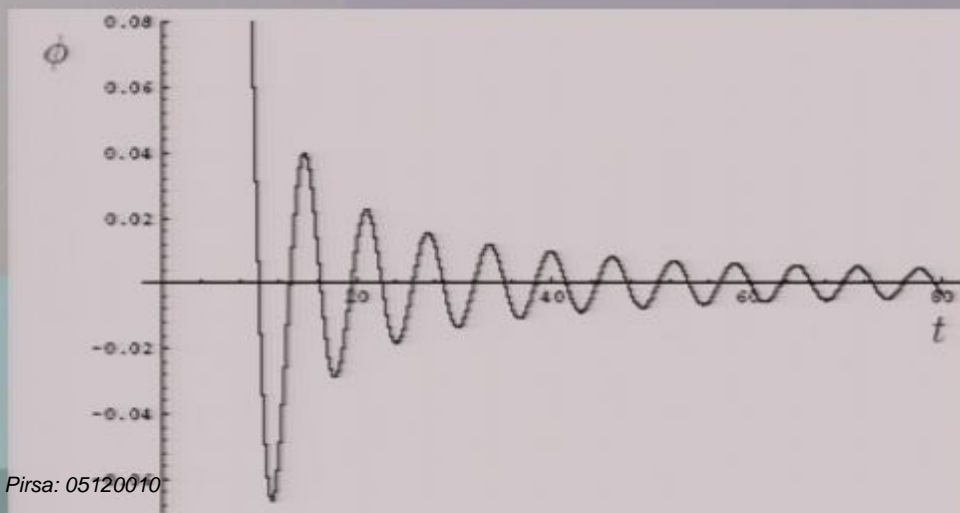


Dynamics of background:

$$H^2 = \frac{8\pi}{3M_P^2} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

$$\phi \sim \frac{M_P}{2} \text{ or } t \sim 10^{-37} \text{ s inflation (slow roll) ends.}$$



The inflaton starts to oscillate near the minimum of its potential as does the effective mass squared of the matter field
(Kofman, Linde, Starobinsky 1994)

Parametric resonance 2

Decomposition into modes: $\hat{\chi}(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \chi_k(t) e^{-i\mathbf{k}\mathbf{x}} + \hat{a}_k^+ \chi_k^*(t) e^{i\mathbf{k}\mathbf{x}} \right)$

Since there is no translation invariance along t , eigenfunctions are not Fourier modes.

They satisfy the equation $\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2(0) - \xi R + g^2\phi^2 \right) \chi_k = 0$

Redefining $X_k(t) = a^{3/2}(t)\chi_k(t)$ one can get rid of the first deriv. term:

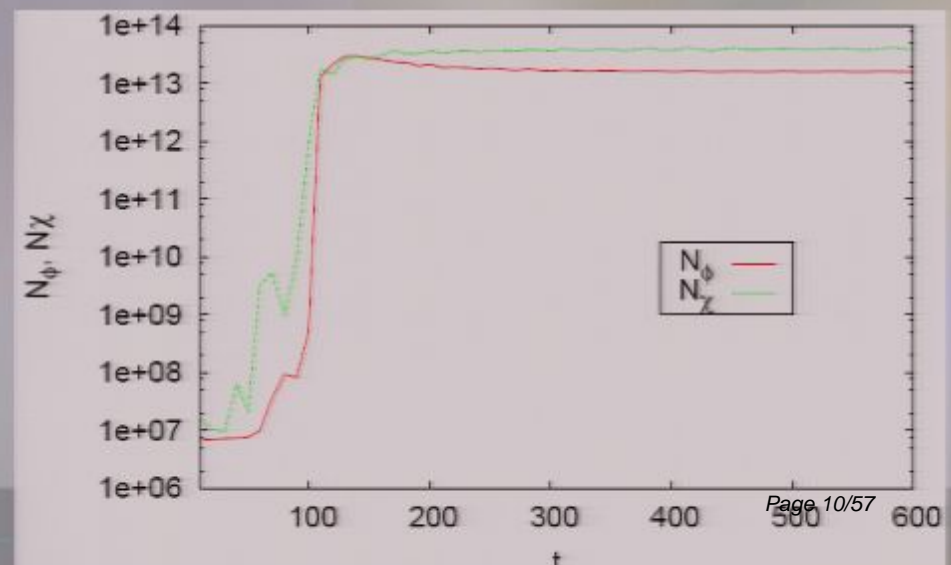
$$\ddot{X}_k + \omega_k^2 X_k = 0 \quad \text{- oscillator with variable frequency} \quad \omega_k^2 = \frac{k^2}{a^2(t)} + g^2\Phi^2 \sin^2 mt$$

If the frequency does not change rapidly in time:

$$\frac{\dot{\omega}_k}{\omega_k^2} \ll 1$$

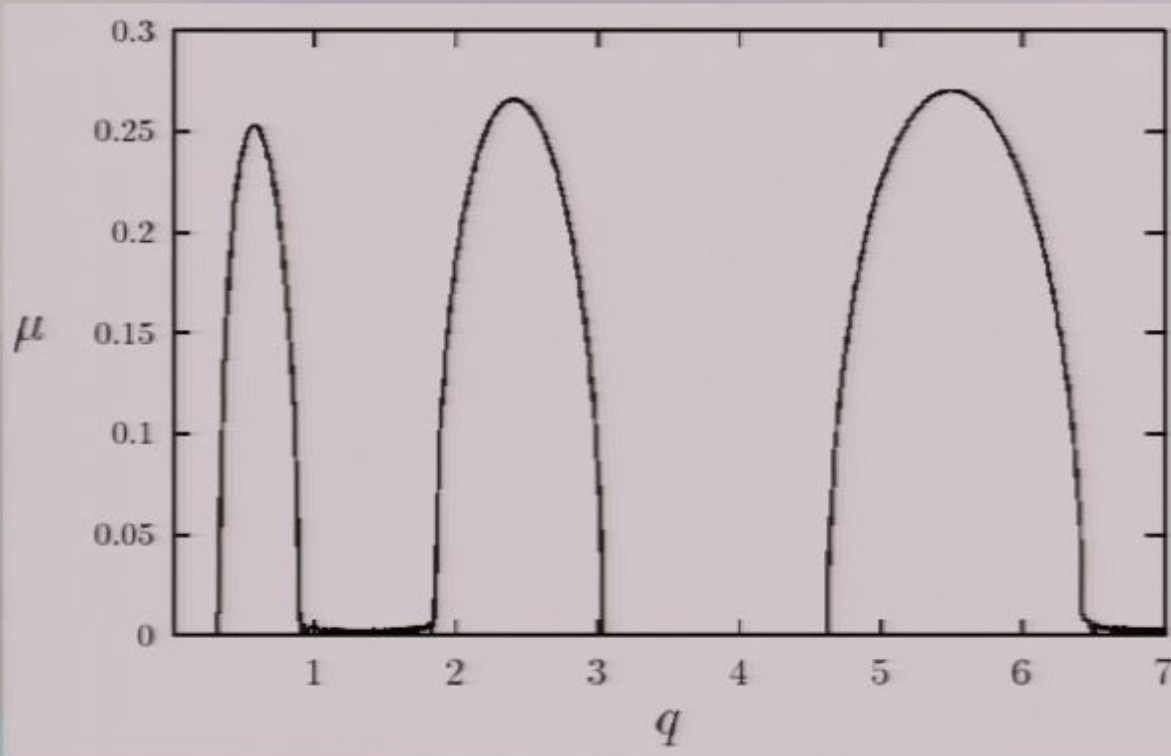
adiabatic invariant conserves
giving a basis for QFT in a
curved spacetime
(number of particles):

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2 \right) - \frac{1}{2}$$



Parametric resonance 3

The parameter controlling preheating is $q = \frac{g^2 \Phi^2}{4m^2}$ where Φ is the amplitude of the inflaton's oscillations. Depending on its value, the exponential rate of the growth of particle numbers will be different (Kofman, Linde, Starobinsky 1997).



Numbers of particles behave as

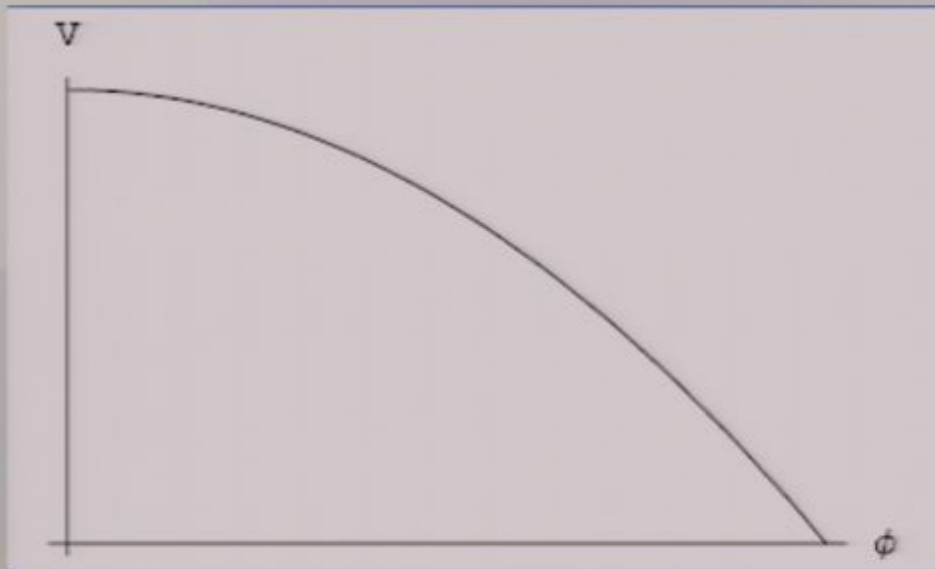
$$n_k \sim \exp(\mu_k m t)$$

Characteristic exponent μ is a multivalued function of k and q in flat spacetime. In curved spacetime the situation is more complicated --- it may also depend on time.

On the figure – μ as a function of q for $k=0$; flat spacetime.

Preheating due to the tachyonic instability

In many models of inflation such as new inflation and hybrid inflation the scalar fields in the end of inflation fall down a slope with a negative curvature. As a result, all modes with wave number k smaller then the curvature of potential (effective mass squared) will be exponentially amplified (Felder et al. 2000).



Example: spontaneous symmetry breaking

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 \equiv \frac{m^4}{4\lambda} - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

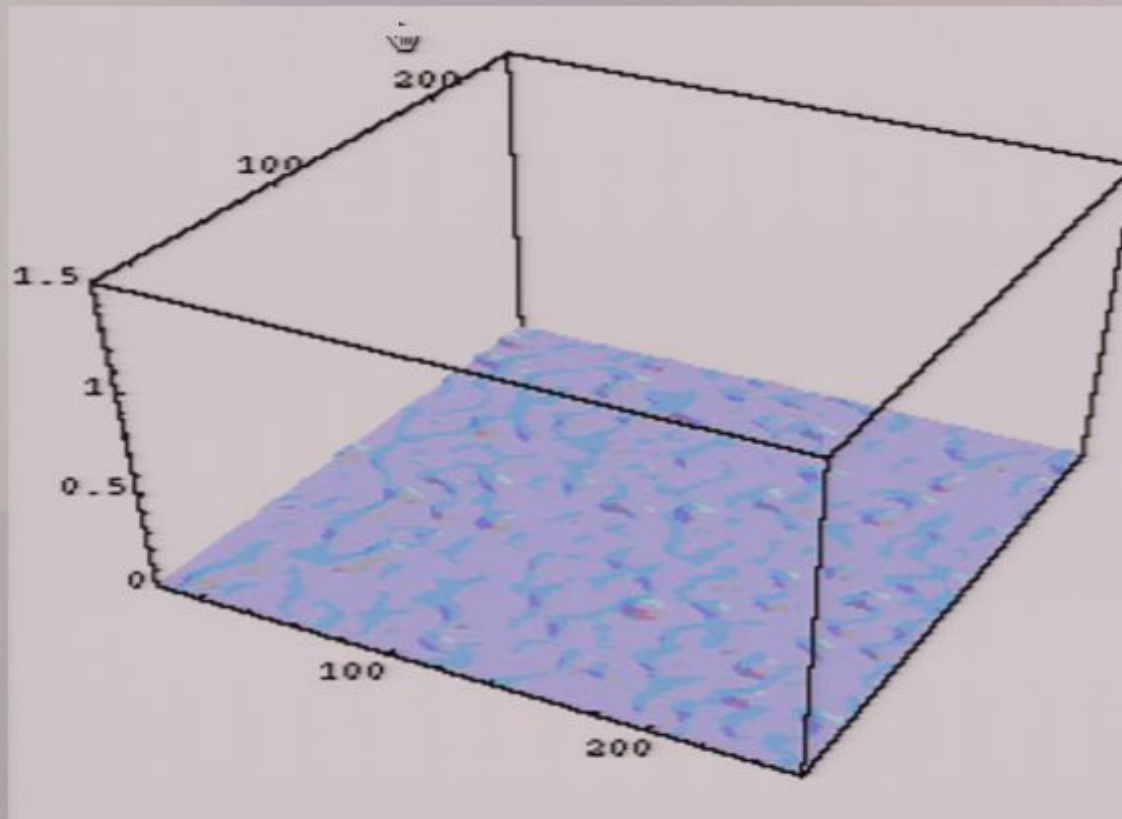
$$\phi_k \sim \exp(t\sqrt{m^2 - k^2})$$

The growth of ϕ modes continues until $\sqrt{\langle \delta\phi^2 \rangle}$ reaches the value $v/2$

At this moment occupation numbers are as large as

$$n_k \sim \exp(2mt_*) \sim \exp\left(\ln \frac{\pi^2}{\lambda}\right) = \frac{\pi^2}{\lambda} \gg 1$$

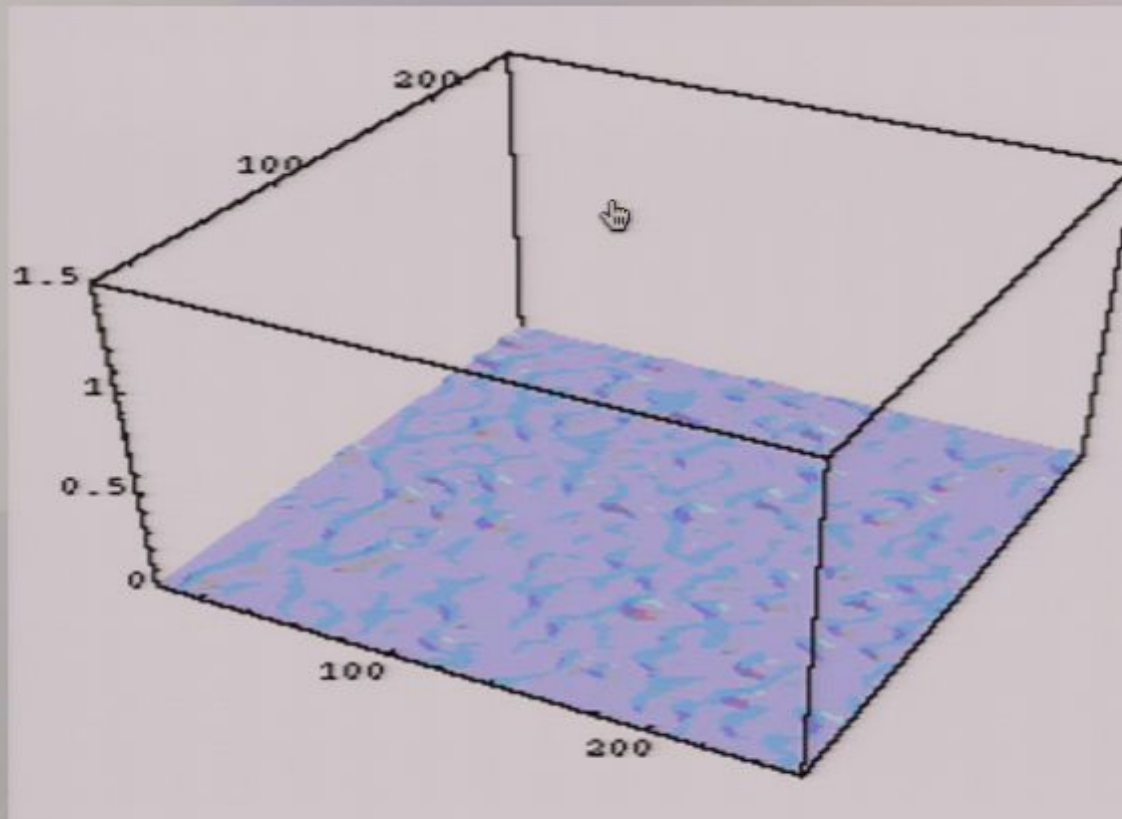
Tachyonic instability at the level of fluctuations



1. Initial gaussian fluctuations of the inflaton and the matter field (no particles)
2. Generation of squeezed state (rapid changes in the amplitude, long range correlation of phases)
3. Correlation of phases is broken; final state is described in terms of particles

Amplitude of the inflaton's fluctuations as a function of the position in space (2d slice of 3d simulations).

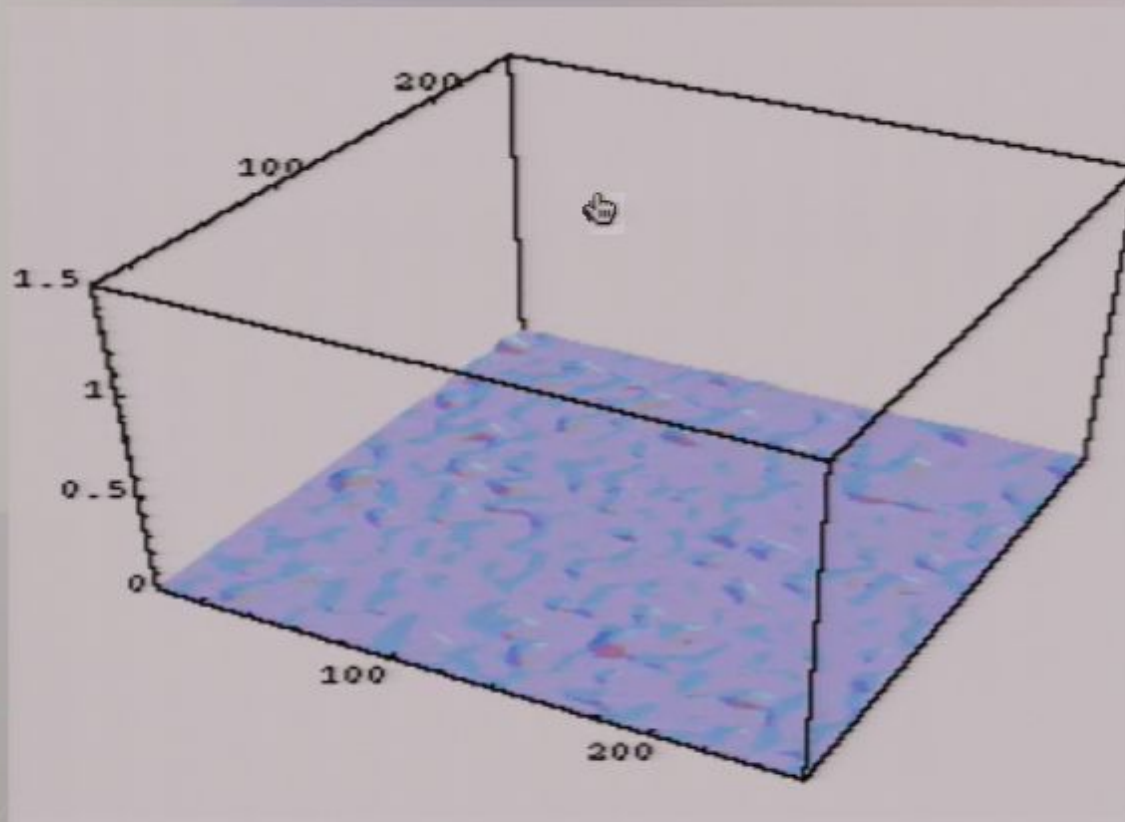
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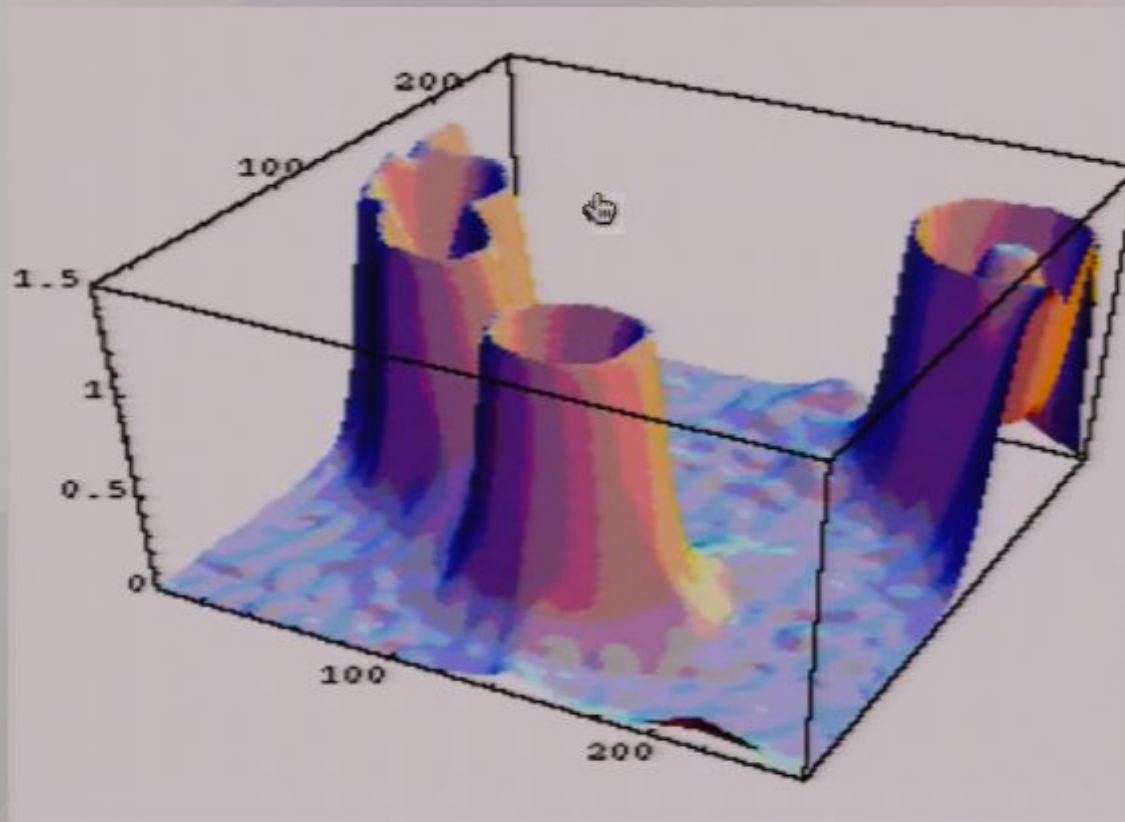
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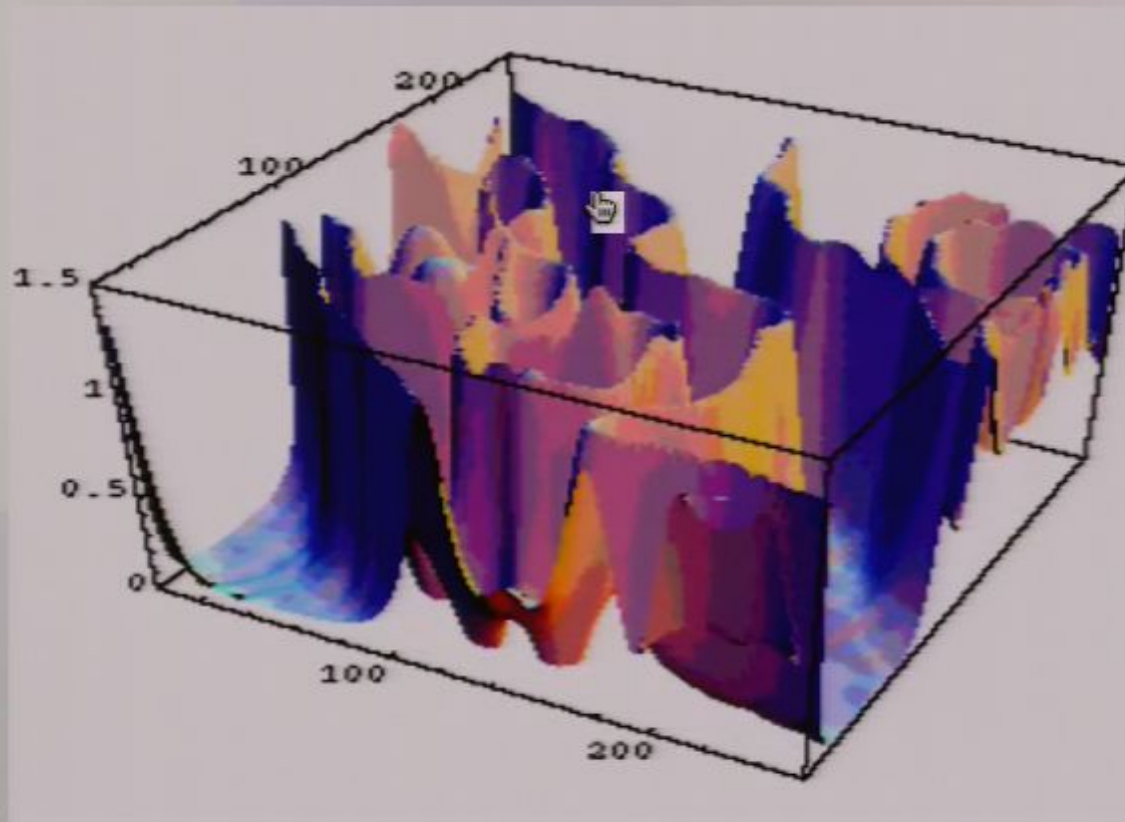
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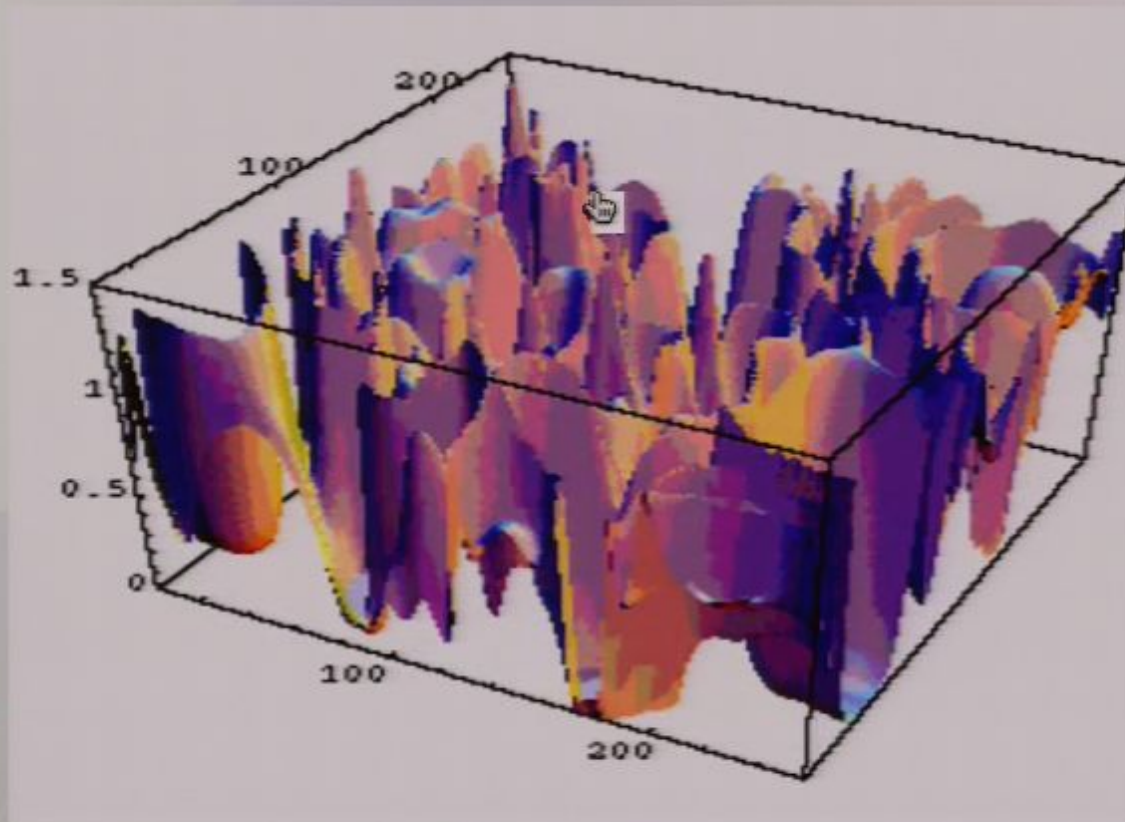
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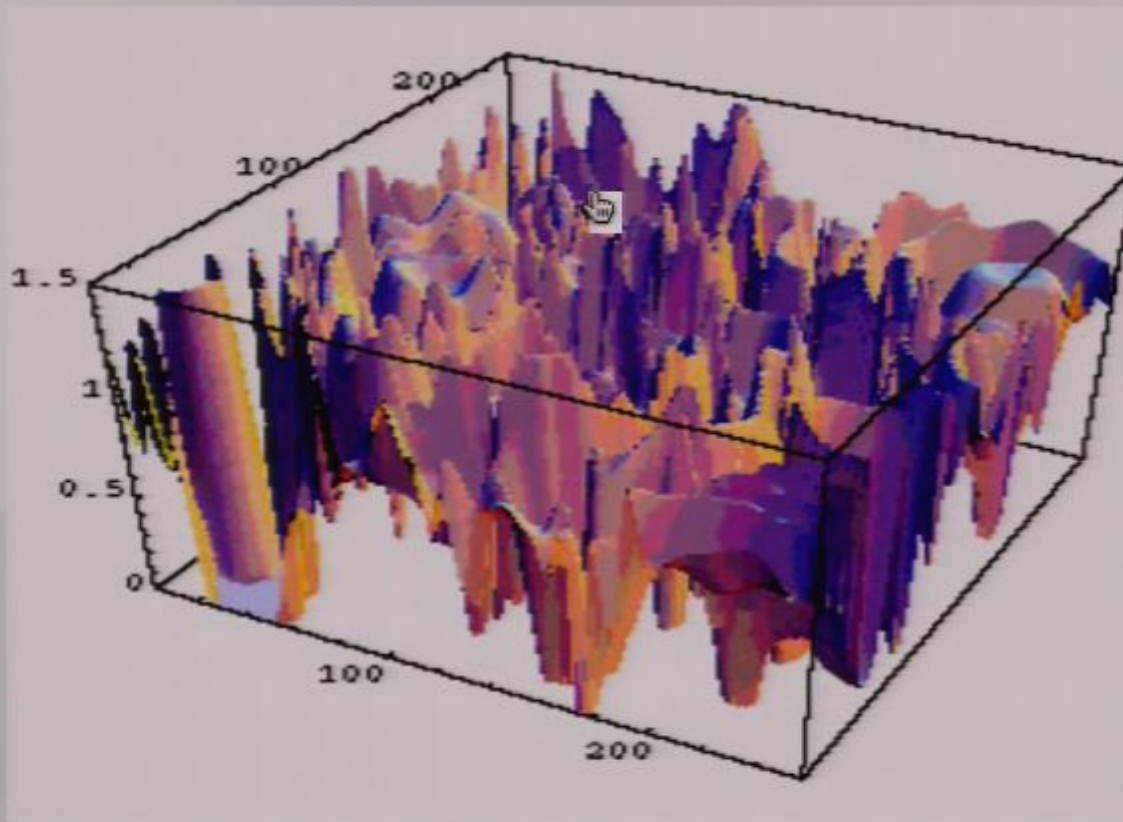
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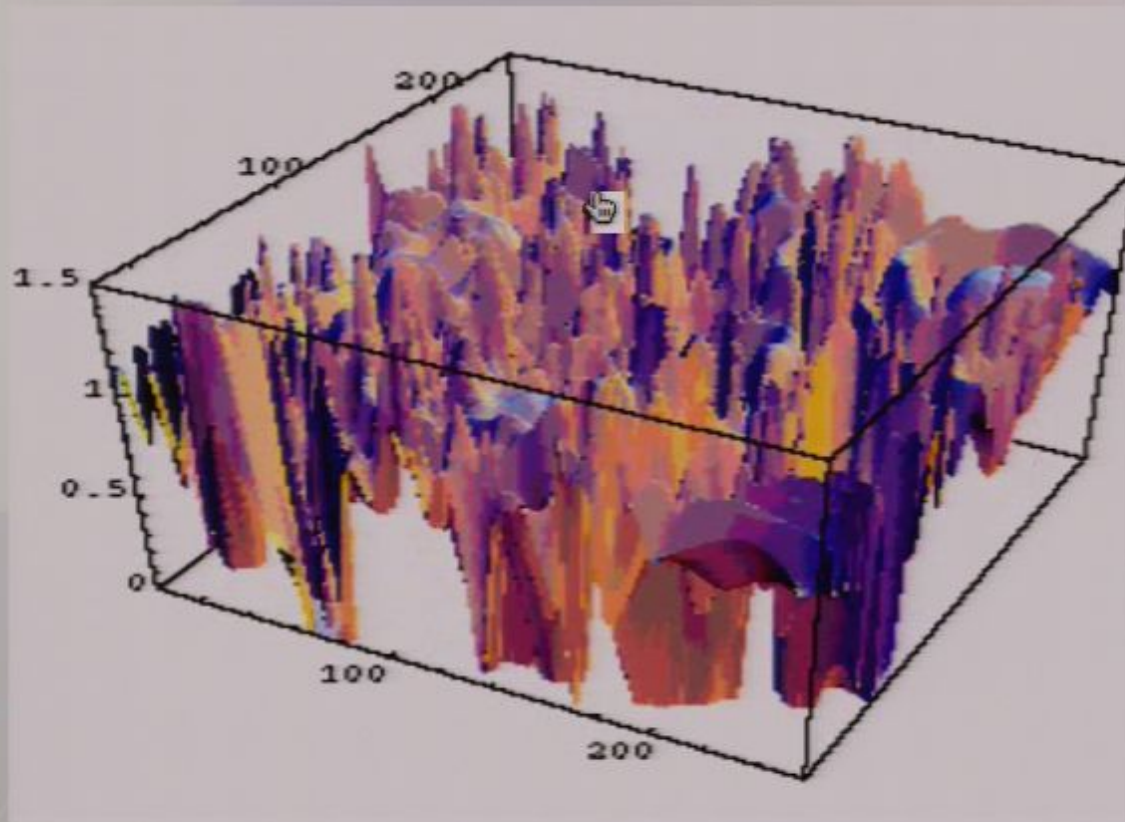
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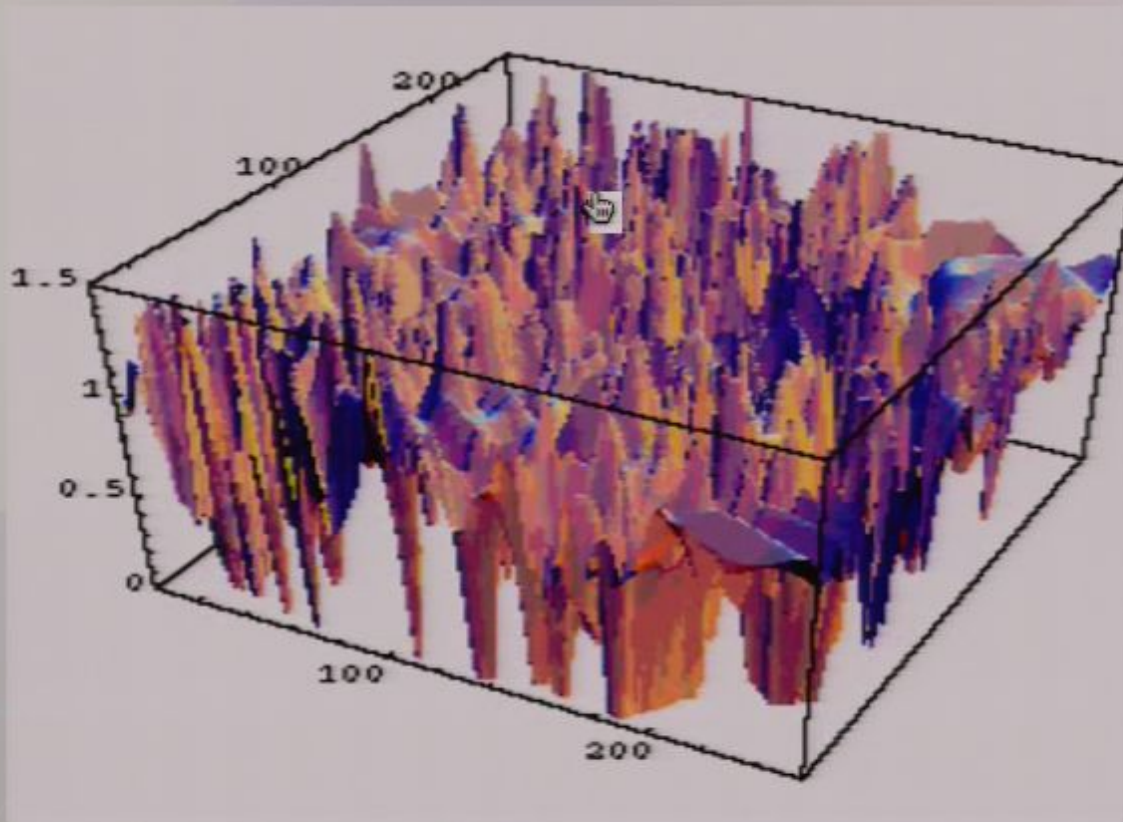
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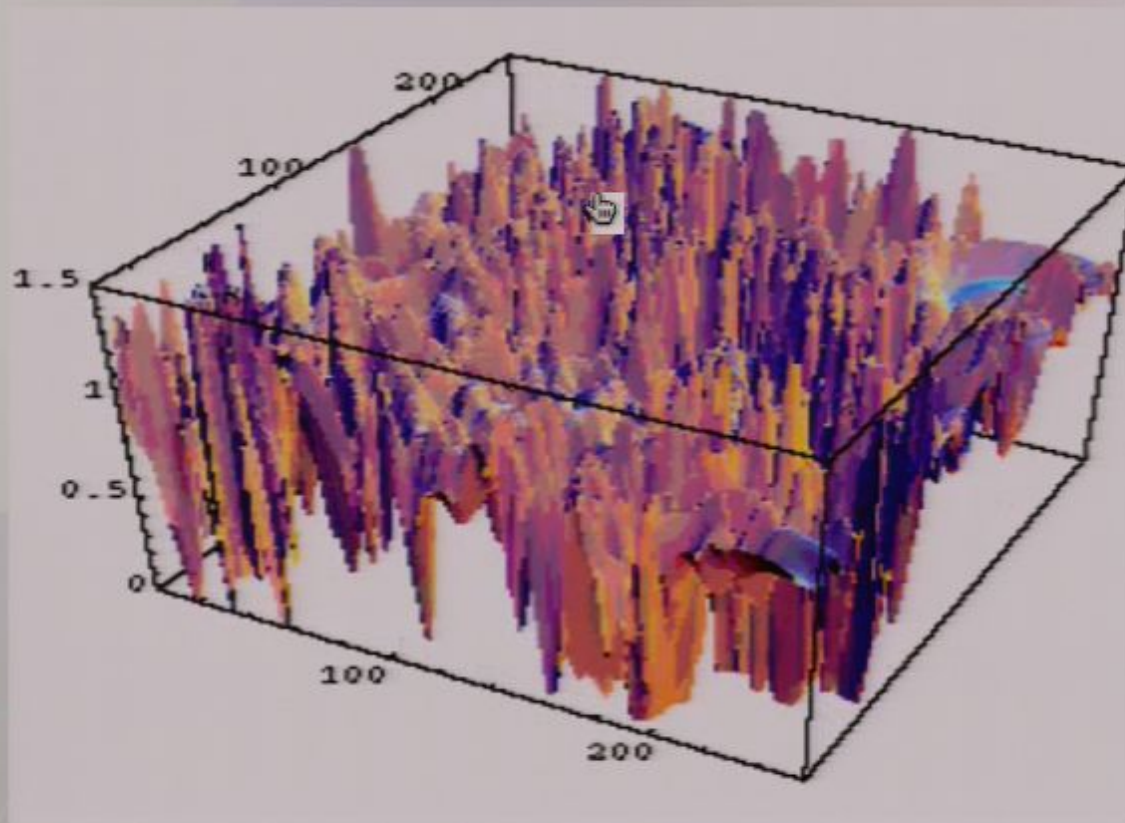
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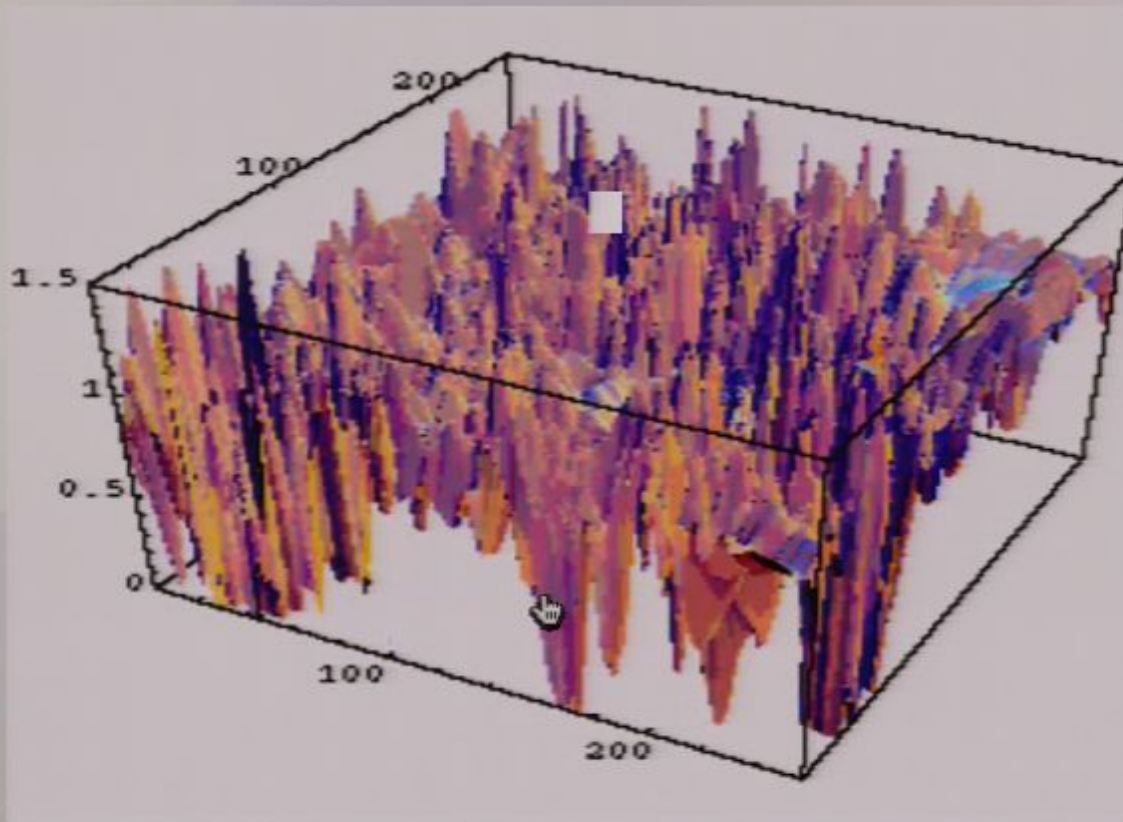
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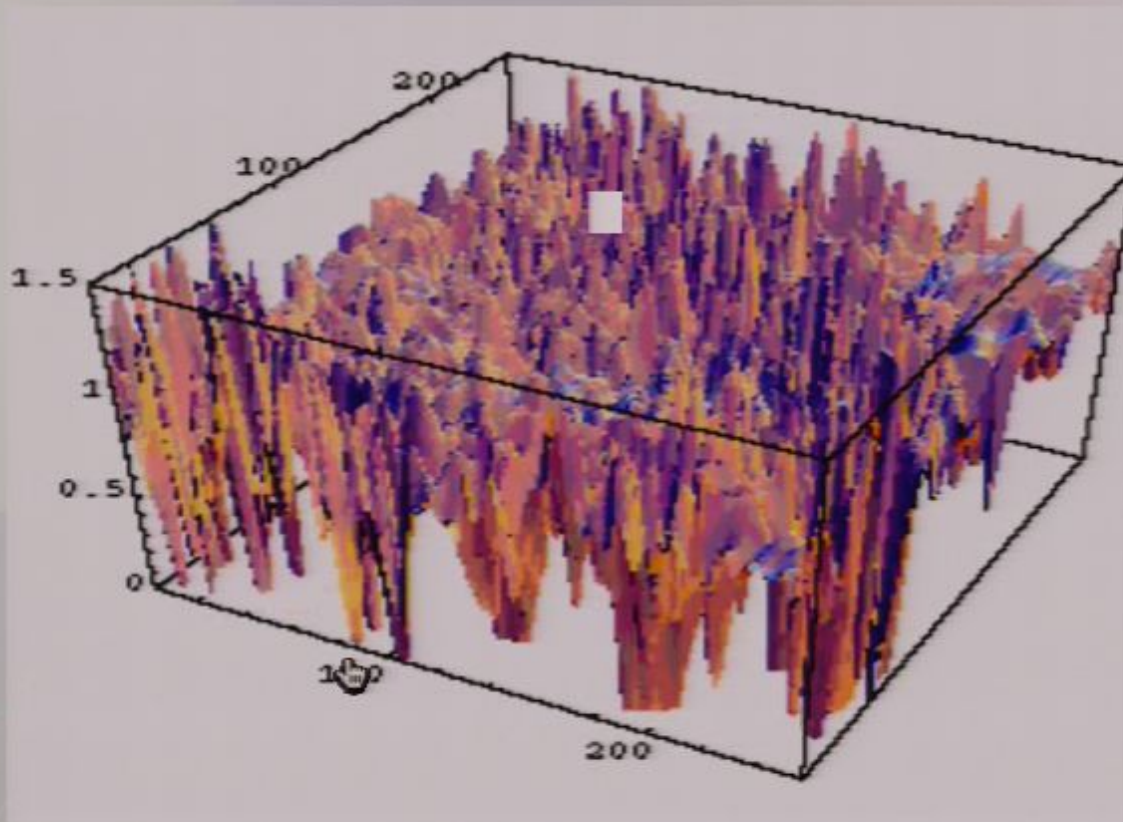
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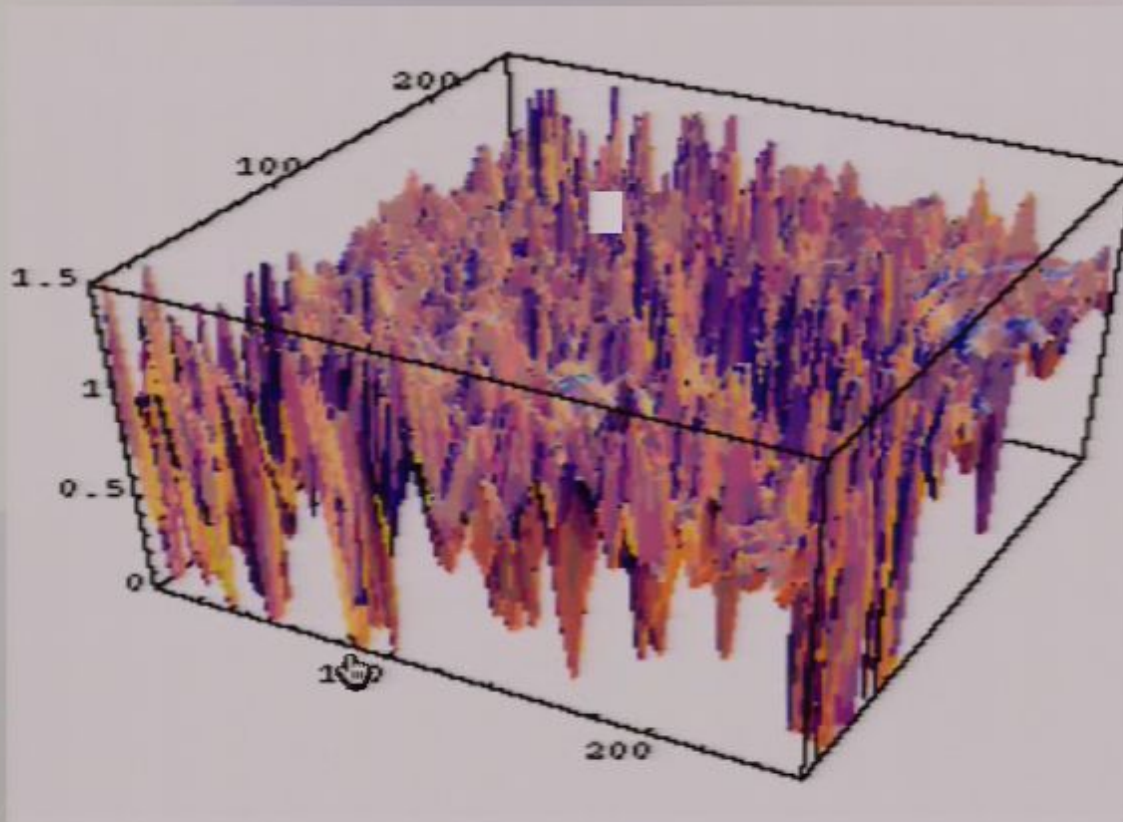
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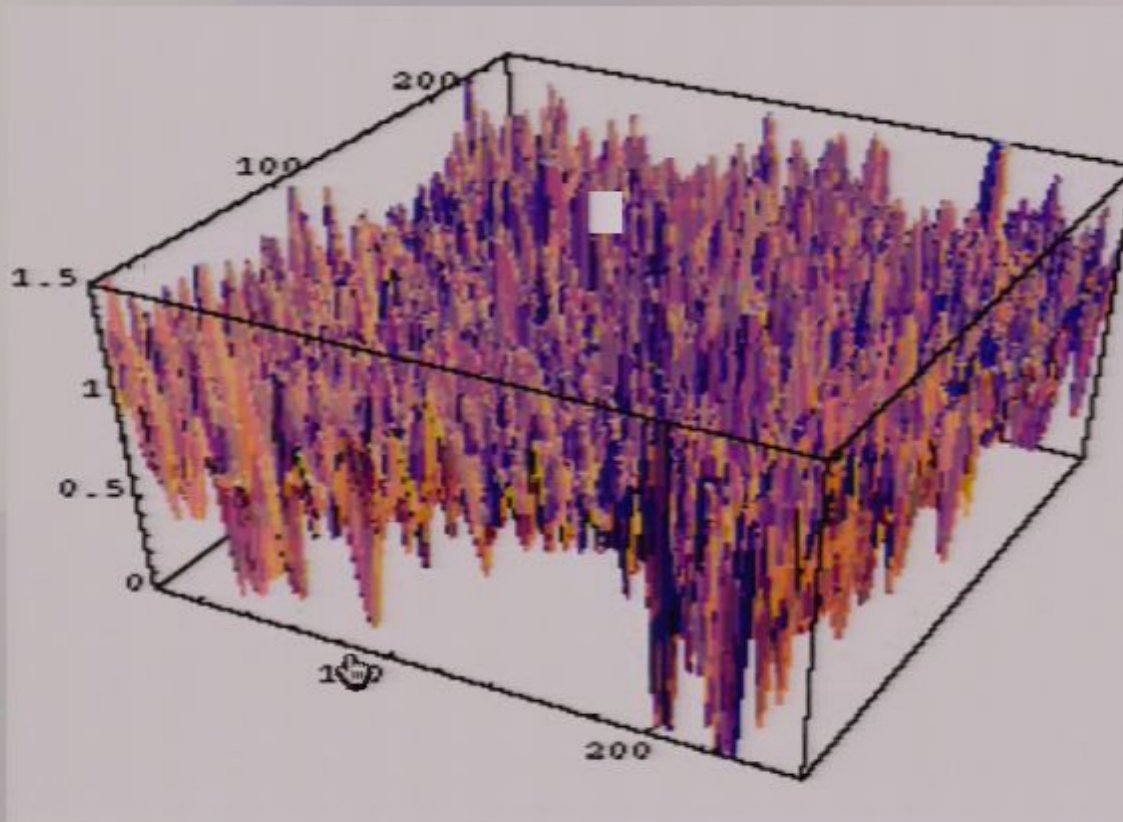
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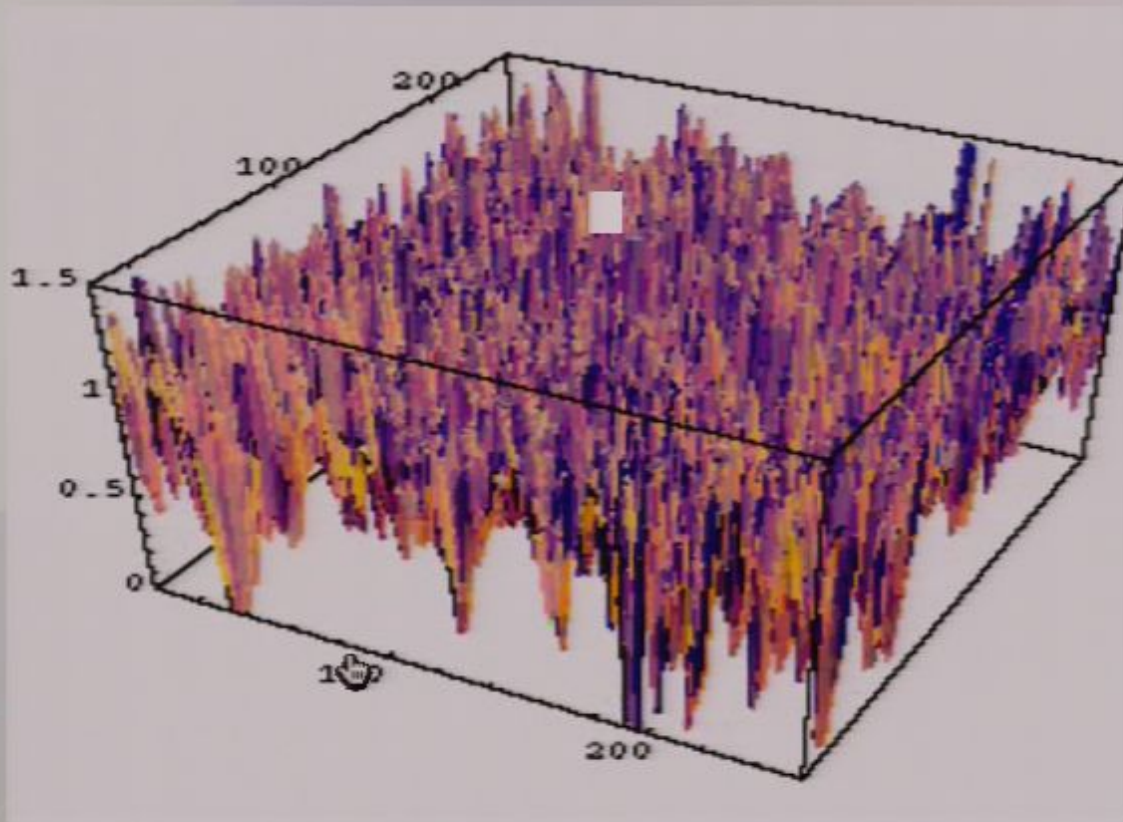
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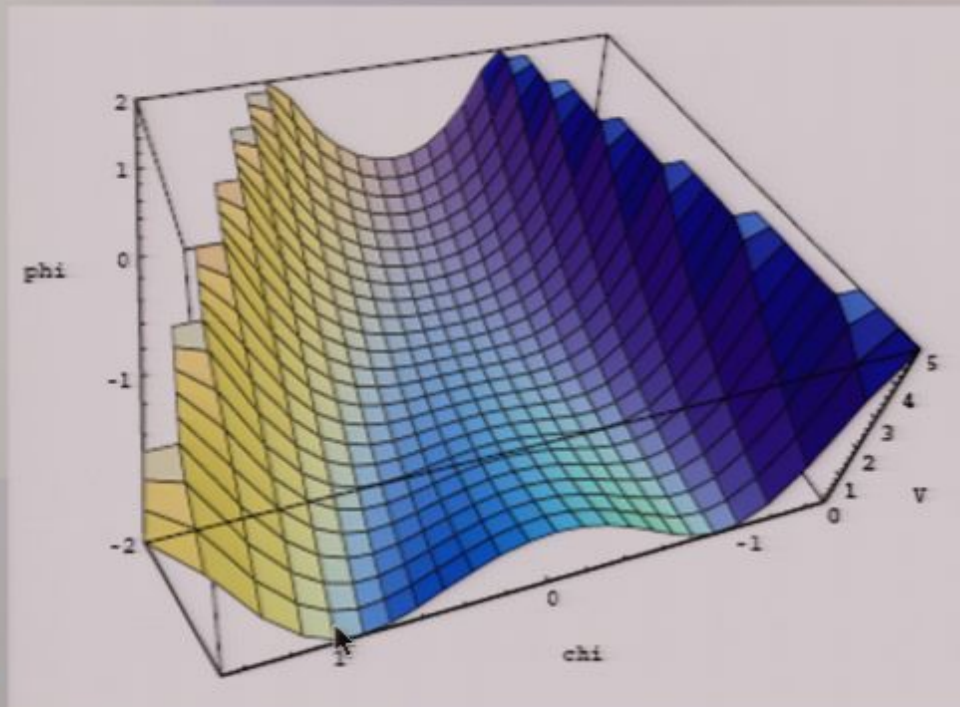


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“Tachyonic resonance”: the model

Motivation: inflaton should decay completely; scenario is the most universal



Potential for $g = 0$, $\lambda = \sigma^2/m^2$

$$V = \frac{m^2}{2} \phi^2 + \frac{\sigma}{2} \phi \chi^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{\lambda}{4} \chi^4$$

Eff. frequency for χ in flat spacetime

$$\omega_k^2(t) = k^2 + \sigma \Phi(t) \sin(mt) + g^2 \Phi(t)^2 \sin^2(mt)$$

The frequency can be negative during the part of the period for the IR mode
Therefore, mixture of parametric resonance and tachyonic effects.

Controlling parameters:

$$q_{3i} = \sigma \Phi_0 / m^2 \quad q_{4i} = g^2 \Phi_0^2 / m^2$$

$$q_{\chi i} = \lambda \Phi_0^2 / m^2$$

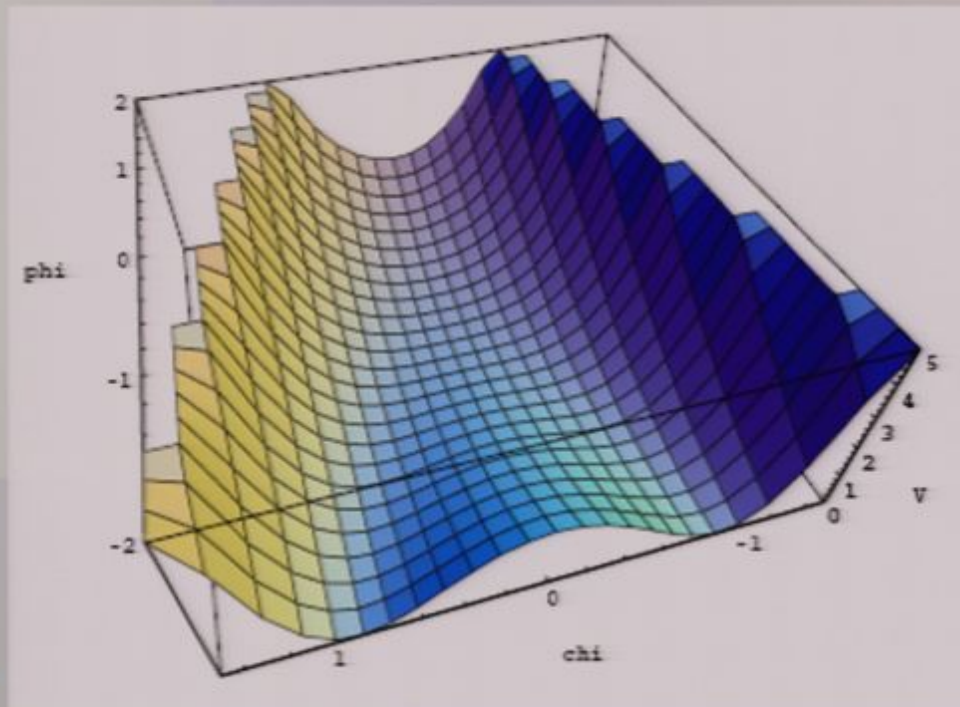
Non-renormalizable interactions: if $g^2 < \frac{\Phi}{M} < 1$, non-renormalizable interactions drive preheating, typically quintic interaction (rescattering is driven by trilinear int.):

$$V = V(\phi) + \chi^2 \left(m_\chi^2 + \sigma \phi + g^2 \phi^2 + \lambda_3 \frac{\phi^3}{M} + \lambda_4 \frac{\phi^4}{M^2} + \dots \right) + \dots$$

13-15
2-2
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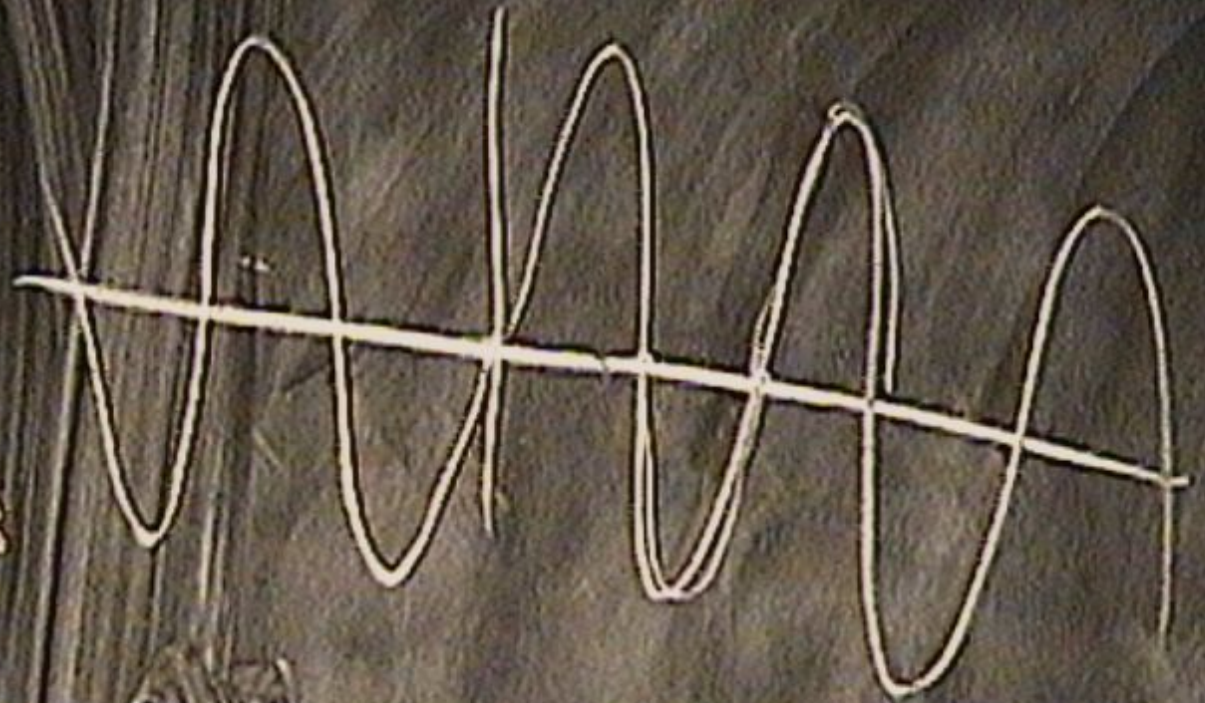
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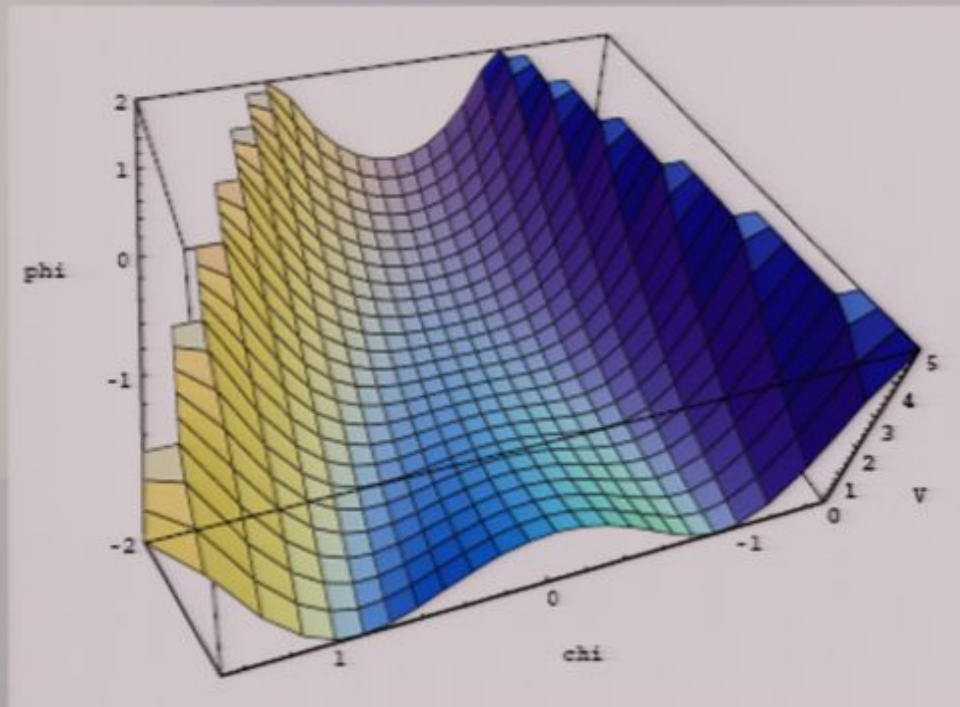
ω^2

ω
 2
 10
 ω
 2
 10



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$$\frac{1}{2}(\dot{\phi})^2 - \lambda \phi^4$$

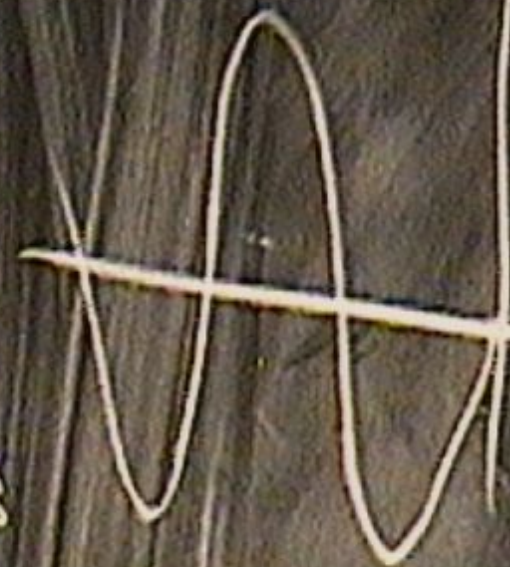
$$\ddot{\phi} + \lambda \phi^3 = 0$$

$$\phi_{n+1} - 2\phi_n + \phi_{n-1} + \lambda \phi_n^3 = 0$$

$$\omega^2$$

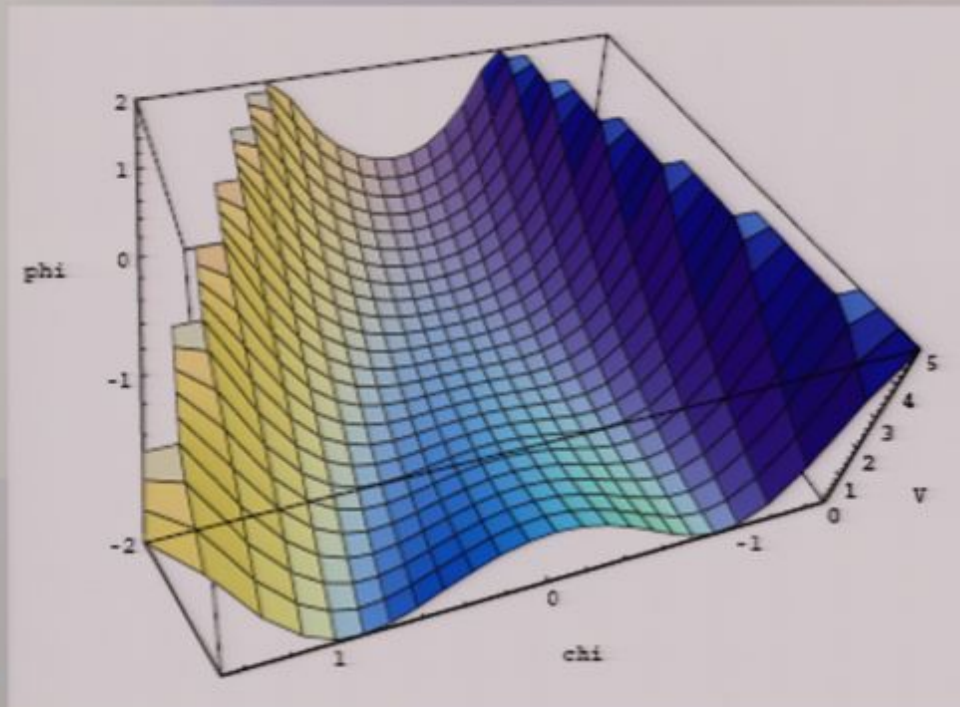
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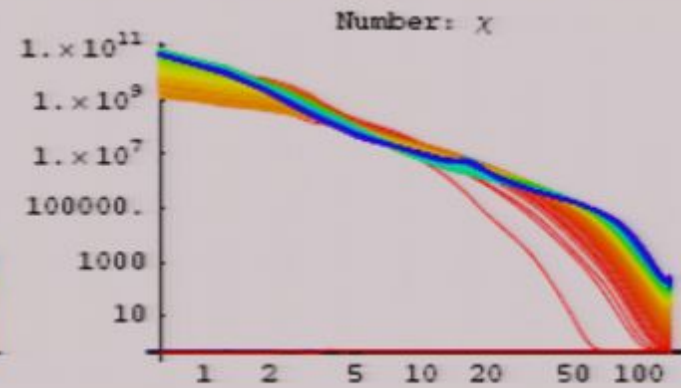
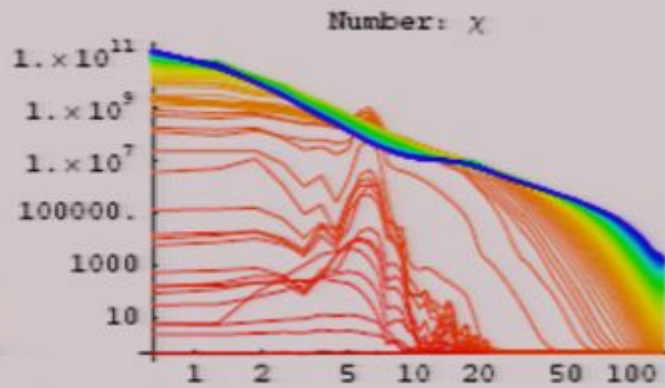
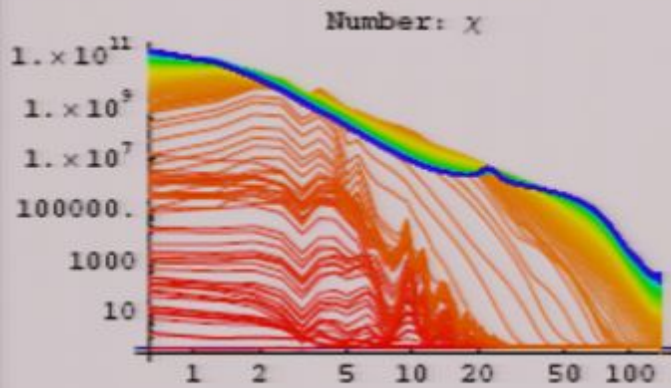
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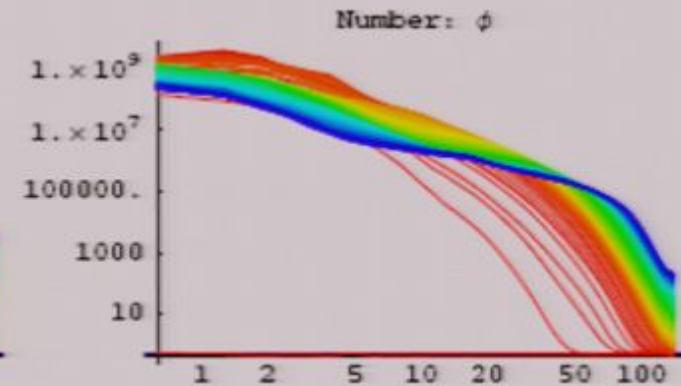
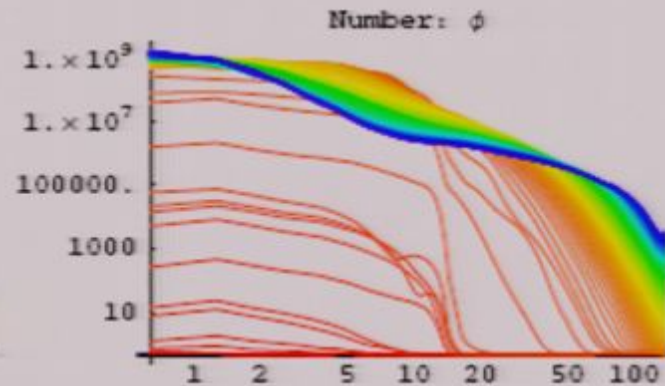
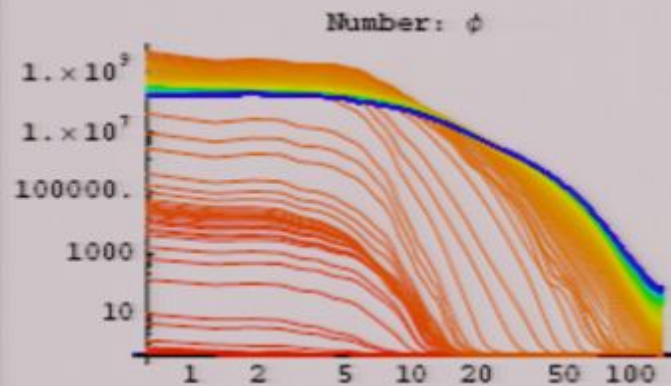
“Tachyonic resonance”: spectra and other observables 1



$$q_{4i} = 10^4, \quad q_{\chi i} = 5 \cdot 10^3, \quad q_{3i} = 0$$

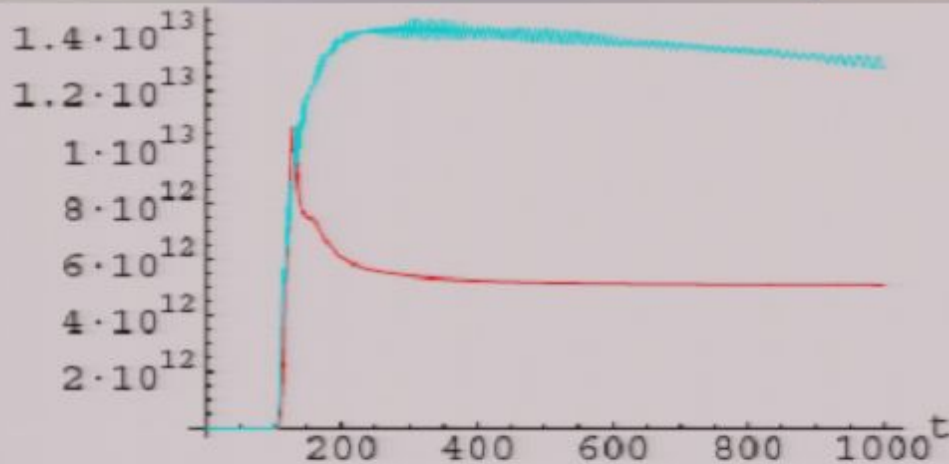
$$q_{4i} = 10^4, \quad q_{\chi i} = 5 \cdot 10^3, \quad q_{3i} = 10^2$$

$$q_{4i} = 0, \quad q_{\chi i} = 10^4, \quad q_{3i} = 10^2$$

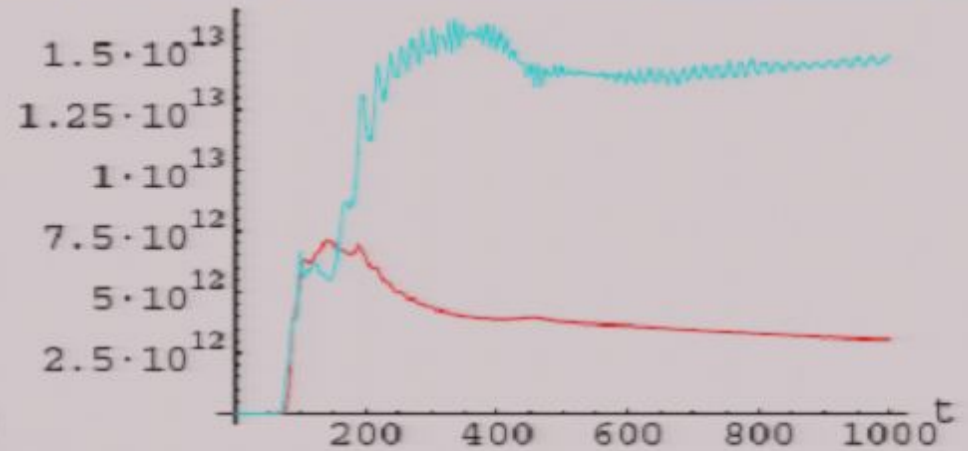


“Tachyonic resonance”: spectra and other observables 2

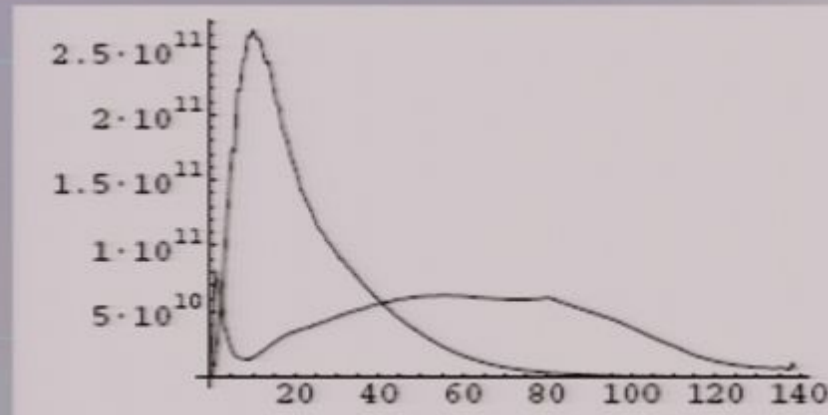
Total number of particles (red – ϕ , green – χ)



$$q_{4i} = 10^4, \quad q_{\chi i} = 5 \cdot 10^3, \quad q_{3i} = 0$$



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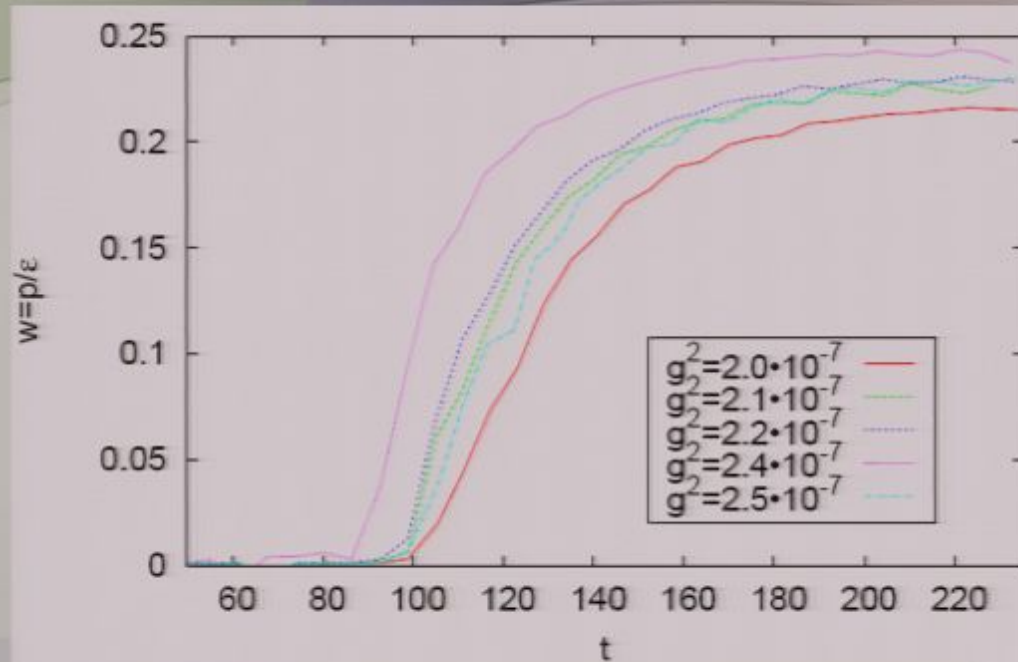
Spectra $k^2 \omega_k n_k / m^3$ (as a function of k/m) of inflatons shown at $mt = 1000$. The curve to the right corresponds to the model with trilinear interaction. Rescattering and thermalization with trilinear interaction is much more effective.

Can we have any information about preheating from observations?

It is believed that the details of the transition between inflation and the hot radiation dominant stage are not relevant, except for the so-called reheating temperature, the temperature of the ultra-relativistic plasma at the time when it reaches thermal equilibrium. The system loses all the information about early postinflationary epochs of its evolution.

True?

Equation of state and response of metric



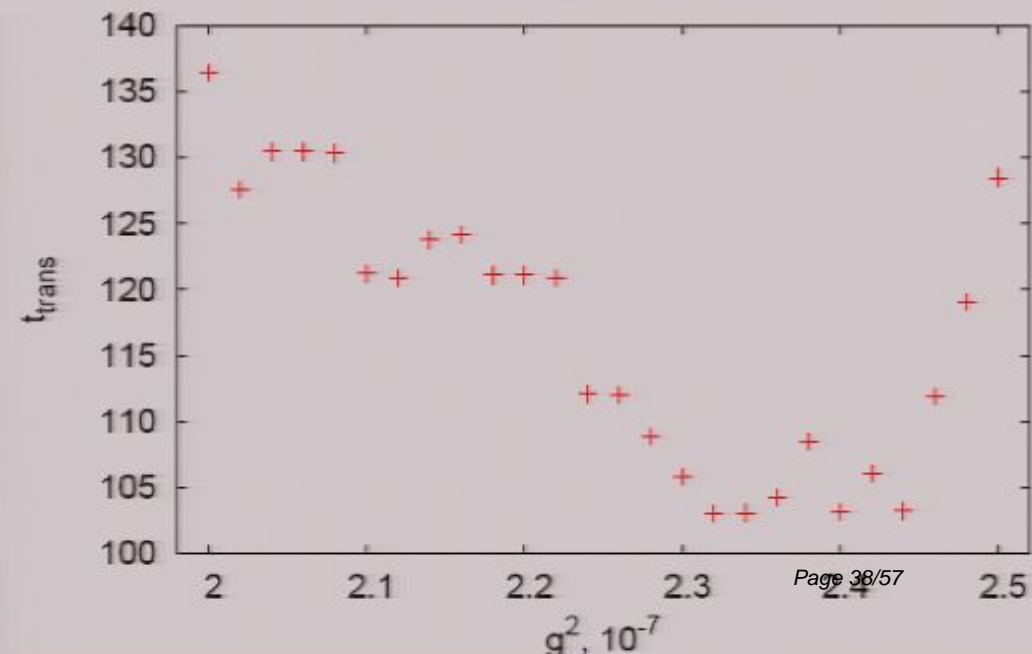
On the fig.: preheating after chaotic inflation driven by the interaction term

$$\mathcal{L}_{int} = -\frac{g^2}{2}\phi^2\chi^2$$

Equation of state defines the regime of the expansion:

$$a(t) \sim t^{\frac{2}{3(1+w)}}$$

In different Hubble patches the lengths of preheating stages are different under certain conditions. This leads to the generation of large scale cosmological perturbations (mechanism which gives contribution into the CMB spectrum) (Podolsky et al., 2005)



$$\omega = 0$$

$$\frac{m^2 \phi^2}{2}$$

$$\omega^2$$

$$\frac{1}{2}(\dot{\phi})^2 - \frac{\lambda}{4}\phi^4$$

$$\ddot{\phi} + \lambda\phi^3 = 0$$

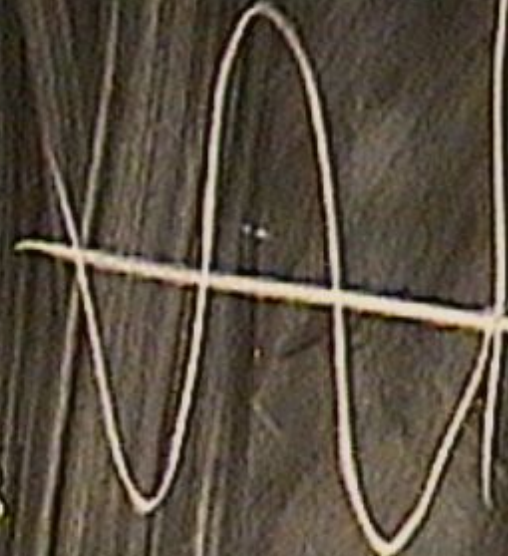
$$\phi_n$$

$$+ \phi_{n-1}$$

$$+ \lambda\phi_n^3 = 0$$

$$\rho \sim \frac{1}{q^3}$$

$$\rho^2 \sim \frac{1}{q^4}$$



$$\frac{1}{2}(\dot{\varphi})^2 - \lambda \varphi^4$$

$$\ddot{\varphi} + \lambda \varphi^3 = 0$$

$$\varphi_{n+1} - 2\varphi_n + \varphi_{n-1} + \lambda \varphi_n^3 = 0$$

$$\frac{m^2 \varphi^2}{2}$$

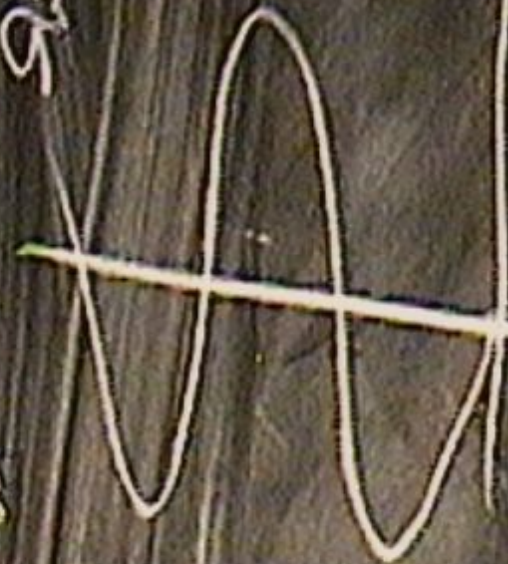
$$\omega = 0$$

$$\omega^2$$

$$\rho \sim \frac{1}{a^3}$$

$$\rho \sim \frac{1}{a^3}$$

$$\rho \sim \frac{1}{a^4}$$

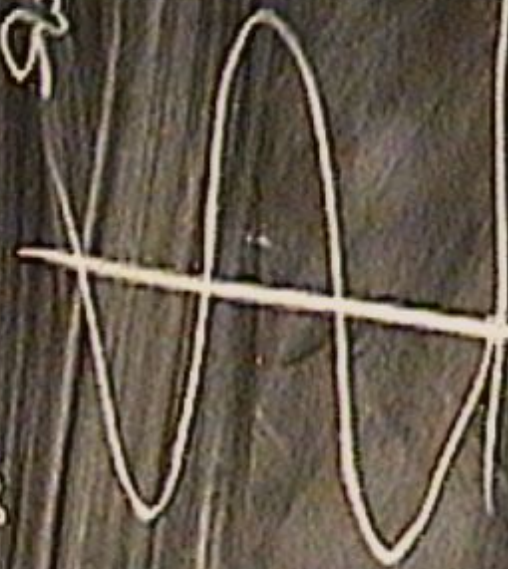


$$m \frac{\omega^2}{2} \quad \omega = 0 \quad \omega \sim \frac{1}{\beta} \quad \omega^3 \quad p \sim \frac{1}{q^3}$$

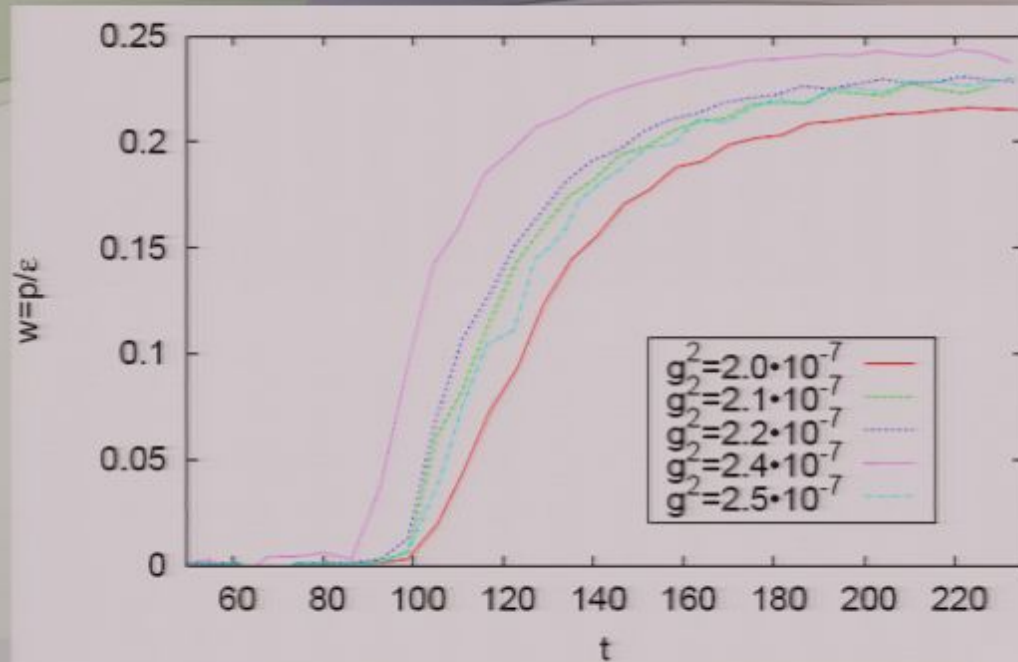
$$\frac{1}{2}(\dot{\varphi})^2 - \lambda \varphi^4$$

$$\ddot{\varphi} + \lambda \varphi^3 = 0$$

$$\varphi_{n+1} - 2\varphi_n + \varphi_{n-1} + \lambda \varphi_n^3 = 0 \quad p \sim \frac{1}{q^3} \quad p \sim \frac{1}{q^4}$$



Equation of state and response of metric



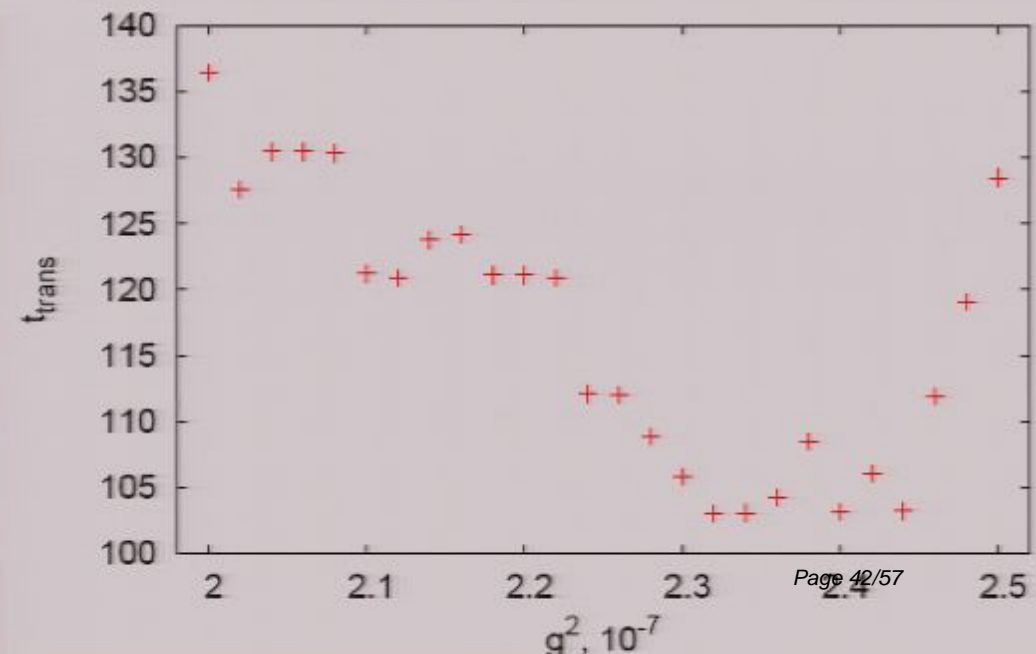
On the fig.: preheating after chaotic inflation driven by the interaction term

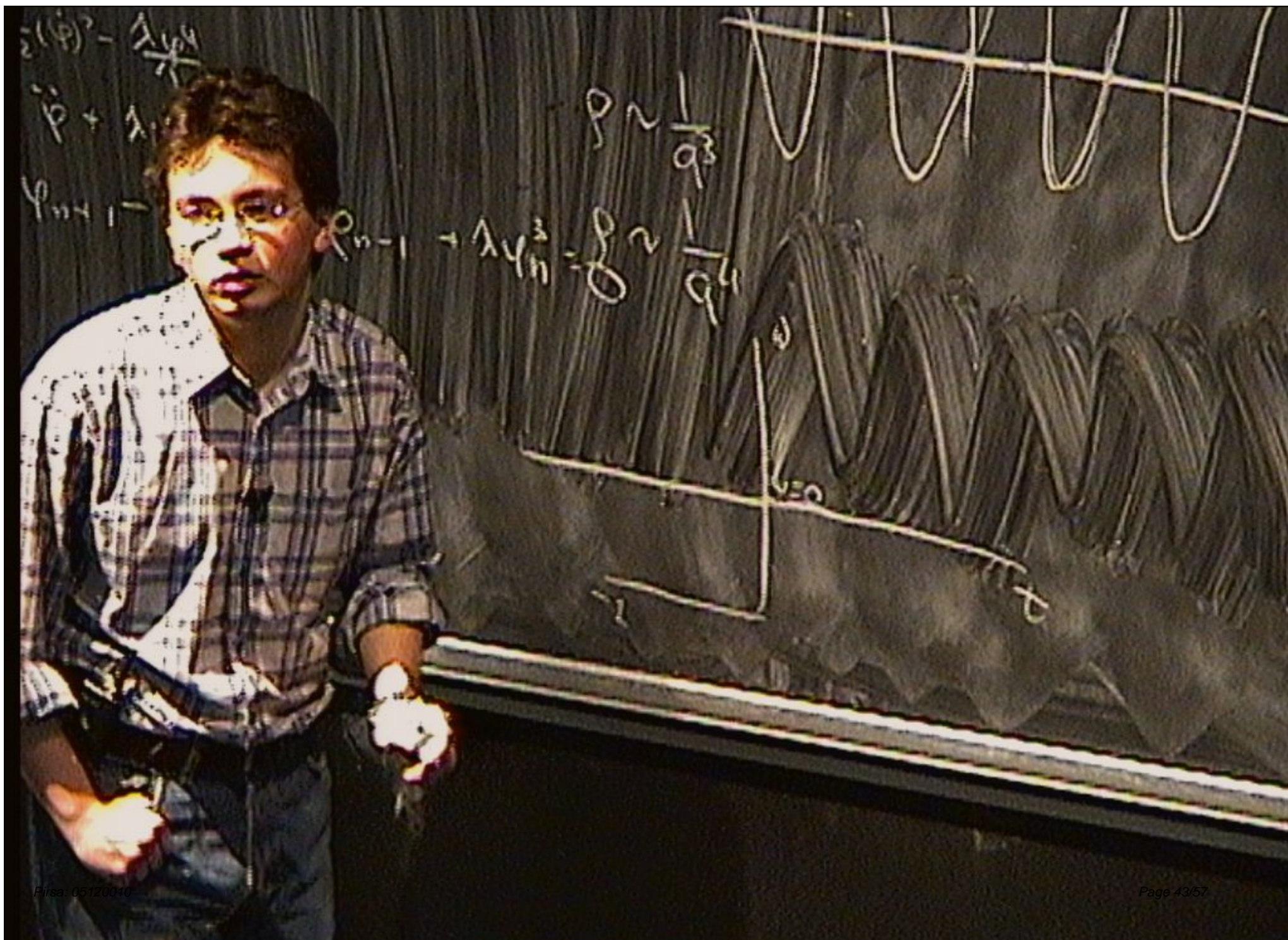
$$\mathcal{L}_{int} = -\frac{g^2}{2}\phi^2\chi^2$$

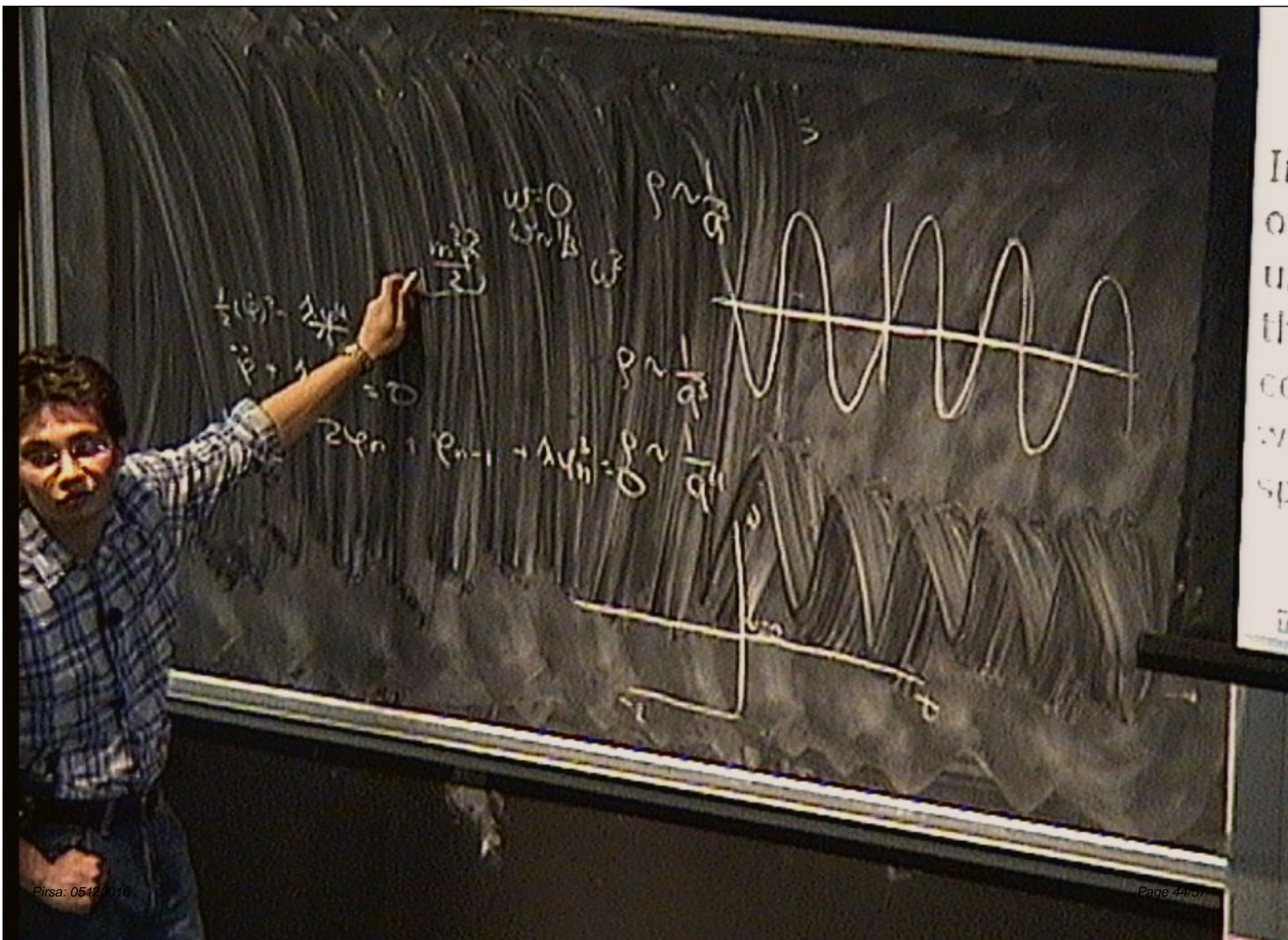
Equation of state defines the regime of the expansion:

$$a(t) \sim t^{\frac{2}{3(1+w)}}$$

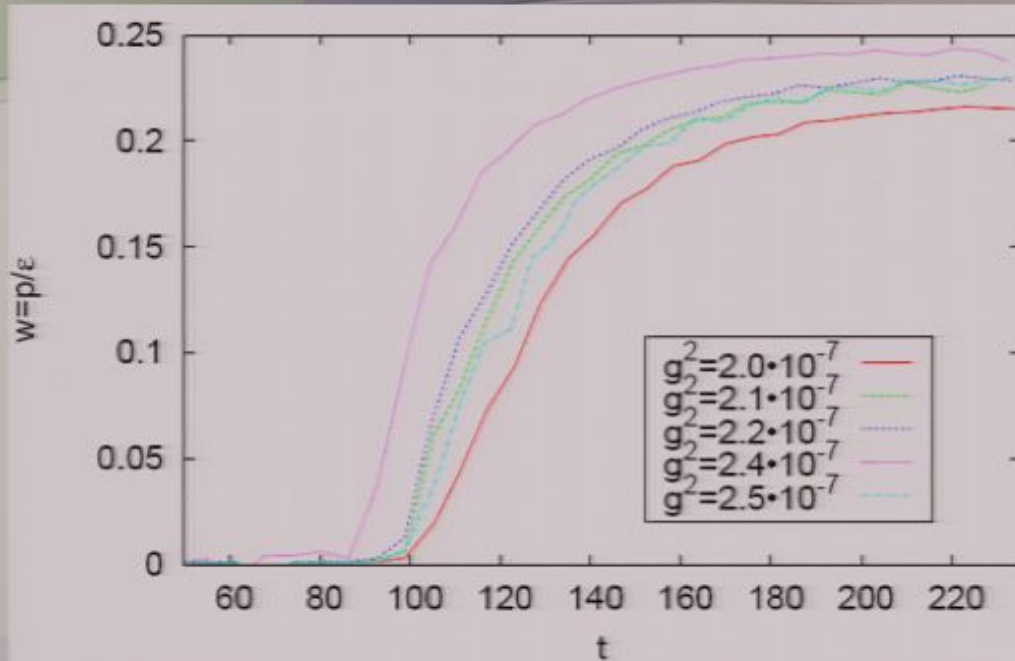
In different Hubble patches the lengths of preheating stages are different under certain conditions. This leads to the generation of large scale cosmological perturbations (mechanism which gives contribution into the CMB spectrum) (Podolsky et al., 2005)







Equation of state and response of metric



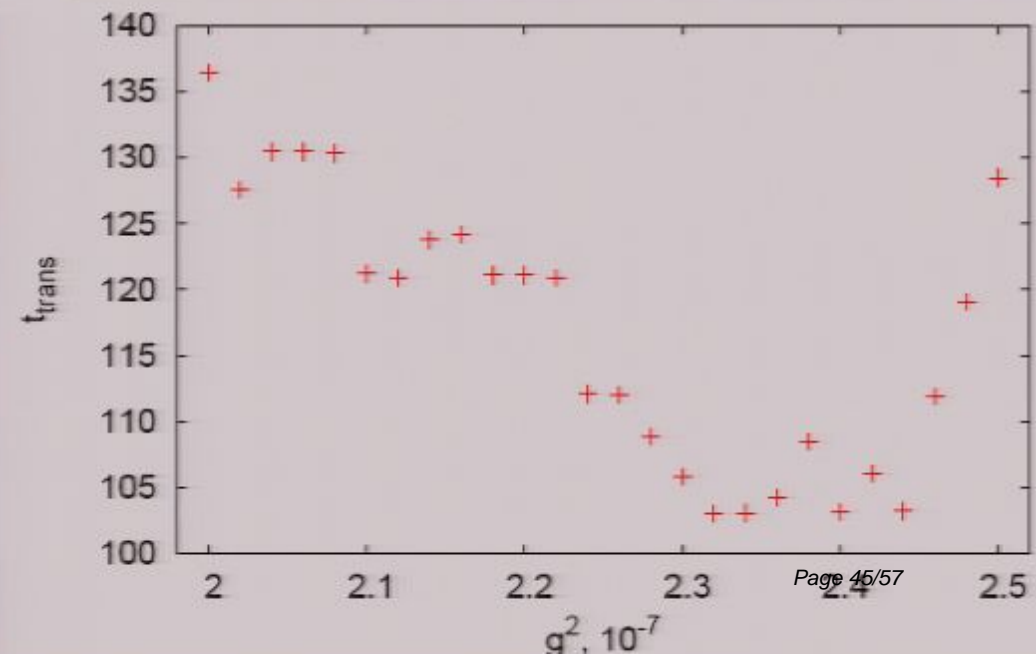
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Number of efoldings and observable wavelength of cosmological perturbations

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N(k)} \frac{a_{\text{end}}}{a_{\text{reh}}} \frac{a_{\text{reh}}}{a_{\text{eq}}} \frac{H_k}{H_{\text{eq}}} \frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0}$$

the relation depends on the whole history of the scale factor

$$N(k) = 62 - \ln \frac{k}{6.96 \times 10^{-5} \text{ Mpc}^{-1}} + \Delta, \text{ where}$$

$$\Delta = -\ln \frac{10^{16} \text{ GeV}}{V_k^{1/4}} + \frac{1}{4} \ln \frac{V_k}{V_{\text{end}}} - \frac{1}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}}$$

here ρ_{reh} is the energy density at the end of reheating,

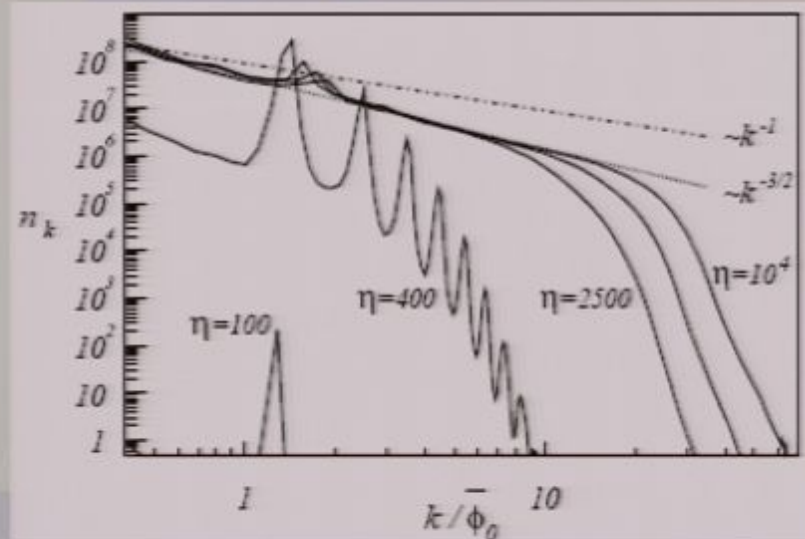
V_k is the value of the inflaton potential at the moment when the mode with the comoving wave number k exists the horizon at inflation, V_{end} is the value of potential at the end of inflation. A better definition for the last term is $-\frac{1}{4} \ln \frac{a_{\text{rd}}}{a_{\text{inf}}}$

and it is quite universal: $a_{\text{rd}}/a_{\text{inf}} \sim 10\text{--}20$ depending on the model of preheating (Podolsky et al., 2005).

Prethermalization

Conformal model (Micha, Tkachev 2004)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4$$



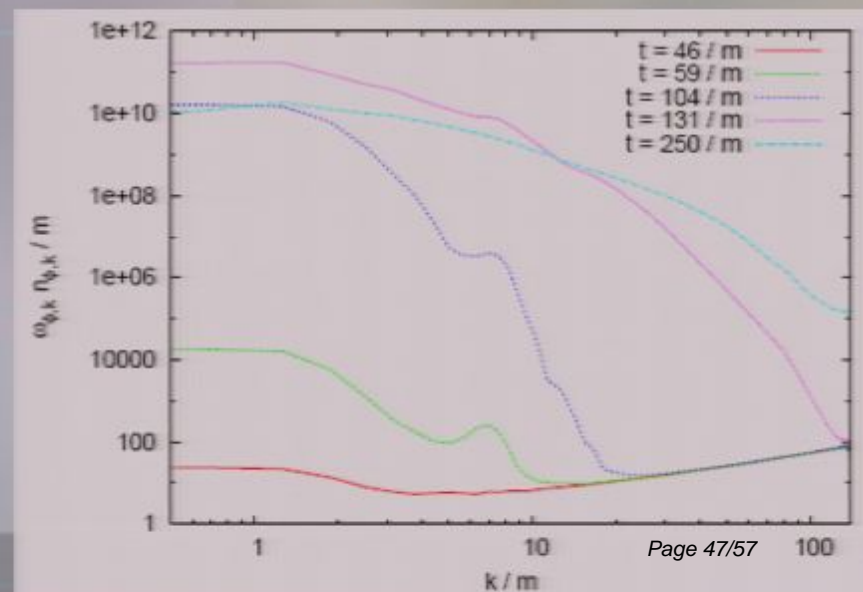
- Self-similar spectra described by the kinetic equation of weak turbulence
- Cascade into UV
- Long thermalization time
- Low reheating temperature

$$T \sim \lambda_\chi^2 M_{\text{Pl}} \sim 100 \text{ eV}$$

Parametric resonance preheating after chaotic inflation (Podolsky et al. 2005)

- Early thermalization of such quantities as effective equation of state
- IR part is close to Rayleigh-Jeans spectrum

$$n_k \approx \frac{T_{\text{eff}}}{\omega_k - \mu}$$



$$\frac{1}{2}(\dot{\psi})^2 = \lambda \psi^4$$

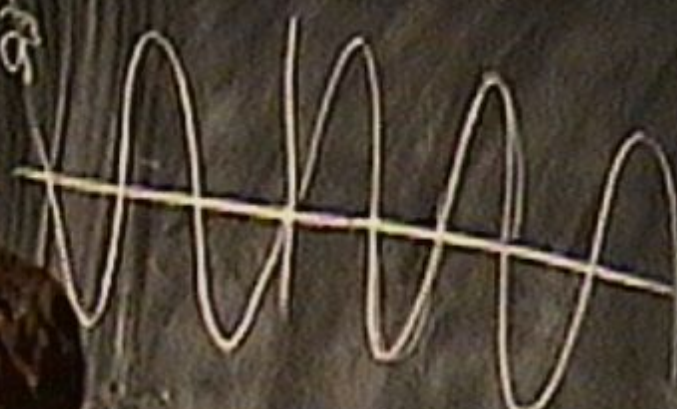
$$\ddot{\psi} + \lambda \psi^3 = 0$$

$$\psi_{n+1} - 2\psi_n + \psi_{n-1} + \lambda \psi_n^3 = 0$$

$$\frac{m^2}{2}$$

$$u=0$$

$$\rho \sim \frac{1}{a}$$



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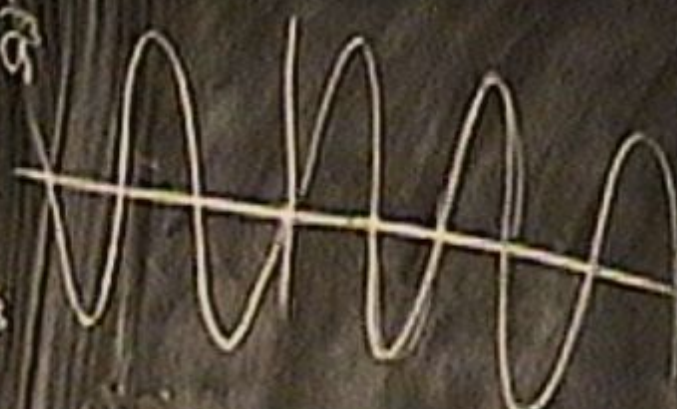


$$\frac{m^2 R^2}{2L}$$

$$\omega = 0 \quad \omega \sim \frac{1}{a}$$

$$\rho \sim \frac{1}{a^3}$$

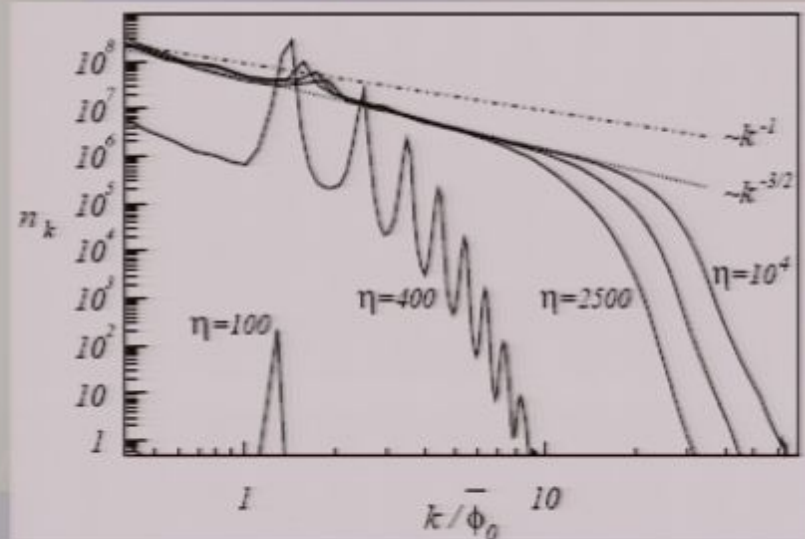
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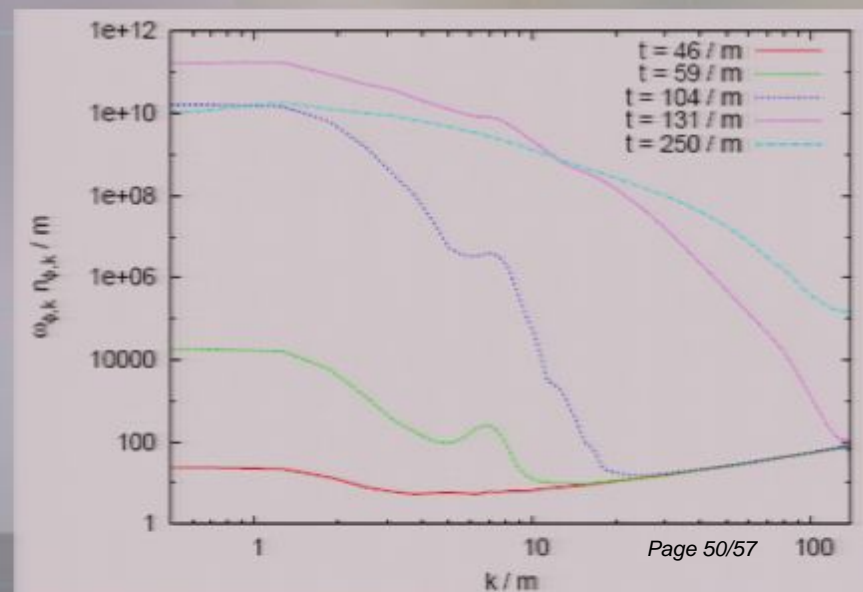
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$$K < 10^{10} \text{ N/m}^2$$

$$\omega = 0$$

$$\omega \sim \frac{1}{L}$$

$$\omega^2$$

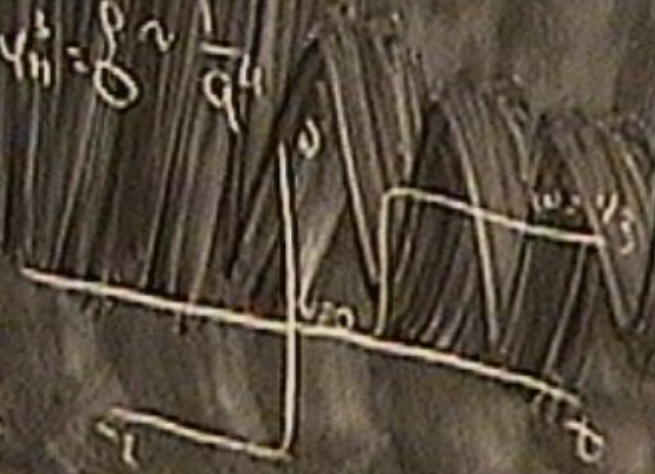
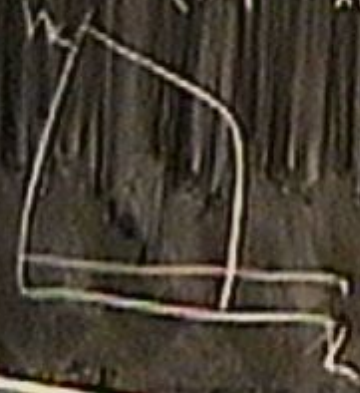
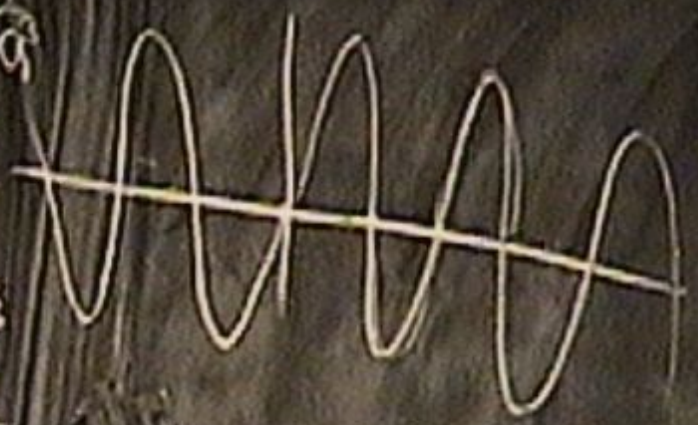
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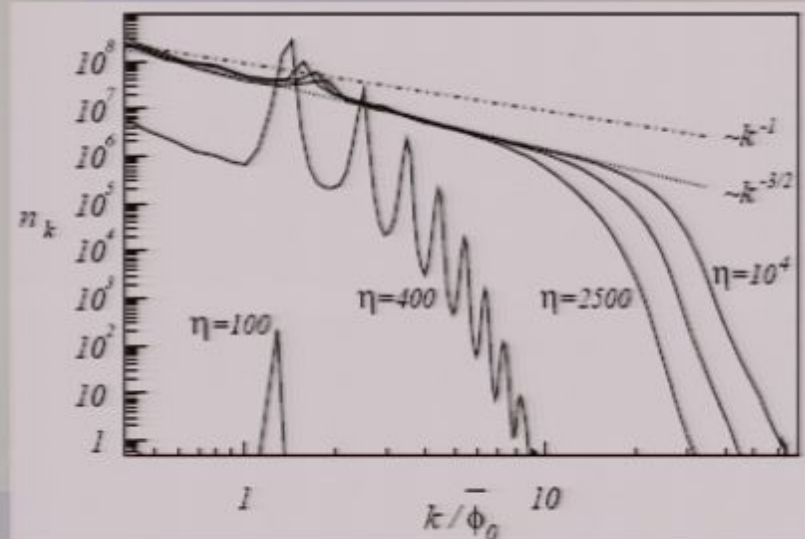
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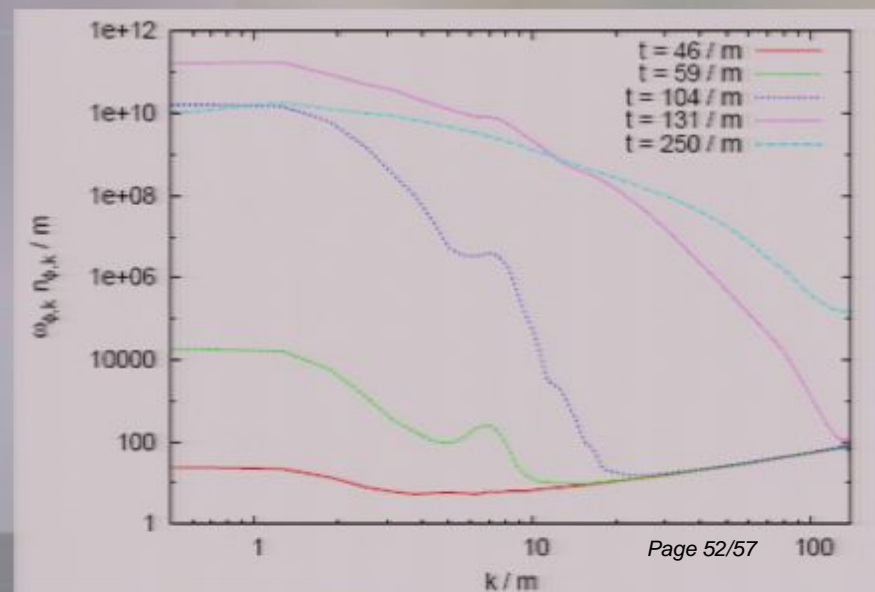
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What's next?

Although we know a lot, after 20 years we are still stuck with the old quasiclassical picture. This language is fine for the earliest stages of preheating, it is not when we want to study rescattering and thermalization beyond Hartree-Fock, as well as backreaction effects.

It turns out that preheating is unique theoretical laboratory to test the power of **non-equilibrium QFT** (coherent fields with very large occupation numbers, strong coupling kinetics). Recently, with the development of powerful non-perturbative methods (Berges et al., 2004) it reached the level when it can be applied to deal with the physics of early postinflationary stages.

Just a note: A lot of problems of QFT in curved spacetime require the use of non-equilibrium QFT.

Example: massless scalar field in the planar patch of dS. **Will the quantum gravity require an essentially non-equilibrium formulation?**

$$\partial_\mu \mathcal{L} = g_{\mu\nu} \bar{\psi}^\nu$$

$$\Delta_\mu = 2$$

$$(\square + m^2)\varphi = 0$$

$$\langle \varphi^2 \rangle \sim \frac{\hbar}{\epsilon^2}$$

$$\Rightarrow \mathcal{L} = \bar{\psi}^\mu \bar{\psi}^\nu g_{\mu\nu}$$

$$R = e^{\psi}$$

$$\Phi = \Phi_0 + \ln u$$

$$\partial_\mu \mathcal{L} = g_{\mu\nu} \bar{\beta}^\nu$$

$$\Rightarrow \mathcal{L} = \bar{\beta}^\mu \bar{\beta}^\nu g_{\mu\nu}$$

$$\Phi = \Phi_0 + \dots$$

$$\Delta_0 = 2$$

$$P = c^4$$

$$\langle \varphi^2 \rangle$$

$$(\square + m^2)\varphi = 0$$

$$\langle \varphi^2 \rangle \sim \frac{H^4}{2\pi^2}$$

$$ds^2 = dt^2 - c^2 d\vec{x}^2$$

$$\langle \varphi^2 \rangle \sim H^2 t$$



$$\partial_\mu \bar{C} = g_{\mu b} \bar{\beta}^b$$

$$\Rightarrow \bar{C} = \bar{\beta}^a \bar{\beta}^b g_{ab}$$

$$\Phi = \Phi_0 + \dots$$

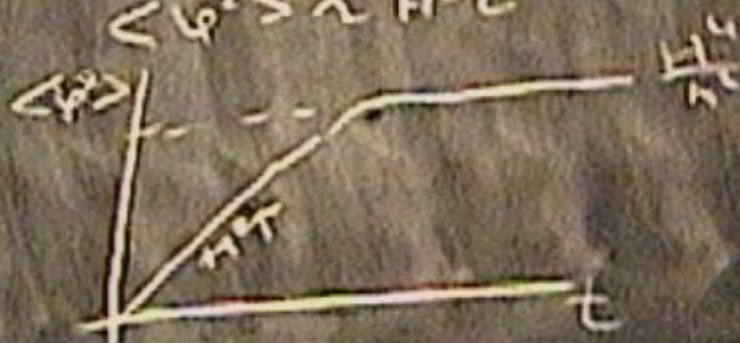
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