

Title: Inflation model building and cosmic microwave background

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Abstract:

**“CMB and inflation model building”  
--particle theorist’s view--  
Kenji Kadota  
Particle Astrophysics Center, Fermilab**

- Can the CMB say anything about inflation?
- An example for inflation model building : Modular Cosmology

K.K. & Ewan Stewart

“Successful Modular Cosmology” (hep-ph/0304127)

“Inflation on Moduli Space and Cosmic Perturbations” (hep-ph/0311240)

Donghui Jeong, K.K., Wan-Il Park & Ewan Stewart

“Modular Cosmology, Thermal Inflation, Baryogenesis and Predictions for Particle Accelerators”  
(hep-ph/0406136)

- Another example (more phenomenology oriented)

K.K. & Jun’ichi Yokoyama

“Right-handed Sneutrino Inflation and Leptogenesis” (hep-ph/0512???)

- Inflationary parameter estimation from the cosmological data

“Parameterizing the power spectrum: Beyond the Truncated Taylor expansion”

Kevork Abazajian, K.K. & Ewan Stewart (astro-ph/0507224)

- Summary and Conclusion

# CMB

China **M**otor **B**us (Hong Kong)

Center for **M**athematical **B**iology

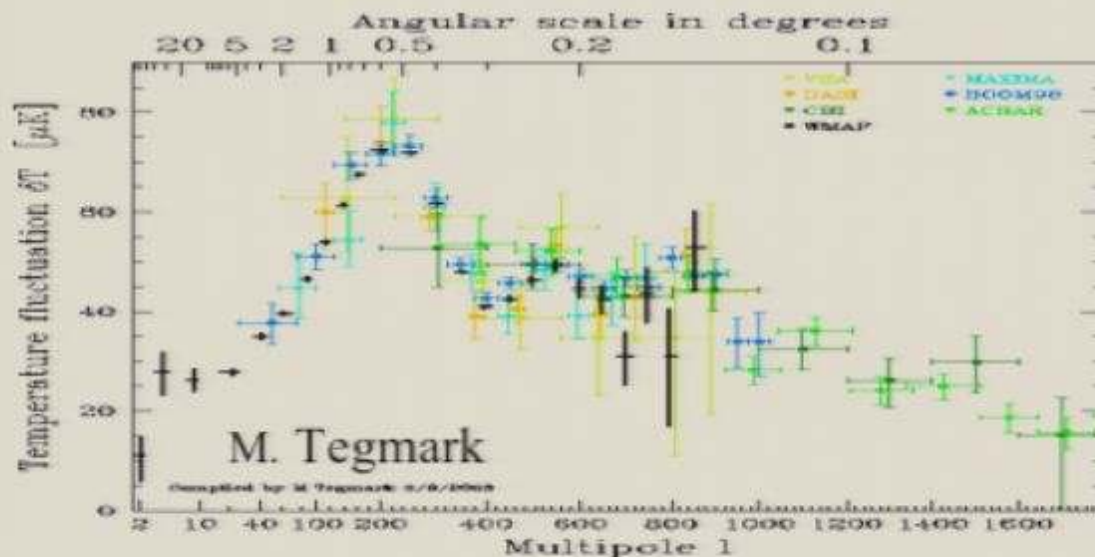
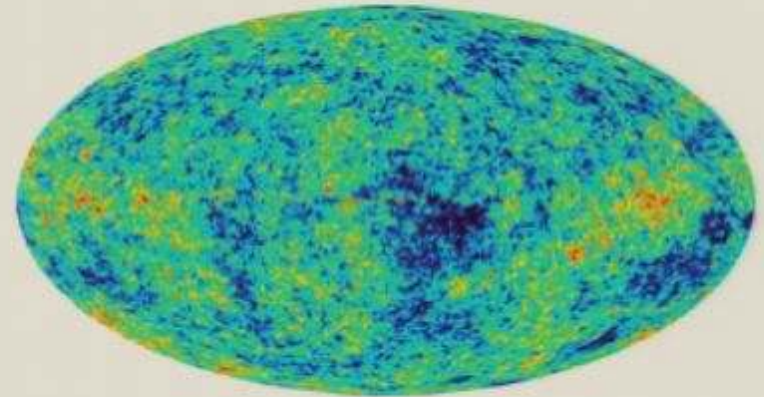
Chase **M**anhattan **B**ank

Central **M**assachusetts **B**andits

Please don't get confused ...

# CMB

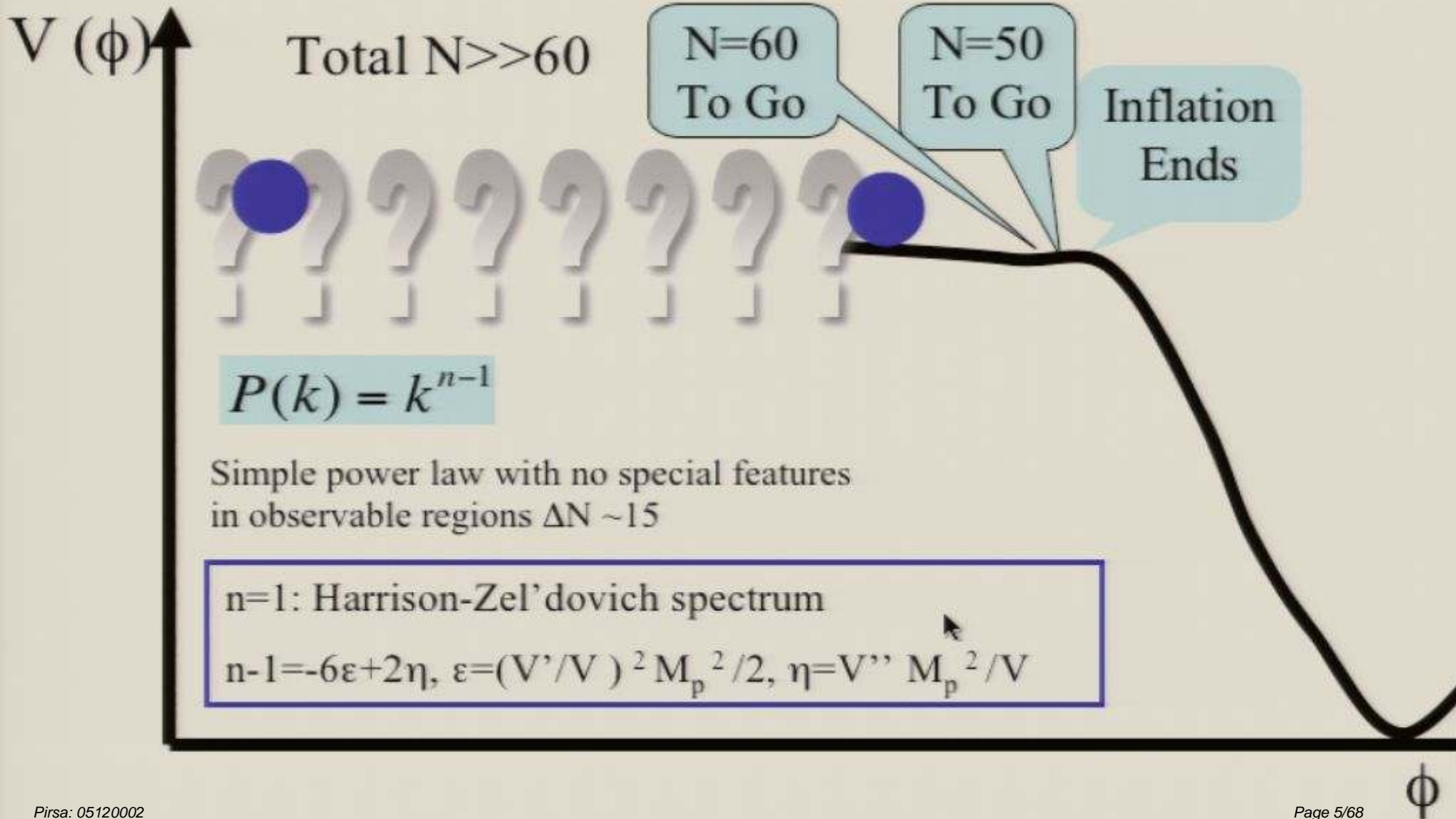
China Motor Bus (Hong Kong)  
Center for Mathematical Biology  
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Please don't get confused ...



- Dominant component of current radiation energy
- 380,000 yrs after Big Bang

( $z \sim 1100$ ,  $T \sim 3000K$ ,  $E \sim 0.1$  eV)

# Can we guess the nature of inflation from the CMB ?



# Can CMB tell anything about type of inflation?



Large field

$$-\epsilon < \eta \leq \epsilon$$

$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$

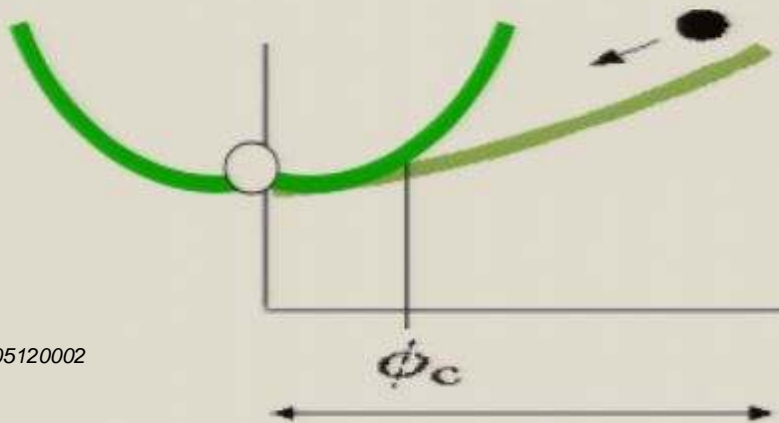
$$V(\phi) = \Lambda^4 \exp(\phi/\mu)$$



Small field

$$\eta < -\epsilon$$

$$V(\phi) = \Lambda^4 [1 - (\phi/\mu)^p]$$



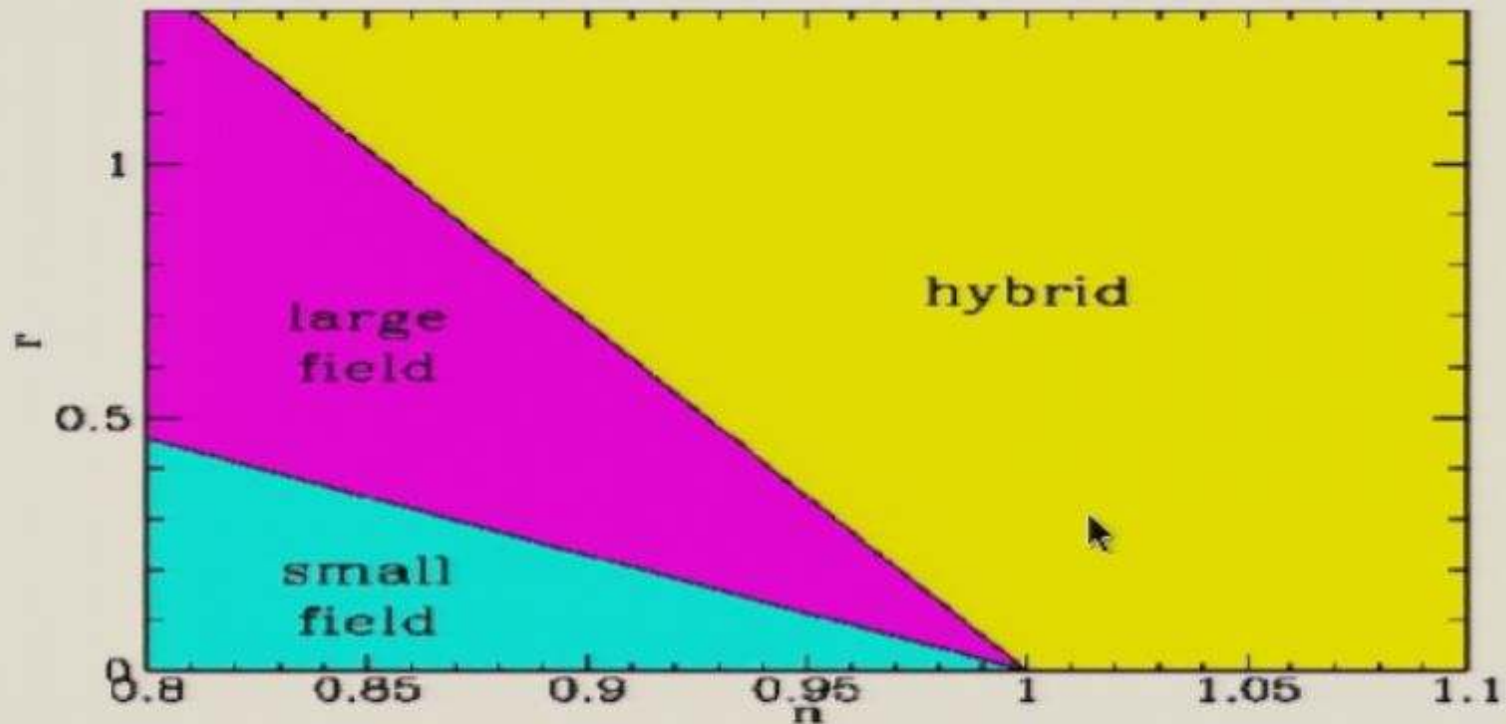
Hybrid

$$0 < \epsilon < \eta$$

$$V(\phi) = \Lambda^4 [1 + (\phi/\mu)^p]$$

# Can CMB tell anything about inflation?

Regions in the  $(r, n)$  plane



# The bottom line

Thousands of inflation models can lead to an identical CMB spectrum.

Constraining inflation models from CMB data alone is

**HARD**

# Inflation Model Building through an example

“Which is the best inflation model?” D. Lyth [hep-th/0311040]

Table 1: Suggested ranking of inflation models

Model	Potential	Problems	$n(k)$
<b>NEW INFLATION</b>			
2-field mod.	various	—	1.00?
bulk mod.	$V_0 - m^2 \phi^2$	(v)	$\sim 0.9$
p.n.g.b.	$V_0 - M\phi^3$	(iv)	0.92
$\sim$ m.s.s.m.	$V_0 - m^2 \phi^2 - \lambda\phi^4$	(iii)	1.00
original	$V_0 + \lambda\phi^4 \log(\phi/Q)$	(ii)	0.94
<b>X</b> modular <b>X</b>	$V_0 - m^2 \phi^2$	(i)	$\sim 0.9$
<b>HYBRID INFLATION</b>			
running	$V_0 - m^2 \phi^2 \log \phi$	—	$n' > 0$
<i>D</i> to <i>F</i>	various	—	?
p.n.g.b.	various	—	?
<i>F</i> -term	$V_0(1 + g^2 \log \phi)$	—	0.99
coll. brane	various	(vi)	$\sim 0.97$
‘supernatural’	$V_0 + m^2 \phi^2$	(v)	$\sim 1.1$
quartic	$V_0 + \lambda\phi^4$	(iv)	1.00
dyn. s. b.	$V_0 + c\phi^{-p}$	(iv)	1.00
<i>D</i> -term	$V_0(1 + g^2 \log \phi)$	(iii)	0.99
$\sim$ m.s.s.m.	$V_0 \pm m^2 \phi^2$	(ii)	1.00
<b>CHAOTIC INFLATION</b>			
quadratic	$m^2 \phi^2$	see note	0.96
<b>X?</b> quartic <b>X?</b>	$\lambda\phi^4$	(i)?(ii)	0.94
<b>X</b> EXTENDED INFLATION <b>X</b>		killed by COBE (1991)	

Need particle theory to judge them!

## References for Table 1

Taken from D. Lyth

NEW

2-field mod.

Kodata & Stewart 2003

Bulk mod.

Banks 1999

p.n.g.b.

Adams, Ross & Sarkar 1997

$\sim$ m.s.s.m.

Dine & Riotto 1997

original

Linde 1982, Albrecht & Steinhardt 1982

modular

Binetruy & Gaillard 1986

HYBRID

running

Stewart 1997

$D$  to  $F$

Copeland, Liddle, Lyth, Stewart & Wands 1998

Gaillard, Lyth & Murayama 1998

$F$  term

CLLSW 1994; Dvali, Schaeffer & Shafi 1998

collid. brane

Dvali & Tye 1998

supernatural

Randall, Soljatic & Guth 1996

p.n.g.b.

Cohn & Stewart 2000

quartic

Linde 1991

dynam. S. B.

Kinney & Riotto 1998

$D$ -term

Stewart 1995; Binetruy & Dvali 1996;

Halyo 1996

$\sim$ m.s.s.m.

Bastero-Gil & King 1997

CHAOTIC

quadratic

Linde 1983;

Arkani-Hamed, Cheng, Creminelli  
& Randall 2003

quartic

Linde 1983

EXTENDED

La & Steinhardt 1989



# FAQ for inflation model building

(Particle theory is important)

- What is the inflaton field?
  - What kind of fields were there in early Universe?
  
- What is the form of the inflaton potential?

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Hidden Sector Supersymmetry Breaking

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Hidden Sector Supersymmetry Breaking

(CMB is important)

# Particle Theory Setup: Moduli (Working Definition)

Flat at tree level

The form of potential :

( $f$  is function with  $O(1)$  coefficients)

gravitational-strength decay

$$V(\phi) = M_s^4 f\left(\frac{\phi}{M_P}\right)$$

## Properties:

$$V_0 \sim M_s^4, \quad m_\phi \sim M_s^2 / M_P, \quad V'' / V \sim O(1)$$

THE energy scale of potential

$$V_0^{1/4} \sim M_s \sim 10^{10-11} \text{ GeV}$$

THE mass of field

$$m_\phi \sim M_s^2 / M_P \sim 10^{2-3} \text{ GeV}$$

Other choices of parameter values would be unnatural fine-tuning

(1)

$$V = e^k ( \dots ) + D\text{-terms}$$
$$V = k' e^k ( \dots ) + \text{others}$$



(11)

$$V = e^k ( \dots ) + D\text{-terms}$$

$$V' = k' e^k ( \dots ) + \text{others}$$

$$V'' = k'' V + \dots$$

$$\frac{V''}{V} = k'' \sim \alpha^{(1)} + \dots$$



$$V = e^k ( \dots ) + D\text{-terms}$$

$$V = k' e^k ( \dots ) + \text{others}$$

$$V'' = k'' V + \dots$$

$$V' = k' \sim \alpha(1) + \dots$$

$$\eta = \frac{V''}{V}$$

# Now Ask:

- Can self-consistent cosmology scenario be realized from this natural setup ?

(without any modifications or unnatural fine-tuning)

- What is the prediction for this particular scenario?

“Successful Modular Cosmology” (K.K.&E. Stewart, 04)

“Inflation on Moduli Space and Cosmic Perturbations” (K.K.&E. Stewart, 04)

Also related work by, for instance,

S. Thomas “Moduli Inflation from Dynamical Supersymmetry Breaking”

Banks, Berkooz, Shenker, Moore and Steinhardt “Modular Cosmology”

“ .... Several speculative explanations of the discrepancy between SUSY breaking scale ( $10^{10}$  GeV) and apparent inflation scale ( $10^{16}$  GeV). ... ”

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## Difficulties in Modular Cosmology & Its Possible Resolutions

- Cosmic perturbations: Fine-tuning necessary??
- Cosmological moduli problem  $\Rightarrow$  Late entropy production  
Thermal inflation
- Baryon Asymmetry  $\Rightarrow$  Leptogenesis at low energy scale ( $H \sim 10^{-25}$ )

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# Cosmic Perturbations

# MODEL



Energy Scale of Potential

$$V_0^{1/4} \sim M_s \sim 10^{10} \text{ GeV}$$

Mass of Field

$$m_\phi \sim M_s^2 / M_p \sim \text{TeV}$$

## Cosmic Perturbations: Match Observation ?

Let's try calculating Cosmic Perturbations now.

Not slow-roll  $m^2 \sim V_0 \sim (TeV)^2$

(Radial Fluctuation has highly suppressed spectrum)

$$\delta\phi, \delta\theta$$

Radial fluctuations (too red spectrum, which is good!)

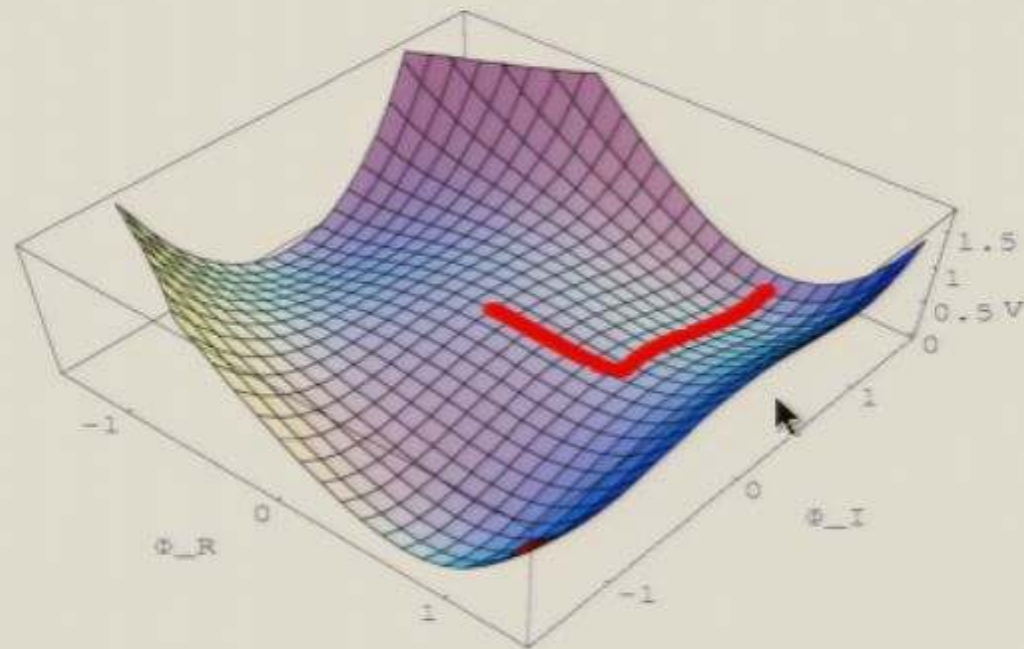
# Match Observations?

Angular Fluctuations

$$\delta\theta \sim \frac{H}{\phi}$$

Still non-flat!

$$V = V_0 - m_\phi^2 |\Phi|^2 + \frac{1}{3} m_\phi^2 (\Phi^3 + h.c.) + m_\phi^2 |\Phi|^4$$



# Points of Enhanced Symmetry

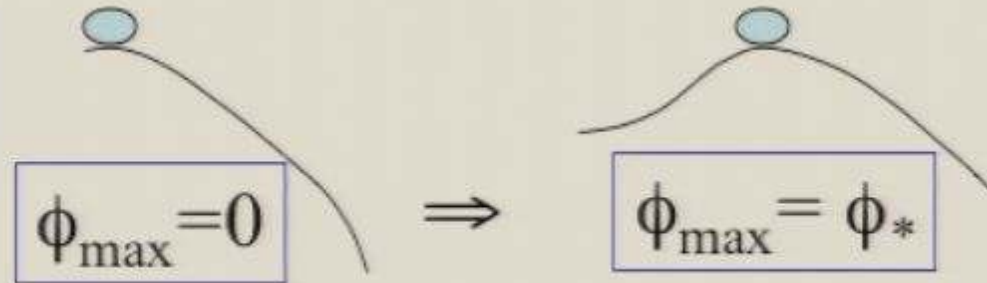
The points in moduli space where symmetry is enhanced  
(generic and robust properties of moduli space in string theory)

## -Properties-

- Fixed points of symmetry group  
(local maximum/minimum or saddle points)
- New interactions:  $\chi^2 |\phi - \phi_0|^2$   
(corresponds to Higgs in low-energy effective theory)  
New interactions show up and consequent  
renormalization of potential

# Flattening Pot'l by Loop Corrections At Enhanced Symmetric Point

$$V = V_0 \left[ 1 - \frac{1}{2} f(\beta \ln \phi) \phi^2 + \dots \right]$$



$$\Delta\phi \Rightarrow \phi_* + \Delta\phi \sim \phi_*$$

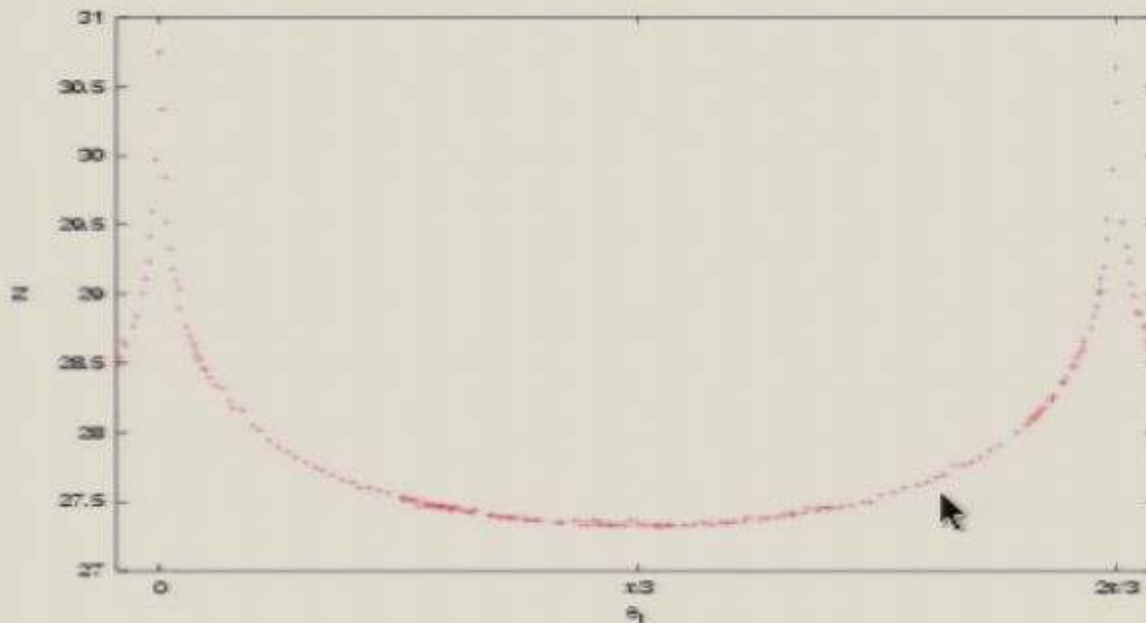
$$H/\phi \Rightarrow H/\phi_*$$

$H/\phi_*$  is now flat

$$R = \frac{\partial N}{\partial \phi} \delta \phi + \frac{\partial N}{\partial \theta} \delta \theta \quad (\text{Sasaki \& Stewart '96})$$

Angular Component Fluctuations

$$\frac{\partial N}{\partial \theta} \delta \theta \sim \frac{\partial N}{\partial \theta} \frac{H}{\phi}$$



$$V = e^k ($$

$$V = k' e^k ($$

$$V'' = k'' V +$$

$$V'' = k'' \sim \alpha$$

$$dN = \int R dt$$

$$= R_1 = R_2$$



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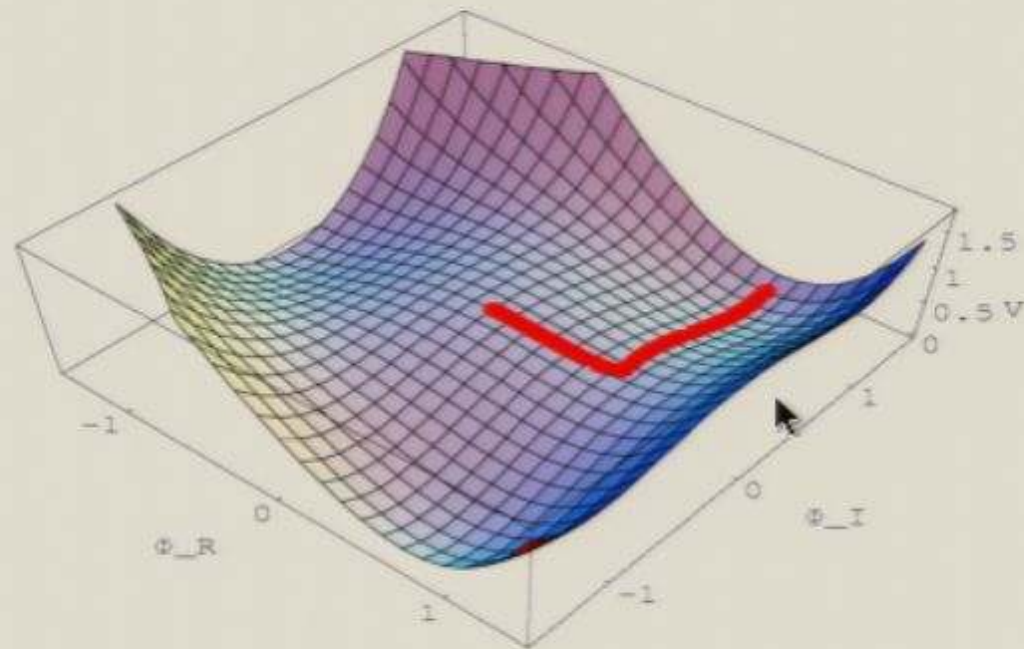
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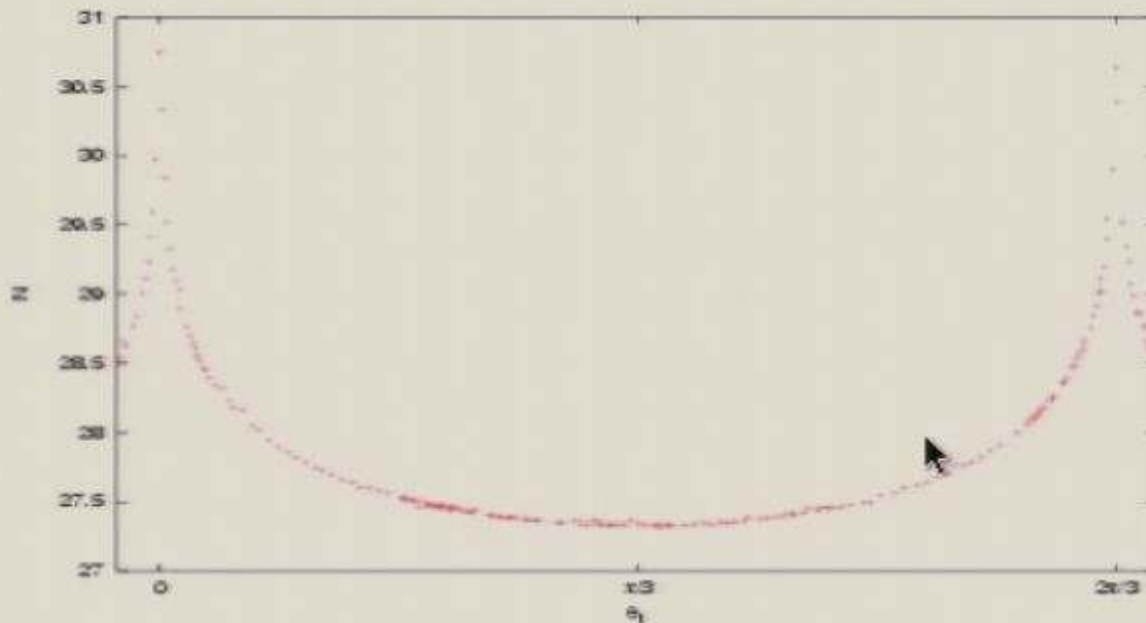
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$$\frac{\partial N}{\partial \theta} \delta \theta \sim \frac{\partial N}{\partial \theta} \frac{H}{\phi}$$



# Predicted Cosmic Perturbations

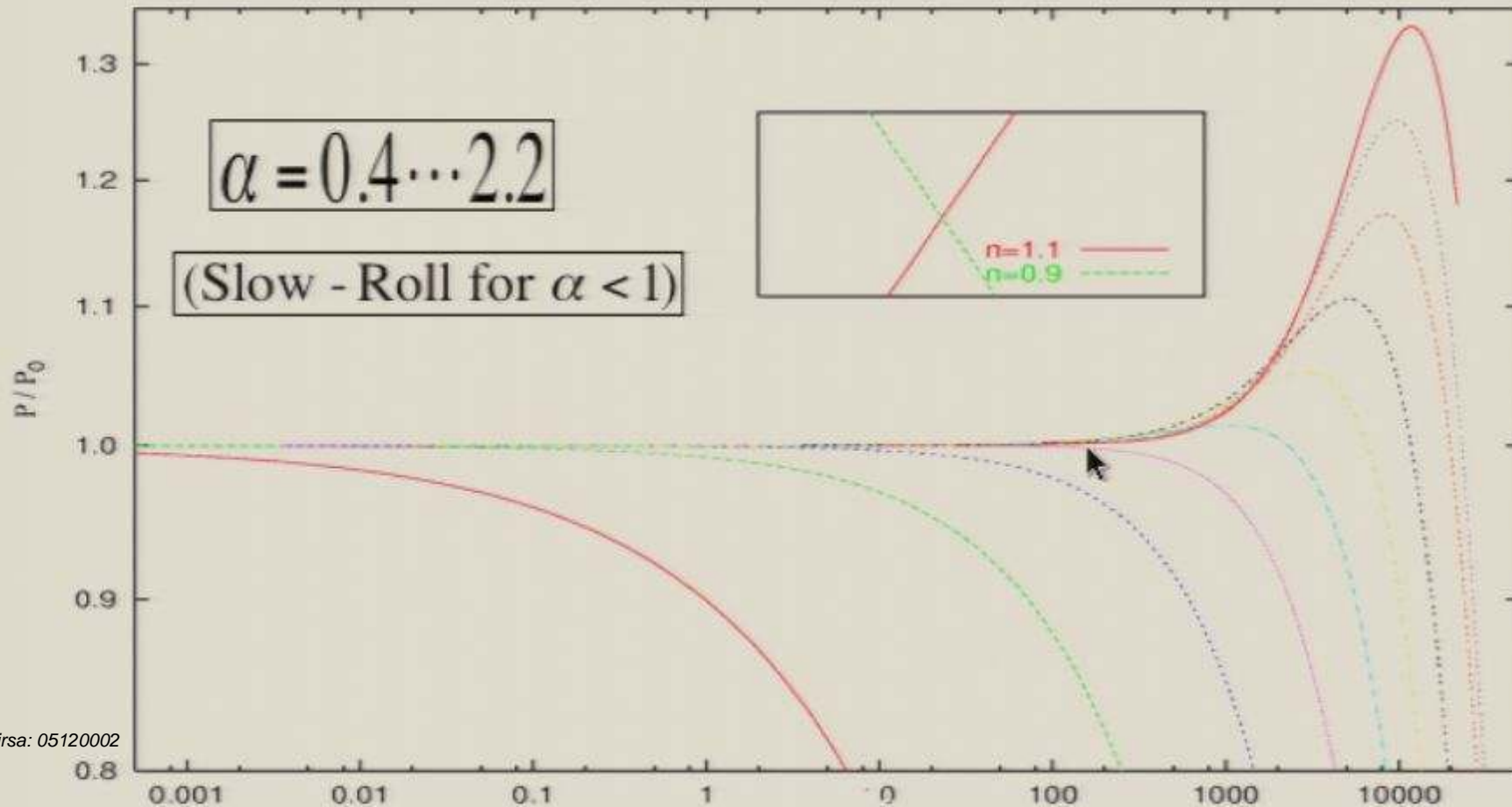
$$n - 1 \equiv \frac{d \ln P}{d \ln k}$$

$$n - 1 = -Ak^{2\alpha}$$

$$n' \equiv \frac{dn}{d \ln k} = -2\alpha Ak^{2\alpha}$$

$$|n - 1| \sim |n'| \sim |n''| \text{ for } \alpha \sim 1$$

Watch Out ! Running of running  
(Can't use slow-roll approximations .  
Low-energy scale inflation preferable



# Difficulties in Modular Cosmology & Its Possible Resolutions

- Cosmic perturbations
- Cosmological moduli problem  $\Rightarrow$  Late entropy production  
Thermal inflation
- Baryon Asymmetry  
 $\Rightarrow$  Leptogenesis at low energy scale ( $H \sim 10^{-25}$ )

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# Thermal Inflation

$$V = V_0 + (g^2 T^2 - m^2) |\sigma|^2 + (A\lambda\sigma^n + h.c.) + |\lambda|^2 |\sigma|^{2n-2}$$

- Flat direction lifted by

$$W_{non-ren} = \frac{\lambda}{nM^{n-3}} \Sigma^n$$

- Inflation for

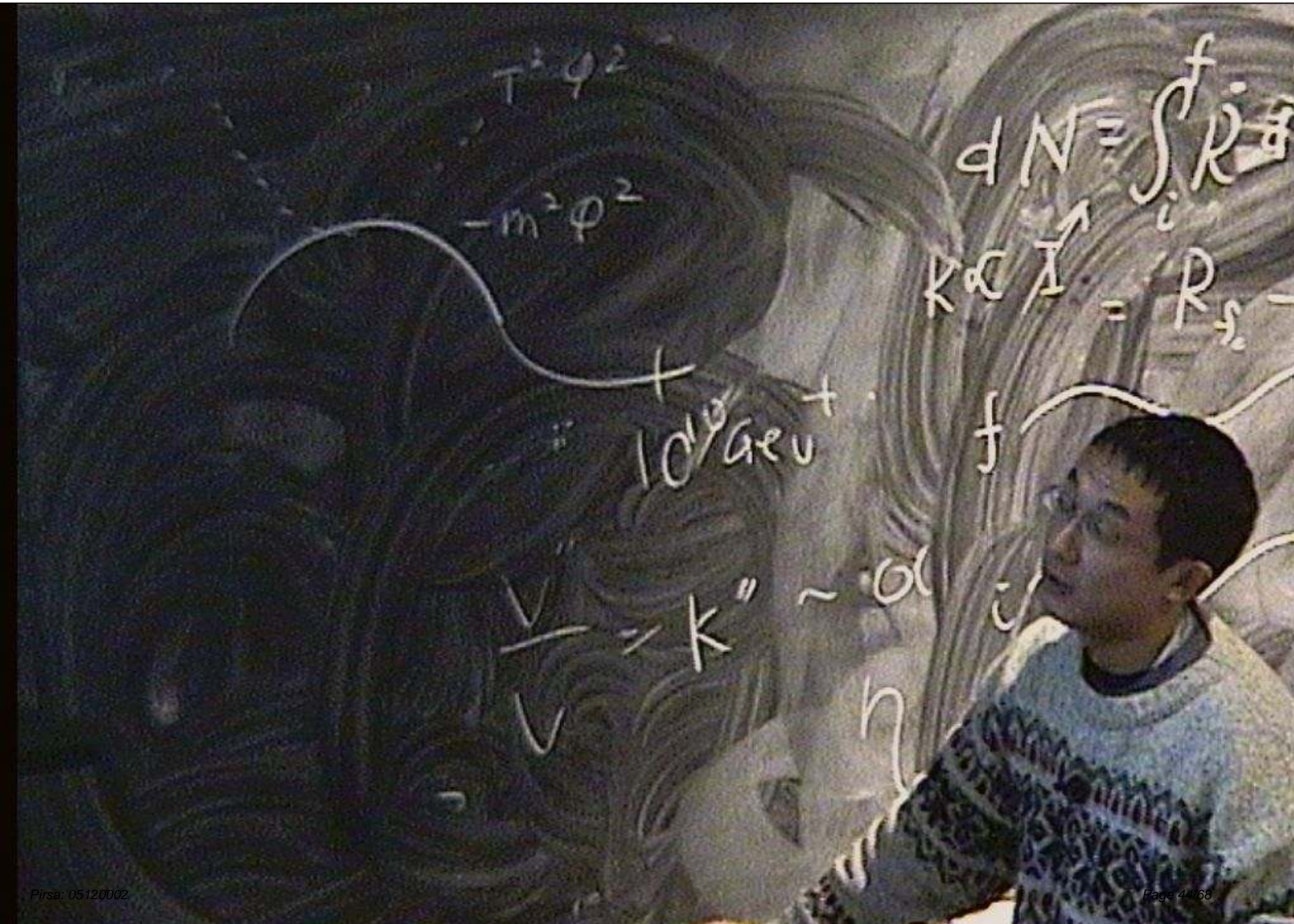
$$m \leq T \leq V_0^{1/4}$$

$$N \sim 5 - 10, T_R \sim GeV$$

(high enough for BBN)

- Finite temperature effects in presence of SUSY flat direction solves Polonyi/Moduli problem.





45

$$T^2 \phi^2 - m^2 \phi^2$$

$$dN = \int_i R_i$$

$$R \propto \vec{A} = R_s$$

$$10^{14} \text{ GeV}$$

$$\frac{v}{\Lambda} = k'' \sim \dots$$

$h$

45-20

$$T^2 \phi^2$$
$$-m^2 \phi^2$$



$$+ \frac{10^{14}}{g e u}$$

$$\frac{v}{v} = k''$$

$$dN = \int R_i \vec{a}$$
$$\vec{A} = R_s$$

$\alpha$

$\eta$

$$T^2 \phi^2$$

$$-m^2 \phi^2$$

$$10^{16} \text{ GeV}$$

$$\frac{v}{\Lambda} = k'' \sim \alpha$$

$h$

$$45 - 20 = 25$$

$$dN = \int R_i^f$$

$$R \propto \vec{\lambda} = R_s$$

$f$

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# “Affleck-Dine” Leptogenesis in thermal inflation scenario

Cosmological moduli problem

Thermal inflation ,  $H \sim 10^{-25}$

⇒ Baryon asymmetry production at low energy scale

“Standard” baryogenesis mechanism doesn't work

E. Stewart, M. Kawasaki and T. Yanagida (hep-ph/9603324)

D. Jeong, K.K., W. Park and E. Stewart (hep-ph/0406136 )

45

$$T^2 \phi^2$$

$$-m^2 \phi^2$$

$$(m^2 - H^2)$$

$$10^{16} \text{ GeV}$$

$$\frac{v}{\Lambda} = k'' \sim \frac{v}{h}$$

$$T^2 \phi^2$$

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$$-m^2 \phi^2$$

$$(m^2 - H^2)$$

$$+ 10^{16} \text{ GeV}^4$$

$$\frac{v}{\Lambda} = k'' \sim \alpha \frac{v}{h}$$

$$T^2 \phi^2$$

45 - 0

$$-m^2 \phi^2$$

$$(m^2 - \cancel{A^2})$$

$$+ 10^{16} \text{ GeV}^4$$

$$\frac{v}{\Lambda} = k'' \sim \alpha$$

$h$

# “Affleck-Dine” Leptogenesis in thermal inflation scenario

$$W = \text{Yukawa} + \phi^2 H_u H_d + \phi^4 + L H_u L H_u$$

1. Relevant fields trapped at the origin (thermally)
2.  $L H_u$  starts rolling first away from the origin [assume:  $m_L^2 + m_{H_u}^2 < 0$ ]

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  3. Flaton  $\phi$  starts rolling away
- F-term  $|\phi^2 H_u|^2$  gives  $L H_u$  positive mass squared

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Now by cross term  $\phi^{*2} H_d^* L^2 H_u$

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➤ Phase of  $L H_u$  : Initially by A-term  $L H_u L H_u$

Now by cross term  $\phi^{*2} H_d^* L^2 H_u$

(For thermal inflation,  $H \sim 10^{-25} \Rightarrow$  Standard AD mechanism doesn't work)

➤ Non-zero  $H_d$  (due to A-term  $\phi^2 H_u H_d$  and cross terms)

Q and L are stabilized via  $|QH_d|^2$  and  $|LH_u|^2$  at MSSM minimum

$m_L^2 + m_{H_u}^2 < 0 \Rightarrow$  Deep non-MSSM minimum.

Charge and Colour Breaking bounds violated.

c.f. A. Kusenko, P. Langacker and G. Segre (hep-ph/9602414)

“Phase transition and Vacuum Tunneling into Charge and color

breaking minima in the MSSM”

# Summary So Far

- ❑ Thousands of inflation models can lead to identical observations
  - ❑ An example for inflation model building :
    - particle theory should be able to tell us
      - what THE inflaton is
      - what THE values of the parameters such as inflation energy scale are
      - what THE inflaton potential should look like, etc
- Based on those setups, particle theory can make “testable” predictions.
- ❑ Cosmological observations can falsify/justify particle theory’s prediction

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# Parameterizing the Power Spectrum: Beyond the Truncated Taylor Expansion

Kevork Abazajian, K.K. and Ewan D. Stewart  
JCAP08(2005)008, [astro-ph/0507224])

# Traditional Approach: Truncated Taylor Expansion of the Power Spectrum

$$\ln P(k) = \ln P_* + (n_* - 1) \ln\left(\frac{k}{k_*}\right) + n'_* \ln^2\left(\frac{k}{k_*}\right) \quad \left[ n - 1 \equiv \frac{d \ln P}{d \ln k}, n' \equiv \frac{dn}{d \ln k} \right]$$

$$\left| (n_* - 1) \ln\left(\frac{k}{k_*}\right) \right| \gg \left| n'_* \ln^2\left(\frac{k}{k_*}\right) \right|$$

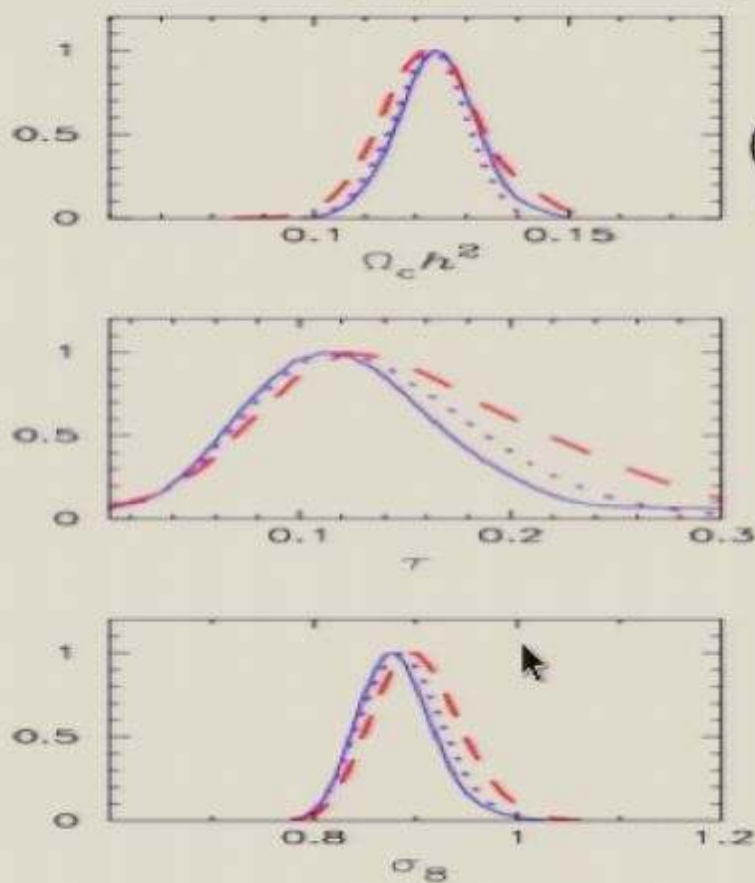
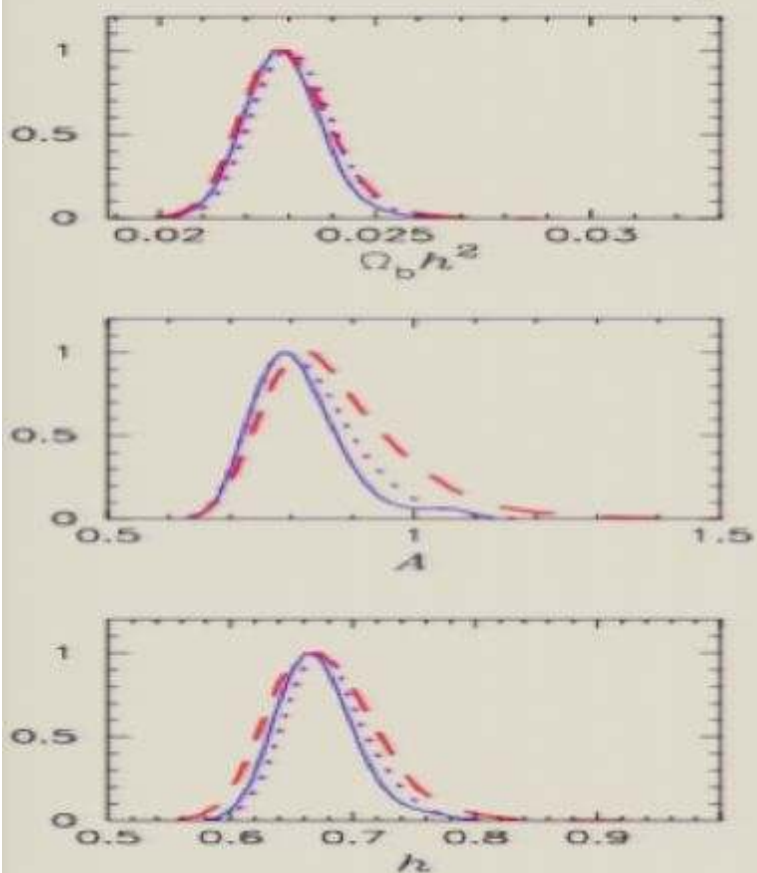
⇐ Not a trivial assumption, in particular for  $|\ln k/k_*| \gtrsim 1$ .

In fact, the current observations are consistent with

$$\left| (n_* - 1) \ln\left(\frac{k}{k_*}\right) \right| \sim \left| n'_* \ln^2\left(\frac{k}{k_*}\right) \right|$$

# Likelihood analysis for the cosmological parameters

- Markov-Chain Monte-Carlo(MCMC) using the data from WMAP, ACBAR, CBI, VSA, SDSS and Lyman- $\alpha$  (covering up to  $k \sim 5h/\text{Mpc}$ )



(Marginalized 1-D PDFs  
Solid: Ours,  
Dashed: Taylor,  
Dotted: Constant n )

Not a big difference within the currently available data, but would change once we get more data at higher  $k$  with better precision

# Improved parameterization and its inflationary motivation

$$\ln P(k) = \ln P_* - Ak^v, n - 1 = -vAk^v$$

General slow-roll formula:  $|n-1| \gtrsim |n'| \gtrsim |n''| \dots$   
 (Stewart '02, Lee et al '05)

$$\ln P = C - B\xi^{-v}$$

$$\left( \xi \equiv -\int \frac{dt}{a} \right)$$

(concrete particle theory motivated example, e.g.

General slow-roll formula gives

Kadota&Stewart '03 )

$$\ln P(k) = C - Ak^v \text{ for } v < 2$$

$$\ln P(k) = C - Ak^2 \text{ for } v \geq 2$$

$$\frac{n'_*}{n_* - 1} = \begin{cases} v & \text{for } v < 2 \\ 2 & \text{for } v \geq 2 \end{cases}$$

Pirsa: 05120002

( $n'_*/(n_*-1) \leq 2$  would be a consistency check for our analytical justification of the form of our parameterization)

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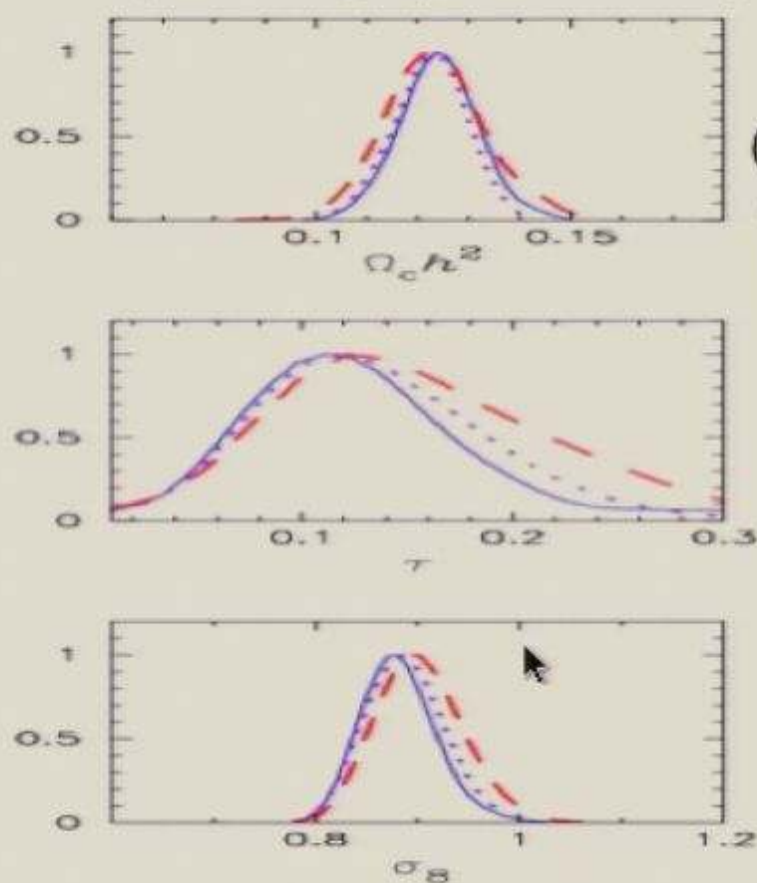
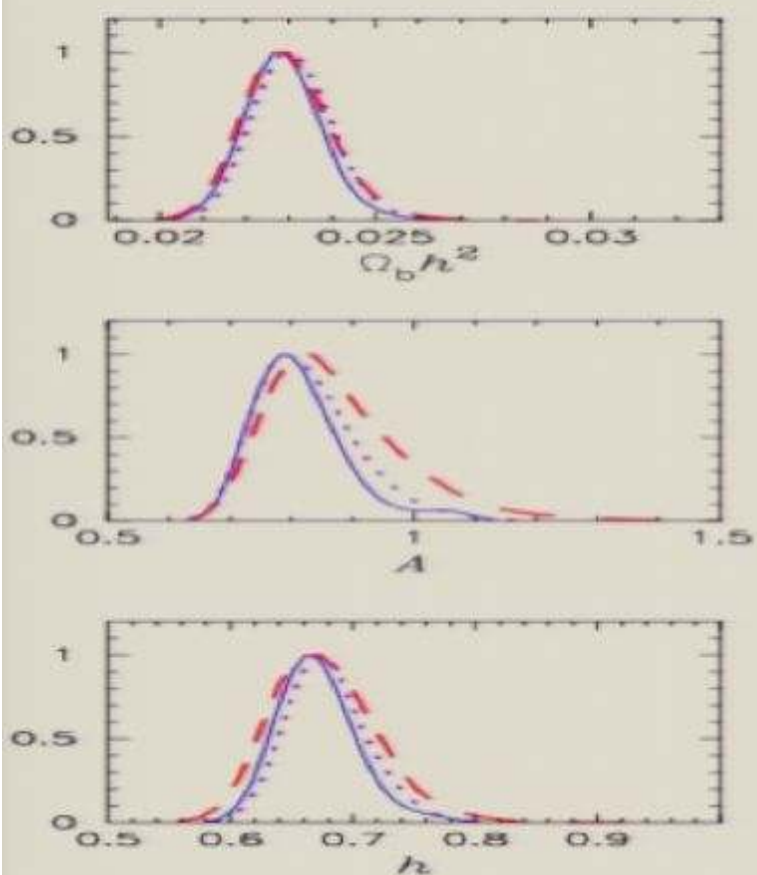
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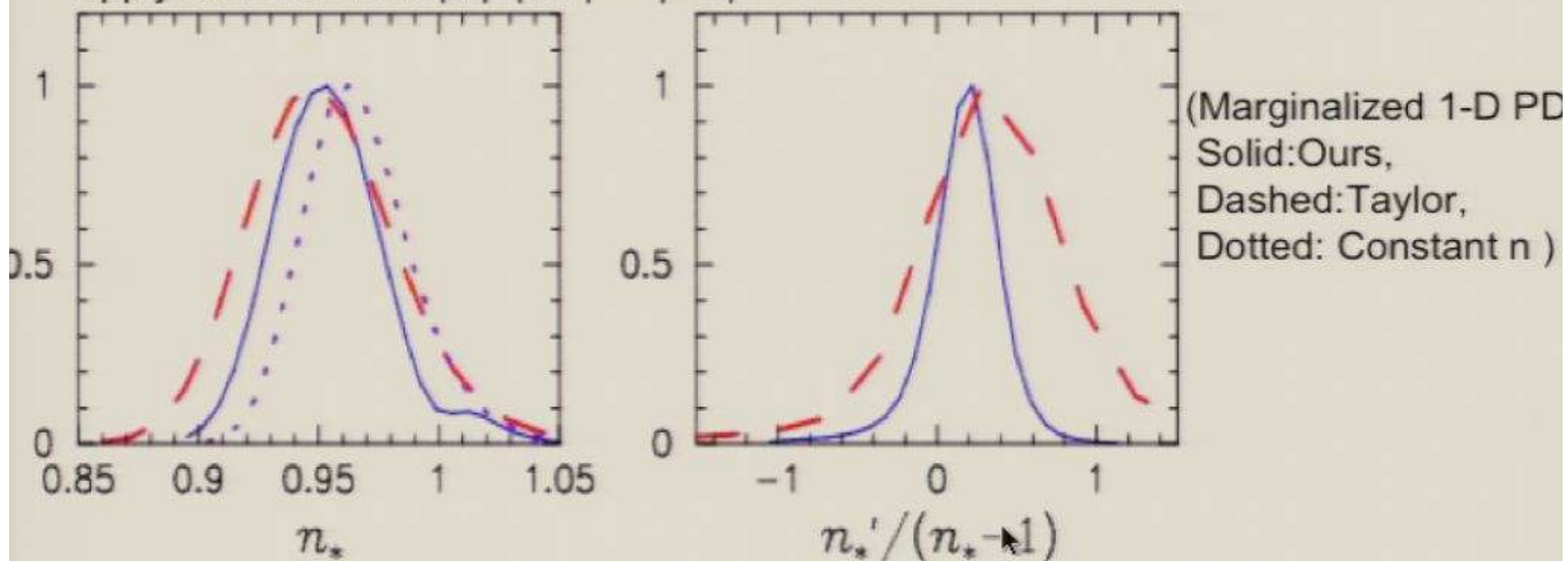


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# Likelihood analysis for the inflationary parameters

Simple single component inflation models where the standard slow-roll approximation apply often leads to  $|n'| \sim |n-1|^2 \ll |n-1|$



(Central values: ours  $n'_s = -0.0087 \pm 0.0084$ , truncated Taylor  $n'_s = -0.019 \pm 0.014$  )

Our improved parameterization and even the truncated Taylor series expansion show that  $|n'| \sim |n-1|$  is still consistent with the data.

Pirsa: 05120002  $\left| (n_s - 1) \ln\left(\frac{k}{k_*}\right) \right| \gg \left| n'_s \ln^2\left(\frac{k}{k_*}\right) \right|$  is not valid (our data covers  $\Delta \ln k \sim 10$ ). Page 66/68

$$(n_* - 1) \ln \frac{k}{k_*}$$

0,05

$$-m^2 \phi^2$$

$$5 \rightarrow k$$

$$(N_k - 1) \ln \frac{k}{k_*} \approx 10^{10} \text{ GeV}^4$$

$$0.05 \rightarrow k_*$$

$$0.05 \frac{V}{V} = k'' \sim$$