

Title: The statistical challenge of cosmic weirdness

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Abstract: In the standard cosmological model, galaxies and large-scale structure grew by a process of gravitational instability from initial perturbations which were of the simplest statistical form imaginable: a statistically homogeneous and isotropic Gaussian random field. One of the properties of such a field is that its Fourier transform has real and imaginary parts which are independently Gaussian and consequently the phases are uniformly random. The same thing applies to the phases of the spherical harmonic coefficients involved when observed fluctuations over the celestial sphere, such as in the cosmic microwave background. Defining anything other than random phases as "weird", I discuss various aspects of cosmic weirdness and the non-randomness they produce in harmonic space. I introduce some novel methods for visualizing weirdness in CMB data and elsewhere, and discuss their relationship to more conventional statistical analyses. If I have time I will also discuss a few other interesting things to do with CMB fluctuations.



The University of  
**Nottingham**

# *The Statistical Challenge of Cosmic Weirdness*

Peter Coles

(University of Nottingham)



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# **“The Essence of Cosmology is Statistics”**

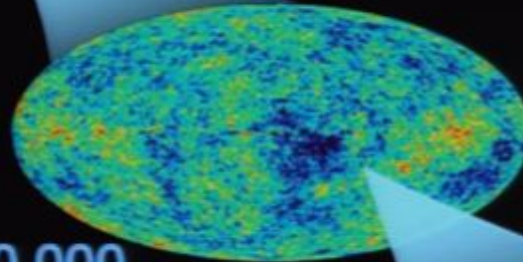
**George McVittie**

**DAWN  
OF  
TIME**



**tiny fraction  
of a second**

**inflation**



**380,000  
years**

**13.7  
billion  
years**



# “CONCORDANCE”





# OUTLINE

- The importance of phase information in cosmology
- Fourier phases in gravitational clustering
- Spherical Harmonic phases
- Illustration using preliminary WMAP data
- Some other funny properties of WMAP





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# How Weird is the Universe?

- **The concordance cosmology is a “first-order” model**
- **In it (and other “first-order” models), the initial fluctuations were a statistically homogeneous and isotropic Gaussian Random Field (GRF)**
- **These are the “maximum entropy” initial conditions having “random phases” motivated by inflation.**
- **Anything else would be weird....**

# Statistical Gaussianity

- **Consider a set of values**  $Y = \{y_i\} = \{y(\underline{x}_i)\}$
- **All the finite-dimensional joint densities of any  $n$  of these must be a multivariate Gaussian:**

$$P_n(y_1, \dots, y_n) = \frac{\|\mathbf{M}\|^{1/2}}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2} \mathbf{Y}^T \mathbf{M} \mathbf{Y}\right)$$

$$\mathbf{M}^{-1} = \mathbf{C} = \langle y_i y_j \rangle$$

# How can you test this?

- It's hard, and it helps if you know what you're looking for.
- There are, however, general methods...e.g.
- Consider the bivariate problem  $\{a_i, b_i\}$
- Do a linear regression using a model
$$a = b + \alpha b^2$$
- Is  $\alpha$  significantly non-zero?

# Statistical Gaussianity

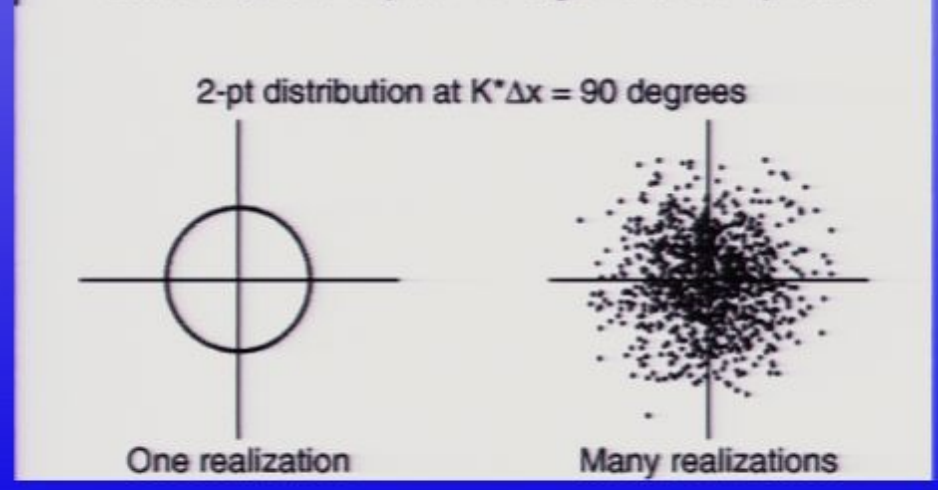
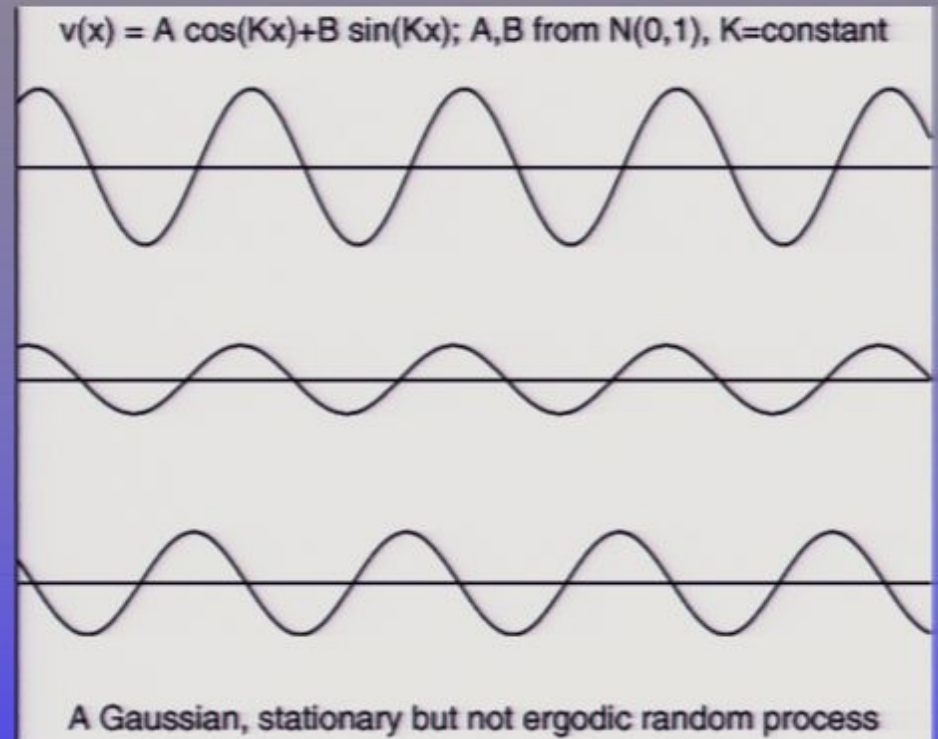
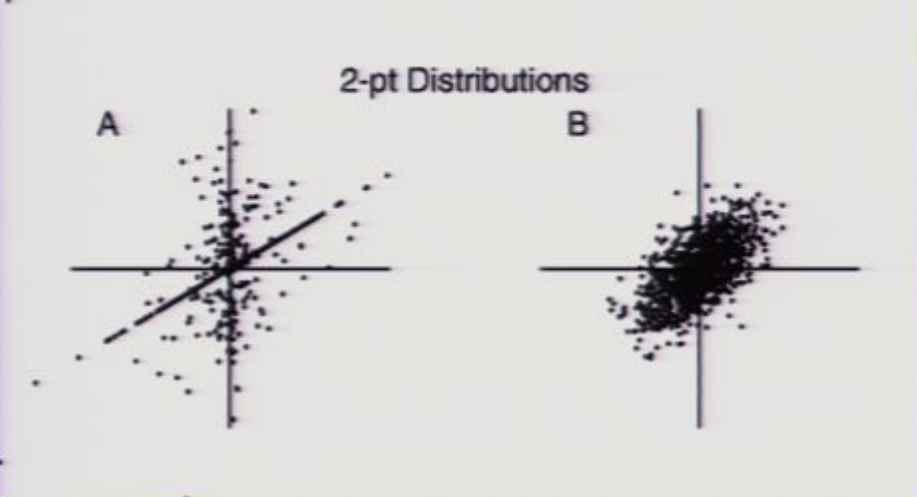
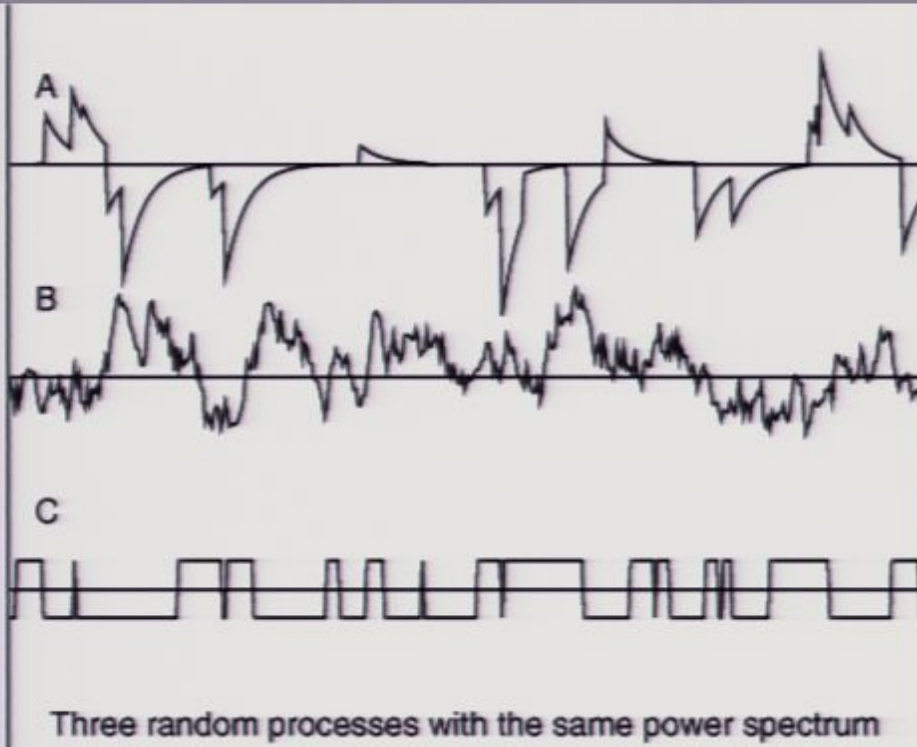
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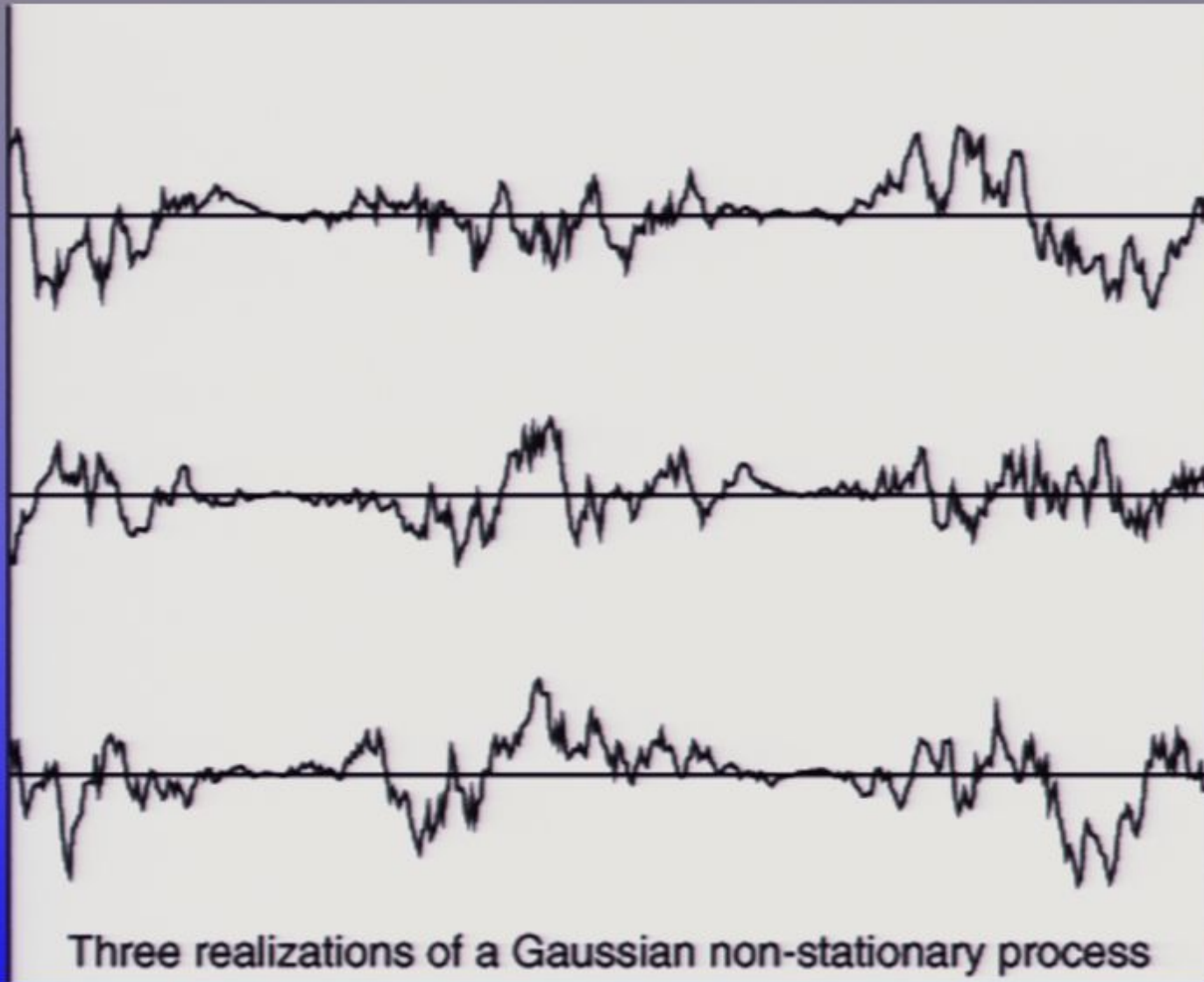
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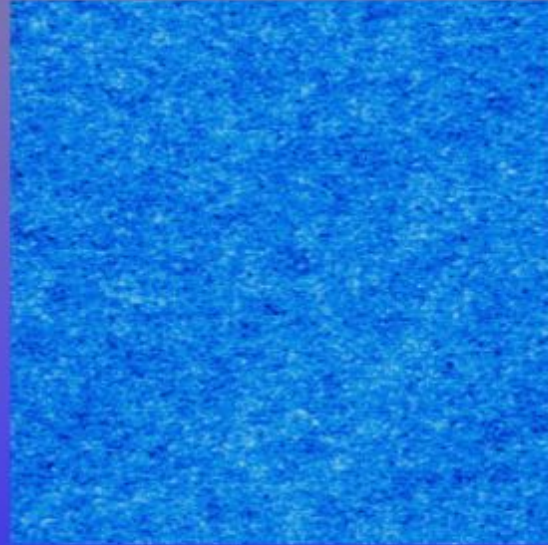
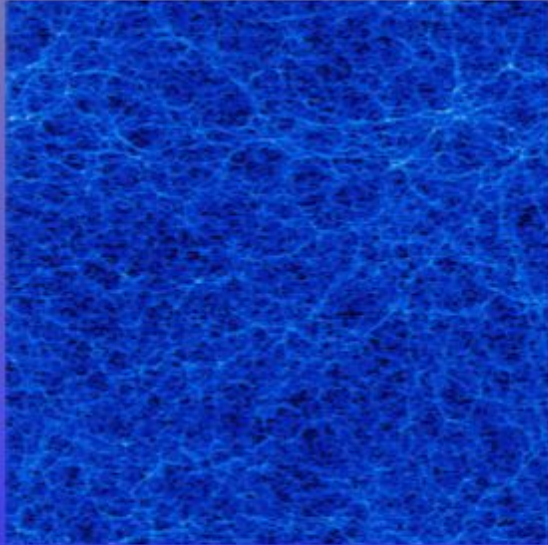
# Fourier Phases

- The usual thing  $\delta(x) = \sum_k \delta(k) \exp(ik \cdot x)$
- where  $\delta(k) = |\delta(k)| \exp[i\varphi_k]$
- In a homogeneous and isotropic GRF then the phases  $\varphi$  are random...
- ..apart from  $\delta(k) = \delta(-k)^*$
- ..as are differences, e.g.  $\varphi_{k_1} - \varphi_{k_2}$



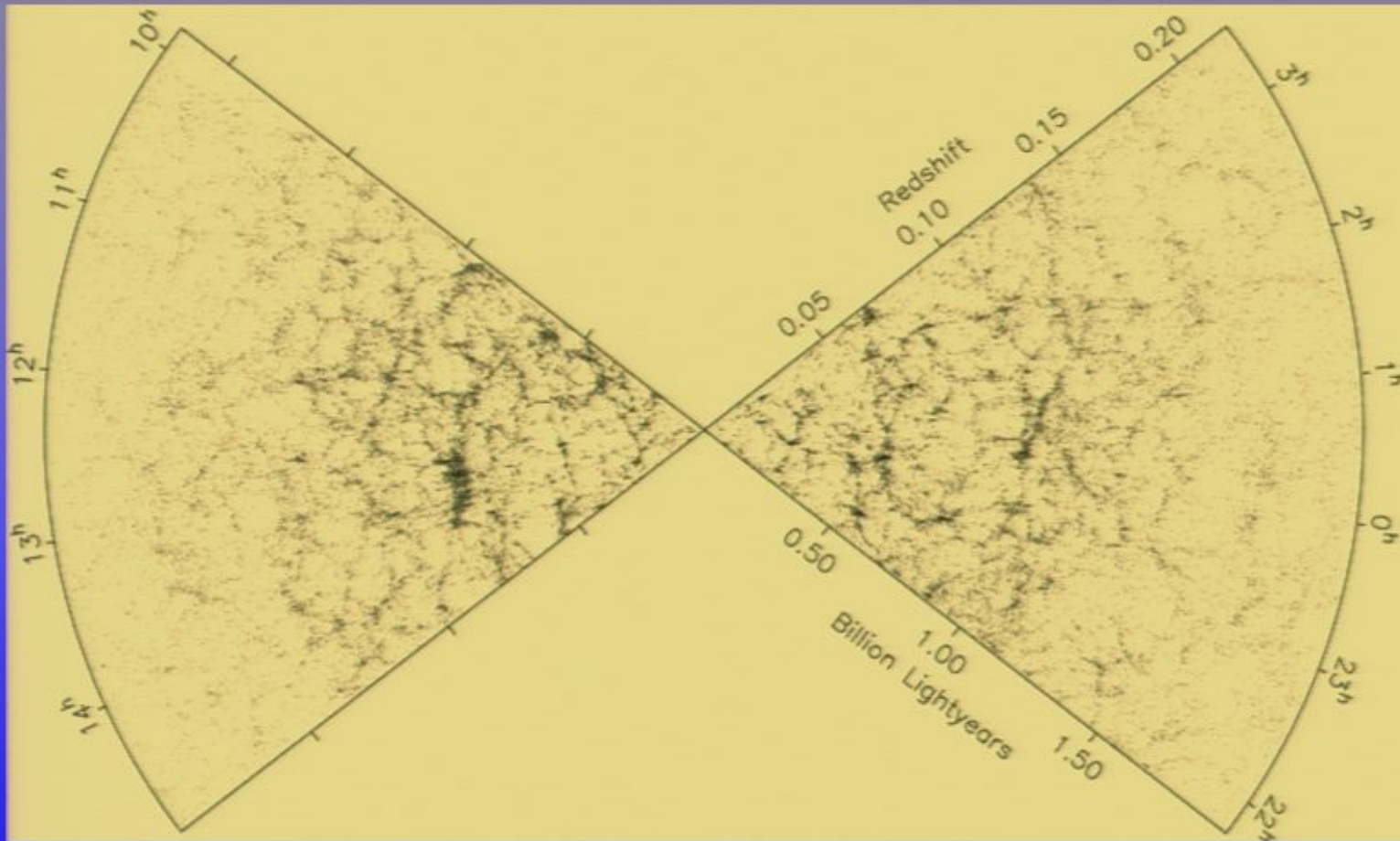


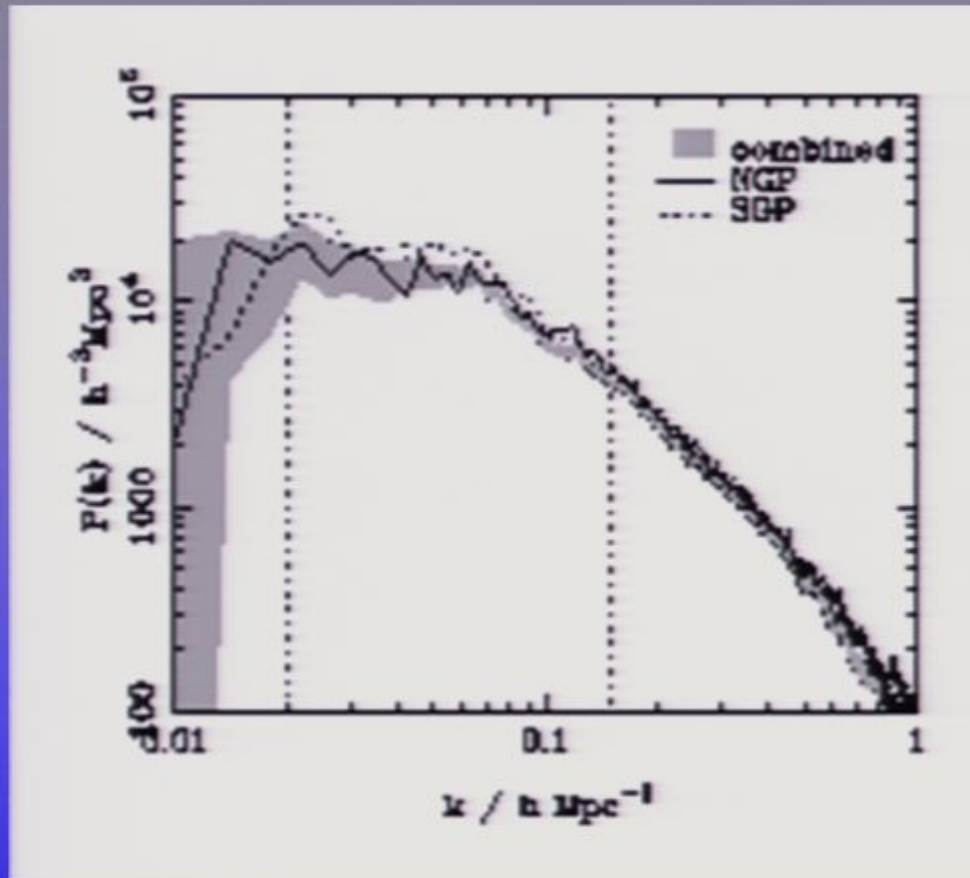




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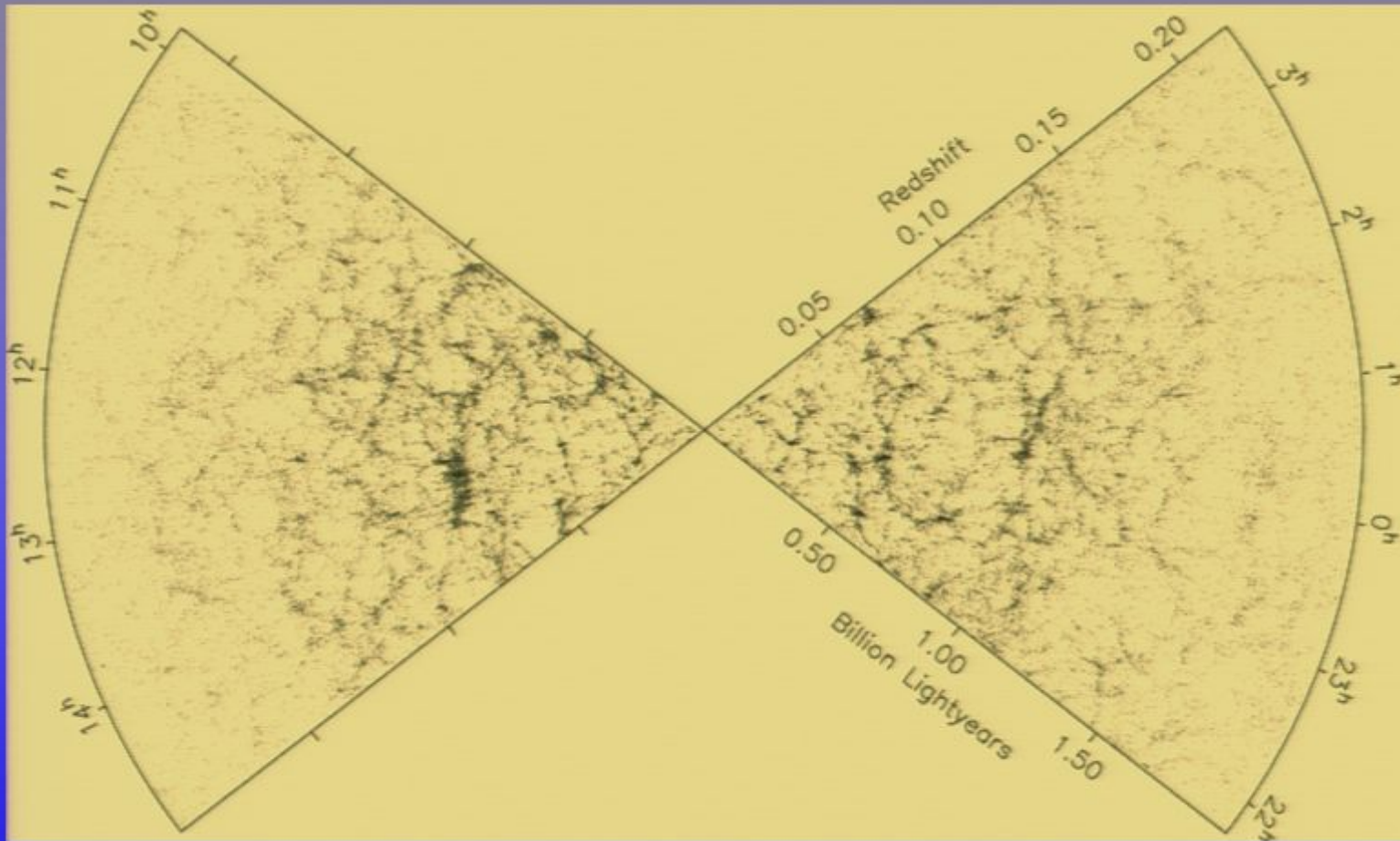
## 2dFGRS Power Spectrum

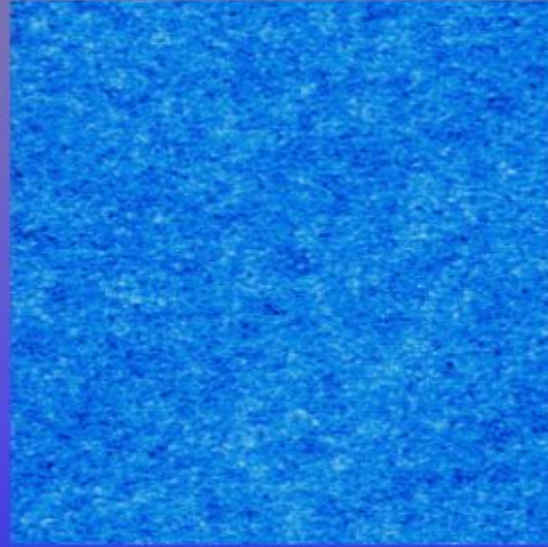
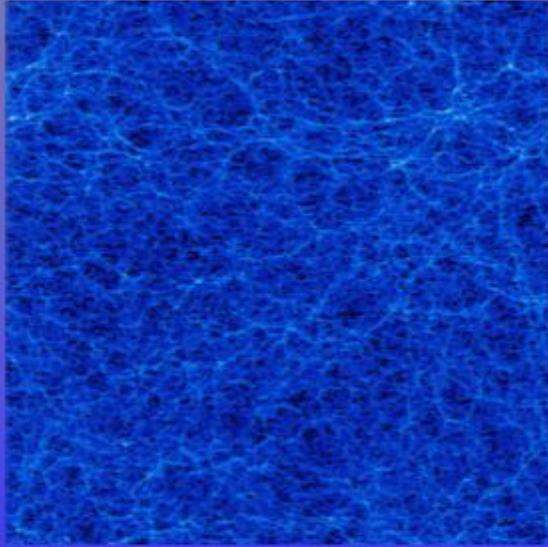
# Weird Statistics..

- **Could be stationary non-Gaussian, such as the quadratic model:**

$$\Phi(\underline{x}) = \Phi_L(\underline{x}) + f_{NL} \left[ \Phi_L^2(\underline{x}) - \langle \Phi_L^2(\underline{x}) \rangle \right]$$

- **Or non-stationary Gaussian: e.g. small Universes (or maybe foregrounds)**

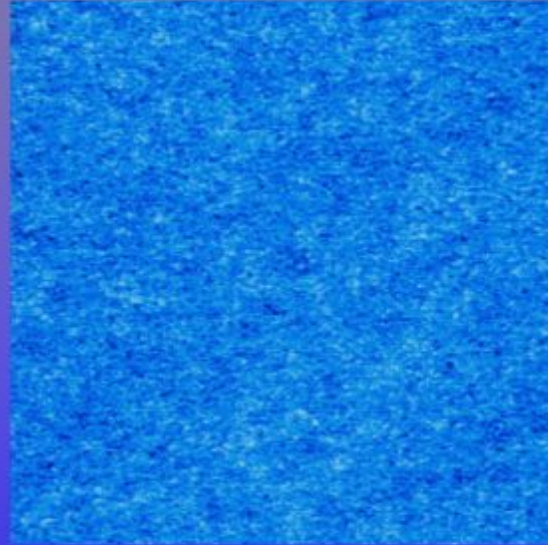
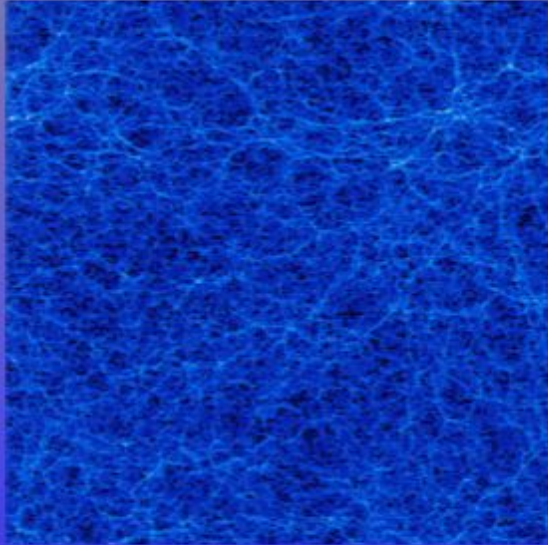


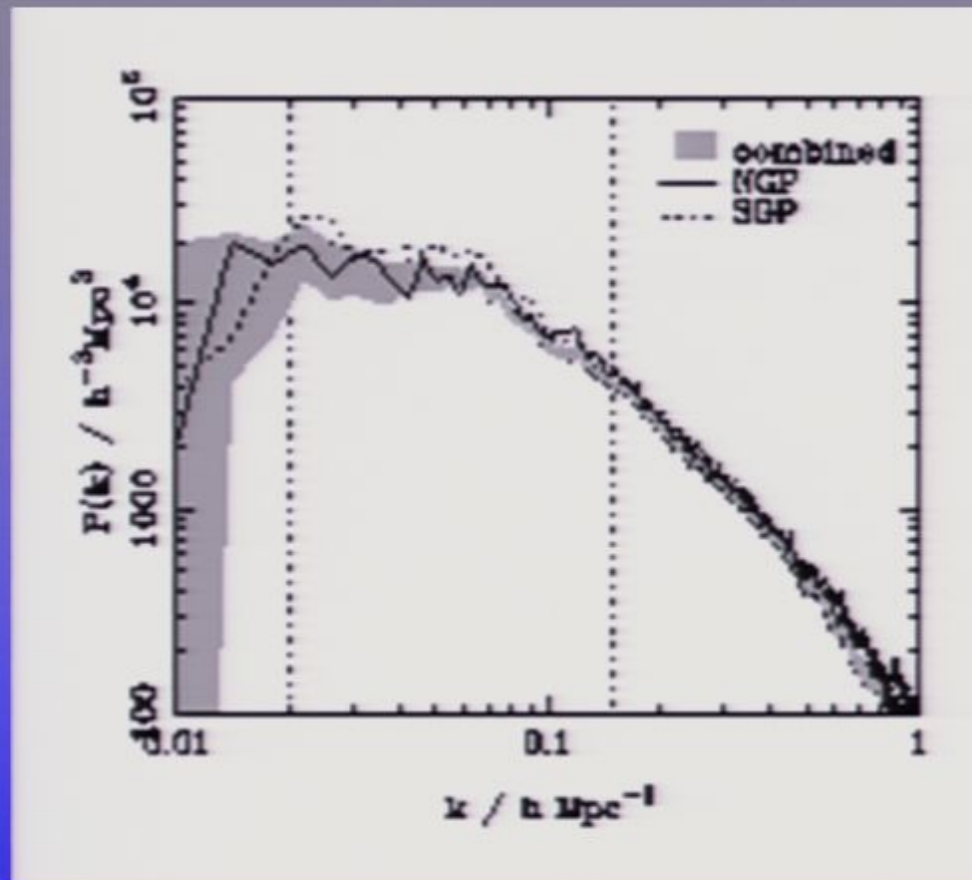




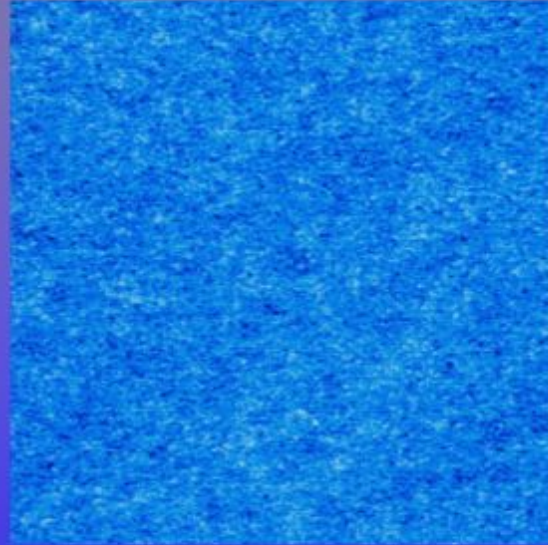
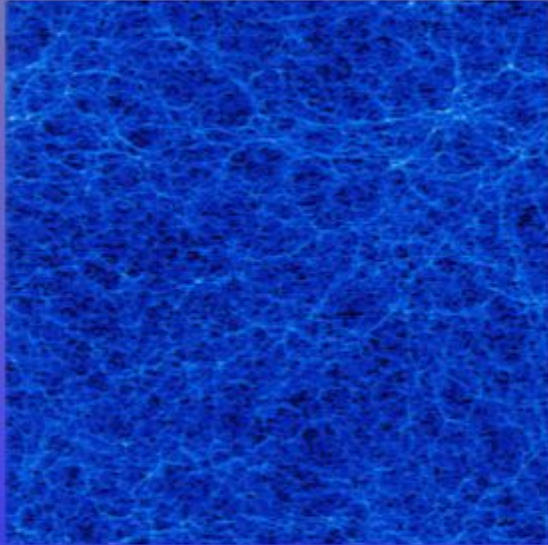
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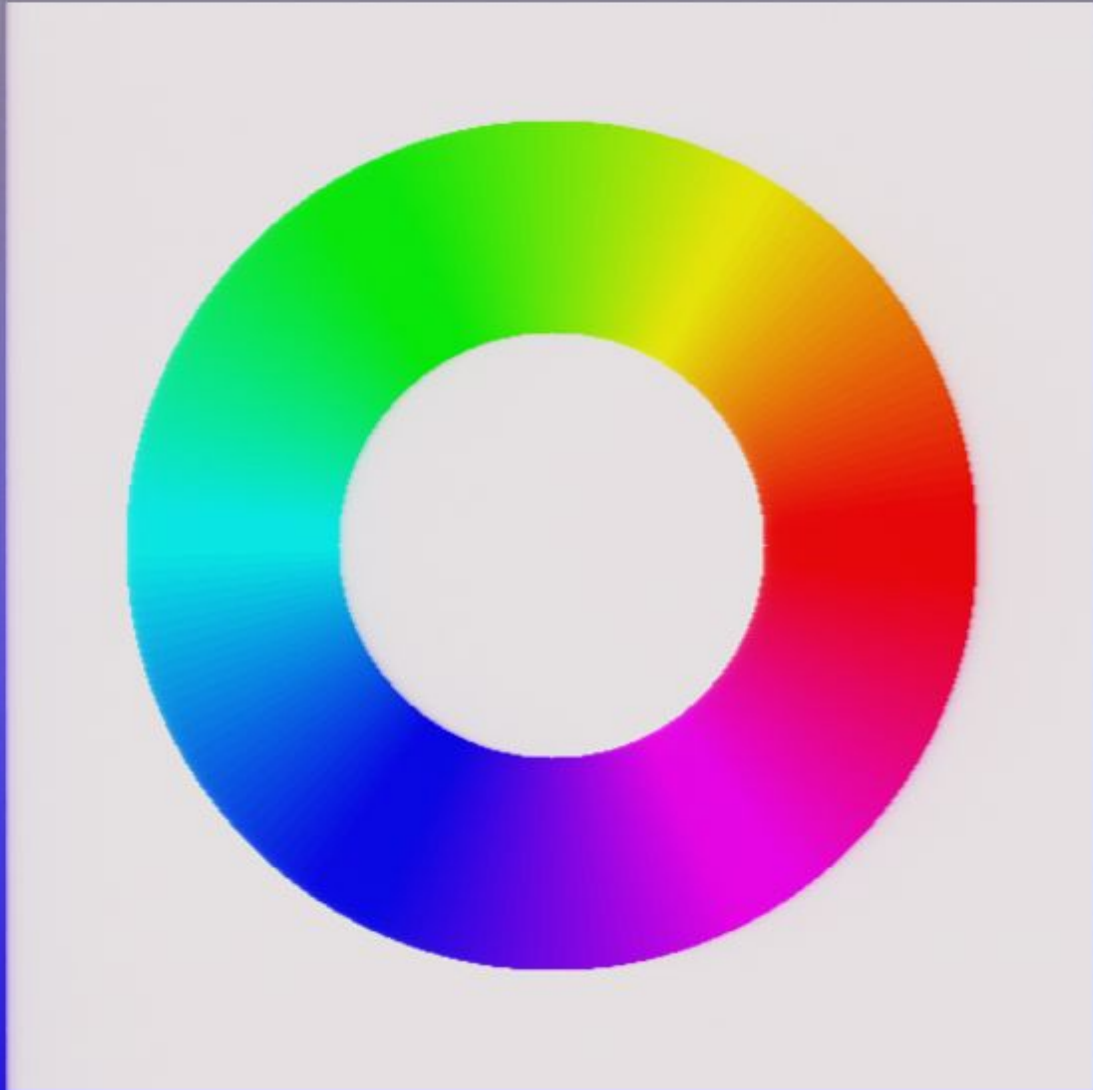
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# Phase Information

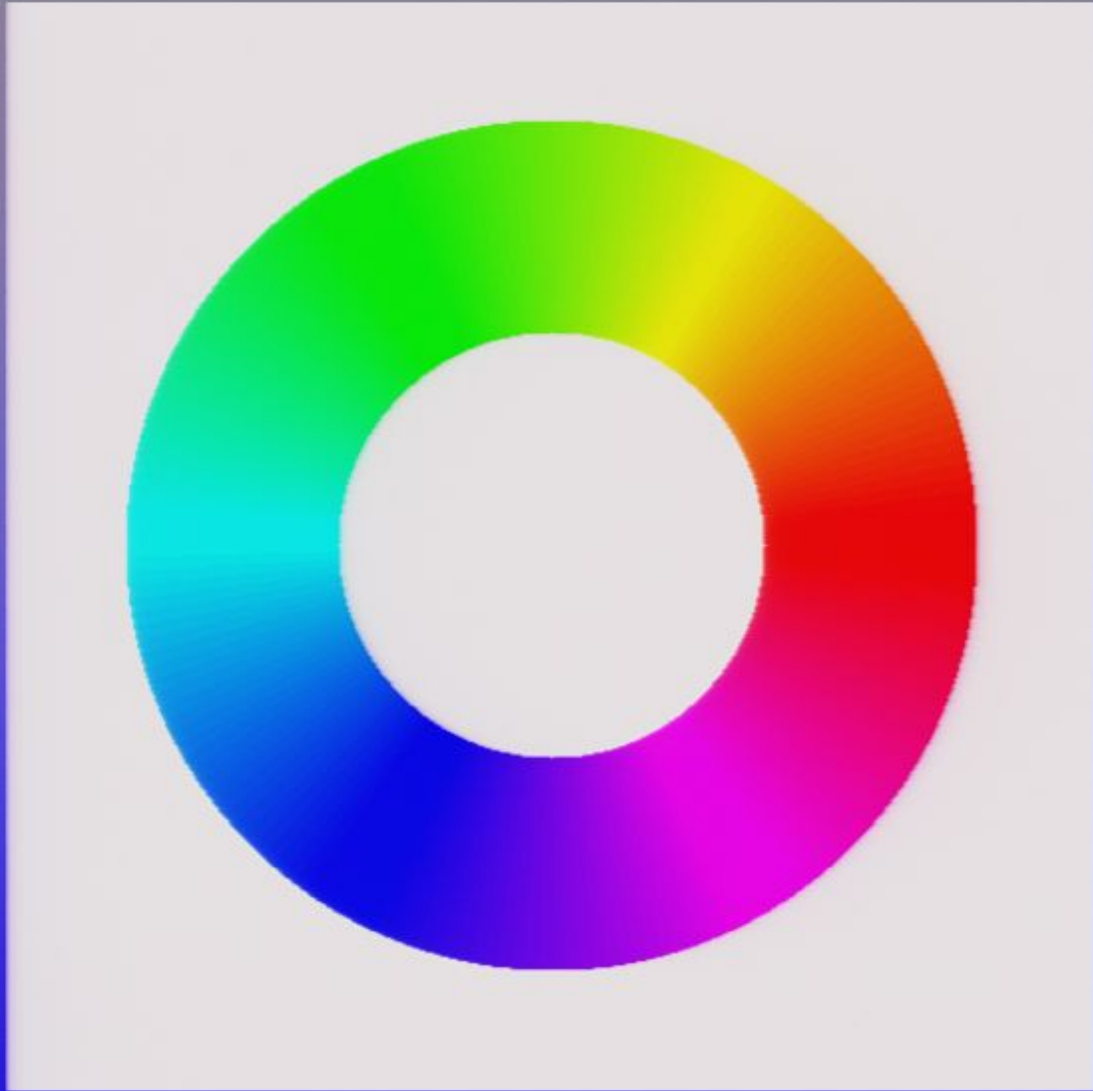
- Weirdness = non-random phases
- Change origin by  $x$ , and  $\varphi_k$  changes by  $kx$ , but phase gradients change by a constant.
- During evolution  $\varphi(t) = \varphi(0) + m \cdot 2\pi$
- ..where  $m$  can be very large. One-point distribution appears random. Phase wrapping.
- Phase signatures can be subtle!



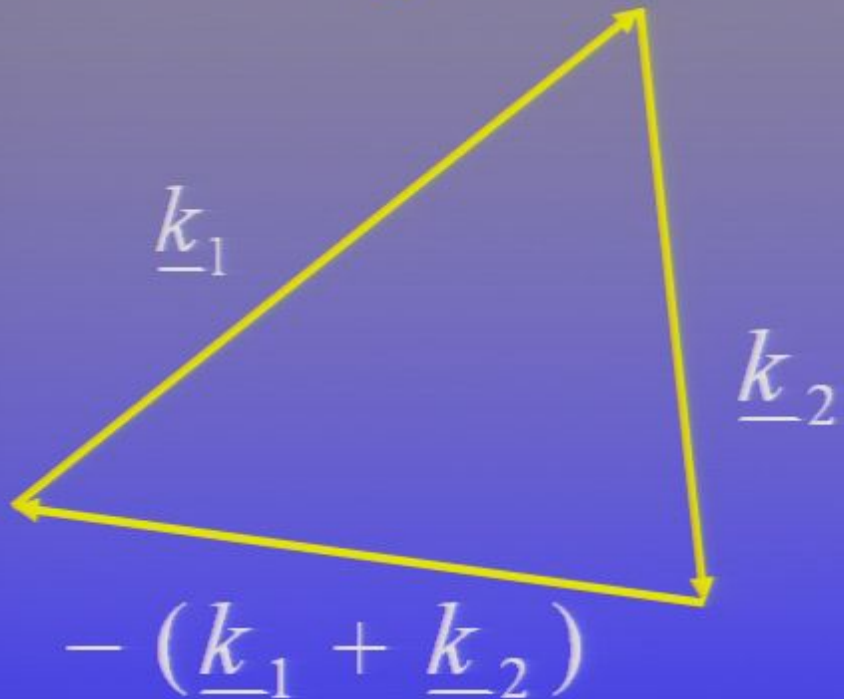


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# Quadratic Phase Coupling

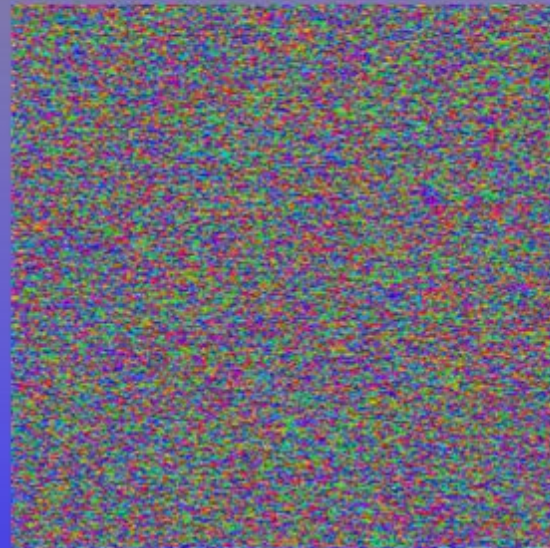
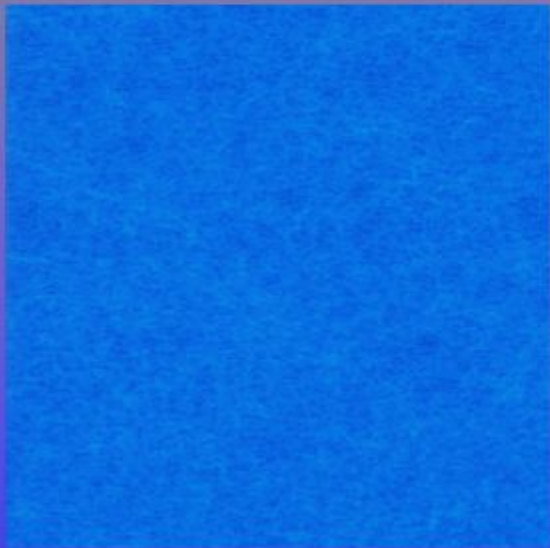


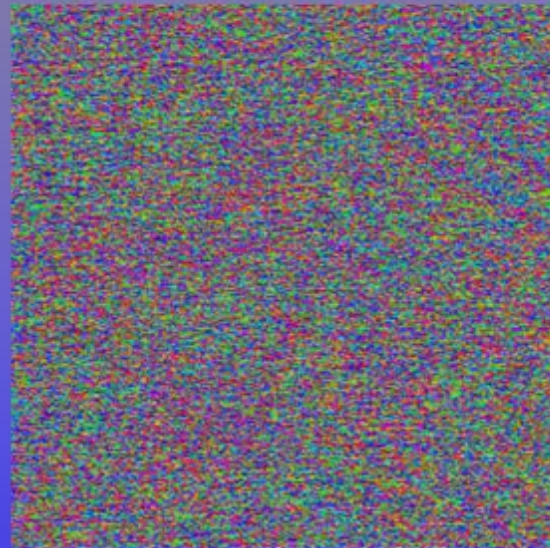
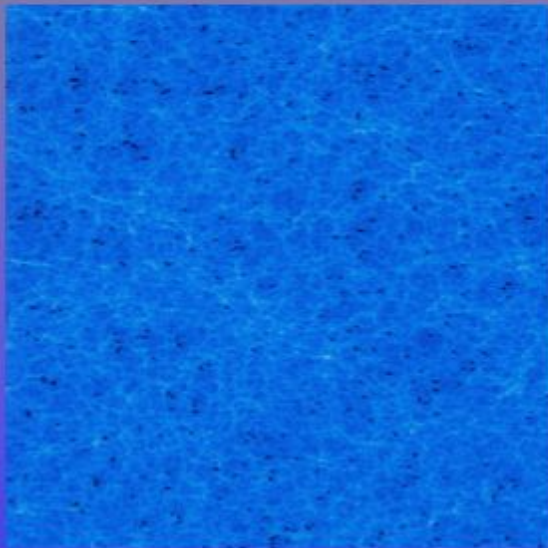
$$\delta_1 \Rightarrow (\underline{k}_1, \phi_1)$$

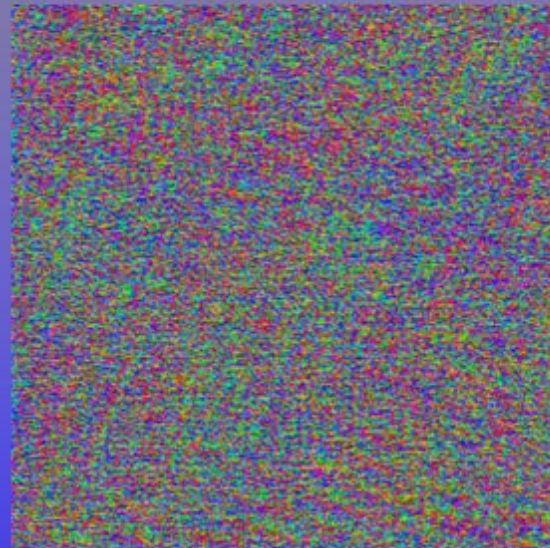
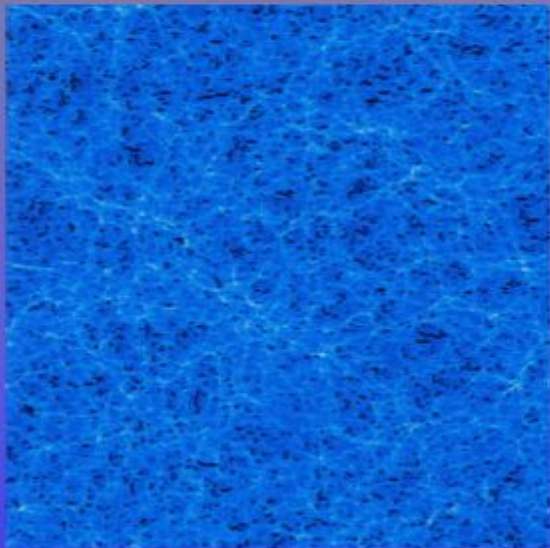
$$\delta_2 \Rightarrow (\underline{k}_2, \phi_2)$$

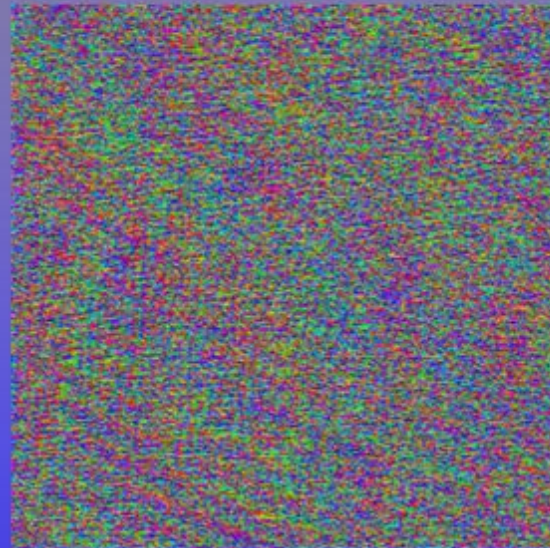
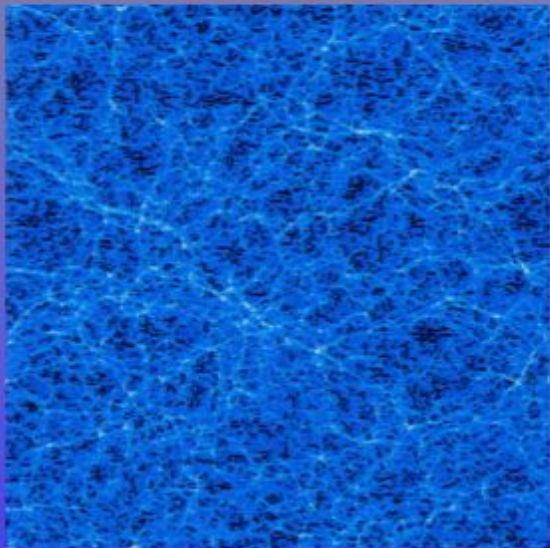
$$(\delta_1 + \delta_2)^2 \Rightarrow (2\underline{k}_1, 2\phi_1) + (2\underline{k}_2, 2\phi_2) + (\underline{k}_1 + \underline{k}_2, \phi_1 + \phi_2)$$

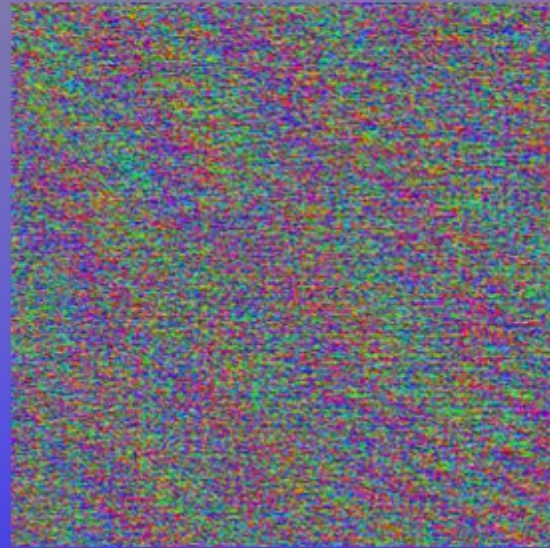
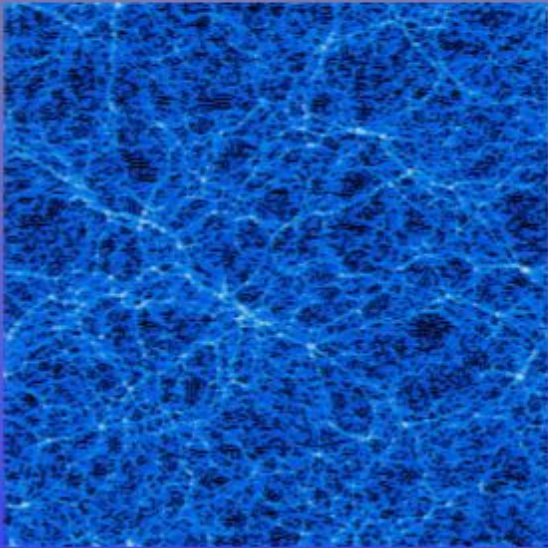
$$\arg(\delta_1 \delta_2 \delta_{-(1+2)}) = \phi_1 + \phi_2 - \phi_{1+2}$$



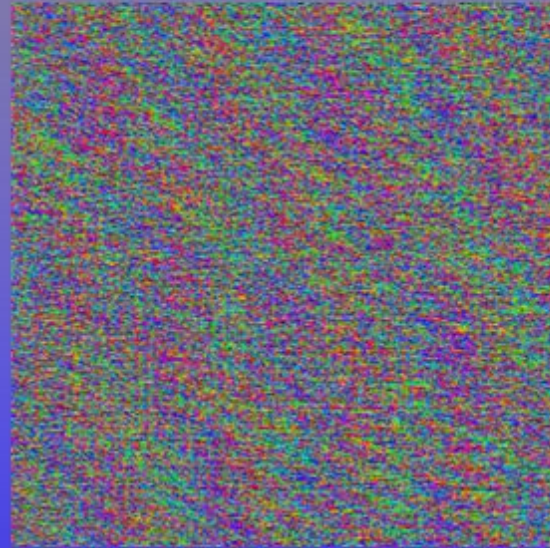
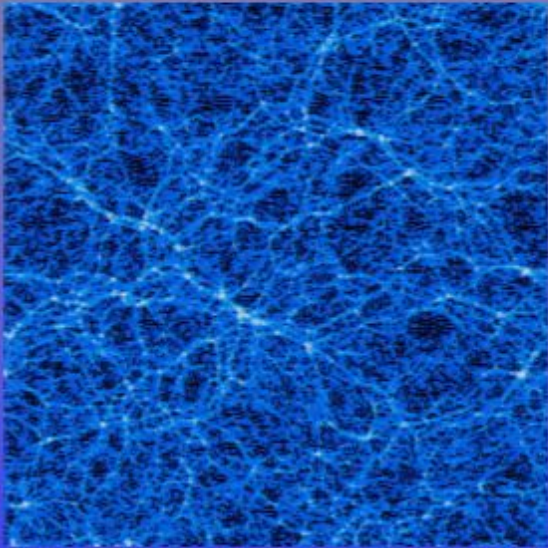


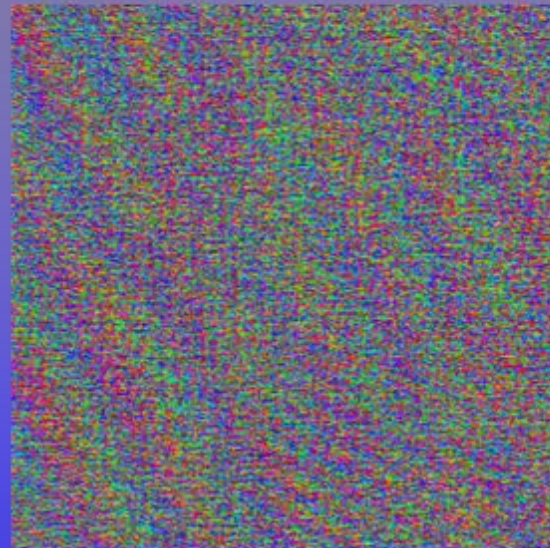
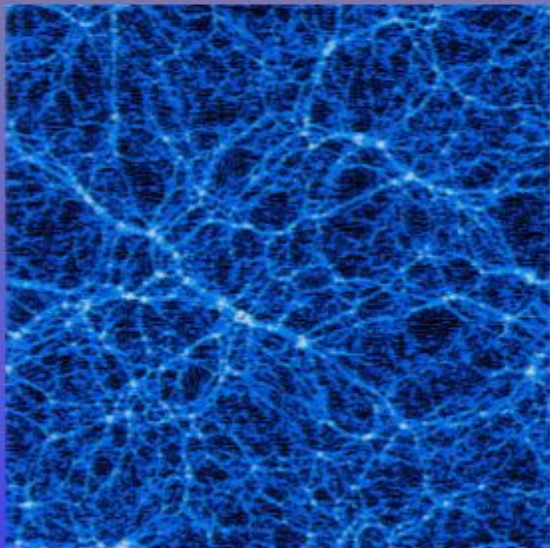


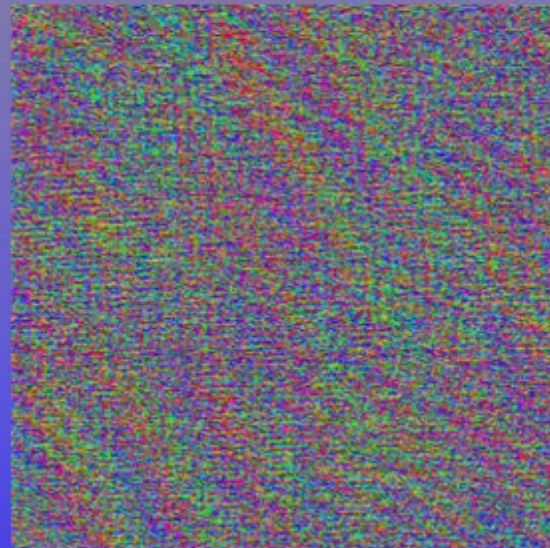
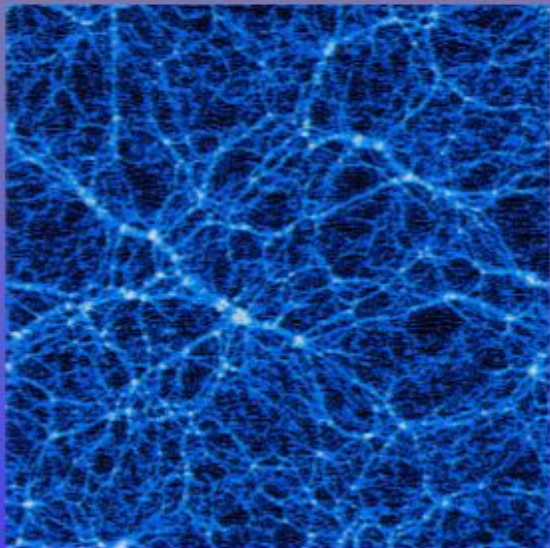


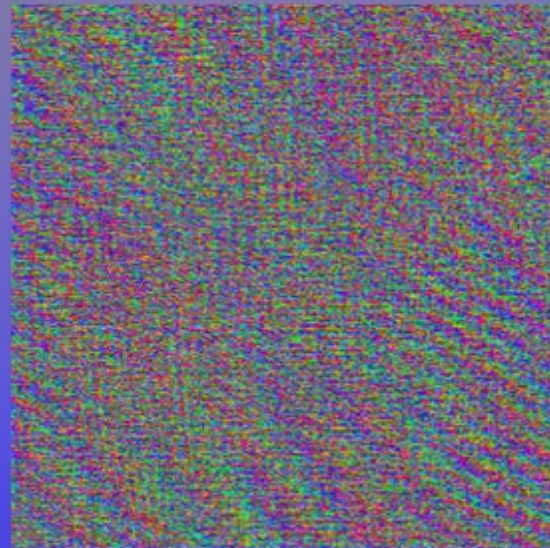
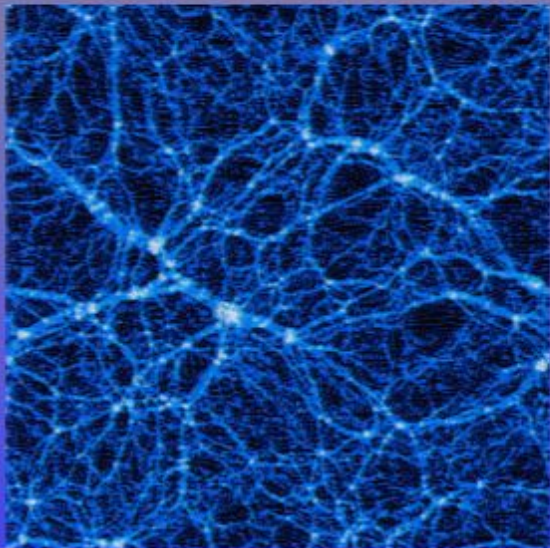


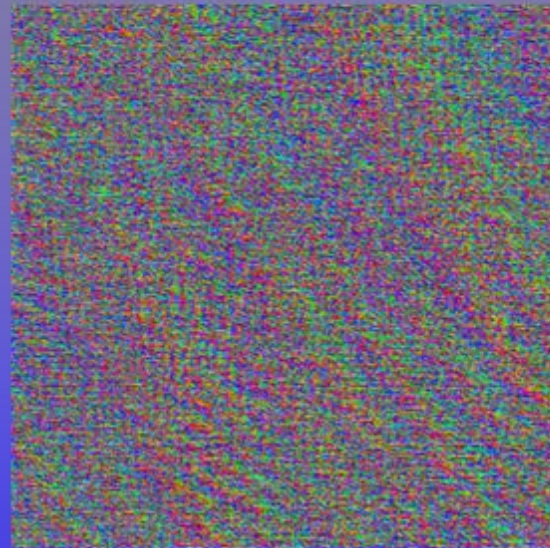
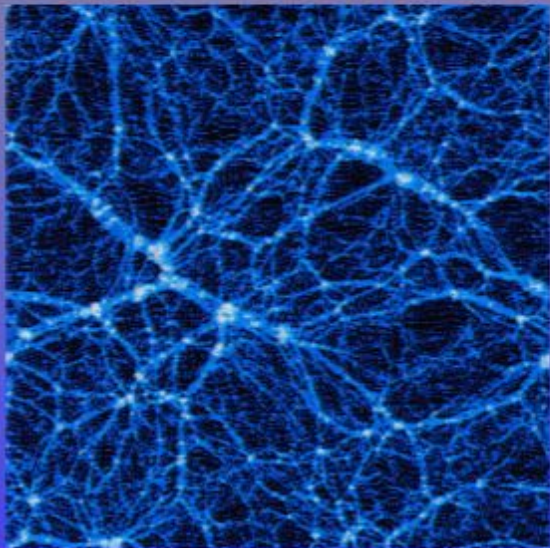


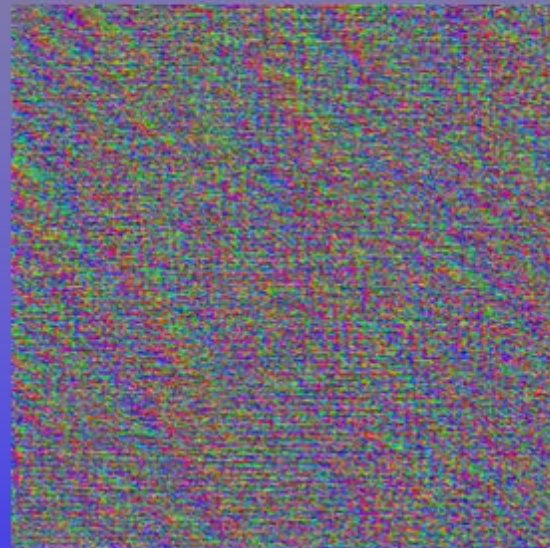
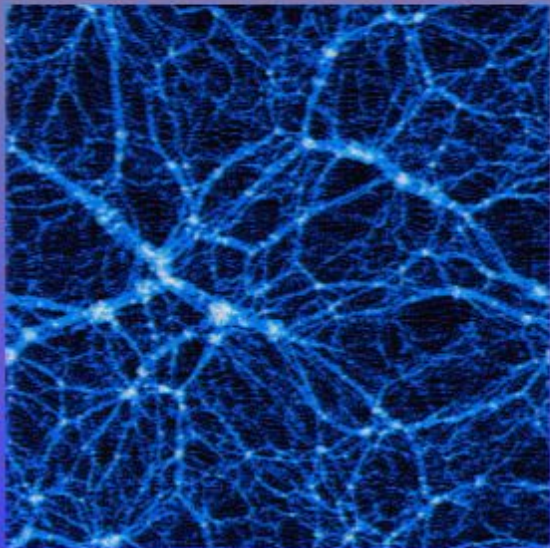


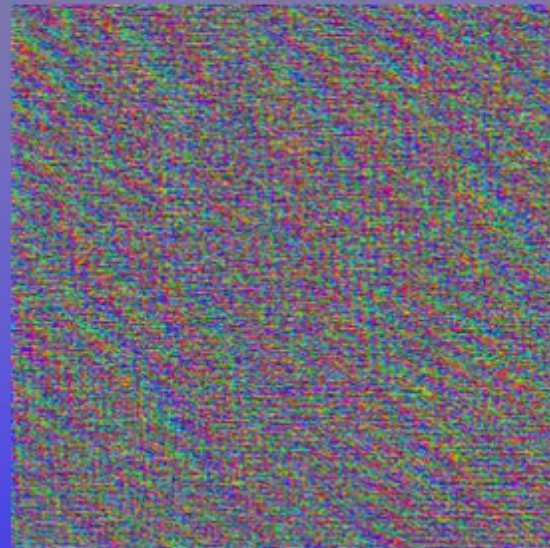
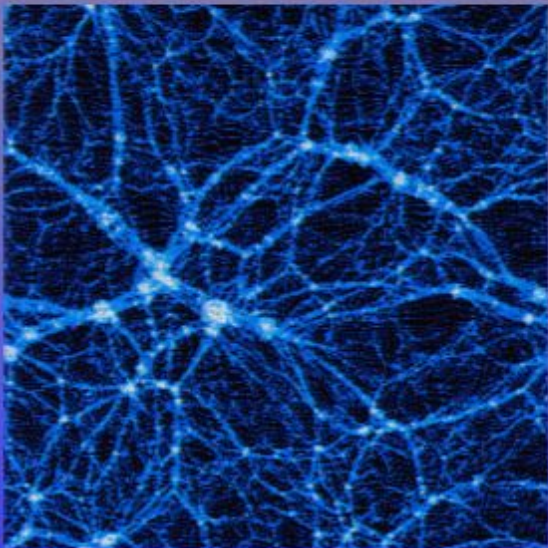


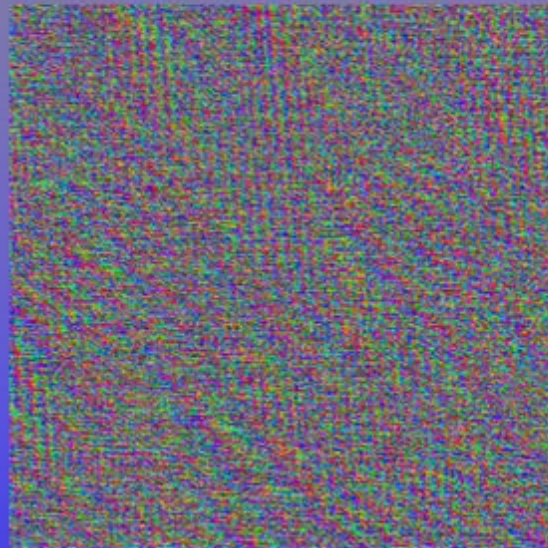
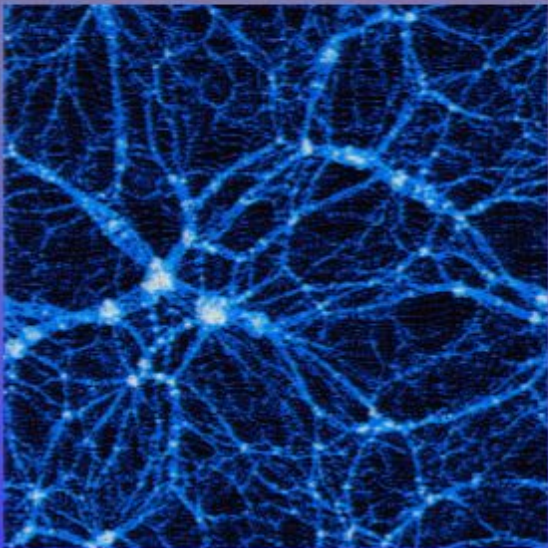




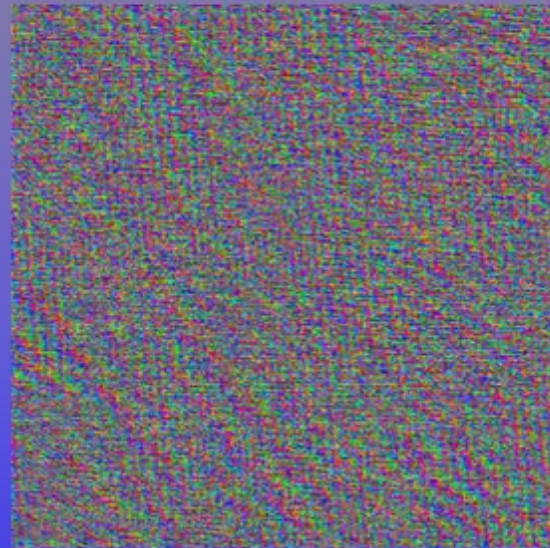
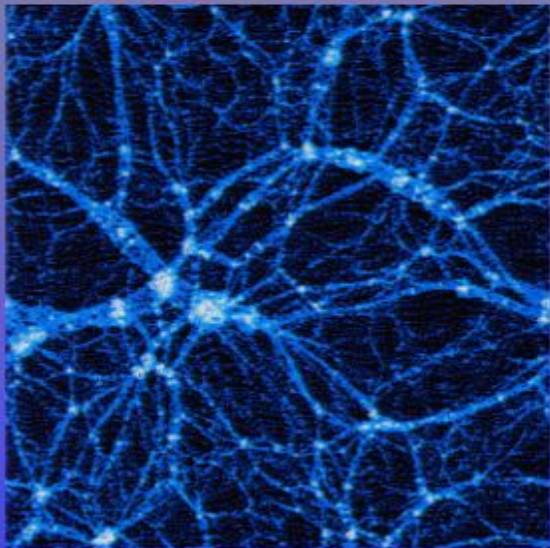


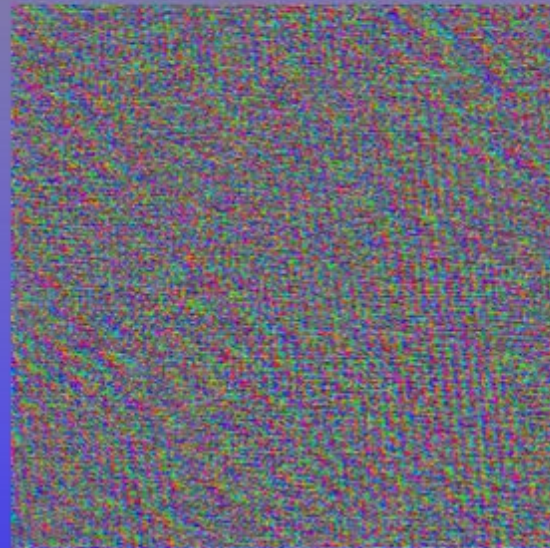
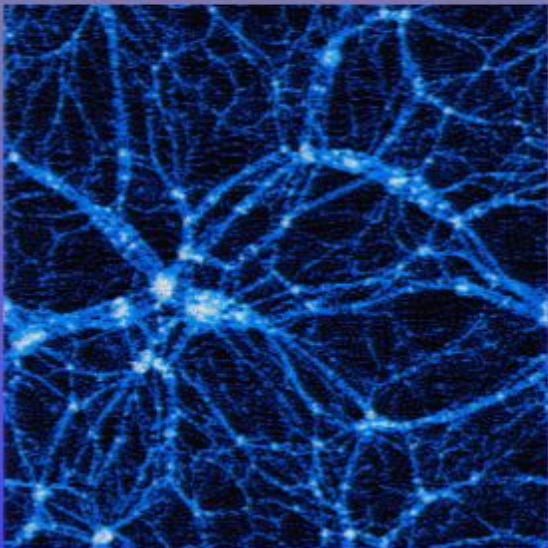


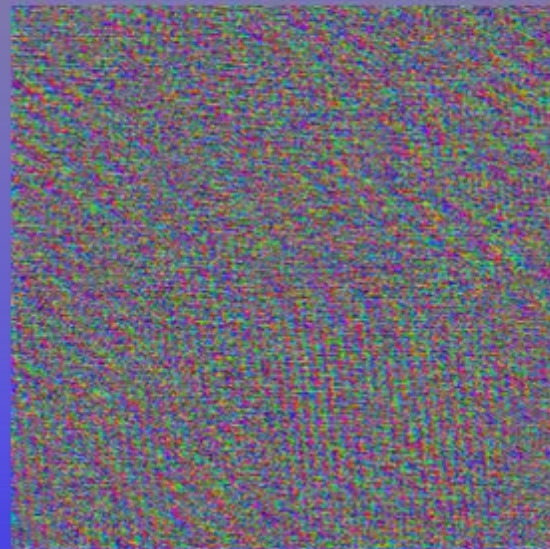
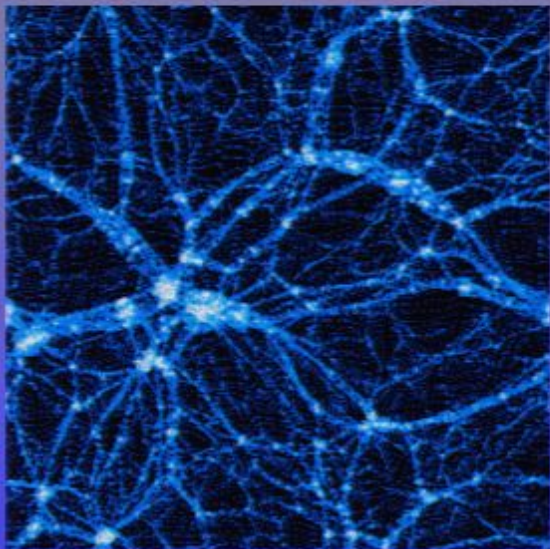


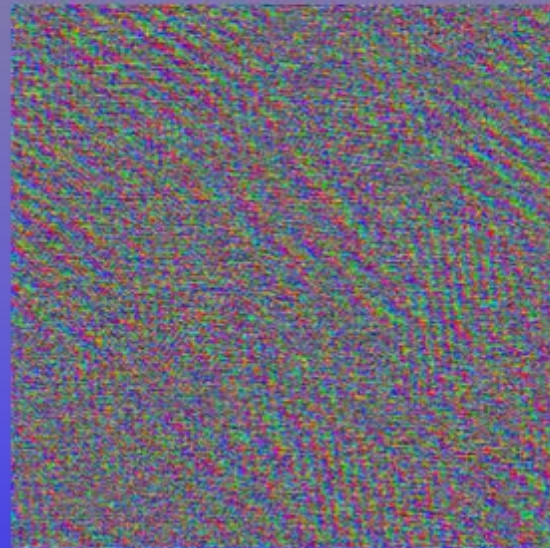
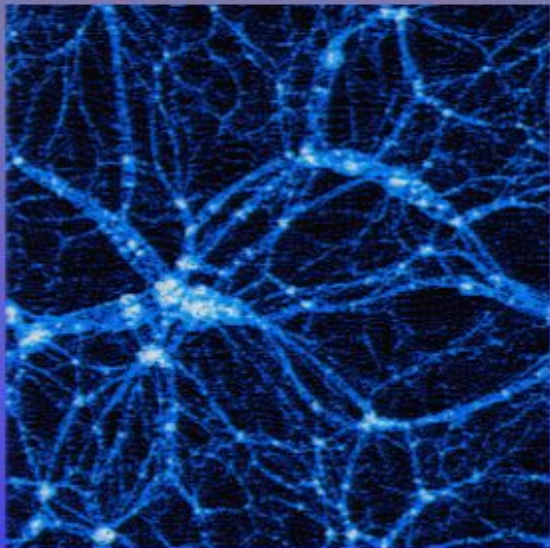


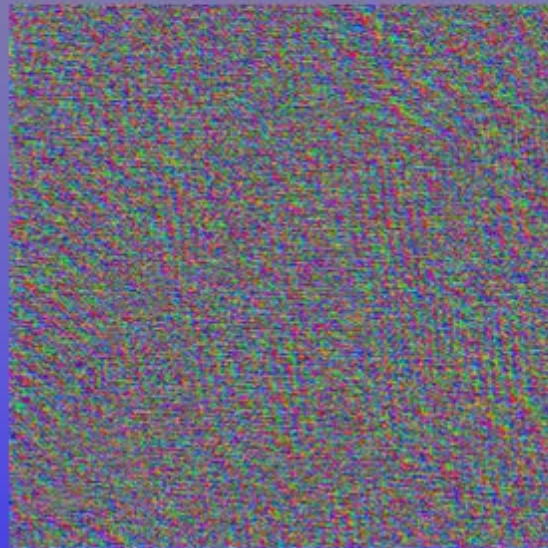
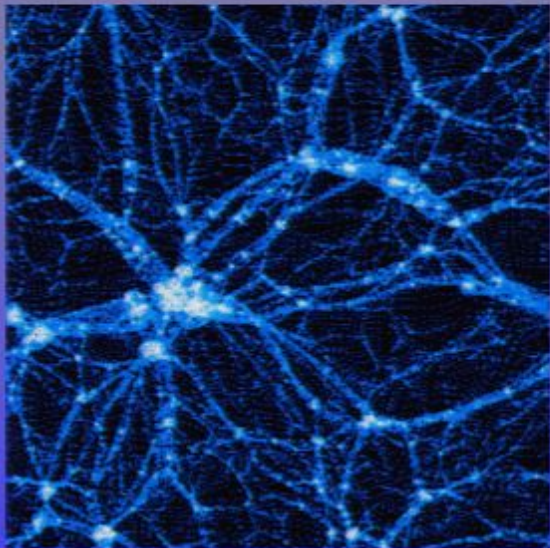


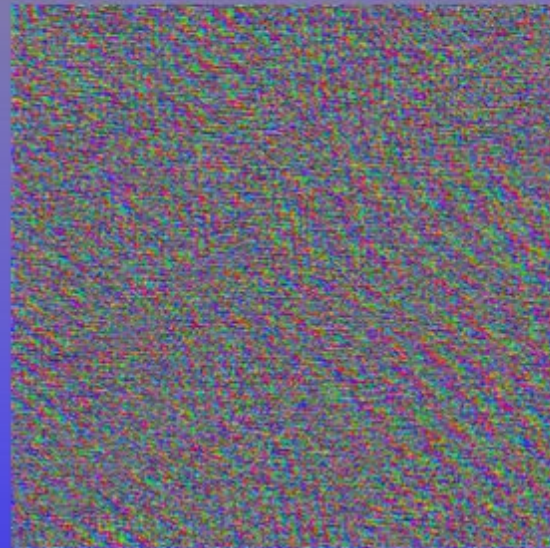
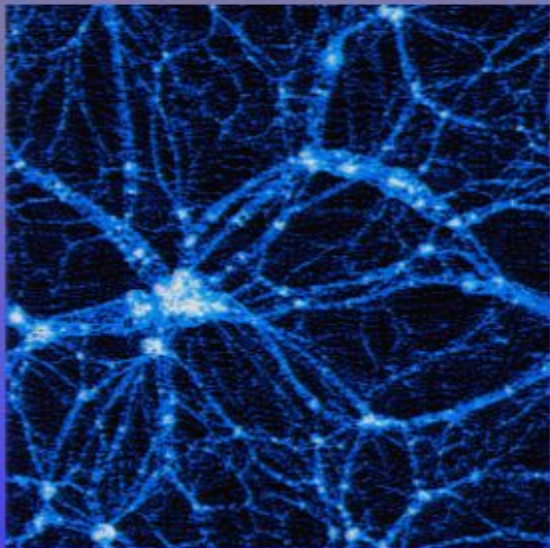


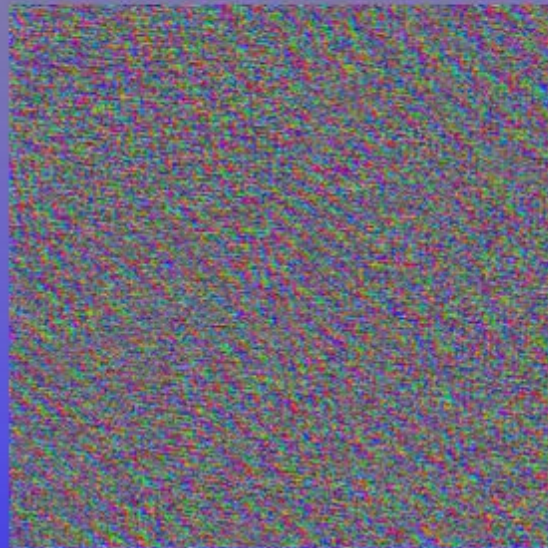
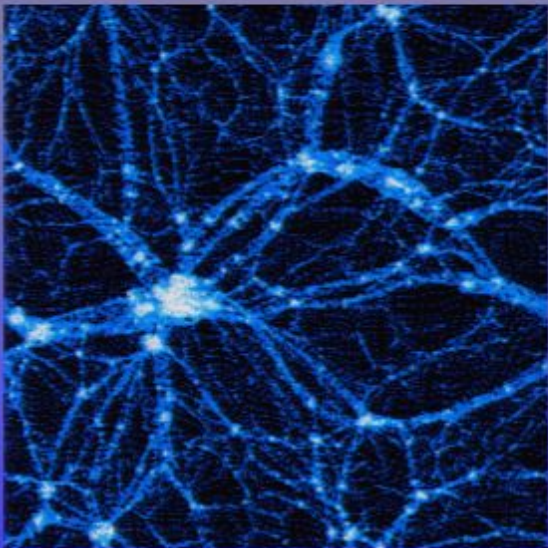


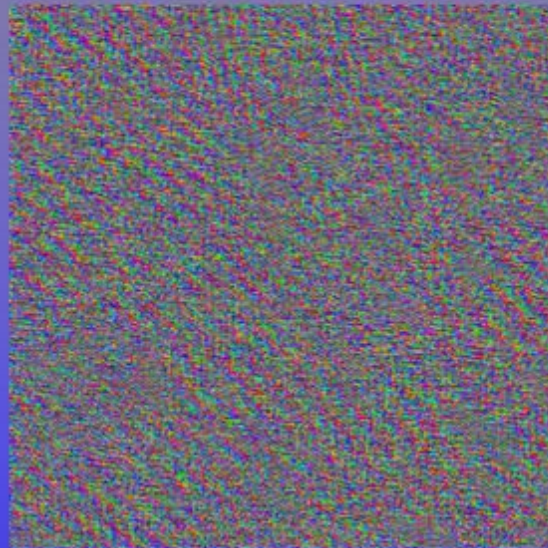
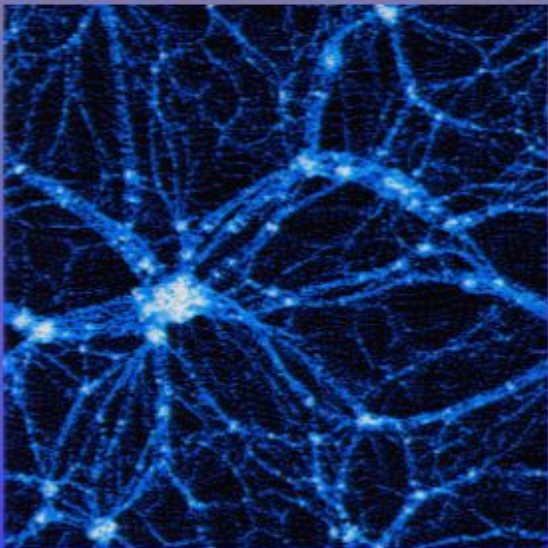




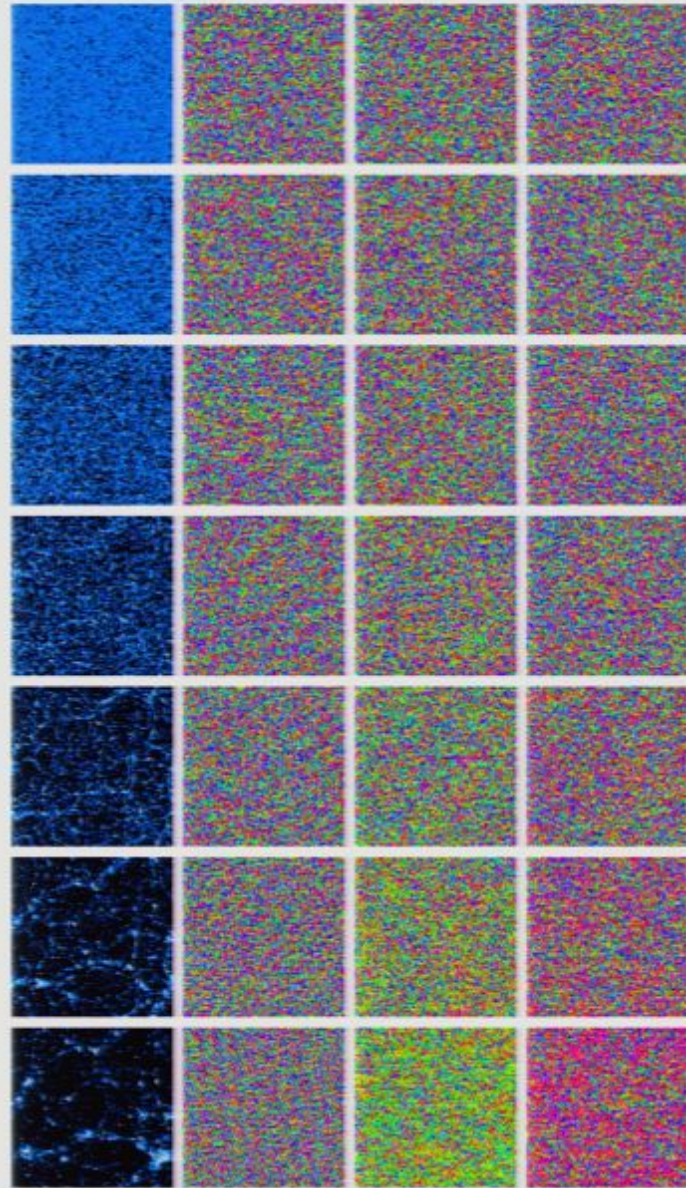












# Some papers on Fourier Phases

- Chiang & Coles, 2000, MNRAS, 311, 809-824
- Coles & Chiang, 2000, Nature, 406, 376-378
- Chiang, Coles & Naselsky, 2002, MNRAS, 337, 488-494
- Watts & Coles 2002, MNRAS, 338, 806
- Watts, Coles & Melott, 2003, ApJL, 589, L61
- For animations, etc, see also:  
<http://www.nottingham.ac.uk/~ppzpc/phases/index.htm>

# Spherical Harmonic Phases

- The usual thing  $\frac{\Delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} a_{l,m} Y_{lm}(\theta, \phi)$
- where  $a_{l,m} = |a_{l,m}| \exp[i\varphi_{l,m}]$
- If the fluctuations are a homogeneous and isotropic GRF then the phases  $\varphi_{l,m}$  are random...
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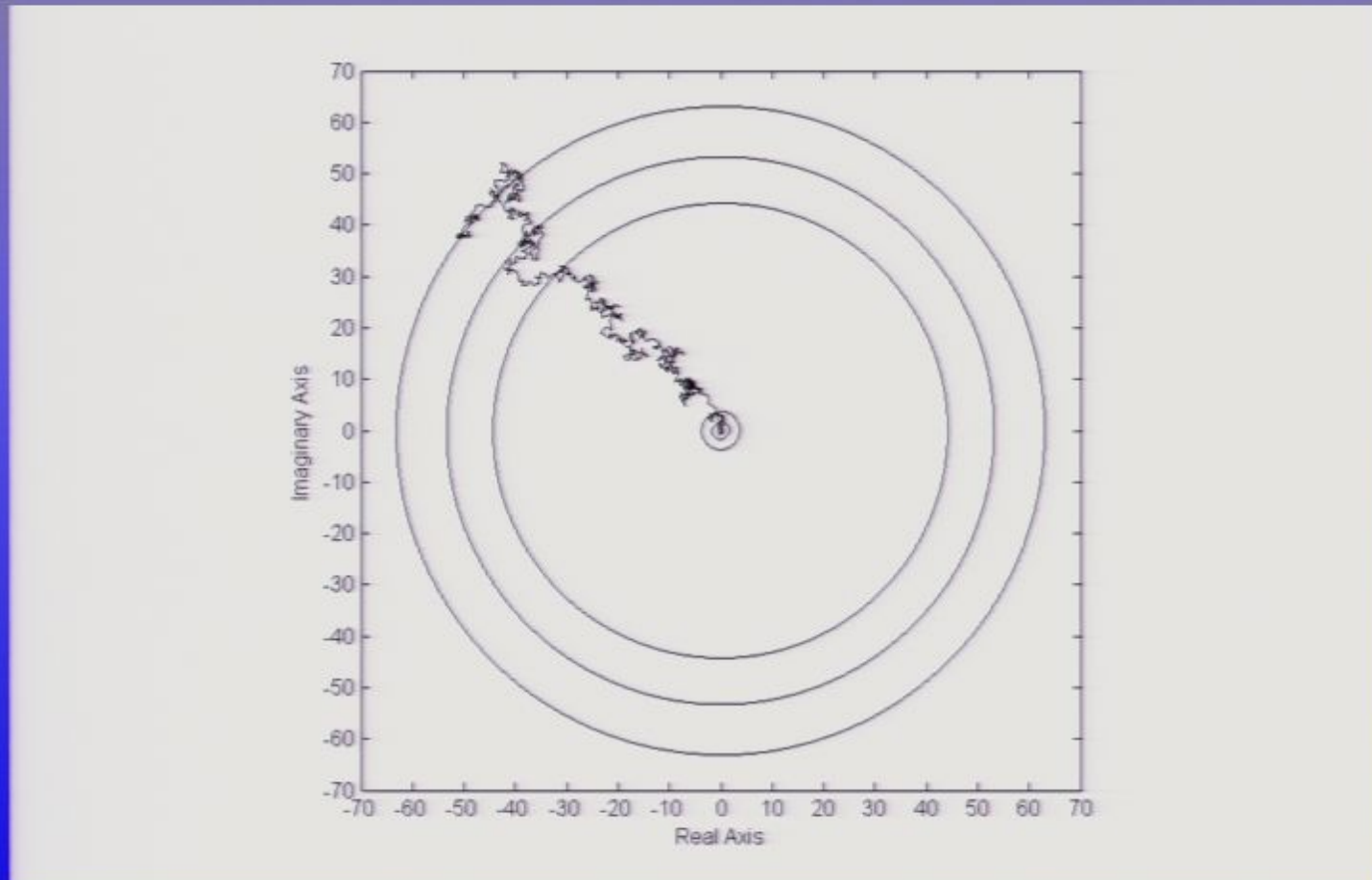
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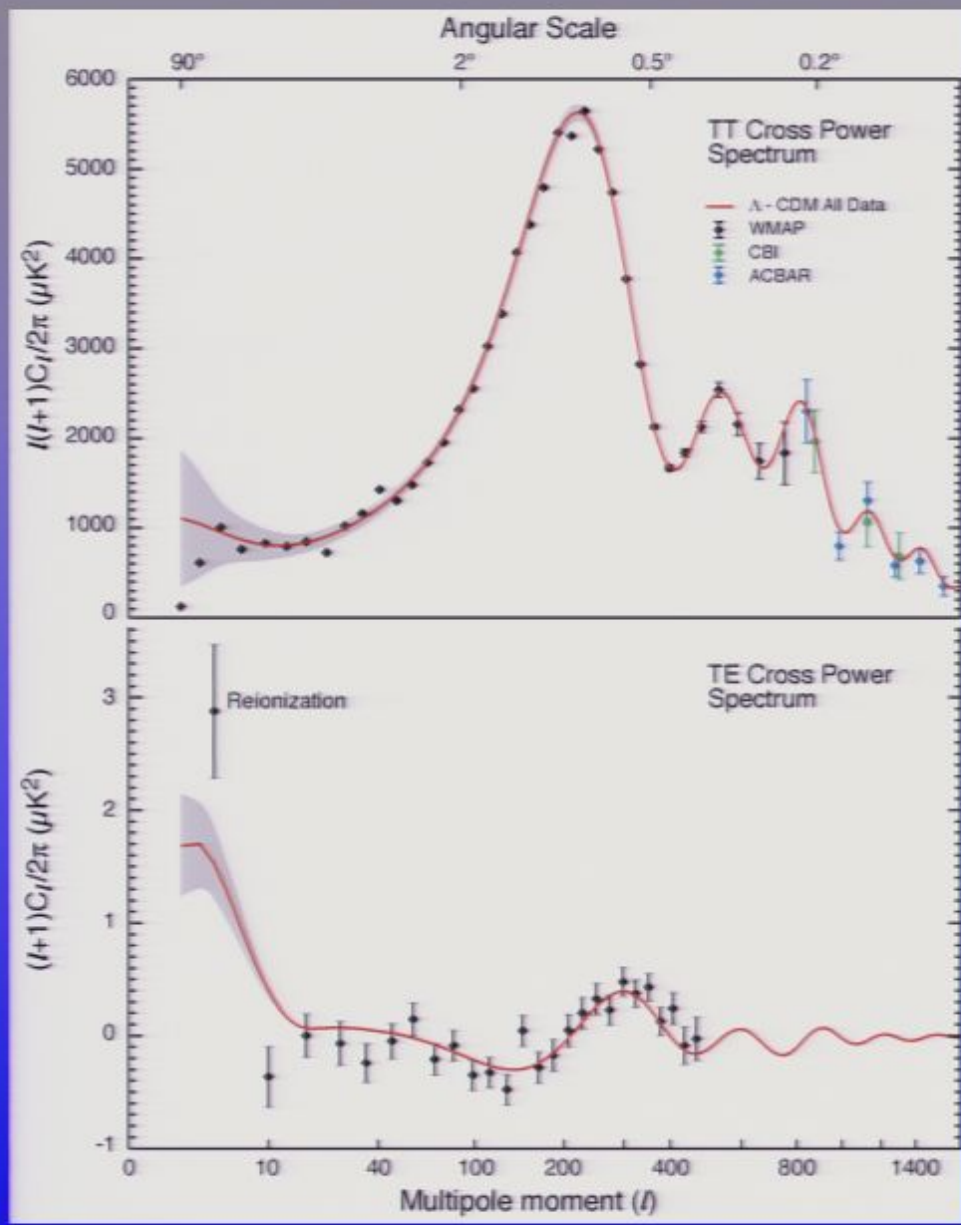
# Spherical Harmonic Phases

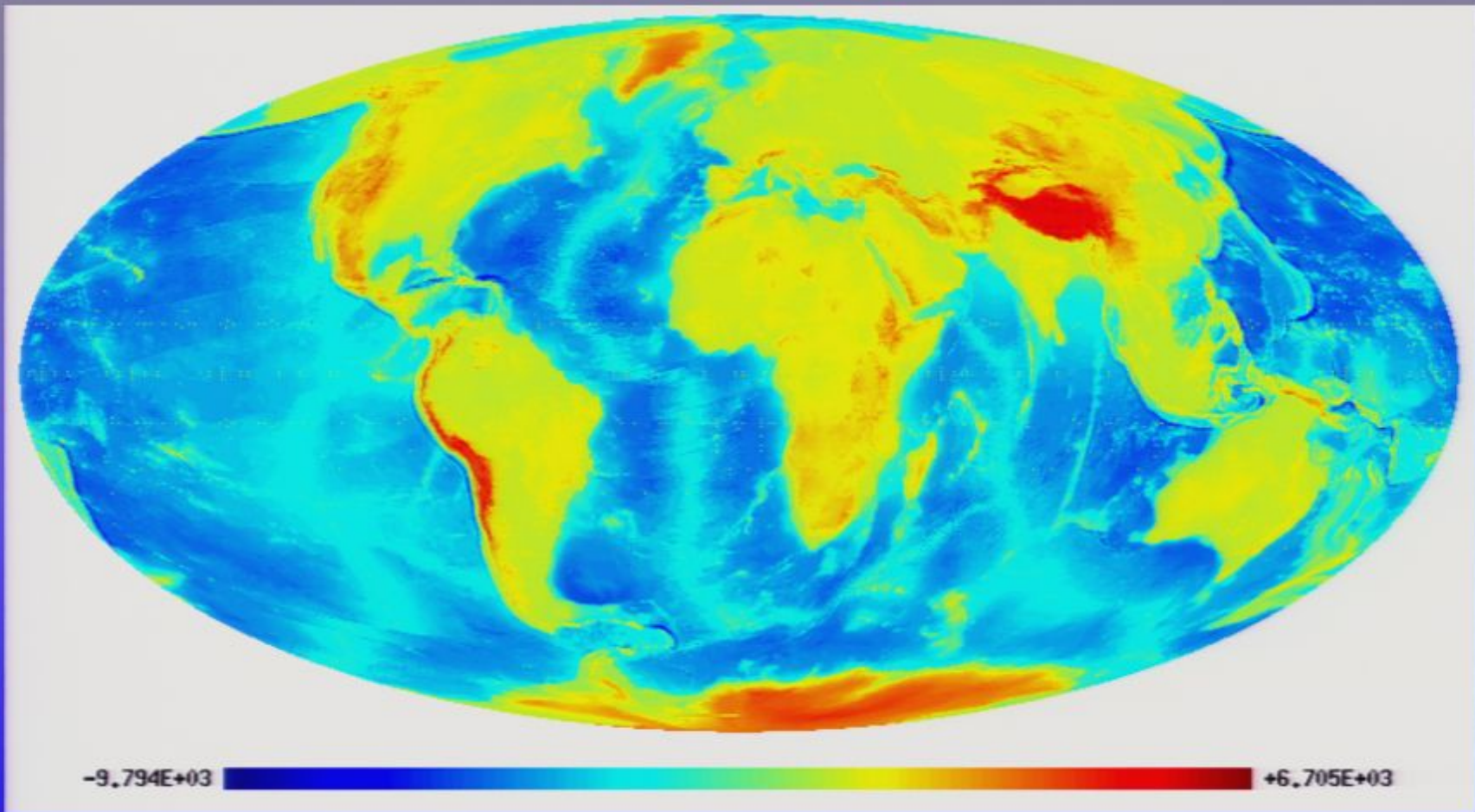
- The usual thing  $\frac{\Delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{l,m} Y_{lm}(\theta, \phi)$
- where  $a_{l,m} = |a_{l,m}| \exp[i\varphi_{l,m}]$
- If the fluctuations are a homogeneous and isotropic GRF then the phases  $\varphi_{l,m}$  are random...
- ..apart from  $a_{l,m}^* = a_{l,-m}$
- ..as are differences, e.g.  $\varphi_{l,m} - \varphi_{l,m-1}$

# Random walks in Spherical Harmonic Space

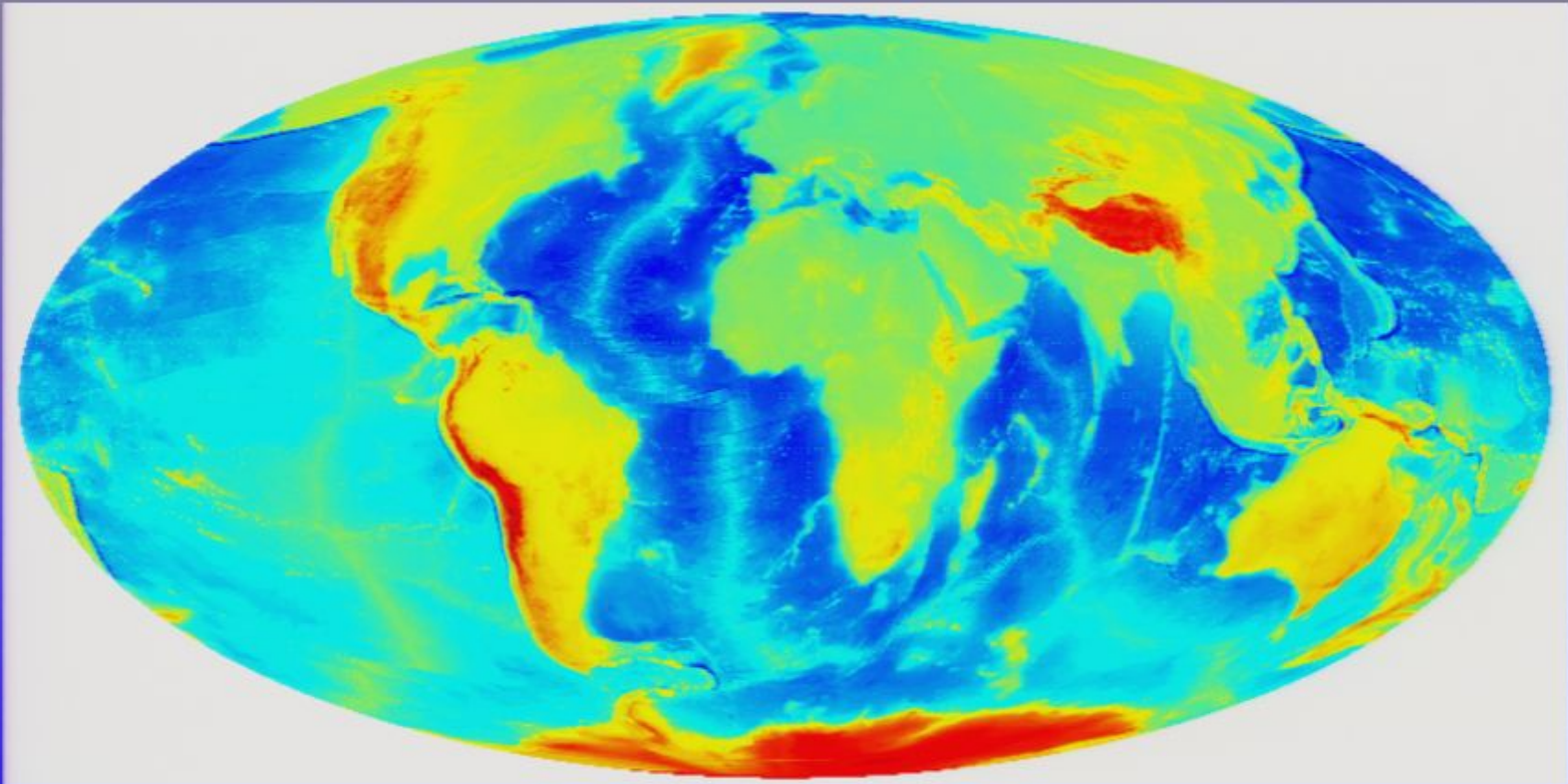
(Stannard & Coles, MNRAS, in press, astro-ph/0410633)

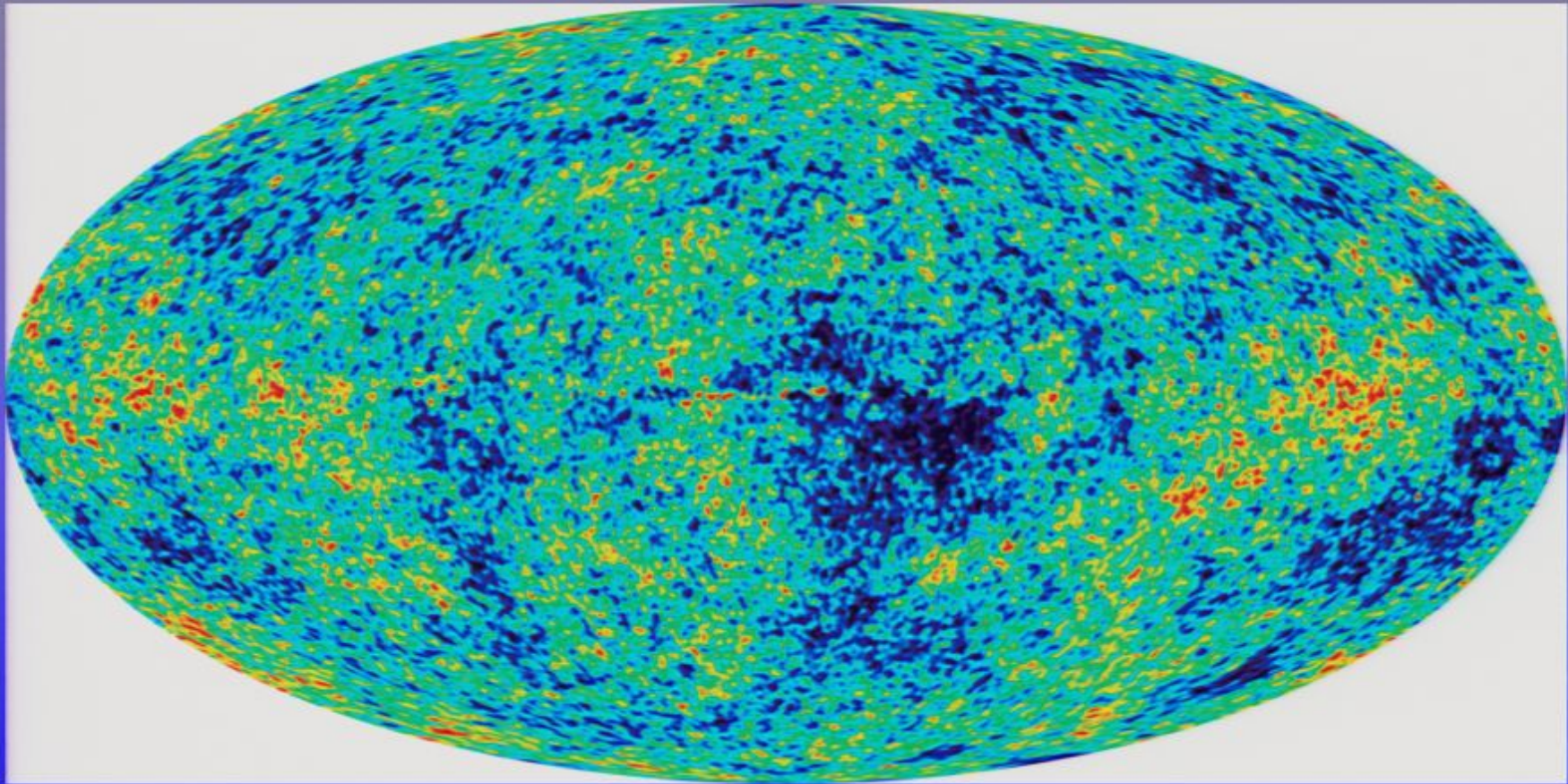


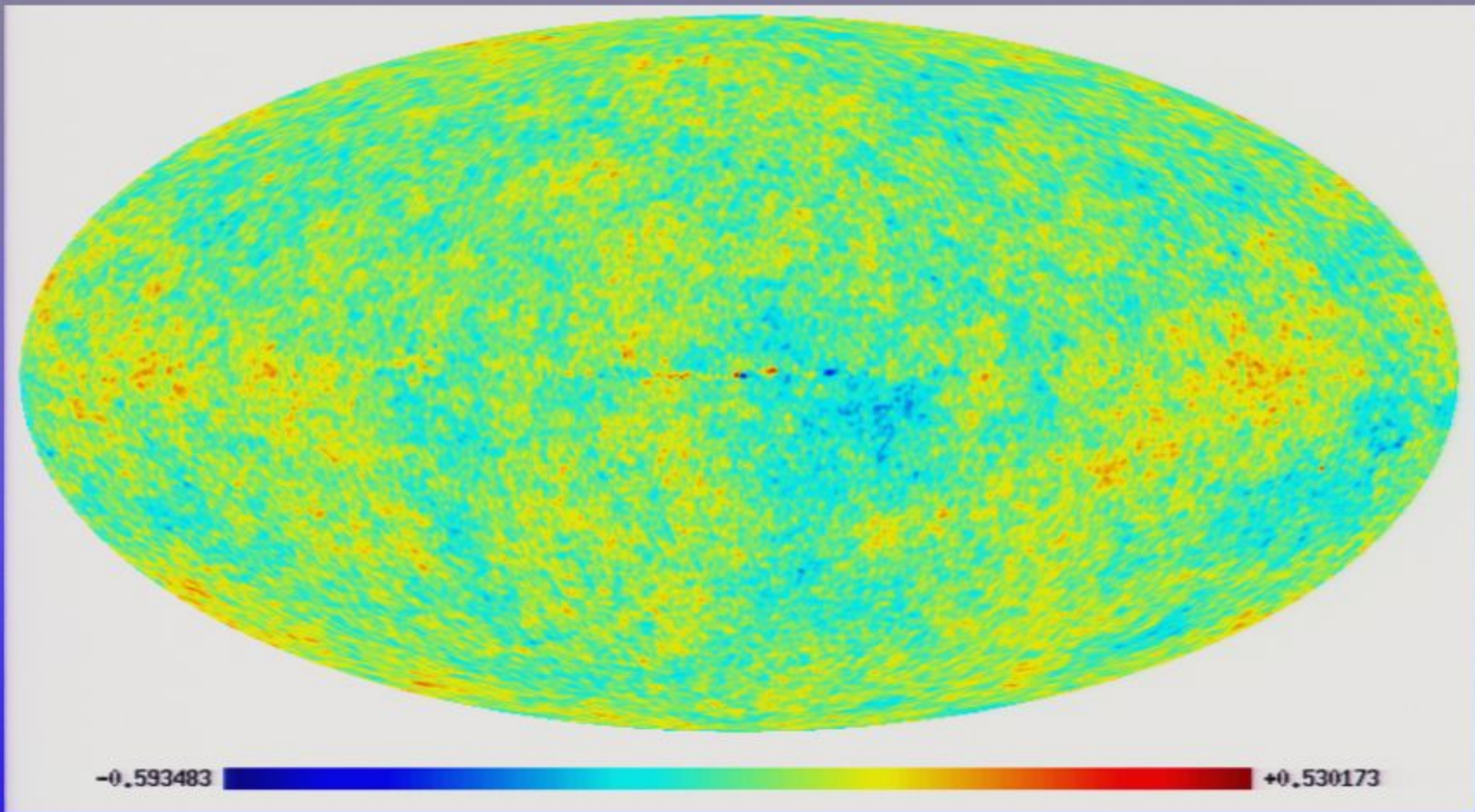


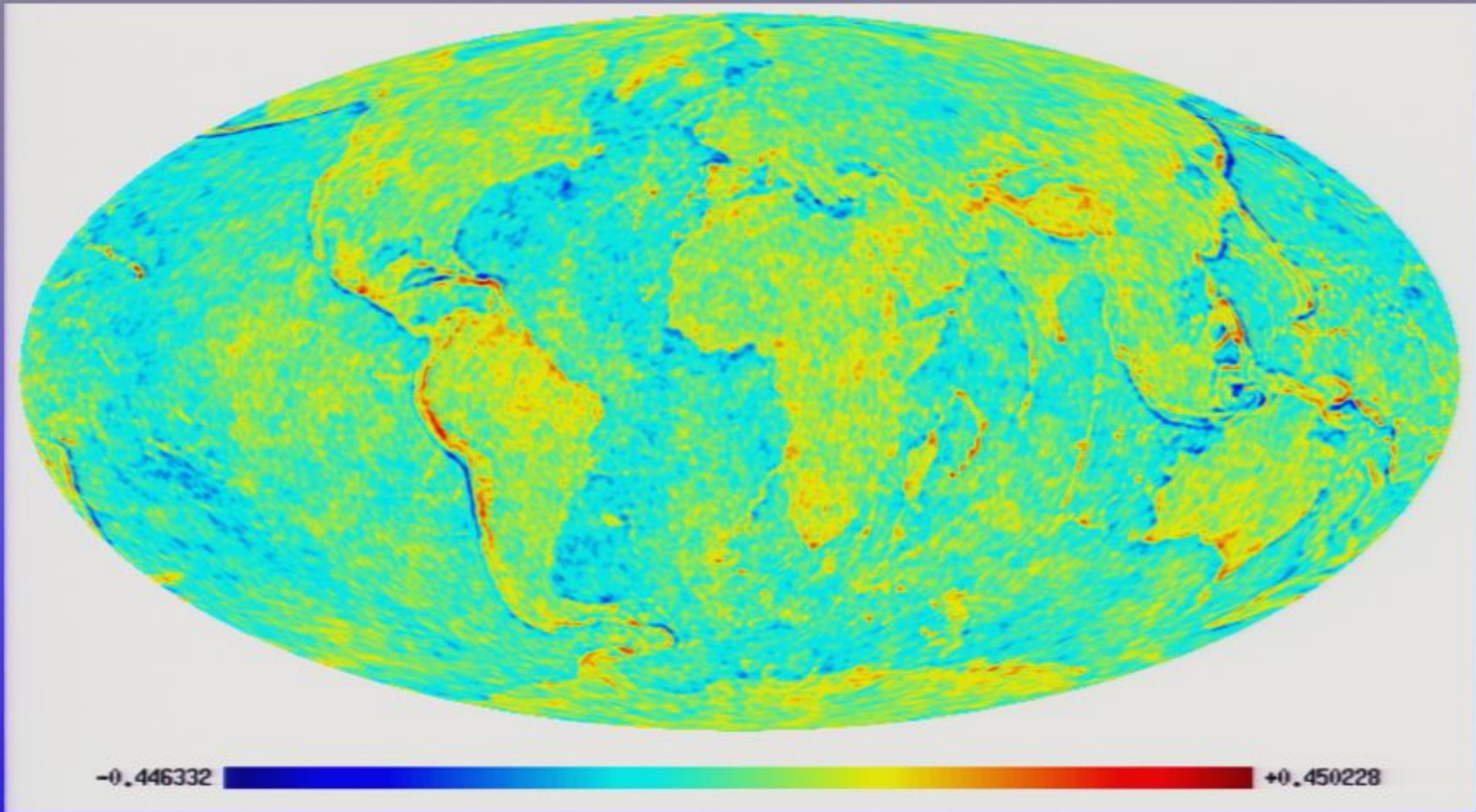


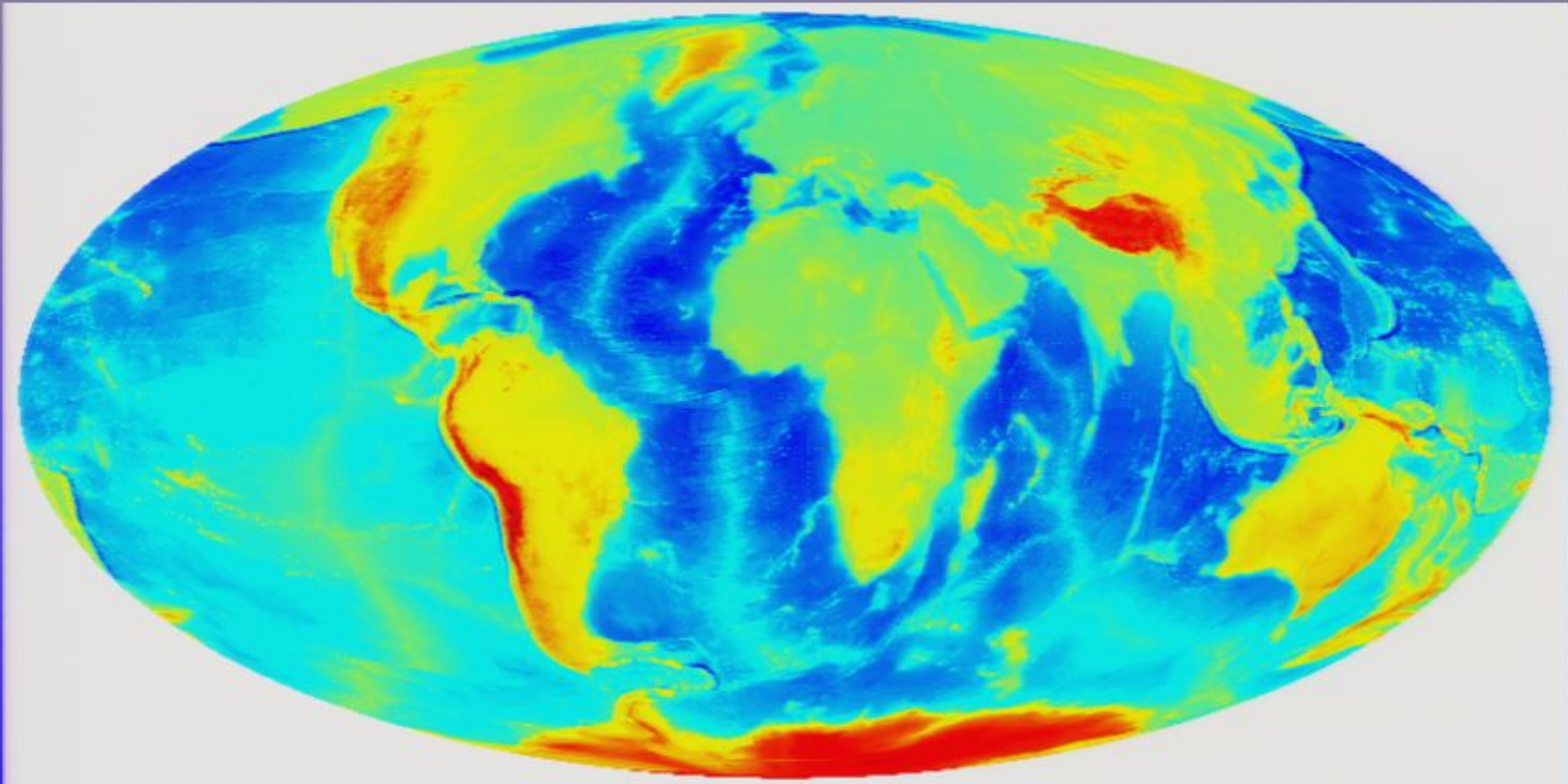


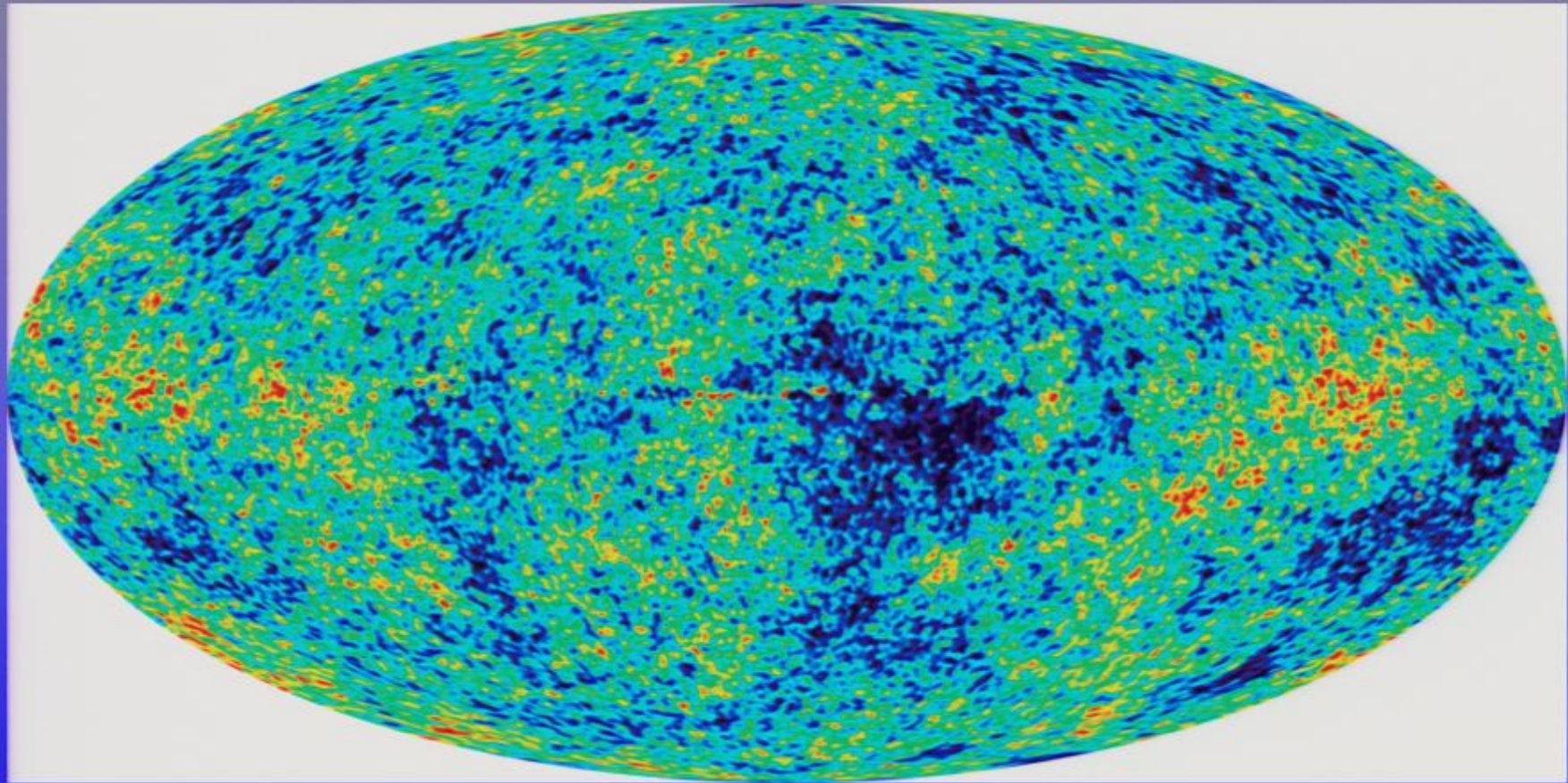


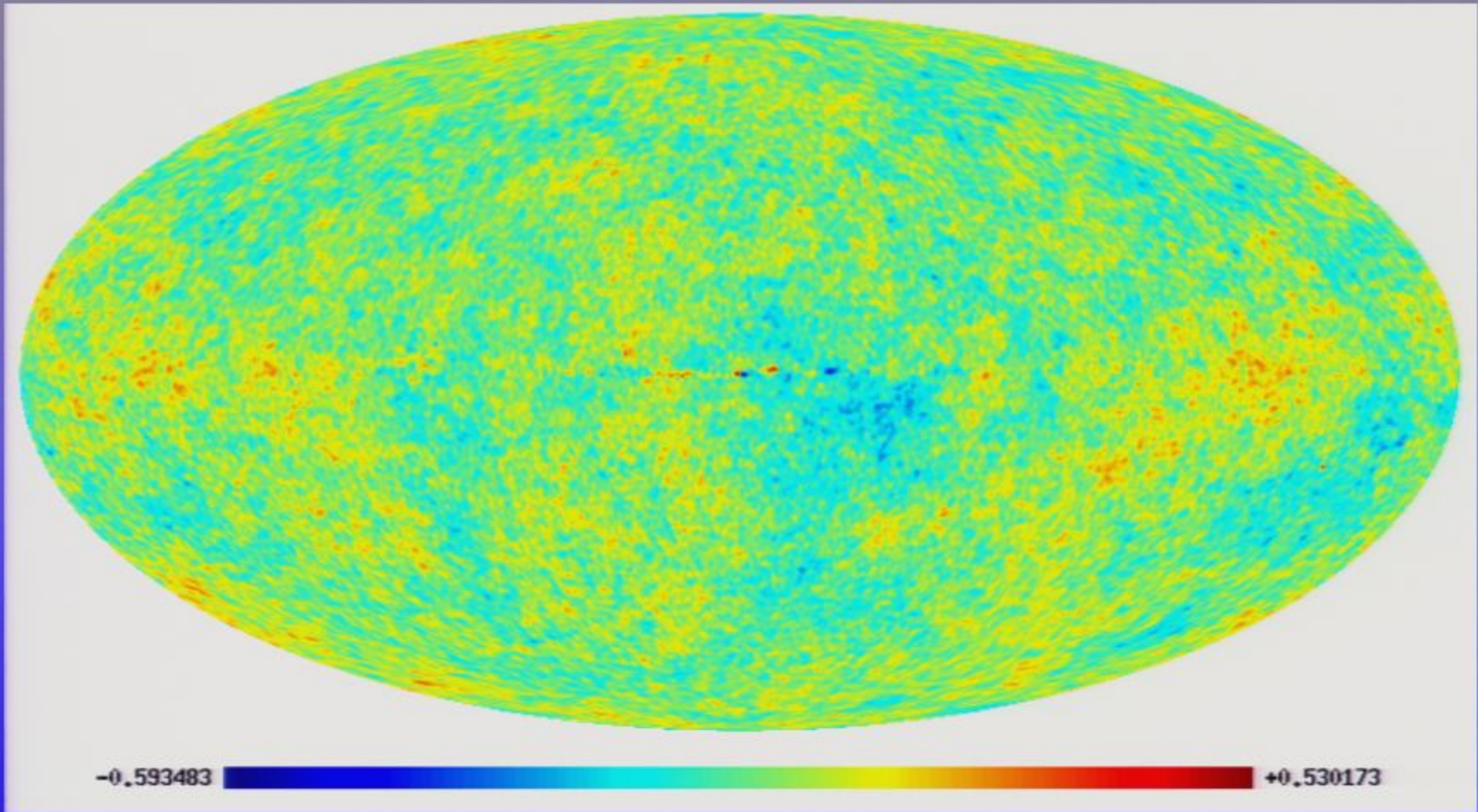


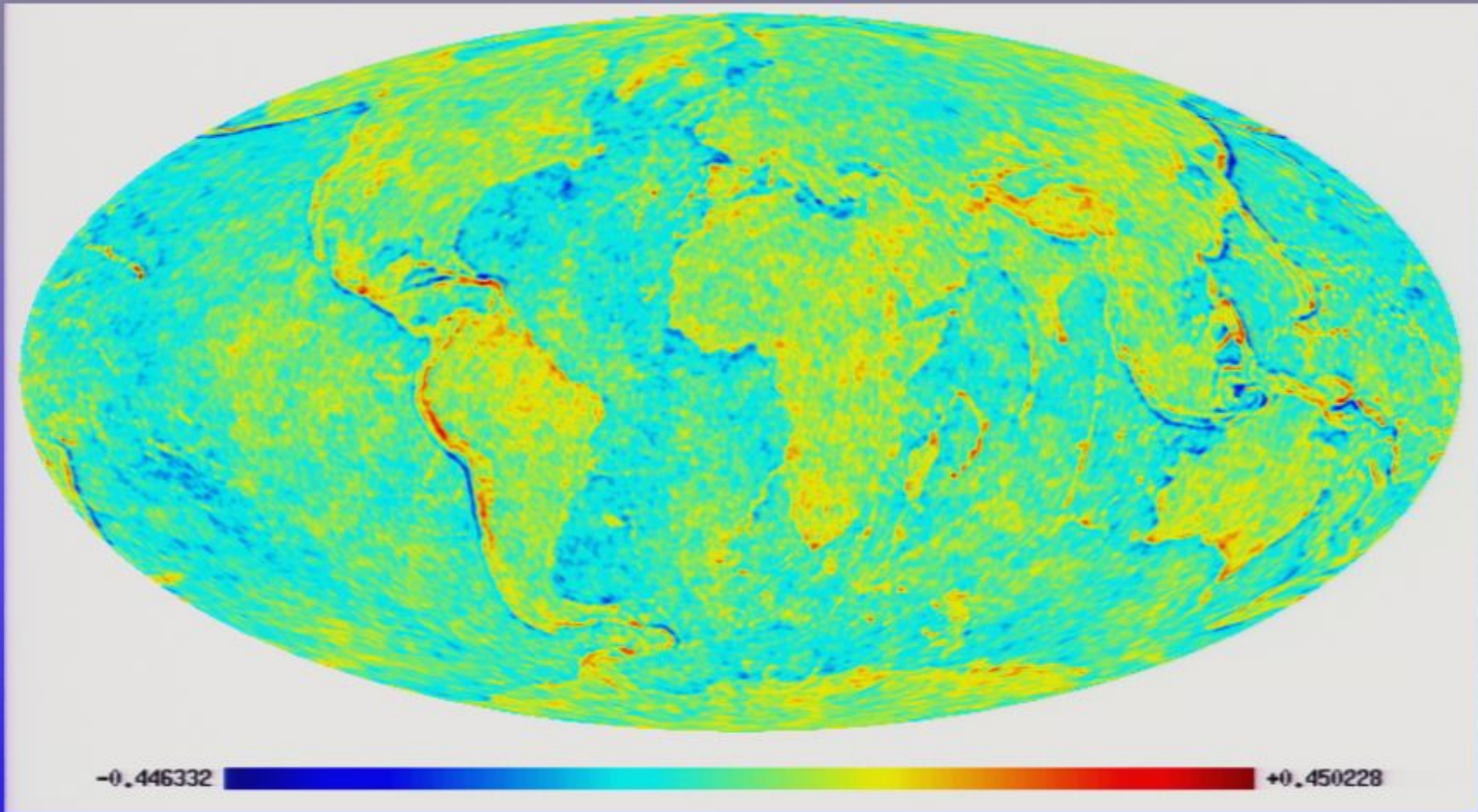




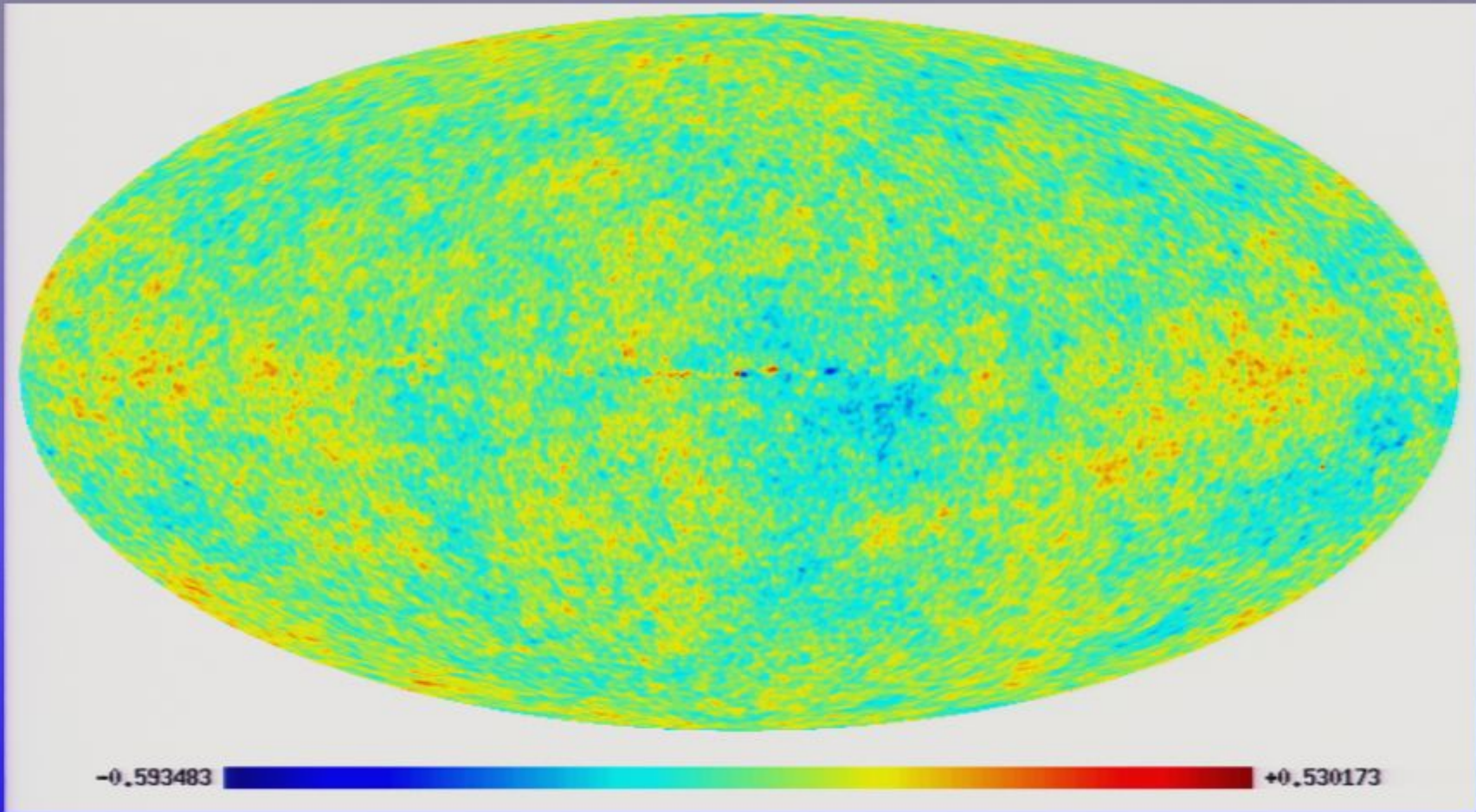


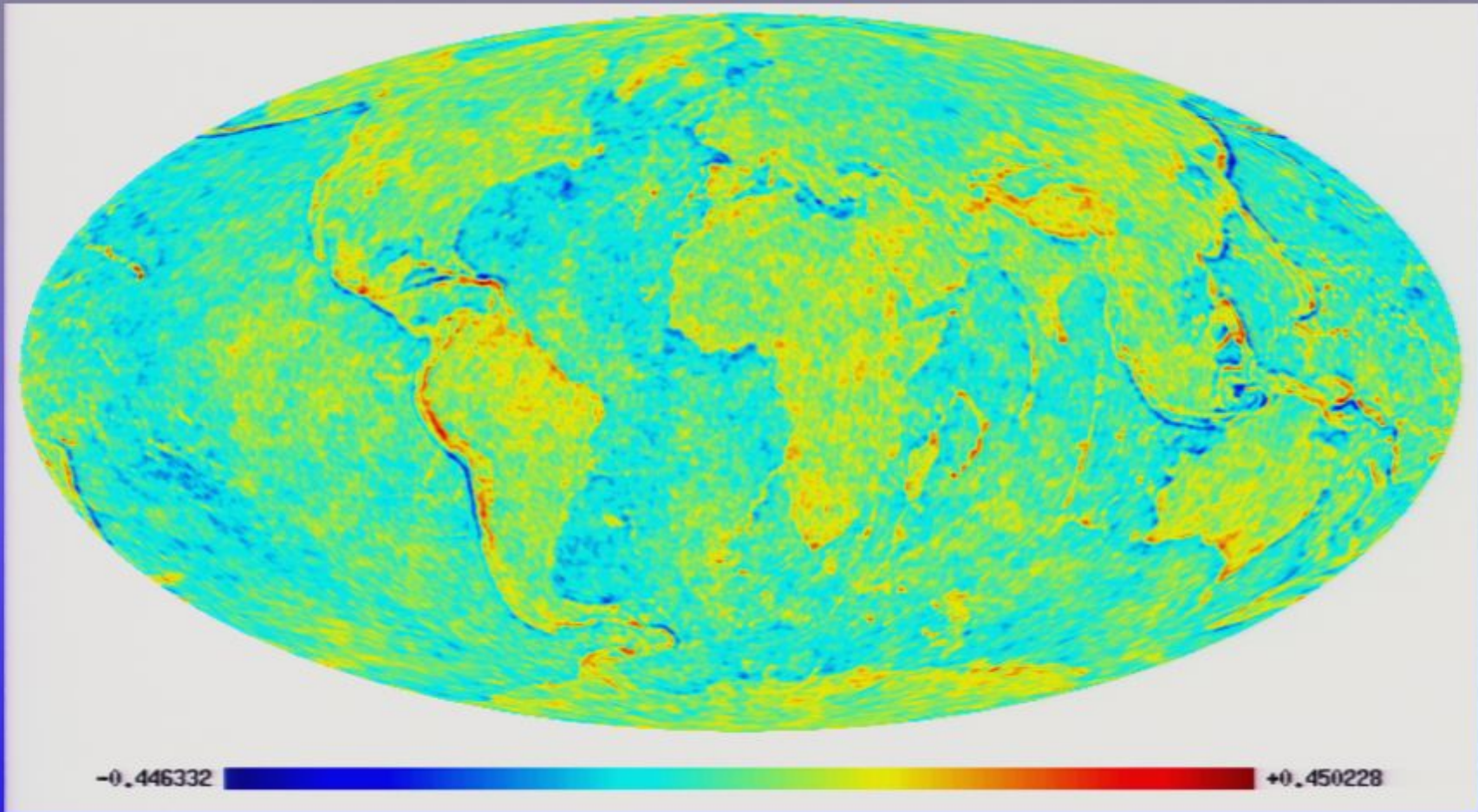


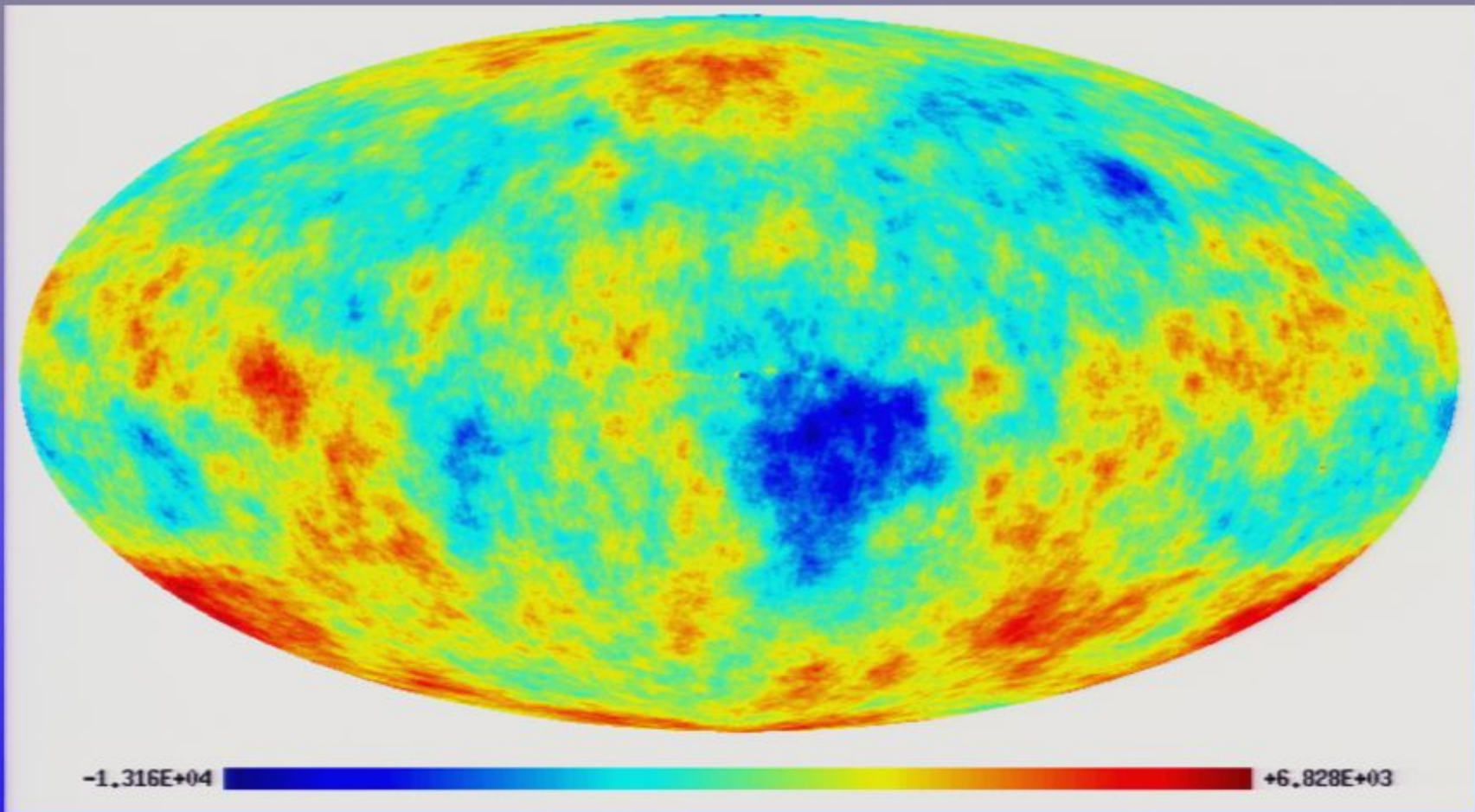


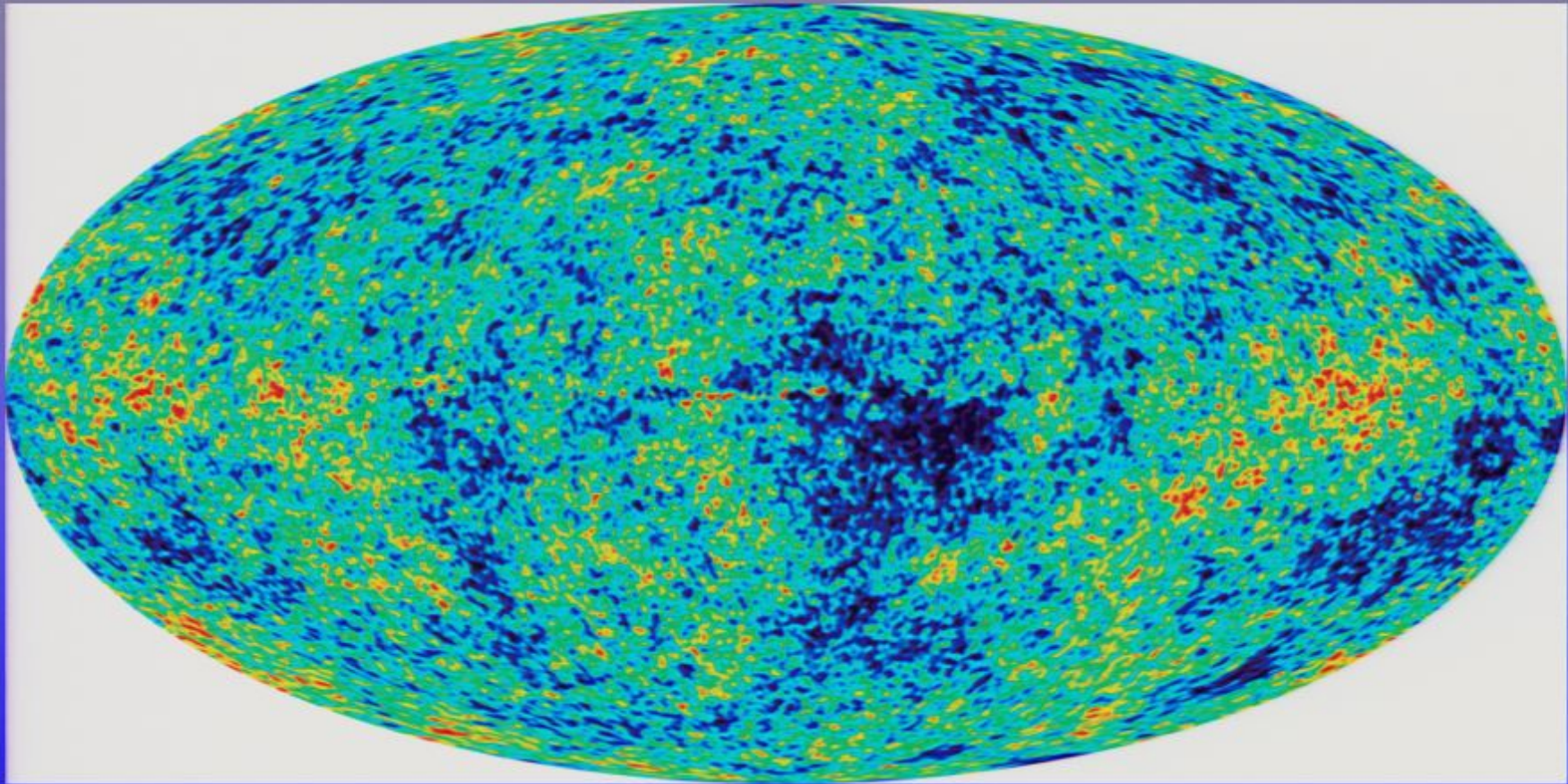


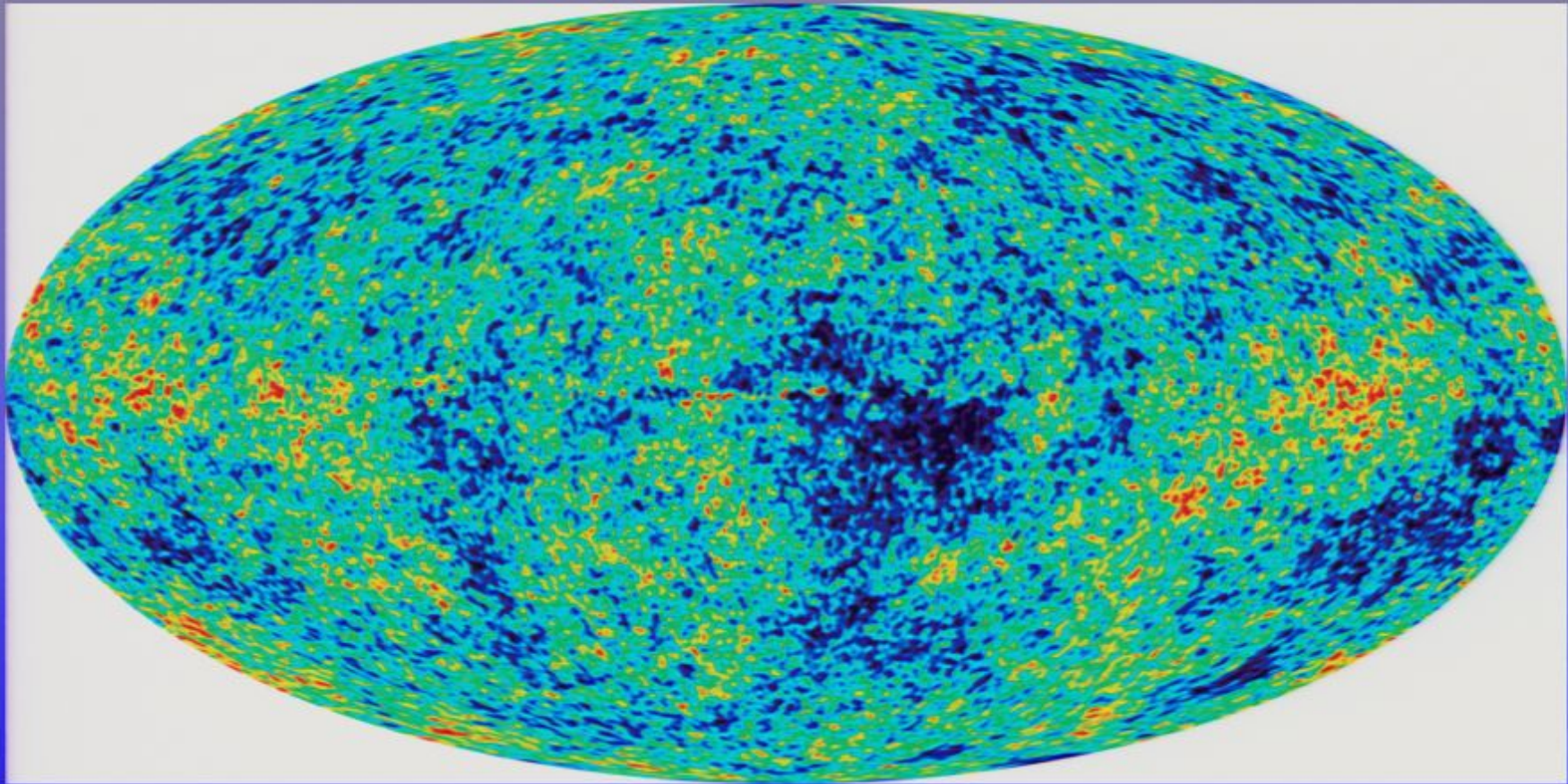


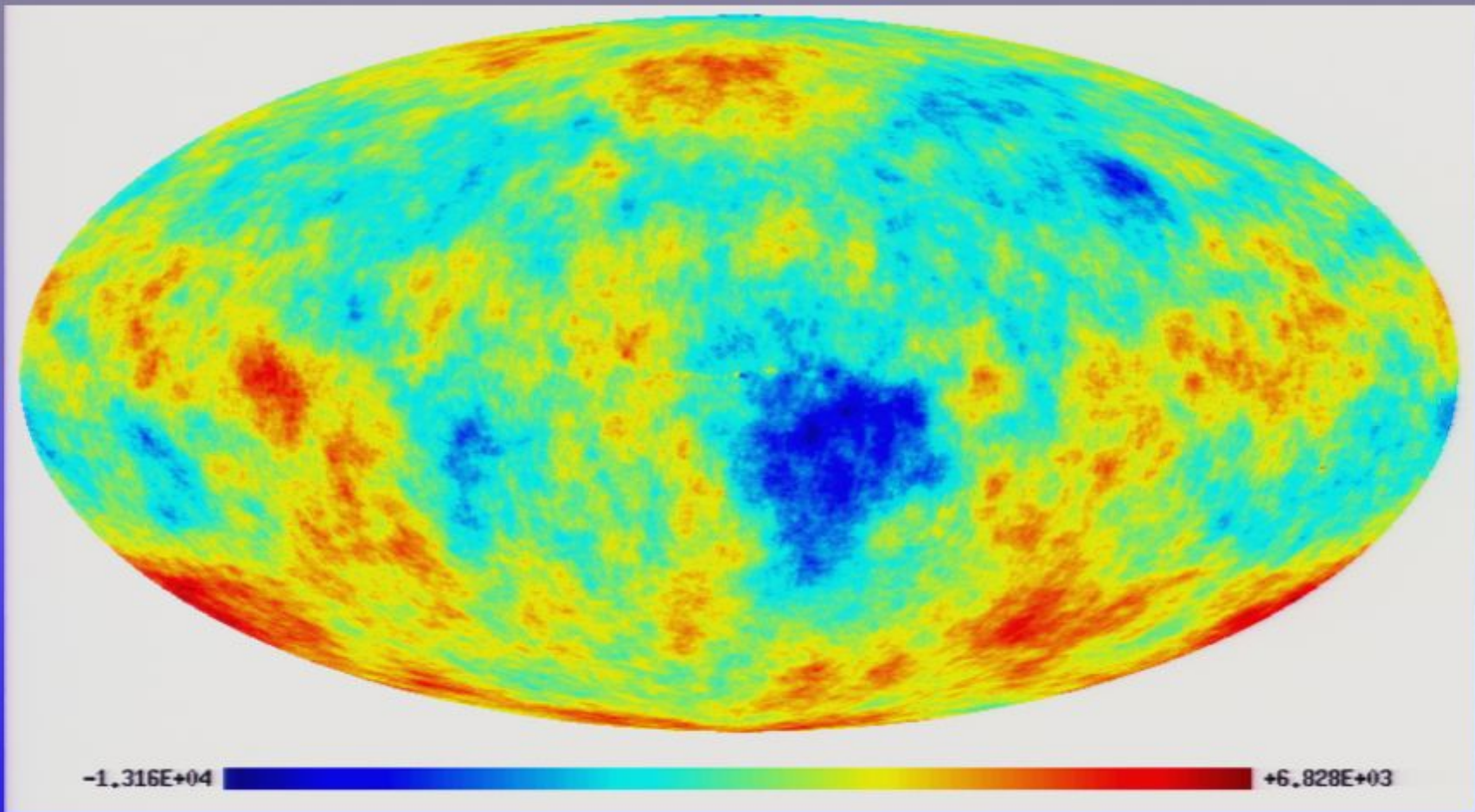


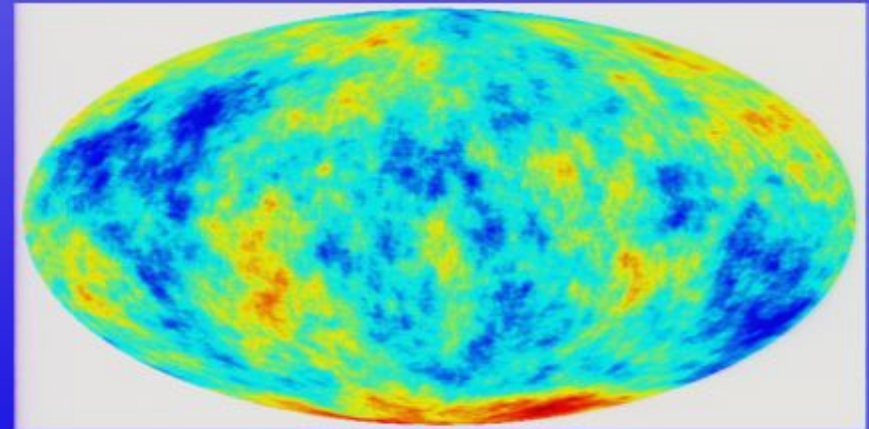
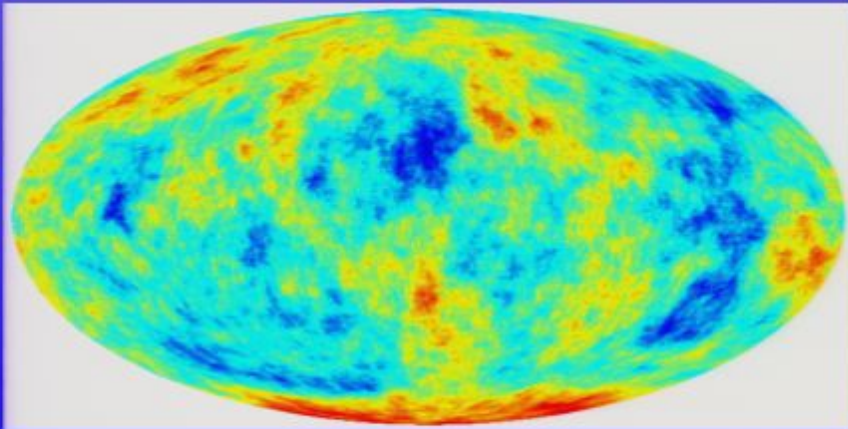
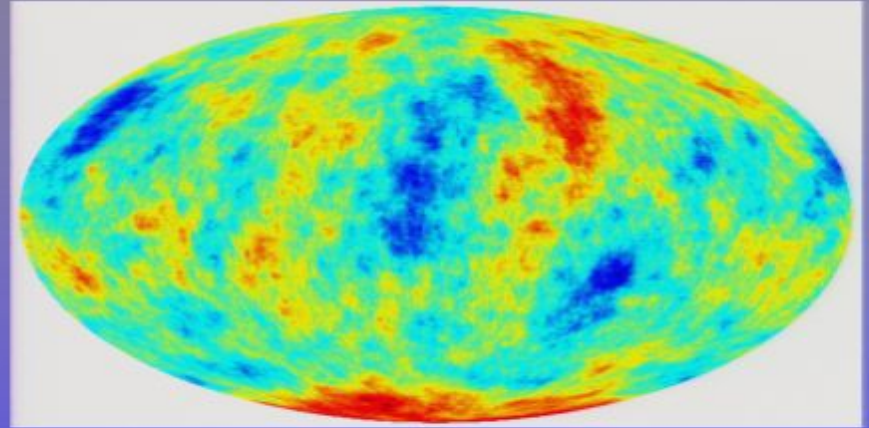
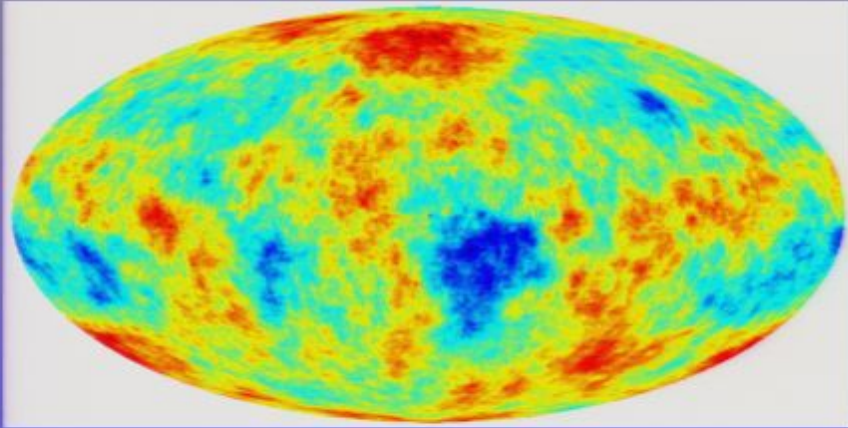












# Comments

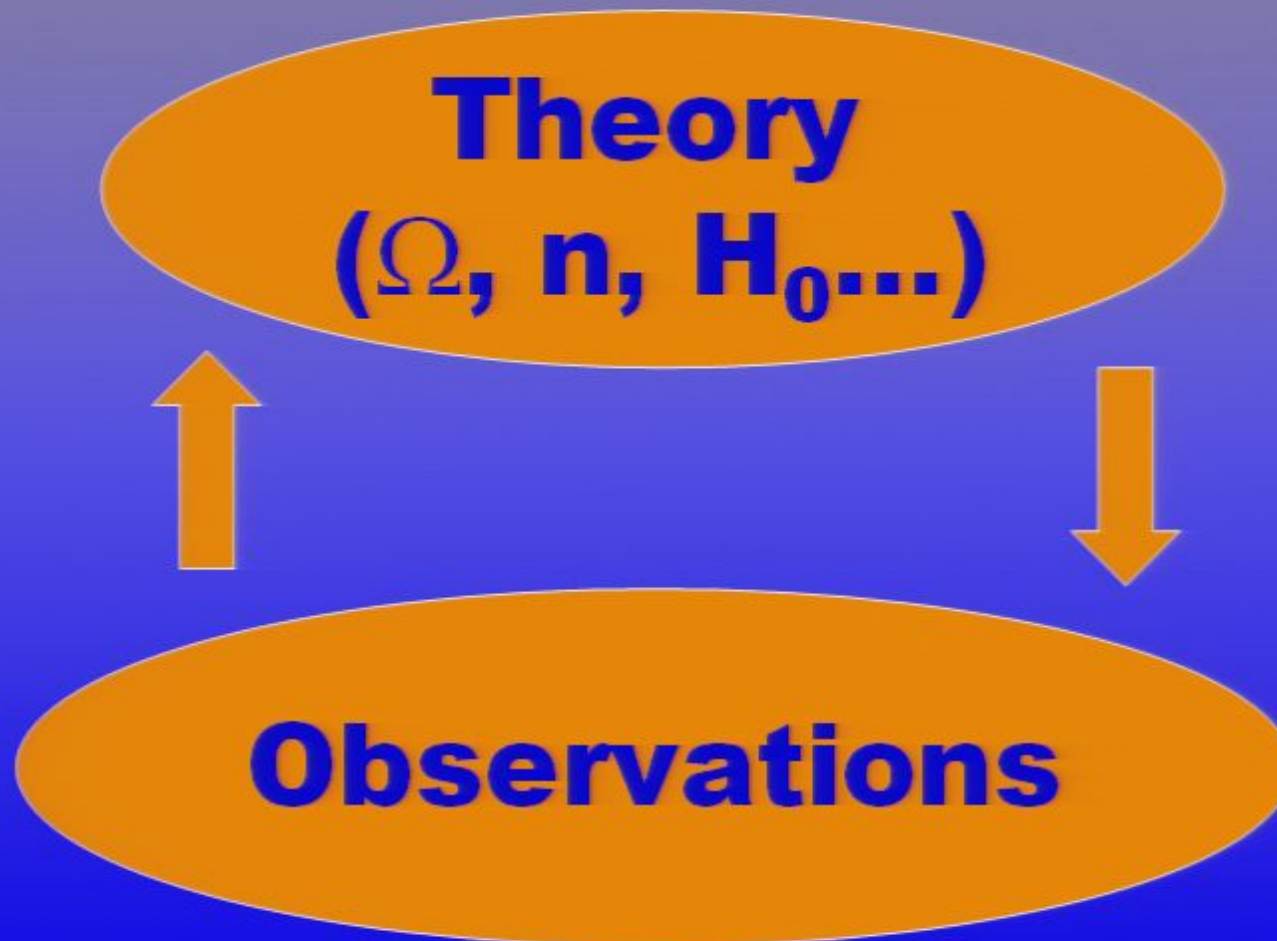
- The simplest statistics are distributions of phases and phase-differences: these are highly sensitive to departures from statistical homogeneity.
- Different signatures are more sensitive to non-Gaussianity in statistically homogeneous fields.
- BUT the phases vary in a complicated way under rotations
- AND they will not be random if there is a mask
- This is OK for a non-parametric approach, since it can all be included in fast Monte Carlo simulations



# Apologies to Bayesians

- This is a frequentist approach...
- If you have a sufficiently well-developed alternative model, be Bayesian and infer parameters of a model (evidence, etc)
- If you don't, you simply have to try rejecting the null hypothesis using non-parametric methods
- It's a question of whether you test in model space or data space!

# Inference vs Hypothesis Testing



# Kuiper's Statistic

- Non-parametric test for uniformity on the unit circle..c.f Kolmogorov-Smirnov

- Define

$$X_i = g_i / 2\pi$$

- Then

$$S_n^+ = \max \left\{ \frac{1}{n} - X_1, \frac{2}{n} - X_2, \dots, 1 - X_n \right\}$$

- and

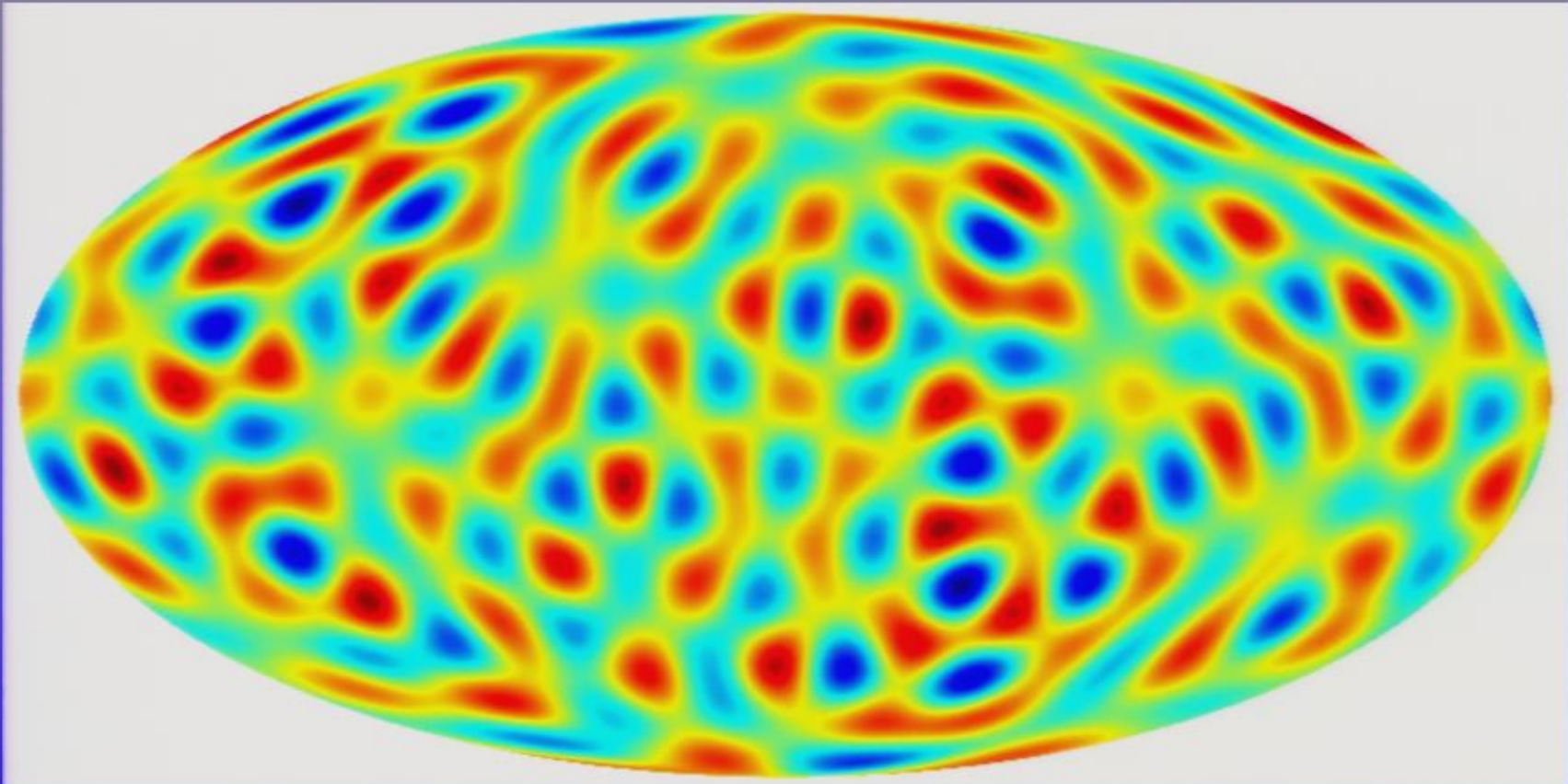
$$S_n^- = \max \left\{ X_1, X_2 - \frac{1}{n}, \dots, X_n - \frac{n-1}{n} \right\}$$

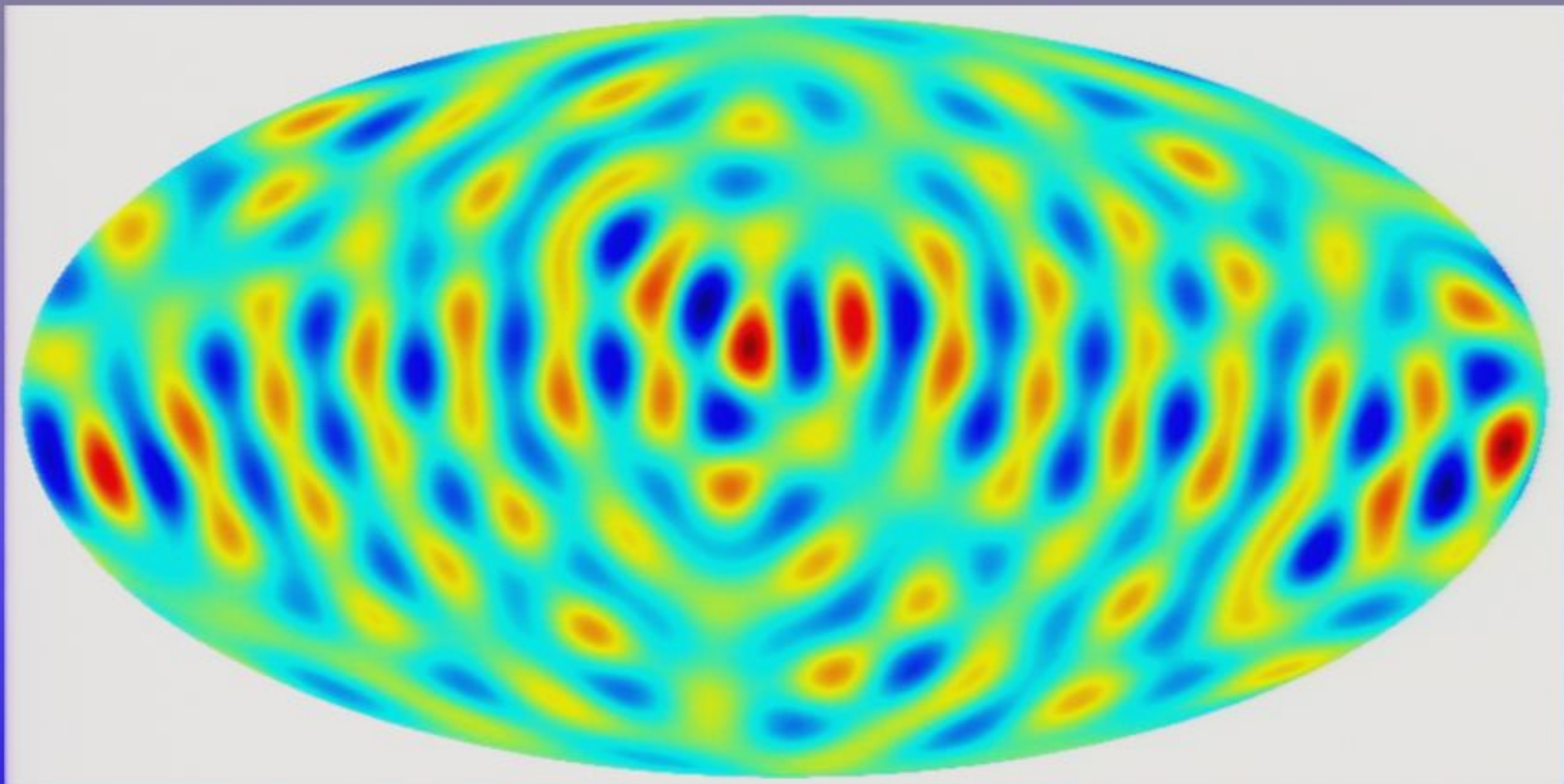
- the statistic is

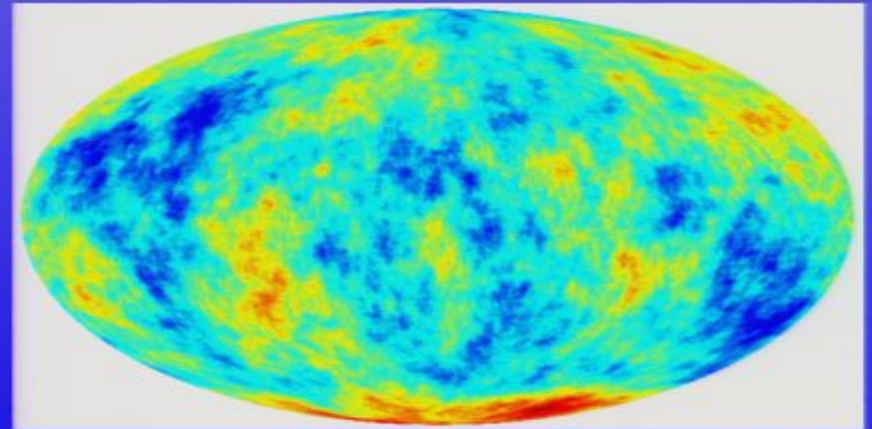
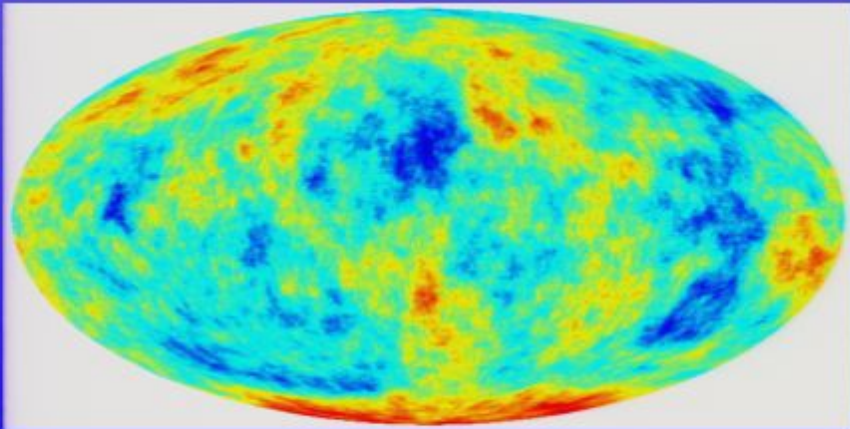
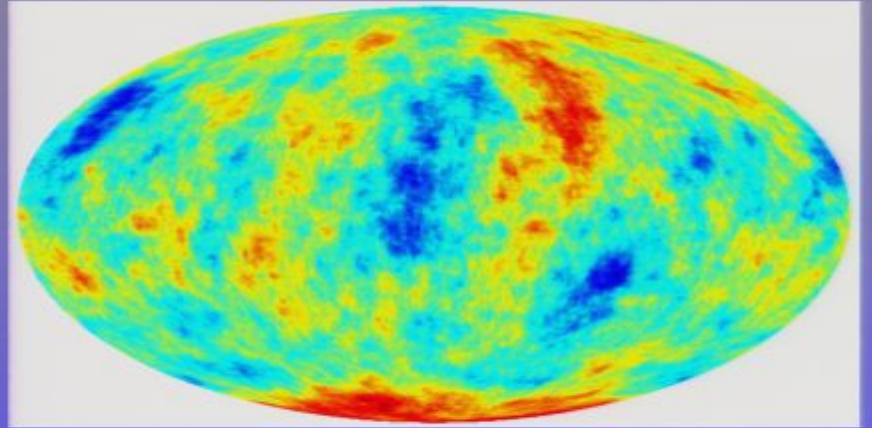
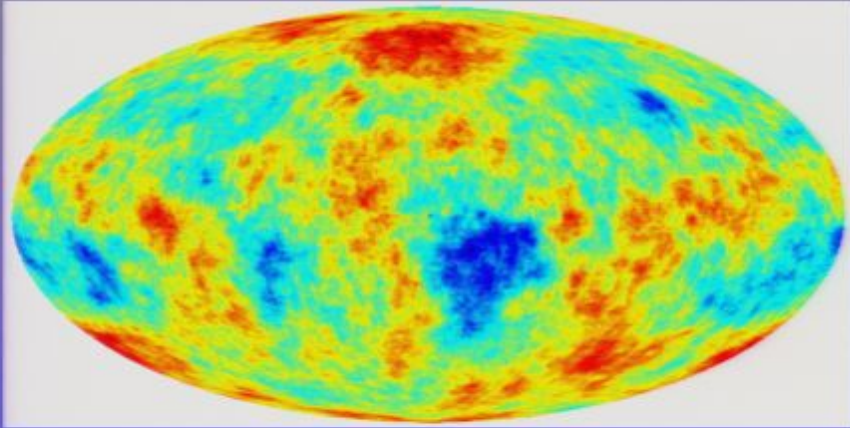
$$V = [S_n^+ + S_n^-] A(n)$$

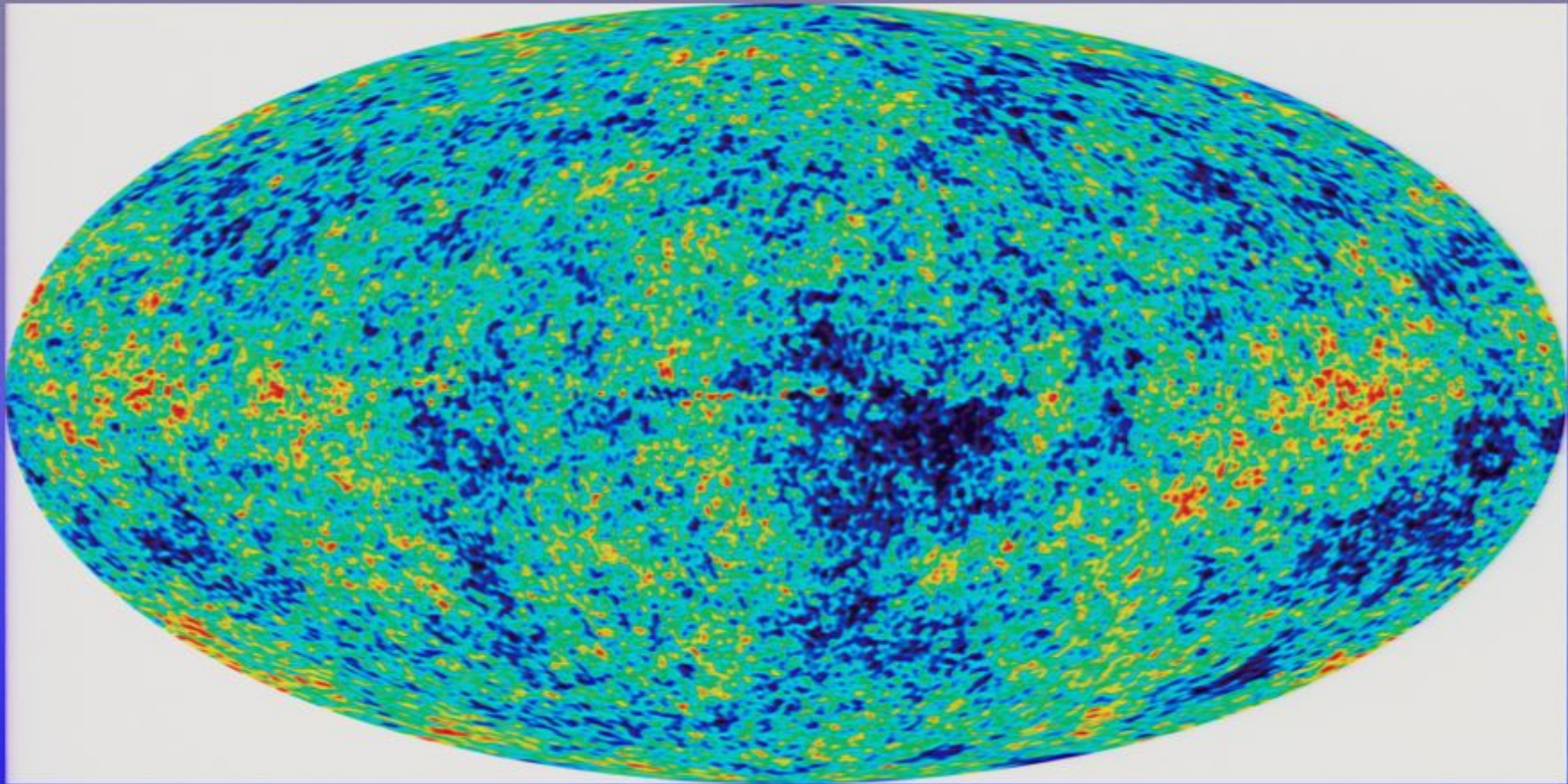
# Results

- Simplest thing is to do this with phase differences
- This is not a very good test of quadratic non-Gaussianity, but is good for statistical isotropy
- The ILC, TOH etc are all weird at  $l=16$
- biggest departure in phase differences at fixed  $l$ , rather than fixed  $m$
- COBE DMR is weird at  $l=10!$
- More details in Coles et al., 2004, MNRAS, 350, 989

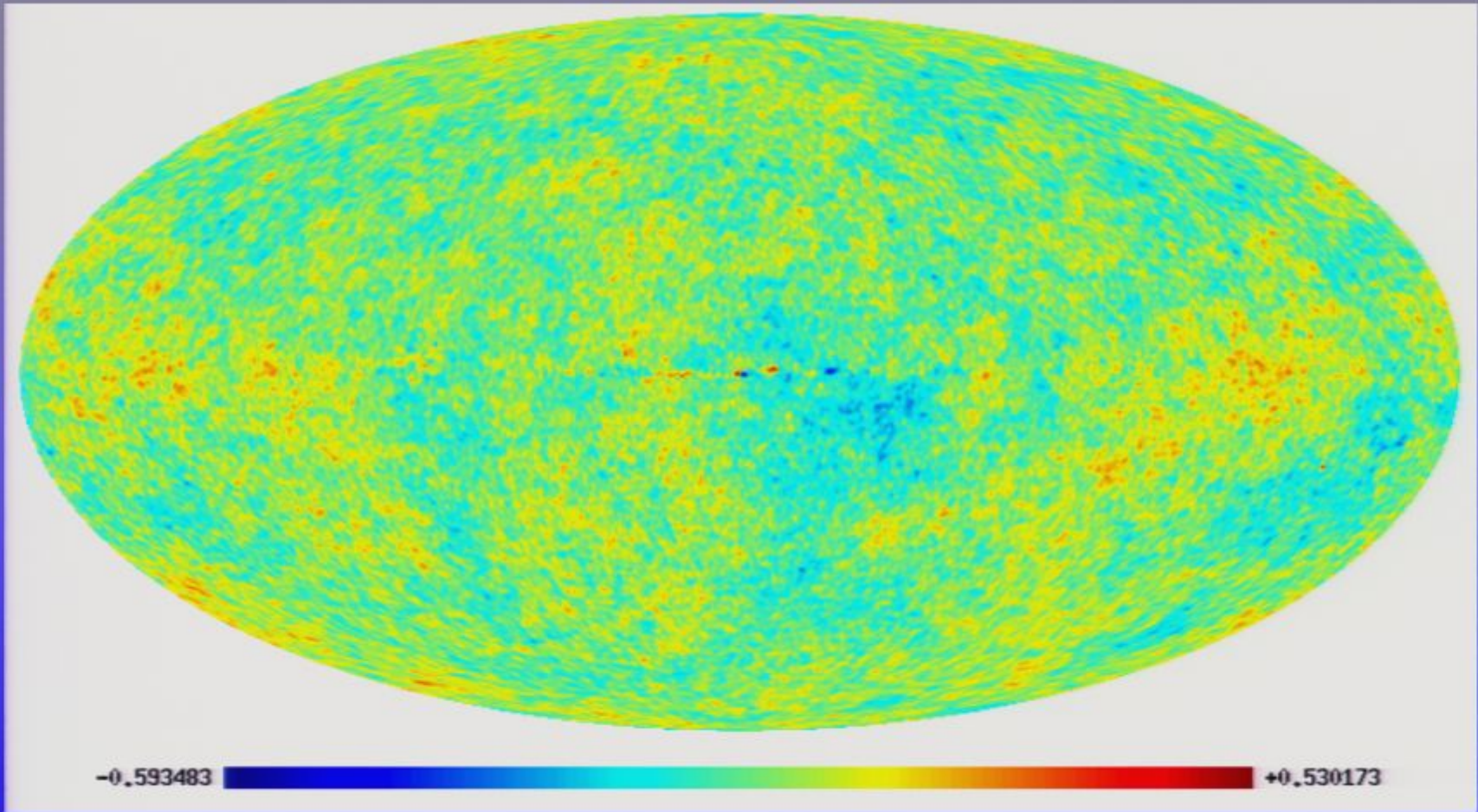




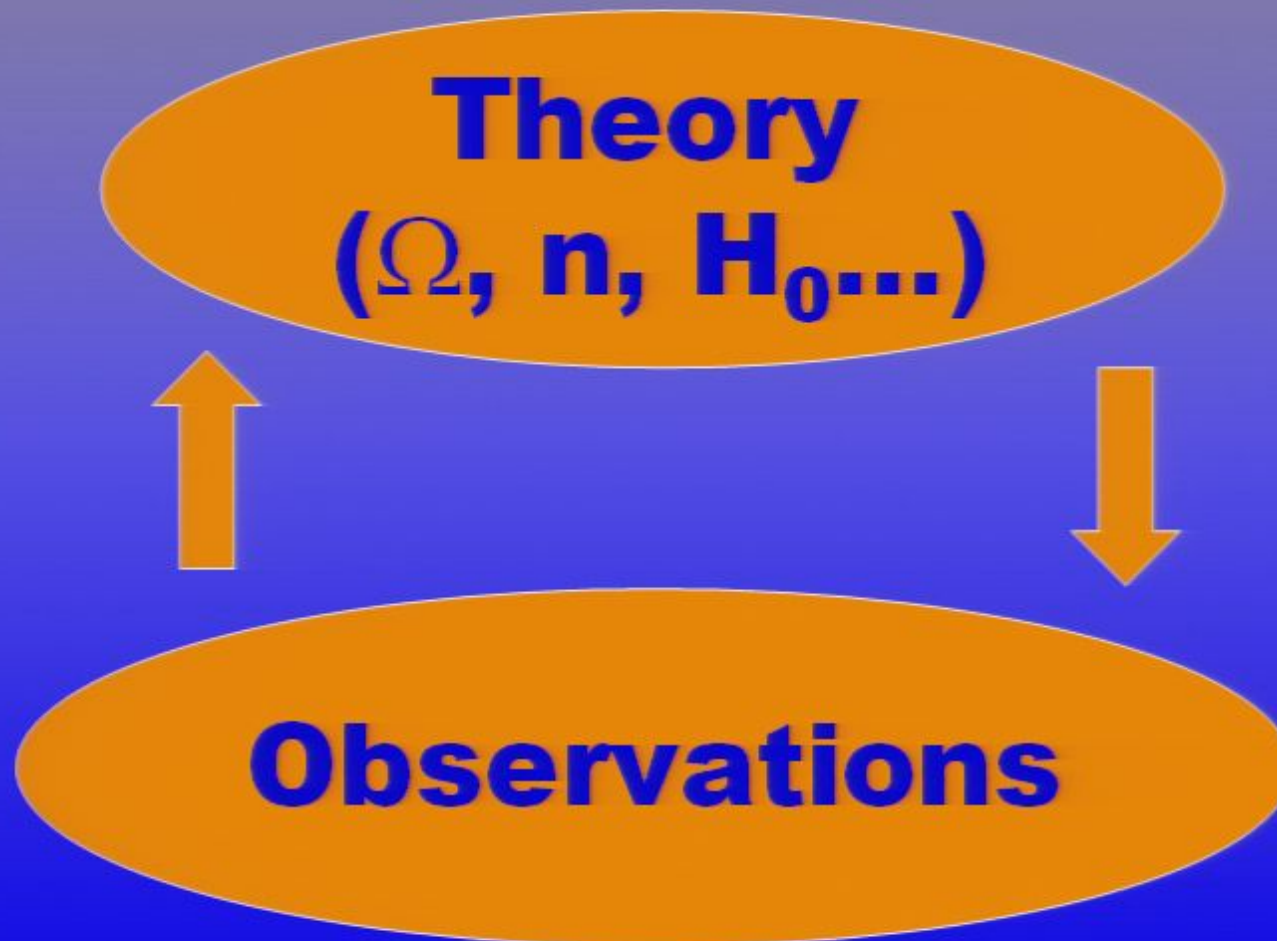


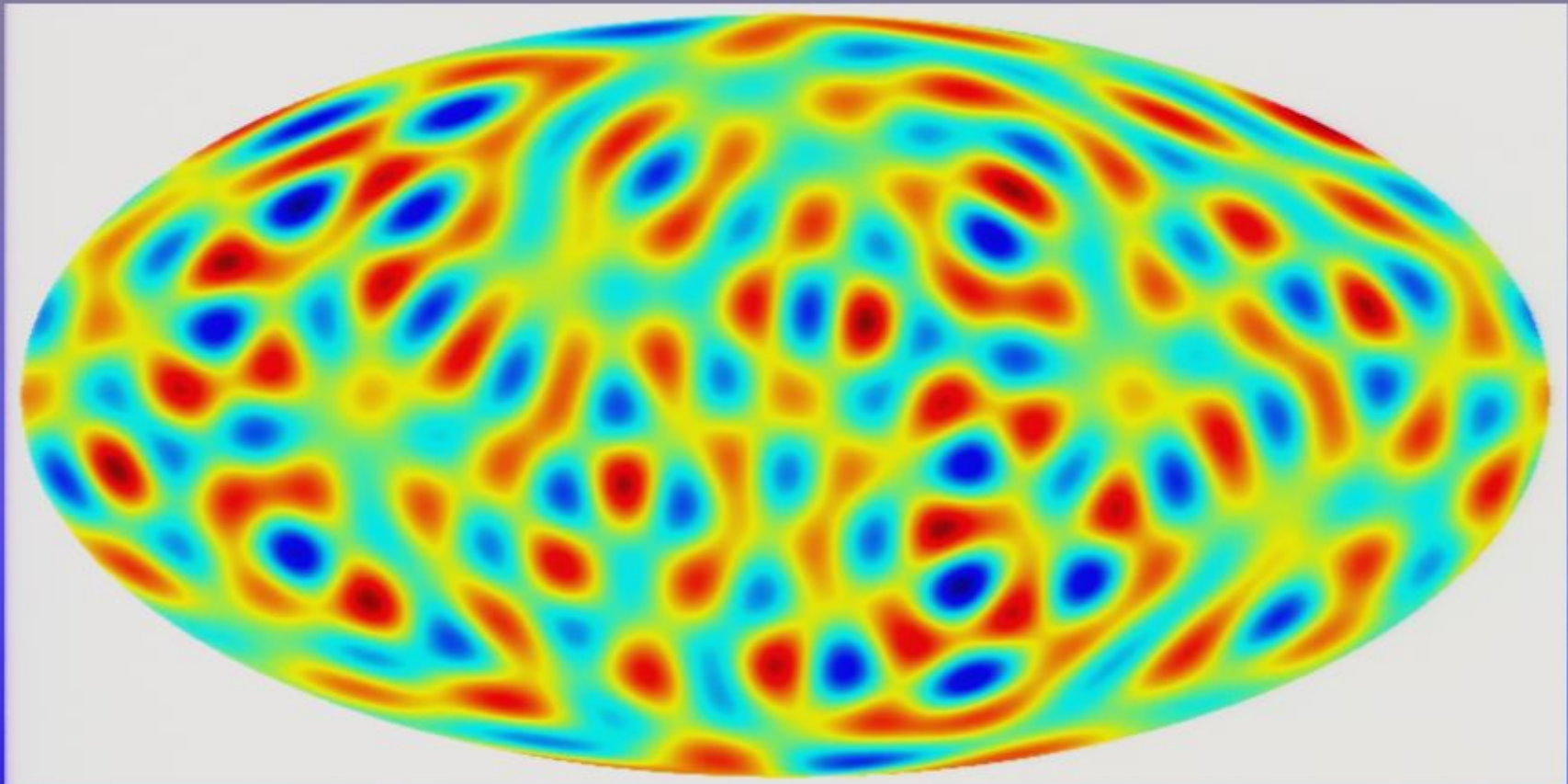






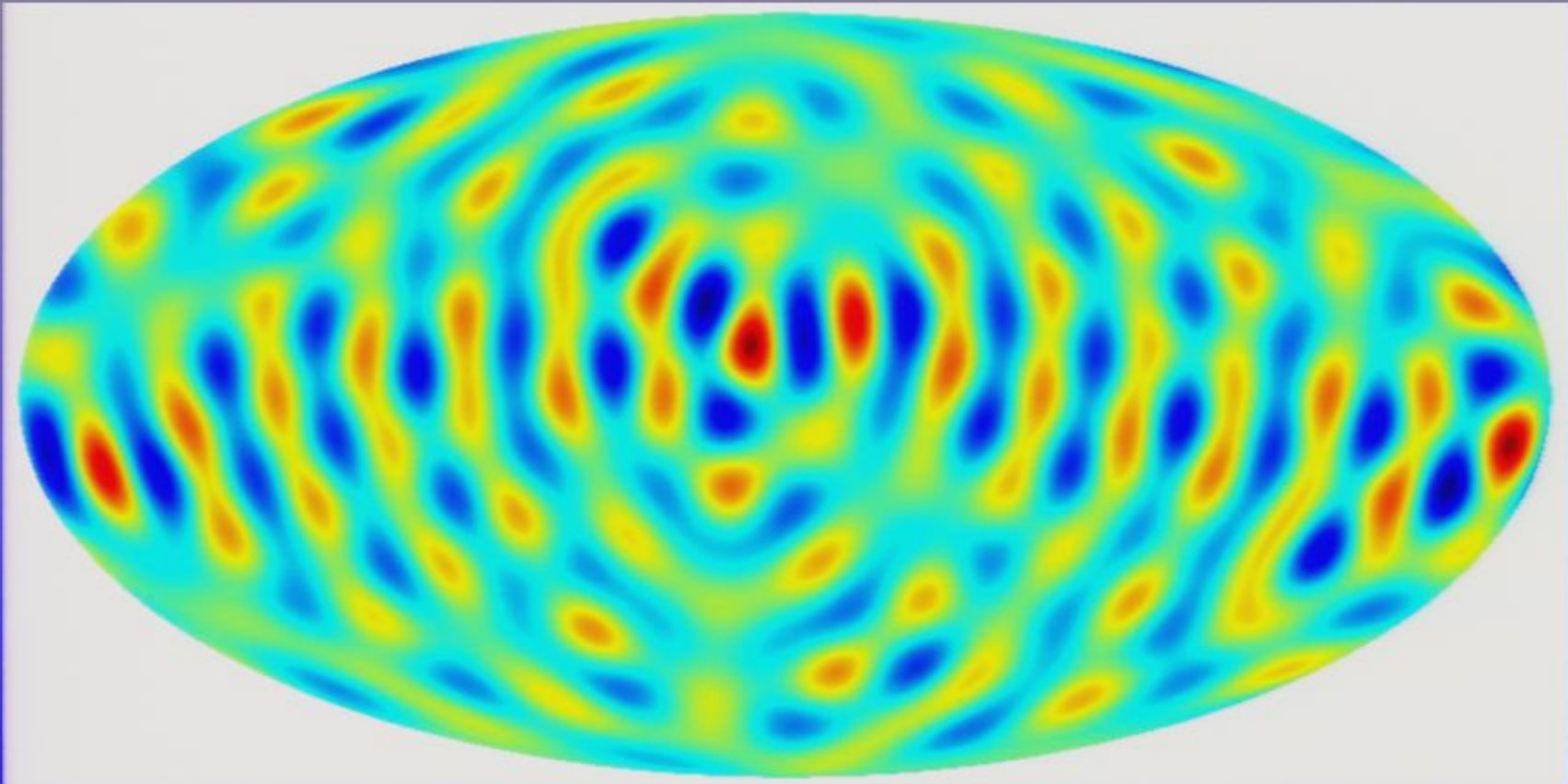
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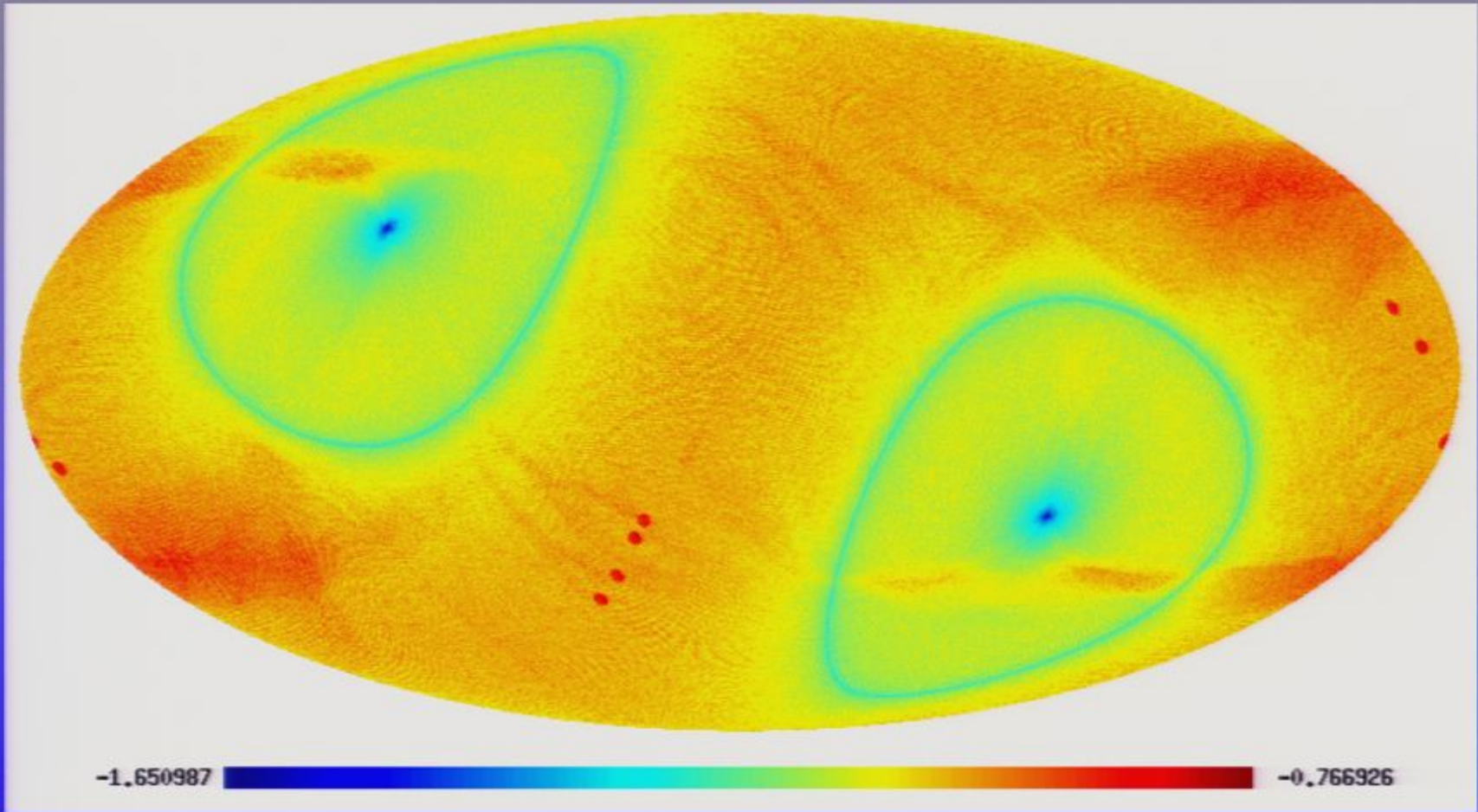


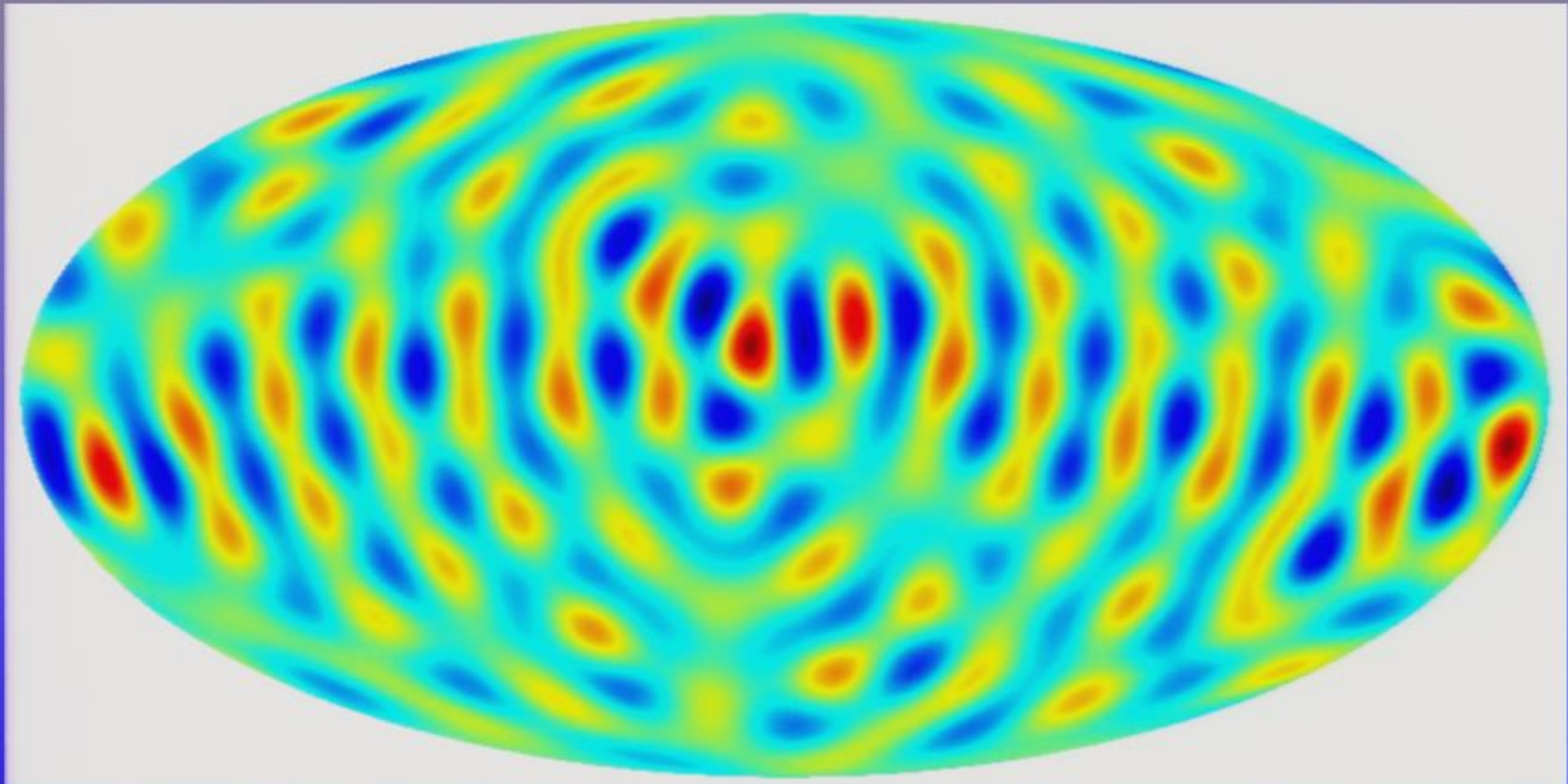


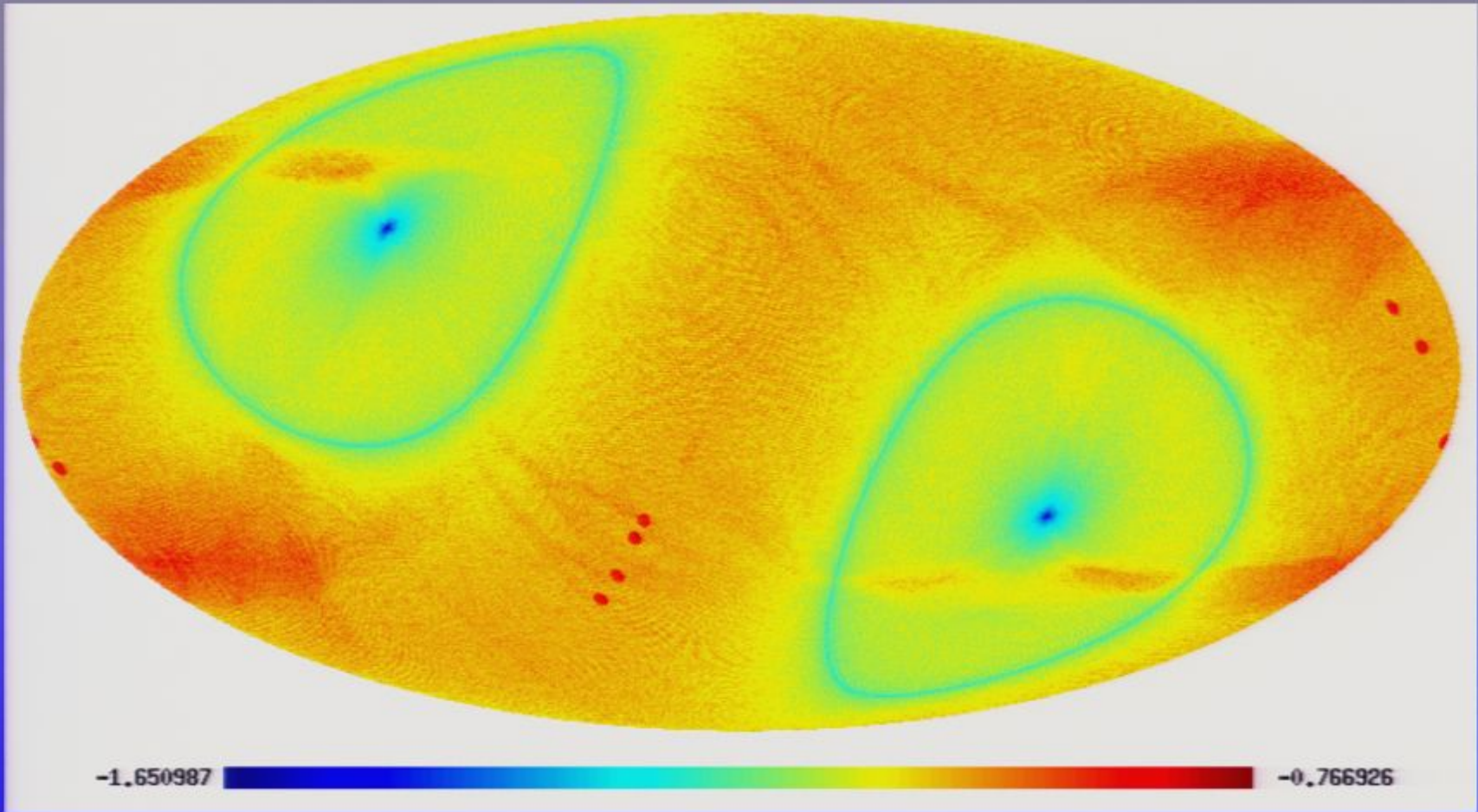
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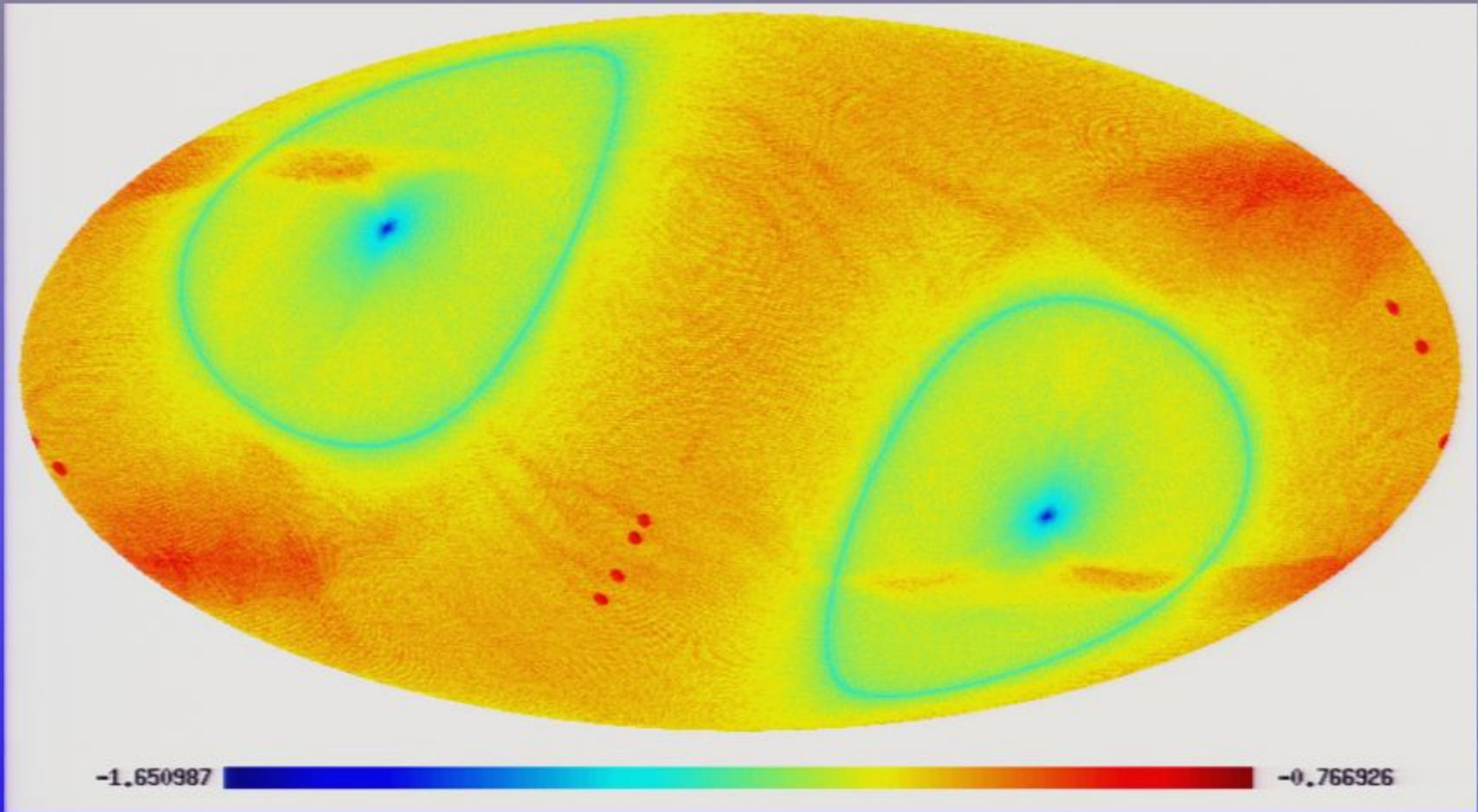






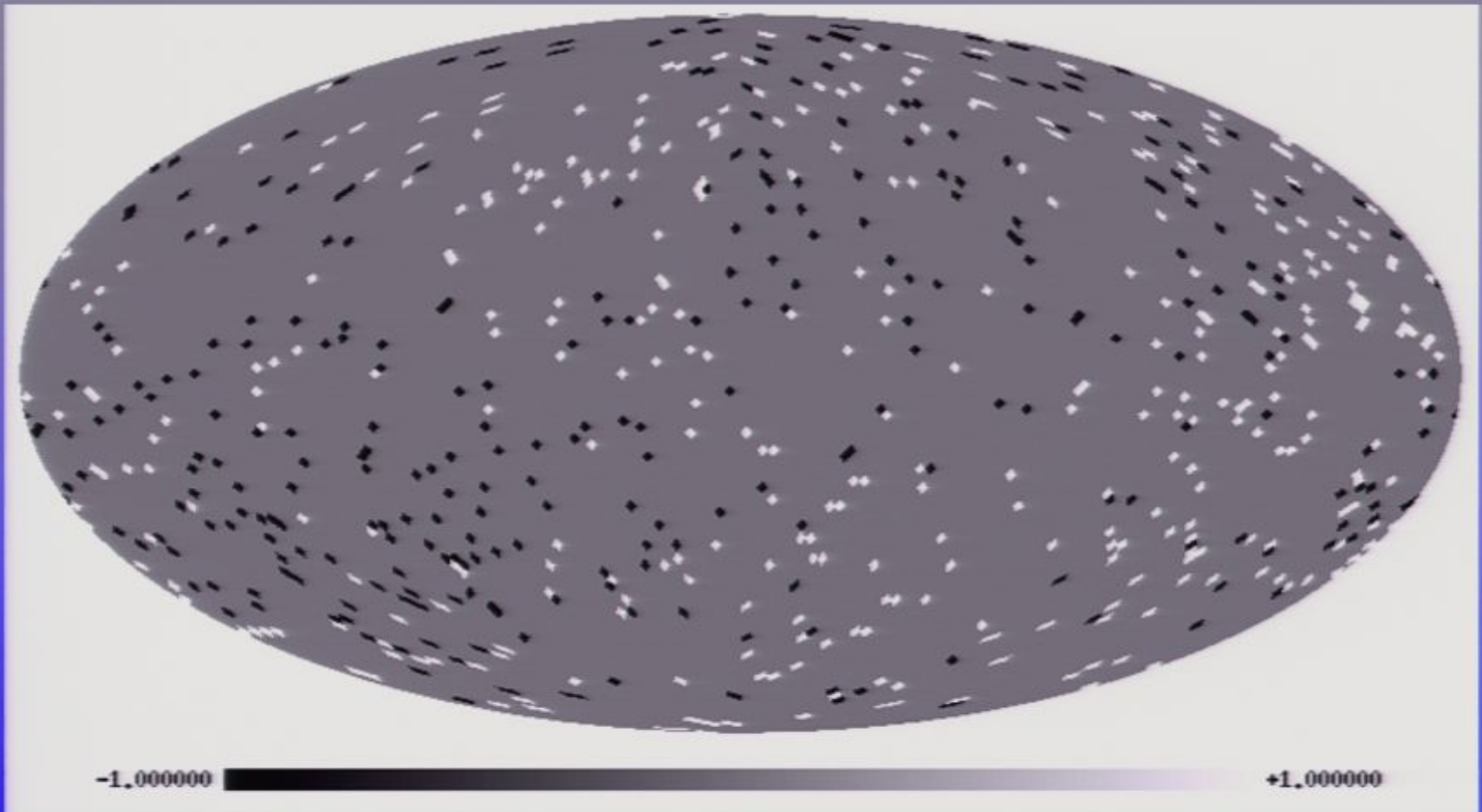
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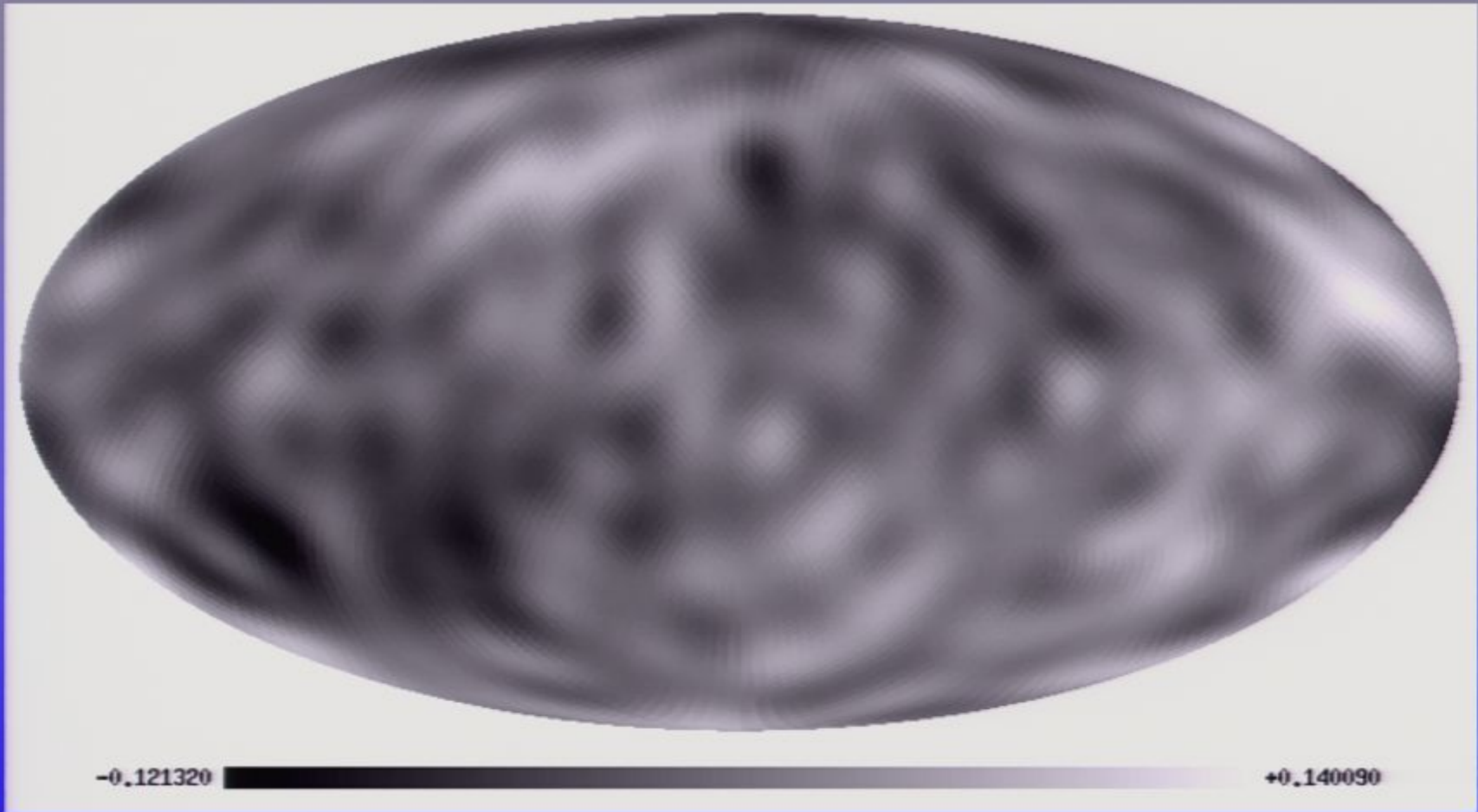
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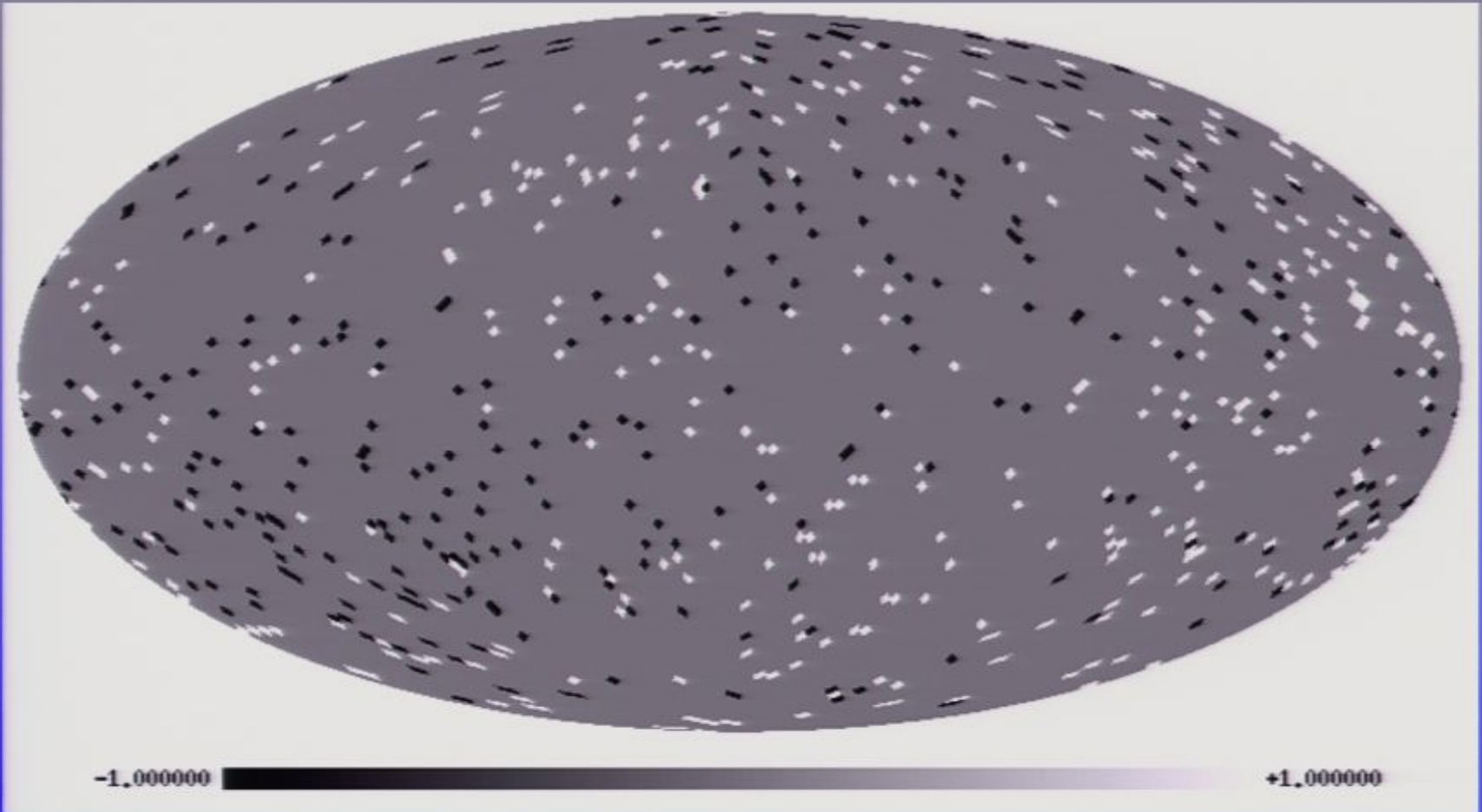


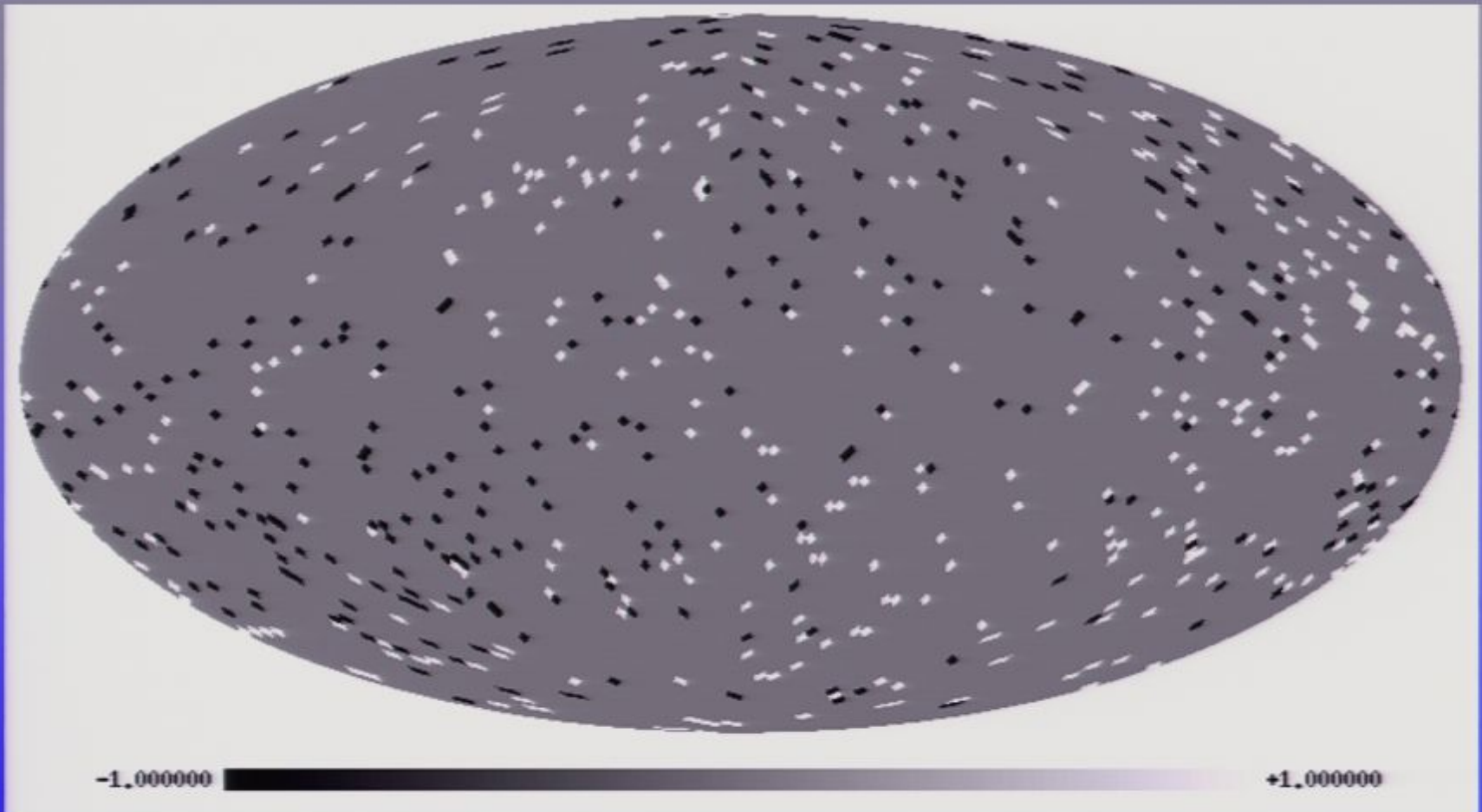
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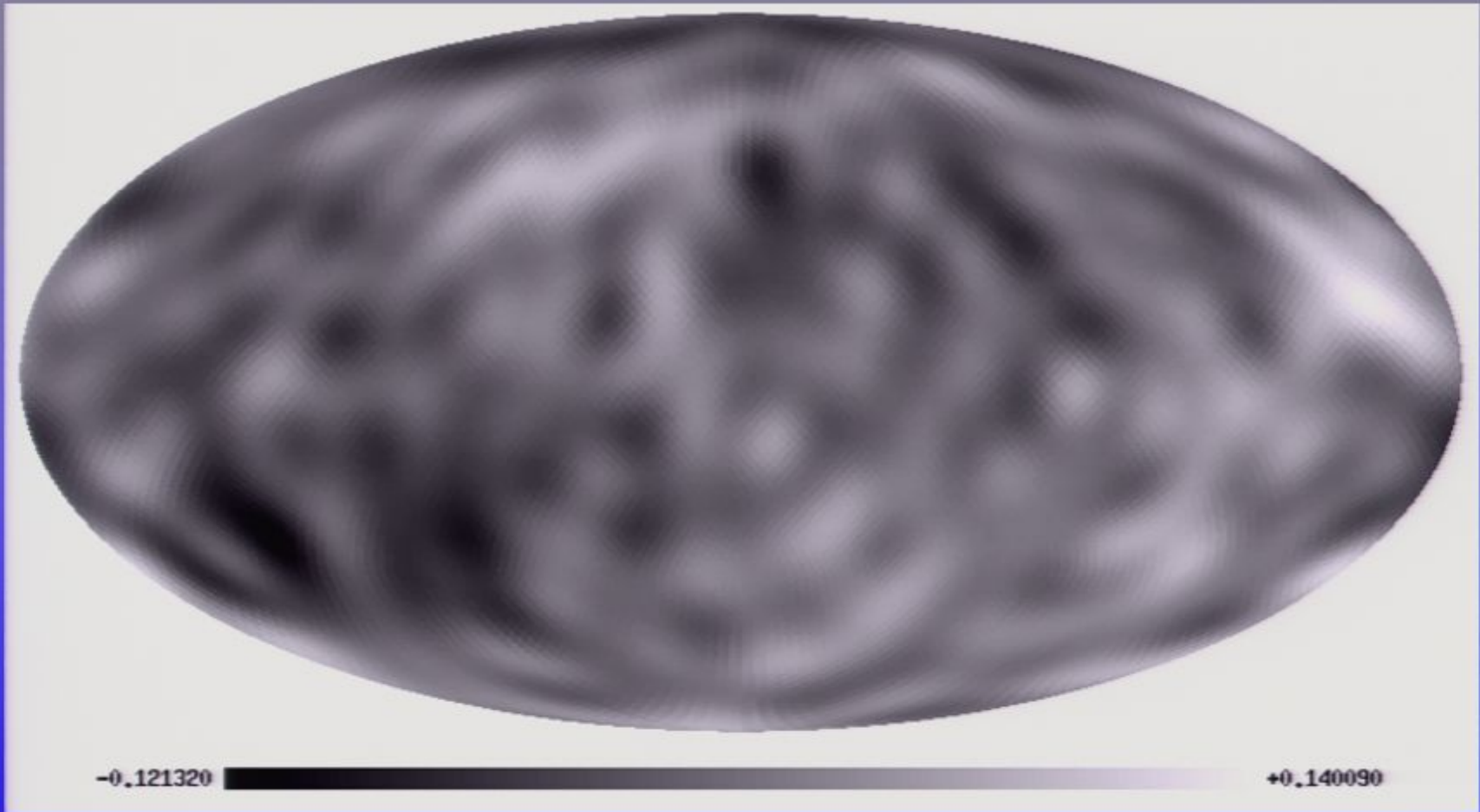


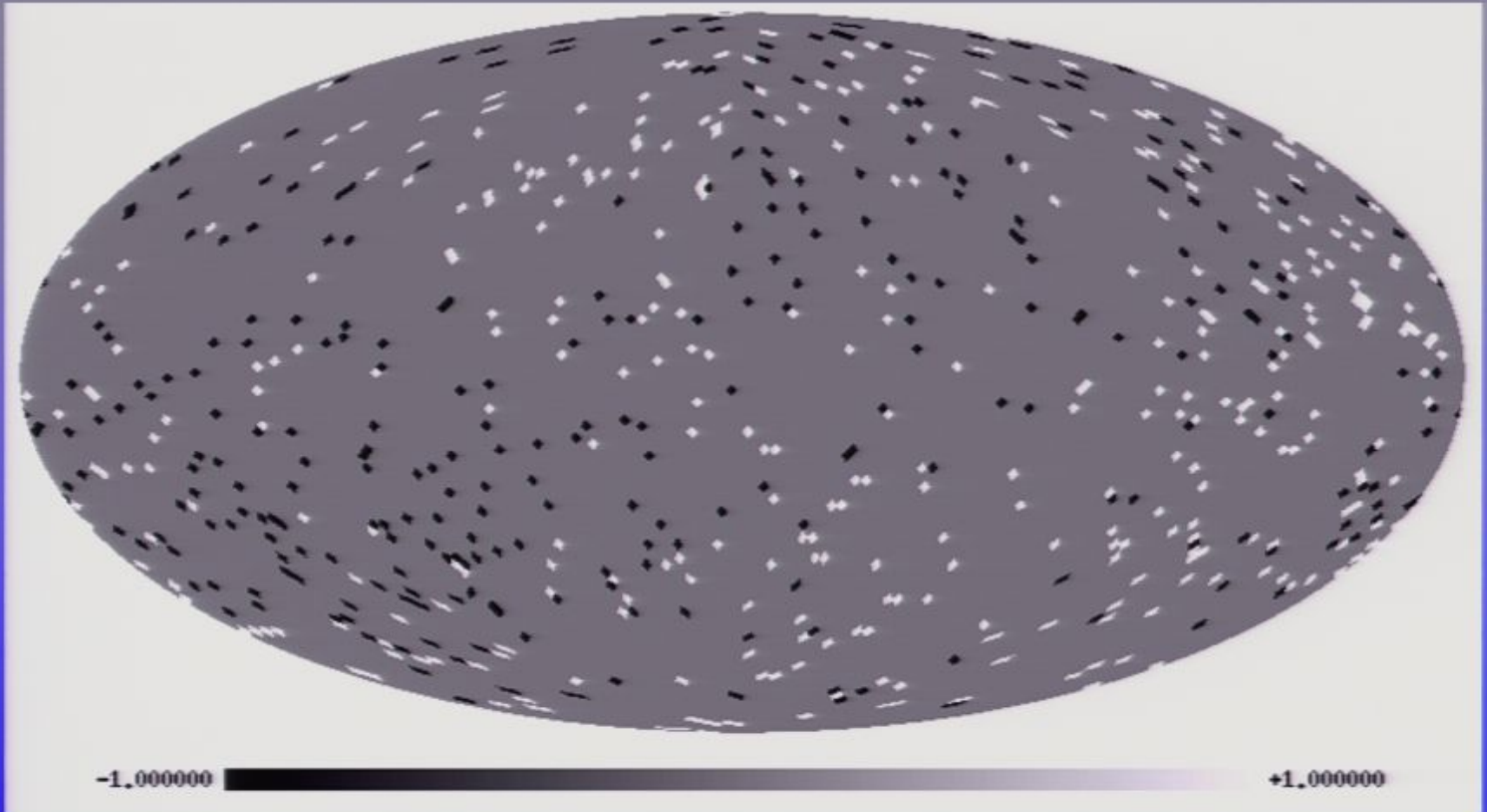






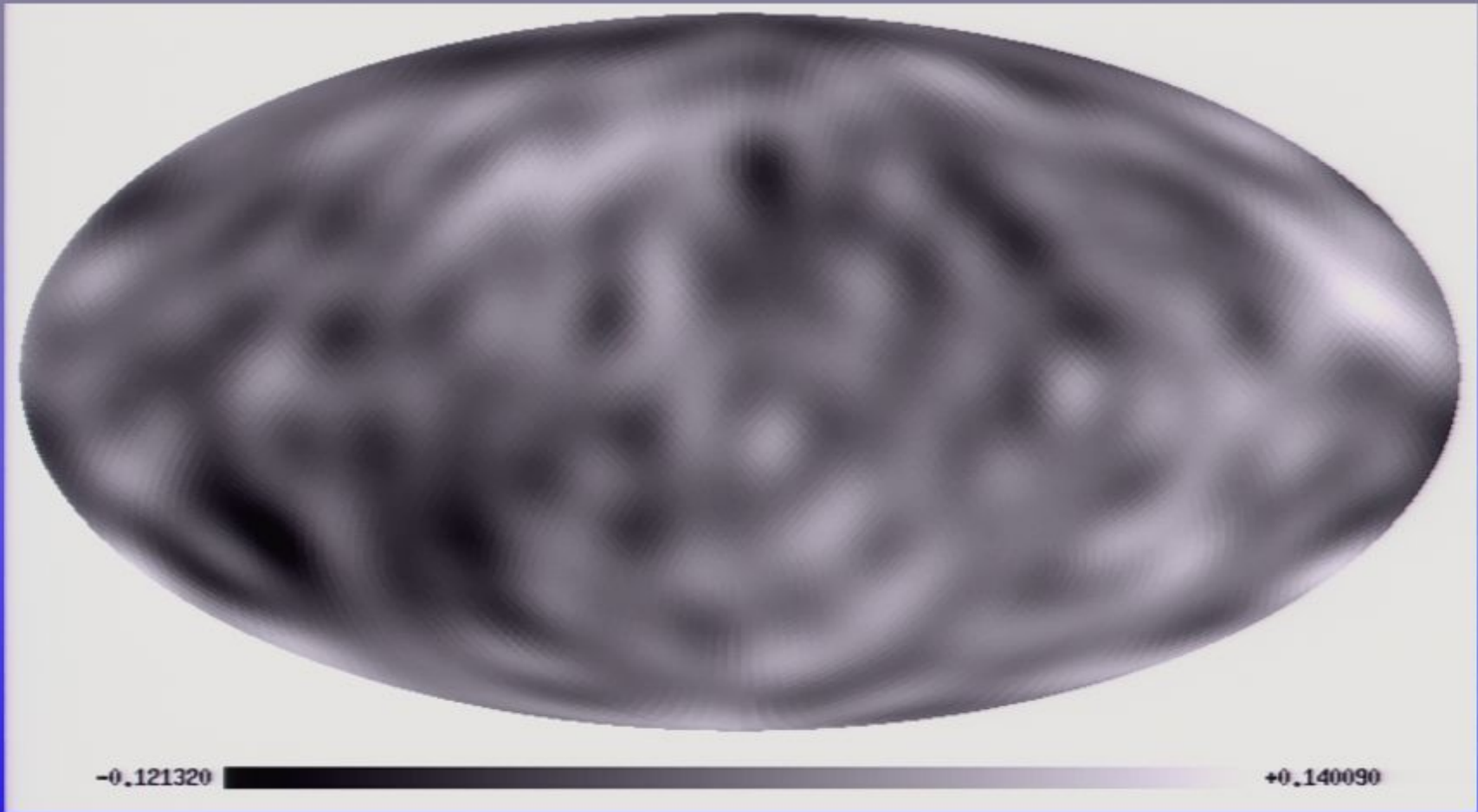






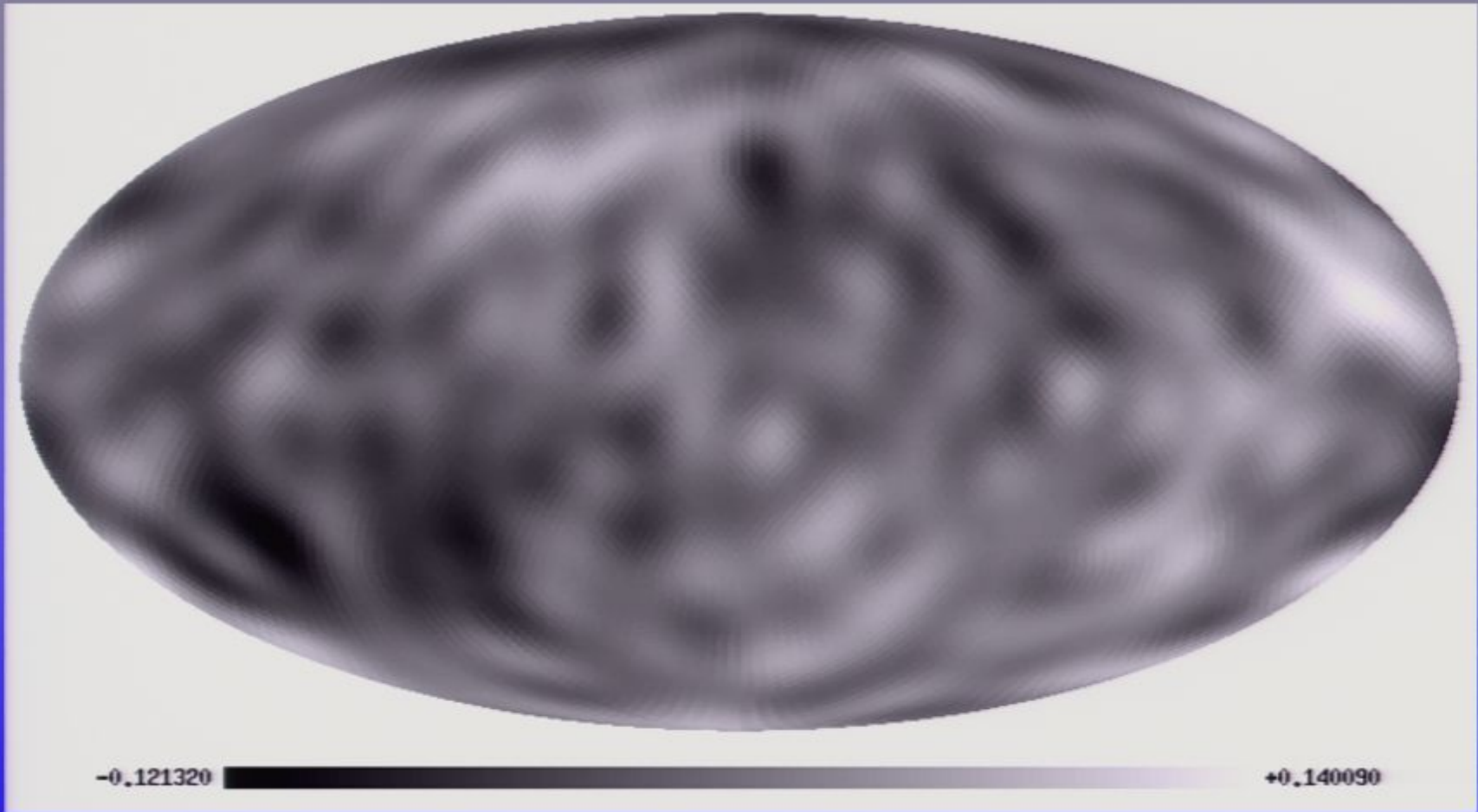
# A Faraday Rotation Map of the Galactic Sky

- The FR catalogues contain information about our Galaxy and the distribution of sources
- Imagine  $\underline{y}$  is a vector made up of the spherical harmonics
- $\underline{y} \cdot \underline{y}^T = \mathbf{I}$  for the whole sky
- $\underline{y} \cdot \underline{y}^T \neq \mathbf{I} = \mathbf{W}$  (say) for a cut or masked sky
- Inverting  $\mathbf{W}$  gives a new set of harmonics which are orthogonal
- Reconstructed map has better statistical properties.



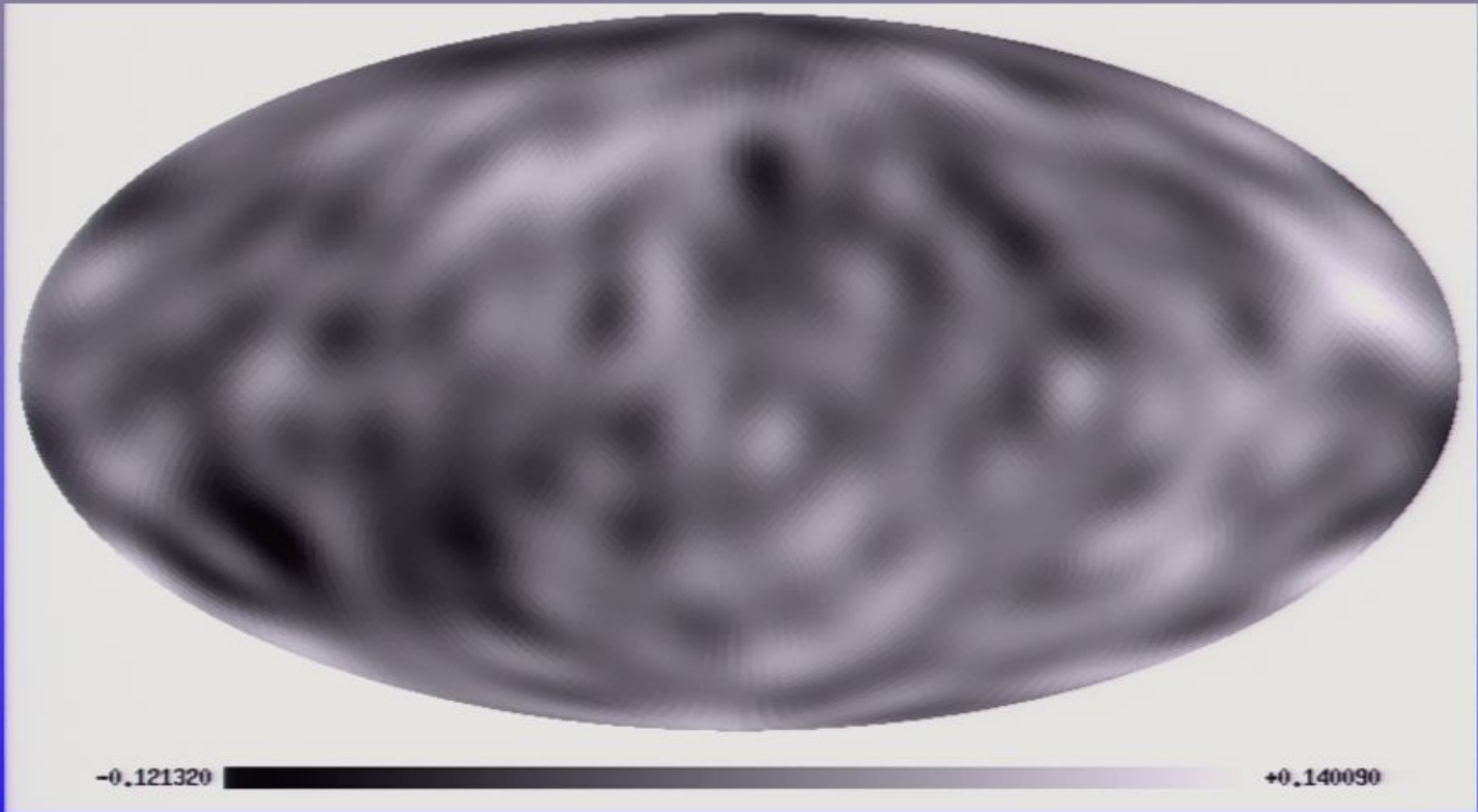
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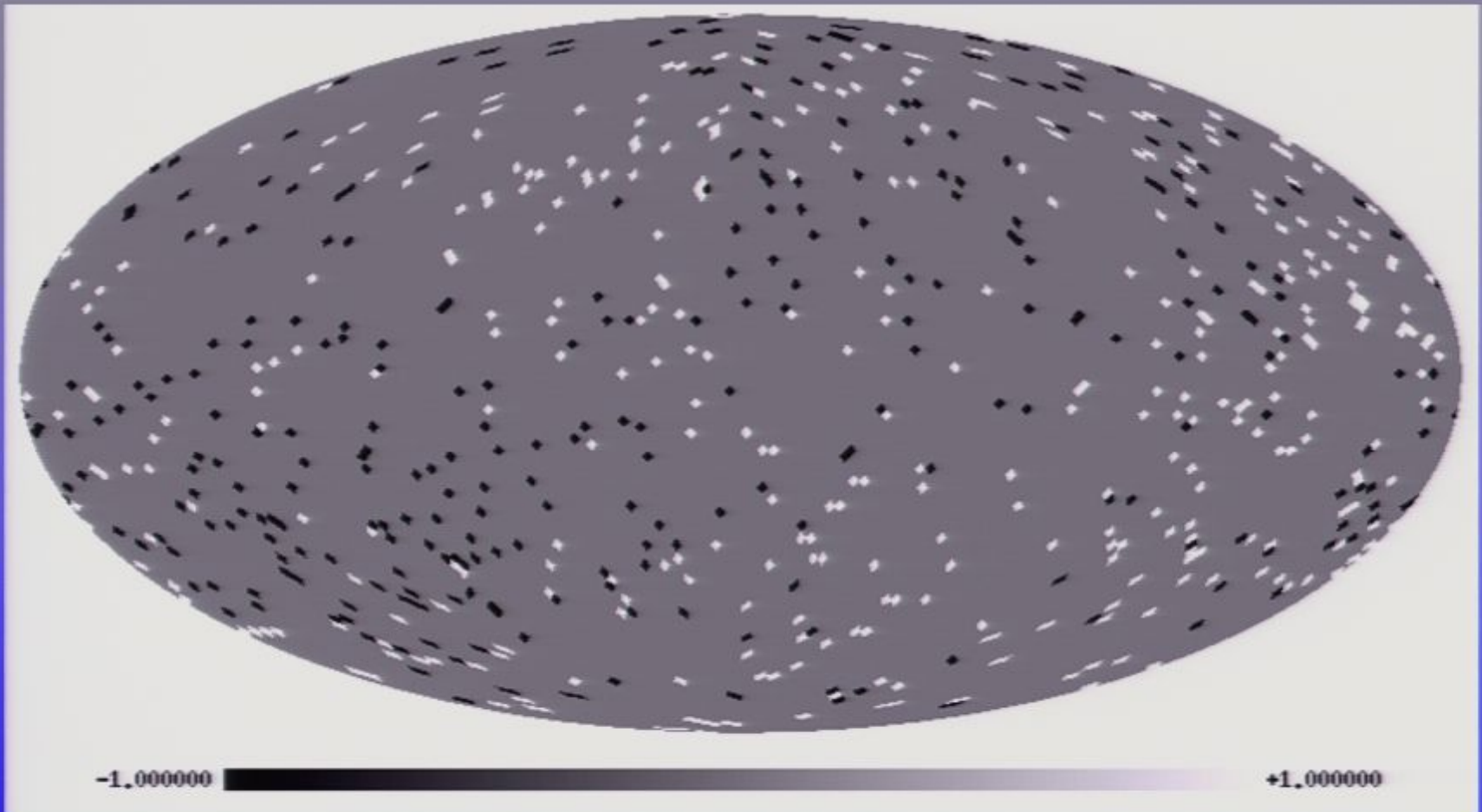


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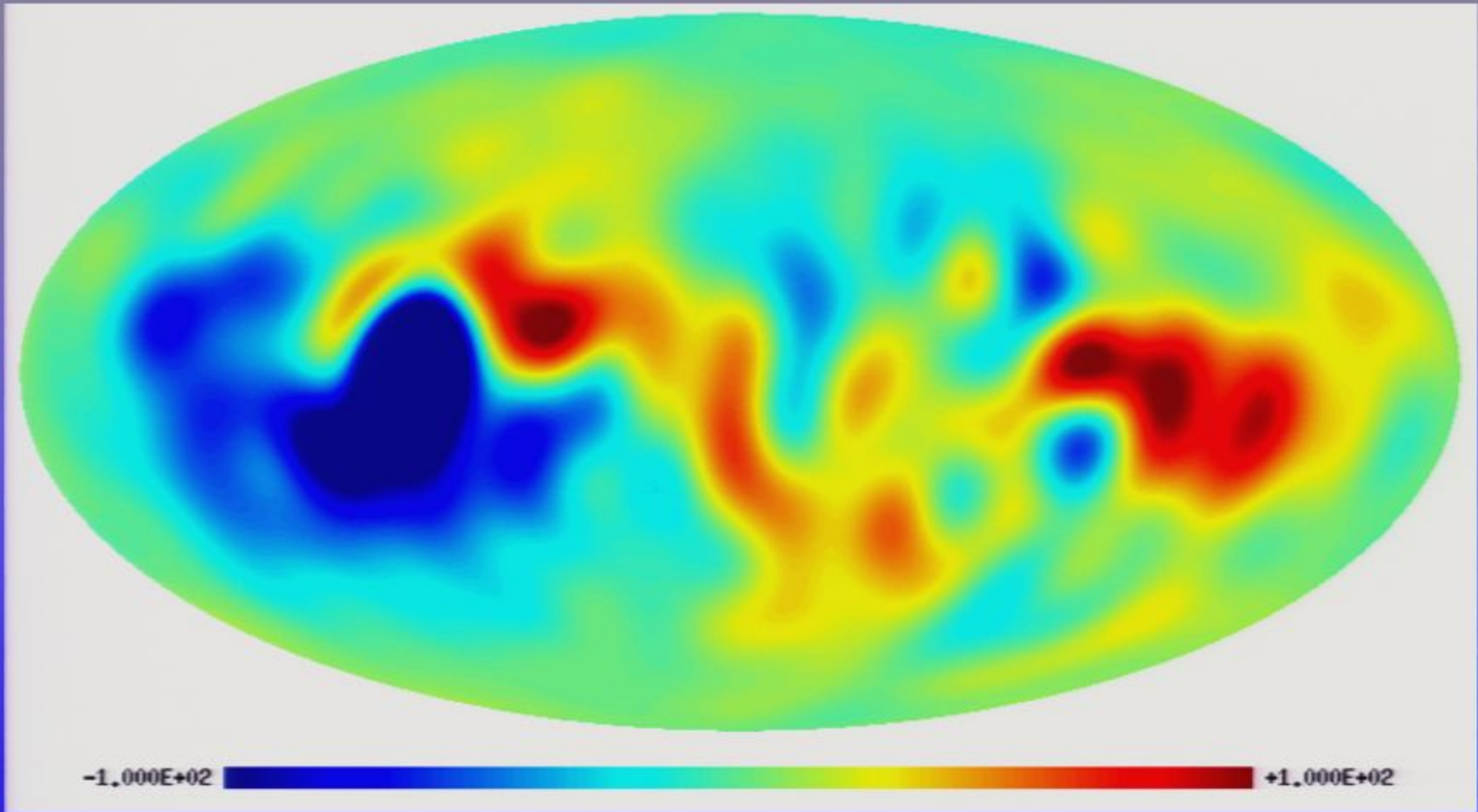




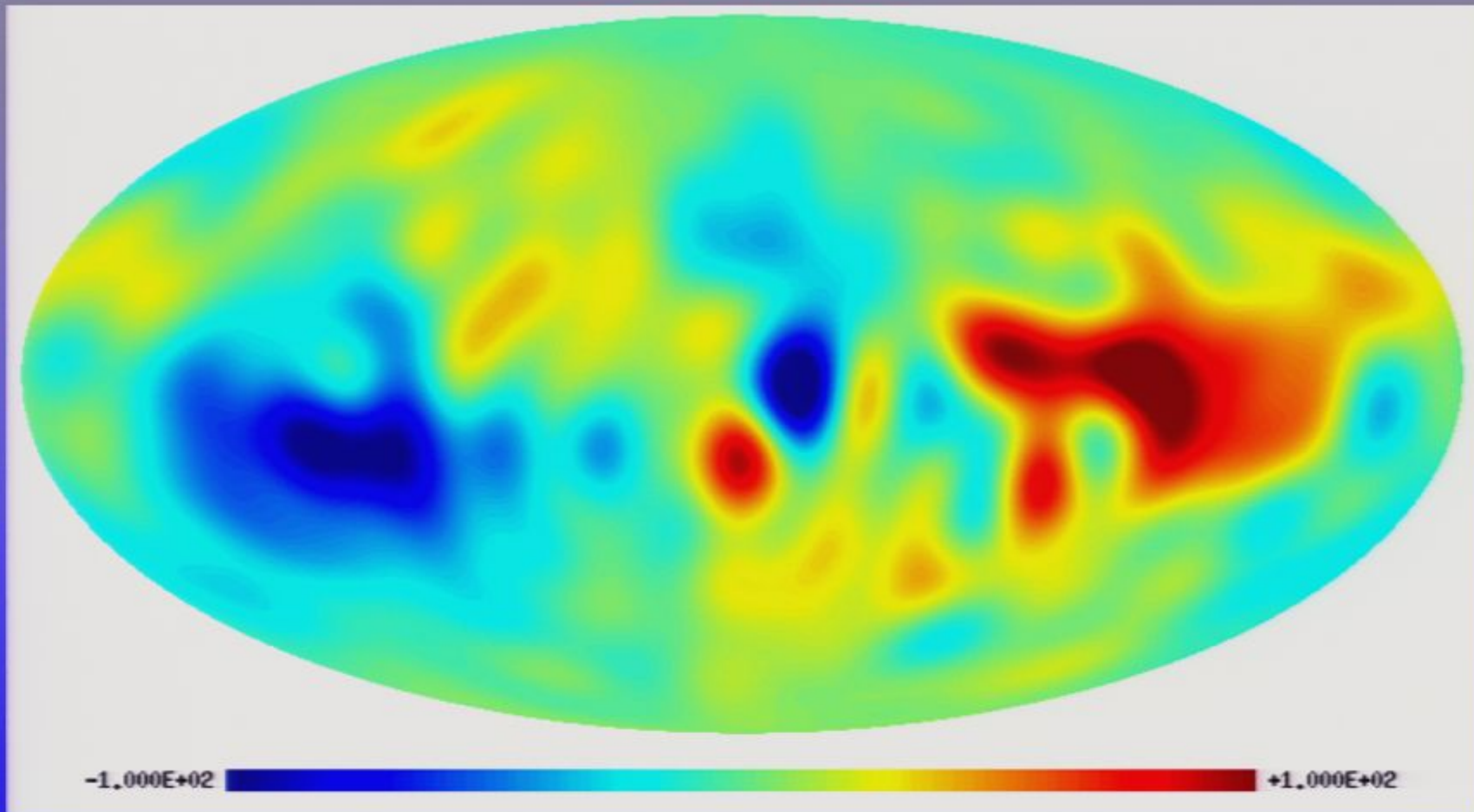


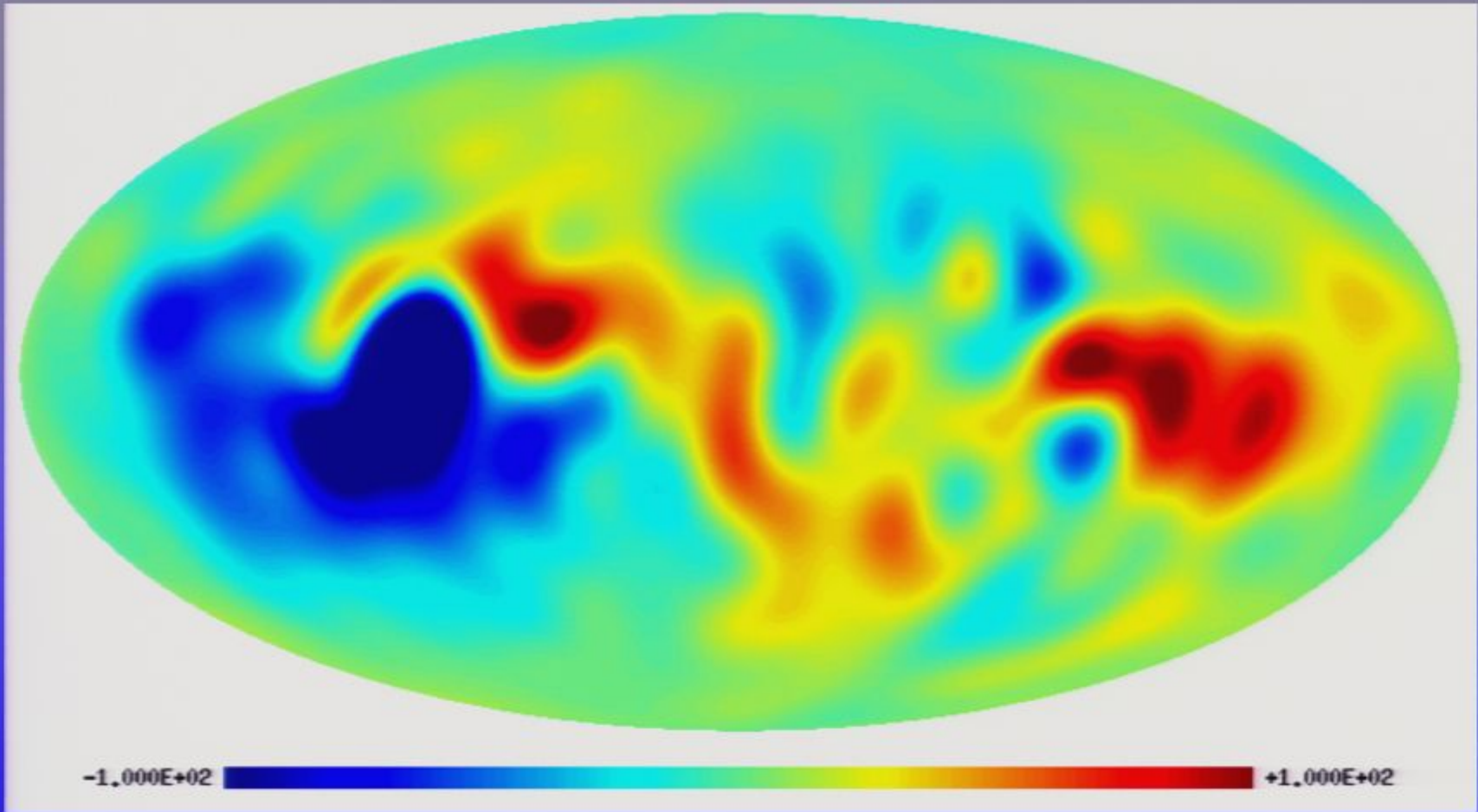
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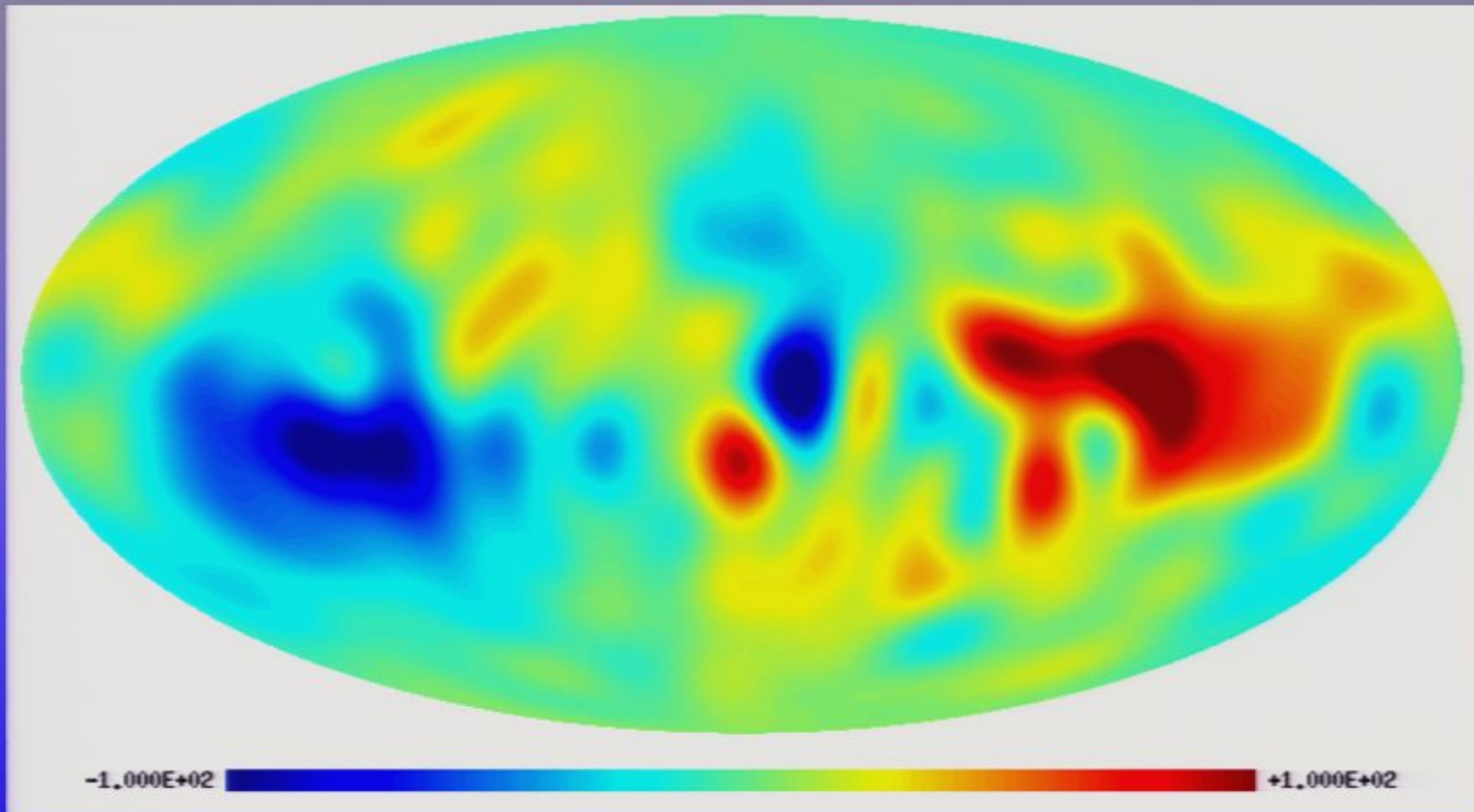


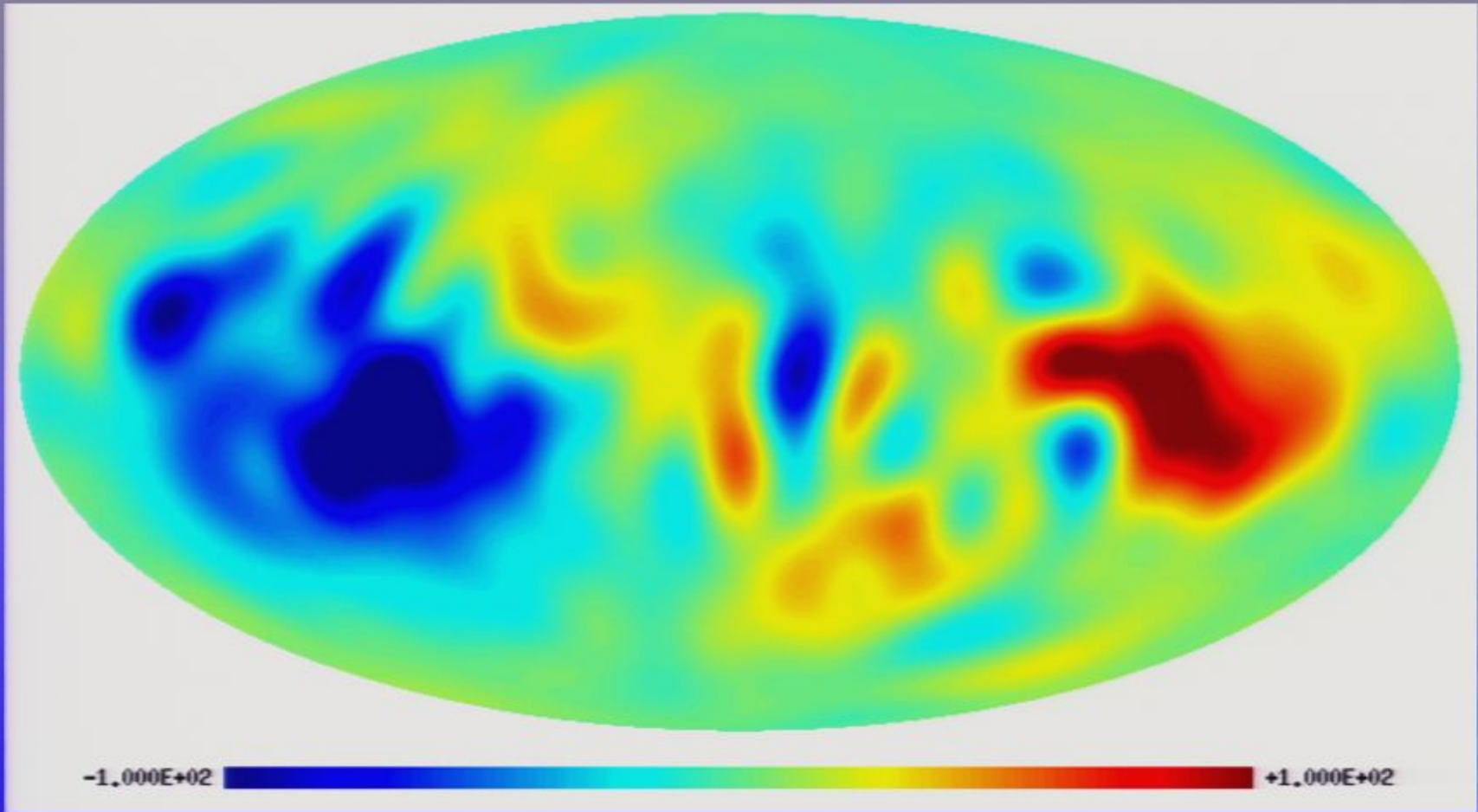
Dineen & Coles, astro-ph/0410636





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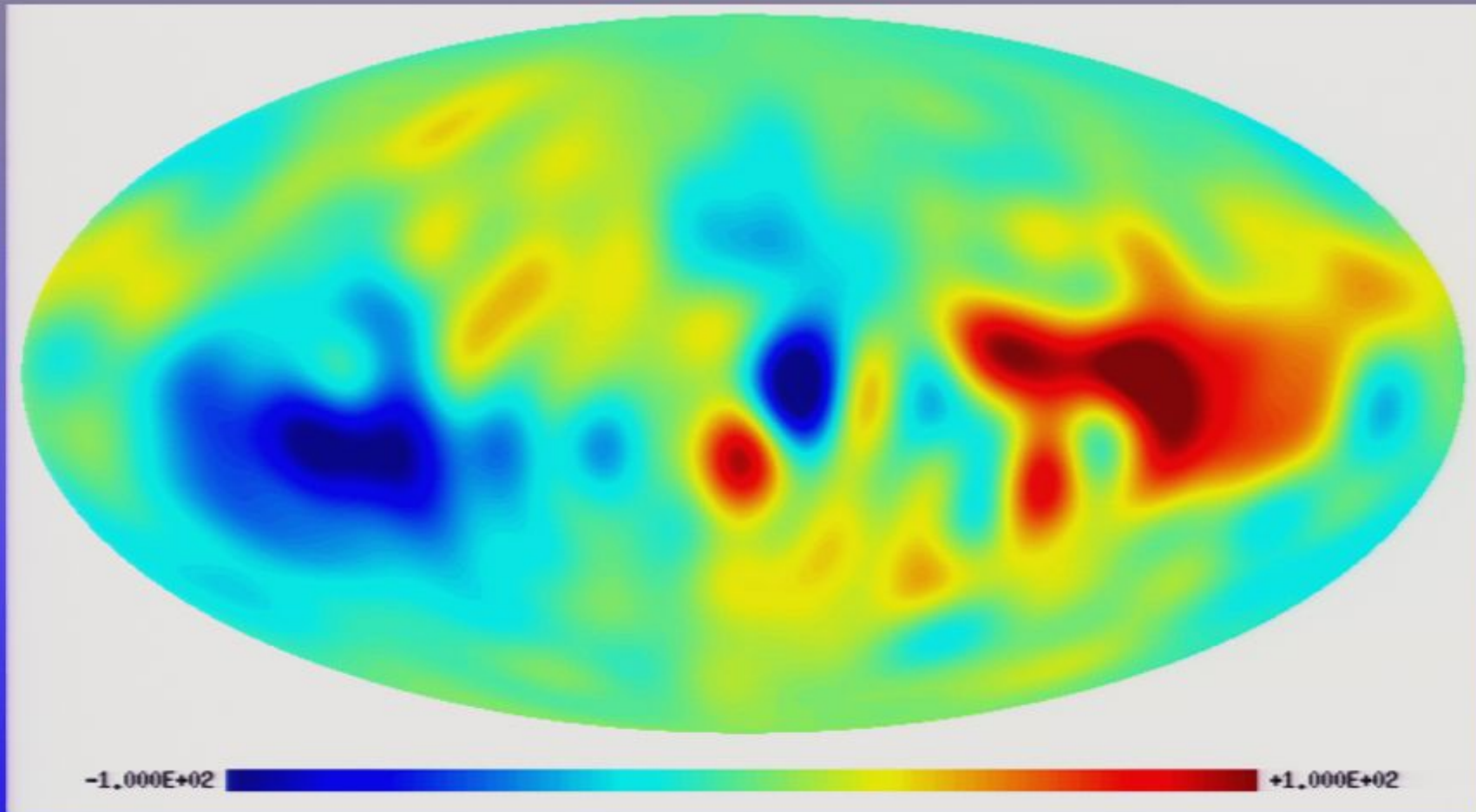


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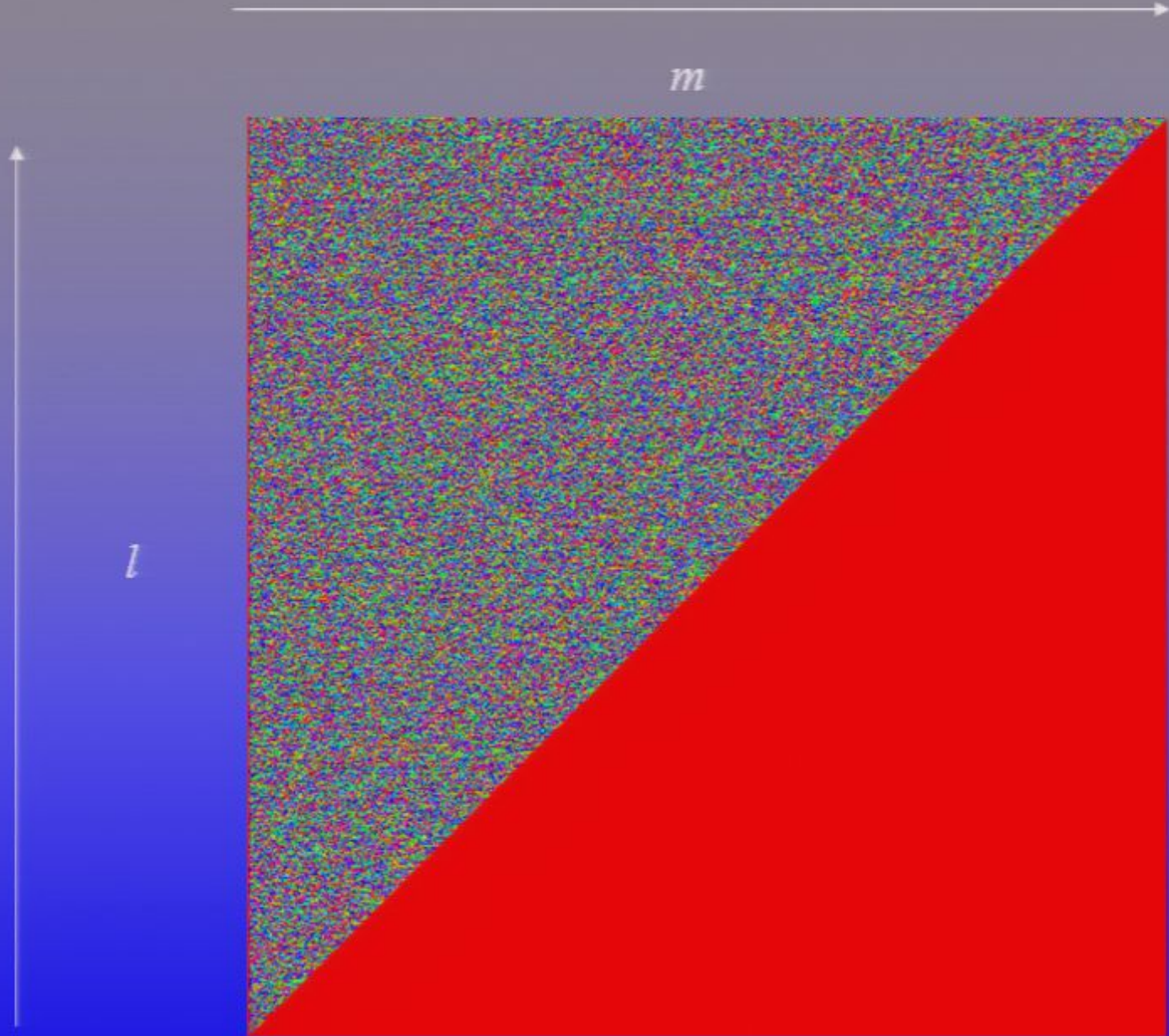


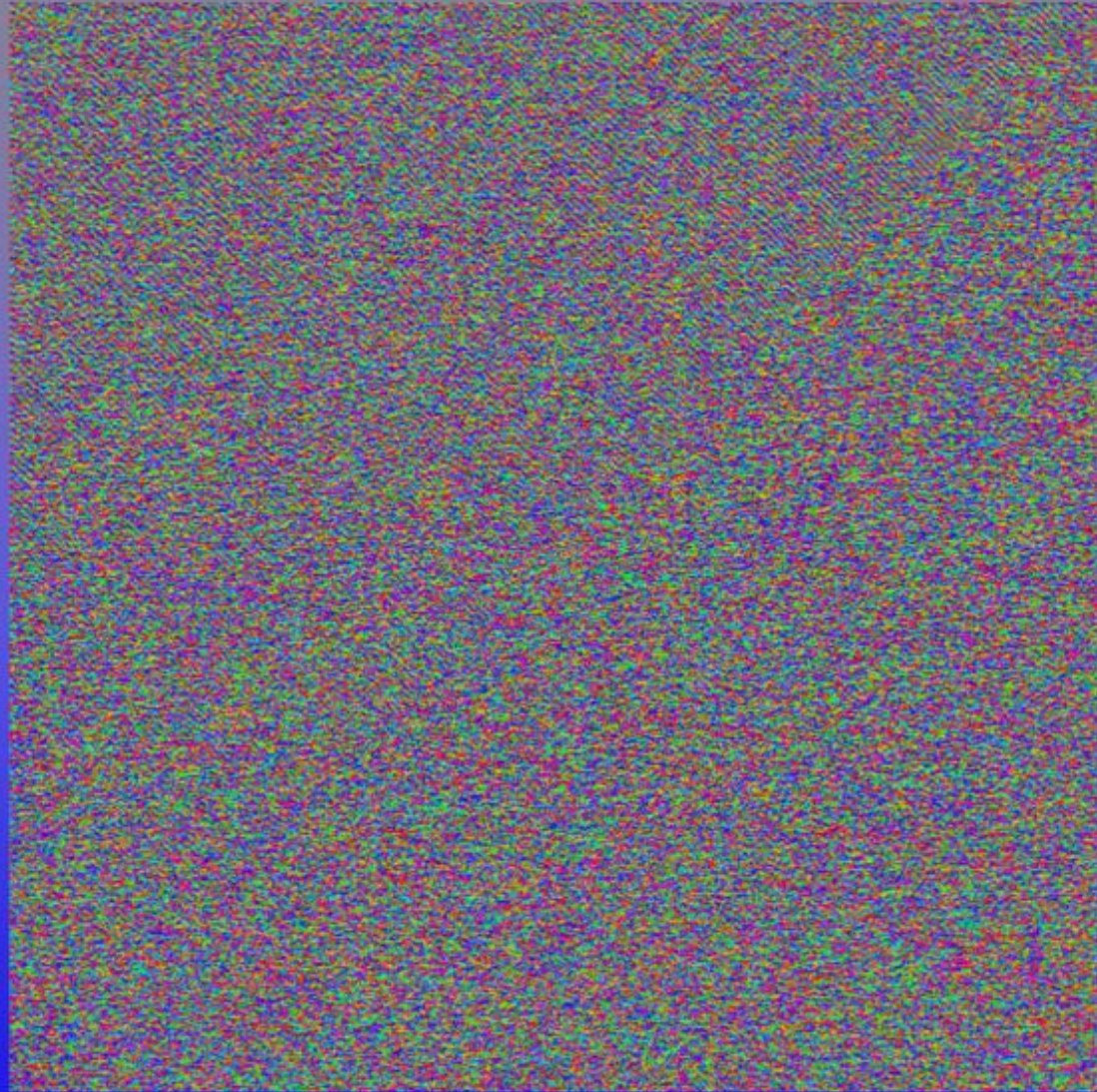
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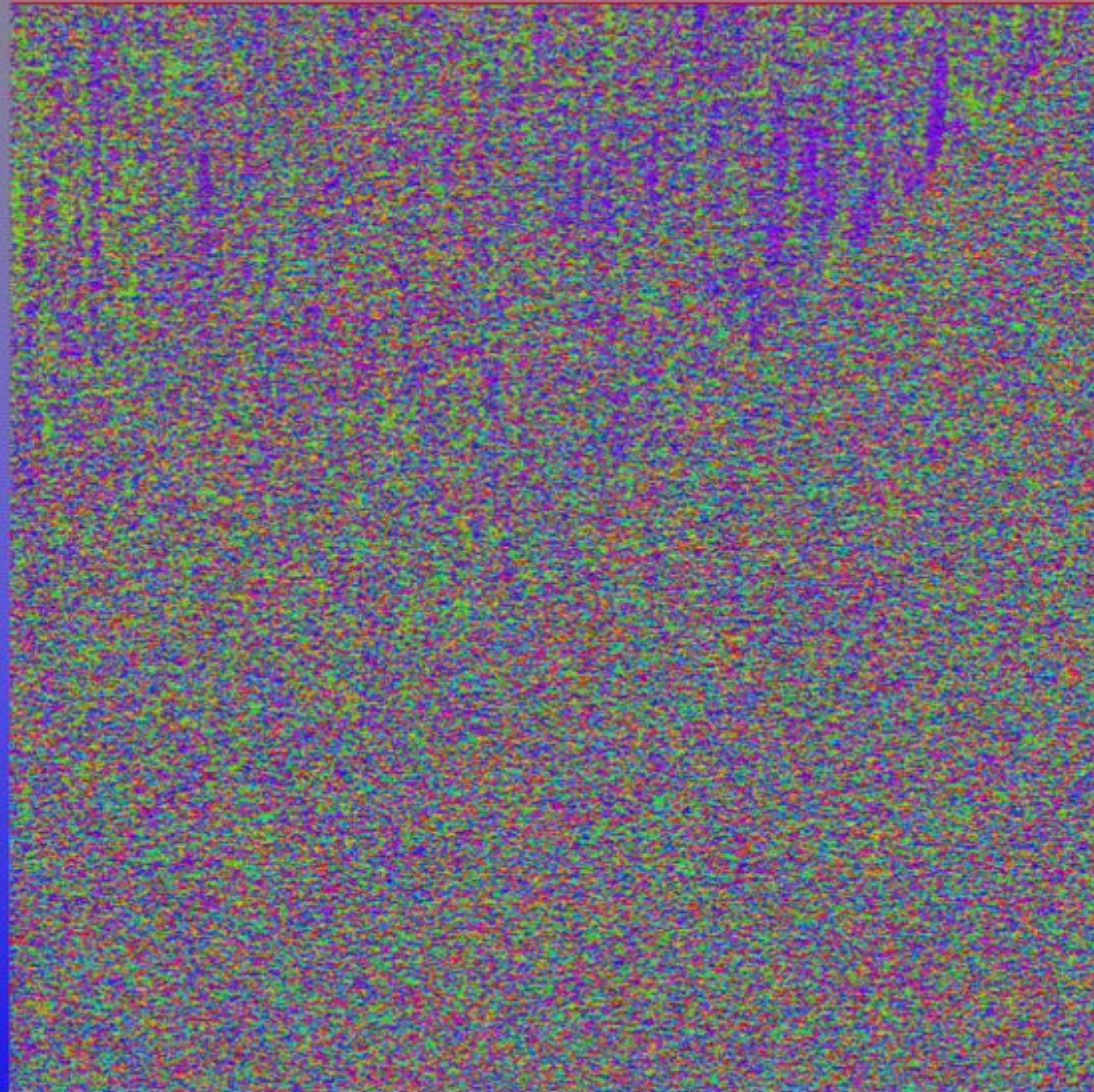


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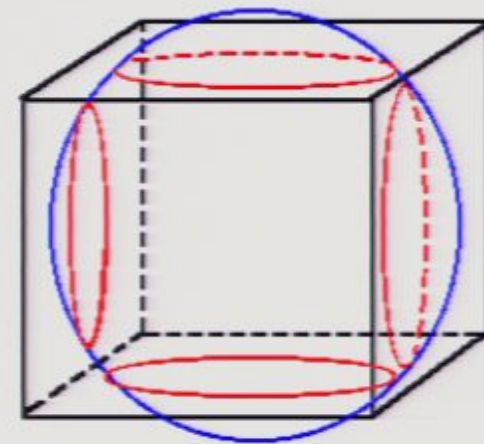
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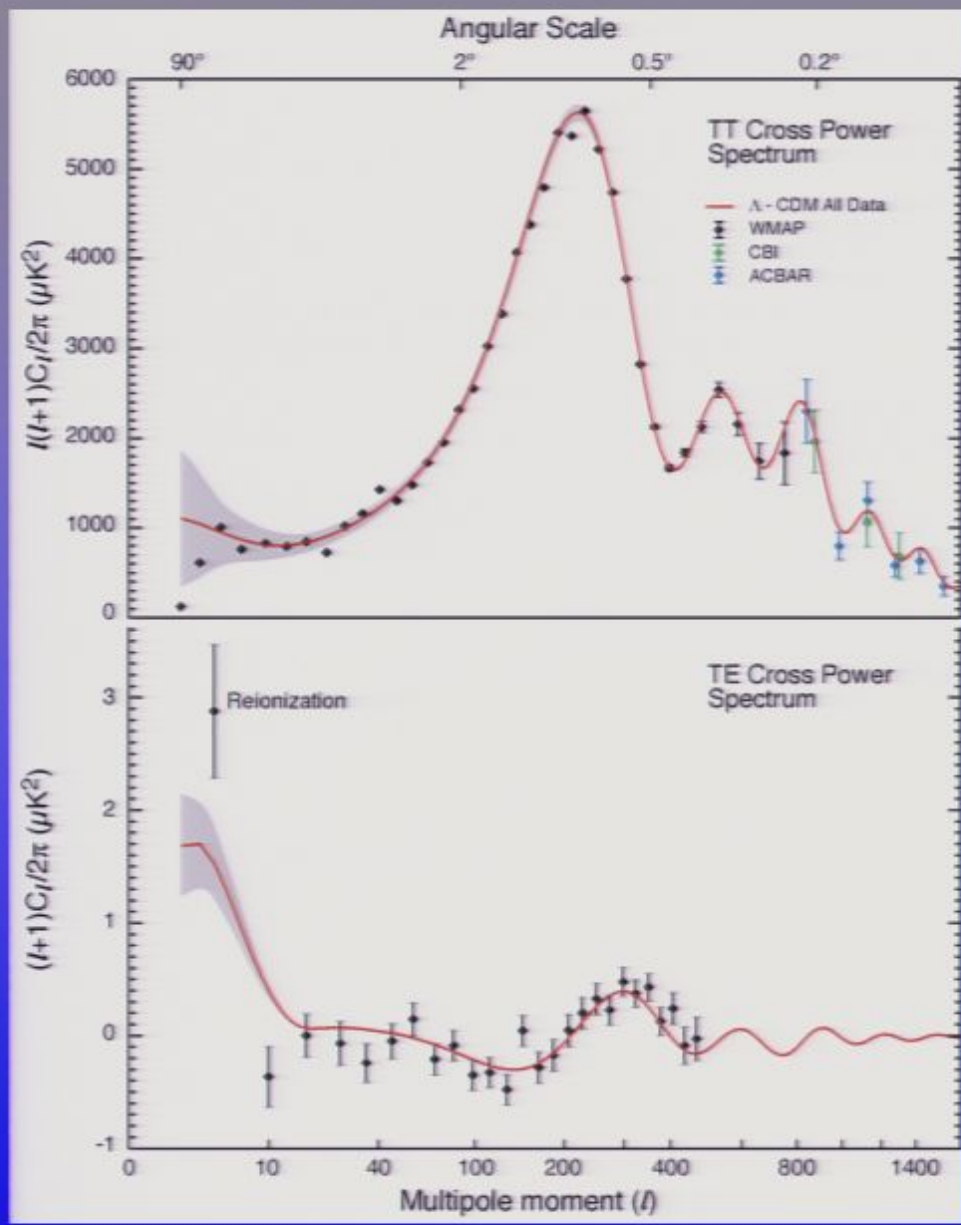
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- GR is a *local* theory
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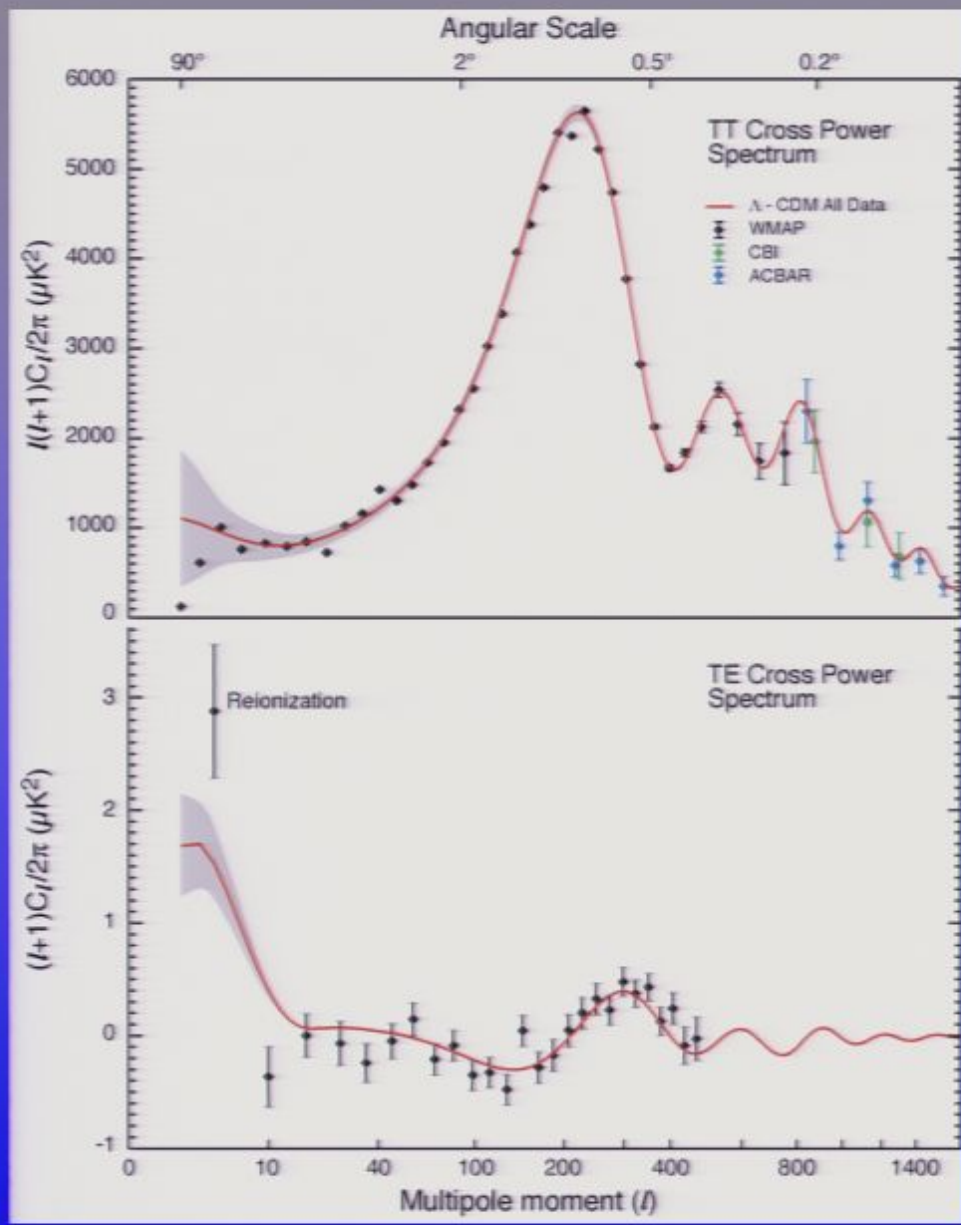
Intersection of the CMB sphere with the (imaginary) walls of the fundamental domain of a 3-torus. The intersections with the front and back walls are not shown.





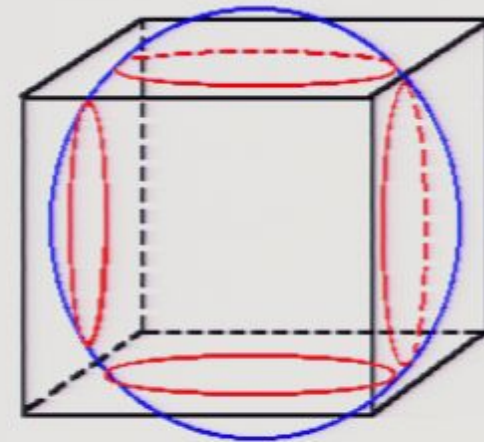
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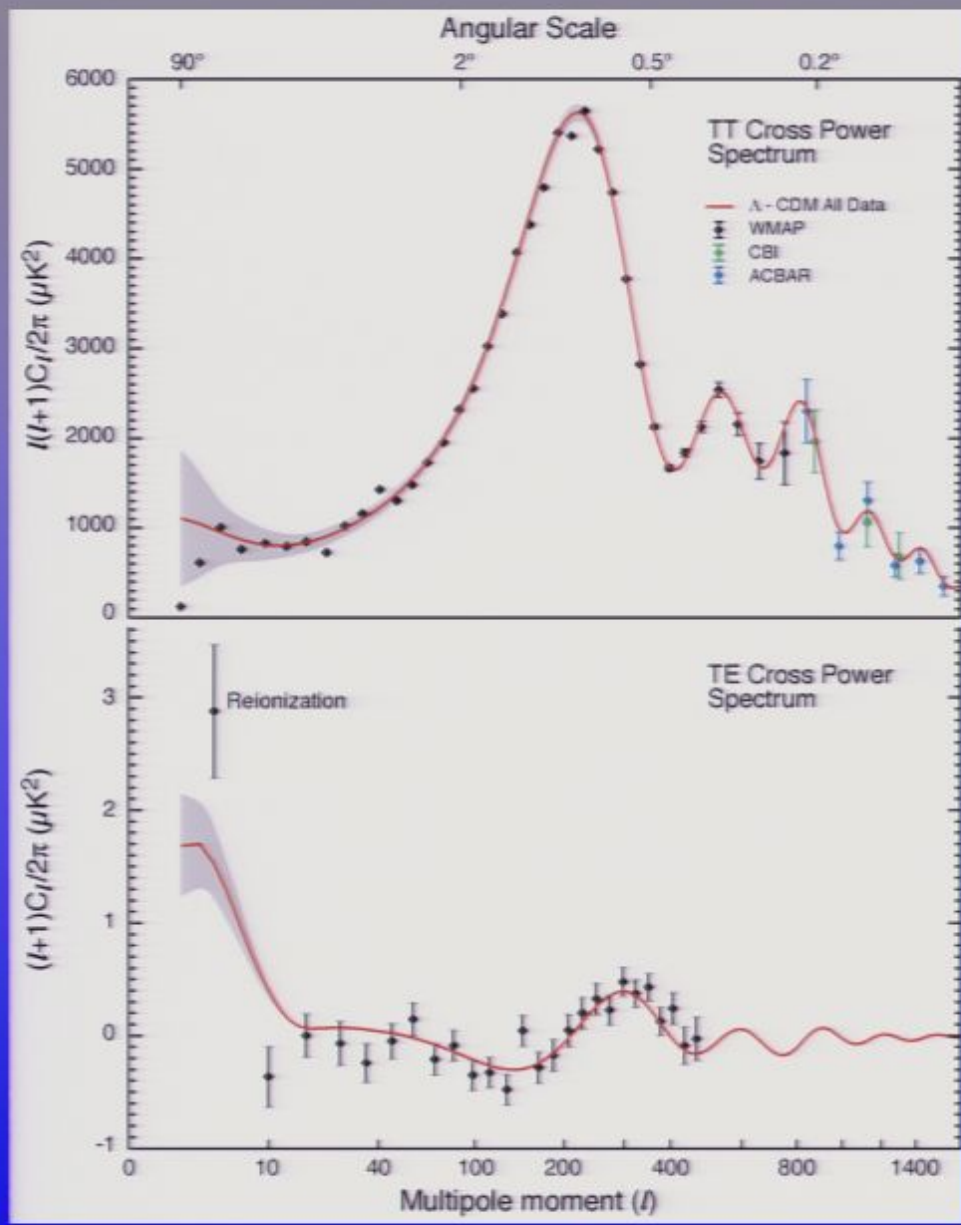


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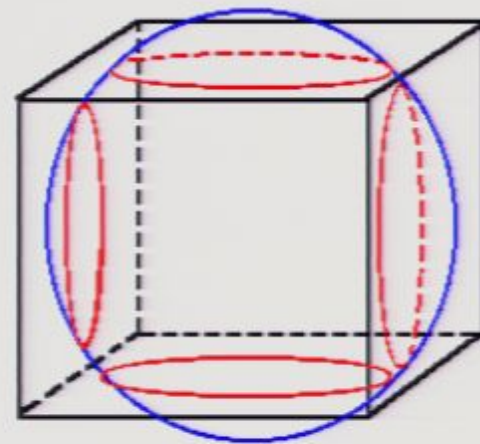


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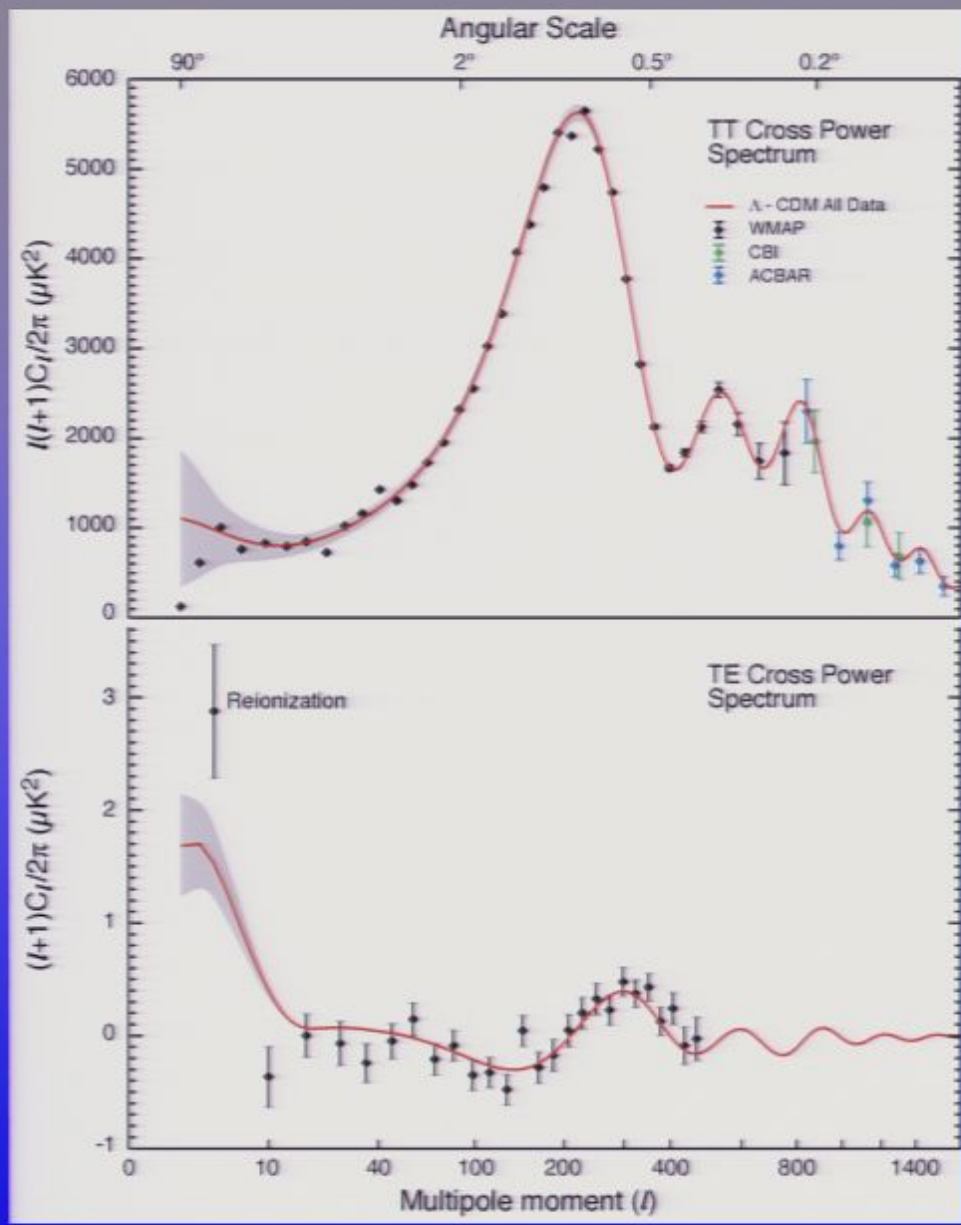


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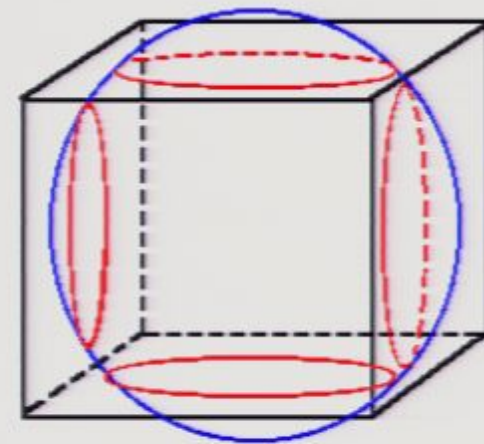


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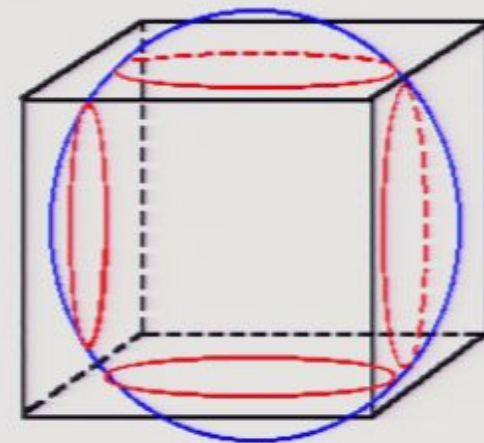


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# SUMMARY

- “Normal” models have Gaussian primordial fluctuations
- The Universe gets weirder as it gets older
- WMAP is weird, but it’s not obviously genetic
- It will be a battle to get foregrounds down, especially for polarisation
- WMAP does not provide evidence for weird topology.

