Title: Emergent Spacetime in Noncritical String and M-Theory

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Abstract:

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work with Cindy Keeler, hep-th/0508024

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see also hep-th/0503006.

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Strings and Fermi Liquids

Intriguing relations between string theory and nonrelativistic Fermi liquids:

- Noncritical strings in two spacetime dimensions are described by a matrix model, equivalent to a theory of nonrelativistic fermions in an inverted harmonic oscillator.
- The noncritical M-theory for such strings can be described as a nonrelativistic Fermi liquid in $2+1$ dimensions.

LLM : 1/2 BPS geometries of Type IIB supergravity mapped to the phase space distribution of nonrelativistic fermions in $1+1$ dimension. (Gauge-gravity duality.)

In all those cases, the role of "more fundamental constituents" of spacetime is played by fermions, which in turn originate as D-branes – usually defined as stringy spacetime defects on which strings can end.

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Stable Fermi surfaces in nonrelativistic Fermi liquids in D dimensions clasified by K-theory, just like charges of D-branes in string theory!

Outline of the talk

I. Introducing noncritical M-theory

Motivation and definition
Two-dimensional Type 0A, 0B as solutions
The space of all vacua

II. (2+1)-dimensional vacuum of M-theory

Exact vacuum energy
Exact free energy at finite temperature
Symmetries, observables, bosonization

III. Spacetime physics as hydrodynamics

Emergent spacetime: Hydrodynamics of the Fermi surface
Time-dependent solutions: *e.g.*, interpolating
(Type 0 vacuum) → (M-vacuum) → (Type 0 vacuum)

IV. Conclusions

I.

Noncritical M-theory for noncritical strings

Motivation

10 years since the second string revolution, string theory is a unique theory, but not (always) of strings.

The starfish diagram:

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. . . but the degrees of freedom of M-theory remain mysterious.

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It would be desirable to have a complete understanding of the starfish diagram (the "whole elephant", or moduli space of solutions), and understand the underlying degrees of freedom in the process.

We will adress this goal in the highly controlled context of two-dimensional noncritical strings.

Type 0A and 0B strings in two dimensions

Type 0B theory:

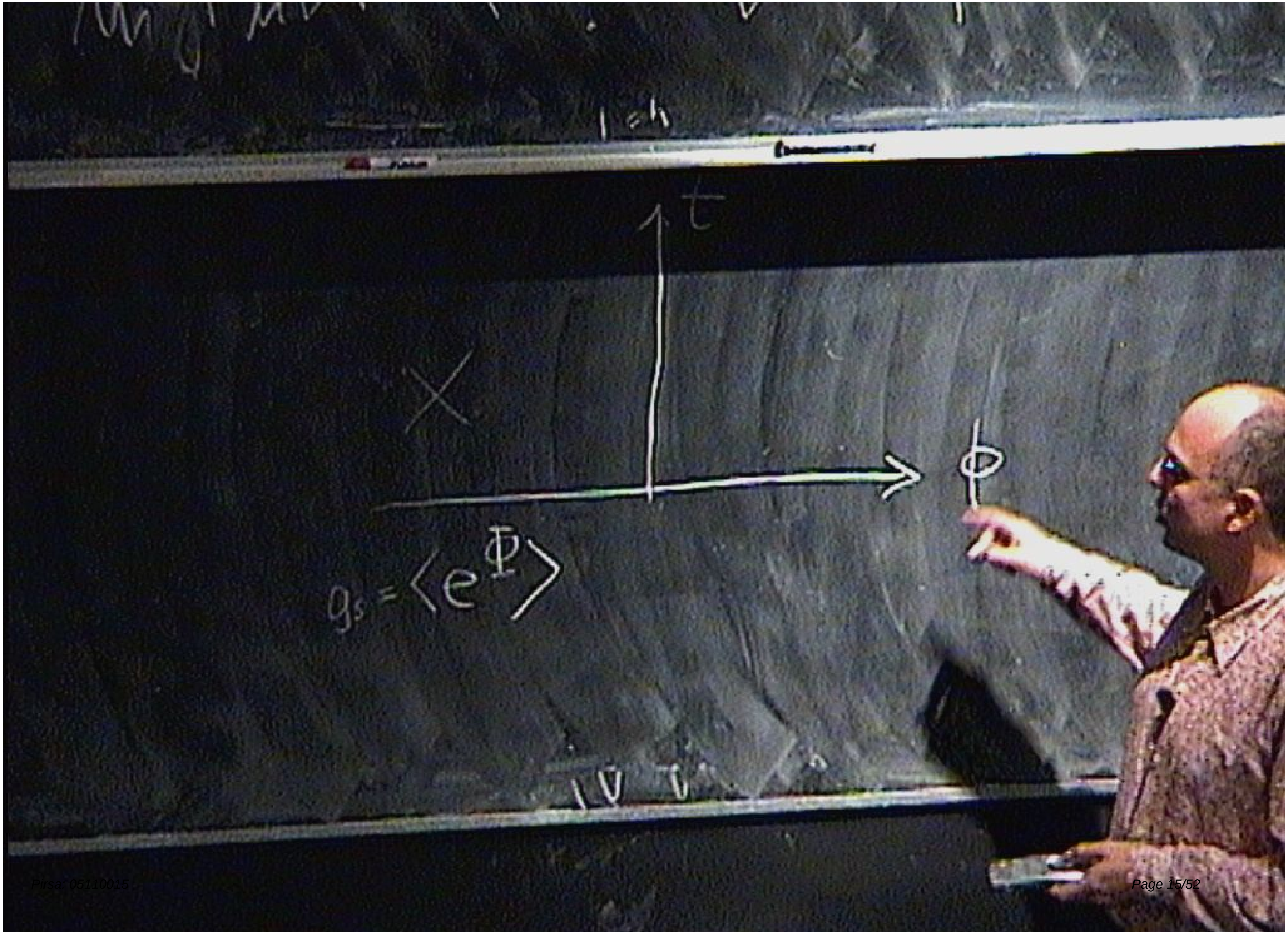
Gauged $U(N)$ matrix model of an $N \times N$ matrix M,

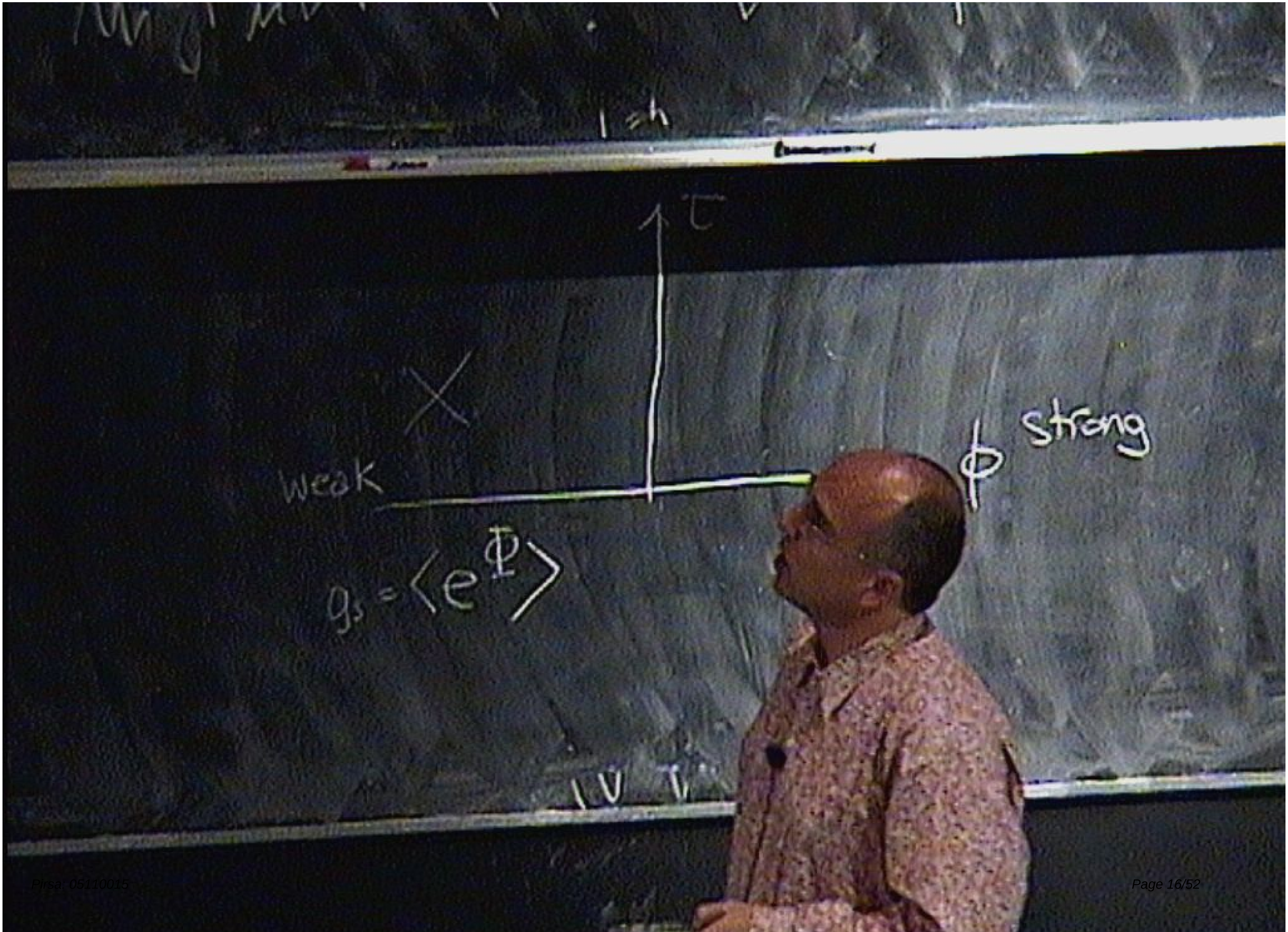
$$S_{0\mathrm{B}} = \beta N \int dt \, \mathrm{Tr} \left(\frac{1}{2}(D_t M)^2 + \frac{1}{4\alpha'} M^2 + \dots \right)$$

in a double scaling limit, involving $N \to \infty$.

Eigenvalues λ of M act as free fermions, theory can be formulated in terms of a second-quantized nonrelativistic Fermi field $\psi(\lambda, t)$ in $1+1$ dimensions. Double-scaling limit: $\epsilon_F \to 0$ with

$$\mu = N \epsilon_F \sim \frac{1}{g_s}$$


t
x
ϕ
$g_s = \langle e^{\Phi} \rangle$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fixed and playing the role of the 0B string coupling.

Type 0A theory:

A $U(N) \times U(N+q)$ quiver matrix model of an $N \times (N+q)$ matrix M,

$$S_{0\mathrm{B}} = \beta N \int dt\, \mathrm{Tr}\left((D_t M)^\dagger D_t M + \frac{1}{2\alpha'} M^\dagger M + \dots \right)$$

in the double scaling limit. N fermions in $1+1$ dimensions, in a potential modified by q.

q has several physical meanings:

- net D0-brane charge (0A has stable D0's & anti-D0's)
- background RR flux, of a RR two-form field strength
- angular momentum on a plane.

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Noncritical M-theory for noncritical strings

We are looking for "noncritical M-theory".
Some desired properties may include:

- D0-charge reinterpreted as momentum along an extra, compact dimension (S^1) of M-theory (*i.e.*, expect the RR 1-form to be a KK gauge field);
- The string coupling g_s related to the radius of the extra S^1;
- Type 0A, 0B, . . . string theories should be solutions;
- Expect 2+1 dimensional solutions, beyond $\hat{c} = 1$ string theory.

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- Expect 2+1 dimensional solutions, beyond $\hat{c} = 1$ string theory.

Such a theory indeed exists.

How to find it: In the matrix model language? In the string effective action language?

Our philosophy: The $\hat{c} = 1$ string theories are fully defined via the double-scaled Fermi theory. Look for the nonperturbative formulation of M-theory in the same language.

The angular dimension on the plane is the extra dimension of M-theory.

String coupling vs. the radius R_3

A parable:

Consider the Einstein-Hilbert action in $D+1$ dimensions,

$$\frac{1}{\ell_{D+1}^{D-1}} \int d^{D+1}X \sqrt{G}\, \mathcal{R}(G),$$

reduce to the string-frame effective action,

$$\int d^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R}(g) + \ldots \right).$$

For $D=2$, we get

$$R_3 = \frac{\ell_3}{g_s^2}$$

Hence, strong string coupling corresponds to small radius.

Nonperturbative noncritical M-theory as a Fermi liquid in 2+1 dimensions

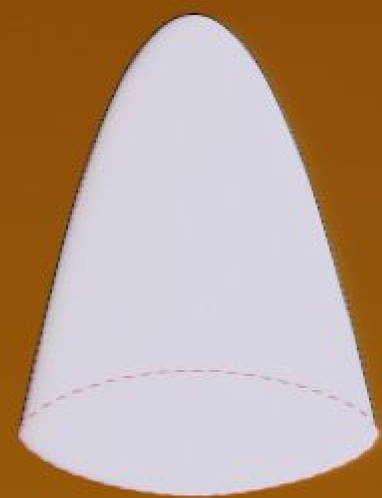
Start with a nonrelativistic Fermi field theory of $\Psi(t,\lambda_1,\lambda_2)$ (a spinless field), on flat R^3 (parametrized by t, λ_i, $i=1,2$), in the upside-down harmonic oscillator potential,

$$S_M = \int dt\, d^2\lambda \left(i\Psi^\dagger\partial_t\Psi + \frac{1}{2}\Psi^\dagger(\Delta + \omega_0^2\lambda^2 + \dots)\Psi \right).$$

Noncritical M-theory:= double-scaling limit of this system.

Careful double-scaling limit involves a regulating cutoff, by non-universal pieces in the potential. We will regulate by placing an infinite wall at some distance $\sqrt{2\Lambda}$ from the origin in the λ_i plane ($\omega_0 = 1$).

The potential looks like this:



The moduli space of all solutions

Theory easily quantized, using info from double-scaled $1+1$ fermions.
Two natural representations:

Cartesian coordinates

Expansion in products of wavefunctions of Type 0B theory,

$$\Psi(t,\lambda_i) = \int d^2E\, a_{\alpha_1\alpha_2}(E_1,E_2)e^{i(E_1+E_2)t}\psi_{\alpha_1}(E_1,\lambda_1)\psi_{\alpha_2}(E_2,\lambda_2)$$

with anticommutation relations:

$$\{a_{\alpha_1\alpha_2}(E_1,E_2), a^\dagger_{\alpha_1'\alpha_2'}(E_1',E_2')\} = \delta^2(E_i - E_i')\delta_{\alpha_1\alpha_1'}\delta_{\alpha_2\alpha_2'}$$

Polar coordinates λ, θ

Expansion in Type 0A wavefunctions,

$$\Psi(t,\lambda,\theta) = \sum_{q\in\mathbf{Z}} e^{iq\theta} \int dE\, a_q(E) e^{iEt} \psi_q(E,\lambda)$$

(again, q is the Type 0A RR-flux).

More on the double-scaling limit

Two parts:

- Introduce cutoff Λ, N fermions, take $N \to \infty$;
- Simultaneously, identify the scaling variable (typically, a product of N with a conserved quantity, such as energy or angular momentum).

General philosophy on the moduli space of solutions:

A state is specified by declaring how each canonical pair of oscillators acts on it. Typically, two such different states are not in each other's Hilbert spaces; represent (excitations of) different vacua.

Most states do not have a "smooth" hydrodynamic description (such as in terms of the bosonic profile of a Fermi surface).

Spacetime is an emergent property: For states that do have a hydrodynamic description (bosonization).

Time-dependent solutions emerge on an equal footing with static solutions.

Thus, the free fermions define the starfish diagram.

Type 0A and 0B string vacua as solutions

Type 0A strings in the linear dilaton vacuum

Pick a value of q. Define $|0\mathrm{A}, \mu\rangle$ by filling only that sector, up to $-\mu$:

$$a_q(E)|0\mathrm{A}, \mu\rangle = 0, \quad E > -\mu,$$
$$a_q^\dagger(E)|0\mathrm{A}, \mu\rangle = 0, \quad E < -\mu,$$

and

$$a_{q'}(E)|0\mathrm{A}, \mu\rangle = 0, \quad q' \neq q, \text{ for all } E.$$

(Note: This is not equivalent to just naively sending $\mu \to \infty$ in sectors of $q' \neq q$ after the double-scaling limit.)

Type 0B strings in the linear dilaton vacuum

Pick a value of E_2. Define $|0\text{B},\mu\rangle$ by filling only that sector, up to $-\mu$:

$$a_{\alpha_1\alpha_2}(E_1,E_2)|0\text{B},\mu\rangle = 0, \quad E_1 > -\mu,$$
$$a^\dagger_{\alpha_1\alpha_2}(E_1,E_2)|0\text{B},\mu\rangle = 0, \quad E_1 < -\mu,$$

and

$$a^\dagger_{\alpha_1\alpha_2}(E_1,E_2')|0\text{B},\mu\rangle = 0, \quad E_2' \neq E_2, \text{ for all } E_1.$$

Comments:

• Unlike in Type 0A, the choice of E_2 does not add a new parameter – shift of E_1 equivalent to shift in μ.

• T-duality between Type 0A and 0B is non-obvious in this M-theory framework (just as it is not obvious between IIA and IIB in critical M-theory).

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Type 0B strings in the linear dilaton vacuum

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$$a_{\alpha_1\alpha_2}(E_1,E_2)|0\mathrm{B},\mu\rangle = 0, \quad E_1 > -\mu,$$
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II.
The (2+1)-dimensional vacuum

The 2+1 dimensional vacuum of noncritical M-theory

The regulated theory at finite N has an obvious solution $|\mathrm{M}, \mu\rangle$: Fill every state up to a uniform Fermi energy $-\mu$, (irrespective of q, etc.).

$$a_q(E)|\mathrm{M}, \mu\rangle = 0, \quad E > -\mu, \text{ all } q$$

$$a_q^\dagger(E)|\mathrm{M}, \mu\rangle = 0, \quad E < -\mu, \text{ all } q.$$

We will refer to this state as the "M-theory vacuum."

This is the noncritial analog, for strings in the linear dilaton background, of the 11-dimensional Minkowski solution of critical M-theory.

Let us explore some of the properties of $|\mathrm{M}, \mu\rangle$.

Vacuum energy: Le energy

$$\ldots n(\mu/\Lambda) + \mathcal{O}(1/\mu^2).$$

For the M-theory vacuum, look t ver connected Riemann surfaces.
identify the scaling variable.

$N \to \infty$ is the semiclassical limi ouville dimension.

ntrivial, and given in terms of

$$N = \int \frac{d^2 p d^2 \lambda}{(2\pi\hbar)^2} \theta$$

The density of states is

$$\rho(\epsilon) = \hbar \frac{\partial N}{\partial \epsilon} \sim \int_{\sqrt{2\epsilon}}^{\sqrt{2 \ldots}}$$

This gives

$$F_0 \sim \frac{\epsilon_F^3}{\hbar^3} + \dots$$

Hence:

- The scaling variable is $\mu \equiv -\epsilon_F N$, just as in $\hat{c} = 1$ string theory;
- The leading log of string theory disappears; instead, the leading ("tree-level") term in the vacuum energy scales as μ^3. (We shall write this as κ^{-2}.)
- The natural expansion parameter is $1/\mu \sim \kappa^{2/3}$. Recall heterotic M-theory in 11d!
- The volume dependence is absent from the leading term in F_0

- Particle-hole duality predicts that ρ should be an even function!

M-theory at finite temperature

The calculation can be extended to Euclidean compactified time. The free energy $\Gamma(\mu, R)$ is given by

$$\frac{\partial^2 \Gamma}{\partial \mu^2} \sim \frac{\frac{1}{2R}\frac{\partial}{\partial \mu}}{\sin\left(\frac{1}{2R}\frac{\partial}{\partial \mu}\right)} \frac{\mu}{\tanh(\pi \mu)}.$$

Unlike the string-theory free energy, this does not exhibit any obvious self T-duality. (Just as in the case of critical M-theory.)

High-T behavior: Another M-theory solution, with $\kappa_{\mathrm{eff}} = \kappa T^{3/2}$ (to be contrasted with field theory: $g_{\mathrm{eff}} = g T^{1/2}$, and string theory: $g_{\mathrm{eff}} = g_s T$).

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Hydrodynamic equations for the Fermi surface

can be derived just as in noncritical string theory.

Choose, for example, $p_1 = P(p_2, \lambda_1, \lambda_2, t)$ as the dependent variable. Then the EoM is

$$\partial_t P = \lambda_1 - (\lambda_2 \partial_{p_2} + p_2 \partial_{\lambda_2}) P - P \partial_{\lambda_1} P.$$

(Of course, sometimes it is useful to switch to another representation, such as in polar coordinates.)

Various static as well as time-dependent solutions can be easily found.

Some static solutions

A simple modification $|\tilde{\mathrm{M}}, \mu\rangle$ of the M-theory vacuum:
Fill up to $-\mu$ for even q, and down to $-\mu$ for odd q.

This is an example of a slightly exotic state, which nevertheless seems hydrodynamical (but slightly outside the EoM for the Fermi surface).

Another example:

Define a family of Fermi surfaces via

$$\frac{1}{2}p_1^2 + \frac{1}{2}p_2^2 - \frac{1}{2}\lambda_1^2 - \frac{1}{2}\lambda_2^2 + \Omega(p_1\lambda_2 - p_2\lambda_1) = -\mu.$$

(Ω: angular velocity). Playing with Ω and μ, one can interpolate between Fermi surfaces set by H and J.

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(Ω: angular velocity). Playing with Ω and μ, one can interpolate between Fermi surfaces set by H and J.

An interesting duality

Consider the Fermi surface given by

$$p_1\lambda_2 - p_2\lambda_1 = q.$$

A linear canonical transformation maps this to

$$\frac{1}{2}\tilde{p}_1^2 + \frac{1}{2}\tilde{\lambda}_1^2 - \frac{1}{2}\tilde{p}_2^2 - \frac{1}{2}\tilde{\lambda}_2^2 = -\tilde{\mu}$$

Under this map, $H \leftrightarrow \tilde{J}$, $J \leftrightarrow \tilde{H}$.

Duality to the Hamiltonian of the thermofield dynamics of free fermions in the rightside-up harmonic oscillator potential.

Contact with the Itzhaki-McGreevy, . . . string theory (and its M-theory lift).

Some time-dependent solutions

Use the time-dependent charges to find time-dependent solutions for the EoM of the Fermi surface.

For example,

$$\frac{1}{2}p_1^2 + \frac{1}{2}p_2^2 - \frac{1}{2}\lambda_1^2 - \frac{1}{2}\lambda_2^2 + c_1(p_1 + \lambda_1)e^{-t} + c_2(p_2 - \lambda_2)e^{t} = -\mu$$

(with c_1, c_2 constants) is a typical such solution.

It represents a Type 0B string theory in the far past, evolving via the M-theory phase, and decaying into another Type 0B theory in the future.

Novelty compared to string theory: well-defined macroscopic initial and final states.

Tachyon scattering can be studied in this time-dependent background. Is it possible to define a unitary S-matrix?

IV.
Conclusions

Lessons for string/M-theory

• Moduli space of solutioı
• Spacetime emergent a:
some states
• DoF of M-theory (nonre
Recall [P.H., hep-th/050:
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Lessons for string/M-theory

- Moduli space of solutions fully defined via fermions
- Spacetime emergent as a hydrodynamic description of only some states
- DoF of M-theory (nonrelativistic fermions/D-branes)

Recall [P.H., hep-th/0502006]: K-theory classifies stable Fermi surfaces.

- Remarkable "moral" similarities with M(atrix) theory
- and holographic field theory.

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Lessons for quantum gravity

- Two successful approaches to quantum gravity in $2+1$ dimensions so far:

Chern-Simons theory, critical string/M-theory. Noncritical M-theory might be a third. Indications that the topological nature of the CS formulation is combined with propagating DoF.

- Vacuum energy ($\approx$ cosmo. constant) has fascinating features;

- The theory is Machian: No fermions, no spacetime.