Title: An origin of light and electrons -- a unification of gauge interaction and Fermi statistics

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Abstract:

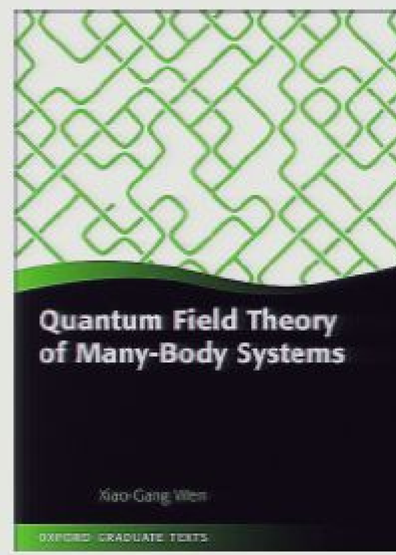
An origin of light and electrons

– a unification of gauge interaction and Fermi statistics

Michael Levin and Xiao-Gang Wen

http://dao.mit.edu/~wen

- Artificial light and quantum orders ...
  PRB **68** 115413 (2003)
- Fermions, strings, and gauge fields ...
  PRB **67** 245316 (2003)
- Strings-net condensation ...
  PRB **71** 045110 (2005)
- *Quantum field theory of many-body systems*
  (Oxford Univ. Press, 2004)



Deep mysteries of nature

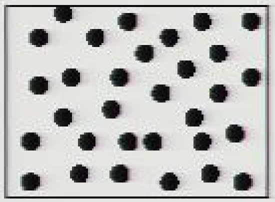
- Identical particles (Why two hydrogen atoms are exactly the same?)
- Gauge interactions (long range, massless gauge bosons)
- Fermi Statistics (Who ordered it?)
- Massless fermions (nearly, $M_f/M_P \sim 10^{-20}$)
- Chiral fermions (Are we edge excitations?)
- Gravity (The correct physical theory allows only integers)

A great-grand unification:
a single structure that explains all the mysteries

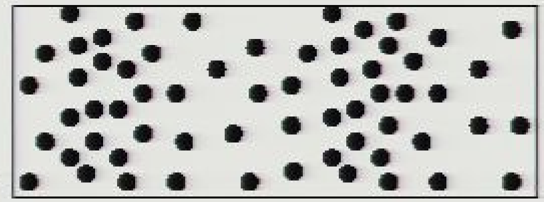
We will discuss a baby-grand unification that explains the first four mysteries from a single structure – local bosonic model.

Where do Maxwell equation and Dirac equation come from?

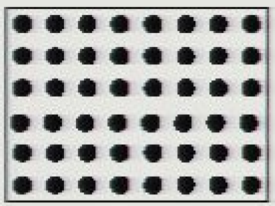
- Eular equation: $\partial_t^2 \rho - v^2 \partial_i^2 \rho = 0 \rightarrow$ massless scalar identical bosons



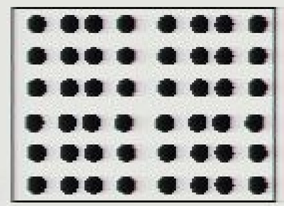
superfluid $\rightarrow$ density fluctuations

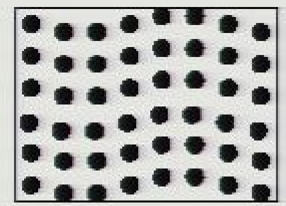


- Navier equation: $\partial_t^2 u^i - T_m^{ijk} \partial_j \partial_k u^m = 0 \rightarrow$ phonons (identical bosons)



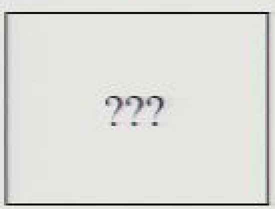
crystal $\rightarrow$ lattice fluctuations





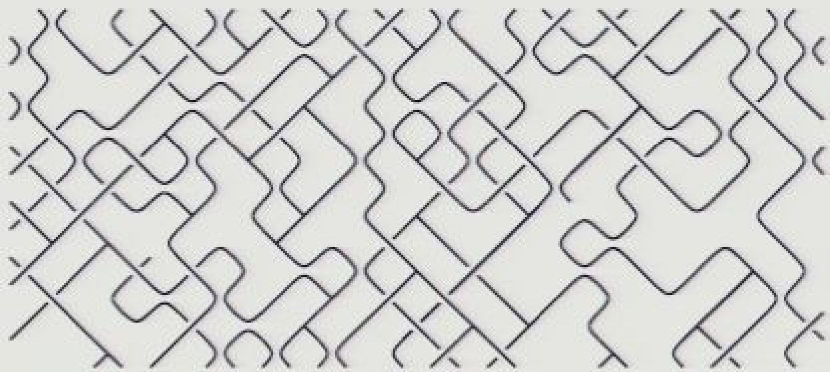
Identical particles $\rightarrow$ vacuum is not empty

- Maxwell equation: $\partial \times \mathbf{E} + \partial_t \mathbf{B} = \partial \times \mathbf{B} - \partial_t \mathbf{E} = 0 \rightarrow$ photons



- Dirac equation: $(\gamma^\mu \partial_\mu - m)\psi = 0 \rightarrow$ fermions

- Both Maxwell equation and Dirac equation can come from local bosonic models or lattice spin models if bosons/spin (a) form Long strings and (b) strings from a quantum liquid (string-net condensed state):

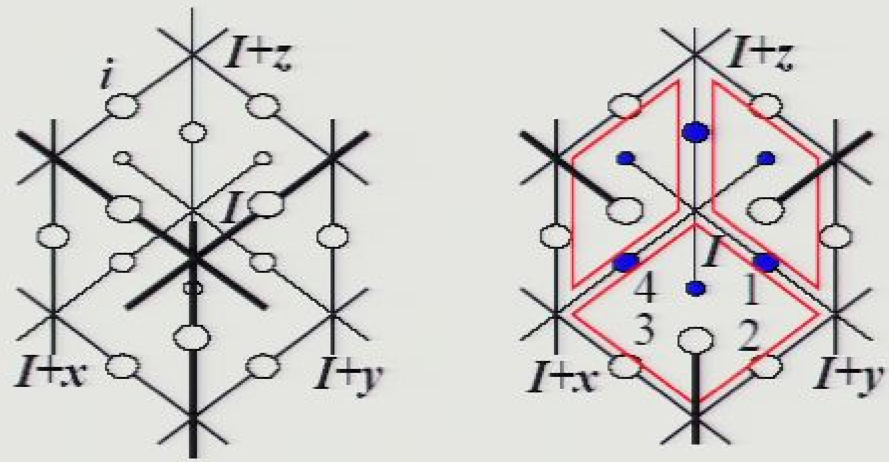


Gauge bosons and fermions can emerge as low energy collective modes of the condensed string-nets

String-net condensation provides a way to unify gauge interactions and Fermi statistics

The appearance of the gauge interaction and Fermi statistics in our nature is not an accident.

A local bosonic model on cubic lattice



A rotor θ_i on every link of the cubic lattice:

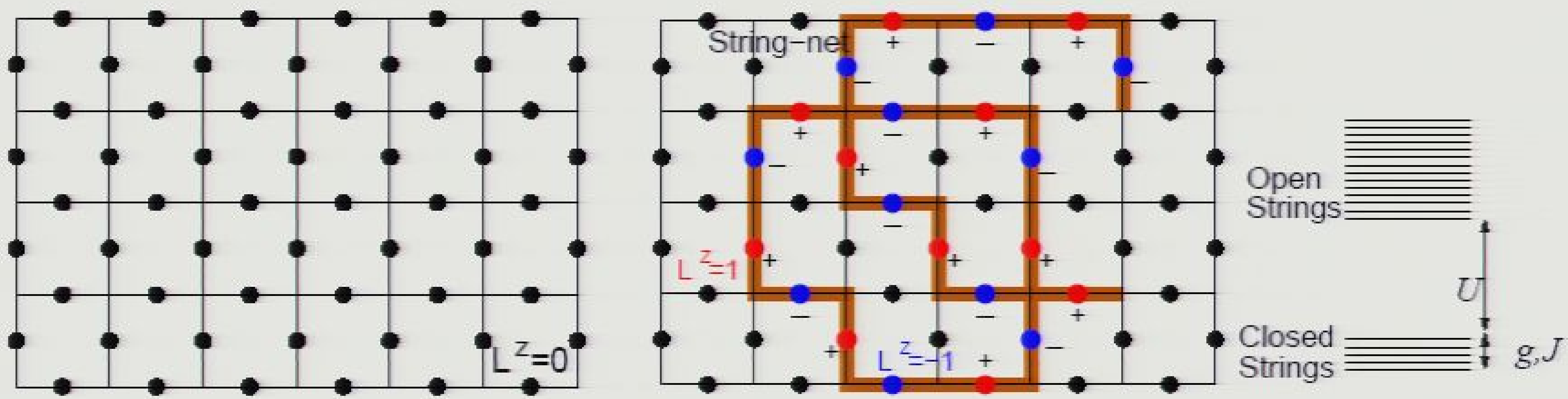
$$H = U \sum_{\mathbf{I}} Q_{\mathbf{I}}^2 - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{i}} (L_{\mathbf{i}}^z)^2$$

$$Q_{\mathbf{I}} = \sum_{\text{i next to } \mathbf{I}} L_{\mathbf{i}}^z, \qquad B_{\mathbf{p}} = L_1^+ L_2^- L_3^+ L_4^-$$

$L^z = i\partial_\theta$: the angular momentum of the rotor

$L^\pm = e^{\pm i\theta}$: the raising/lowering operators of L^z

What is string-net



Physical meaning of the three terms:

- the U-term $\to$ closed strings. Open ends cost energy.
- J-term $\to$ string tension
- the g-term $\to$ strings can fluctuate

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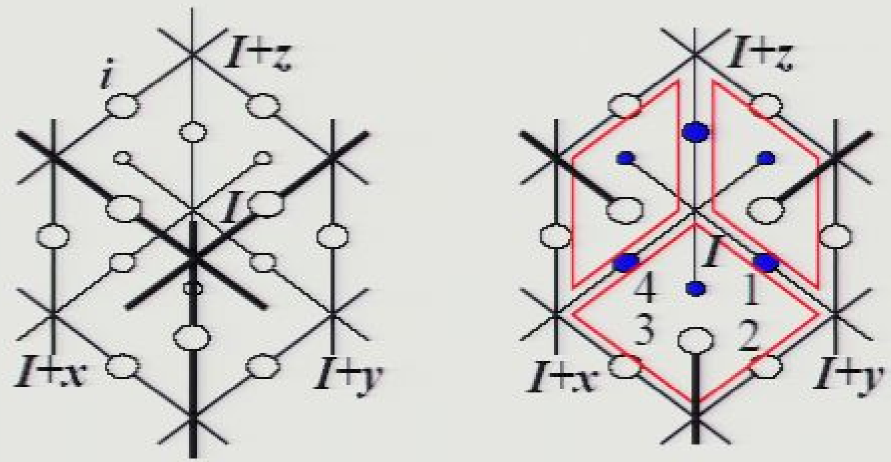

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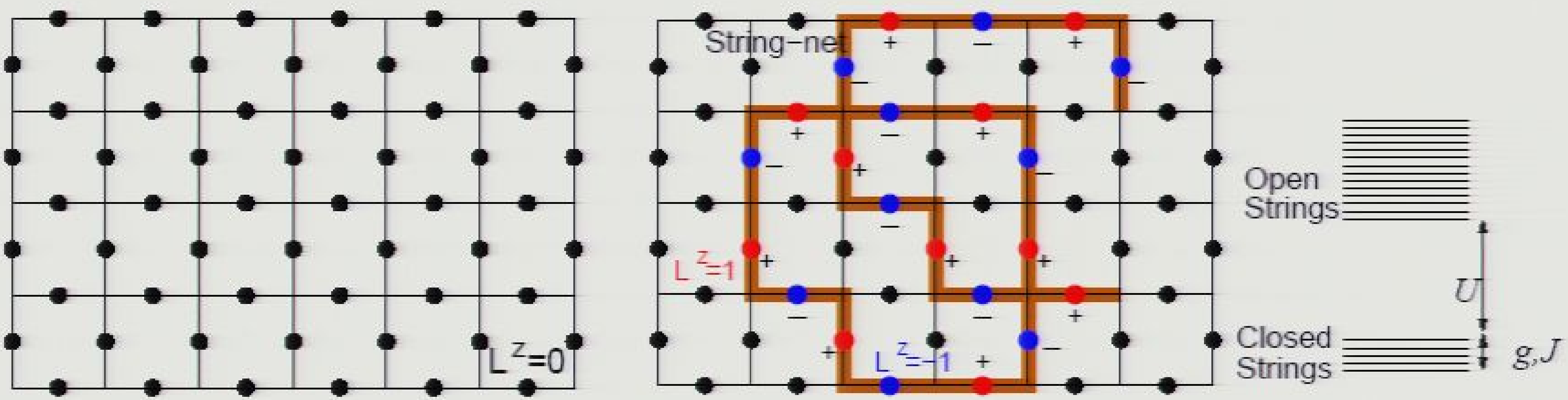

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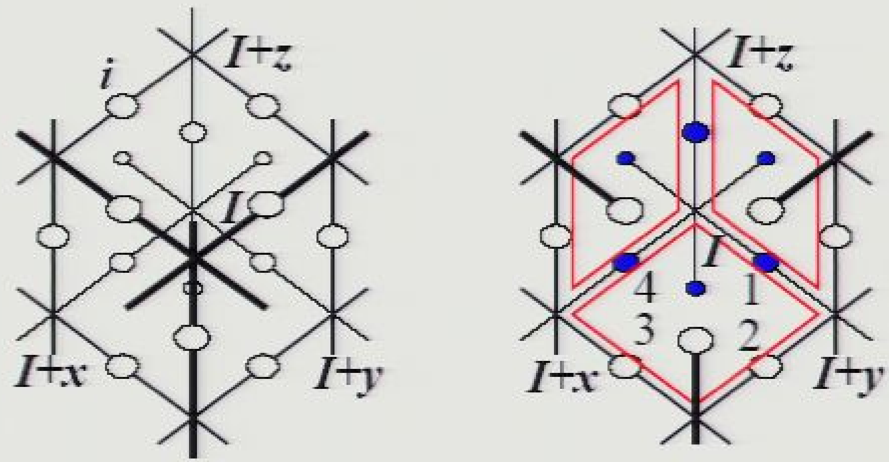

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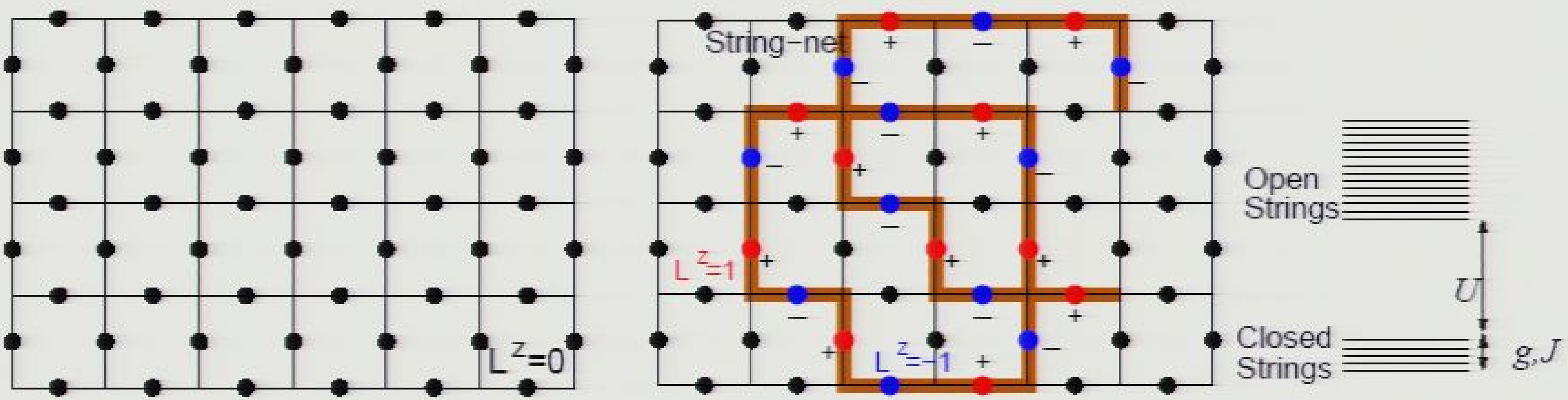

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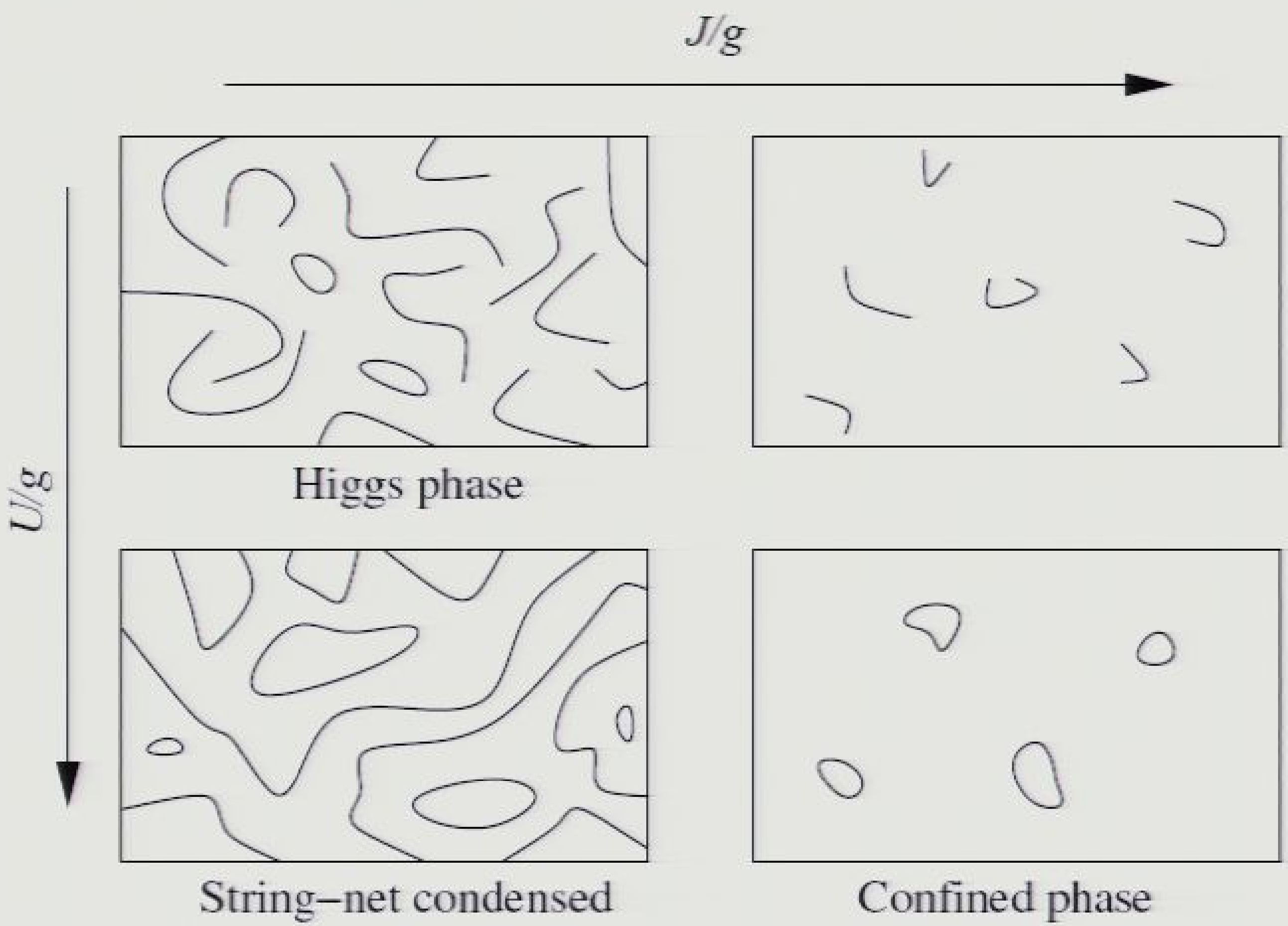
What is string-net condensation

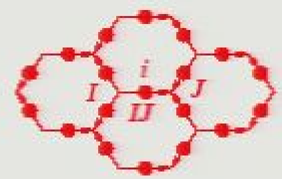


$$|\text{string-net condensed}\rangle = \sum_{\text{all closed-string-nets}} |\text{string-net}\rangle$$

Fluctuations of condensed string-nets = $U(1)$ gauge bosons

- When $U=\infty$, the rotor model can be mapped to $U(1)$ lattice gauge model, with $\theta_{\mathbf{i}}$ on link $\mathbf{IJ}$ as the $U(1)$ gauge potential:

$$\theta_{\mathbf{i}} = a_{\mathbf{IJ}},$$



- For finite U and with other perturbations:

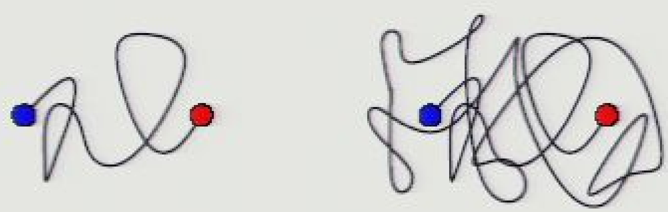
$$L_{\text{eff}} = L_{U(1)}(a_{\mathbf{IJ}}) + \epsilon_1 a_0^2 + \epsilon_2 \cos(a_{\mathbf{IJ}})$$

Since $a_{\mathbf{IJ}} = \theta_{\mathbf{i}}$ is compact, the ϵ-terms do not generate a mass for the $U(1)$ gauge bosons, if $\epsilon_{1,2}$ are small enough.

Gauge bosons and "gauge symmetry" can be emergent

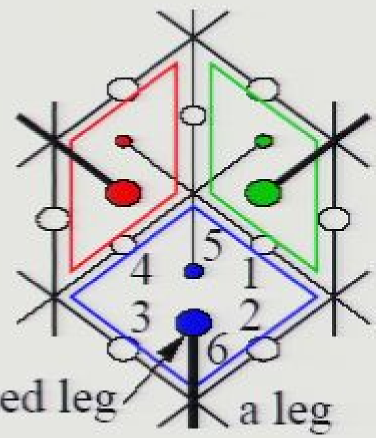
Ends of open strings = gauge charges

- Strings are unobservable in string condensed state.
- Ends of strings behave like independent particles.



- $L^z_{\mathbf{i}} \sim \dot\theta_{\mathbf{i}} = \dot a_{\mathbf{IJ}}$ correspond to electric field/flux.
  Ends of condensed strings are gauge charges
  They also carry fractional rotor-angular momentum ($L^z = \pm 1/2$)

Can get fermions for free (almost) Levin & Wen 04



Just add some legs

- Dressed-string model:

$$H = U \sum_{\mathbf{I}} Q_{\mathbf{I}} - g \sum_{\mathbf{p}} (B_{\mathbf{p}} + h.c.) + J \sum_{\mathbf{i}} (L_{\mathbf{i}}^z)^2$$

$$B_{\mathrm{p}} = L_1^+ L_2^- L_3^+ L_4^- (-1)^{L_5^z + L_6^z}$$

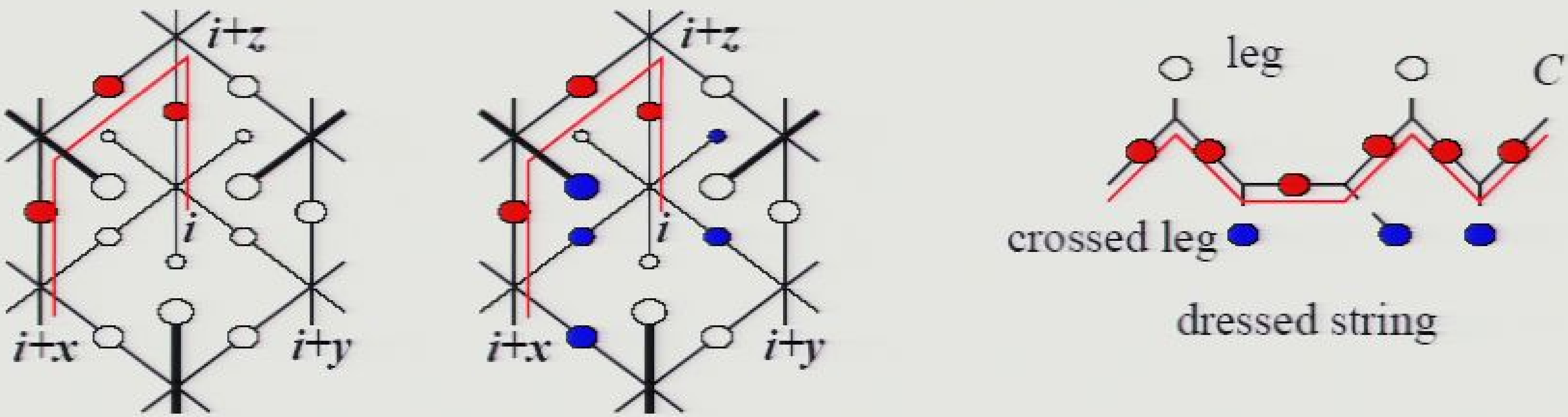
- Different ground state wave function for the string-net condensed state

$$|\text{string-net condensed}\rangle = \sum_{\text{all closed-string-nets}} \pm|\text{string-net}\rangle$$

which leads to different statistics for the ends of condensed strings.

String operators – creation operators of gauge charges

- A pair of gauge charges is created by an open string operator which commute with the Hamiltonian except at its two ends.
  Strings cost no energy and is unobservable.



- In simple-string model – simple-string operator

$$L^{+}_{i_1} L^{-}_{i_2} L^{+}_{i_3} L^{-}_{i_4} \dots$$

- In dressed-string model – dressed-string operator

$$(L^{+}_{i_1} L^{-}_{i_2} L^{+}_{i_3} L^{-}_{i_4} \dots) \prod_{\text{i on crossed legs of } C} (-1)^{L^z_i}$$

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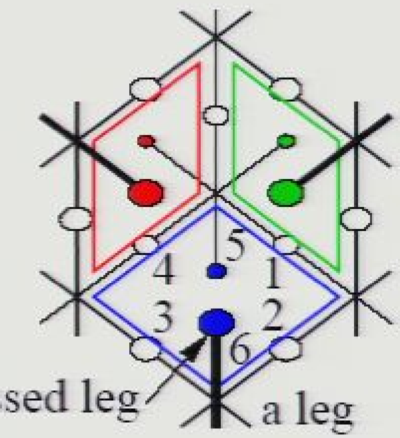

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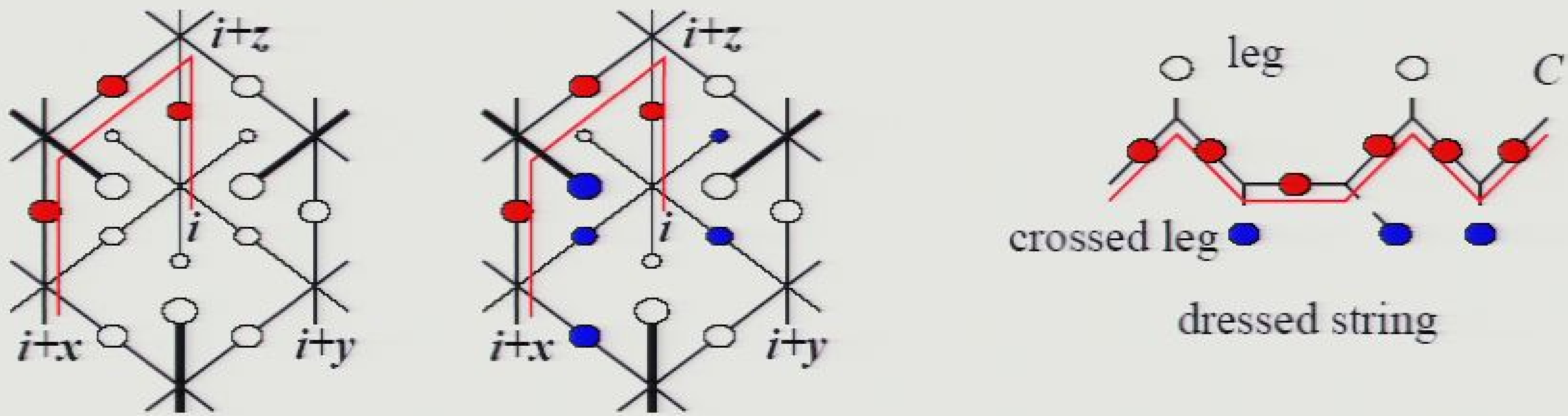

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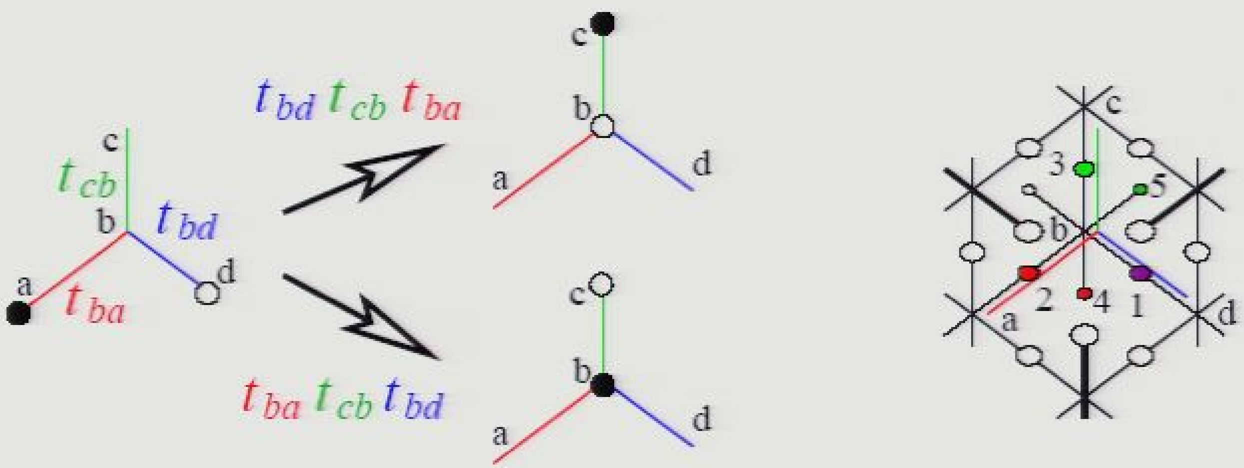
A dressed-string operator creates a pair of fermions

- The statistics is determined by particle hopping operators Levin & Wen 02:



- An open string operator is a hopping operator of the gauge charges. Open string operator determine the statistics.
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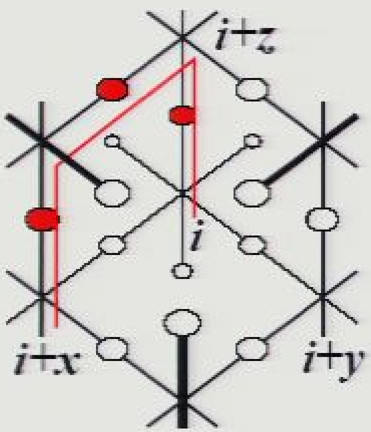

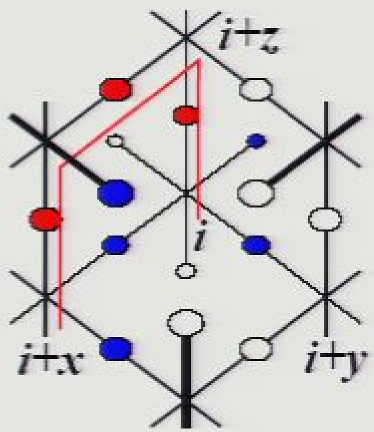

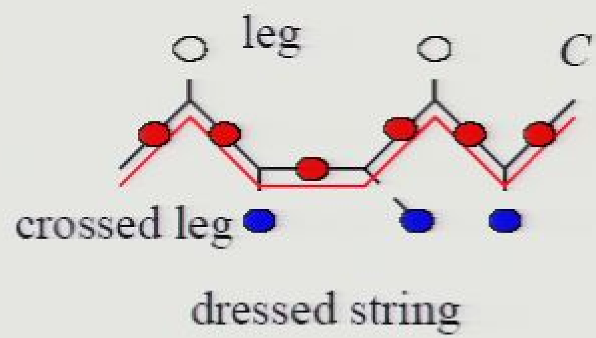

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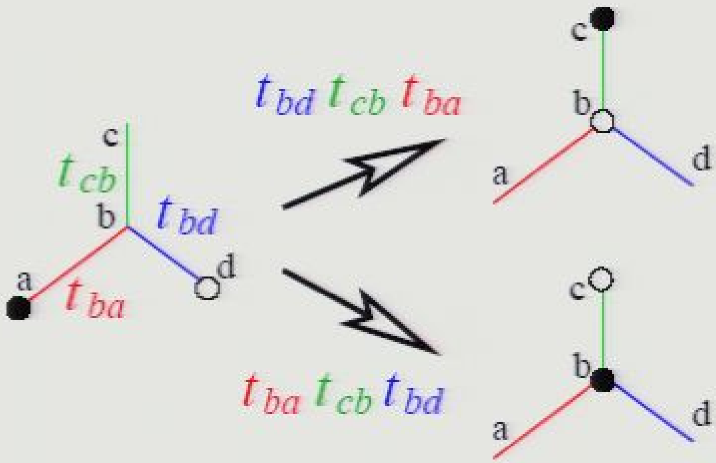

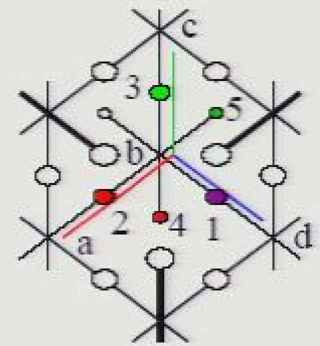

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What make fermions massless?

- Consider the hopping Hamiltonian for a *single* end of string

$$\hat{H} = \sum_{\mathbf{ij}} (\hat{t}_{\mathbf{ij}} + h.c.)$$

$\hat{H}$ may realize translation symmetry only projectively.

- The translation $\hat{T}^{(2)}_{\mathbf{a}}$ of the *two* ends of a string satisfies the translation algebra

$$\hat{T}^{(2)}_{\mathbf{a}} \hat{T}^{(2)}_{\mathbf{b}} = \hat{T}^{(2)}_{\mathbf{b}} \hat{T}^{(2)}_{\mathbf{a}}, \qquad \mathbf{a}, \mathbf{b} = \mathbf{x}, \mathbf{y}, \mathbf{z}$$

The translation $\hat{T}_{\mathrm{a}}$ of the *one* ends of a string satisfies

$$\hat{T}_{\mathrm{a}} \hat{T}_{\mathbf{b}} = \eta \hat{T}_{\mathbf{b}} \hat{T}_{\mathrm{a}}, \qquad \eta = \pm 1$$

- $\eta = -1 \to \pi$-flux through each square $\to$ massless fermions
- The string-net wave function $\Phi(X) = (-1)^{N_X}$ given rise to the π-flux, where N_X=number of spares enclosed by the closed string X.

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Comparison with superstring theory

Superstring theory

- gauge boson = small open string of size l_P.
- fermion comes from "super world sheet" $(\sigma^1, \sigma^2, \theta^\alpha)$.
- graviton = small closed string of size l_P.

Fermions do not have to carry gauge charges.

String-net theory

Every thing comes from local bosonic model — **locality principle**

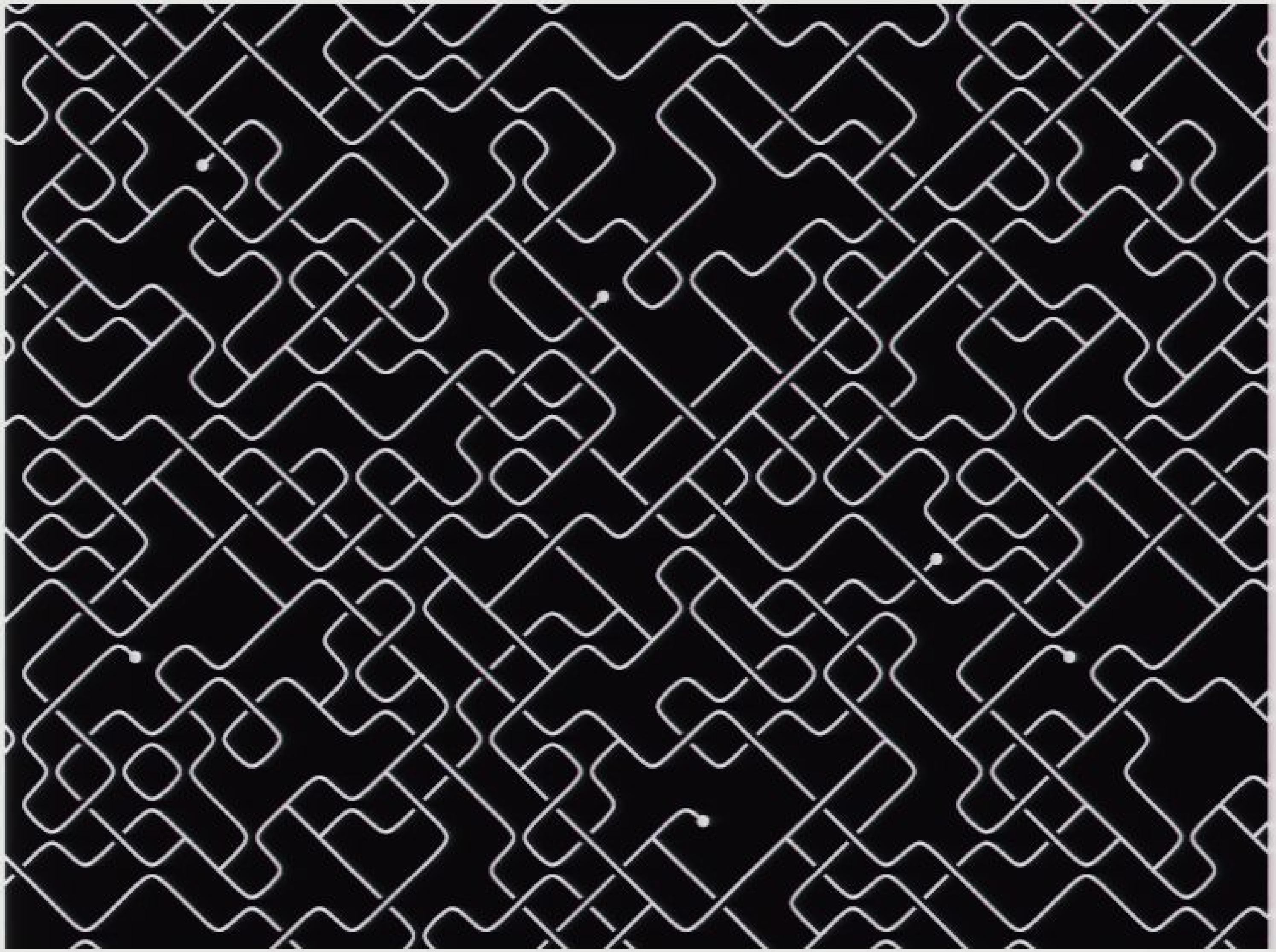
1. $\mathcal{H}_{tot} = \mathcal{H}_i \otimes \mathcal{H}_j \otimes ....$
2. Local operator = operators acting within $\mathcal{H}_i$.
3. Hamiltonian = sum of local operators.

- gauge boson = fluctuations of large string-nets that fill the space.
- fermion = one end of open string.
- graviton = ???.

Fermions (including composite fermions) must carry gauge charges.

A picture of our vacuum

- a recipe for making an artificial vacuum in condensed matter



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