Title: Emergence of quantum spacetime from causal dynamical triangulations

Date: Nov 18, 2005 11:40 AM

URL: http://pirsa.org/05110012

Abstract:

①

Mission Statement:

To find a consistent theory of quantum gravity which describes the dynamical behaviour of spacetime geometry on all scales and reproduces Einstein's theory of General Relativity on large scales.

Preferably, it should also predict new observable physical phenomena.

"Causal Dynamical Triangulations" (CDT) tries to achieve this with minimal ingredients:

① superposition principle

gravity which describes the dynamical behaviour of spacetime geometry on all scales and reproduces Einstein's theory of General Relativity on large scales. Preferably, it should also predict new observable physical phenomena.

"Causal Dynamical Triangulations" (CDT) tries to achieve this with minimal ingredients:

1. superposition principle
2. (micro-) causality
3. minimal background structure
4. no extra dimensions
5. no new fields
6. no SUSY or other new symmetries

"Sum over histories":

$$Z = \sum_{\text{geometries } T} e^{iS(T)}$$

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We <u>DO</u> it,

cosmol. term

$$S \equiv S^{\text{Einstein}} = \kappa \int d^4x \sqrt{-g}\,(R - 2\Lambda).$$

AND we produce some RESULTS.

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How can we succeed where Hawking has failed?

③

A to-do list

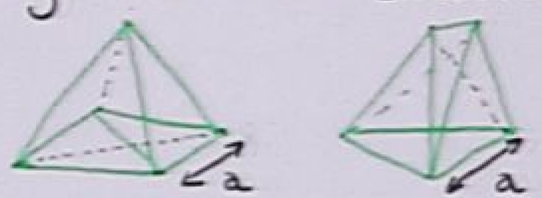
necessary cond.s!
- do Lorentzian, not purely Euclidean path integral
- put in causality conditions

no acausal "baby universes"

- construct non-perturbative Wick rotation

most approaches fail here
- set up regularized path integral such that it actually <u>exists</u>

(here: in terms of piecewise flat "Regge geometries" constructed from



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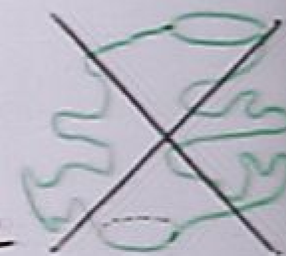

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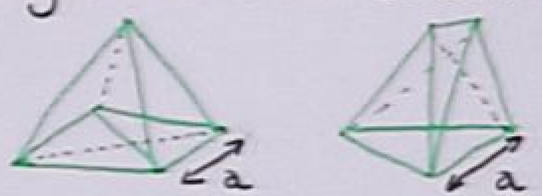

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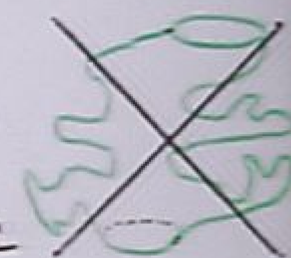

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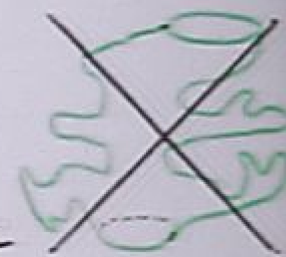

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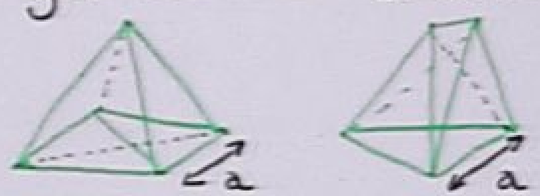

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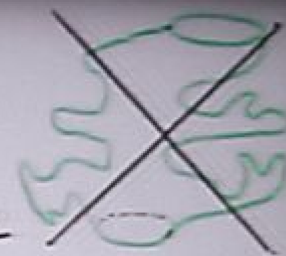

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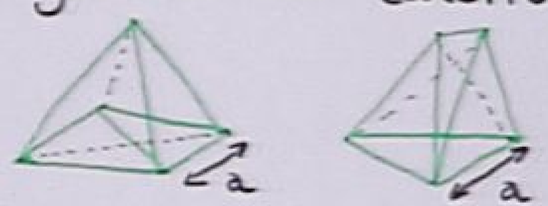

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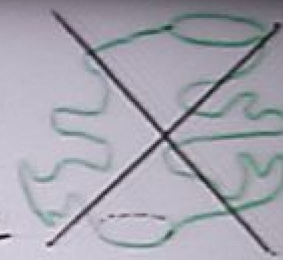

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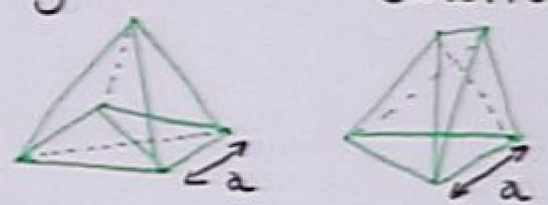

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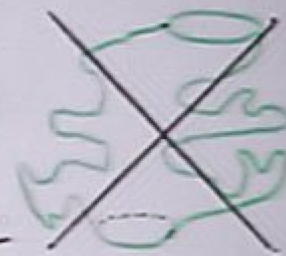

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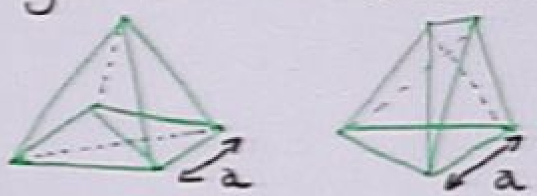

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THE news:

Unlike in previous Euclidean formulations, an extended (quantum) spacetime emerges as ground state of the statistical CDT ensemble in the continuum limit.

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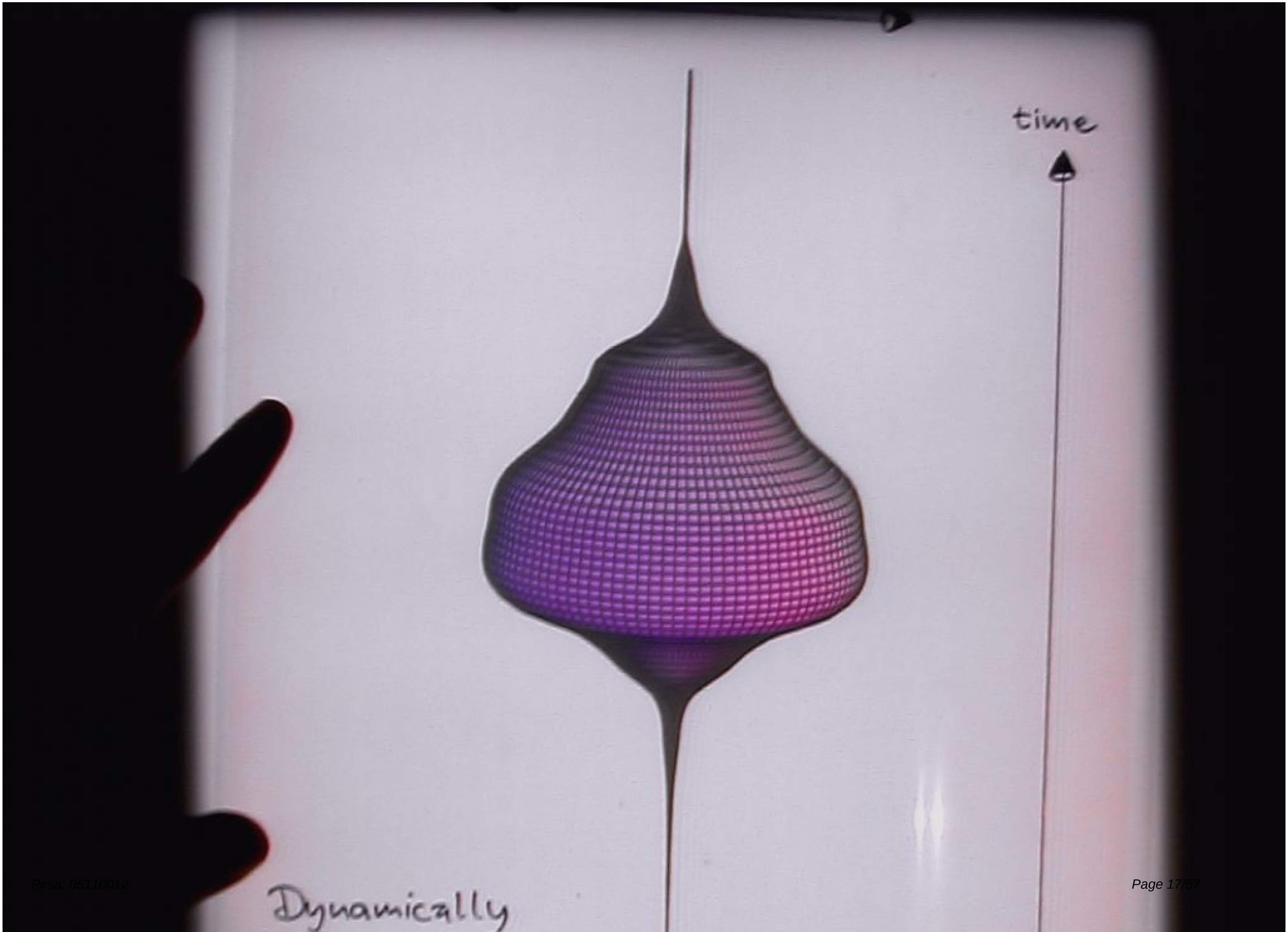
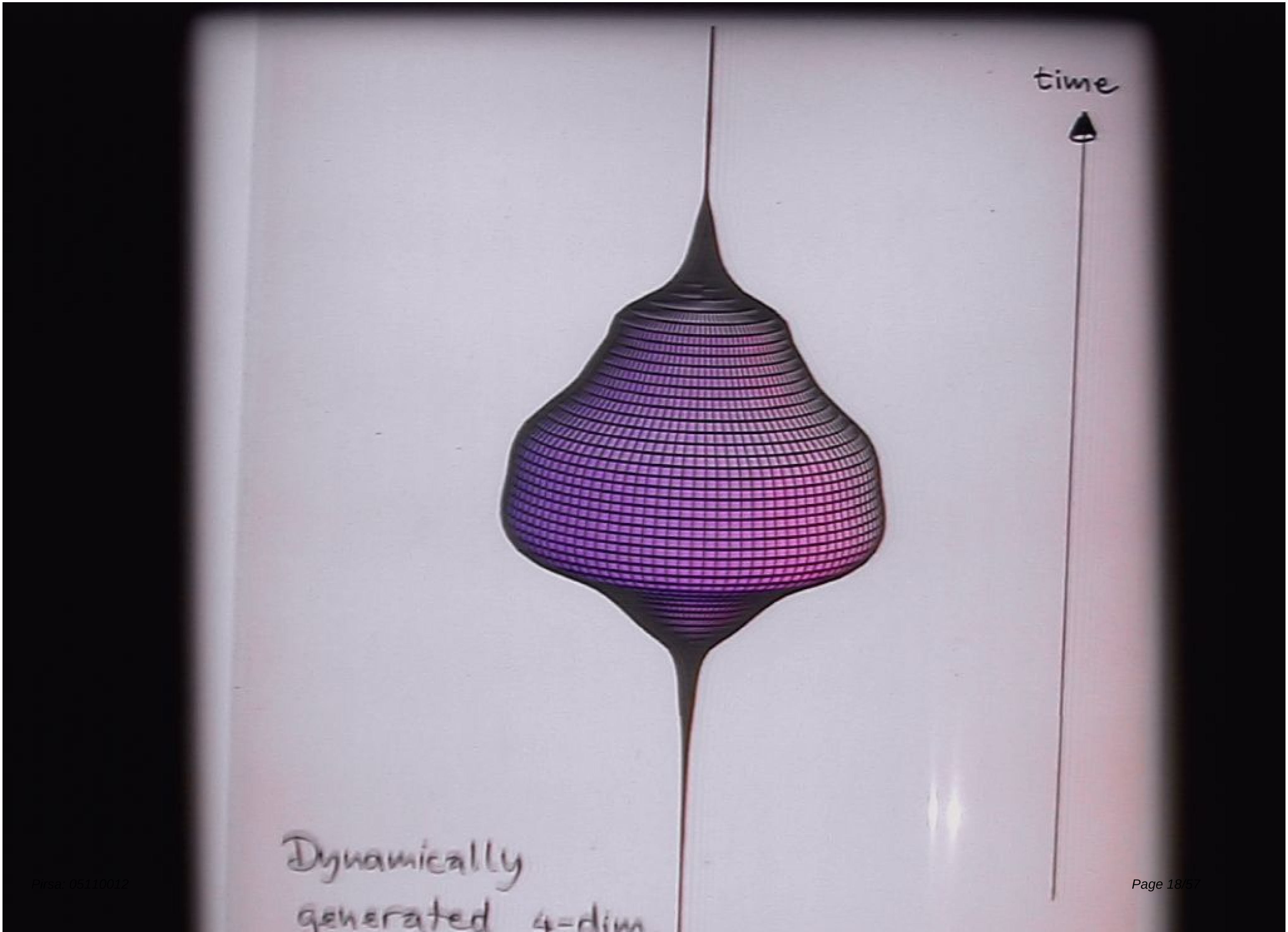
CDT produces concrete evidence for

- classical behaviour on large scales
- highly fluctuating and curved quantum


time
Dynamically


time
Dynamically
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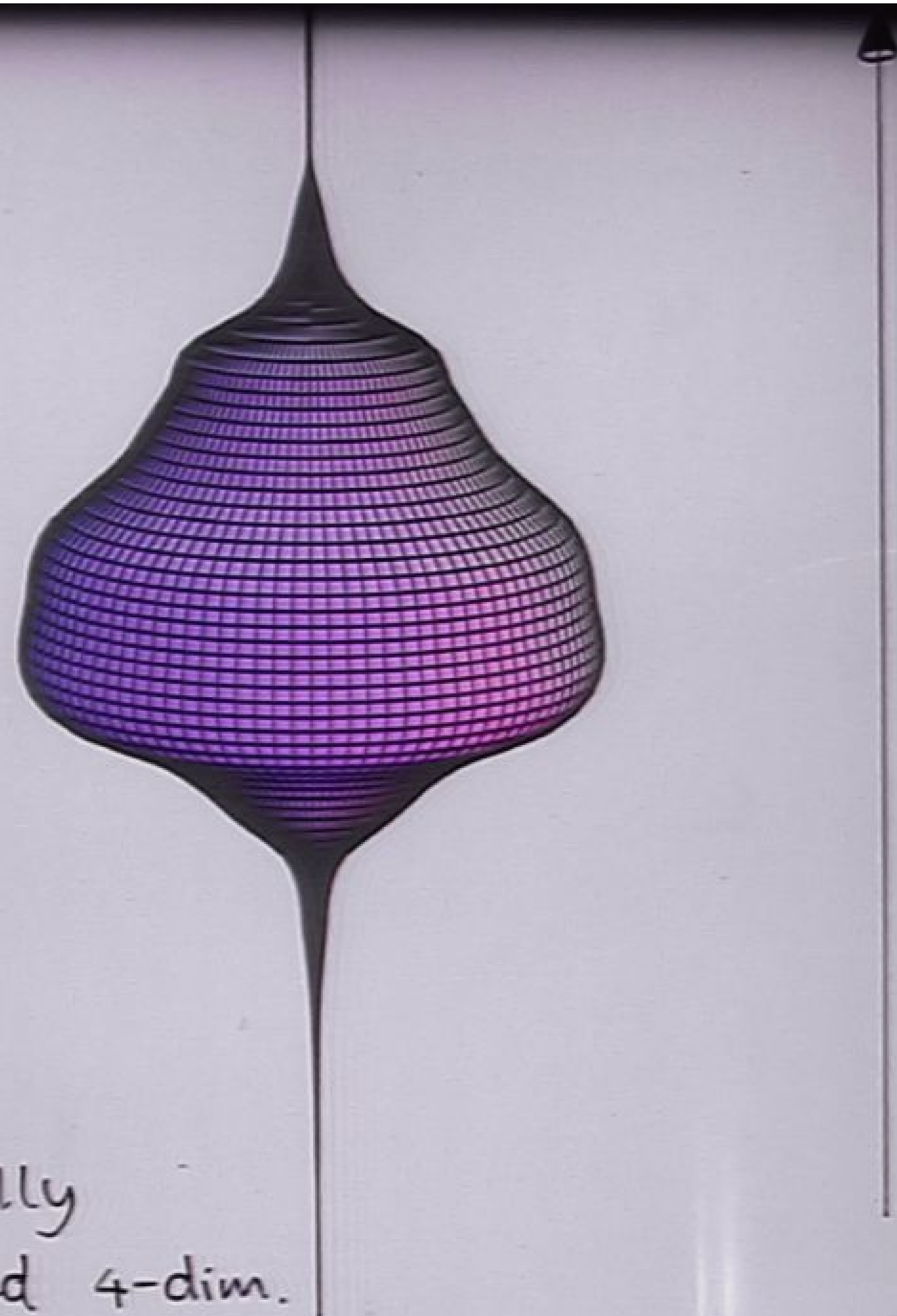

Dynamically
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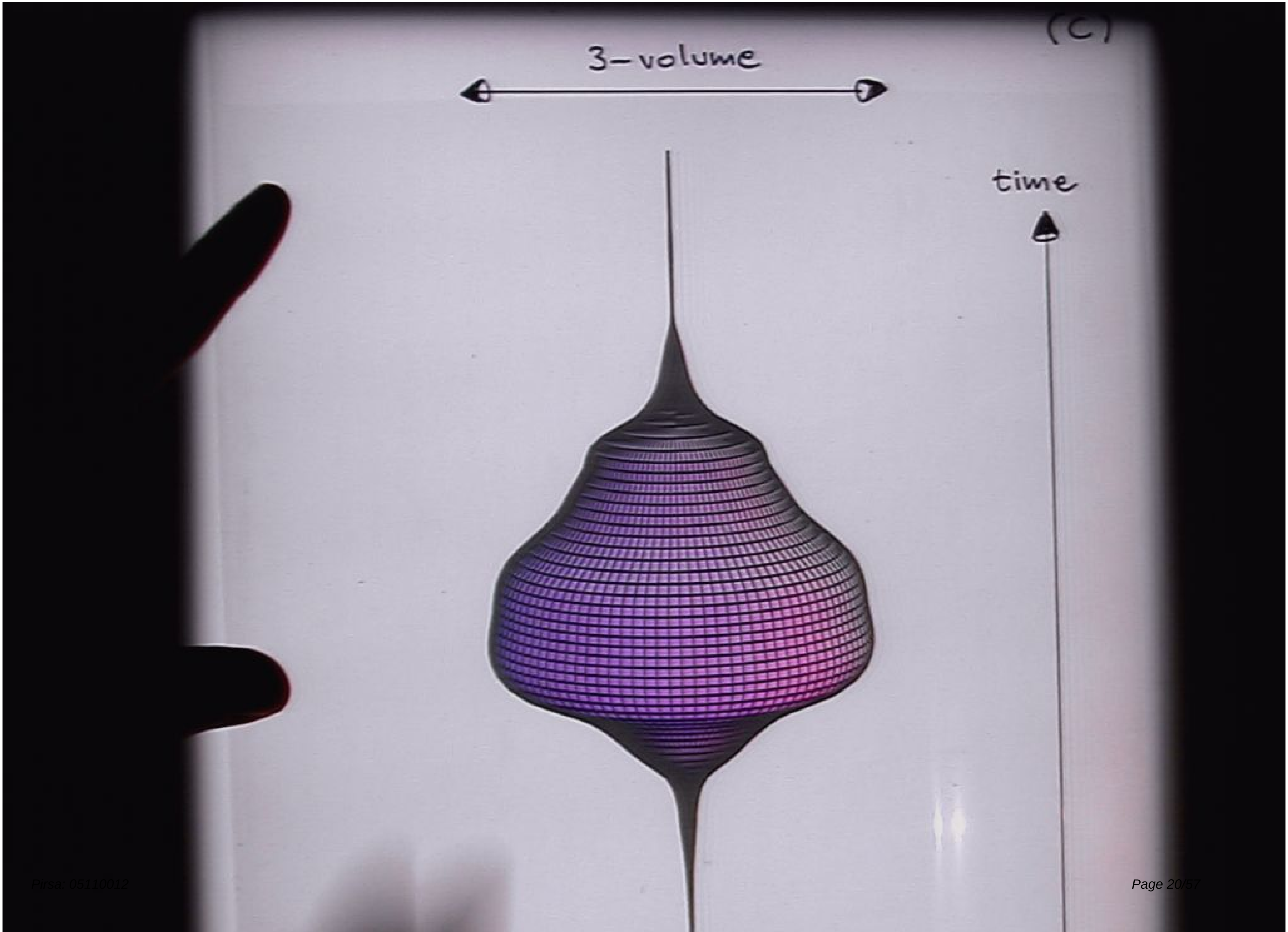
quantum universe, obtained from a

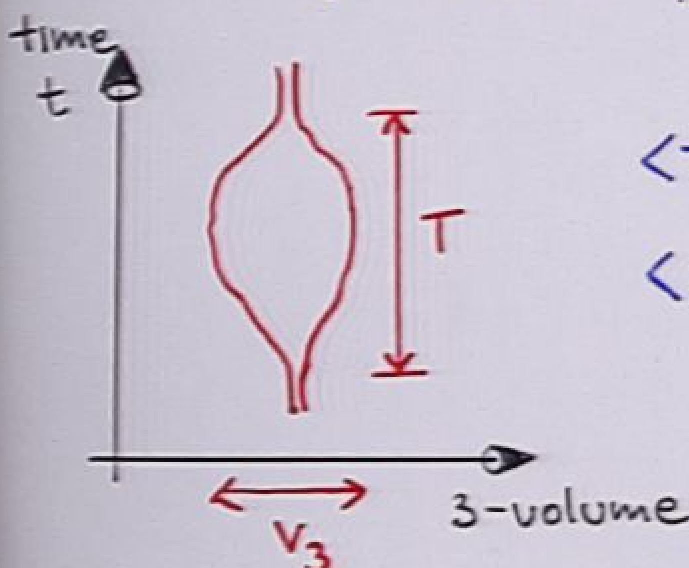

(c)
3-volume
time

Last year's breakthrough

(hep-th/0404156, PRL '04; /0411152, PLB '04)

CDT generates a stable "universe" which is four-dimensional on large scales:

- simulate at various 4-volumes
  $V_4 := \#(\text{4-simplices})$



$$\left.\begin{array}{l}\langle T\rangle \sim V_4^{1/D}\\ \langle V_3\rangle \sim V_4^{3/D}\end{array}\right\}$$ and $D=4$ within measuring accuracy

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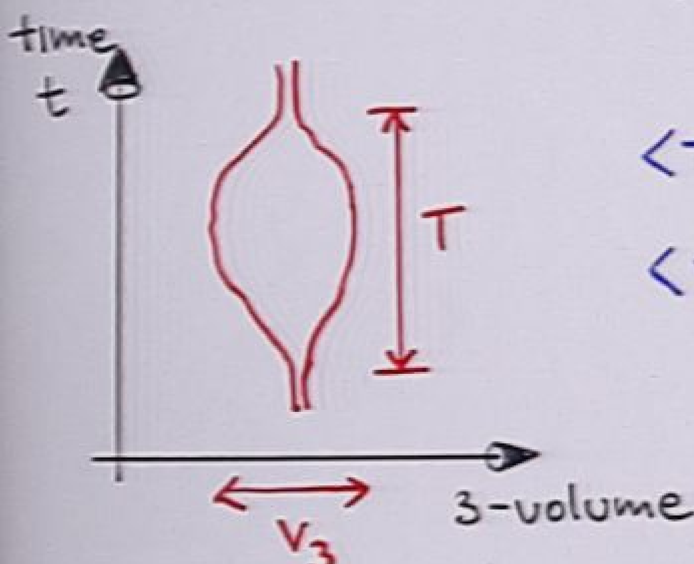

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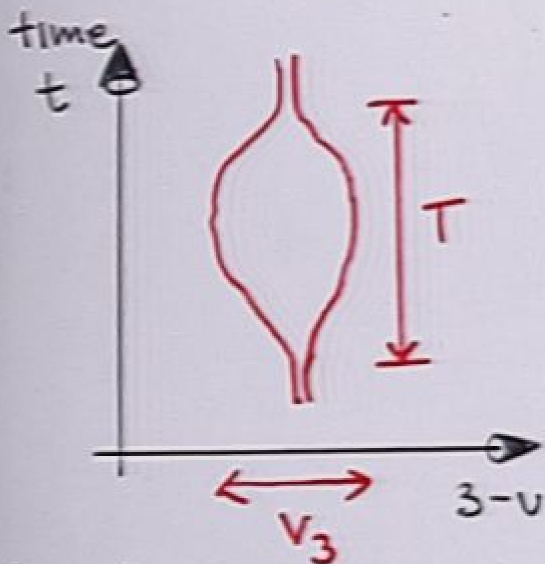

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- shape ($\equiv V_3(t)$) on large scales described by cosmological minisuperspace actio.

→ we are getting some things right!

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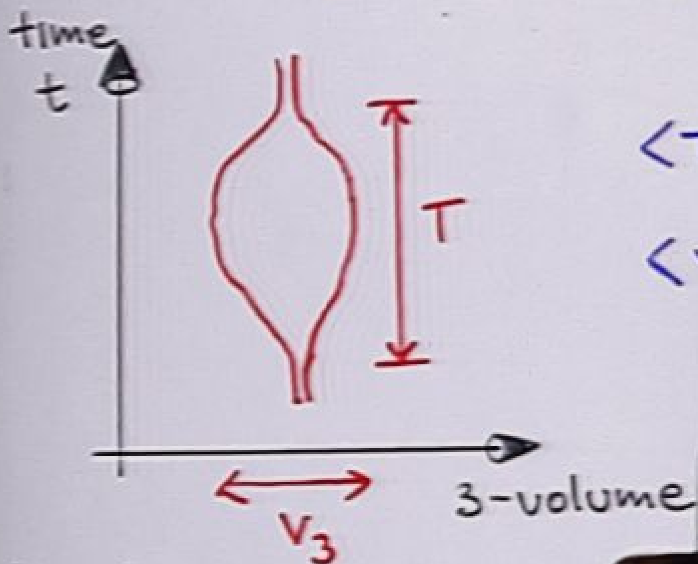

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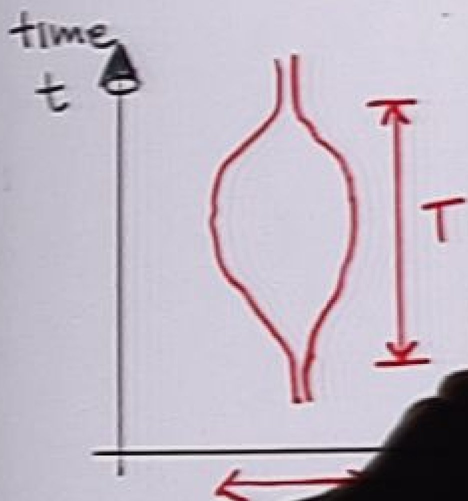

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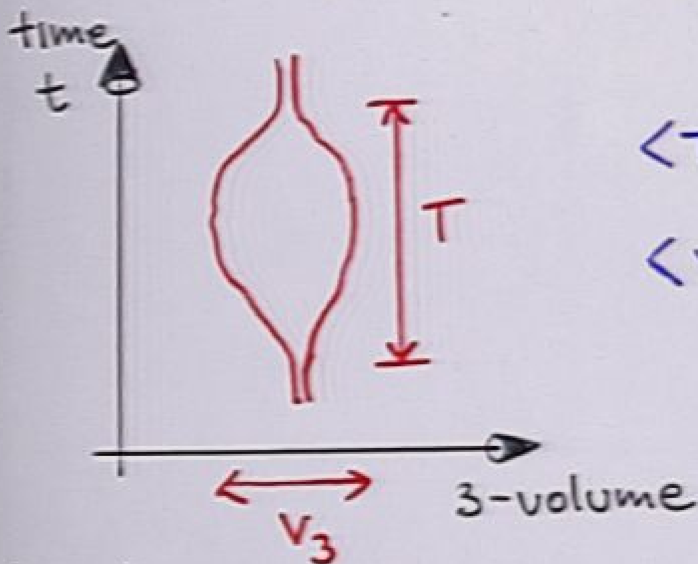

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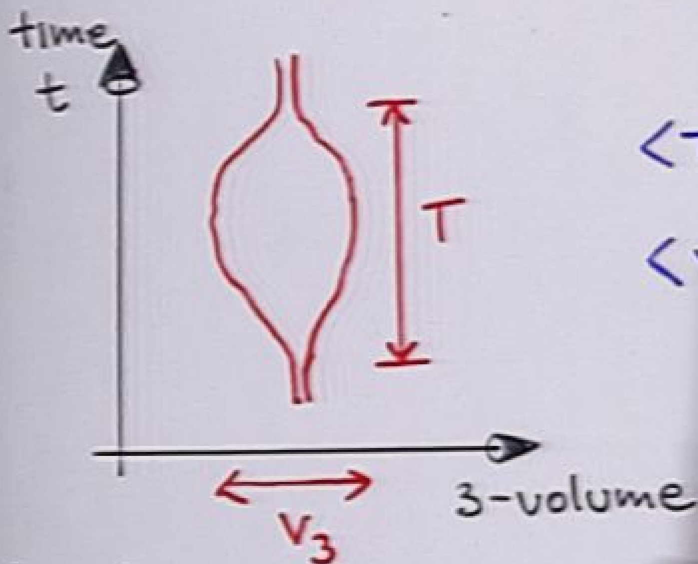

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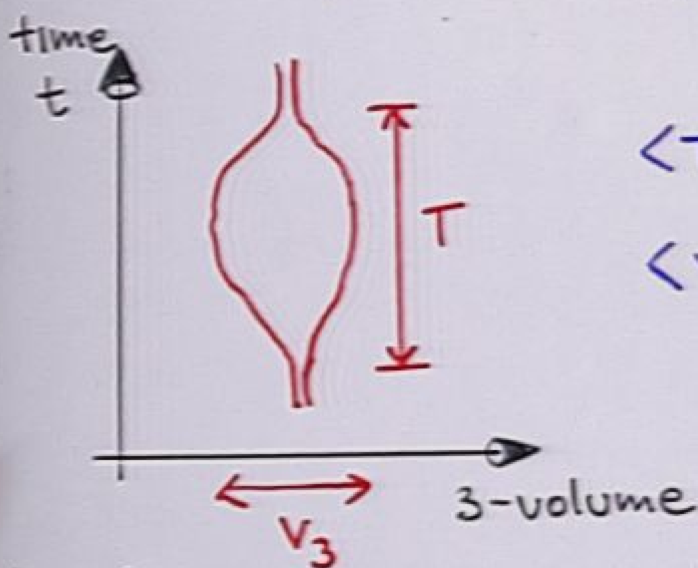

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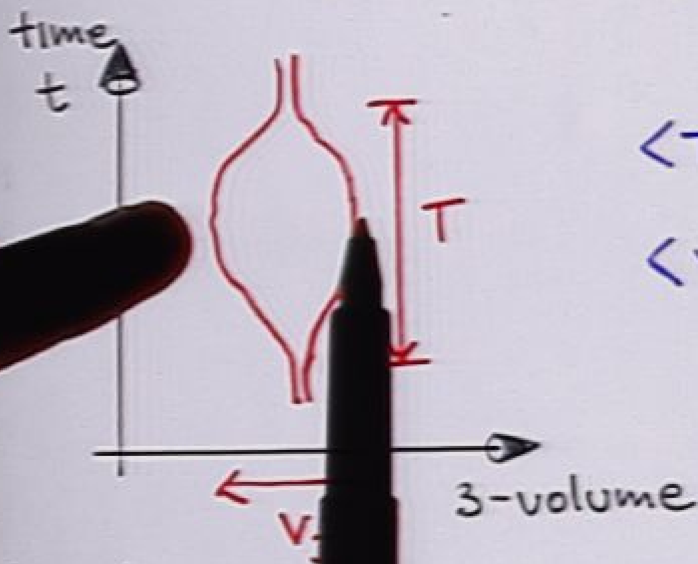

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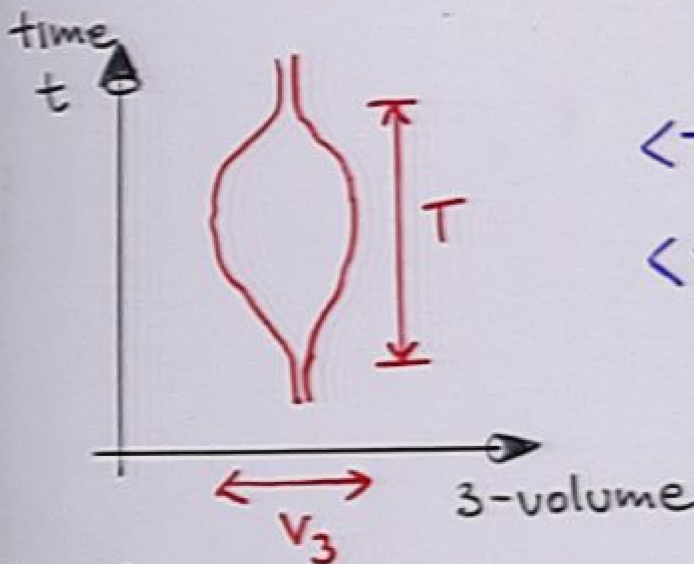

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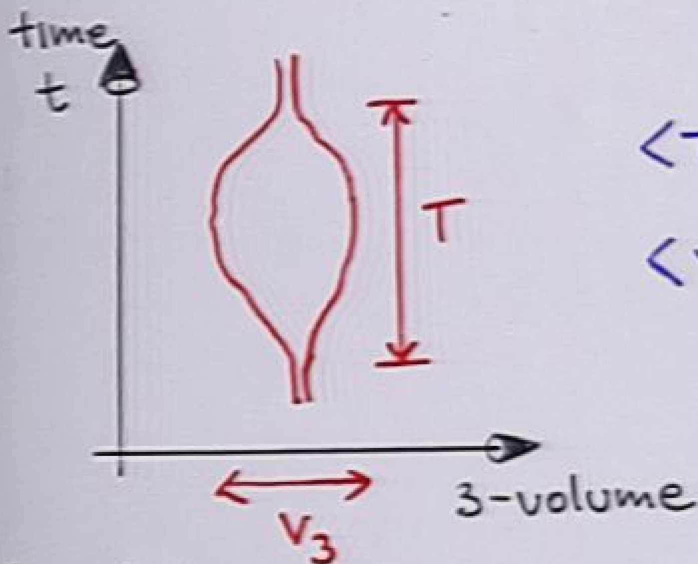

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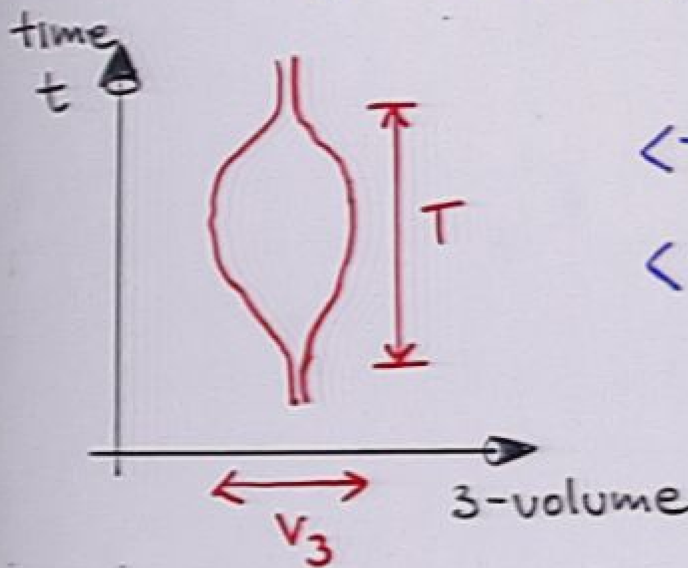

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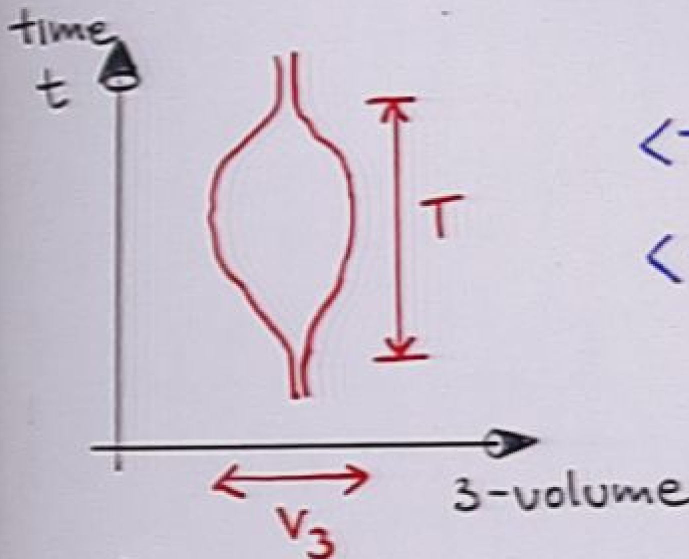

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→ interesting new physics should be contained in deviations from

This year's news ⑥

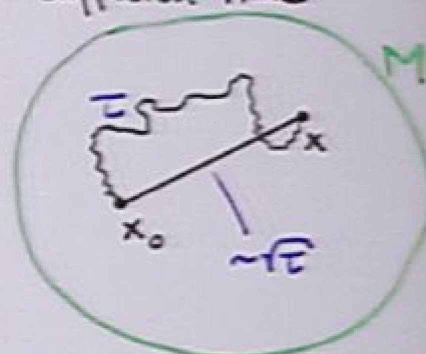
(hep-th/0505113, PRL'05; /0505154, PRD '05)

Study diffusion process / random walks on the ensemble of CDT geometries.

c.f. diffusion in d-dimensional flat space:

$$\partial_\tau P(x,\tau) = \nabla^2 P(x,\tau)$$ diffusion time

$$P(x,x_0;\tau) = \frac{e^{-(x-x_0)^2/4\tau}}{(4\pi\tau)^{d/2}}$$



for $P(x,x_0;\tau=0) = \delta^d(x-x_0)$

average return probability:

$$\mathcal{R}_V(\tau) := \frac{1}{V(M)} \int_M d^dx\, P(x,x;\tau) \overset{\text{here}}{=} \frac{1}{(4\pi\tau)^{\cdot}}$$

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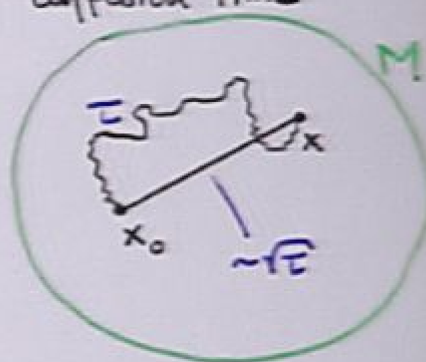

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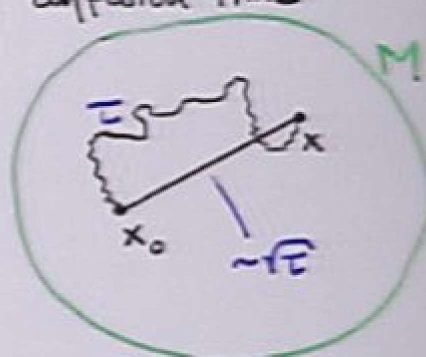

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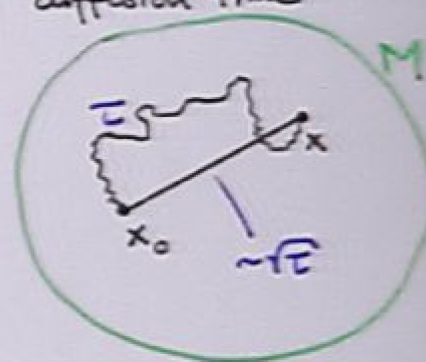

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$$\partial_\tau P(x,\tau) = \nabla^2 P(x,\underbrace{\tau}_{\text{diffusion time}})$$

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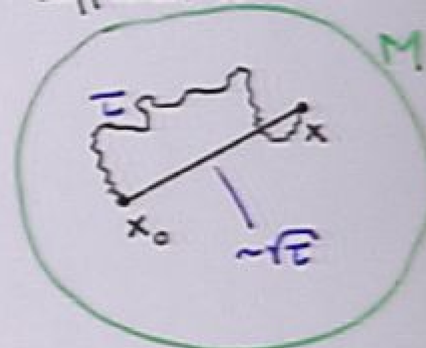

for $P(x,x_0;\tau=0) = \delta^d(x-x_0)$

average return probability:

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$\mathcal{R}_V(\tau)$

⑥

This year's news

(hep-th/0505113, PRL'05; /0505154, PRD '05)

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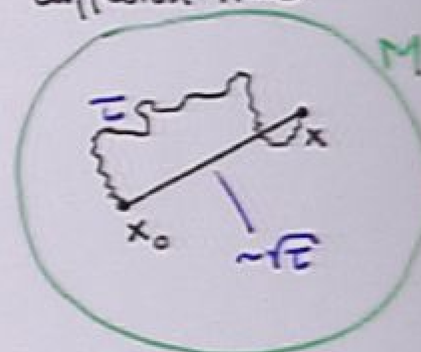

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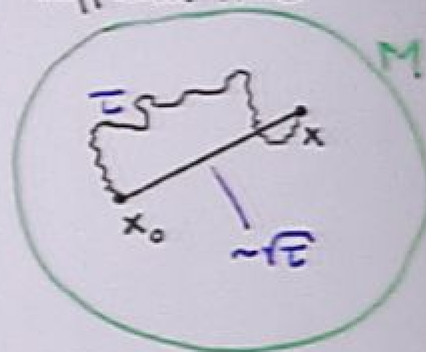

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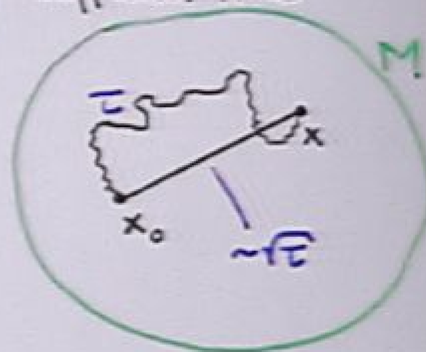

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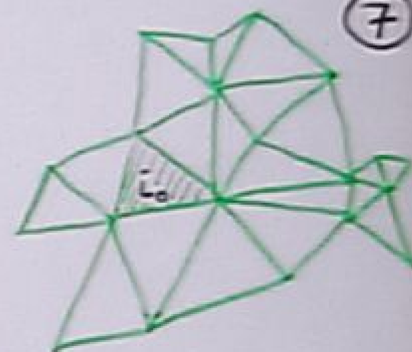
Ex. 2-dim. triangulation T ⑦



$$P_T(i, i_0; \overset{\text{discrete diffusion time}}{\sigma} = 0) = \delta_{i, i_0}$$

define evolution

$$P_T(j, i_0; \sigma+1) = \frac{1}{3} \underbrace{\sum_{k \to j}}_{\text{nearest neighbours of } j} P_T(k, i_0; \sigma)$$

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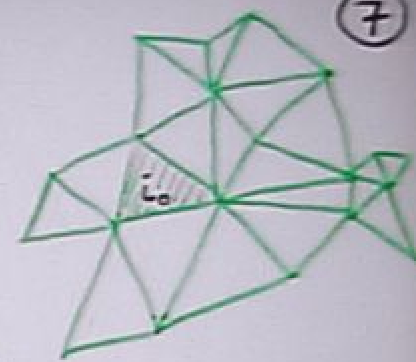

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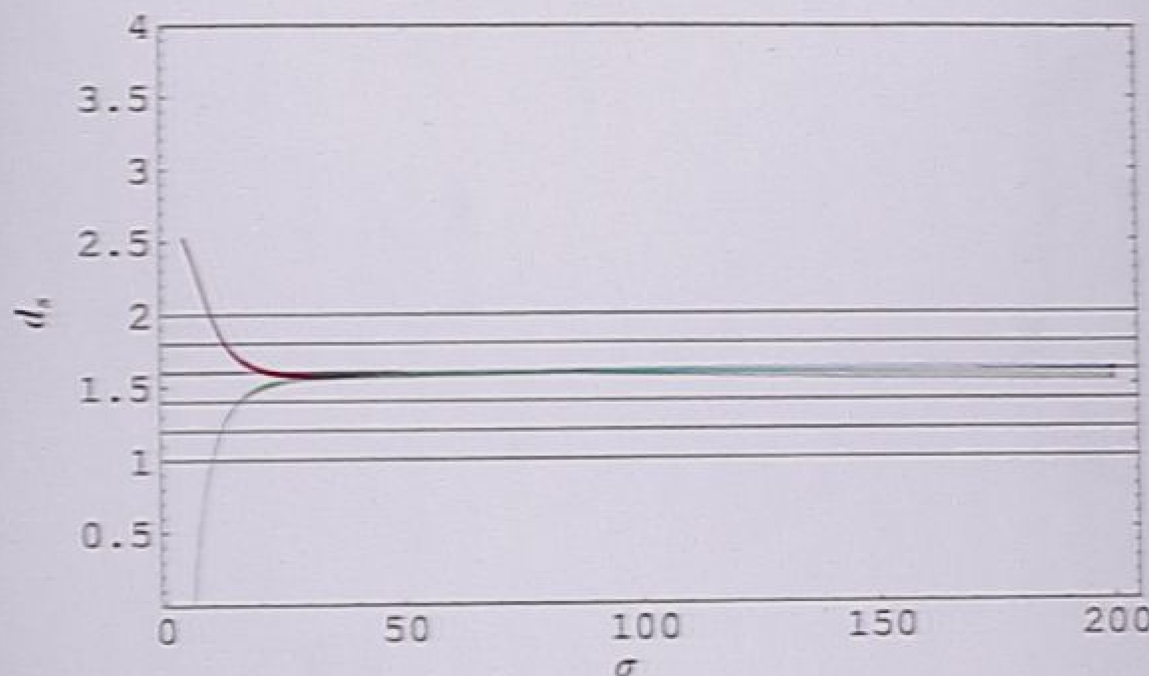
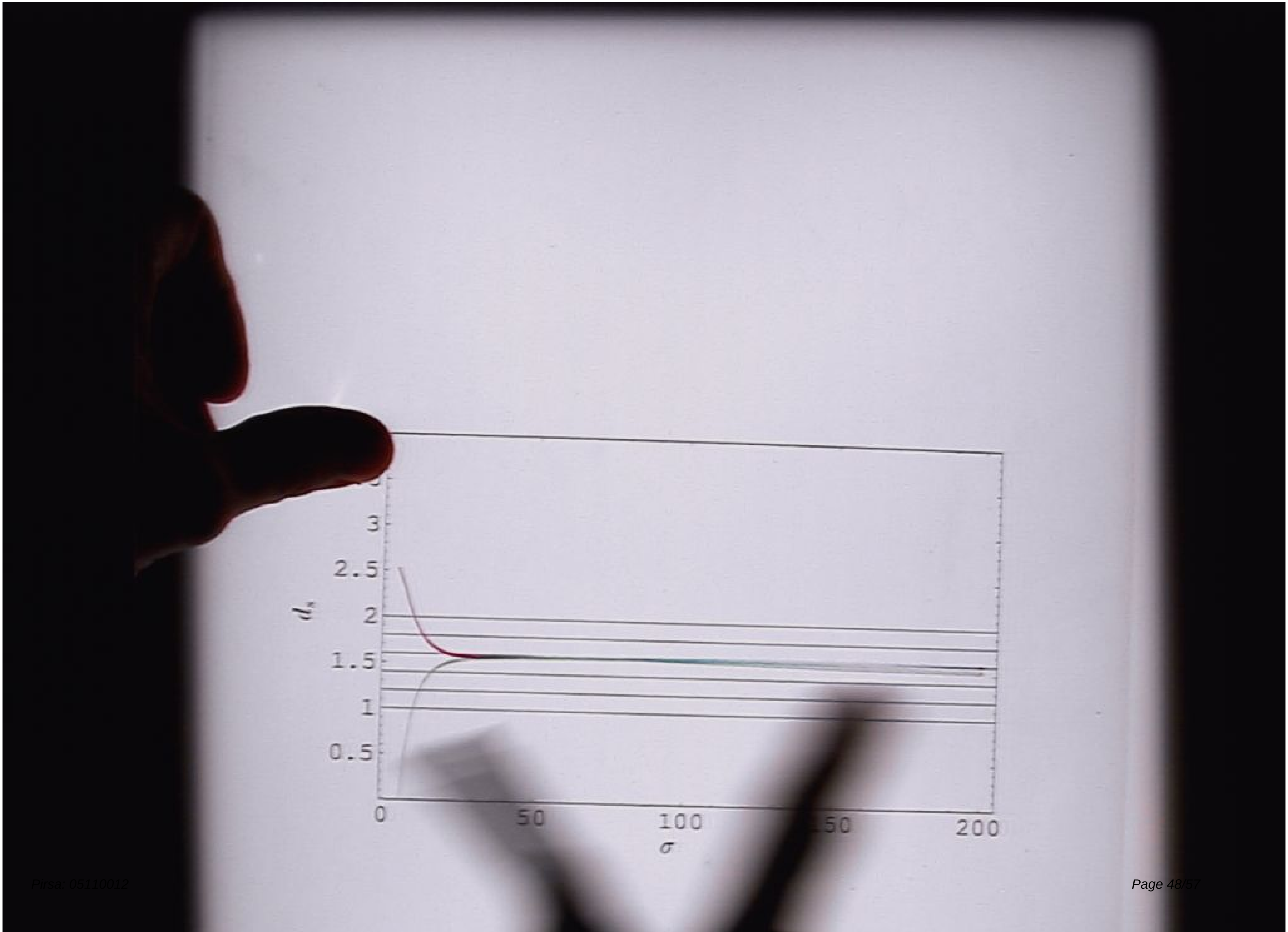


4
3.5
3
2.5
2
1.5
1
0.5
d_s
0
50
100
150
200
σ


3
2.5
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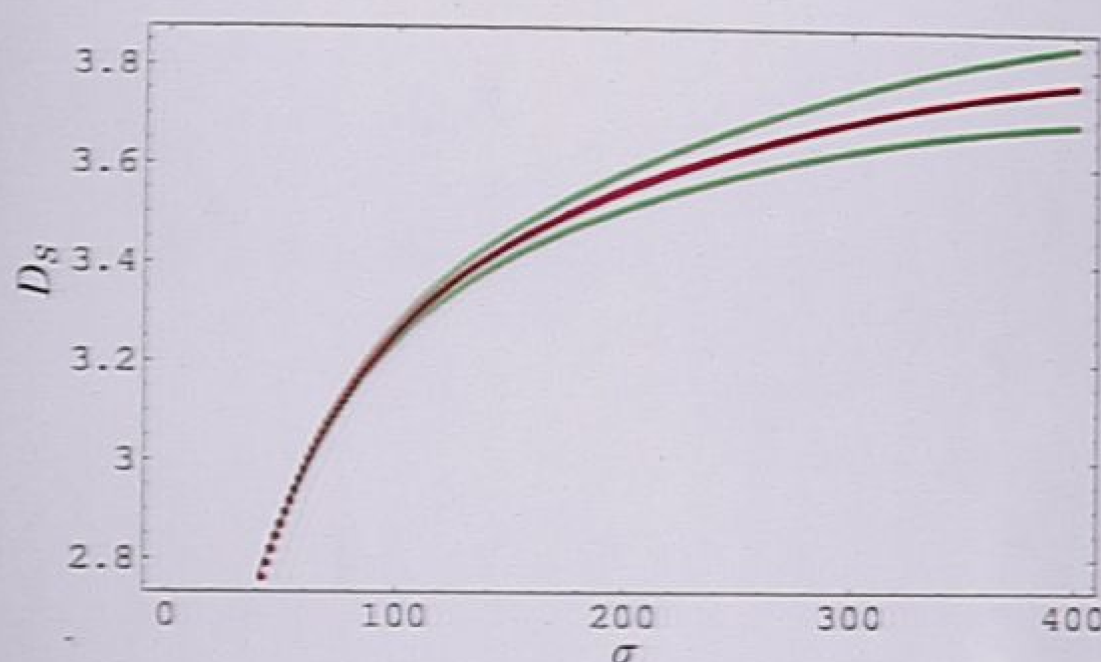
3.8
3.6
3.4
3.2
3
2.8
D_S
0
100
200
300
400
σ

8

- typical behaviour of $D_s(\sigma)$ on random geom.s → PICT
- by contrast, curve for CDT → PICT

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$D_s(\sigma) \xrightarrow{\sigma \to 0} a - \frac{b}{c} = 1.82 \pm 0.25$ short-dist[illegible]

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$\overset{?}{\Longrightarrow}$ nonperturb. QG has "built-in" UV regulator

$$\langle \mathcal{R}(\sigma) \rangle \underset{\text{dim. full}}{\rightsquigarrow} \frac{1}{\sigma^2 \left(1 + \frac{l^2_{Planck}}{\sigma}\right)}$$

- result robust under relative scaling

⑨

- short-distance structure does not resemble any classical manifold
- evidence of fractality from looking at slices $t = \text{const}$.
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  - critical exponents coincide with those of an "exotic branched polymer"
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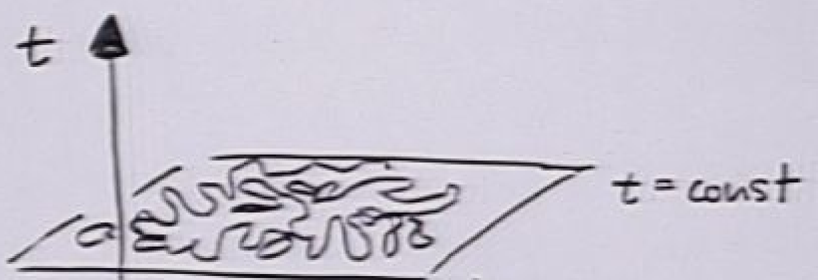

Conclusions & Outlook

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→ a new paradigm for Quantum Gravity ?