

Title: A talk about Nothing

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Abstract:

A talk about Nothing

$$\langle ds^2 \rangle = 0$$



A talk about Nothing

$$\langle ds^2 \rangle = 0$$



$$S(t) = \int \frac{G}{4\pi} dx dy$$

A talk about Nothing

$$\langle ds^2 \rangle \neq 0$$

strategy:

- perturbative $\mathcal{N} = 0$
- not seriously using gap
- connect gapped phase
to unconfined phase
by Lüscher evolution
- $\mathcal{N} = 1$
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A talk about Nothing

$$\langle \psi^2 \rangle = 0$$

II. XY model
III.

Strategy:

- perturbative $\beta = 0$
- take seriously non-perturbative
- construct gapped phase
+ non-perturbative phase
by renormalization evolution

$$\begin{aligned} & \psi \rightarrow \psi \\ & \psi \rightarrow \psi + \psi \end{aligned}$$

A talk about Nothing

$$\langle \psi^2 \rangle = 0$$

I. XY model

III. Spontaneous ordering

IV. "truly at the end of the line"

Wolff / 0502021

Adams et al.

Wolff / 0506130

Strategy:

participative $2n = 0$

• take seriously wrong up

• correct 3d phase

to "normal" phase

by Liouville evolution

$$x \rightarrow x + \epsilon$$

$$x \rightarrow x + \epsilon$$

II. $\sigma \sim 0 + \pi$

$$Z_{-1} = \frac{L^2}{4\pi g_3} \int \partial_0 \bar{\partial}_0 \hat{z}$$

II. $\sigma = 0 + 2\pi i$

$$Z_{-1} = \frac{L^2}{4\pi g_s} \int \partial \bar{\phi} \bar{\phi} \, d\bar{z}$$

$$\psi = e^{i\phi} |14\rangle$$

$$\frac{L^2}{4\pi g_s} = p_c = 141^2$$

II. $\theta = 0 + 2\pi$

$$Z_{n1} = \frac{L^2}{\pi g_s} \int \partial\theta \bar{\partial}\theta \, d\tau$$

$$Q_{nm} = \exp i(n\theta + m\bar{\theta})$$

$$\psi = e^{i\theta} |\psi\rangle$$

$$\frac{L^2}{\pi g_s} = p_c = 141^2$$

$$\begin{aligned} \theta &= \theta_1 + \theta_2 \\ \bar{\theta} &= \bar{\theta}_1 + \bar{\theta}_2 \end{aligned}$$

$$\begin{aligned} \bar{\partial}\theta_1 &= 0 \\ \partial\bar{\theta}_2 &= 0 \end{aligned}$$

II. $\theta \sim \theta + 2\pi$

$$Z_{n1} = \frac{L^2}{\pi g_s^2} \int \partial\theta \bar{\partial}\theta \, d^2z$$

$$\psi = e^{i\phi} |\psi|$$

$$\frac{L^2}{\pi g_s^2} = p_s = |\psi|^2$$

$$\phi_{nm} = \exp i(n\theta + m\bar{\theta})$$

11.1) $\Delta_{nm} = \left(\frac{n}{L}\right)^2 + (mL)^2$

if $L < L_c = \sqrt{L_s}$, $\Delta_{0,1} = L^2 < 1$

$$\begin{aligned} \theta &= \theta_L + \theta_R \\ \bar{\theta} &= \bar{\theta}_L - \bar{\theta}_R \end{aligned}$$

$$\begin{aligned} \bar{\partial}\theta_L &= 0 \\ \partial\bar{\theta}_R &= 0 \end{aligned}$$

II. $\theta \rightarrow \theta + 2\pi$

$$Z_{n1} = \frac{L^2}{\pi R^2} \int \partial\theta \bar{\partial}\theta \, d^2z$$

$$\psi = e^{i\theta} |\psi|$$

$$\frac{L^2}{\pi R^2} = p_r = 14 \text{ }^2$$

$$O_{nm} = \exp i(n\theta + m\bar{\theta})$$

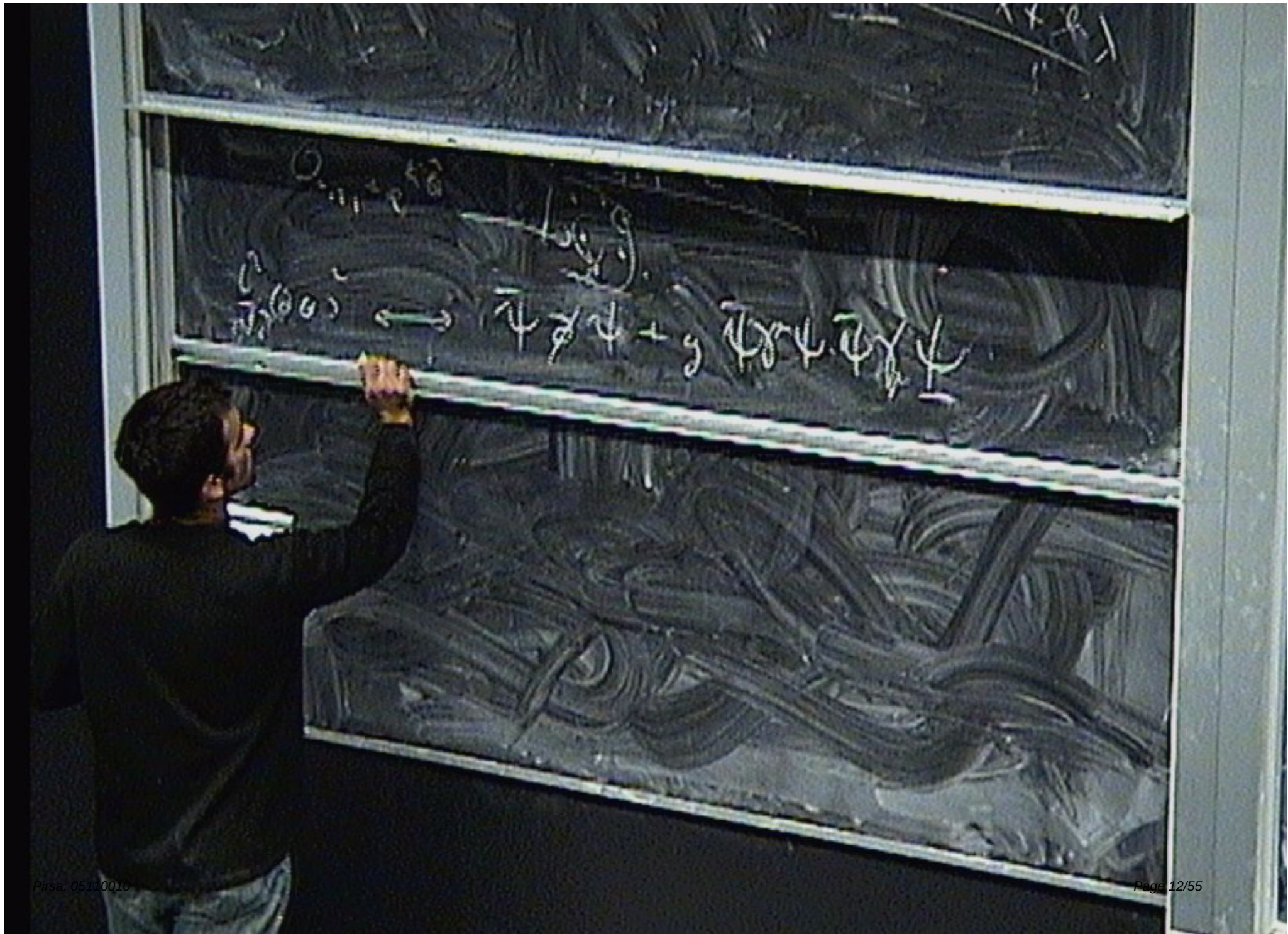
1) $\Delta_{nm} = \left(\frac{n}{L}\right)^2 + (mL)^2$

if $L < L_c = \sqrt{L_s}$, $\Delta_{0,1} = L^2 < 1$

$$O_{0,1} = e^{i\bar{\theta}}$$

$$\begin{aligned} \theta &= \theta_L + \theta_R \\ \bar{\theta} &= \theta_L - \theta_R \end{aligned}$$

$$\begin{aligned} \bar{\partial}\theta_L &= 0 \\ \partial\theta_R &= 0 \end{aligned}$$



$$\left(\frac{1}{\sqrt{2}}\psi\right) \leftrightarrow \psi\psi + g\psi\psi\psi\psi$$

$$1 - \frac{1}{2} = 2\frac{1}{2}$$

$$\langle \frac{1}{\sqrt{2}}(00) \rangle \longleftrightarrow \psi \phi \psi + g \psi \phi \psi \psi \psi$$

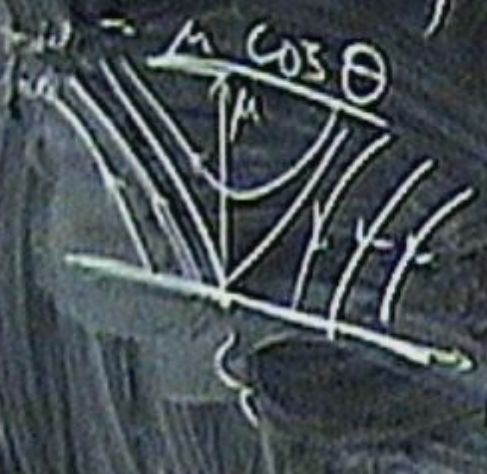
$$r(\vec{e}_1 \cdot \vec{e}_2 + \vec{e}_1 \cdot \vec{e}_3) \longleftrightarrow \mu \bar{\psi} \psi$$

$1 - \frac{2}{3} = 2 \frac{L^2}{L^2}$
 $\mu \cos \theta$


$$\frac{1}{m}(\partial_\mu \partial^\mu) \longleftrightarrow \bar{\psi} \not{\partial} \psi + g \bar{\psi} \not{A} \psi$$

$$m \left(e^{i\theta} + e^{-i\theta} \right) = 2 \frac{L^2}{L^2}$$

$$m \bar{\psi} \psi$$



$$\frac{1}{m} \frac{\partial^2}{\partial x^2} \psi \leftrightarrow \bar{\psi} \not{\partial} \psi + g \bar{\psi} \not{A} \psi$$

$$m \left(e^{i\theta} + e^{-i\theta} \right) \leftrightarrow m \bar{\psi} \psi$$



$$\frac{1}{m}(\partial_\mu \partial^\mu) \longleftrightarrow \bar{\psi} \not{\partial} \psi + g \bar{\psi} \not{A} \psi$$

$$m(e^{i\theta} + e^{-i\theta}) = 2 \frac{L^2}{L^2}$$

$$m \bar{\psi} \psi$$

$$\Delta(\frac{p}{L}) = \frac{2\pi}{L}$$

III RS compactification

IIA on Σ_4

$$P_{\text{nu}} = P_{\text{nu}}$$

III RS compactification

IIA on Σ_4

$$\beta_{\text{IR}} = \beta_{\text{UV}}$$

constant axion vev
is a local min.

III RS compactification

IIA on Σ_4

$$\beta_{uv} = P_{uv}$$

• constant negative curvature
is a local min

$$\frac{1}{\epsilon} \int_{\Sigma} R \quad V_H\left(\frac{\bar{\phi}}{f}\right) \sim \frac{1}{\ell_s^2} \left(\frac{g_s}{V_2}\right)^{2/3} (2h-2)$$

III RS compactification

IIA on Σ_4

$$\beta_{\text{IR}} = \beta_{\text{UV}}$$

→ constant negative curvature
is a local min.

$$V_{\text{IR}}(\frac{\Phi}{f}) \sim \frac{1}{g_s^2} \left(\left(\frac{g_s}{V_5} \right)^{2/3} \right) (e^{4-z})$$

all threads $\frac{V_2 \rightarrow \infty}{g_s \rightarrow 0}$ slowly of $\frac{g_s}{V_2} \ll g_s^2$

III IRS compactification

IIA on Σ_4

$$\beta_{\text{IR}} = \beta_{\text{UV}}$$

a constant negative curvature
is a local min.

$$C = 2 + \ln \alpha' R + \mathcal{O}(\alpha' R)^{-1}$$



$$V_{\text{eff}}\left(\frac{g_s}{V_4}\right) = \frac{1}{\partial g_s} \left(\left(\frac{g_s}{V_4} \right)^{2/5} \right) \quad (\text{check})$$

all kinds $V_4 \rightarrow \infty$
 $g_s \rightarrow 0$, study of $\frac{g_s}{V_4} \propto g_s^2$

III RS compactification

IIA on Σ_4

$$\beta_{uv} = \beta_{uv}$$

constant negative curvature
is a local min.

$$C = 2\pi^2 + b\alpha' R + O(\alpha'^2 R^2)$$



$$\frac{1}{\epsilon} \int_{\Sigma_4} V_{eff}(\frac{\phi}{f}) \sim \frac{1}{\epsilon} \left(\left(\frac{g_s}{V_5} \right)^{2/3} (2b-2) \right)$$

all time, $V_5 \rightarrow \infty$
 $g_s \rightarrow 0$, slowly of $\frac{g_s}{V_5} \ll 1$

III RS competition

IIA in Σ_L

$$\beta_{\text{IR}} = \beta_{\text{UV}}$$

constant negative curvature
is a local min

$$C = 2 + b\alpha' R + O(\alpha'^2 R^2)$$



IR

2nd order approximation

$$V_{\text{eff}}\left(\frac{g_s}{V_2}\right) \sim \frac{1}{g_s^2} \left(\left(\frac{g_s}{V_2} \right)^{2/3} \right) (2h-2)$$

all kinds $V_2 \rightarrow \infty$
 $g_s \rightarrow 0$, slowly of $\frac{g_s}{V_2} \ll 1$



6) R

all time: $V_2 \rightarrow \infty$
 $g_5 \rightarrow 0$

$$g_s \left(\frac{1}{V_2} \right) (2h-2)$$

study of $\frac{g_s}{V_2} \ll g_s$



$$\frac{1}{V_2} \ll \left(\frac{g_s}{V_2} \right)^2$$

$$ds^2 = dx^2 + L(x)^2 d\theta^2$$

$$\omega^1 \omega^2 = -\frac{1}{2} + \frac{L^2}{2g_s^2}$$

6) R

all kinds $\frac{1}{2} \rightarrow \infty$
 $g \rightarrow 0$

$$g_s \left(\left(\frac{1}{V_s} \right) \right) (chz)$$

study of $\frac{g_s}{\frac{1}{2}} \ll g_s$



$$ds^2 = dx^2 + L(x) d\theta^2$$

$$\frac{1}{V_s} \ll \left(\frac{g_s}{V_s} \right)^2$$

$$\omega^2 n^2 = -\frac{1}{2} + \frac{L^2}{2g_s^4}$$

6) 5R

all kinds: $V_2 \rightarrow \infty$
 $g_s \rightarrow 0$

$$\mathcal{L}_s \left(\left(\frac{V_2}{V_1} \right) \right) (2h-2)$$

study of $\frac{ds}{V_2} \ll \mathcal{L}_s$



$$ds^2 = dx^2 + L(x)^2 d\theta^2$$

$$\frac{1}{V_2} \ll \left(\frac{L_1}{V_2} \right)^2$$

$$\omega^2 = -\frac{1}{2} + \frac{L^2}{2L_1^2}$$

deriv.



6) 5R

all kinds $V_2 \rightarrow \infty$
 $g_5 \rightarrow 0$

$$O_s \left(\left(\frac{1}{V_2} \right) \right) (ch 2)$$

study of $\frac{ds}{V_2} \ll O_s$



$$ds^2 = dx^2 + L(x) d\theta^2$$

$$\frac{1}{V_2} \ll \left(\frac{L^2}{V_2} \right)^{1/2}$$

$$\omega^2 = -\frac{1}{2} + \frac{L^2}{2g_5^2}$$

claim

discussing. Dyontherm
 2)



6) R

all time: $V_2 \rightarrow \infty$
 $g_5 \rightarrow 0$

$$g_s \left(\left(\frac{V_2}{V_5} \right) \right) (2h2)$$

slowly of $\frac{g_s}{V_2} \ll g_s$



$$ds^2 = dx^2 + L(x)^2 d\theta^2$$

$$\frac{1}{L(x)} \ll \left(\frac{L(x)}{V_2} \right)^2$$

$$\omega^2 n^2 = -\frac{1}{2} + \frac{L^2}{2g_s^2} + \dots$$

claim:

discriminant: Δ positive

2) $G_0 \rightarrow 0$

3)

really 2 type 0



$G_2 \otimes \mathbb{R}$

all kinds: $V_2 \rightarrow \infty$
 $g_2 \rightarrow 0$

$$g_s^2 \left(\frac{1}{V_2} \right) \quad (ch 2)$$

study of $\frac{g_s}{V_2} \ll g_s^2$



$$ds^2 = dx^2 + L(x)^2 d\theta^2$$

$$\frac{1}{L(x)} \ll \left(\frac{L(x)}{V_2} \right)^{1/2}$$

$$\omega^2 n^2 = -\frac{1}{2} + \frac{L^2}{2g_s^2}$$

claim:



discriminant: 1) $g_s^2 \ll 1$

2) $G_2 \rightarrow 0$, really g_s type 0

$$3) \text{tr}(-1)F = \chi(\mathbb{Z}_4) = 2 - 2i$$

6) 5R

all kinds $\frac{1}{g_s} \rightarrow \infty$
 $g_s \rightarrow 0$

$$g_s \left(\left(\frac{1}{V_2} \right) \right) (2h-2)$$

slowly $\frac{g_s}{V_2} \ll g_s^2$



$$ds^2 = dx^2 + L(x)^2 d\theta^2$$

$$\frac{1}{L(x)} \ll \left(\frac{L(x)}{V_2} \right)^2$$

$$\omega^1 \omega^1 = -\frac{1}{2} + \frac{L^2}{2L_1^4}$$

claim:



discriminant: Δ must be non-zero

- 2) $G_0 \rightarrow 0$, null, 2d type 0
- 3) $\text{tr}(-1)F = \chi(\Sigma_4) = 2-2h$

with null, distinct

all hands $V_2 \rightarrow \infty$
 $g_2 \rightarrow 0$

slowly of $\frac{d}{dt} \ln \frac{V_2}{V_1} \ll \frac{1}{\tau_2}$



$$(-1) = \chi(\tau_4) = 2 - 2/$$

Constructing checks

2d vch. take $\sum_{n=0}^{\infty} \frac{1}{\tau_2^n} |F_n + \tau F_0|^2$

$$\int H \otimes H \rightarrow \int_{\mathbb{R}^2} \frac{1}{\tau_2^n} |F_n + \tau F_0|^2$$

all kinds $V_2 \rightarrow \infty$
 $g \rightarrow 0$

study of $\frac{J_1}{V_2} \ll 1$



consistency check

2d ferm. time step

$$\int H \otimes H \rightarrow \int d^2x \left(\frac{1}{t_2} |\underline{F}_1 + i \underline{F}_2|^2 \right)$$

class

$\vec{J}_1(t) \rightarrow$

"classically confined"

(light
sublattice)

all fields $V_2 \rightarrow \infty$
 $g_5 \rightarrow 0$

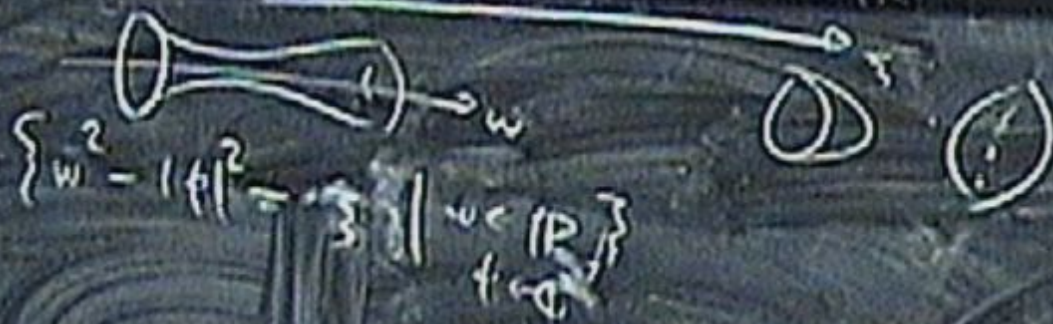
slowly $\delta \frac{g_5}{V_2} \ll g_5^2$



GLSM

UV

IR



all times $V_2 \rightarrow \infty$
 $g_2 \rightarrow 0$

slowly of $\frac{g_2}{V_2} \ll g_2^2$



GLSM

III

IR



$$\{w^2 - |t|^2 = -3\} \cap \{w \in \mathbb{R}\}$$

first attempt: as $U(1)$

(2.2) $\delta = \int d^3x W + h.c.$
 $W = \int d^3x \phi_1 \eta +$
 $F_2 \phi_1 \eta +$



$$\tau_Q = -2 = \beta_3$$

all kinds $V_2 \rightarrow \infty$
 $g_5 \rightarrow 0$

slowly $\delta \frac{J_1}{V_2} \ll g_5^2$



GLSM

UV

IR



$$\{w^2 - |t|^2 = -3\} \mid w \in \mathbb{R}$$

first attempt: as $U(1)$

$$W = \frac{1}{2}(X^2 - Y^2)$$

(2.2)

$$\delta J = \int d^4x W + \text{h.c.} + \gamma_+ \phi_- P_2$$

$$W = \frac{1}{2} P_- \phi_+ \gamma_+$$

$$P_+ = \phi_+ \gamma_+$$



$$\delta Q = -2 = \beta_3$$

$$\delta J = \frac{1}{2} \int d^4x (P_- P_+ + \phi_+ \gamma_+ \bar{\phi}_+ \gamma_+)$$

dist. vec. : $\Sigma = \underline{\sigma} + \sigma \lambda_a + O^2(F+1T) + \dots$

$Z = 101^2 |41|$

$\int d\sigma_1 d\sigma_2 \hat{U} + h..$

$\tilde{W} = \pm \Sigma + Q \tau Q \Sigma$

dust vacuum : $\Sigma = \underline{\sigma} + \sigma \lambda_a + \sigma^2 (F + iD) + \dots$

$\mathcal{L} = 10 \log_{10} |H|$

$\int d\sigma_+ d\sigma_- \hat{W} + h..$

$\hat{W} = \underline{t\Sigma} + \underline{\omega_T \tau \ln \Sigma}$

sign.

$\theta_+ \rightarrow e^{i\omega_+} \theta_+$
 $\theta_- \rightarrow e^{i\omega_-} \theta_-$

$\tilde{\theta} \rightarrow e^{i(\omega_+ - \omega_-)} \tilde{\theta}$

dust vacuum: $\Sigma = \sigma + \sigma \lambda_0 + \sigma^2 (F + iD) + \dots$

$\mathcal{L} = 16\pi^2 |\dot{\phi}|^2$

$\int d\sigma_+ d\sigma_- \hat{W} + h.c.$

$\hat{W} = \underline{\pm \Sigma} + \underline{Q_+ \bar{\Sigma} Q_- \Sigma}$

sign.

$\sigma_+ \rightarrow e^{i\alpha_+} \sigma_+$
 $\sigma_- \rightarrow e^{i\alpha_-} \sigma_-$

$\bar{\Sigma} \rightarrow e^{i(\alpha_+ - \alpha_-)} \bar{\Sigma}$

$\text{chiral iso. } \sigma \rightarrow \sigma \quad \mathbb{Z}_2 \subset U(1) \quad \text{and } Q_+ Q_- \bar{\Sigma}$

$0 = \hat{W} \Rightarrow \sigma = \pm \epsilon^{1/2}$

dist vacua: $\Sigma = \underline{\sigma} + \sigma \lambda_a + \sigma^2 (F + iD) + \dots$

$\mathcal{L} = |\sigma|^2 |\mathcal{H}|^2$ $\int d\sigma_1 d\sigma_2 \hat{W} + h.c.$

$\tilde{W} = \underline{\pm \Sigma} + \underline{Q_T \tau Q_L \Sigma}$

sign. $\sigma_+ \rightarrow e^{i\alpha} \sigma_+$ $\sigma_- \rightarrow e^{i\alpha} \sigma_-$ $\tau \rightarrow e^{i(\alpha_+ - \alpha_-)} \tau$

$U(1) \subset U(2)$ $Q_L \rightarrow e^{i\alpha} Q_L$ $Q_R \rightarrow e^{i\alpha} Q_R$

$0 = \tilde{W} \Rightarrow \sigma = \pm i |\sigma| \frac{1}{2} \rightarrow \text{only } \sigma = 0$

$\sigma = \pm 1/2 \rightarrow \text{2d type 0}$

IV. metric

$$ds^2 = -dt^2 + L^2(r) d\phi^2 + r^2 ds_{\perp}^2$$

$$\sigma = \pm \epsilon^{1/2} \rightarrow \text{2d type 0}$$

IV. friction

$$ds^2 = -dt^2 + L^2(r) d\theta^2 + ds_{\perp}^2 \quad \text{w/ } \Lambda B C \text{ on } \theta$$

$$0 = \dot{t}$$

$$\text{closed: } \dot{t} \ll 1$$

$$L \gg 0$$



$$Z \sim \int (dx) \sim \int_{AB} G_{\mu\nu} dx^\mu dx^\nu$$

$$\delta = \pm \epsilon^{1/2} \rightarrow \text{2d type 0}$$

IV. friction

$$ds^2 = -dt^2 + L^2(r) d\theta^2 + ds_{\perp}^2 \quad \text{w/ } ABC \text{ on } \theta$$

$$0 = \dot{t}$$

$$\text{closed: } \dot{t} \ll 1$$

$$L \gg 0$$

$$Z = \int (dx) \cdot \left[\int_{AB} \left(G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \frac{1}{2} \frac{\dot{x}^\mu \dot{x}^\mu}{\cos \theta} \right) \right]$$



$\delta = \pm e^{i\pi/2} \rightarrow \text{2d type 0}$

IV. trick

$$ds^2 = -dt^2 + L^2(r) d\theta^2 + ds_{\perp}^2 \quad \text{w/ } \Lambda B \subset \text{on } \theta$$

$$0 = \dot{t}$$

closed: $i \ll 1$
 $L \gg 0$



$$Z \sim \int (dx) \sim i \int_{AB} \left[G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \mu \epsilon \frac{k x^0}{\cos \theta} \right]$$

$\vec{F}(x^0) \sim \epsilon \frac{k}{r}$
 $T(x^0) \sim \epsilon \frac{k}{r}$

$$\delta = \pm \epsilon^{1/2} \rightarrow \text{2d type 0}$$

$$\frac{\partial}{\partial t} \mathcal{Z}_t = \int \frac{dx_0}{(2\pi)} (dx) \int (dx_1) \frac{e^{i(S_{\text{int}} + \int dx_1 \cdot e^{kx_0})}}{i e^{kx_0}}$$

$$X^0 = X_0^0 + \hat{X}_0^0$$

$$\delta = \pm \epsilon^{1/2} \rightarrow \text{2d type 0}$$

$$\frac{2}{\hbar} Z_1 = \int dx_0 (dx) \int (dx_1) \mu e^{i(S_{\text{int}} + \int dx_1 e^{ikx_0})}$$

$$x^0 = x_0^0 + \tilde{x}_0^0$$

$$= \int (dx_1) dx_1 \frac{1}{\hbar} \int dx_0^0 e^{-ikx_0^0} e^{-ikx_0^0} e^{-ikx_0^0}$$

$$\int_0^\infty \frac{dx_1}{k} e^{-\gamma x_1} = +\frac{1}{\gamma}$$

$$r = \pm e^{ikl/2} \rightarrow \text{2d type 0}$$

$$\frac{\partial}{\partial t} Z_T = \int \underline{dx_0} (dx) \int (dx_1) \mu e^{i(S_{int} + \int dx_1 e^{ikx_0})}$$

$$X^0 = X_0^0 + \hat{X}^0$$

$$= \int (dx_1) dx_2 \int \underline{dx_0} e^{-ikx_0} e^{ikx_0} e^{S_{int}}$$

$$= \frac{1}{\Lambda k} \int (dx_1) dx_2 e^{\int_{S_{int}} \frac{dx_1}{k} e^{-\gamma S} = + \frac{1}{\Lambda k}}$$

$$\sigma = \pm i \frac{1}{2} \rightarrow \text{2d type 0}$$

$$\frac{\partial}{\partial t} Z_t = \int \underline{dx_0} (dx) \int (dx_1) \mu e^{i(S_{int} + \int dx_1 e^{ikx_1})}$$

$$X^0 = X_0^0 + \hat{X}^0$$

$$= \int (dx_1) dx_2 \int \underline{dx_0} e^{-ikx_0} e^{ikx_0} e^{iS_{int}}$$

$$= \frac{1}{\mu k} \int (dx_1) dx_2 e^{iS_{int}} \int \frac{dy}{k} e^{-ys} = + \frac{1}{\mu k}$$

$\sigma = \pm i k \frac{1}{2} \rightarrow 2d \text{ type 0}$

$$\frac{\partial}{\partial t} Z_I = \int dx_0 (dx) \int dx_1 \mu e^{i(S_{int}/\hbar) + i\hat{H} e^{ikx_0}}$$

$$X^0 = X_0^0 + \hat{X}^0$$

$$= \int (dx) dx_1 \int dx_0 e^{-ikx_0} e^{ikx_0} e^{S_{int}}$$

$$= \frac{1}{\hbar k} \int dx_1 dx_0 e^{-ikx_0} e^{ikx_0} e^{S_{int}} \quad Z_{T^2} = \frac{1}{k} \ln(1/\mu_k) Z_{tot} \quad \mu_k = e^{X_k}$$

$$\delta = \pm \epsilon^{1/2} \rightarrow \text{2d type 0}$$

$$\frac{\partial}{\partial \alpha} Z_I = \int d\vec{x}_0 (d\vec{k}) \int dX_I \left\{ \mu e^{i(S_{\text{int}}/\hbar)} \right\} \frac{\partial}{\partial \alpha} e^{i k \cdot \vec{x}_0}$$

[illegible]

$\bar{e} \rightarrow R$
 all kinds $V_2 \rightarrow \infty$
 $g_s \rightarrow 0$, slowly to $\frac{g_s}{V_2} \ll g_s^2$



∇ : other probes



$= \int d^4x \mathcal{L}_{\text{probe}}(x)$
 $\mathcal{F}_{\mu\nu}$

$$W = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$$

$$= \mathcal{F}_3 = Q_T$$

$$\mathcal{L}(\mathcal{P}_2 \mathcal{P}_2 + \phi_2^2 \mathcal{Y}_2 + \phi_2^2 \mathcal{Y}_2)$$

$\epsilon \rightarrow R$
 all kinds $\frac{1}{s} \rightarrow \infty$
 $g \rightarrow 0$, study of $\frac{g}{\sqrt{s}} \ll g_s$



∇ : other probes

$$\begin{aligned}
 & \text{Diagram: a horizontal line with a loop on top} \sim \int \frac{d^3x}{(2\pi)^3} e^{-L|\mathbf{x}|} \\
 & = \int d^3x e^{-L|\mathbf{x}|} \langle \mathbf{x} | \mathbf{x} \rangle
 \end{aligned}$$

(with $L^{-D/2}$ and $\int d^3x$ in the original image)

$$W = \frac{1}{2}(\mathbf{x} - \mathbf{y})^2$$

$$\begin{aligned}
 & = \frac{1}{2} \mathbf{P}_3 \cdot \mathbf{Q}_T \\
 & = \frac{1}{2} (\mathbf{P}_1 \cdot \mathbf{P}_2 + \mathbf{P}_1 \cdot \mathbf{Q}_T + \mathbf{P}_2 \cdot \mathbf{Q}_T)
 \end{aligned}$$

