Title: Effective theories for loop quantization

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Abstract:

EFFECTIVE CONFIGURATIONS FOR LOOP QUANTIZATION.

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OUTLOOK

1. Introduction

2. Preliminary notions

3. Implementation of Wilson's Renormalization Group and the continuum limit

4. Explicit example: 2-D. Ising field theory.

5. Conclusions.

I.INTRODUCTION

- A current interest in Loop Quantum Gravity is to understand more the dynamics of the theory. In this sense, we propose a new strategy to define the dynamics in loop quantized theories as a continuum limit of effective theories, following Wilson renormalization group ideas, as it is done in Lattice Gauge Theories (LGT).
- To do so, we postulate an "extended notion of scale" and implement Wilson's Renormalization group based on this notion.
- As it is done in LGT , working with a truncation we construct a continuum limit inverting the renormalization group flow in the space of the coupling constants .
- If the continuum limit exists it defines the dynamics of a loop quantized theory.
- Moreover, Loop quantization was originally motivated by its application to theories free of a background metric, but the formulation is intended to be general and applicable to background dependent

understand more the dynamics of the theory. In this sense, we propose a new strategy to define the dynamics in loop quantized theories as a continuum limit of effective theories, following Wilson renormalization group ideas, as it is done in Lattice Gauge Theories (LGT).

- To do so, we postulate an "extended notion of scale" and implement Wilson's Renormalization group based on this notion.

- As it is done in LGT , working with a truncation we construct a continuum limit inverting the renormalization group flow in the space of the coupling constants .

- If the continuum limit exists it defines the dynamics of a loop quantized theory.

- Moreover, Loop quantization was originally motivated by its application to theories free of a background metric, but the formulation is intended to be general and applicable to background dependent theories as well. Therefore, the application of this new strategy to metric dependent theories can be reduced to the standard lattice regularization.

II. PRELIMINARY NOTIONS.

1. Cellular decompositions

- Take a two dimensional manifold M.

  A cellular decomposition C of a manifold is a presentation of it as a union of disjoint cells

  $$\mathrm{M} = \cup_{c_\alpha \in C} c_\alpha$$

  where the cells are the image of open disks of dimensions from zero to dim M.

- We are concerned with *generic* cellular decompositions, that is, if we take a set of points $\{p_1 \cdots p_n\} \in \mathrm{M}$, then each of them is contained in a cell of maximal dimension of C.

- The cellular decompositions satisfy three properties:

  i Partial Order Relation

  ii Directed towards refinement

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  U open subset of M $\exists\, C_0$

$$H_p \cong H^p \cong H^{d-p}$$

iii) U_{open} M $\exists$ C_0

$C \subset C_0 \Rightarrow C \subset U$

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2. The abstract Lattice

For a given cellular dec. C, denote the set of its cells by $L(C)$. We can embed $L(C)$ into M mapping every element $\alpha \in L(C)$ to a point in the corresponding cell $c_\alpha \in C$.

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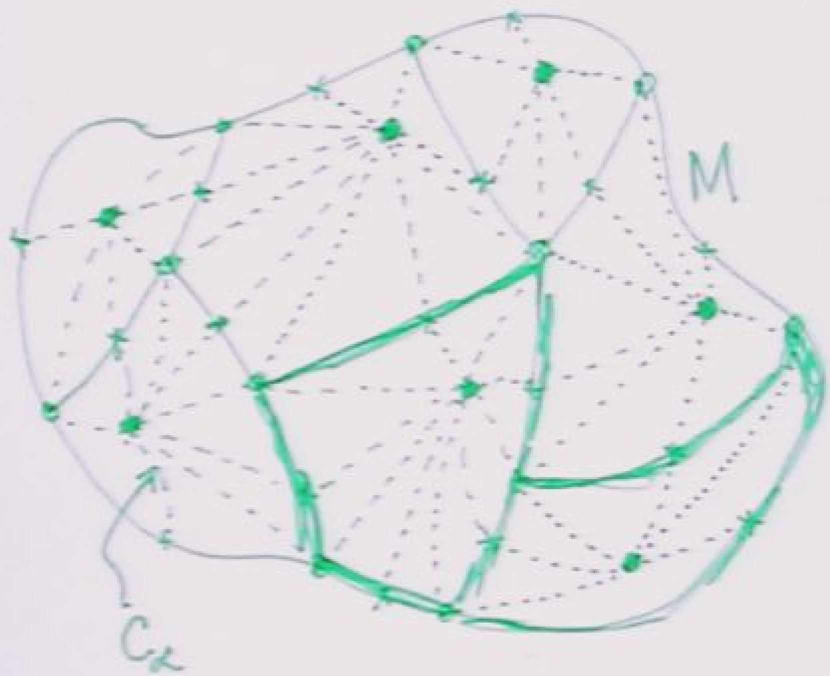

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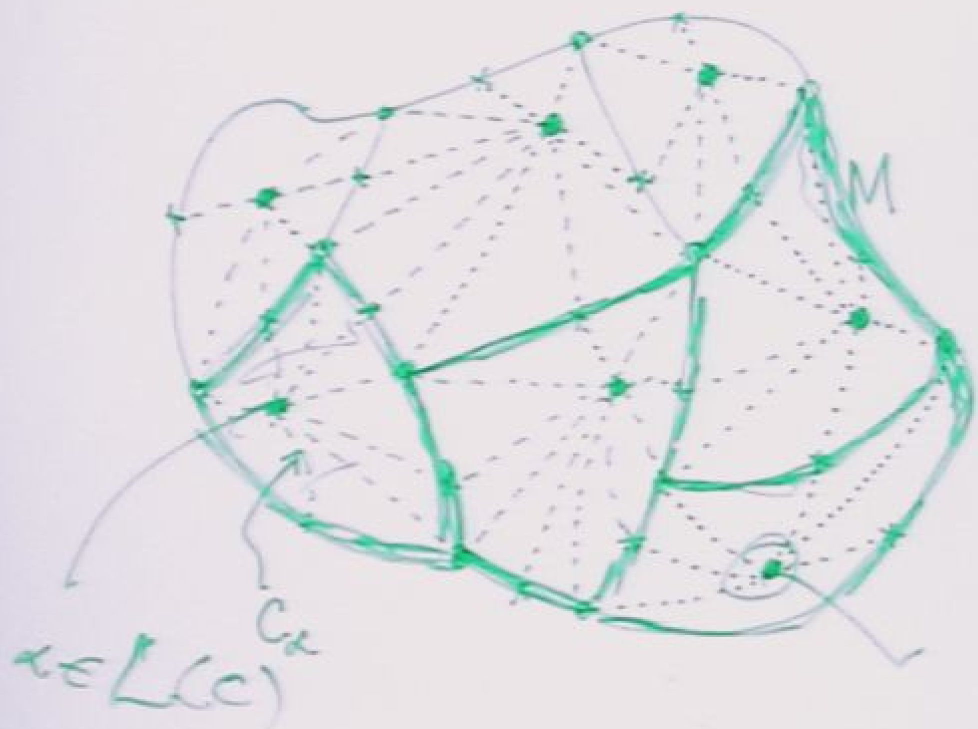

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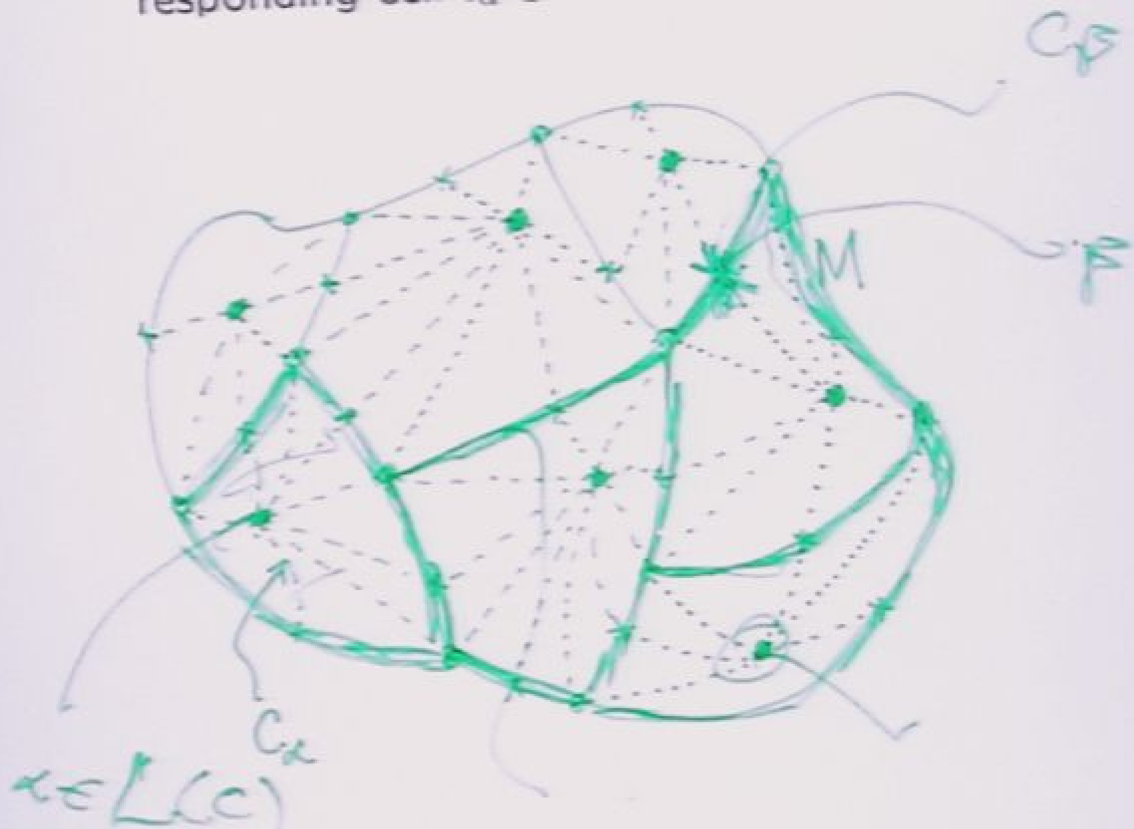

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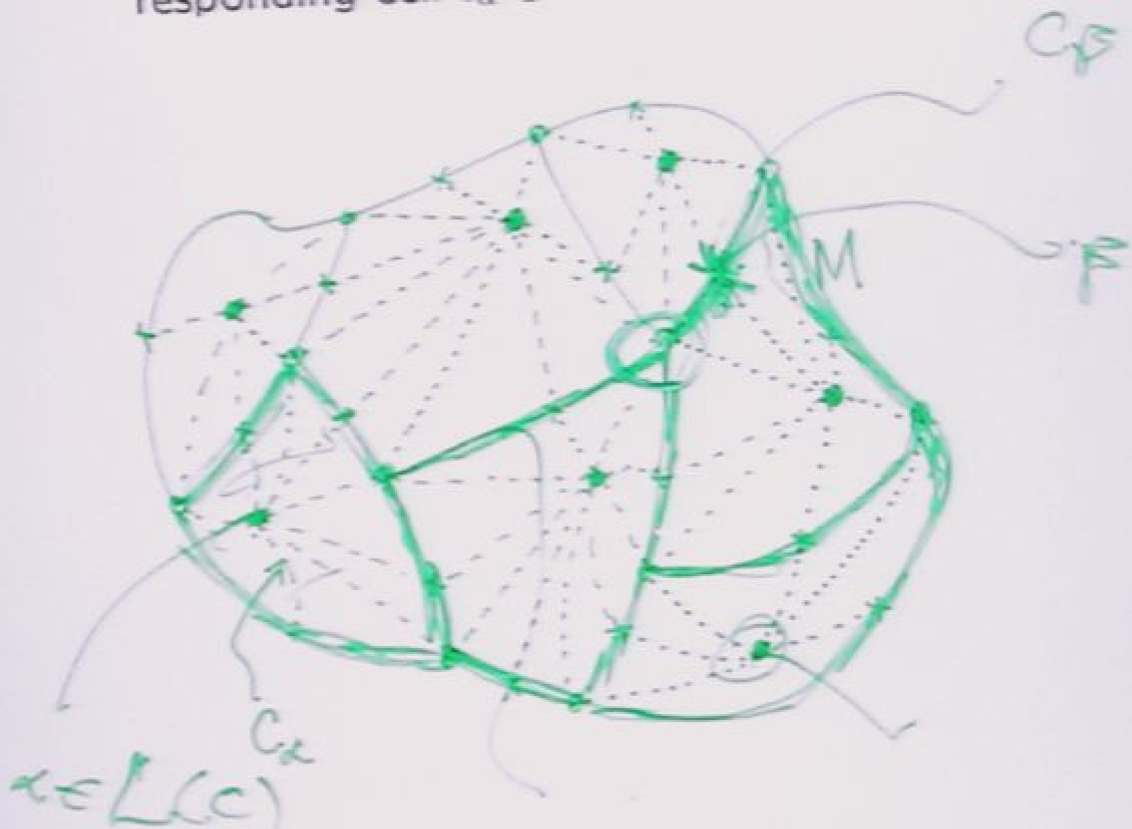

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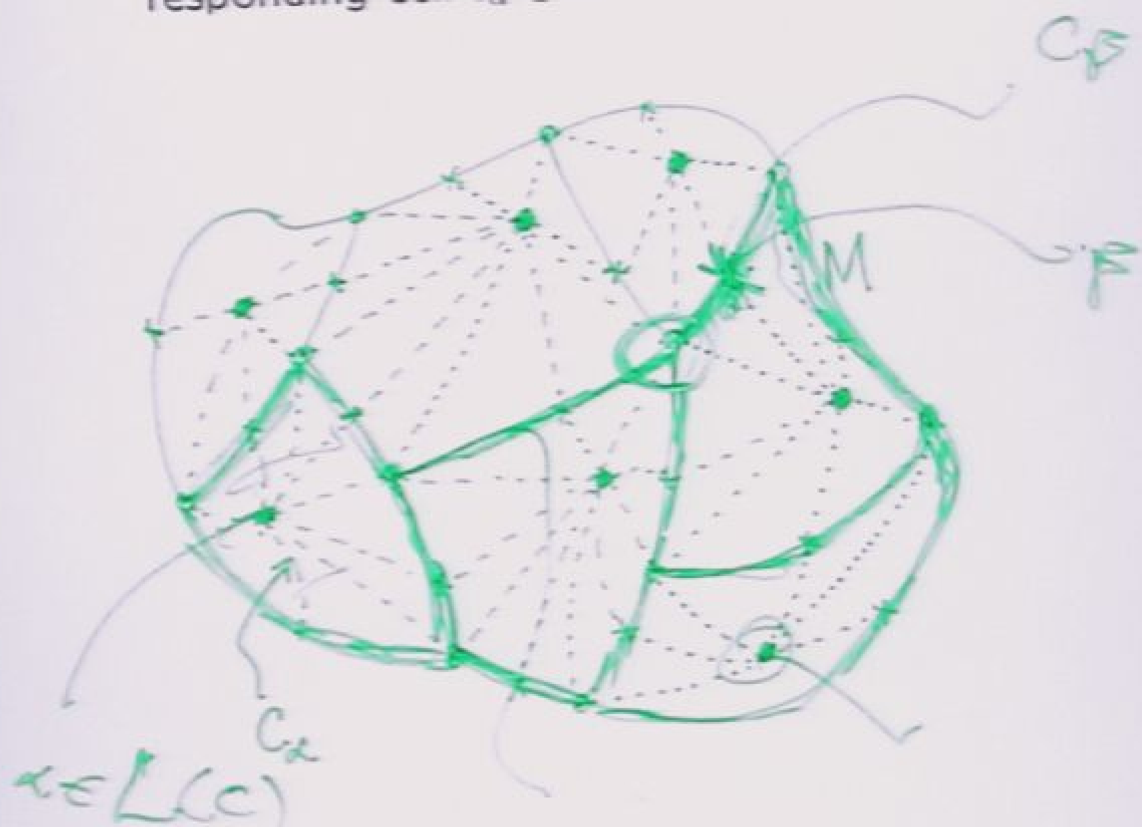

Implementation of Wilson's Renormalization Group and the continuum limit

- Here we work in the Euclidean description of QFT. The general prescription is given for spin systems, sigma models and scalar fields. (**For gauge systems see J.Martinez, C. Meneses, J.A. Zapata. Geometry of C-flat conections, carse grainning and the continuum limit. Jul 2005. 29pp. Published in J.Math.Phys.46:102301,2005 e-Print Archive: hep-th/0507039**)

- Let be $\overline{\mathcal{A}}_{\mathrm{M}}$ the space of Euclidean histories. Its elements $s \in \overline{\mathcal{A}}_{\mathrm{M}}$ assign an element of a compact group to any point of spacetime $p \in \mathrm{M}$, $s(p) \in G$ without continuity requirement.

- The algebra of observables is Cyl($\overline{\mathcal{A}}_{\mathrm{M}}$) which are functions that depend on the histories restricted to finitely many points.

** *Our primary goal is to construct physical measures μ_{M} on $\overline{\mathcal{A}}_{\mathrm{M}}$, that encode the dynamics of the theory.*

- Define the space of effective Euclidean histories at "scale" C as the space of C-constant configurations, $\mathcal{A}_C$ in the following way:

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- $\mathcal{A}_C$ is in one to one correspondence with the configuration space on the lattice $\mathcal{A}_{L(C)}$.

- The effective theory at "scale" C is given by the partition function on the space of effective histories.

- Given two scales $C_1 \leq C_2$ there are two ways to relate the corresponding effective theories. one *inclusion* map i_{C_1,C_2} in the direction of refinement and a *coarse graining* map π_{C_2,C_1}

$$(\mathcal{A}_{C_1}, \mu_{C_1}) \underset{\pi_{C_2C_1}}{\overset{i_{C_1C_2}}{\rightleftarrows}} (\mathcal{A}_{C_2}, \mu_{C_2})$$

Actually, we have $i_C : \mathcal{A}_C \to \bar{\mathcal{A}}_M$. which induces a *regularization map* that brings any observable of the continuum to scale C,

$$i_C^* : \mathrm{Cyl}(\bar{\mathcal{A}}_M) \to \mathrm{Cyl}(\mathcal{A}_C)$$

- Because of this regularization property on our families of effective configurations we have

$$\lim_{C \to M} \mathcal{A}_C = \bar{\mathcal{A}}_M,$$

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- Coarse graining map $\pi_{C_2 C_1}$ models an average procedure by means of integrating out the degrees of freedom that do not have an impact at scale C_2.

Once that we choose an embedding

$$\mathrm{Emb}_{1,2} : L(C_1) \to L(C_2)$$

define

$$\pi_{C_2C_1} \doteq \mathrm{Emb}^{*}_{1,2} : \mathcal{A}_{L(C_2)} \to \mathcal{A}_{L(C_1)}$$

Coarse graining from the continuum is done similarly. The measure μ_M on $\bar{A}_M$ is coarse grained to act on effective observables at scale C. The result of coarse graining is a measure in $\mathcal{A}_C$ denoted by $\pi_{C*}\mu_M$.

** Renormalization

- Consider a sequence of increasingly finer scales $\{C_i\}$. The observables of any two effective theories of the sequence are linked by a regularization map and a coarse graining map.

- These coarse graining maps let us use a microscopic effective theory to evaluate expectation values of macroscopic observables by integrating out degrees of freedom.

$$\mu_{C_i} \approx \pi_{C_{i+1}C_i *}\mu_{C_{i+1}} \qquad (1)$$

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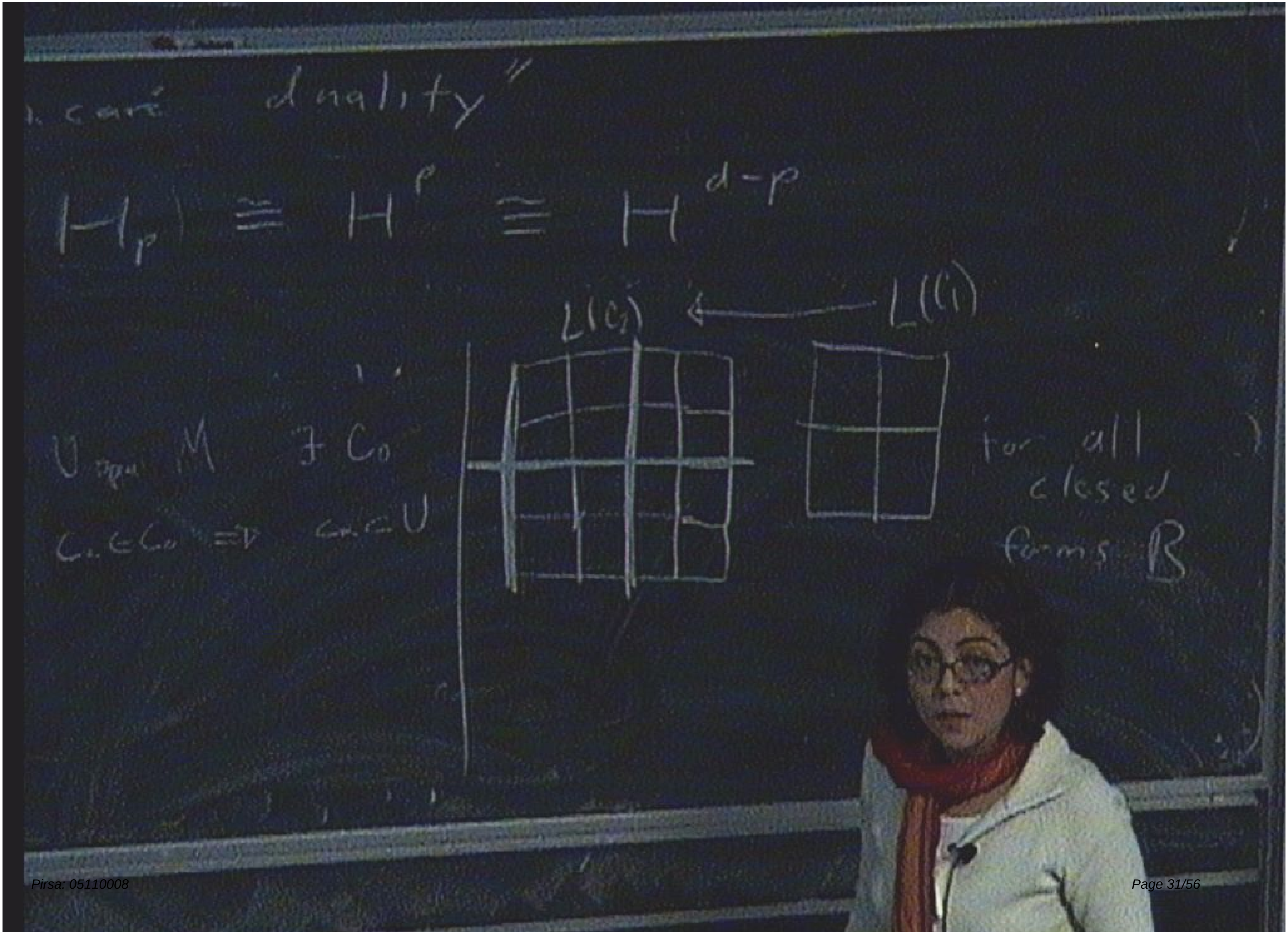
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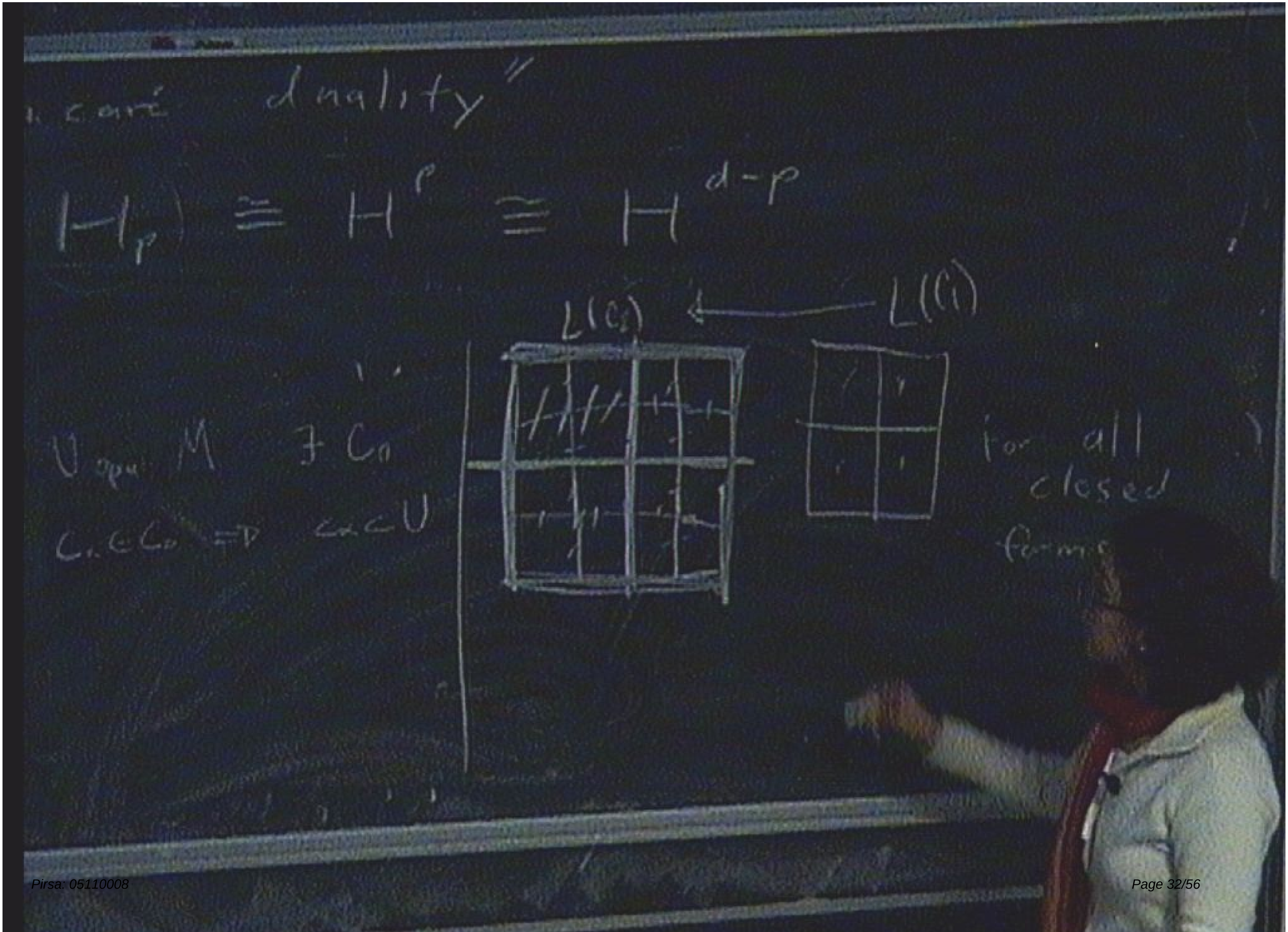
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(b) On the other hand, if we work at a truncation in which only measures of a certain functional form are considered the relation is the best approximation among the measures of the desired type of the coarse graining of the measure $\mu_{\beta(C_{i+1})}$ at the finer scale.

- Asking that some physically important correlation functions be equal when calculated using the coarse graining of the C_{i+1} effective theory or directly the C_i effective theory is called a *renormalization prescription*. This equation(s) is solved to determine the coupling constant(s) $\beta(C_{i+1})$ in terms of $\beta(C_i)$.

- The resulting flow in the space of coupling constants is called a *renormalization group flow*.

- If the renormalization group transformation can be invertible, then we can consider a flow towards the continuum (taking a subfamily of generic cell.dec.).

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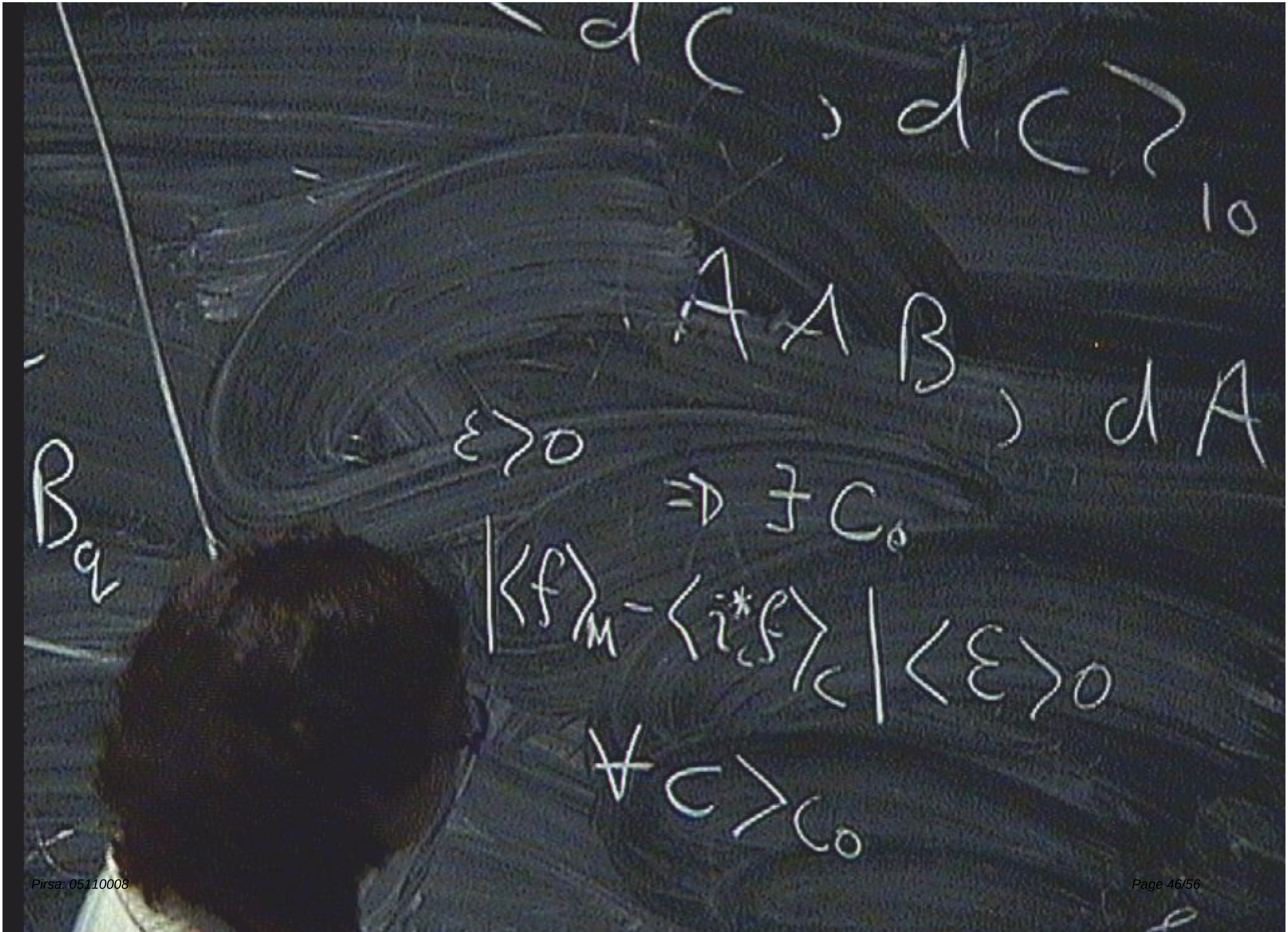
- Take $f \in \text{Cyl}(\bar{\mathcal{A}}_M)$ and calculate $\langle i_C^\star f\rangle_C$; our effective theories define a theory in the continuum only if

$$\langle f\rangle_M = \lim_{C\to M} \langle i_C^\star f\rangle_C \tag{2}$$

- If the limit in equation (2) exists for every cylindrical function we have given a constructive definition of a functional μ_M in $\bar{\mathcal{A}}_M$.

Notice that our continuum limit is not a projective limit of measures. The definition relies on the regularization maps $i_C^\star : \text{Cyl}(\bar{\mathcal{A}}_M) \to \text{Cyl}(\mathcal{A}_C)$ that are completely novel in loop quantization. The only arrows before are projection maps that could be considered somehow analogous to our coarse graining maps $\pi_C^\star : \text{Cyl}(\mathcal{A}_C) \to \text{Cyl}(\bar{\mathcal{A}}_M)$.


d C , d C ⟩ 10
A ∧ B , d A
B q
ε > 0
⇒ ∃ C0
|⟨f⟩M − ⟨i*c f⟩c| < ε > 0
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$d\mathcal{C},\ d\mathcal{C}\rangle_{10}$

$A\,A\,B,\ dA$

$\varepsilon > 0$

$\Rightarrow \exists\, C_0$

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$\forall\, C > C_0$

B_q

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$$
\begin{array}{ccccccc}
 & & & & & & (\overline{\mathcal{A}_{\mathcal{M}}}, \mu_{\mathcal{M}}) \\
 & & & & & \nearrow\!\!\!\!\nearrow & \downarrow \\
 & & & & & & \vdots \\
 & & & \nearrow & & & \\
 & & (\mathcal{A}_{C_2}, \mu_{C_2}^{(2)}) & \longleftrightarrow & \cdots & \longleftrightarrow & (\mathcal{A}_{C_2}, \mu_{C_2}^{ren}) \\
 & \nearrow & \downarrow & & & & \downarrow \\
(\mathcal{A}_{C_1}, \mu_{C_1}^{(1)}) & \longleftrightarrow & (\mathcal{A}_{C_1}, \mu_{C_1}^{(2)}) & \longleftrightarrow & \cdots & \longleftrightarrow & (\mathcal{A}_{C_1}, \mu_{C_1}^{ren})
\end{array}
$$

Where the terminology of the diagram is:

- $\downarrow$ for the exact renormalization group tansformation $\mu_{C_i}^{(k)} = \pi_{C_{i+1}C_i*}\mu_{C_{i+1}}^{(k)}$.
- $\longleftrightarrow$ indicates that both measures coincide when they calculate some physically important correlation functions.
- $\nearrow$ means that the measure $\mu_{C_{i+1}}$ will be chosen as to produce a $\pi_{C_{i+1}C_i*}\mu_{C_{i+1}}$ which agrees with μ_{C_i} when they calculate some physically important correlation functions.
- $\mu_{C_i}^{ren} = \mu_{C_i}^{(\infty)}$ is the completely renormalized measure at scale C_i.
- $\nearrow\!\!\!\!\nearrow$ is the continuum limit that defines $\mu_{\mathcal{M}}$ after the measures $\mu_{C_i}^{(i)}$ that solve the relations in the triangle have been found.

Explicit example

- Euclidean histories $\bar{A}_M$ is the space of spin fields on $\mathbb{R}^2$ $\bar{A}_{\mathbb{R}^2}$.

- The system that has a given physical coherence length ξ_{phys}.

- We use families of cellular decompositions $\{C_{m,t}\}_{m\in N}$ called Cartesian regular cellular decomposition of sizes $\{a_m = 1/2^m\}_{m\in N}$.

- We will use n-point functions to characterize the measure in the continuum. At scale $C_{m,t}$ the n-point functions are

$$\langle s(\alpha_1)\cdots s(\alpha_n)\rangle_C = \frac{1}{Z_{C_{m,t}}}\sum_s \frac{s(\alpha_1)\cdots s(\alpha_n)}{M(\beta_{C_{m,t}})^n}\exp[-\beta_{C_{m,t}}\sum_{(\alpha\beta)} s(\alpha)s($$

- A single step coarse graining map is defined as $\pi_{m+1,m} = \text{Emb}^*_{m,m+1}$ with $\text{Emb}_{m,m+1} : L(C_{m,t}) \to L(C_{m+1,t})$ defined by

$$\text{Emb}_{m,m+1}(\alpha(M,N)) \doteq \alpha(2M,2N)$$

.

- The coarse graining of n-point functions is simply

$$\langle \pi^*_{m+1,m}[s(\alpha_1^m)\cdot\ldots\cdot s(\alpha_n^m)]\rangle_{C_{m+1,t}} = \langle s(\alpha_1^{m+1})\cdot\ldots\cdot s(\alpha_n^{m+1})\rangle_{C_{m+1,t}} \quad (3)$$

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with $\alpha^{m+1} = \mathrm{Emb}_{m,m+1}(\alpha^m)$.

- *Renormalization prescription:*

$$a_m \xi_m = a_m |z^2 + 2z - 1|^{-1} [z(1 - z^2)]^{1/2} = \xi_{\text{phys}}$$

where $z = \tanh(\beta_{C_{m,l}})$.

Theorem 1 (McCoy, Tracy, Wu) *Choose $\beta_{C_{m,l}}$ as to satisfy the above renormalization prescription. Then a measure $\mu_{\mathbb{R}^2,\xi_{phys}}$ in $\bar{A}_{\mathbb{R}^2}$ is defined by its n-point functions that are calculated as a continuum limit*

$$\lim_{C_{m,l} \to \mathbb{R}^2} \langle s(p_1) \cdot \ldots \cdot s(p_n) \rangle_{C\beta_{C_{m,l}}} = \langle s(p_1) \cdot \ldots \cdot s(p_n) \rangle_{\mathbb{R}^2 \xi_{phys}}.$$

- Once we have constructed the theory in the continuum we can go back scale $C_{m,t}$ and calculate corrections to the effective theory.

- The completely renormalized theory at scale $C_{m,t}$ is determined by the coarse graining of n-point functions from the continuum

$$\langle \pi_m^*[s(\alpha_1^m)\cdot \ldots \cdot s(\alpha_n^m)] \rangle_{\mathbb{R}^2, \xi_{m,n}} = \langle s(\alpha_1^{m+1})\cdot \ldots \cdot s(\alpha_n^{m+1}) \rangle_{C_{m+1,t}}^{\text{ren}} \tag{4}$$

  with $p = \text{Emb}_m(\alpha^m)$.

- The n-point functions in the continuum defined above satisfy the Osterwalder-Schrader axioms.

- Rotational invariance is restored in the continuum limit.

- We can construct a covariant Hamiltonian quantum theory following an Osterwalder-Schrader type constuction on the space $\bar{A}_M$.

V. CONCLUSION

- The new notion of effective theories in the context of loop quantization allows us to define a loop quantized theory in the continuum, where the dynamics is systematically constructed by our strategy.
- In our example, the Euclidean path integral of the theory is rigorously defined as a measure in the space $\overline{\mathcal{A}_{\mathbb{R}^2}}$.
- Explicit knowledge of the relation between this quantum field theory and the standard realization of the 2d Ising field theory [phi^4 Landau-Ginzburg model] in terms of linear scalar fields would certainly be desirable.
- This strategy can be applied as well to gauge theories and topological theories.
- **Main goal**: the case of quantum gravity for which the loop quantization was first realized.

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