Title: Microstates for BPS black holes and black rings

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Abstract:

Microstates for BPS Black Holes and Black Rings

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Outline

1. The fuzzball hypothesis
2. The Bena-Warner ansatz
3. Solving the equations for a three-charge system
   - Solve ansatz
   - Lay out conditions for a causal, smooth spacetime
4. Examples
5. Geometric transitions and quantum foam
6. Discussion and conclusions

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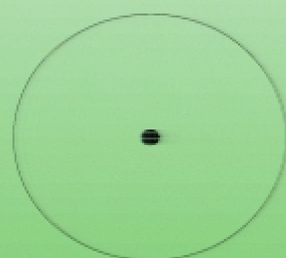

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- Each microstate looks the same asymptotically. Closer in we see differences
- Our three-charge solutions will replace a core region of singular brane sources with a geometric transition to a bubbling foam of two-cycles threaded by flux
- The intricate geometry of these cycles will distinguish individual microstates
- Along the way we will find rules for arranging the cycles

The Bena-Warner ansatz

- Bena and Warner have laid out an anzatz for 1/8 BPS solutions with three charges in five dimensions. Each charge comes from wrapping membranes on three separate T^2s
- The 5D space is time fibred over a hyperkahler base space, HK

$$ds_{11}^2 = -(Z_1 Z_2 Z_3)^{-2/3}(dt+k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_{HK}^2 + ds_{T^6}^2,$$
$$ds_{T^6}^2 = (Z_1 Z_2 Z_3)^{1/3}\left(Z_1^{-1}(dz_1^2+dz_2^2) + Z_2^{-1}(dz_3^2+dz_4^2) + Z_3^{-1}(dz_5^2+dz_6^2)\right).$$

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- The C-field is given by

$$\begin{aligned} C_{(3)} &= -(dt+k)\left(Z_1^{-1}\, dz_1 \wedge dz_2 + Z_2^{-1}\, dz_3 \wedge dz_4 + Z_3^{-1}\, dz_5 \wedge dz_6\right) \\ &\quad + 2\, a_1 \wedge dz_1 \wedge dz_2 + 2\, a_2 \wedge dz_3 \wedge dz_4 + 2\, a_3 \wedge dz_5 \wedge dz_6. \end{aligned}$$

Bena-Warner ansatz continued

Define $G_i = da_i$. The BW ansatz solves the EOM if

$$\begin{aligned} G_i &= \star G_i, \\ d \star dZ_i &= 2s^{ijk} G_j \wedge G_k, \\ dk + \star dk &= 2G_i Z_i. \end{aligned}$$

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Solving the EOM: The HK metric

- We relax the hyperkahler condition to allow a psuedo-hyperkahler HK so long as the total space is smooth
- We write the metric in Gibbons-Hawking form as

$$ds^2_{HK} = H^{-1}\sigma^2 + H(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)$$

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- Our full solution will be completely encoded by a set of 8 harmonic functions on $\mathbf{R}^3$: H, h_i, M_i and K

The function H

- When we relax the hyperkahler condition we can have more general candidates for H, which is harmonic on $\mathbf{R}^3$

$$H = \sum_{p=1}^{N} \frac{n_p}{r_p}, \qquad r_p = |\vec{r}_p| = |\vec{x} - \vec{x}_p|, \qquad \sum_{p=1}^{N} n_p = 1$$

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- We now have a set of non-singular two-cycles S^{pq}, coming from the fiber σ over each interval from $\vec{x}_p$ to $\vec{x}_q$

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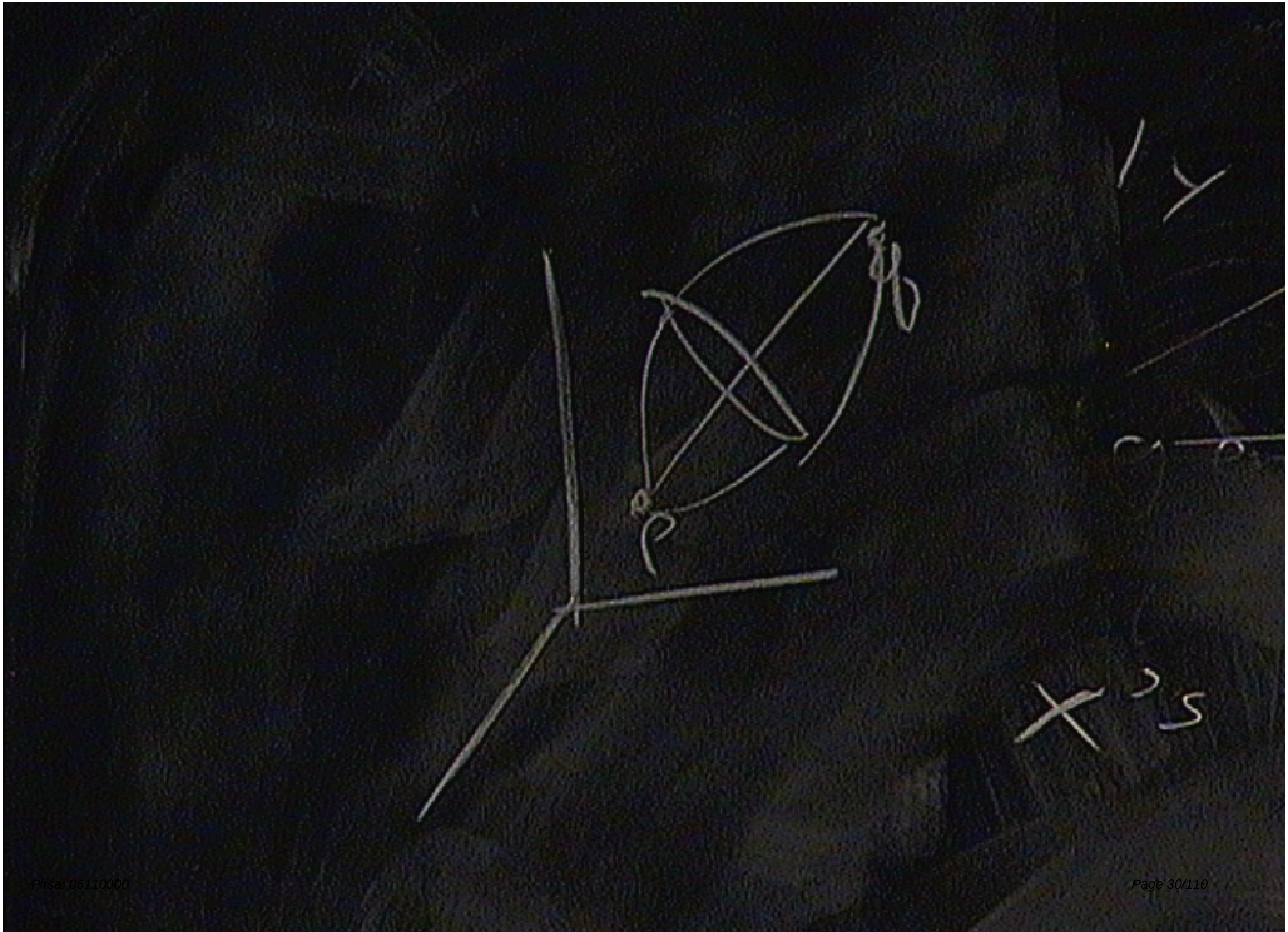

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The dipole fields

- The M5-brane charge can be read off from the components of $\star dC_{(3)}$ which have a leg in the time direction. These are proportional to the G_i
- We need these to fall off faster than $1/r$ so there is no net M5-brane charge at infinity
- We will also want to ensure that the G_i have no singularities except at $H = 0$

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$$G_i = d(h_i/H) \wedge \sigma - H \star_3 d(h_i/H)\,, \quad h_i = \sum_{p=1}^{N} \frac{d_i^{(p)}}{4r_p}\,, \quad \sum_{p=1}^{N} d_i^{(p)} = 0$$

Small aside on notation

- For convenience we define the following quantities:

$$\Pi^r = \prod_{p=1}^{N} r_p, \quad \Pi_p^r = \prod_{q \neq p} r_q, \quad \Pi_{ps}^r = \prod_{s \neq q \neq p} r_q, \qquad f = \sum_{p=1}^{N} n_p \Pi_p^r$$

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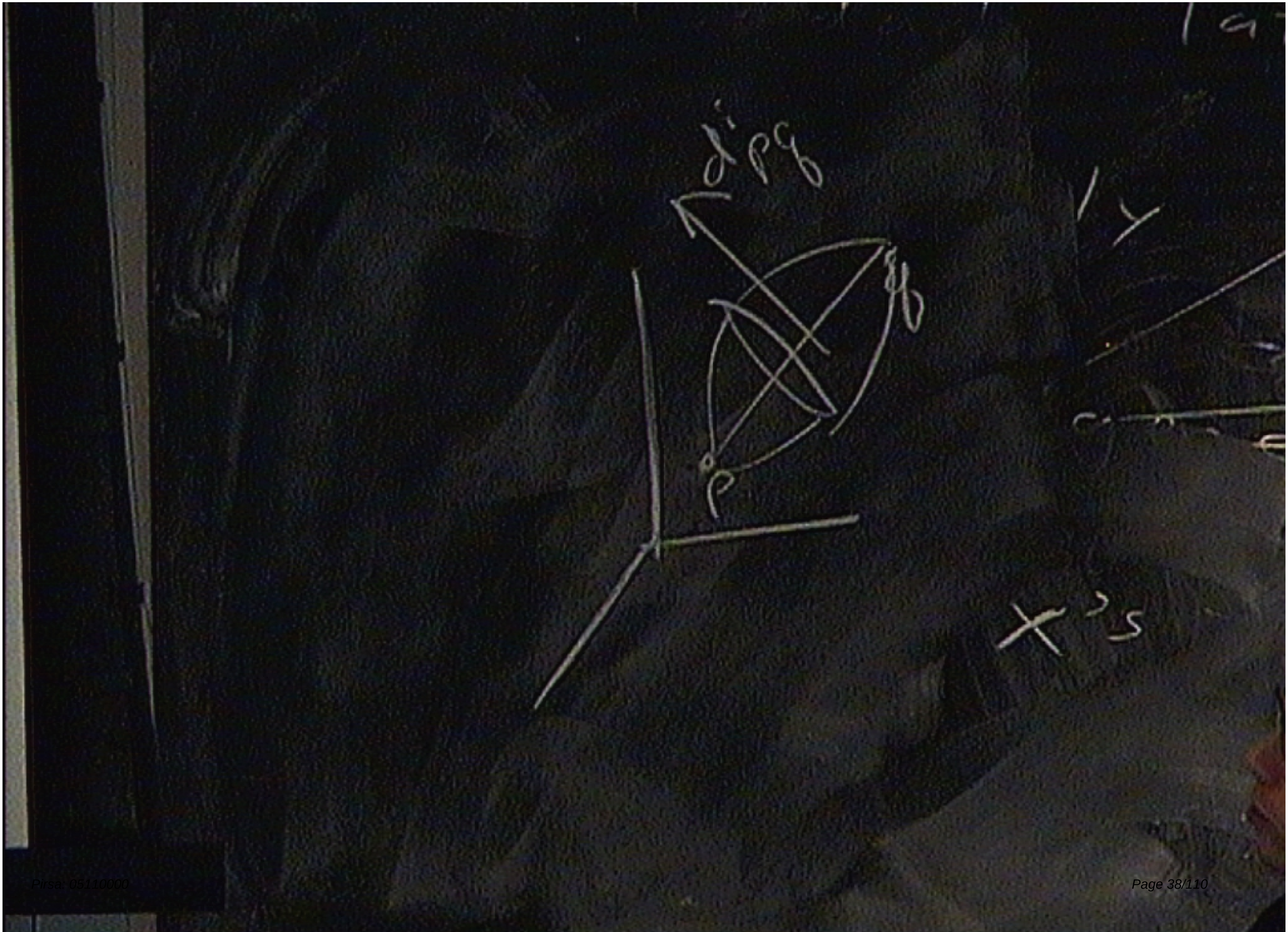
- We'll also find it very useful to define a global Dipole moment and more locally relative dipole moments

$$d_i^{pq} = n_p d_i^{(q)} - n_q d_i^{(p)} \Rightarrow \quad d_i^{(p)} = -\sum_{q=1}^{N} d_i^{pq}, \quad \vec{D}_i = -\frac{1}{2} \sum_{pq} d_i^{pq} \left(\vec{x}_p - \vec{x}_q\right)$$

$$Q_i = \sum_{p,q} Q_i^{pq}, \qquad \text{where} \qquad Q_i^{pq} = -\frac{s^{ijk} d_j^{pq} d_k^{pq}}{4 n_p n_q}$$



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- Correcting a bit to satisfy the equations of motion we find

$$Z_i \;=\; M_i + 2\,H^{-1} s^{ijk} h_j\, h_k, \quad M_i = 1 + \sum_{p=1}^{N} \frac{Q_i^{(p)}}{4r_p}, \quad \sum_{p=1}^{N} Q_i^{(p)} = Q_i$$

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where we have defined a new harmonic function K and its partner function $\bar{K}$

$$K = \sum_{p=1}^{N} \left(\frac{\ell_p}{r_p} \right) , \qquad \bar{K} = K - 4H^{-2} h_1 h_2 h_3$$

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$$\ell_p = \frac{d_1^{(p)} d_2^{(p)} d_3^{(p)}}{16 n_p^2}, \qquad k_0|_{r_p=0} = 0$$

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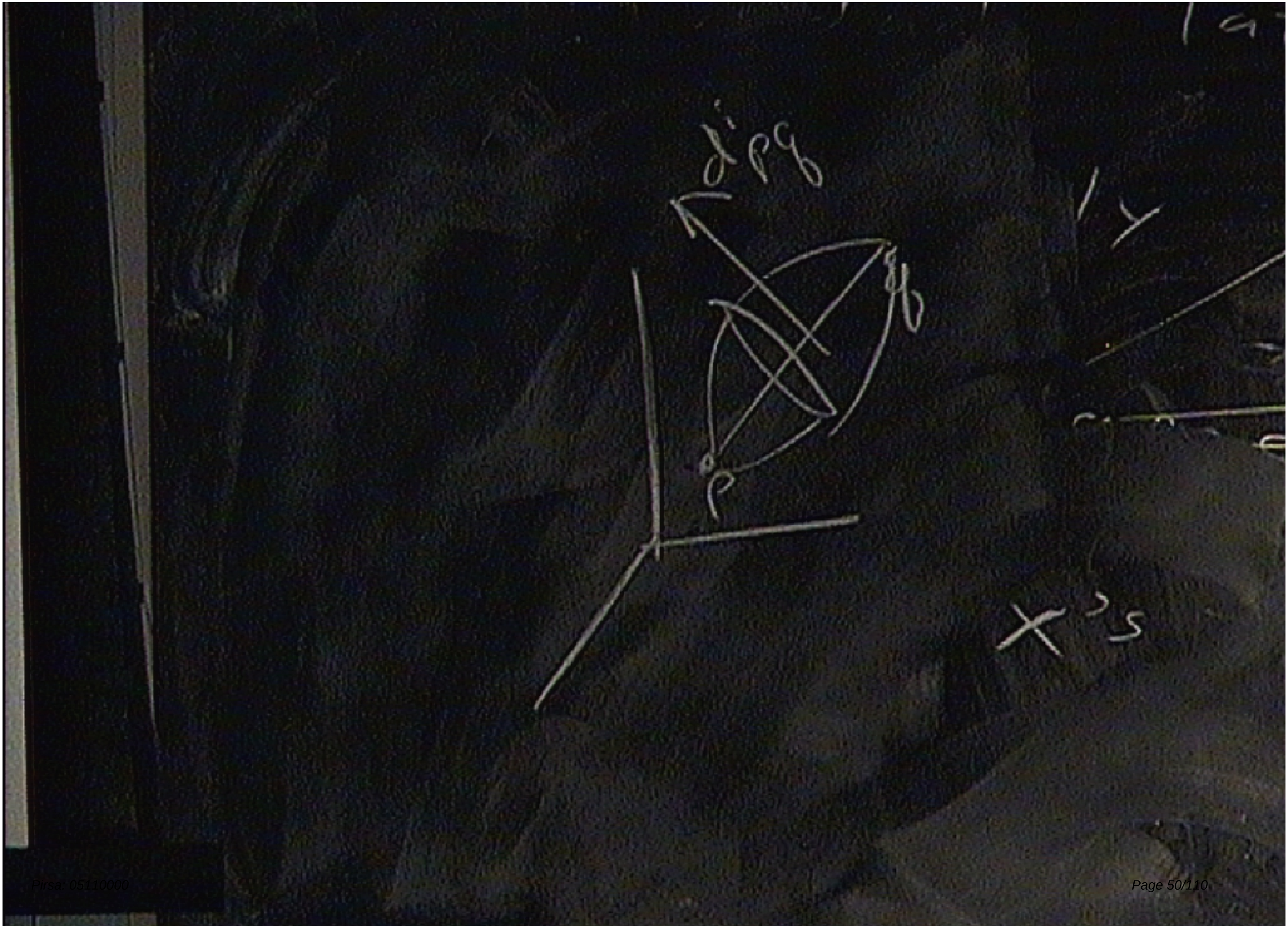

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- The first condition removes the poles in $\bar{K}$ and the second condition insures that $d^2(k_a dx^a) = 0$. The second condition can be solved giving

$$0 = \sum_i d_i^{(p)} + \sum_q \frac{1}{4 n_p^2 n_q^2} \frac{1}{r_{pq}} \prod_i d_i^{pq}$$

where $r_{pq} = |\vec{x}_p - \vec{x}_q|$
- This condition puts at most $N - 1$ constraints on the relative pole positions

The angular momentum continued

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- We can do this if we use the freedom to add any closed form to $k_a dx^a$ and demand

$$\ell_p = \frac{d_1^{(p)} d_2^{(p)} d_3^{(p)}}{16 n_p^2}, \qquad k_0|_{r_p=0} = 0$$

- The first condition removes the poles in $\bar{K}$ and the second condition insures that $d^2(k_a dx^a) = 0$. The second condition can be solved giving

$$0 = \sum_i d_i^{(p)} + \sum_q \frac{1}{4 n_p^2 n_q^2} \frac{1}{r_{pq}} \prod_i d_i^{pq}$$

where $r_{pq} = |\vec{x}_p - \vec{x}_q|$

- This condition puts at most $N-1$ constraints on the relative pole positions

The angular momentum continued

- We can also integrate the EOM to get an expression for $k_a dx^a$ (too complex to display here)
- The 2-form dk naturally splits into a self-dual and anti-self-dual part

$$
\begin{aligned}
dk_L = (dk + \star\, dk)/2 &= Z_i G_i \\
dk_R = (dk - \star\, dk)/2 &= (dK \wedge \sigma + H \star_3 dK) + (h_i/H)\,(dM_i \wedge \sigma + H \star_3 dM_i) \\
&= (d\bar{K} \wedge \sigma + H \star_3 d\bar{K}) + (h_i/H)\,(dZ_i \wedge \sigma + H \star_3 dZ_i)
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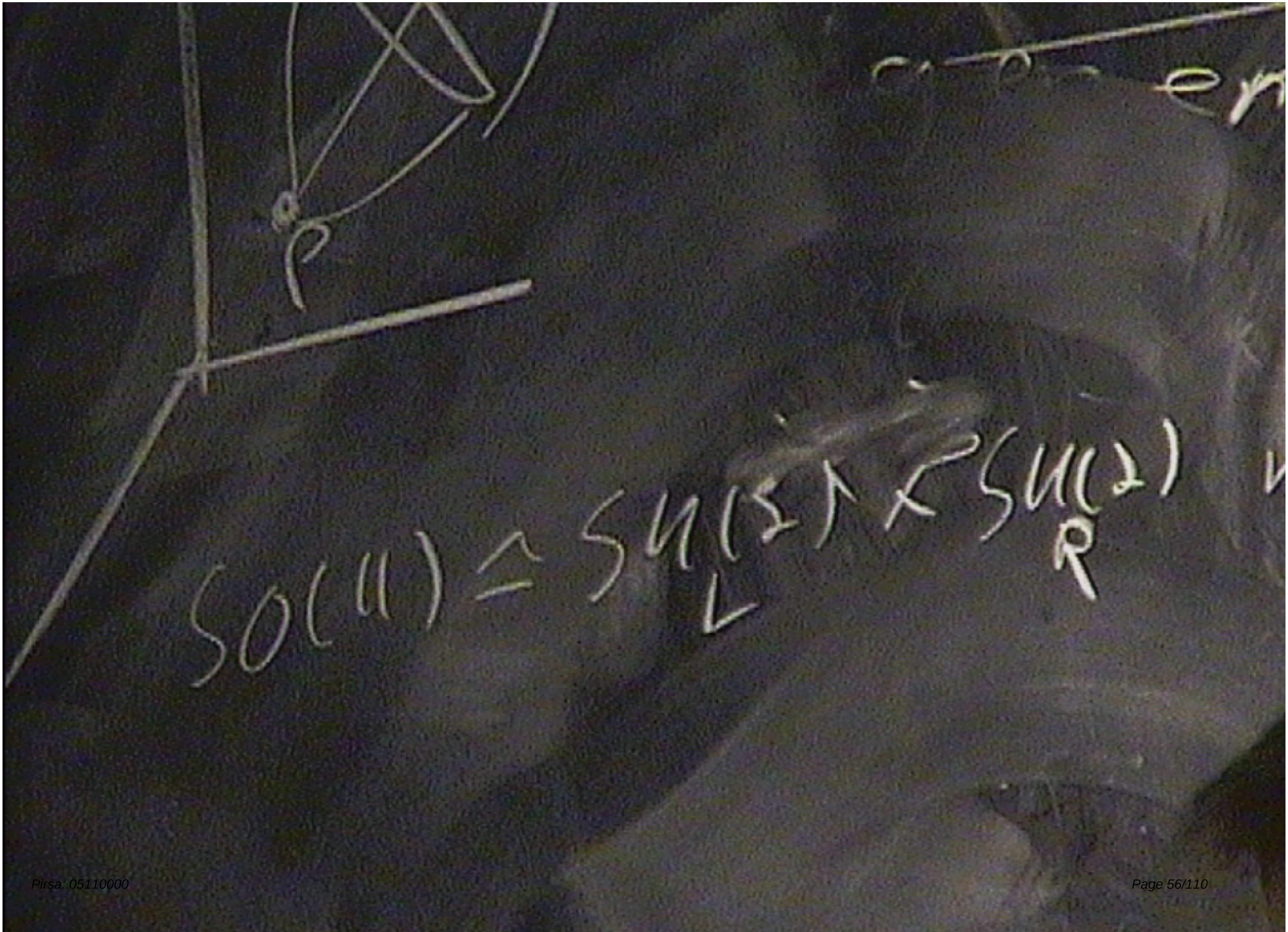
SO(4) ≃ SU(2) × SU(2)
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- Using the asymptotics of these forms we can read off the angular momenta at infinity to be

$$J_L = \frac{4G_5}{\pi} j_L = |4 \sum_i \vec{D}_i| \qquad J_R = \frac{4G_5}{\pi} j_R = 16 \sum_{p=1}^{N} \ell_p$$

where the $j_{L,R} \in \mathbf{Z}$

Small aside on notation

- For convenience we define the following quantities:

$$\Pi^r = \prod_{p=1}^{N} r_p, \quad \Pi^r_p = \prod_{q \neq p} r_q, \quad \Pi^r_{ps} = \prod_{s \neq q \neq p} r_q, \qquad f = \sum_{p=1}^{N} n_p \Pi^r_p$$

Hence, H can be written as f/Π^r.

- We'll also find it very useful to define a global Dipole moment and more locally relative dipole moments

$$d_i^{pq} = n_p d_i^{(q)} - n_q d_i^{(p)} \Rightarrow d_i^{(p)} = -\sum_{q=1}^{N} d_i^{pq}, \quad \vec{D}_i = -\frac{1}{2} \sum_{pq} d_i^{pq} (\vec{x}_p - \vec{x}_q)$$

$$Q_i = \sum_{p,q} Q_i^{pq}, \qquad \text{where} \qquad Q_i^{pq} = -\frac{s^{ijk} d_j^{pq} d_k^{pq}}{4 n_p n_q}$$

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- One can easily show that in general, the metric and C-field are regular as $H \to 0$, so this puts no further constraints on our solution

Zeros of the Z_i

- To avoid singularities we need the determinant of the metric and its inverse to be well defined and non-vanishing

$$\sqrt{-g_{11}} = (Z_1 Z_2 Z_3)^{1/3}\, H\, \sqrt{g_{\mathbf{R}^3}}$$

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- We see that to avoid singularities we need $Z_i \neq 0$. Our simple tactic for enforcing this is to everywhere demand

$$Z_i H > 0 \qquad \forall i \in 1, 2, 3$$

- We can rewrite this condition as

$$4f - s^{ijk} \sum_{p,q} \frac{d_j^{pq} d_k^{pq} \Pi_{pq}^r}{4 n_p n_q} > 0 \qquad \forall i \in 1, 2, 3$$

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- For a spacetime to be stably causal it must admit a globally defined, smooth function whose gradient is everywhere timelike. We call this a *time function*
- Our candidate function is simply the coordinate t, which is a time function if

$$-g^{\mu\nu}\partial_\mu t \partial_\nu t = -g^{tt} = (Z_1 Z_2 Z_3)^{-1/3} H^{-1} \left((Z_1 Z_2 Z_3) H - H^2 k_0^2 - g_{\mathrm{R}^3}^{ab} k_a k_b \right) > 0$$

- In general, this is a complicated function and we have not analyzed this in detail. It is possible that this will place further constraints on the relative pole positions

Horizons

- Given our time function t, we can show there are no event horizons. The vector ∂_r has norm

$$g_{rr} = (Z_1 Z_2 Z_3)^{-2/3}\left((Z_1 Z_2 Z_3)H - k_r^2\right) \geq -g^{tt}$$

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- Consider the vector field

$$\xi = \left(\frac{g_{rr}}{-g^{tt}} \right)^{1/2} g^{t\mu} \partial_\mu + \epsilon \, \partial_r$$

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- If we choose $0 < \epsilon < 1$ then ξ is always timelike and trajectories generated by it always eventually reach asymptotic infinity. Hence, there are no event horizons

Topology of the σ fibration

- The R^3 base metric has orbifold points with identification on the σ fiber
- For a given two-sphere we calculate the c_1 of the σ fibration $U(1)$ bundle by integrating $d\sigma$ over it

$$\int_{S^2} d\sigma \quad = \quad \int_{S^2} \star_3 dH = \int_{B^3} d \star_3 dH = \sum_p \int_{B^3} n_p \, \delta^3(\vec{r} - \vec{r_p})$$

- This yields an integer which counts the poles inside the two sphere. If this integer is zero, the topology of the σ-fiber over this S^2 is $S^2 \times S^1$, if the integer is ± 1 then the topology is that of S^3. Any larger integer m will give the topology S^3/Z_m.
- This topology receives no corrections in the full metric

Topology of the gauge fields:Membrane probes

- Consider a probe membrane $\mathcal{M}_i$ wrapped on the torus T_i. This effectively yields a charged particle in the 5D reduced space with charge

$$e_i = V_i \tau_2 = \frac{V_i}{(2\pi)^2 \ell_P^3}$$

which experiences a gauge field

$$\mathcal{A}_i = 2a_i - Z_i^{-1}(dt + k), \qquad F_i = d\mathcal{A}_i$$

- The compact two-cycles in our geometry, S^{pq}, are represented by line segments on R^3 between two points $\vec{x}_p$ and $\vec{x}_q$, where H blows up, along with the fiber σ
- For quantum consistency of the probe charge's wavefunction we require the field strength be an integral cohomology class on the universal cover of S^{pq}, i.e. $n_p \cdot n_q$ time our original cycle. We thus define an integer for each two-cycle

$$m_i^{(pq)} = n_p\, n_q \frac{e_i}{2\pi} \int_{\vec{x}_p}^{\vec{x}_q} \int_\sigma F_i d\tau\, ds = 2 n_p\, n_q\, e_i\, \mathcal{A}_i \Big|_p^q = e_i d_i^{pq}$$

Topology of the gauge fields:Dirac strings

- Near each point p the local geometry is a cone over $S^3/\mathbf{Z}_{n_p}$
- This has $\pi_1 = \mathbf{Z}_{n_p}$ and implies that we have the possibility of a Dirac string for each gauge field.
- Consider the gauge field near one of the orbifold points, we see that the Dirac string phase is

$$2\pi \left(4e_i h_i/H\right)\Big|_p = 2\pi\, m_i^{(p)}/n_p$$

- This implies a quantization of dipole and conserved membrane charge

$$d_i^{(p)} = m_i^{(p)}/e_i, \quad N_i = \frac{\pi}{4e_i G_5} Q_i = -\sum_{p,q} s^{ijk} \frac{m_j^{pq} m_k^{pq}}{4 n_p n_q}$$

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Summary of conditions

- Our solution is completely parameterized by a set of poles on $\mathbb{R}^3$ with quantized residues n_p and quantized fluxes $m_i^{pq} = n_p m_i^{(q)} - n_q m_i^{(p)}$
- These and the quantities that depend on them must satisfy the following conditions for us to have a smooth (up to orbifold points) and regular solution free of CTCs and horizons to 11D SUGRA with three membrane charges and 4 supersymmetries:

$$
\begin{aligned}
1) &\quad \sum_i d_i^{(p)} + \sum_q \frac{1}{4n_p^2 n_q^2} \frac{1}{r_{pq}} \prod_i d_i^{pq} = 0, \\
2) &\quad Z_i H > 0 \qquad \forall i \in 1,2,3, \\
3) &\quad (Z_1 Z_2 Z_3) H - H^2 k_0^2 - g_{\mathbb{R}^3}^{ab} k_a k_b > 0
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- We also choose dipoles that are diagonal in the three $U(1)$'s so that the dipole vector $\vec{D}_i$ are all parallel so we can compare with BPS black rings. We choose $d_i^{(1)} = d_i^{(2)} = d$ and $d_i^{(3)} = -2d$

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- One can show analytically that this condition is satisfied at each of the poles and asymptotic infinity. A numerical analysis confirms it is satisfied everywhere for any value of r_{12}

3-pole solution continued

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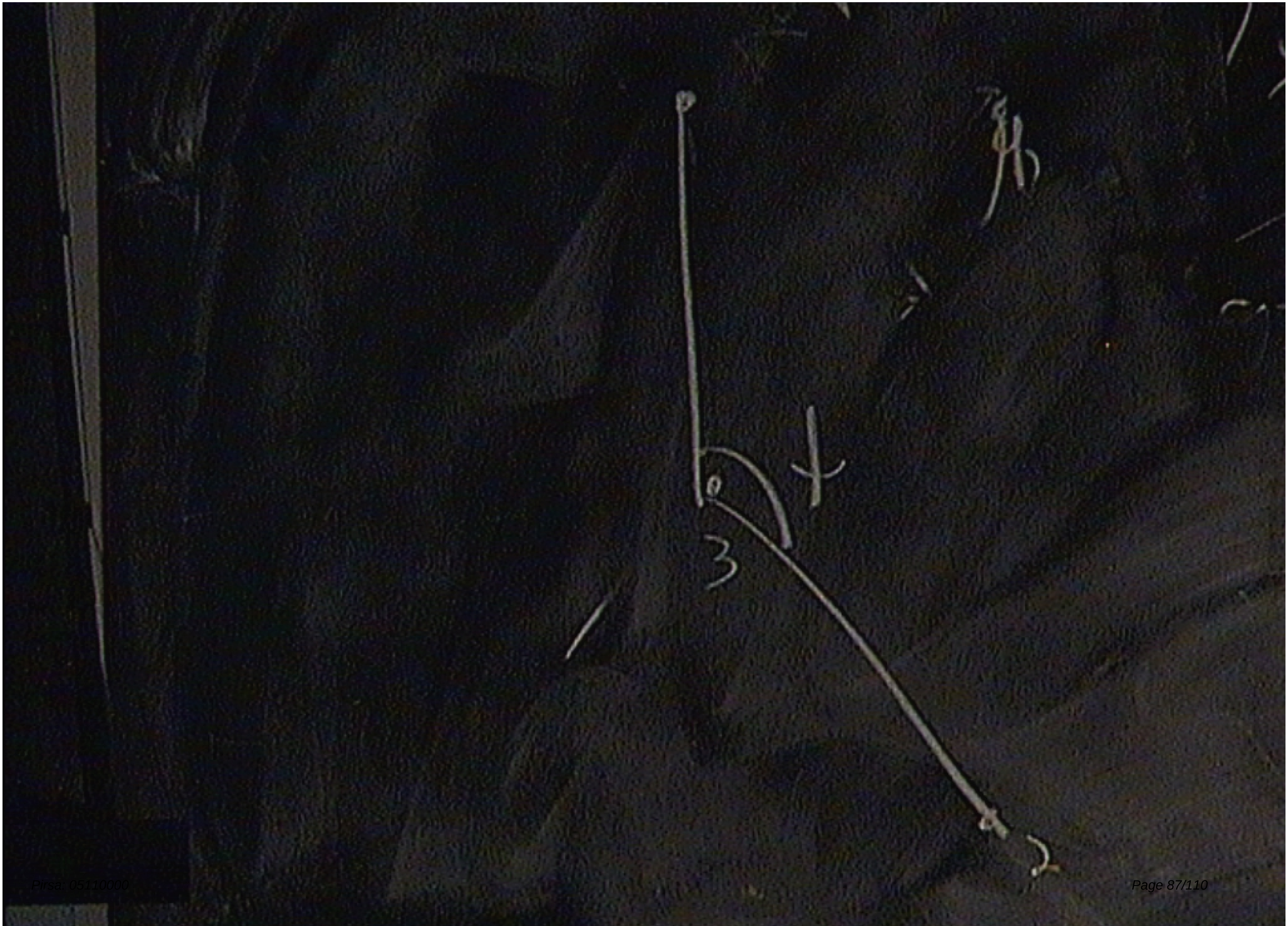
$$\vec{D}_i = \frac{d^3}{12}[\hat{z}(1 + \cos\psi) + \hat{x}\sin\psi]$$

- One can now easily read off the asymptotic charges

$$Q_1 = Q_2 = Q_3 = 2d^2, \;\; J_R^2 = 36d^6, \;\; J_L^2 = 2d^6(1 + \cos\psi)$$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- We have seen that our solutions replace a core region, which naively would have a naked singularity or a horizon with a core region containing a “foam” of new topologically nontrivial cycles. This draws interesting parallels with work by Vafa et. al. on geometric transitions and melting crystal space-time foam
- These cycles actually live in the *non-compact* space, which is a new phenomenon
- Because the n_p can be positive or negative, we actually have BPS solutions with branes *and* anti-branes. This can still be supersymmetric because of flux
- We anticipate that higher pole solutions can be microstates for black holes and rings
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- We have demonstrated a solution generating technique for general $U(1)$ invariant, BPS, three-charge microstates
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- These solutions replaced a singular core region with an intricate geometry of two-cycles threaded by electric and magnetic flux
- How do we invert our conditions so that we can find and count all microstates for given conserved charges?
- What are the dual CFT states? How can the CFT encode our microscopic variables?
- These solutions with some modification can be reduced to four-dimensions. What are the relations to the OSV conjecture on the black hole partition function and topological strings?
- Our cycles live in the non-compact space (though they also affect the size of the torii cycles), what is their relation to situations where the cycles live in the compact space?

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const.

$$H' = \delta H + H$$

$$L^2 \sim \frac{1}{\delta H} \qquad M'$$

$$\delta H + H$$

$$h^i = \delta h^i + h^i$$

$$L^2 \sim \frac{1}{\delta_H}$$

$$M^i = \delta M + M$$

$$K = \delta K + M$$

The angular momentum continued

- We need k to be a regular, globally defined 1-form that only has singularities where $H = 0$ and falls off like $1/r$ asymptotically
- We can do this if we use the freedom to add any closed form to $k_a dx^a$ and demand

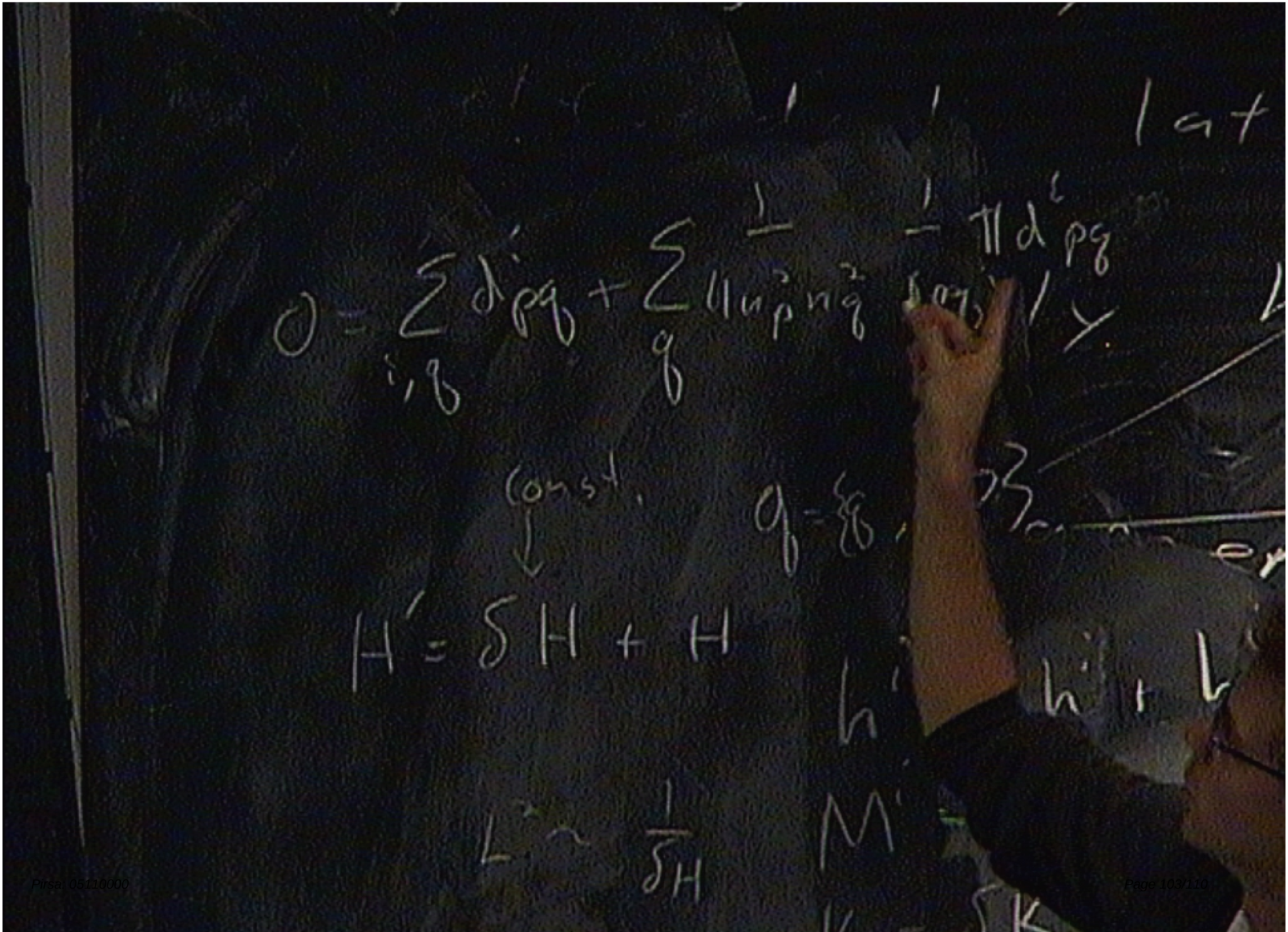
$$\ell_p = \frac{d_1^{(p)} d_2^{(p)} d_3^{(p)}}{16 n_p^2}, \qquad k_0|_{r_p=0} = 0$$

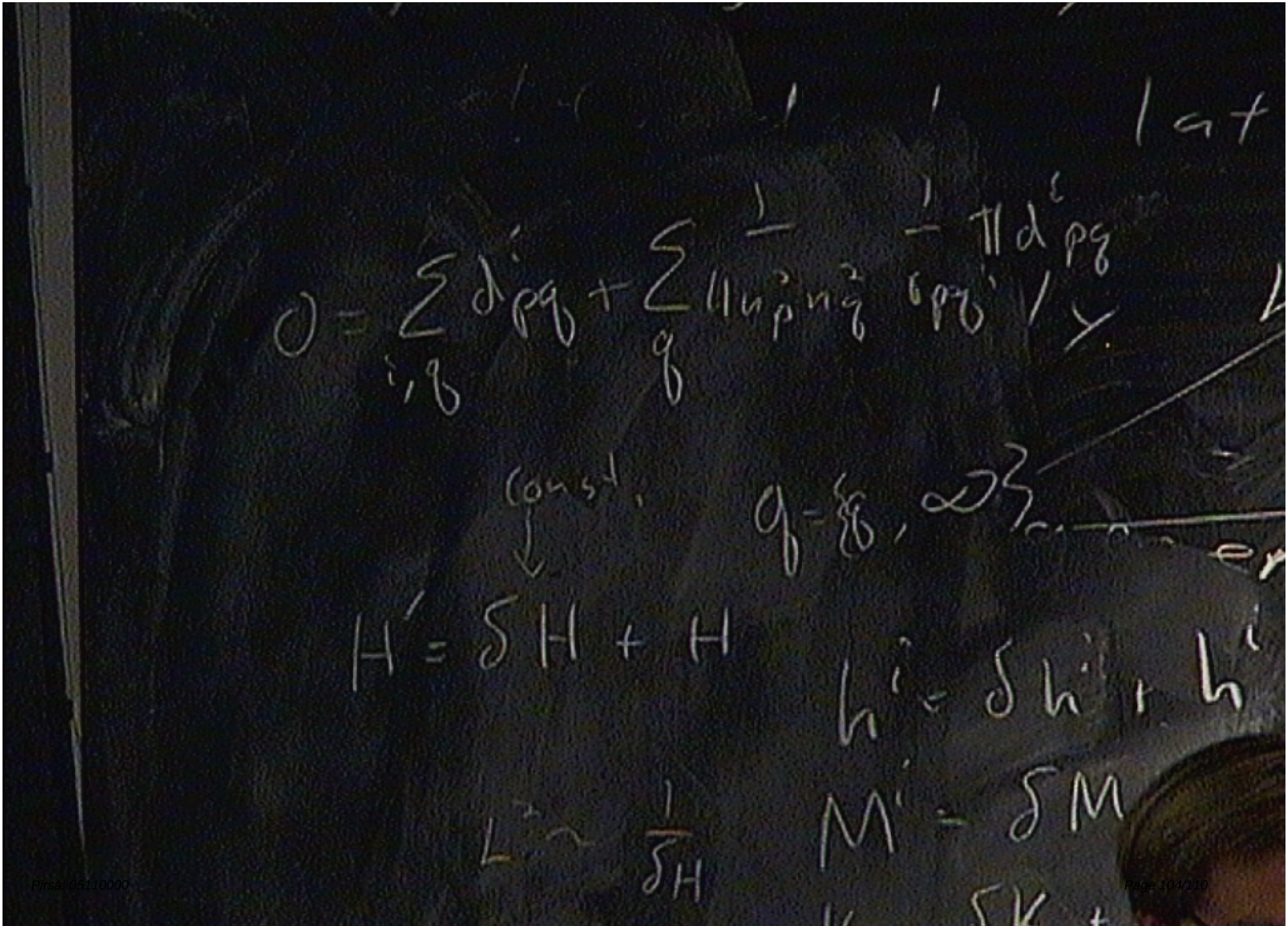
- The first condition removes the poles in $\bar{K}$ and the second condition insures that $d^2(k_a dx^a) = 0$. The second condition can be solved giving

$$0 = \sum_i d_i^{(p)} + \sum_q \frac{1}{4 n_p^2 n_q^2} \frac{1}{r_{pq}} \prod_i d_i^{pq}$$

where $r_{pq} = |\vec{x}_p - \vec{x}_q|$

- This condition puts at most $N - 1$ constraints on the relative pole positions


lat
const.
H = δH + H
L ~ 1/δH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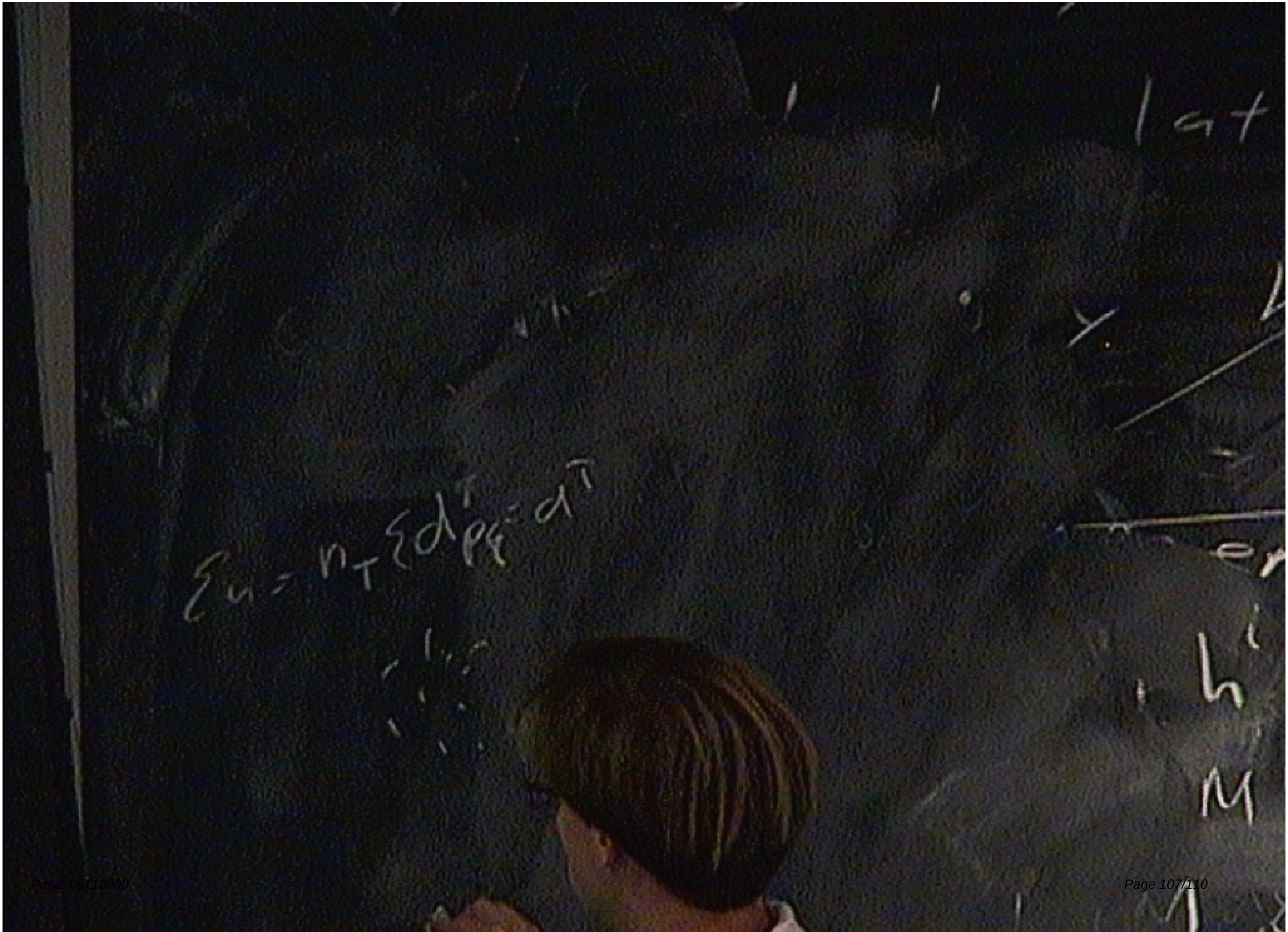
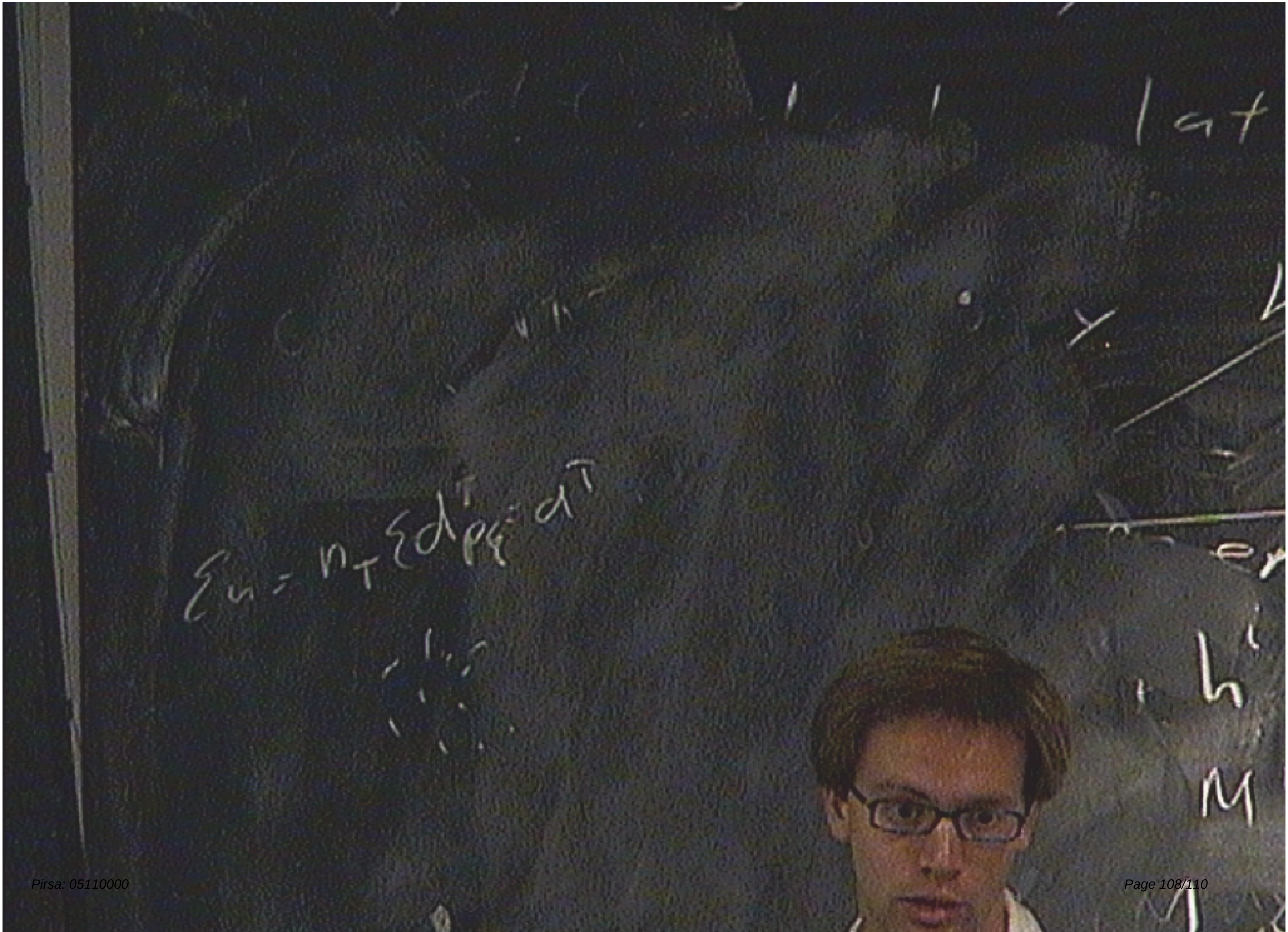
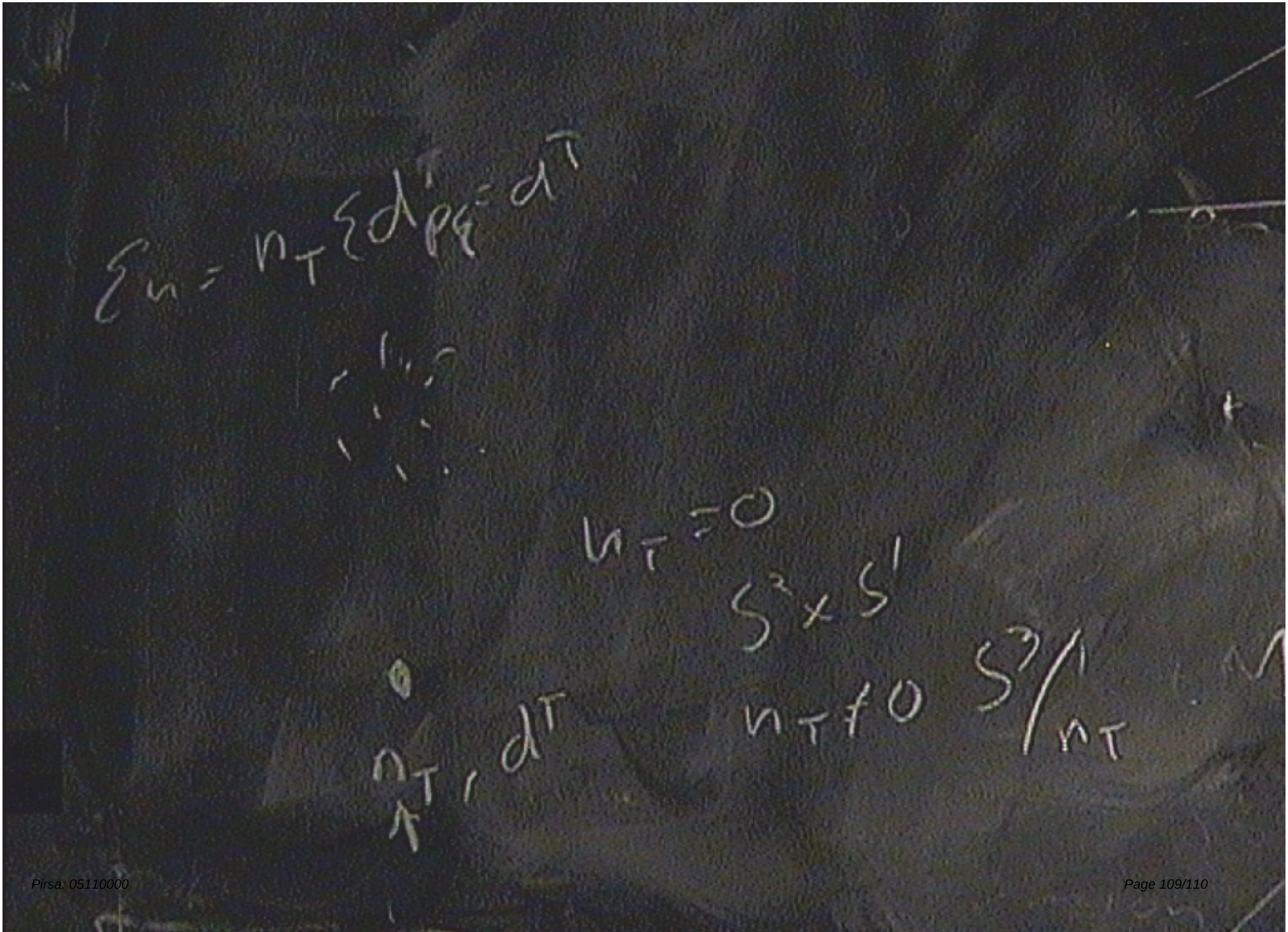
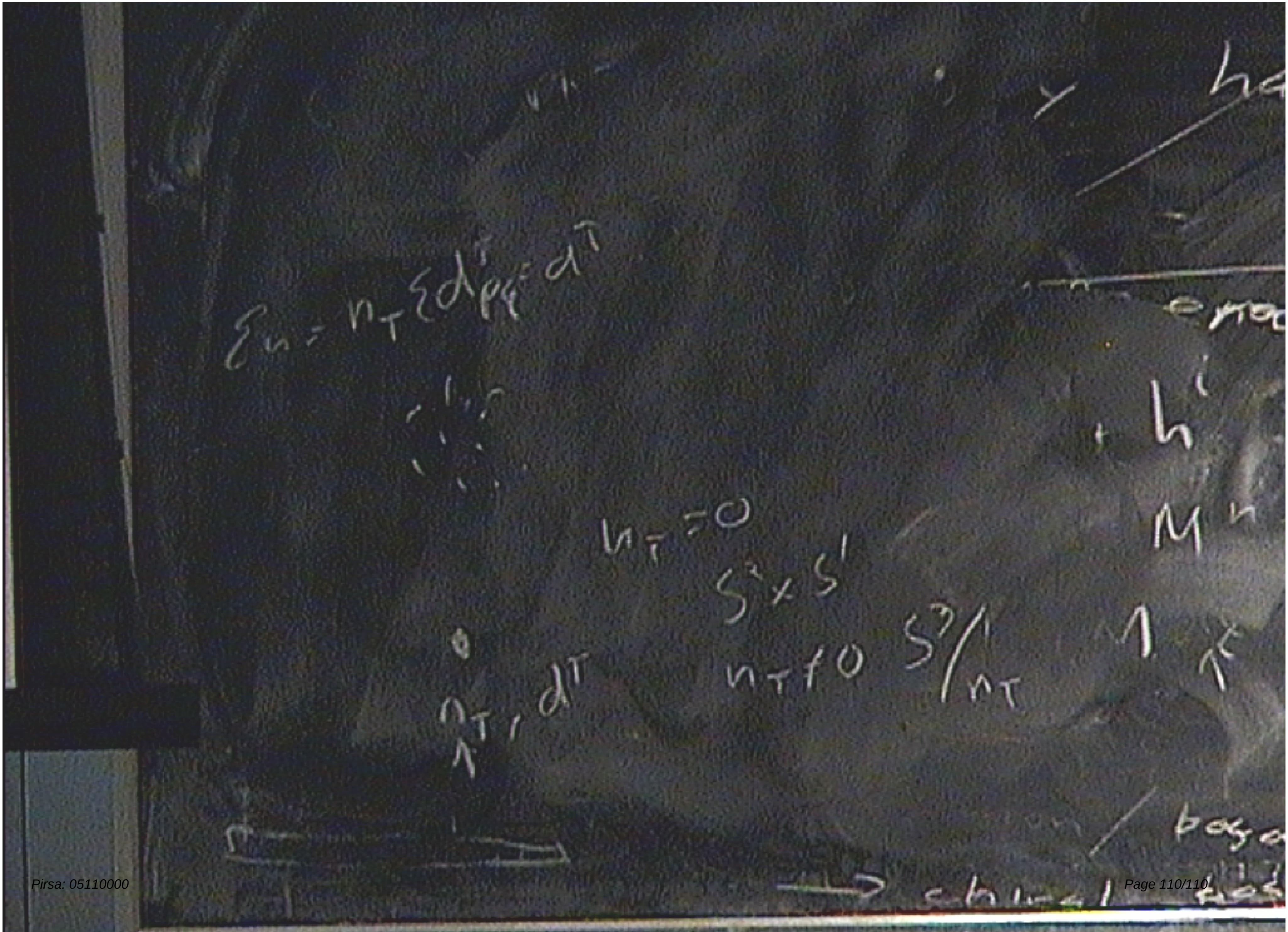






$E_n = n_T \Sigma d^T_{PF} = d^T$
$n_T = 0$
$S^3 \times S^1$
$n_T \neq 0 \quad S^3/n_T$
n_T, d^T



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