Title: Spherically symmetric solutions of massive gravity

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Abstract:

DGP gravity and Cosmology

APC/PI Workshop
on Cosmological Frontiers in
Fundamental Physics

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Astroparticules
et Cosmologie

Waterloo 2005

1/ DGP model (in 5D) or « brane induced gravity » and « massive gravity »

2/ Cosmology and phenomenology

Why being interested in these models of « massive gravity »?

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Changing the dynamics of gravity ?

Dark matter or dark energy ?

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Changing the dynamics
of gravity ?

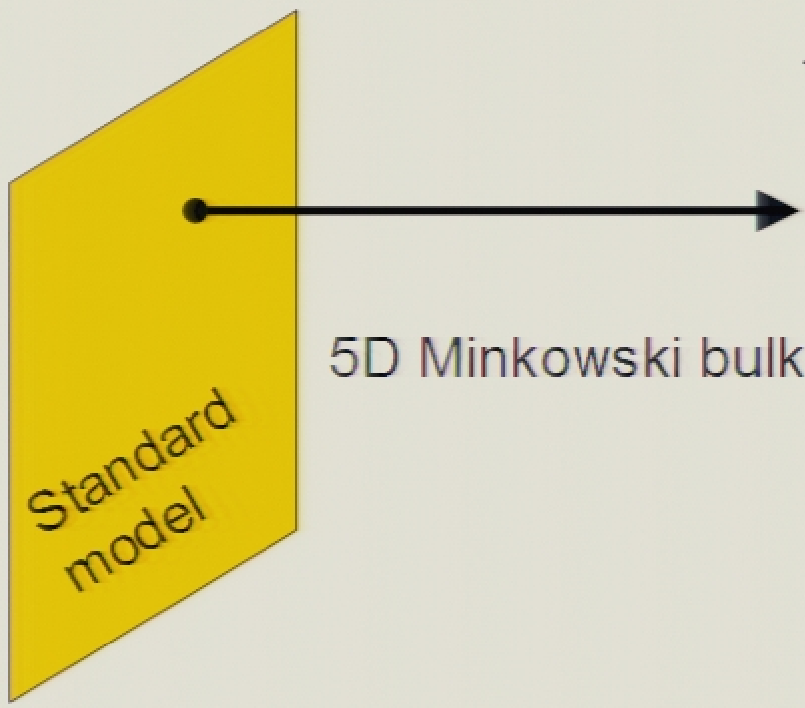
Historical example the success/failure of both approaches: Le Verrier and

- The discovery of Neptune
- The non discovery of Vulcan… but that of GR

1. The DGP model (or brane-induced gravity).

Dvali, Gabadadze, Porrati

$$S = M_{(5)}^3 \int d^5x\sqrt{g}\left(\tilde{R} + \cdots\right) + \int_{\text{brane}} d^4x\sqrt{g}\mathcal{L}_{\text{matter}}$$



$$+\int_{\text{brane}} d^4x\sqrt{g}\left(M_P^2\, R + \cdots\right)$$

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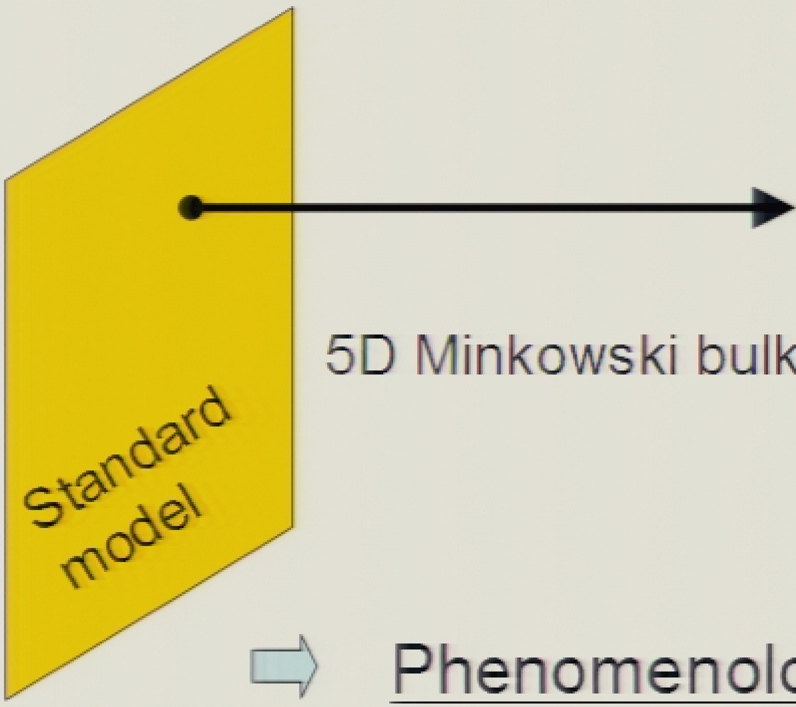

5D Minkowski bulk

$$+ \int_{\text{brane}} d^4x\sqrt{g}\left(M_P^2\, R + \cdots\right)$$

⇨ Phenomenological interest
large distance modification of gravity …

⇨ Theoretical interest
Consistent (?) non linear massive gravity …

1. 1. scalar toy model

$$S = M_{(5)}^3 \int d^4x \, dy \, (\partial_A \phi)^2 + M_{(4)}^2 \int_{\rm brane} d^4x \, (\partial_\mu \phi)^2$$

$\Rightarrow$ Green equation $\left(M_{(5)}^3 \partial_{(5)}^2 + M_{(4)}^2 \delta(y) \partial_{(4)}^2 \right) G_R = \delta^4(x)\delta(y)$

$$\Downarrow$$

$$G_R = \frac{1}{2M_{(5)}^3} \frac{1}{p + r_c p^2} e^{-p|y|}$$

5D potential at large distances

$$V(r) \sim \frac{1}{r^2}$$

$\longleftrightarrow$

4D potential at small distances

$$V(r) \sim \frac{1}{r}$$

Transition $\quad r \sim r_c \sim \dfrac{M_{(4)}^2}{M_{(5)}^3}$

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In terms of Kaluza-Klein modes

Wavefunction suppression on the brane

$$V(r) \propto \frac{1}{M_{(5)}^3} \int_0^\infty \overbrace{\frac{dm}{4 + m^2 r_c^2}} \frac{e^{-mr}}{r}$$

Only modes with $m \ll 1/r_c$ contribute (I)

•If suppression (I) does operate (if $1/r_c \ll 1/r$) : the number of contributing mode is frozen: 4D potential at small distances

•If suppression (I) does not operate (if $1/r_c \gg 1/r$) : 5D potential (at large r)

1.2 Back to the DGP model :

- Newtonian potential on the brane behaves as

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→ Leads to the van Dam-Veltman-Zakharov discontinuity on Minkowski background!

vDVZ discontinuity
and
Pauli-Fierz action

Pauli-Fierz action: second order action for a massive spin two $h_{\mu\nu}$

$$\int d^4x \sqrt{g} \underbrace{(R - 2\Lambda)}_{\text{second order in h}} + m^2 (h_{\mu\nu} h^{\mu\nu} - h^2)$$

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second order in h

Only Ghost-free (quadratic) action for a massive spin two Pauli, Fierz

(NB: breaks explicitly gauge invariance)

On a Minkowski background:

propagator for $m=0$ $\quad D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{2p^2} + \mathcal{O}(p)$

propagator for $m\neq 0$ $\quad D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{3(p^2-m^2)} + \mathcal{O}(p)$

e.g. amplitude between two non relativistic sources:

$$\left.\begin{array}{l}\hat{T}^{\mu}_{\nu} \propto \mathrm{diag}(\hat{m}_1, 0, 0, 0) \\ \hat{S}^{\mu}_{\nu} \propto \mathrm{diag}(\hat{m}_2, 0, 0, 0)\end{array}\right\} \quad \mathcal{A} \sim \tfrac{2}{3}\hat{m}_1\hat{m}_2 \quad \text{Instead of} \quad \mathcal{A} \sim \tfrac{1}{2}\hat{m}_1\hat{m}_2$$

Rescaling of Newton constant $G_{\mathrm{Newton}} = \frac{4}{3} G_{(4)}$

defined from Cavendish experiment

appearing in the action

but amplitude between an electromagnetic probe and a non-relativistic source is the same as in the massless case (the only difference between massive and massless case is in the trace part) ⇨ wrong light bending! (factor ¾)

An other look at the vDVZ discontinuity: Schwarzschild-type solution

$$ds^2 = -e^{\nu(\rho)}dt^2 + e^{\lambda(\rho)}d\rho^2 + e^{\mu(\rho)}\rho^2 d\Omega_2^2$$

$$\nu(r) = -\frac{r_S}{r}(1 \quad + \ldots$$

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This coefficient equals +1
in Schwarschild solution

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$$\lambda(r) = +\frac{1}{2}\;\;\frac{r_S}{r}(1-\frac{21}{8}\epsilon + \ldots$$

$$\text{with} \quad \epsilon = \frac{r_S}{m^4 r^5}$$

Vainshtein '72

Introduces a new length scale r $_v$ in the problem below which the perturbation theory diverges!

For the sun: bigger than solar system! ⇐ $\text{with} \quad r_v = (r_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein's answer (1972):

There exists an other perturbative expansion at smaller distances, reading:

$$\left.\begin{aligned}\nu(r) &= -\frac{r_S}{r}\left\{1+\mathcal{O}\left(r^{5/2}/r_v^{5/2}\right)\right\}\\ \lambda(r) &= +\frac{r_S}{r}\left\{1+\mathcal{O}\left(r^{5/2}/r_v^{5/2}\right)\right\}\end{aligned}\right\}$$

with $r_v^{-1} \propto m^{4/5}$

This goes smoothly toward Schwarschild as m goes to zero

No warranty that this solution can be matched with the other for large r! Boulware, Deser '72

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This polarization can be described by the following action:

Arkani-Hamed and Schwartz

$$\frac{1}{2}(\nabla\phi)^2 - \frac{1}{M_P}\phi T + \frac{1}{\Lambda^5}\left\{(\nabla^2\phi)^3 + \underbrace{\ldots}_{\text{Other cubic terms omitted}}\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$

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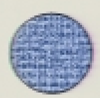
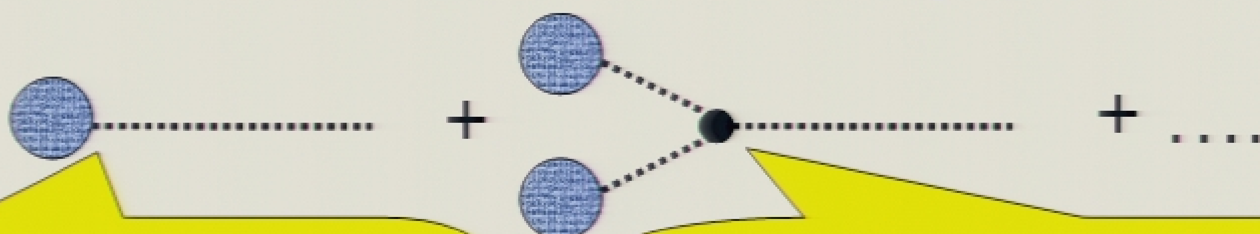
E.g. around a heavy source:  of mass M



Interaction M/M_P of the external source with ϕ

The cubic interaction above generates O(1) coorrection at $r = r_v \equiv (r_S m^{-4})^{1/5}$

Besides the vDVZ problem

At non linear level, the most simple generalization of Pauli-Fierz action propagates 6 instead of 5 degrees of freedom, the energy of the sixth d.o.f. having no lower bound!

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(See also Creminelli, Nicolis, Papucci, Trincherini)

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The action for the scalar polarization

$$\frac{1}{2}(\nabla\phi)^2 - \frac{1}{M_P}\phi T + \frac{1}{\Lambda^5}\left\{(\nabla^2\phi)^3 + \quad ... \quad \right\}$$

Leads to order 4 E.O.M. $\Rightarrow$, it describes two scalar fields, one being ghost-like

Namely, the action

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Can be rewritten as

$$\frac{1}{2}(\nabla\phi)^2 - \underbrace{\frac{1}{2}(\nabla\psi)^2}_{\text{ghost}} - \frac{1}{M_P}\phi T + \frac{1}{M_P}\psi T \pm \underbrace{\frac{2}{3\sqrt{3}}\psi^{3/2}\Lambda^{5/2}}_{\text{Encodes the cubic self-interaction}}$$

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… and one can argue that this ghost is the Boulware-Deser ghost.

What is going on in DGP model ?

- vDVZ discontinuity
- Ghosts and strong coupling

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⇒ Homogeneous cosmology of DGP model

2.1 Homogeneous cosmology of DGP model

C.D. '01

Brane position

One solves $$G^{(5)}_{AB} = \kappa^2_{(5)} \delta^\mu_A \delta^\nu_B \delta(y) \underbrace{\left(T_{\mu\nu} - \frac{1}{\kappa^2_{(4)}} G^{(4)}_{\mu\nu} \right)}_{T^{eff}_{\mu\nu}}$$

2.1 Homogeneous cosmology of DGP model

C.D. '01

Brane position

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Energy momentum tensor
of brane « real matter »

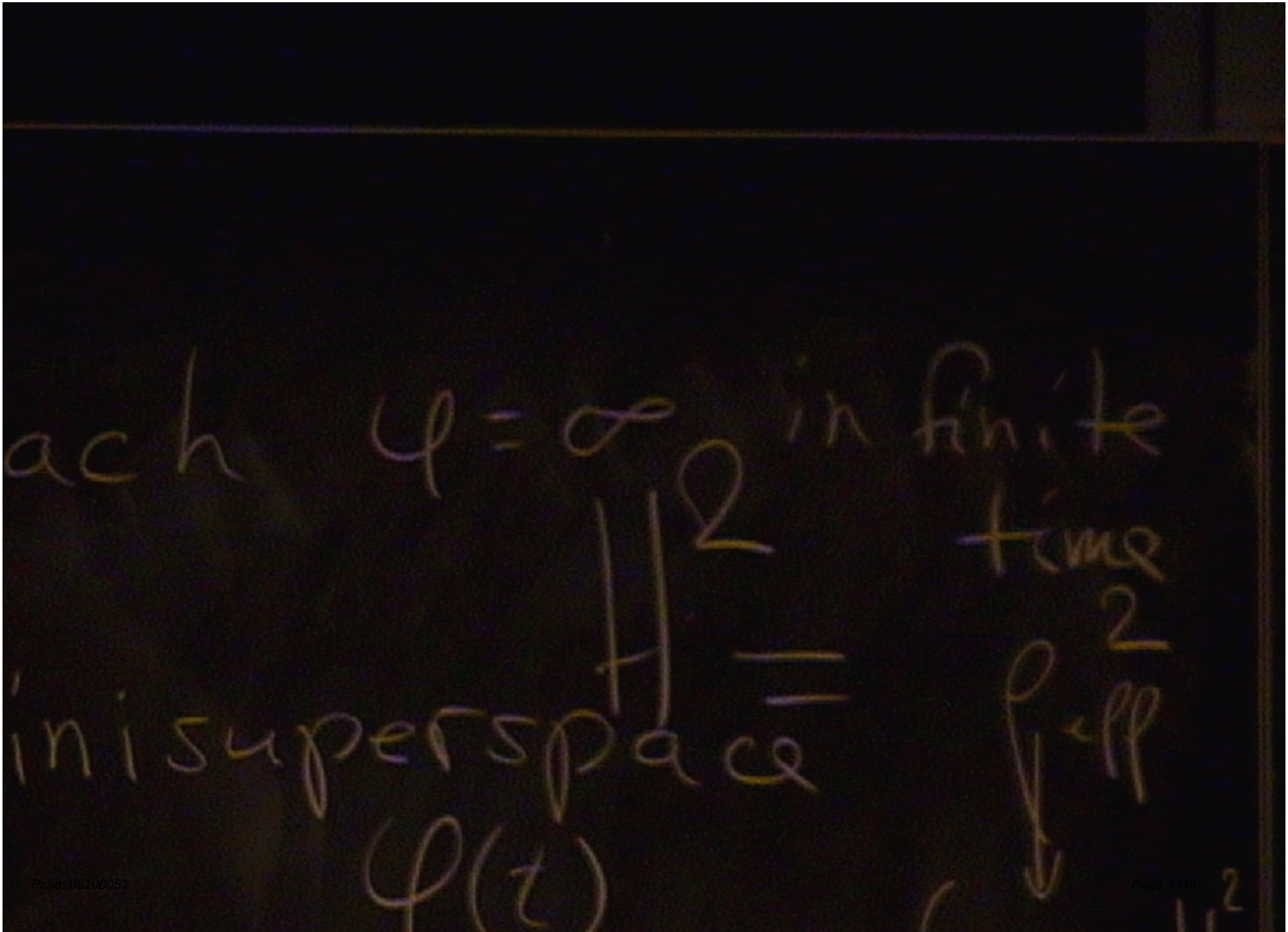
Effective energy
momentum tensor
of induced gravity

ach $\varphi = \sigma$ infinite time

$H^2 = \rho^2$

inisuperspace

$\varphi(t)$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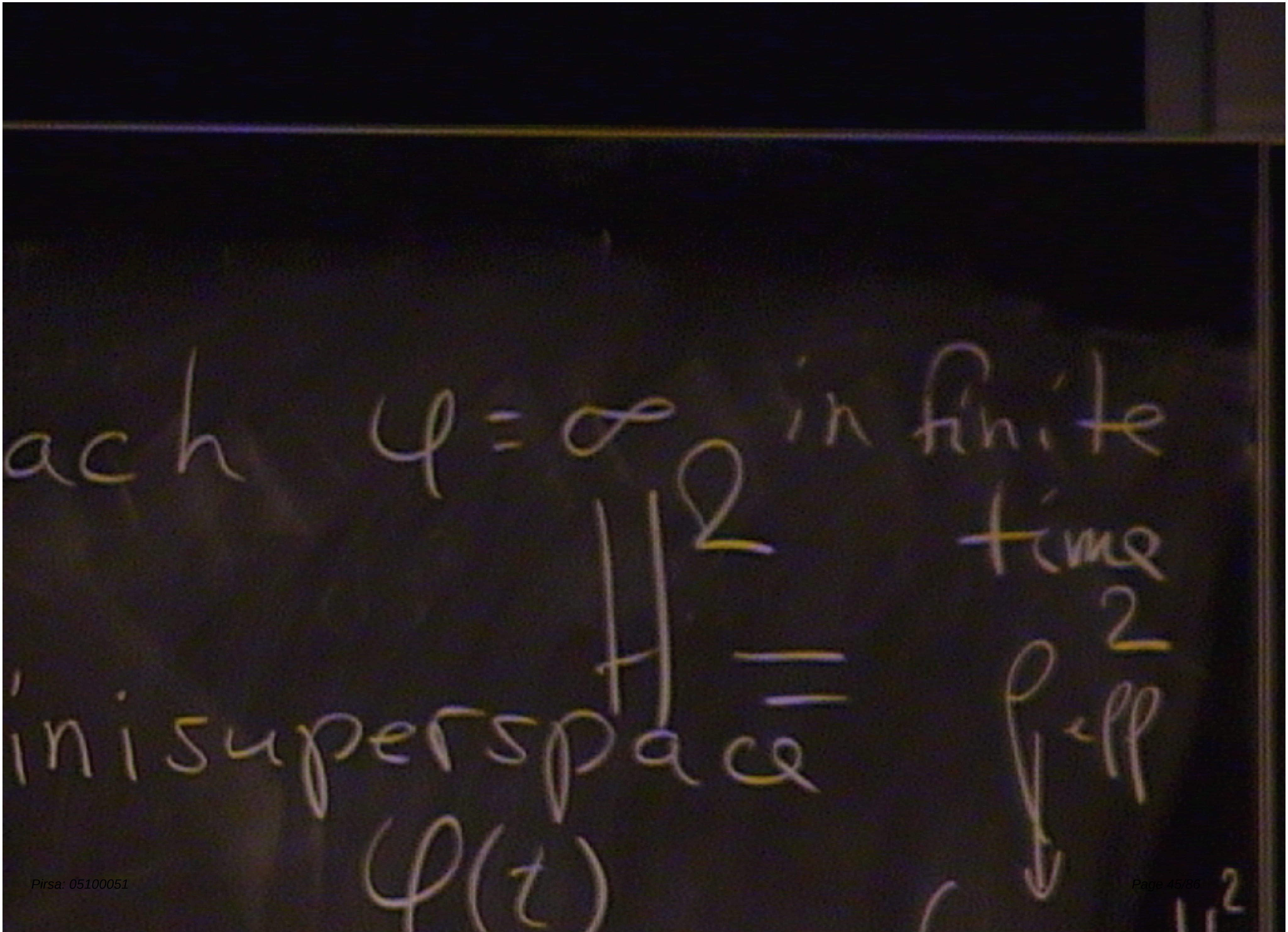
ach φ = σ infinite
H² time
H² =
inisuperspace
φ(t)

ach $\varphi = \infty$ in finite time

H^2

$H = \rho_{eff}$

inisuperspace

$\varphi(t)$

Those solutions have exactly Minkowskian bulk (vanishing bulk c.c. and Weyl tensor) and induced metric of the form FLRW:

$ds^2 = -dt^2 + a^2(t)\, dx^i\, dx^j\, \gamma_{ij}$.

Energy density of brane localized matter

This yields the Friedmann equations:

$$\begin{cases} \sqrt{H^2 + \frac{k}{a^2}} &= \frac{\epsilon}{2r_c} + \sqrt{\frac{\rho_{(M)}}{3M_P^2} + \frac{1}{4r_c^2}} \\ \text{with } \epsilon &= \pm 1 \\ \dot{\rho} &= -3H\,(P+\rho) \end{cases}$$

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Analogous to standard (4D) Friedmann equations

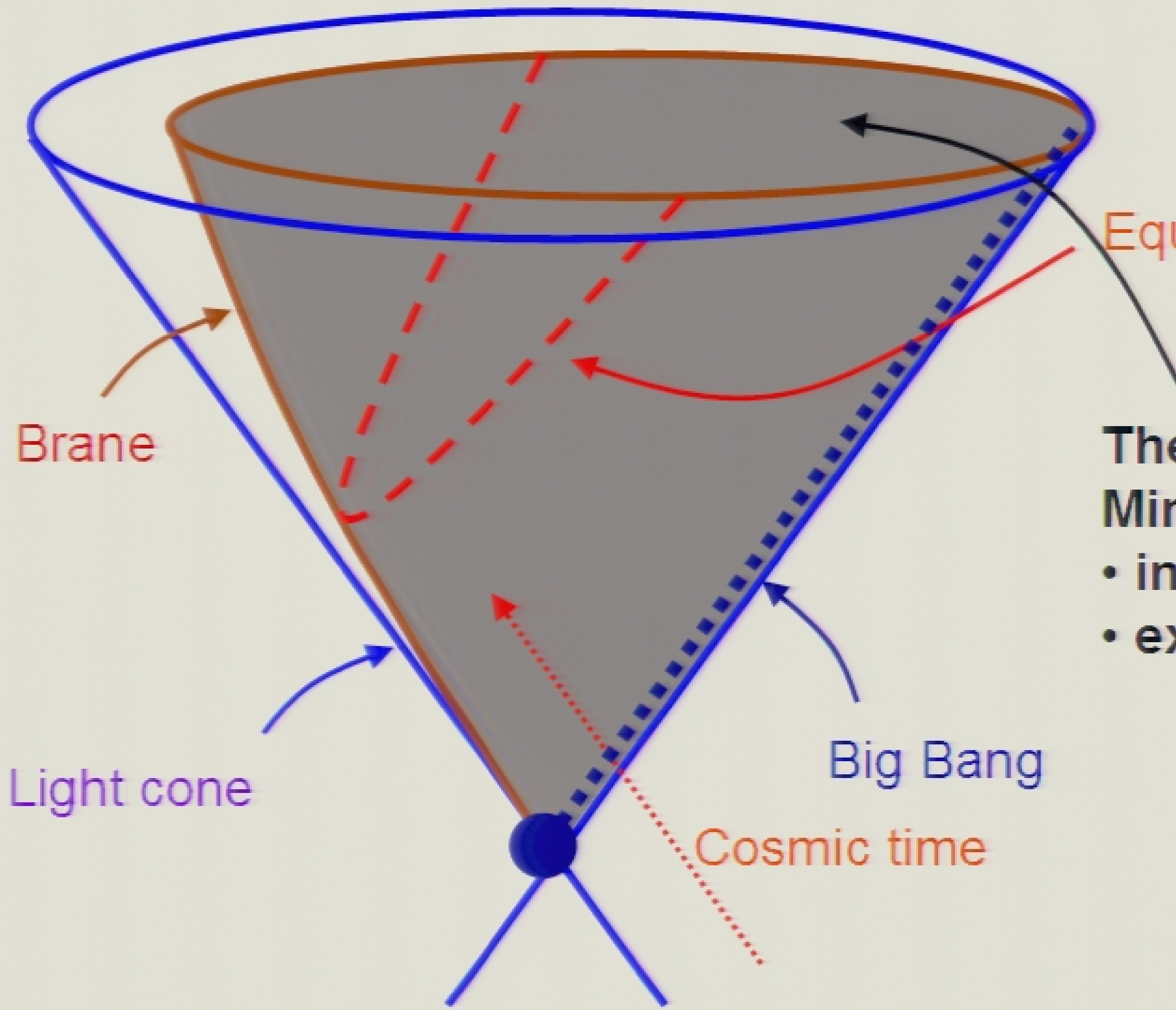
$$H^2 + \frac{k}{a^2} = \frac{\rho_{(M)}}{3M_P^2}$$

for small Hubble radii $H^{-1} \ll r_c$

Global Structure of cosmological solutions and the ε degeneracy

e.g.: brane with flat spatial section and a **Big-Bang**

Ishihara, Deruelle, Dolezel, C.D.; Lue



The bulk is a slice of 5D Minkowski space-time

- **interior slice if ε = - 1**
- **exterior slice if ε = +1**

This generalizes the well known global structure of Ipser and Sikivie's; Vilenkin's domain wall space-time

Late time cosmology

$$\sqrt{H^2 + \frac{k}{a^2_{(b)}}} = \frac{\epsilon}{2r_c} + \sqrt{\frac{\kappa^2_{(4)}\rho_{(M)}}{3} + \frac{1}{4r_c^2}}$$

Depending on the sign of ϵ

$\epsilon = -1$

$$H^2 + \frac{k}{a^2} = \frac{\rho^2_{(M)}}{36M^6_{(5)}}$$

$\epsilon = +1$

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Late time deviation from standard cosmology

Late time cosmology

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Brane cosmology in 5D Minkowski bulk with no R term on the brane (i.e.: solution to 5D Einstein-Hilbert Action)

$\epsilon = +1$

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One virtue of DGP model:
can get accelerated universe by large distance modification of gravity (C.D ('01); C.D., Dvali, Gabadaze ('02)).

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Brane cosmology in 5D Minkowski bulk with no R term on the brane (i.e.: solution to 5D Einstein-Hilbert Action)

Self accelerating solution (asympotes de Sitter space even with zero matter energy density)

Some phenomenology of homogeneous DGP cosmology

The brane (first) Friedmann equation

$$H^2 = -\frac{k}{a_{(b)}^2} + \left\{ \frac{\epsilon}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\kappa_{(4)}^2 \rho_{(M)}}{3}} \right\}^2$$

Can be rewritten as

$$H^2(z) = H_0^2 \left\{ \Omega_k^2 (1+z)^2 + \left(\underbrace{\epsilon\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c}}}_{} + \Omega_M (1+z)^3 \right)^2 \right\}$$

with $\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}$

Acts as a cosmological constant if ε = +1

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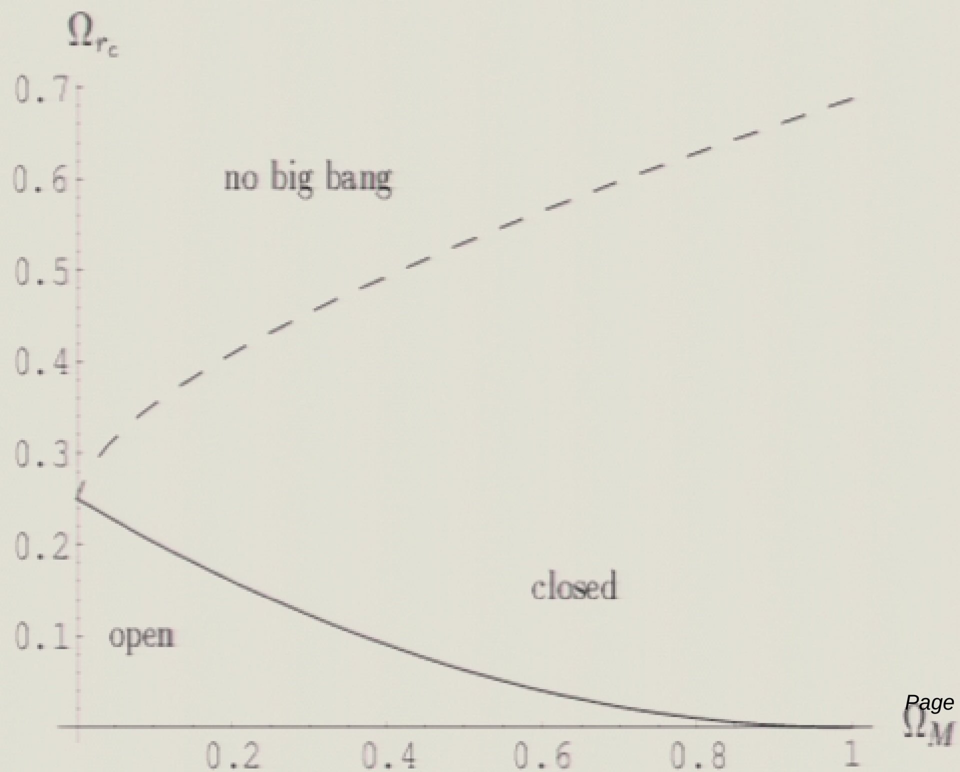
$$H^2 = -\frac{k}{a_{(b)}^2} + \left\{ \frac{\epsilon}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\kappa_{(4)}^2 \rho_{(M)}}{3}} \right\}^2$$

Can be rewritten as

$$H^2(z) = H_0^2 \left\{ \Omega_k^2 (1+z)^2 + \left(\epsilon \sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_M (1+z)^3} \right)^2 \right\}$$

with $\Omega_{r_c} = \frac{1}{4 r_c^2 H_0^2}$

Phase diagram
with ε = +1



The self-accelerating phase fits well various observables

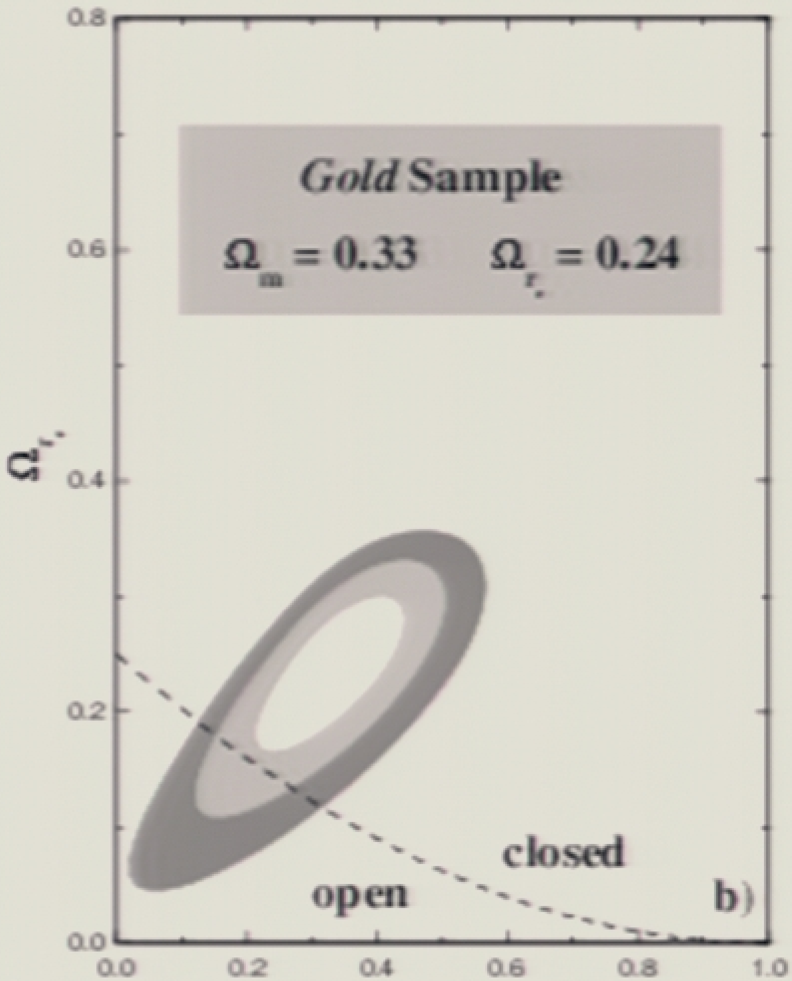


TABLE I: Recent estimates of the crossover radius r_c

Method	Reference	r_c[a]
SNe Ia + CMB	[10]	1.4
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Angular size	[11]	0.94
Gravitational lenses	[12]	1.76
High-z galaxies	[13]	≤ 2.04
SNe Ia + X-ray	[21]	1.09
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X-ray[c]:		
Arbitrary curvature	This paper	0.90
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[a] In units of H_o^{-1}.
[b] Most recent SNe Ia data
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From Alcaniz and Zhu '04

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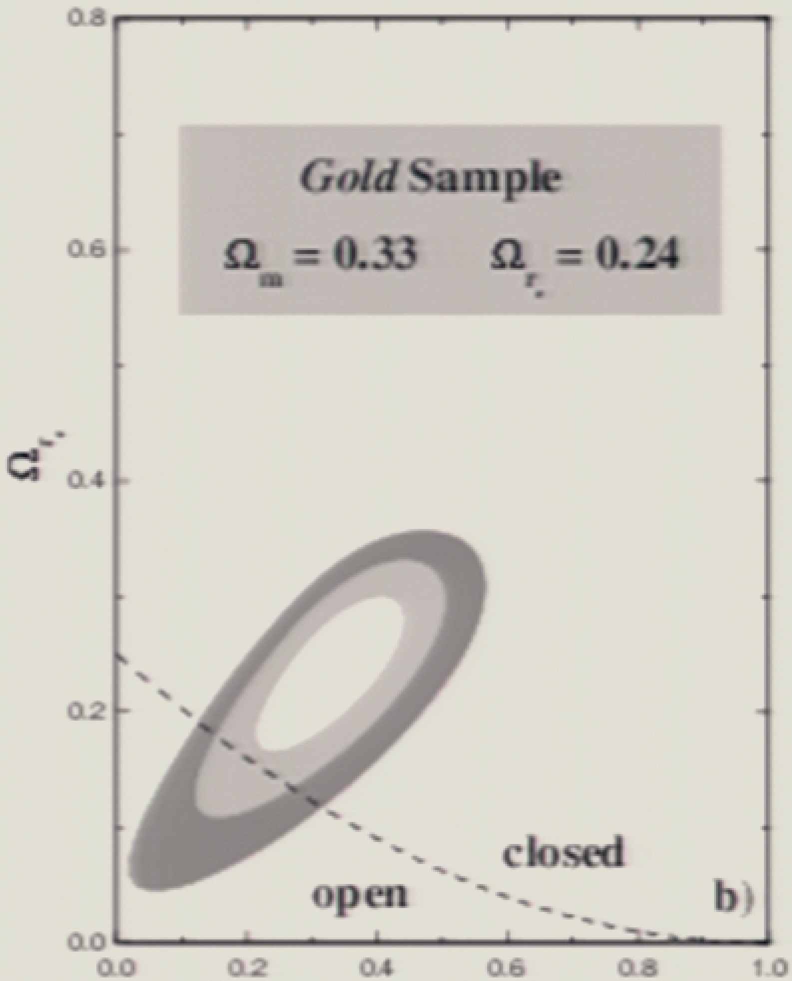

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Yields $r_c \sim H_0^{-1}$, $\Omega_M \sim 0.2$[illegible] for a flat Universe

2. 2 Back to the van Dam-Veltman-Zakharov discontinuity…

- Exact cosmological solutions provide an explicit example of interpolation between theories with different tensor structure for the graviton propagator.

C.D.,Gabadadze, Dvali, Vainshtein (2002)

$$H^2 = \frac{\rho}{3M_P^2} \quad \xleftarrow[\text{small } r_c]{\text{large } r_c} \quad H^2 = \frac{\rho^2}{36 M_{(5)}^6}$$

Solution of 4D GR with cosmic fluid

$$S_0 = M_P^2 \int d^4x \sqrt{-g} R$$

Solution of 5D GR with a brane source

$$S_5 = M_{(5)}^3 \int d^5x \sqrt{-g} R$$

↳ Comes in support of a « Vainshtein mechanism » [non perturbative recovery of the « massless » solutions] at work in DGP…… Recently an other exact solution found by Kaloper for localized relativistic source showing the same recovery…..

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Gruzinov, Porrati, Lue, Lue & Starkman, Tanaka

Potential: 4D | 4D | 5D

Tensor structure: 4D $r_V = \left(r_c^2 r_s\right)^{1/3}$ 5D r_c 5D

Vainshtein radius for DGP model

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$$3K^{(b)}/r_c + K^{(b)}_{\mu\nu} K^{\mu\nu}_{(b)} - K_{(b)} K^{(b)} = T/M_P^2$$

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Non linearities responsible for the « Vainsthein radius »

Tanaka; Nicolis Rattazzi;Damour

$$3K^{(b)}/r_c + K^{(b)}_{\mu\nu} K^{\mu\nu}_{(b)} - K_{(b)} K^{(b)} = T/M_P^2$$

With $K^{(b)}_{\mu\nu} = \partial_\mu \partial_\nu \phi$,

this yields the equations of motion for ϕ

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$$6\,\partial^2\phi \underbrace{- \frac{1}{\Lambda^3_{DGP}} \left(\partial^\mu \partial^\nu \phi\right)^2 + \frac{1}{\Lambda^3_{DGP}} \left(\partial^2 \phi\right)} = -\frac{T}{4M_P}$$

Generates the correct correction to Schwarzschild metric, with the strong coupling scale Λ_{DGP}

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However, equations are second order !

No ghost à la Boulware, Deser

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Lue & Starkman

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At distances much smaller than r_V^*, the correction to the Schwarzschild metric were found to be (Lue, Starkman)

$$\nu_{DGP}(r) = -\frac{r_s}{r}\left\{1+\epsilon\left(\frac{r}{r_V}\right)^{3/2}\right\}$$

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Perturbative study of Schwarzschild type solutions of DGP model on a de Sitter background

Potential: 4D ... 5D

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For « massive gravity »

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Other consequences: non standard growth of LSS, yields $\sigma_8 < 0.8$ (at two sigma level) Lue, Scoccimarro, Starkman

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Need for a better understanding of those issues

2. 3 The dark side of DGP gravity…

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- $r_c = \frac{M_P^2}{2M_{(5)}^3}$ and $r_c \sim H_0^{-1}$

$\Rightarrow M_{(5)} \sim 10 - 100 \quad MeV$

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Dvali, Gabadadze, Kolanovic, Nitti; Kiritsis, Tetradis, Tomaras; Antoniadis, Minasian, Vanhove; Kohlprath; Kohlprath, Vanhove

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⇨ But it has a dark side if not dark energy !

- Interesting playground to investigate « massive gravity » (a candidate for a consistent theory of « massive gravity ») as well as the phenomenology of large distance modification of gravity… … Cosmological solutions and other arguments in agreement with a Vainshtein recovery of solutions to ordinary G.R. but issues related to the way it does work in detail.