Title: Singularities in String Theory

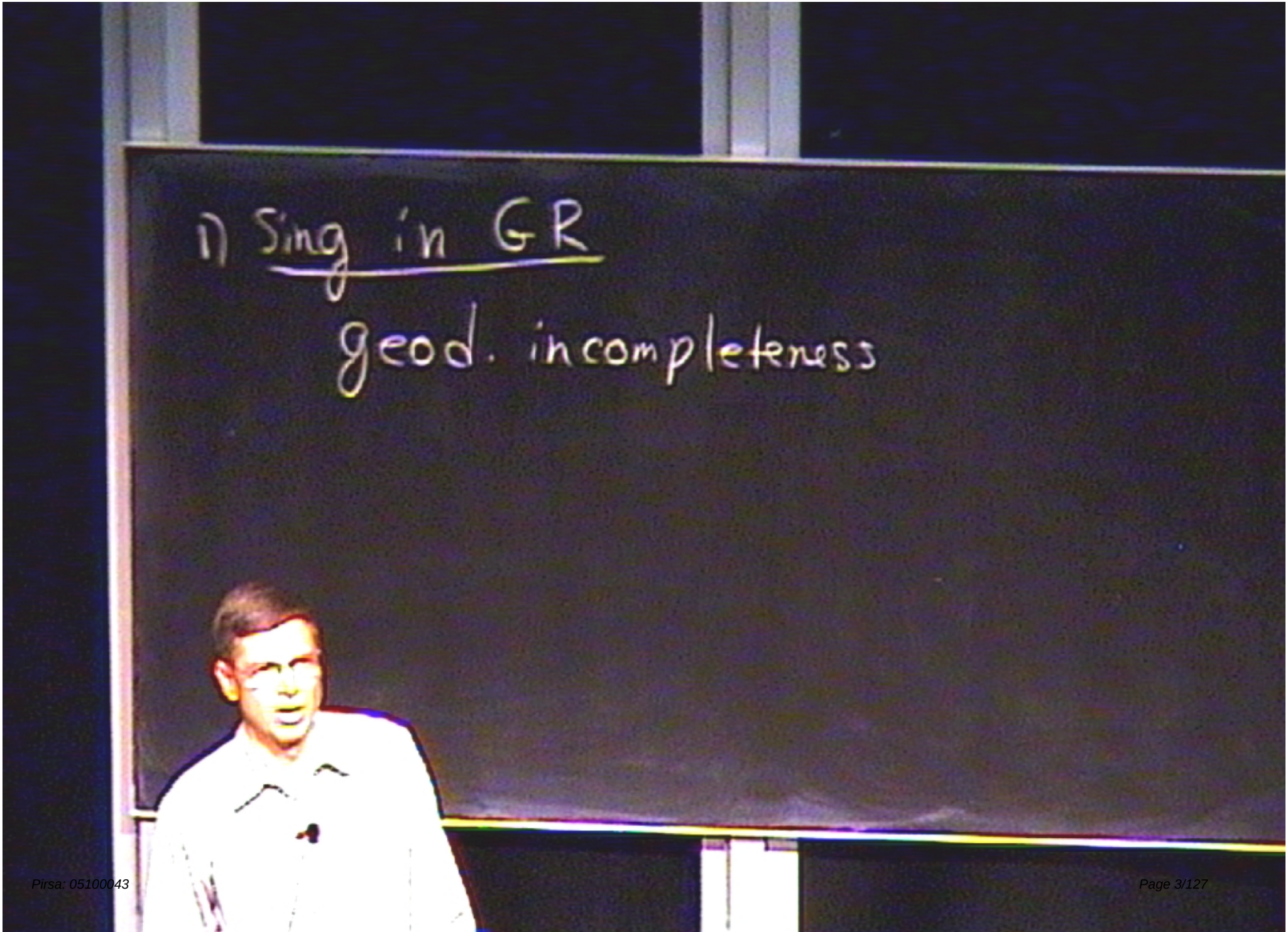
Date: Oct 25, 2005 09:30 AM

URL: http://pirsa.org/05100043

Abstract:

Outline

1) Singularities in GR
2) Sing in Loop QG
3) Intro to String Theory
4) Sing. resolved by string theory
5) Sing not " " " "
6) Cosmology
7) BH sing.


1) Sing in GR
geod. incompleteness

1) Sing in GR

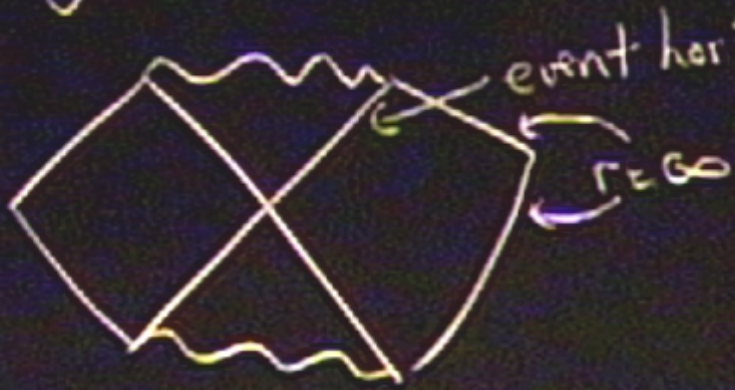
geod. incompleteness



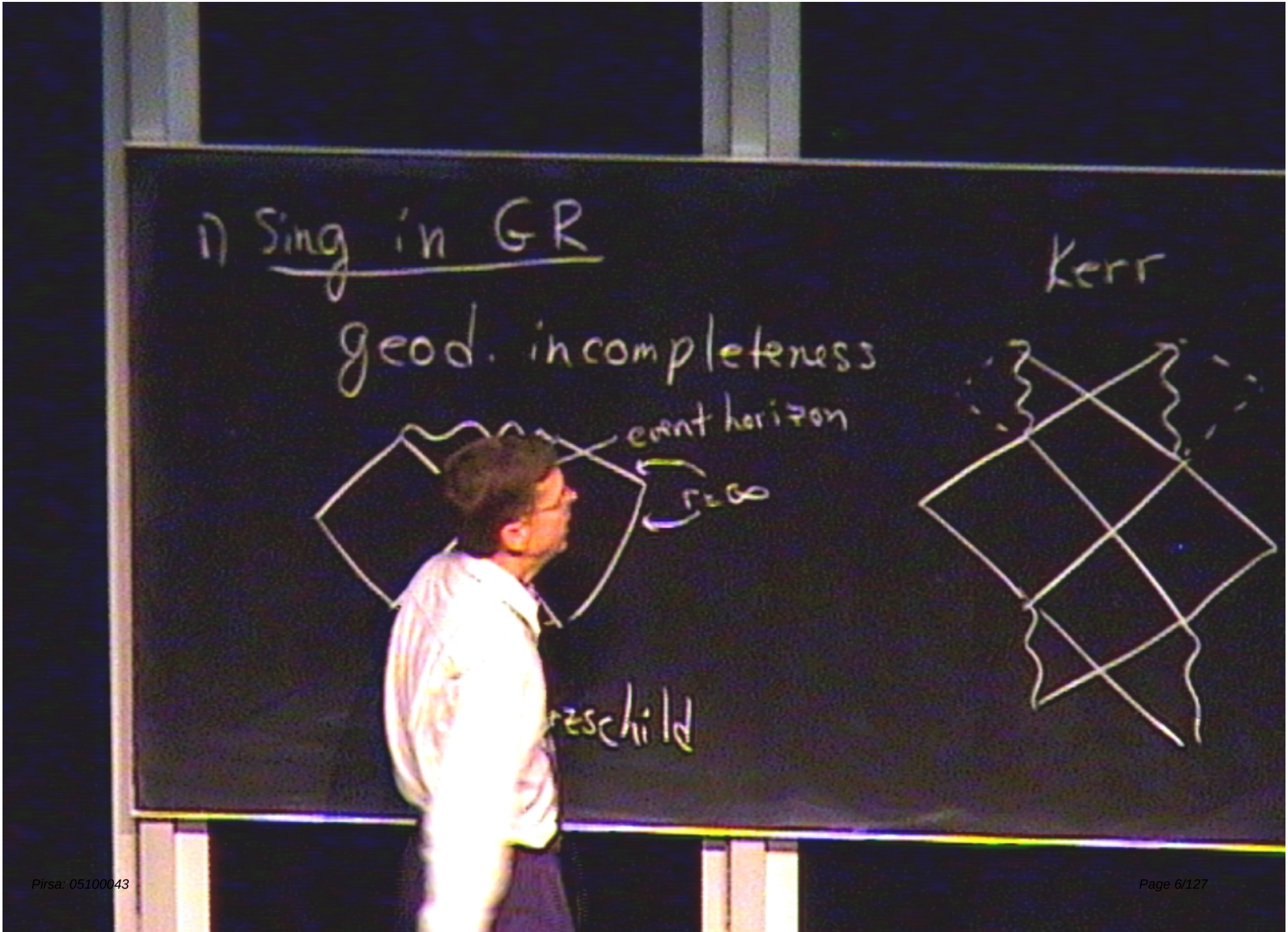
Schwarzschild

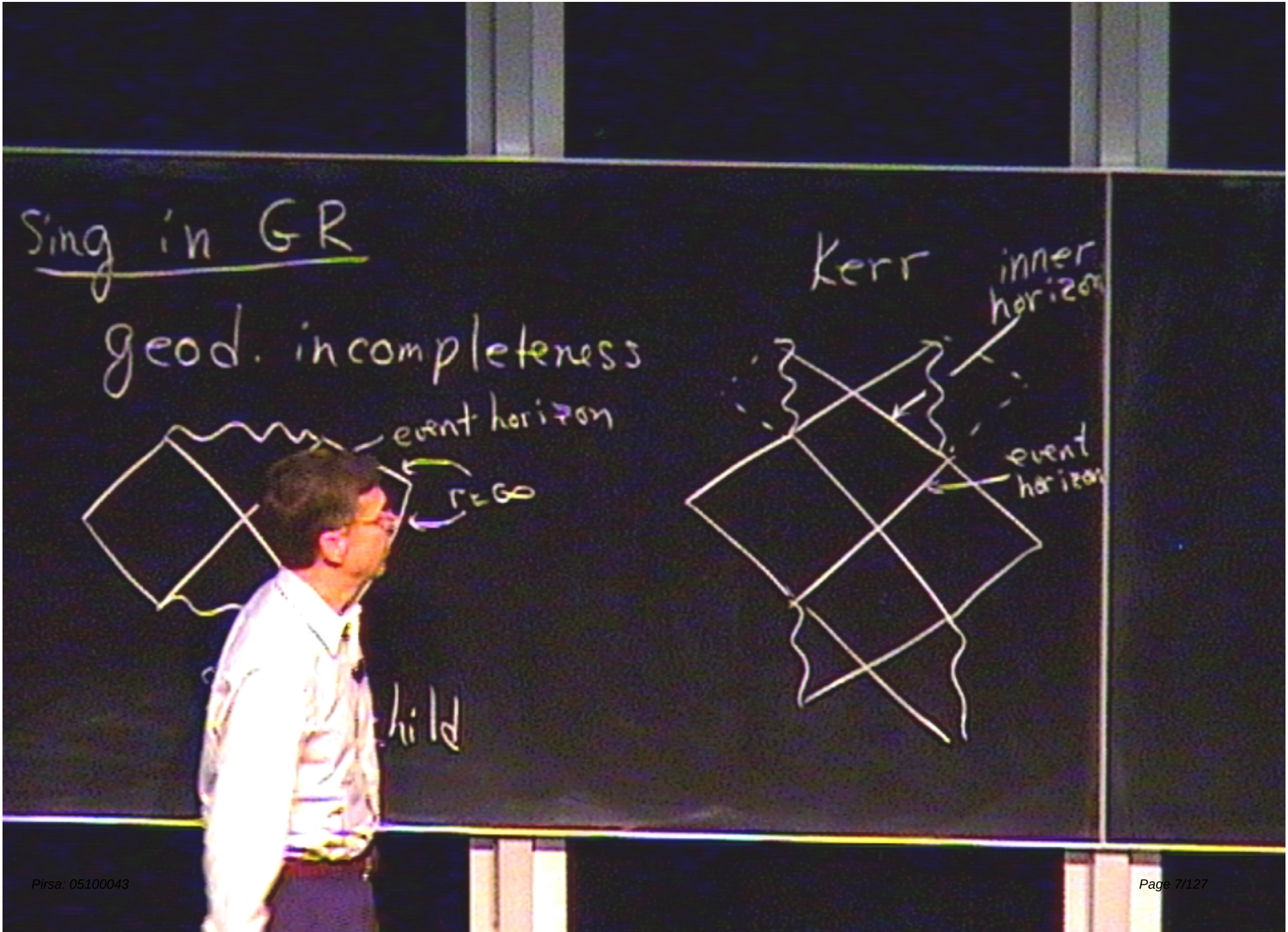
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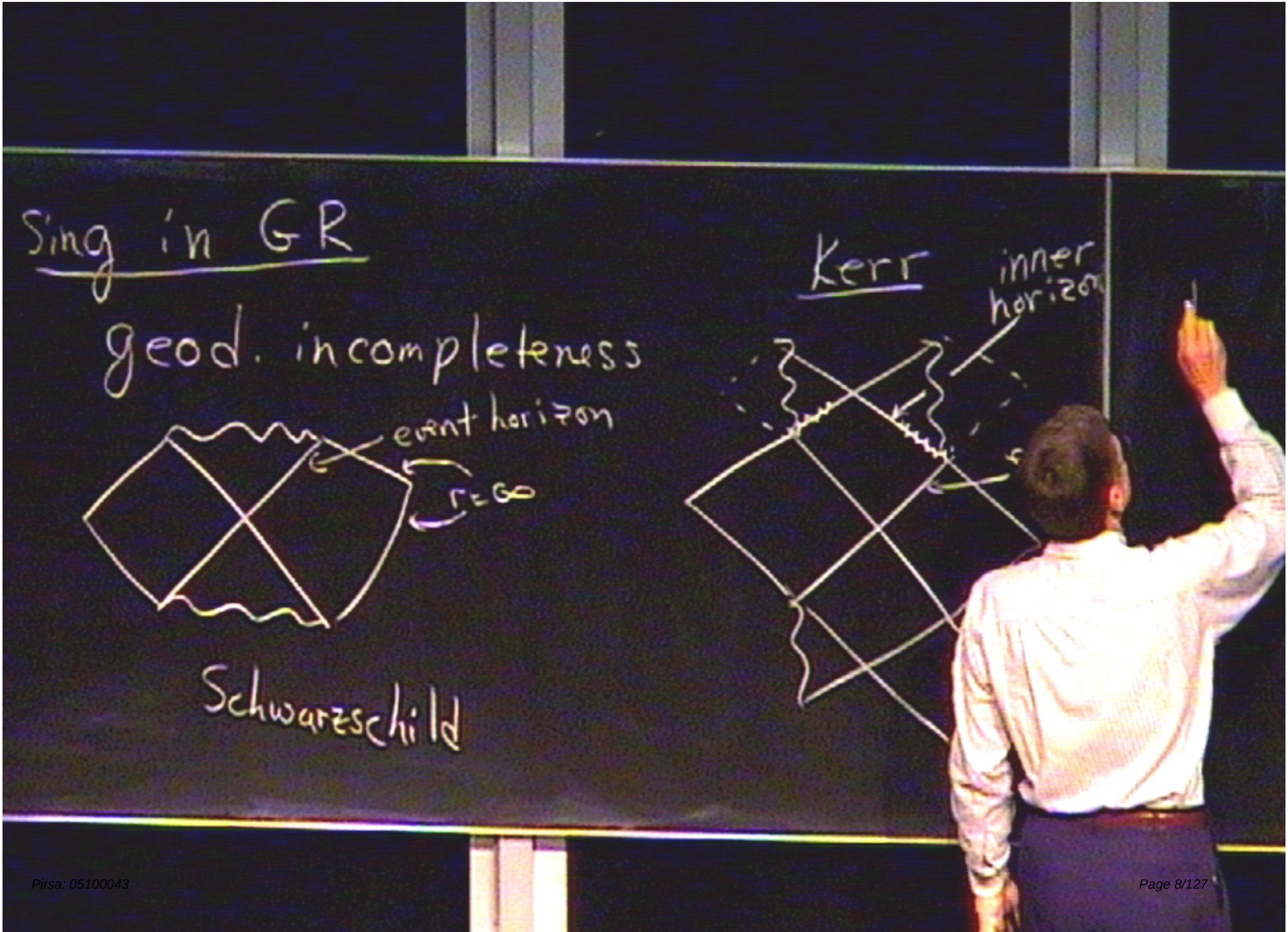
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Schwarzschild

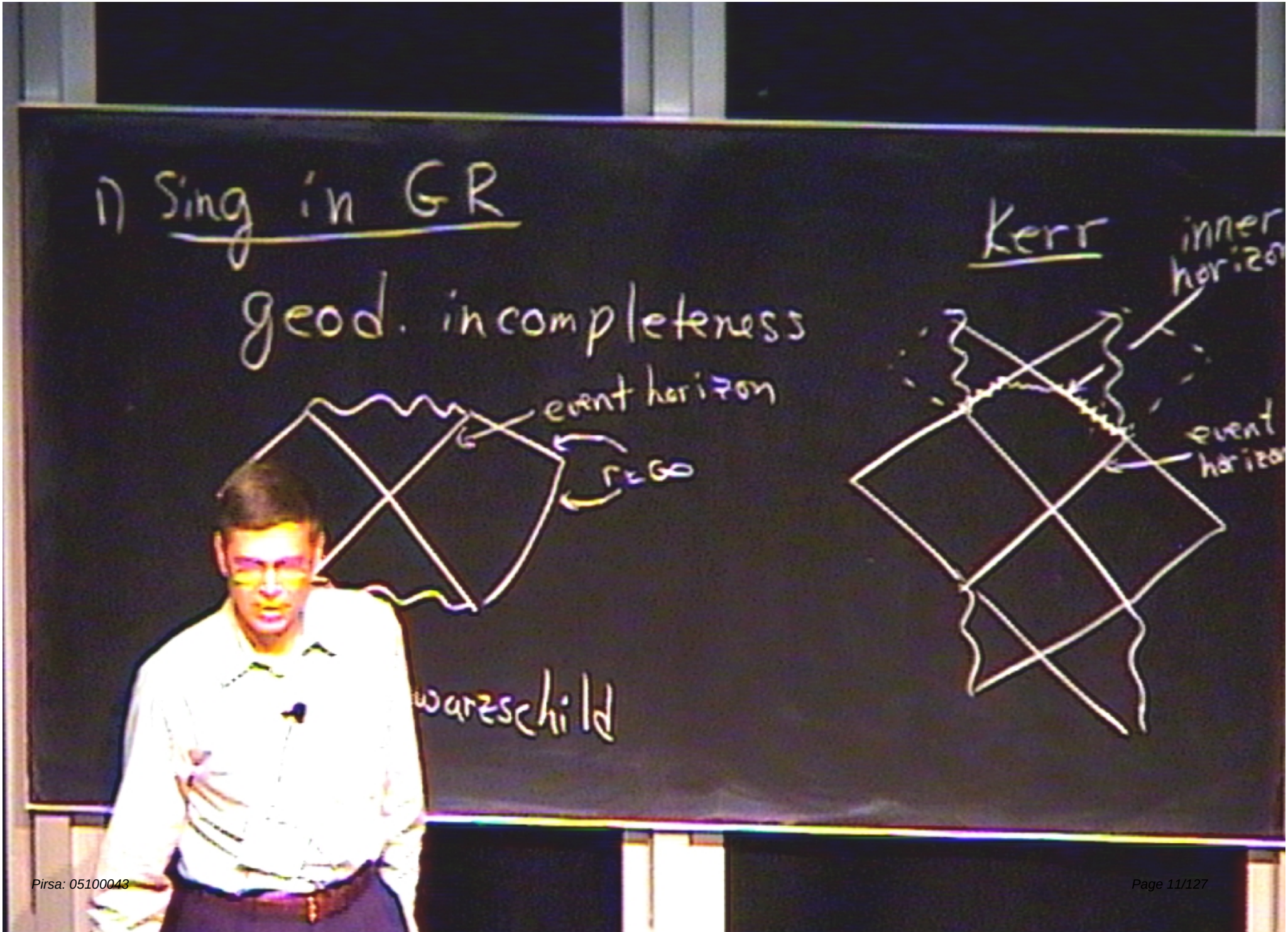

1) Sing in GR
Kerr
geod. incompleteness
event horizon
r=2G
reschild

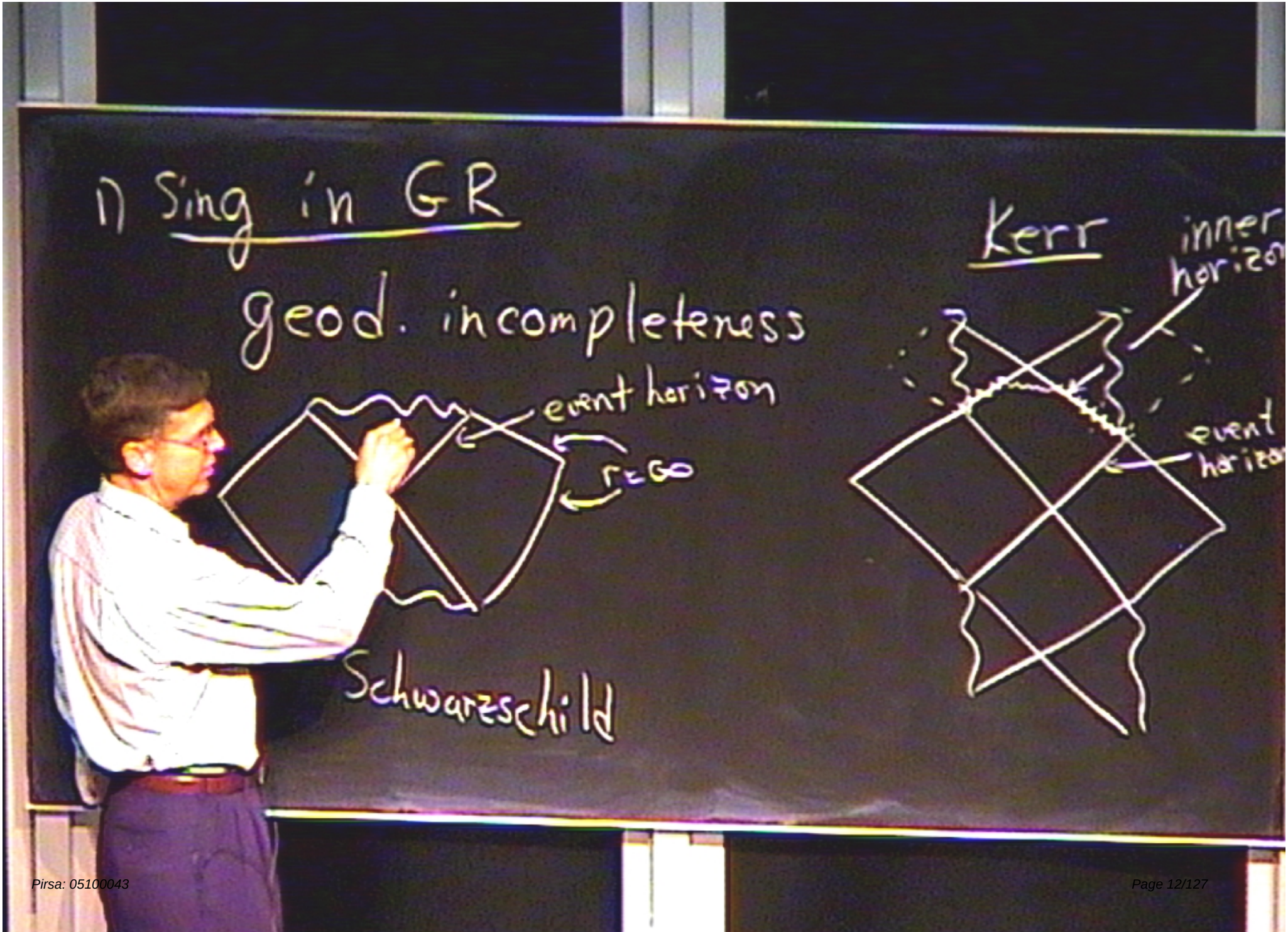

Sing in GR
geod. incompleteness
event horizon
r=0
Kerr
inner horizon
event horizon

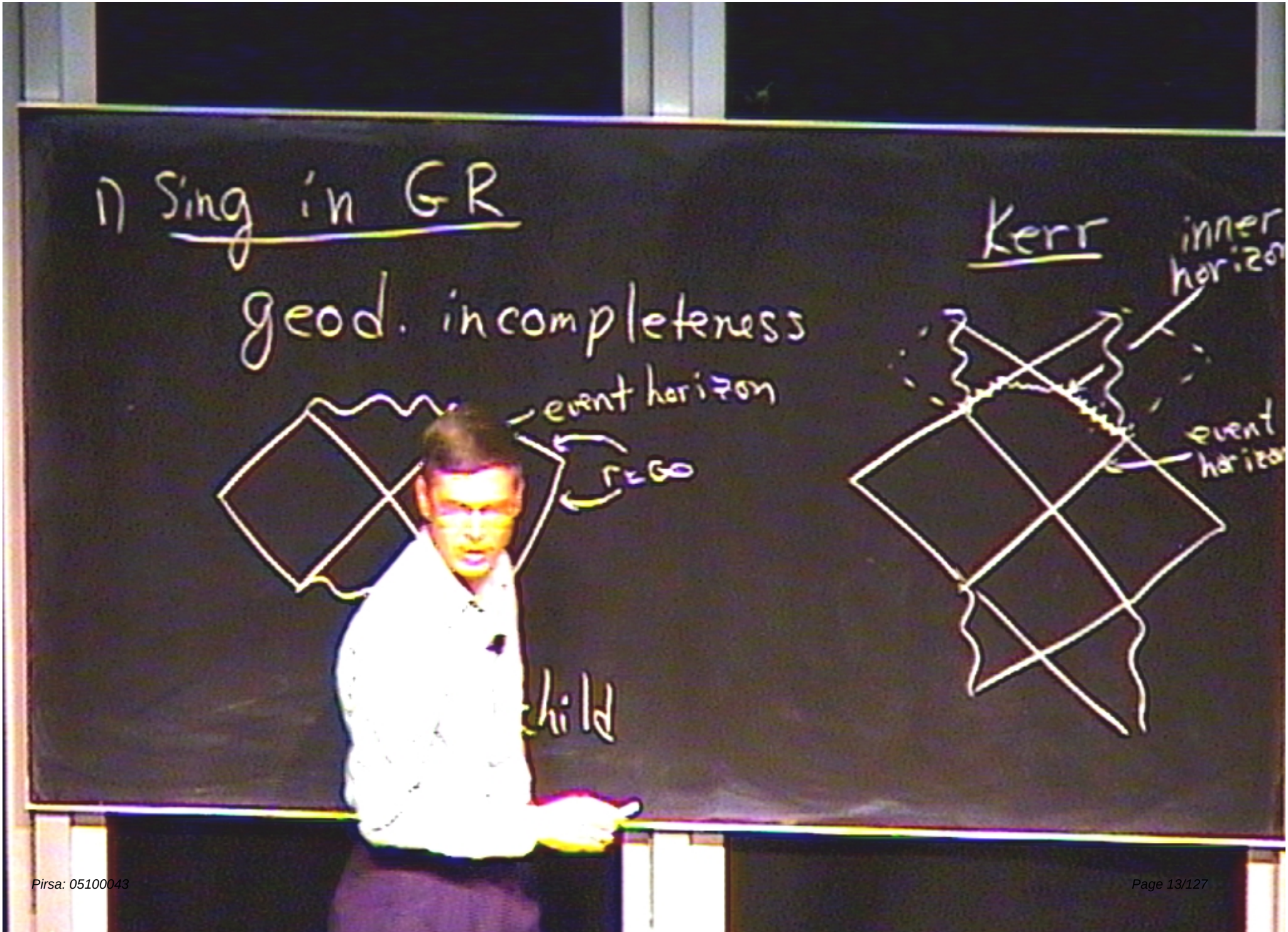

Sing in GR
geod. incompleteness
event horizon
r=0
Schwarzschild
Kerr
inner
horizon

inner horizon → null sing.

inner horizon → null sing. a weak
total tidal force finite


1) Sing in GR
geod. incompleteness
event horizon
r=0
warzschild
Kerr
inner
hori
event
horiz

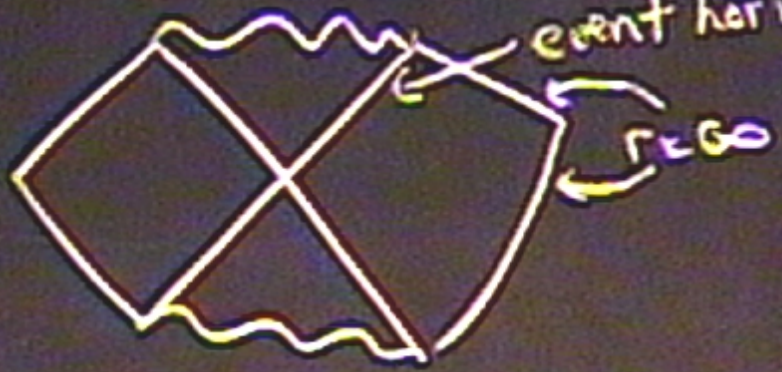

n Sing in GR
geod. incompleteness
event horizon
r=GO
Schwarzschild
Kerr
inner
hori:zon
event
horizon


Sing in GR
geod. incompleteness
event horizon
r=GO
child
Kerr
inner
hori
event
horiz

inner horizon → null sing. & weak
total tidal force finite
Schw. sing is unstable to J≠0 perturb.

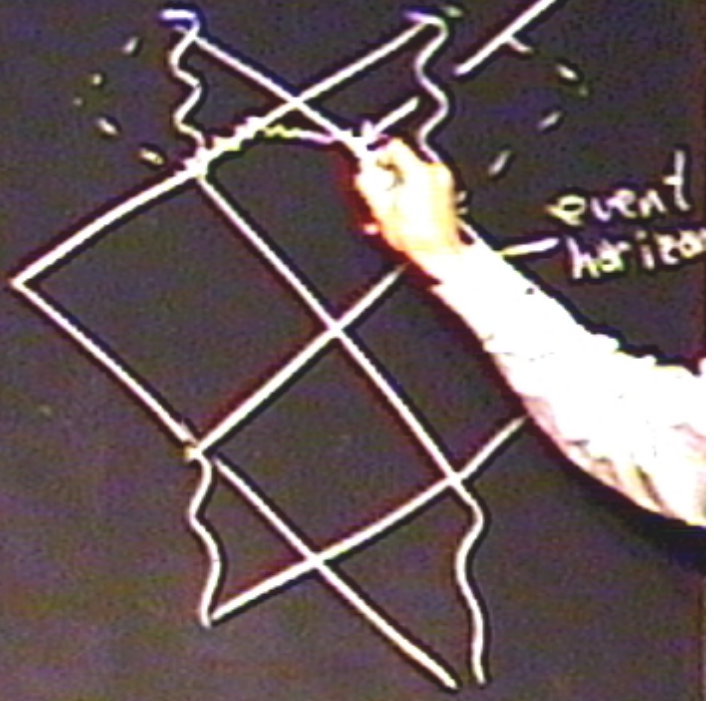
Sing in GR

geod. incompleteness



Schwarzschild

Kerr



inner horizon → null sing. a weak
total tidal force finite

Schw. sing is unstable to $J \neq 0$ perturb.

Cosmological sing.

inner horizon → null sing. & weak
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Cosmological sing. BKL

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different spatial pts decouple

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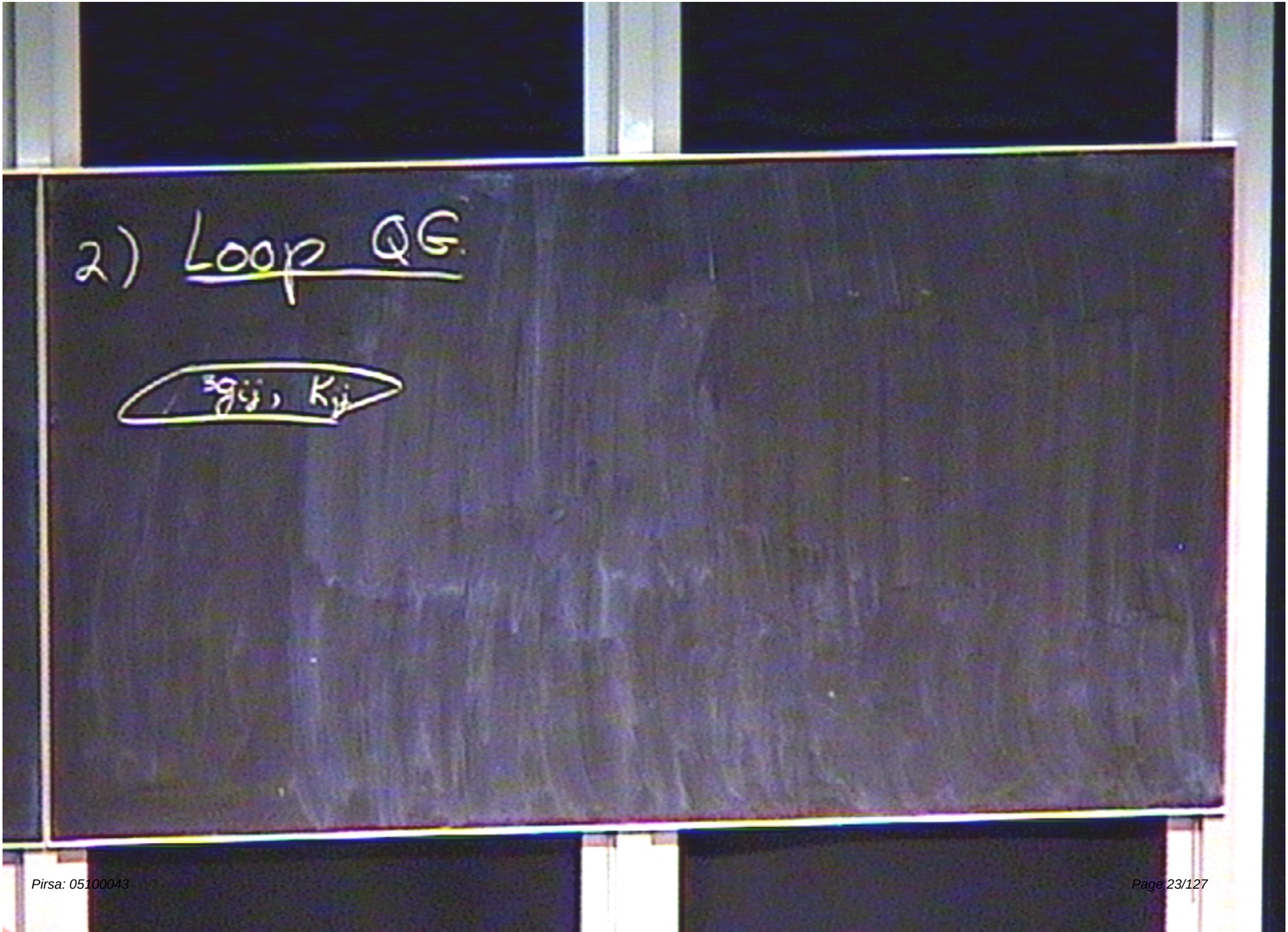
inner horizon → null sing. a weak
total tidal force finite
Schw. sing is unstable to J≠0 perturb.

Cosmological sing. BKL



different spatial pts decouple
anisotropic (Kasner) expansion many times

2) Loop QG


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g_{ij}, K_{ij}

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$\Psi[{}^3g_{ij}]$

${}^3g_{ij}$, K_{ij}

2) Loop QG

$^3g_{ij}$, K_{ij} $\Rightarrow$ $H\Psi[^3g_{ij}]$

2) Loop QG

$H\Psi[{}^3g_{ij}] = 0$

${}^3g_{ij}$, K_{ij}

2) Loop QG

$^3g_{ij}$, K_{ij} $\quad H\Psi[{}^3g_{ij}] = 0$

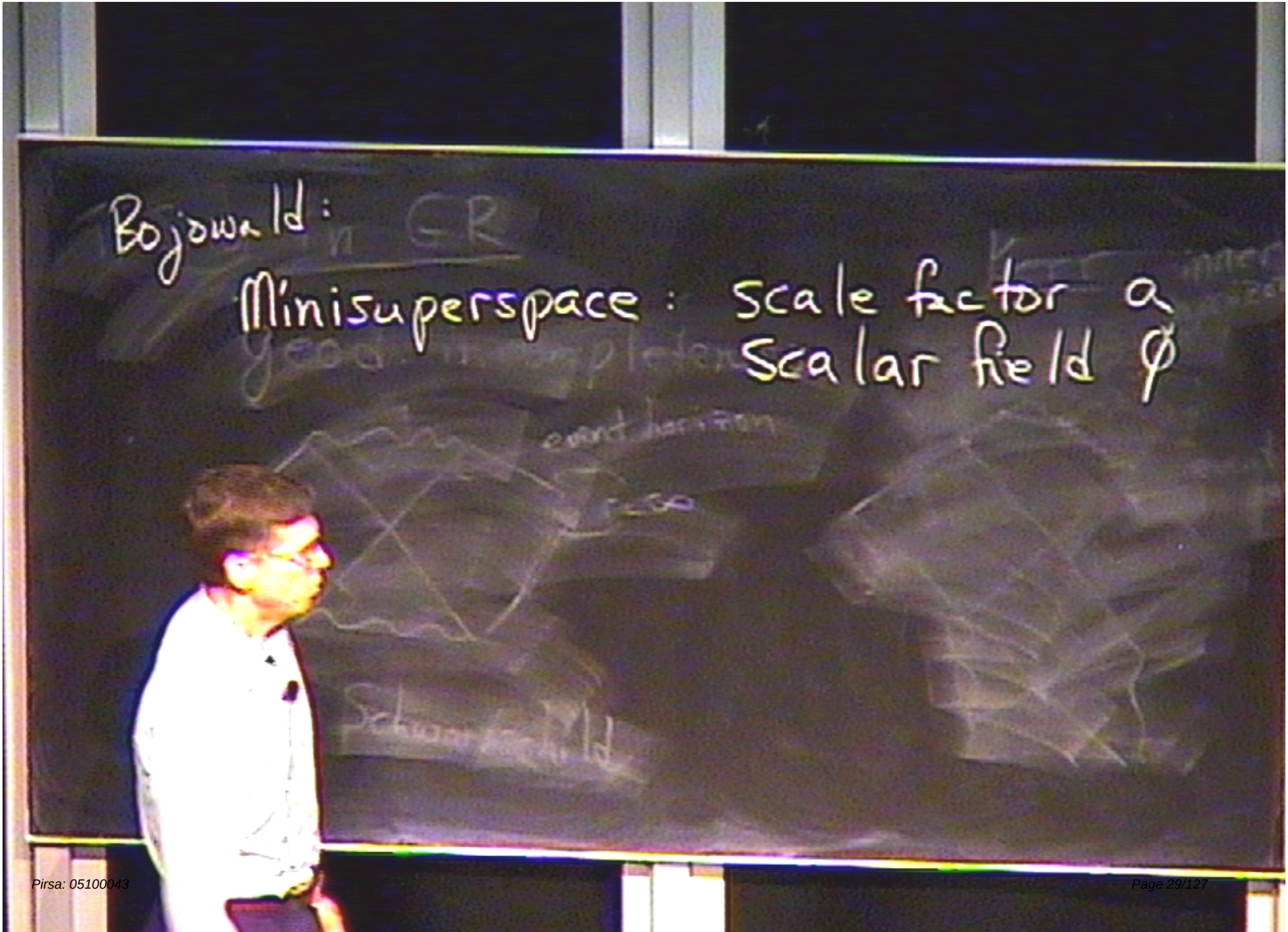
Ashtekar: A_i

2) Loop QG

$^3g_{ij}$, K_{ij} $\quad H\Psi[{}^3g_{ij}] = 0$

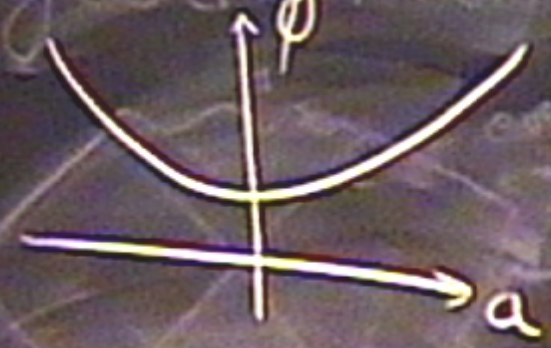
Ashtekar: (A_i^a, E^i_a) $\quad H\Psi[A_i^a] = 0$

A_i^a: connection; E^i_a: triad


Bojowald:
Minisuperspace: Scale factor a
Scalar field φ

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Minisuperspace: Scale factor a
Scalar field ϕ



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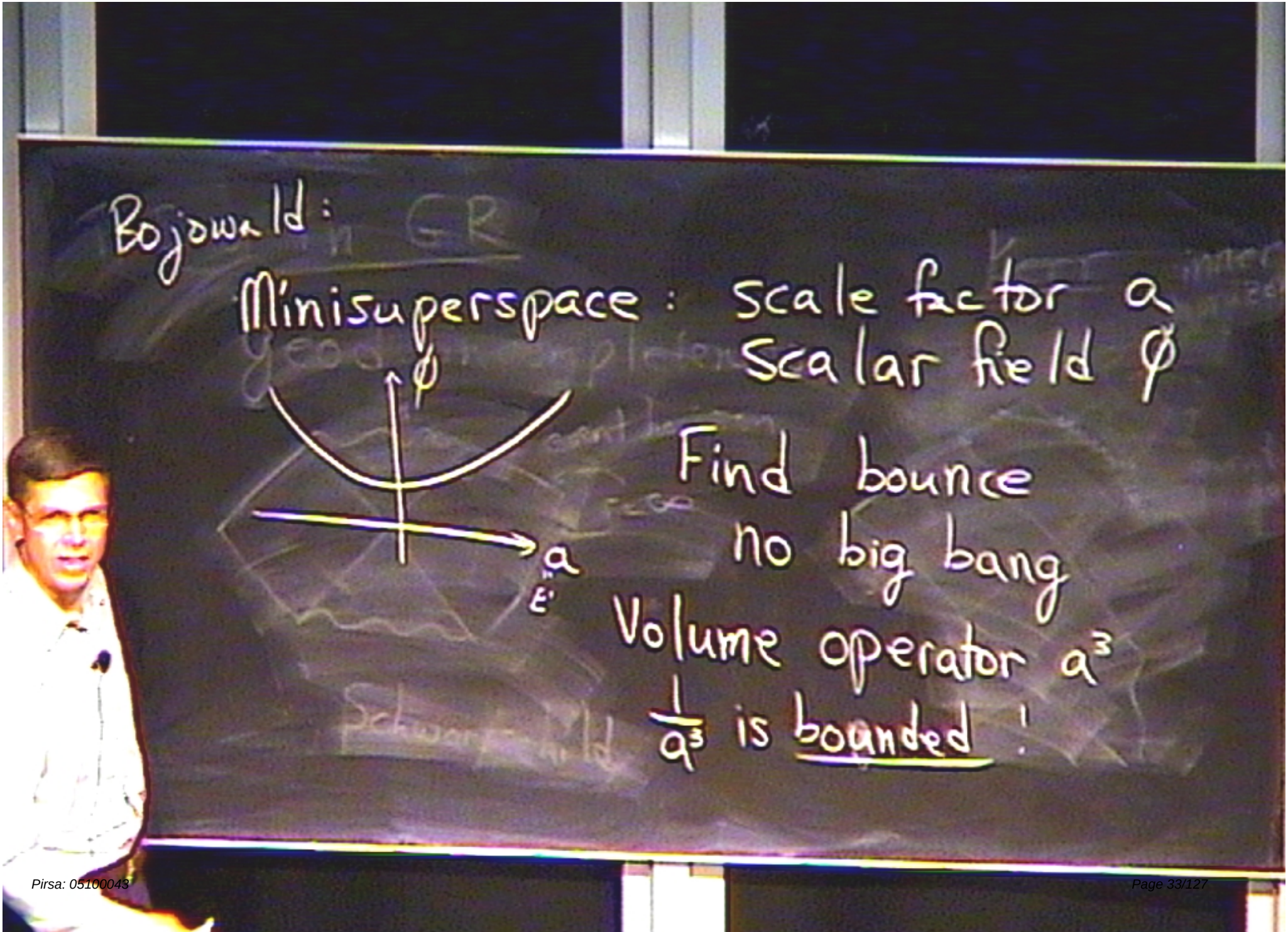
Find bounce
no big bang

Bojowald:

Minisuperspace: Scale factor a
Scalar field ϕ



Find bounce
no big bang
Volume operator a^3
$\frac{1}{a^3}$ is bounded !


Bojowald:
Minisuperspace: Scale factor a
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φ
a
Find bounce
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3) String theory

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In 4D

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In 4D, BH

1) horizon is spherical
2) characterized by $M, \vec{J}$
3)

3) String theory

In 4D BH:

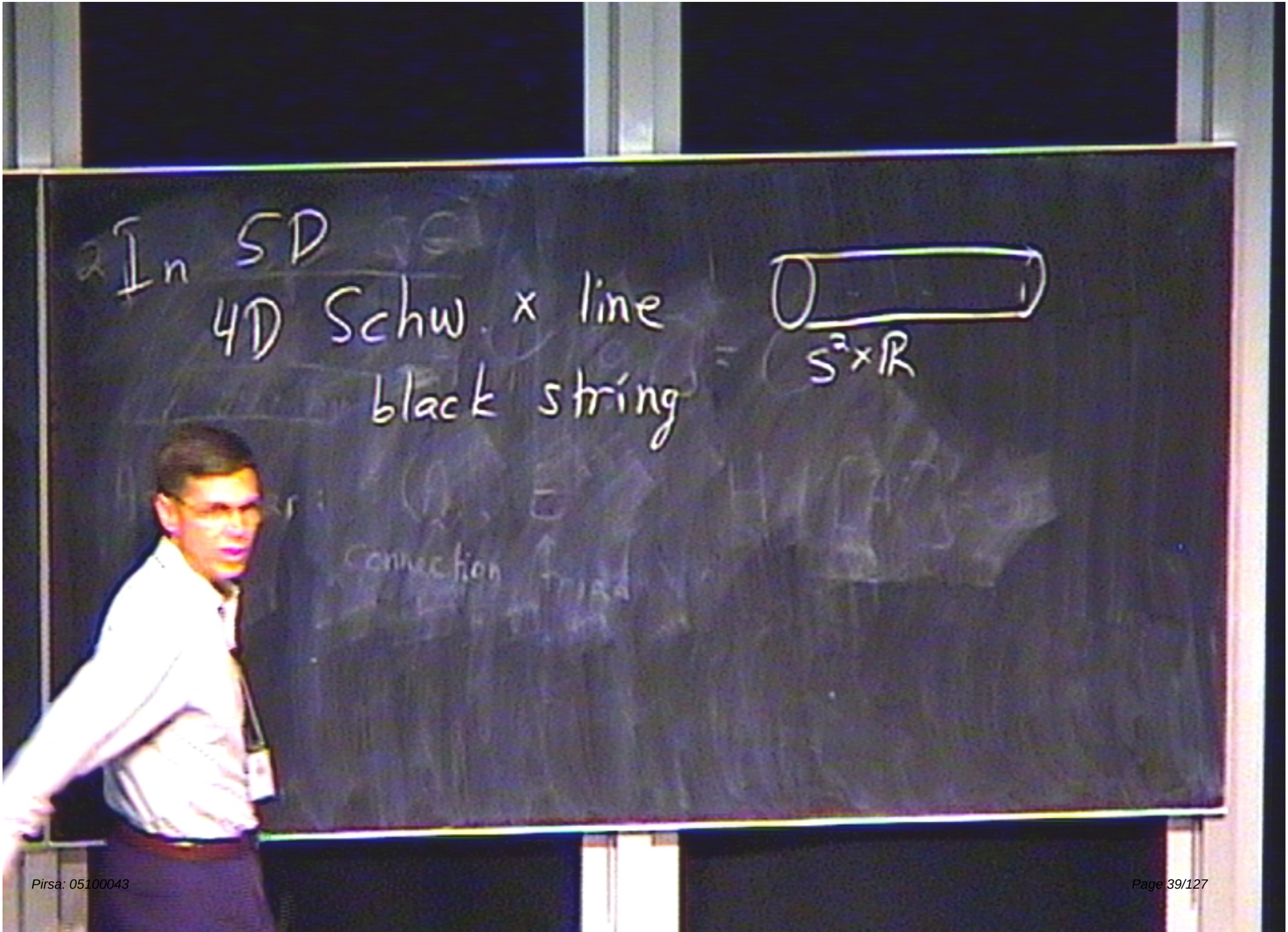
1) horizon is spherical
2) characterized by $M, \vec{J}$
3) stable

3) String theory

In 4D BH:

1) horizon is spherical
2) characterized by $M, \vec{J}$
3) stable

All fail in $D>4$: (Lastest)


In 5D
4D Schw. x line
black string
$S^2 \times R$

In 5D

4D Schw. x line



black string ← unstable

In 5D

4D Schw. x line



$S^2 \times \mathbb{R}$

black string ← unstable

black ring



$S^2 \times S^1$

In 5D

4D Schw. x line



$S^2 \times \mathbb{R}$

black string ← unstable

black ring



$S^2 \times S^1$

For some M, J

In 5D

4D Schw. x line



$S^2 \times \mathbb{R}$

black string ← unstable

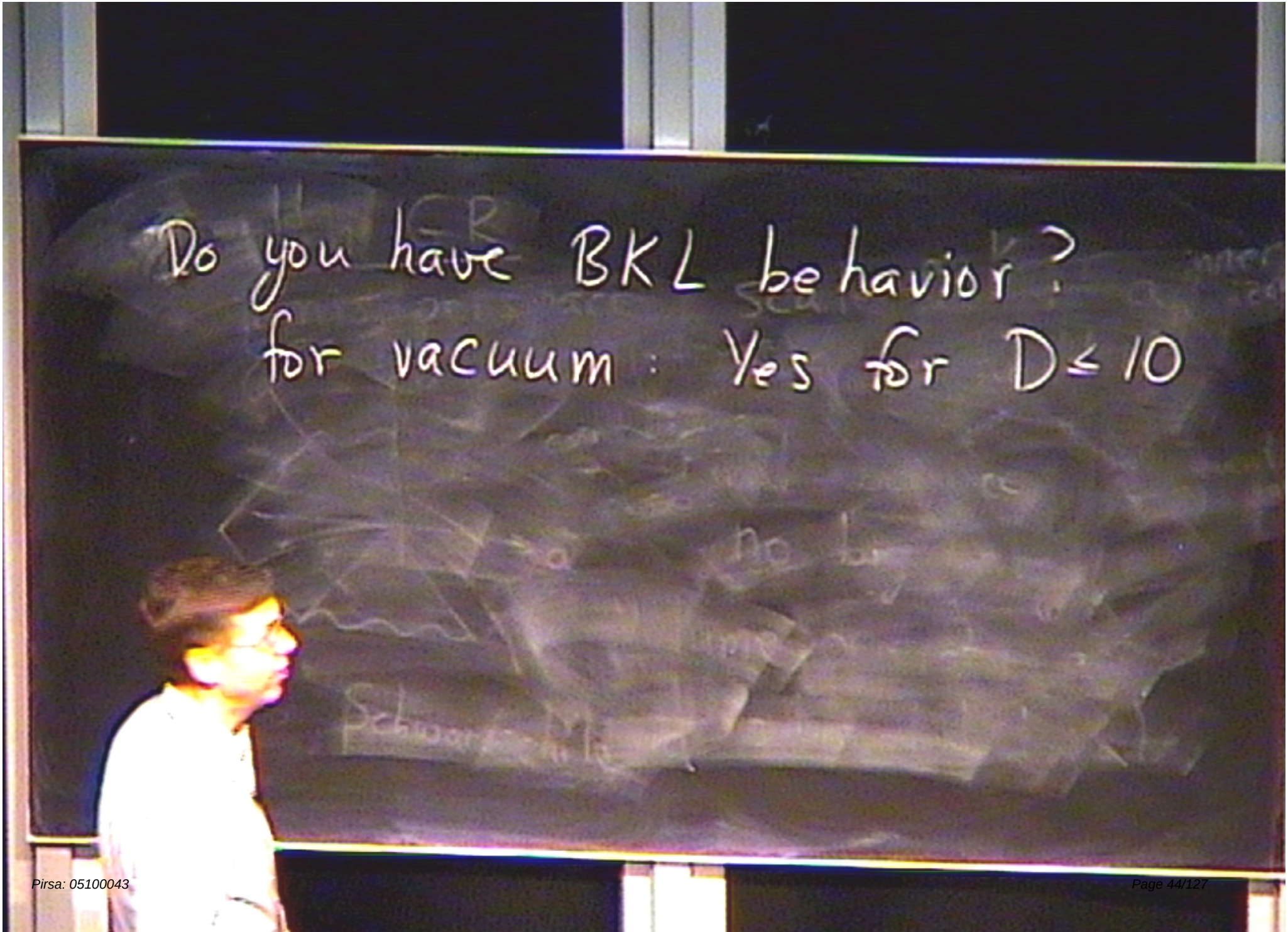
black ring

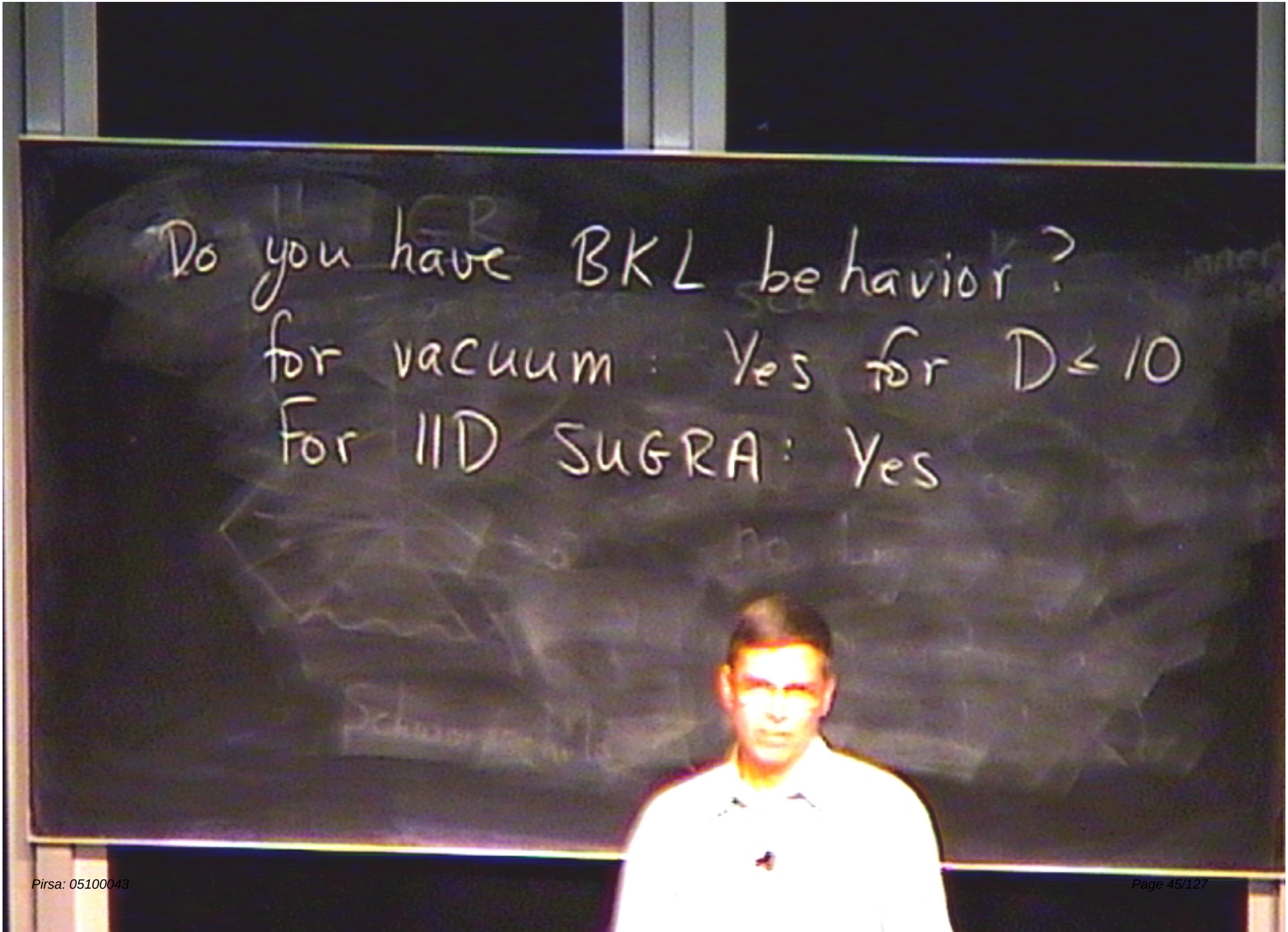


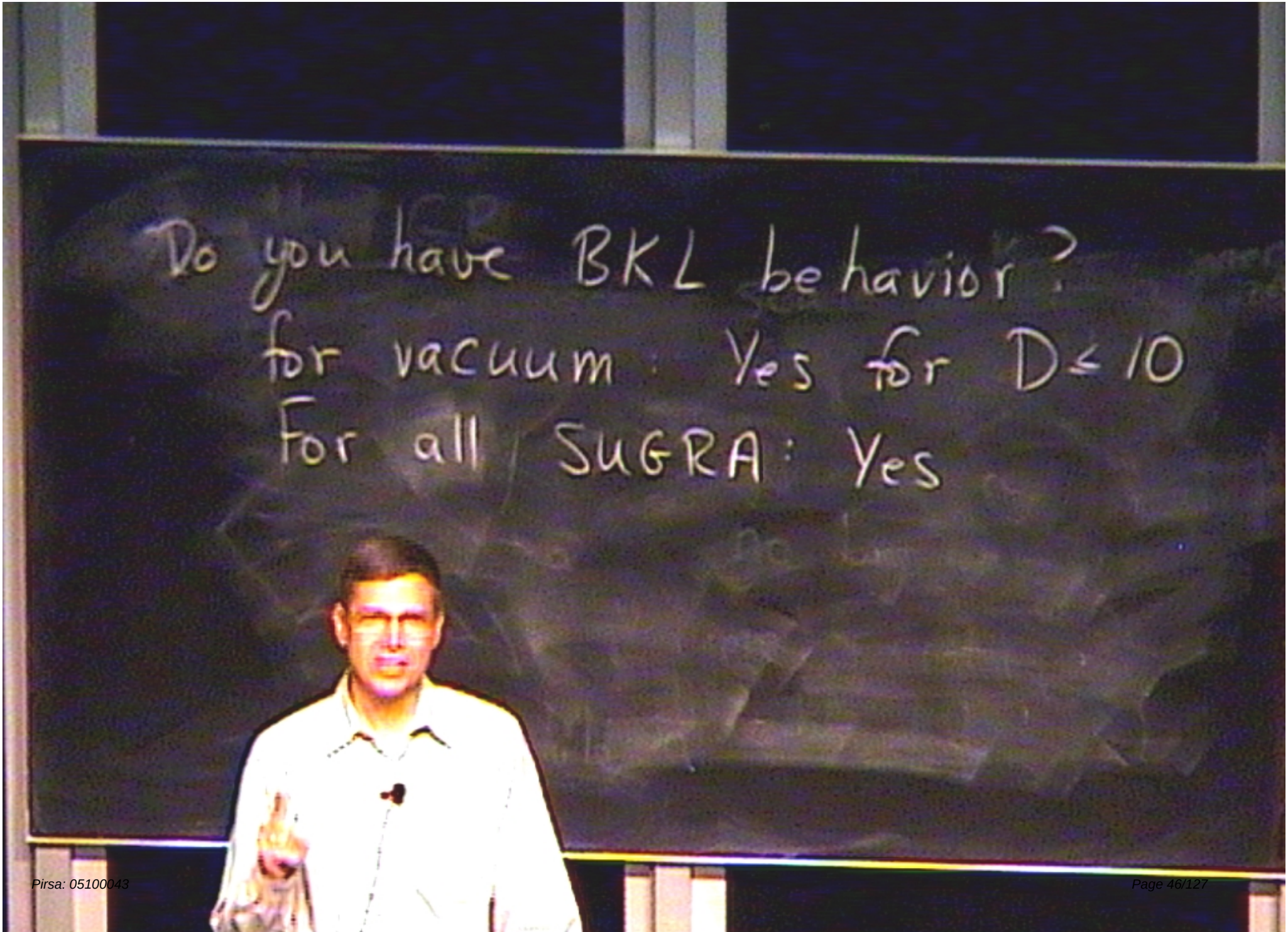
$S^2 \times S^1$

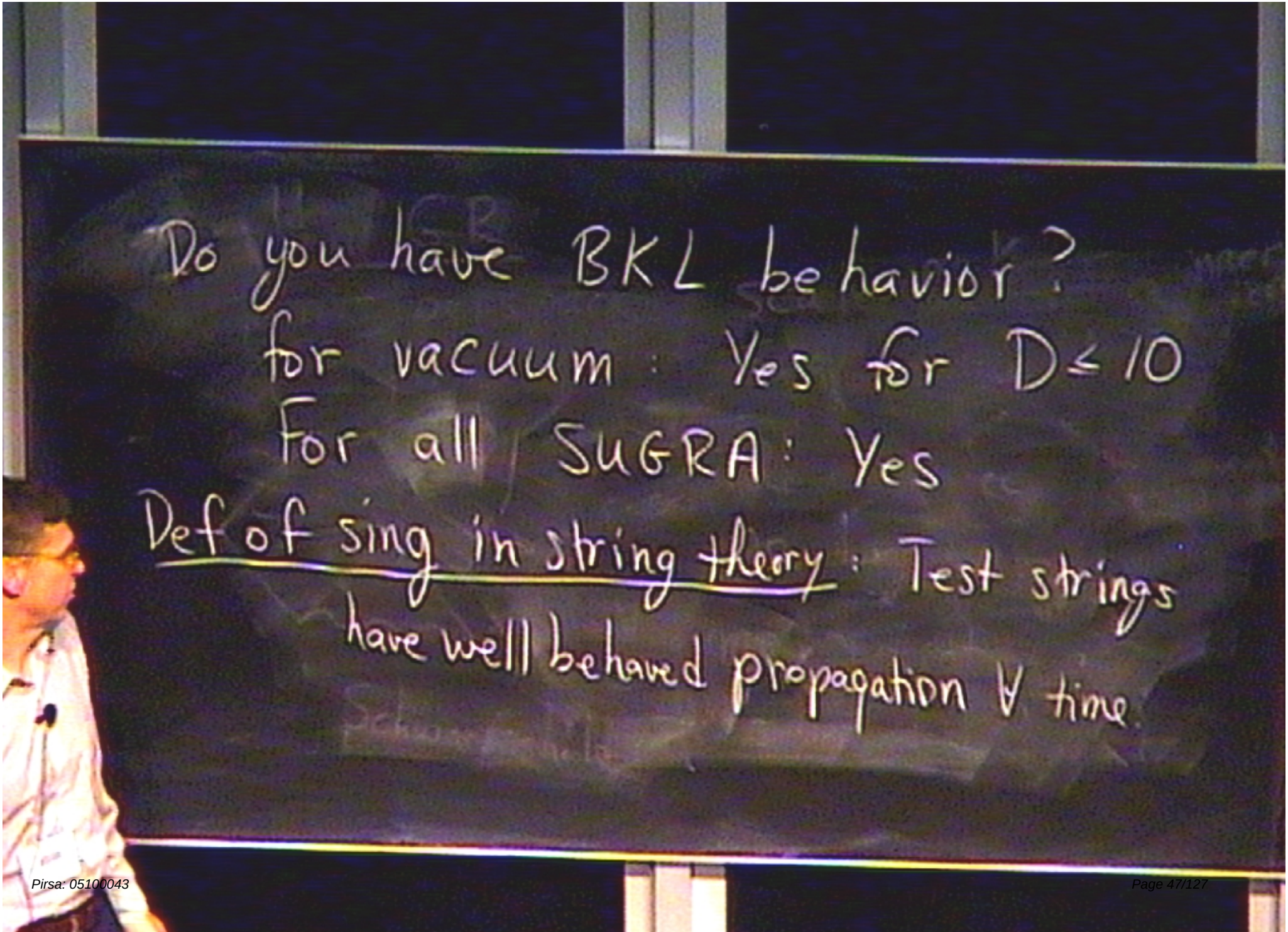
For some M, J there are S^3 BH

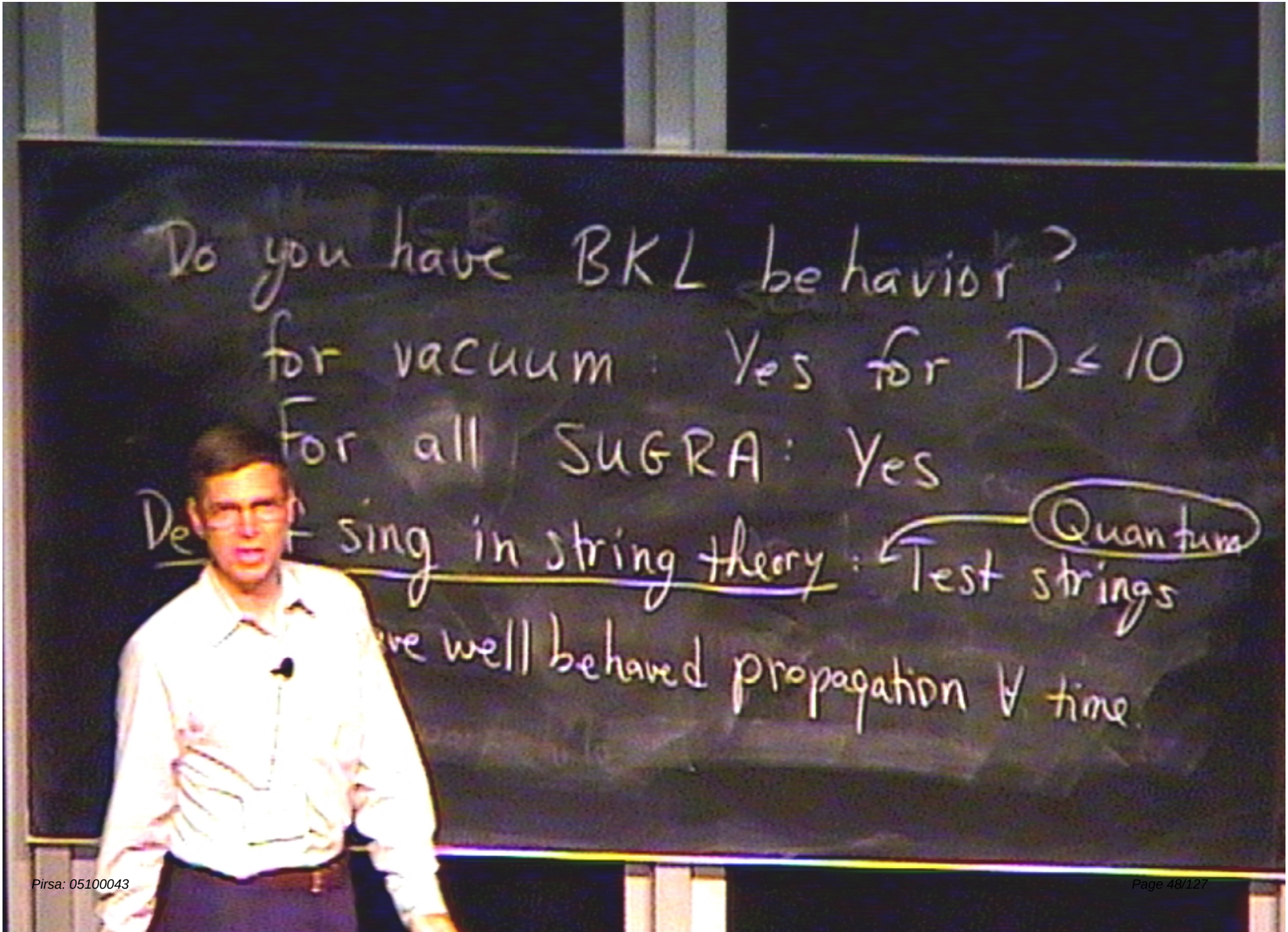
& $S^2 \times S^1$ black ring


Do you have BKL behavior?
for vacuum: Yes for D≤10


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for vacuum: Yes for D≤10
For 11D SUGRA: Yes


Do you have BKL behavior?
for vacuum: Yes for D≤10
For all SUGRA: Yes


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for vacuum: Yes for D≤10
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Def of sing in string theory: Test strings
have well behaved propagation ∀ time

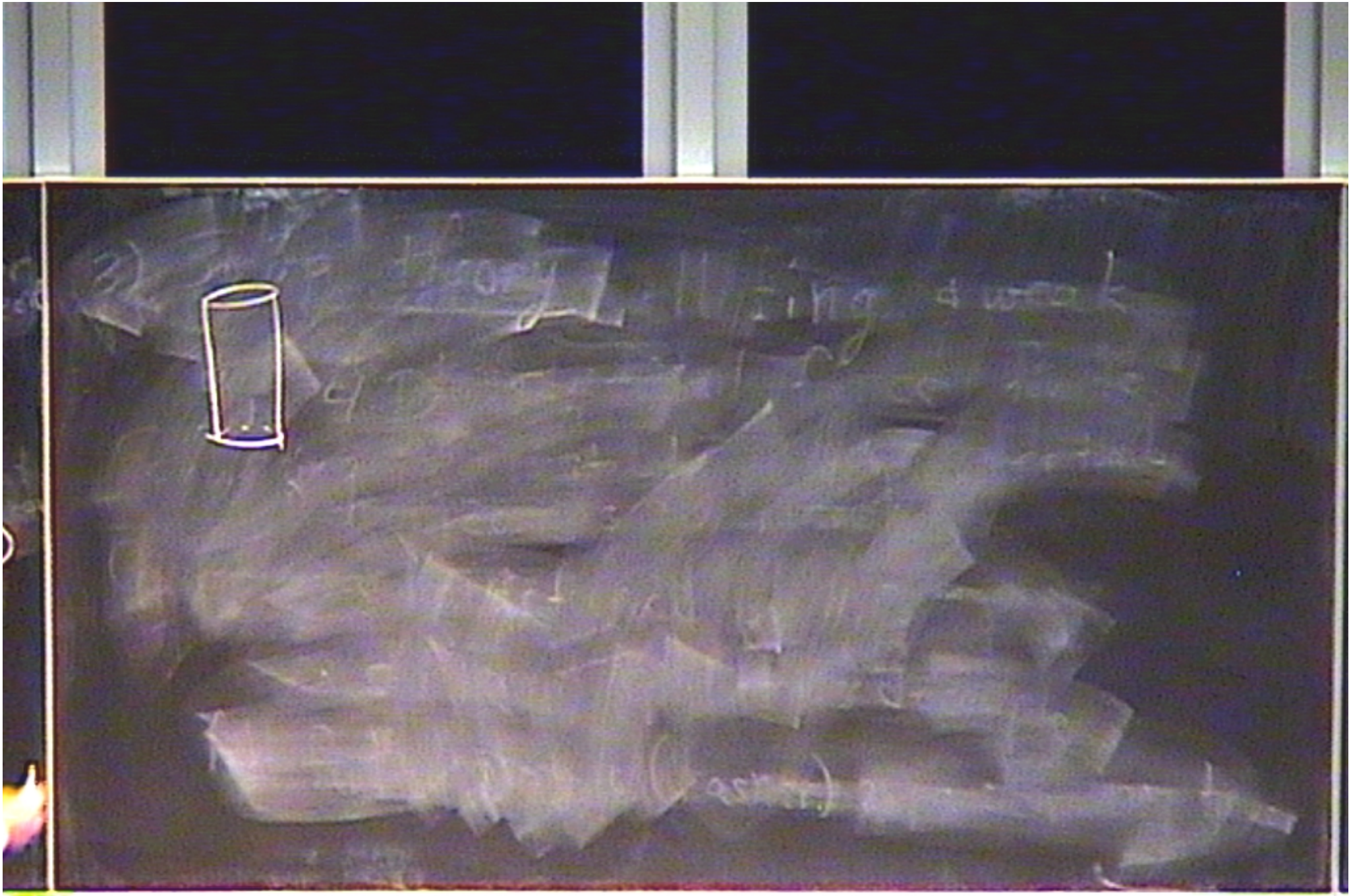

Do you have BKL behavior?
for vacuum: Yes for D≤10
For all SUGRA: Yes
Quantum
Test strings
well behaved propagation ∀ time

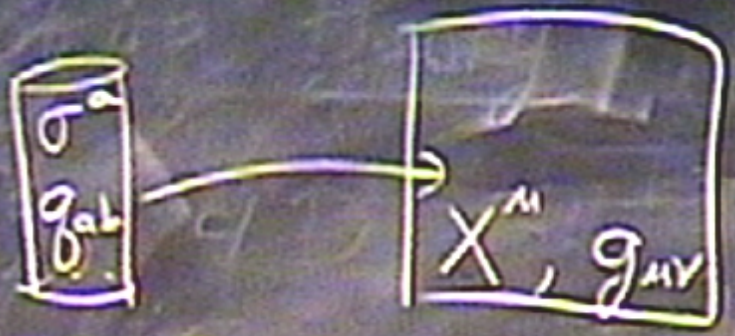
Do you have BKL behavior?

for vacuum: Yes for $D \leq 10$

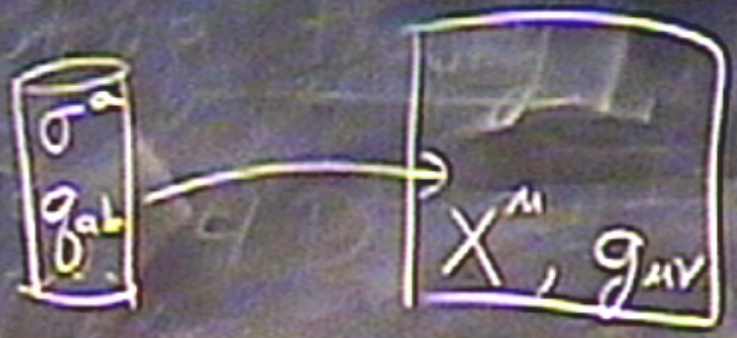
For all SUGRA: Yes

Def of sing in string theory: (Quantum) Test strings do not have well behaved propagation $\forall$ time.



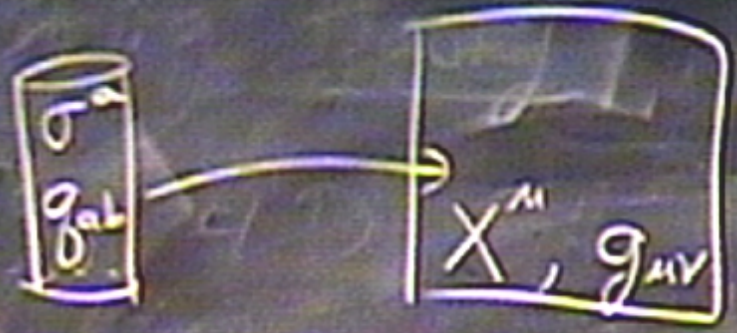


$$S = \frac{1}{l_s^2}$$



$$S = \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{g}\, g^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

σ-model Inv. under $g_{ab} \to \Omega^2(\sigma) g_{ab}$



$$S = \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{g}\, g^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

σ-model Inv. under $g_{ab} \to \Omega^2(\sigma) g_{ab}$. For some $g_{\mu\nu}$ sym. holds QM

Geom. diff spacetimes can be equiv
in string theory.

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T-duality: If metric is indep of φ

Change of variables $g_{44} \to \frac{1}{g_{44}}$

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Eg. 

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E.g.  Circle of radius R

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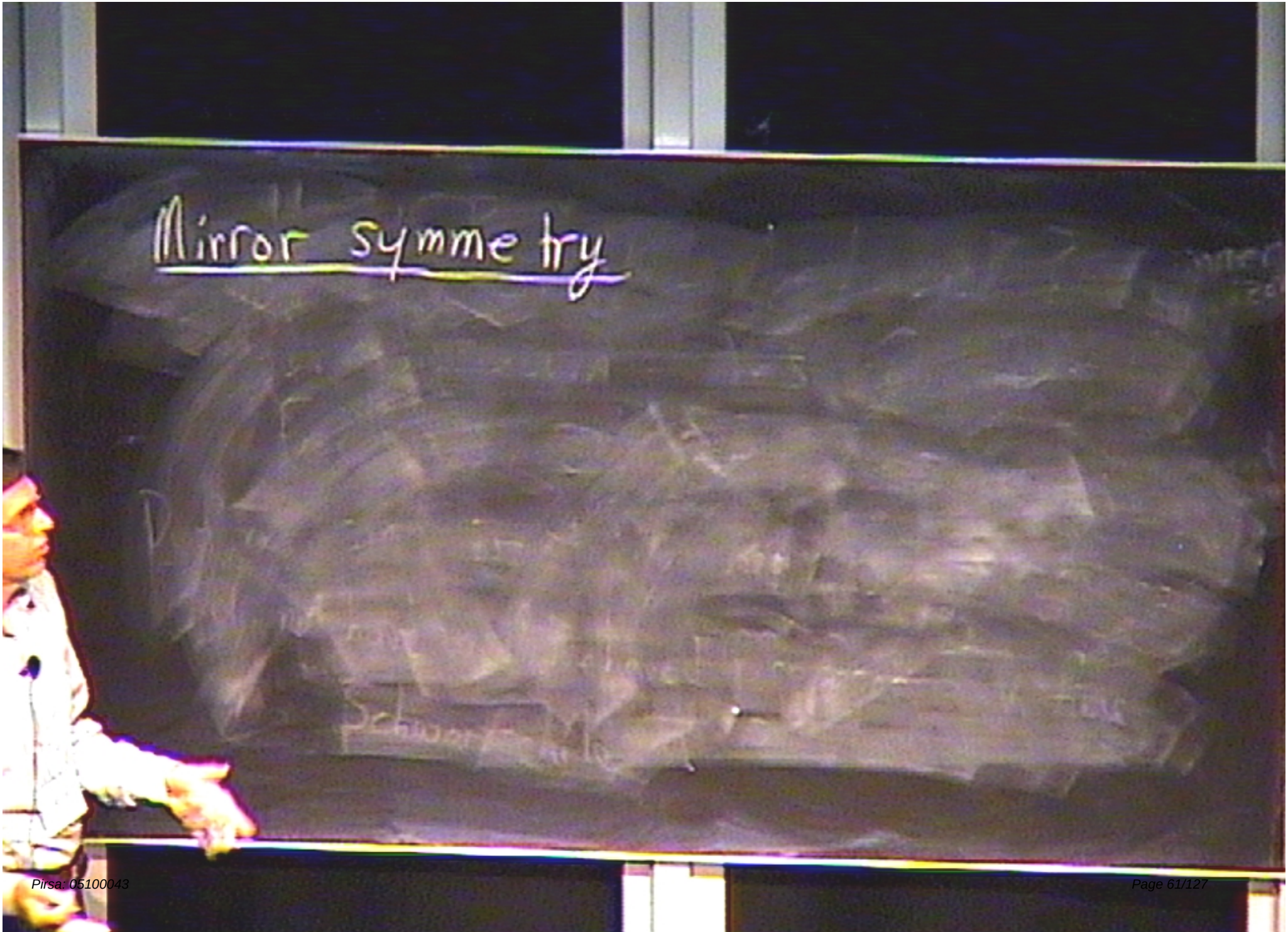
Geom. diff spacetimes can be equiv in string theory.

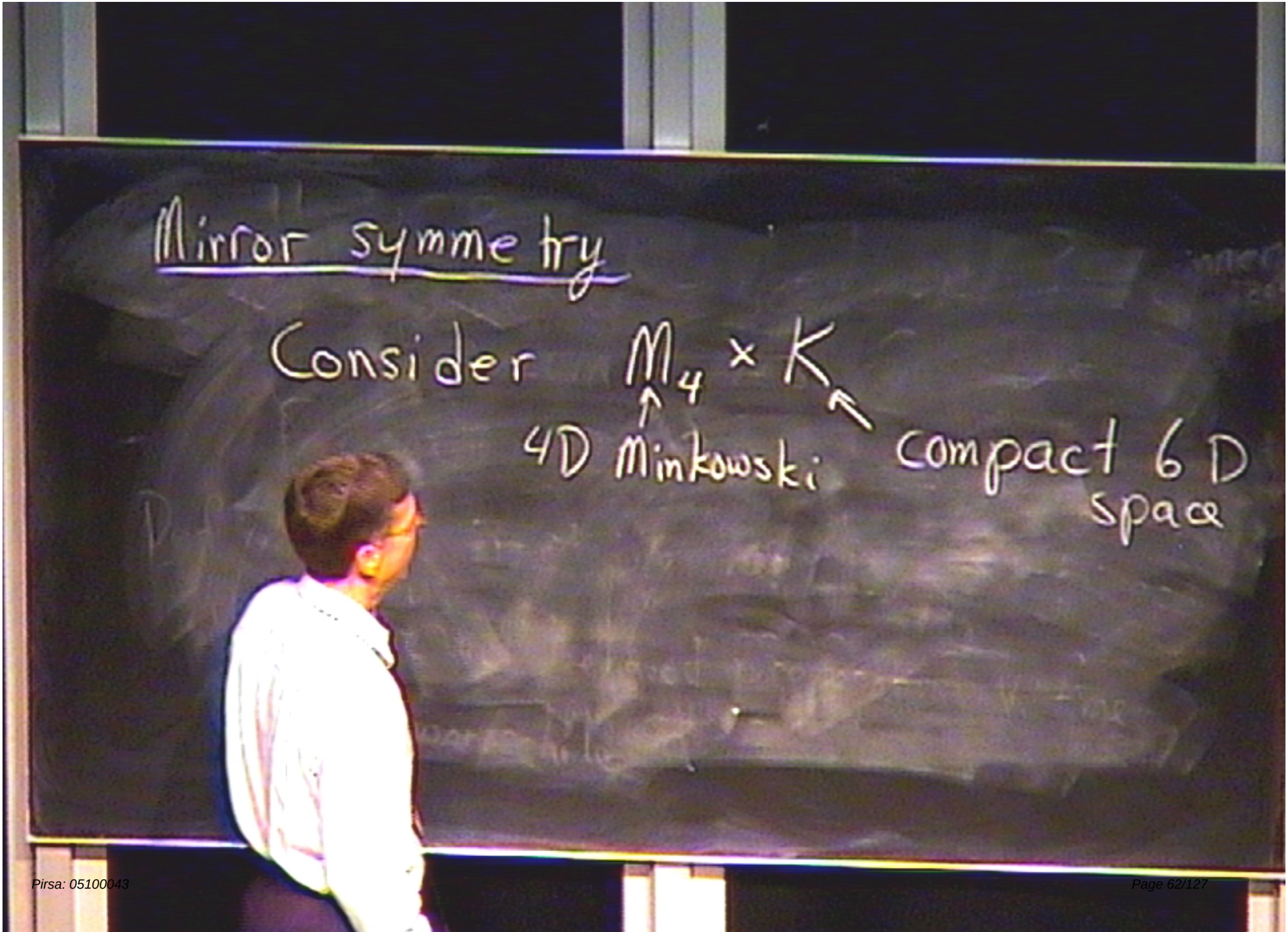
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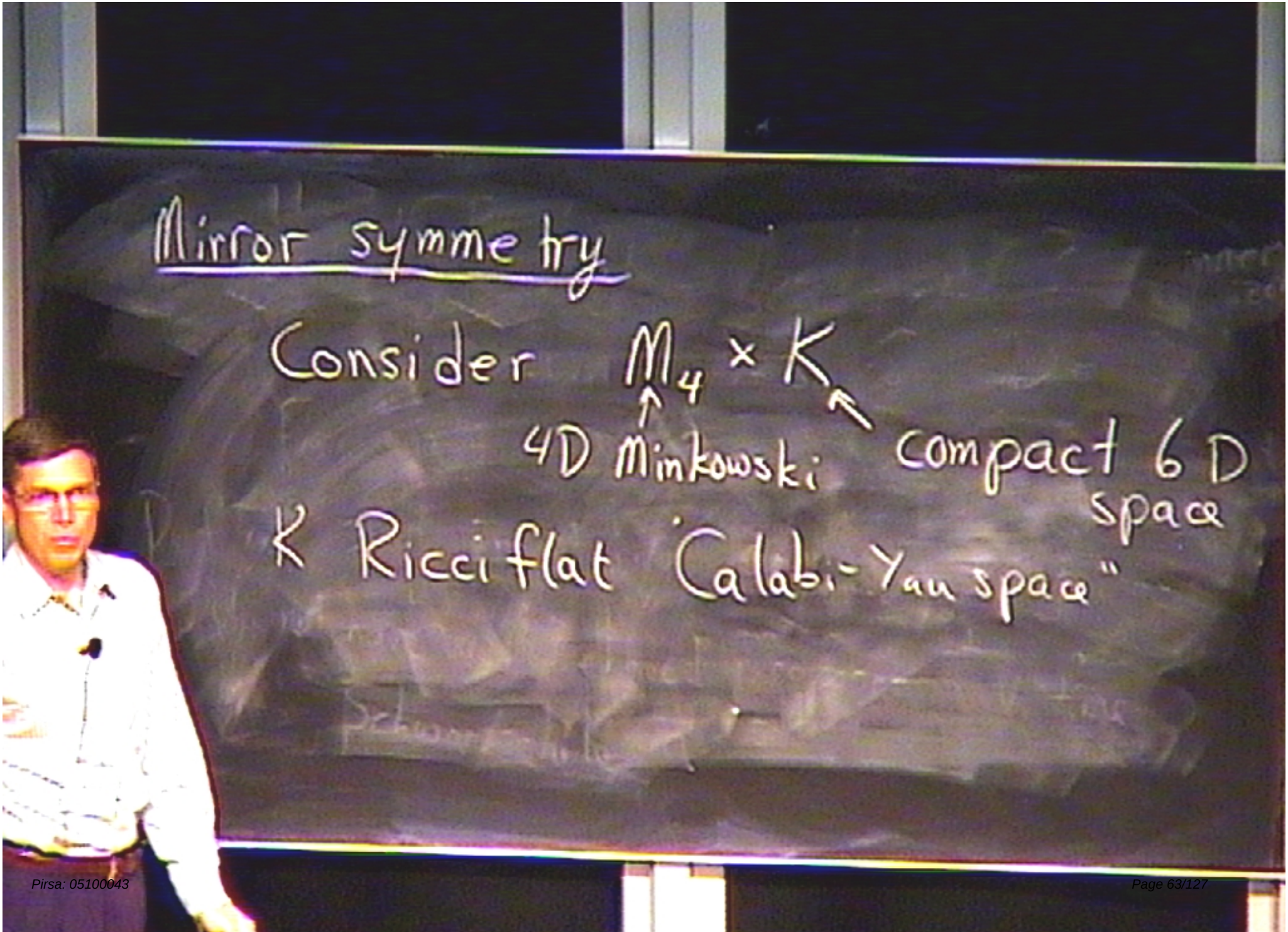
Change of variables $g_{44} \to \frac{1}{g_{44}}$

E.g.  Circle of radius R

$\equiv$ Circle of radius l_s^2/R.


Mirror symmetry


Mirror symmetry
Consider $M_4 \times K$
4D Minkowski
compact 6D space


Mirror symmetry
Consider $M_4 \times K$
4D Minkowski
compact 6D space
K Ricci flat "Calabi-Yau space"

Mirror symmetry

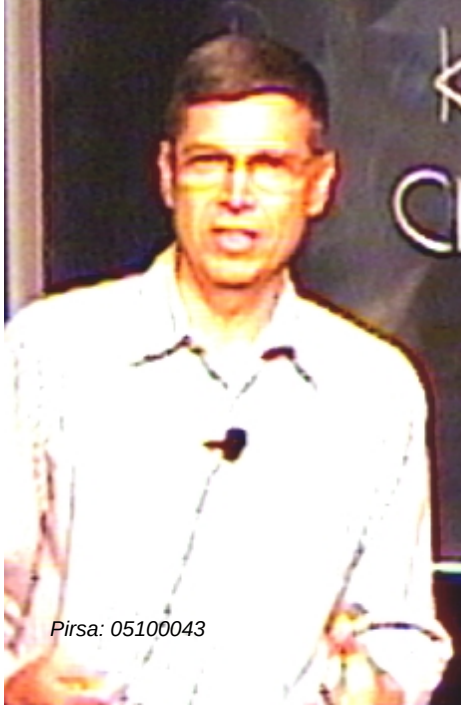
Consider $M_4 \times K$

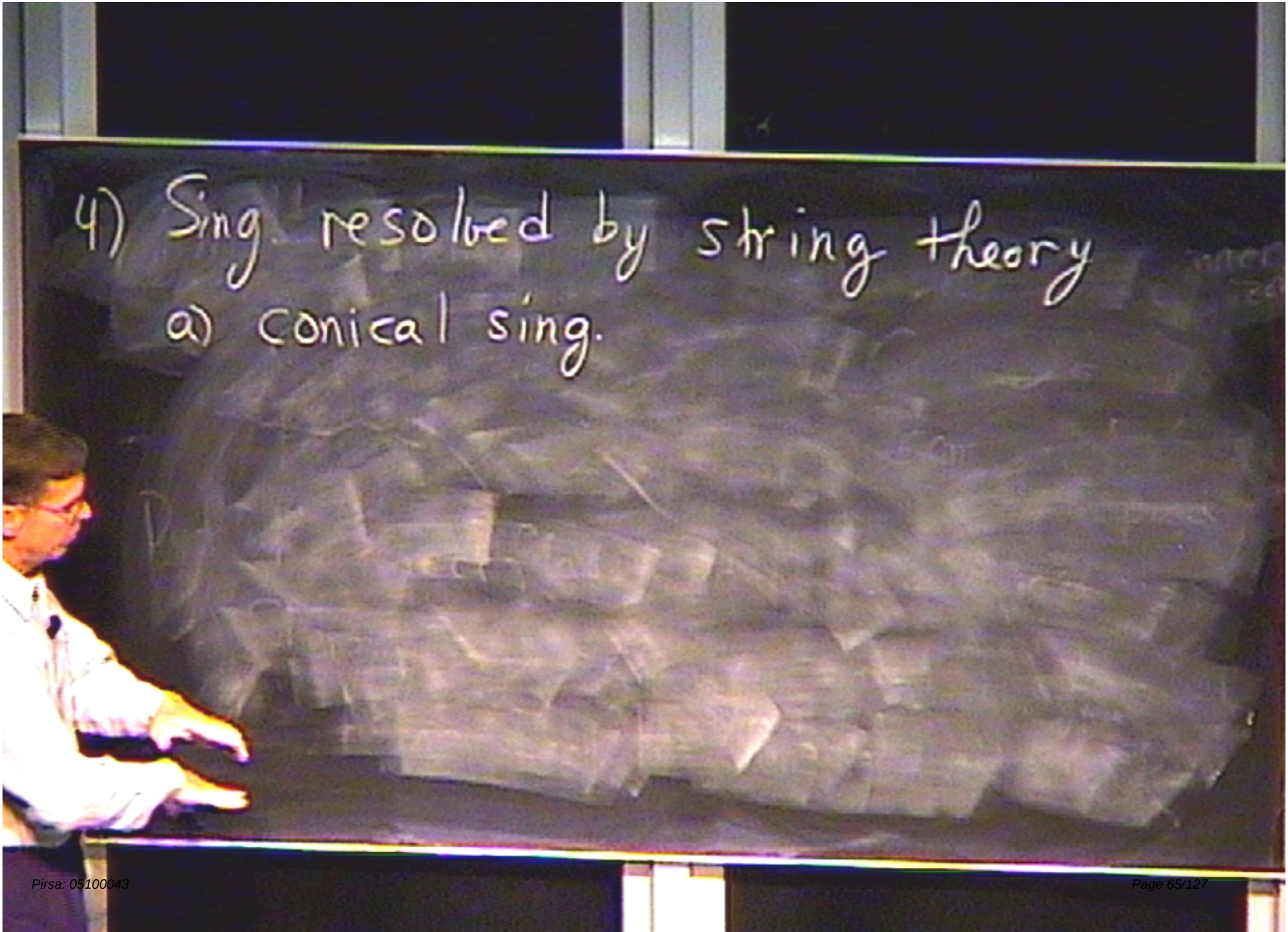
M_4: 4D Minkowski

K: compact 6D space

K Ricci flat "Calabi-Yau space"

Changing sign of $U(1)$ current in σ-model has effect of changing $M_4 \times K \to M_4 \times \tilde{K}$




4) Sing. resolved by string theory
a) conical sing.

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strings extended objects → O.k.

4) Sing. resolved by string theory

a) conical sing.



Strings extended objects → O.k.

$$ds^2 = dr^2 + r^2 d\varphi^2$$

4) Sing. resolved by string theory

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Strings extended objects → O.k.

b) $ds^2 = dr^2 + r^2 d\varphi^2 \xrightarrow[T\text{-duality}]{} ds^2 = dr^2 + \frac{1}{r^2} d\varphi^2$

4) Sing. resolved by string theory

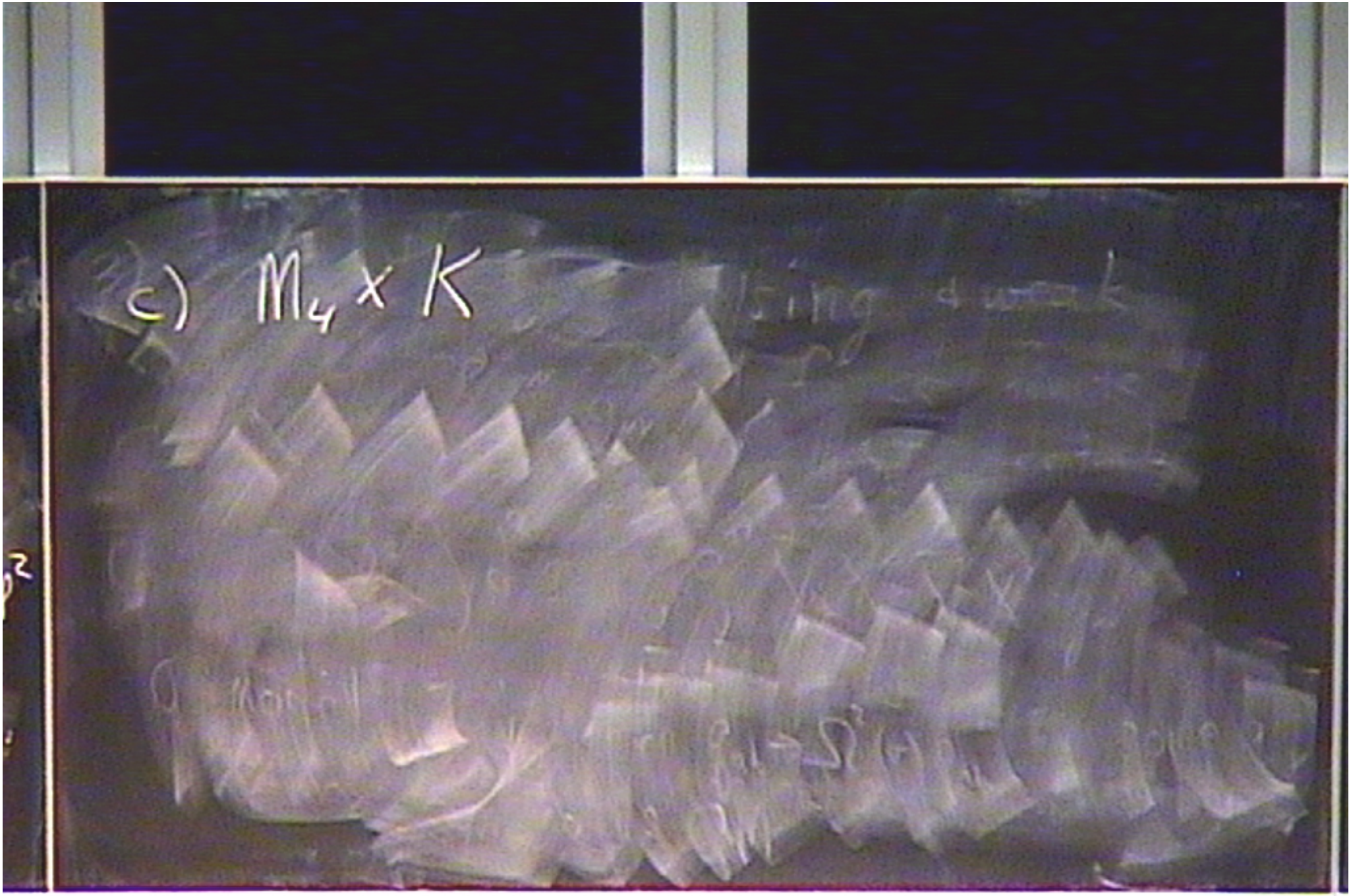
a) conical sing.



Strings extended objects → O.k.

b) $ds^2 = dr^2 + r^2 d\varphi^2 \xrightarrow{T\text{-duality}} ds^2 = dr^2 + \frac{1}{r^2} d\varphi^2$

Curvature sing. can be resolved. Singularity at $r=0$


c) $M_4 \times K$

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there are typically lots of Ricci flat metrics on K

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$S^2 \subset K$

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$S^2 \subset K \longrightarrow$ pt.

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$S^2 \subset K \longrightarrow$ pt. Curv. sing.

In mirror description: on $M_4 \times \hat{K}$ nothing is sing.

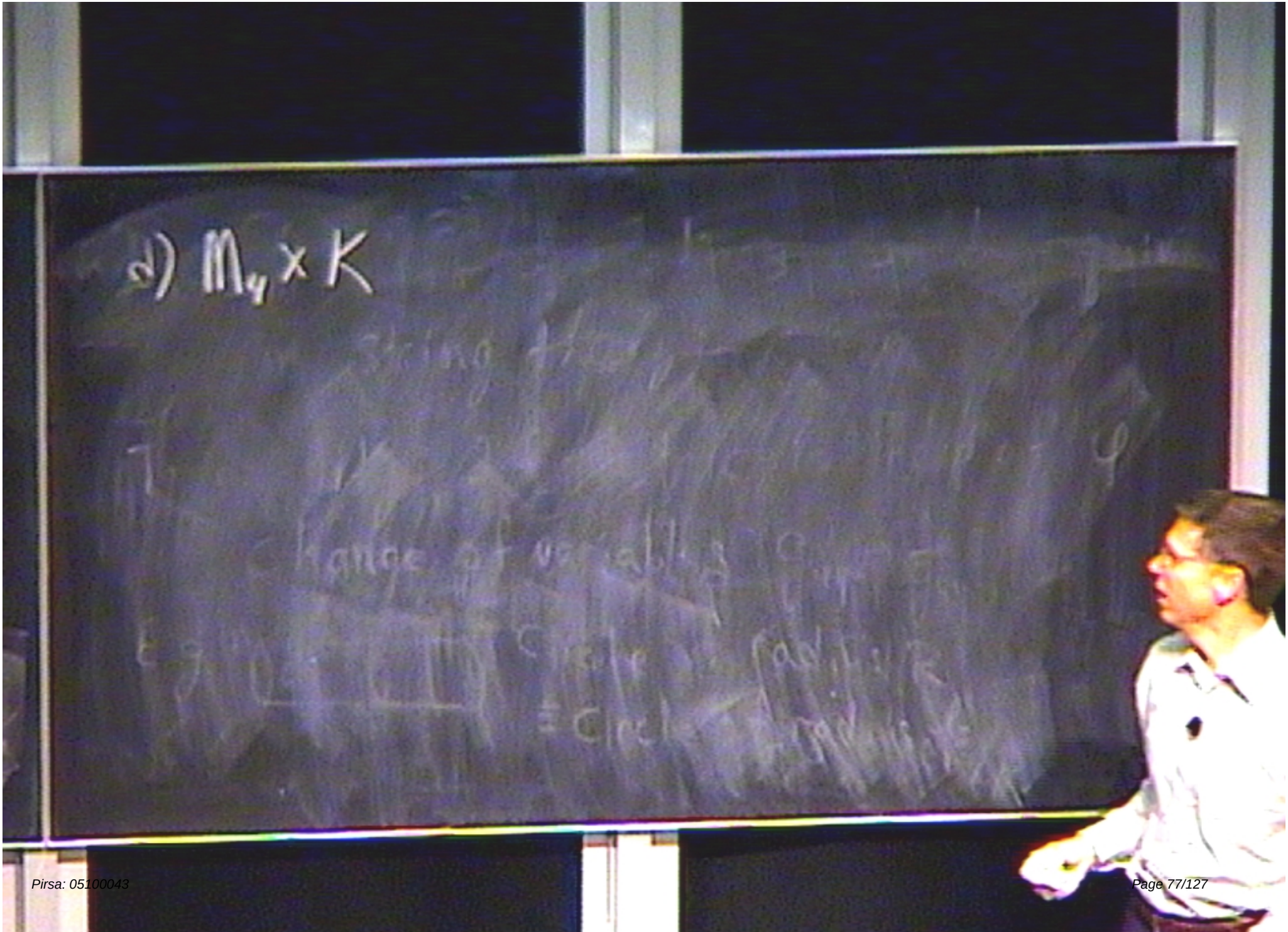
c) $M_4 \times K$

there are typically lots of Ricci flat metrics on K



$S^2 \subset K \longrightarrow$ pt. Curv. sing.

In mirror description: on $M_4 \times \hat{K}$ nothing is sing $\Rightarrow$ Change topology $K \to K'$


d) $M_4 \times K$

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Change metric on k s.t. $S^3 \to pt$

d) $M_4 \times K$

Change metric on K s.t. $S^3 \to pt$

Use other extended objects

branes

d) $M_4 \times K$

Change metric on k s.t. $S^3 \to pt$

Use other extended objects

branes - wrap 3-brane around S^3

d) $M_4 \times K$

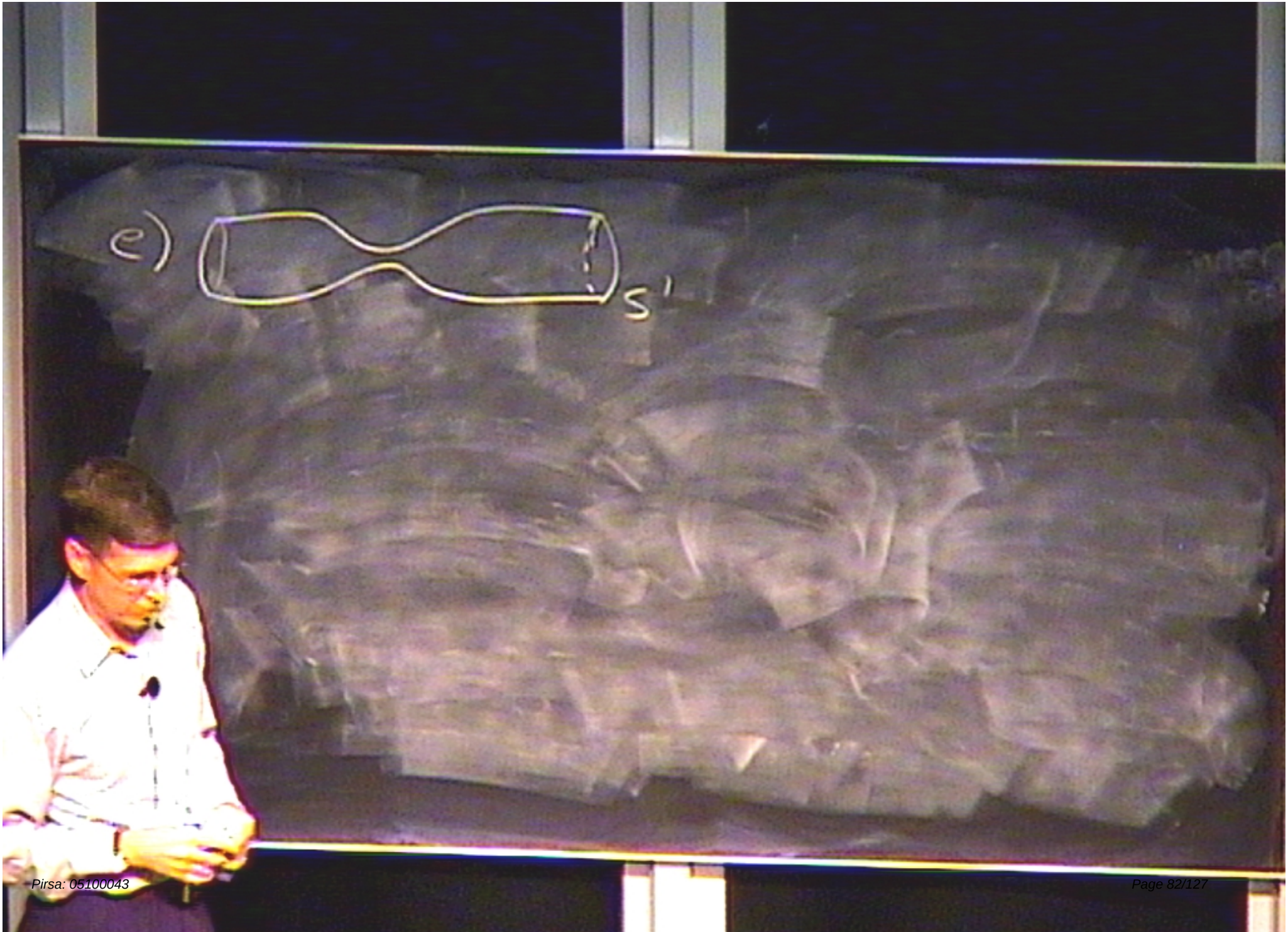
Change metric on K s.t. $S^3 \to pt$

Use other extended objects

branes - wrap 3-brane around S^3

Light when $S^3 \to pt$, when included

evol. is non-singular.


e)
S'

e)  S^1

If fermions are antiperiodic

wound strings are massless $R \sim l_s$

are tachyonic for $R < l_s$

c)  S^1

If fermions are antiperiodic

Wound strings are massless $R \sim l_s$

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Silverstein et al Neck pinches off



c)  S^1

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e)  S^1

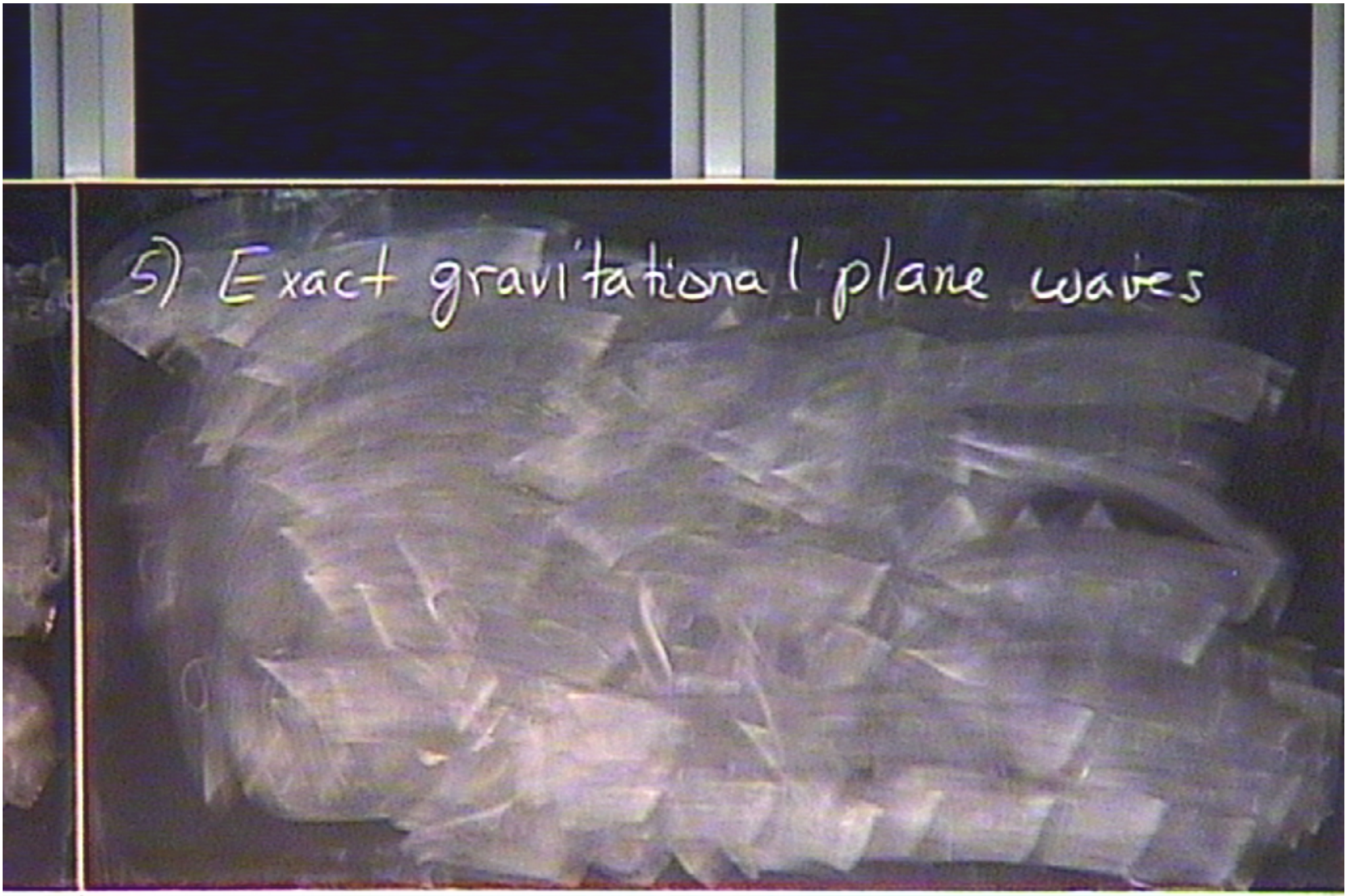
If fermions are antiperiodic

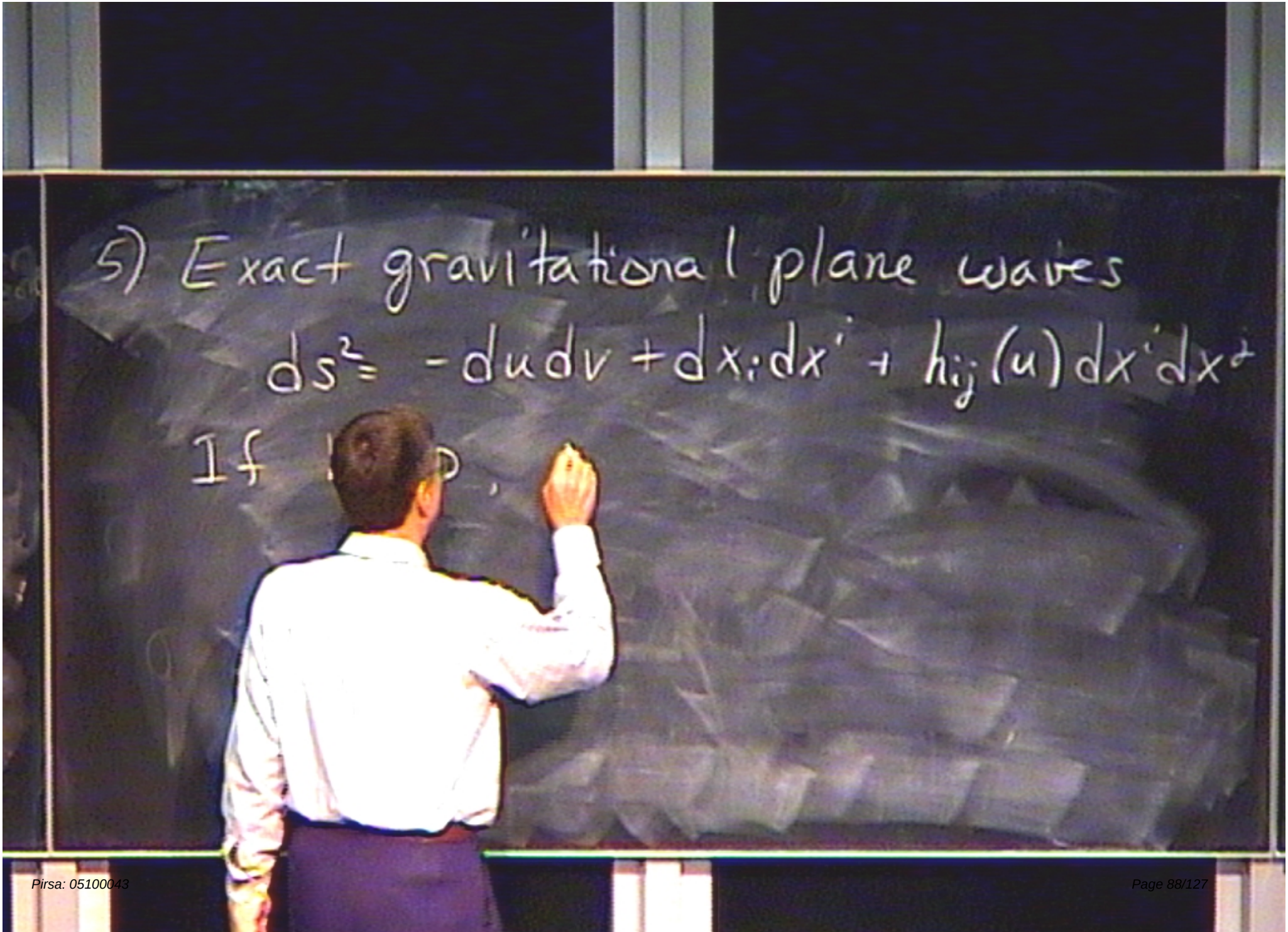
Wound strings are massless $R \sim l_s$

are tachyonic for $R < l_s$

Silverstein et al Neck pinches off




5) Exact gravitational plane waves


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$ds^2 = -dudv + dx_i dx^i + h_{ij}(u)\,dx^i dx^j$
If

5) Exact gravitational plane waves

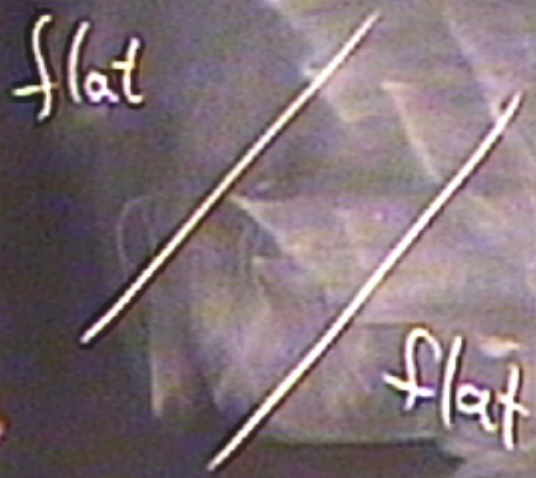
$$ds^2 = -du\,dv + dx_i\,dx^i + h_{ij}(u)\,dx^i\,dx^j$$

If $h'_{ij} = 0$, $R_{\mu\nu} = 0$

5) Exact gravitational plane waves

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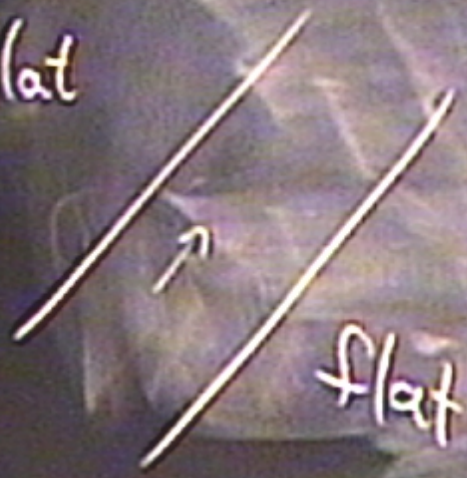
If $h'_{ij} = 0$, $R_{\mu\nu} = 0$ arb dep on u



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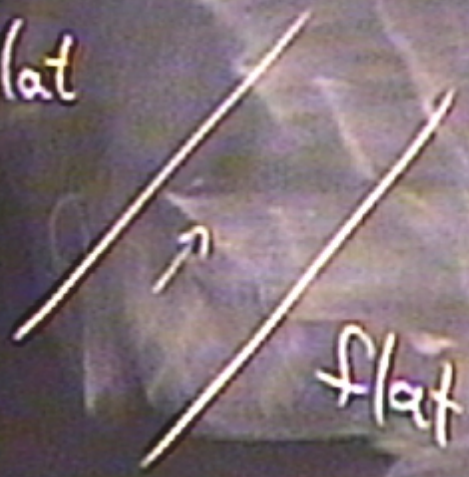


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Exact soln to string theory

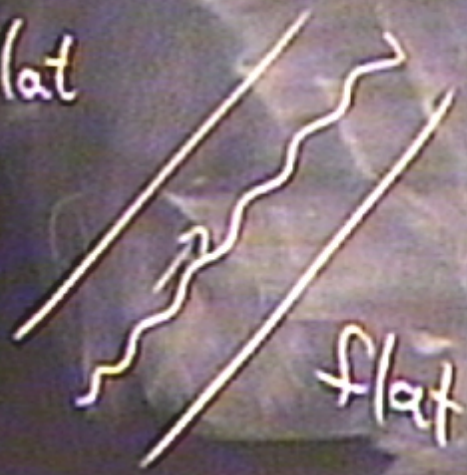


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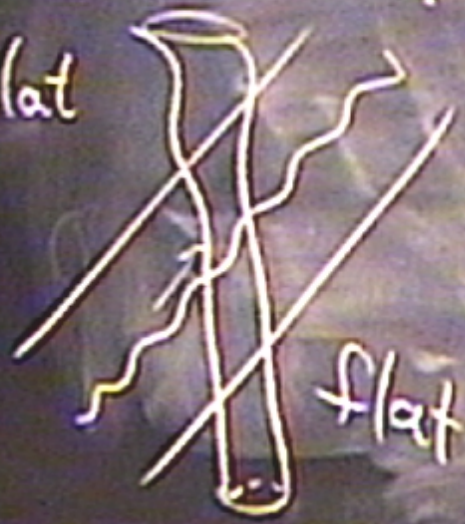


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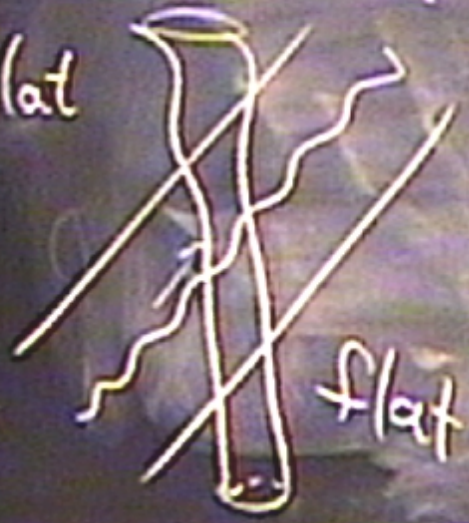
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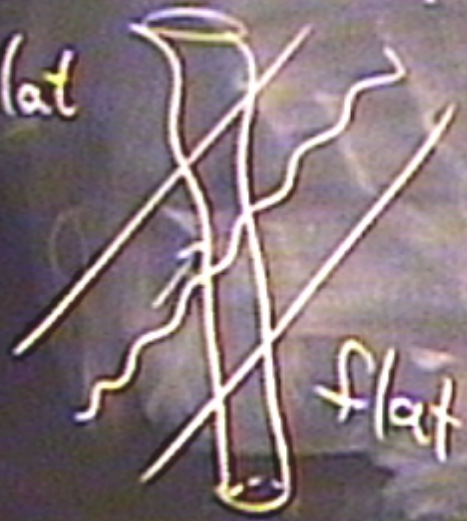
Exact soln to string theory

String ∞ excited for sing. wave

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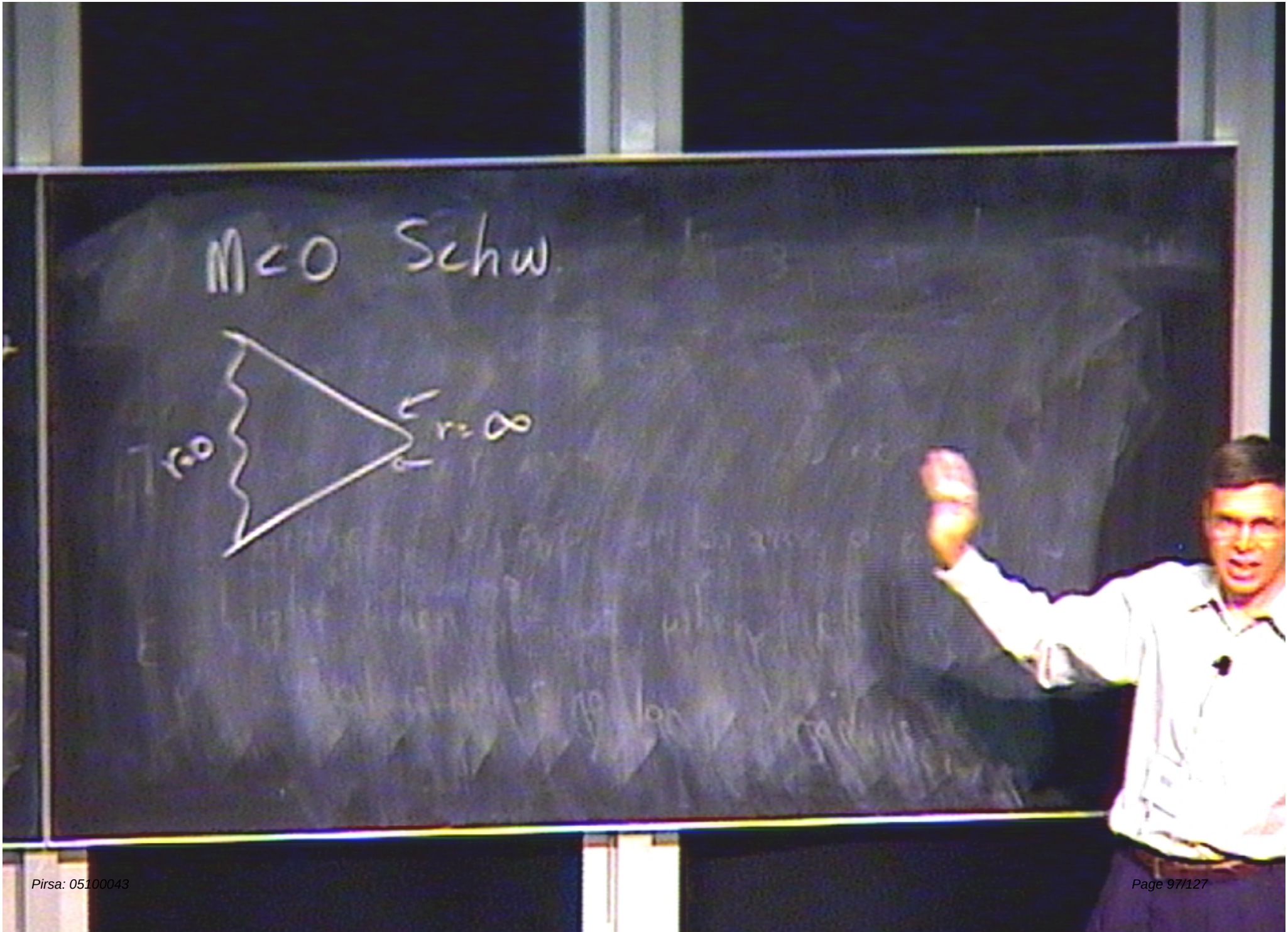
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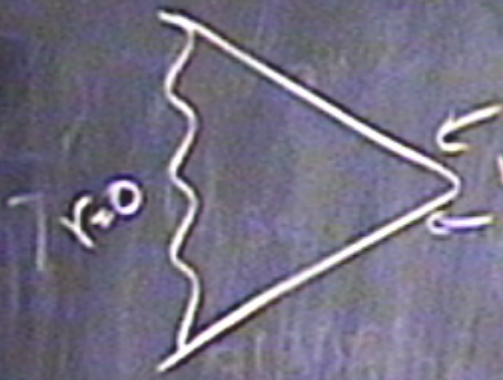
Exact soln to string theory

String ∞ excited for sing. wave

Sing. not removed.

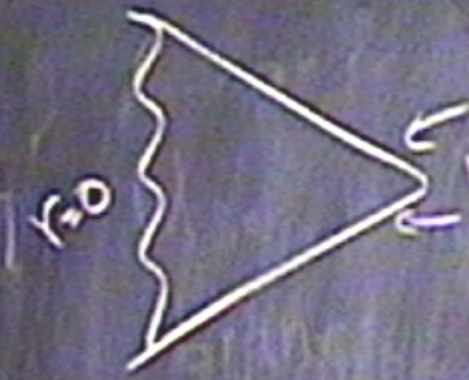

M<0 Schw
r=0
r=∞

M<0 Schw.



Exact string soln which is like this at large r

M<0 Schw.

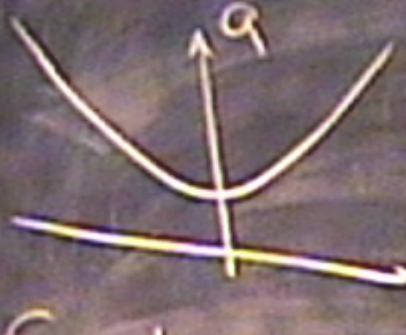


Exact string soln which is like this at large r

If r=0 sing. is removed, arb neg. energy solns → unstable.

6) Cosmology

a) Bounce: Veneziano: pre-big bang"



GR breaks down
Assume bounce

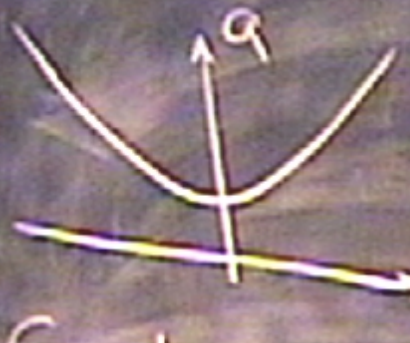
Cyclic universe:

3-brane



6) Cosmology

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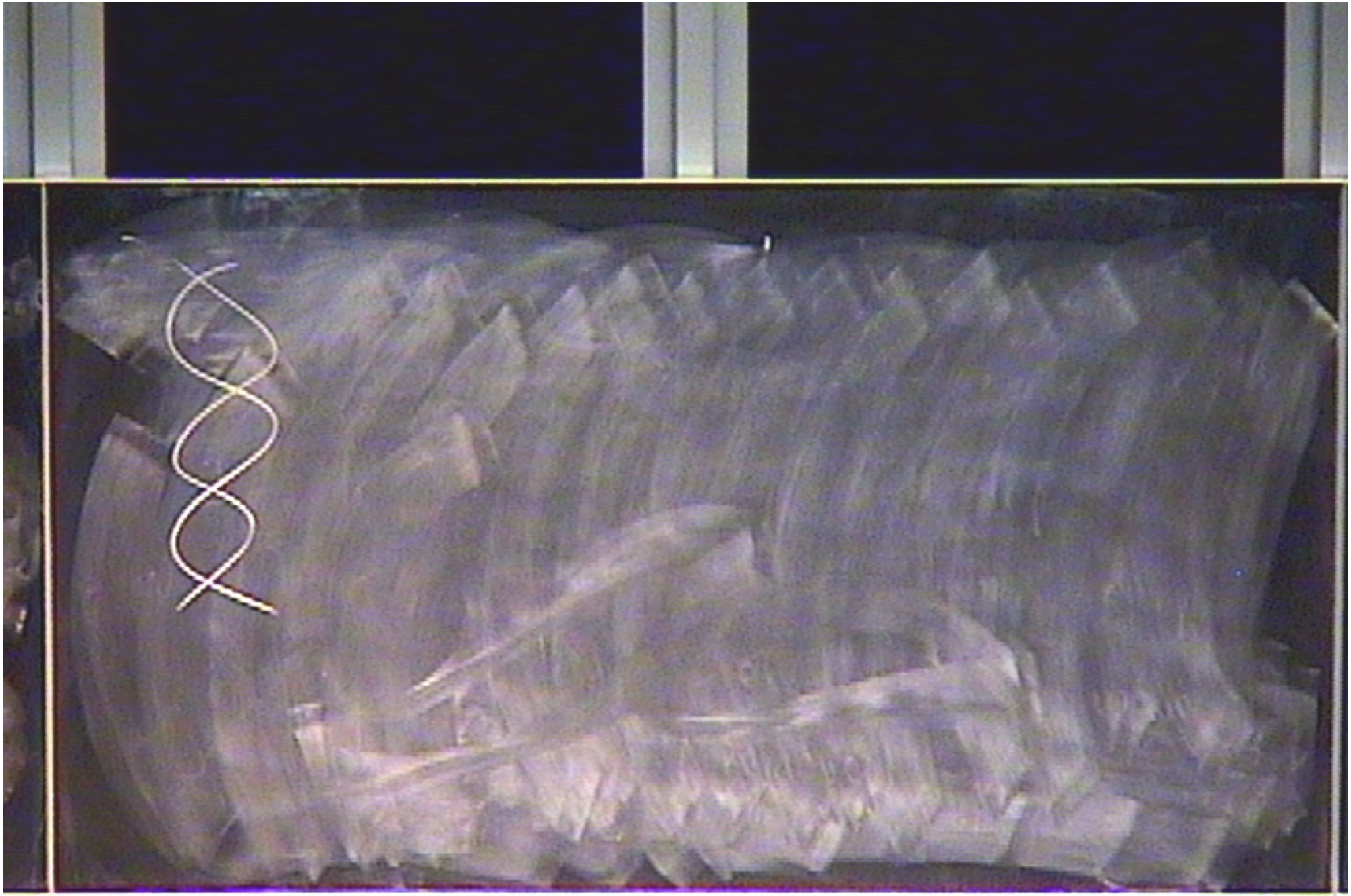
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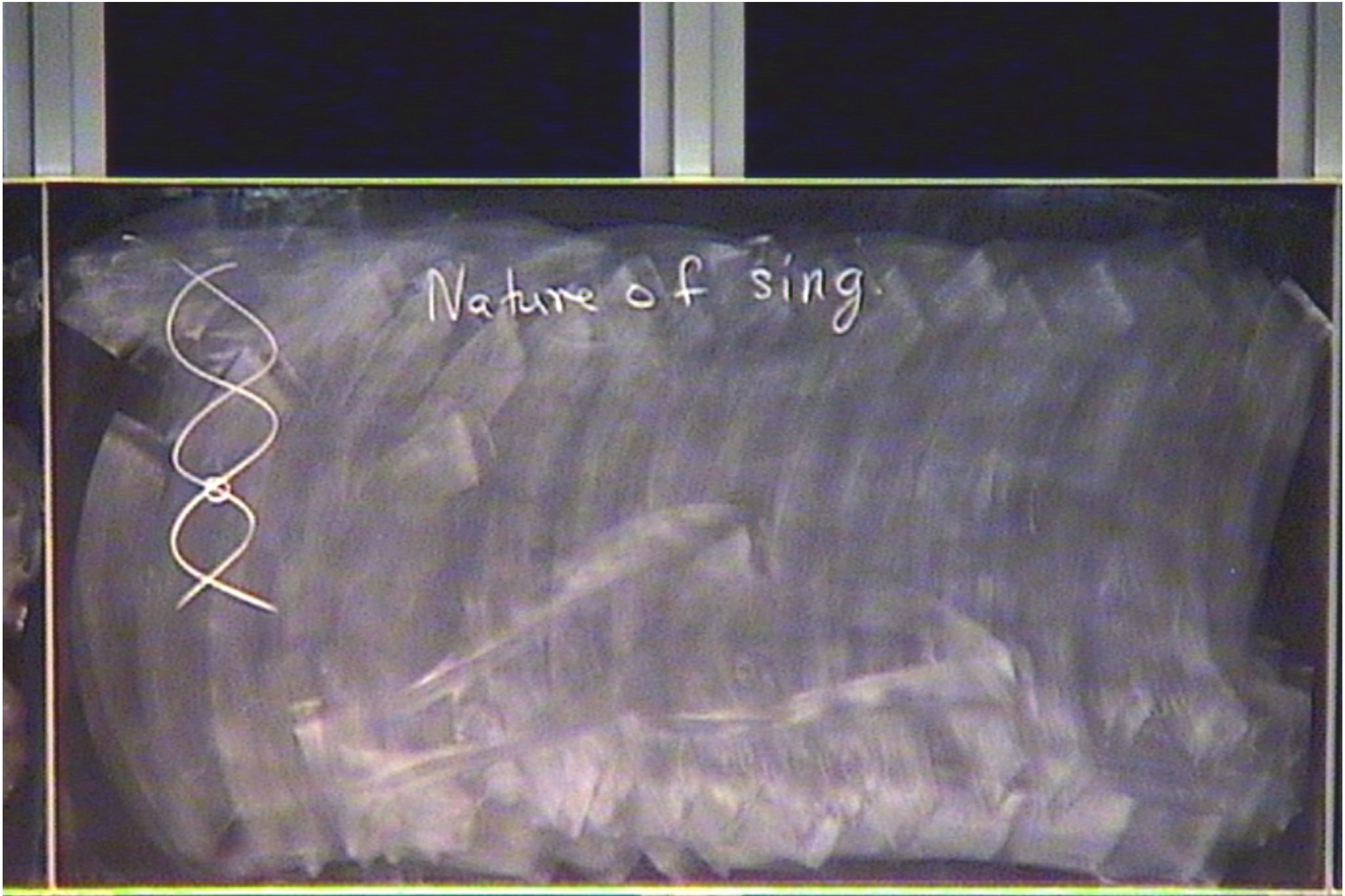
Cyclic universe:

3-brane



Big bang: collision of branes




Nature of sing.

Nature of sing: Milne universe







$ds^2 = -dt^2 + t^2 d\varphi^2$

Nature of sing: Milne universe







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perturbation → curv sing.

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b) No bounce

Nature of sing: Milne universe





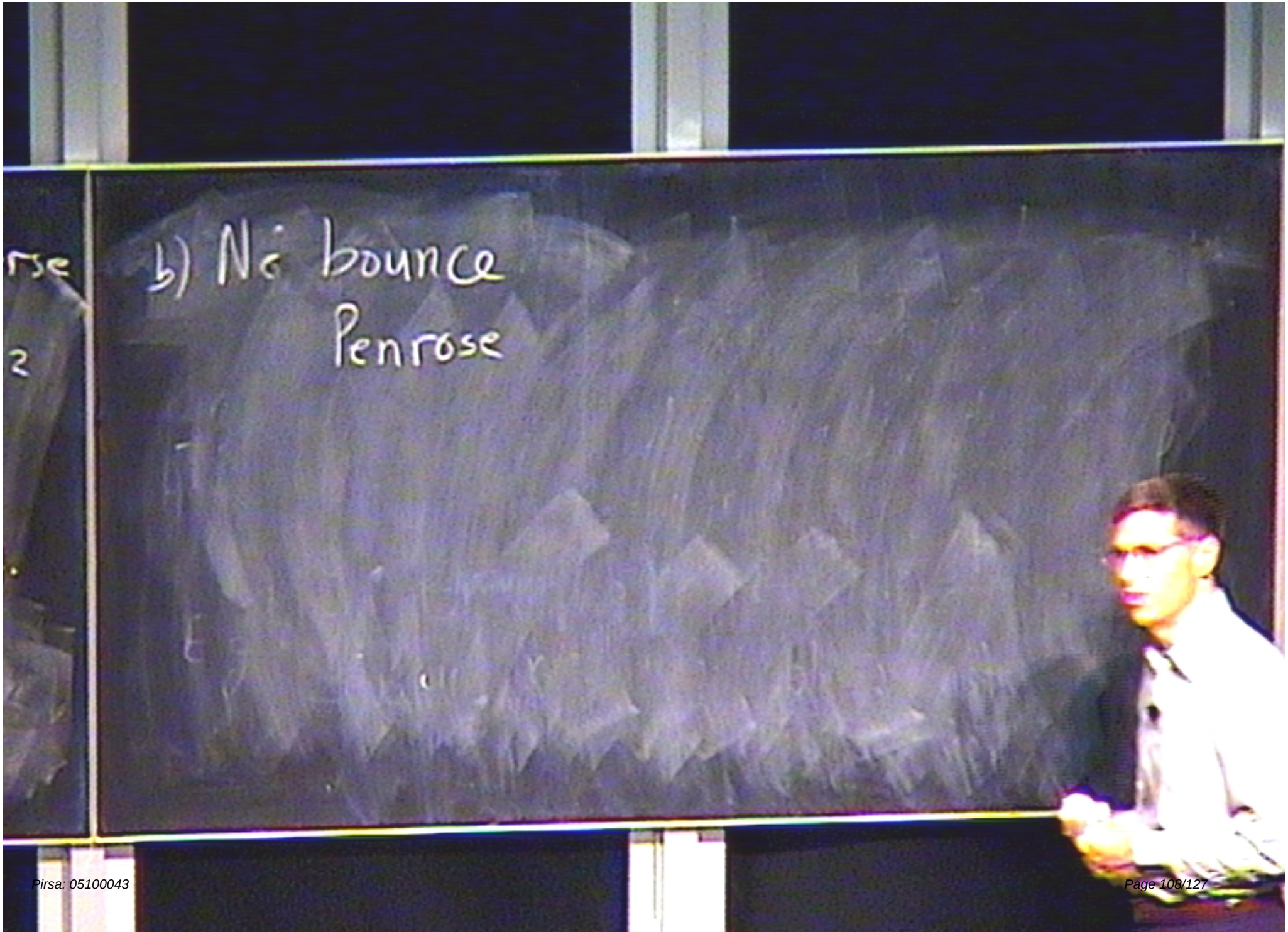


$ds^2 = -dt^2 + t^2 d\varphi^2$

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String gas $-dt^2 + a^2(t)\,dx^2$

Cosmology $(T\text{-}dual)\ \frac{1}{a^2}$


b) No bounce
Penrose

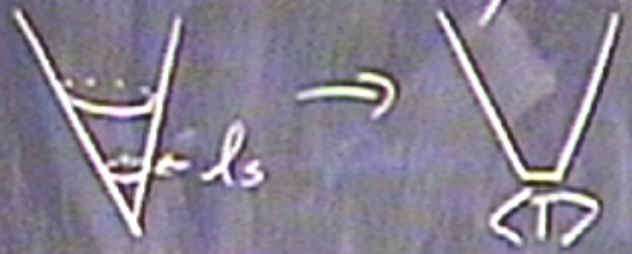
b) No bounce

Penrose: Initial state very special

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Silverstein & McGreevy: Antiperiodic fermions



Nature of sing: Milne universe





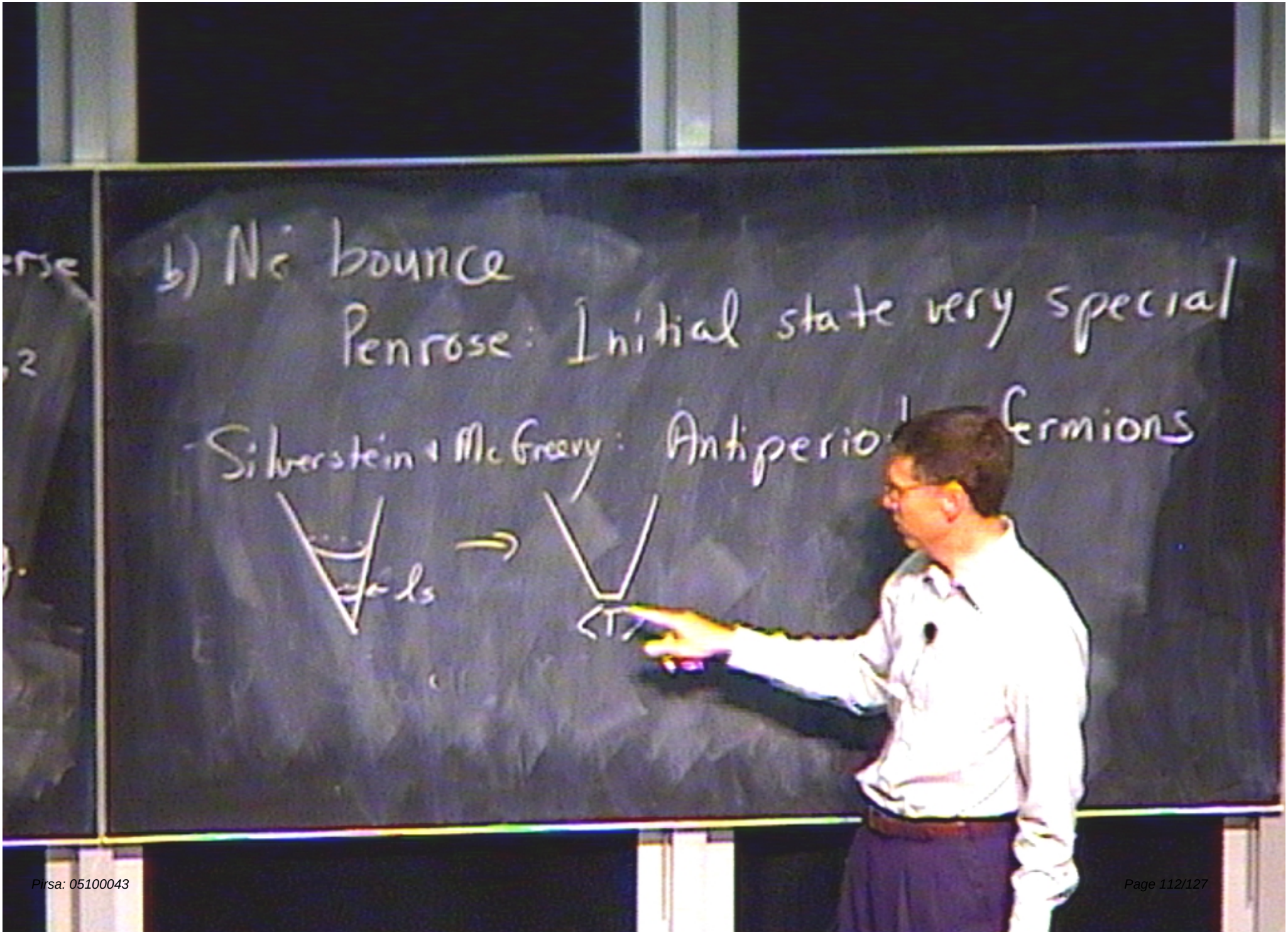


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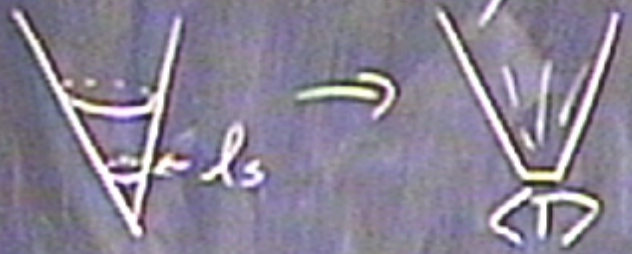
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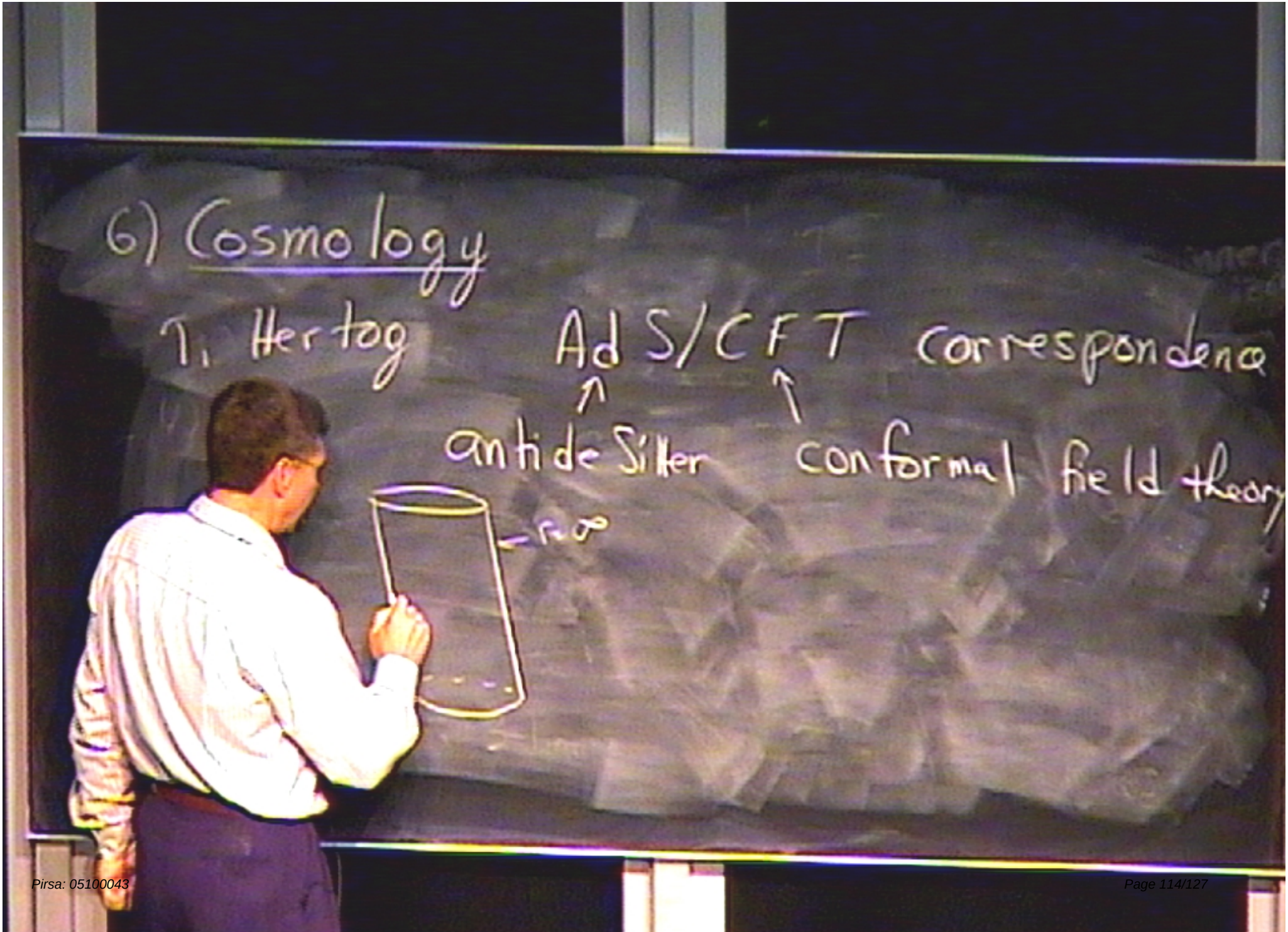
Penrose: Initial state very special

- Silverstein + McGreevy: Antiperiodic fermions



found 1-parameter family of states

get thermal distrib of particles with temp T

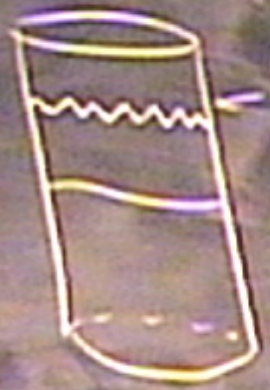

6) Cosmology
T. Hertog
AdS/CFT correspondence
↑ ↑
antide Sitter
conformal field theory
r→∞

6) Cosmology

T. Hertog

AdS/CFT Correspondence

↑ antide Sitter ↑ conformal field theory



6) Cosmology

T. Hertog

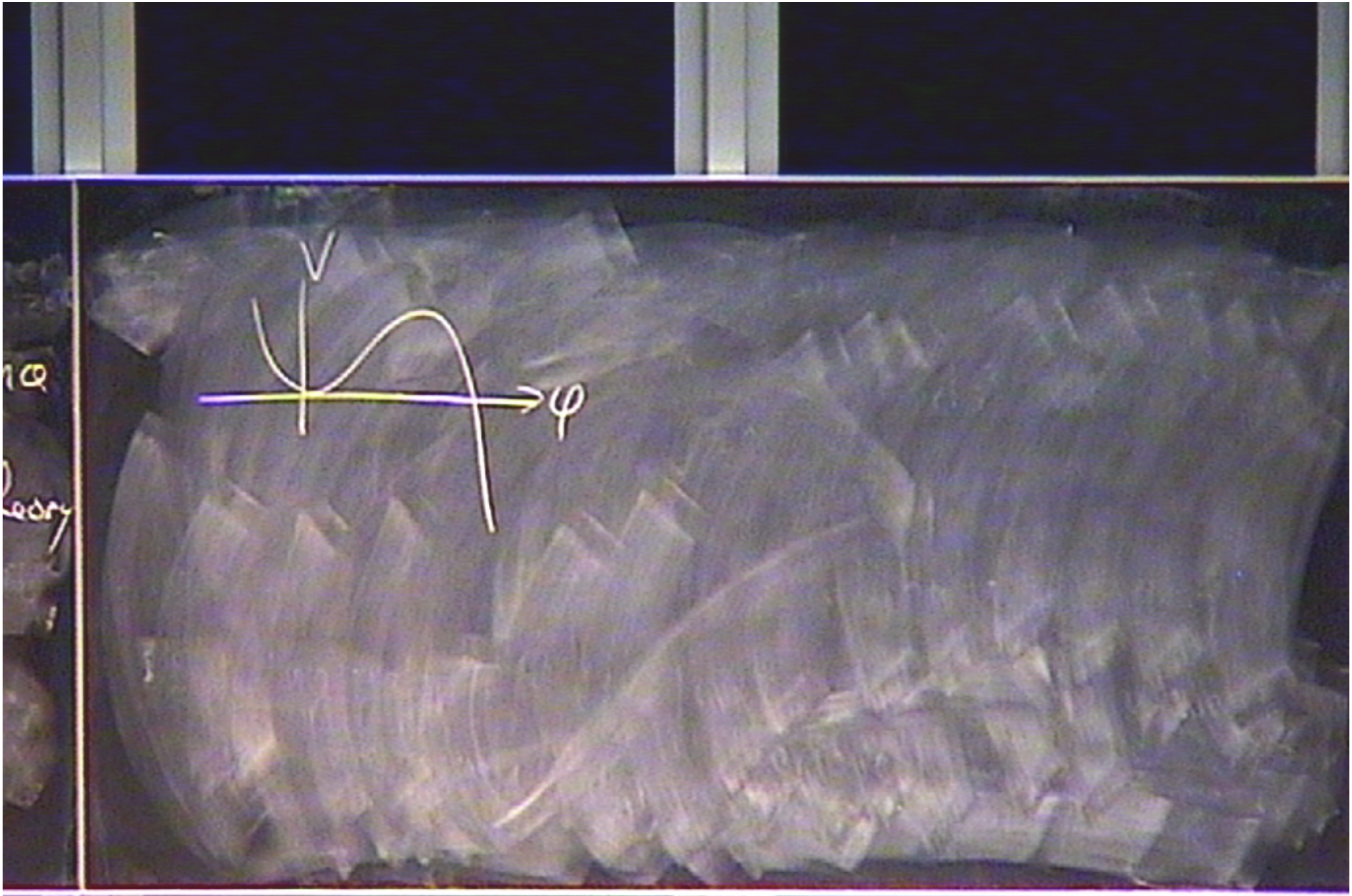
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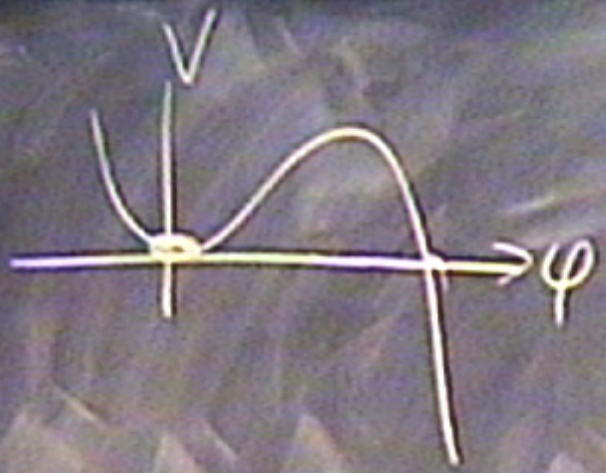
↑ antide Sitter ↑ conformal field theory



CFT: $\int d^3x (\nabla\varphi)^2 + V$

$V = \frac{1}{2} m^2 \varphi^2 - \frac{1}{3} k \varphi^6$


V
φ



reach $\varphi = \infty$ in finite time

6) Cosmology

1. Hertog

AdS/CFT correspondence

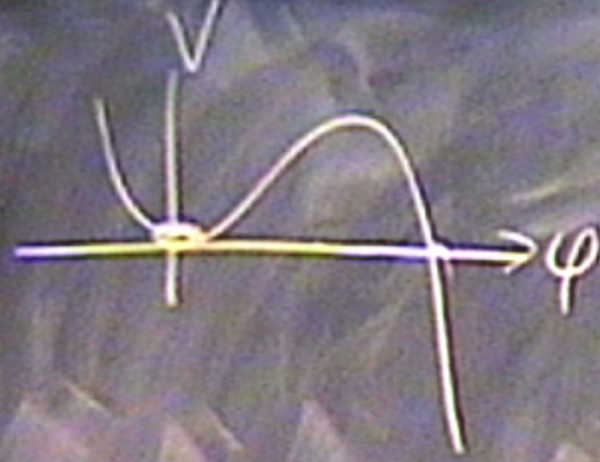
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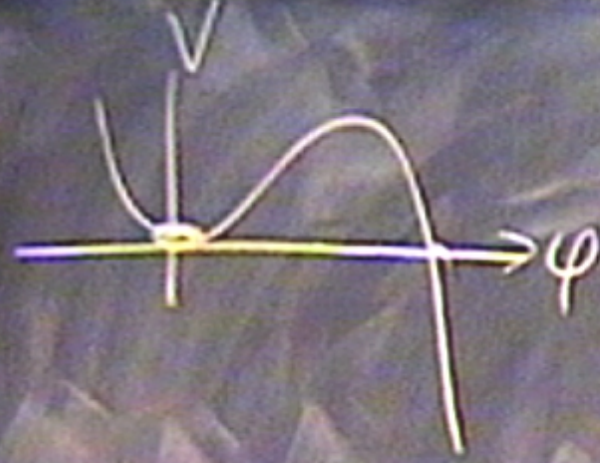
$S^3 \times \mathbb{R}$



reach $\varphi = \infty$ in finite time

In minisuperspace

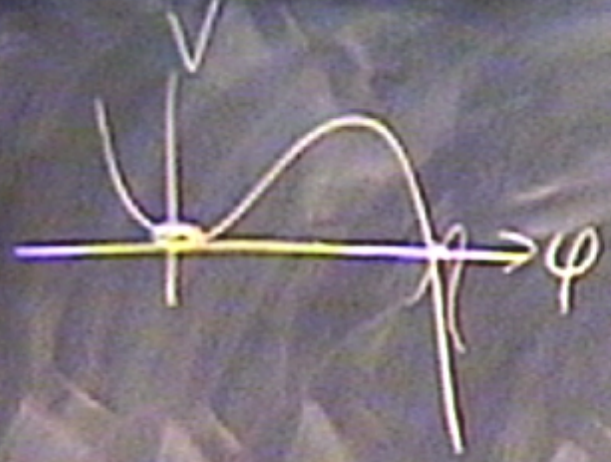
$\varphi(t)$



reach $\varphi = \infty$ in finite time

In minisuperspace $\varphi(t)$

Get QM: need extra input



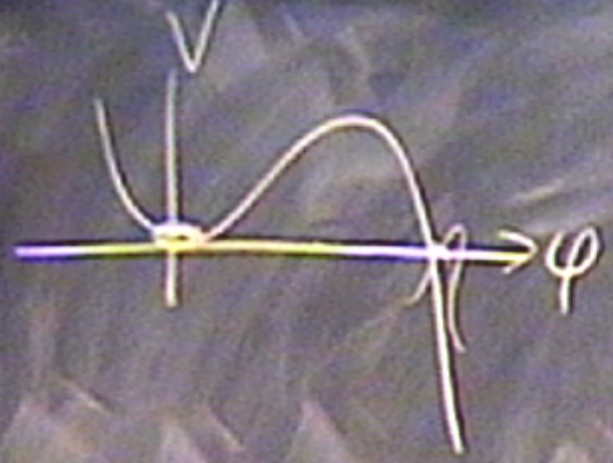
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In minisuperspace

$\varphi(t)$

Get QM: need extra input

$\langle x \rangle \to \infty$

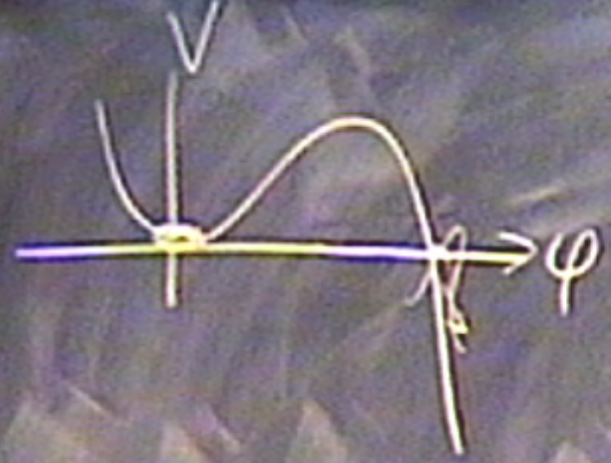


reach $\varphi = \infty$ in finite time

In minisuperspace $\varphi(t)$

Get QM: need extra input

$\langle x \rangle \to \infty$ & returns → bounce!



reach $\varphi = \infty$ in finite time

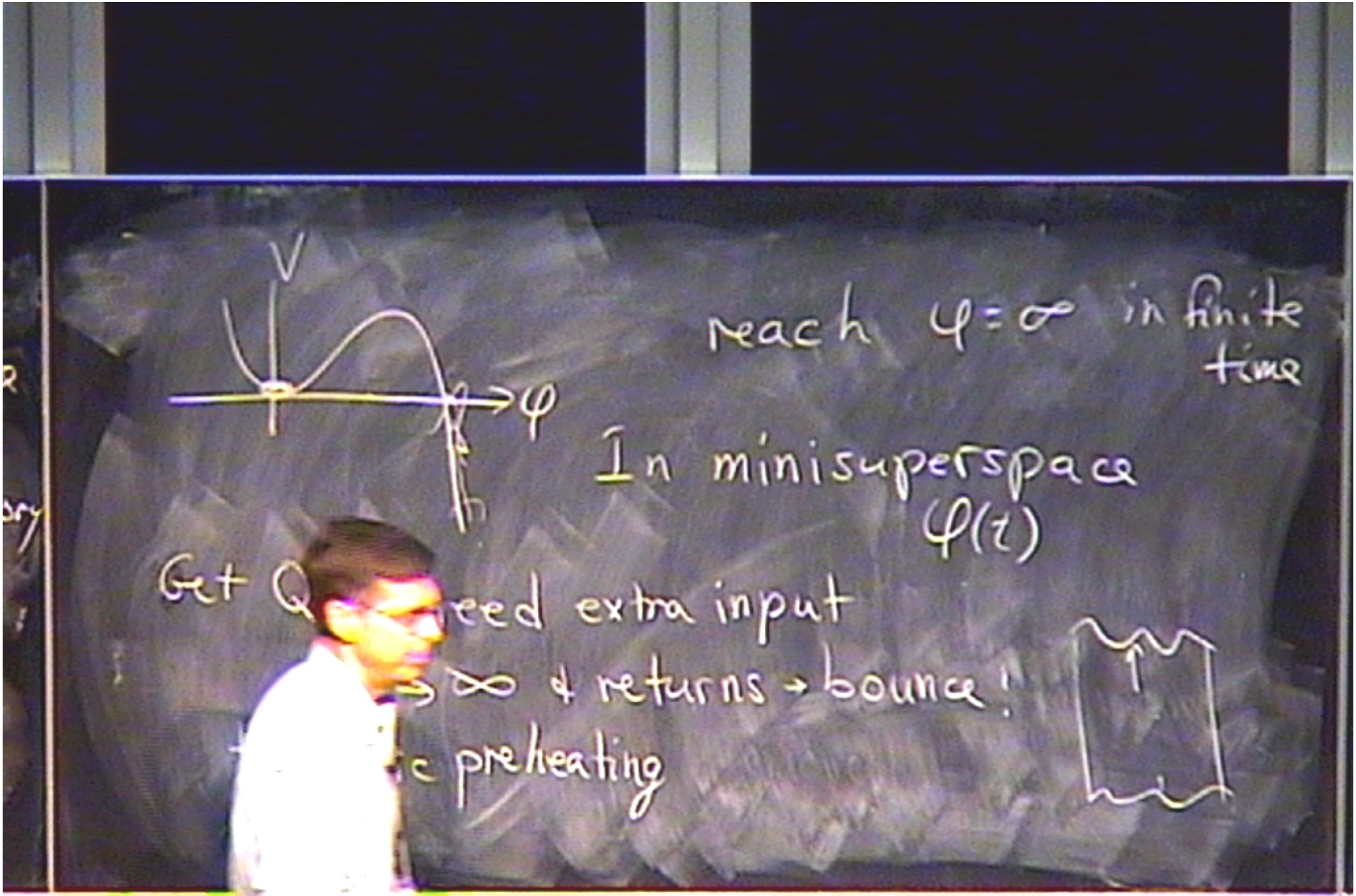
In minisuperspace

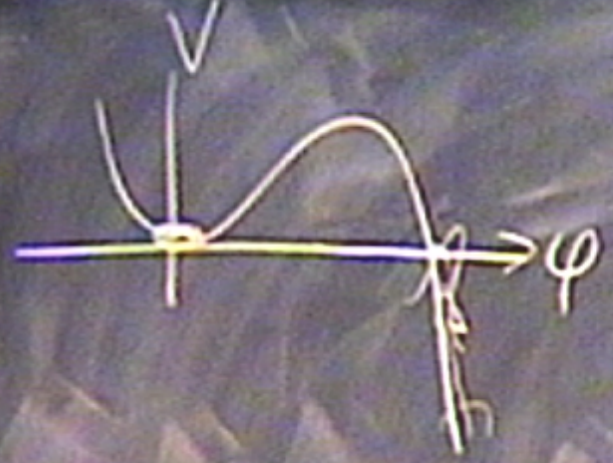
$\varphi(t)$

Get QM: need extra input

$\langle x \rangle \to \infty$ & returns → bounce!

tachyonic preheating


V
φ
reach φ = ∞ in finite time
In minisuperspace
φ(t)
Get Q
eed extra input
∞ & returns → bounce!
preheating



reach $\varphi = \infty$ in finite time

In minisuperspace
$\varphi(t)$

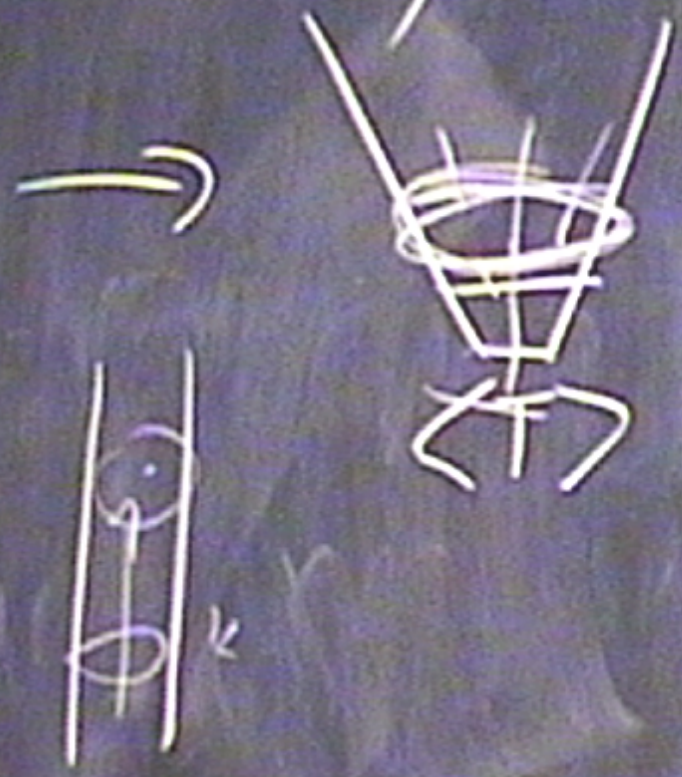
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tachyonic preheating



in & McGreevy: Antiperiodic



found 1-param

of states

get thermal dis

particles wi