Title: Cosmological Model Selection and the Inflationary Cosmology

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Abstract:

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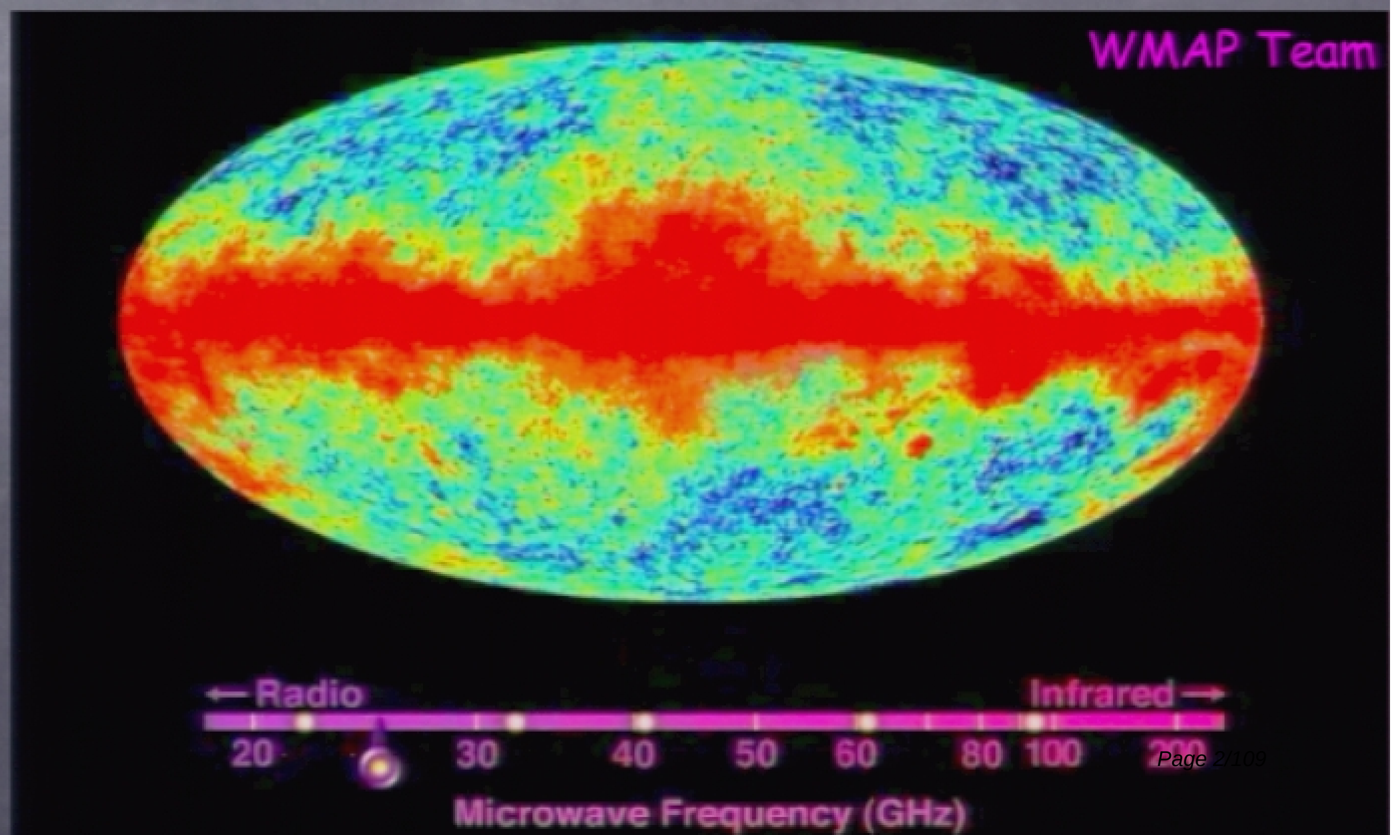
Cosmological model selection and inflation

Andrew Liddle
October 2005



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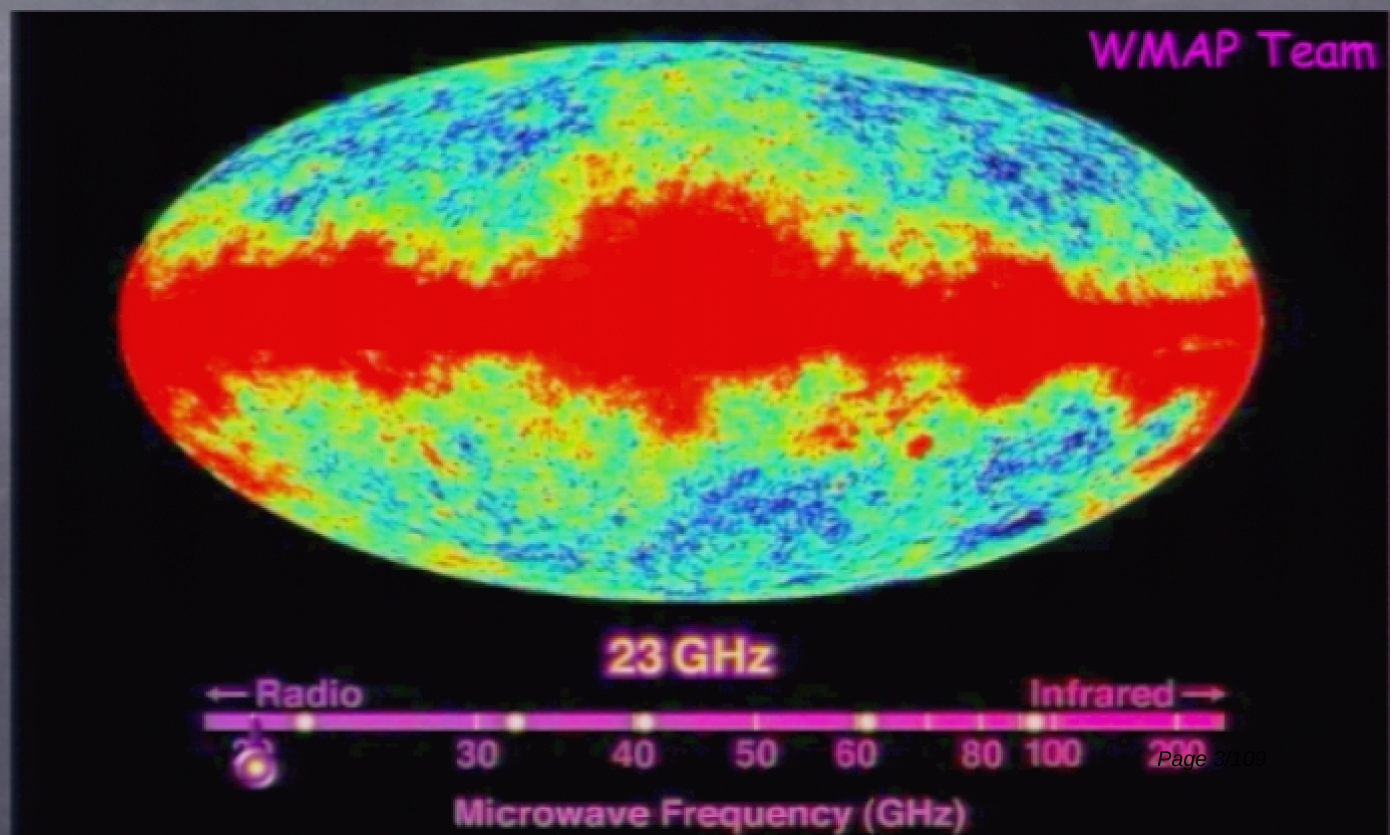

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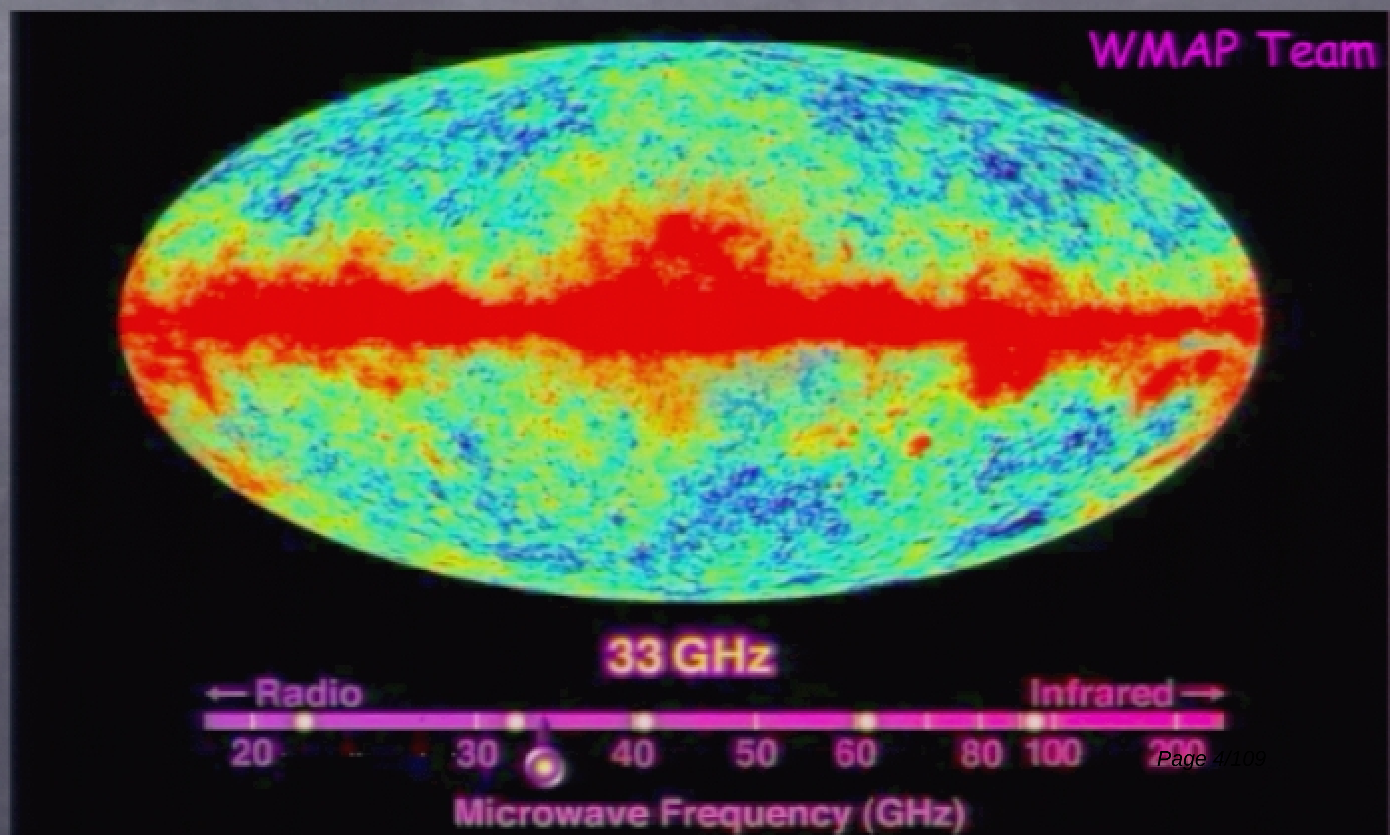

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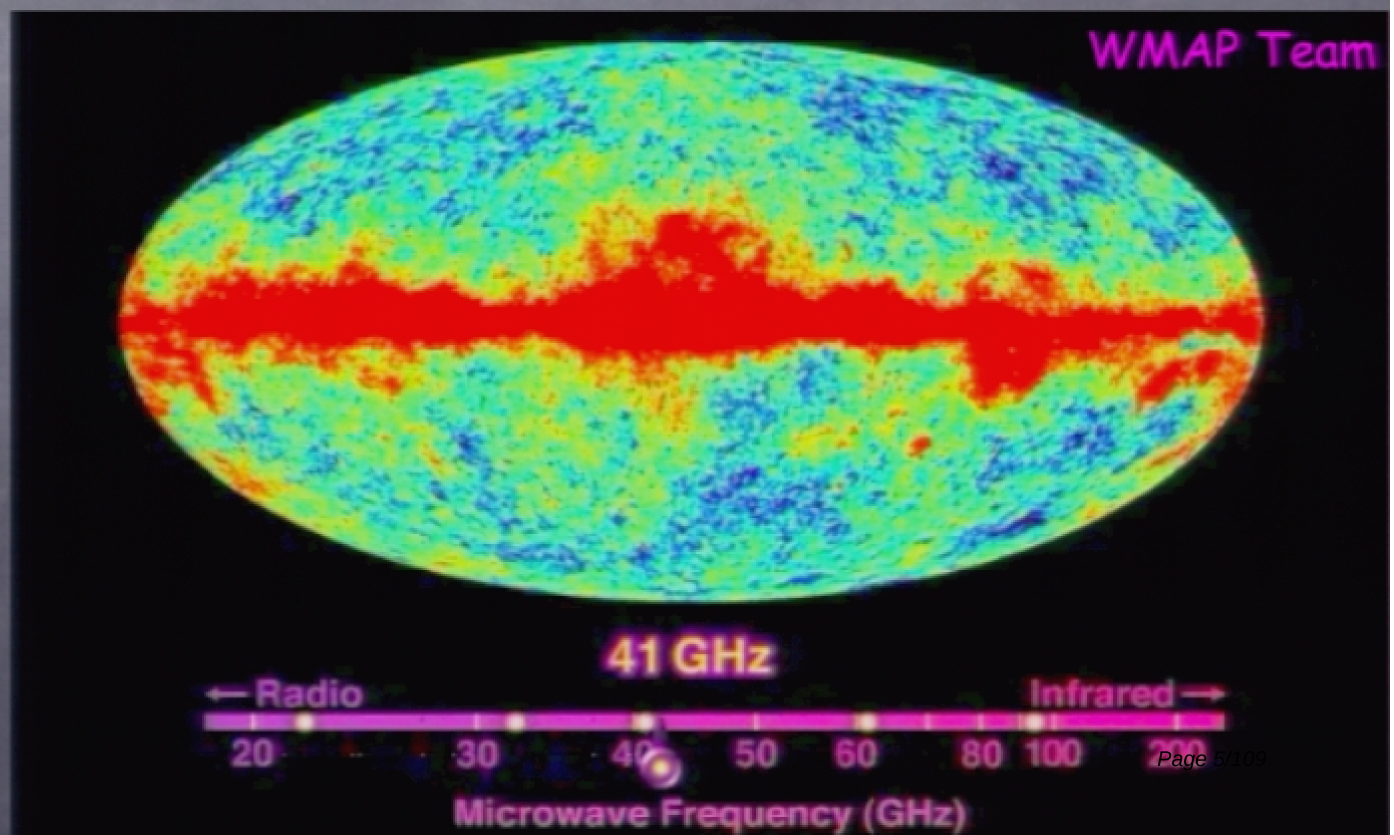

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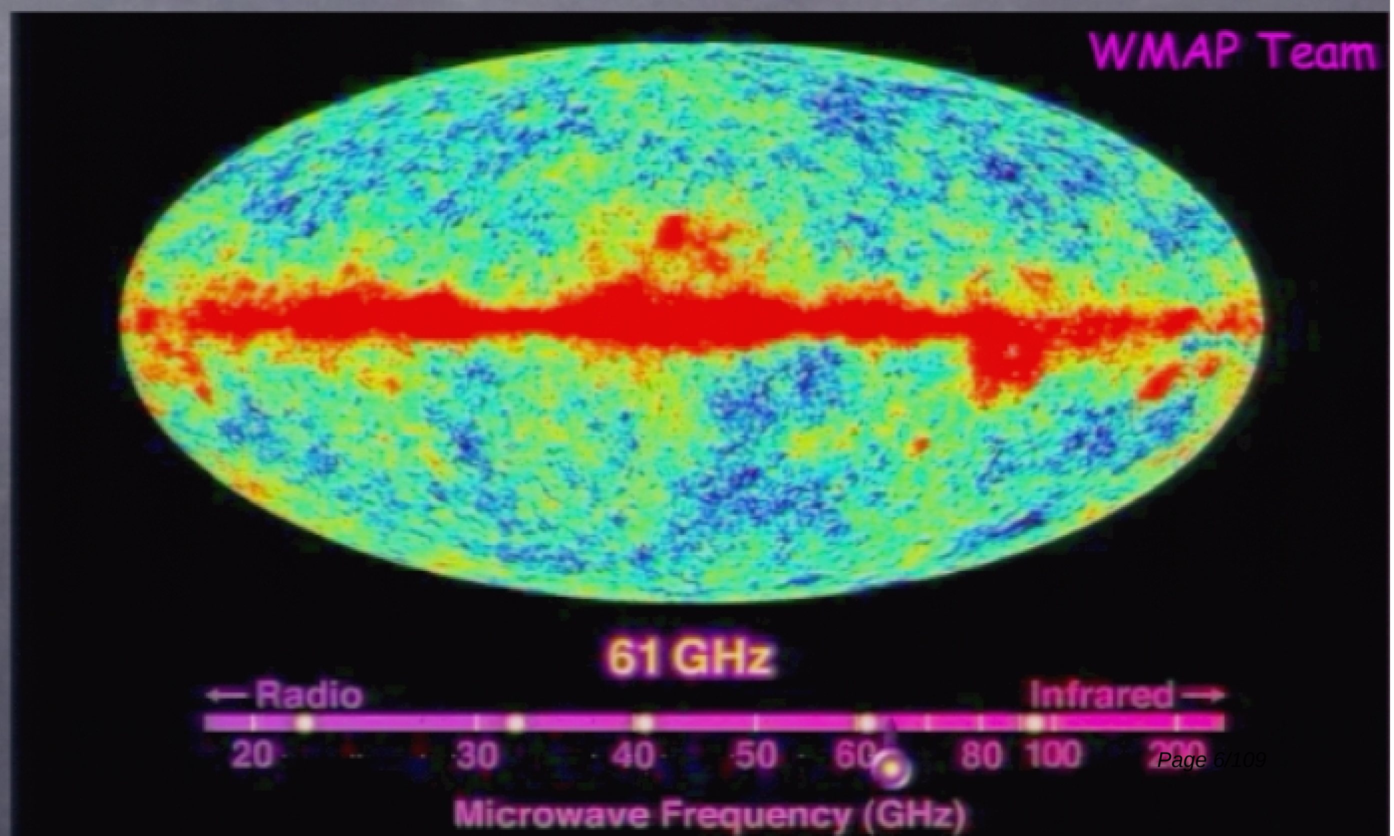

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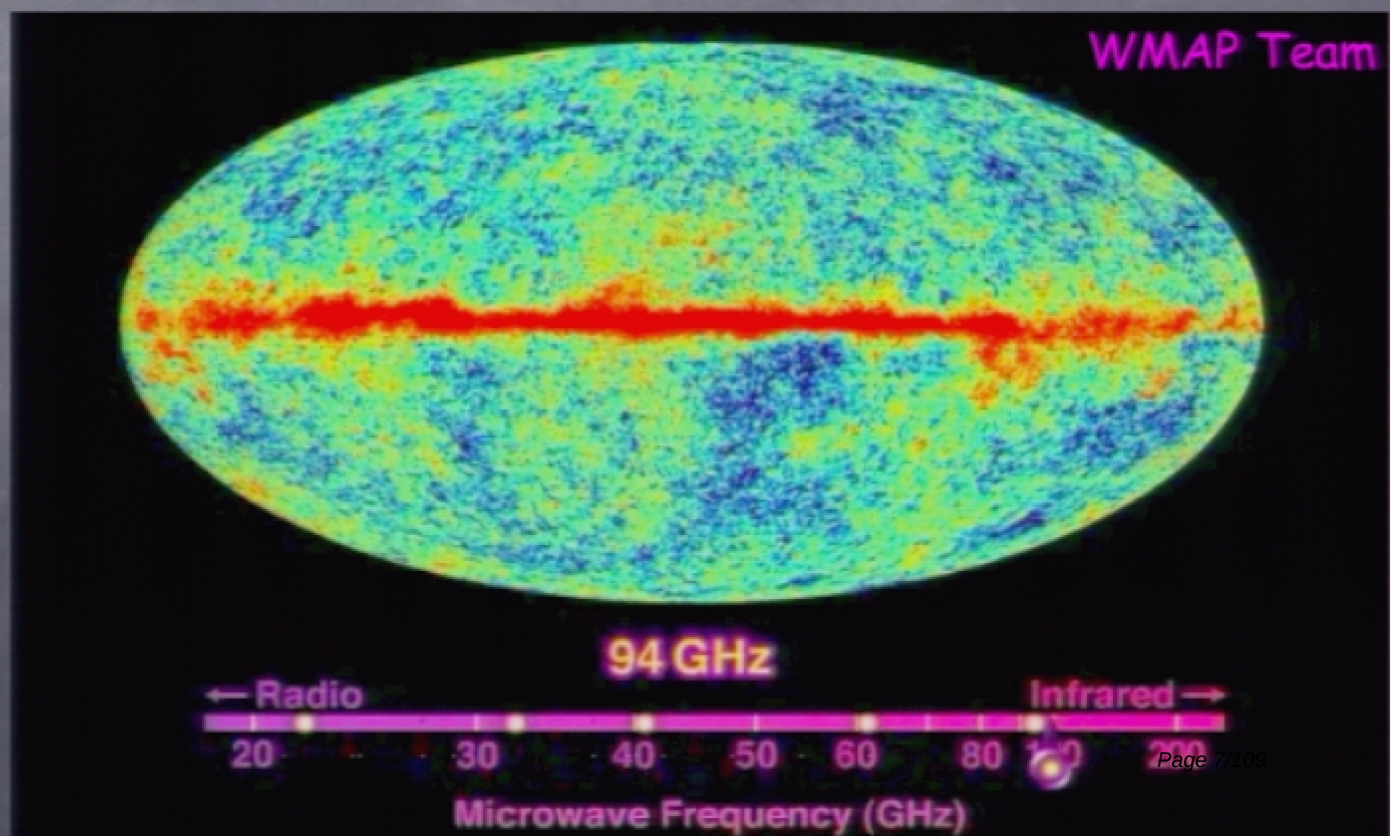

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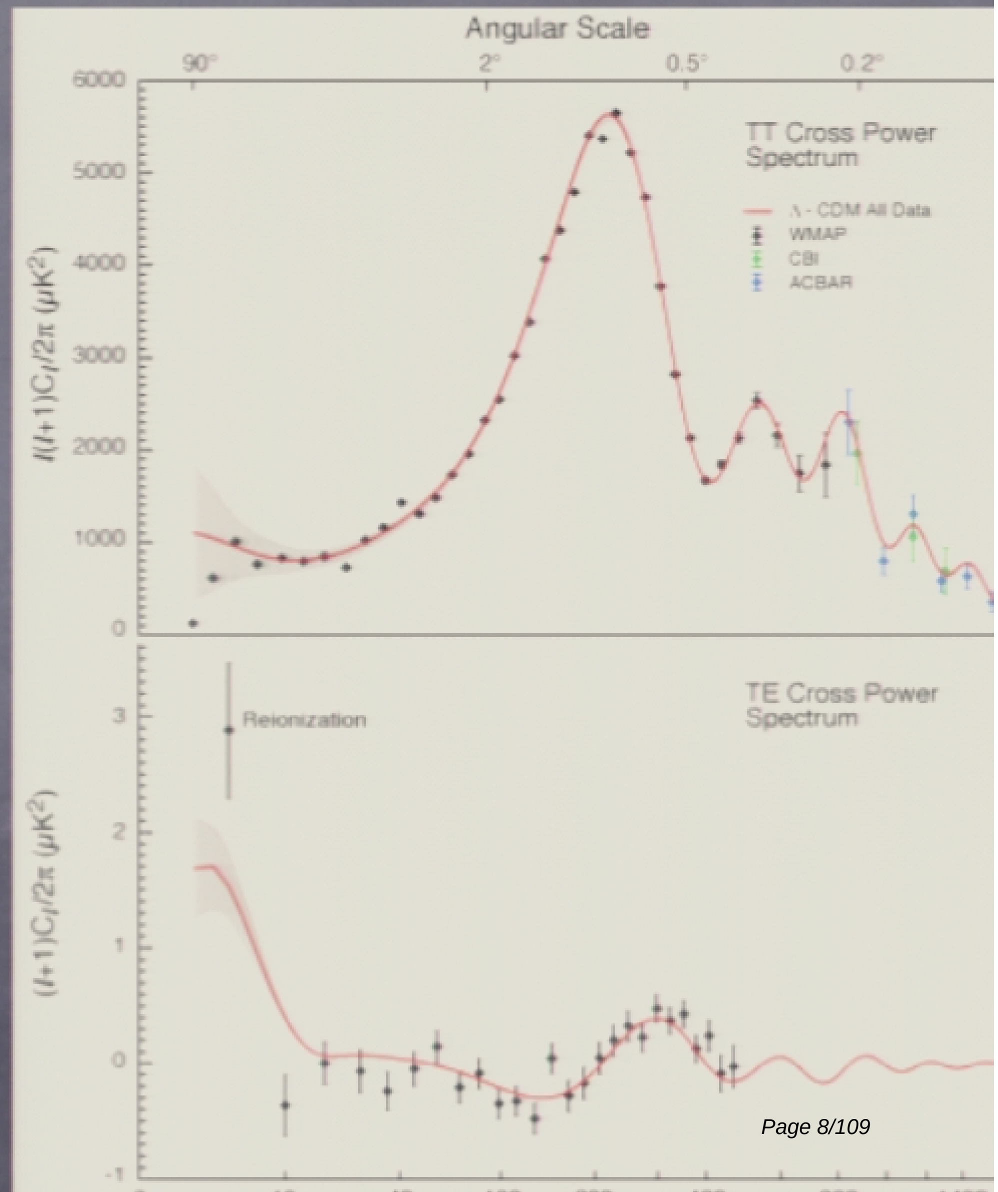
Conclusions from WMAP:

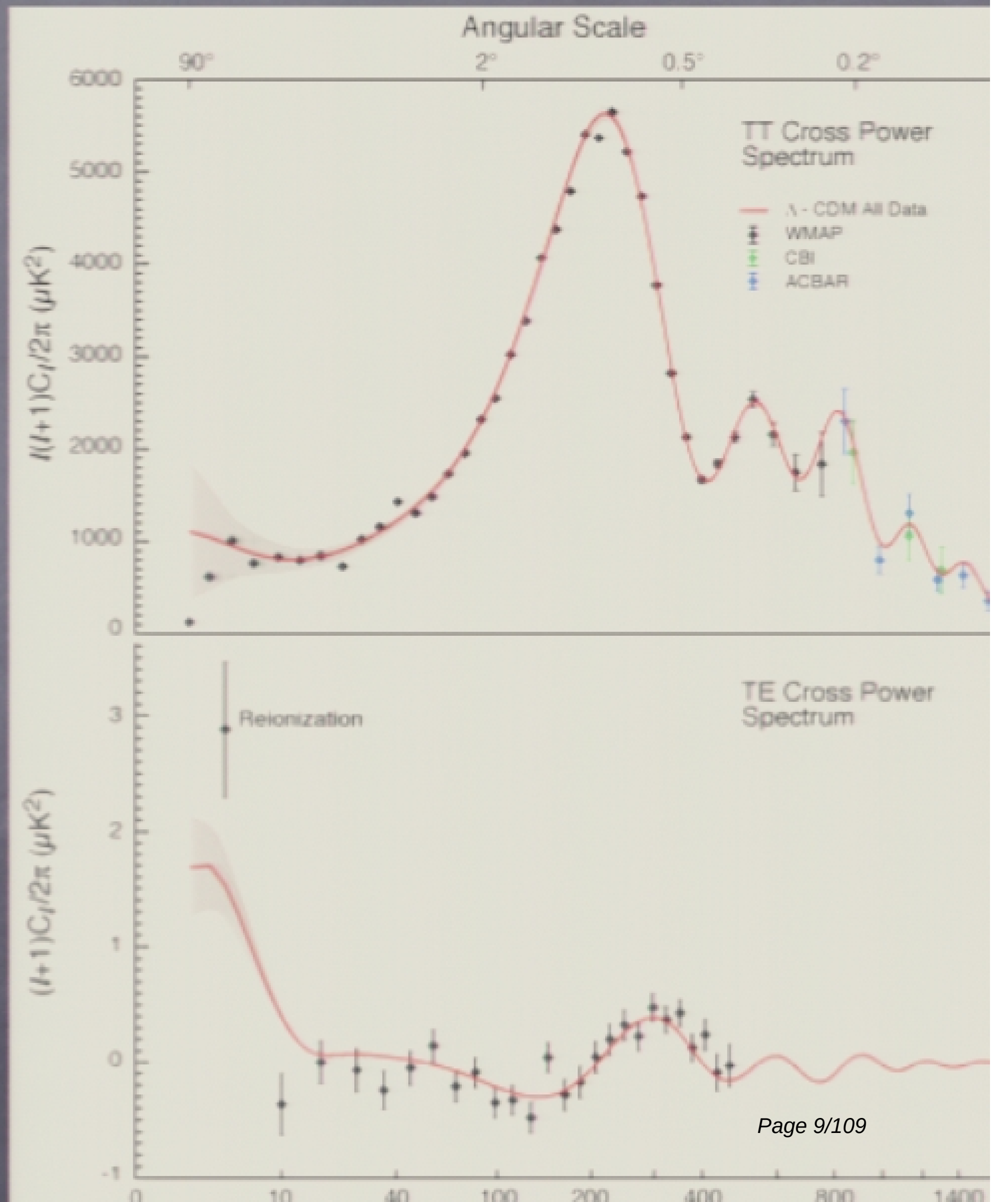
If you want to explain this data, the simplest way is ...



Conclusions from WMAP:

If you want to explain this data, the simplest way is ...

- A spatially-flat Universe
- Dark matter and dark energy
- Initial perturbations which are gaussian, adiabatic and nearly scale-invariant, e.g. as given by inflation.



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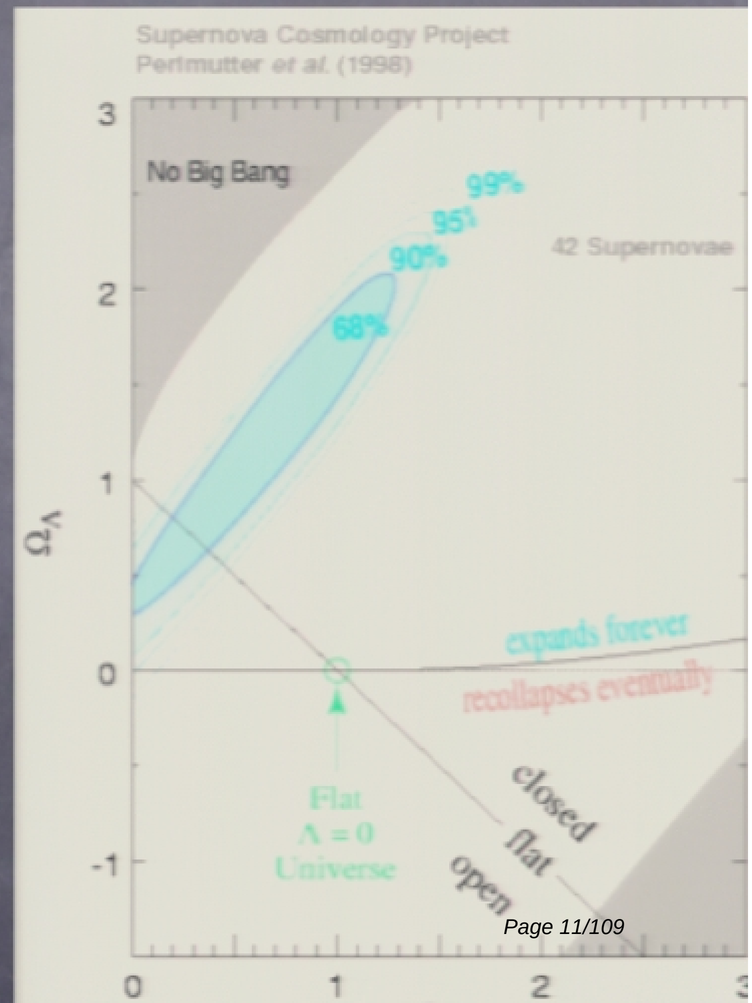

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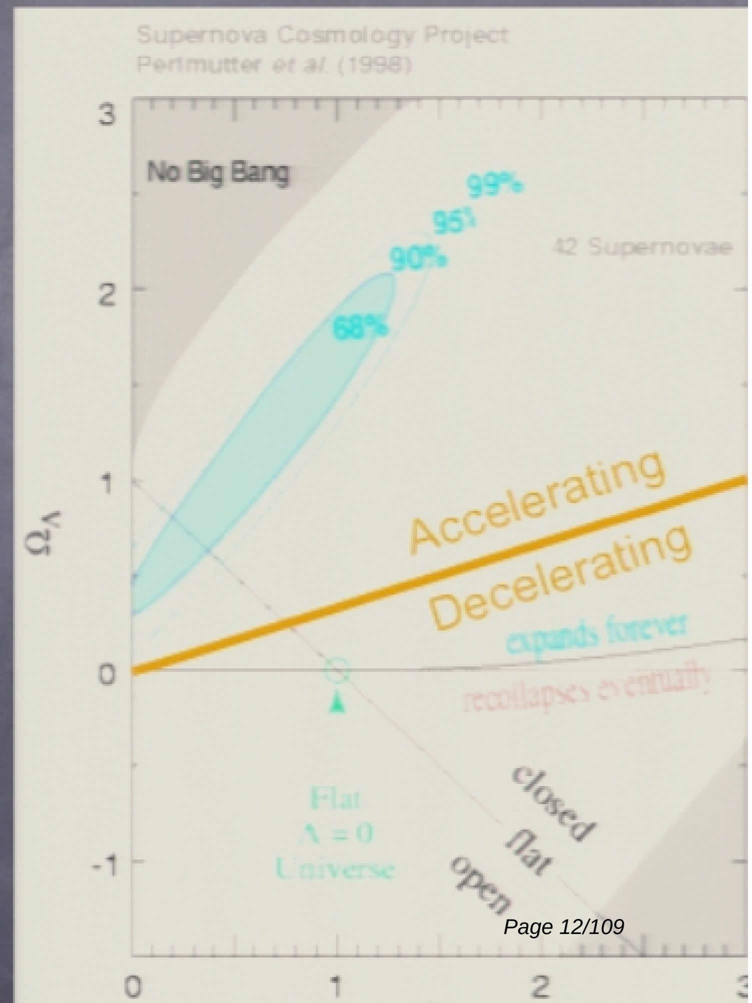

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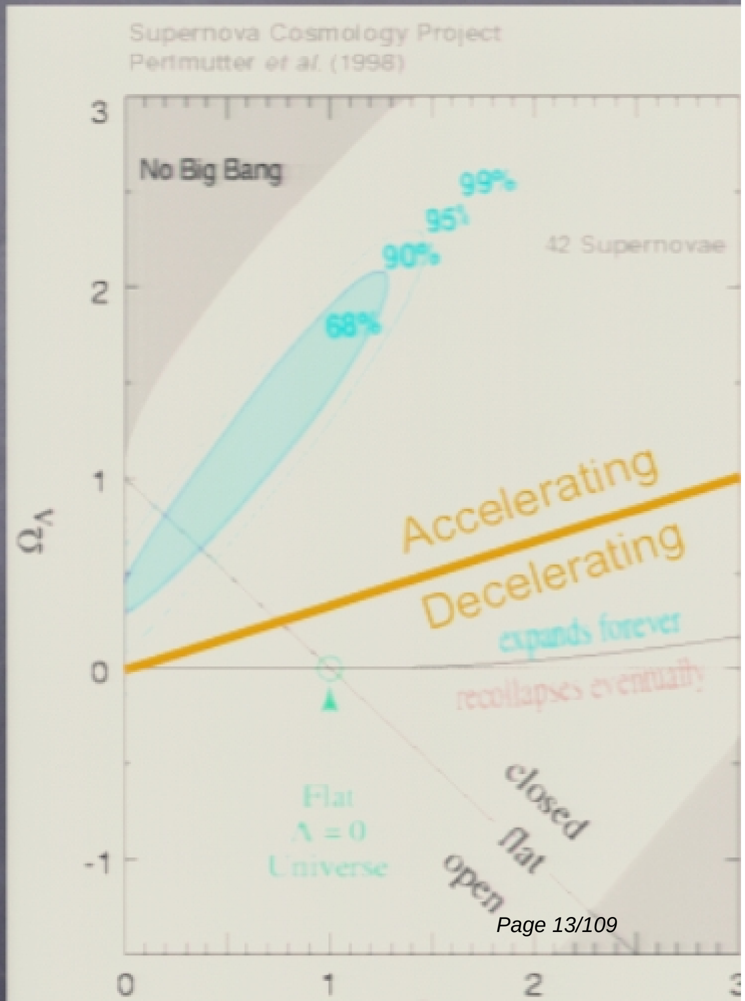
Early Universe inflation is the most plausible exlanation we have for the origin of structure.



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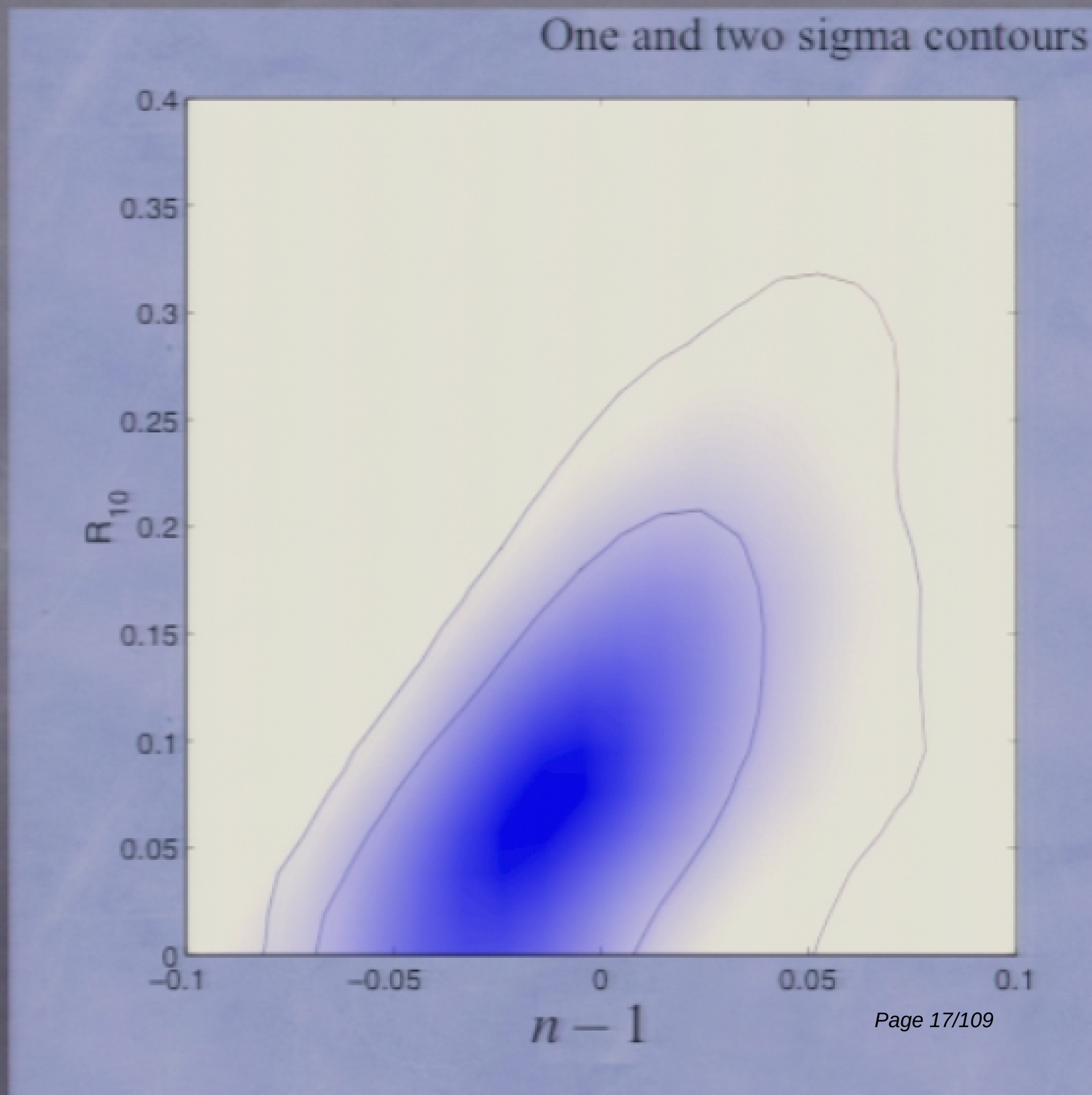
WMAP does not provide any evidence against any of these, and gives support to all but the gravitational waves. As such, it gives strong general support to the inflationary paradigm (but not uniquely to inflation).

Current constraints

Leach & Liddle, PRD, astro-ph/0306305

Comparison with observations:

- Fit to data compilation of WMAP, other CMB experiments (VSA, CBI and ACBAR), and 2dF galaxy survey.
- Use CAMB plus CosmoMC plus WMAP likelihood code plus slow-roll inflation module.



Current status of single-field inflation models

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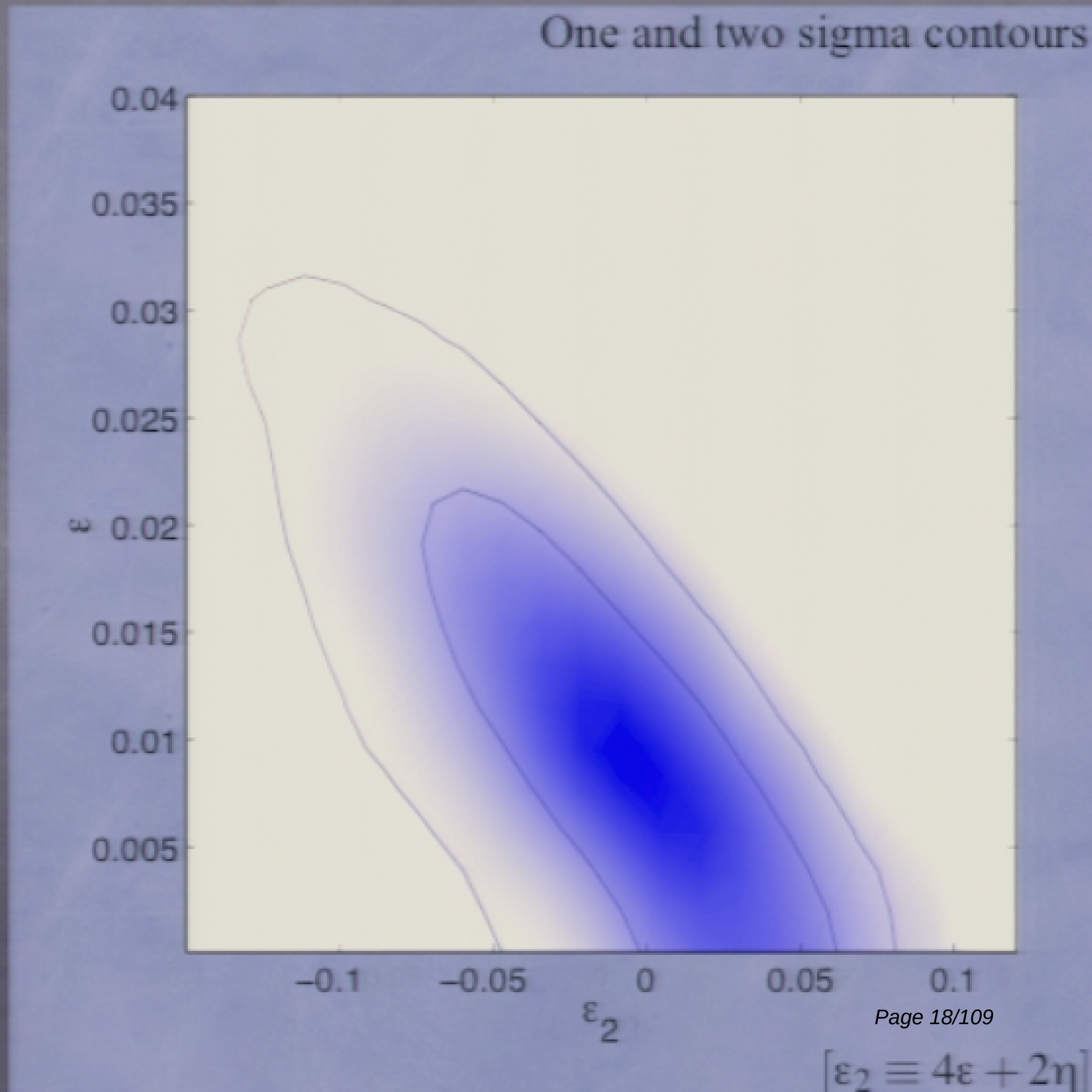

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- Gravitational waves: Not detected: $r < 0.43$ (for a power-law fit to the data)
- Growing mode: Good fit to data assuming no decaying mode. Temperature-polarization anti-correlation.
- Spatial flatness: $\Omega_{\text{tot}} = 1.02 \pm 0.02$

Conclusion: the simplest inflation models are doing very well!!!

More complicated models

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Even if effects from these more complex models are never seen, they introduce degeneracies in interpretting observations.

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- The set of cosmological parameters used to define the cosmological model.

There have been a variety of choices made for both of these.

WMAP: Spergel et al

Table 7. Best Fit Parameters: Power Law Λ CDM

	WMAP	WMAPext [16]a	WMAPext+2dFGRS	WMAPext+ 2dFGRS+ Lyman α
A	0.9 ± 0.1	0.8 ± 0.1	0.8 ± 0.1	$0.75^{+0.08}_{-0.07}$
n_s	0.99 ± 0.04	0.97 ± 0.03	0.97 ± 0.03	0.96 ± 0.02
τ	$0.166^{+0.076}_{-0.071}$	$0.143^{+0.071}_{-0.062}$	$0.148^{+0.073}_{-0.071}$	$0.117^{+0.057}_{-0.053}$
h	0.72 ± 0.05	0.73 ± 0.05	0.73 ± 0.03	0.72 ± 0.03
$\Omega_m h^2$	0.14 ± 0.02	0.13 ± 0.01	0.134 ± 0.006	0.133 ± 0.006
$\Omega_b h^2$	0.024 ± 0.001	0.023 ± 0.001	0.023 ± 0.001	0.0226 ± 0.0008
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Table 8. Best Fit Parameters for the Running Spectral Index ΛCDM Model

	WMAP	WMAPext	WMAPext+2dFGRS	WMAPext+ 2dFGRS+ Lyman α
A	0.92 ± 0.12	0.9 ± 0.1	0.84 ± 0.09	$0.83^{+0.09}_{-0.08}$
n_s	$0.93^{+0.07}_{-0.07}$	0.91 ± 0.06	$0.93^{+0.04}_{-0.05}$	0.93 ± 0.03
$dn_s/d\ln k$	-0.047 ± 0.04	-0.055 ± 0.038	$-0.031^{+0.023}_{-0.025}$	$-0.031^{+0.016}_{-0.017}$
τ	0.20 ± 0.07	0.20 ± 0.07	0.17 ± 0.06	0.17 ± 0.06
h	0.70 ± 0.05	0.71 ± 0.06	0.71 ± 0.04	$0.71^{+0.04}_{-0.03}$
$\Omega_m h^2$	0.14 ± 0.02	0.14 ± 0.01	0.136 ± 0.009	$0.135^{+0.008}_{-0.009}$
$\Omega_b h^2$	0.023 ± 0.002	0.022 ± 0.001	0.022 ± 0.001	0.0224 ± 0.0009
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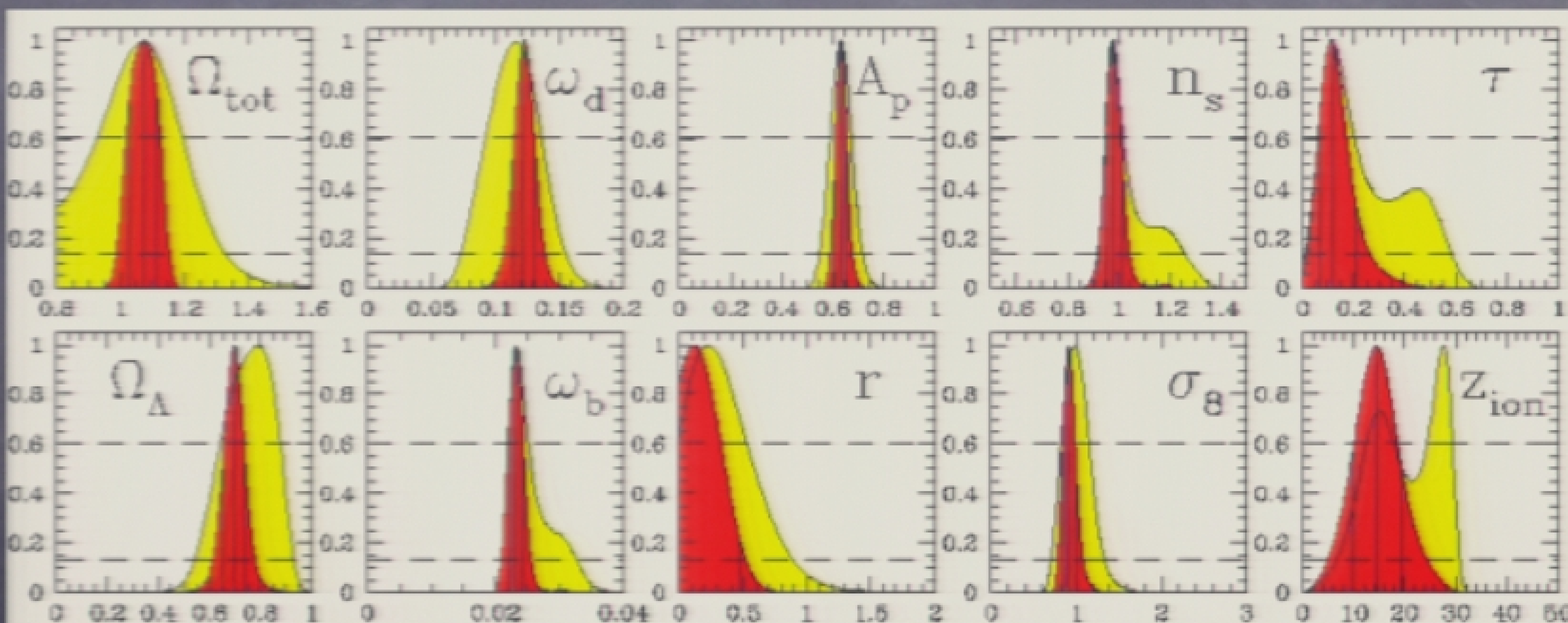

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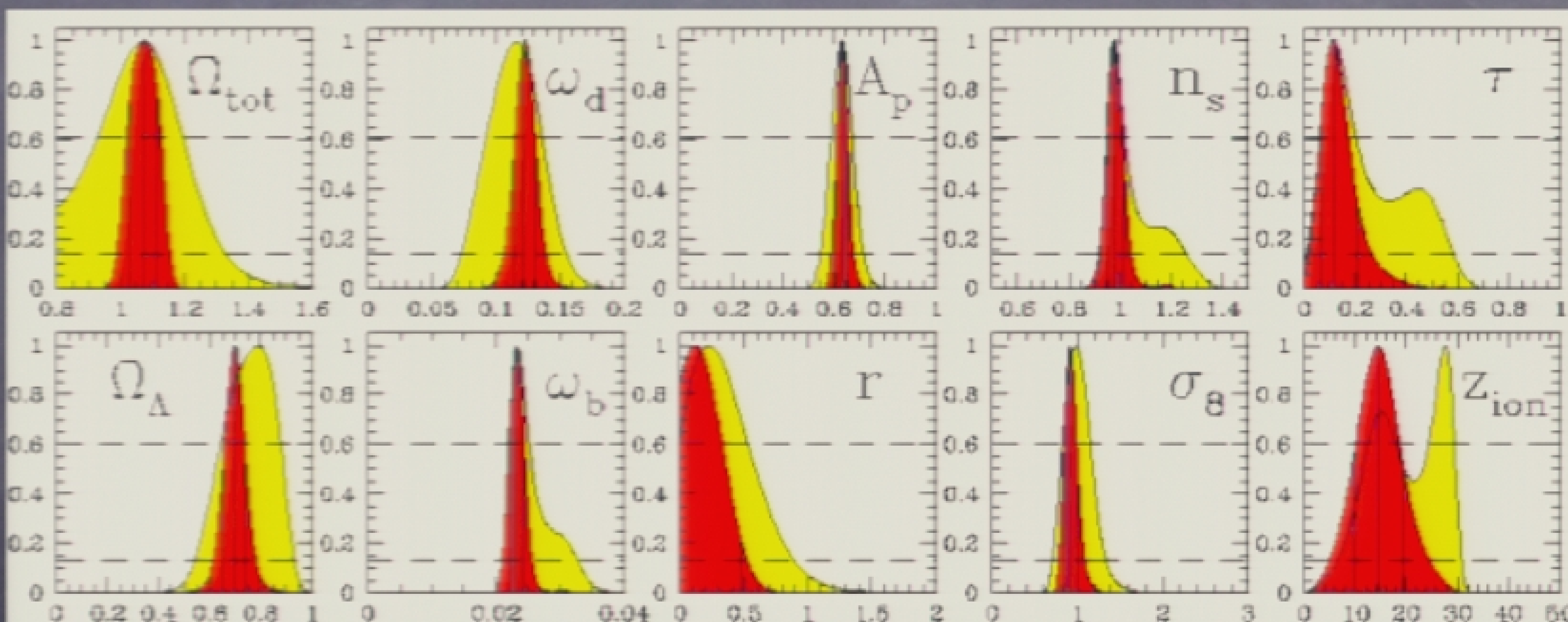

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The maximum likelihood gives the best values for the parameters, and the neighbouring behaviour gives the confidence limits.

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Ω_{m}	matter density
Ω_{b}	baryon density
Ω_{r}	radiation density
h	hubble parameter
A	adiabatic density perturbation amplitude
τ	reionization optical depth
b	bias parameter (or parameters)

There are many, many ways in which this base cosmological model can be extended.

Table 2. Candidate parameters: those which might be relevant for cosmological observations, but for which there is presently no convincing evidence requiring them. They are listed so as to take the value zero in the base cosmological model. Those above the line are parameters of the background homogeneous cosmology, and those below describe the perturbations. Of the latter set, the first six refer to adiabatic perturbations, the next three to tensor perturbations, and the remainder to isocurvature perturbations.

Ω_k	spatial curvature
$N_\nu - 3.04$	effective number of neutrino species (CMBFAST definition)
m_{ν_i}	neutrino mass for species 'i'
	[or more complex neutrino properties]
m_{dm}	(warm) dark matter mass
$w + 1$	dark energy equation of state
dw/dz	redshift dependence of w
	[or more complex parametrization of dark energy evolution]
$c_S^2 - 1$	effects of dark energy sound speed
$1/r_{\mathrm{top}}$	topological identification scale
	[or more complex parametrization of non-trivial topology]
$d\alpha/dz$	redshift dependence of the fine structure constant
dG/dz	redshift dependence of the gravitational constant
$n - 1$	scalar spectral index
$dn/d\ln k$	running of the scalar spectral index
k_{cut}	large-scale cut-off in the spectrum
A_{feature}	amplitude of spectral feature (peak, dip or step) ...
k_{feature}	... and its scale
	[or adiabatic power spectrum amplitude parametrized in N bins]
f_{NL}	quadratic contribution to primordial non-gaussianity
	[or more complex parametrization of non-gaussianity]
r	tensor-to-scalar ratio
$r + 8n_{\mathrm{T}}$	violation of the inflationary consistency equation
$dn_{\mathrm{T}}/d\ln k$	running of the tensor spectral index
$\mathcal{P}_S$	CDM isocurvature perturbation ...
n_S	... and its spectral index ...
$\mathcal{P}_{S\mathcal{R}}$	... and its correlation with adiabatic perturbations ...
$n_{S\mathcal{R}} - n_S$	... and the spectral index of that correlation
	[or more complicated multi-component isocurvature perturbation]
$G\mu$	cosmic string component of perturbations

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We need a way of penalizing use of extra parameters - an implementation of Ockham's razor.

Model Selection Statistics

Liddle, MNRAS, astro-ph/0401198

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- Bayesian information criterion (Schwarz 1978)
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- Bayesian evidence (Jeffreys 1961 etc)

$$E = \int d\theta\,\mathcal{L}(\theta)\,\mathrm{pr}(\theta)$$

θ = parameter vector, pr = prior

The preferred model is the one which minimizes the information criterion, or maximizes the evidence.

The Bayesian evidence is the most powerful of these. It is a full implementation of Bayesian inference, and literally gives the probability of the data given the model (note not the probability of particular parameter values). If multiplied by the prior model probability it gives the posterior model probability. However it can be hard to calculate, being a highly-peaked multi-dimensional integral.

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The Akaike Information Criterion was derived using information theory techniques. It gives an approximate minimization of the so-called Kullback-Leibler information entropy, which is a measure of the difference between two probability distributions.

Model selection techniques are essential when considering whether or not new data requires the addition of new parameters to describe it.

A simple example: spatial curvature

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Assuming that the density is the only parameter, with a uniform prior from 0.1 to 2, and likelihood

$$\mathcal{L} = \mathcal{L}_0 \exp\left(-\frac{(\Omega - 1.02)^2}{2 \times 0.02^2}\right)$$

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Assuming that the density is the only parameter, with a uniform prior from 0.1 to 2, and likelihood

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- Flat: $\text{Evidence} = \mathcal{L}(\Omega = 1) = 0.6\mathcal{L}_0$
- Curved: $\text{Evidence} = \frac{1}{1.9}\int \mathcal{L}(\Omega) d\Omega \simeq 0.03\mathcal{L}_0$

According to the evidence, the flat model is a better description of the data, with odds of about 20:1 **against** the curved model.

Note that this assumes flat and curved were thought equally likely before the data came along.

A simple example: spatial curvature

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Notes:

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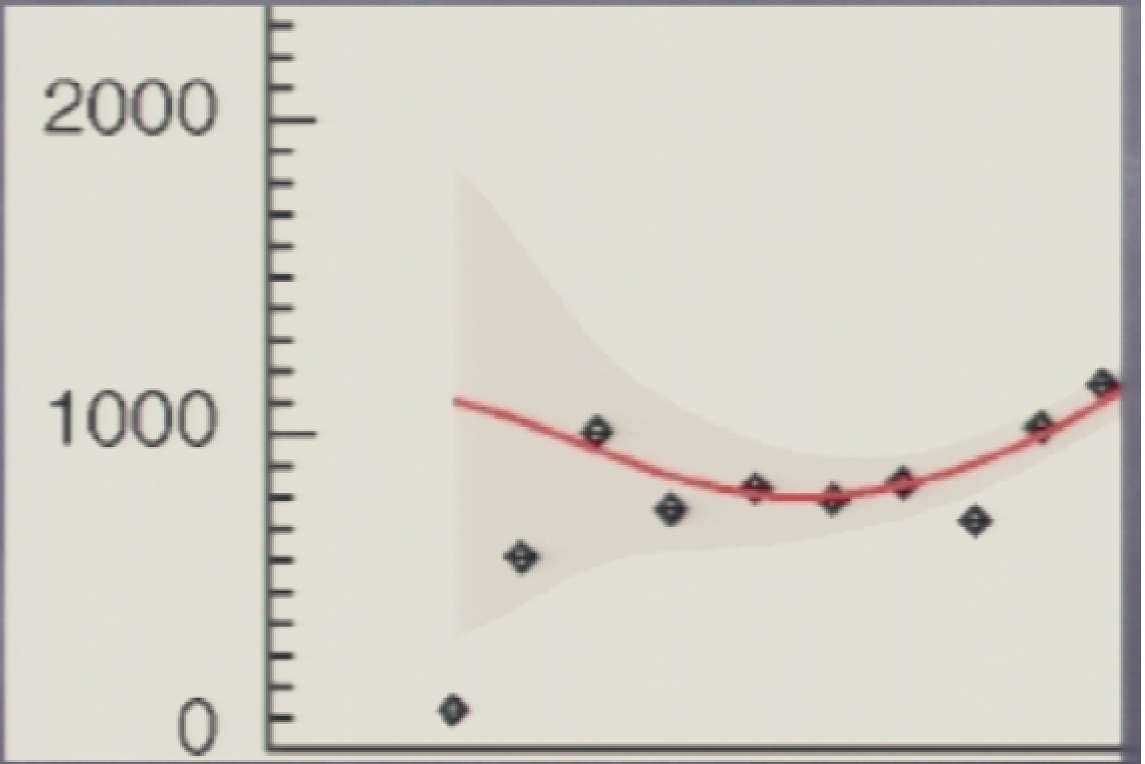
New physics from low quadrupole??

[Argument roughly following Niarchou et al]



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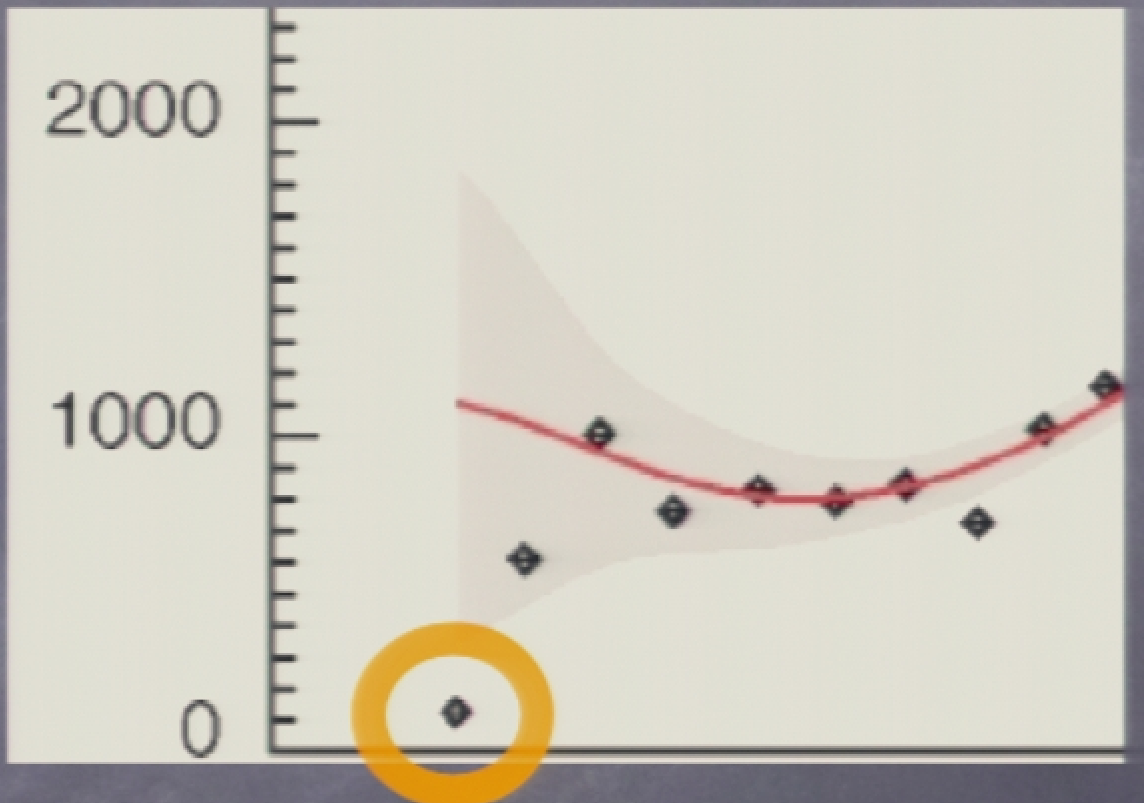

If you want to explain this with new physics, you have to introduce new parameters, for which you will be penalized. As the discrepancy is only at the 95% level, the gain in fit will never compensate for this penalty.

How do we compare different cosmological models (i.e. different choices of fundamental parameters)? Can we say which model is best?

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- **Inappropriate "a posteriori" reasoning:** choosing "interesting" features from the data and assessing their significance via Monte Carlo analyses.
- **Neglect of model dimensionality:** using parameter estimation rather than model selection.

Model Selection and Isocurvature Modes

Beltran, Garcia-Bellido, Lesgourgues, Liddle, Slosar, PRD, astro-ph/0501477

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We consider the three observationally-distinct classes of isocurvature mode, CDI, NID and NIV. Only one type of mode is permitted per model, but with arbitrary spectral index and correlation to adiabatic: 4 extra parameters. We compare with two adiabatic models, one with $n=1$ and one with n varying.

Model Selection and Isocurvature Modes

The Bayesian Evidence was computed using a technique called thermodynamic integration. This is an MCMC method where the chains are heated in order to fully explore the prior space (parameter estimation chains sample the posterior which is usually localized to a small fraction of the prior).

We tested several variants on this scheme. Accurate determination of the evidence required approximately 10^7 likelihood evaluations per model, making it a supercomputer class problem.

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Jeffreys Scale:

$\Delta \ln E < 1$	Not worth more than a bare mention
$1 < \Delta \ln E < 2.5$	Substantial evidence
$2.5 < \Delta \ln E < 5$	Strong to very strong evidence
$5 < \Delta \ln E$	Decisive evidence

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AD-HZ	0.0 ± 0.1	
AD-n	0.0 ± 0.1	
[illegible]	-1.0 ± 0.2	-1.0 ± 0.2
NID	-1.0 ± 0.2	-2.0 ± 0.2
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Mukherjee, Parkinson and Liddle, astro-ph/0508461

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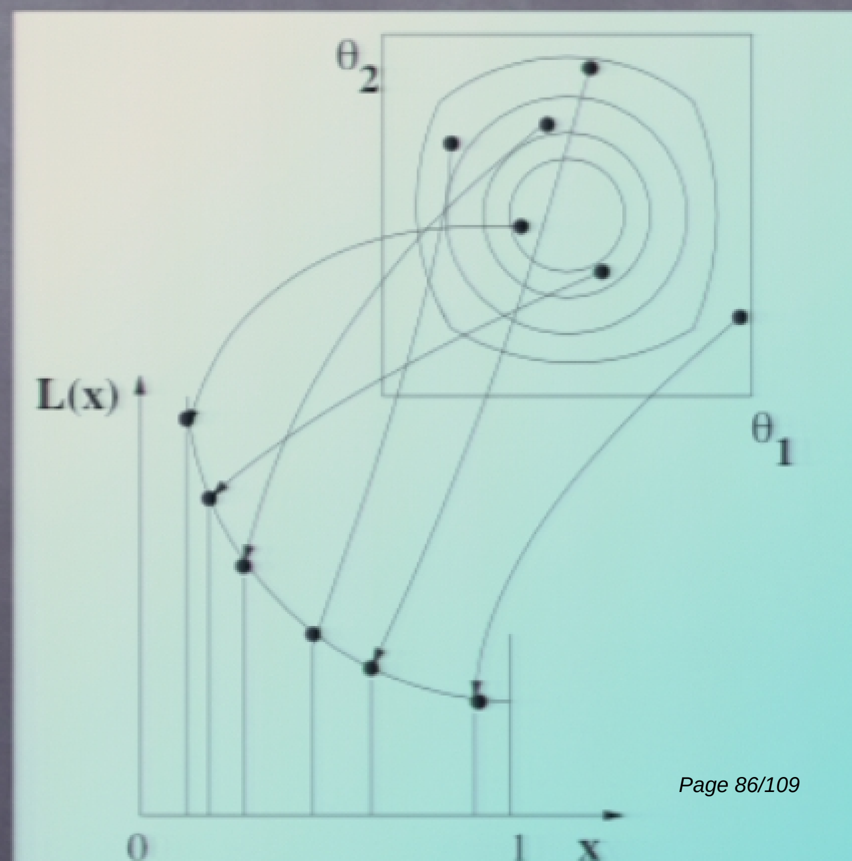
where X is the fractional prior mass.

This can then be evaluated using Monte Carlo samples to trace the variation of likelihood with prior mass, peeling away thin nested isosurfaces of equal likelihood.



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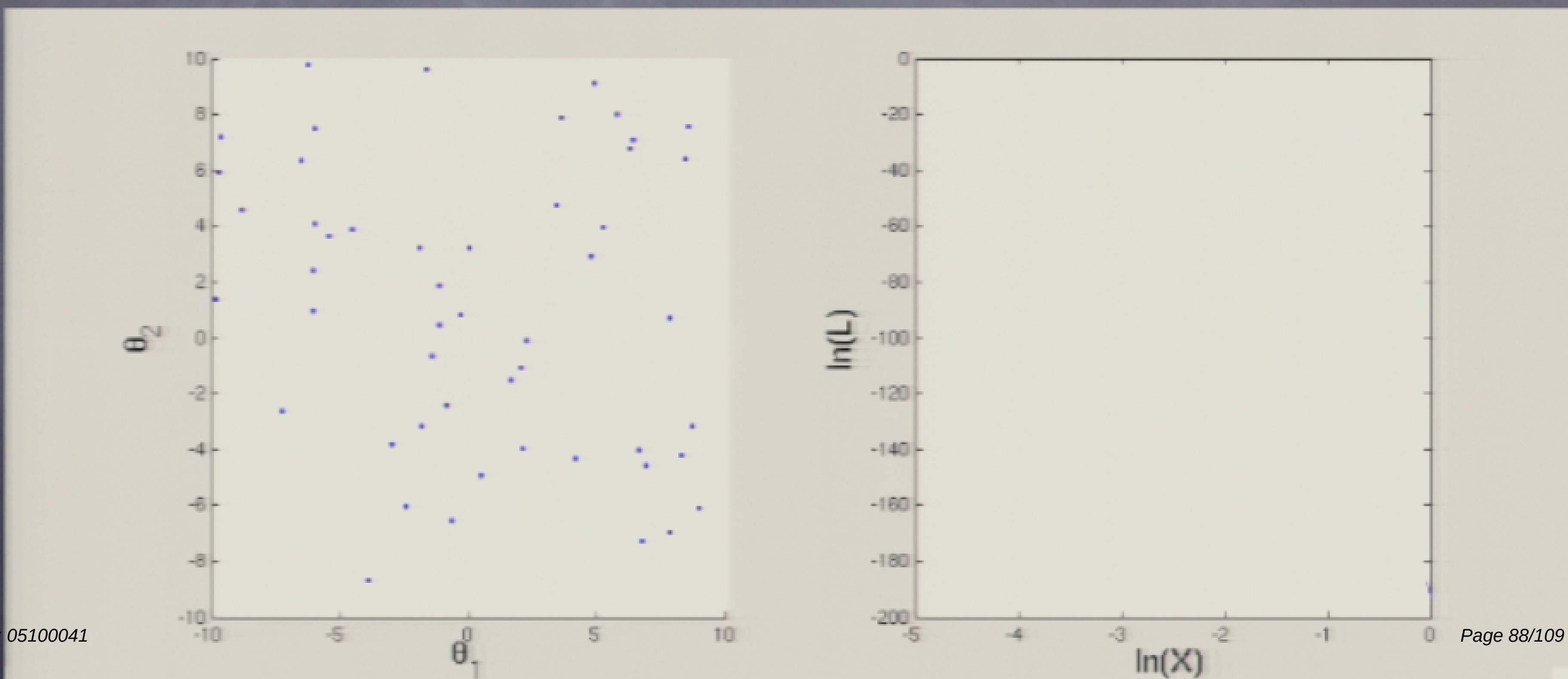

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Model	ΛCDM+HZ	ΛCDM+n_s	ΛCDM+n_s (wide prior)	HZ+w	$w+n_s$
n_s	1	$0.8-1.2$	$0.6-1.4$	1	$0.8-1.2$
w	-1	-1	-1	$-\frac{1}{3}--1$	$-\frac{1}{3}--1$
e.f	1.5	1.7	1.7	1.7	1.8
$N_{\rm like}(\times10^4)$	8.4	17.4	16.7	10.6	18.0
$\ln E$	0.00 ± 0.08	-0.58 ± 0.09	-1.16 ± 0.08	-0.45 ± 0.08	-1.52 ± 0.08

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At the moment the more complex models are not excluded, but they are mildly disfavoured against the simplest model.

Model selection for survey comparison/design

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- Bayes factor approach:
  simulate data at each point in parameter plane;
  compute Bayes factor (ie evidence ratio) of full model versus eg LambdaCDM at each point.

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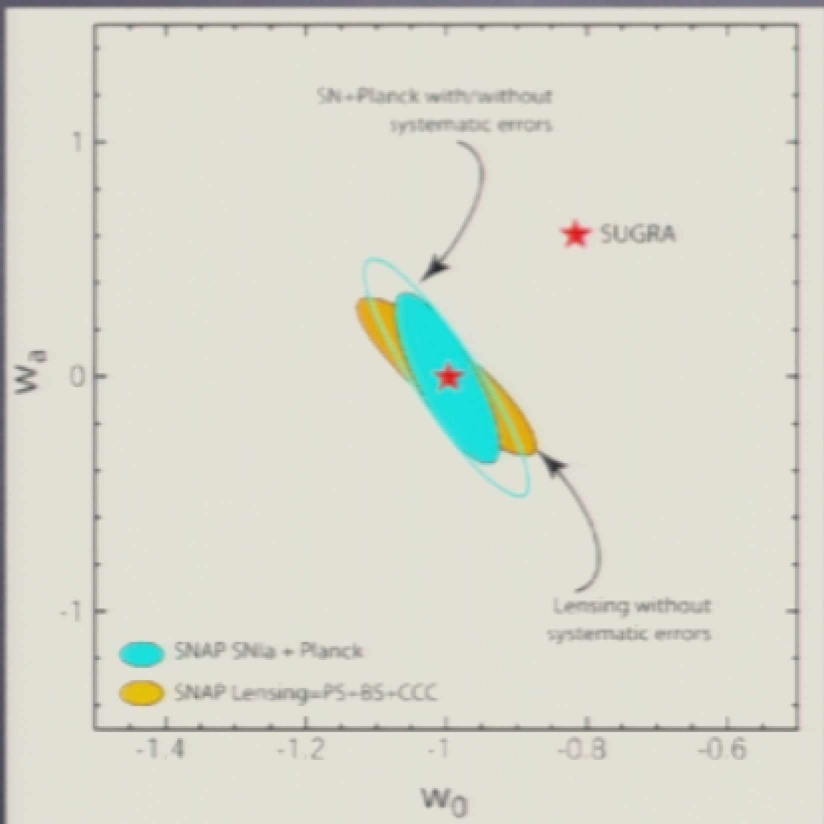

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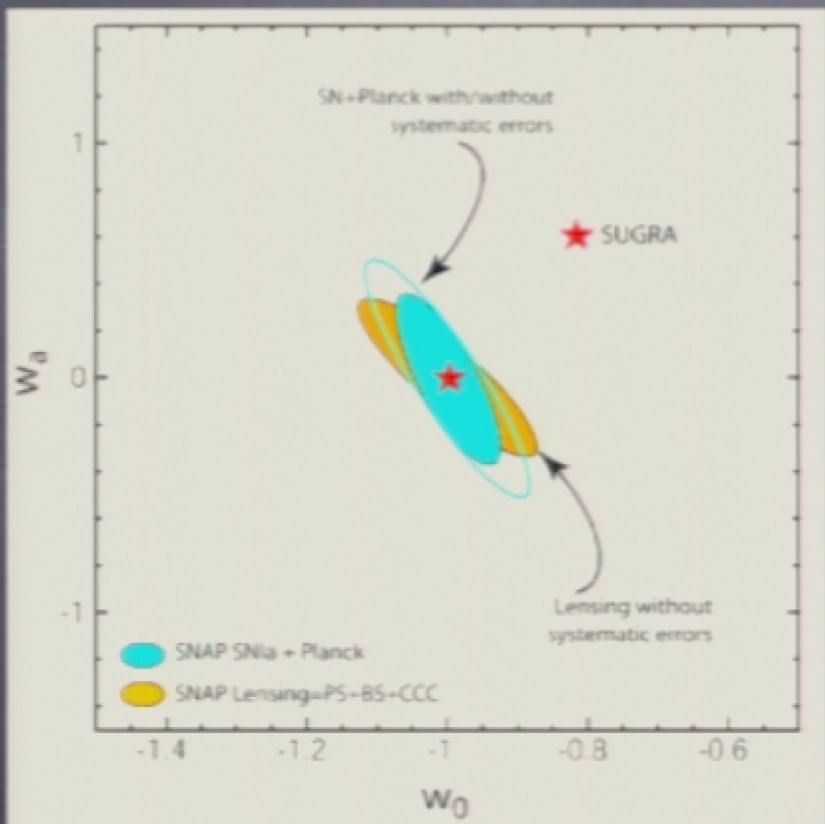

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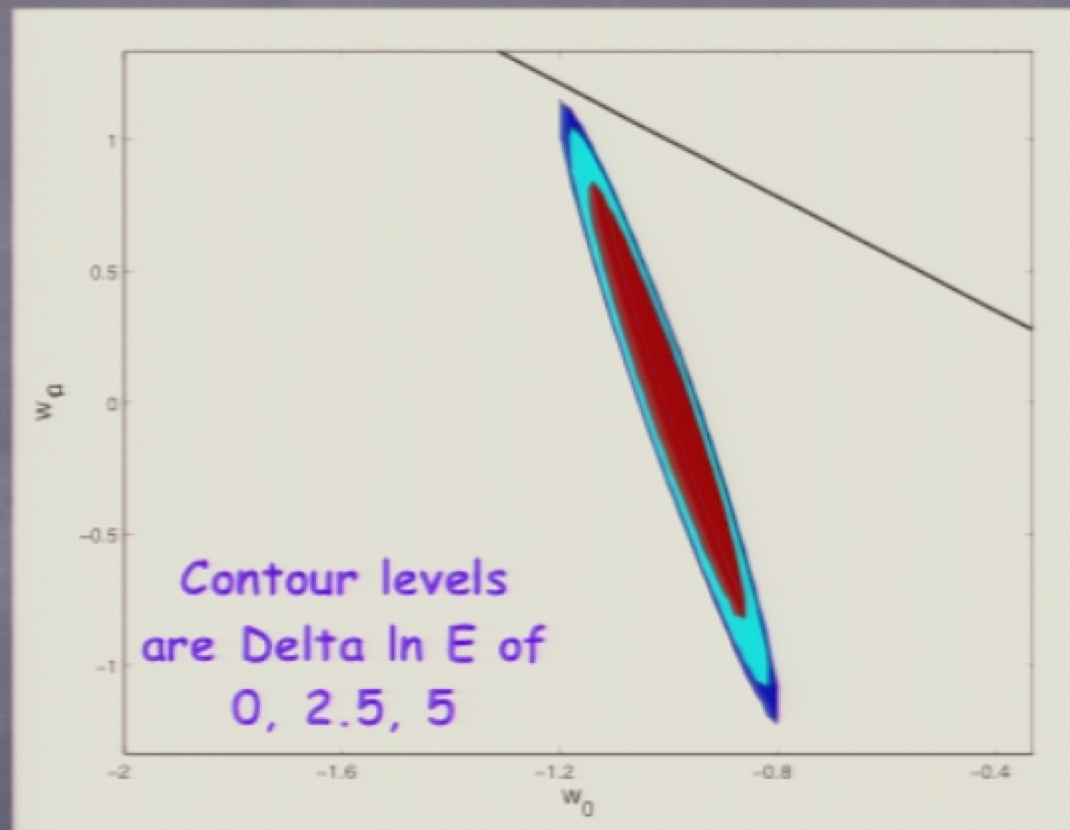

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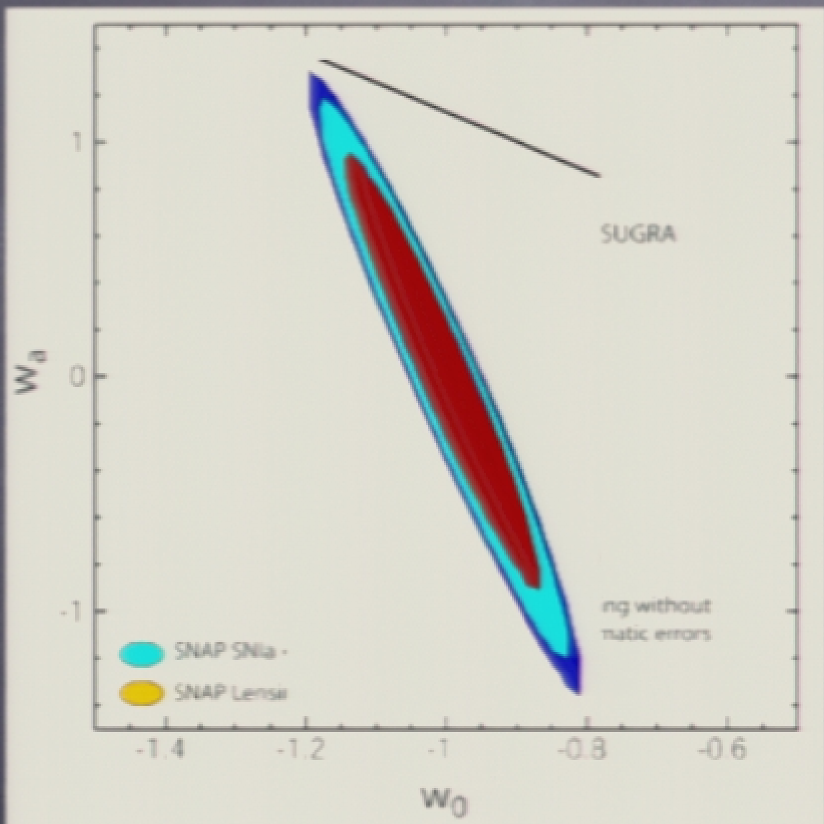

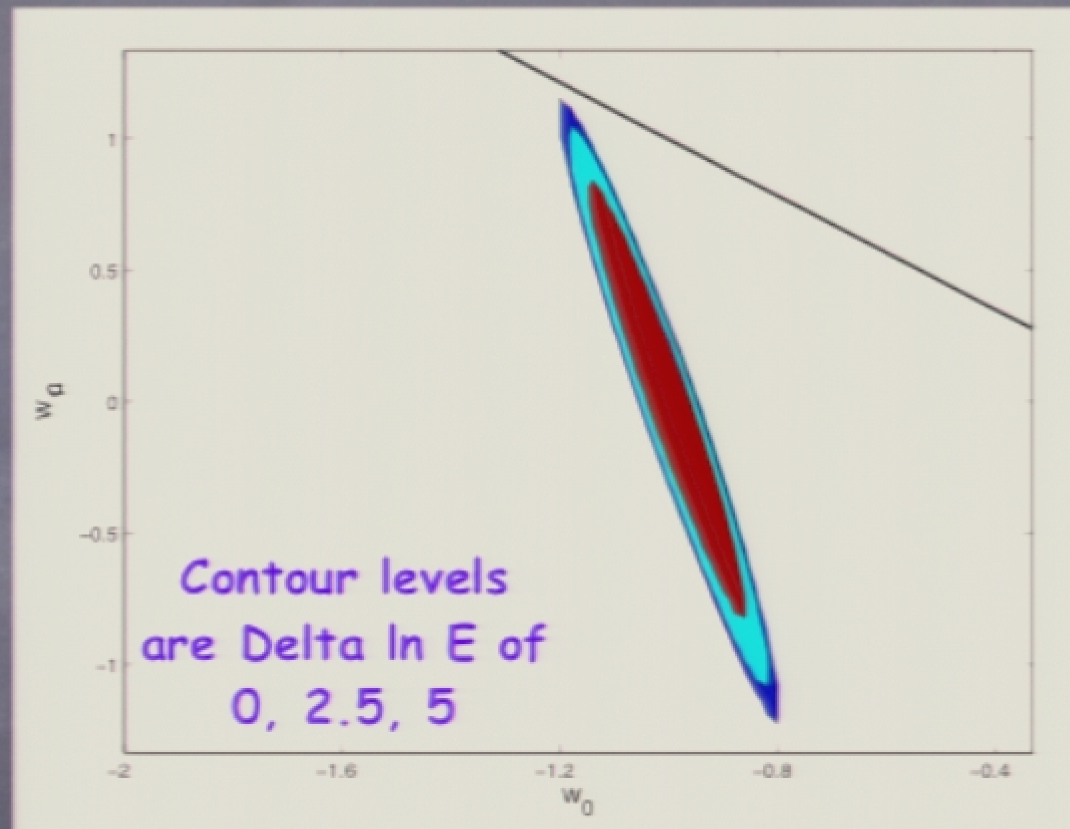

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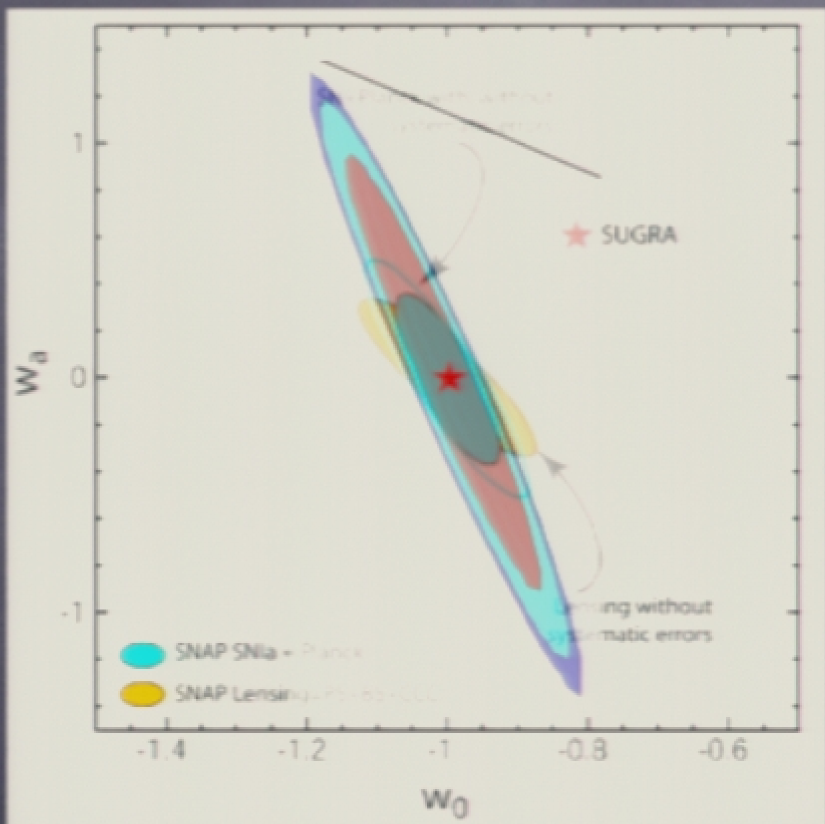

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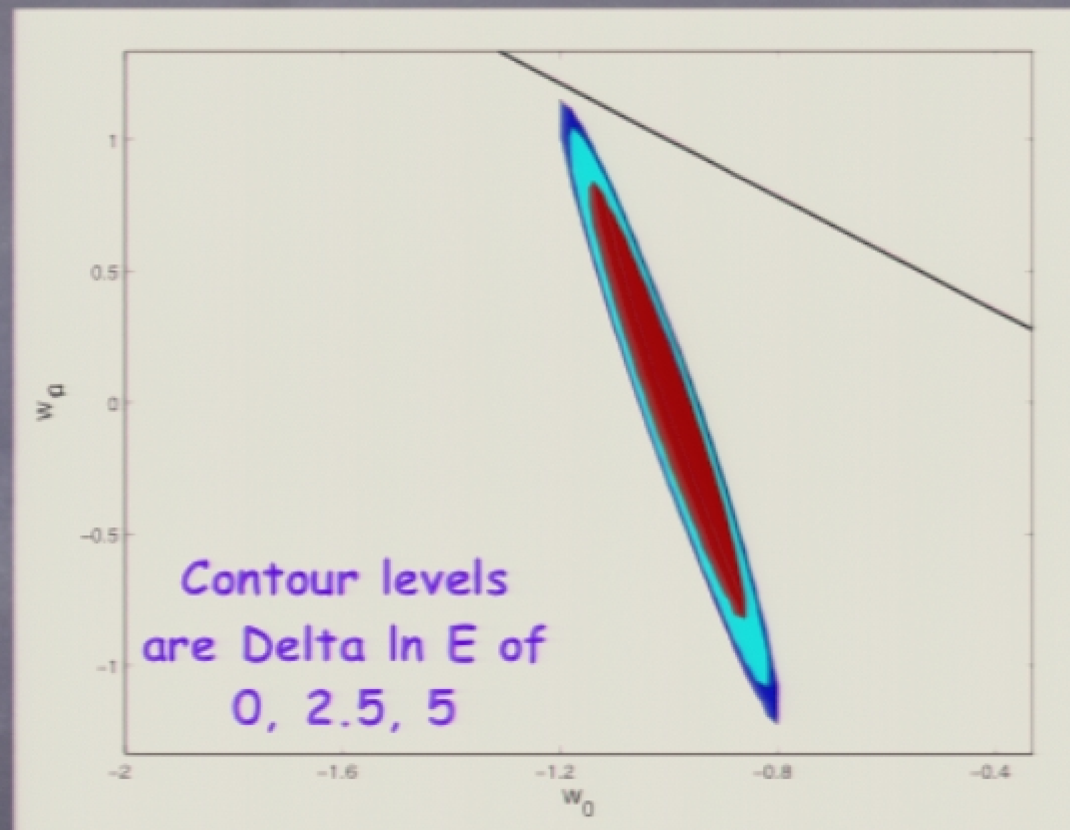

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Conclusions

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- The Bayesian evidence is the most powerful available tool. It is challenging to compute but nested sampling makes it feasible.
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