Title: Probing the Geometry of Space - Mathematics circa 1900

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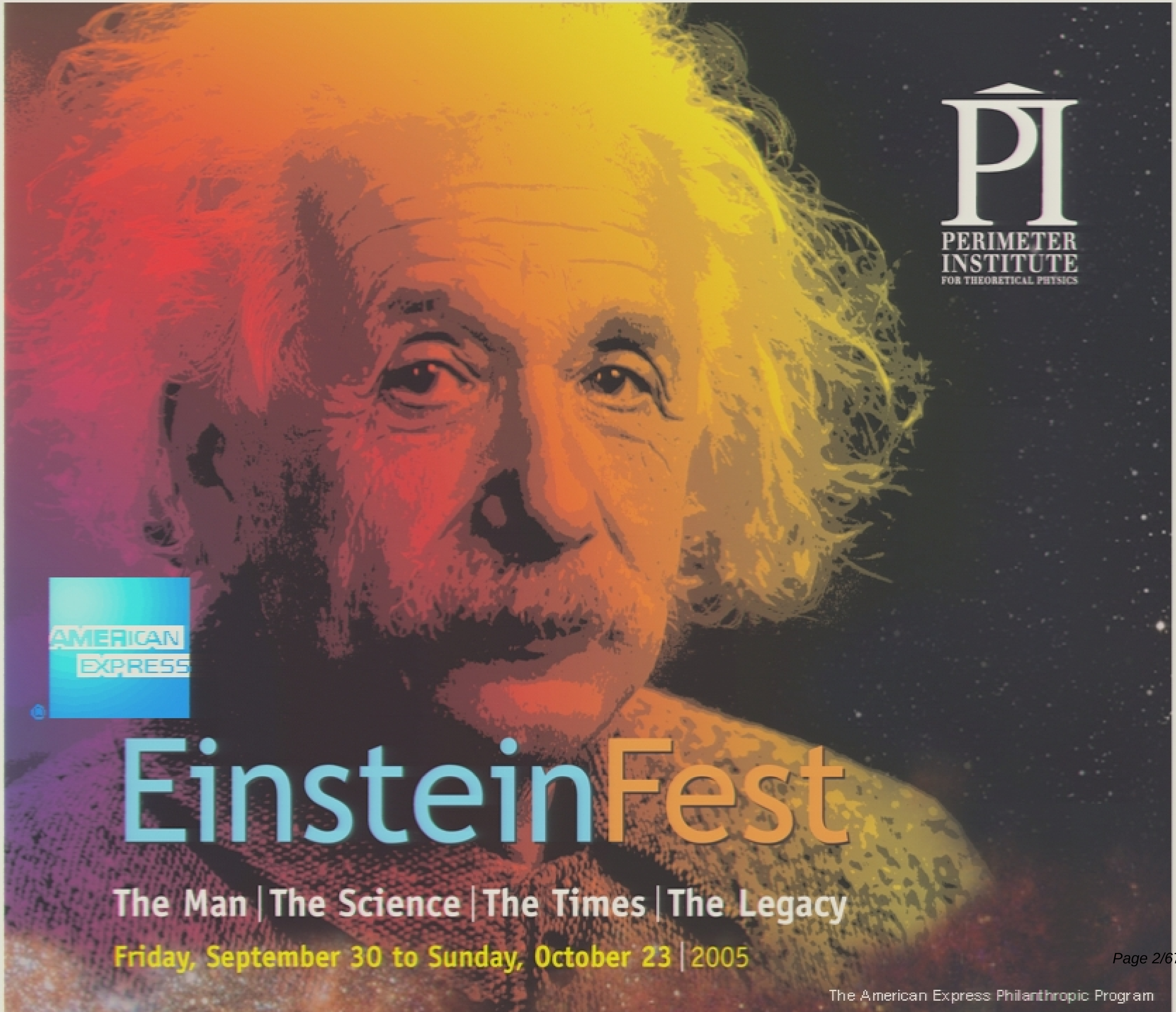
Abstract: One of the most hotly debated topics of the late nineteenth century concerned the geometry of physical space, an issue that arose with the discovery of non-Euclidean geometries. Lobachevsky and Bolyai opened the way, but it was not until the 1860s that scientists began to take this revolutionary theory seriously. Assuming the free mobility of rigid bodies, Helmholtz concluded that the geometry of space was Euclidean or else of constant curvature (either positive of negative). In 1899 these cases were tested by the astronomer Karl Schwarzschild who used data on stellar parallax to estimate the minimum size of the universe. Many argued that the notion of a curved space was nonsensical, whereas PoincarÃ©, the most prominent mathematician of the era, thought that the geometry of space could never be determined absolutely. These classical debates played a major role in the discussions spawned by EinsteinÂ's general theory of relativity. <kw> David Rowe, geometry, space-mathematics, Euclid, Carl Friedrich Gauss, Gaussian curvature, Gaussian Theory, Einstein, differential geometry, playfair, spherical triangles, platonic solids, Netwon, non-Euclidean geometry, Riemann, Ricci, Poincare, Klein</kw>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Euclidean Geometry and Physical Space

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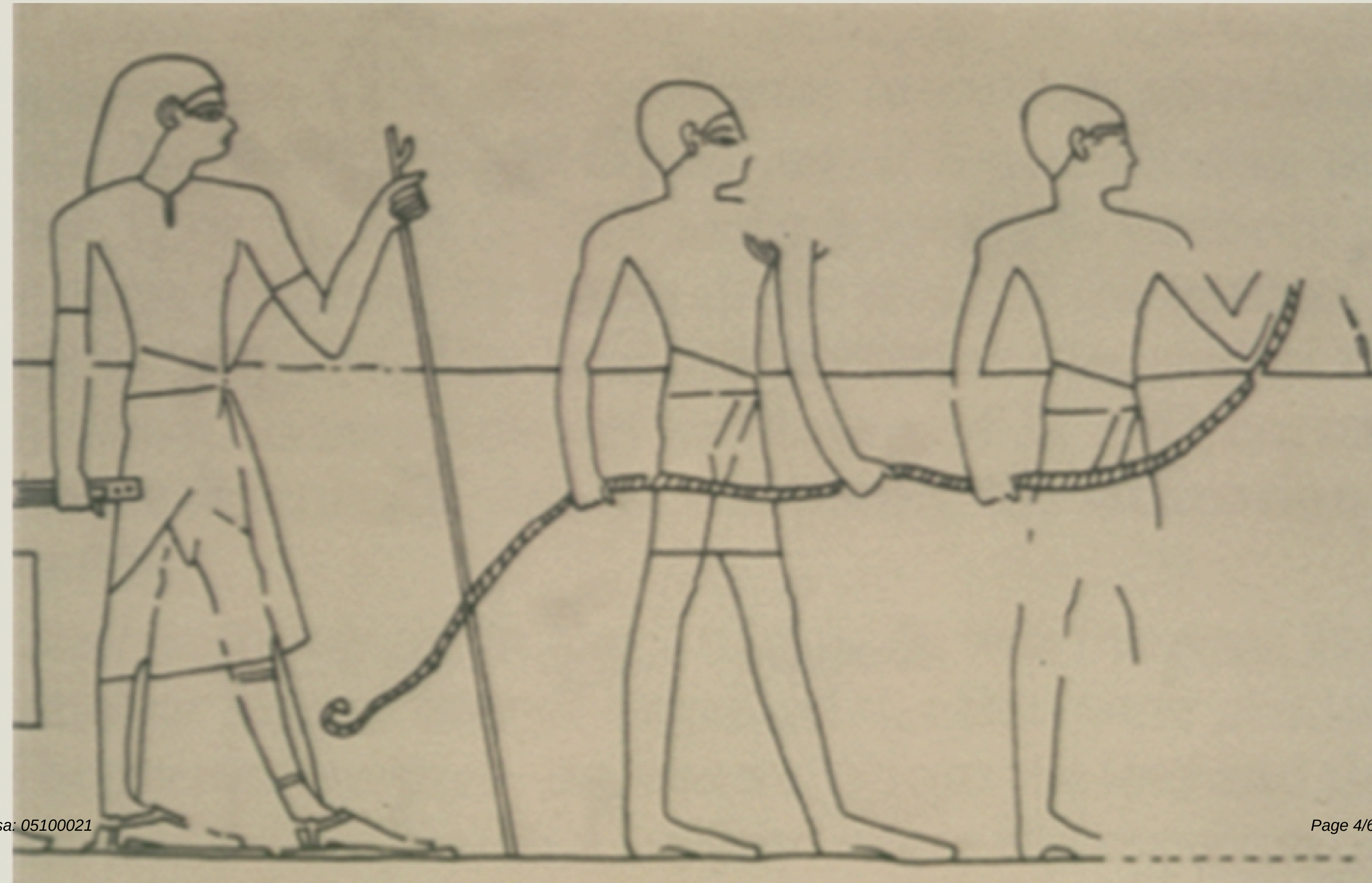
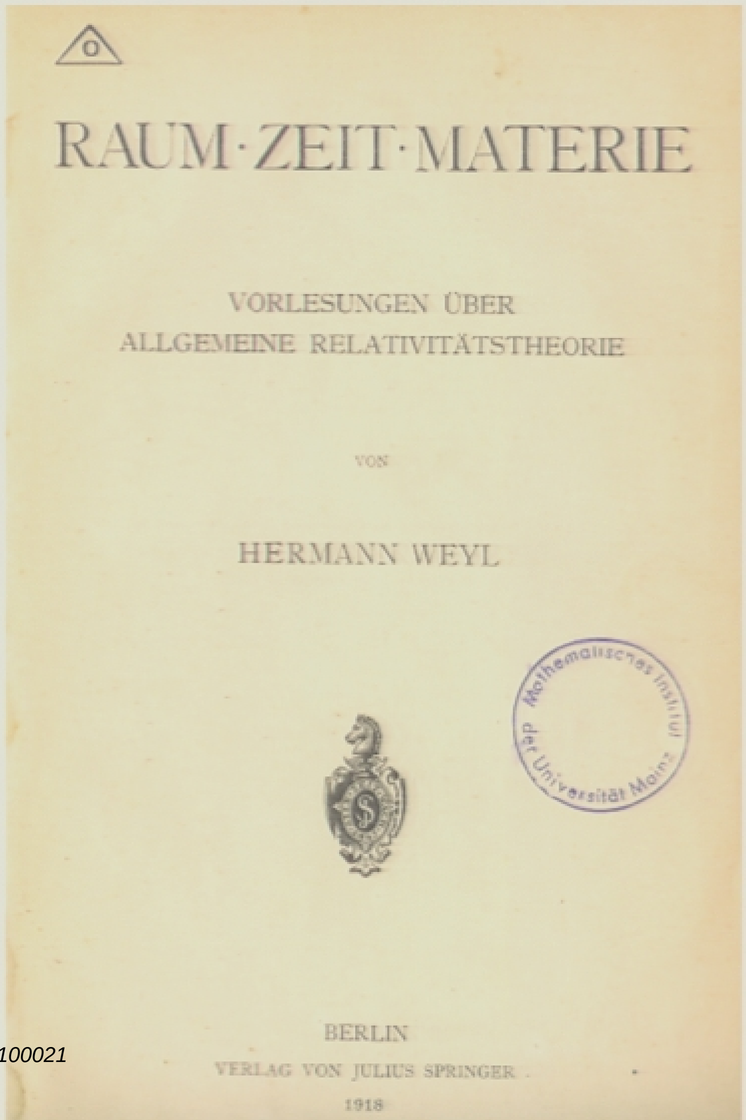


An Early Modern Edition of Euclid's *Elements*

Frege vs. Hilbert

Hermann Weyls *Raum-Zeit-Materie*, 1. Auflage, 1918


RAUM·ZEIT·MATERIE

VORLESUNGEN ÜBER
ALLGEMEINE RELATIVITÄTSTHEORIE

VON

HERMANN WEYL

BERLIN
VERLAG VON JULIUS SPRINGER
1918

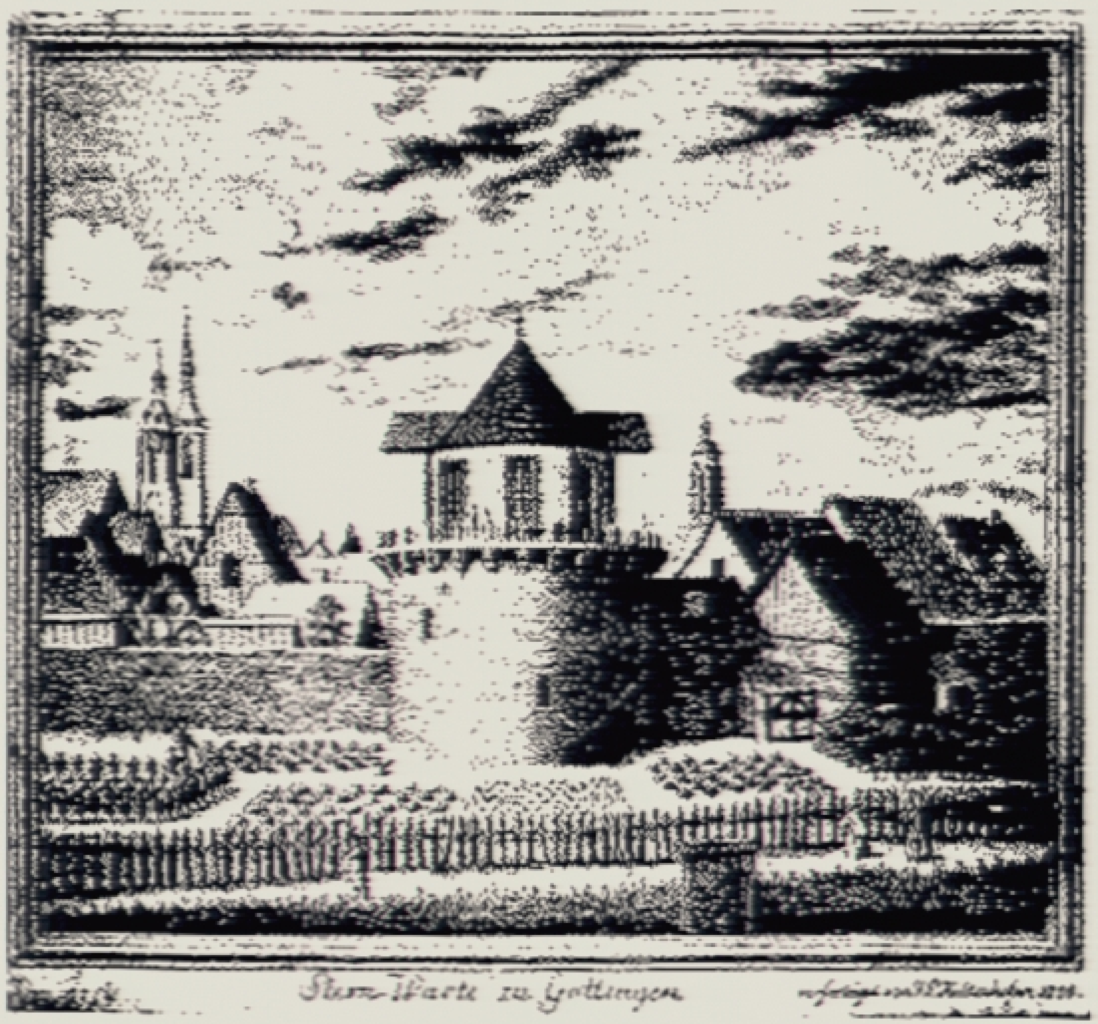
Carl Friedrich Gauss (1807-1855)

Gauss, after a Hard Day of Work

Gauss, after a Hard Day of Work

Gauss, before the Introduction of the Euro

The Old Sternwarte in Göttingen

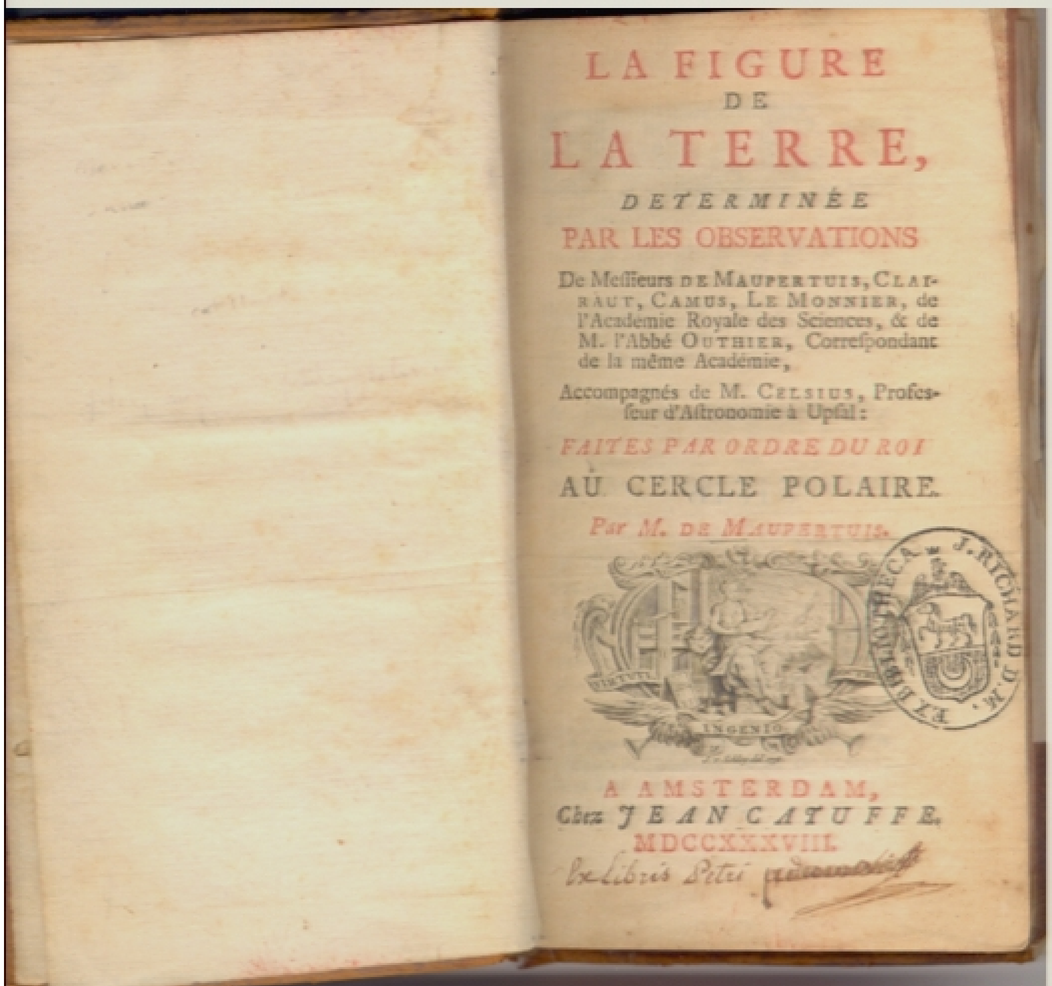


The New Göttingen Sternwarte

A Look Inside Gauss' Workshop


Tobias Mayer.
URANOGRA

Pierre-Louis Moreau de Maupertuis: "the earth-flattener"

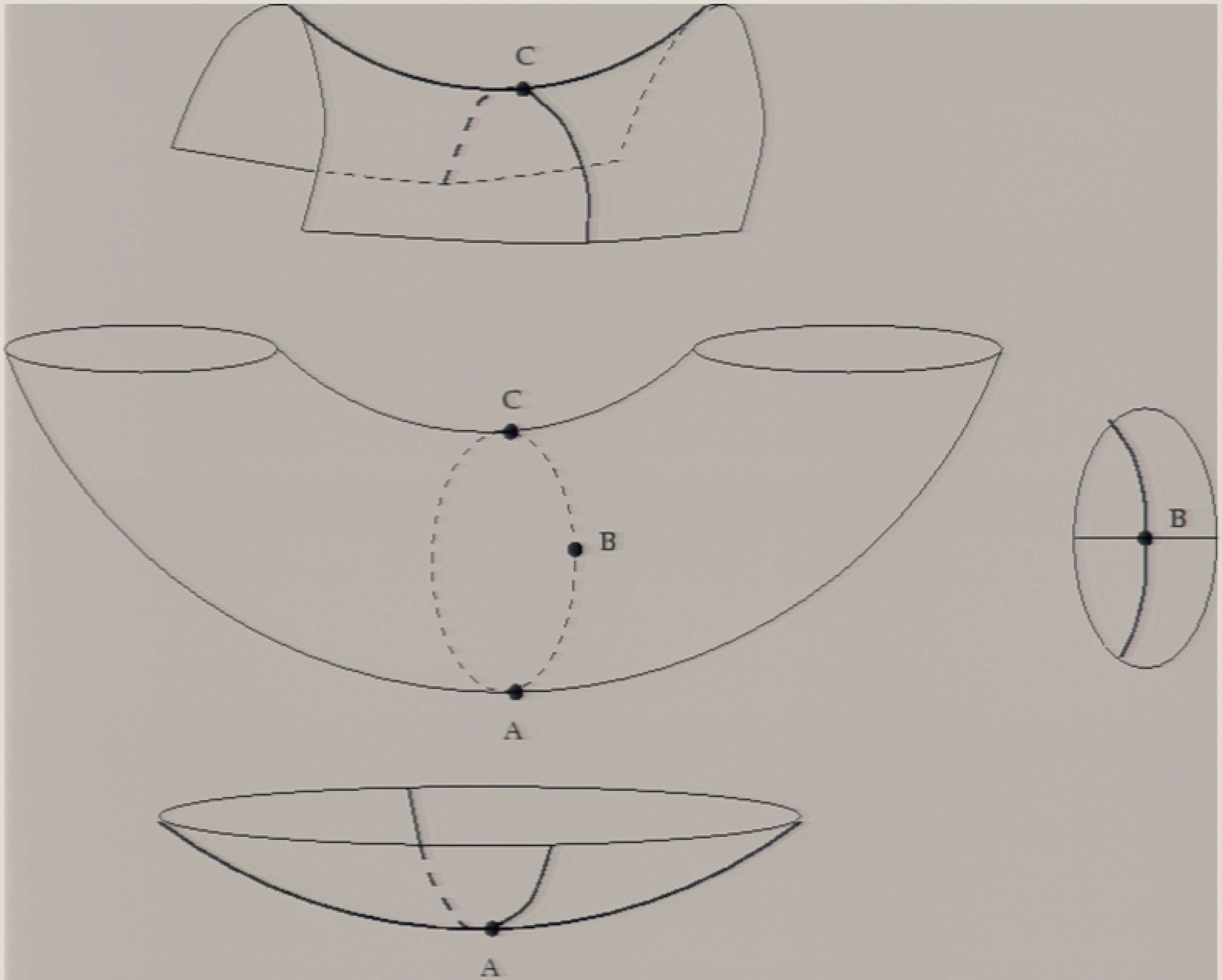


LA FIGURE
DE
LA TERRE,
DETERMINÉE
PAR LES OBSERVATIONS
De Messieurs DE MAUPERTUIS, CLAIRAUT, CAMUS, LE MONNIER, de l'Académie Royale des Sciences, & de M. l'Abbé OUTHIER, Correspondant de la même Académie,
Accompagnés de M. CELSIUS, Professeur d'Astronomie à Upsal:
FAITES PAR ORDRE DU ROI
AU CERCLE POLAIRE.
Par M. DE MAUPERTUIS.

A AMSTERDAM,
Chez JEAN CATUFFE.
MDCCXXXVIII.

Ex libris Petri

MAUPERTUIS, M. de : La figure de la terre, determinée par les observations.
Am 8.



The thick lines denote the geodesics of extremal curvature

Gauss' Theorema Egregium:

If two surfaces are isometric then they have the same Gaussian curvature at corresponding points.

Gaussian Curvature for a Sphere

On a sphere the surface normal at each point passes through the sphere's center and each plane section determines a great circle. So

$$\kappa_1 = \kappa_2 = \frac{1}{r},$$

and the Gaussian curvature is therefore

$$\kappa = \kappa_1 \cdot \kappa_2 = \frac{1}{r^2}$$

Karl Geiser's Lectures on Surface Theory at the ETH

Einstein's Recollections, Kyoto 1923

I was sitting in a chair in the patent office in Bern, when all of a sudden I was struck by a thought: “If a person falls freely, he will certainly not feel his own weight.”

I was startled. This simple thought made a really deep impression on me. My excitement motivated me to develop a new theory of gravitation. My next thought was: “When a person falls, he is accelerating. His observations are nothing but observations in an accelerated system.”

This remained an unsolved problem for me until 1912. In that year, I suddenly realized that there was good reason to believe that the Gaussian theory of surfaces might be the key to unlock the mystery. I realized at that point the great importance of Gaussian surface coordinates. However, I was still unaware of the fact that Riemann had given an even more profound discussion of the foundations of geometry. I happened to remember that Gauss’s theory had been covered in a course I had taken during my student days with a professor of mathematics named Geiser. From this I developed my ideas, and I arrived at the notion that geometry must have physical significance.

Einstein and Mathematicians from Zurich and Göttingen

A Group Photo from 1917

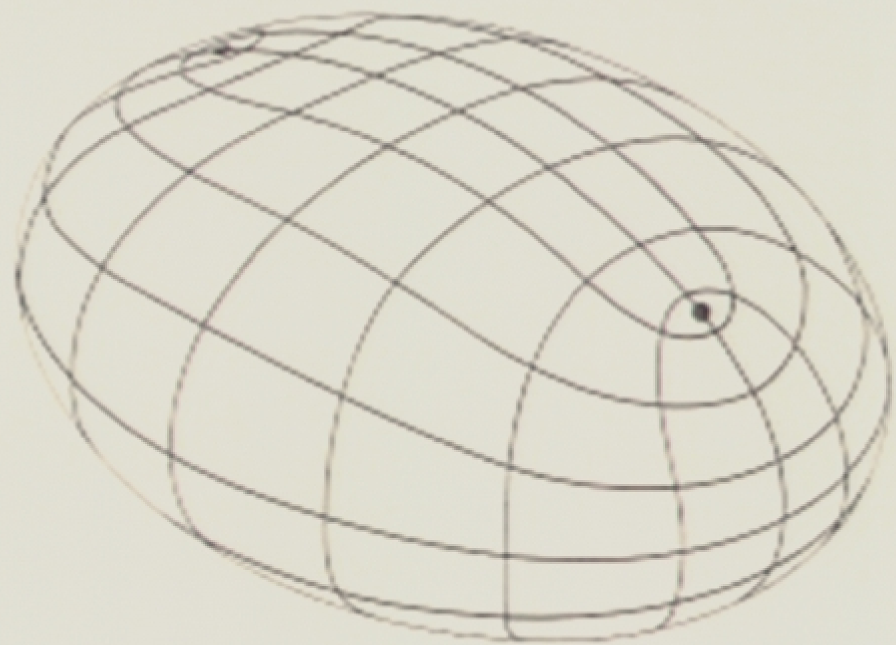
Carathéodory, Grossmann, Hilbert, Geiser, Weyl und Bernays

What Einstein Meant about his Insight from 1912

- Analogy between the gravitational problem and theory of surfaces
- Theorem: the force-free motion of a point mass along a surface takes place along geodesic curves (recorded by Grossmann in his Notes for C. F. Geiser's Course, WS 1897/98)
- Derived 1912 in Einstein's Scratch Notebook

Differential Geometry of Surfaces

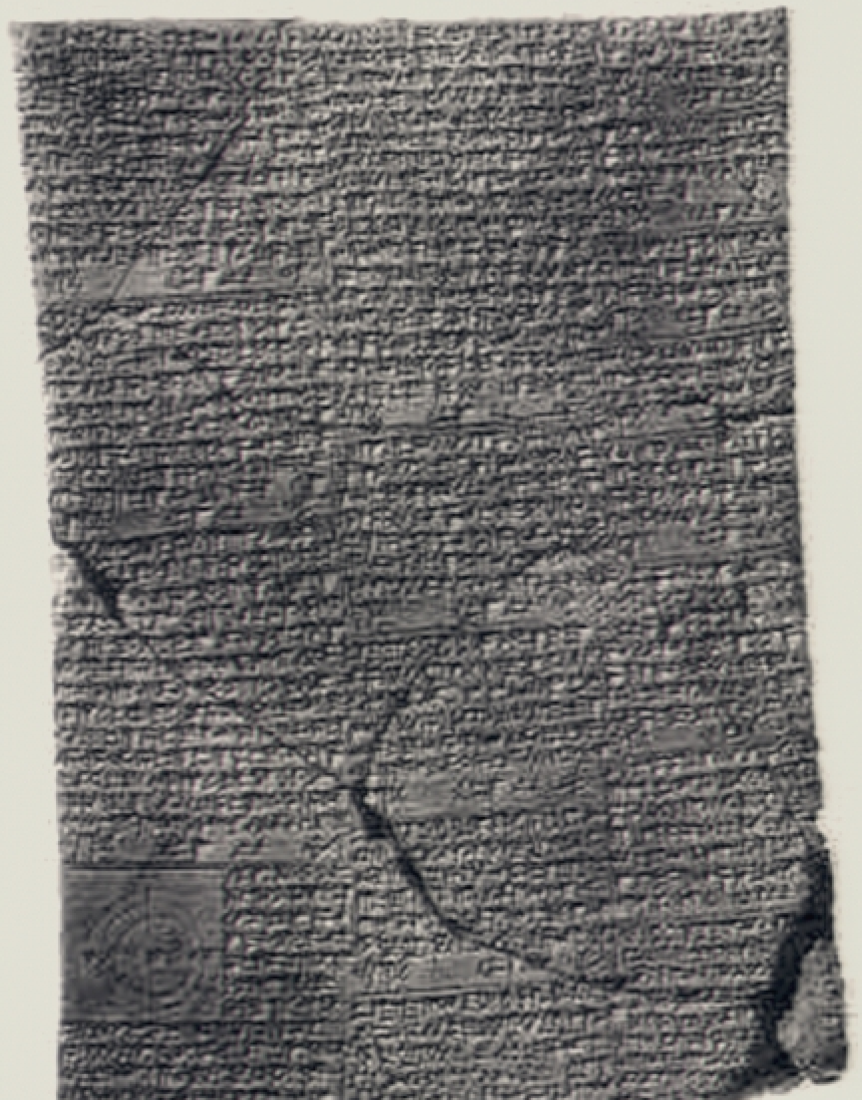
- Geodesics of ellipsoid are given by four systems of confocal curves
- These correspond to the lines of curvature on the surface
- Which intersect orthogonally (Dupin's Theorem)




albedo@tekom.odessa.ua Arseny Kostenko

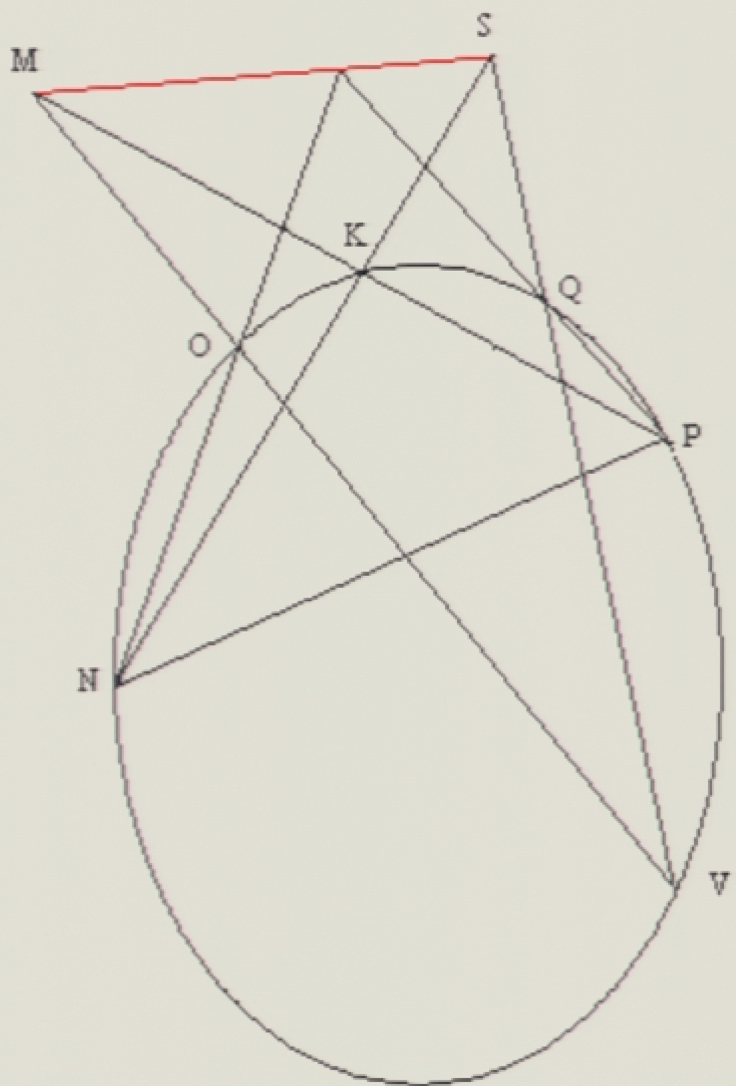
Rhind Papyrus

Babylonian Cuneiform Tablets



Euclid's Proof of I. 47

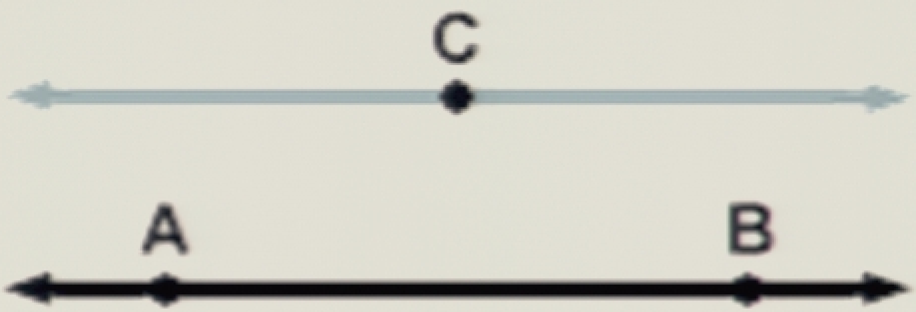
Pascal's Configuration



Lewis Carroll and Edith, Lorina, and Alice Liddell

Alice's Adventures in Wonderland

Playfair vs. Euclid



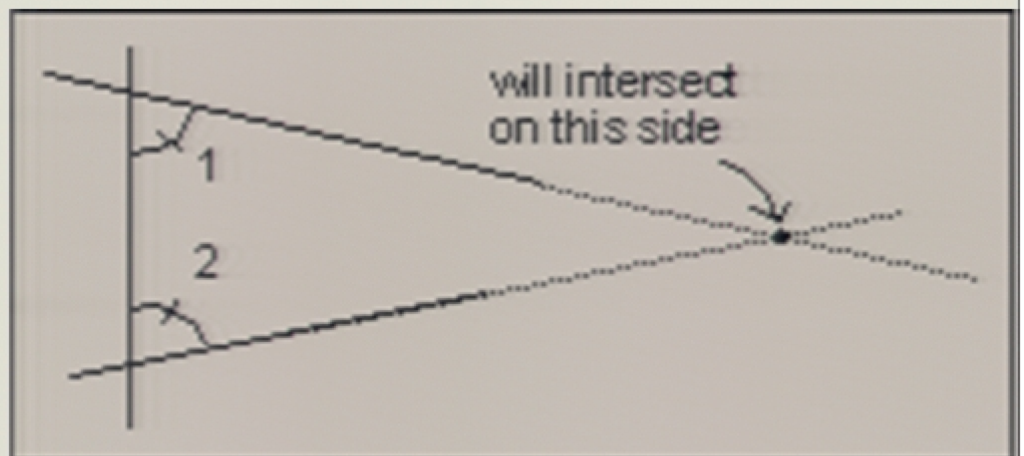
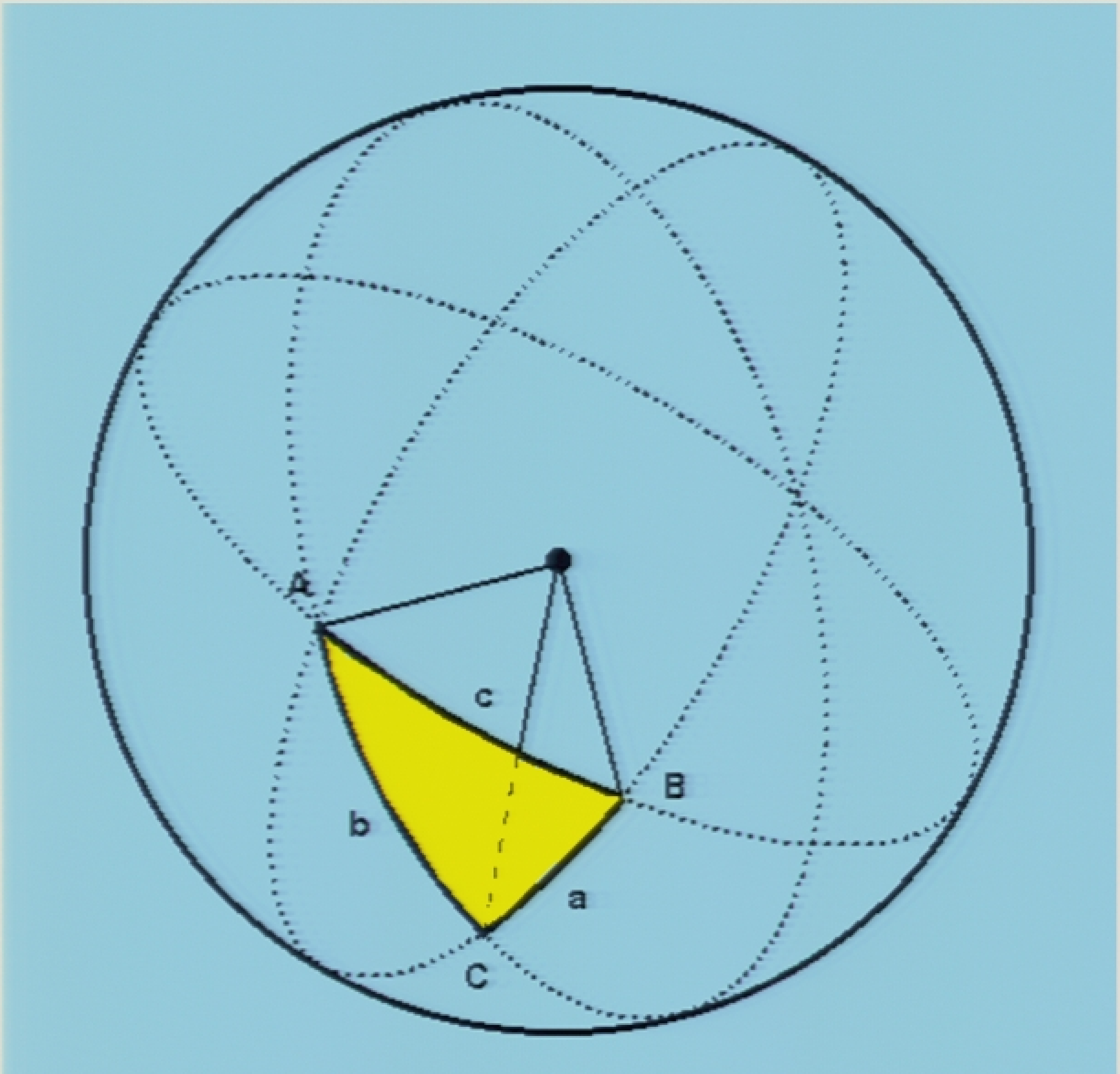
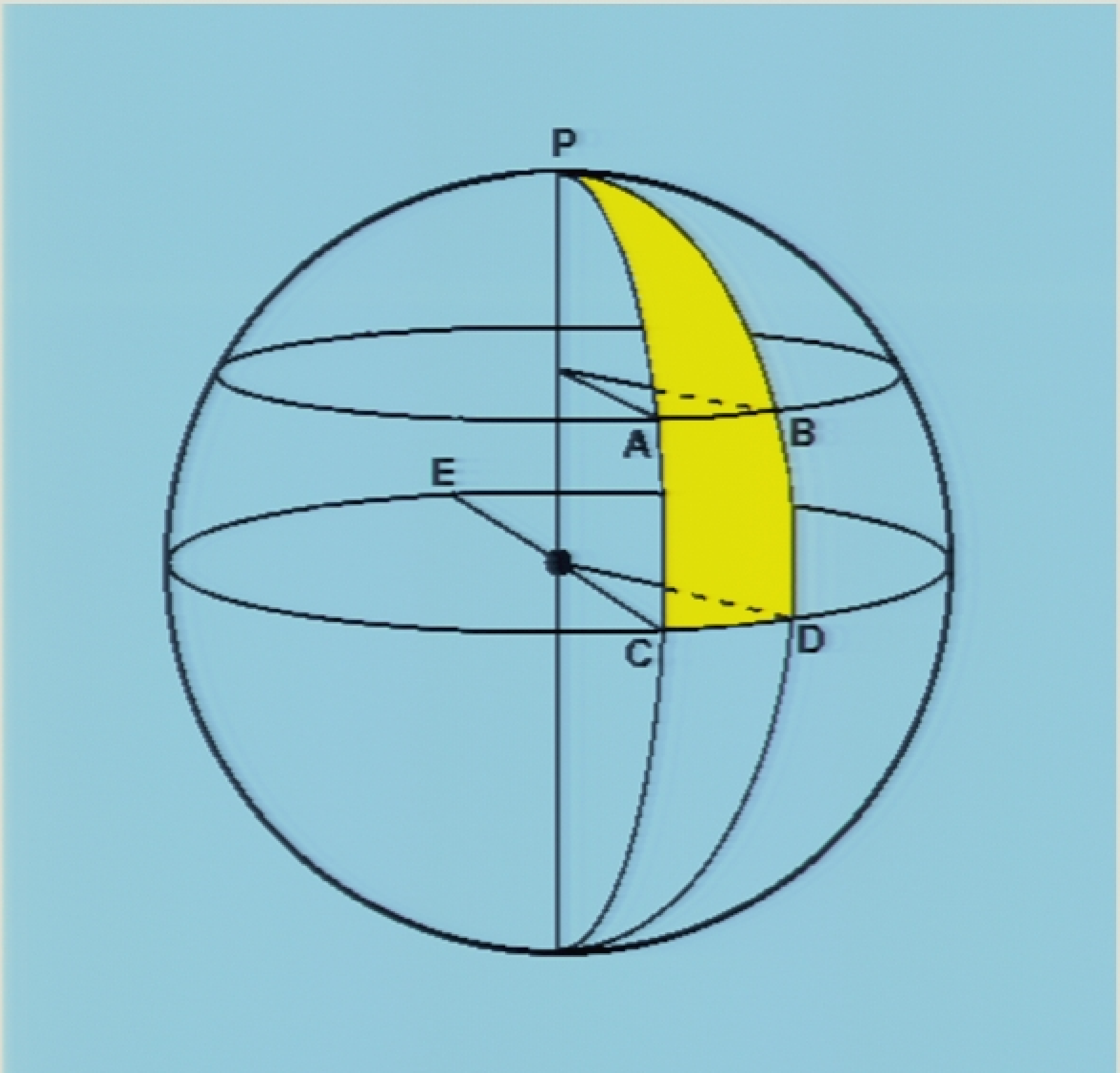


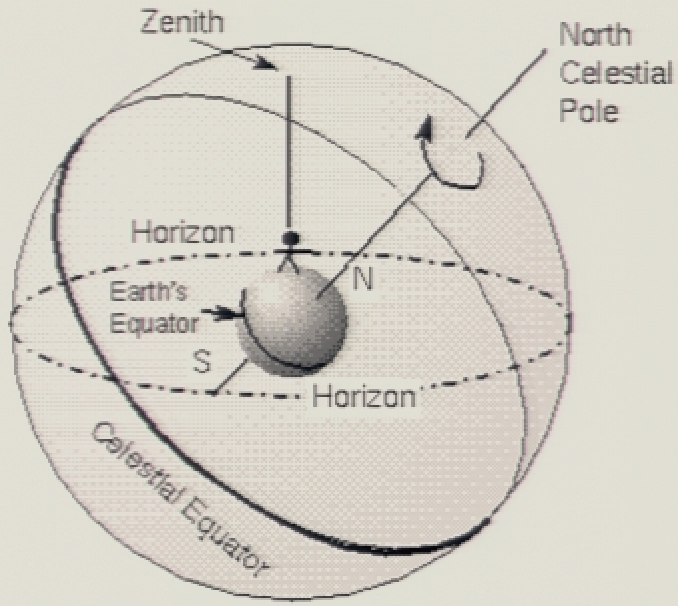
Figure 7

Spherical Triangles

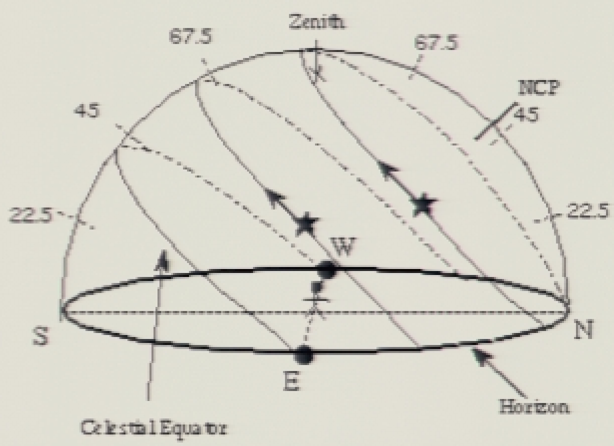


Area decreases with angle size

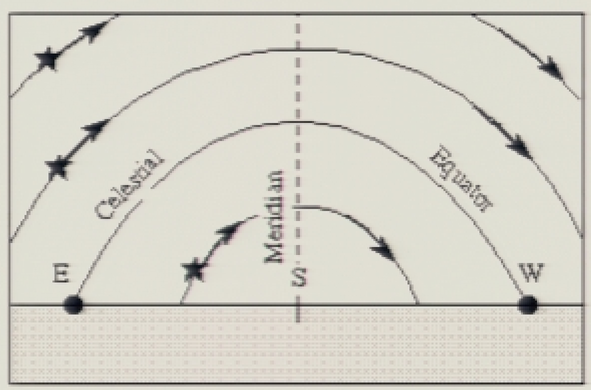




The celestial sphere for an observer in Seattle.
The angle between the zenith and the NCP = the angle between the celestial equator and the horizon.
That angle = 90° – observer's latitude.

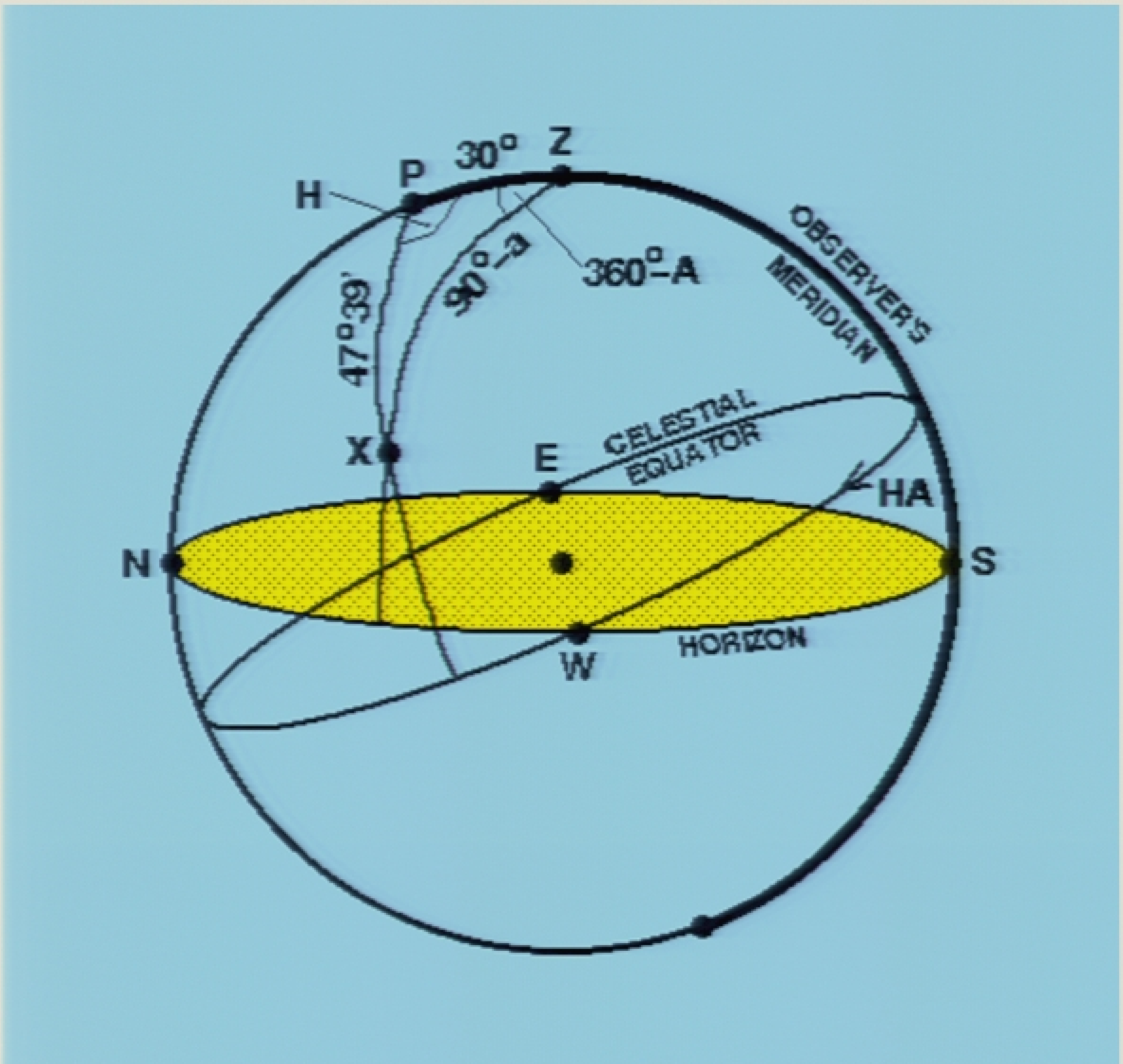


Stars motion at Seattle. Stars rotate parallel to the Celestial Equator, so they move at an angle with respect to the horizon here. Altitudes of 1/4, 1/2, and 3/4 the way up to the zenith are marked.

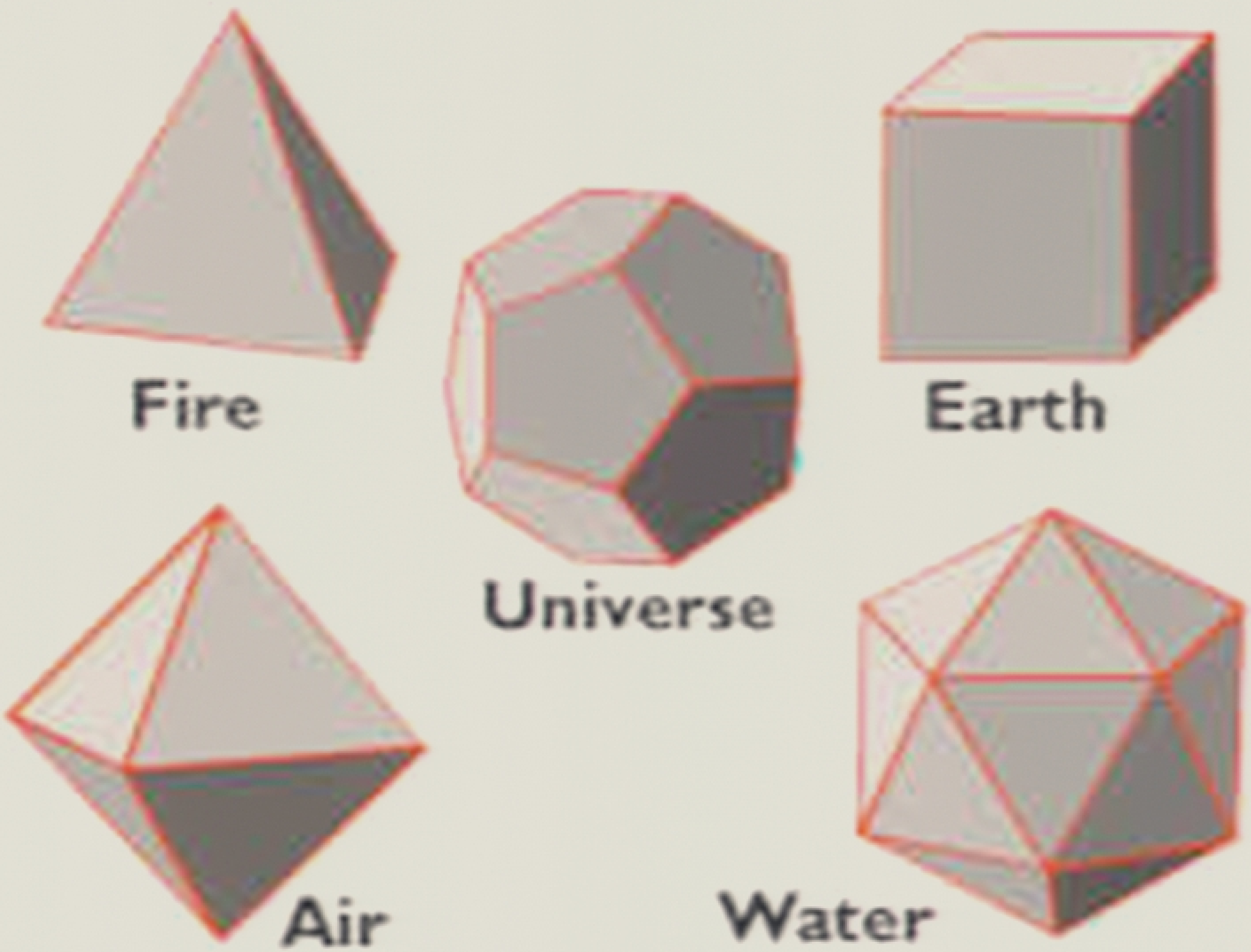


Your view from Seattle. Stars rise in the East half of the sky, reach maximum altitude when crossing the meridian (due South) and set in the West half of the sky. The Celestial Equator goes through due East and due West.

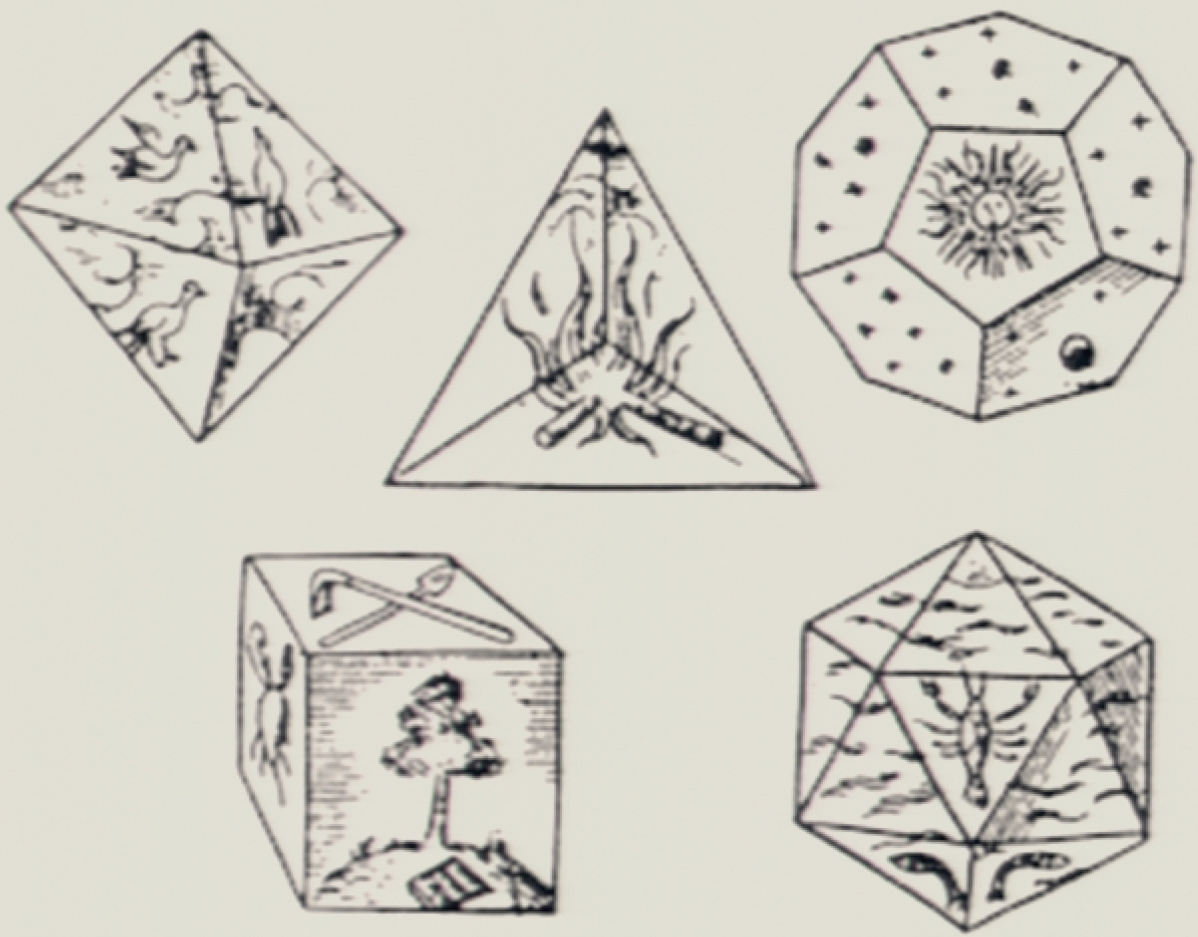
Ancient Sphaerics



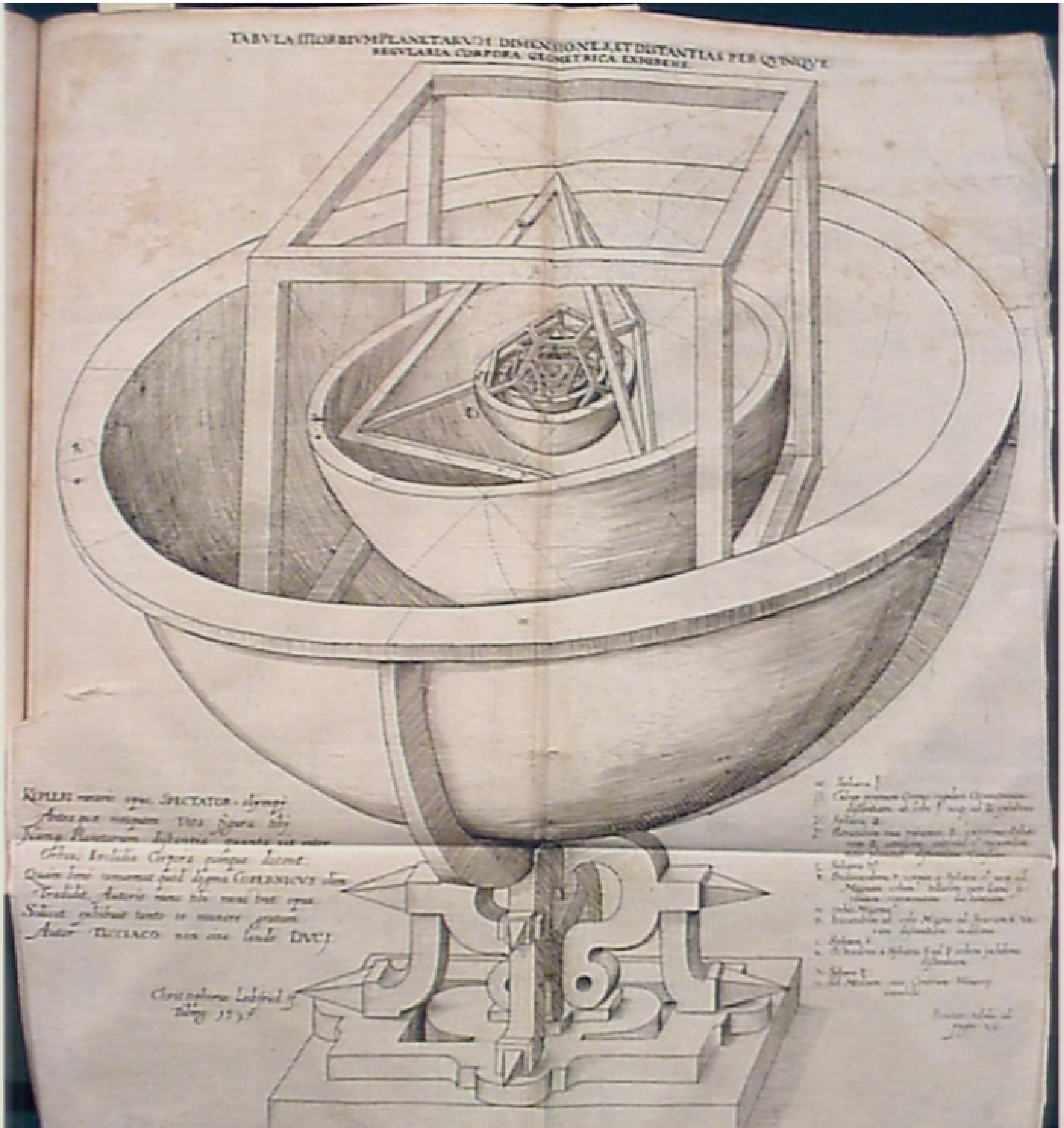
Platonic Solids



Kepler's *World Harmonies*



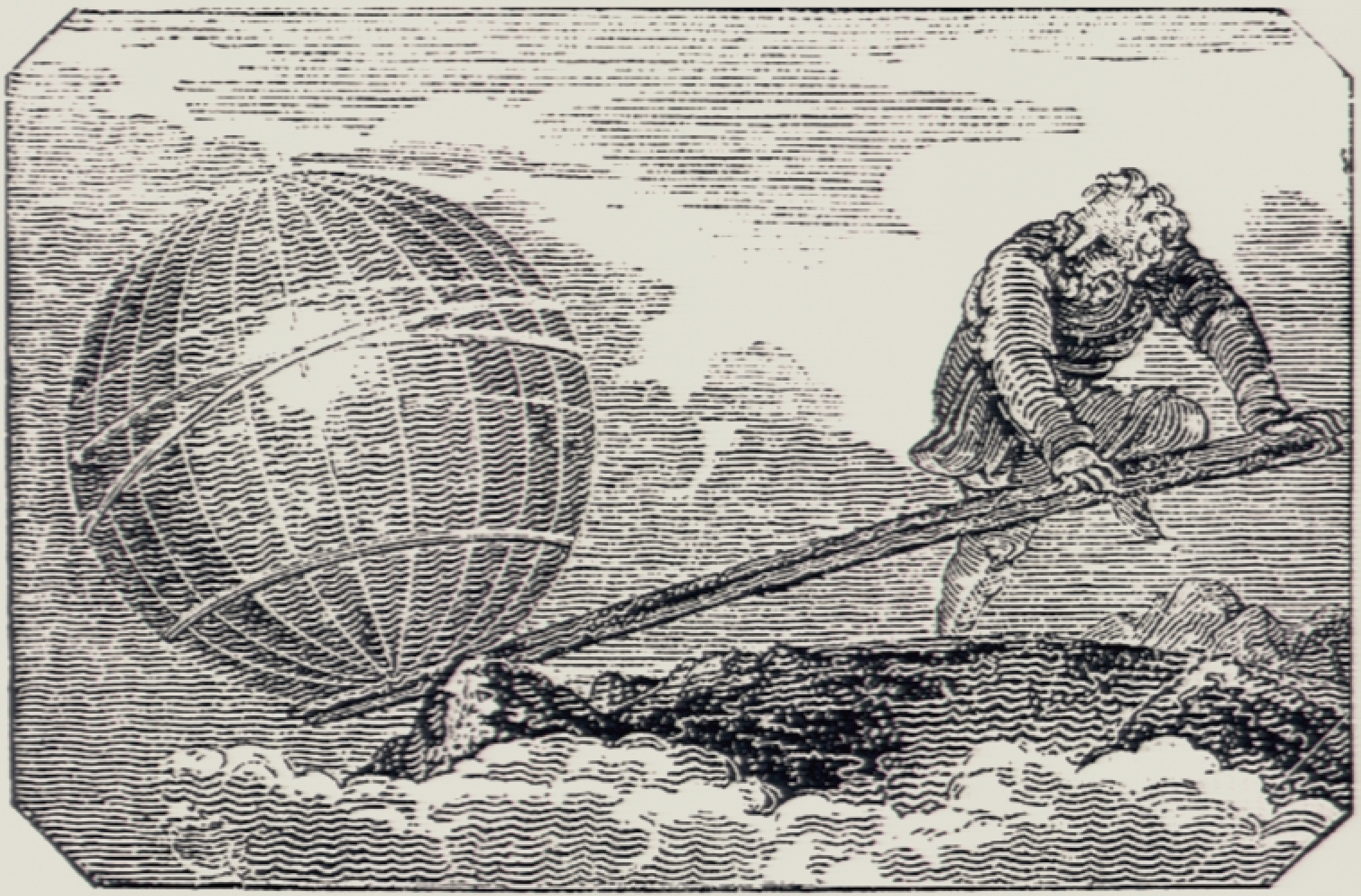
Kepler's *Mysterium Cosmographicum*



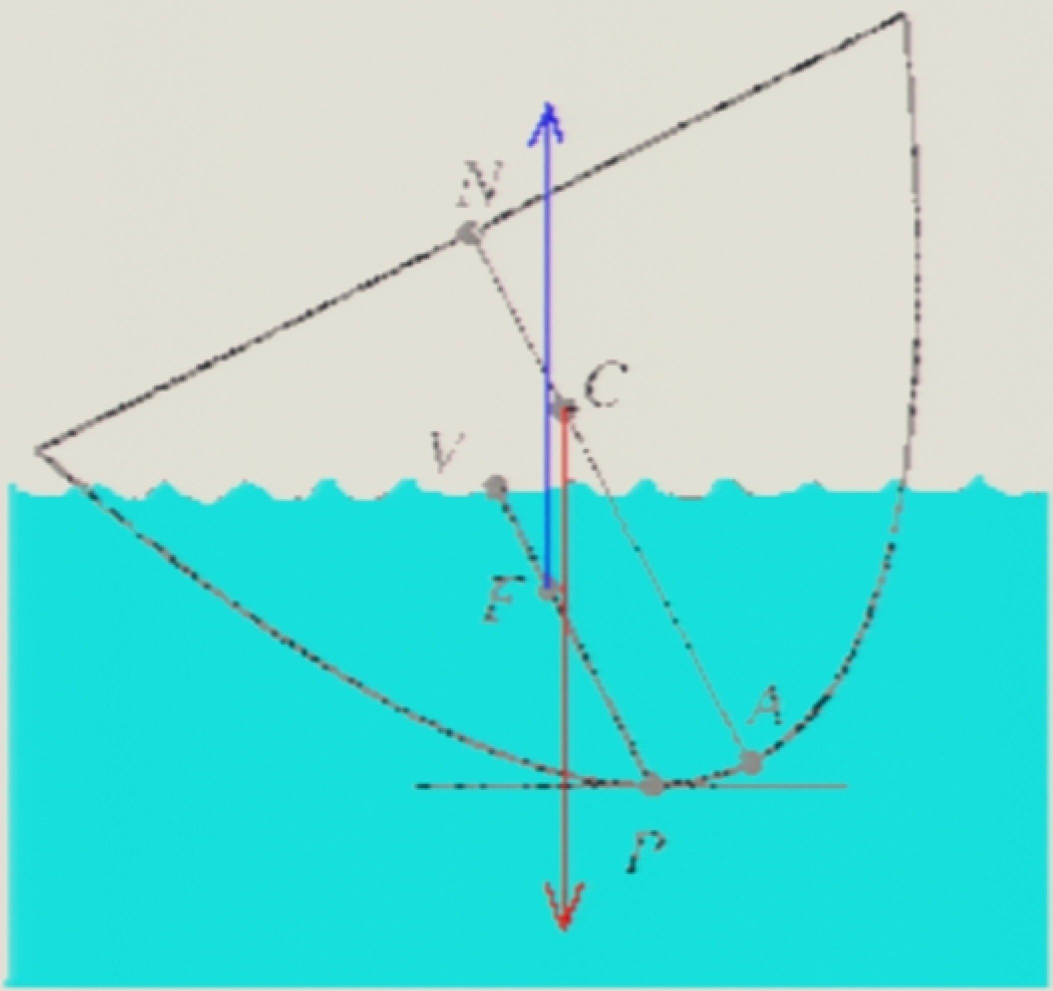
Archimedes and the Law of the Lever

Beweis des Hebelgesetzes

“Give me a place to stand and I’ll move the world!”

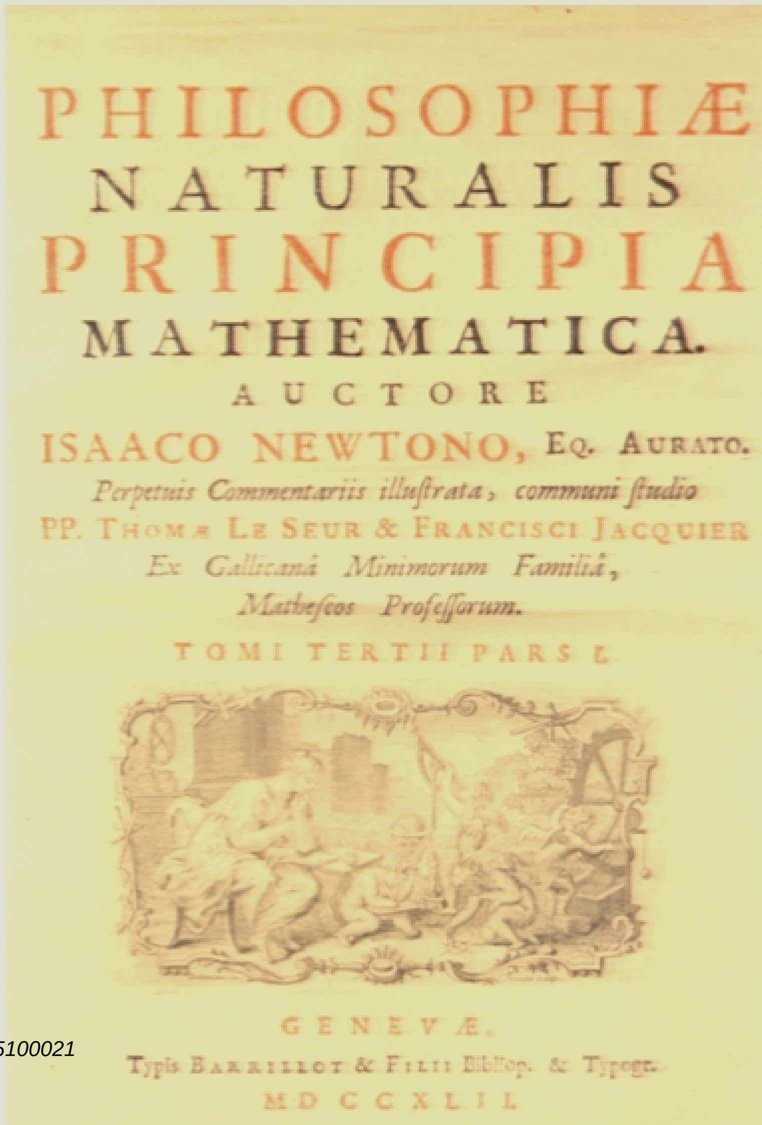


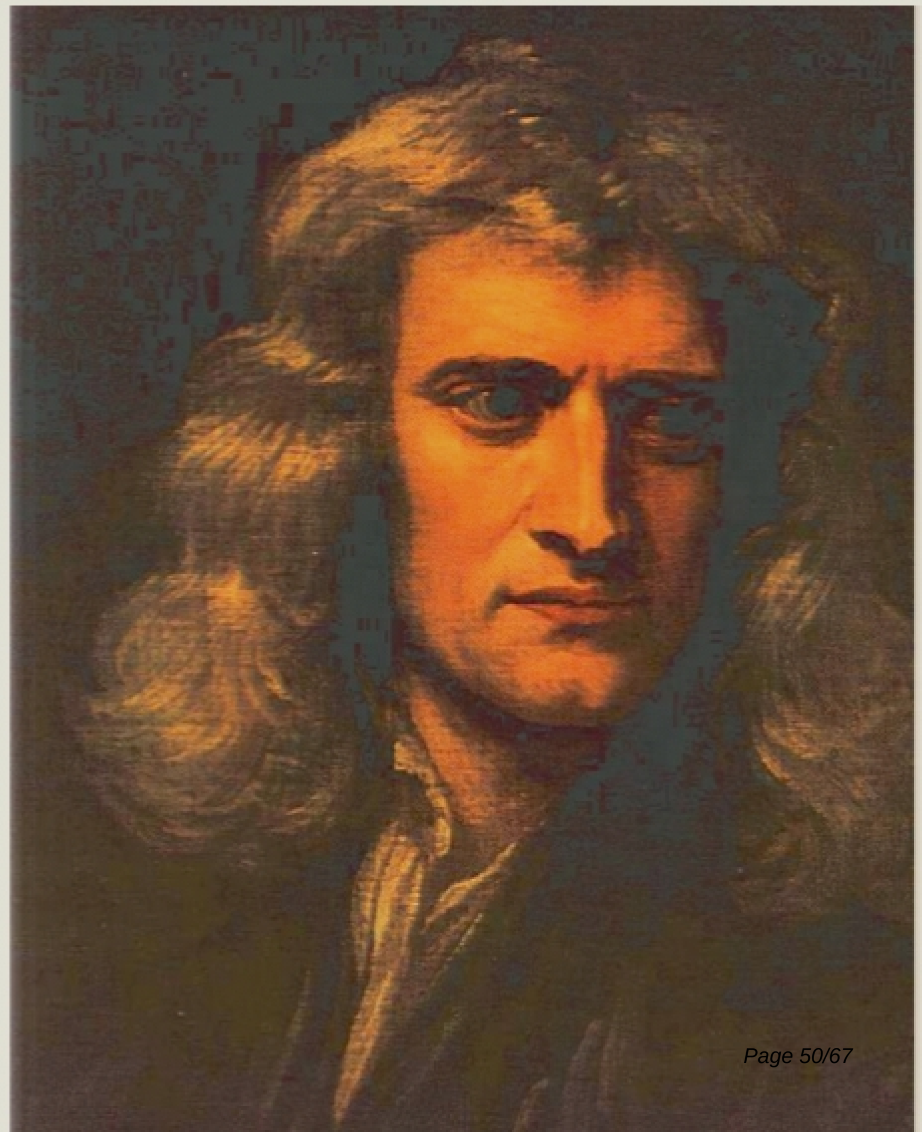
“On Floating Bodies”



King Gustav Adolf's: Vasa

Newton's *Principia* (1687)



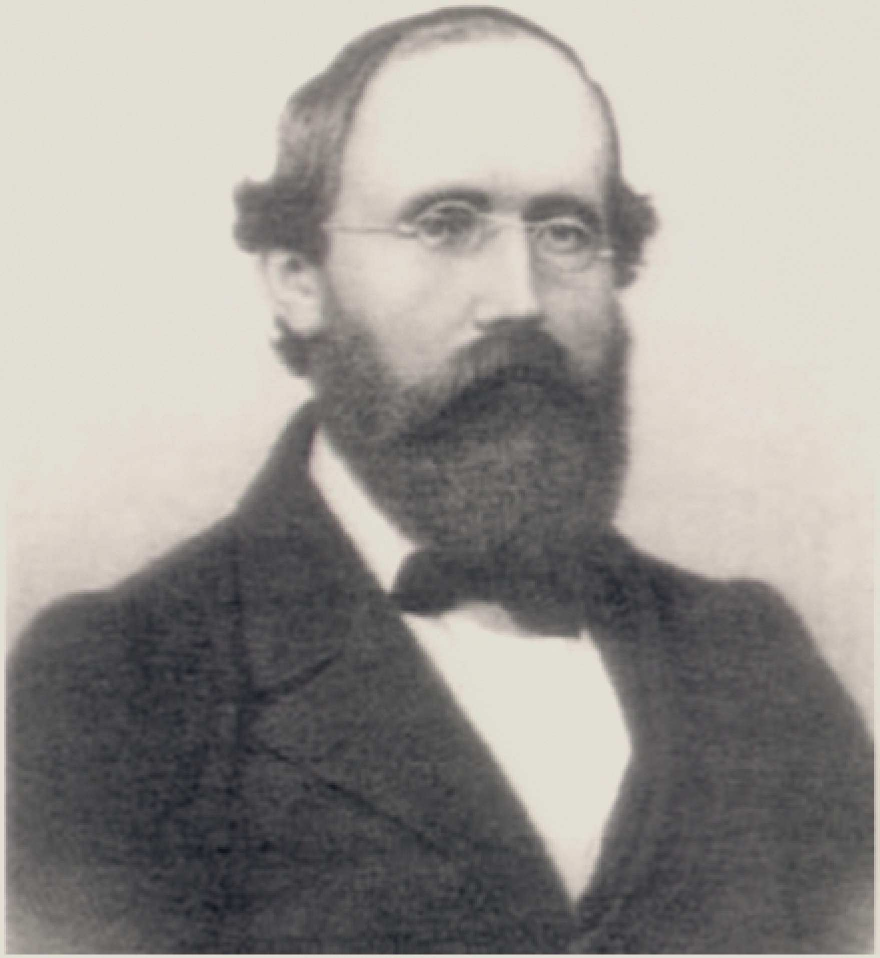


Gauss on non-Euclidean Geometry:

The assumption that the sum of the three angles of a triangle is less than 180 leads to a curious geometry, quite different from ours [i.e. Euclidean geometry] but thoroughly consistent, which I have developed to my entire satisfaction, so that I can solve every problem in it excepting the determination of a constant, which cannot be fixed a priori. .... the three angles of a triangle become as small as one wishes, if only the sides are taken large enough, yet the area of the triangle can never exceed, or even attain a certain limit, regardless of how great the sides are.

Nicholas Lobachevsky and Janos Bolyai

Bernhard Riemann (1826-1866)



Habilitationsvortrag, “On the Hypotheses underlying Geometry” (1854, *published 1868*))

On Riemann's Famous Lecture

- Introduced a vague notion of manifolds, inspired by Riemann surfaces, etc.
- Described sectional curvature as method to extend Gauss's results to higher dimensions
- Distinguished topological from metrical properties of manifolds
- Suggested the possibility that space might be finite, but unbounded

Felix Klein as a Young Admirer of Riemann

- Came in Contact with Riemann's Ideas through Alfred Clebsch (1839-1872) in Göttingen
- Competed as self-appointed Champion of Riemann with leading members of the Weierstrass school

Felix Klein on Riemann’s Habilitationsvortrag of 1854:

“I still have a vivid recollection of the extraordinary impression that Riemann’s ideas then [ca. 1870] made on the young mathematicians. Much appeared to us as dark and difficult and yet of unfathomable depth.” (Felix Klein, 1917)

Other Readers of Riemann’s Lecture

- E. B. Christoffel, a Berlin-Trained Analyst
- Classic Paper in 1869 on Equivalence of Quadratic Differential Forms
- Introduced Christoffel Symbols and Riemann-Christoffel Curvature Tensor

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. . .Read by Members of the Olympia Academy in Bern. . .

- Conrad Habicht, Maurice Solovine, and Einstein, ca. 1902

- Solovine later compiled a list of the works they read by:

- Mach, Hume, Helmholtz, Poincaré, Dedekind, et al.

Who Else Read Riemann's Lecture?

Gregorio Ricci (1853-1925)

Inventor of the Ricci-Calculus

Twice passed over by the Accademia dei Lincei when he submitted this work in prize competitions

Who Knew About the Ricci-Calculus?

Tullio Levi-Civita
(1873-1941)

Ricci's Student in Padua
(1890-1894)

In 1901 they wrote a fundamental paper on the Ricci-Calculus for *Mathematische Annalen*

Who Read the Paper by Ricci and Levi-Civita?

Marcel Grossmann
(1878-1936)

Studied Mathematics at the ETH Zürich

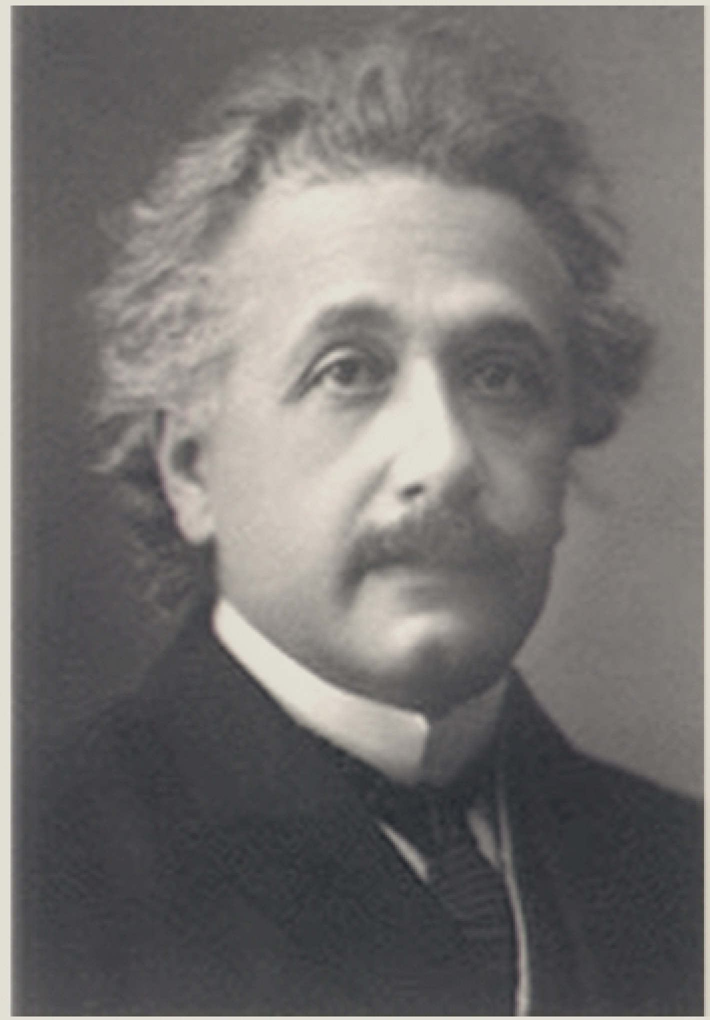
1900 became Assistent of Wilhelm Fiedler

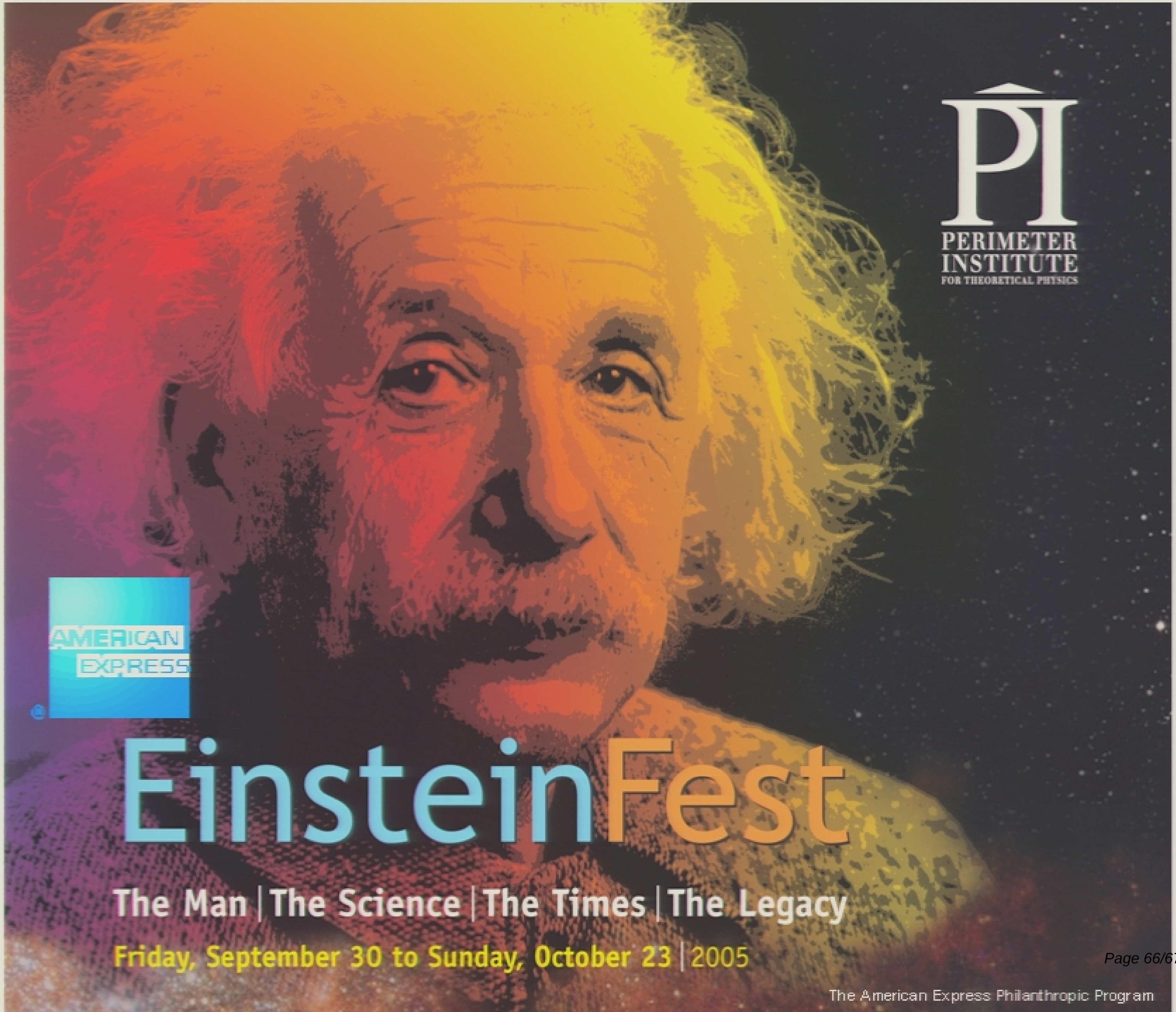
Grossmann teaches Einstein the Ricci-Calculus

In 1912 Einstein joined Grossmann at the ETH

Einstein’s quest for a general theory of relativity had reached an impasse

Einstein: „Grossmann, Du muß mir helfen, sonst werde ich verrückt.“

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