Title: Complementarity, Entanglement - and No End to Uncertainty

Date: Oct 12, 2005 02:00 PM

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Abstract:

Background/Motivation

Whither Quantum Foundations?

Is quantum mechanics final?
(Measurement Problem!?)

Suspicion: Puzzles got shifted, not resolved

From Wave–Particle Duality [continuum vs discrete]

via COMPLEMENTARITY (causality vs space-time)

to quantum-classical ambivalence

(cf. also: peaceful (but uneasy) coexistence of quantum theory and relativity)

Issues:

- quantum mechanics determines limits of applicability of classical concepts

- hence renders classical physics an approximation

- quantum world on classical space-time background
  – (Mackey) quantization
  – (remnant of correspondence principle)

- necessity of classical language (Bohr) for description of experiments

- Quantum Measurement:
  – from INDETERMINATE values
  to definite outcomes
  – linear dynamics → non-separability, non-locality

Personal perspective... (Whither Paul Busch?)

Starter

Ongoing ARGUMENT:

Which Quantum Principle is "deeper" —

COMPLEMENTARITY or INDETERMINACY?

- unresolved since Bohr-Heisenberg conflict (1927) – vividly debated in 1990s

- at the heart of Copenhagen "Interpretation"

- Well-posed question?
  "Deep"? Are CP and UP independent?

- Could add more:
  Superposition Principle, Entanglement (non-separability), nonlocality

"Quantum Principle"

(a) [postulate used to deduce/formulate theory]

(b) → principal (distinctive) feature of theory

Plan

I. Review

- New Ideas and Experiments
- Old Misconceptions, New Controversy

II. Discussion

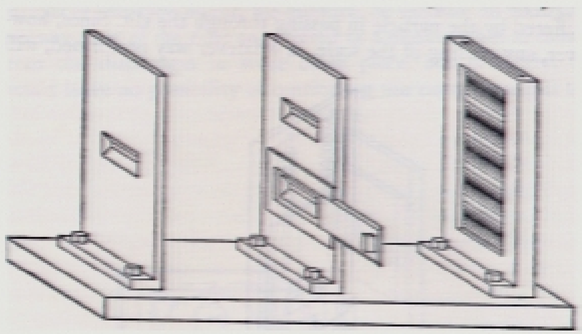
- Roots of Controversy
- Critique

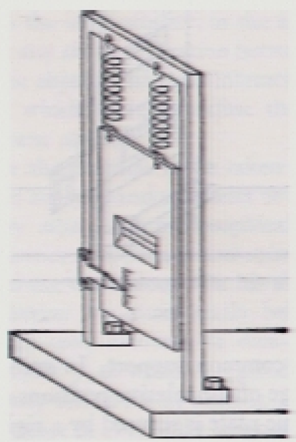
III. Conclusion – Coherent Account

Warm-Up

"Complementarity"

Bohr...





From Bohr's contribution in P. Schilpp, "Albert Einstein – Philosopher-Scientist", 1949

....complementarity in a nutshell:

- wave-particle duality
- mutual exclusion:
  path knowledge vs. interference
  in 2-slit or Mach-Zehnder interferometer
- mutual exclusion of corresponding set-ups
- limitation of applicability of
  classical physical concepts

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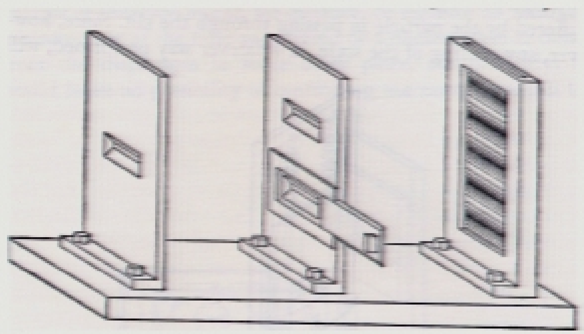

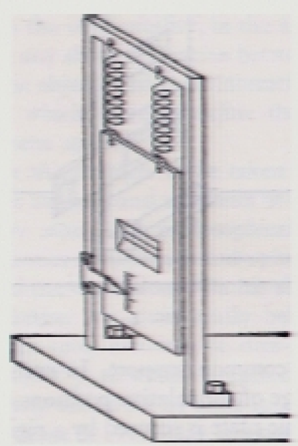

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“Uncertainty”

[The New Yorker, 1940]

- Heisenberg’s Uncertainty Relation

- limitation of applicability of
  classical physical concepts

Preliminary (Typical) Definitions

Complementarity

"We say that two observables are 'complementary' if precise knowledge of one of them implies that all possible outcomes of measuring the other one are equally probable." [SEW 1991]

Principle of Complementarity

"For each degree of freedom the dynamical variables are a pair of complementary observables.
...less precise version in practical terms is:
No matter how the system is prepared, there is always a measurement whose outcome is utterly unpredictable." [SEW 1991]

Source [SEW 1991]: M.O. Scully, B.-G. Englert, H. Walther, "Quantum optical tests of complementarity" Nature 351, 9 May 1991

Uncertainty Relation

$$\mathrm{Var}_\psi Q \cdot \mathrm{Var}_\psi P \;\geq\; \frac{\hbar^2}{4}$$

$$\begin{aligned}\mathrm{Var}_\rho A \cdot \mathrm{Var}_\rho B &\\ \geq \tfrac{1}{4}\left|\left\langle [A,B]_- \right\rangle_\rho\right|^2 &\\ +\tfrac{1}{4}\left|\left\langle \{A-\langle A\rangle_\rho, B-\langle B\rangle_\rho\}_+ \right\rangle_\rho\right|^2 &\end{aligned}$$

Uncertainty Principle

Non-controversial statement:

The value distributions of canonically conjugate pairs of variables, if measured (separately) on equally prepared systems, satisfy Heisenberg's uncertainty relation.

Controversial statements:

The values of noncommuting pairs of quantities CANNOT be simultaneously known/determined/measured with arbitrary accuracy.

Noncommuting pairs of quantities CAN be measured simultaneously IF – and ONLY IF – imprecisions satisfy Heisenberg uncertainty relation.

Complementarity versus Uncertainty – Some Typical Assessments

"Complementarity distinguishes the world of quantum phenomena from the realm of classical physics." [SEW 1991]

Julian Schwinger, "Quantum Mechanics" (2001) (ed. B.-G. Englert):
highlights Complementarity as the distinctive feature of quantum physics
and plays down Uncertainty Principle to Uncertainty Relation

Asher Peres, "Quantum Theory - Concepts and Methods" (1993):
states Principle of Complementarity
but (almost) ignores Uncertainty Principle
(though not the Uncertainty Relation)

Richard Feynman, "The Feynman Lectures of Physics Vol. 3":
on interference in 2-slit experiment:

"In reality it contains the only mystery."

... but his analysis refers to Uncertainty Principle, avoiding reference to Complementarity

Leslie Ballentine, "Quantum Mechanics – A Modern Perspective" (1998):
allows for Uncertainty Principle
but does not mention Complementarity

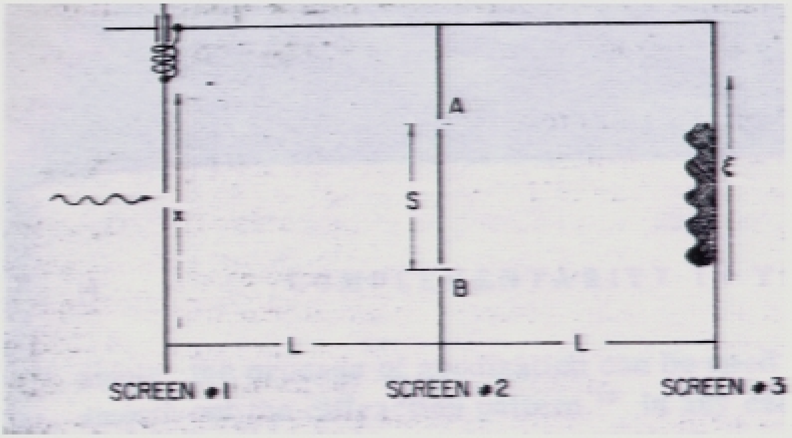
Text book survey...

... two opposing camps, and a 'neutral' party

→ where does the uneasiness come from?
→ where does it lead?

Path Marking – Traditional Schemes

Bohr vs Einstein – recoiling diaphragm



From: Wootters, Zurek, Phys. Rev. D 19 (1979) 473

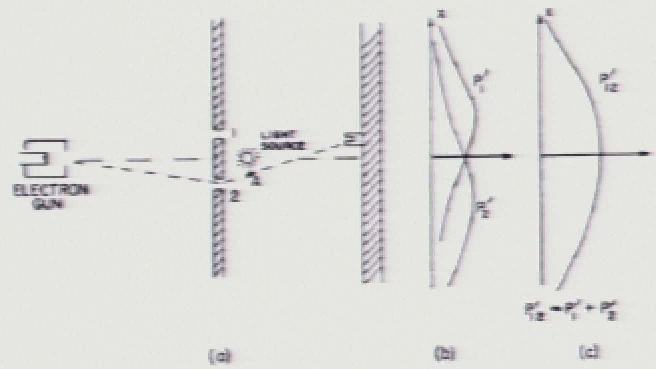
Momentum transfer

→ path determination

→ position uncertainty,
via uncertainty relation $\delta x \cdot \delta p \gtrsim h$

→ smearing out of interference fringes

Feynman – light scattering



From: Feynman Lectures Vol 3 (1965)

Localisation through light scattering

→ uncontrollable momentum kick,
due to uncertainty principle

→ loss of interference fringes

Path Marking and Erasure – Novel Schemes

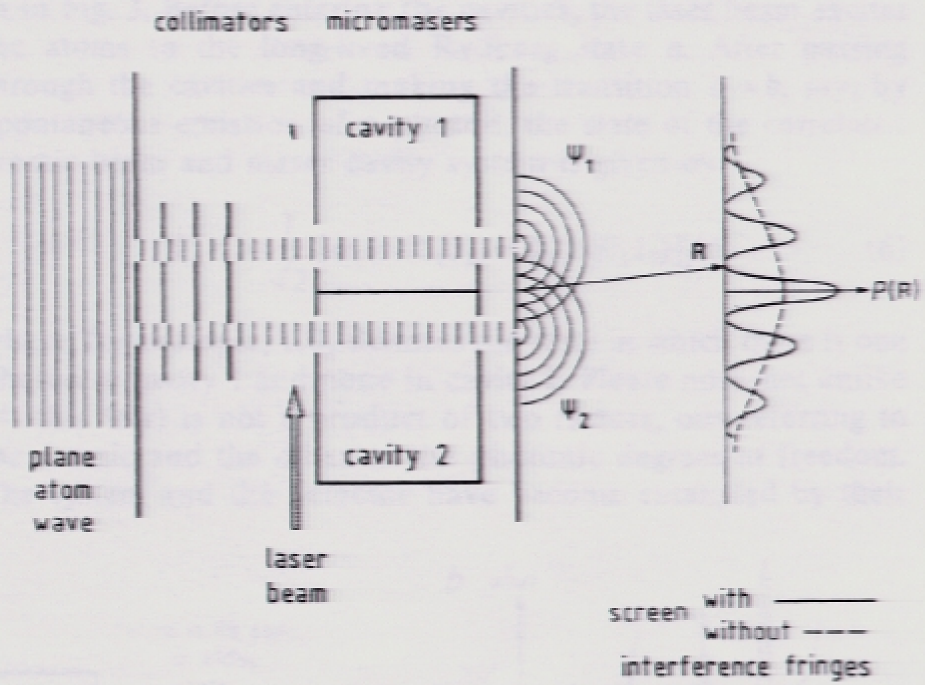


FIG. 3 Two-slit experiment with atoms. A set of wider slits collimates two atom beams which illuminate the narrow slits where the interference pattern originates. The collimation of the atomic beams would actually be done using atomic optics. One could, for instance, employ six-pole fields operating either on the magnetic dipole moment, or in the case of Rydberg atoms on the field-induced electric dipole moment. This set-up is supplemented by two high-quality micromaser cavities and a laser beam to to provide which-path information.

Scully, Englert, Walther, NATURE · VOL 351 · 9 MAY 1991

Path marking through entanglement destroys interference:

$$\Psi = \frac{1}{\sqrt{2}}[\psi_1 + \psi_2] \otimes \phi_0$$
$$\longrightarrow \Psi' = \frac{1}{\sqrt{2}}[\psi_1 \otimes \phi_1 + \psi_2 \otimes \phi_2]$$

$$\mathcal{P}(x|\Psi') = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2$$

ERASURE: Path remains indeterminate

$$\Psi' = \frac{1}{\sqrt{2}}[\psi_1 \otimes \phi_1 + \psi_2 \otimes \phi_2]$$

$$= \frac{1}{\sqrt{2}}[\psi_+ \otimes \phi_+ + \psi_- \otimes \phi_-]$$

where

$$\psi_\pm = \frac{1}{\sqrt{2}}[\psi_1 \pm \psi_2], \qquad \phi_\pm = \frac{1}{\sqrt{2}}[\phi_1 \pm \phi_2]$$

Compare: EPR

Novel features:

- Utilisation of entanglement
- No significant change of component wave functions ψ_1, ψ_2
- Possibility of erasure

Controversial claims:

- No significant momentum transfer
- Uncertainty relation not needed to enforce complementarity

Debate 1991-1995

"In the first two of these examples [2-slit scheme with Bohr's recoiling slit or Feynman's light microscope], Heisenberg's position-momentum uncertainty relation

$$\delta x\, \delta p \geq \frac{\hbar}{2}$$

makes it impossible to determine which hole the electron (or photon) passes through without at the same time disturbing the electrons (photons) enough to destroy the interference pattern."

[SEW 1991]

Rebuttal:

"The authors then conclude that the principle of complementarity is simply a consequence of Heisenberg's uncertainty relation and not a more fundamental concept. We disagree. The principle of complementarity is much deeper than the uncertainty relation, although it is frequently enforced by $\delta x \delta p \geq \hbar/2$."

[SEW 1991]

Both parties criticise each other of physical mistakes and misunderstandings in their analyses of the scheme, and the debate seems to end inconclusively.

A reconciliation is offered by another party a year later.

Reaction:

"We have shown that the loss of interference from a double slit in the presence of a welcher Weg detector is physically caused by momentum kicks, the magnitude of which are determined by the uncertainty principle. We therefore conclude that the principle of complementarity is a consequence of the Heisenberg uncertainty relation. The source of these momentum kicks in the micromaser detector suggested by Scully et al... is the repeated emission and reabsorption of microwave photons by the atom."

[Storey, Tan, Collett, Walls, Nature 367, 17 Feb 1994]

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Proposed Reconciliation:

"In his debates with Einstein, Bohr used the simple picture of uncontrolled classical momentum kicks to show how the uncertainty principle enforced complementarity... Scully et al. have shown that this naive realist interpretation of the uncertainty principle does not work in general. There is room for Scully et al. to claim that complementarity is more fundamental than the uncertainty principle; but there is also room for claims by Storey et al. that one can always consider complementarity as being enforced by the uncertainty principle, if instead the latter is interpreted in terms of the more subtle idea of momentum-kick amplitudes.

[Wiseman, Harrison, Nature 377, 19 Oct 1995]

– do not question the link between momentum kicks and uncertainty principle

– Salomonic solution, or ... authority problem?

Debate continues 1998-...

- Experimental realisation: Dürr, Nonn, Rempe, "Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer", Nature 395, 3 Sep 1998

"... A microwave field is used to store the which-way information in internal atomic states. We study the mechanical effect of the which-way detection on the atomic centre-of-mass motion separately, and ... find that the 'classical' momentum kicks are much too small to wash out the interference pattern. Instead, correlations between the which-way detector and the atomic motion destroy the interference fringes. We show that the back action onto the atomic momentum implied by Heisenberg's position-momentum uncertainty relation cannot explain the loss of interference...
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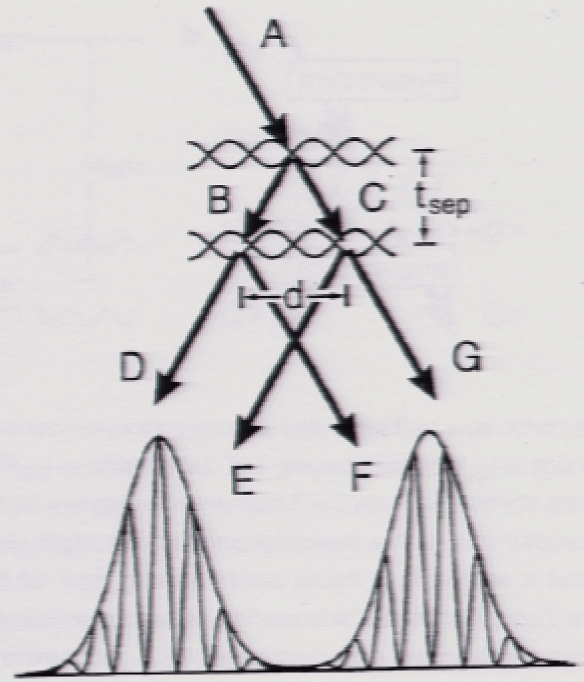

Figure 1 Scheme of the atom interferometer. The incoming atomic beam A is split into two beams: beam C is transmitted and beam B is Bragg-reflected from a standing light wave. The beams are not exactly vertical, because a Bragg condition must be fulfilled. After free propagation for a time t_{sep} the beams are displaced by a distance d. Then the beams are split again with a second standing light wave. In the far field, a spatial interference pattern is observed.

Durr, Nonn, Rempe, Nature 395 3 Sep 1998

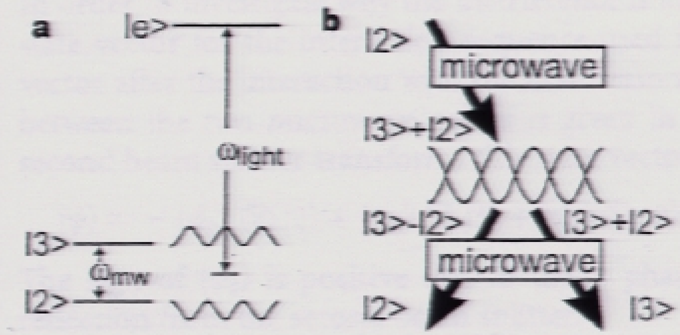

Figure 3 Storage of which-way information. **a**, Left, simplified level scheme of ^{85}Rb. The excited state ($5^2P_{3/2}$) is labelled $|e\rangle$. The ground state ($5^2S_{1/2}$) is split into two hyperfine states with total angular momentum $F = 2$ and $F = 3$, which are labelled $|2\rangle$ and $|3\rangle$, respectively. Right, the standing light wave with angular frequency ω_{light} induces a light shift for both ground states which is drawn as a function of position. **b**, The beam splitter produces a phase shift that depends on the internal and external degree of freedom. A Ramsey scheme, consisting of two microwave $\pi/2$ pulses, converts this phase shift into a population difference (see text).

Subsequent 'resolutions'

Björk et al, Phys. Rev. A 60, Sep 1999:
– complementarity and uncertainty relation "are intimately connected"

Luis, Phys. Rev. A 64, June 2001:
"Very simple duality relations assessing complementarity for two-dimensional systems are obtained... These relations fully explain the enforcement of complementarity in situations in which the standard position-momentum uncertainty relation plays no role."

Dürr, Rempe, "Can wave-particle duality be based on the uncertainty relation?", Am. J. Phys. 68, Nov 2000:

"The explanations for the loss of interference fringes involving *only* the uncertainty relation are (so far) limited to a few special schemes. In other words: There are several other schemes for which no such explanation is known ... On the other hand, explanations involving *only* correlations apply to all which-way schemes known so far. This leads to the conclusion that wave-particle duality is connected to correlations more closely than to the uncertainty relation."

Kim, Mahler, "Uncertainty rescued: Bohr's complementarity for composite systems", Phys. Lett. A 269 (2000) 287-292:

– use uncertainty relation including covariance term for observables of 'system+probe'

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II. Discussion

Issues of the controversy

1. Complementarity $\leftrightarrow$ Wave Particle Duality
2. Complementarity $\leftrightarrow$ Uncertainty
3. Mechanisms enforcing Complementarity
   Uncertainty $\leftrightarrow$ kicks?!
   entanglement – vs. uncertainty?

Roots of Controversy

Critique

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Bohr, Heisenberg, Pauli

– exerted great 'flexibility' in the use of terms

"...Bohr's principle of complementarity, the sharp formulation of which, moreover, I have been unable to achieve despite much effort which I have expended on it." [A Einstein 1949]

"In the later literature, there have been attempts to give a very precise meaning to this concept of complementarity. But it is at least not in the spirit of our discussion in the Copenhagen of 1927 if the unavoidable lack of precision in our language shall be described with extreme precision." [W Heisenberg 1976]

von Weizsäcker's 1955 analysis and interpretation of Bohr's conceptions of complementarity were categorically rejected by Bohr. [M Jammer 1974]

Bohr scholarship $\cdots\rightarrow$ "Bohr-bashing"

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Bohr – Complementarity Interpretation

(1928,1935)

"The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively." [Bohr 1928]

Complementarity of –

(C1) space-time description and causal description
observation vs. (state) definition

(C2) wave picture and particle picture

(C3) wave mechanics and particle mechanics

(C4) definition of energy-momentum (plane wave)
and
definition of space-time localization (packet)

(C5) position and momentum (1935)
mutual exclusion of experimental setups
e.g. for measurements of position or momentum

Limitation of classical physical concepts and modes of description

Origin: Quantum Postulate

— discontinuity/individuality of quantum phenomena

observation → non-negligible interaction, state change

→ "Heisenberg effect" (state reduction?)

"Non-separability" in *statu nascendi*?

Emphasis of "Phenomenon"

(inseparability of object and measuring instrument)

Relational state concept [Jammer 1974]

"Now, the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed." [Cf. Heisenberg's "cut".]

"In particular, it became clear to me that the statistical interpretation of the theoretical results always enters at the point where one divides a closed system into two parts, which then are interpreted as observed object and measuring instrument, respectively, and then asks what can be said about one part without knowledge of the other."

[W Pauli, Letter to Bohr, 17 Oct 1927]

Bohr's Program:

"Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a "complementarity" theory the consistency of which can be judged only by weighing the possibilities of definition and observation." (Sec. 1, 1928)

And the very last sentences of the 1928 paper:

"Already the formulation of the relativity argument implies essentially the union of the space-time co-ordination and the demand of causality characterizing the classical theories. In the adaptation of the relativity requirement to the quantum postulate, we must therefore be prepared to meet with a renunciation as to visualization in the ordinary sense going still further than in the formulation of the quantum laws considered here. Indeed, we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of Nature. The hindrances met with on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception. In the quantum theory we meet this difficulty at once in the question of the inevitability of the feature of irrationality characterizing the quantum postulate. I hope, however, that the idea of complementarity is suited to characterize the situation, which bears a deep-going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object."

Far-sighted! (but Heisenberg went further with the modulation of "visualizability")

If you still find Bohr obscure – remember his dictum of the complementarity of clarity and truth.

And yet, Bohr's "Complementarity Theory" Program was an inspiration to part of the research community.
e.g., Ludwig (1950s-1980s); Lahti (1980s); Schwinger

Heisenberg – Uncertainty Interpretation

(1927,1930)

"One may view the world with the p-eye and one may view it with the q-eye but if one opens both eyes simultaneously then one gets crazy." [W Pauli, Letter to Heisenberg, 19 Oct 1926]

"Es wird Tag in der Quantenmechanik." [W Pauli 1927]

"It is the theory which decides what we can observe."

Cloud chamber particle tracks are only "approximate" trajectories
– in accordance with the uncertainty relations.

- uncertainty relation for preparations as consequence of formalism
- inaccuracy of simultaneous measurement
- mutual disturbance of position measurements and momentum measurements

Heisenberg's Program (1927):

"Quantum kinematics and mechanics shows far reaching differences from the ordinary theory. The applicability of classical kinematics and mechanical concepts, however, can be justified neither from our laws of thought nor from experiment. The basis for this conclusion is relation (1), $p_1 q_1 \sim h$... Of course we would also like to be able to derive, if possible, the quantitative laws of quantum mechanics directly from the physical foundations - that is, essentially, from relation (1)... As soon as one accepts that all quantum-theoretical quantities are "in reality" matrices, the quantitative laws follow without difficulty."

Geometric quantization,
quantum mechancis on phase space,...

Heisenberg 1955

Quantum mechanics as the mathematics of the possible, not actual

Environment-induced superselection through interaction of apparatus with classical environment

POTENTIALITY ontology (1950s)

Heisenberg on Complementarity

- particle picture (Ψ function) and wave picture (quantized field) are equivalent alternative options [not mutually contradicting necessities]

- matrix mechanics and wave mechanics are equivalent representations of the particle picture

- accepts view that uncertainty relation is an expression of complementarity of position and momentum

Bohr on Uncertainty

Uncertainty Relation is –

- expression of limitations of possibilities of definition and measurement
- expression of limitation of application of classical physical concepts
- confirmation of the consistency of the quantum formalism with the description of experience
- expression of the inevitable state change due to measurement

Pauli as mediator (1927, 1933, 1958, 1980)

"With Heisenberg's uncertainty principle and Bohr's fundamental discussions thereon the initial phase of development of the theory came to a preliminary end."

"This solution [of the problems of the wave-particle duality of light and matter] is obtained at the cost of abandoning the possibility of treating physical phenomena objectively, i.e. by abandoning the classical space-time and causal description of nature which essentially rests upon our ability to separate uniquely the observers and the observed."

- complementarity of position and momentum
- uncertainty relation as expression of this complementarity

"We might call modern quantum theory 'The Theory of Complementarity'..."

All three – Bohr, Heisenberg, and Pauli – speak about atomic objects (electrons, photons) as unsharply defined individuals whose locations and momenta are approximately defined through the extension of their wave packets, in obeyance to the uncertainty relation.

Three approaches...

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Three approaches...

Critique

Recall: Issues of the controversy

(1) Complementarity $\leftrightarrow$ Wave Particle Duality

SEW used 'value complementarity'

(2) Complementarity $\leftrightarrow$ Uncertainty

Long story cut short:

Position-momentum UR is *not* sufficient to enforce 'strict' complementarity in 2-slit experiment.
Wootters, Zurek (1979); Hilgevoord, Uffink (1980s)

But: value complementarity (for Mach-Zehnder) is limiting case of suitable uncertainty relations.

Shift of meaning:

from strict complementarity

to graded (or quantitative) complementarity

– simultaneous, but partial, path and and interference information (knowledge)

(3) Mechanisms enforcing Complementarity

Uncertainty → kicks?!

entanglement – vs. uncertainty?

Traditional Conflation:

uncertainty ⟷ (classical) momentum kick (disturbance)

Novel alternative:

uncertainty ⟷ (quantum) momentum transfer

– need for mechanistic, causal explanation?

Physical Mechanism (Einstein-Bohr):

path determination ⟷ momentum transfer

Novel Mechanism:

path determination via entanglement

Recall Questions:

Complementarity – more fundamental than Uncertainty?

Entanglement – more fundamental than Uncertainty?

Work from 1980s

Critique of disturbance theory of uncertainty by Brown, Redhead (1980) is completely ignored

Much of the work of 1980's on relation between complementarity and uncertainty is largely ignored

Hilgevoord, Uffink (1983-1985, 1988): New mathematical expression of the uncertainty principle:
– alternative measure of width
– concept of fine structure
– show that standard uncertainty relation does not in general suffice to make Bohr's defense of complementarity compelling – discuss graded duality relations
– however: they use term uncertainty principle for a relation that they then regard as an expression of complementarity – without giving an independent definition of either notion

Graded complementarity in 1980s:
'simultaneous partice/wave knowledge',
or: 'simultaneous path/interference information'

– Deutsch (1984): entropic uncertainty relation
– Mittelstaedt, Prieur, Schieder (1987): experiment
– Greenberger, Yasin (1988)
– more widely noted since mid-1990s

Incomplete Review: B.-G. Englert, J.A. Bergou, "Quantitative quantum erasure", Optics Communications 179 (2000) 337-355

Quantitative complementarity in erasure context:

$$\mathcal{P}(\rho)^2 + \mathcal{C}(\rho)^2 \leq 1$$

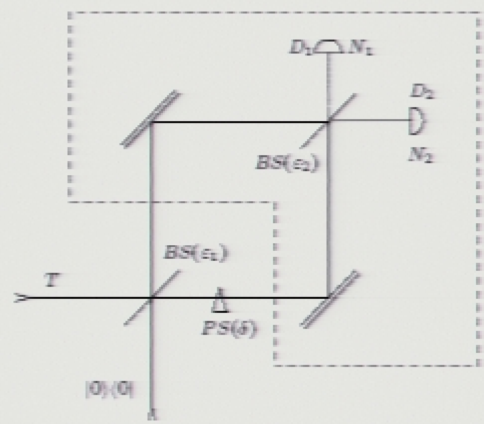
where $\mathcal{P}(\rho)$ is path predictability (minimally available path knowledge) and $\mathcal{C}(\rho)$ is the coherence (maximally available visibility).

Connection with joint measurability not noted by Englert but by Björk et al (1999) and by de Muynck (2000).

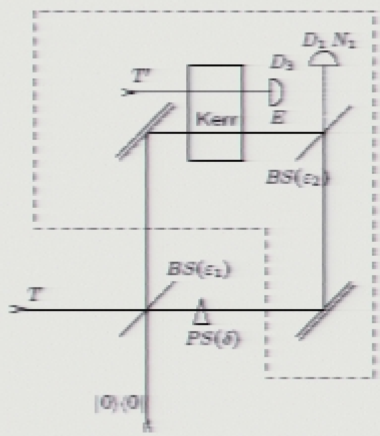
Measurement Complementarity

Distinction between Preparation and Measurement in discussions of complementarity and uncertainty remains largely unnoticed.

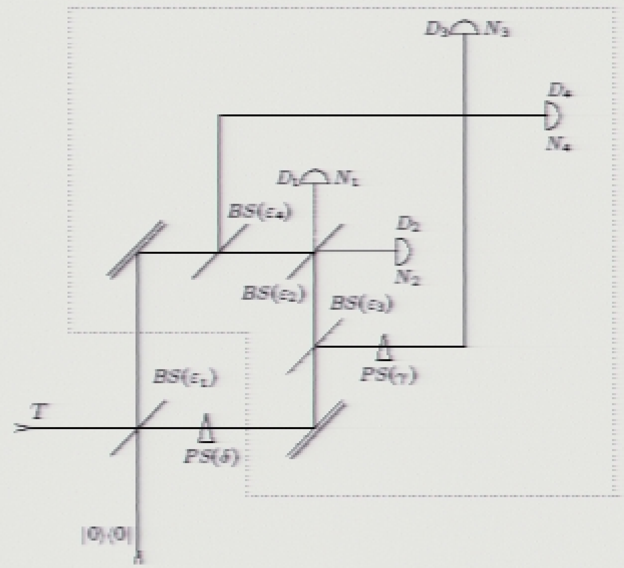
Mach-Zehnder Interference setup



Imperfect path marking setup



Busch, Grabowski, Lahti,
"Operational Quantum Physics" (1995)
see also: Busch (1985,1987),
Martens and de Muynck (1990), de Muynck (2000),
Busch and Shilladay (2003)



Busch, Found. Phys. 1987

Joint measurability of 'qubit' observables

$$E_{\pm} = \frac{1}{2}[I \pm \mathrm{a}\cdot\sigma], \qquad F_{\pm} = \frac{1}{2}[I \pm \mathrm{b}\cdot\sigma]$$

where $|\mathrm{a}| \leq 1,\ |\mathrm{b}| \leq 1$

E, F jointly measurable if and only if:

$$\frac{1}{2}|\mathrm{a}+\mathrm{b}| + \frac{1}{2}|\mathrm{a}-\mathrm{b}| \leq 1$$

For $\mathrm{a} \perp \mathrm{b}$, this is equivalent to:

$$|\mathrm{a}|^2 + |\mathrm{b}|^2 \leq 1$$

and hence

$$U(E) + U(F) \geq 1$$

where

$$U(E) = \min_{\rho} Var(E,\rho) = 1 - |\mathrm{a}|^2$$
$$U(F) = \min_{\rho} Var(F,\rho) = 1 - |\mathrm{b}|^2$$

are measures of the unsharpness of E, F.

Conclusion – Coherent Account

Complementarity versus Uncertainty

Uncertainty:

– in preparation: latitude in definition of values of observables;

– in measurement: imprecision in individual measurement outcomes.

Uncertainty Principle:

– for preparations: In any quantum state ρ, the values of most observables A are indeterminate – especially if $\rho A - A\rho \neq O$;

– for measurements: Pairs of noncommuting observables can be measured jointly if and only if (?) the measurement imprecisions are not too small.

Uncertainty Relation (as manifestation of UP):

Any trade-off relation between measures of indeterminacy or imprecision of pairs of (non-commuting) observables.

Complementary pair of observables:

setups for preparation or measurement of definite values of the two observables are mutually exclusive (encompassing, broad definition)

– preparation version: if the value of one observable is definite, the other observable is completely uncertain, and vice versa (or some variants of this statement);

– measurement version: non-commuting observables cannot be measured together.

Complementarity Principle:

There are complementary pairs of observables.

Logical Relations

Complementarity is based on the notion of uncertainty.
Highlights algebraic aspect: non-commutativity.

Uncertainty is based on the existence of superpositions.
Highlights operational aspect (canonical classical embedding of qm).

(Strict) Complementarity expresses a limitation: strict exclusion of the simultaneous application of possible preparation or measurement procedures.

Uncertainty Principle expresses a reconciliation of the mutually exclusive preparation or measurement options – at a price: trade-off for uncertainties or imprecisions, uncertainty relations.

CP and UP 'meet' in 'graded/quantitative duality' and 'quantitative erasure'

Entanglement:
an instance of superposition of pairwise orthogonal product states, hence an instance of uncertainty.

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In quantum mechanics, the notions and principles of Uncertainty, Complementarity, Entanglement, Superposition are not completely independent, nor do they stand in any simple logical relation.

CP and UP are consequences of the QM formalism. In order to study the logical relations of these ideas, a more general theory/language is needed in which CP and UP can be formulated as contingent facts. This was made very clear and carried out by Pekka Lahti in the 1980s.

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