

Title: The entanglement interpretation of black hole entropy

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Abstract: We show that the entropy resulting from the counting of microstates of non extremal black holes using field theory duals of string theories can be interpreted as arising from entanglement. The conditions for making such an interpretation consistent are discussed. First, we interpret the entropy (and thermodynamics) of spacetimes with non degenerate, bifurcating Killing horizons as arising from entanglement. We use a path integral method to define the Hartle-Hawking vacuum state in such spacetimes and discuss explicitly its entangled nature and its relation to the geometry. If string theory on such spacetimes has a field theory dual, then, in the low-energy, weak coupling limit, the field theory state that is dual to the Hartle-Hawking state is a thermofield double state. This allows the comparison of the entanglement entropy with the entropy of the field theory dual, and thus, with the Bekenstein-Hawking entropy of the black hole. As an example, we discuss in detail the case of the five dimensional anti-de Sitter, black hole spacetime.

Entanglement interpretation of BH entropy

Ram Brustein



אוניברסיטת בן-גוריון

R.B., M. Einhorn, A. Yarom,
[hep-th/0508217](#)

Series of papers with
Amos Yarom, BGU
(also David Oaknin)

français

english

What does BH entropy count?

- What is entanglement entropy ?
 - ❖ How is it related to BH entropy ?
 - ❖ How to evaluate entanglement entropy ?

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 - ❖ How are the two methods related ?

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What's new ?

- Extension of PI methods to more space-times (BHs, dS)
- Application to FT/Gravity duals

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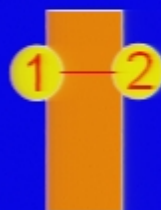
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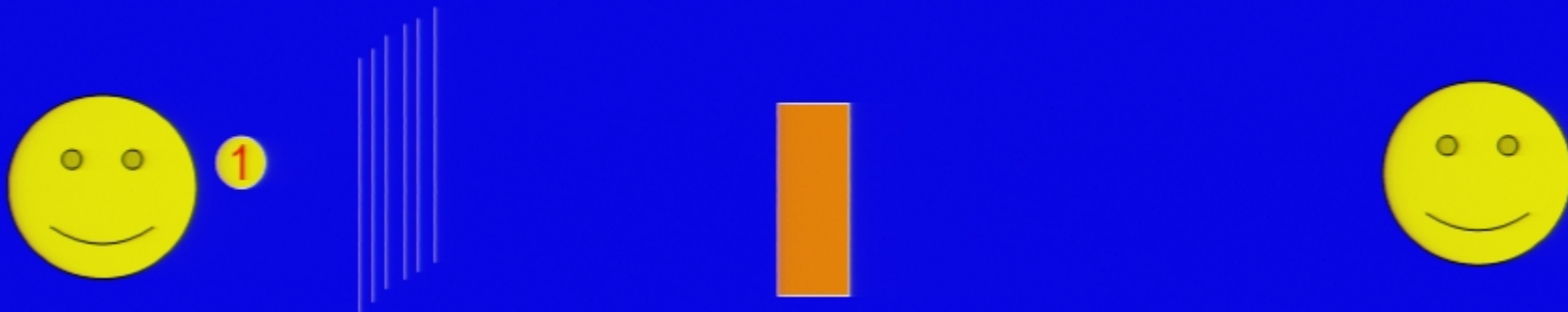


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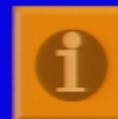
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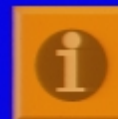
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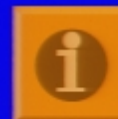
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If : thermal & time translation invariance then TFD:

$$H_{in} = -H_{out} \quad \& \quad |\psi\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta}{2} E_i} |E_i\rangle |E_i\rangle$$

purification

Entanglement in space-time

$$ds^2 = -f(r)d\tau^2 + dr^2/f(r) + q(r)dx_{\perp}^2$$

Examples: Minkowski, de Sitter, Schwarzschild, non-rotating BTZ BH, can be extended to rotating, charged, non-extremal BHs

“Kruskal” extension

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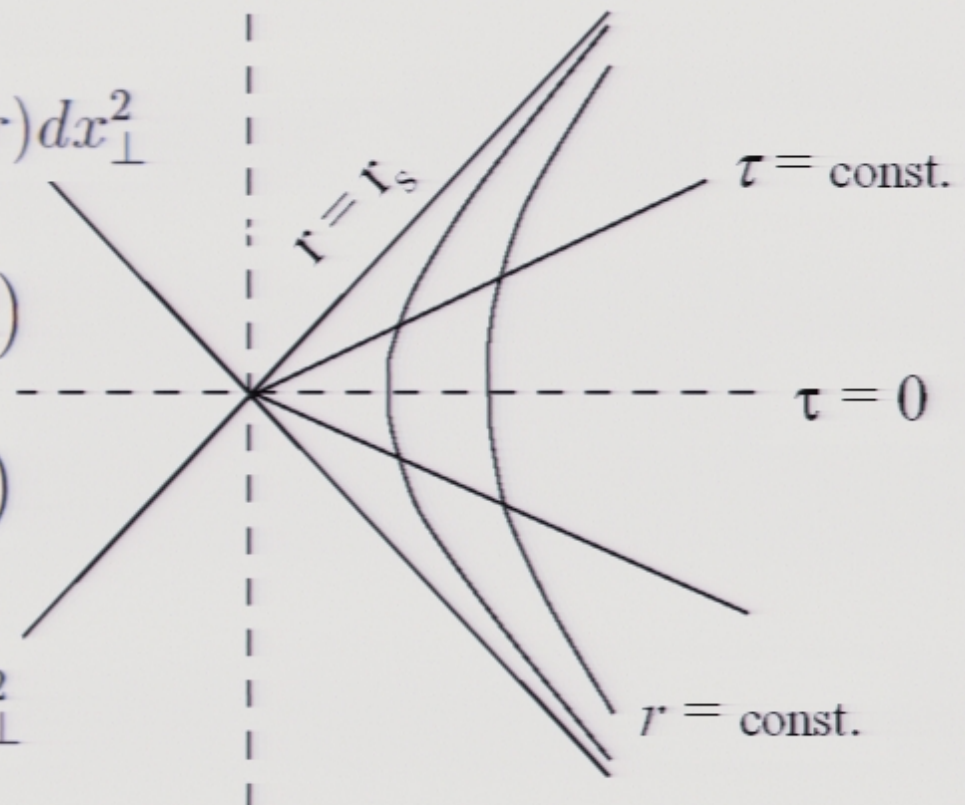
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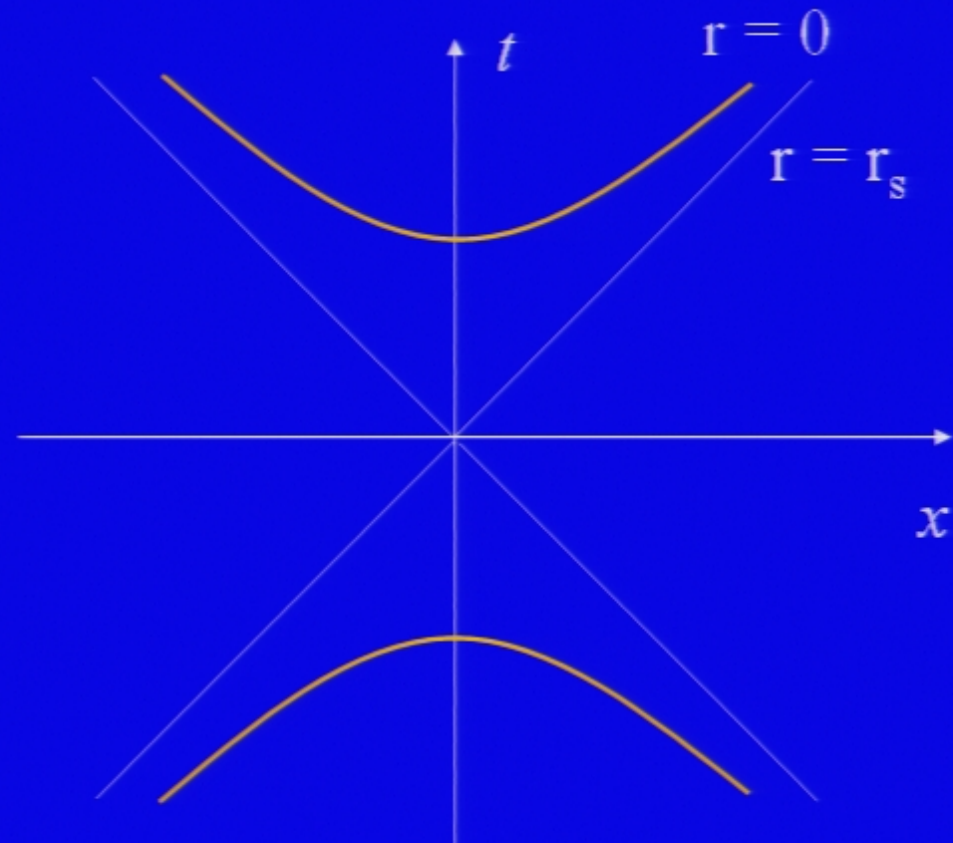
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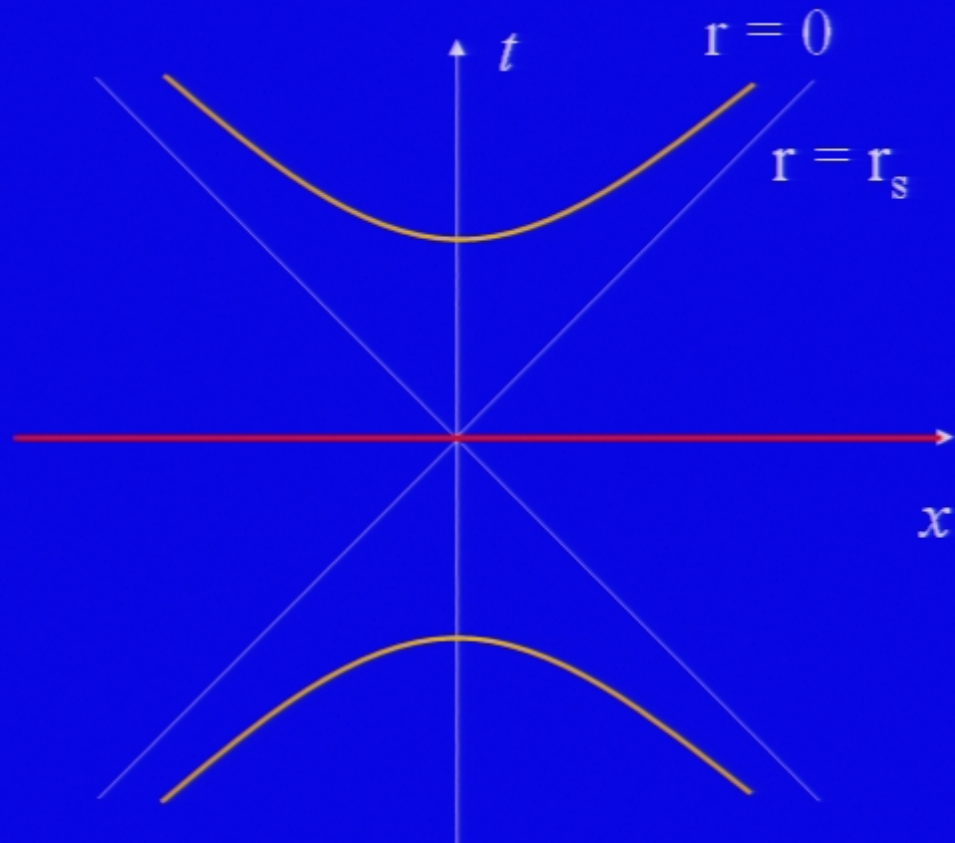
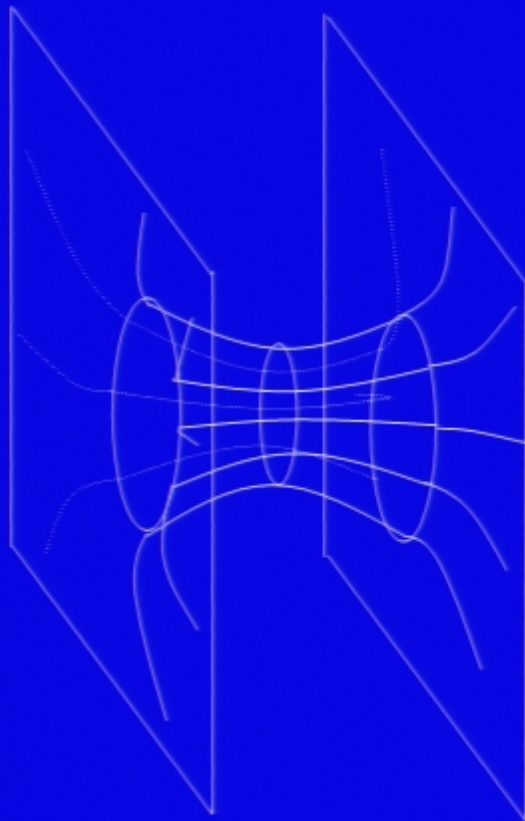
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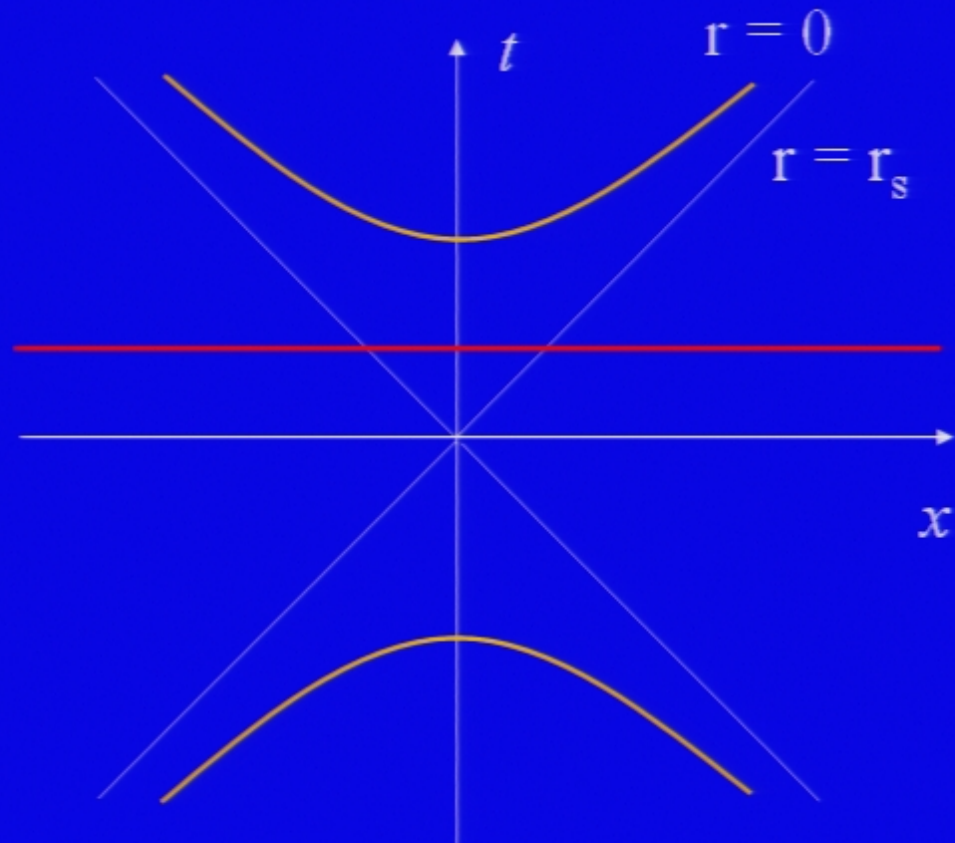
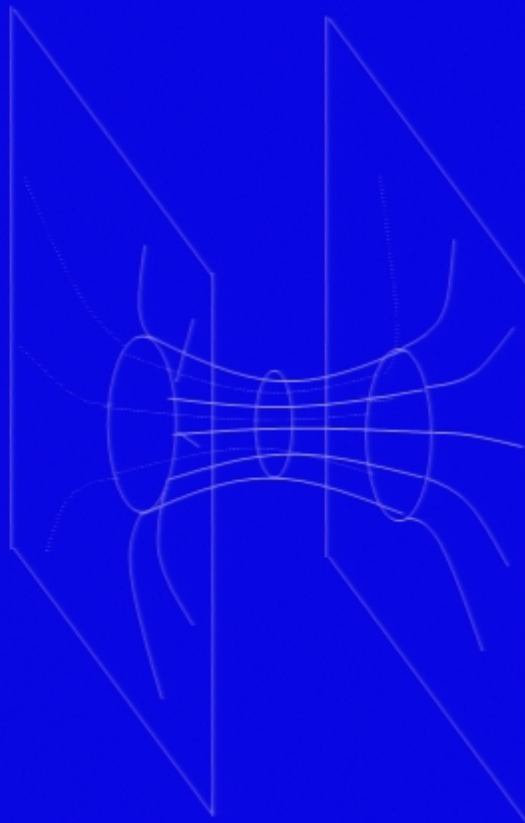
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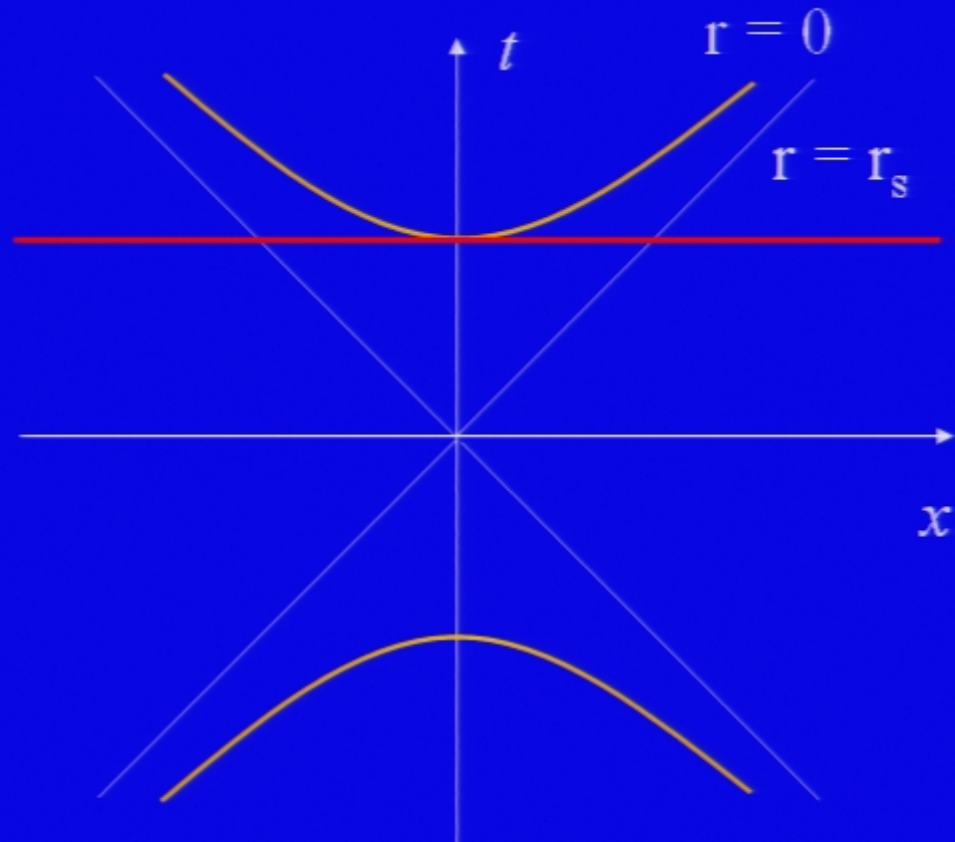
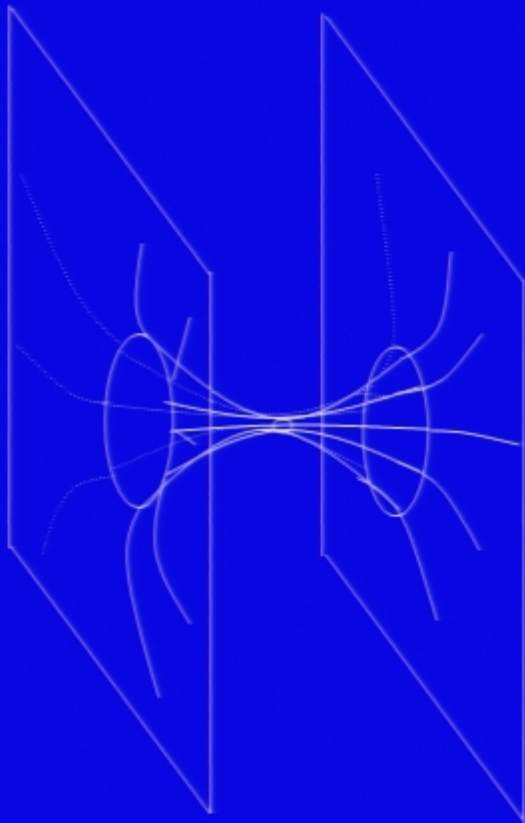
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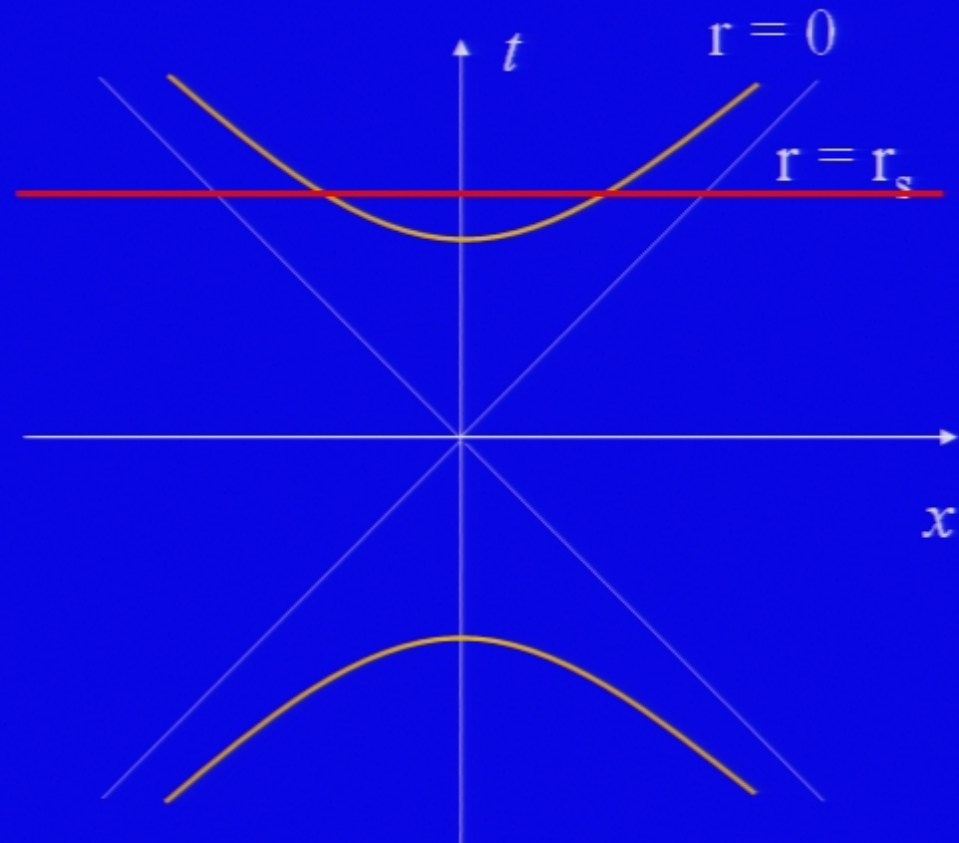
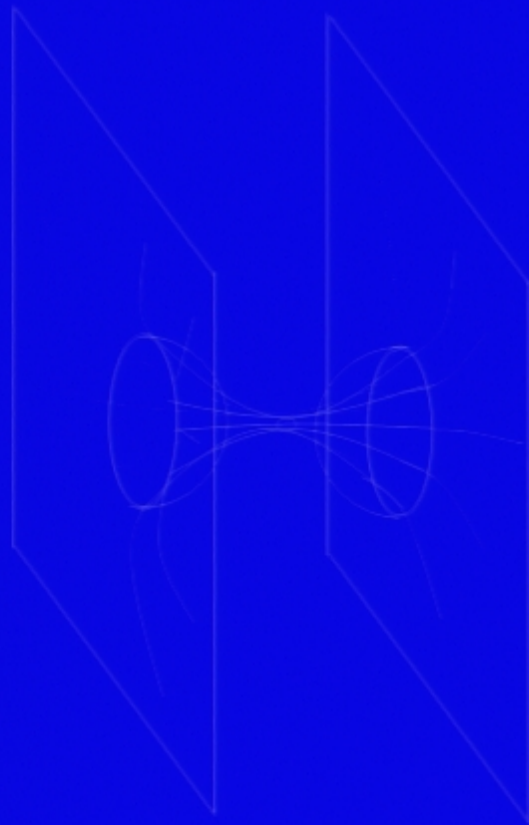
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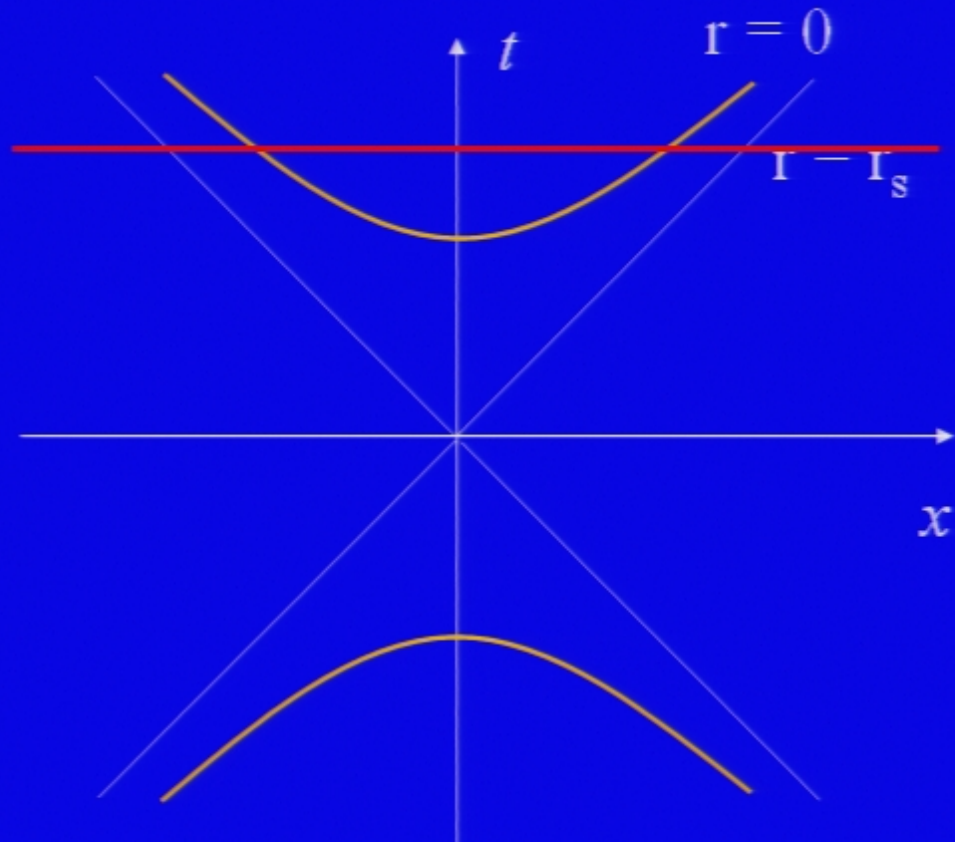
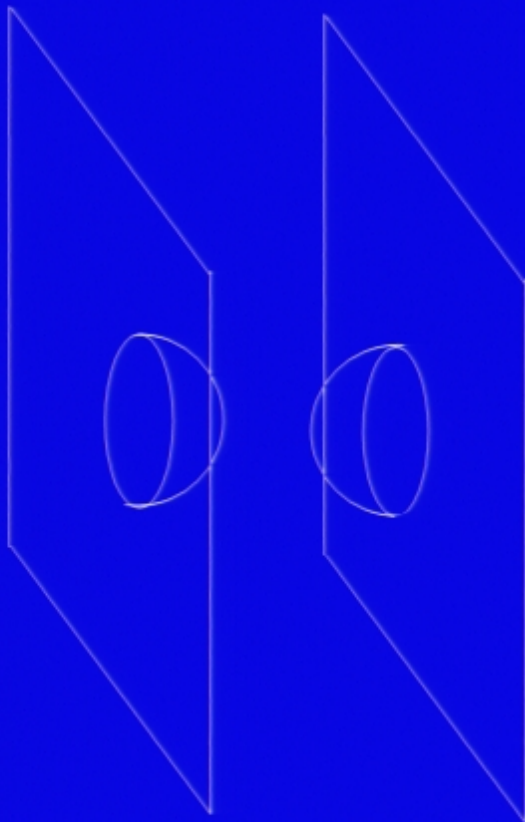
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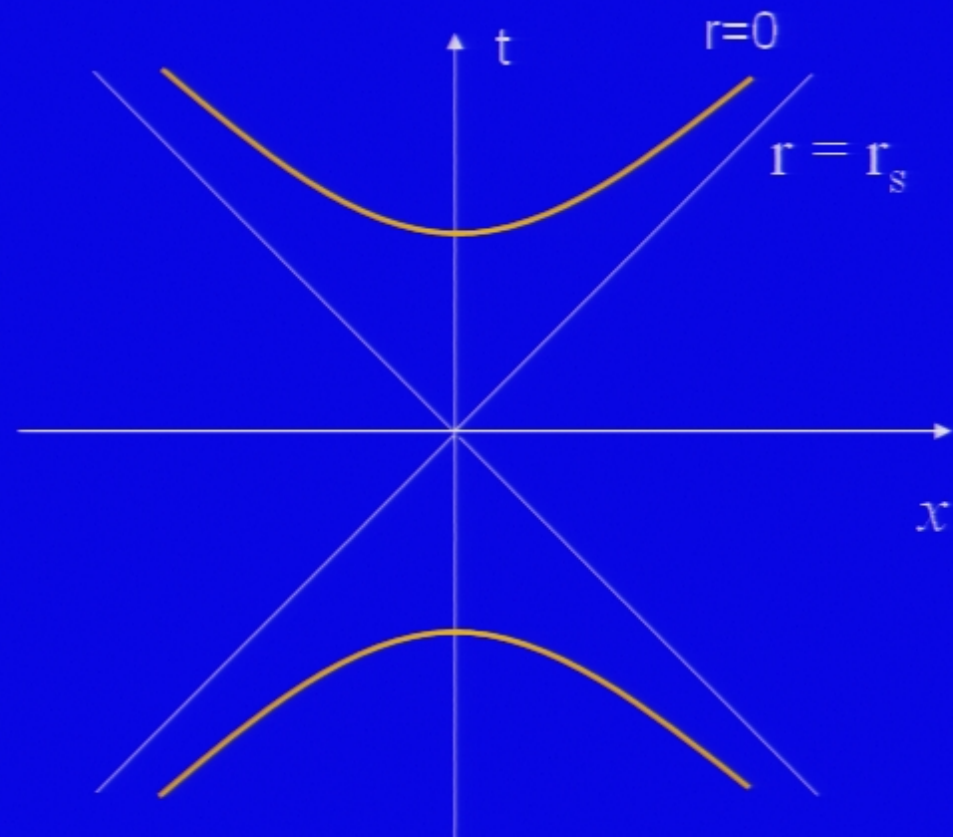
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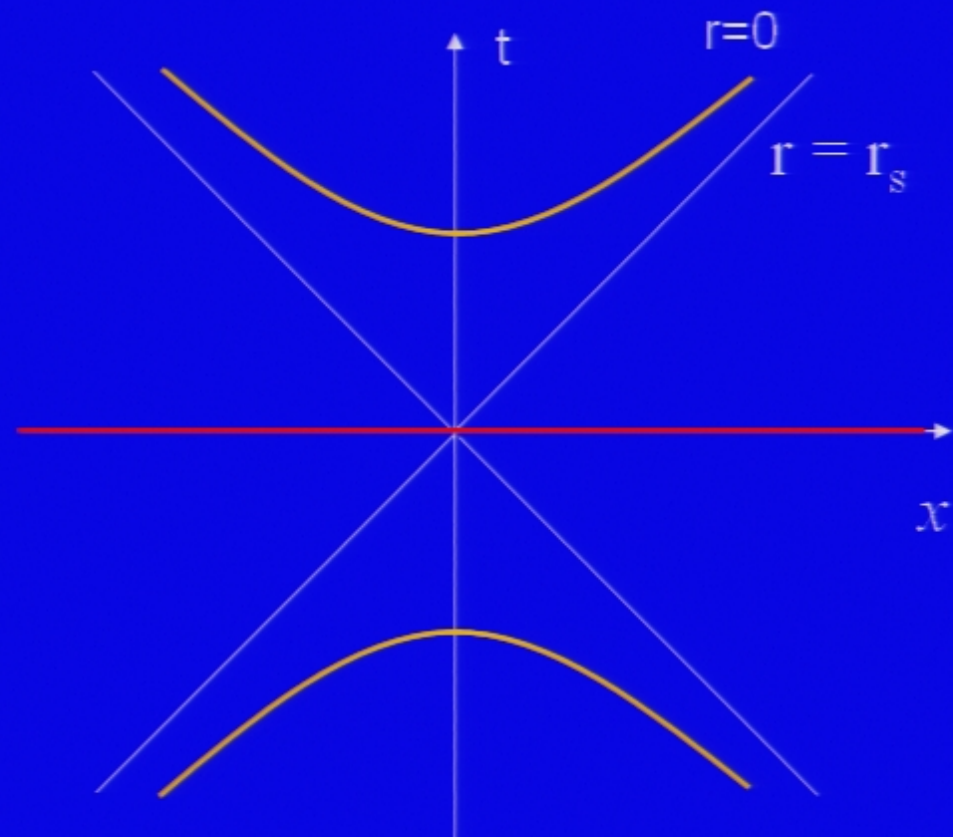
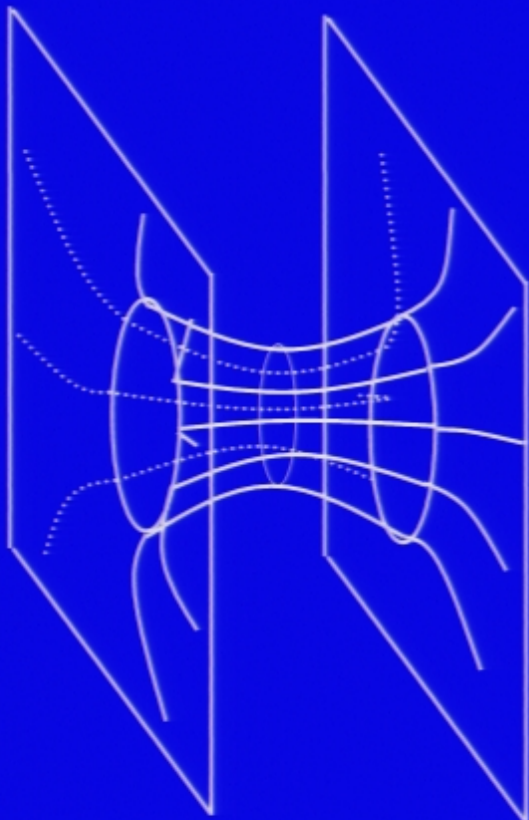
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The vacuum state



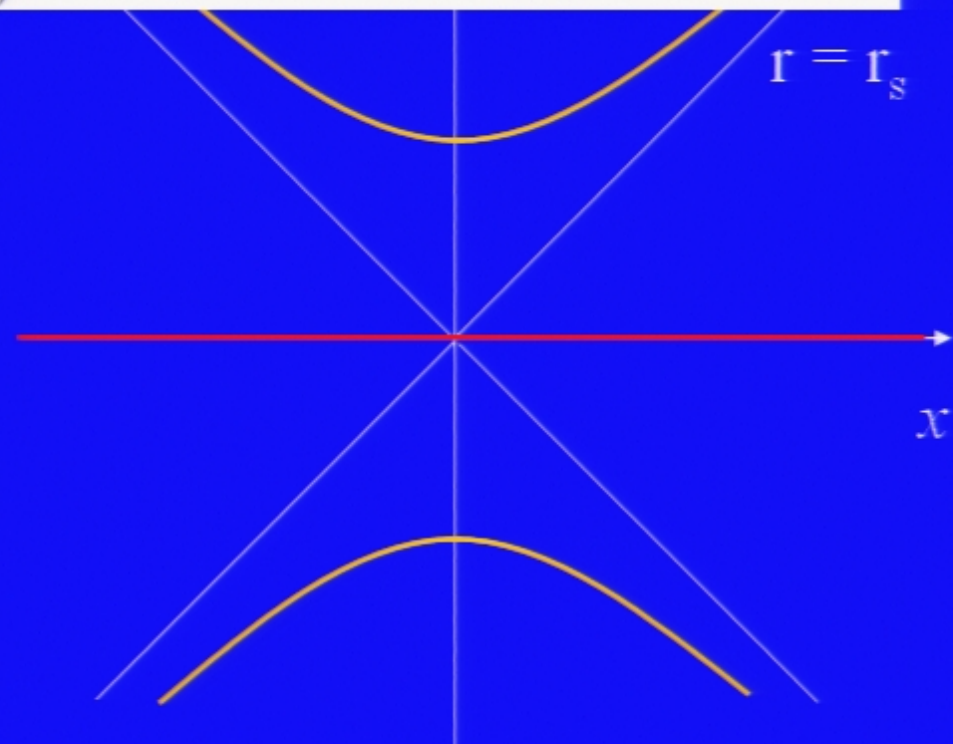
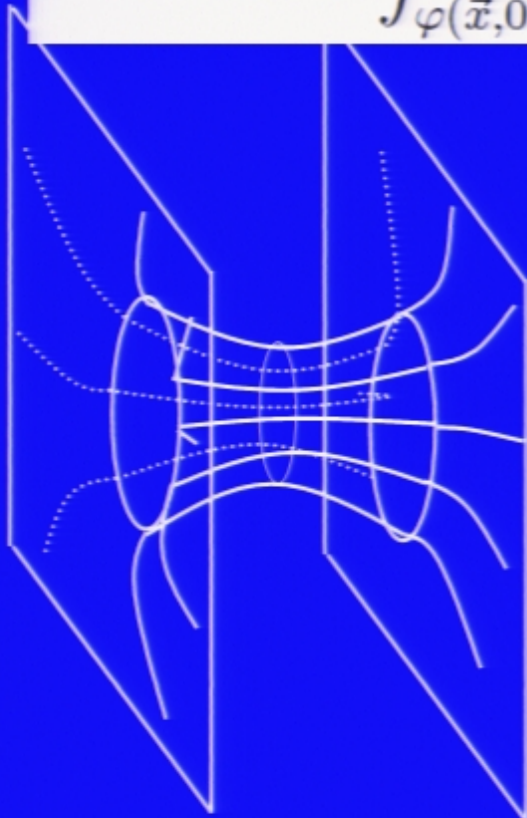
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The vacuum state

$|0\rangle$

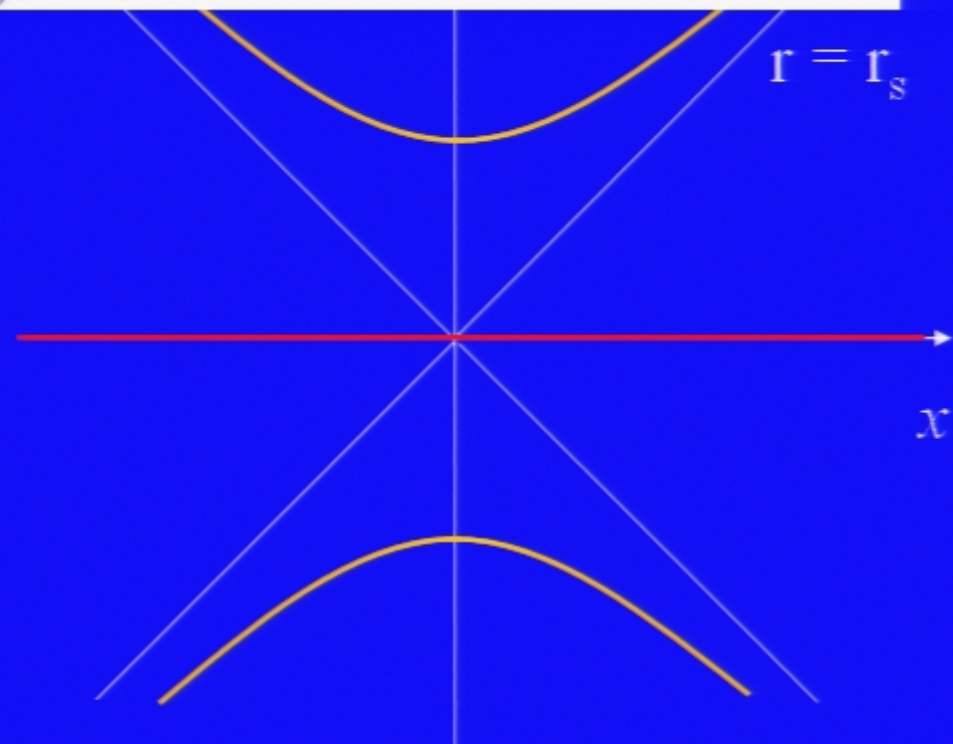
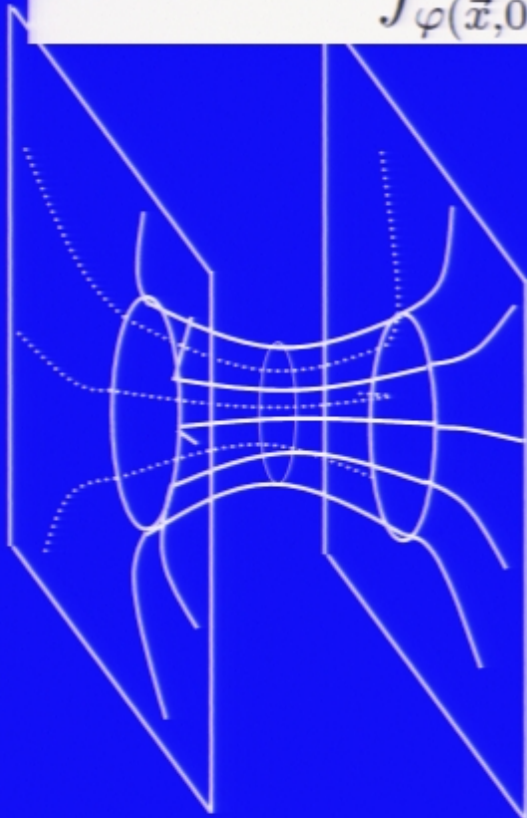
$$\langle 0 | \psi(\vec{x}) \rangle = \int_{\varphi(\vec{x}, 0) = \psi(\vec{x})} \exp \left[- \int_0^\infty \dots \int \mathcal{L} d^d x d\tau \right] D\varphi$$



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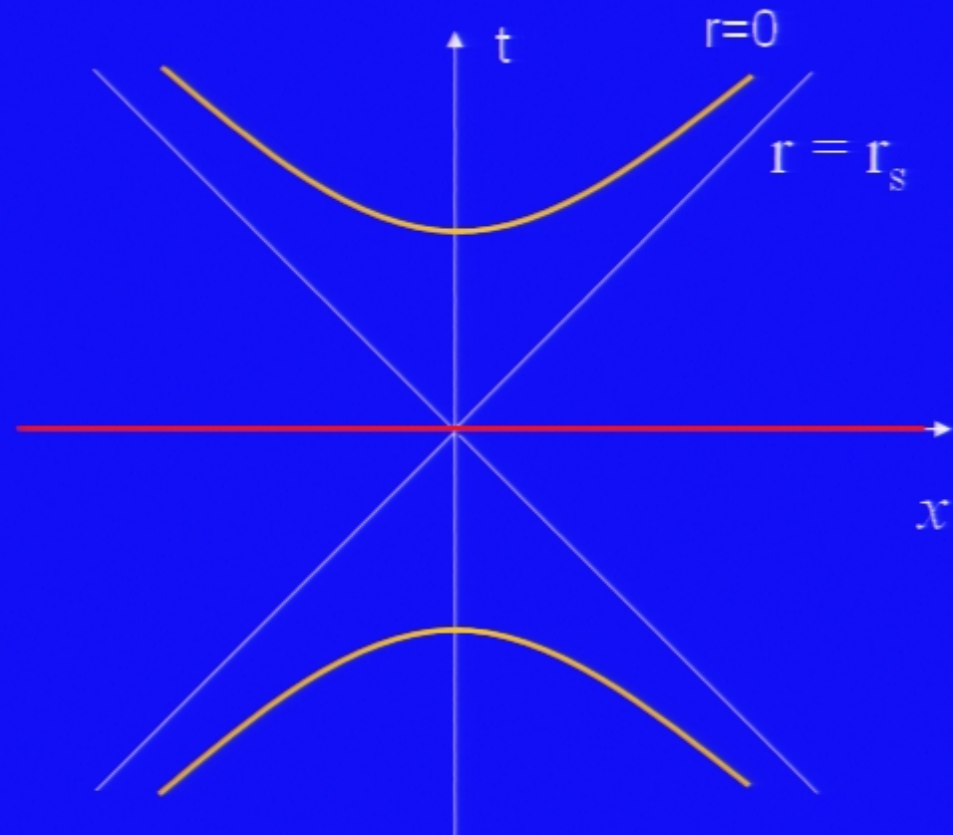
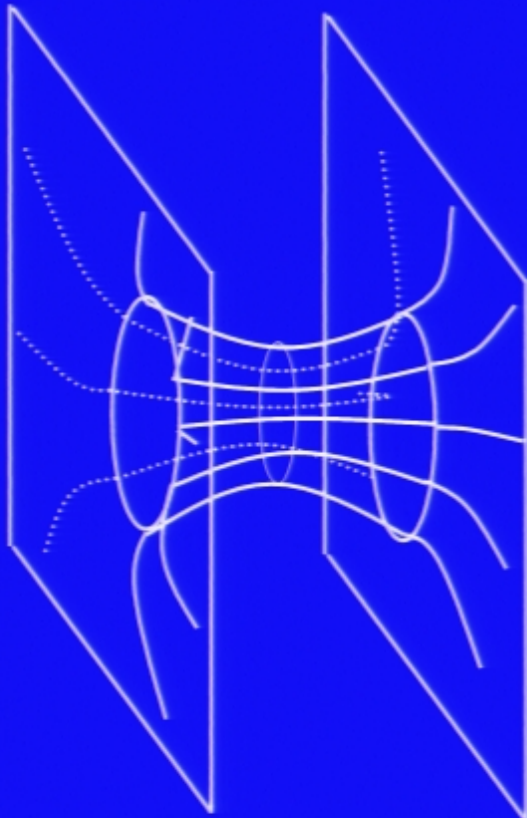
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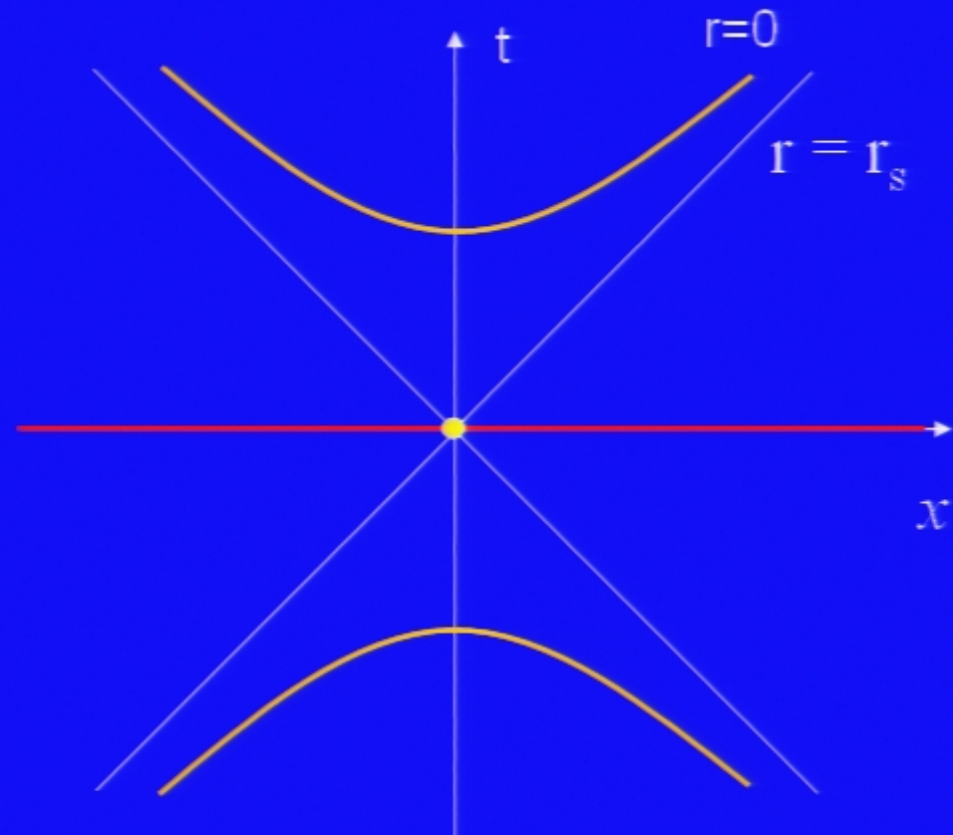
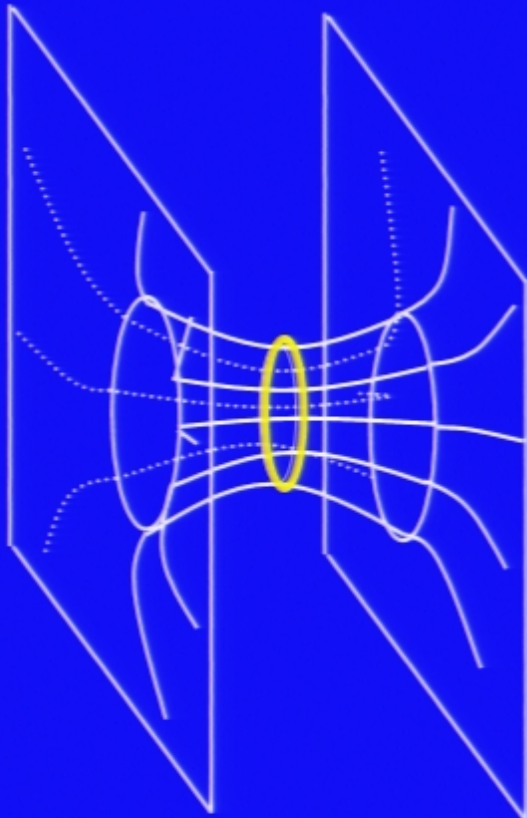


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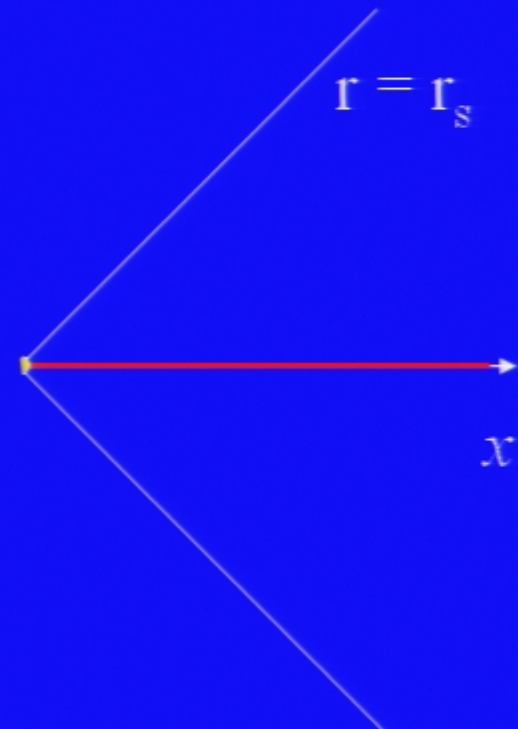
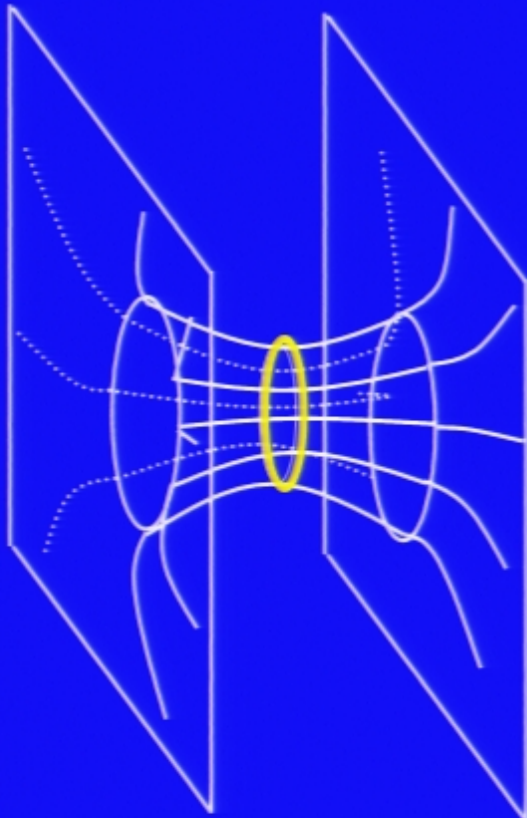
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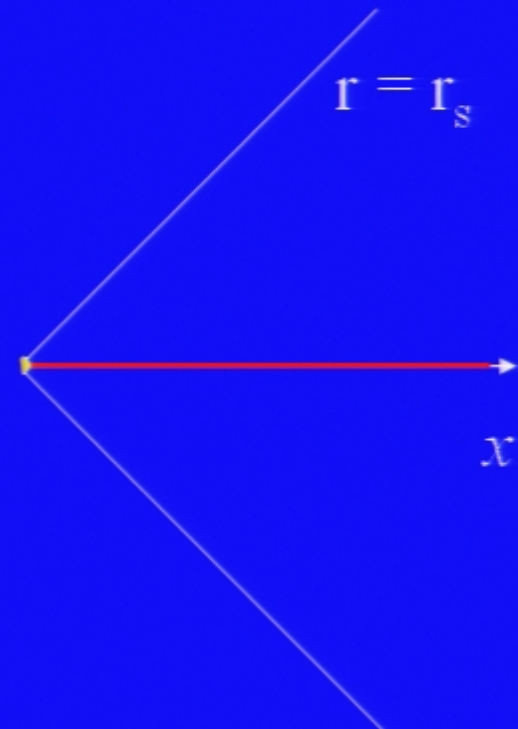
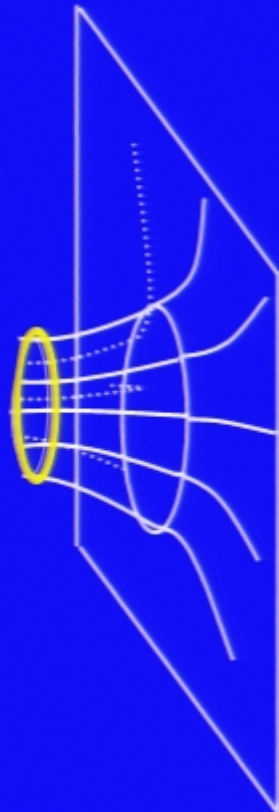
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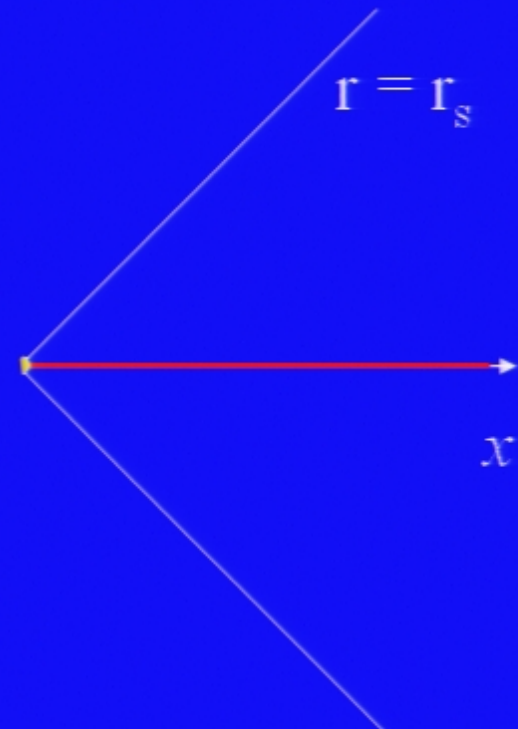
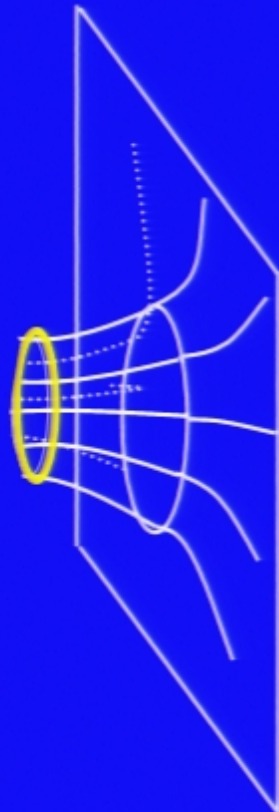
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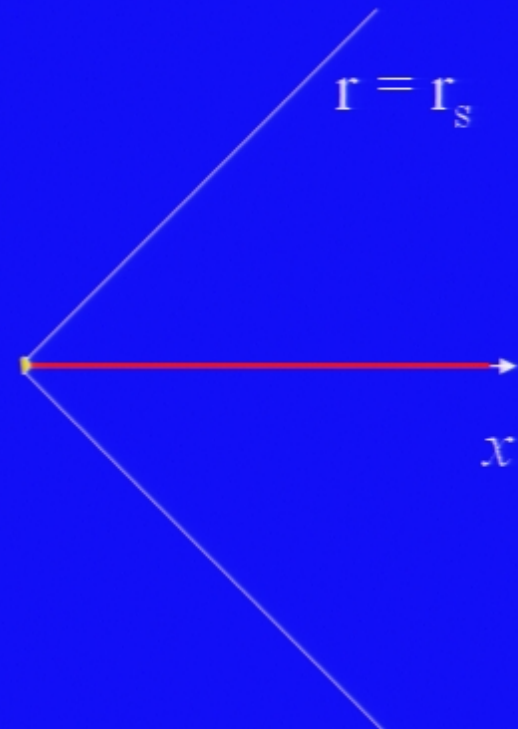
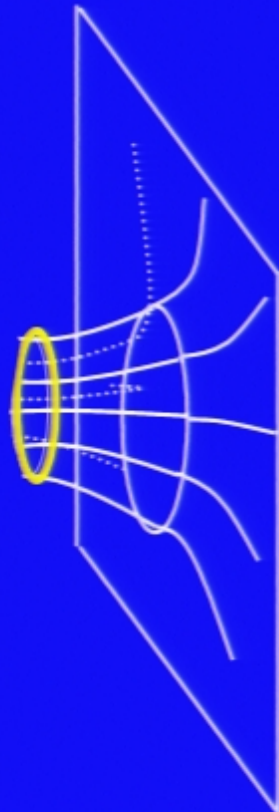
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The vacuum state

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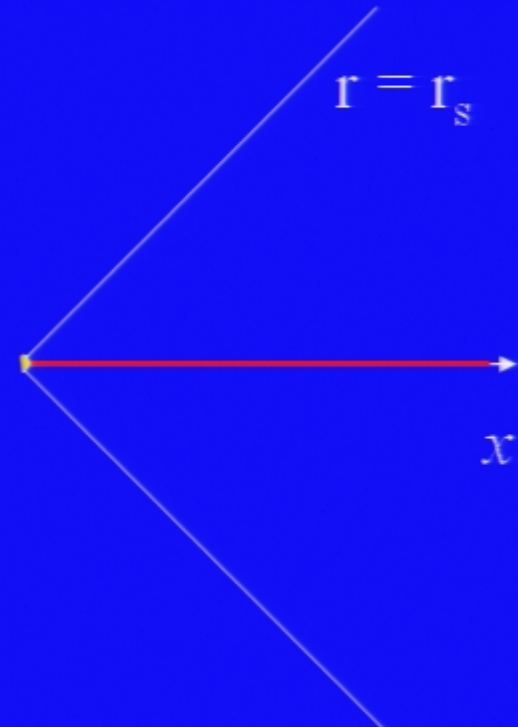
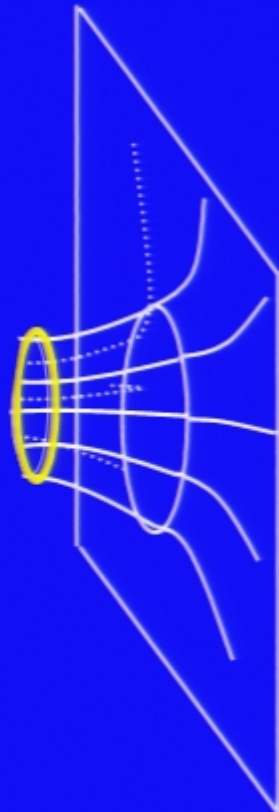
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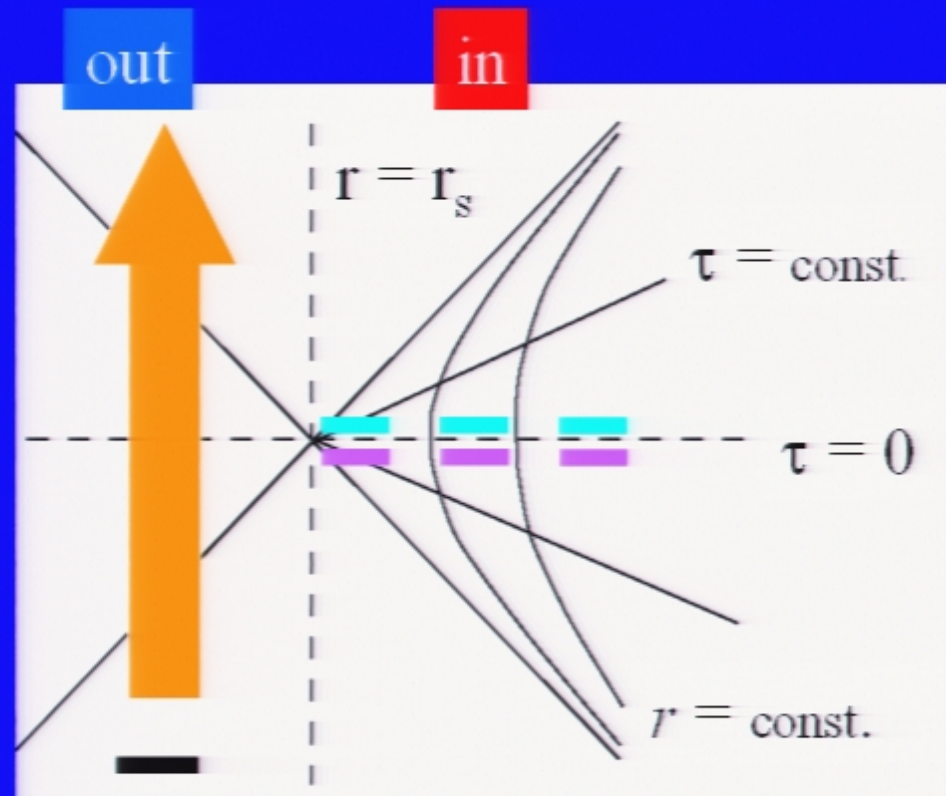
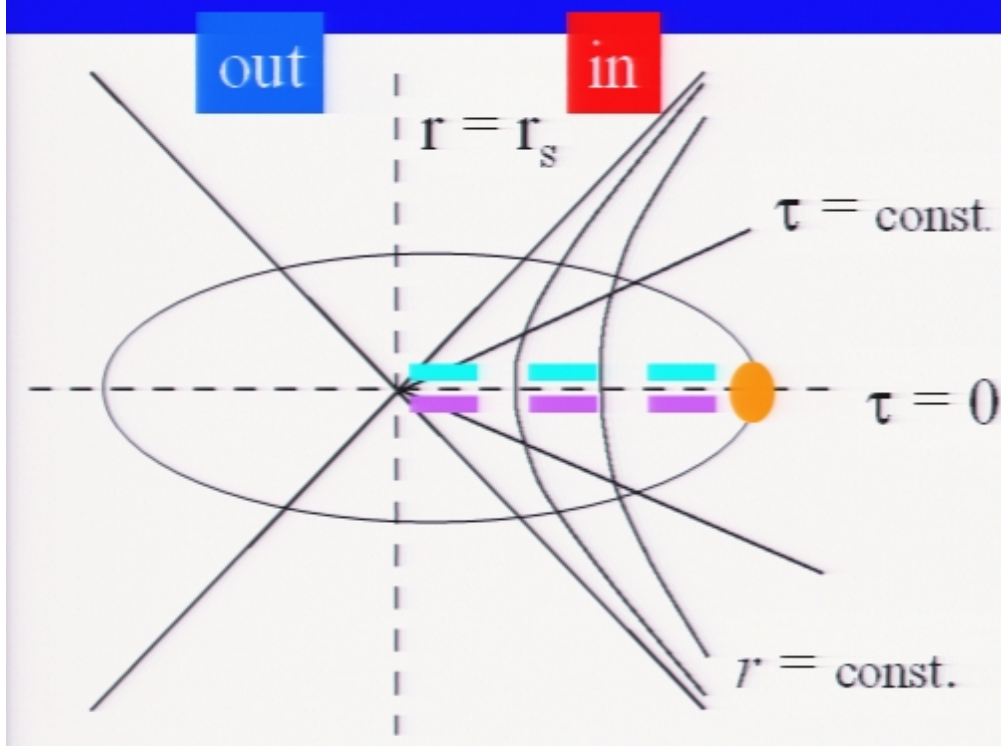
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Two ways of calculating ρ_{in}

R.B., M. Einhorn and A. Yarom



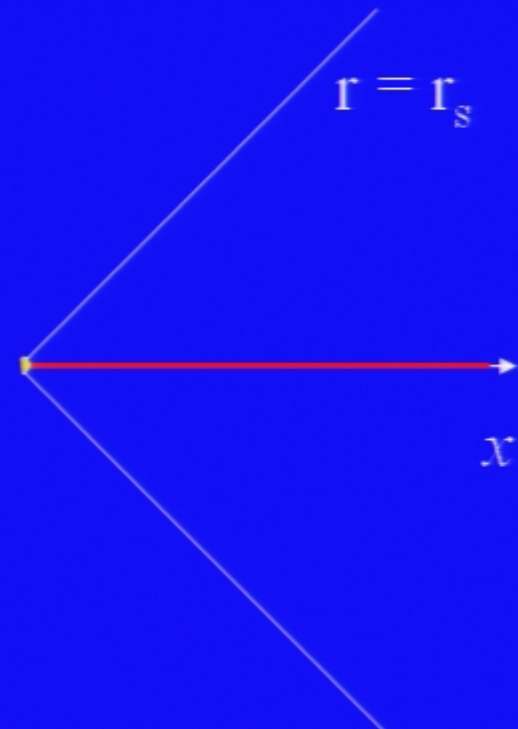
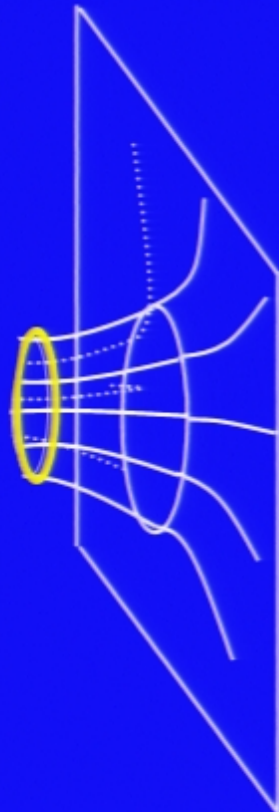
Construct the HH vacuum: the invariant regular state

Kabat & Strassler (flat space)
Jacobson

The vacuum state

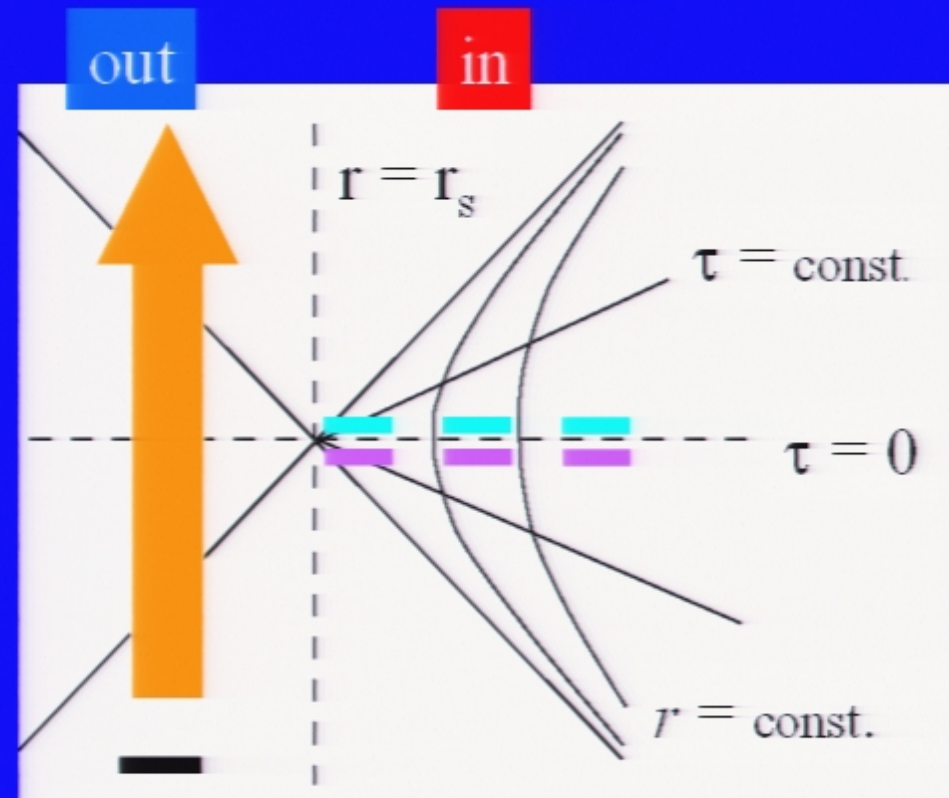
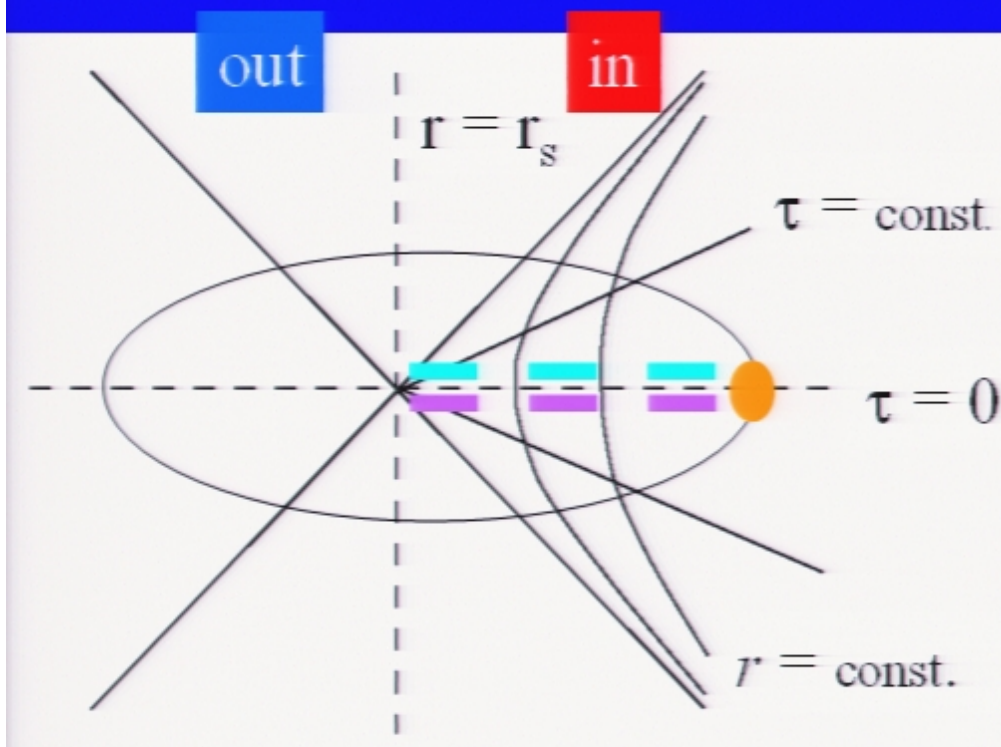
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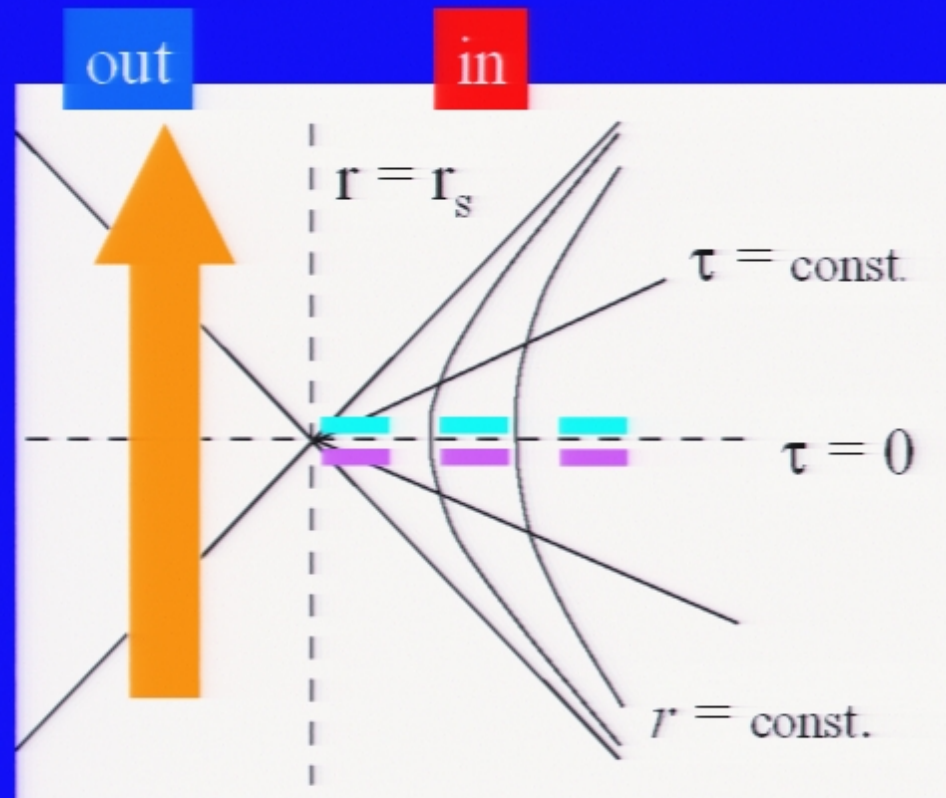
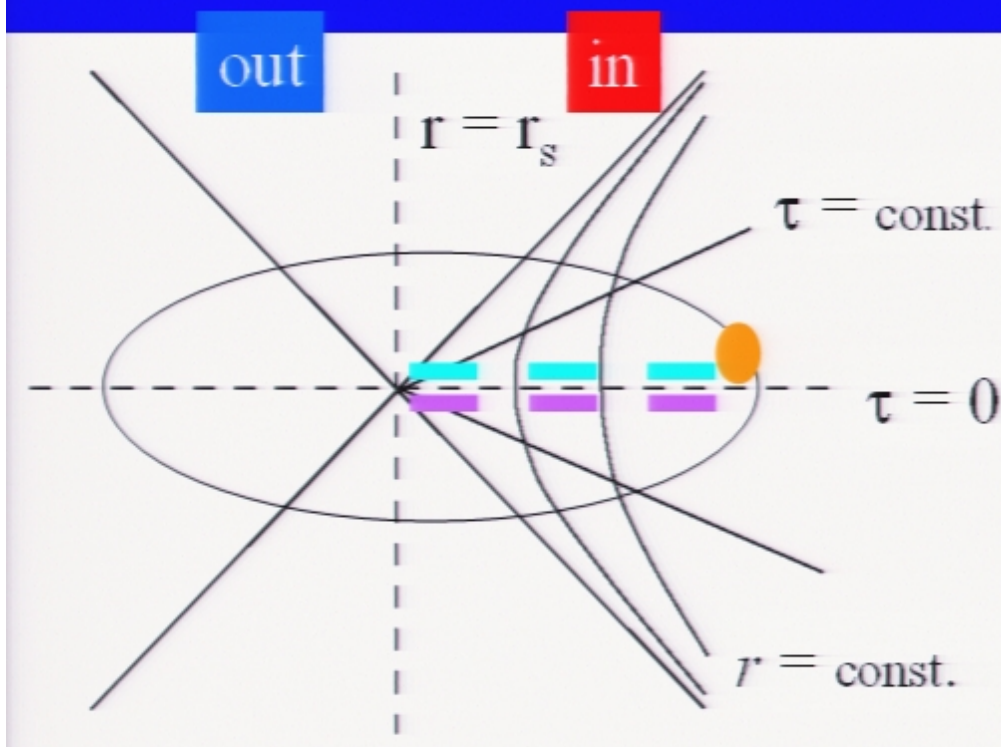


Construct the HH vacuum: the invariant regular state

Kabat & Strassler (flat space)
Jacobson

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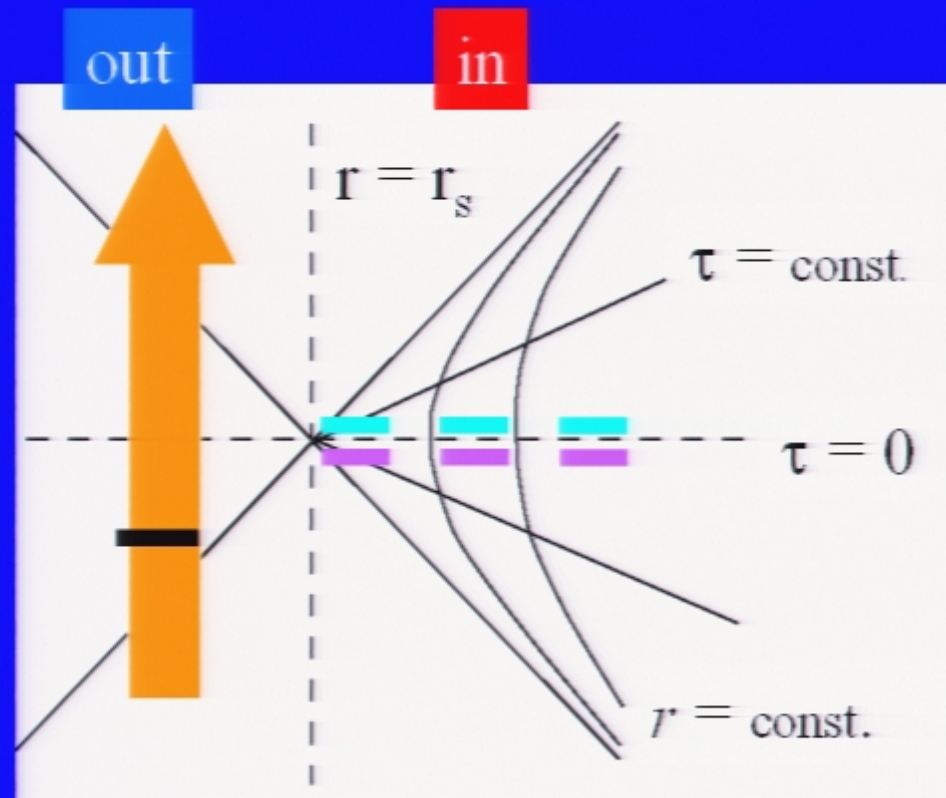
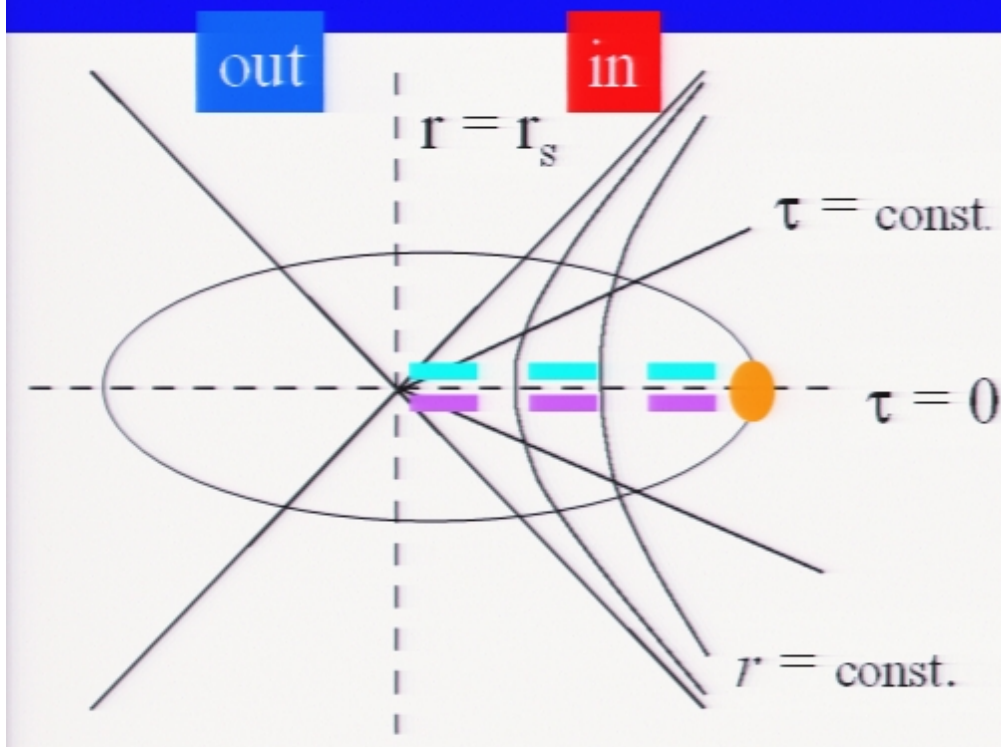


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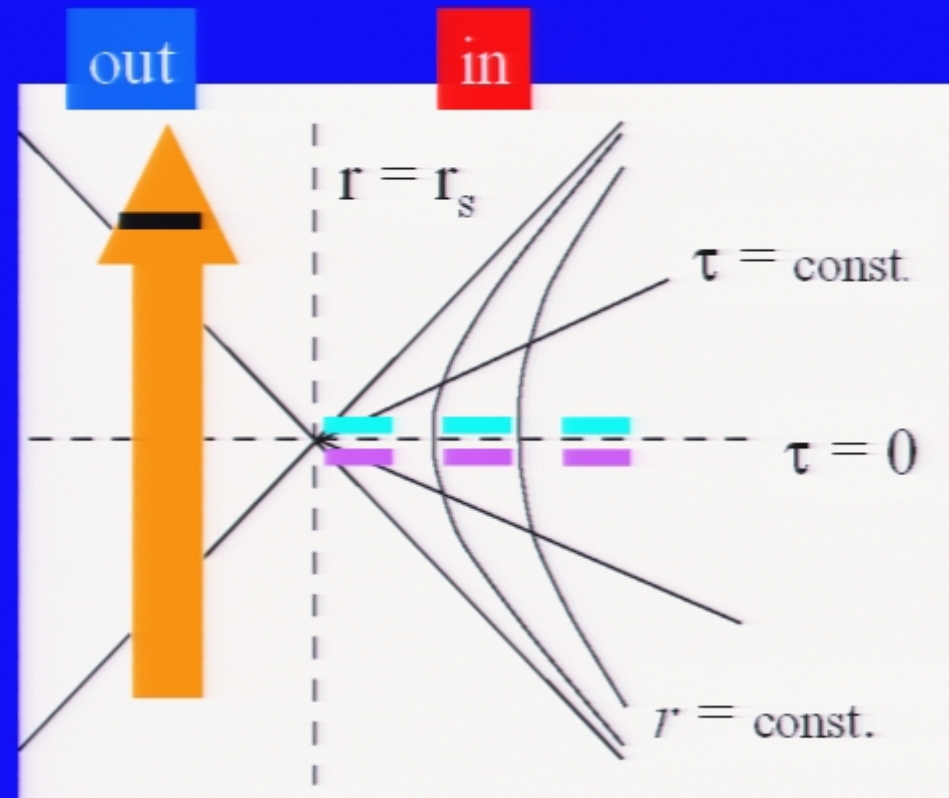
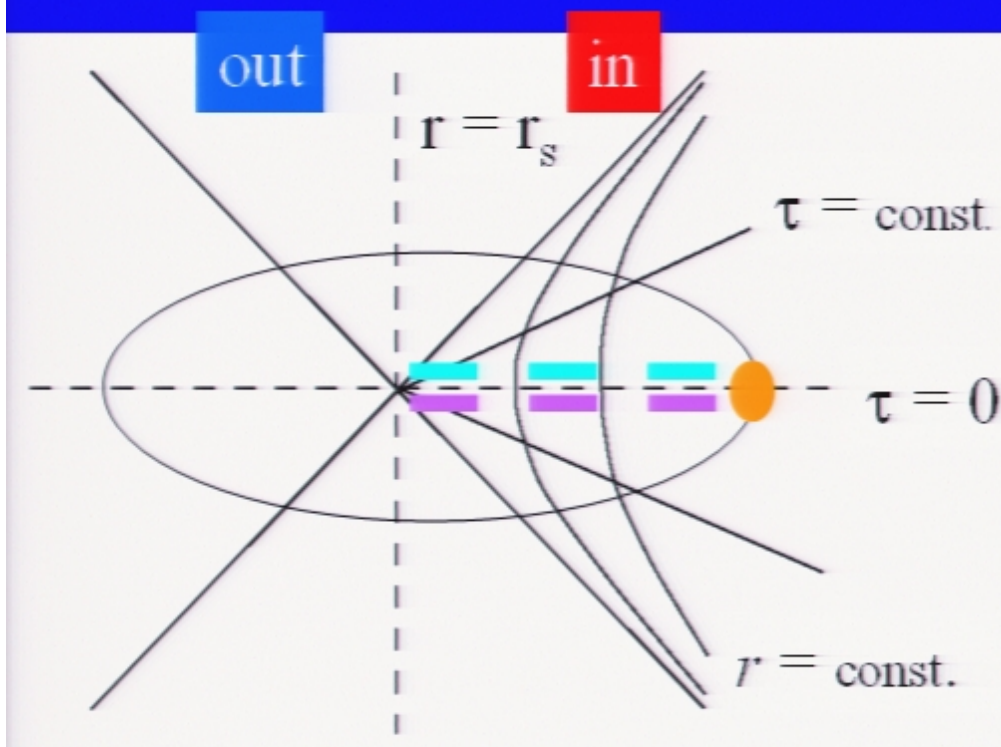


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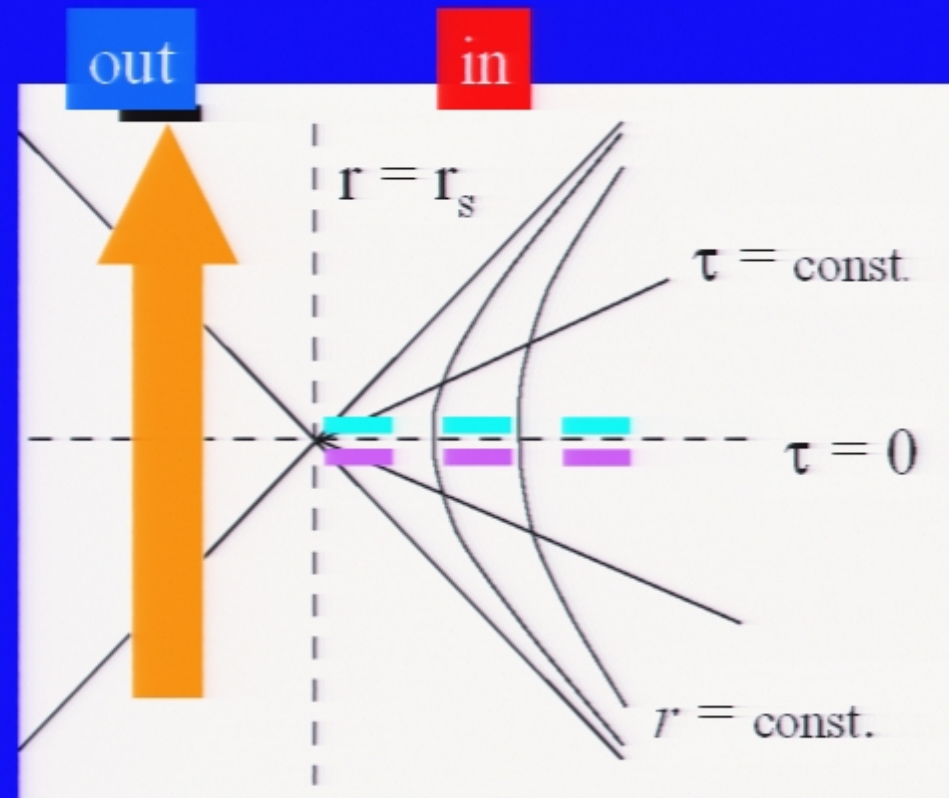
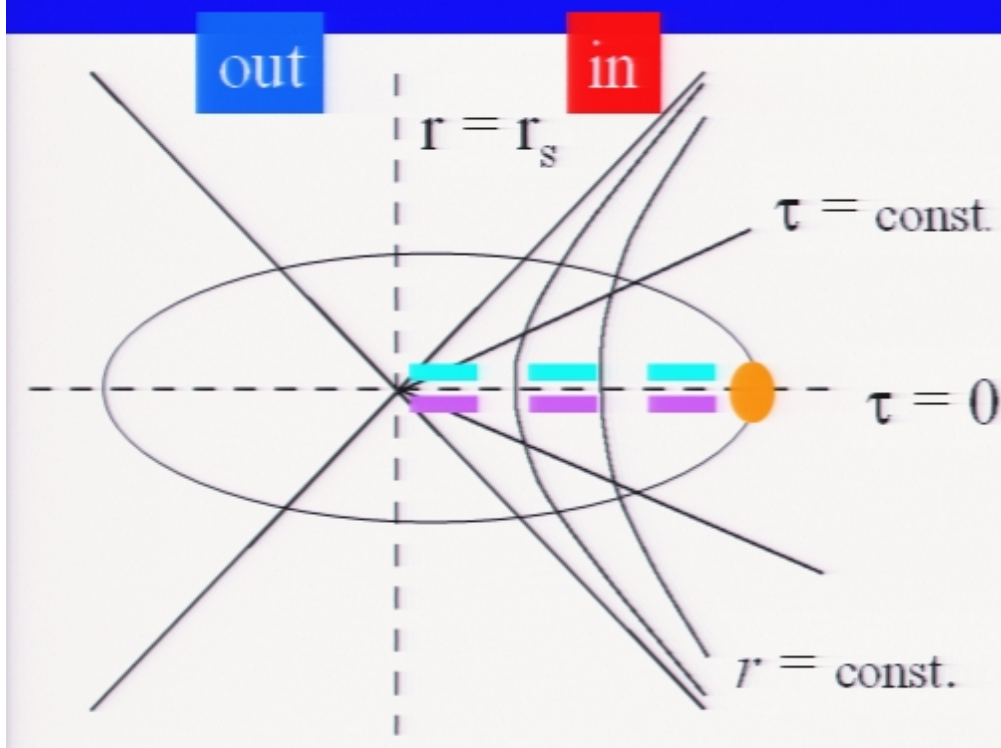


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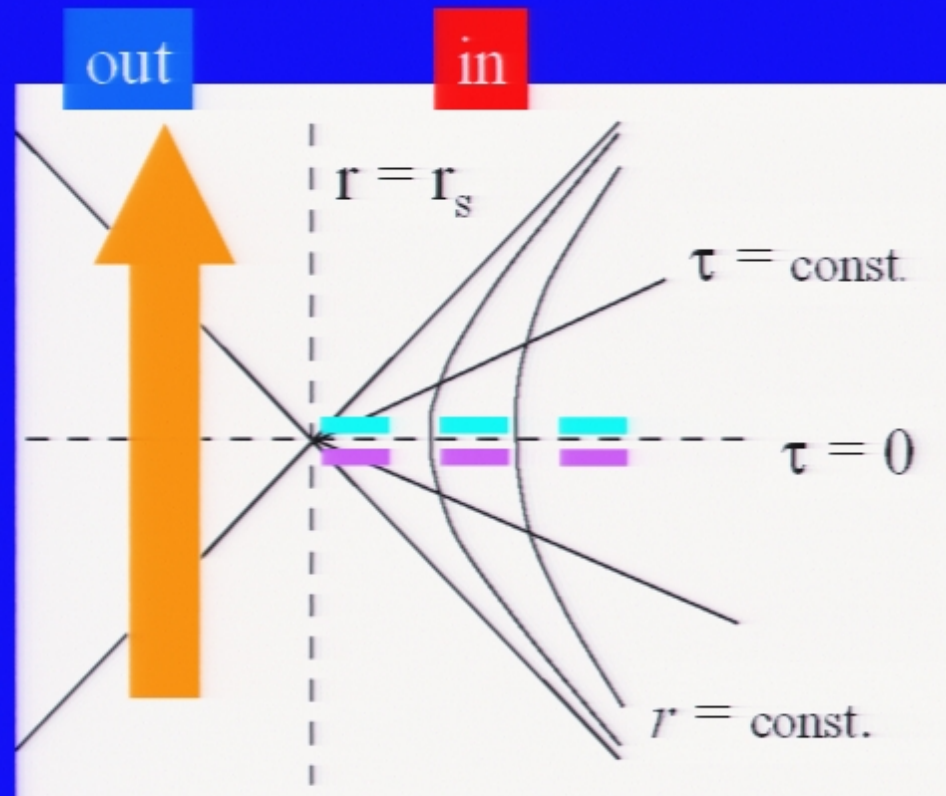
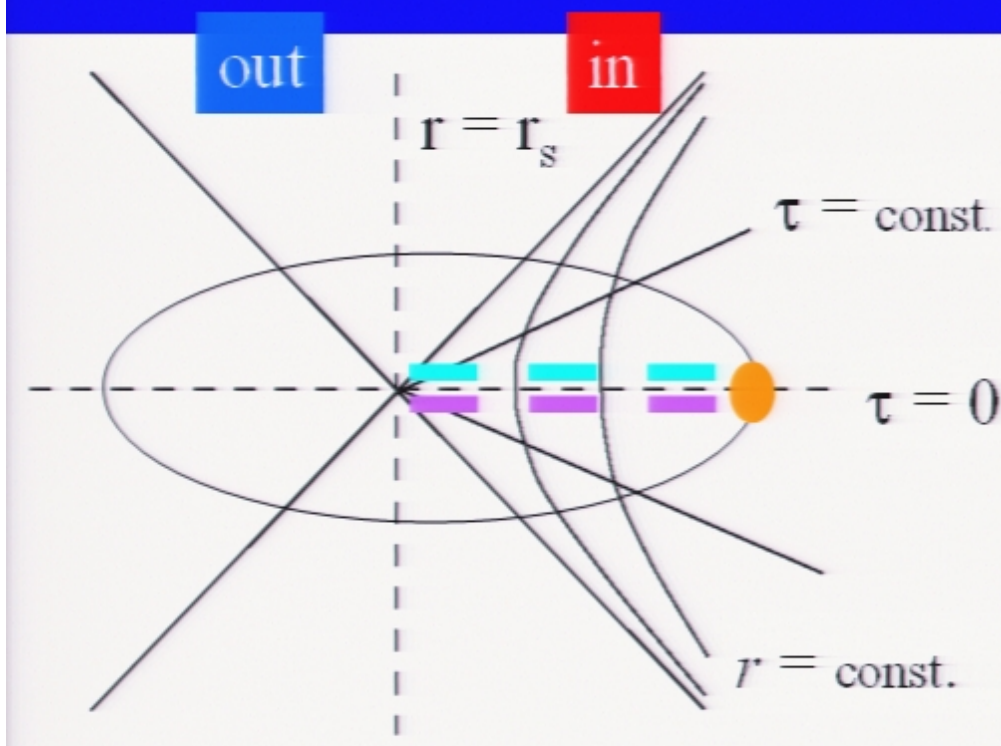


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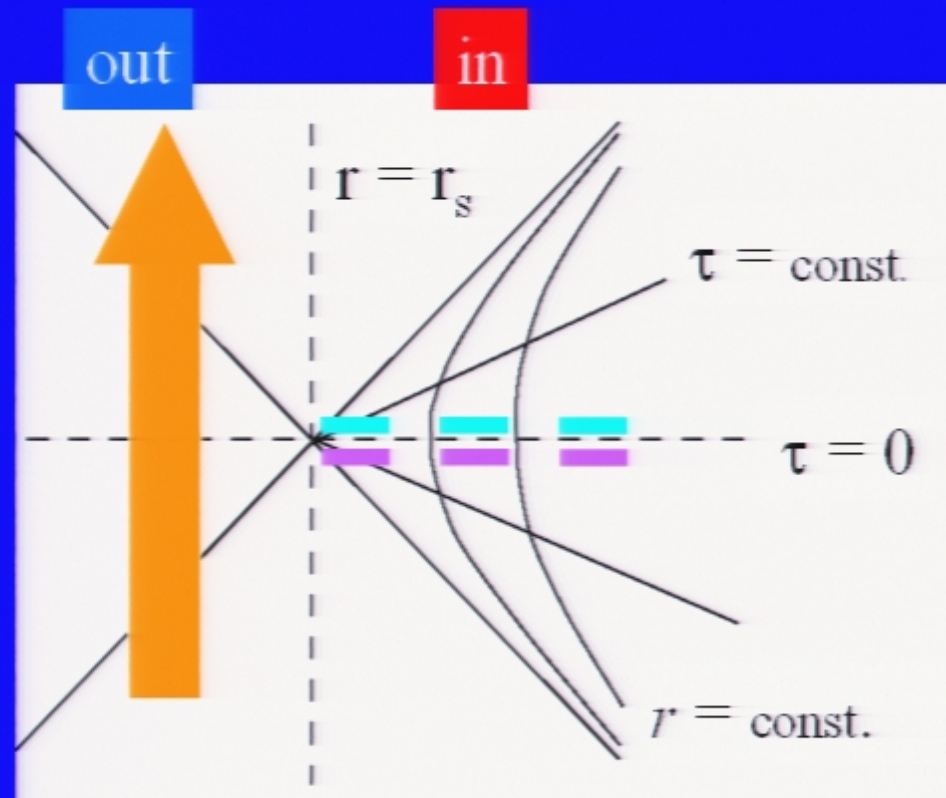
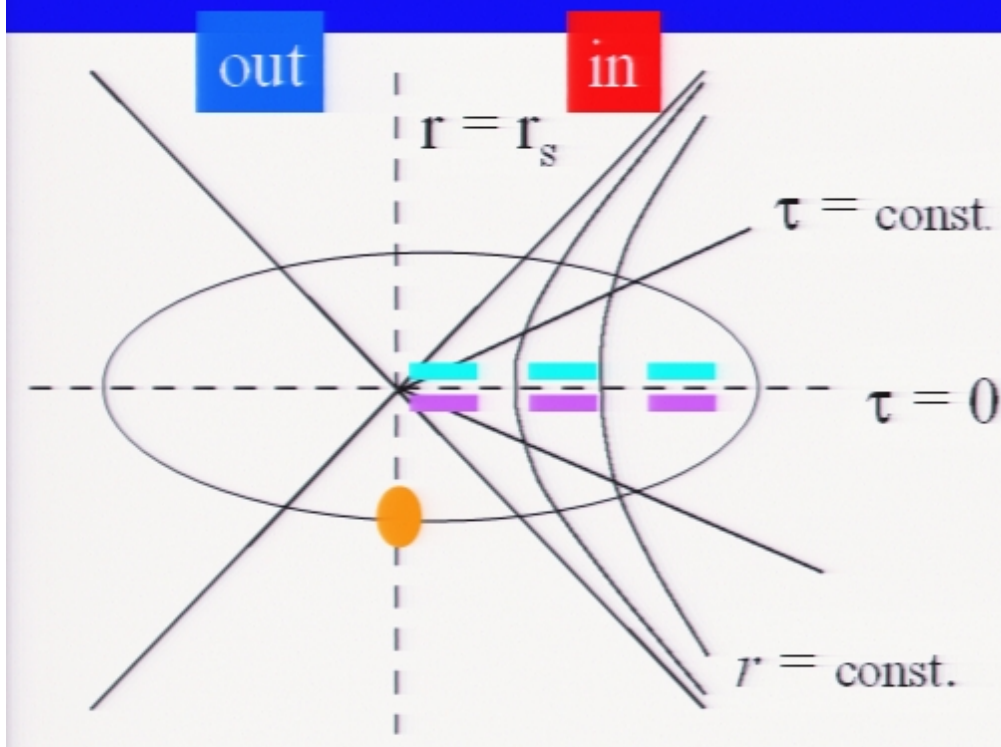


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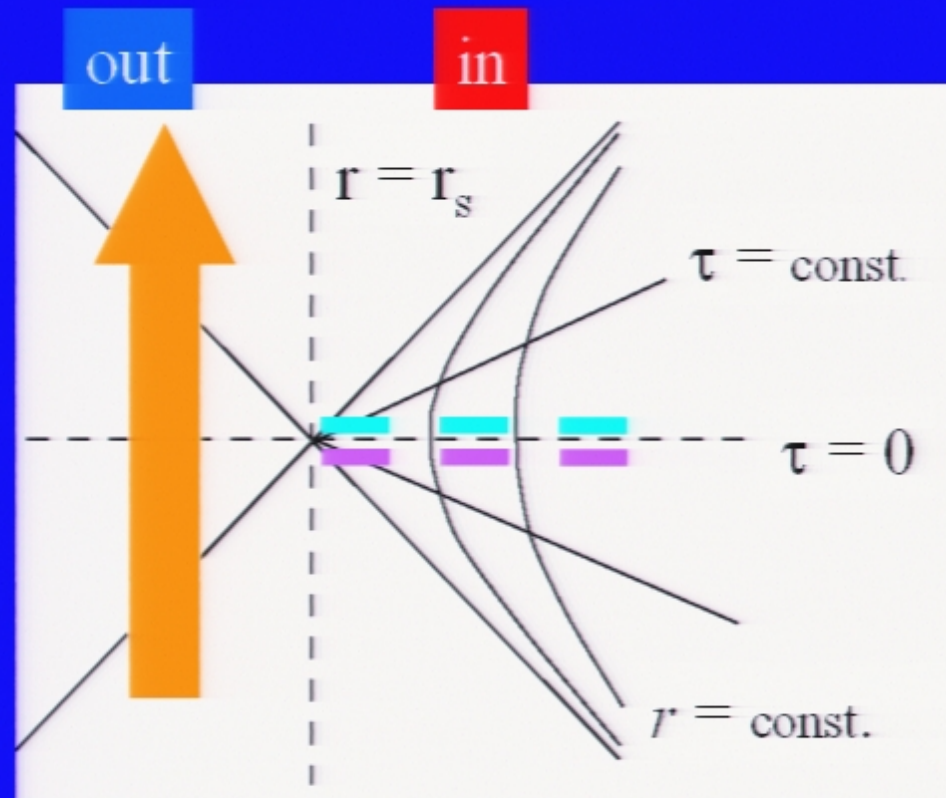
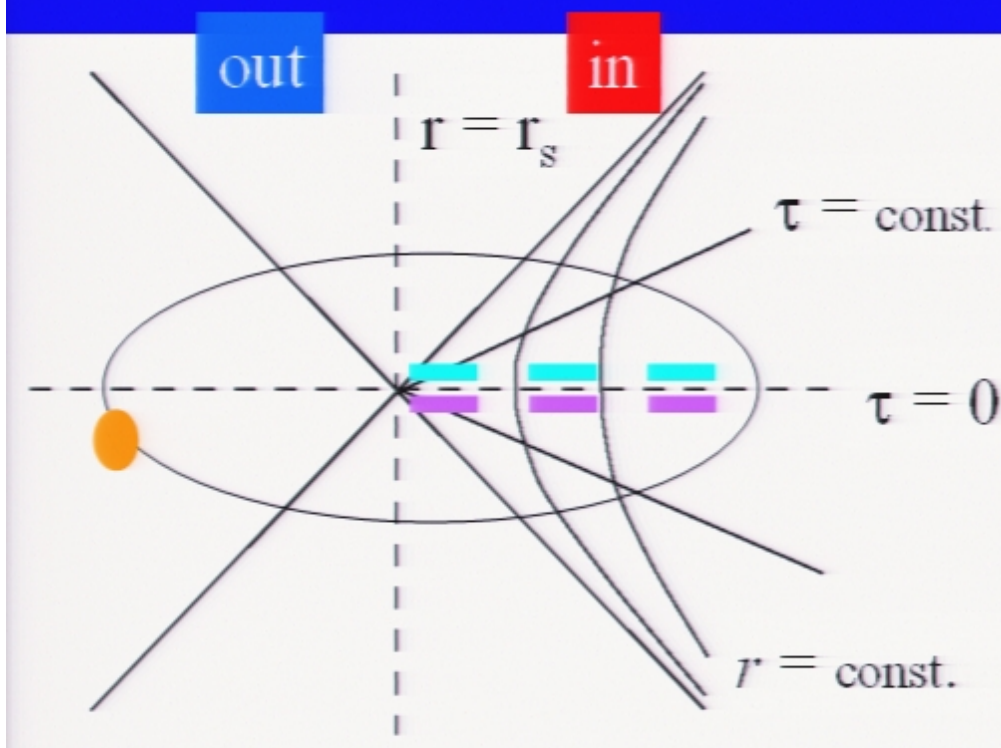


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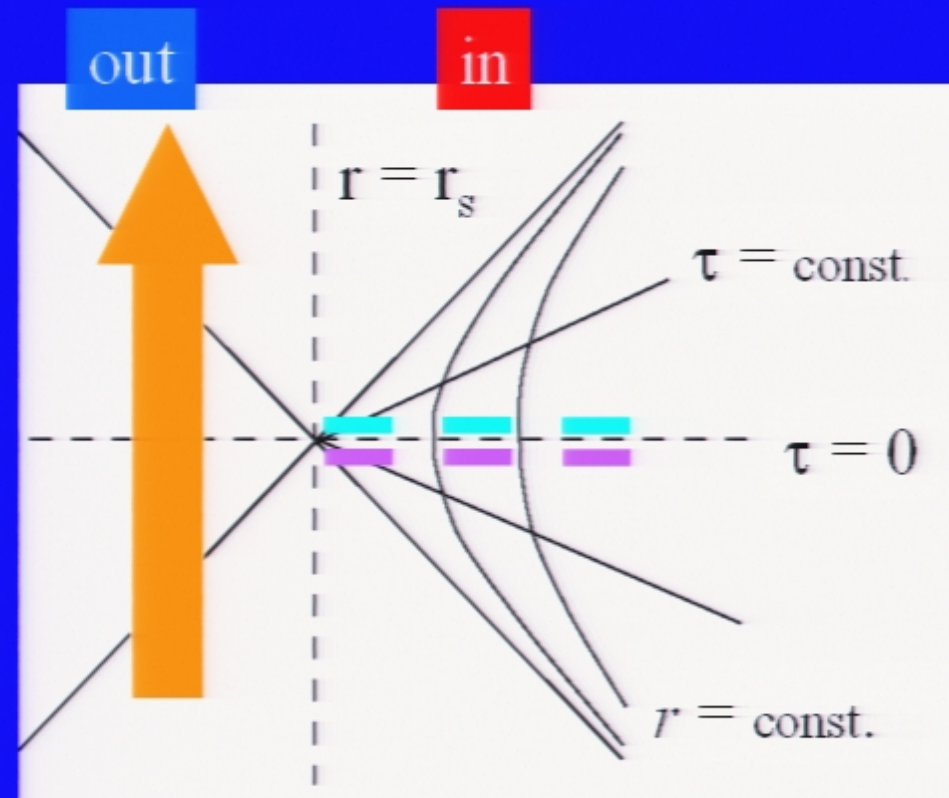
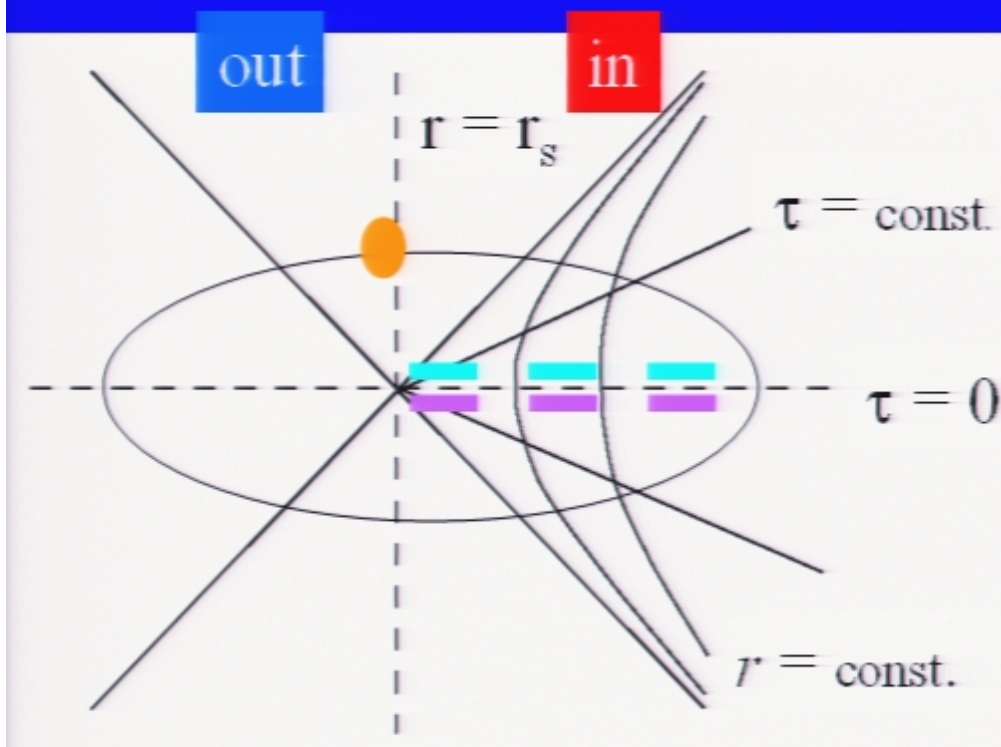


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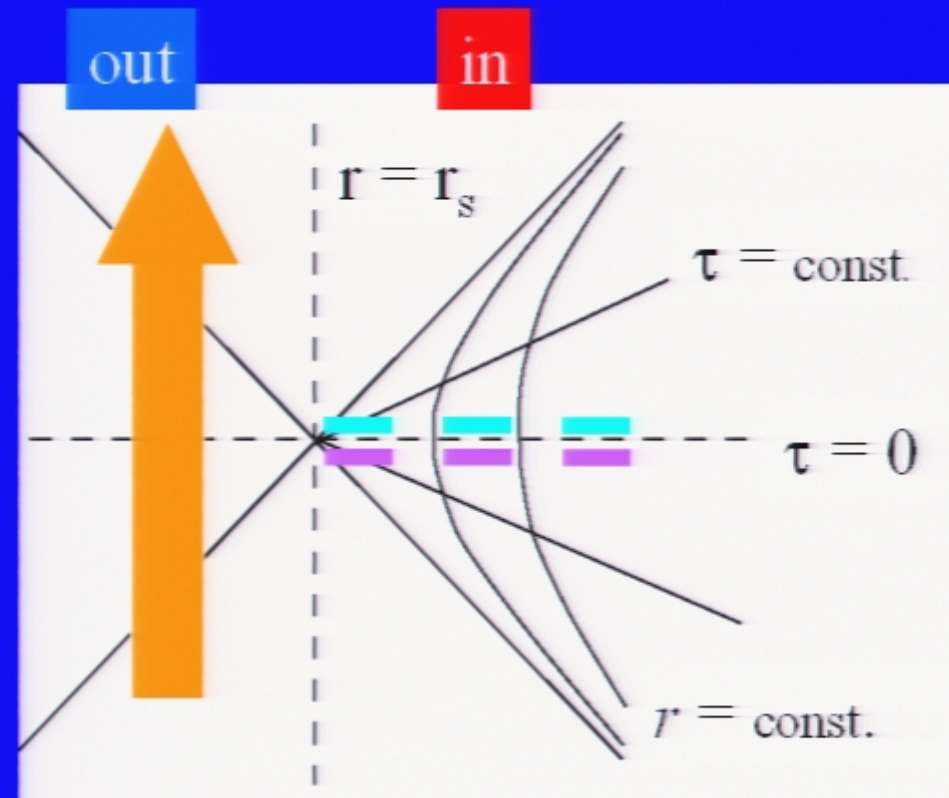
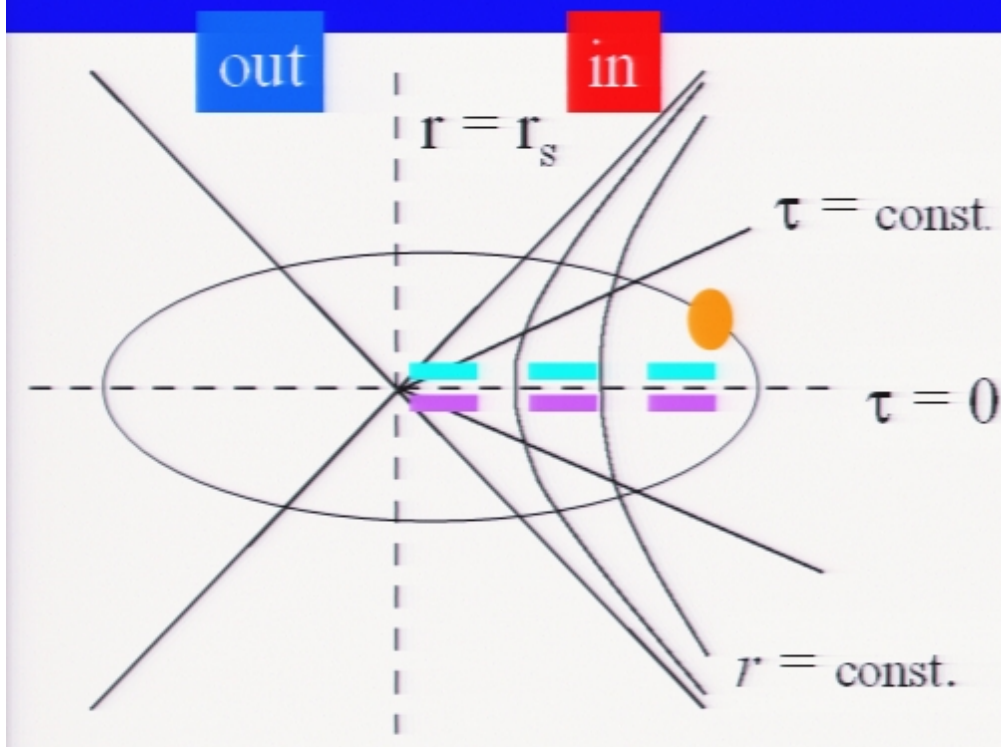


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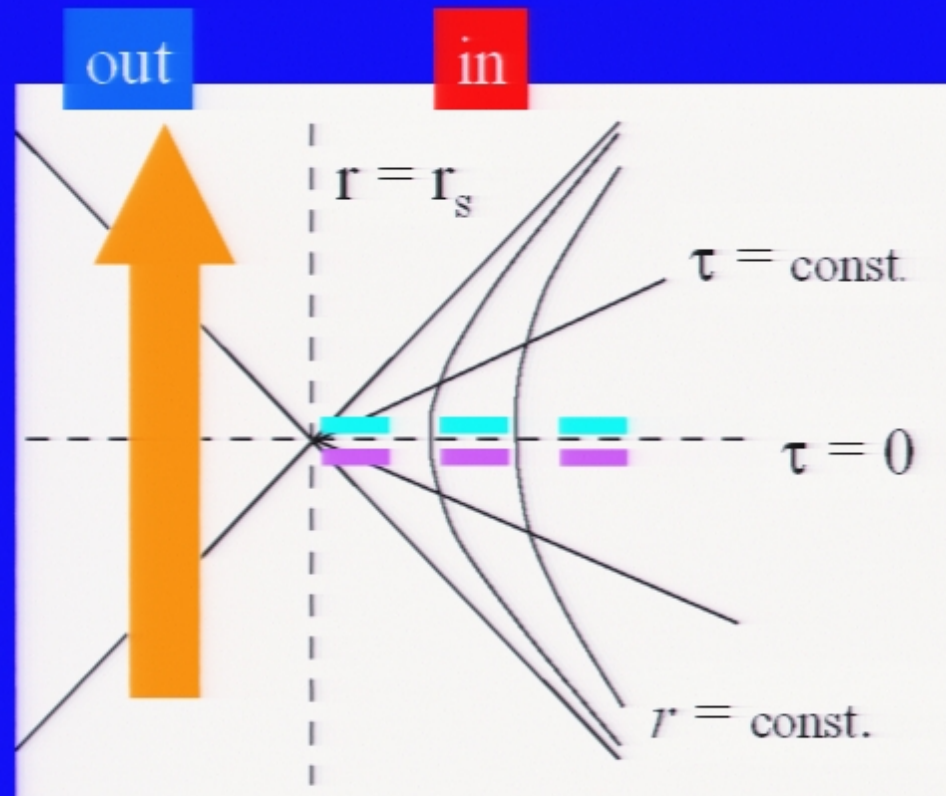
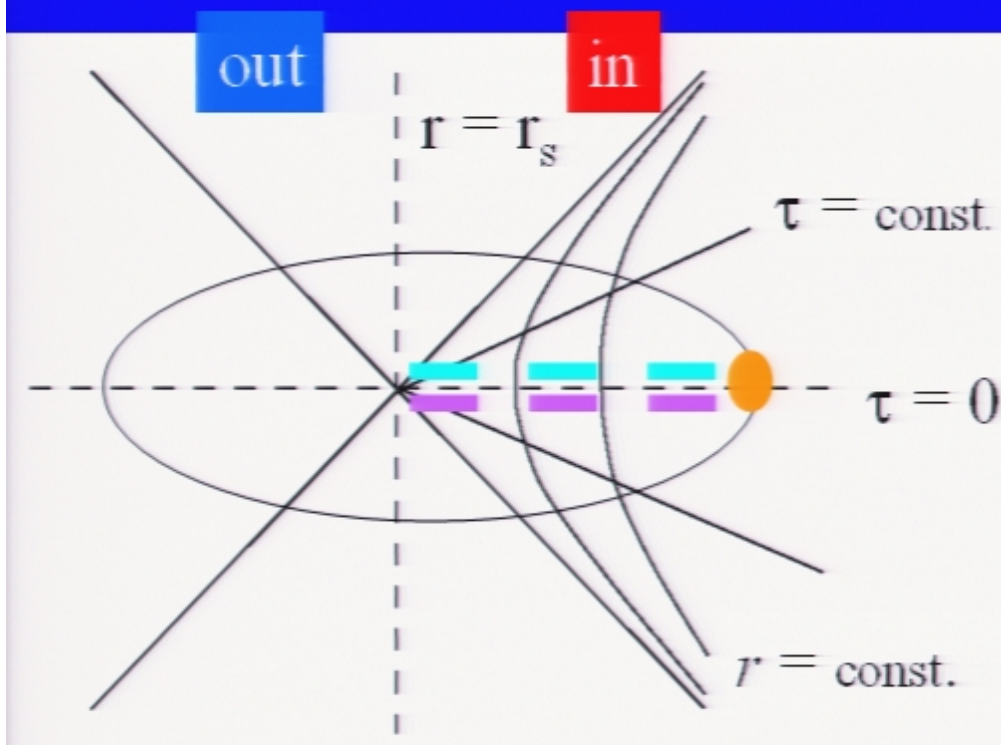


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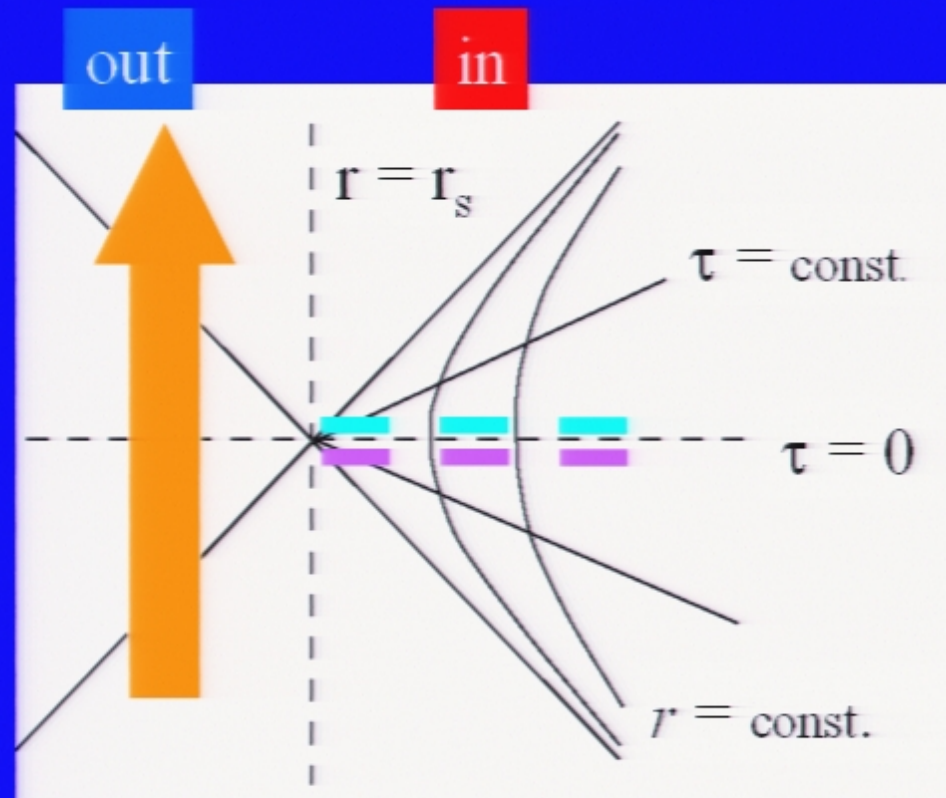
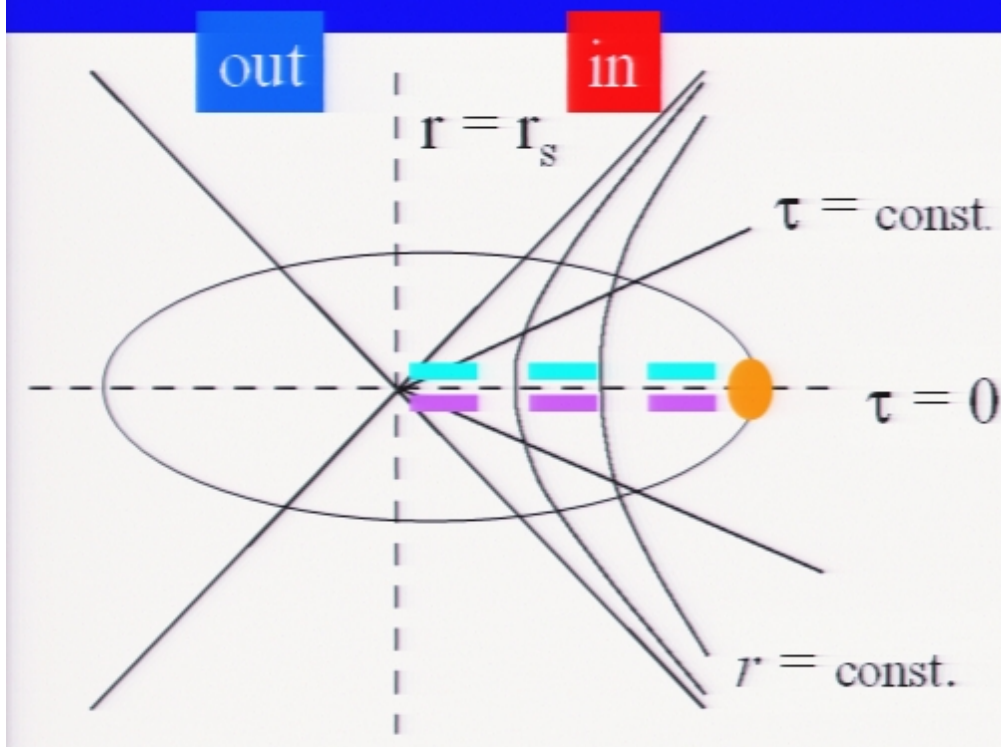


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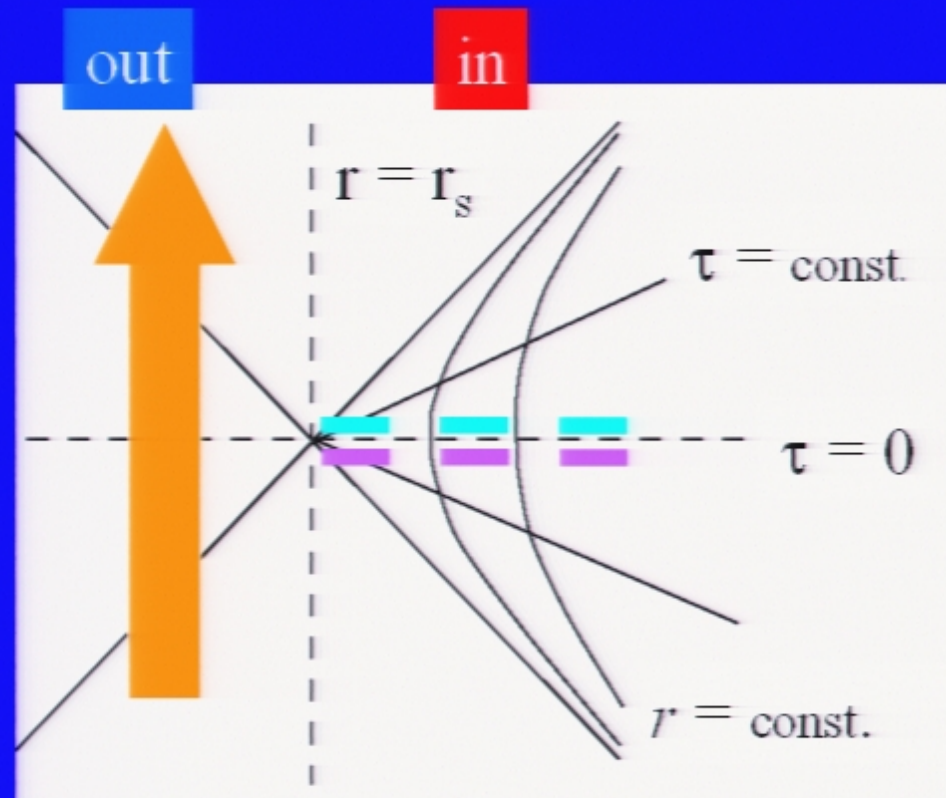
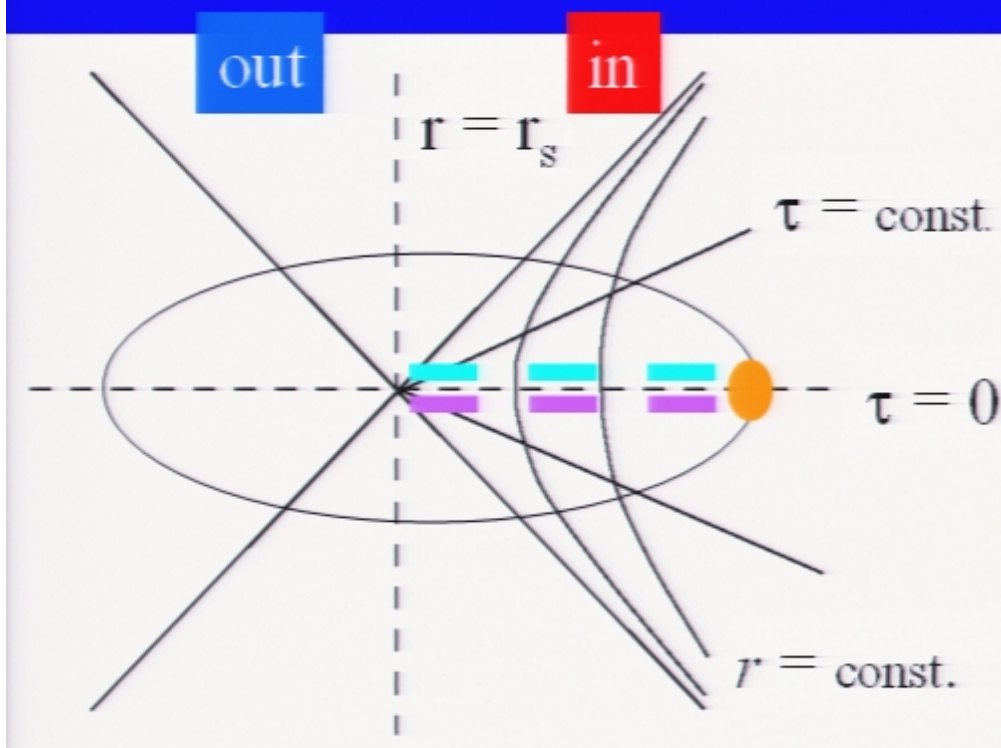


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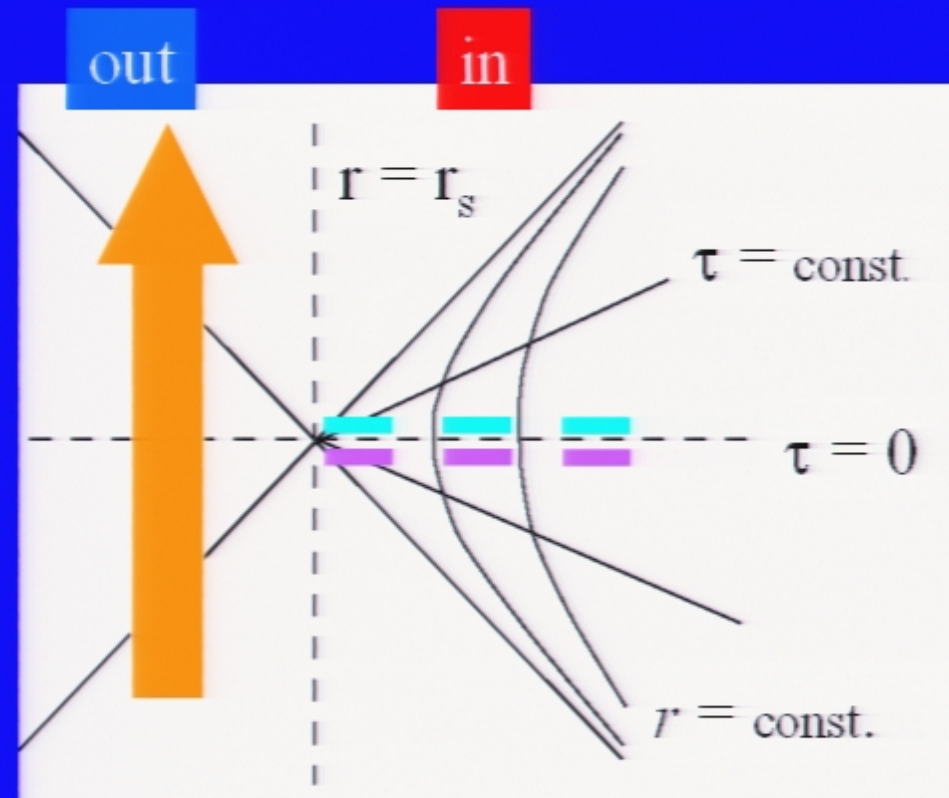
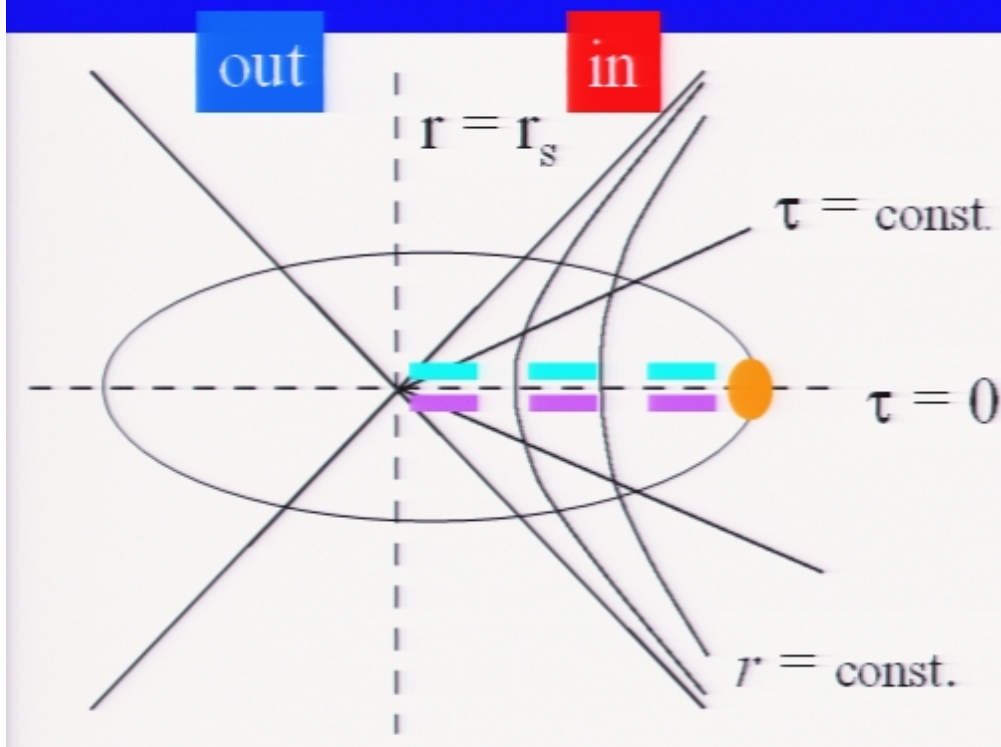


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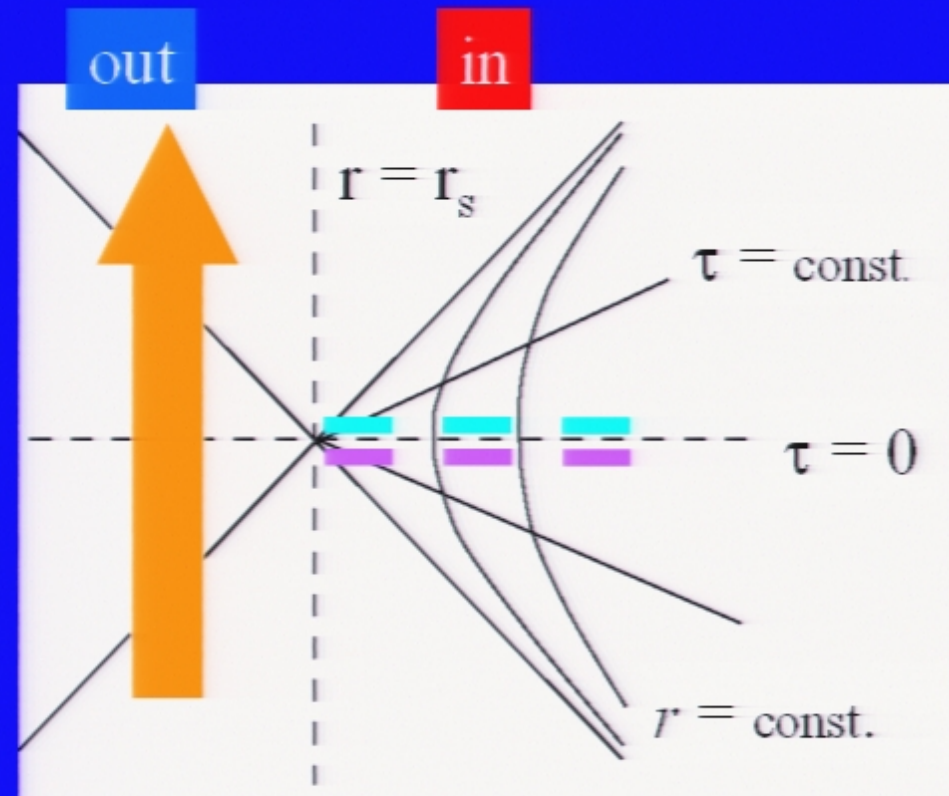
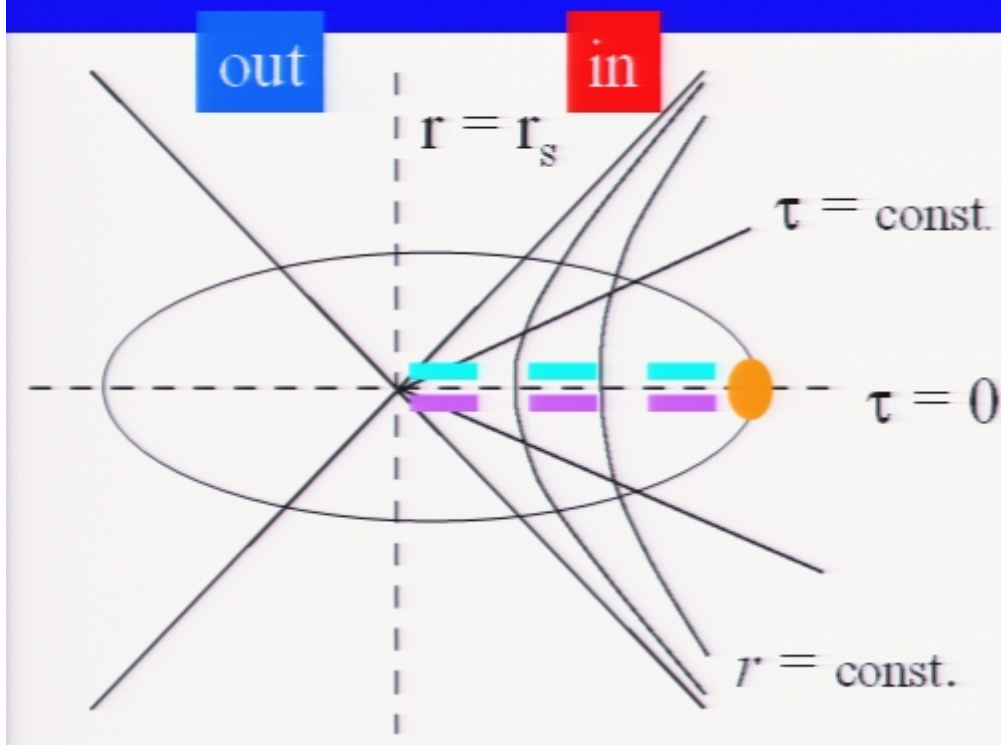


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Results*:

* Method works for more general cases 

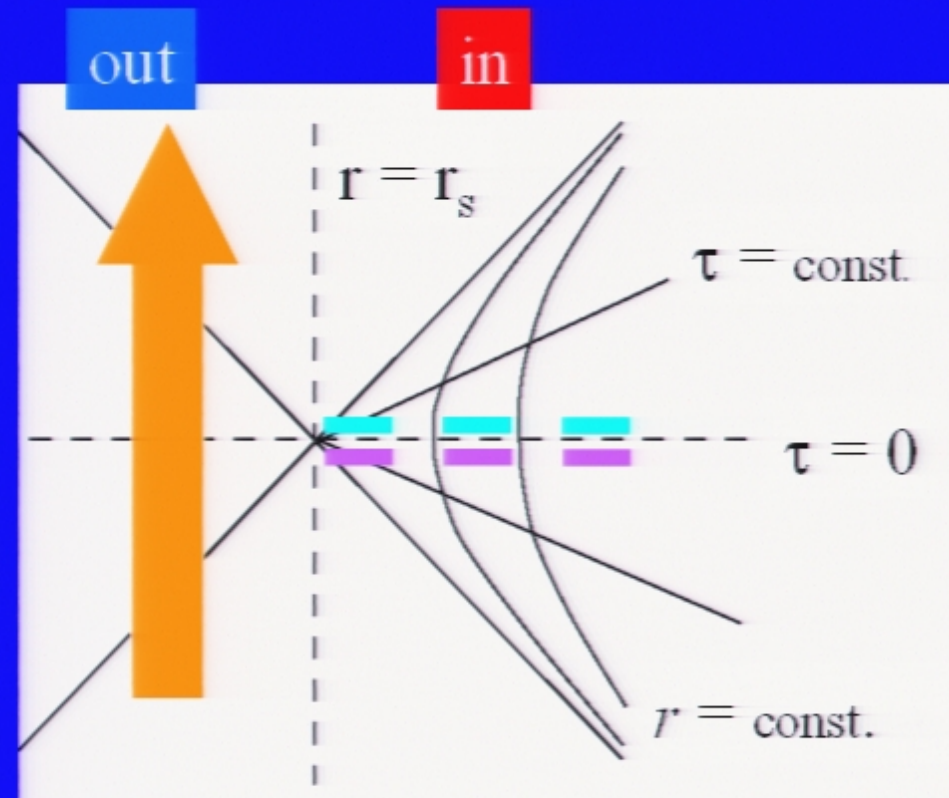
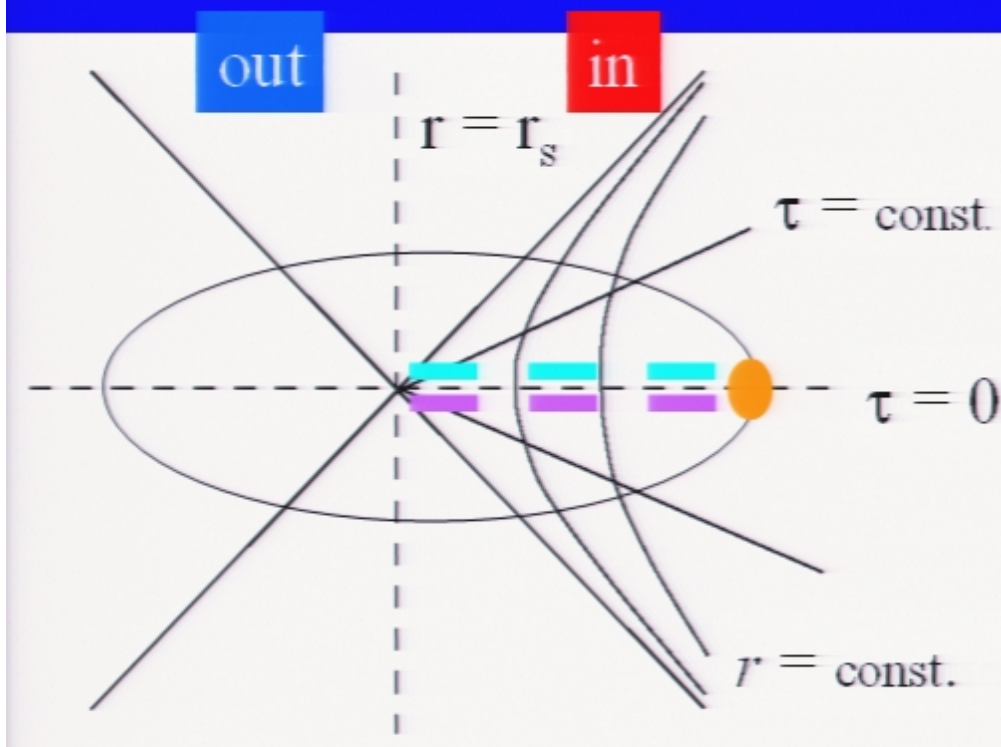
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$$\Omega = \frac{1}{\sqrt{g_{00}^E}}$$

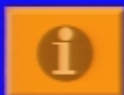
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If

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2. The actions are equal
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Entanglement entropy

$$S_{in} = -Tr(\rho_{in} \ln \rho_{in})$$

$$T = \frac{f'(r_s)}{4\pi}$$

de Alwis & Ohta

$$S = \frac{q(r_s)^{\frac{d-1}{2}} (d+1) \Gamma\left(\frac{d+1}{2}\right) \zeta(d+1)}{2^{2d-1} \pi^{\frac{3d+1}{2}} (d-1)} \delta^{-(d-1)} \int d\vec{x}_\perp$$



S is divergent

δ – proper length short distance cutoff

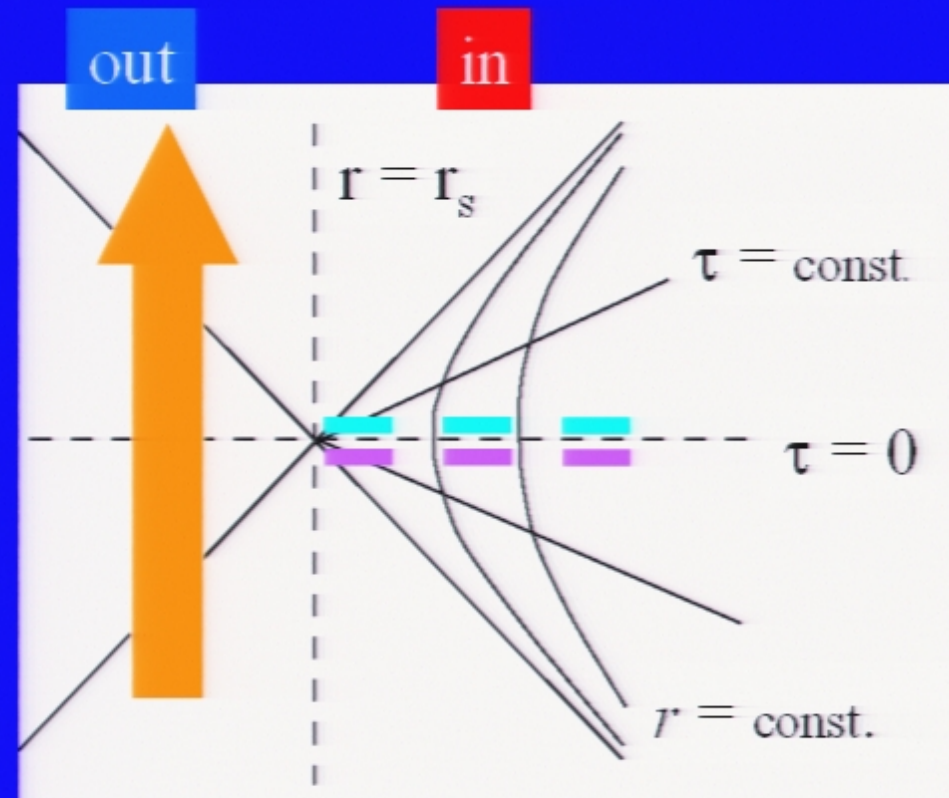
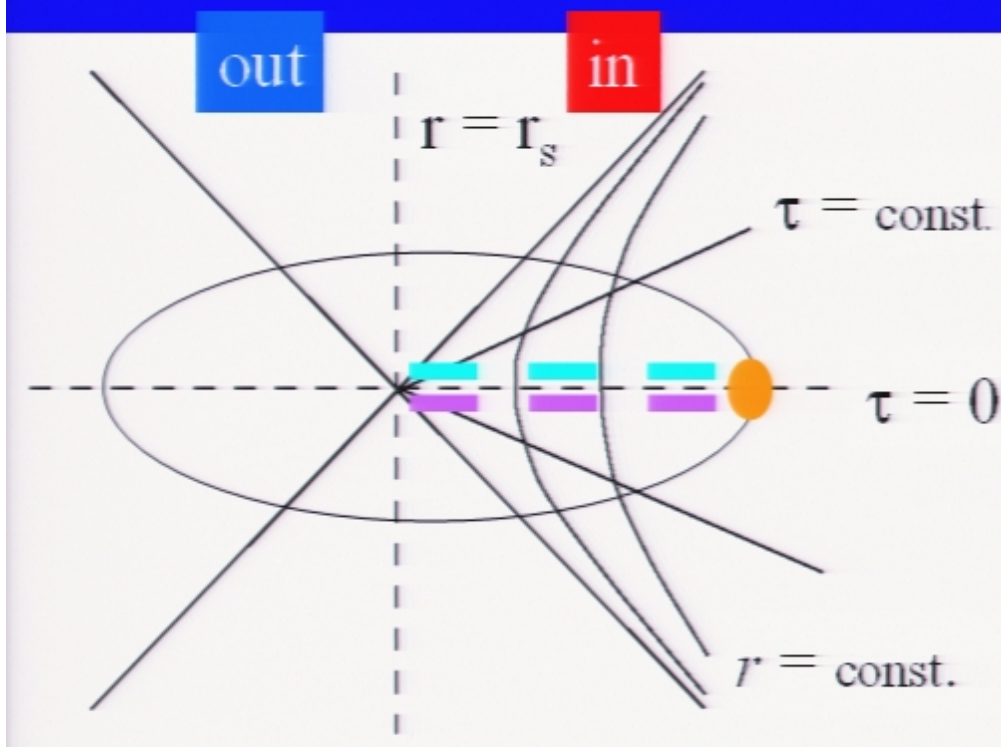
Naïve origin:

divergence of the optical volume near the horizon

More later ... *not* brick wall

Two ways of calculating ρ_{in}

R.B., M. Einhorn and A. Yarom



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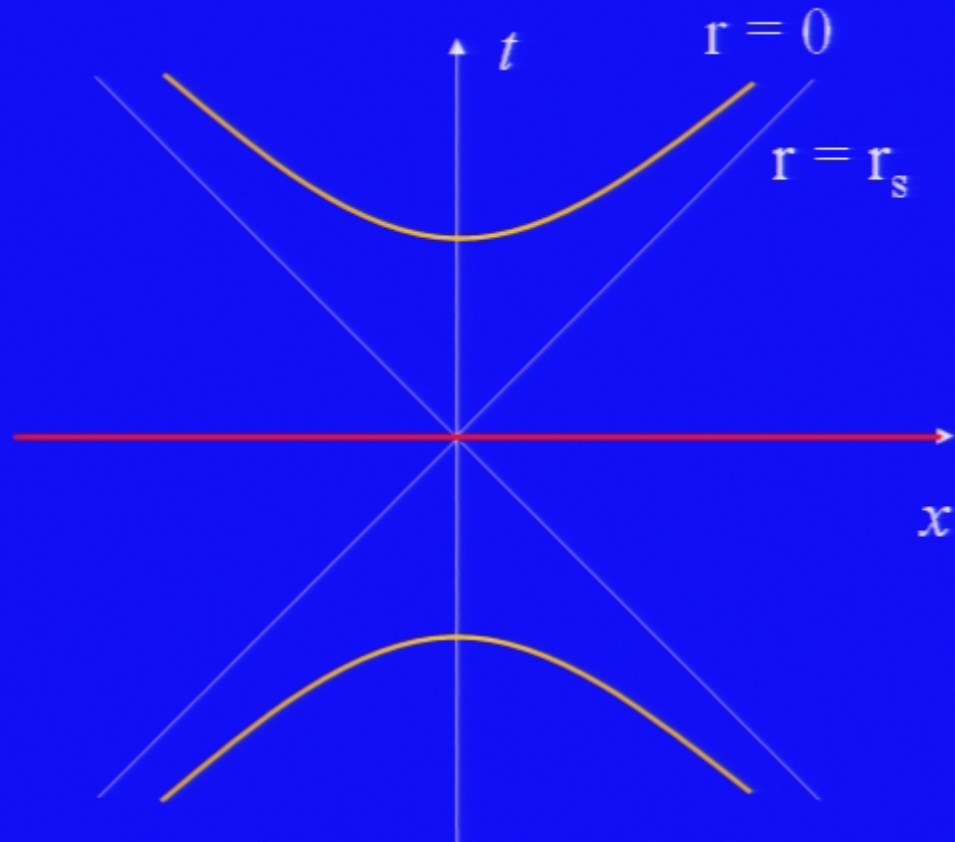
“Kruskal” extension

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + q(r)d\Omega^2$$

$$ds^2 = h(r)(-dt^2 + dx^2) + q(r)d\Omega^2$$

$$x = \sqrt{g(r)} \cosh(\tau/a)$$

$$t = \sqrt{g(r)} \sinh(\tau/a)$$



“Kruskal” extension

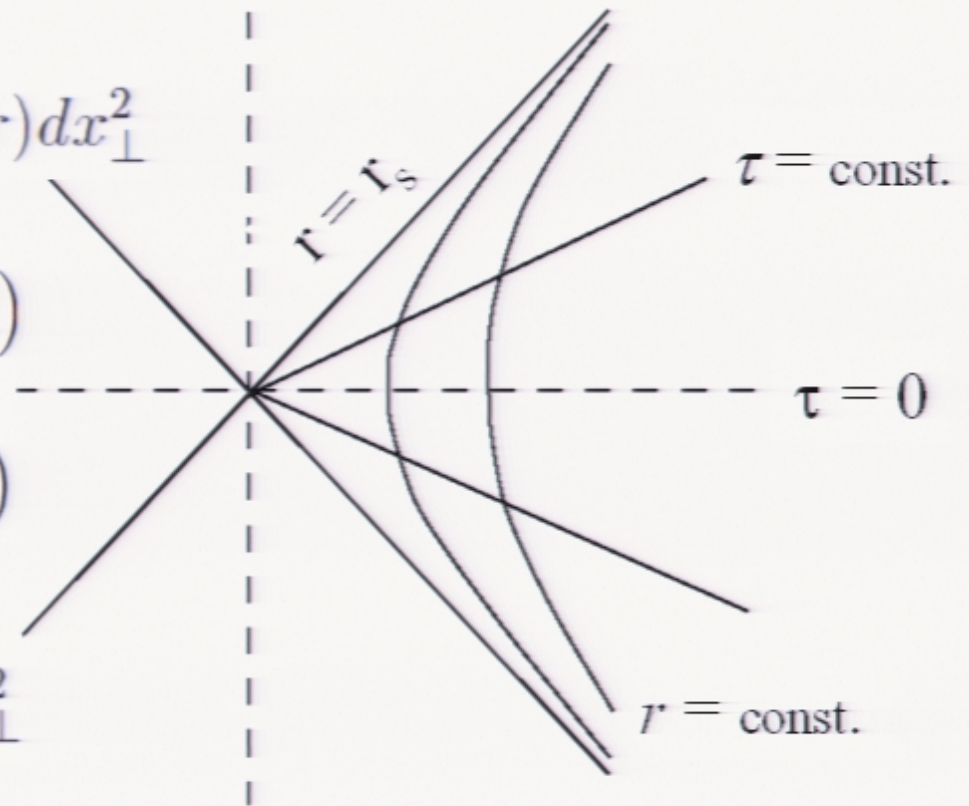
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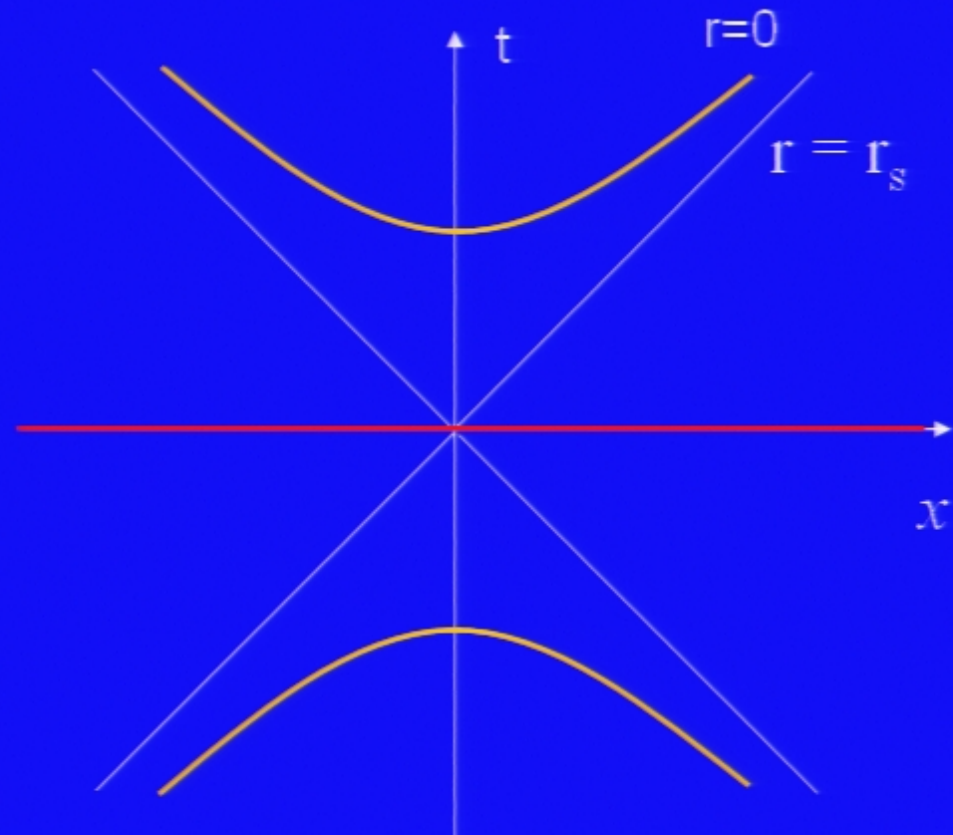
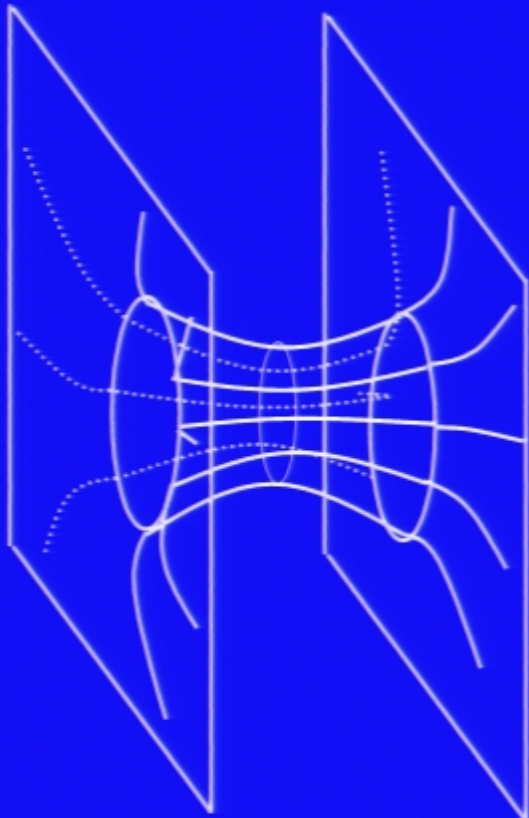
$$ds^2 = h(r)(-dt^2 + dx^2) + q(r)dx_{\perp}^2$$

$$h(r) = \frac{1}{\xi_0} f(r) e^{-f'(r_s) \int^r \frac{dr'}{f(r')}}$$



Examples: Minkowski, de Sitter, Schwarzschild, non-rotating BTZ BH, can be extended to rotating, charged, non-extremal BHs

The vacuum state



How to relate them?

Entanglement entropy

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How to relate them?

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$$S_{BH} = \frac{A}{4G_N}$$

How to relate them?



?

$$S_{BH} = \frac{A}{4G_N}$$

$$S_{BH} \propto N \frac{A}{\delta^{(d-1)}}$$

How to relate them?

How to relate them?



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BH entropy in string theory



BH entropy in string theory



$(C)FT$

BH entropy in string theory



$(C)FT$

$$T_{\text{BH}} = T_{\text{FT}}$$

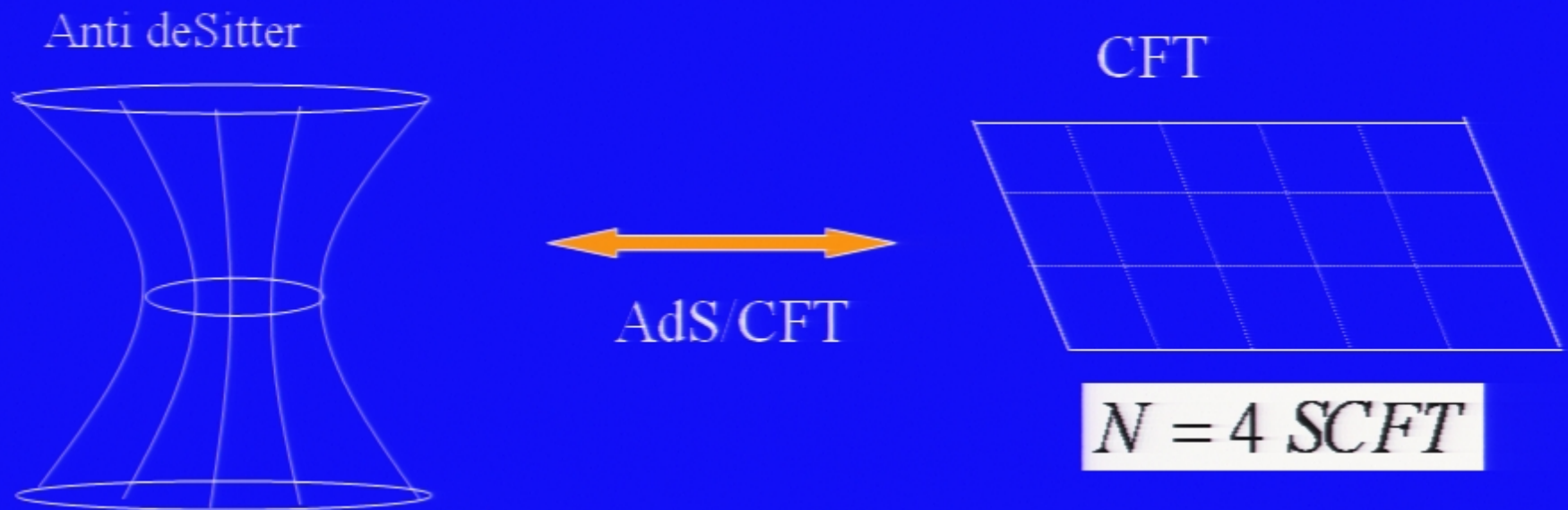
$$S_{\text{BH}} = S_{\text{FT}}(T_{\text{BH}})$$

AdS BH Entropy

Gubser, Klebanov, Peet

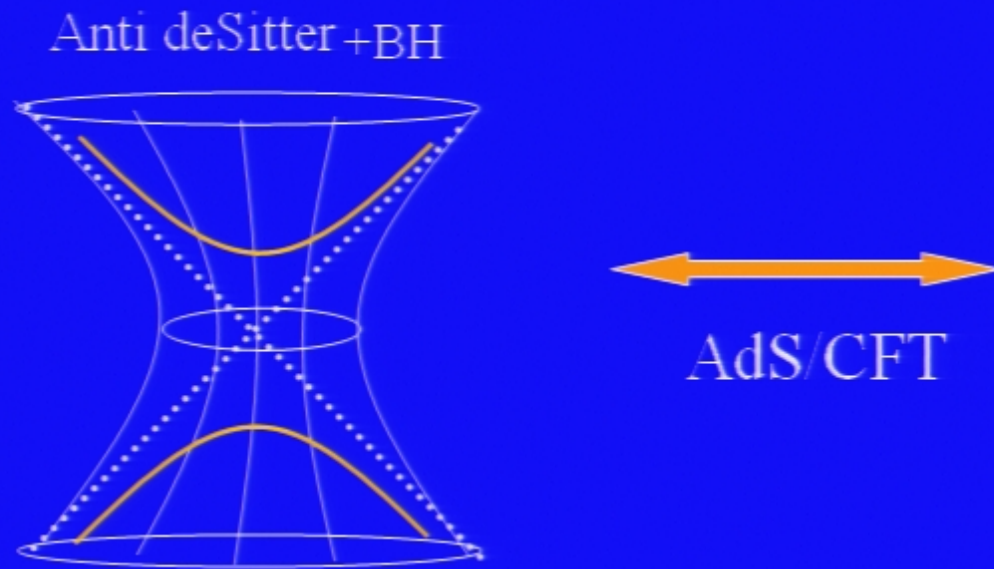
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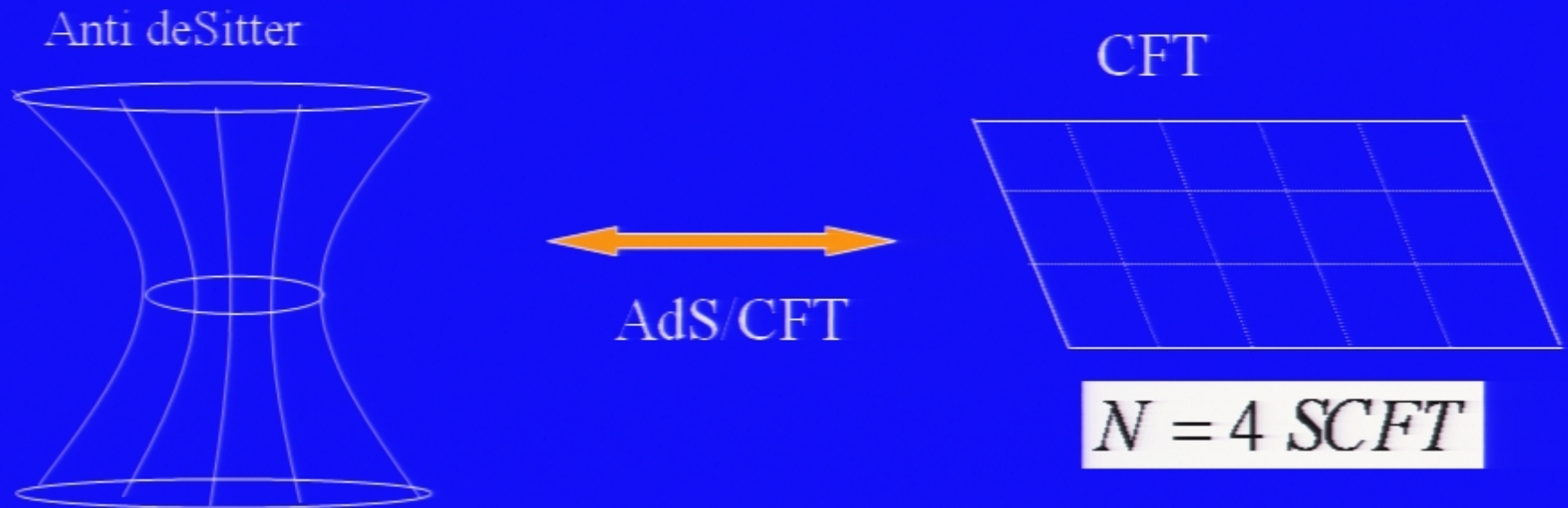
Gubser, Klebanov, Peet

Anti deSitter + BH



AdS BH Entropy

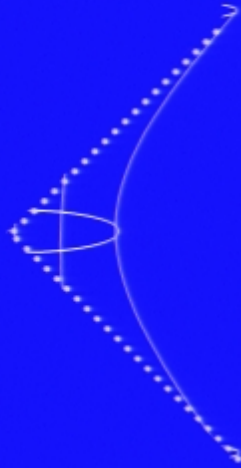
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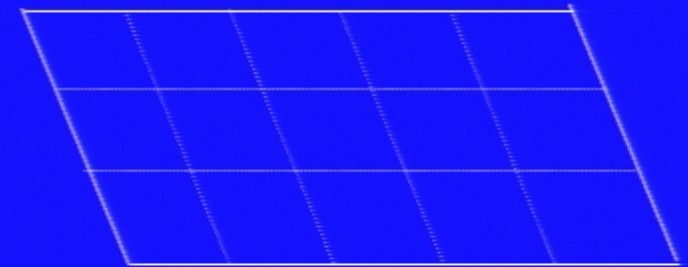
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AdS/CFT

CFT, $T > 0$

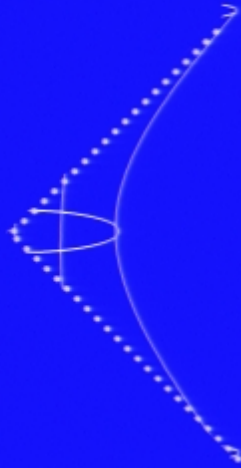


$N = 4$ SCFT

AdS BH Entropy

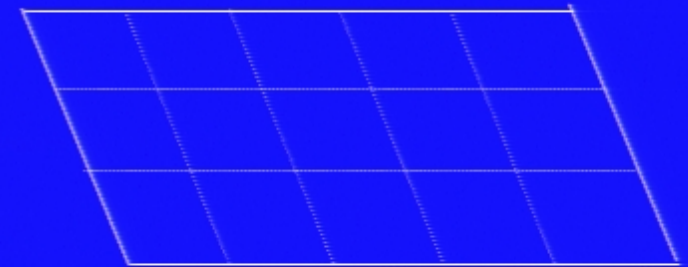
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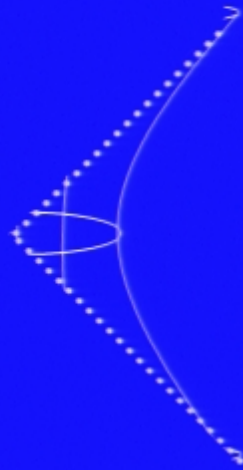


$N = 4$ SCFT

AdS BH Entropy

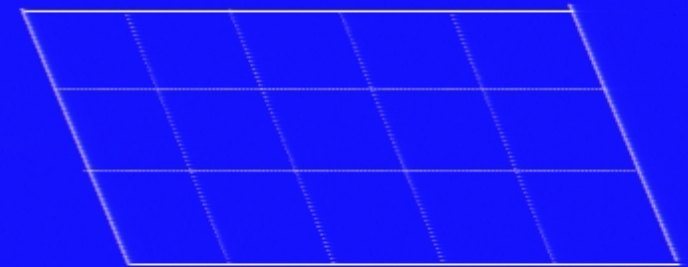
Gubser, Klebanov, Peet

Anti deSitter+BH



$$S_{\text{BH}} = A/4$$

CFT, $T > 0$



$N = 4$ SCFT

$$S = A/3$$

↔
AdS/CFT

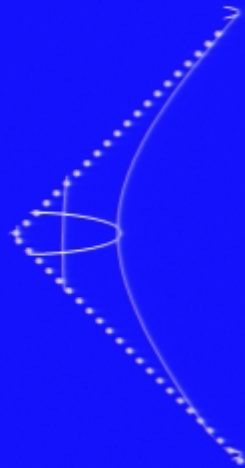
$$R^4 = \lambda_{\text{YM}}$$

AdS BH Entropy

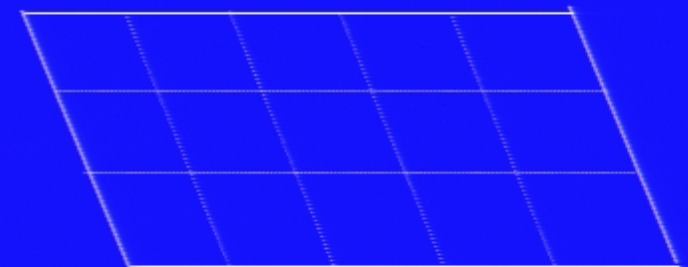
Gubser, Klebanov, Peet

Anti deSitter+BH

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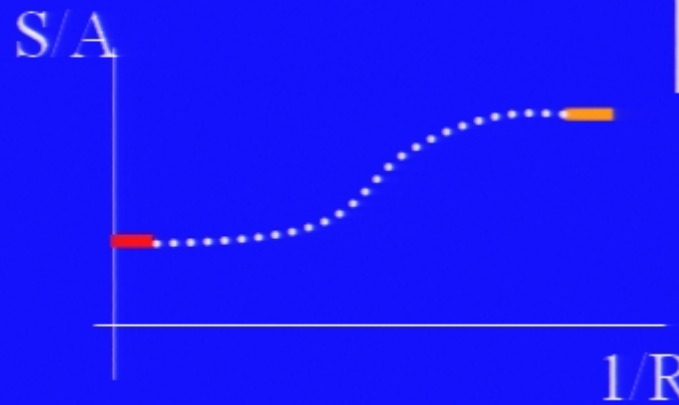


AdS/CFT



$N = 4$ SCFT

$$S_{\text{BH}} = A/4$$



$$S = A/3$$

Semiclassical gravity:

$$R \gg l_s$$

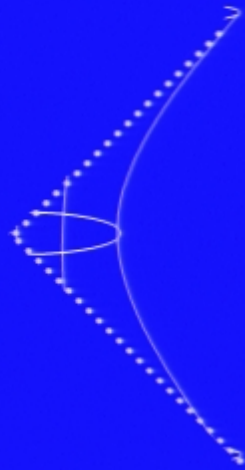
Free theory:
 $\lambda \rightarrow 0$

$$R^4 = \lambda_{\text{YM}}$$

AdS BH Entropy

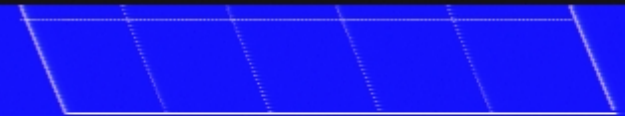
Gubser, Klebanov, Peet

Anti deSitter+BH



What is entanglement entropy? ✓
 What is entanglement entropy of BH's ✓
 How does string theory evaluate BH entropy? ✓
 How are the two methods related to each other?

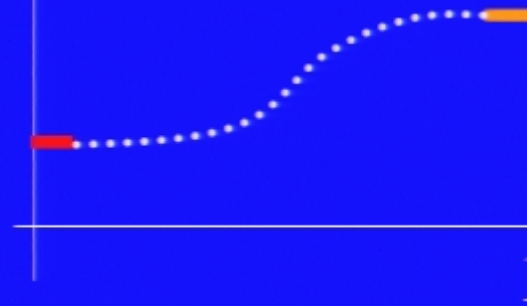
AdS/CFT



$N = 4$ SCFT

S/A

$$S_{\text{BH}} = A/4$$



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$1/R$

Semiclassical gravity:

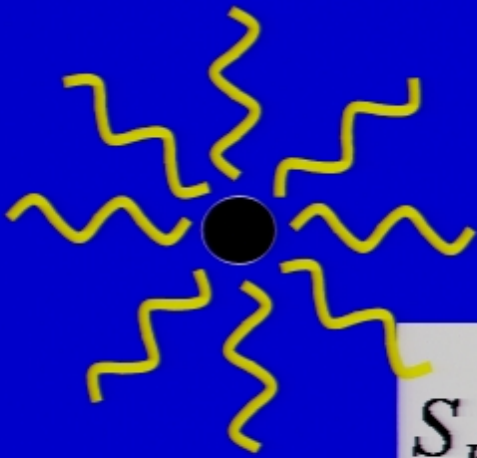
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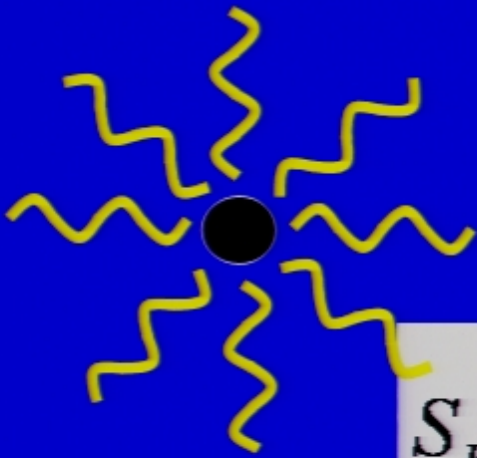


$$S_{BH} = \frac{A}{4G_N}$$



$$S_{BH\ ent.} \propto N \frac{A}{\delta^{(d-1)}}$$

How to relate them?

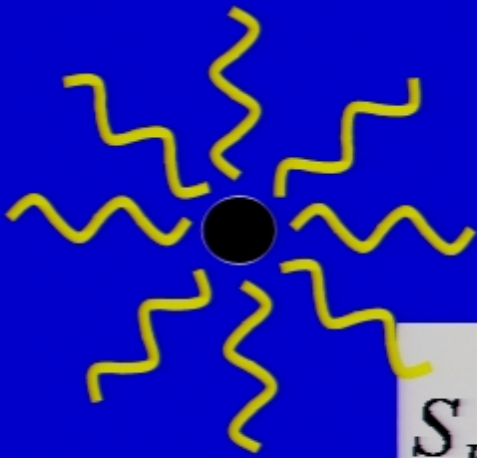


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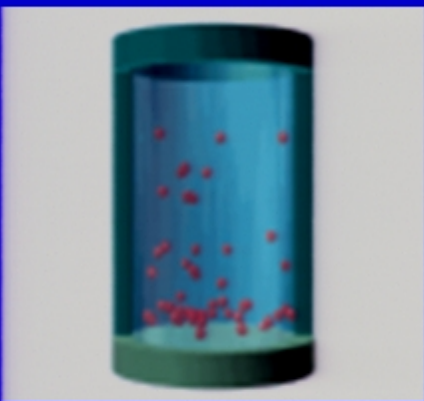
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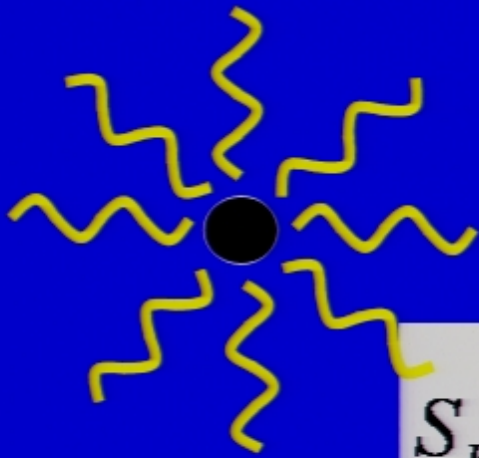
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||



$$S_{Thermal} = \frac{A}{4G_N}$$

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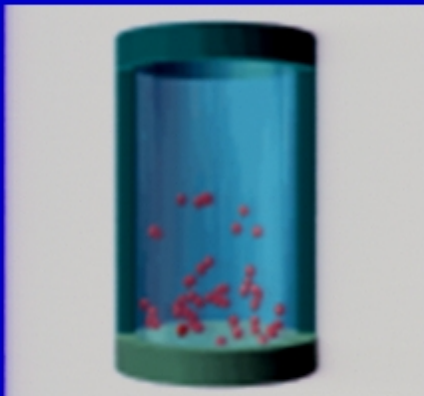


||

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Entanglement & FT/Gravity duality

R.B., M. Einhorn and A. Yarom

Extension of Maldacena's "eternal BHs in AdS"

- $Z_{\text{string}}(X) = Z_{\text{FT}}(M)$
- Hilbert spaces isomorphic

$$\mathbf{H}_W \cong \tilde{\mathbf{H}}_W$$

- $Z_{\text{string}}(X_W) = Z_{\text{FT}}(M_W)$
- Hilbert spaces isomorphic

$$\mathbf{H} \cong \tilde{\mathbf{H}}$$

Assume:

if one side of the duality is in a thermal state so is the other

(Periodic time on one side is mapped to periodic time on the other side)

$$X \longleftrightarrow M$$

$$X_W \longleftrightarrow M_W$$

Entanglement & FT/Gravity duality

$$X \longleftrightarrow M$$

$$X_w \longleftrightarrow M_w$$

$$g_s \rightarrow 0, \alpha' \rightarrow 0$$

- $Z_{\text{string}}(X) \sim e^{-I_{\text{SUGRA}}}$
- Some limit in the FT (In AdS/CFT $N \rightarrow \infty, \lambda_{\text{YM}} \rightarrow \infty$)

The duality holds in the limit

Entanglement & FT/Gravity duality

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$$\mathbf{H} = \mathbf{H}_W \otimes \mathbf{H}_W$$

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The duality holds in the limit

$$\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_W \otimes \tilde{\mathbf{H}}_W$$



$$\mathbf{H} = \mathbf{H}_W \otimes \mathbf{H}_W$$

- Bulk
 - Thermal state

$$\rho_R = \text{Tr}_L |0\rangle\langle 0|$$

$$e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)}$$

- Bulk
 - Thermal state

$$\rho_R = \text{Tr}_L |0\rangle\langle 0|$$

$$e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)}$$

- Boundary
 - Thermal state

$$\tilde{\rho}_W = \frac{1}{\tilde{Z}} \sum_i e^{-\beta \tilde{E}_i} |\tilde{E}_i\rangle\langle \tilde{E}_i|$$

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$$e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)}$$

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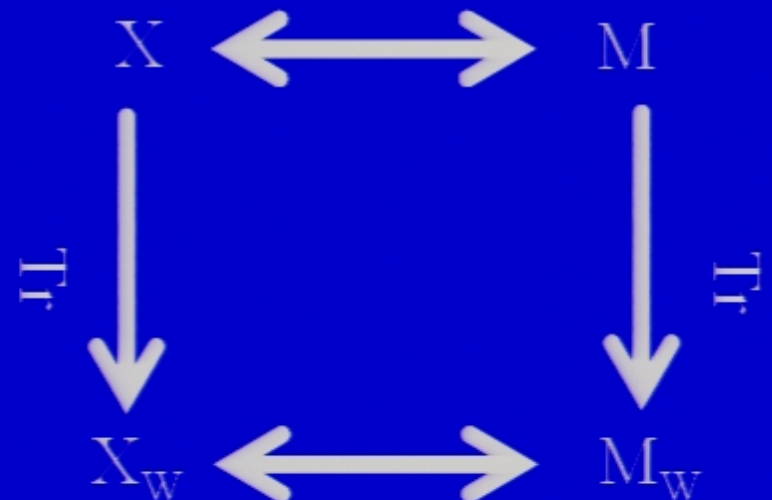
$$Z_{FT}(M) = e^{-I(X)} \xrightarrow{\text{Tr}} e^{-I(X_W)} = Z_{FT}(M_W)$$

$$g_s \rightarrow 0, \alpha' \rightarrow 0$$

$$\sum_i \langle \psi_{Li} | 0 \rangle \langle 0 | \psi_{Li} \rangle \cong \sum_i \langle \tilde{\psi}_{Li} | \tilde{0} \rangle \langle \tilde{0} | \tilde{\psi}_{Li} \rangle$$

$$\rho_W = \text{Tr}_L |0\rangle \langle 0| \cong \text{Tr}_L |\tilde{0}\rangle \langle \tilde{0}|$$

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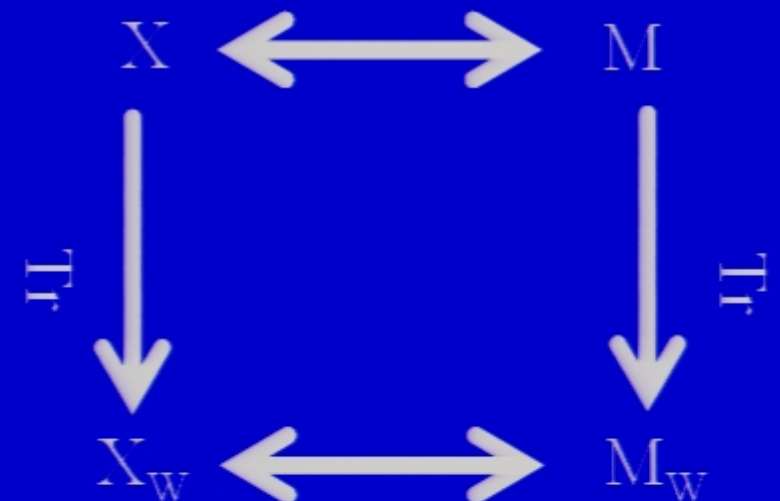
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$$\tilde{\rho}_W = \text{Tr}_L |\tilde{0}\rangle \langle \tilde{0}|$$

$$|\tilde{0}\rangle \text{ is TFD on } \tilde{\mathbf{H}}_W \otimes \tilde{\mathbf{H}}_W$$



If the trace on each side gives a thermal state with the same temp. then the state is a TFD

Entanglement & FT/Gravity duality

A chain of equalities

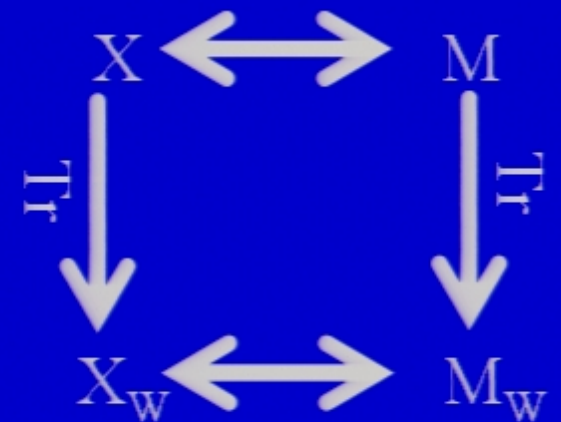
Entanglement & FT/Gravity duality

A chain of equalities

1. $S_{\text{entanglement, gravity}} = S_{\text{entanglement, FT}}$
2. $S_{\text{entanglement, FT}} = S_{\text{thermal, FT}} : T > 0$

Interpretation

$$g_s \rightarrow 0, \alpha' \rightarrow 0$$



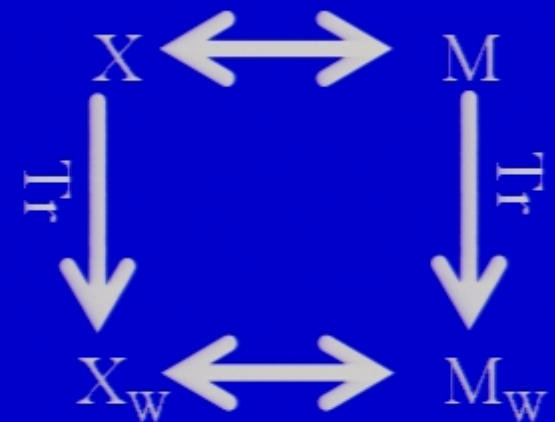
$$S(M_w) = \frac{A}{4G_N}$$



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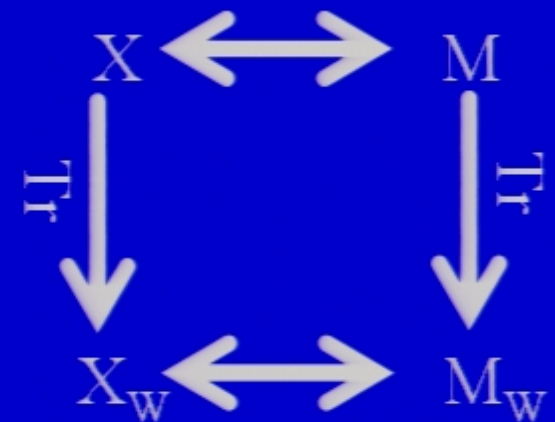
BH entropy is due (entirely) to entanglement

$S(M_W)$ is due to entanglement $\rightarrow$

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Interpretation

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→

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Both entropies diverge in the limit $G_N \rightarrow 0$

Use FT to define and compare at finite G_N

Eternal AdS-BH (5D)

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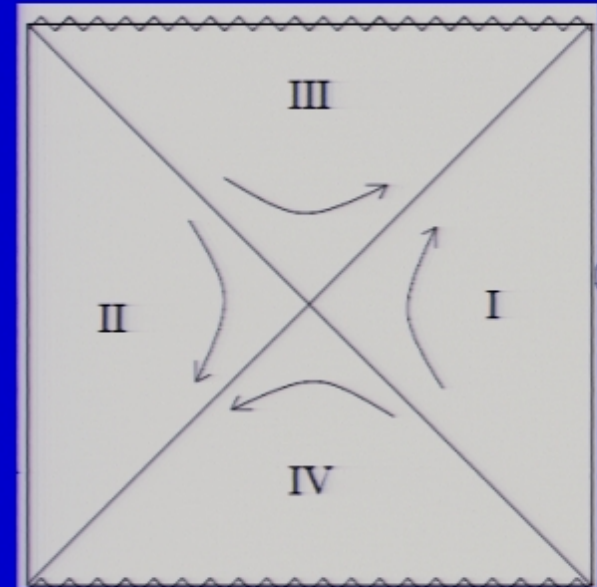
$$ds^2 = - \left(\frac{r^2}{R^2} - \frac{b^4 R^2}{r^2} \right) dt^2 + \left(\frac{r^2}{R^2} - \frac{b^4 R^2}{r^2} \right)^{-1} dr^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2 + dx_3^2)$$

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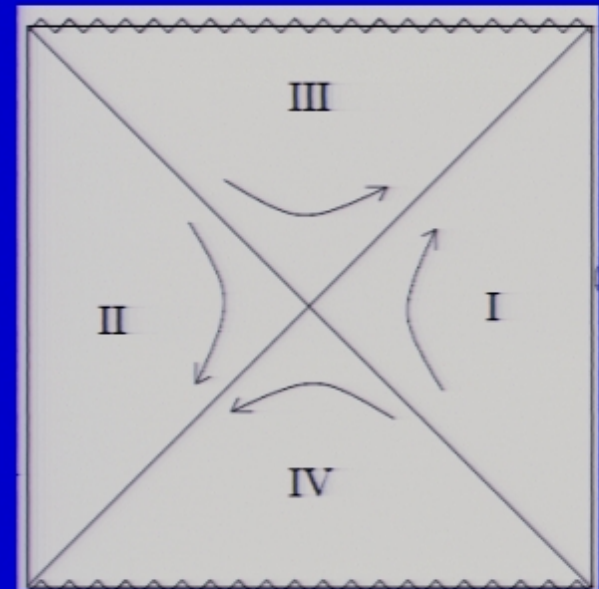
Entanglement entropy is
the entropy in a canonical
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$$S_{BH} = \frac{A}{4G_5} = \frac{1}{4} \frac{C R^5 A}{\alpha'^4 g_s^2}$$

$$S_{ENT} = NA\delta^{-3}$$



Number of fields



Eternal AdS-BH (5D)

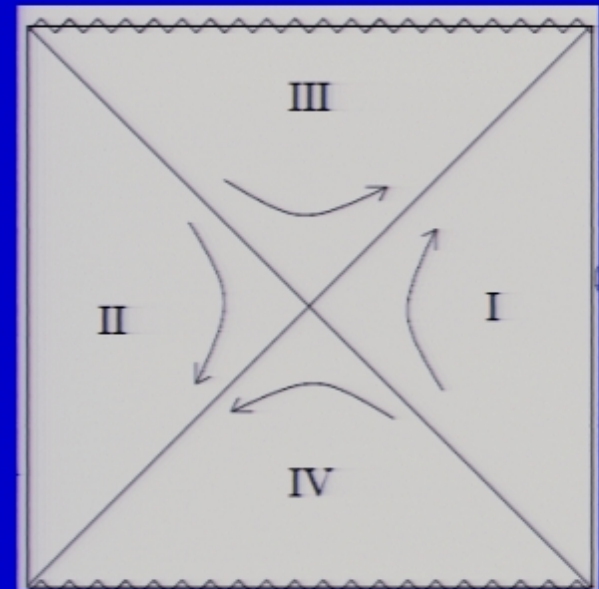
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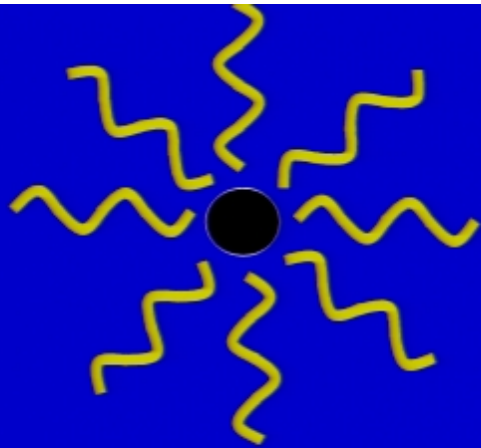
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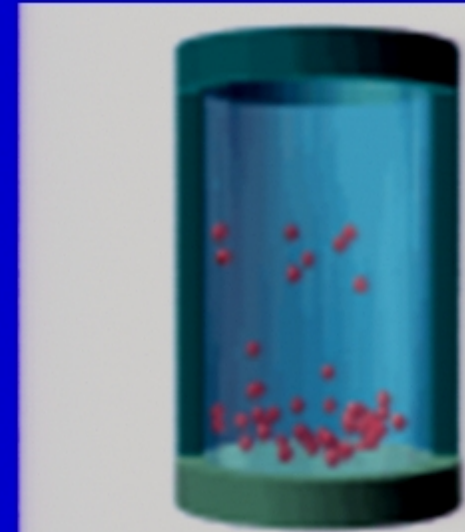
Number of fields



At the moment cannot compute the bulk regulator in terms of the boundary regulator → free constant



$$S_{ENT} = C N \frac{A}{\delta^3}$$



$$S_{ENT} = S_{BH} = \frac{A}{4G_N^{(5)}}$$

$$G_N^{(5)} = \frac{l_s^8 g_s^2}{R^5}$$

$$\begin{aligned} g_s &\rightarrow 0 \\ l_s &\rightarrow 0 \end{aligned}$$

$$\Rightarrow G_N^{(5)} \rightarrow 0$$

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KK modes

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Extensions, Consequences

1. Rotating and charged BHs, RB, Einhorn, Yarom
2. Extremal BHs (on FT side): Marolf and Yarom, TBP
3. Information paradox : RB, Einhorn, Yarom, in progress
4. Gibbons-Hawking, Wald vs. entanglement
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Summary

1. Method of calculating ρ in a wedge
2. FT/Gravity duality: The dual state to the global HH vacuum is a TFD
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$\rho_W = \text{Tr}_L[\rho_{\text{total}}] = \text{Tr}_L[|\psi\rangle\langle\psi|]$

$\tilde{\rho}_W = \text{Tr}_L[|\tilde{0}\rangle\langle\tilde{0}|]$

$|\tilde{0}\rangle \rightarrow \text{TFD} \rightarrow \tilde{H}_L \otimes \tilde{H}_R$

If the system is closed, given density matrix with the same energy, density matrix is TFD

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A chain of equalities

- $S_{\text{entanglement gravity}} = S_{\text{entanglement FT}}$
- $S_{\text{entanglement FT}} = S_{\text{thermal FT}} : T > 0$
- $S_{\text{thermal FT}} : T > 0 = S_{\text{BH}}$

$\Rightarrow S_{\text{entanglement gravity}} = S_{\text{BH}}$

20

$S(M_{\text{ext}}) = \frac{A}{4G_*} \rightarrow S(X_{\text{ext}}) = \frac{A}{4G_*}$

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 $S(M_{\text{ext}})$ is due to entanglement $\rightarrow$
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assembly of the FT: $r_h \sim bM$

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$S_{\text{ENT}} = N A \delta^{-3}$

Number of fields

As the momenta increase, compare the bulk regulator in terms of the boundary regulator $\rightarrow$ fine constant

22

$S_{\text{ext}} = C N \frac{A}{g_*^2}$

$S_{\text{ext}} = S_{\text{BH}} = \frac{A}{4G_N^{(5)}}$

$G_N^{(5)} = \frac{l_p^2 g_*^2}{R^5}$ $g_* \rightarrow 0$ $l_p \rightarrow 0 \Rightarrow G_N^{(5)} \rightarrow 0$

23

$S_{\text{ext}} = C N \frac{A}{g_*^2} \rightarrow S_{\text{ext}} = \frac{A}{4G_N^{(5)}}$

$\delta = l_p = l_p g_*^{1/4}$ $N = \frac{R^5}{g_*^5}$

KK modes

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24

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- Rotating and charged BHs, e.g. Sen, York
- Extremal BHs (on FT side): Maldacena, York, TSP
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26

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BTZ BH

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$r = \sqrt{\frac{R^2}{2} \coth^2 \rho}$
 $r = \frac{R}{2} \tanh \rho$

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4 5 6

4. *Estuaries and A. Yano*

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* Method work for men general case

$$\langle w_{\pm} | e^{-\sigma_3 \beta} | w_{\pm} \rangle = \int_{\substack{(z_1, z_2) \in \mathbb{R}_+ \times \mathbb{R}_+ \\ \sigma(z_1, z_2) = \pm 1}} D\varphi \| \mathbb{E}^* \exp \left[- \int_0^{\sigma} d\eta d\zeta d\eta' d\zeta' L \right]$$

1. The boundary conditions are $\Omega = \frac{1}{\sqrt{2\pi}}$ and

Then $Q_{\beta} = e^{-\beta H} e^{\beta F}$

 $\bar{H}_{\text{tot}}$ - generator of (im. t) time translations
$$T = \frac{f'(r_*)}{4\pi}$$

S is divergent 5 - proper length, shortest distance cut off
Naïve origin:
divergence of the optical volume near the horizon
More later... *not* brick wall

More later...**not** brick wall

More later... *not* brick wall

Diagram illustrating a black hole emitting Hawking radiation (wavy lines). A red circle with a question mark is placed near the radiation lines, indicating a question about the entropy of the radiation.

Equations shown below the diagram:

$$S_m = \frac{A}{4G}$$

$$S_m \propto N \frac{A}{2^4 \epsilon}$$

$$S_{\text{me}} = \frac{A}{4G}$$

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Author's address: *University of Twente, Enschede, The Netherlands*



$$S_{\text{coil}} = \frac{A}{4G_p}$$

$$S_{\text{filament}} \propto N \frac{A}{G^{(d-1)}}$$

$$S_{\text{filament}} = \frac{A}{4G_p}$$

?

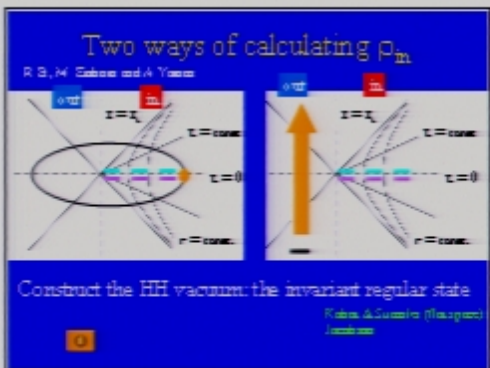
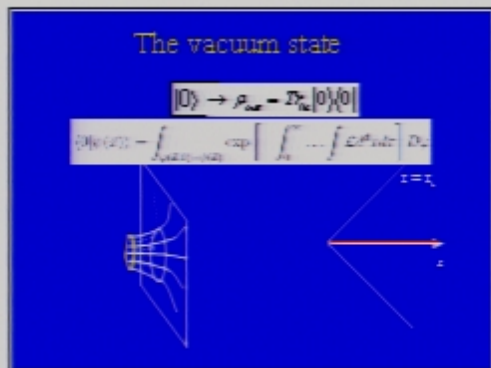
Page 142

$$S_{\text{BH}} = \frac{A}{4G_N} \quad S_{\text{BH}} \propto N \frac{A}{\phi^{(d-1)}}$$

$$S_{\text{trans}} = \frac{A}{4G_p}$$

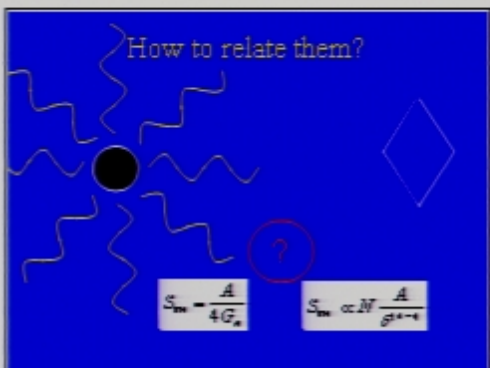
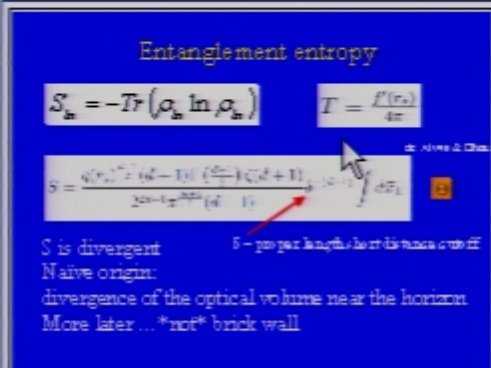
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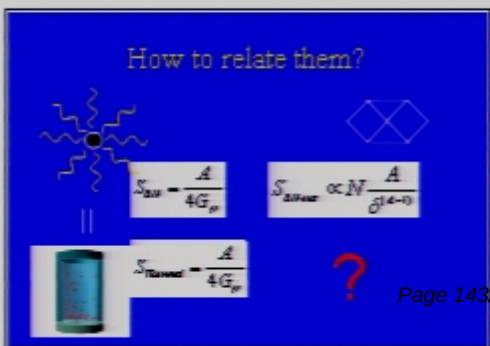
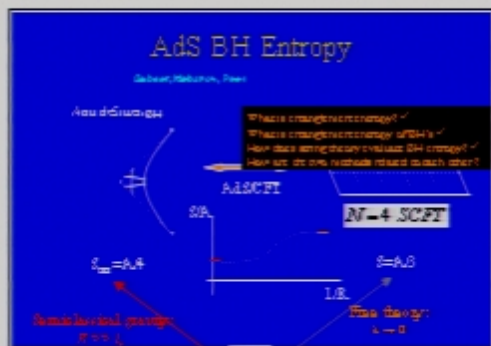
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Entanglement entropy

$$S_{in} = -Tr(\rho_{in} \ln \rho_{in})$$

$$T = \frac{f'(r_s)}{4\pi}$$

de Alwis & Qhta

$$S = \frac{q(r_s)^{\frac{d-1}{2}} (d+1) \Gamma\left(\frac{d+1}{2}\right) \zeta(d+1)}{2^{2d-1} \pi^{\frac{3d+1}{2}} (d-1)} \delta^{-(d-1)} \int d\vec{x}_{\perp}$$



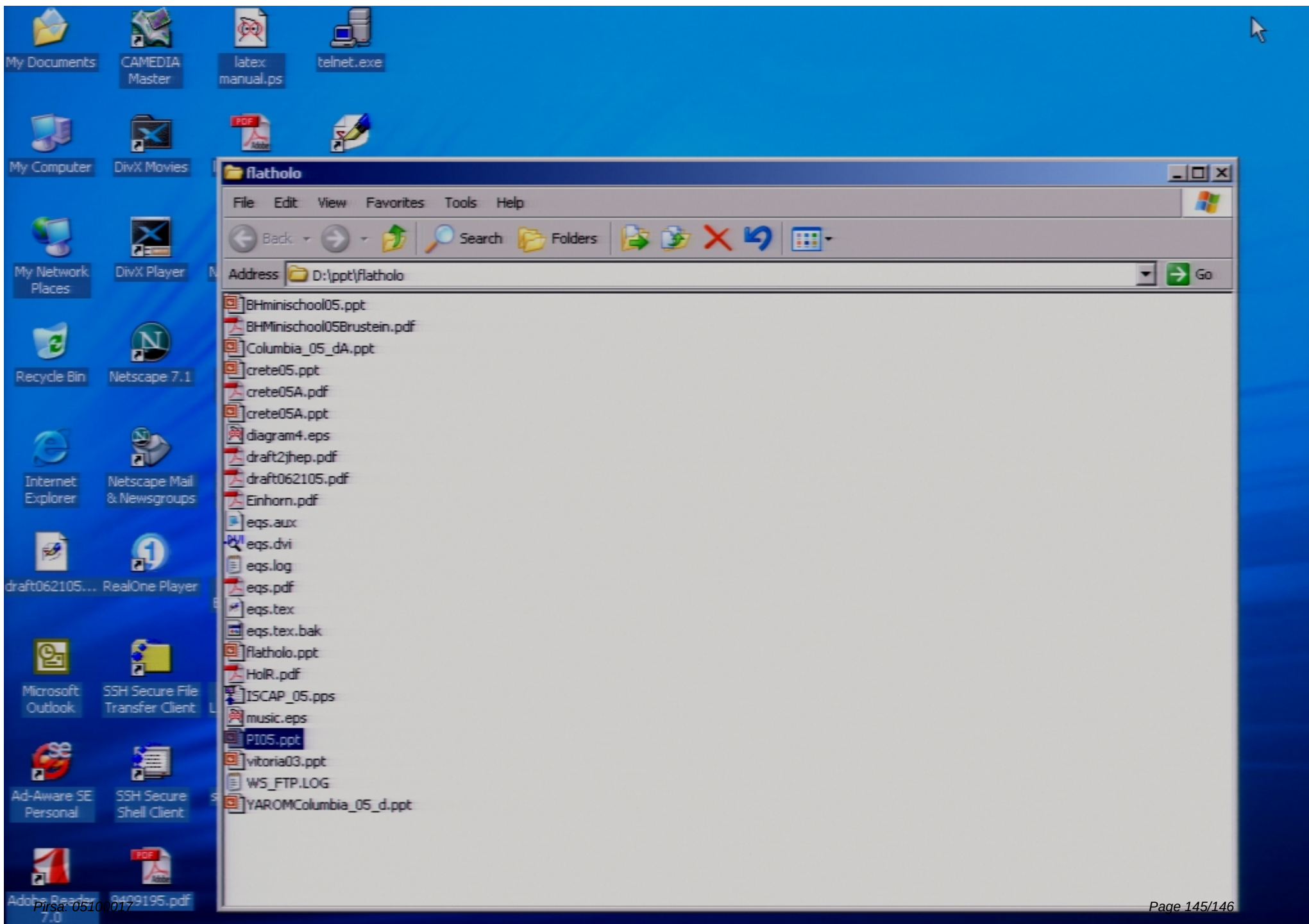
S is divergent

Naïve origin:

divergence of the optical volume near the horizon

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δ – proper length short distance cutoff



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