Title: Bohm Trajectories, Feynman Paths ans Subquantum Dynamical Processes

Date: Oct 05, 2005 03:30 PM

URL: http://pirsa.org/05100011

Abstract:

Publicon File Edit Format Insert Cell View Window Help Wed 3:35 PM

PIQuDos05.6

Bohm Trajectories, Feynman Paths and Subquantum Dynamical Processes

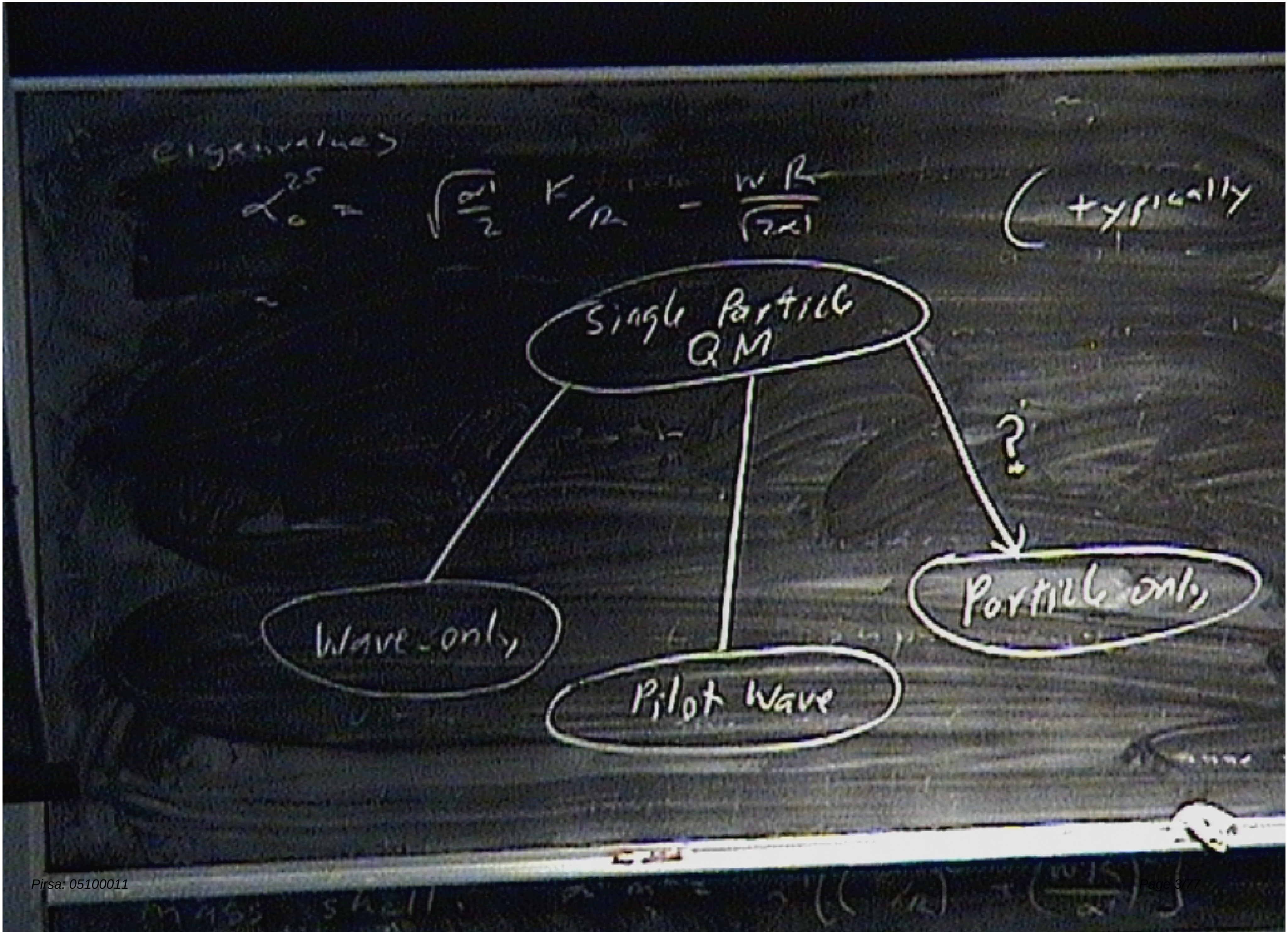
Garnet Ord

Ryerson University/PI

Abstract

Bohmian trajectories and Feynman paths are conceptually different objects from radically different views of quantum mechanics. Both offer different `particle pictures' in a subject that is based on wave mechanics. Some recent models of subquantum dynamical processes underlying the Dirac equation suggest that there may be an unexplored link between the two concepts via the quantum potential. We sketch the qualitative ideas involved and view some simple implementations that quantitatively illustrate the suggested link.

October 2005


eigenvalues
typically
Single Particle
QM
?
Wave-only
Pilot Wave
Particle-only

Publicon File Edit Format Insert Cell View Window Help Wed 3:37 PM 9%

PIQuDos05.6

Outline

1 What is this talk about?

The domain of this talk is propagation in elementary, single particle quantum mechanics. The tools are simple but non-standard for discussions of quantum mechanics. We shall use only classical statistical mechanics. The target of the talk are solutions of equations like:

$$i \frac{\partial \psi}{\partial t} = H \psi \qquad (1)$$

Where H is a simple Hamiltonian. From a quantum mechanical perspective, ψ is important as an element of a probability calculus. However there is no agreement within the physics community as to whether ψ represents anything in an external physical reality.

In classical statistical mechanics the task is simply to count recognizable objects. If we can arrive at Eqn. (1) using only statistical mechanical tools, we will have a context in which ψ itself is recognizable in its own right.

Publicon File Edit Format Insert Cell View Window Help Wed 3:41 PM 9%) Wec

PIQuDos05.6

2. What is the relevance to interpretations of quantum mechanics?

Roughly speaking, there are two categories of 'pictures' in the interpretation of quantum mechanics, based on whether there is an external reality that contains objects resembling classical particles.

1. The wave-only picture

- Here the objects of study are waves. Particles are wave-packets and are a derived concept.

2. The pilot-wave picture

- The object of study here is a real (smooth) particle trajectory.
- Agrees with intuitive ideas of particles and paths.
- Explains nodes and quantum interference through the quantum potential.
- Solutions of wave equation determine particle paths through the quantum potential.
- Waves are a necessary but adjunct concept.

3. A particle-only picture

- Is there a picture in which the particle is the object of study and waves are a derived concept? (complementary to 1 above)?
- We shall establish one.

2. What is the relevance to interpretations of quantum mechanics?

Roughly speaking, there are two categories of 'pictures' in the interpretation of quantum mechanics, based on whether there is an external reality that contains objects resembling classical particles.

1. The wave-only picture

- Here the objects of study are waves. Particles are wave-packets and are a derived concept.
- This picture is fine up to measurement. Measurement poses the problem of wavefunction collapse.
- "Theory as explanation" very questionable.
- External reality??

2. The pilot-wave picture

- The object of study here is a real (smooth) particle trajectory.
- Agrees with intuitive ideas of particles and paths.
- Explains nodes and quantum interference through the quantum potential.
- Solutions of wave equation determine particle paths through the quantum potential.
- Waves are a necessary but adjunct concept.

3. A particle-only picture

Publicon File Edit Format Insert Cell View Window Help Wed 3:42 PM

PIQuDos05.6

2. What is the relevance to interpretations of quantum mechanics?

Roughly speaking, there are two categories of 'pictures' in the interpretation of quantum mechanics, based on whether there is an external reality that contains objects resembling classical particles.

1. The wave-only picture

- Here the objects of study are waves. Particles are wave-packets and are a derived concept.

2. The pilot-wave picture

- The object of study here is a real (smooth) particle trajectory.
- Agrees with intuitive ideas of particles and paths.
- Explains nodes and quantum interference through the quantum potential.
- Solutions of wave equation determine particle paths through the quantum potential.
- Waves are a necessary but adjunct concept.

3. A particle-only picture

- Is there a picture in which the particle is the object of study and waves are a derived concept? (complementary to 1 above)?
- We shall establish one.

3. Example: Nodes in the double slit experiment

1. The experiment with electrons.

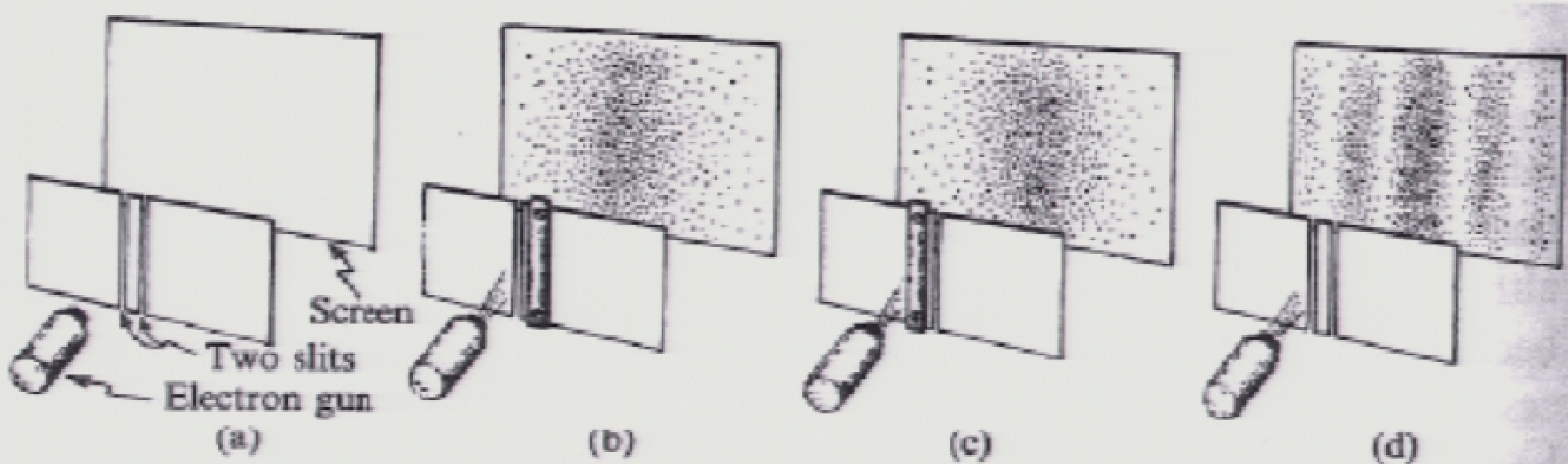


Fig. 21.4 (a) Arrangement for the two-slit experiment. One electron is emitted at a time, aimed at the screen through the pair of slits. (b) Pattern on the screen when the right-hand slit is covered. (c) The same, when the left-hand slit is covered. (d) Interference occurs when both slits are open. Some regions on the screen cannot now be reached despite the fact that they can be with just one or the other slit open

Publicon File Edit Format Insert Cell View Window Help Wed 3:45 PM 9%) Wec

PlQuDos05.6

2. Pilot-wave

How do you obtain nodes in a particle-based theory? Bohms theory uses solutions of the wave equation to construct a quantum potential. The potential establishes path-rich and path-poor areas in a wave pattern.

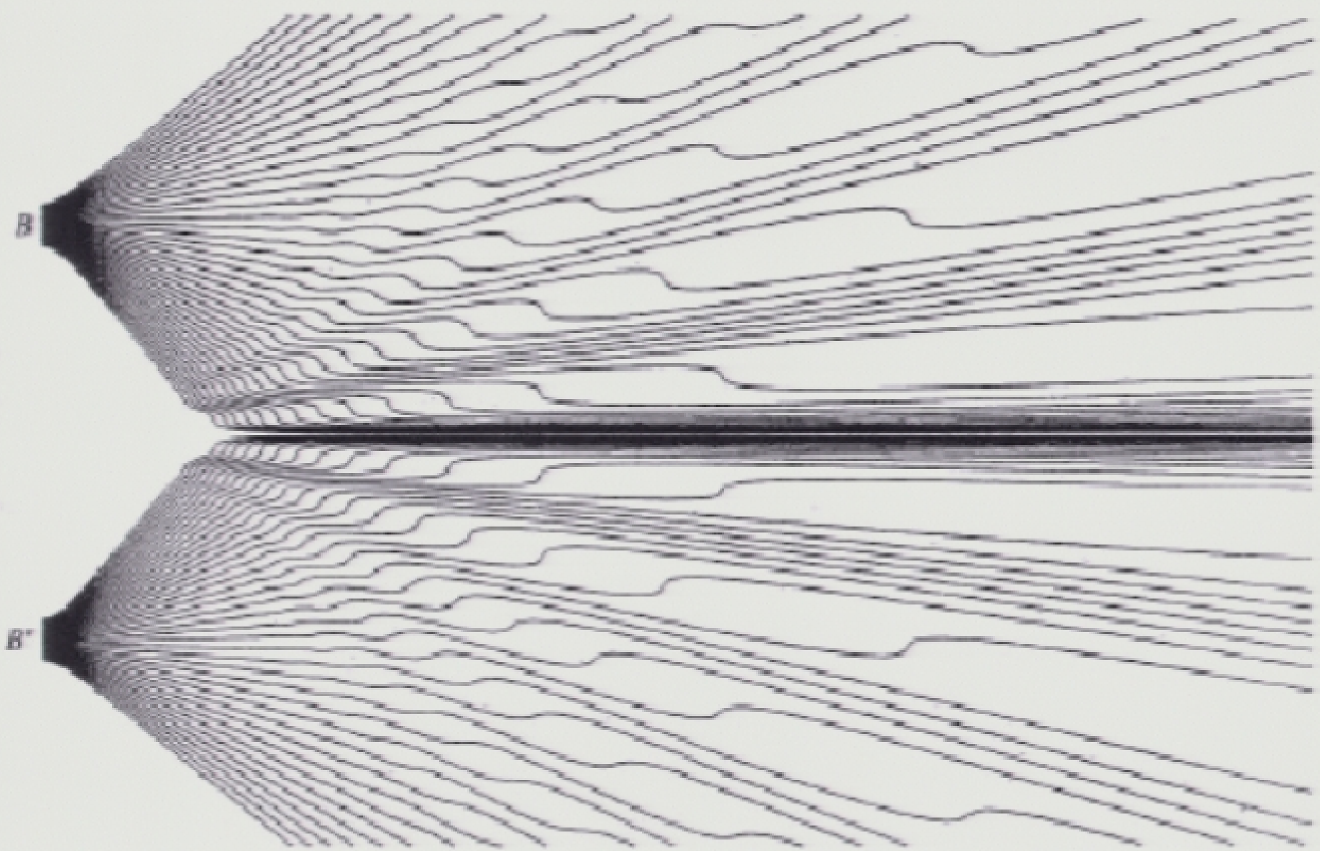


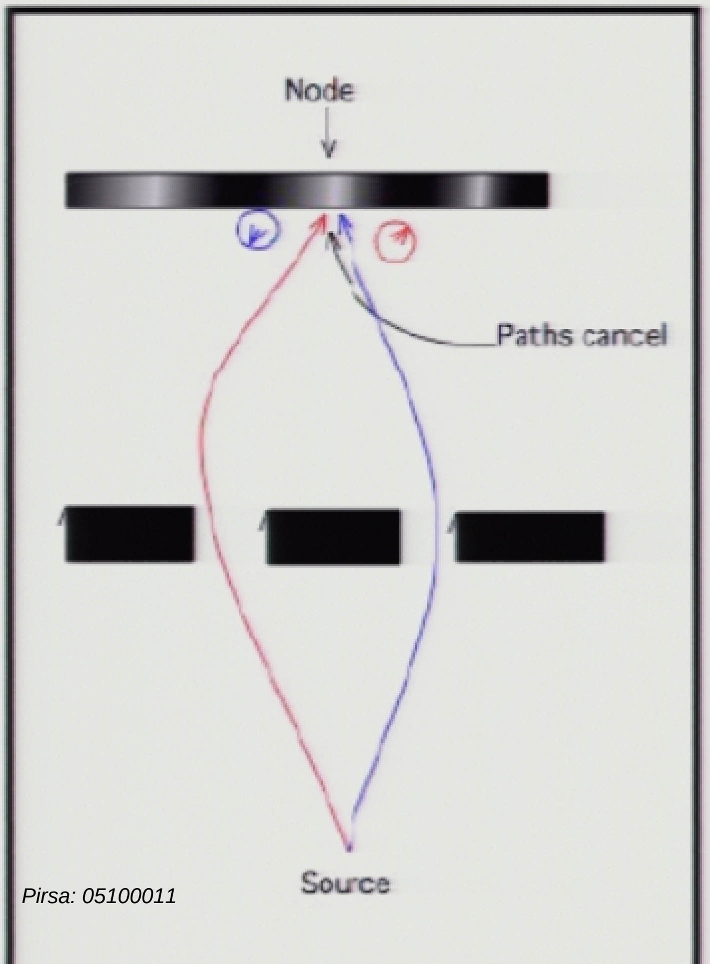
Fig. 5.5 Trajectories for two Gaussian slits with a uniform distribution of initial positions at each slit (from Philippidis et al. (1979)).

Publicon File Edit Format Insert Cell View Window Help Wed 3:46 PM

PIQuDos05.6

3. Feynman Paths

What about Feynman paths? These are not considered real, as Bohm paths are, however it is interesting to see how wave patterns are built. In particular, in terms of Feynman paths, nodal regions are not path-poor, they are path-paired where the opposite phase of pair members cancel.



Publicon File Edit Format Insert Cell View Window Help Wed 3:47 PM

PIQuDos05.6

4. Whence Phase?

- The physical origin of phase is unknown ... it is a wave concept grafted onto the particle paradigm.

  (Compare Feynman phase factor $e^{i S[x(t)]}$ with the Wiener integral Boltzman factor $e^{-S[x(t)]}$. The Boltzman factor is the result of just counting trajectories, the Feynman phase appears to be counting wave amplitudes.)

- The function of phase is to propagate subtraction ... this appears to be outside the statistical mechanics of classical particles.

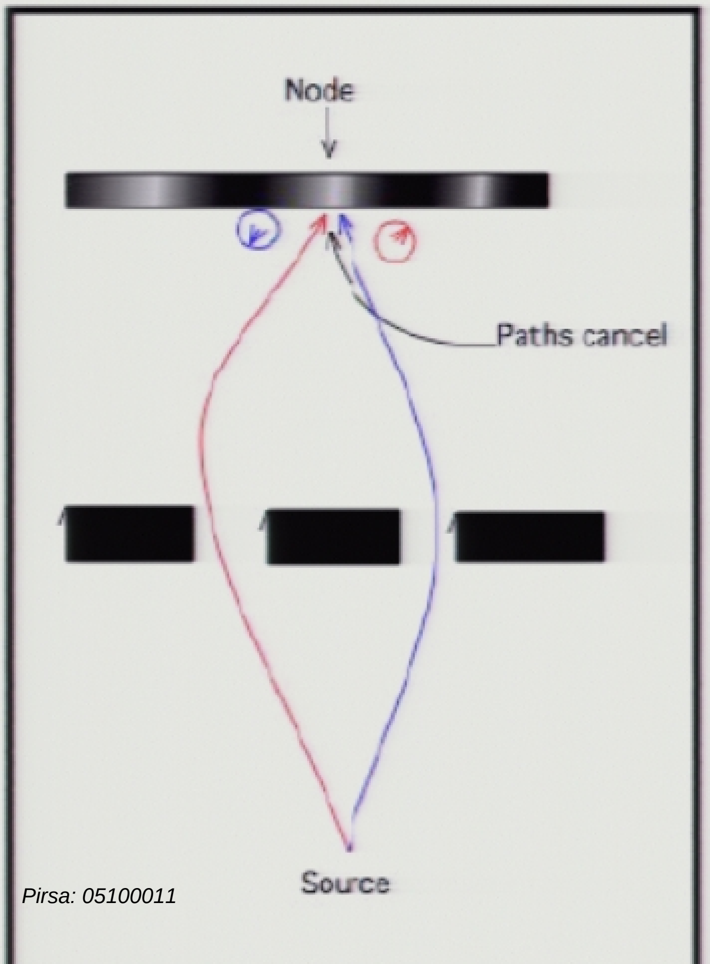
  (eg. If we wanted the lighting lowered in this room we would request that the lights be dimmed, we would not expect that a 'darkness projector' be turned up to reduce the ambient light.)

Publicon File Edit Format Insert Cell View Window Help Wed 3:49 PM 9%) Wec

PIQuDos05.6

3. Feynman Paths

What about Feynman paths? These are not considered real, as Bohm paths are, however it is interesting to see how wave patterns are built. In particular, in terms of Feynman paths, nodal regions are not path-poor, they are path-paired where the opposite phase of pair members cancel.



5.Counting with negative integers in classical statistical mechanics.

1. Simple paths

Suppose we assume paths (world-lines) that are continuous, begin before $t = 0$ and end beyond the time scale of interest. These simple paths, where x is a single-valued function of t, are counted with the natural numbers N. The diffusion equation may be shown to be a continuum limit of a counting process for a particular kind of such paths.

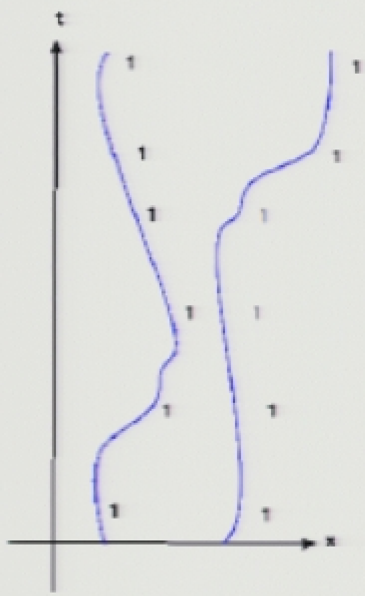


FIG. 4: Two simple paths. Only positive integers required to count the number of paths.

Publicon File Edit Format Insert Cell View Window Help Wed 3:51 PM

PIQuDos05.6

2. Non-simple paths

Suppose however that paths are continuous and traverse a particular time interval of interest, but this time they can double back! Now x is not a single-valued function of t anymore. If we want to count such paths, we have to count taking into account the direction of traversal! In the figure below, the path is colour coded to indicate direction of traversal, blue for forward in t, red for backwards in t. If we associate a +1 with blue and a -1 with red, the sum of all contributions at fixed t will count the number of paths. Note we now need the integers Z to do our counting.

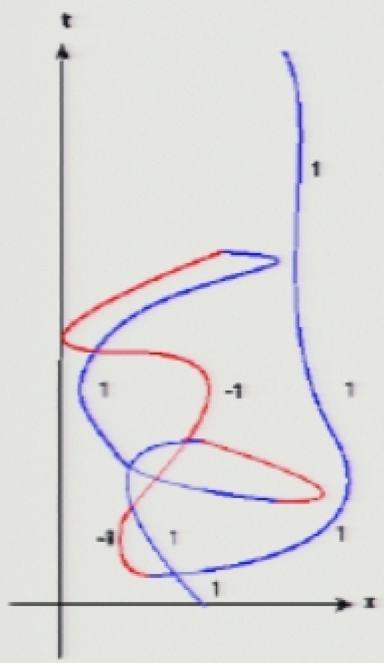


FIG. 5: Continuous paths with reversed segments require the negative integers for counting paths. At fixed t the number of continuous paths is the number of blue contributions minus the number of red.

Publicon File Edit Format Insert Cell View Window Help Wed 3:52 PM

PIQuDos05.6

6. The Dirac Equation for Accountants

Can we construct a single non-simple path in such a way that it mimics solutions of the Dirac equation? If we can, we have a chance to understand quantum propagation in a way that is similar to Einstein's explanation of diffusion in terms of Brownian motion.

Dirac Version

The Dirac Equation is usually produced by arguments that begin by requiring a PDE of the Schrödinger form ($\hbar = 1$)

$$i\frac{\partial \psi}{\partial t} = H\,\psi. \text{ (Input waves here)} \tag{2}$$

This is followed by the relativistic requirement that

$$E^2 = m^2 + p^2 \text{ (Input Special Relativity here)} \tag{3}$$

where m is the rest mass of the electron and p is the momentum. Combining these requirements lead Dirac to propose

$$i\frac{\partial \psi}{\partial t} = (\alpha.p + \beta\, m)\,\psi. \tag{4}$$

2. Non-simple paths

Suppose however that paths are continuous and traverse a particular time interval of interest, but this time they can double back! Now x is not a single-valued function of t anymore. If we want to count such paths, we have to count taking into account the direction of traversal! In the figure below, the path is colour coded to indicate direction of traversal, blue for forward in t, red for backwards in t. If we associate a +1 with blue and a -1 with red, the sum of all contributions at fixed t will count the number of paths. Note we now need the integers Z to do our counting.

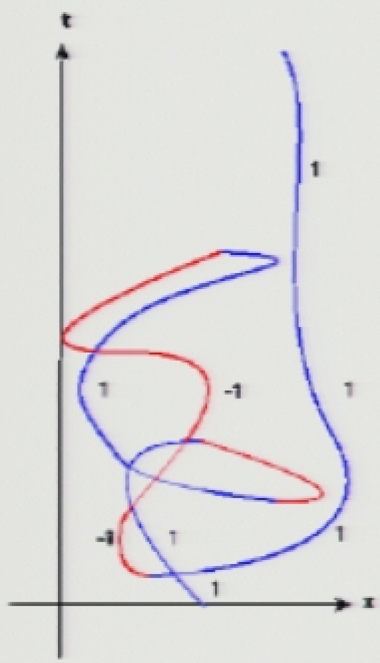


FIG. 5: Continuous paths with reversed segments require the negative integers for counting paths. At fixed t the number of continuous paths is the number of blue contributions minus the number of red.

6. The Dirac Equation for Accountants

Can we construct a single non-simple path in such a way that it mimics solutions of the Dirac equation? If we can, we have a chance to understand quantum propagation in a way that is similar to Einstein's explanation of diffusion in terms of Brownian motion.

Dirac Version

The Dirac Equation is usually produced by arguments that begin by requiring a PDE of the Schrödinger form ($\hbar = 1$)

$$i \frac{\partial \psi}{\partial t} = H\,\psi. \text{ (Input waves here)} \qquad (2)$$

This is followed by the relativistic requirement that

$$E^2 = m^2 + p^2 \text{ (Input Special Relativity here)} \qquad (3)$$

where m is the rest mass of the electron and p is the momentum. Combining these requirements lead Dirac to propose

$$i \frac{\partial \psi}{\partial t} = (\alpha.p + \beta\, m)\,\psi. \qquad (4)$$

Can we construct a single non-simple path in such a way that it mimics solutions of the Dirac equation? If we can, we have a chance to understand quantum propagation in a way that is similar to Einstein's explanation of diffusion in terms of Brownian motion.

Dirac Version

The Dirac Equation is usually produced by arguments that begin by requiring a PDE of the Schrödinger form ($\hbar = 1$)

$$i \frac{\partial \psi}{\partial t} = H\,\psi. \text{ (Input waves here)} \qquad (2)$$

This is followed by the relativistic requirement that

$$E^2 = m^2 + p^2 \text{ (Input Special Relativity here)} \qquad (3)$$

where m is the rest mass of the electron and p is the momentum. Combining these requirements lead Dirac to propose

$$i \frac{\partial \psi}{\partial t} = (\alpha.p + \beta\, m)\,\psi. \qquad (4)$$

Iterating Equation 2 , using Equation 3 suggested the usual anticommutation relations for the matrices α and β. Much as the original argument was brilliant and insightful, there was no sense in which the resulting equation described a wavefunction ψ that had a *prior physical meaning*.

6. The Dirac Equation for Accountants

Can we construct a single non-simple path in such a way that it mimics solutions of the Dirac equation? If we can, we have a chance to understand quantum propagation in a way that is similar to Einstein's explanation of diffusion in terms of Brownian motion.

Dirac Version

The Dirac Equation is usually produced by arguments that begin by requiring a PDE of the Schrödinger form ($\hbar = 1$)

$$i \frac{\partial \psi}{\partial t} = H \psi. \text{ (Input waves here)} \quad (2)$$

This is followed by the relativistic requirement that

$$E^2 = m^2 + p^2 \text{ (Input Special Relativity here)} \quad (3)$$

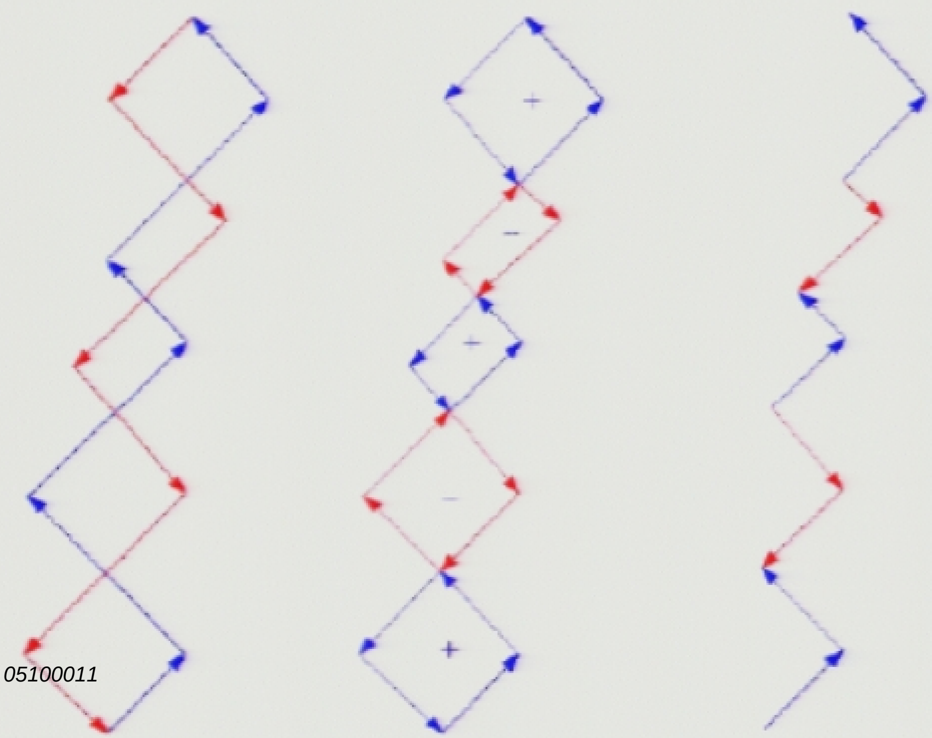
where m is the rest mass of the electron and p is the momentum. Combining these requirements lead Dirac to propose

$$i \frac{\partial \psi}{\partial t} = (\alpha.p + \beta m) \psi. \quad (4)$$

Accountant Version

Here we imagine deriving the Dirac equation in 1+1 dimension for an accountant who is of course familiar with simple arithmetic, notably counting with integers and using rational numbers when a reason for division of integers is explained.

Equation 1 would have little meaning for our accountant. $i=\sqrt{-1}$ is outside common arithmetic as are Real numbers and the calculus. However, counting configurations of a stochastic process on a lattice is within the domain of accountancy tools. Consider the stochastic walk considered in Fig. 6



Publicon File Edit Format Insert Cell View Window Help Wed 3:59 PM

PIQuDos05.6

Accountant Version

Here we imagine deriving the Dirac equation in 1+1 dimension for an accountant who is of course familiar with simple arithmetic, notably counting with integers and using rational numbers when a reason for division of integers is explained.

Equation 1 would have little meaning for our accountant. $i = \sqrt{-1}$ is outside common arithmetic as are Real numbers and the calculus. However, counting configurations of a stochastic process on a lattice is within the domain of accountancy tools. Consider the stochastic walk considered in Fig. 6

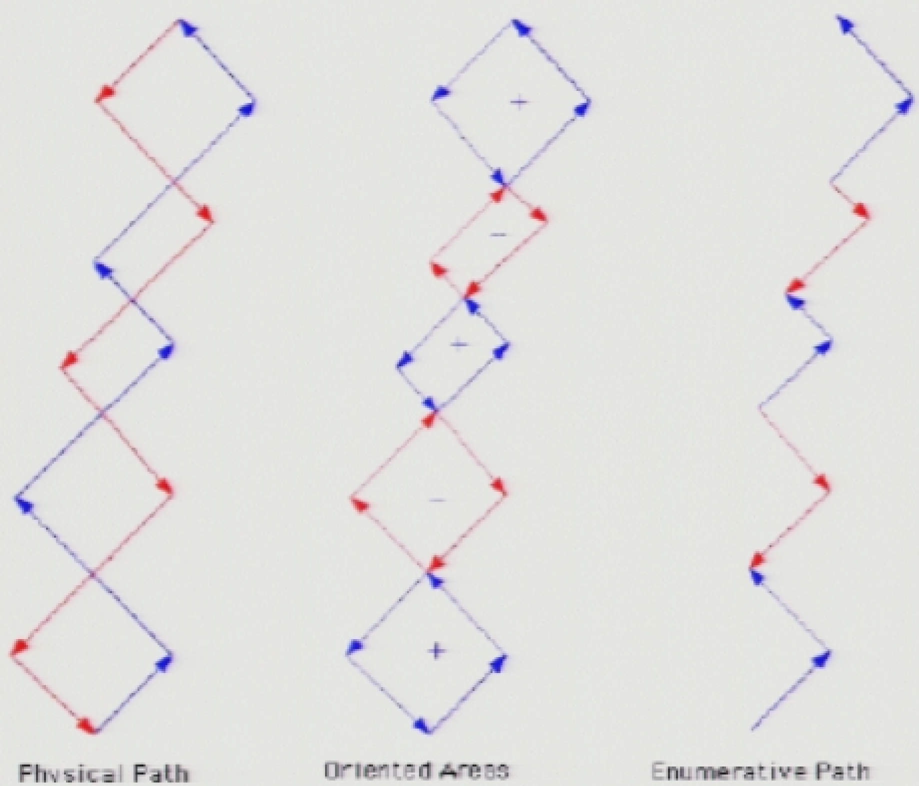


FIG. 6: On the left is a single `entwined path' in the (x, y) plane. The colour indicates the direction of traversal, blue for

Publicon File Edit Format Insert Cell View Window Help Wed 4:03 PM 9%) Wec

PIQuDos05.6

Walk description

Oriented Areas.

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must

Publicon File Edit Format Insert Cell View Window Help Wed 4:03 PM

PIQuDos05.6

Oriented Areas.

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_\pm(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must obey the difference equation:

$$u_-(x, y+\epsilon) = (1-\epsilon m)\, u_-(x+\epsilon, y) + \epsilon m\, u_+(x-\epsilon, y) \qquad (6)$$

Walk description

Oriented Areas.

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must

Publicon File Edit Format Insert Cell View Window Help Wed 4:05 PM

PIQuDos05.6

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_\pm(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must obey the difference equation:

$$u_-(x, y+\epsilon) = (1-\epsilon m)\, u_-(x+\epsilon, y) + \epsilon m\, u_+(x-\epsilon, y) \qquad (6)$$

The positive sign for the second term reflects the fact that the change of direction for a `left turn' on an enumerative path does *not* change orientation.

Publicon File Edit Format Insert Cell View Window Help Wed 4:05 PM

PIQuDos05.6

on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must obey the difference equation:

$$u_-(x, y+\epsilon) = (1-\epsilon m)\, u_-(x+\epsilon, y) + \epsilon m\, u_+(x-\epsilon, y) \qquad (6)$$

The positive sign for the second term reflects the fact that the change of direction for a `left turn' on an enumerative path does *not* change orientation.

Walk description

Oriented Areas.

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must

Publicon File Edit Format Insert Cell View Window Help Wed 4:06 PM

PlQuDos05.6

Solutions of the difference equations.

Remove decay and rewrite

Write

$$u_{\pm}(x, n\,\epsilon) = w_{\pm}(x, n\,\epsilon)\,(1 - \epsilon\, m)^{n} \qquad (7)$$

then $w_{\pm}$ satisfies the equations:

$$\begin{aligned} w_{+}(x, y+\epsilon) &= w_{+}(x-\epsilon, y) - \epsilon\, m\, w_{-}(x+\epsilon, y) \\ w_{-}(x, y+\epsilon) &= w_{-}(x+\epsilon, y) + \epsilon\, m\, w_{+}(x-\epsilon, y) \end{aligned} \qquad (8)$$

to lowest order in ϵ.

Solutions Rational, counting process mundane.

Solutions of the difference equations.

Remove decay and rewrite

Write

$$u_{\pm}(x, n\,\epsilon) = w_{\pm}(x, n\,\epsilon)\,(1 - \epsilon\, m)^{n} \qquad (7)$$

then $w_{\pm}$ satisfies the equations:

$$\begin{aligned} w_{+}(x, y+\epsilon) &= w_{+}(x-\epsilon, y) - \epsilon\, m\, w_{-}(x+\epsilon, y) \\ w_{-}(x, y+\epsilon) &= w_{-}(x+\epsilon, y) + \epsilon\, m\, w_{+}(x-\epsilon, y) \end{aligned} \qquad (8)$$

to lowest order in ϵ.

Solutions Rational, counting process mundane.

Continuum Limit.

We abandon the limitations of arithmetic and approximate the solutions of (8) by taking a continuum limit. If we subtract $w_\pm(x, y)$ from both sides of (7), divide by ϵ and take the limit as $\epsilon \to 0^+$ we find that

$$\partial_y w_+(x,y) = -\partial_x w_+(x,y) - m\, w_-(x,y)$$

$$\partial_y w_-(x,y) = \partial_x w_-(x,y) + m\, w_+(x,y)$$

or writing

$$w = \begin{pmatrix} w_+ \\ w_- \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_y$$

we have

$$\partial_y w = -\sigma_z\, \partial_x w - i\, m\, \sigma_y\, w \qquad (9)$$

This may be recognized as a form of the Dirac equation where $c = \hbar = 1$ and $y = t$ (Not $i\,t$!!!). Note that if we iterate this equation to get a second order form we have

$$\partial_y^2 w = \partial_x^2 w - m^2\, w \qquad (10)$$

which is the Klein-Gordon equation.

Note, this means that the solutions of these 'wave equations' are a continuum limit of a pattern formed by

Publicon File Edit Format Insert Cell View Window Help Wed 4:08 PM

PIQuDos05.6

If we subtract $w_{\pm}(x, y)$ from both sides of (7), divide by ϵ and take the limit as $\epsilon \to 0^{+}$ we find that

$$\partial_y w_+(x, y) = -\partial_x w_+(x, y) - m\, w_-(x, y)$$

$$\partial_y w_-(x, y) = \partial_x w_-(x, y) + m\, w_+(x, y)$$

or writing

$$w = \begin{pmatrix} w_+ \\ w_- \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_y$$

we have

$$\partial_y w = -\sigma_z \partial_x w - i\, m\, \sigma_y w \qquad (9)$$

This may be recognized as a form of the Dirac equation where $c = \hbar = 1$ and $y = t$ (Not $i\,t$!!!). Note that if we iterate this equation to get a second order form we have

$$\partial_y^2 w = \partial_x^2 w - m^2 w \qquad (10)$$

which is the Klein-Gordon equation.

Note, this means that the solutions of these 'wave equations' are a continuum limit of a pattern formed by the a counting process for a single 'space-time' trajectory!

Publicon File Edit Format Insert Cell View Window Help Wed 4:08 PM

PIQuDos05.6

What we have Not Done

Notice that to obtain the Dirac and Klein-Gordon equations above we have not invoked :

(A) the uncertainty principle
(B) quantization or Schrödinger's equation
(C) complex numbers
(D) special relativity
(E) any `interpretation' regarding quantum mechanics and the nature of reality.

We have simply taken the output of an `accounting argument' and written it in a language familiar in the context of relativistic quantum mechanics.

About the equation.

Now (9) *is* just a continuum limit of (8) written in a familiar form. (Notice that there has been no analytic continuation forced on the system. w is real and the i in (8) is present only because σ_y is imaginary.) The point here is that we can regard (8) either as a fundamental equation about the 'wavefunction' of an electron, without knowing exactly what a wavefunction represents in the physical world, *or* we can take (8) as the continuum limit of an equation describing an equilibrium distribution of a simple stochastic process. The continuum language that we use does not tell us whether we are describing a `Dirac wavefunction in one dimension' or a `spacetime that maintains an accountancy ledger for the EP stochastic process'.

7. What is different about this ‘derivation’ of the Dirac Equation?

In the context of quantum mechanics, all ‘ derivations’ of quantum mechanics from particle mechanics involve a formal analytic continuation (FAC), either explicit or forced by a global requirement. For example:

- the usual $\{p \to -i\,\hbar\,\partial_x, E \to i\,\hbar\,\partial_t\}$ is an explicit FAC that takes us from real dynamical variables to complex operators.
- Nelson's work is a forced analytic continuation. There is no explicit invocation of complex numbers, however Nelson's argument cleverly forces the diffusion equation into the complex domain by requiring reversibility.

The above derivation of the Dirac equation has no such analytic continuation, either explicit or globally forced. So what makes it work???

Publicon File Edit Format Insert Cell View Window Help Wed 4:09 PM 9%) Wec

PIQuDos05.6

Continuum Limit.

We abandon the limitations of arithmetic and approximate the solutions of (8) by taking a continuum limit. If we subtract $w_{\pm}(x, y)$ from both sides of (7), divide by ϵ and take the limit as $\epsilon \to 0^{+}$ we find that

$$\partial_y w_+(x, y) = -\partial_x w_+(x, y) - m\, w_-(x, y)$$

$$\partial_y w_-(x, y) = \partial_x w_-(x, y) + m\, w_+(x, y)$$

or writing

$$w = \begin{pmatrix} w_+ \\ w_- \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_y$$

we have

$$\partial_y w = -\sigma_z\, \partial_x w - i\, m\, \sigma_y\, w \qquad (9)$$

This may be recognized as a form of the Dirac equation where $c = \hbar = 1$ and $y = t$ (Not $i\,t$!!!). Note that if we iterate this equation to get a second order form we have

$$\partial_y^2 w = \partial_x^2 w - m^2\, w \qquad (10)$$

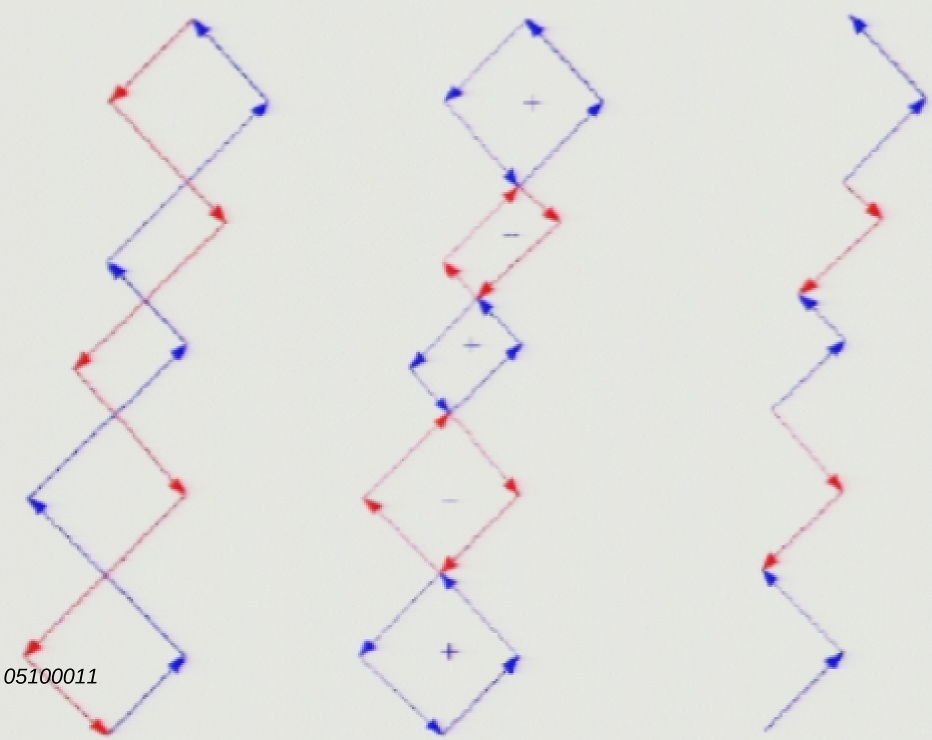
which is the Klein-Gordon equation.

Note, this means that the solutions of these 'wave equations' are a continuum limit of a pattern formed by

Accountant Version

Here we imagine deriving the Dirac equation in 1+1 dimension for an accountant who is of course familiar with simple arithmetic, notably counting with integers and using rational numbers when a reason for division of integers is explained.

Equation 1 would have little meaning for our accountant. $i = \sqrt{-1}$ is outside common arithmetic as are Real numbers and the calculus. However, counting configurations of a stochastic process on a lattice is within the domain of accountancy tools. Consider the stochastic walk considered in Fig. 6



Walk description

Oriented Areas.

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \qquad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must

Enumerative Paths

Counting Oriented Areas

Now our task to count oriented rectangles is reduced to counting the contributions of enumerative paths on the lattice. We can do this using the structure of the walks to deduce what the equilibrium pattern, assuming there is one, must be.

Let us label the lattice sites by $x = m\epsilon$, and $y = n\epsilon$ where m and n are integers. We consider a two component density $u_{\pm}(x, y)$ where u_+ counts the number of $(1, 1)$ and $(-1, -1)$ directed links and u_- counts the number of $(-1, 1)$ and $(1, -1)$ directed links *by orientation*. We need a 2-component density here because our enumerative paths continually shuffle orientation counts between the two directions.

Now any link at $(x, y+\epsilon)$ in the $\pm(1, 1)$ direction either follows a link of the same direction and colour at $(x-\epsilon, y)$ or follows a link of the opposite direction and colour at $(x+\epsilon, y)$. The former occurs with probability $(1-\epsilon m)$, the latter with probability ϵm. Thus if an equilibrium density is reached it must satisfy:

$$u_+(x, y+\epsilon) = (1-\epsilon m)\, u_+(x-\epsilon, y) - \epsilon m\, u_-(x+\epsilon, y) \quad (5)$$

Notice here the subtraction involved in the second term. This is because whenever our enumerative path `turns right' it switches orientation, thus changing the sign of its contribution. Since this happens for all paths, it must happen for the equilibrium distribution. We can similarly deduce that the u_- density must obey the difference equation:

$$u_-(x, y+\epsilon) = (1-\epsilon m)\, u_-(x+\epsilon, y) + \epsilon m\, u_+(x-\epsilon, y) \quad (6)$$

The positive sign for the second term reflects the fact that the change of direction for a `left turn' on an enumerative path does *not* change orientation.

What we have **Not** Done

Notice that to obtain the Dirac and Klein-Gordon equations above we have not invoked :

(*A*) the uncertainty principle
(*B*) quantization or Schrödinger' *s* equation
(*C*) complex numbers
(*D*) special relativity
(*E*) any `interpretation' regarding quantum mechanics and the nature of reality.

We have simply taken the output of an `accounting argument' and written it in a language familiar in the context of relativistic quantum mechanics.

About the equation.

Now (9) *is* just a continuum limit of (8) written in a familiar form. (Notice that there has been no analytic continuation forced on the system. w is real and the i in (8) is present only because σ_y is imaginary.) The point here is that we can regard (8) either as a fundamental equation about the 'wavefunction' of an electron, without knowing exactly what a wavefunction represents in the physical world, *or* we can take (8) as the continuum limit of an equation describing an equilibrium distribution of a simple stochastic process. The continuum language that we use does not tell us whether we are describing a `Dirac wavefunction in one dimension' or a `spacetime that maintains an accountancy ledger for the EP stochastic process'.

Publicon File Edit Format Insert Cell View Window Help Wed 4:14 PM

PIQuDos05.6

7. What is different about this ‘derivation’ of the Dirac Equation?

In the context of quantum mechanics, all ‘ derivations’ of quantum mechanics from particle mechanics involve a formal analytic continuation (FAC), either explicit or forced by a global requirement. For example:

- the usual $\{p \to -i\hbar\partial_x, E \to i\hbar\partial_t\}$ is an explicit FAC that takes us from real dynamical variables to complex operators.
- Nelson’s work is a forced analytic continuation. There is no explicit invocation of complex numbers, however Nelson’s argument cleverly forces the diffusion equation into the complex domain by requiring reversibility.

The above derivation of the Dirac equation has no such analytic continuation, either explicit or globally forced. So what makes it work???

8. Constructing Complex Numbers

Historically and to a certain extent logically, the evolution of number systems is something like:

$\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$

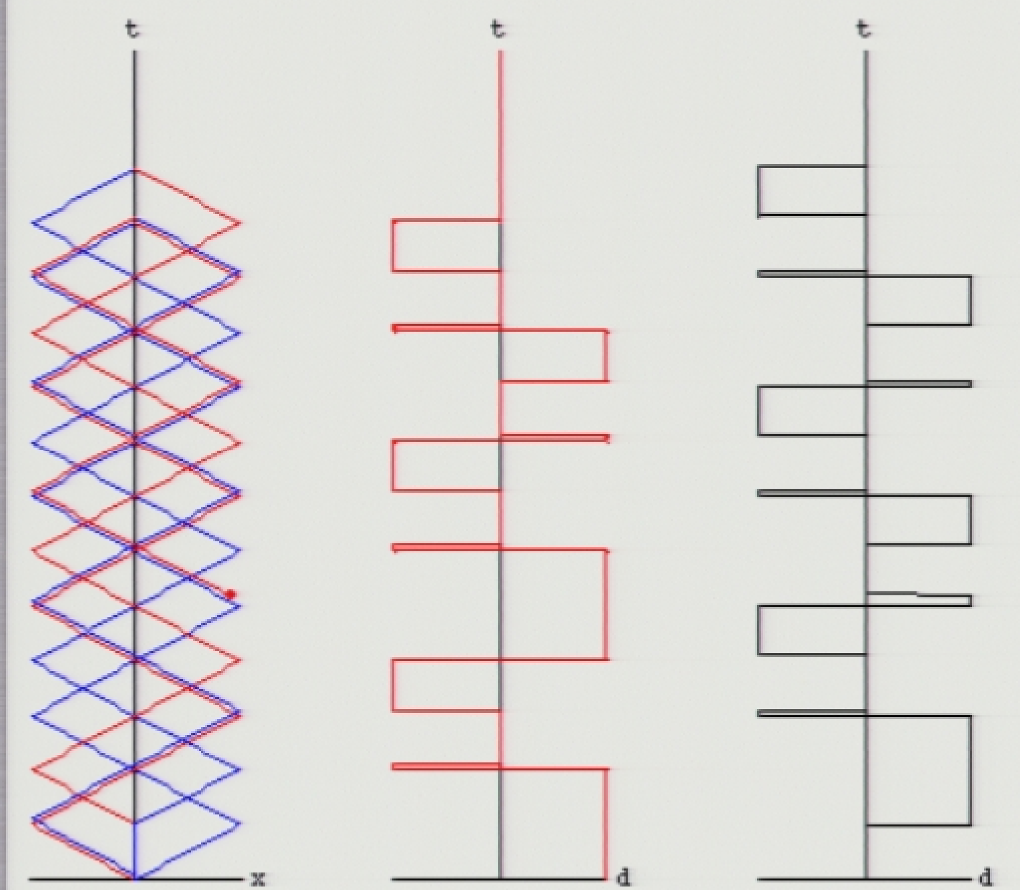
$\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are appropriate number systems for the counting problems of classical statistical mechanics. However in making the transition from classical mechanics to quantum mechanics, we habitually start in $\mathbb{R}$ (we need a smooth continuum for classical mechanics) and we are forced into $\mathbb{C}$ by 'wave-particle duality'.

In the 'Accountant version' we started with a counting problem for non-simple curves. This forced us to use Integers for the primary number system. The geometry (counting oriented areas) required that the counting have a 2-component structure with a built-in periodicity. The continuum limit of the counting process then contained the algebra of the complex numbers, not just the Reals. The logical structure is something like:

$$\mathbb{Z}_2 \to \mathbb{Q}_2 \to \mathbb{C}\ (\to \mathbb{R})$$

where $\mathbb{Z}_2$ and $\mathbb{Q}_2$ are essentially subsets of $\mathbb{C}$ with discrete phase and modulus. The difference here from other derivations of the Dirac equation is that the Algebra of complex numbers is built by a combination of the local geometry of the trajectory and the counting process that detects oriented areas. The natural extension of the counting process is to $\mathbb{C}$, not $\mathbb{R}$. Unfortunately the connection between the 'subquantum dynamical process' and the resulting 'wavefunction' is lost in the continuum limit because the process itself is below the resolution of the resulting partial differential equation. (compare thermodynamics and stat. mech.)

evolve_smooth6-10.mov



00:00:10

8

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

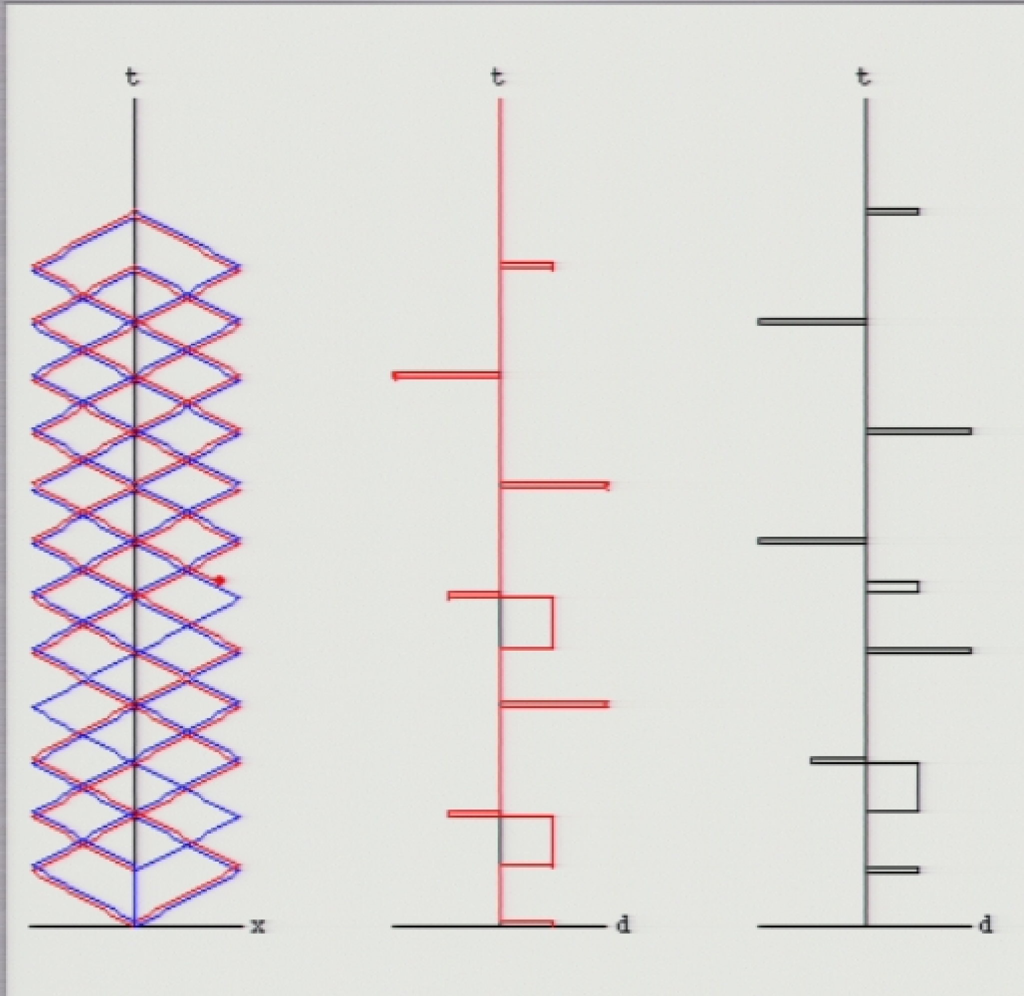
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:21 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:13

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

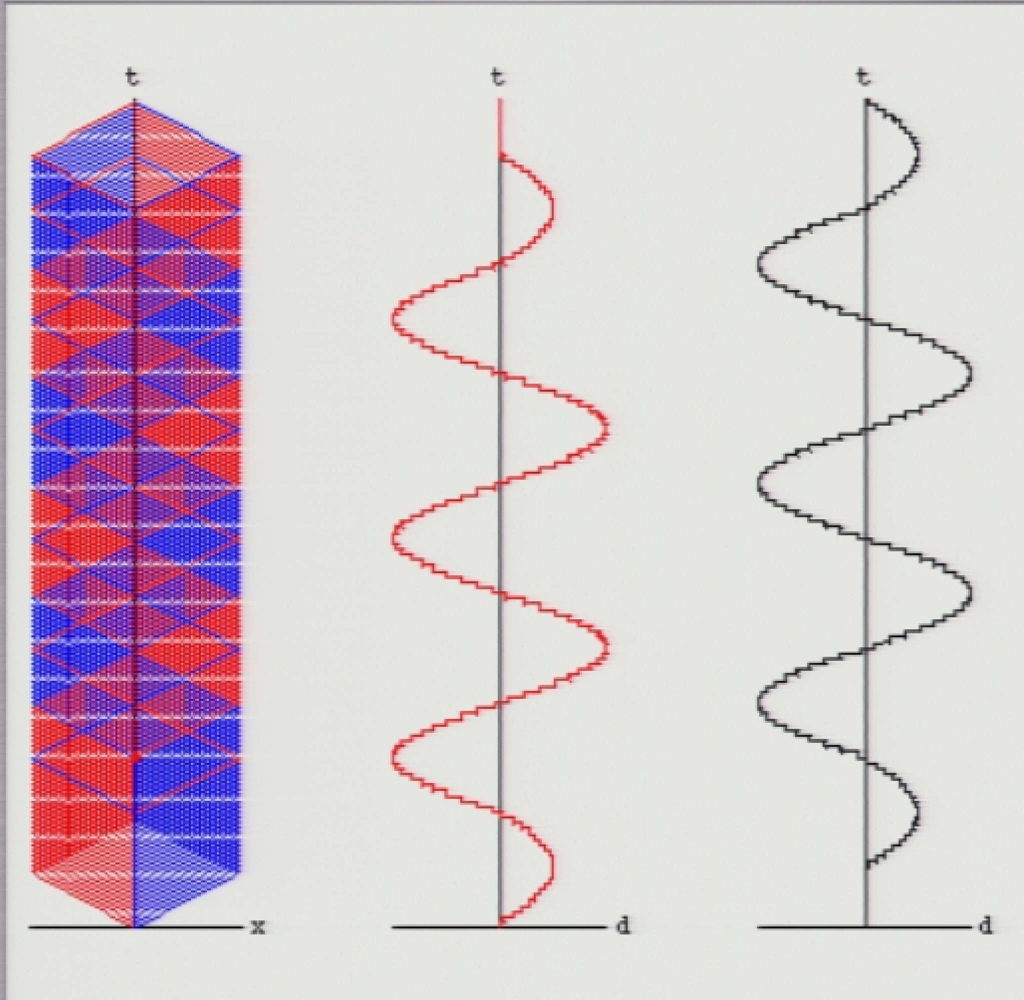
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
s that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:21 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:17

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

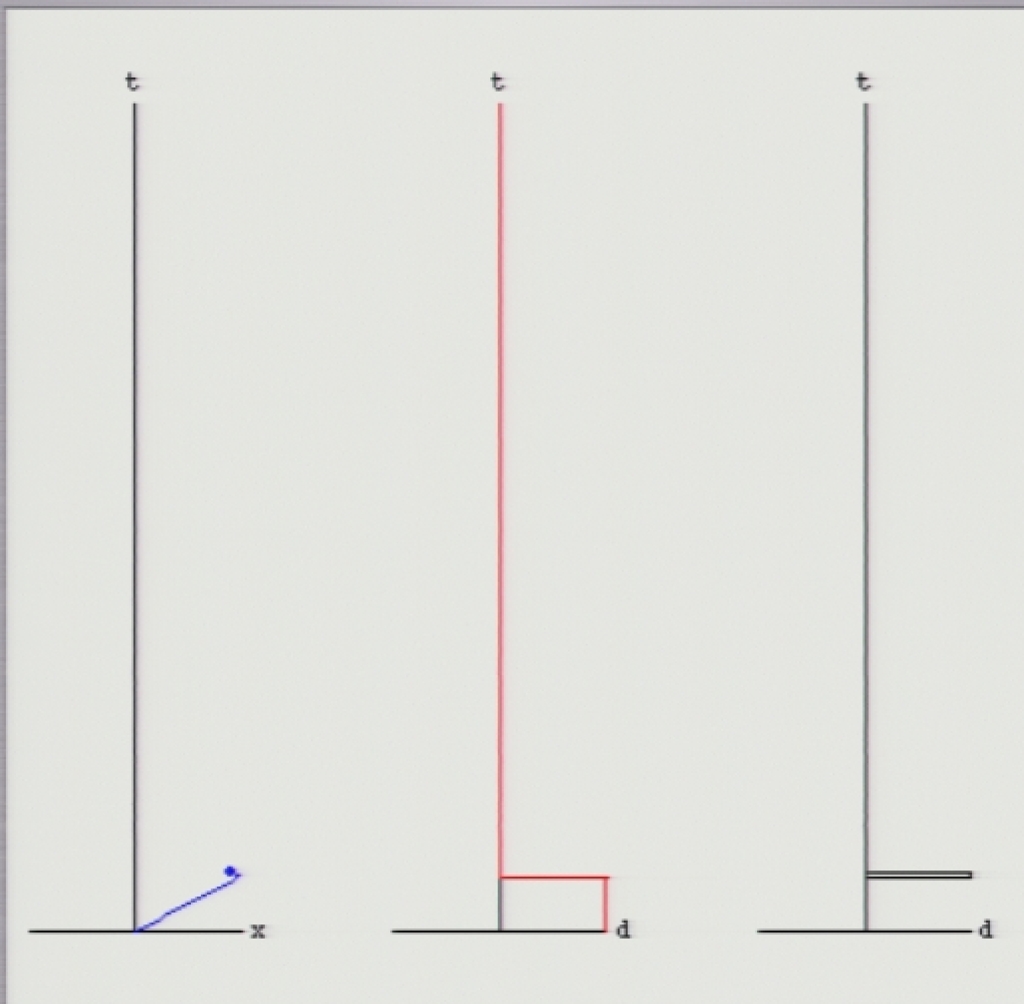
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:22 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:00

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

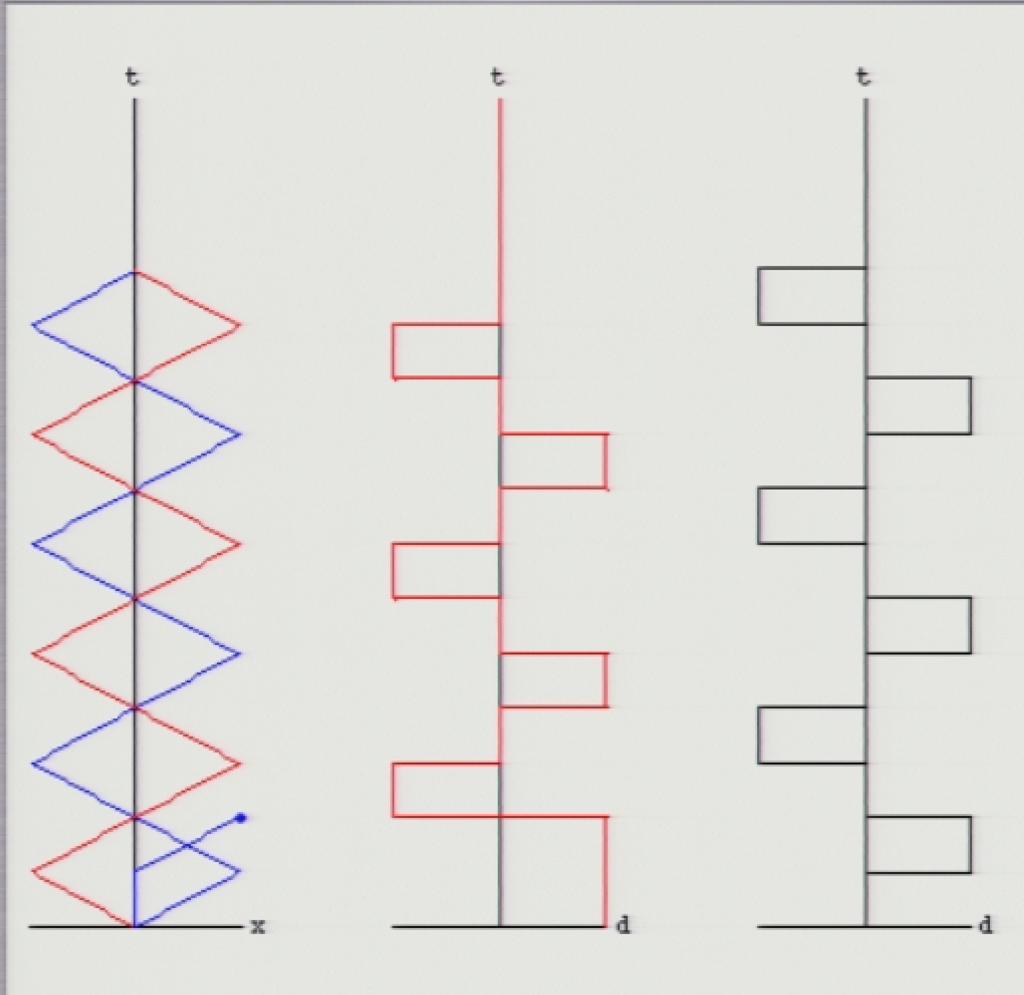
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help (9! Wed 4:22 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:03

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by ‘wave-particle duality’.

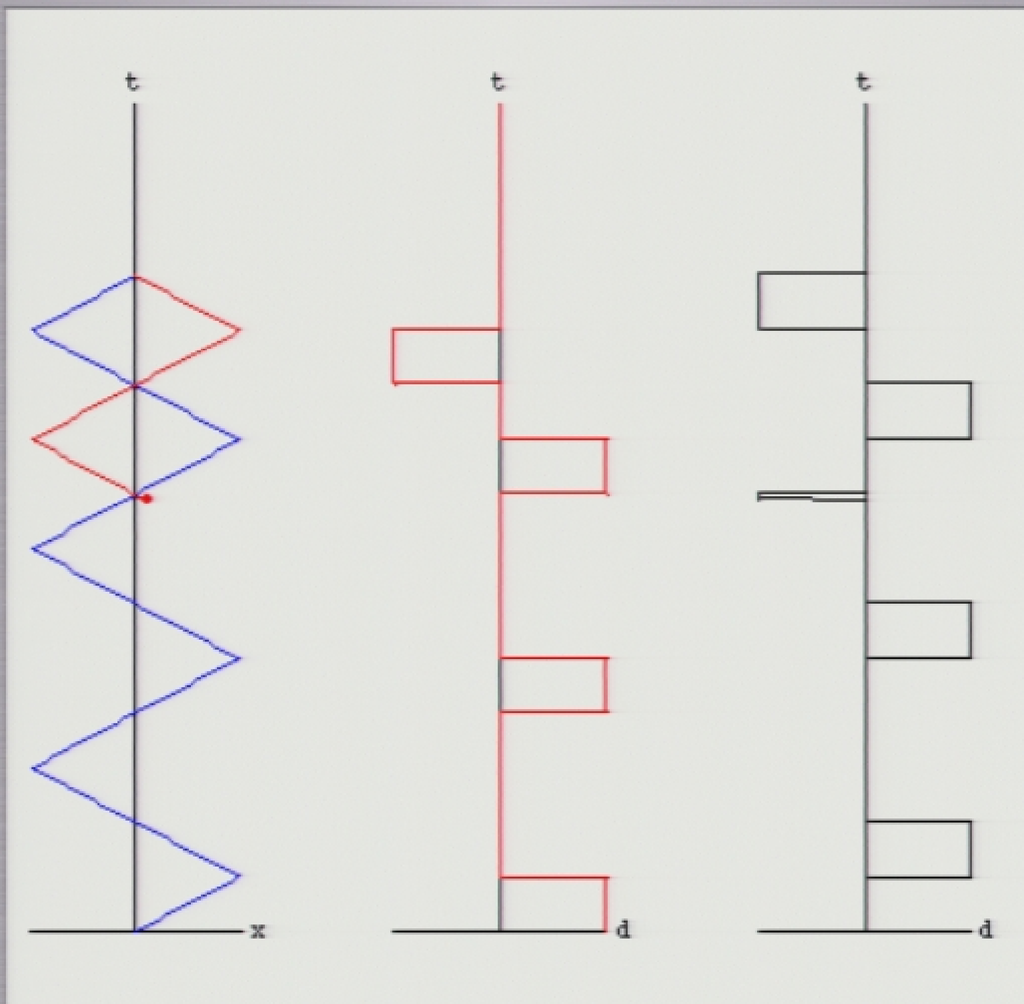
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
s that detects oriented areas. The natural
ly the connection between the ‘subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:22 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:02

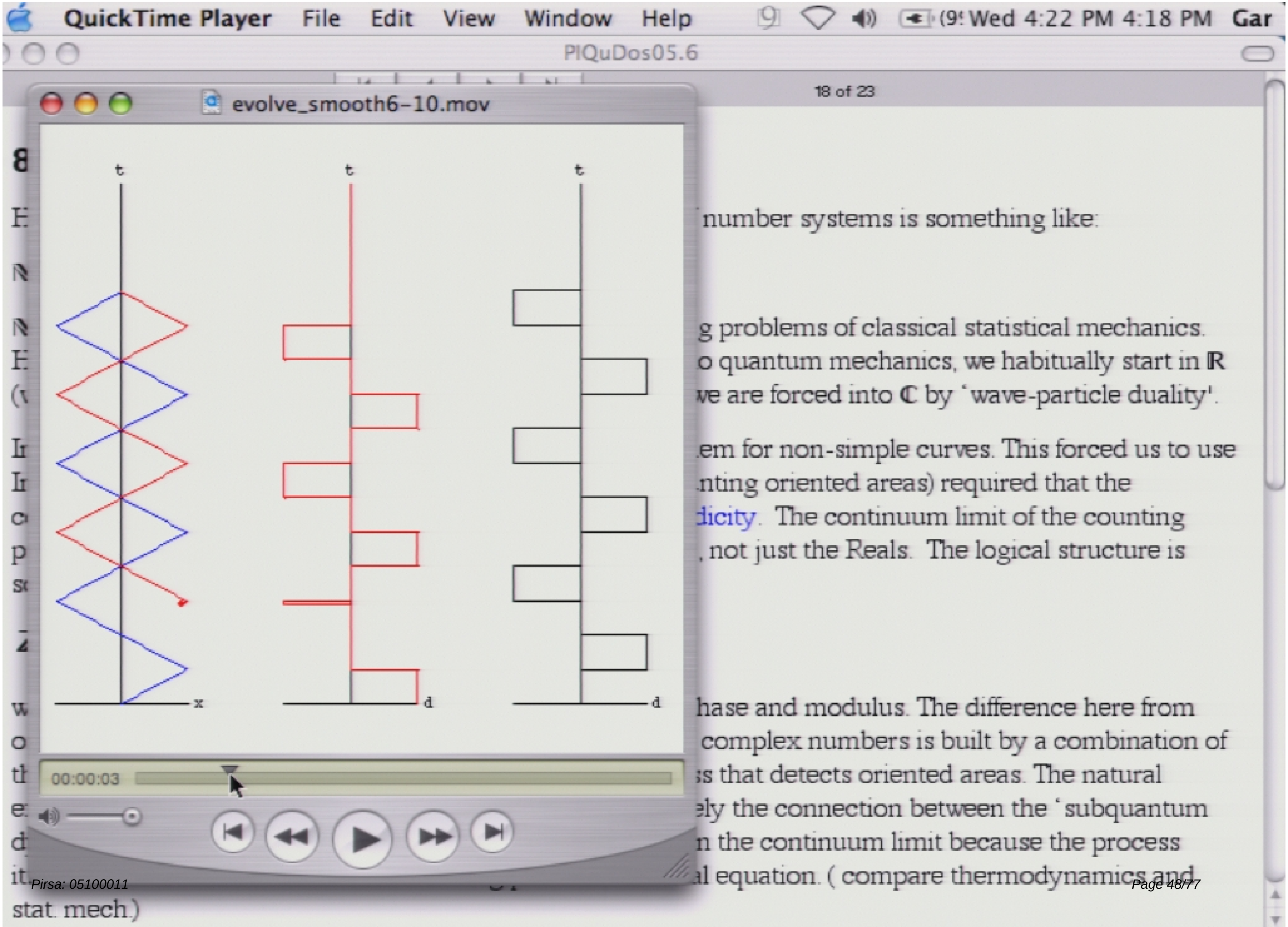
8

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in $\mathbb{R}$
we are forced into $\mathbb{C}$ by 'wave-particle duality'.

em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

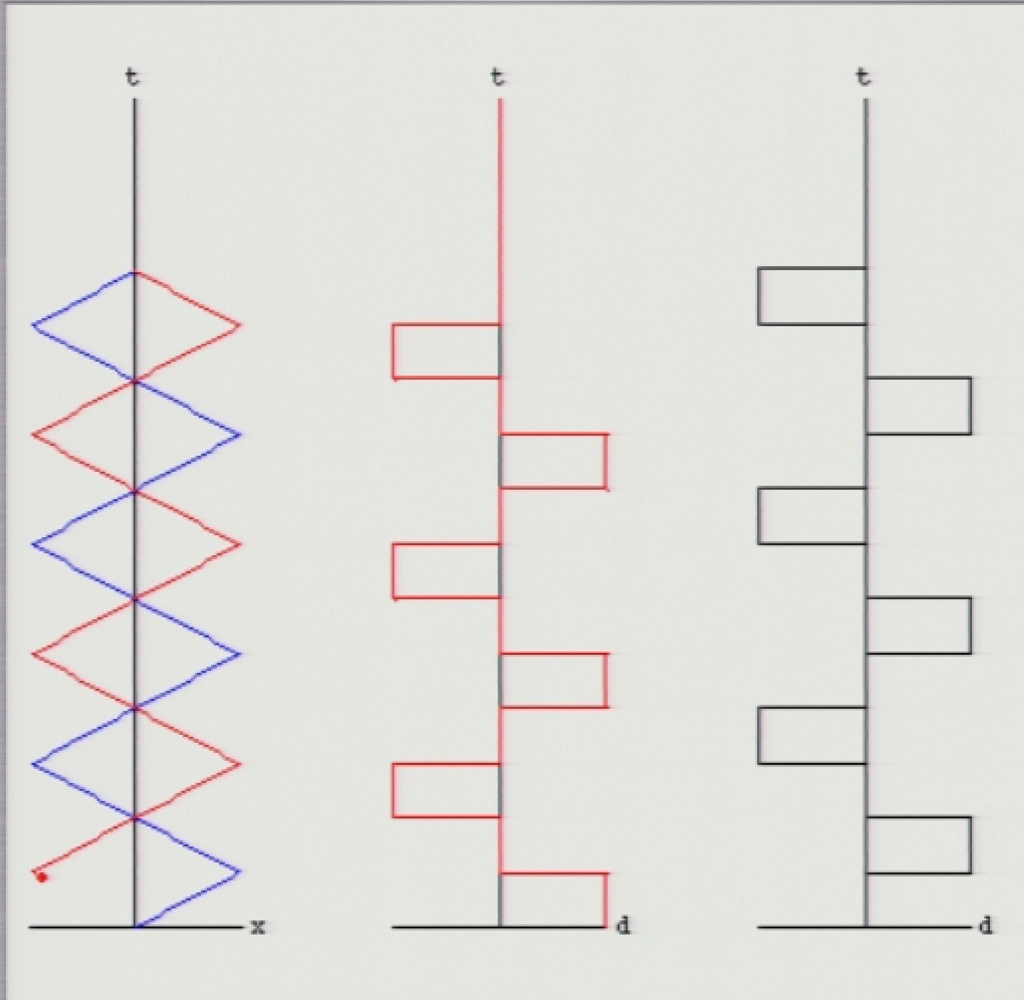
hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)


QuickTime Player File Edit View Window Help
Wed 4:22 PM 4:18 PM Gar
PIQuDos05.6
18 of 23
evolve_smooth6-10.mov
t
t
t
x
d
d
00:00:03
number systems is something like:
g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is
hase and modulus. The difference here from
complex numbers is built by a combination of
s that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:22 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:03

8

E number systems is something like:

N

N g problems of classical statistical mechanics.
E o quantum mechanics, we habitually start in ℝ
(v we are forced into ℂ by 'wave-particle duality'.

Ir em for non-simple curves. This forced us to use
Ir nting oriented areas) required that the
c dicity. The continuum limit of the counting
p , not just the Reals. The logical structure is
s

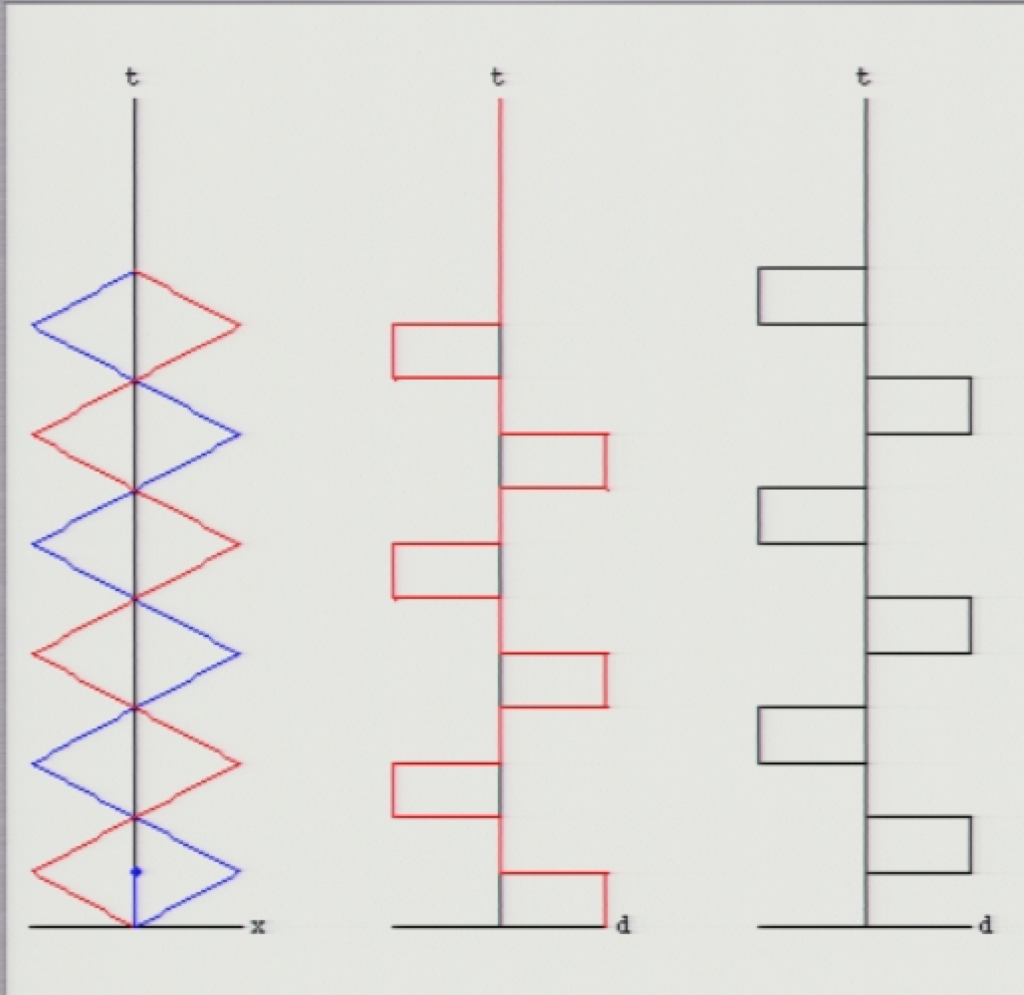
Z

w hase and modulus. The difference here from
o complex numbers is built by a combination of
tl s that detects oriented areas. The natural
e ely the connection between the 'subquantum
d n the continuum limit because the process
it al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help (9 Wed 4:22 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:03

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

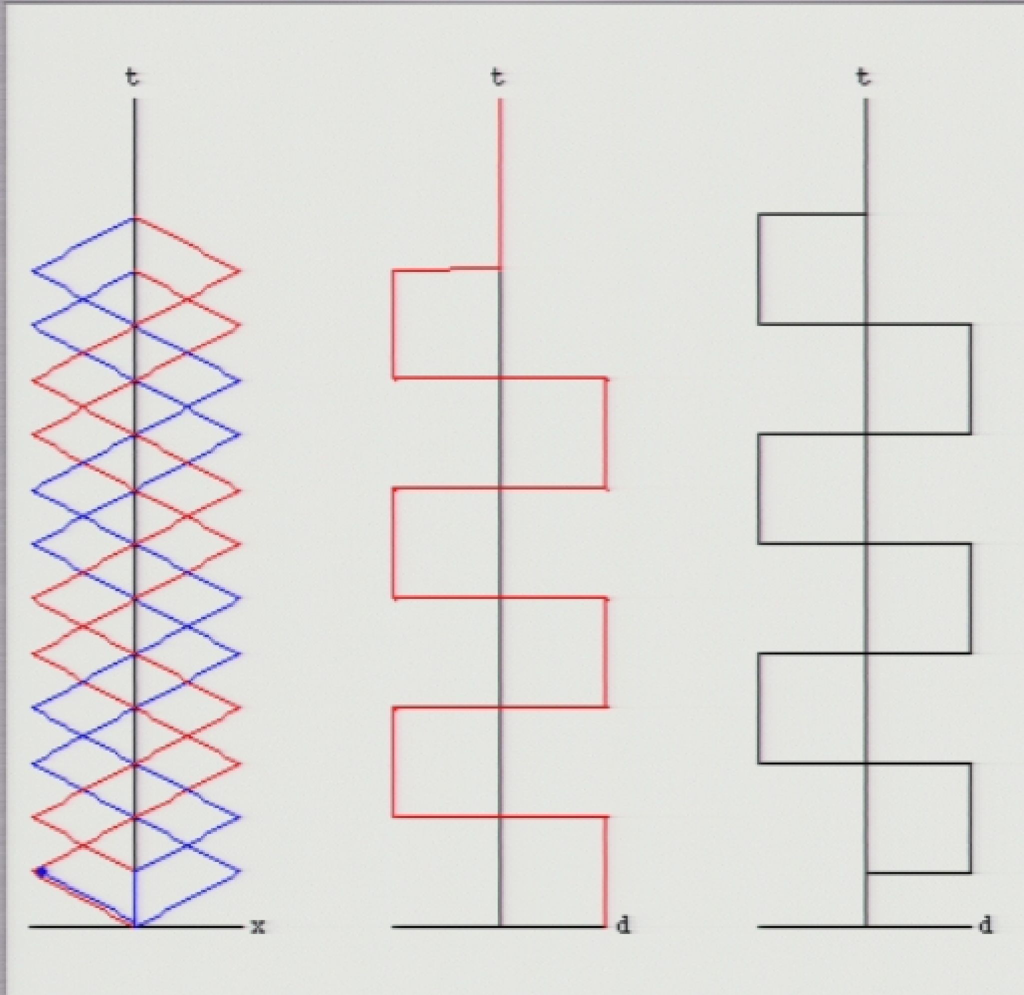
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:23 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:07

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in $\mathbb{R}$
we are forced into $\mathbb{C}$ by 'wave-particle duality'.

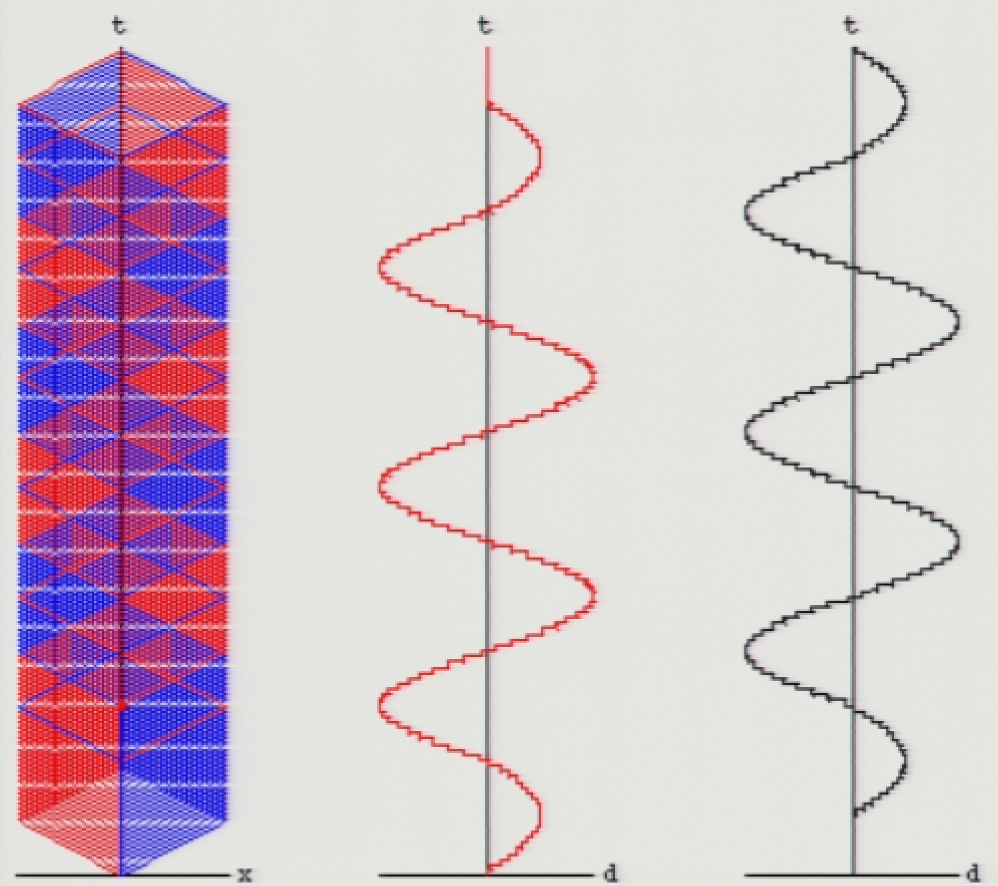
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
s that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:26 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:17

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by ‘wave-particle duality'.

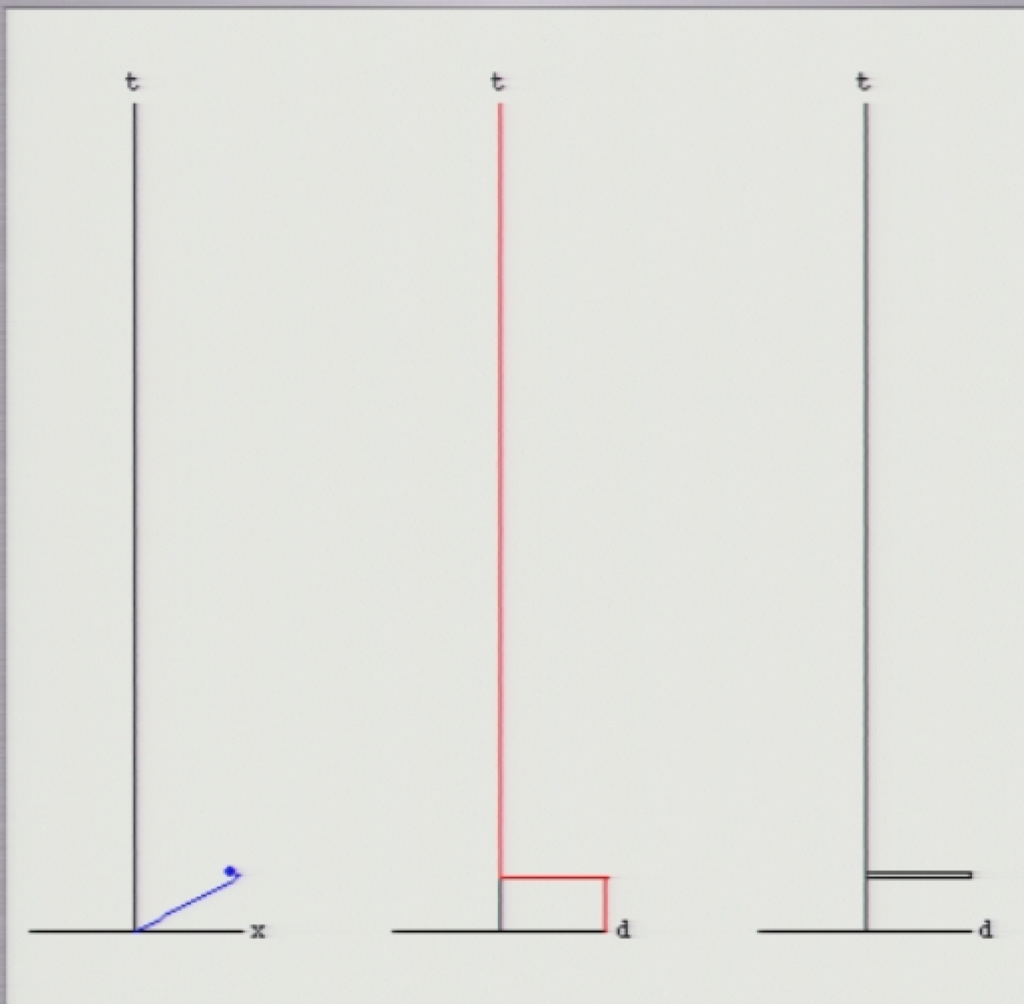
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the ‘subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:28 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:00

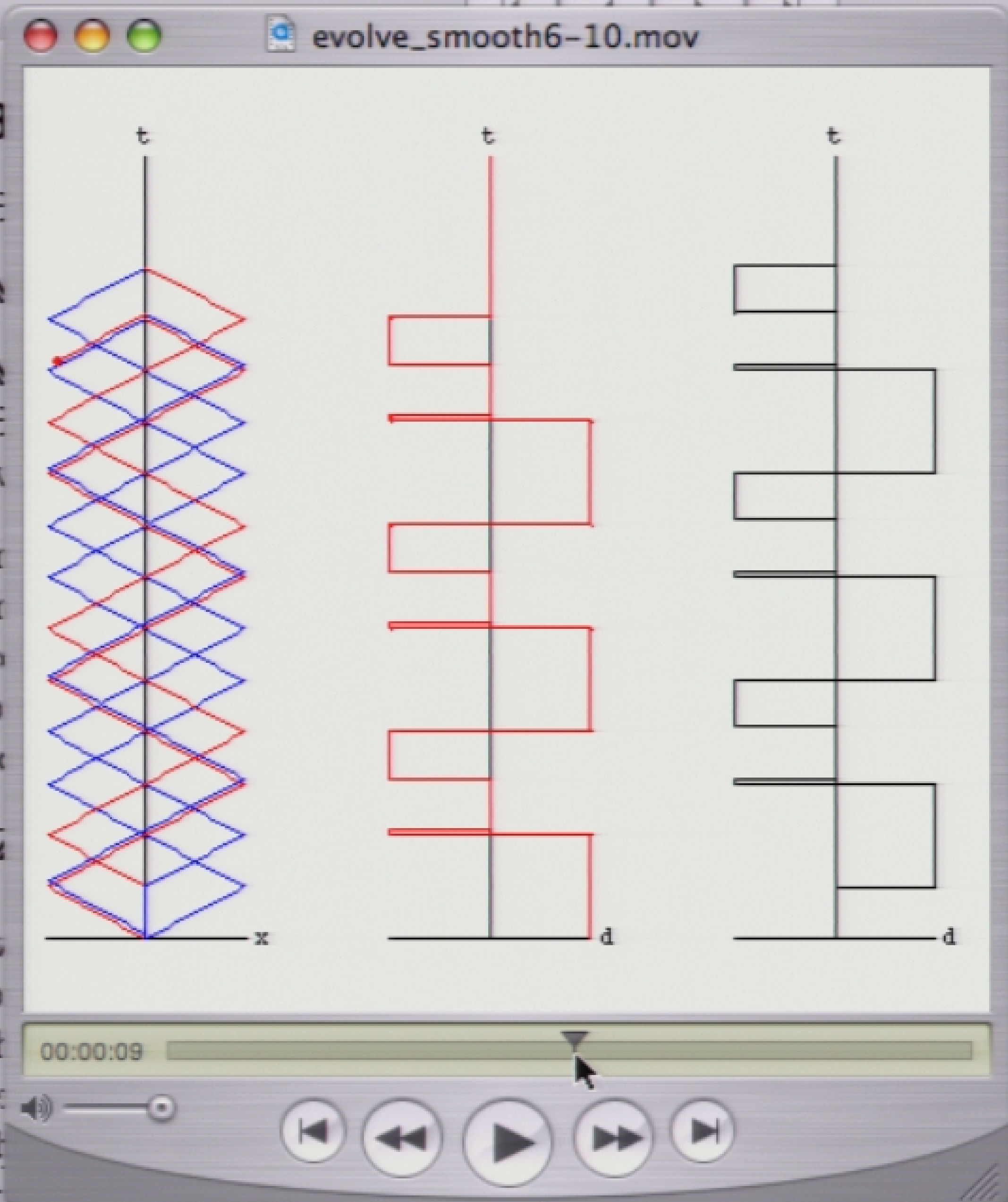
number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

8



number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

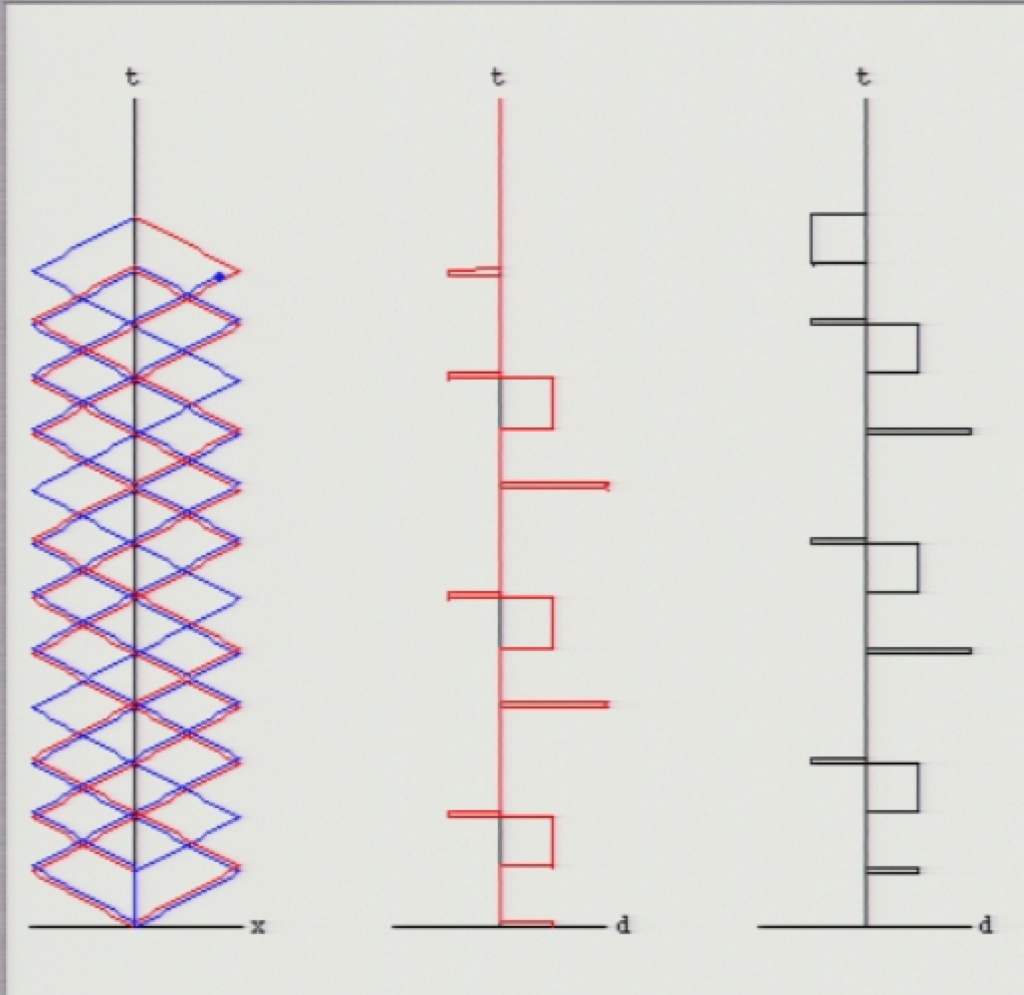
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
s that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:28 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:12

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

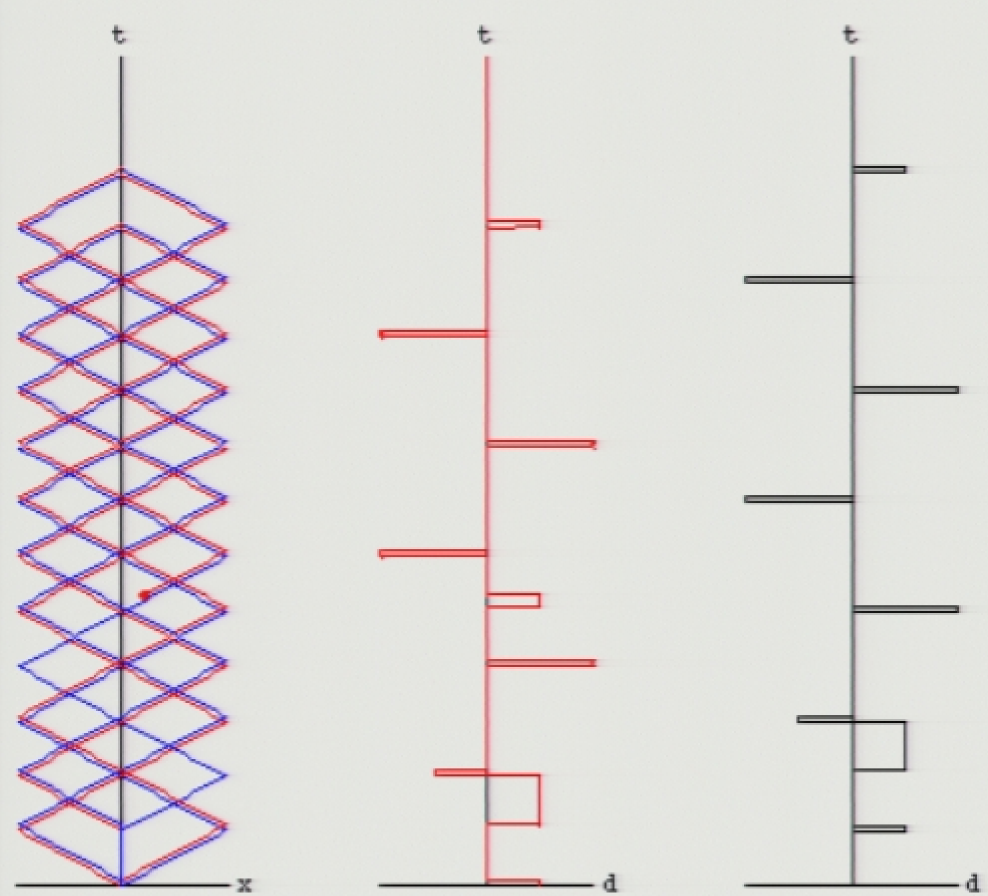
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:28 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:13

8

H number systems is something like:

ℕ

ℕ g problems of classical statistical mechanics.
H o quantum mechanics, we habitually start in ℝ
(v we are forced into ℂ by 'wave-particle duality'.

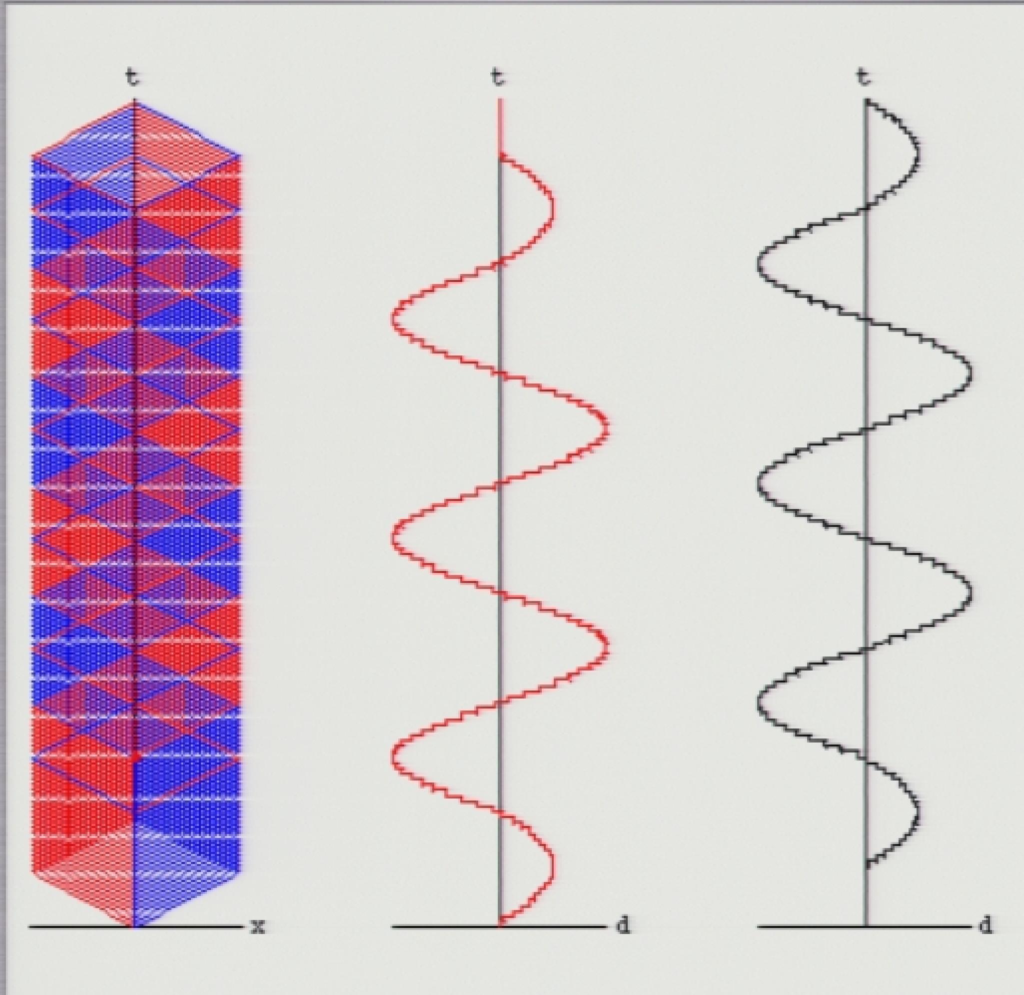
In em for non-simple curves. This forced us to use
In nting oriented areas) required that the
c dicity. The continuum limit of the counting
p , not just the Reals. The logical structure is
s

w hase and modulus. The difference here from
o complex numbers is built by a combination of
th s that detects oriented areas. The natural
e ely the connection between the 'subquantum
d n the continuum limit because the process
it al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:28 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:17

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

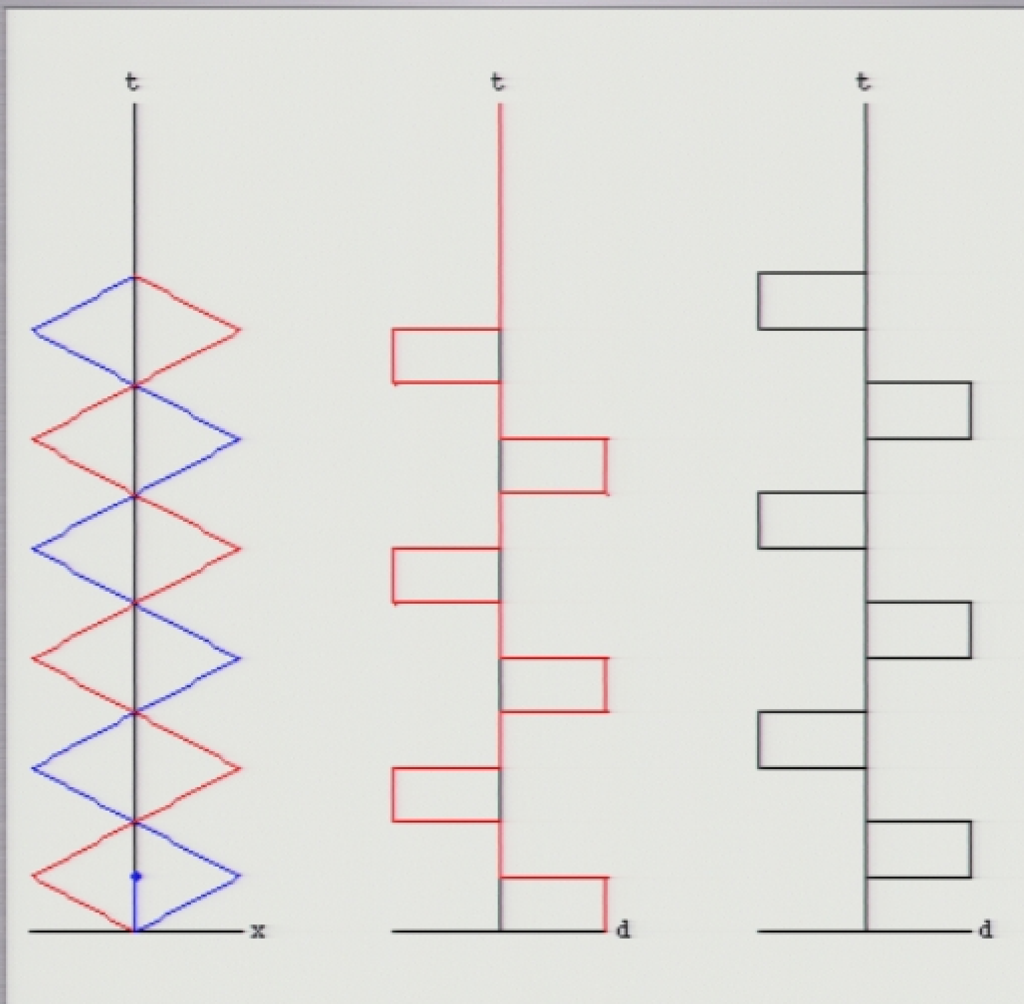
em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

QuickTime Player File Edit View Window Help Wed 4:29 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



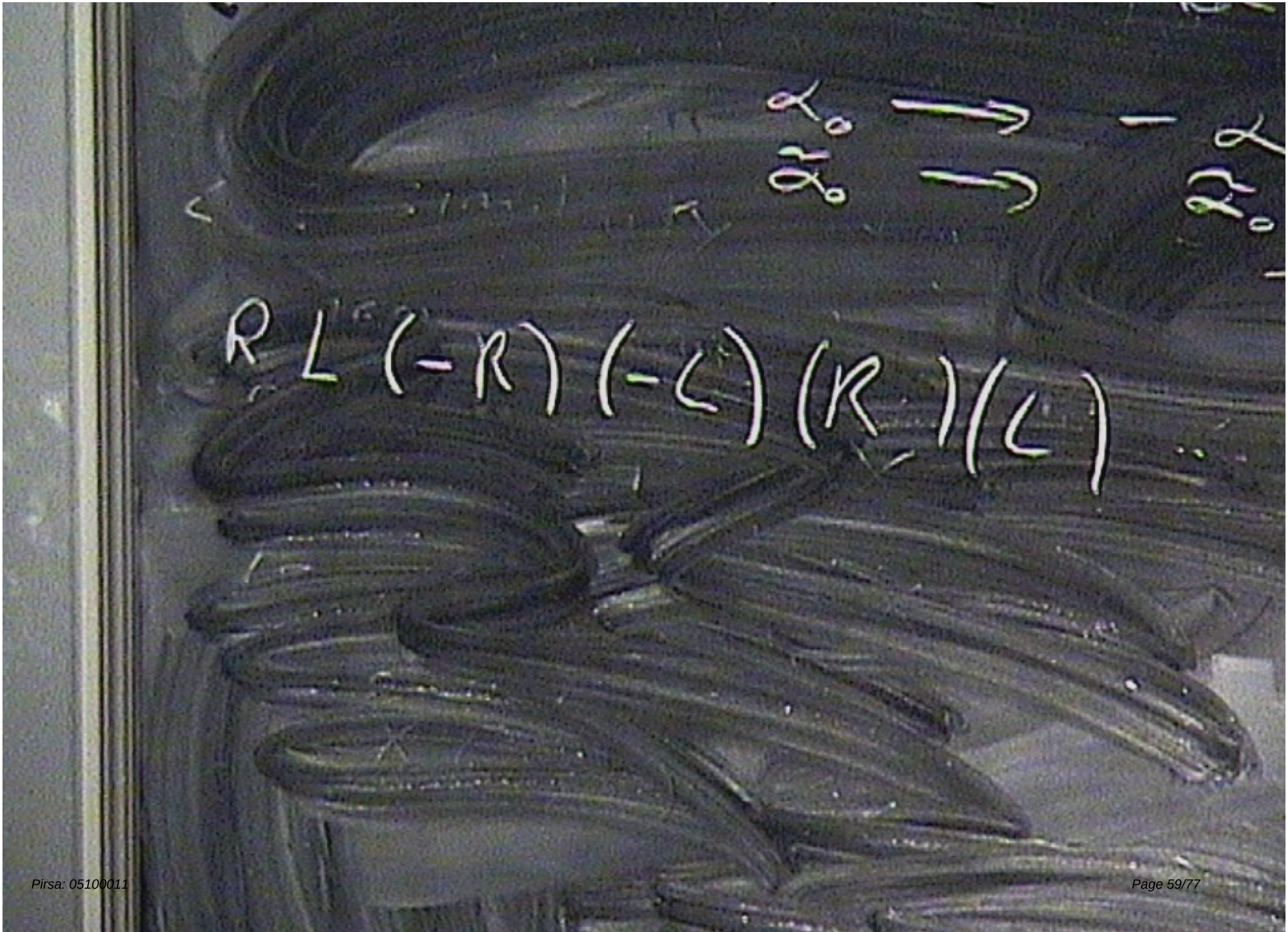
00:00:03

number systems is something like:

g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)

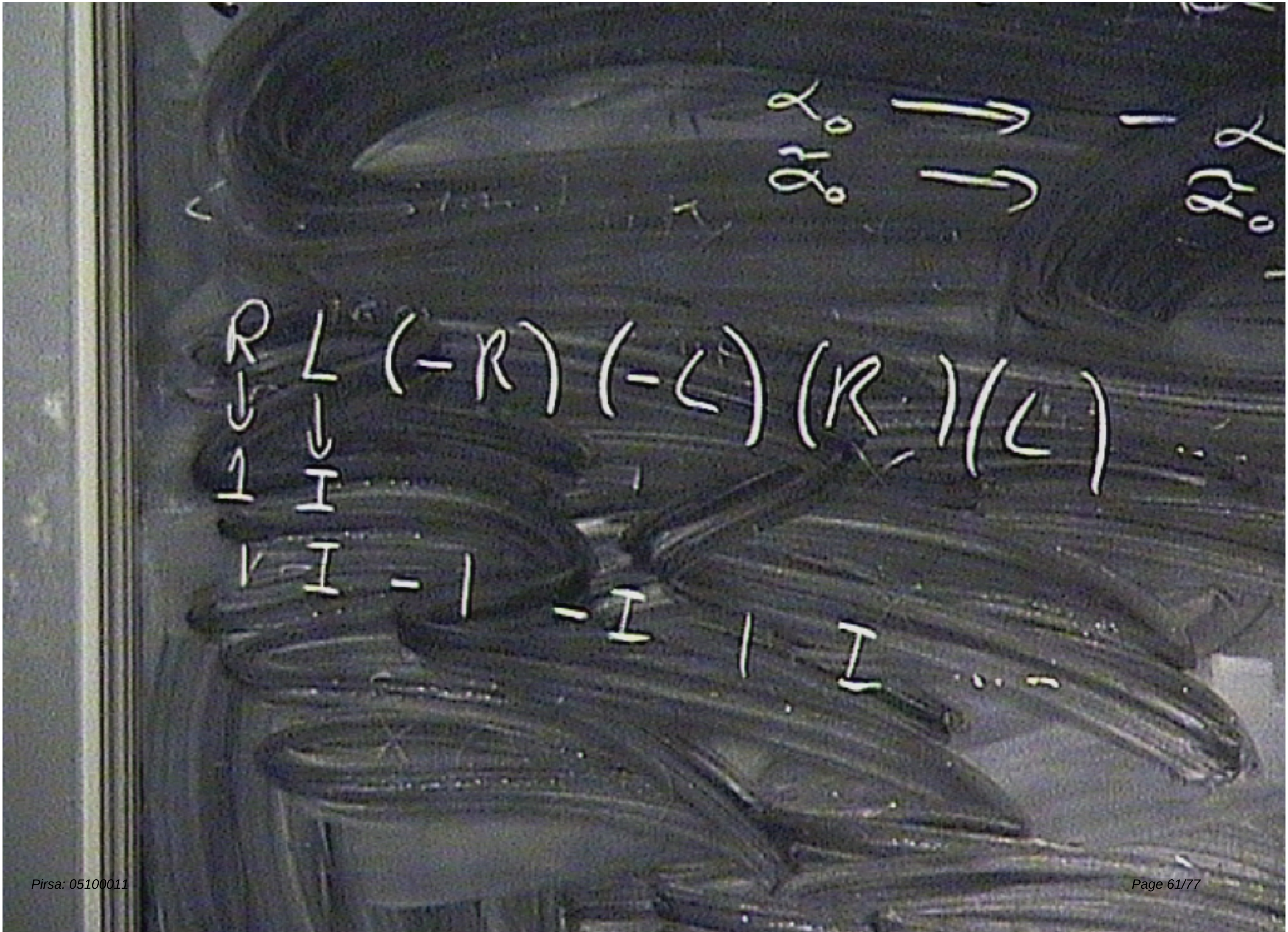

α₀ ⟼ − α
α̃₀ ⟼ α̃₀
RL(−R)(−L)(R)(L) ..

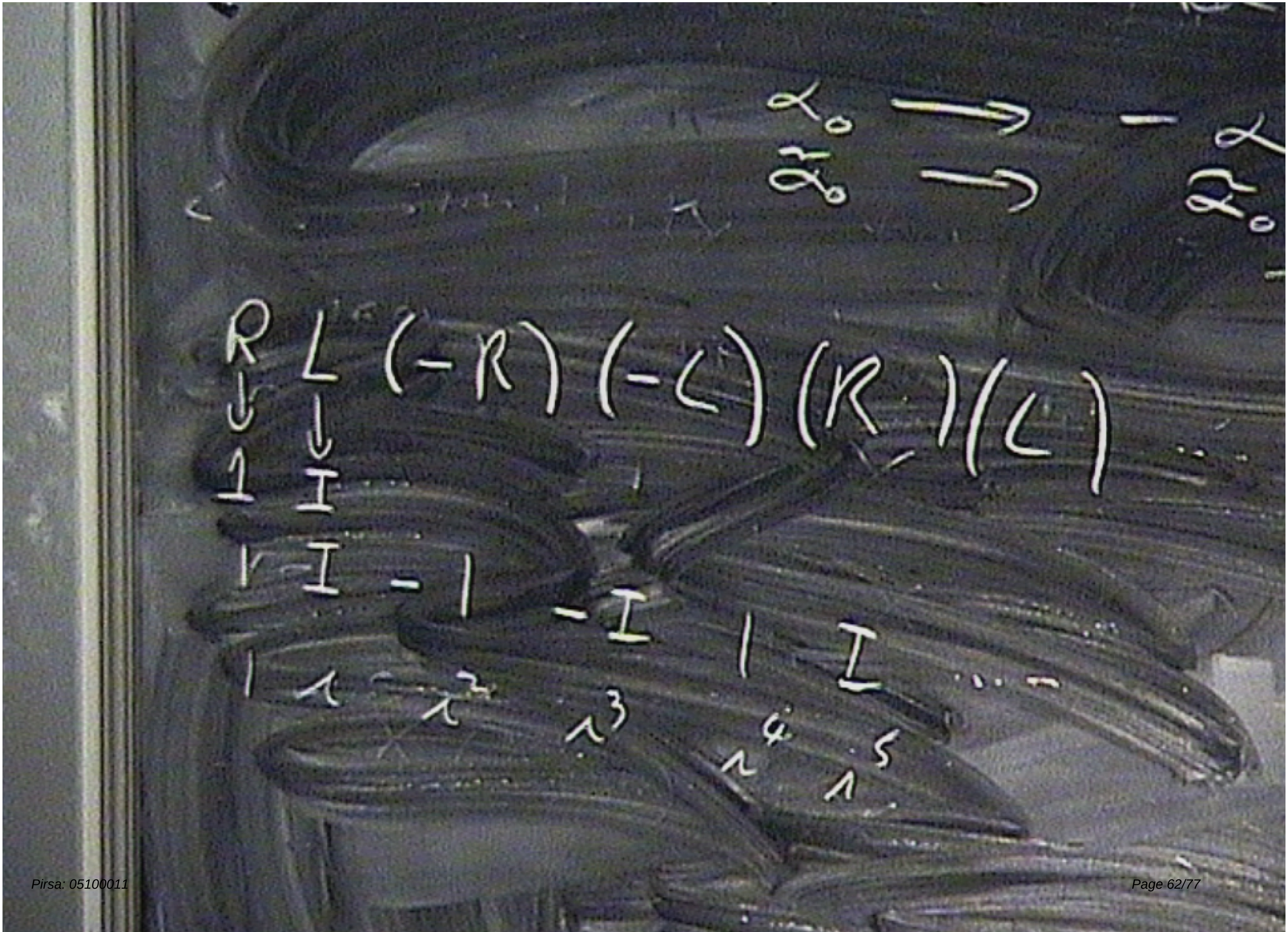
$\alpha_0 \longrightarrow -\alpha$

$\ell_0 \longrightarrow \ell_0$

$$R \; L \; (-R)(-L)(R)(L) \ldots$$

$R \downarrow 1$, $L \downarrow I$


α₀ ⟹ − α
α̃₀ ⟹ α̃₀
R L (−R) (−L) (R) (L) …
1 I
I −1 −I 1 I …



$\alpha_0 \longrightarrow -\alpha$
$\rho_0 \longrightarrow \rho_0$
R L (−R) (−L) (R) (L) ...
1 I
1 I −1 −I 1 I ...
1 λ λ^2 λ^3 λ^4 λ^5

$Q_0 \longrightarrow Q_0$

$$R \quad L \quad (-R) \quad (-L) \quad (R) \quad (L) \quad \ldots$$

$$\downarrow \quad \downarrow$$

$$1 \quad I$$

$$1 \quad I \quad -1 \quad -I \quad 1 \quad I \quad \ldots$$

$$1 \quad \lambda \quad \lambda^2 \quad \lambda^3 \quad \lambda^4 \quad \lambda^5$$

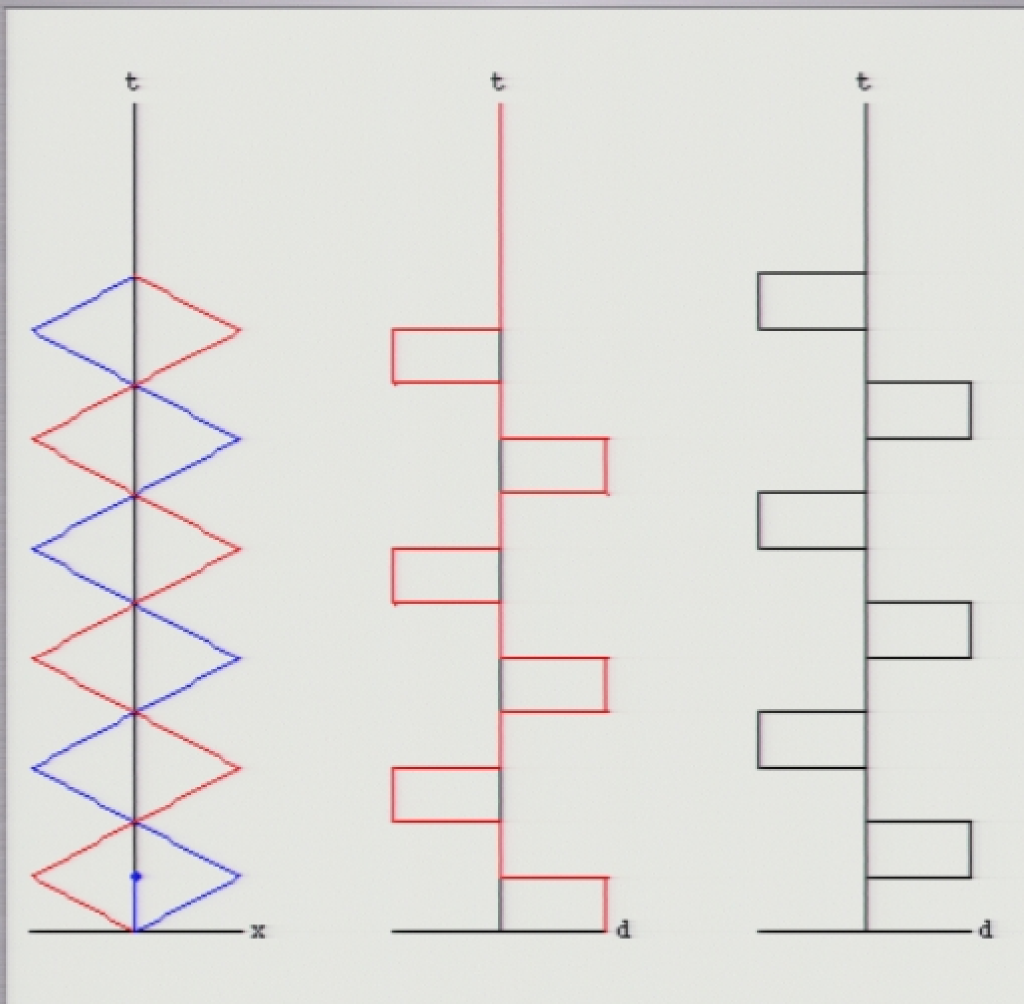
$$1 \quad \lambda \quad -1 \quad -\lambda \quad 1 \quad \lambda$$

CRW

QuickTime Player File Edit View Window Help Wed 4:30 PM 4:18 PM Gar

PIQuDos05.6

evolve_smooth6-10.mov



00:00:03

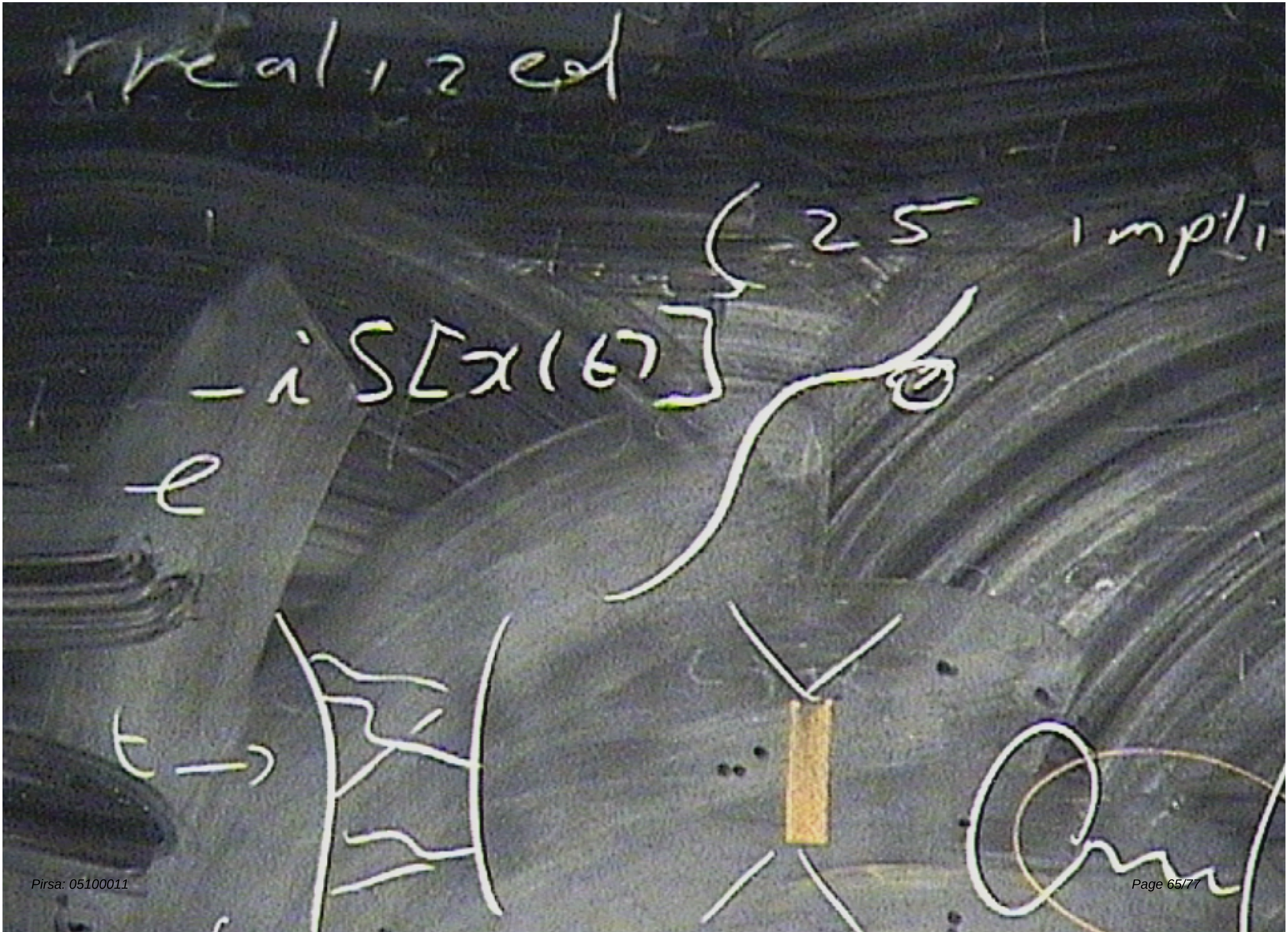
8

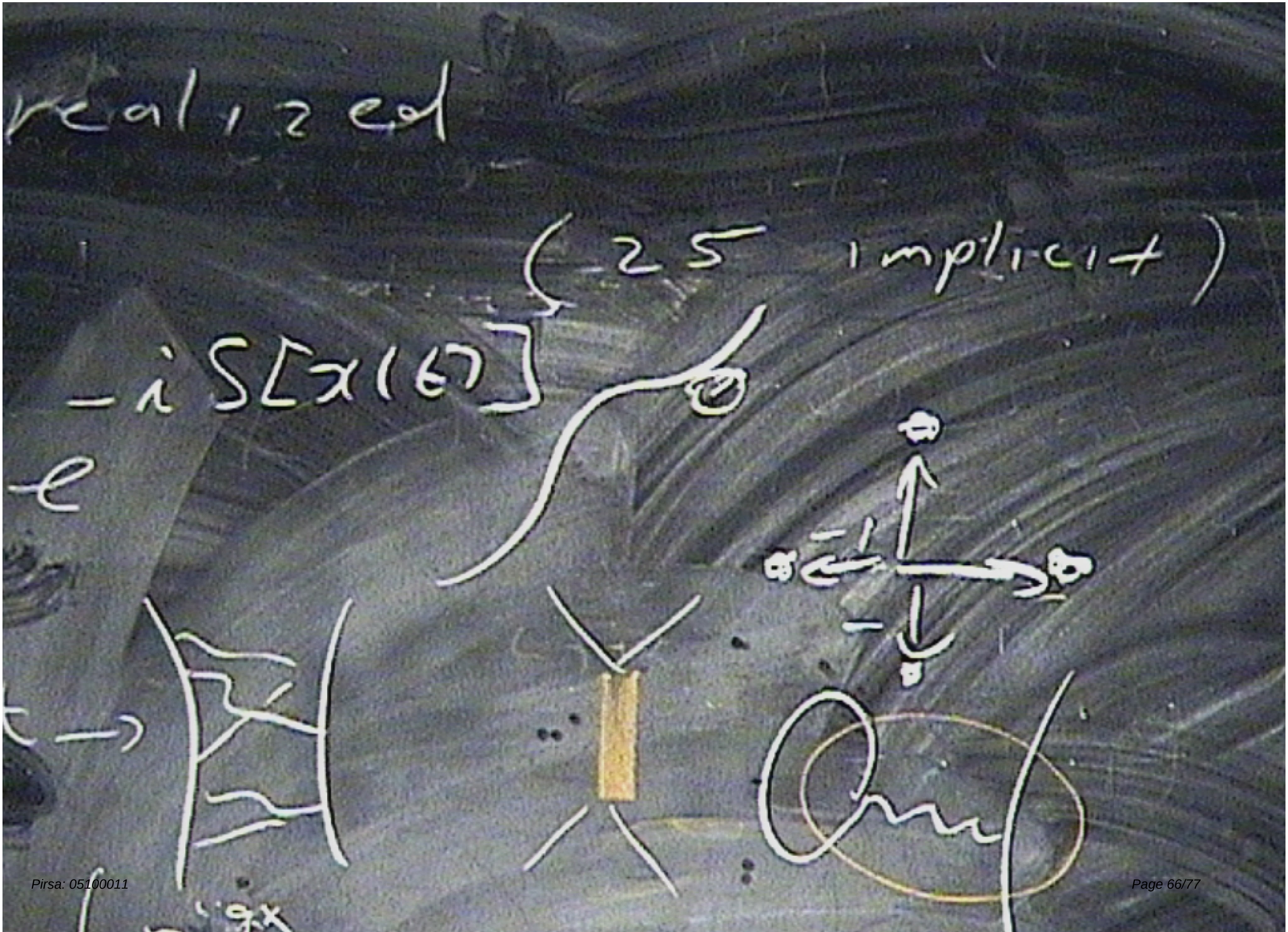
number systems is something like:

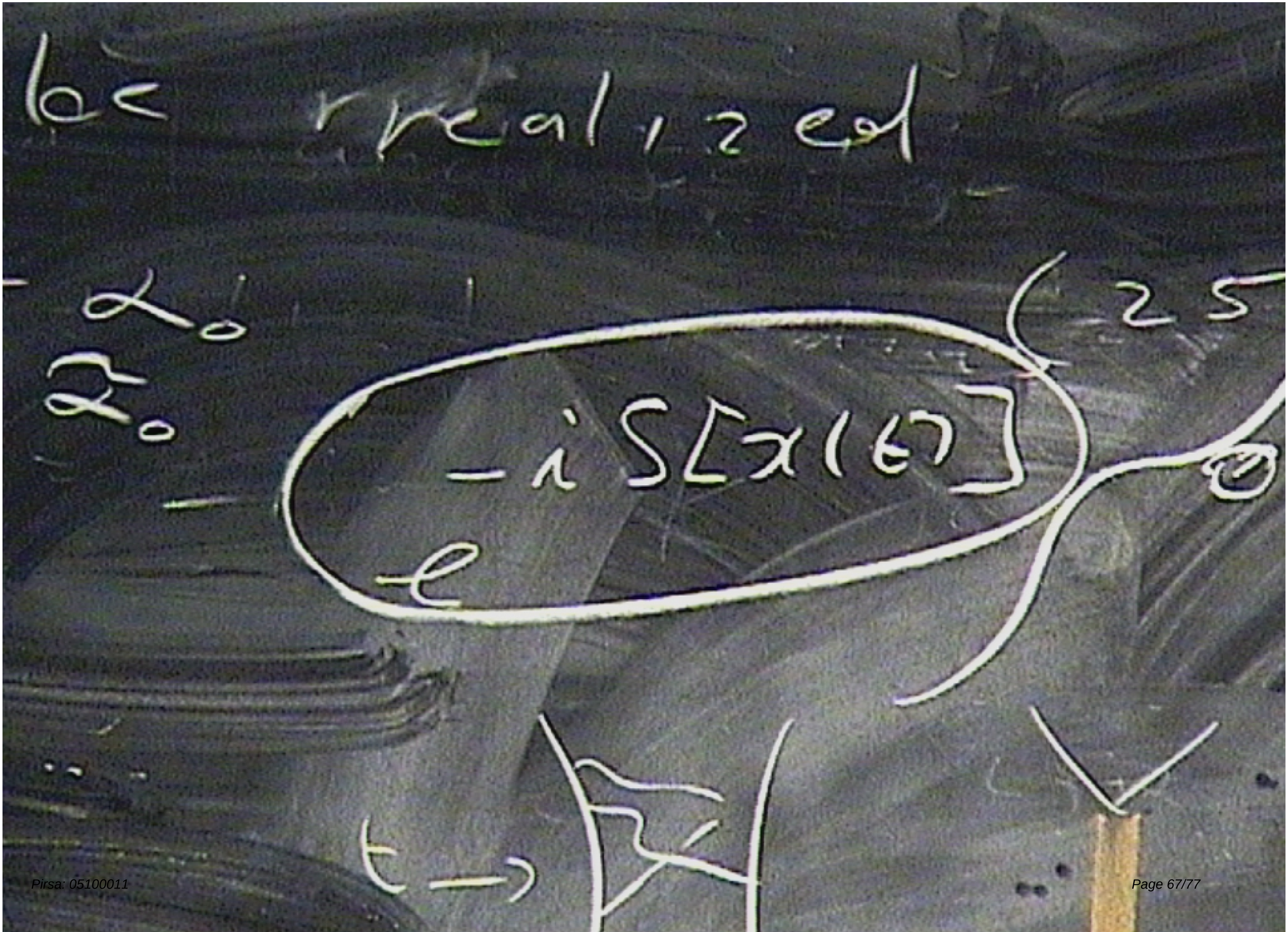
g problems of classical statistical mechanics.
o quantum mechanics, we habitually start in ℝ
we are forced into ℂ by 'wave-particle duality'.

em for non-simple curves. This forced us to use
nting oriented areas) required that the
dicity. The continuum limit of the counting
, not just the Reals. The logical structure is

hase and modulus. The difference here from
complex numbers is built by a combination of
ss that detects oriented areas. The natural
ely the connection between the 'subquantum
n the continuum limit because the process
al equation. (compare thermodynamics and
stat. mech.)


realized
25
impli
-iS[x(t)]
e
t→


realized
(2S implicit)
−iS[x(t)]
e


be realized
e
-iS[x(t)]

QuickTime Player File Edit View Window Help (9: Wed 4:32 PM 4:18 PM Gar

PIQuDos05.6

8. Constructing Complex Numbers

Historically and to a certain extent logically, the evolution of number systems is something like:

$\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$

$\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are appropriate number systems for the counting problems of classical statistical mechanics. However in making the transition from classical mechanics to quantum mechanics, we habitually start in $\mathbb{R}$ (we need a smooth continuum for classical mechanics) and we are forced into $\mathbb{C}$ by 'wave-particle duality'.

In the 'Accountant version' we started with a counting problem for non-simple curves. This forced us to use Integers for the primary number system. The geometry (counting oriented areas) required that the counting have a 2-component structure with a built-in periodicity. The continuum limit of the counting process then contained the algebra of the complex numbers, not just the Reals. The logical structure is something like:

$$\mathbb{Z}_2 \to \mathbb{Q}_2 \to \mathbb{C}\ (\to \mathbb{R})$$

where $\mathbb{Z}_2$ and $\mathbb{Q}_2$ are essentially subsets of $\mathbb{C}$ with discrete phase and modulus. The difference here from other derivations of the Dirac equation is that the Algebra of complex numbers is built by a combination of the local geometry of the trajectory and the counting process that detects oriented areas. The natural extension of the counting process is to $\mathbb{C}$, not $\mathbb{R}$. Unfortunately the connection between the 'subquantum dynamical process' and the resulting 'wavefunction' is lost in the continuum limit because the process itself is below the resolution of the resulting partial differential equation. (compare thermodynamics and stat. mech.)

9. Questions ...

Q. How robust is this? Does the Dirac equation depend on minute details in the geometry and/or the stochastic process?

A. Apparently not. The geometry has to be periodic, and there has to be a strict pairing of forward and return paths. The dynamical process can be stochastic, but it is vastly more efficient to have the dynamics deterministic and input the stochastic element at the initial conditions.

Q. The argument was for 1+1 dimensions. What about 3+1.

A. This seems to be a fairly straight-forward extension ... but it has not received a critical appraisal by anyone else yet.

Q.What is the analog of superposition in this context?

A. Concatenation. Any two space-time patterns A and B formed by entwined paths from the same origin form the pattern A+B when the trajectories are concatenated.

Q. It is known that the real difficulty with quantum mechanics is 'measurement'. The toy model only covers propagation. Why would this be an advantage given we already understand propagation in terms of waves.

Publicon File Edit Format Insert Cell View Window Help (9! Wed 4:34 PM 4:32 PM Gar

PIQuDos05.6

A. Apparently not. The geometry has to be periodic, and there has to be a strict pairing of forward and return paths. The dynamical process can be stochastic, but it is vastly more efficient to have the dynamics deterministic and input the stochastic element at the initial conditions.

Q. The argument was for 1+1 dimensions. What about 3+1.

A. This seems to be a fairly straight-forward extension ... but it has not received a critical appraisal by anyone else yet.

Q.What is the analog of superposition in this context?

A. Concatenation. Any two space-time patterns A and B formed by entwined paths from the same origin form the pattern A+B when the trajectories are concatenated.

Q. It is known that the real difficulty with quantum mechanics is 'measurement'. The toy model only covers propagation. Why would this be an advantage given we already understand propagation in terms of waves.

A. The problem with waves is that you are hard pressed to figure out why measurement favours particles. If, as in the toy model, the underlying dynamical process produces a particle-like trajectory, the hope is that the measurement problem will be lessened.

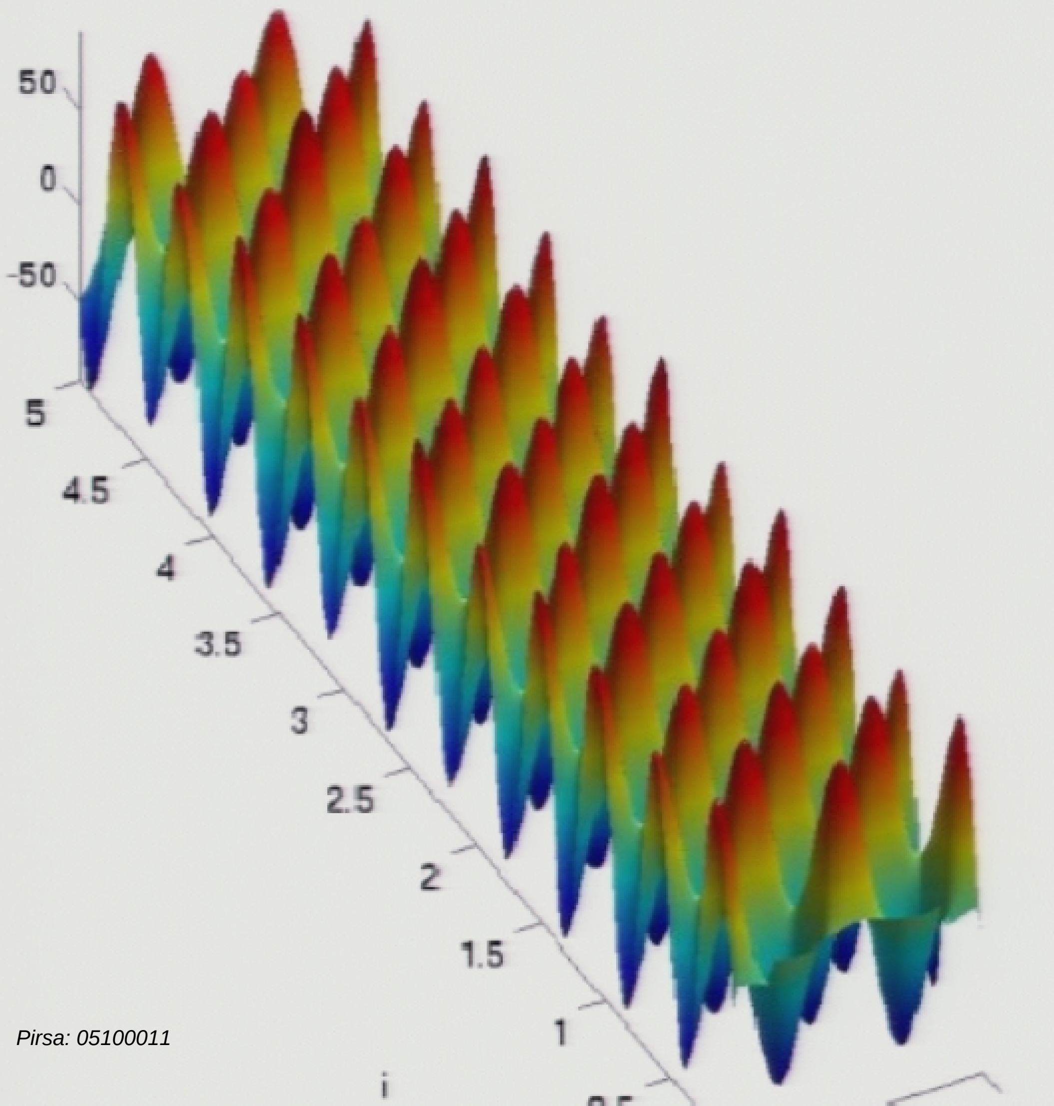
Q. What about relevance to the Bohm picture.

A. At this point they could be connected in a rather artificial way. A (deterministic) entwined path can 'write' the appropriate wavefunction on a region of spacetime given the correct initial condition. To mimic Bohm we could instruct the particle to so write the appropriate wavefunction and at then finally exit the spacetime region via the Bohm path determined by the previously written wavefunction.

Q. What about multi-particle quantum mechanics?

A. Still to be looked at.

10. Pictures



Publicon File Edit Format Insert Cell View Window Help (9! Wed 4:37 PM 4:32 PM Gar

PIQuDos05.6

Conclusions

- Developed a particle-only picture of propagation
- Waves appear as a derived concept
- No physics involved ... only path counting and geometry
- Derived Dirac equation in 1+1 dimension

  This assumed:
  - Path is non-simple stochastically produced entwined curve and counting is for oriented areas
  - Path is long enough to produce an equilibrium distribution
- Self-interference appears to be generic and robust (there will be many models that will produce 'waves')
- Deterministic versions that satisfy the uncertainty principle through initial conditions are vastly more efficient than completely stochastic versions
- There is much to explore!!!

References

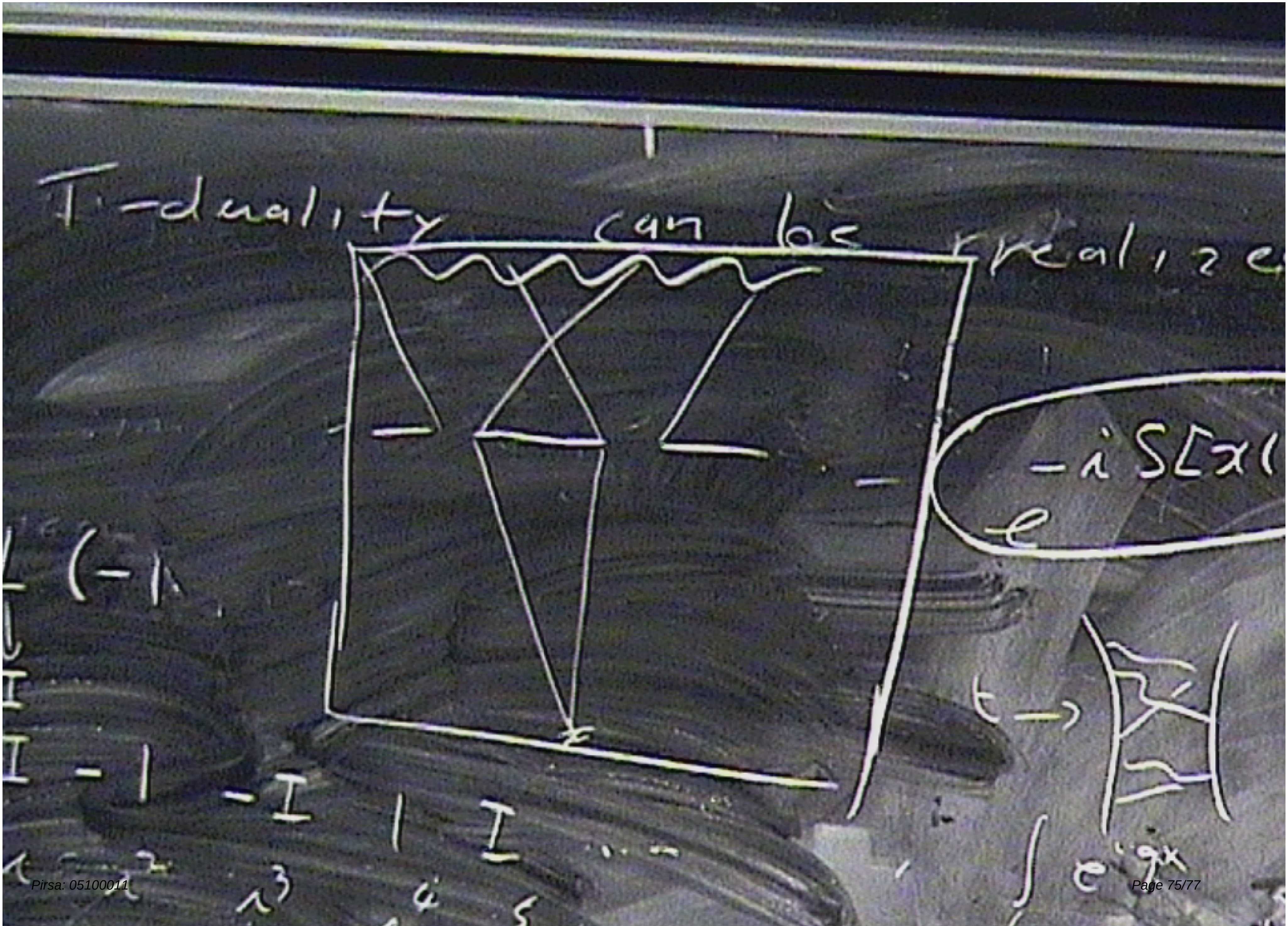
[1] R. Penrose (2004). *The road to Reality*. London: Jonathan Cape.

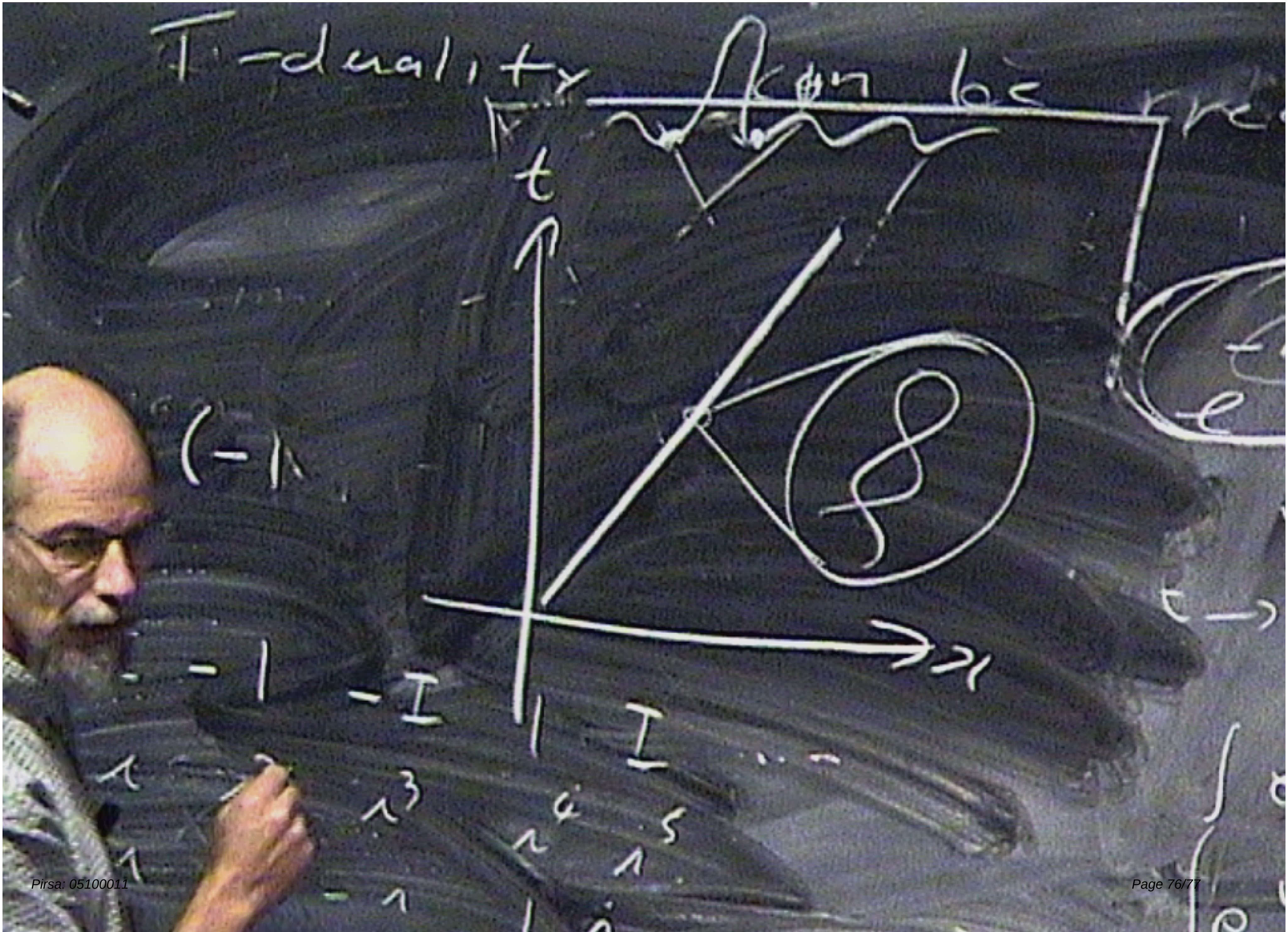
[2] P. Holland, (1993). *The quantum theory of motion*. : Cambridge University Press.

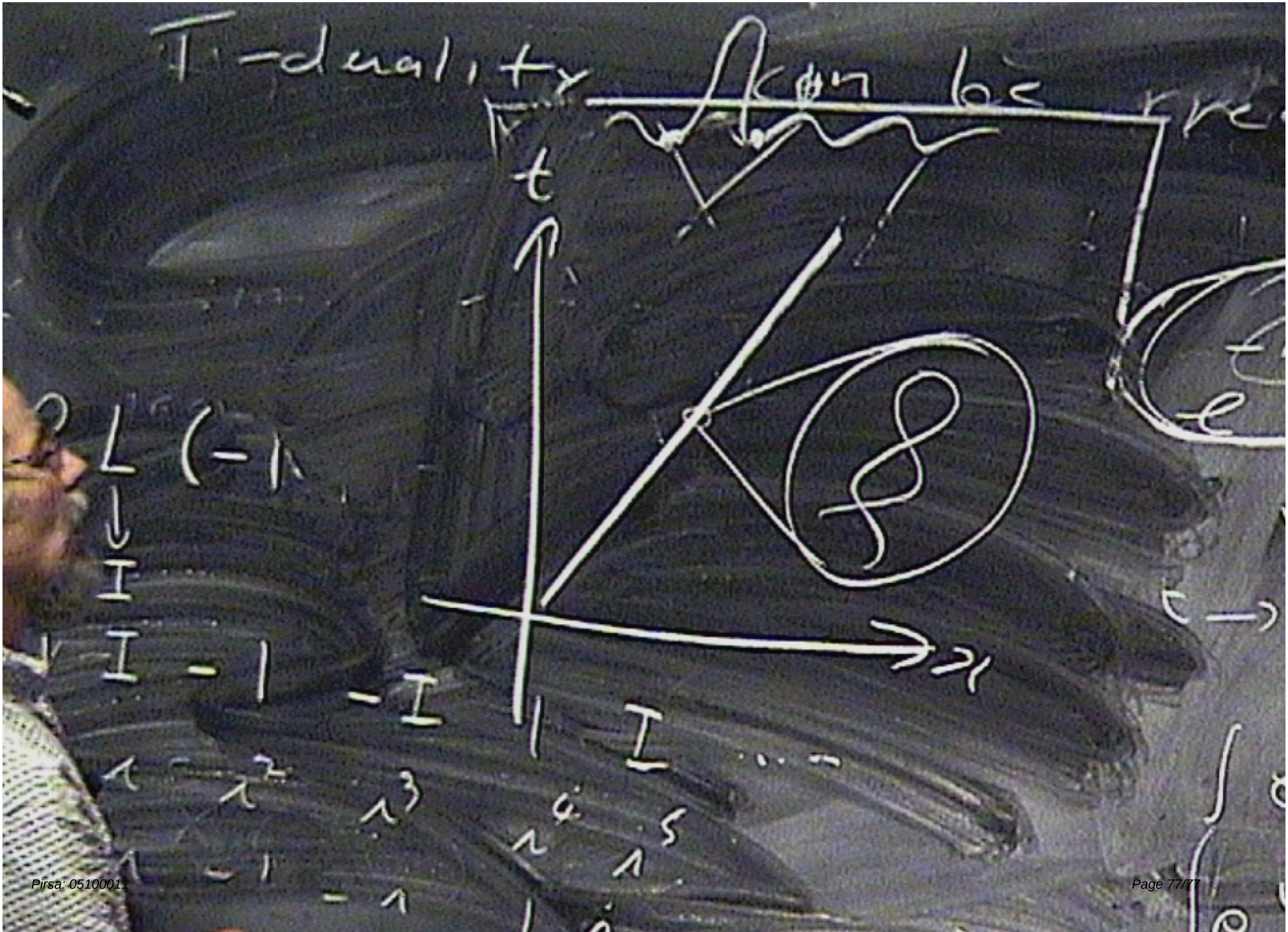
G.N. Ord and J. A. Gualtieri. The feynman propagator from a single path. Phys. Rev. Lett, 89, 2002.

G.N. Ord and R.B. Mann. Entwined pairs and Schrödinger's equation. Annals of Physics, 308(2):478–492, 2003.

G.N. Ord and R.B. Mann. Entwined paths, difference equations and the Dirac equation. Phys. Rev. A, 67, 2003.


T-duality can be realized
$-iS[x($
e


T-duality
t
x


T-duality