Title: Unitarity in the Presence of Closed Timelike Curves

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Abstract: TBA

UNITARITY IN THE PRESENCE OF CLOSED TIMELIKE CURVES

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Based on hep-th/0506104 with Miguel S. Costa

Perimeter Institute, 4th Oct. 2005

Introduction

- Can quantum field theory (or quantum gravity, or string theory) make sense in a spacetime with closed timelike curves?

- Free theory (single particle states) usually OK (possible quantization of energy eigenvalues)

- Unitarity breaks down when interactions are turned on perturbatively

- Will conjecture and partly show that, in some cases, unitarity is restored at discrete values of the coupling constant

A Spacetime with Closed Timelike Curves: an Orbifold of $\mathbb{M}^3$

- $\mathbb{M}^3$ with coordinates

$$ds^2 = -2dx^+dx^- + dx^2$$

- Killing direction

$$\kappa = i\left(J_{+x} + E^{-1}P_-\right)$$

parameterized by the energy scale E.

- Introduce new coordinates

$$y^+ = x^+ + Exx^- + \frac{E^2}{3}\left(x^-\right)^3$$
$$y^- = x^-$$
$$y = x + \frac{E}{2}\left(x^-\right)^2$$

- Metric and Killing direction become

$$ds^2 = -2dy^-dy^+ + 2Ey(dy^-)^2 + dy^2$$
$$\kappa = E^{-1}\frac{\partial}{\partial y^-}$$

- Consider the orbifold

$$\mathbb{M}^3/\Omega \qquad\qquad (\Omega = e^{\kappa})$$
$$y^- \sim y^- + E^{-1}$$

The Geometry of the Orbifold

- y^+ is light–cone time and $\frac{\partial}{\partial y^+} = \frac{\partial}{\partial x^+}$ is a null Killing vector

- Orbifold generator $\frac{\partial}{\partial y^-}$ null at $y = 0$ and timelike for $y < 0$

$$H \equiv \left(\frac{\partial}{\partial y^-}\right)^2 = 2Ey$$

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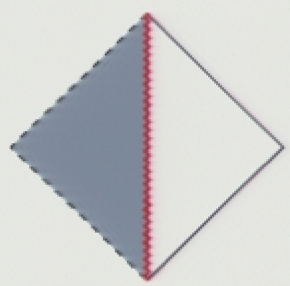
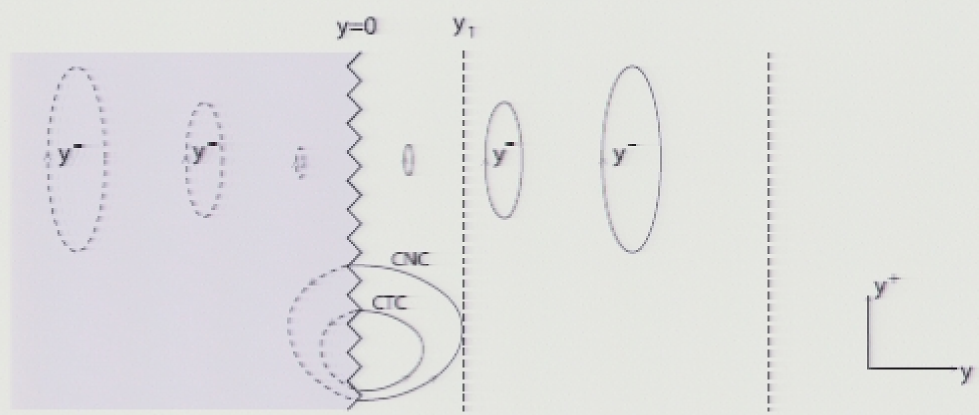
- Two dimensional reduction

$$ds^2 = ds_2^2 + H\left(dy^- - H^{-1}dy^+\right)^2$$
$$ds_2^2 = -H^{-1}\left(dy^+\right)^2 + dy^2$$

with $y > 0$ and Carter–Penrose diagram with chronological singularity at $y = 0$



- In $3D$ CTC's have to go into the $y < 0$ region



- Polarization surfaces

$$(\mathrm{x} - \Omega^w \mathrm{x})^2 = 0$$
$$y = \frac{1}{24E} w^2$$

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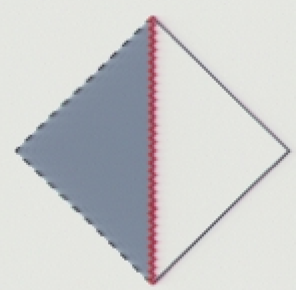

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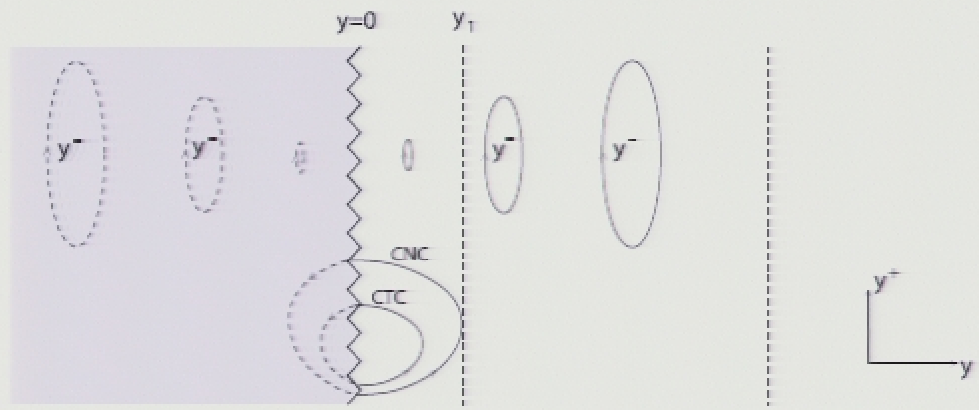


- Polarization surfaces

$$(\mathrm{x} - \Omega^w \mathrm{x})^2 = 0$$
$$y = \frac{1}{24E} w^2$$

- Horizon is $w \to \infty$ limit of polarization surfaces

BTZ Black Hole Inside the Horizon

- Extremal BTZ black hole

$$AdS_3 \ / \ \Omega$$

with

$$\ell M_{bh} = J \qquad\qquad (AdS \text{ radius } \ell)$$

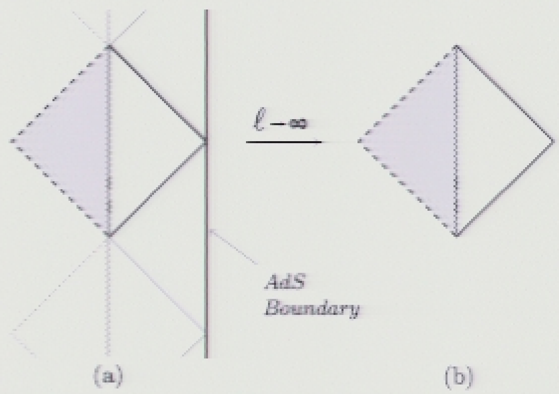
- Dual CFT state

$$L_0 + \tilde{L}_0 \ = \ \ell M_{bh} + \frac{\pi}{4}\ell M$$

$$L_0 - \tilde{L}_0 \ = \ J \qquad\qquad \left(M = \frac{1}{2\pi G} \text{ Planck mass}\right)$$

In a unitary CFT with fermions we expect half–integral spins

$$2J \in \mathbb{Z}$$



- Flat space limit

$$\ell \to \infty \qquad\qquad J \text{ fixed}$$

Obtain previous orbifold of $\mathbb{M}^3$ with

$$\frac{M}{4\pi E} = 2J \in \mathbb{Z}$$

- With fixed geometry E we expect a unitary evolution at discrete values of the gravitational coupling M, in the presence of CTC's

Quantization of M from $O8$ charge quantization

- Type IIA on

$$\mathbb{M}^3/\Omega \times T^7$$

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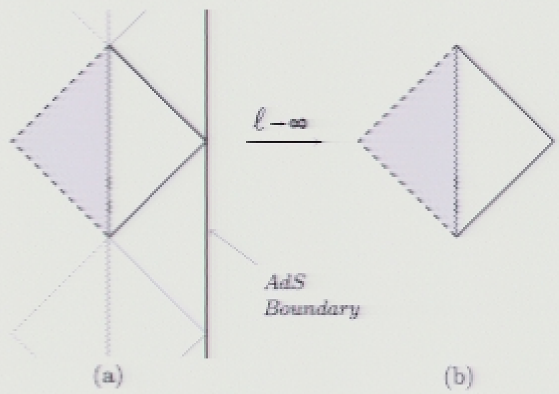

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Quantization of M from $O8$ charge quantization

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$$\mathbb{M}^3/\Omega \times T^7$$

- Uplift to M−theory on

$$\mathbb{M}^3/\Omega \times T^7 \times S^1 \qquad \begin{pmatrix} R_{11} = l_s g_s \\ l_P^3 = g_s l_s^3 \end{pmatrix}$$

Compactify on Ω and T−dualize on $T^7 \times S^1$ to obtain $O8$−plane geometry in Type IIA with

$$\hat{l}_s^2 = E l_P^3 \qquad \hat{g}_s^2 = (2\pi)^{14} \frac{E^5 l_P^{21}}{R_{11}^2 V_7^2}$$

and tension

$$\tau = -\frac{4E}{2\hat{\kappa}^2} \in \tau_{D8}\mathbb{Z}$$

- In terms of the original orbifold $O8$–charge quantization implies as before

$$\frac{M}{4\pi E} \in \mathbb{Z}$$

Scattering Theory

- External Ω–invariant scalar states labelled by $\lambda = m^2$ and by the conserved momenta

$$p_+ \quad \rightarrow \quad \frac{\partial}{\partial y^+}$$

$$p_- \in 2\pi E\mathbb{Z} \quad \rightarrow \quad \frac{\partial}{\partial y^-} \qquad \text{(KK momentum)}$$

- Wavefunctions are sums over action of Ω on plane waves $\phi_{\mathbf{k}}(\mathbf{x})$

$$\int ds \;\; e^{i\,s\frac{p_-}{E}} \;\; \phi_{\mathbf{k}}(\Omega^s \mathbf{x})$$

- Convenient basis

$$V_{\lambda,p_+,p_-}(\mathbf{x}) \;=\; \frac{1}{|p_+|}\int dk\, e^{i(p_+x^+ + kx + k_-x^-)} \times$$
$$\times \exp \frac{i}{2Ep_+^2}\left[\left(2p_+p_- - \lambda\right)k - \frac{k^3}{3}\right]$$

$$\left(k_- = \frac{\lambda + k^2}{2p_+}\right)$$

Decay exponentially in $y < 0$ region

- Propagator: method of images (Δ is the Feynman propagator)

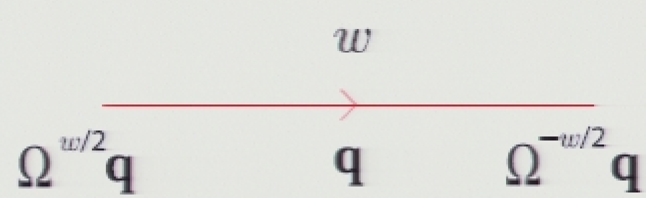
$$\left\langle \Phi\left(\mathrm{x}\right) \Phi\left(\mathrm{x}'\right)\right\rangle = \sum_{w} \Delta\left(\Omega^{w}\mathrm{x}, \mathrm{x}'\right)$$

Summand given by integral

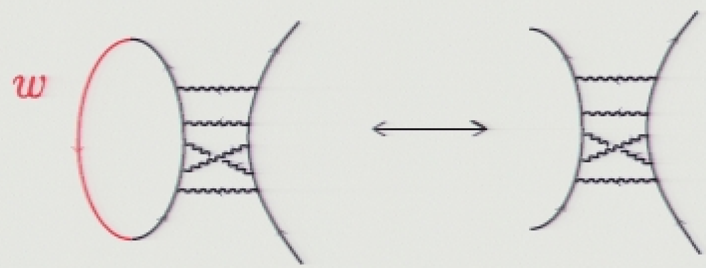
$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \;\; \phi_{\Omega^{-w/2}\mathbf{q}}\left(\mathrm{x}\right) \;\; \phi^{\star}_{\Omega^{w/2}\mathbf{q}}\left(\mathrm{x}'\right)$$

of

$$\frac{-i}{\mathrm{q}^2 + m^2 - i\epsilon} e^{\frac{i}{E}\left(wq_{-} + \frac{1}{24}w^3 q_{+}\right)}$$



- Feynman rules for graphs in the "winding representation" (consider in particular a massless scalar coupled to a massless spin j particle)



  - Assign $3D$ momenta $\mathbf{k}_i$ to external states
  - Assign winding numbers w to propagators
  - Compute graph $\Gamma_w(\mathbf{k}_i)$ using usual vertices and modified propagator

– Average over momenta $\mathbf{k}_i$ to get to V_{λ,p_+,p_-} basis for external states

$$\delta_{\Sigma_i p_{-i}} \int dk_i \;\; \delta\left(c_i k_i\right) \;\; e^{i\chi(k_i)} \;\; \Gamma_w\left(\mathbf{k}_i\right)$$

with $k_{i+} = p_{i+}$, $\mathbf{k}_i^2 = -\lambda$ and

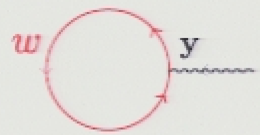
$$\chi\left(k_i\right) = \Sigma_i \frac{1}{2Ep_{+i}^2}\left[\left(2p_{+i}p_{-i} - \lambda_i\right)k_i - \frac{k_i^3}{3}\right]$$

– Sum over "inequivalent" winding choices

Note : equivalent winding choices yield different expressions in the intermediate steps by the same final result.

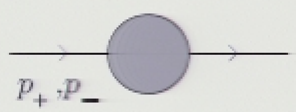
- Cutting rules break down due to winding propagators ($w \neq 0$) going on–shell ! Breakdown of perturbative unitarity.

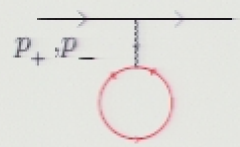
- Tadpole



diverges on polarization surfaces at $y_w = w^2/24E$ (for $j =$ 0 it is proportional to $\Delta\left(\Omega^w \mathbf{y}, \mathbf{y}\right) \propto (y - y_w + i\epsilon)^{-1/2}$) and $\mathrm{Im}\,(\cdots) \neq 0$ for $y < y_w$

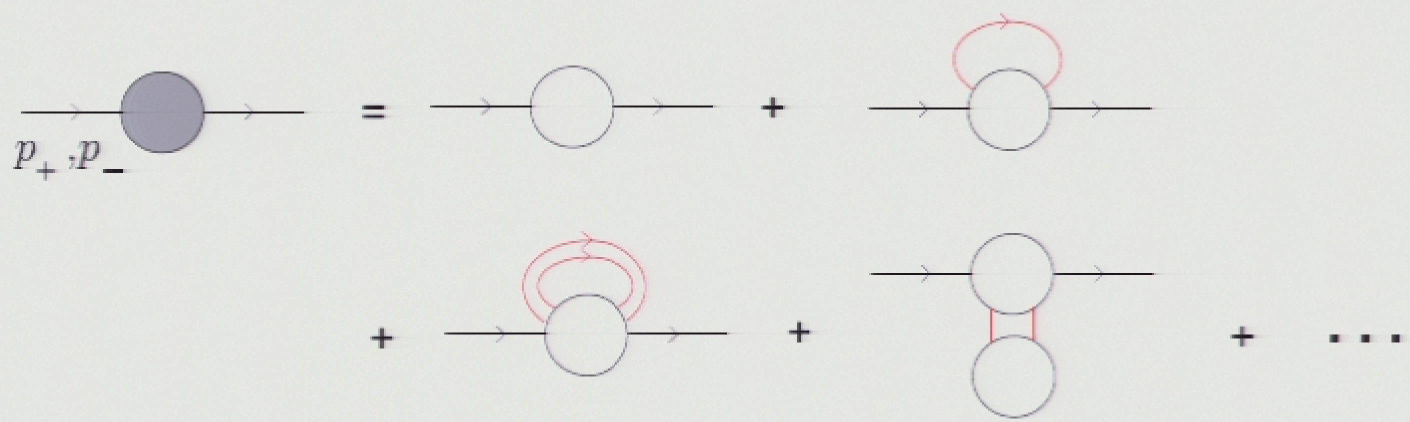
- Consider 2pt function. Leading coupling to tadpole

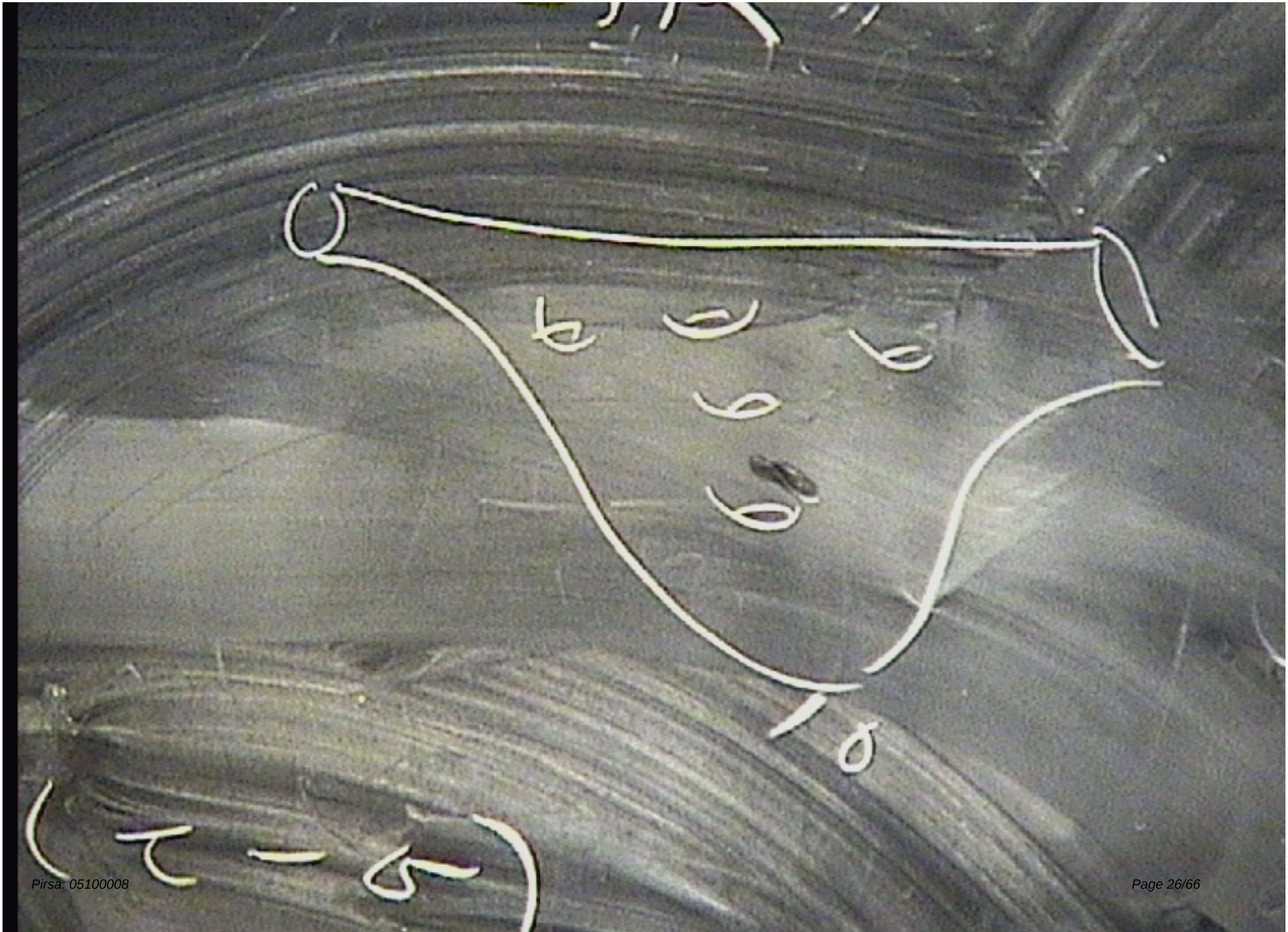




gives a non–vanishing $\mathrm{Im}\,(\cdots)$ part with no decay channel

- Need to partially resum perturbation series
  - leading order in winding
  - all order in parent amplitudes

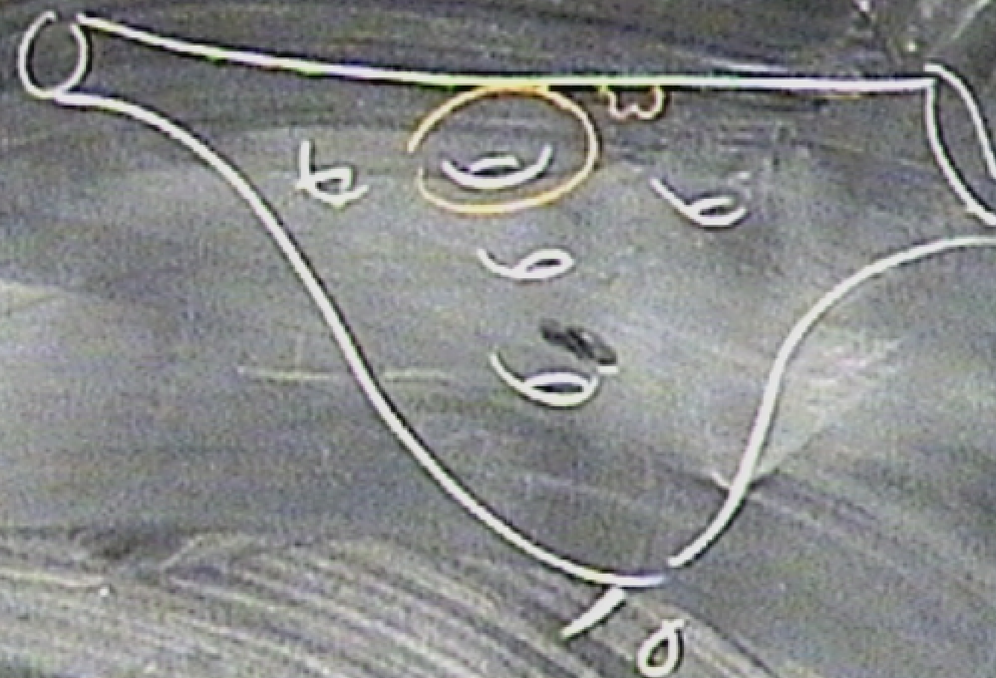




act on $X_{L,R}$:

X_R

X_L



$- X_R(\tau - \sigma)$

$\tau + 2\alpha' K_\mu p^\mu \sigma + \cdots$

gives a non–vanishing Im$(\cdots)$ part with no decay channel

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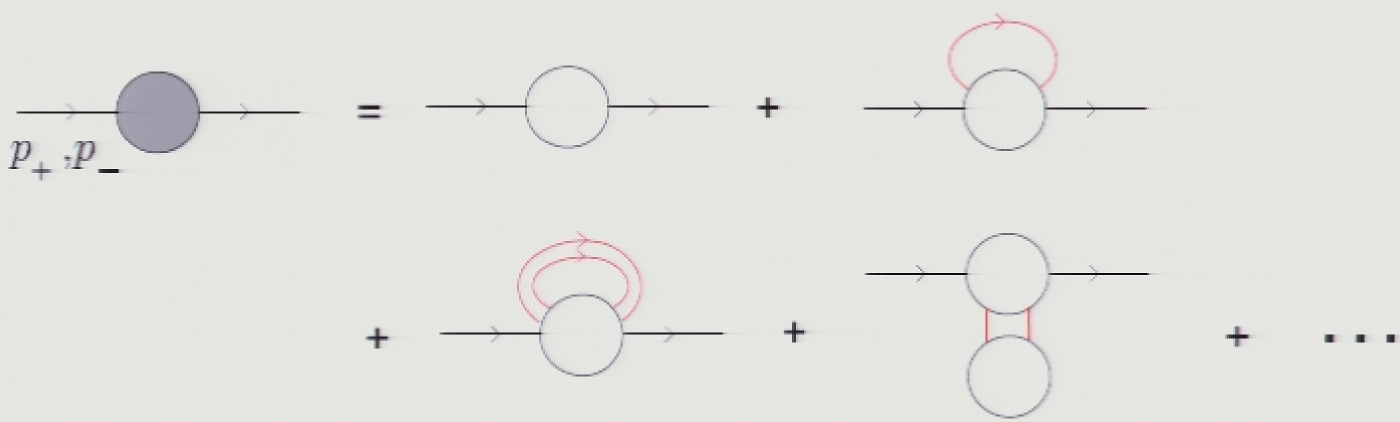
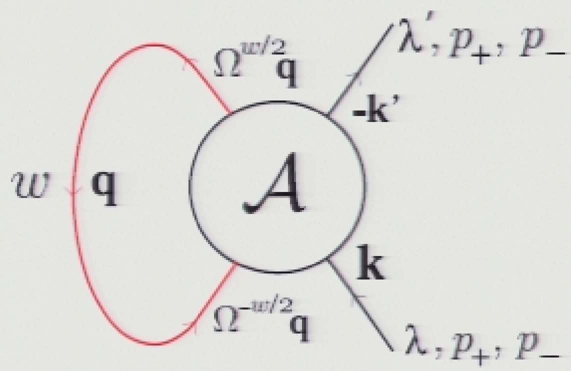


Two Point Function: Leading Winding Contribution



- Special kinematical limit

p_- fixed

$p_+ \to 0$

$\left(\lambda, \lambda' = 0 \text{ for simplicity}\right)$

- External states given by saddle points of phase

$$\chi(k) = \frac{1}{Ep_+^2}\left(p^2 k - \frac{k^3}{3}\right)$$

$$\left(p \equiv \sqrt{2p_+p_-}\right)$$

Saddle at $k = p$ where external momenta are given by

$$\mathbf{k} = \left(p_+, p_-, p\right) \qquad \mathbf{k}' = \left(-p_+, -p_-, p\right)$$

(and other saddle at $k = -p$)

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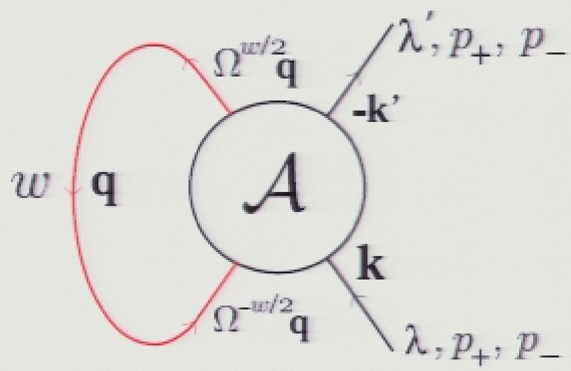

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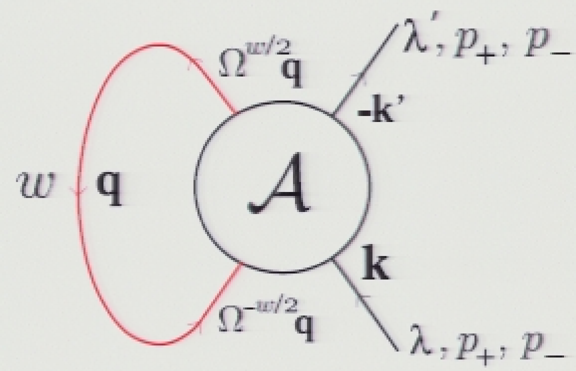

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- Loop integral over $\mathbf{q}$

$$\int d^3\mathbf{q} \quad \frac{-i}{\mathbf{q}^2 - i\epsilon} \quad e^{\frac{i}{E}\left(wq_- + \frac{1}{24}w^3 q_+\right)} \times$$
$$\times \quad \delta^3\left(\Omega^{-w/2}\mathbf{q} - \Omega^{w/2}\mathbf{q} + \mathbf{k} + \mathbf{k}'\right) \times$$
$$\times \quad \mathcal{A}\left(s\left(\mathbf{q}\right), t\left(\mathbf{q}\right), u\left(\mathbf{q}\right)\right)$$

- Two non–trivial δ–functions fix $q_+ = 2p/w$ and $q = 0$ leaving one integral over q_-

- In the limit

$$p_+ \ll \frac{E^2}{w^2 p_-}$$

the Mandelstam invariants read

$$\begin{aligned} s &= -\left(\mathrm{k}+\Omega^{-w/2}\mathbf{q}\right) \simeq \frac{4p}{w}\left(q_- + p_-\right) \\ t &= -\left(\mathrm{k}+\mathrm{k}'\right) = -4p^2 \\ u &= -\left(\mathrm{k}-\Omega^{w/2}\mathbf{q}\right) \simeq \frac{4p}{w}\left(q_- - p_-\right) \end{aligned}$$

Then

$$t \quad \text{fixed}$$

$$u = u(s) = s - \frac{8pp_-}{w}$$

- Two point function is sum of contributions at the two saddles $k = \pm p$

$$c\,\Gamma_+ + c^\star\,\Gamma_-$$

with c a constant (dependent on $p_{\pm}, w$) and

$$\Gamma_{\pm} \simeq \int \frac{ds}{2\pi i} \mathcal{A}\left(s,t,u\left(s\right)\right) \; e^{\pm i \frac{w^2}{4pE} s} \quad \times$$
$$\times \quad \frac{1}{|w|\, s - 4pp_{-} + i\epsilon} \qquad \text{(winding propagator)}$$

- Reality of the two point function equivalent to

$$\Gamma_{-}^{\star} = \Gamma_{+}$$

Graviton Dominance in Sub–Planckian Eikonal Scattering in 3D

- Massless scalar s–channel scattering in $3D$

$$1 + i\mathcal{A} = 4\sqrt{s}\sum_n e^{in\theta} e^{2i\delta_n(s)} \qquad \begin{pmatrix} \sin^2\theta = -t/s \\ \mathrm{Im}\,\delta_n \geq 0 \end{pmatrix}$$

- Small angle

$$\theta \ll 1 \qquad\qquad |t| \ll s$$

$$\textstyle\sum_n \rightarrow \int dx$$

with $x = 2n/\sqrt{s}$ impact parameter. One obtains

$$1 + i\mathcal{A} \simeq 2s \int dx \;\; e^{ix\sqrt{-t}} e^{2i\delta(s,x)}$$

- Leading order in couplings

$$\mathcal{A}_{\text{tree}}(s,t) \simeq 4s \int dx \;\; e^{ix\sqrt{-t}}\,\delta(s,x)$$

- Numerous derivations
  - High energy behavior of Feynman diagrams
  - *"Classical Field"* techniques
- Interaction mediated by spin j particle

$$\mathcal{A}_{\text{tree}} \simeq -4M^{3-2j}\;\frac{s^j}{t}$$

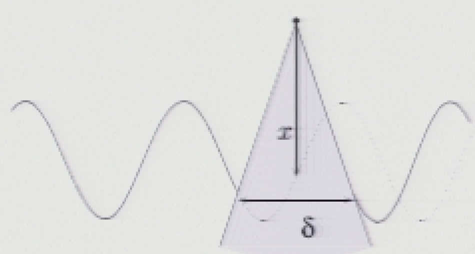
– Planck mass $M = \frac{1}{2\pi G}$. Often choose units so that $M = 1$

– Unit couplings in Planck units

– Phase shift

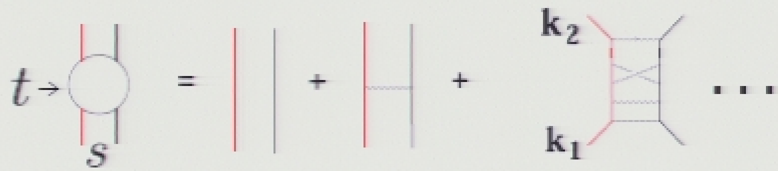
$$2\delta \simeq -s^{j-1}\,|x|$$

negative and linear with impact parameter (easily understood for $j = 2$ as scattering in a conical singularity)



- Full Eikonal Amplitude (sum of high energy behavior of crossed ladder graphs)

$$1 + i\mathcal{A} \simeq -4i \frac{s^j}{t + s^{2j-2} - i\epsilon}$$



- 't Hooft Poles. δ–function contributions at poles changes the free propagation result $4\pi s\,\delta(\sqrt{-t})$ to

$$2\pi s\,\delta(\sqrt{-t} - s^{j-1}) + 2\pi s\,\delta(\sqrt{-t} + s^{j-1})$$

For $j = 2$ scattering angle $\theta \sim \sqrt{s}$ due to conical singularity.

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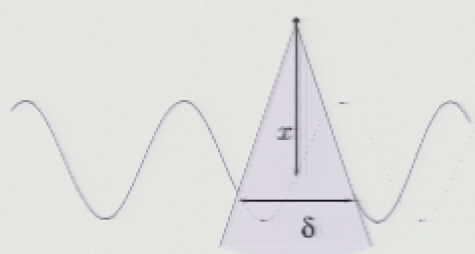

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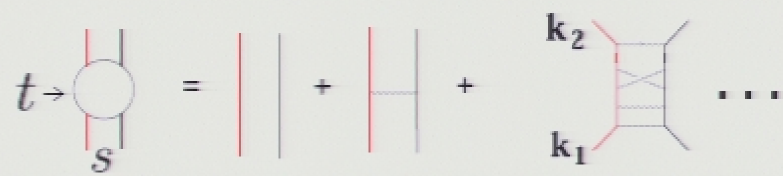

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For $j = 2$ scattering angle $\theta \sim \sqrt{s}$ due to conical singularity.

- Sub Planckian scattering

$$s^2 \lesssim |t| \ll s \ll 1$$

Then

$$\begin{array}{ccccc} j=0 & & j=1 & & j=2 \\ \downarrow & & \downarrow & & \downarrow \\ \frac{1}{t+s^{-2}} \sim s^2 & \ll & \frac{s}{t+1} \sim s & \ll & \frac{s^2}{t+s^2} \sim s^2/t \end{array}$$

  - $j = 2$ graviton dominates
  - $j = 2$ 't Hooft pole reliable

- Closer look at $j = 2$ amplitude. Can write it as

$$\mathcal{A}_{\pm} \simeq \mp \frac{(s-u)^2}{2\sqrt{-t}} \left[\frac{1}{s \mp \sqrt{-t} \mp i\epsilon} + \frac{1}{u \mp \sqrt{-t} \mp i\epsilon} \right]$$

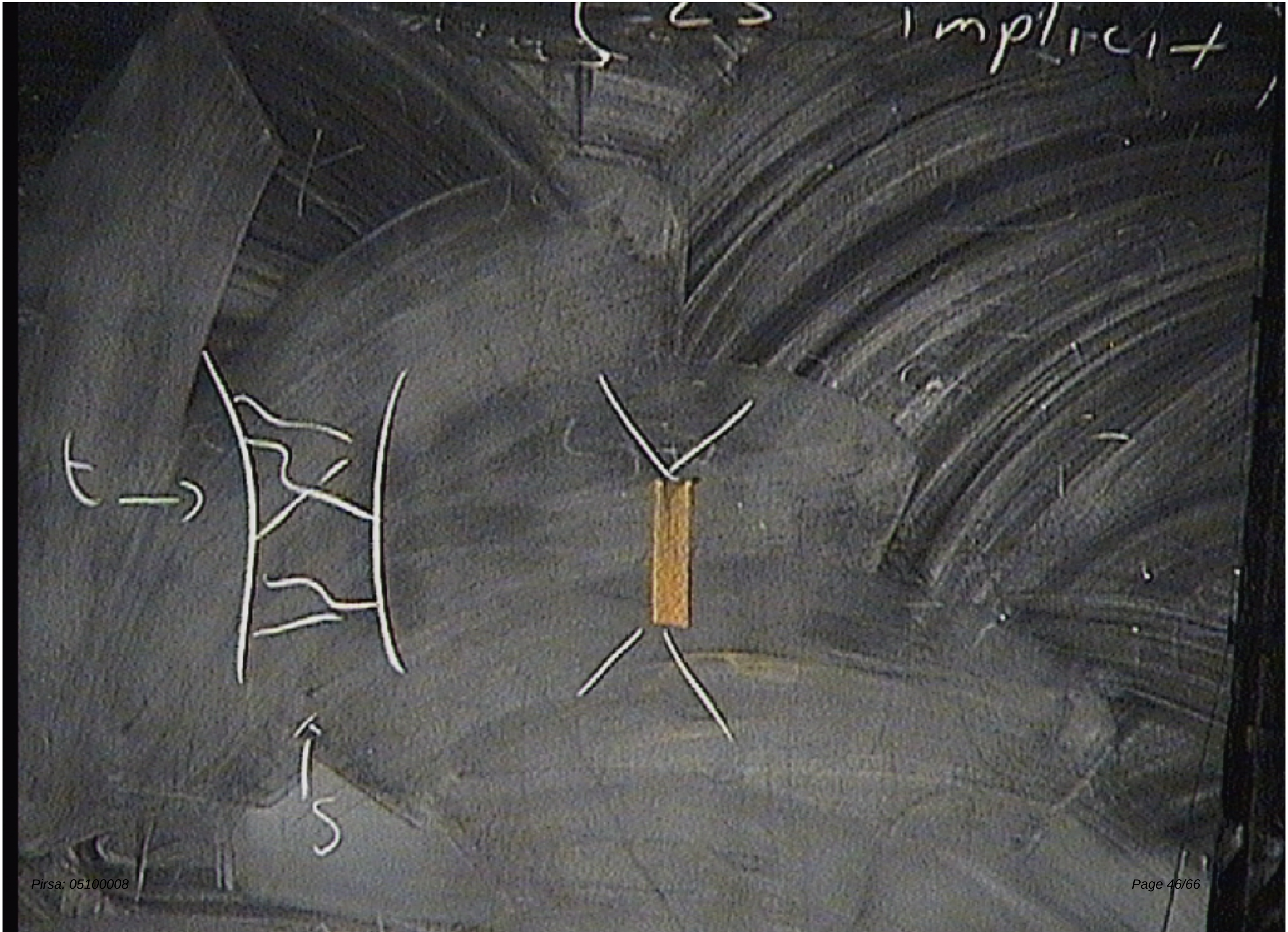
- *Pseudo*–particle exchange of mass$^2 \pm\sqrt{-t}$ in s, u channels.

- Off–shell generalization for the kinematics

$$\mathbf{k}_1^2 = \mathbf{k}_2^2 \neq 0$$
$$s, u \gg |t|$$

The tree–level result is

$$\mathcal{A}_{\text{tree}} \simeq -\frac{(s-u)^2}{t}$$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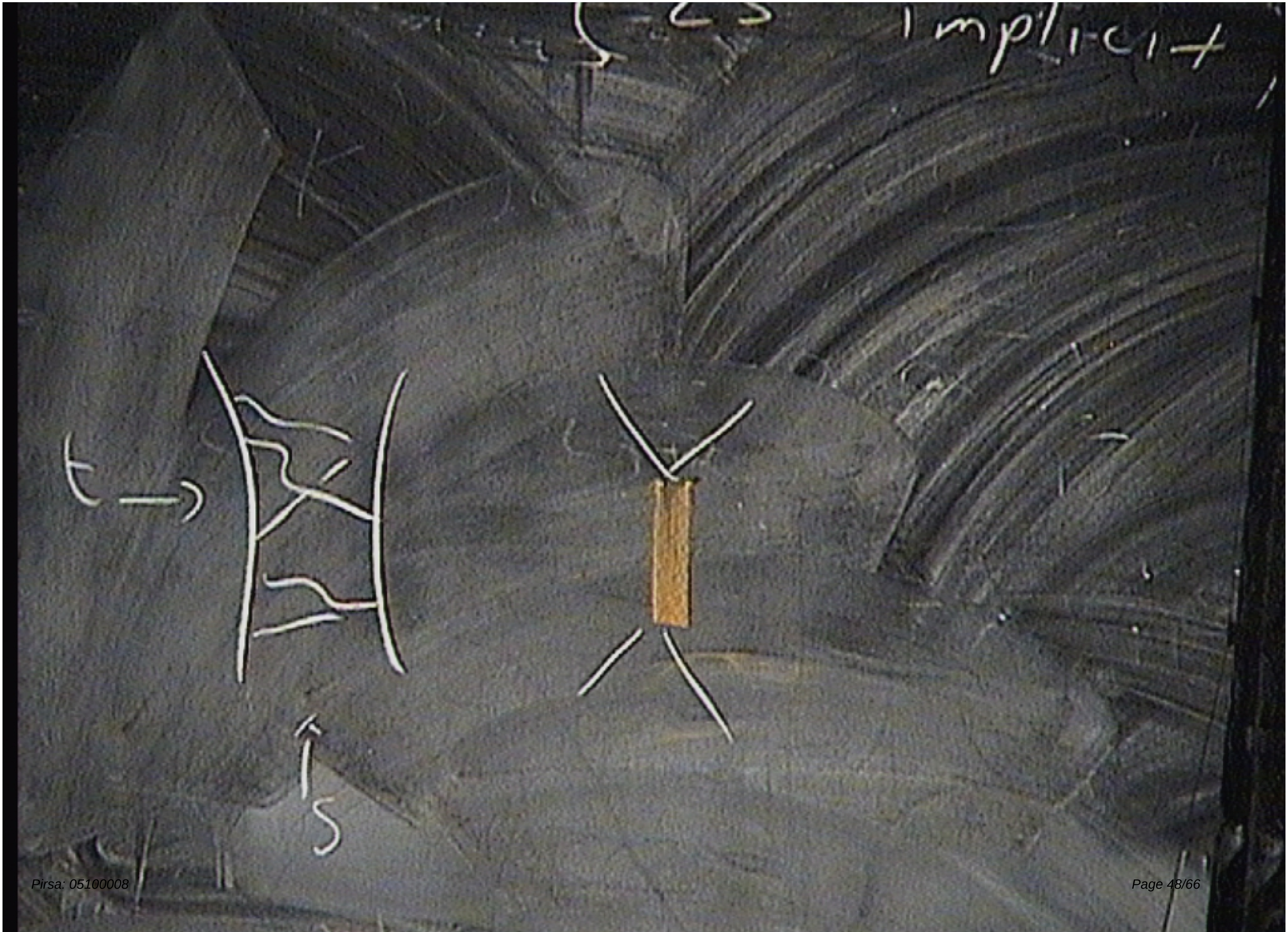
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- Amplitude $\mathcal{A}(s,u,t)$ symmetric in s,u with poles at

$$\begin{aligned} s &= \pm\left(\sqrt{-t}+i\epsilon\right) \\ u &= \pm\left(\sqrt{-t}+i\epsilon\right) \end{aligned}$$

- Example

$$\mathcal{A} = a_+\mathcal{A}_+ + a_-\mathcal{A}_-$$

If $a_+ + a_- = 1$ then compatible with

– on–shell eikonal

– off–shell tree level

Applying the Eikonal Methods

- Recall

$$\Gamma_{\pm} \simeq \int \frac{ds}{2\pi i} \; \mathcal{A}\left(s,t,u\left(s\right)\right) \quad \frac{e^{\pm i \frac{w^2}{4pE} s}}{|w|\, s - 4pp_- + i\epsilon}$$

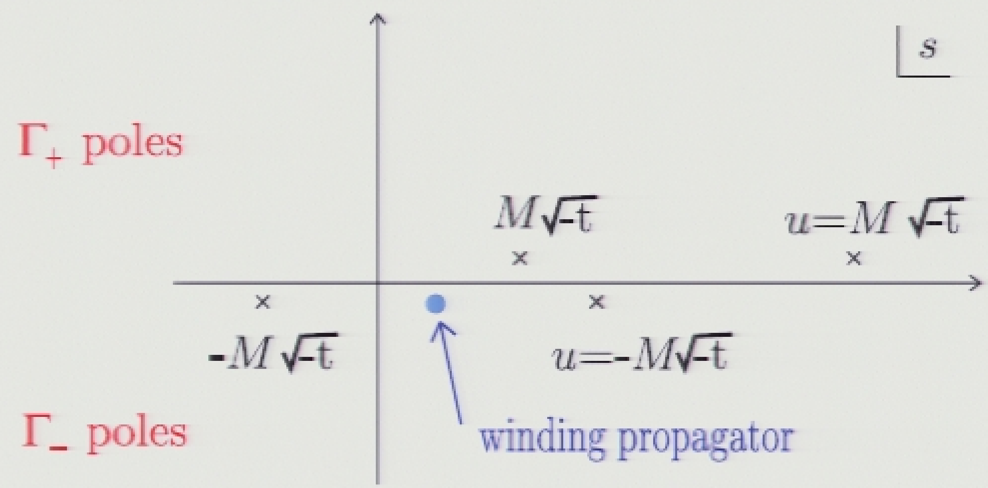
with

$$t = -4p^2 \quad \text{fixed} \qquad\qquad u\left(s\right) = s - \frac{8pp_-}{w}$$

- We are in the eikonal regime

$$|t| \ll s, u \ll M^2$$

- Complex s–plane



- Consider for simplicity the example (results hold in general)

$$\mathcal{A} = a_+ \mathcal{A}_+ + a_- \mathcal{A}_-$$

Applying the Eikonal Methods

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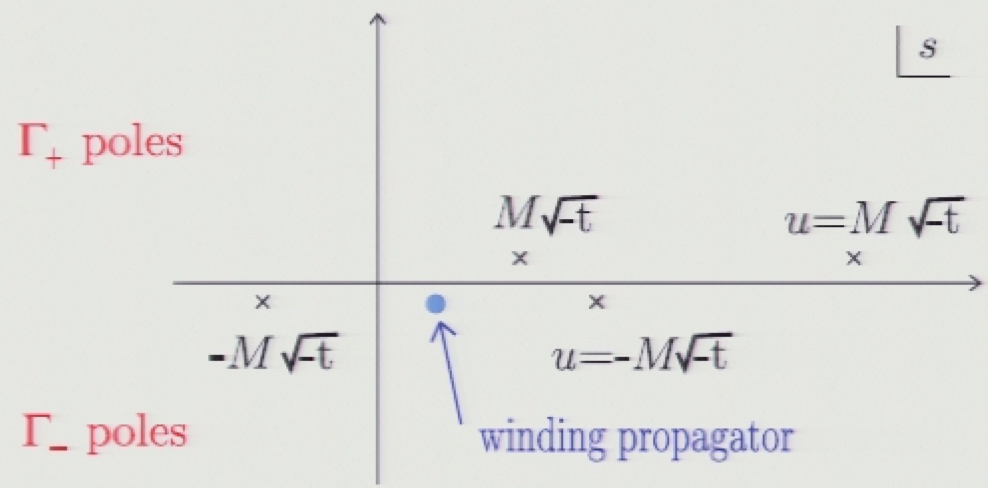

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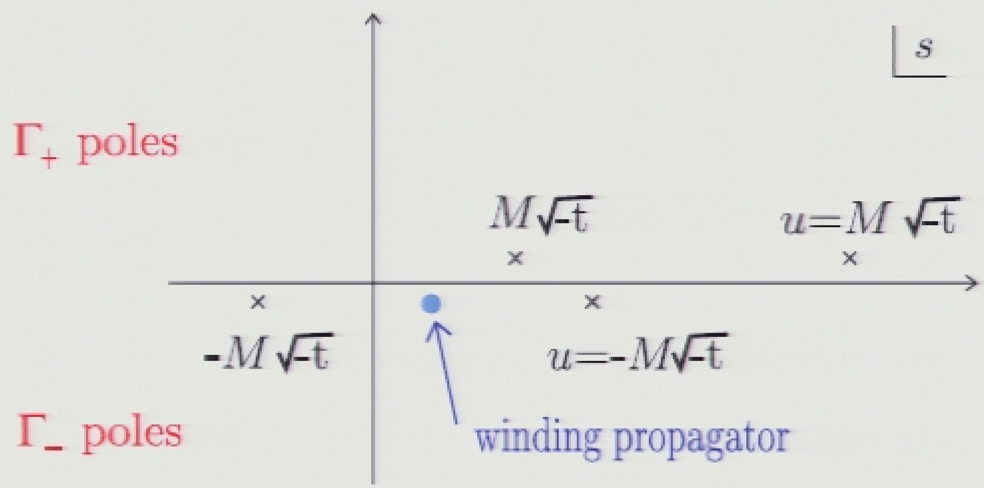

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$$\mathcal{A} = a_+ \mathcal{A}_+ + a_- \mathcal{A}_-$$

One has for all w

$$\Gamma_+ \sim -a_+ e^{i\frac{w^2 M}{2E}}$$
$$\Gamma_- \sim a_- e^{i\frac{w^2 M}{2E}} - 1$$

Unitarity ($\Gamma_+^\star = \Gamma_-$) implies

$$\frac{M}{2E} \in 2\pi\mathbb{Z} \qquad a_+ + a_- = 1$$

Notes

- Tree level result for $\mathcal{A}$ gives

$$\Gamma_+ \sim 0 \qquad \Gamma_- \sim -1$$

- Amplitude $\mathcal{A}(s,u,t)$ symmetric in s,u with poles at

$$\begin{aligned} s &= \pm\left(\sqrt{-t}+i\epsilon\right) \\ u &= \pm\left(\sqrt{-t}+i\epsilon\right) \end{aligned}$$

- Example

$$\mathcal{A} = a_+\mathcal{A}_+ + a_-\mathcal{A}_-$$

If $a_+ + a_- = 1$ then compatible with

– on–shell eikonal

– off–shell tree level

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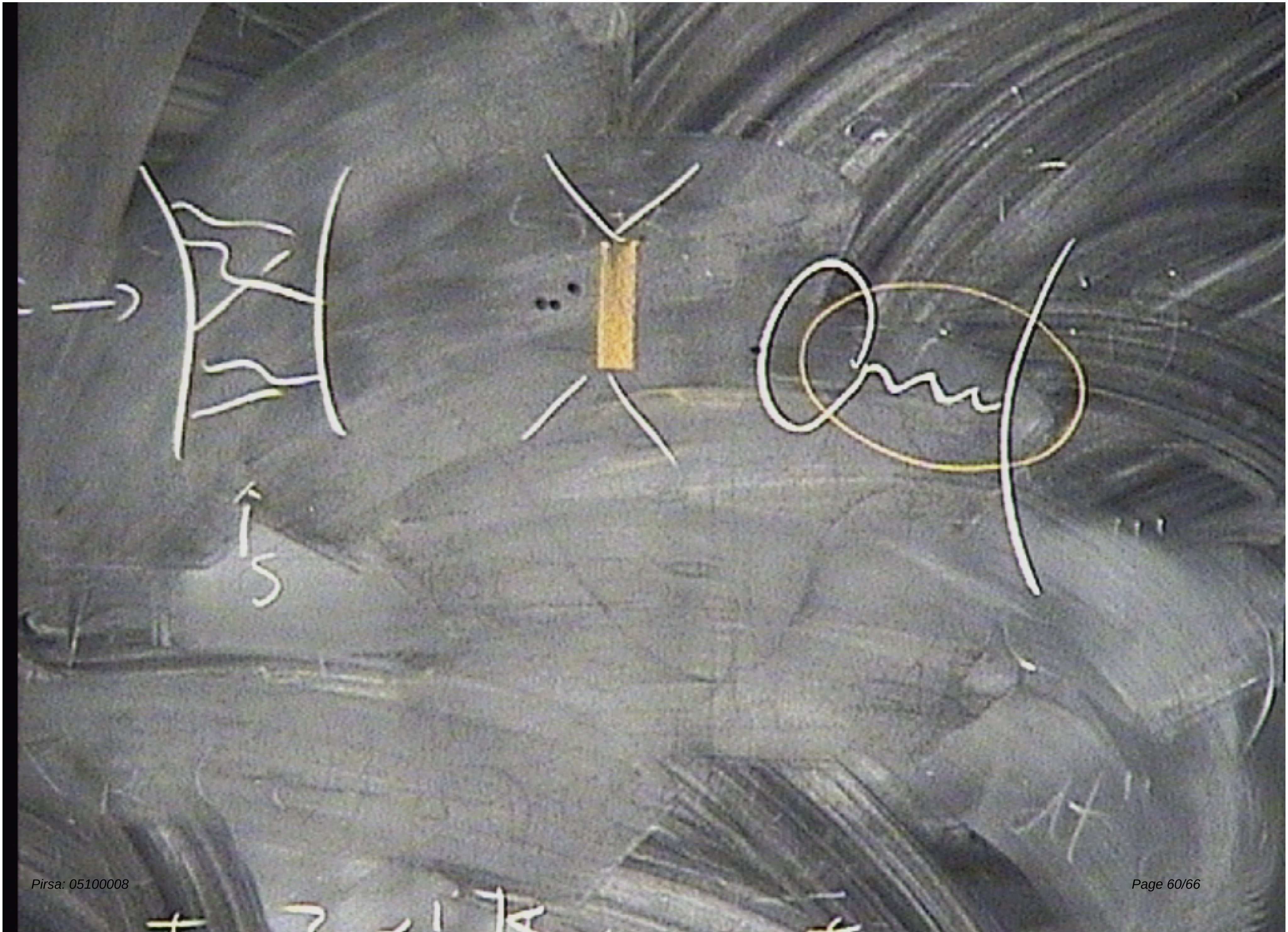

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Unitarity violation due to coupling to tadpole

- Eikonal scattering is transverse, with no p_+ exchange. All decay channels would require splitting the incoming p_+ in the various decay constituents

- Results obtained at energies $\ll 1/\sqrt{\alpha'}$. Independent of string theory (or other high energy completion of the theory) as long as we have a theory of gravity (spin 2 particle exchanged)

- Higher dimensional nature of interaction irrelevant in the low–energy limit, well below the compactification scale. Interactions truly $3D$

Things to Do

- Compute off–shell extension of the eikonal approximation (either by Feynman diagram or by classical field techniques) Problematic due to IR divergences which require resummation

- Extend results to $n \to m$ scattering (possibly using old work of 't Hooft in 3D quantum gravity)

$0 = (\tilde{L}_0 - 1)|\psi\rangle$

$)^2 + 2N_R - 2$

$)^2 + 2N_L - 2$

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