Title: Pilot-wave theory for the standard model

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Abstract:

... description ... of the particle

Find a probab. description of $V(t)$

$$f_{1|1}(x_2, t_2 | x_1, t_1)$$

$x(t)$

$|\psi\rangle$ $\quad i\frac{d}{dt}|\psi\rangle = H|\psi\rangle$

$\psi(x)$

... description ... of the particle

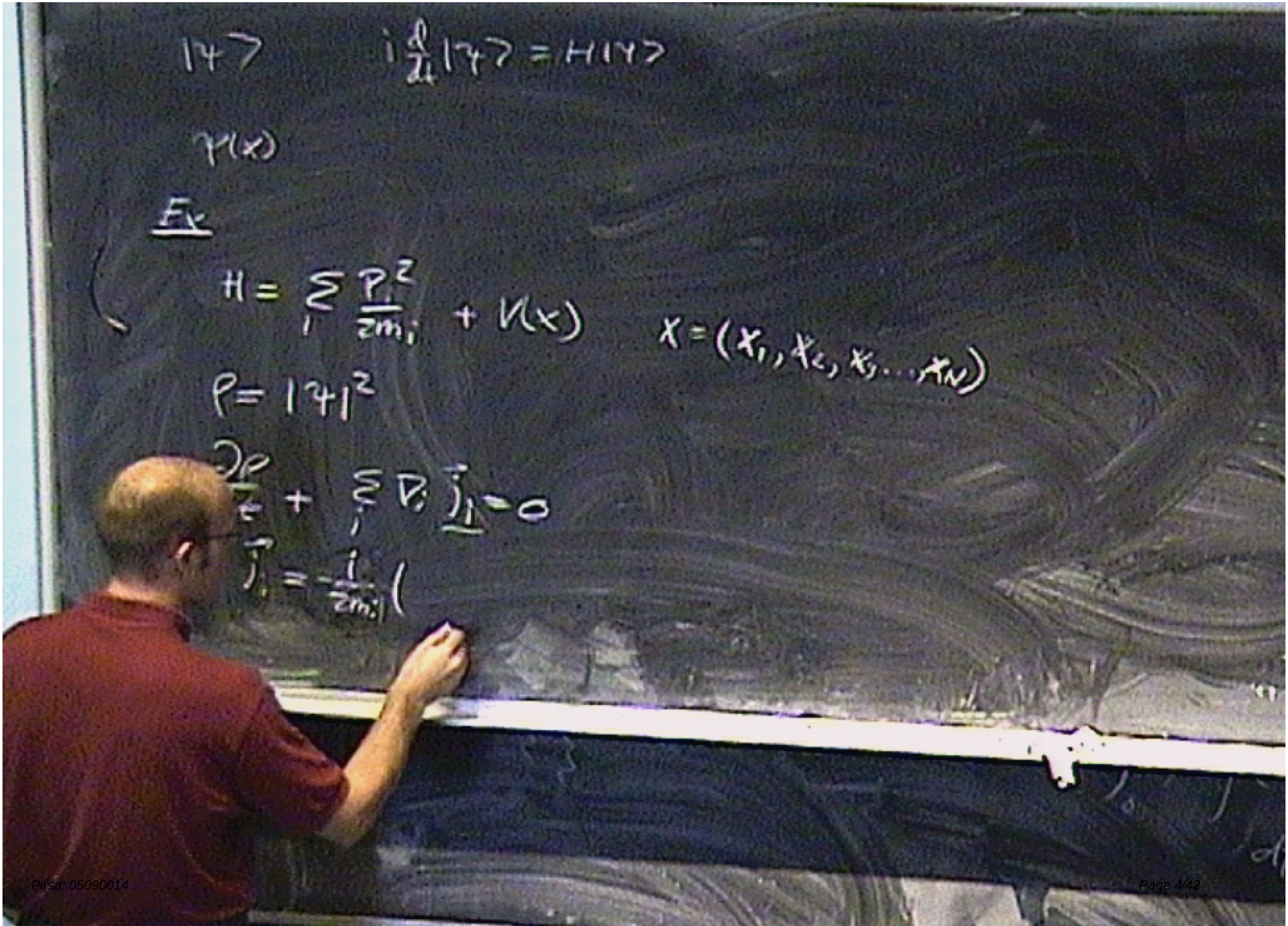
Find a probab. description

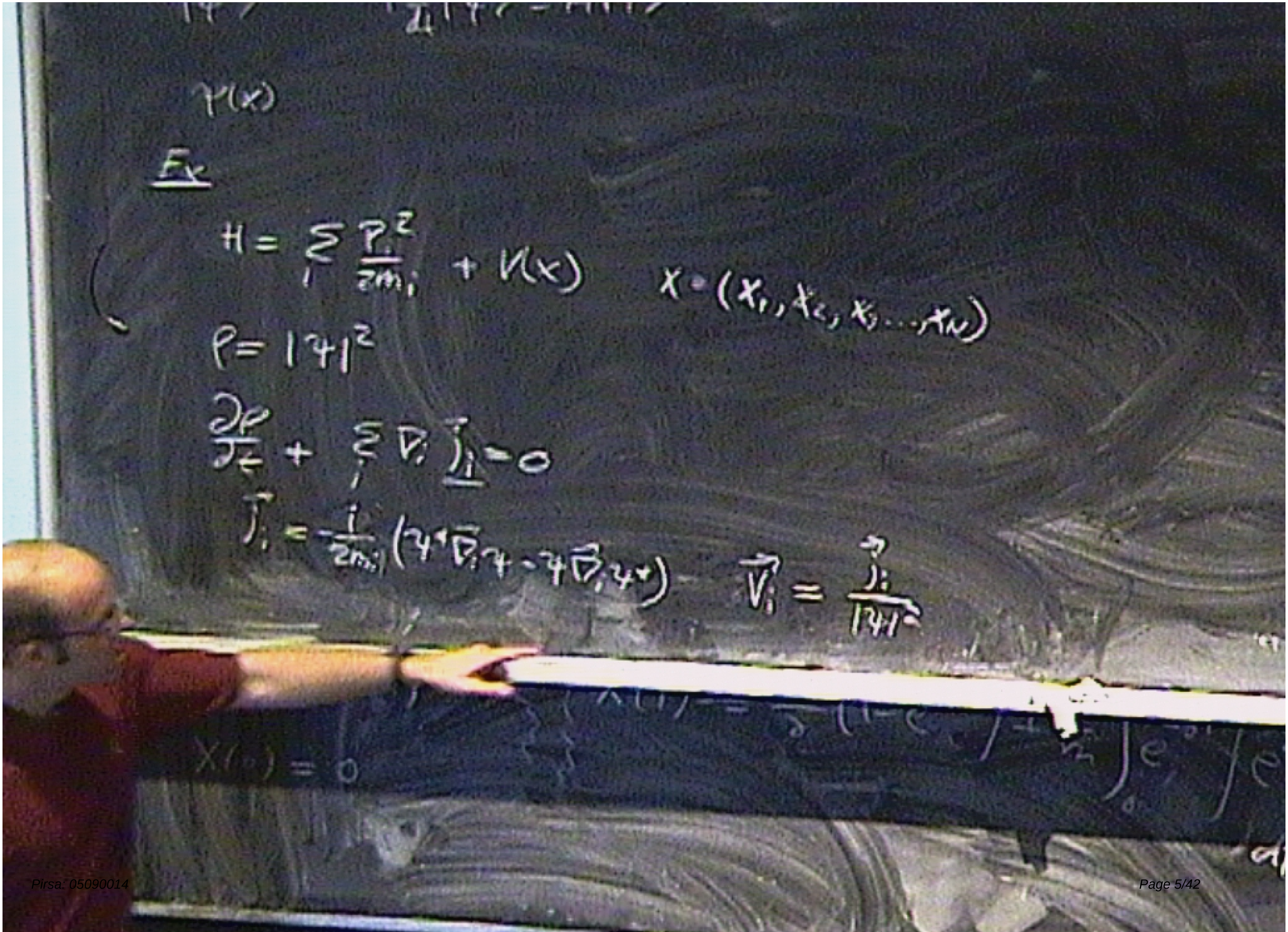
$|\psi\rangle \qquad i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$

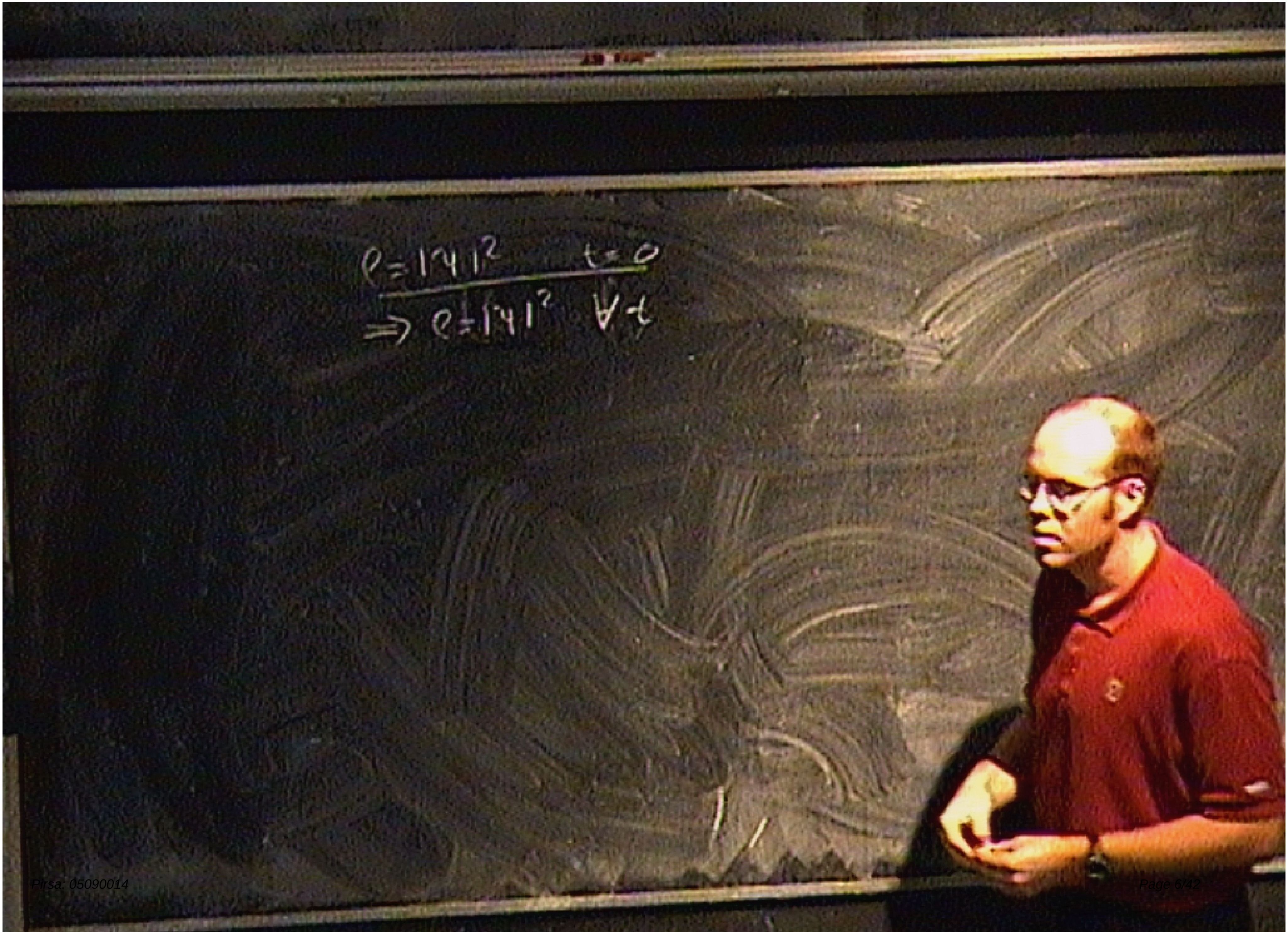
$\psi(x)$

Ex

$$H = \sum_i \frac{P_i^2}{2m_i} + V(x) \qquad X = (x_1, x_2, x_3 \dots x_N)$$


$|\psi\rangle$
$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle$
$\psi(x)$
Ex
$H = \sum_i \frac{P_i^2}{2m_i} + V(x)$
$X = (x_1, x_2, x_3 \ldots x_N)$
$\rho = |\psi|^2$
$\frac{\partial \rho}{\partial t} + \sum_i \nabla_i \cdot \vec{j}_i = 0$
$\vec{j}_i = -\frac{i}{2m_i}($


$\psi(x)$
Ex
$H = \sum_i \frac{P_i^2}{2m_i} + V(x)$
$x = (x_1, x_2, x_3 \ldots, x_N)$
$\rho = |\psi|^2$
$\frac{\partial \rho}{\partial t} + \sum_i \nabla_i \vec{j}_i = 0$
$\vec{j}_i = -\frac{i}{2m_i}(\psi^* \vec{\nabla}_i \psi - \psi \vec{\nabla}_i \psi^*)$
$\vec{V}_i = \frac{\vec{j}_i}{|\psi|^2}$


$\rho = |\psi|^2 \quad t=0$
$\Rightarrow \rho = |\psi|^2 \quad \forall t$

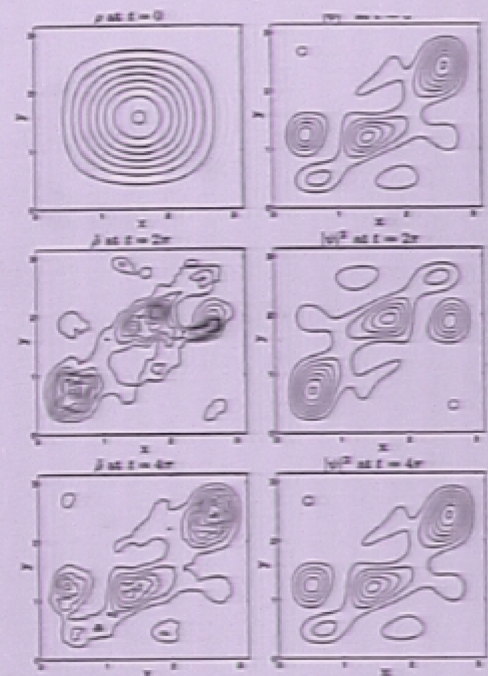
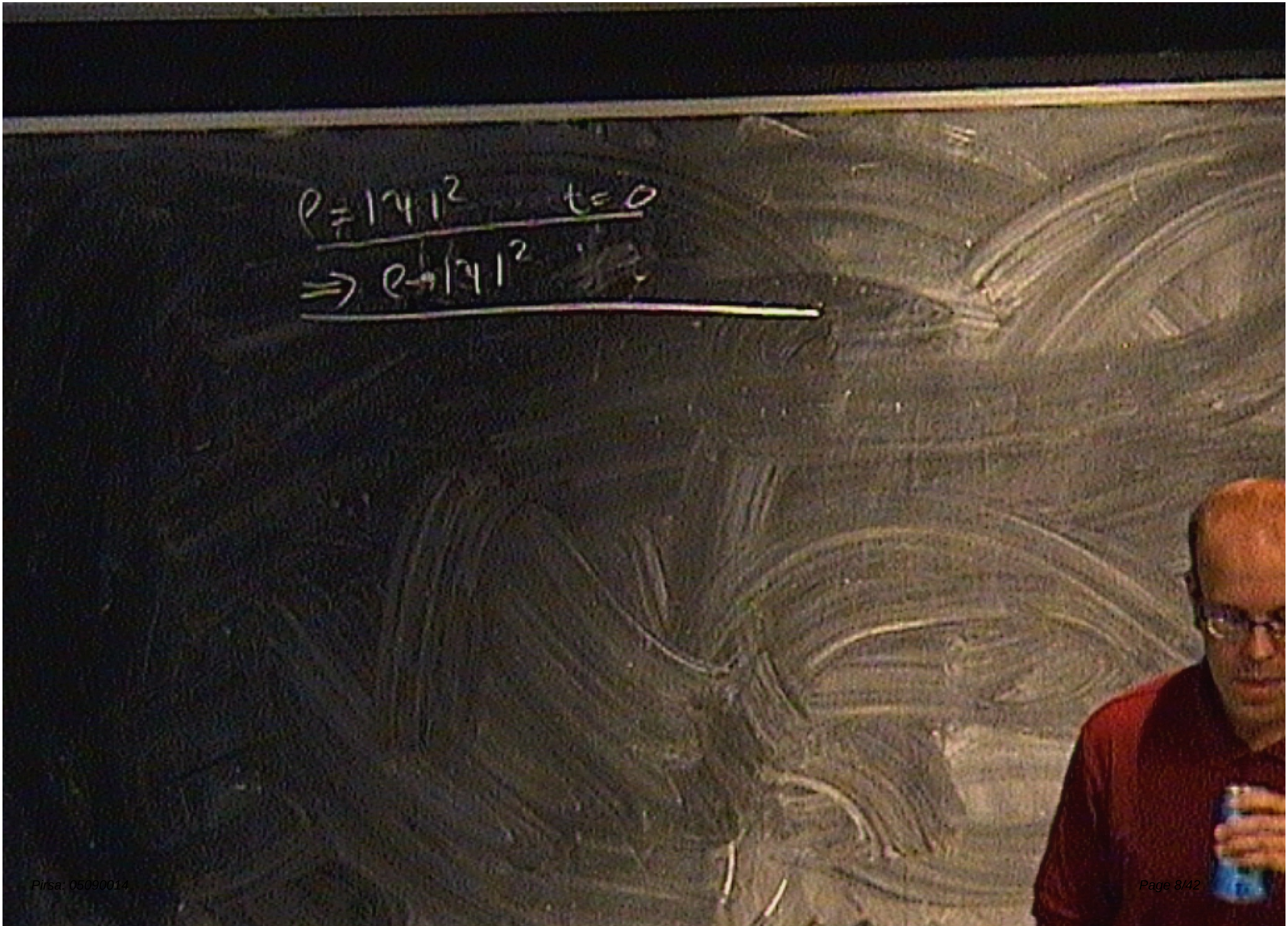


Figure 8: Smoothed $\tilde{\rho}$, compared with $|\psi|^2$, at times $t = 0$, 2π and 4π. The same data as in Fig. 7, displayed as contour plots.


$\rho \neq |\psi|^2 \quad t=0$
$\Rightarrow \rho \to |\psi|^2$

A Pilot-Wave Model for Quantum Electrodynamics

Ward Struyve and Hans Westman

Perimeter Institute for Theoretical Physics
31 Caroline Street North
N2L 2Y5, Waterloo, Ontario, Canada.

Figure 8: Smoothed $\tilde{\rho}$, compared with $|\psi|^2$, at times $t = 0, 2\pi$ and 4π. The same data as in Fig. 7, displayed as contour plots.

A Pilot-Wave Model for Quantum Electrodynamics

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Introduction

- Bohm's pilot wave model for the electromagnetic field (1952).
- Extension: scalare fields, massive spin-1 fields, non-abelian gauge theories (Bohm, Hiley, Holland, Kaloyerou, Valentini, Struyve).
- No working model for fermionic quantum field theory, *with fields as beables*.
- Two attempts: Holland '88, Valentini '92.
- Problems:

→ Valentini's model lacks a probability density for his beables (Struyve 2005).

→ Hollands model do not exhibit effective collapse which is essential for a pilot-wave model in order to reproduce the quantum statistics.

- Do we really need beables for the fermionic degrees of freedom?
- Bell's model for spin: no beable for the spin degree of freedom. Spin is a property of the wavefunction: $\psi_a(\mathbf{x})$.
- Our model for quantum electrodynamics: no beables for the fermionic degrees of freedom. Fermions are a property of the wavefunctional: $\psi_f(q_1, q_2)$.
- All fermionic fields are *without exception* gauge-coupled to bosonic fields.

Outline of the talk

- General frame work.
- Bell's model for spin.
- Effective collapse and equivalence with quantum theory.
- Review of Bohm's 1952 pilot-wave model of the electromagnetic field.
- The model for quantum electrodynamics (QED).

General frame work

- Suppose we have two Hilbert spaces $\mathcal{H}_i$, $i = 1, 2$ with bases $B(\mathcal{H}_i) = \{|o_i\rangle \big| o_i \in O_i\}$, where the O_i are some label sets.
- Consider now the product Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. The set

$$B(\mathcal{H}_1 \otimes \mathcal{H}_2) = \left\{ |o_1, o_2\rangle \Big| |o_1, o_2\rangle = |o_1\rangle \otimes |o_2\rangle ; |o_i\rangle \in B(\mathcal{H}_i), i = 1, 2 \right\} \quad (1)$$

then forms a basis for the product space.

- In this basis a quantum state $|\psi\rangle$ can be expressed as

$$|\psi\rangle = \sum_{o_1, o_2} \psi(o_1, o_2) |o_1, o_2\rangle. \quad (2)$$

- The corresponding density matrix reads

$$\widehat{\rho} = |\psi\rangle\langle\psi| = \sum_{\substack{o_1, o_2 \\ \bar{o}_1, \bar{o}_2}} \psi^*(\bar{o}_1, \bar{o}_2) \psi(o_1, o_2) |o_1, o_2\rangle\langle\bar{o}_1, \bar{o}_2|. \quad (3)$$

- In the basis $B(\mathcal{H}_1 \otimes \mathcal{H}_2)$ the coefficients of the density matrix are

$$\rho(o_1, o_2; \bar{o}_1, \bar{o}_2) = \psi^*(\bar{o}_1, \bar{o}_2) \psi(o_1, o_2). \quad (4)$$

- Suppose now we want to introduce beables only corresponding to the degree of freedom o_1. One can do this by considering the reduced density matrix

$$\widehat{\rho}_1 = \mathrm{Tr}_2 \widehat{\rho} = \sum_{o_1, \bar{o}_1, o_2} \psi^*(\bar{o}_1, o_2) \psi(o_1, o_2) |o_1\rangle\langle\bar{o}_1|. \tag{5}$$

In the basis $B(\mathcal{H}_1)$ this matrix has coefficients

$$\rho_1(o_1; \bar{o}_1) = \sum_{o_2} \psi^*(\bar{o}_1, o_2) \psi(o_1, o_2). \tag{6}$$

- The probability of finding the system in the state $|o_1\rangle$ is given by

$$\rho(o_1) = \rho_1(o_1; o_1) \tag{7}$$

- Potentially one can interpret $\rho(o_1)$ as a density of beables corresponding to the degree of freedom o_1. Given the Schrödinger equation, the velocity field for these beables can then be found by considering the continuity equation for the density $\rho(o_1)$. Holland '93, Squires '94, Goldstein *et al.*'05.

with

$$\mathbf{j} = \sum_a \left(\frac{\hbar}{2mi} (\psi_a^* \boldsymbol{\nabla} \psi_a - \psi_a \boldsymbol{\nabla} \psi_a^*) - \frac{e}{mc} \mathbf{A} \psi_a^* \psi_a \right) . \tag{17}$$

- The guidance equation is then given by

$$\frac{d\mathbf{x}}{dt} = \frac{\mathbf{j}}{\rho}. \tag{18}$$

- This is Bell's model for spin.

Effective collapse and quantum statistics

- Suppose the state has evolved into the superposition

$$\psi_a(\mathbf{x}) = \psi_a^{(1)}(\mathbf{x}) + \psi_a^{(2)}(\mathbf{x}). \tag{19}$$

- If we have

$$\psi_a^{(1)}(\mathbf{x})\psi_{a'}^{(2)}(\mathbf{x}) = 0 \qquad \forall \mathbf{x}; a, a' = -1, 1. \tag{20}$$

  we say that the states are non-overlapping.

- From (20) it follows that the velocity field takes the form

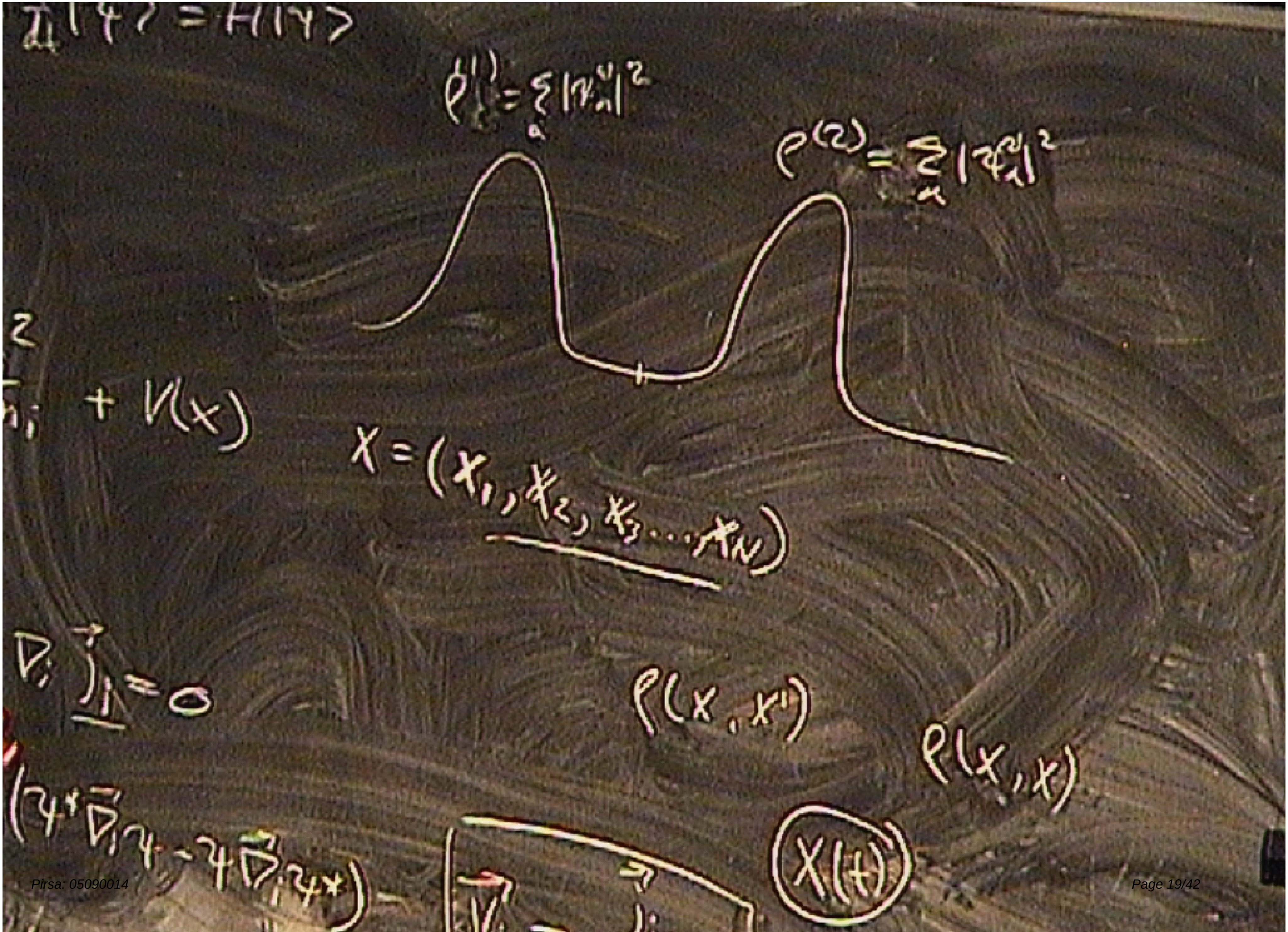
$$\frac{d\mathbf{x}}{dt} = \frac{J^{(1)}}{\rho^{(1)}} \quad or \quad \frac{d\mathbf{x}}{dt} = \frac{J^{(2)}}{\rho^{(2)}}. \tag{21}$$

  with

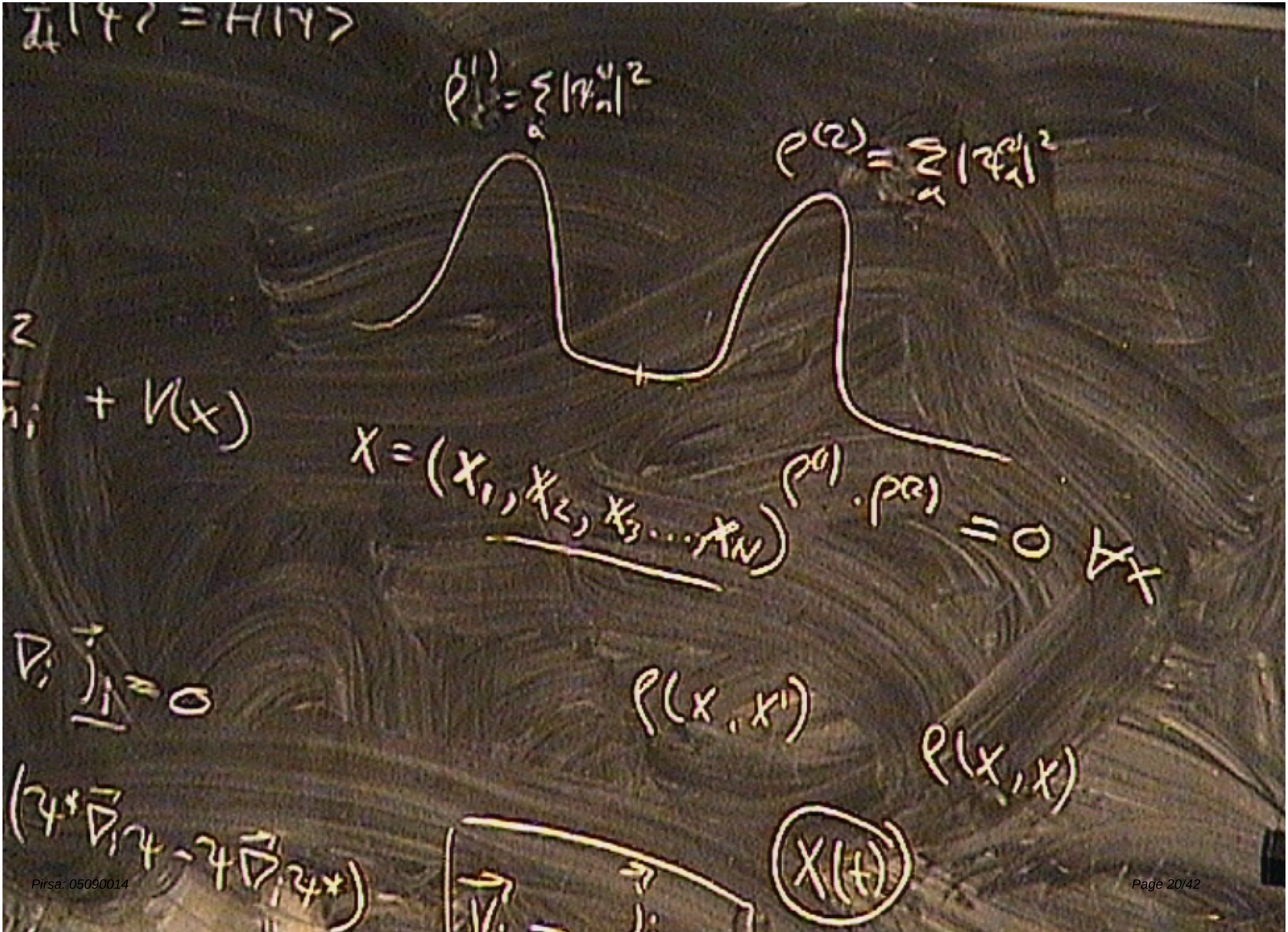
$$\mathbf{j}^{(i)} = \sum_a \left(\frac{\hbar}{2mi} \left((\psi_a^{(i)})^* \boldsymbol{\nabla} \psi_a^{(i)} - \psi_a^{(i)} \boldsymbol{\nabla} (\psi_a^{(i)})^* \right) - \frac{e}{mc} \mathbf{A} (\psi_a^{(i)})^* \psi_a^{(i)} \right) \tag{22}$$

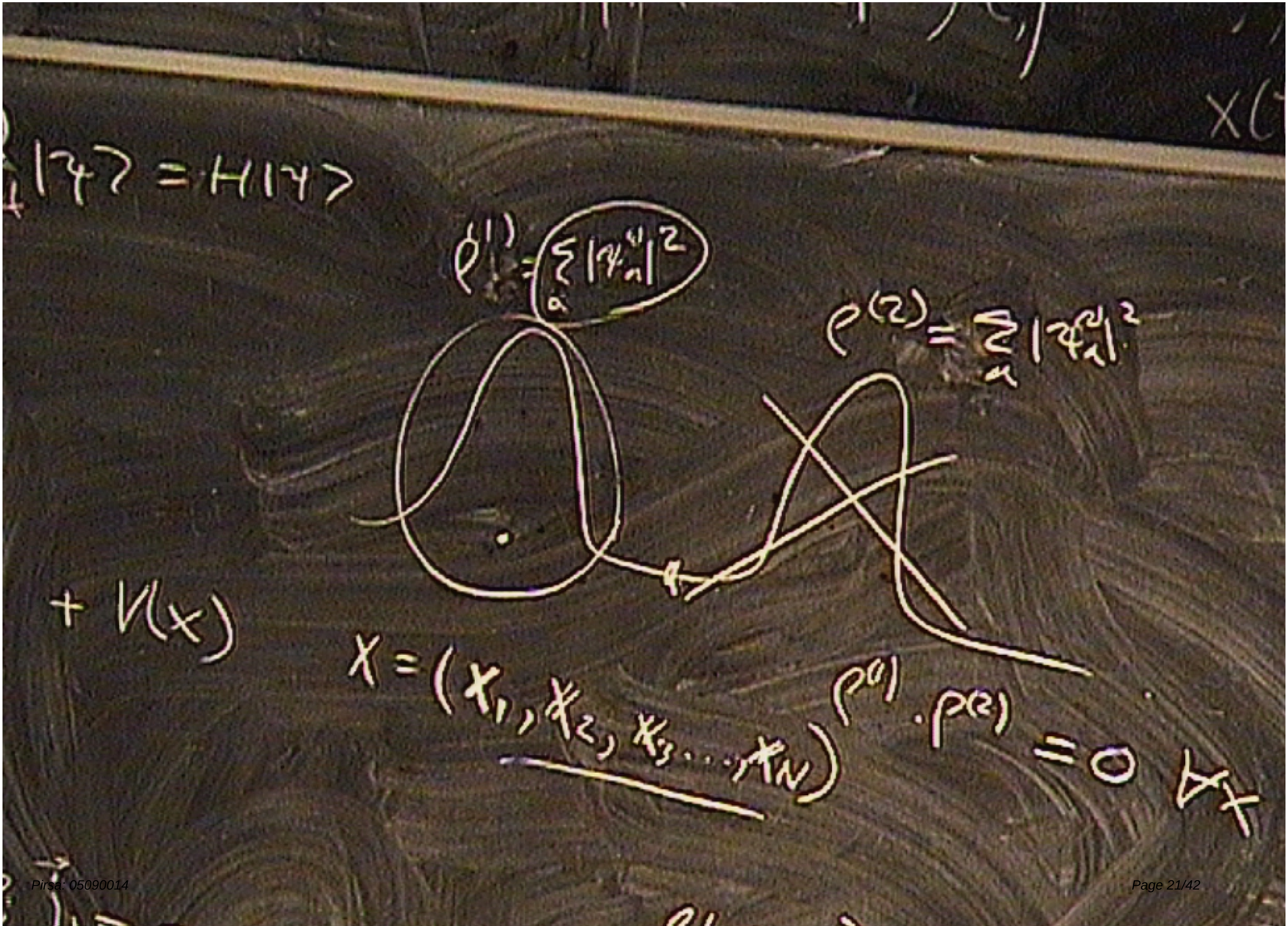
$$\rho^{(i)} = \sum_a \left| \psi^{(i)} \right|^2 \tag{23}$$

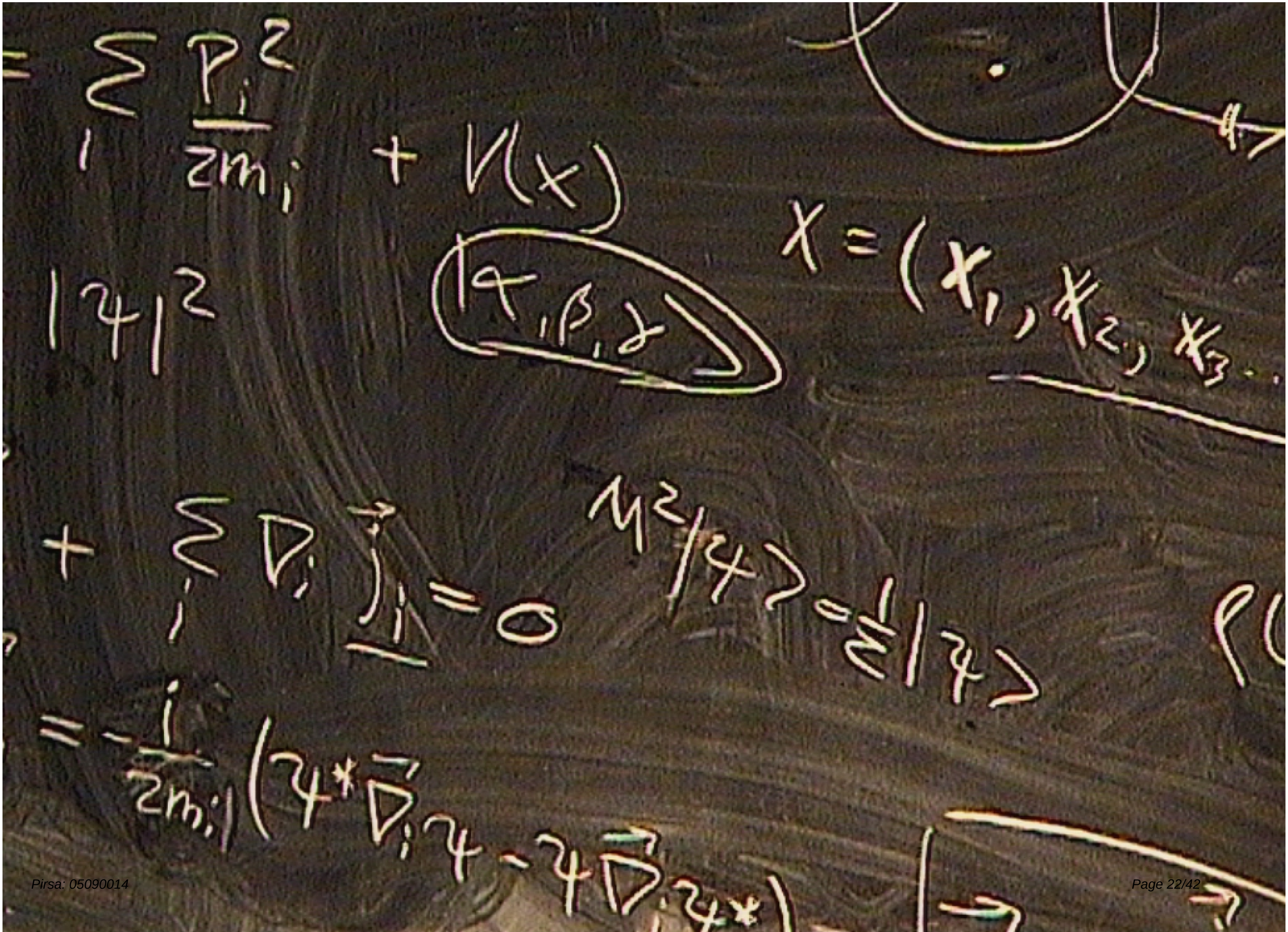
- Hence we see that the particle beable $\mathbf{x}$ is always effectively guided by either $\Psi^{(1)}$ or $\Psi^{(2)}$.

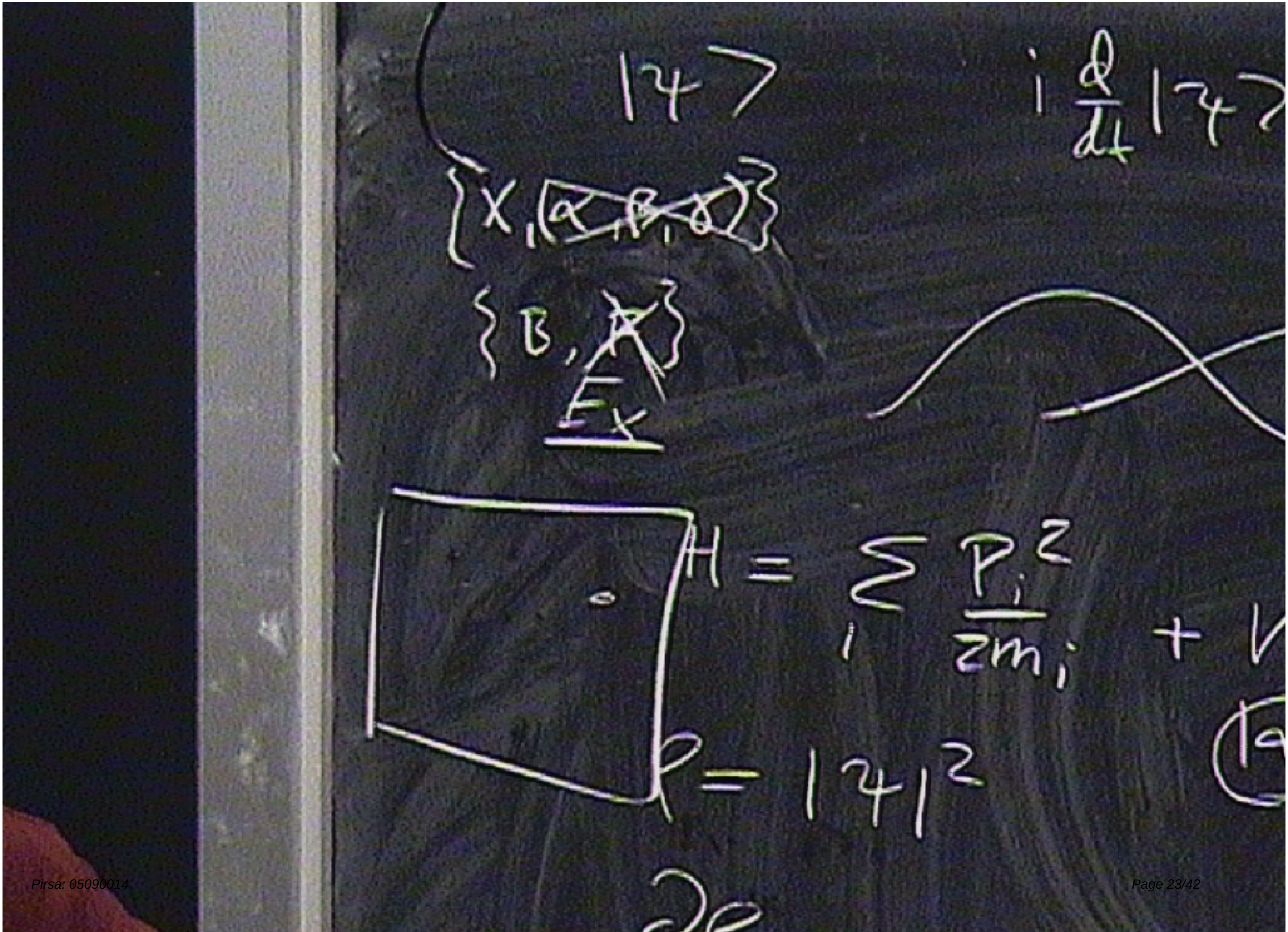


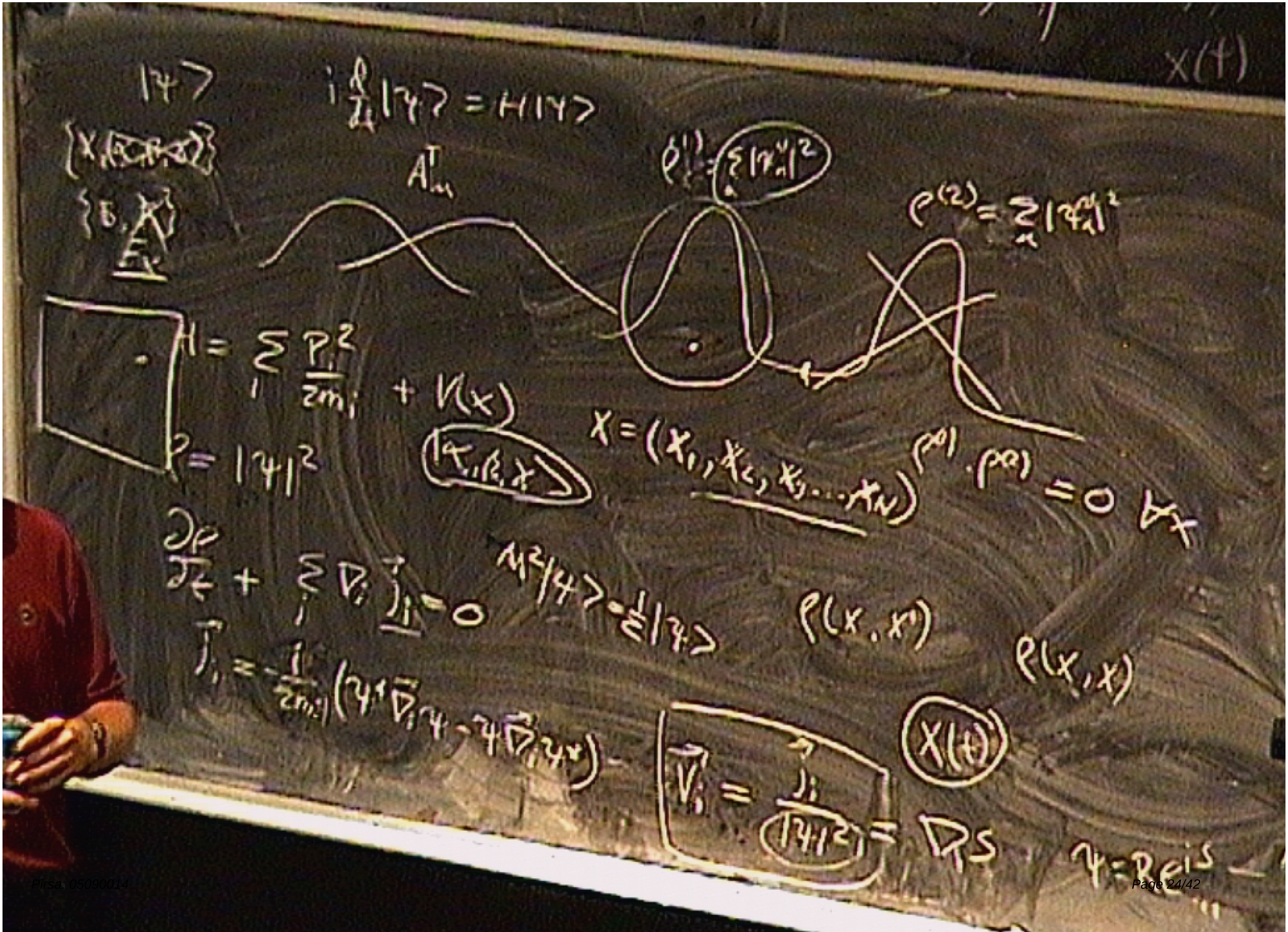
$\rho^{(1)} = \sum_\alpha |\psi^{(1)}_\alpha|^2$
$\rho^{(2)} = \sum_\alpha |\psi^{(2)}_\alpha|^2$
$+ V(x)$
$X = (x_1, x_2, x_3 \ldots x_N)$
$\rho(x, x')$
$\rho(x, x)$
$X(t)$


iħ d/dt |ψ⟩ = H|ψ⟩
ρ(1) = Σ |ψ_n|²
ρ(2) = Σ |ψ_n|²
+ V(x)
X = (x1, x2, x3 ... xN)
ρ(1) · ρ(2) = 0 ∀x
∇i · ji = 0
ρ(x, x')
ρ(x, x)
X(t)


$|\psi\rangle = H|\psi\rangle$
$\rho^{(1)} = \sum_\alpha |\psi_n^\alpha|^2$
$\rho^{(2)} = \sum_\alpha |\psi_n^\alpha|^2$
$+ V(x)$
$X = (x_1, x_2, x_3 \ldots x_N)$
$\rho^{(1)} \cdot \rho^{(2)} = 0 \;\forall x$


= Σ_i p_i^2 / 2m_i + V(x)
|ψ|^2
|α, β, γ⟩
x = (x_1, x_2, x_3 ...
+ Σ_i ∇_i j_i = 0
M^2|ψ⟩ = ε|ψ⟩
= -i/2m_i (ψ* ∇_i ψ - ψ ∇_i ψ*)


|ψ⟩
i d/dt |ψ⟩
{x, α, β, γ}
{B, E}
H = Σ P_i^2 / 2m_i + V
ρ = |ψ|^2


|ψ>
i ∂/∂t |ψ> = H|ψ>
H = Σ_i P_i^2/2m_i + V(x)
ρ = |ψ|^2
X = (x_1, x_2, x_3 ... x_N)
ρ(x, x')
ρ(x, x)
X(t)
x(t)
= ∇S
ψ = Re^iS

Pilot-wave model for the free electromagnetic field

- EM field quantized in the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$, $A_0 = 0$)

$$\widehat{H}_B = \frac{1}{2} \int d^3x \left(\widehat{\mathbf{\Pi}}^T \cdot \widehat{\mathbf{\Pi}}^T - \widehat{\mathbf{A}}^T \cdot \nabla^2 \widehat{\mathbf{A}}^T \right) \quad (24)$$

$$\nabla \cdot \widehat{\mathbf{A}}^T \equiv \nabla \cdot \widehat{\mathbf{\Pi}}^T \equiv 0 \quad (25)$$

$$[\widehat{A}_i^T(\mathbf{x}), \widehat{\Pi}_j^T(\mathbf{y})] = i \left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right) \delta(\mathbf{x} - \mathbf{y}) \quad (26)$$

- Fourier modes

$$\begin{aligned} \widehat{A}_i^T(\mathbf{x}) &= \frac{1}{(2\pi)^{3/2}} \sum_{l=1}^{2} \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} \varepsilon_i^l(\mathbf{k}) \widehat{q}_l(\mathbf{k}) \\ \widehat{\Pi}_i^T(\mathbf{x}) &= \frac{1}{(2\pi)^{3/2}} \sum_{l=1}^{2} \int d^3k e^{-i\mathbf{k}\cdot\mathbf{x}} \varepsilon_i^l(\mathbf{k}) \widehat{\pi}_l(\mathbf{k}) \end{aligned} \quad (27)$$

$$[\widehat{q}_l(\mathbf{k}), \widehat{\pi}_{l'}(\mathbf{k}')] = i\delta_{ll'}\delta(\mathbf{k} - \mathbf{k}'), \quad [\widehat{q}_l(\mathbf{k}), \widehat{q}_{l'}(\mathbf{k}')] = [\widehat{\pi}_l(\mathbf{k}), \widehat{\pi}_{l'}(\mathbf{k}')] = 0 \quad (28)$$

- **Functional Schrödinger picture**

→ Basis for the Hilbert space $\mathcal{H}_B$

$$B(\mathcal{H}_B) = \left\{ |q_1, q_2\rangle \middle| \widehat{q}_l(\mathbf{k})|q_1, q_2\rangle = q_l(\mathbf{k})|q_1, q_2\rangle, l = 1, 2 \right\} \tag{29}$$

→ Operators

$$\begin{aligned} \langle q_1, q_2, f|\widehat{q}_l(\mathbf{k})|q_1', q_2', f'\rangle &= q_l(\mathbf{k})\delta(q_1 - q_1')\delta(q_2 - q_2') \\ \langle q_1, q_2, f|\widehat{\pi}_l(\mathbf{k})|q_1', q_2', f'\rangle &= -i\frac{\delta}{\delta q_l(\mathbf{k})}\delta(q_1 - q_1')\delta(q_2 - q_2') \end{aligned} \tag{30}$$

→ States

$$|\Psi(t)\rangle = \int \mathcal{D}q_1\mathcal{D}q_2\Psi(q_1, q_2, t)|q_1, q_2\rangle \tag{31}$$

→ Functional Schrödinger equation for $\Psi(q_1, q_2, t) = \langle q_1, q_2|\Psi(t)\rangle$

$$i\partial_t\Psi(q_1, q_2, t) = \frac{1}{2}\int d^3k \left(-\frac{\delta^2}{\delta q_l(\mathbf{k})\delta q_l^*(\mathbf{k})} + k^2 q_l(\mathbf{k})q_l^*(\mathbf{k}) \right) \Psi(q_1, q_2, t) \tag{32}$$

- **Pilot-wave model (Bohm 1952)**

$\longrightarrow$ Continuity equation for $|\Psi(q_1, q_2, t)|^2 = |\langle q_1, q_2|\Psi(t)\rangle|^2$

$$\partial_t |\Psi(q_1, q_2, t)|^2 + \sum_{l=1}^{2} \int d^3k \frac{\delta}{\delta q_l(\mathbf{k})} J_l(\mathbf{k}; q_1, q_2, t) = 0 \qquad (33)$$

with

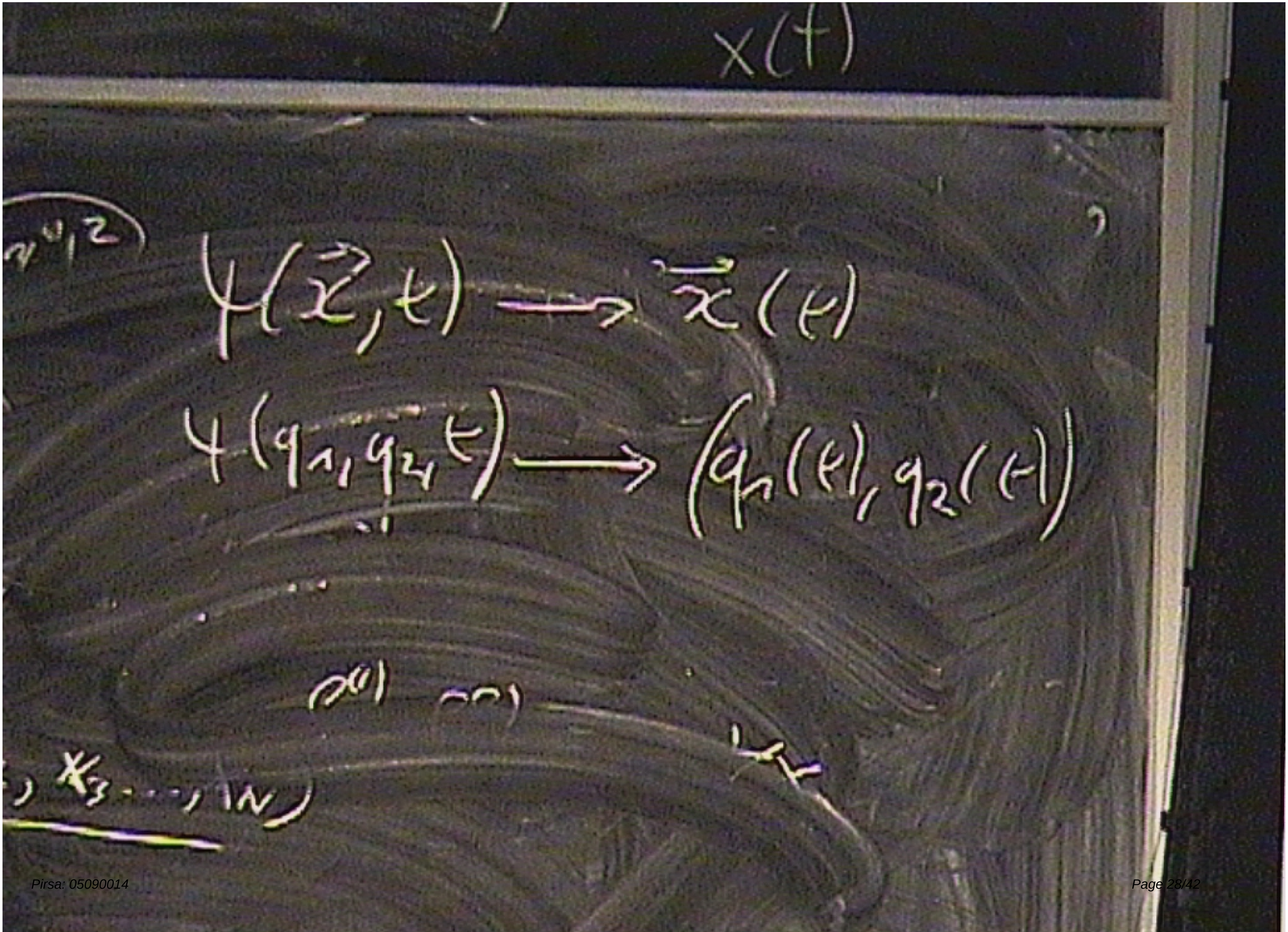
$$J_l(\mathbf{k}; q_1, q_2, t) = \mathrm{Im}\left(\Psi^*(q_1, q_2, t) \frac{\delta}{\delta q_l^*(\mathbf{k})} \Psi(q_1, q_2, t)\right) \qquad (34)$$

$\longrightarrow$ Introduce field beables $q_l(\mathbf{k}, t)$, for $l = 1, 2$

$\longrightarrow$ Guidance equation

$$\frac{\partial q_l(\mathbf{k}, t)}{\partial t} = \frac{J_l(\mathbf{k}; q_1, q_2, t)}{|\Psi(q_1, q_2, t)|^2} \qquad (35)$$

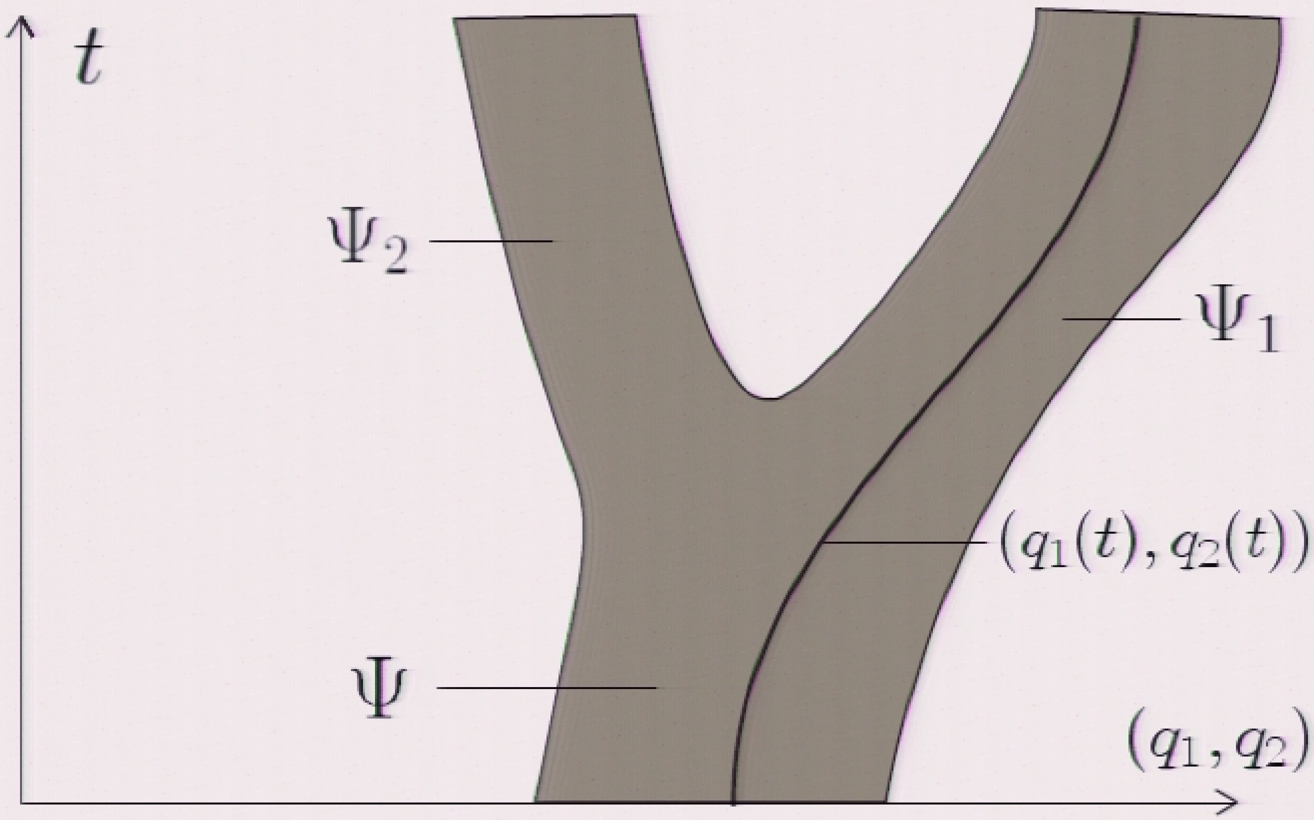
$\longrightarrow$ Field beables are distributed according to $|\Psi(q_1, q_2, t)|^2$


x(t)
ψ(x⃗, t) ⟶ x⃗(t)
ψ(q1, q2, t) ⟶ (q1(t), q2(t))

- **Reproducing quantum theory**

→ Branching of the wavefunction: $\Psi \to \Psi_1 + \Psi_2, \quad \Psi_1\Psi_2 \equiv 0$:

→ Effective collapse $\Psi_1 + \Psi_2 \to \Psi_1$



→ Macroscopically distinct states are non-overlapping in the configuration space of fields

- **Similar pilot-wave models for the other bosonic fields in the Standard Model:**

→ The weak interaction bosons $Z^0_\mu, W^+_\mu, W^-_\mu$ (massive spin-1 fields)

→ The strong interaction field A^a_μ

→ The Higgs field and spontaneous symmetry breaking

The model for quantum electrodynamics

- QED in the Coulomb gauge ($\boldsymbol{\nabla} \cdot \mathbf{A} = 0$, $A_0 = -\frac{1}{\nabla^2} j^0$)

$$\widehat{H} = \underbrace{\frac{1}{2}\int d^3x \left(\widehat{\mathbf{\Pi}}^T \cdot \widehat{\mathbf{\Pi}}^T - \widehat{\mathbf{A}}^T \cdot \nabla^2 \widehat{\mathbf{A}}^T\right)}_{free\ EM\ field}$$
$$+ \underbrace{\int d^3x \widehat{\psi}^\dagger \left(-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla}\right) \psi}_{free\ Dirac\ field}$$
$$\underbrace{-\int d^3x \widehat{\mathbf{A}}^T \cdot \widehat{\mathbf{j}}}_{interaction\ term} + \underbrace{\frac{1}{2}\int d^3x d^3y \frac{\widehat{j}^0(\mathbf{x})\widehat{j}^0(\mathbf{y})}{4\pi|\mathbf{x}-\mathbf{y}|}}_{Coulomb\ potential} \qquad (36)$$

$$[\widehat{A}_i^T(\mathbf{x}), \widehat{\Pi}_j^T(\mathbf{y})] = i\left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}\right)\delta(\mathbf{x}-\mathbf{y}) \qquad (37)$$

$$\{\widehat{\psi}_a(\mathbf{x}), \widehat{\psi}_b^\dagger(\mathbf{y})\} = \delta_{ab}\delta(\mathbf{x}-\mathbf{y}) \qquad (38)$$

• Representation and Schrödinger equation

⟶ Product Hilbert space $\mathcal{H} = \mathcal{H}_B \otimes \mathcal{H}_F$

⟶ Basis for bosonic and fermionic Hilbert spaces

$$\begin{aligned} B(\mathcal{H}_B) &= \left\{ |q_1, q_2\rangle \middle| \widehat{q}_l(\mathbf{k})|q_1, q_2\rangle = q_l(\mathbf{k})|q_1, q_2\rangle, l = 1, 2 \right\} \\ B(\mathcal{H}_F) &= \{|f\rangle\} \end{aligned} \tag{39}$$

⟶ Product basis

$$B(\mathcal{H}) = \left\{ |q_1, q_2\rangle \otimes |f\rangle = |q_1, q_2, f\rangle \right\} \tag{40}$$

⟶ States

$$|\Psi(t)\rangle = \sum_f \int \mathcal{D}q_1 \mathcal{D}q_2 \Psi_f(q_1, q_2, t)|q_1, q_2, f\rangle \tag{41}$$

⟶ Schrödinger equation for $\Psi_f(q_1, q_2, t) = \langle q_1, q_2, f|\Psi(t)\rangle$

$$i\partial_t \Psi_f(q_1, q_2, t) = \sum_{f'} \widehat{H}_{ff'}(q, -i\delta/\delta q)\Psi_{f'}(q_1, q_2, t) \tag{42}$$

- **The pilot-wave model**

→ Density matrix

$$\rho_{f;f'}(q_1, q_2; q'_1, q'_2) = \Psi^*_{f'}(q'_1, q'_2, t)\Psi_f(q_1, q_2, t) \tag{43}$$

→ By tracing out over the fermionic degrees of freedom we obtain the reduced density matrix

$$\rho(q_1, q_2; q'_1, q'_2, t) = \sum_f \rho_{f;f}(q_1, q_2; q'_1, q'_2, t) = \sum_f \Psi^*_f(q'_1, q'_2, t)\Psi_f(q_1, q_2, t) \tag{44}$$

→ Continuity equation for $\rho(q_1, q_2, t) = \rho(q_1, q_2; q_1, q_2, t) = \sum_f |\Psi_f(q_1, q_2, t)|^2$

$$\partial_t \rho(q_1, q_2, t) + \sum_{l=1}^{2} \int d^3k \frac{\delta}{\delta q_l(\mathbf{k})} J_l(\mathbf{k}; q_1, q_2, t) = 0 \tag{45}$$

with

$$J_l(\mathbf{k}; q_1, q_2, t) = \sum_f \mathrm{Im}\left(\Psi^*_f(q_1, q_2, t)\frac{\delta}{\delta q^*_l(\mathbf{k})}\Psi_f(q_1, q_2, t)\right) \tag{46}$$

→ Introduce field beables $q_l(\mathbf{k}, t)$, for $l = 1, 2$

→ Guidance equation

$$\frac{\partial q_l(\mathbf{k}, t)}{\partial t} = \frac{J_l(\mathbf{k}; q_1, q_2, t)}{\rho(q_1, q_2, t)} \tag{47}$$

→ Beables are distributed according to $\rho(q_1, q_2, t) = \sum_f |\Psi_f(q_1, q_2, t)|^2$

• Reproducing quantum theory

→ We have **effective collapse** when

$$\Psi_f(q_1, q_2, t) = \Psi_f^{(1)}(q_1, q_2, t) + \Psi_f^{(2)}(q_1, q_2, t) \tag{48}$$

$$\Psi_f^{(1)}(q_1, q_2, t)\Psi_{f'}^{(2)}(q_1, q_2, t) \equiv 0, \qquad \forall f', f \tag{49}$$

→ Example:

$$|\Psi\rangle = |\Psi_1^{(s)}\rangle|\Psi_1^{(r)}\rangle + |\Psi_2^{(s)}\rangle|\Psi_2^{(r)}\rangle \tag{50}$$

with

$$|\Psi_1^{(s)}\rangle \text{ and } |\Psi_2^{(s)}\rangle \rightarrow \textbf{different states of the system} \tag{51}$$

$$|\Psi_1^{(r)}\rangle \text{ and } |\Psi_2^{(r)}\rangle \rightarrow \textbf{radiation coming from the system} \tag{52}$$

E.g. light scattering of a needle.

In our representation:

$$\begin{aligned} \Psi_f(q_1, q_2, \tilde{q}_1, \tilde{q}_2, t) \; = \; & \Psi_{1,f}^{(s)}(q_1, q_2, t)\Psi_1^{(r)}(\tilde{q}_1, \tilde{q}_2, t) \\ & + \Psi_{2,f}^{(s)}(q_1, q_2, t)\Psi_2^{(r)}(\tilde{q}_1, \tilde{q}_2, t) \end{aligned} \tag{53}$$

If $\Psi_1^{(r)}$ and $\Psi_2^{(r)}$ are macroscopically distinct then we have effective collapse

→ Introduce field beables $q_l(\mathbf{k}, t)$, for $l = 1, 2$

→ Guidance equation

$$\frac{\partial q_l(\mathbf{k}, t)}{\partial t} = \frac{J_l(\mathbf{k}; q_1, q_2, t)}{\rho(q_1, q_2, t)} \tag{47}$$

→ Beables are distributed according to $\rho(q_1, q_2, t) = \sum_f |\Psi_f(q_1, q_2, t)|^2$

Pilot-wave model for the free electromagnetic field

- EM field quantized in the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$, $A_0 = 0$)

$$\widehat{H}_B = \frac{1}{2} \int d^3x \left(\widehat{\mathbf{\Pi}}^T \cdot \widehat{\mathbf{\Pi}}^T - \widehat{\mathbf{A}}^T \cdot \nabla^2 \widehat{\mathbf{A}}^T \right) \tag{24}$$

$$\nabla \cdot \widehat{\mathbf{A}}^T \equiv \nabla \cdot \widehat{\mathbf{\Pi}}^T \equiv 0 \tag{25}$$

$$[\widehat{A}_i^T(\mathbf{x}), \widehat{\Pi}_j^T(\mathbf{y})] = i \left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right) \delta(\mathbf{x} - \mathbf{y}) \tag{26}$$

- Fourier modes

$$\widehat{A}_i^T(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \sum_{l=1}^{2} \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}} \varepsilon_i^l(\mathbf{k}) \widehat{q}_l(\mathbf{k})$$

$$\widehat{\Pi}_i^T(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \sum_{l=1}^{2} \int d^3k e^{-i\mathbf{k}\cdot\mathbf{x}} \varepsilon_i^l(\mathbf{k}) \widehat{\pi}_l(\mathbf{k}) \tag{27}$$

$$[\widehat{q}_l(\mathbf{k}), \widehat{\pi}_{l'}(\mathbf{k}')] = i\delta_{ll'}\delta(\mathbf{k} - \mathbf{k}'), \quad [\widehat{q}_l(\mathbf{k}), \widehat{q}_{l'}(\mathbf{k}')] = [\widehat{\pi}_l(\mathbf{k}), \widehat{\pi}_{l'}(\mathbf{k}')] = 0 \tag{28}$$

- **Reproducing quantum theory**

→ We have **effective collapse** when

$$\Psi_f(q_1, q_2, t) = \Psi_f^{(1)}(q_1, q_2, t) + \Psi_f^{(2)}(q_1, q_2, t) \tag{48}$$

$$\Psi_f^{(1)}(q_1, q_2, t)\Psi_{f'}^{(2)}(q_1, q_2, t) \equiv 0, \qquad \forall f', f \tag{49}$$

→ Example:

$$|\Psi\rangle = |\Psi_1^{(s)}\rangle|\Psi_1^{(r)}\rangle + |\Psi_2^{(s)}\rangle|\Psi_2^{(r)}\rangle \tag{50}$$

with

$$|\Psi_1^{(s)}\rangle \text{ and } |\Psi_2^{(s)}\rangle \to \textbf{different states of the system} \tag{51}$$

$$|\Psi_1^{(r)}\rangle \text{ and } |\Psi_2^{(r)}\rangle \to \textbf{radiation coming from the system} \tag{52}$$

E.g. light scattering of a needle.

In our representation:

$$\begin{aligned}\Psi_f(q_1, q_2, \tilde{q}_1, \tilde{q}_2, t) \;=\; & \Psi_{1,f}^{(s)}(q_1, q_2, t)\Psi_1^{(r)}(\tilde{q}_1, \tilde{q}_2, t) \\ & +\Psi_{2,f}^{(s)}(q_1, q_2, t)\Psi_2^{(r)}(\tilde{q}_1, \tilde{q}_2, t)\end{aligned} \tag{53}$$

If $\Psi_1^{(r)}$ and $\Psi_2^{(r)}$ are macroscopically distinct then we have effective collapse

- **Conclusion**

→ We presented a pilot-wave model for quantum electrodynamics

  → Beables correspond to bosonic degrees of freedom of the quantum state, not to fermionic degrees

→ One can extend this pilot-wave model to the Standard Model of high energy physics

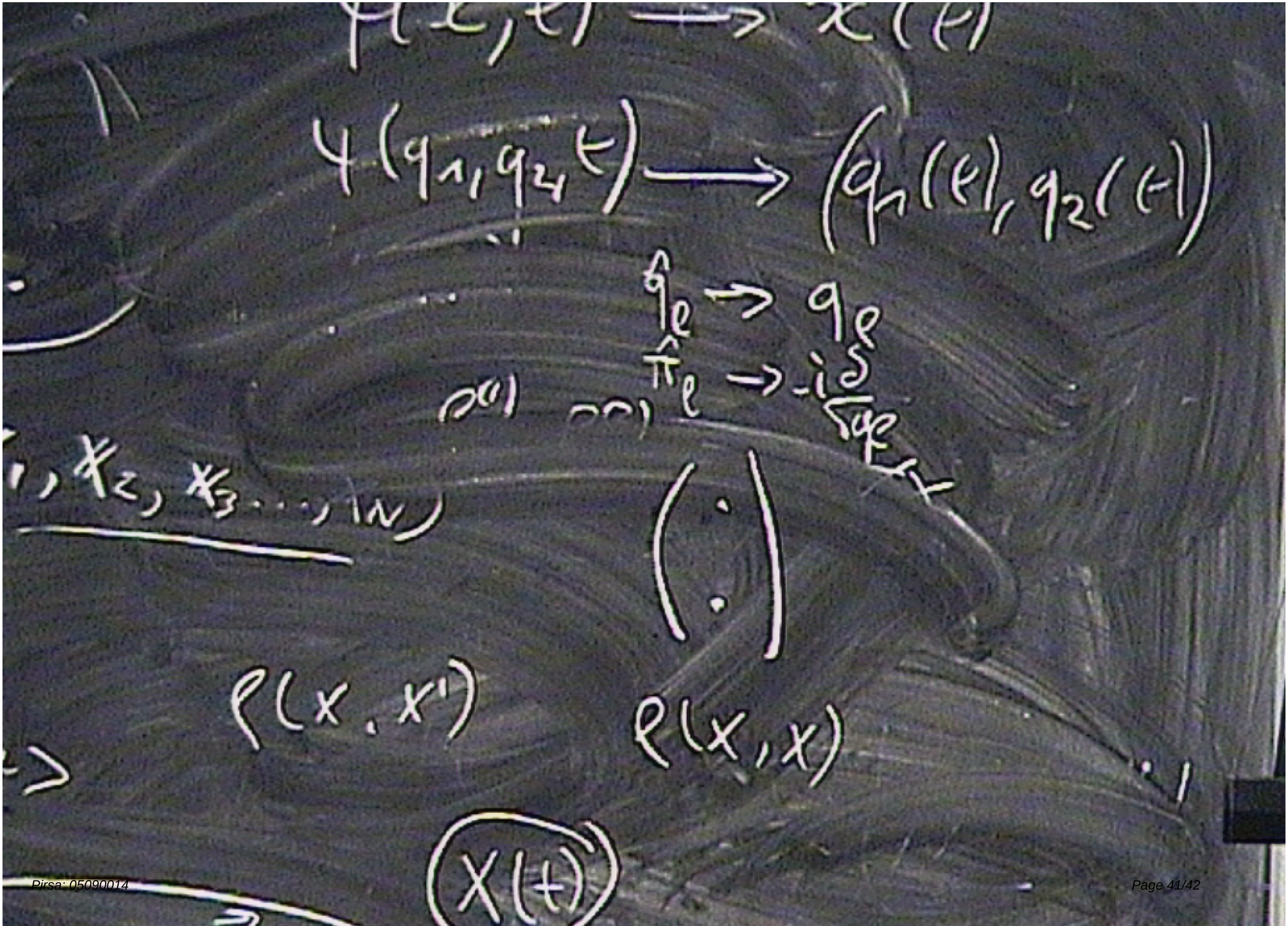
The model for quantum electrodynamics

- QED in the Coulomb gauge ($\boldsymbol{\nabla} \cdot \mathbf{A} = 0$, $A_0 = -\frac{1}{\nabla^2} j^0$)

$$\widehat{H} = \underbrace{\frac{1}{2}\int d^3x \left(\widehat{\boldsymbol{\Pi}}^T \cdot \widehat{\boldsymbol{\Pi}}^T - \widehat{\mathbf{A}}^T \cdot \nabla^2 \widehat{\mathbf{A}}^T\right)}_{free\ EM\ field}$$
$$+ \underbrace{\int d^3x \widehat{\psi}^\dagger \left(-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla}\right) \psi}_{free\ Dirac\ field}$$
$$\underbrace{-\int d^3x \widehat{\mathbf{A}}^T \cdot \widehat{\mathbf{j}}}_{interaction\ term} + \underbrace{\frac{1}{2}\int d^3x d^3y \frac{\widehat{j}^0(\mathbf{x})\widehat{j}^0(\mathbf{y})}{4\pi|\mathbf{x}-\mathbf{y}|}}_{Coulomb\ potential} \tag{36}$$

$$[\widehat{A}_i^T(\mathbf{x}), \widehat{\Pi}_j^T(\mathbf{y})] = i\left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}\right)\delta(\mathbf{x}-\mathbf{y}) \tag{37}$$

$$\{\widehat{\psi}_a(\mathbf{x}), \widehat{\psi}_b^\dagger(\mathbf{y})\} = \delta_{ab}\delta(\mathbf{x}-\mathbf{y}) \tag{38}$$


ψ(q_1, q_2, t) ⟶ (q_1(t), q_2(t))
q̂_ℓ → q_ℓ
π̂_ℓ → -i ∂/∂q_ℓ
ρ(x, x')
ρ(x, x)
X(t)

The model for quantum electrodynamics

- QED in the Coulomb gauge ($\boldsymbol{\nabla} \cdot \mathbf{A} = 0, A_0 = -\frac{1}{\nabla^2} j^0$)

$$
\begin{aligned}
\widehat{H} \;=\; & \underbrace{\frac{1}{2}\int d^3x \left(\widehat{\mathbf{\Pi}}^T \cdot \widehat{\mathbf{\Pi}}^T - \widehat{\mathbf{A}}^T \cdot \nabla^2 \widehat{\mathbf{A}}^T \right)}_{free\ EM\ field} \\
& \underbrace{+ \int d^3x \widehat{\psi}^\dagger \left(-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} \right) \psi}_{free\ Dirac\ field} \\
& \underbrace{- \int d^3x \widehat{\mathbf{A}}^T \cdot \widehat{\mathbf{j}}}_{interaction\ term} \;+\; \underbrace{\frac{1}{2}\int d^3x d^3y \frac{\widehat{j}^0(\mathbf{x}) \widehat{j}^0(\mathbf{y})}{4\pi |\mathbf{x}-\mathbf{y}|}}_{Coulomb\ potential}
\end{aligned}
\tag{36}
$$

$$
[\widehat{A}_i^T(\mathbf{x}), \widehat{\Pi}_j^T(\mathbf{y})] = i \left(\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right) \delta(\mathbf{x}-\mathbf{y}) \tag{37}
$$

$$
\{\widehat{\psi}_a(\mathbf{x}), \widehat{\psi}_b^\dagger(\mathbf{y})\} = \delta_{ab}\delta(\mathbf{x}-\mathbf{y}) \tag{38}
$$