Title: Synchronization: A universal concept in nonlinear sciences

Date: Sep 28, 2005 02:00 PM

URL: http://pirsa.org/05090013

Abstract: Synchronization phenomena are abundant in nature, science, engineering and social life. Synchronization was first recognized by Christiaan Huygens in 1665 for coupled pendulum clocks; this was the beginning of nonlinear sciences. First, several examples of synchronization in complex systems are presented, such as in organ pipes, fireflies, epilepsy and even in the (in)stability of large mechanical systems as bridges. These examples illustrate that, literally speaking, subsystems are able to synchronize due to interaction if they are able to communicate. Second, general physical mechanisms for synchronization and de-synchronization phenomena in coupled complex systems are presented and conditions for synchronizability are discussed. It is explained that diffusion properties give a crucial insight into this problem. I will show that the general concepts of curvature and recurrence are helpful to uncover complex synchronization. Third, applications of these new techniques are given. They range from El Nino Â– Monsoon interactions via electrochemical oscillators and lasers to cognitive processes during reading and to neuroscience.

5 Alexander Dolgov, Dark energy from a particle physics perspective
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19 Reka Albert Lessons from modeling the dynamics of genetic regulato
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Synchronization: A Universal Concept in Nonlinear Sciences

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Synchronization

Take home messages:

- Synchronization is **not a state** but a **process** of adjusting rhythms due to interaction.
- When subsystems (e.g. people, animals, cells, neurons) **synchronize**, they also can **communicate**.

Nonlinear Sciences

Start in 1665 by Christiaan Huygens

Pendulum Clocks

- Christiaan Huygens:

  Pendelum clocks hanging at the same wooden beam (half-timber house)

  It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each pendulum in opposite swings were so much in agreement that they never receded the last bit from each other...Further, if this agreement was disturbed by some interference, it reastablished itself in a short time...after a careful examination I finally found that the cause of this is due to the motion of the beam, even though this is hardly perceptible (Huygens, 1673)

Examples: Sociology, Biology, Acoustics, Mechanics

- Hand clapping (common rhythm)
- Ensemble of doves (wings in synchrony)
- Organ pipes standing side by side – quenching or playing in unison (Lord Rayleigh, 19th century)
- Fireflies in south east Asia (Kämpfer, 17th century)

Example Mechanics

London´s Millenium Bridge

- pedestrian bridge
- 325 m steel bridge over the Themse
- Connects city near St. Paul´s Cathedral with Tate Modern Gallery

Big opening event in 2000 -- movie

Bridge Opening

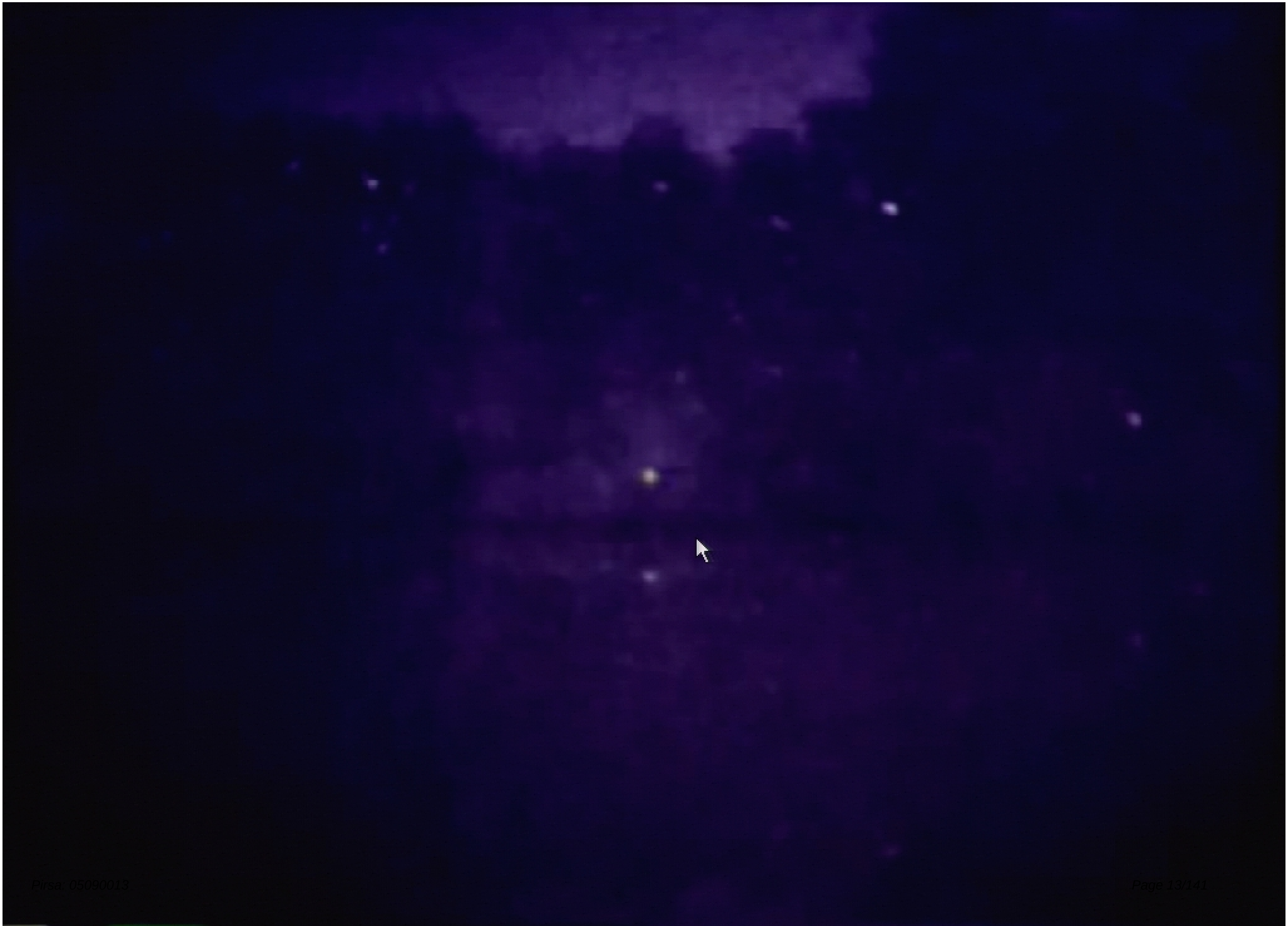
Bridge Opening

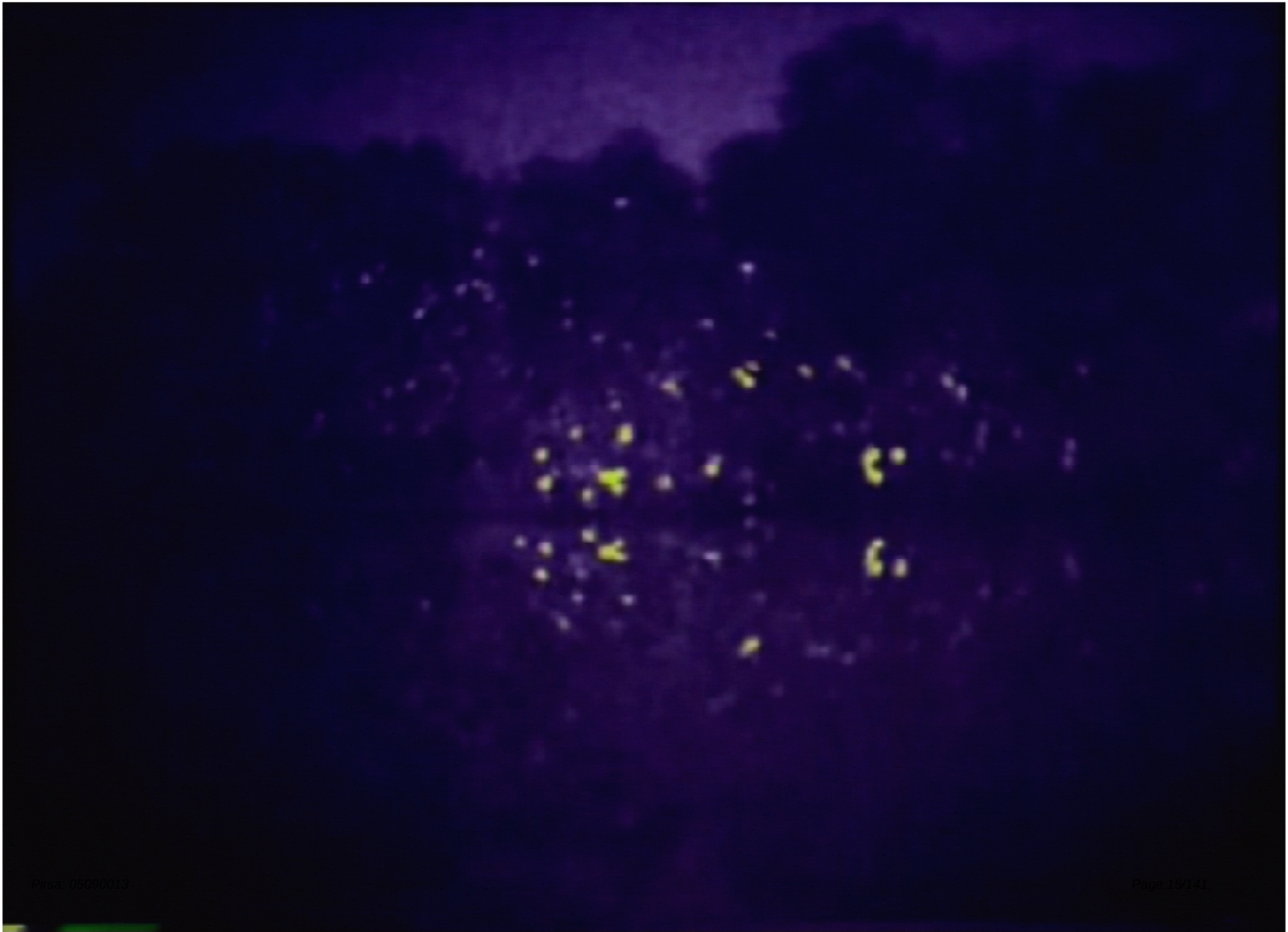
Bridge Opening

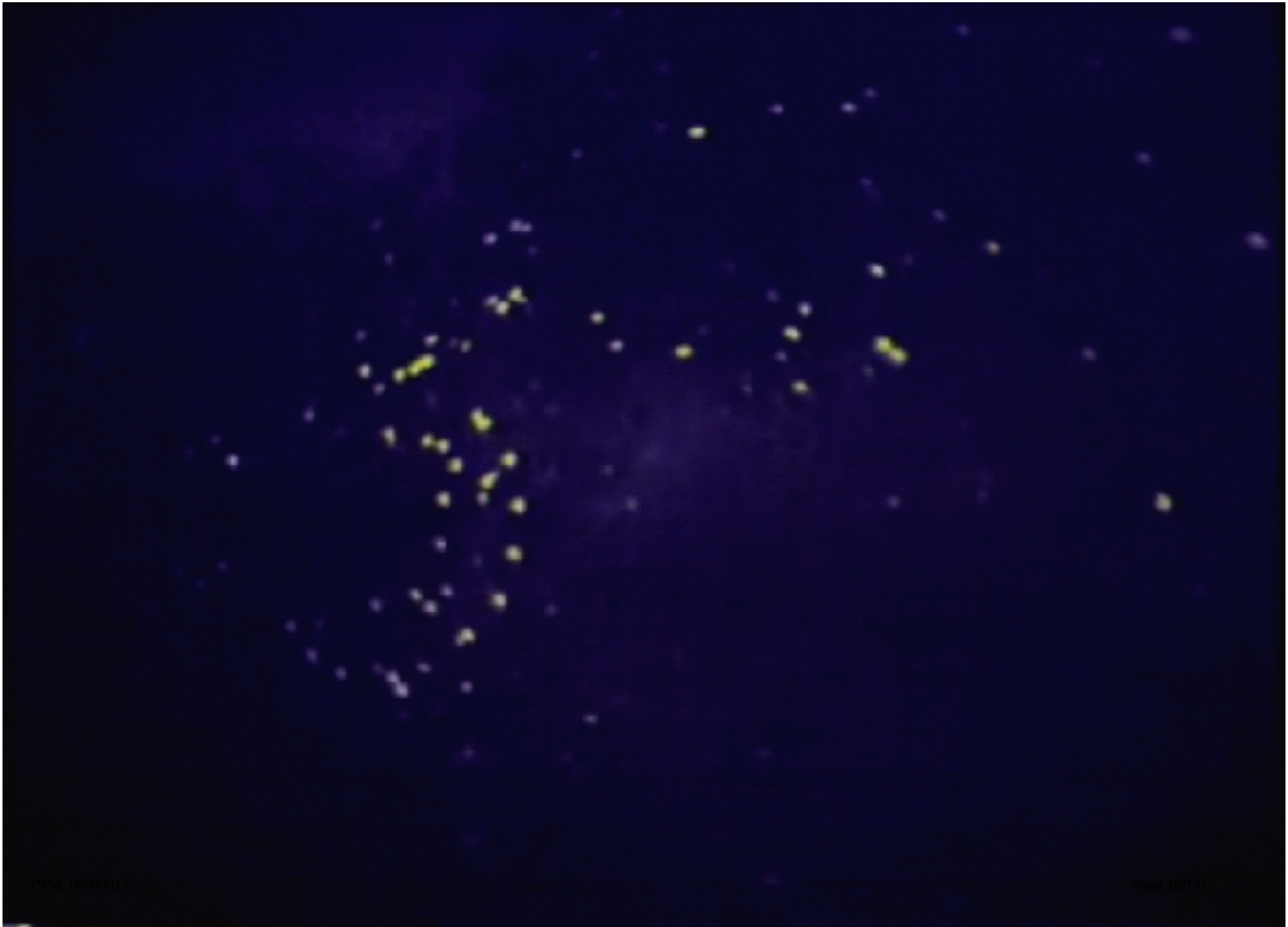
Bridge Opening

- Unstable modes always there
- Mostly only in vertical direction considered
- Here: extremely strong unstable lateral Mode
  – If there are sufficient many people on the bridge we are beyond a threshold and synchronization sets in

  (Kuramoto-Synchronizations-Transition)







Phase Synchronization in Complex Systems

Most systems not simply periodic

➔ Synchronization in complex (non-periodic) systems

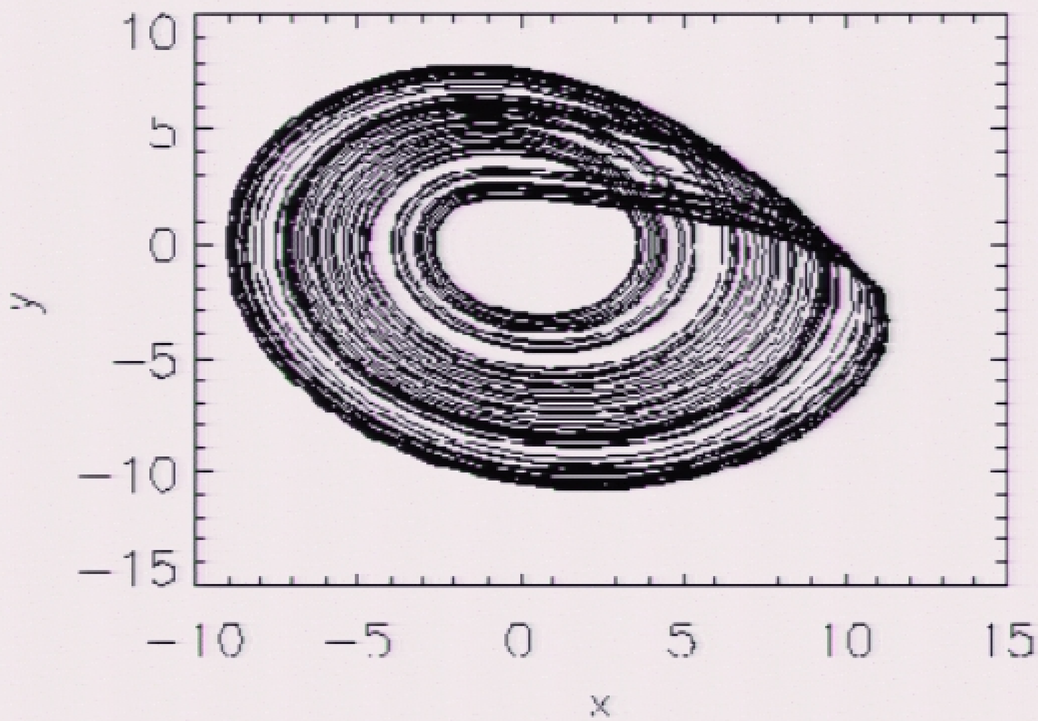
Interest in **Phase Synchronization**

How to retrieve a phase in complex dynamics?

Phase Definitions in Coherent Systems

Rössler Oscillator – 2D Projection

Phase-coherent



phase dynamics in periodic systems

- Linear increase of the phase

$$\varphi(t) = t\ \omega$$

$\omega = 2\Pi / T$ – frequency of the periodic dynamics

T – period length

➔ $\varphi(t)$ increases 2Π per period

$$d\varphi(t) / dt = \omega$$

Phase Definitions

Analytic Signal Representation (Hilbert Transform)

$$\psi(t) = s(t) + j\tilde{s}(t) = A(t)e^{j\phi(t)}$$

$$\tilde{s}(t) = \frac{1}{\pi}\,\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{s(\tau)}{t-\tau}\,d\tau$$

Direct phase

$$\phi(t) = \arctan\left(y(t)/x(t)\right)$$

Phase from Poincare´ plot

$$\phi(t) = 2\pi k + 2\pi\frac{t-\tau_k}{\tau_{k+1}-\tau_k} \qquad (\tau_k < t < \tau_{k+1})$$

(Rosenblum, Pikovsky, Kurths, Phys. Rev. Lett., 1996)

Phase Definitions in Coherent Systems

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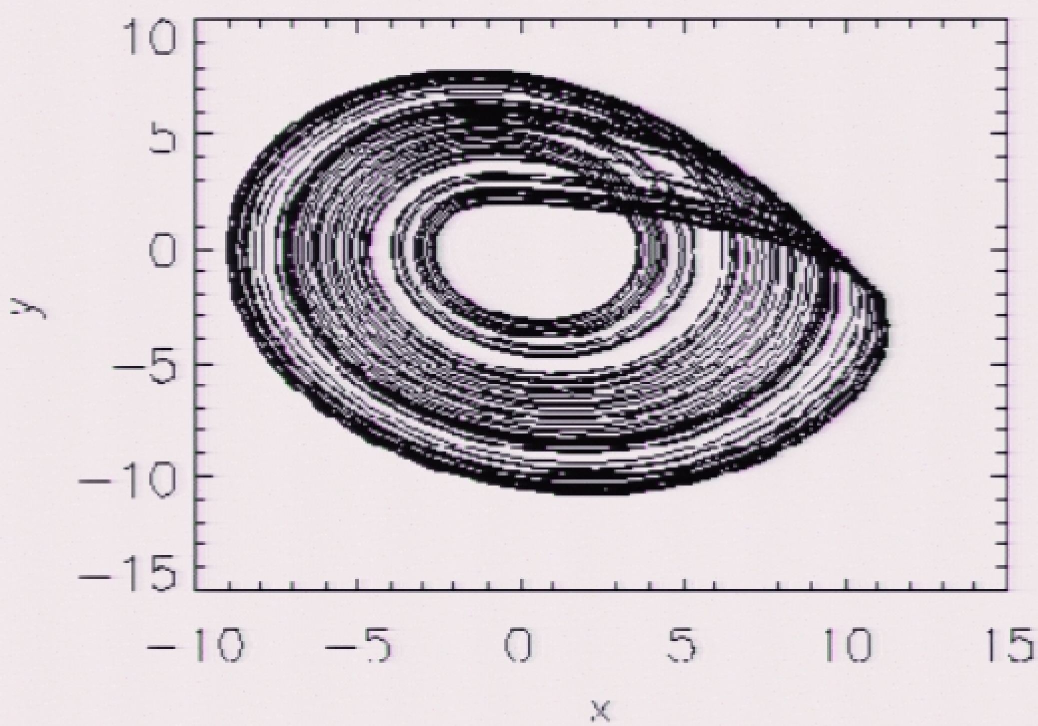

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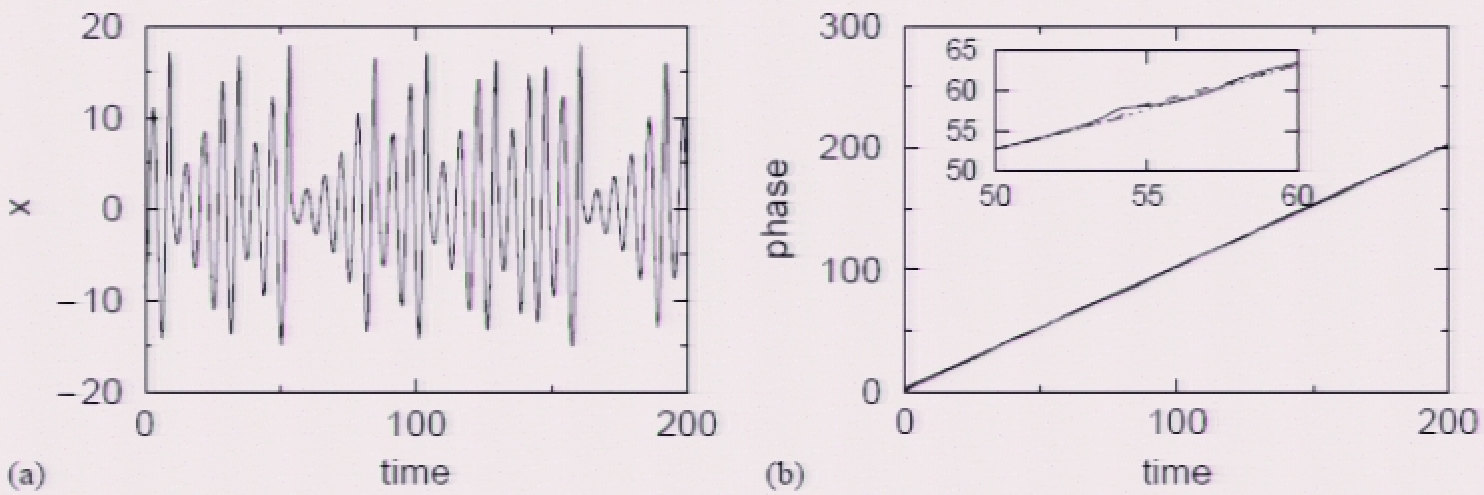
Phase for coherent chaotic oscillators



:. 3.3. (a) Chaotic signal $x(t)$ of the chaotic Rössler oscillator. (b) Phase of the chaotic signal. Solid line: phase of . (3.5); dashed line: phase of Eq. (3.7); and dotted line: phase of Eq. (3.8).

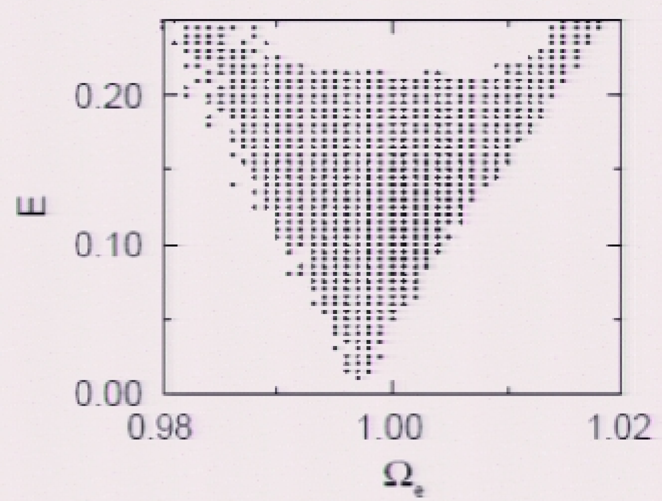
Phase dynamics and phase synchronization phenomena very similar in periodic and phase-coherent chaotic systems, e.g. one zero Lyapunov exponent becomes negative

Synchronization due to periodic driving

$$\dot{x} = -\omega y - z + E\sin(\Omega_e t)$$

$$\dot{y} = \omega x + a y\ ,$$

$$\dot{z} = f + z(x - c)$$

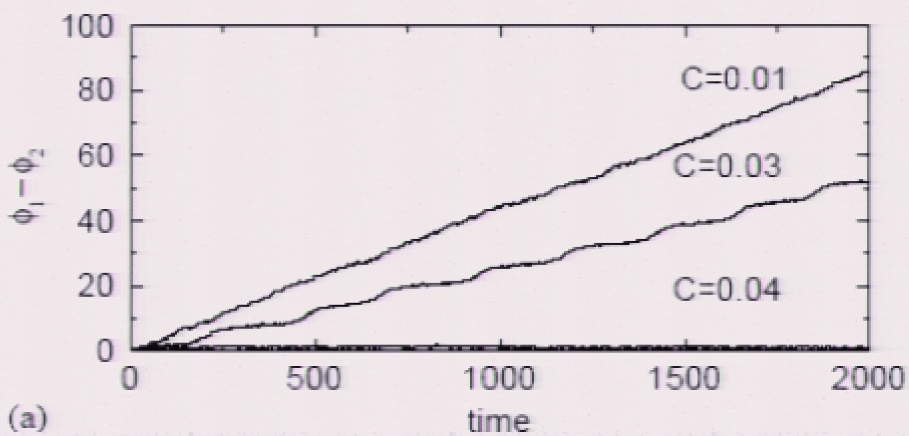


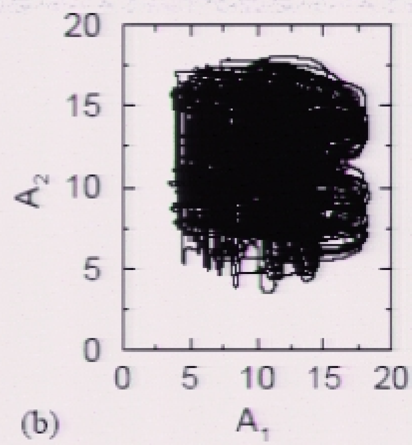
Synchronization of two coupled non-identical chaotic oscillators

$$\dot{x}_{1,2} = -\omega_{1,2} y_{1,2} - z_{1,2} + C(x_{2,1} - x_{1,2}) ,$$

$$\dot{y}_{1,2} = \omega_{1,2} x_{1,2} + a y_{1,2} ,$$

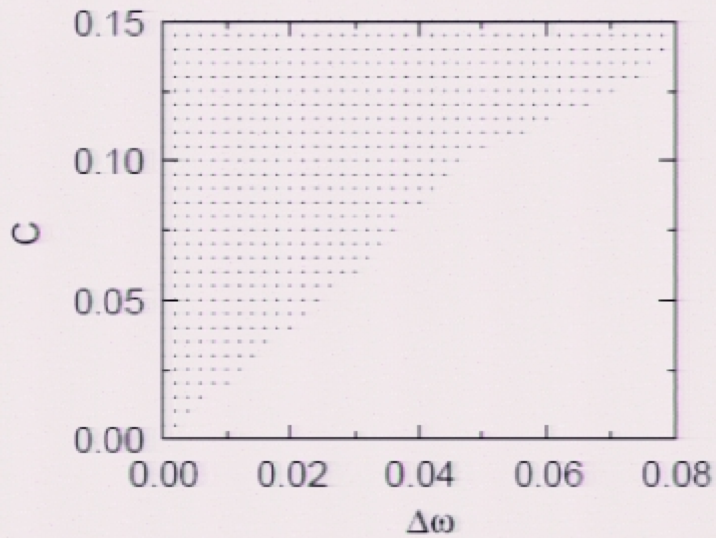
$$\dot{z}_{1,2} = f + z_{1,2}(x_{1,2} - c) ,$$





Phases are synchronized – Amplitudes almost uncorrelated

Synchronization of two coupled chaotic oscillators



Synchronization region

C – coupling strength

Δω – parameter mismatch

Two coupled non-identical oscillators

$$\phi = \arctan(y/x), \quad A = (x^2 + y^2)^{1/2} \ ,$$

we get

$$\dot{A}_{1,2} = aA_{1,2}\sin^2\phi_{1,2} - z_{1,2}\cos\phi_{1,2} + C(A_{2,1}\cos\phi_{2,1}\cos\phi_{1,2} - A_{1,2}\cos^2\phi_{1,2}) \ ,$$

$$\dot{\phi}_{1,2} = \omega_{1,2} + a\sin\phi_{1,2}\cos\phi_{1,2} + z_{1,2}/A_{1,2}\sin\phi_{1,2}$$

$$- C(A_{2,1}/A_{1,2}\cos\phi_{2,1}\sin\phi_{1,2} - \cos\phi_{1,2}\sin\phi_{1,2}) \ ,$$

$$\dot{z}_{1,2} = f - cz_{1,2} + A_{1,2}z_{1,2}\cos\phi_{1,2} \ .$$

Equation for the slow phase θ: $\phi_{1,2} = \omega_0 t + \theta_{1,2}$.

Averaging yields (Adler-like equation):

$$\frac{\mathrm{d}}{\mathrm{d}t}(\theta_1 - \theta_2) = 2\Delta\omega - \frac{C}{2}\left(\frac{A_2}{A_1} + \frac{A_1}{A_2}\right)\sin(\theta_1 - \theta_2)$$

Synchronization threshold

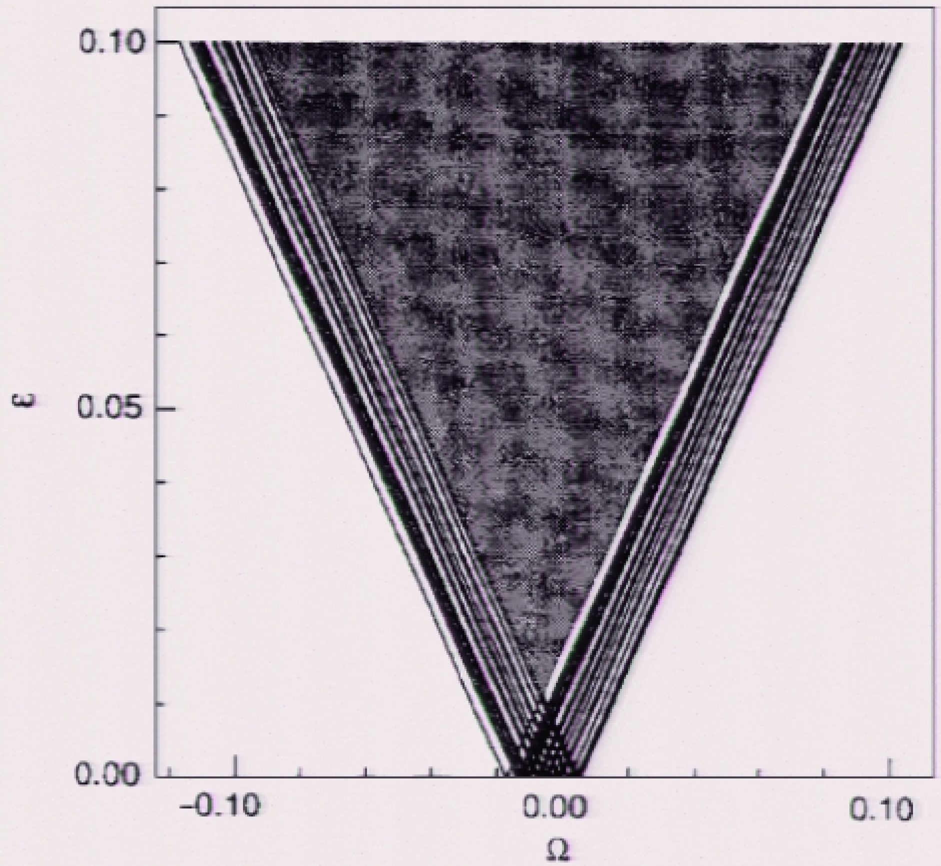
Fixed point solution (by neglecting amplitude fluctuations)

$$\theta_1 - \theta_2 = \arcsin \frac{4\Delta\omega A_1 A_2}{C(A_1^2 + A_2^2)}$$

Fixed point stable (synchronization) if coupling is larger than

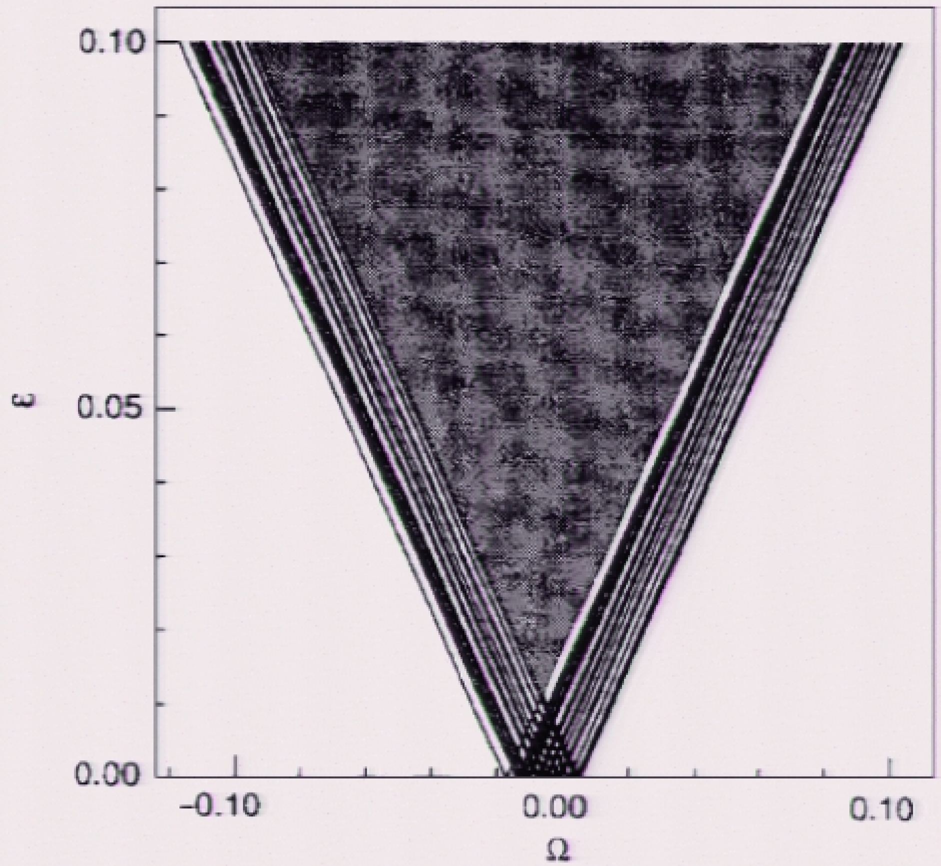
$$C_{\mathrm{PS}} = 4\Delta\omega A_1 A_2/(A_1^2 + A_2^2).$$

Understanding synchronization by means of unstable periodic orbits



Phase-locking regions for periodic orbits with periods 1-5; overlapping region – region of full phase synchronization (dark, = natural frequency of chaotic system – ext force)

Understanding synchronization by means of unstable periodic orbits



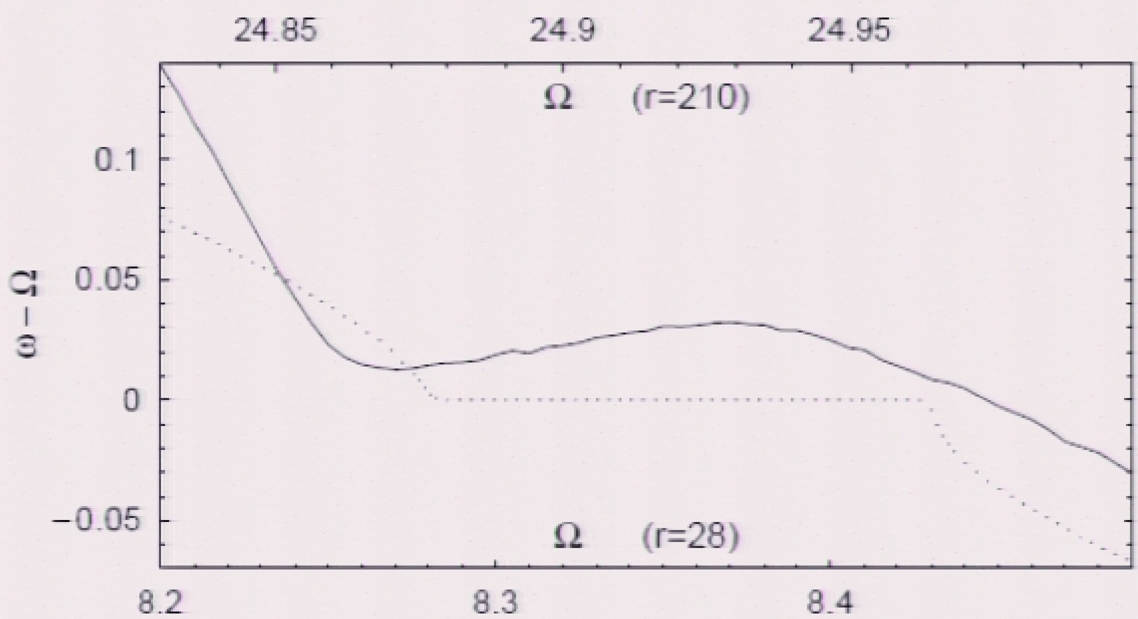
Phase-locking regions for periodic orbits with periods 1-5; overlapping region – region of full phase synchronization (dark, = natural frequency of chaotic system – ext force)

Imperfect Phase Synchronization

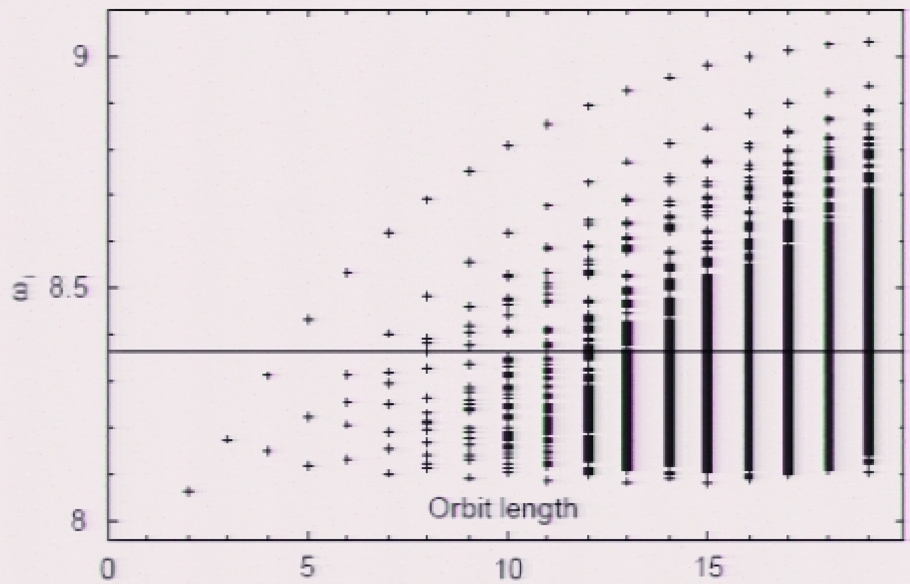
$$\dot{x} = 10(y - x)\,,$$

$$\dot{y} = rx - y - xz\,,$$

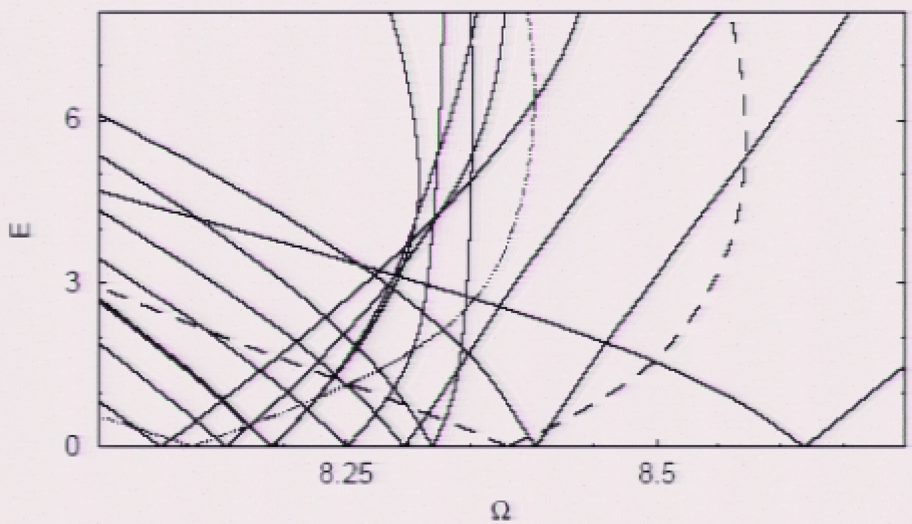
$$\dot{z} = xy - 2.667z + E\cos(\Omega t)$$



Unstable Periodic Orbits – usual Lorenz system



Synchronization regions of UPOs in the usual Lorenz system

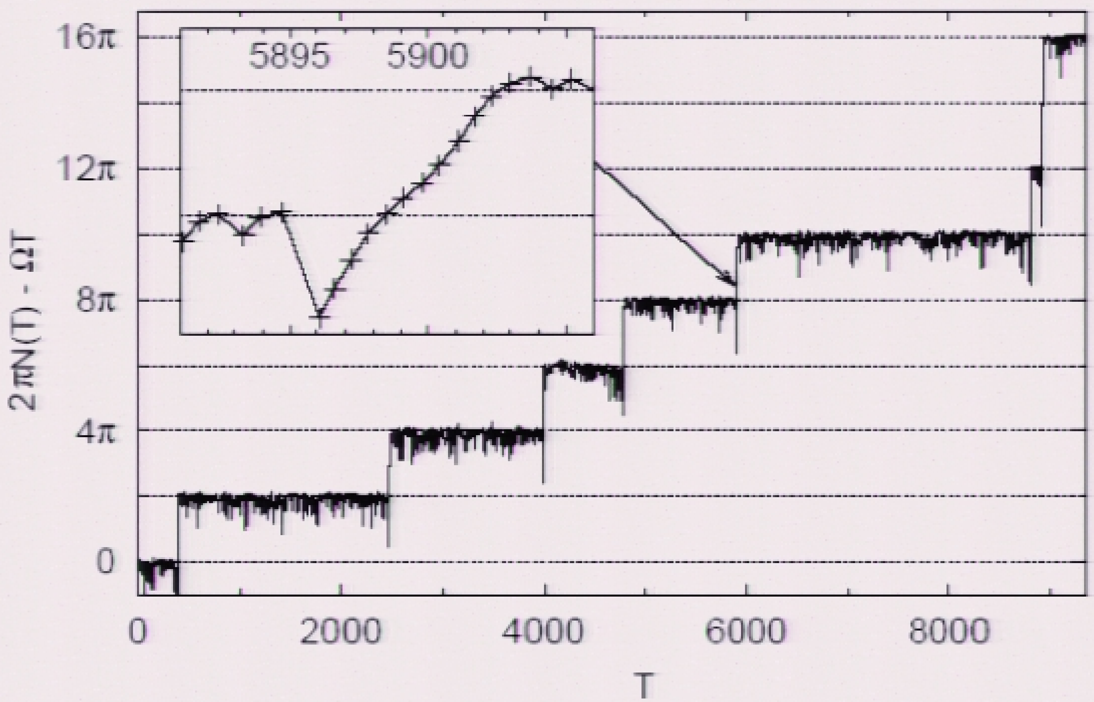


Solid lines – 1:1 synchronization

Dashed line – 14:15 synchronization

Dotted line – 18:20 synchronization

Phase „jumps“ in the forced usual Lorenz model



Problems with phase jumps

- Phase synchronization: difference of phases of two (more) subsystems is bounded
- Phase jumps of +/- 2Π occur due to
  - at the borderline of synchro region
  - influence of noise (Stratonovich)
  - broad variety of unstable periodic orbits, as in the Lorenz system (deterministic effect)

➔ imperfect phase synchronization

(Zaks, Park, Rosenblum, Kurths: Phys Rev Lett 1999)

Cyclic relative phase

- How to consider this problem?
- Phase synchronization in a statistical sense
- Instead of the strong condition

$$|n\phi_1(t) - m\phi_2(t) - \delta| < \text{const}$$

We consider the cyclic relative phase

$$\Psi_{n,m} = \varphi_{n,m} \bmod 2\pi$$

And analyze the frequency distribution of the cyclic relative phase

Statistical description of phase synchronization

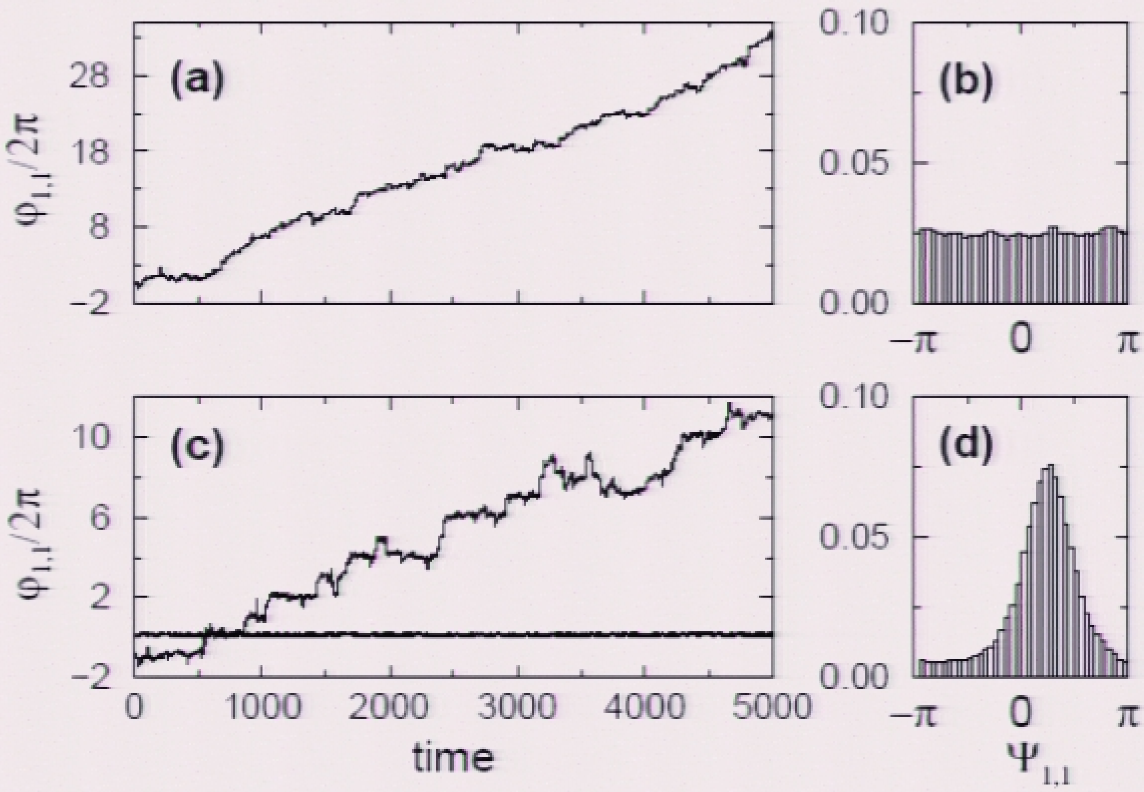


Fig. 3. Relative phase $\varphi_{1,1} = \phi_1 - \phi_2$ and distribution of $\Psi_{1,1} = \varphi_{1,1} \bmod 2\pi$ for the case of uncoupled (a,b) and coupled (c,d) non-identical chaotic systems perturbed by noise. The horizontal line in (c) corresponds to the absence of noise: in this case the phase difference fluctuates around some constant value due to influence of chaotic amplitudes. These fluctuations are rather small (barely seen in this scale), and no phase slips are observed; this fact is explained by the high phase coherence properties of the Rössler attractor. In contrast to the noisy case, here we observe both frequency and phase locking.

Applications in various fields

- Electronic circuits (Parlitz...)
- Plasma tubes (Rosa)
- Driven or coupled lasers (Roy, Arecchi...)
- Electrochemistry (Hudson)
- Controlling (Belykh)
- Convection (Maza...)
- Climate (Fraedrich, Maraun...)
- Epilepsy (Lehnertz)

Synchronization in physiological systems

- Cardio-vascular system (Schäfer et al., Nature 1998)

- Brain and muscle activity data (Parkinsonian patients) (Tass et al., Phys. Rev. Lett. 1998)

Cardio-respiratory System

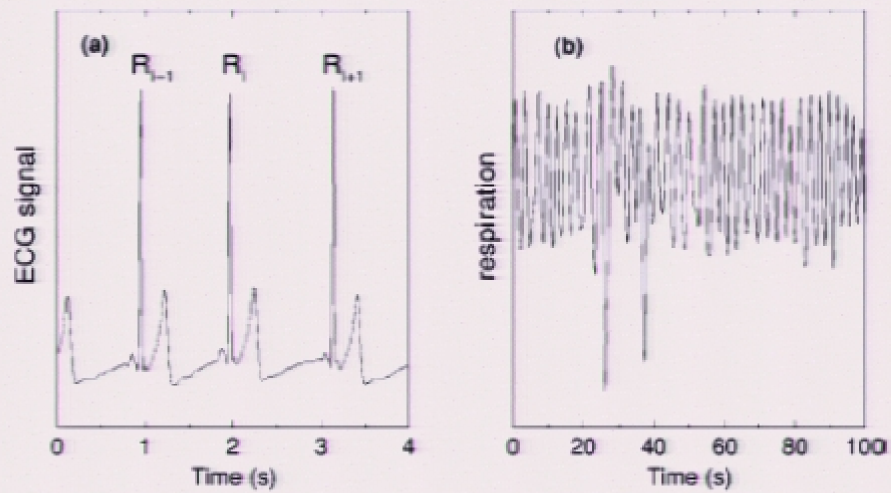
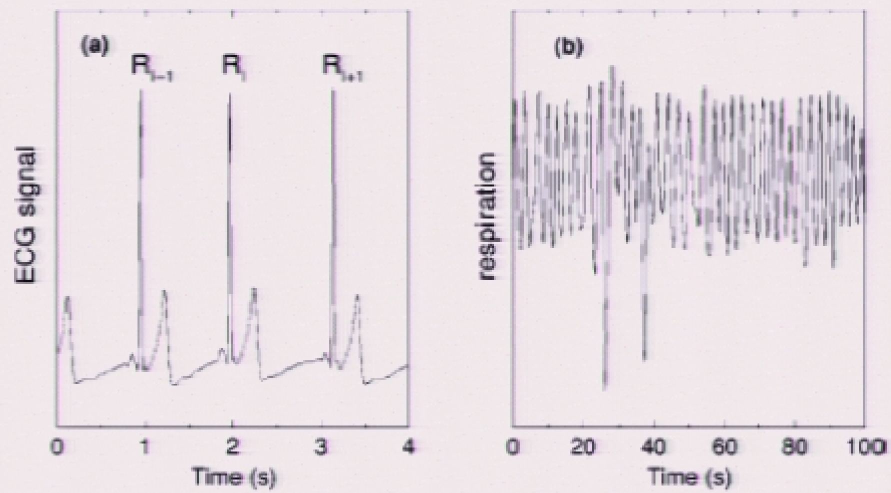


FIG. 1. Short segments of an electrocardiogram with the R peaks marked (a) and of a respiratory signal (b); both signals are in arbitrary units.

Cardio-respiratory System



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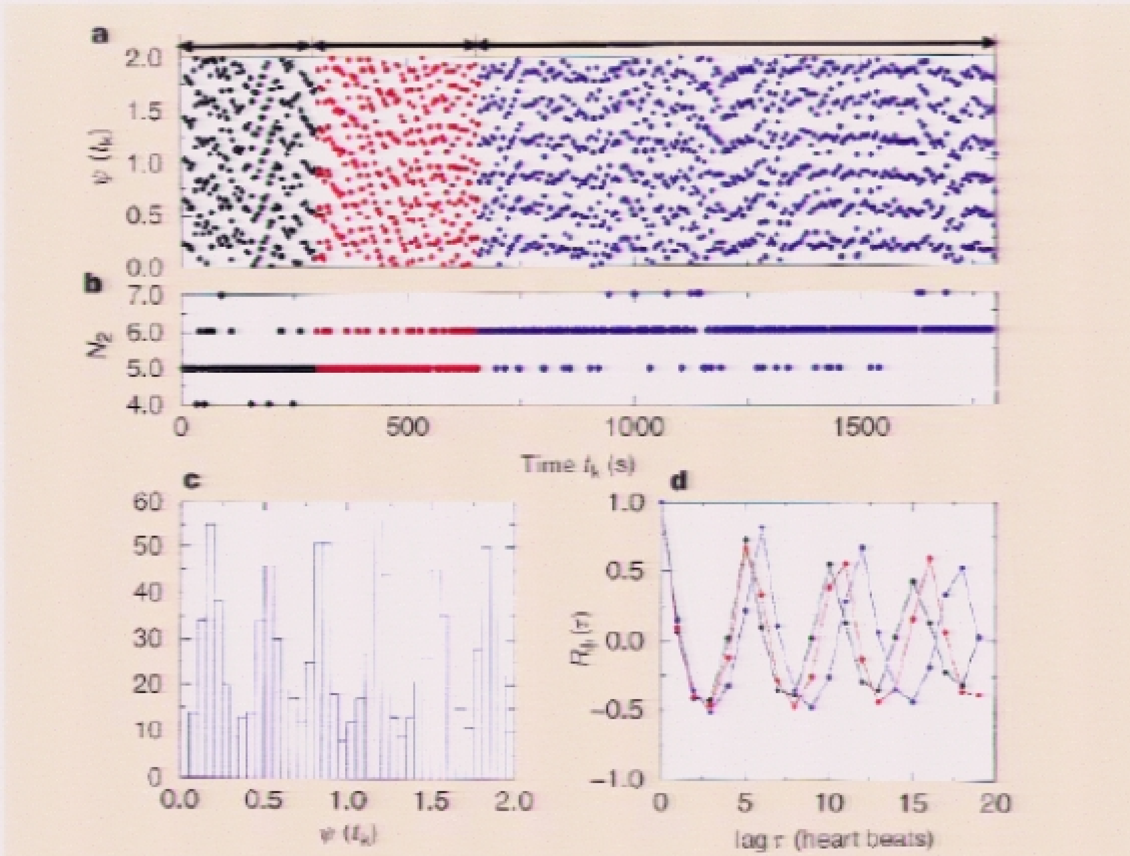

Figure 1 Analysis of cardiorespiratory cycles. a, Cardiorespiratory synchrogram, showing the transition (red) from 5:2 frequency locking (black) to 3:1 phase locking (blue). Each point shows the normalized relative phase of a heartbeat within two adjacent respiratory cycles $\psi(t_k)=(\phi_r(t_k) \bmod 4\pi)/2\pi$. b, Number of heartbeats within two adjacent respiratory cycles. c, Histogram of phases. The six horizontal stripes in the blue region of the CRS result in six well-pronounced peaks in the distribution of phases. d, Autocorrelation function of phases $R_\psi(\tau)=\Sigma_k(\psi(t_k)-\langle\psi\rangle)(\psi(t_{k+\tau})-\langle\psi\rangle)/\Sigma_k(\psi(t_k)-\langle\psi\rangle)^2$. The coloured curves correspond to respective regions.

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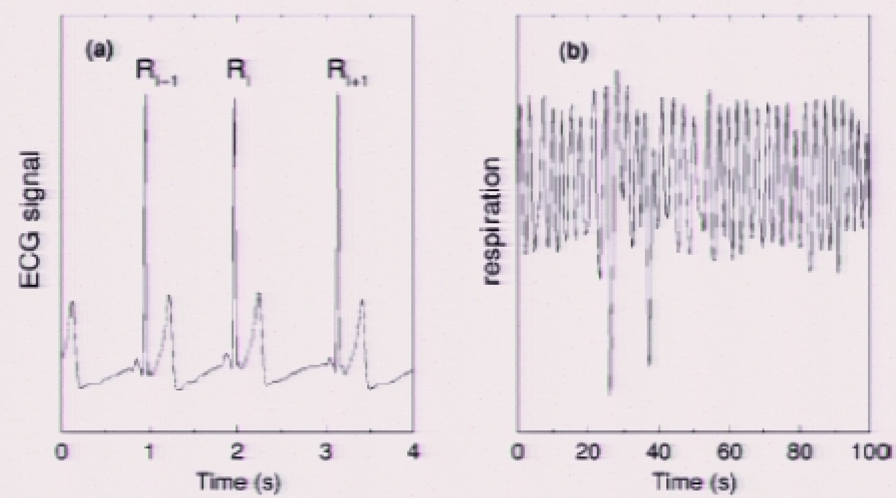
Schäfer, Rosenblum, Abel, Kurths: Nature, 1998

Cardio-respiratory System



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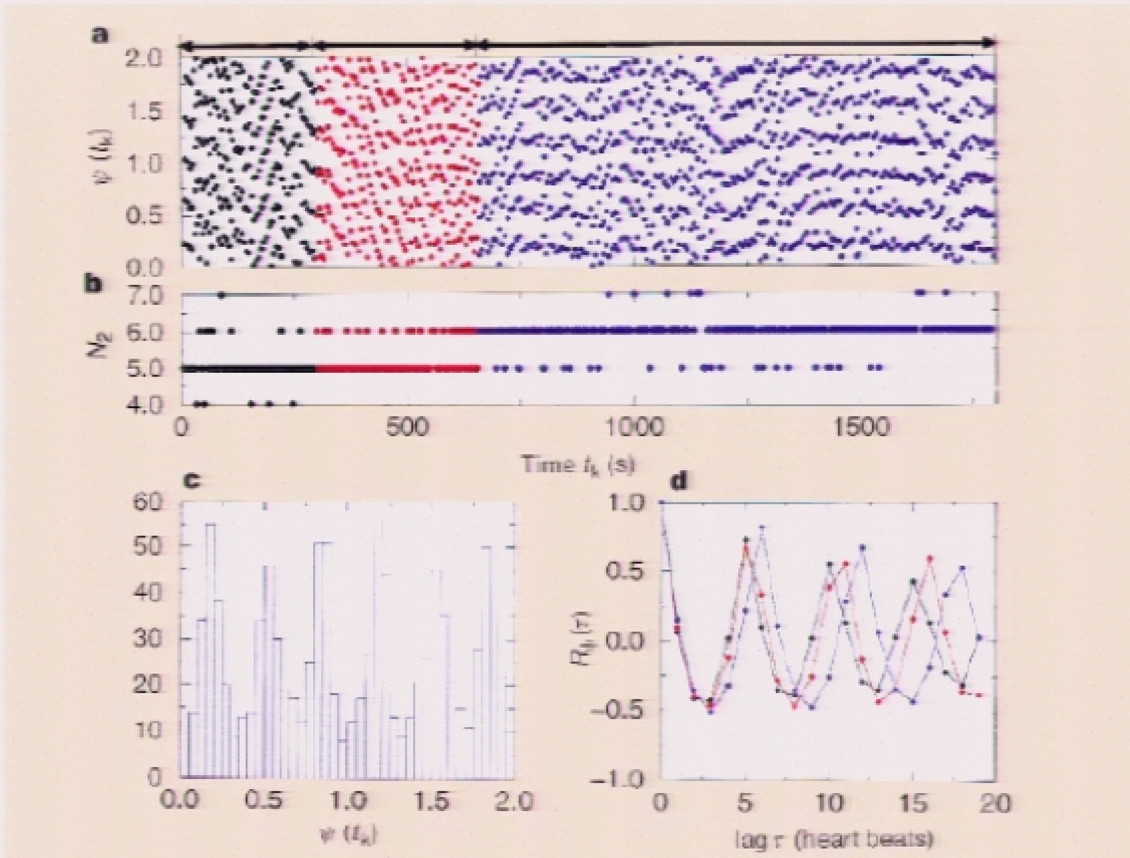

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Schäfer, Rosenblum, Abel, Kurths: Nature, 1998

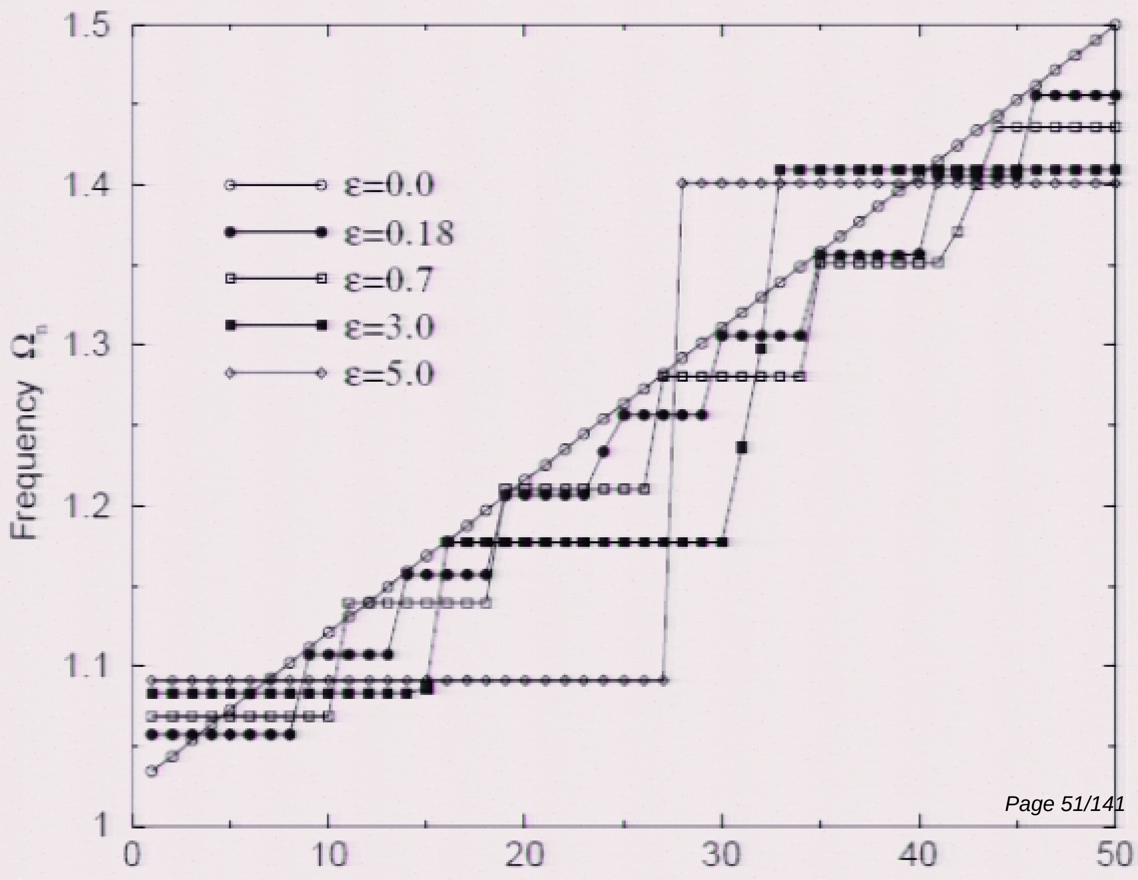
Synchronization in spatially extended systems

$$\dot{x}_n = -\omega_n y_n - z_n \ ,$$

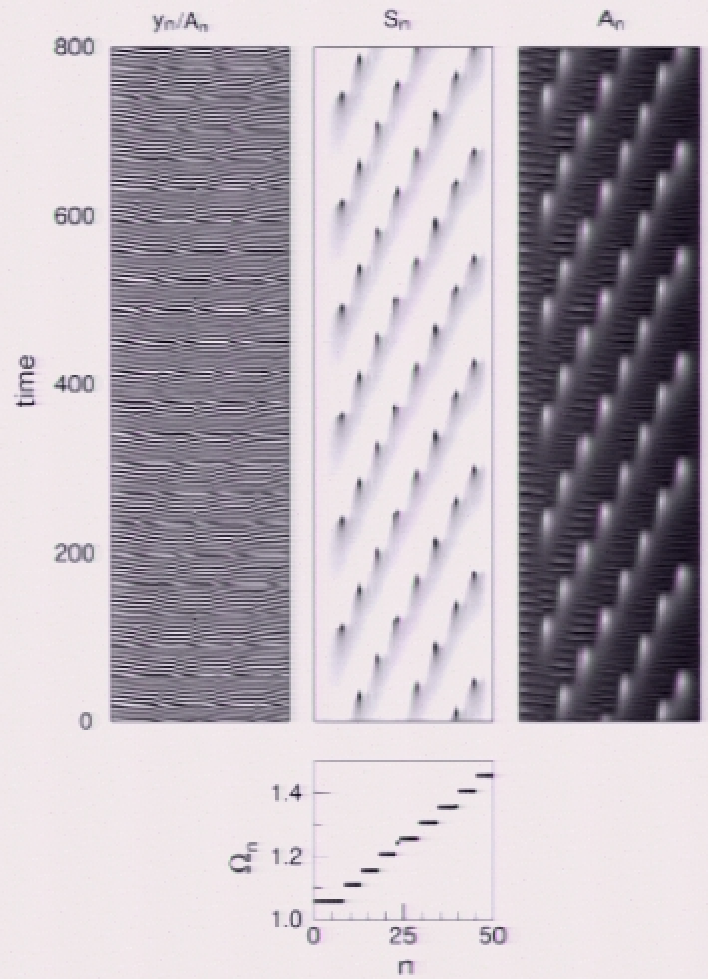
$$\dot{y}_n = \omega_n x_n + a y_n + \epsilon(y_{n+1} - 2y_n + y_{n-1}) \ ,$$

$$\dot{z}_n = 0.4 + (x_n - 8.5) z_n \ .$$

Chain of diffusively coupled Roessler oscillators



Clustered phase synchronization



$$s_n = \sin^2\left(\frac{\phi_{n+1}(t) - \phi_n(t)}{2}\right) \qquad A_n = \sqrt{x_n^2 + y_n^2}\ .$$

Example: Chain of globally coupled spiking-bursting maps

$$\begin{cases} x(i,n+1) = \frac{\alpha_i}{1+x(i,n)^2} + y(i,n) + \frac{\varepsilon}{N}\sum_{j=1}^{N} x(j,n) \\ y(i,n+1) = y(i,n) - \sigma_i x(i,n) - \beta_i, \end{cases}$$

Rulkov Map

- x fast variable
- y slow variable
- α randomly distributed in [4.1, 4.4]

➔ Spiking and bursting dynamics

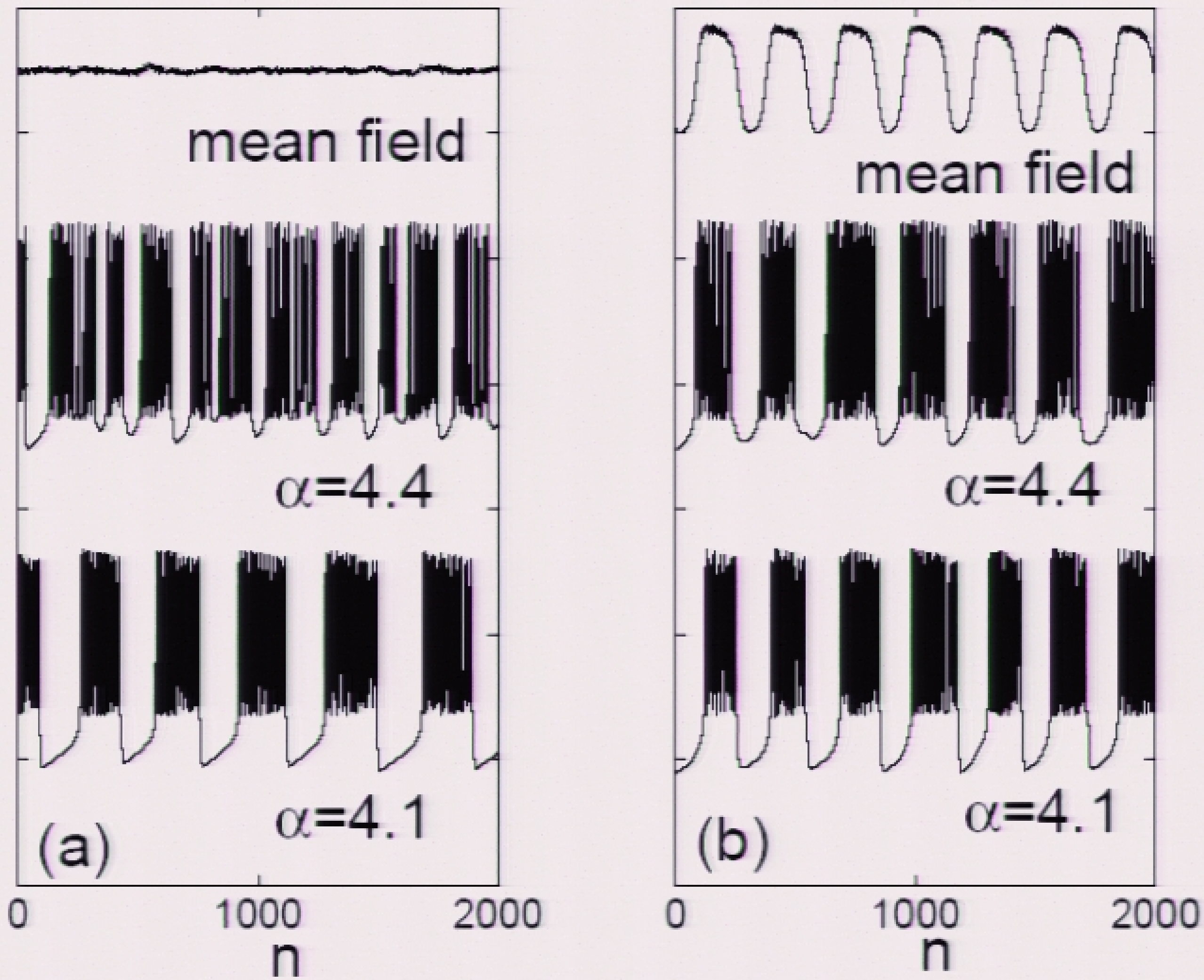


FIG. 1: Realizations of $x(i, n)$ of two neurons and the mean field from (a) an uncoupled ensemble ($\varepsilon = 0$) and (b) a coupled ensemble ($\varepsilon = 0.04$, synchronization of bursts is achieved), in the absence of external signal ($d = 0$). Different values of α_i are implemented, $\sigma = \beta = 0.001$. Here

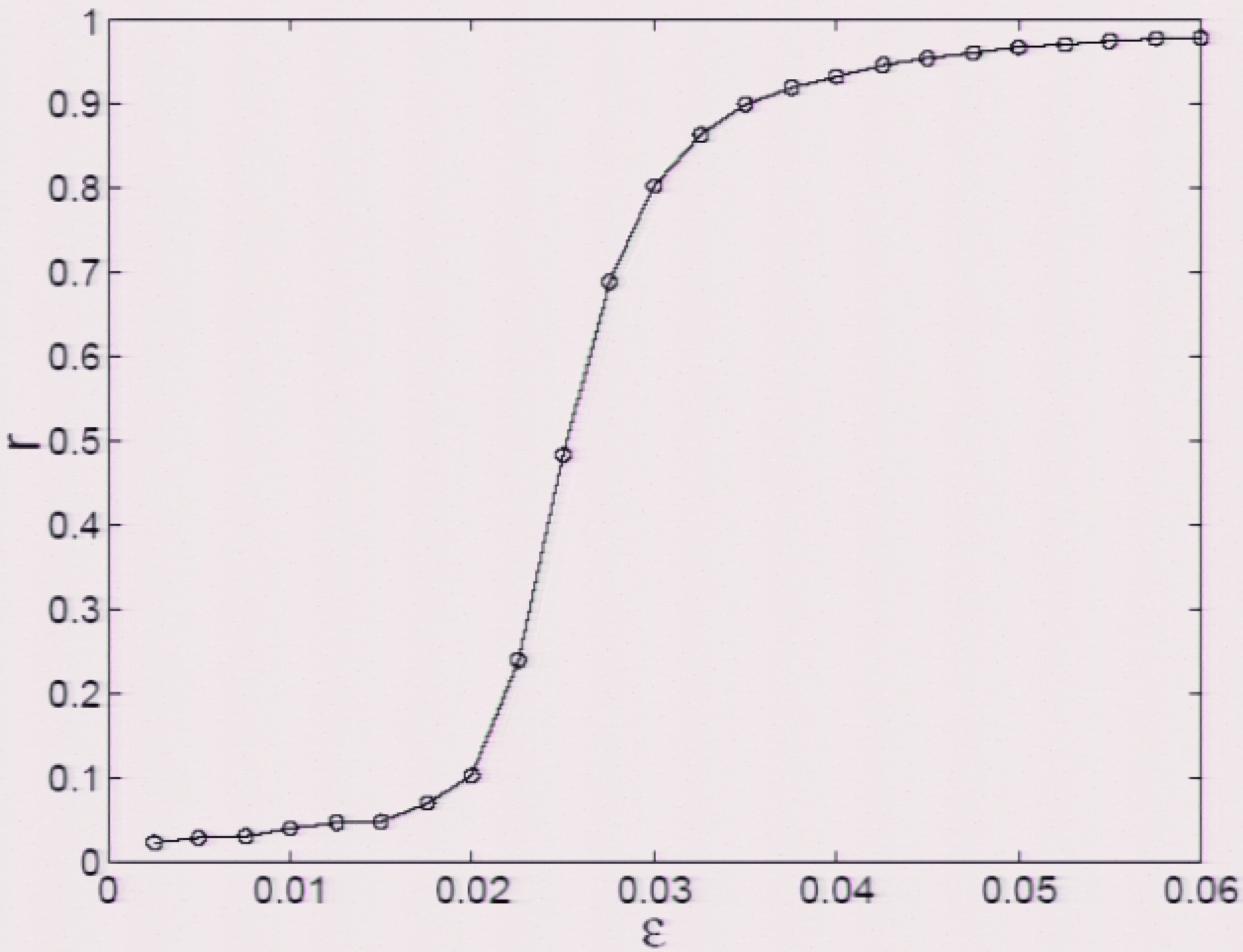


FIG. 3: The order parameter r vs. mean field coupling coefficient ε indicates a second-order phase transition to CPS of bursting ($N = 1000$).

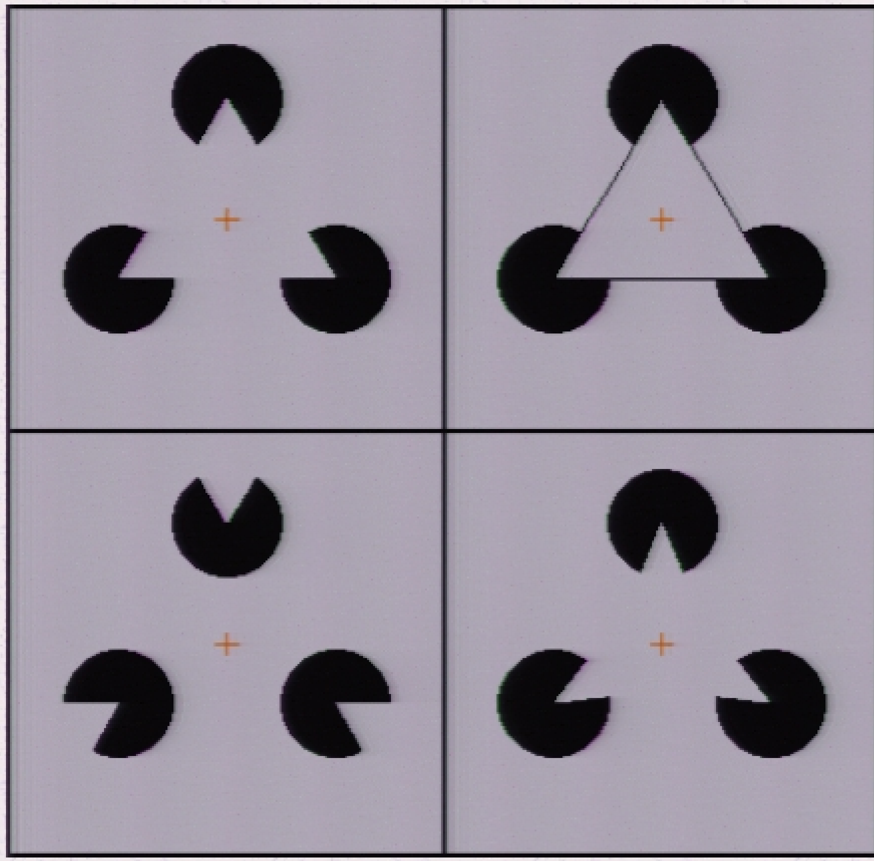
Synchronization only of bursts and not of spikes!

Ivanchenko, Osipov, Shalfeev, Kurths: Phys Rev Lett 92, 234101 (2004).

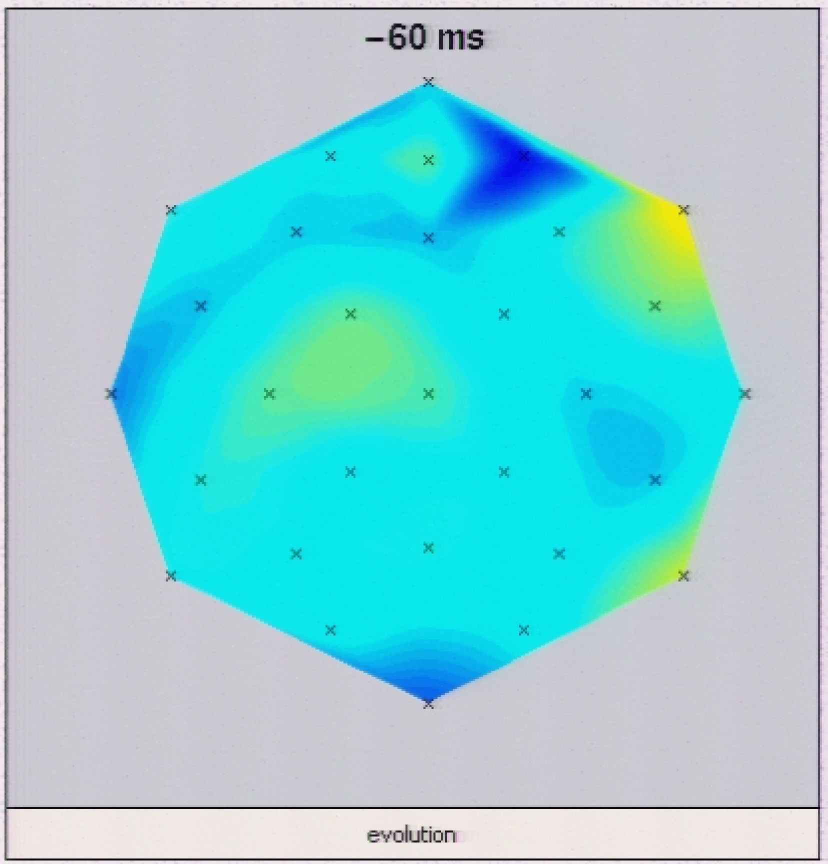
Cognitive Processes

Processing of visual stimuli

- Kanizsa-figure as stimulus (virtual figure vs. control figure)
- EEG-measurements (500 MHz, 30 channels)
- Multivariate synchronization analysis
- Synchronization analysis (Allefeld, Kurths, Int. J. Bif. Chaos 2004)



EEG (Cognition)



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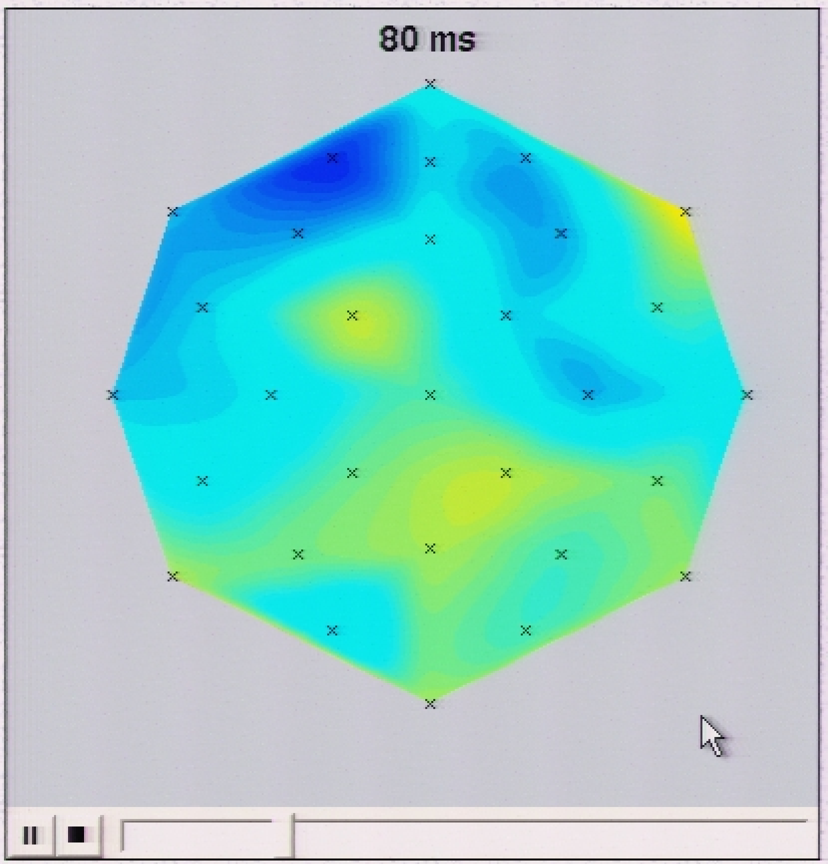

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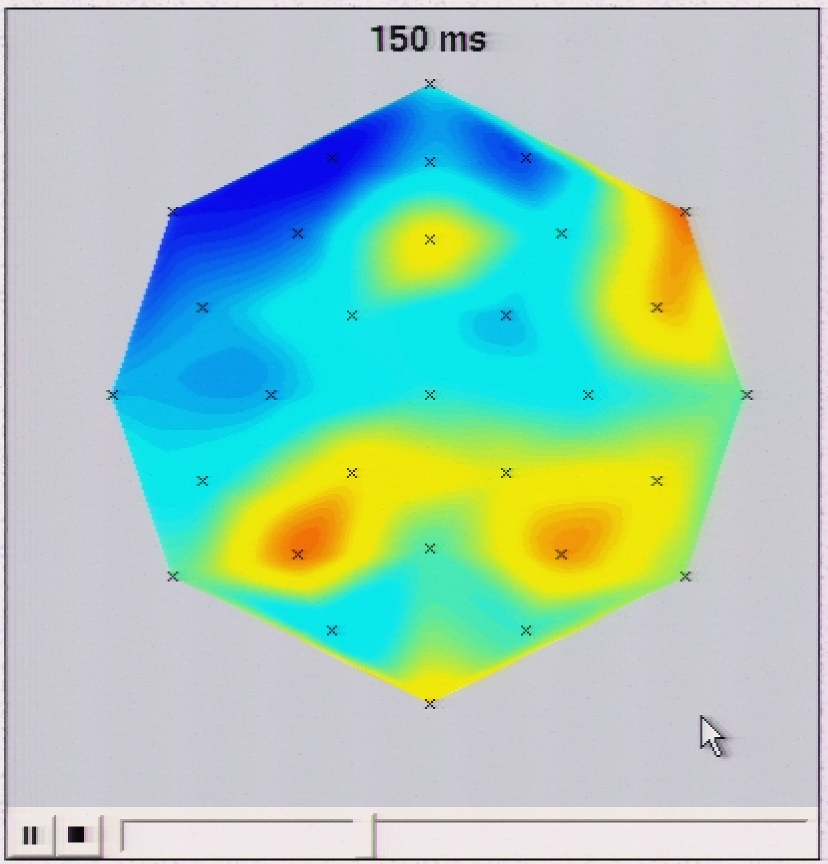

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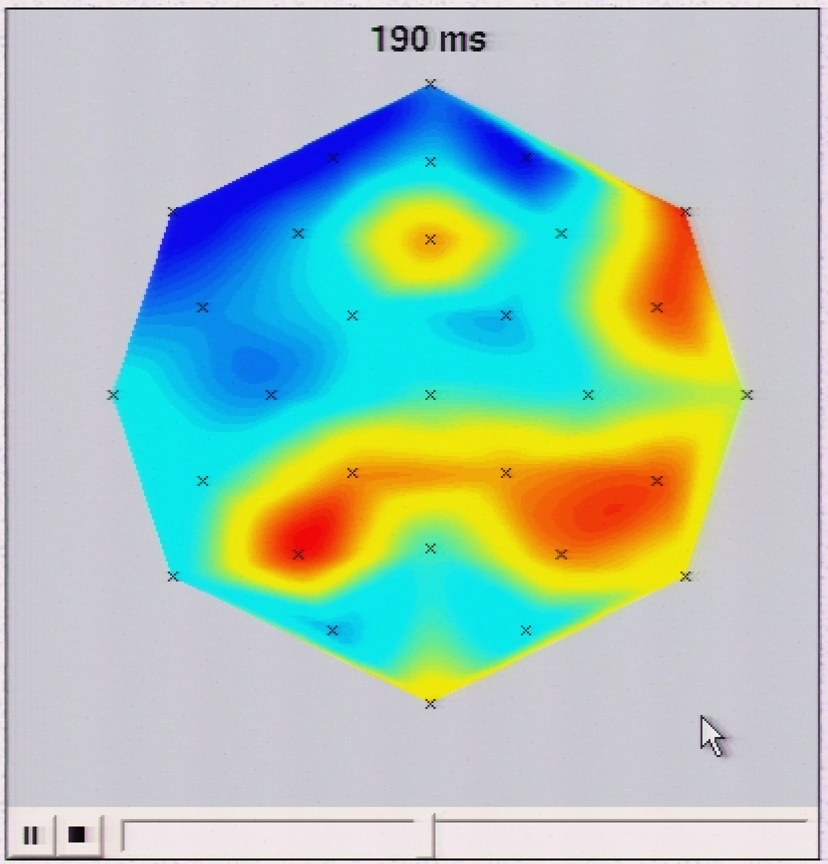

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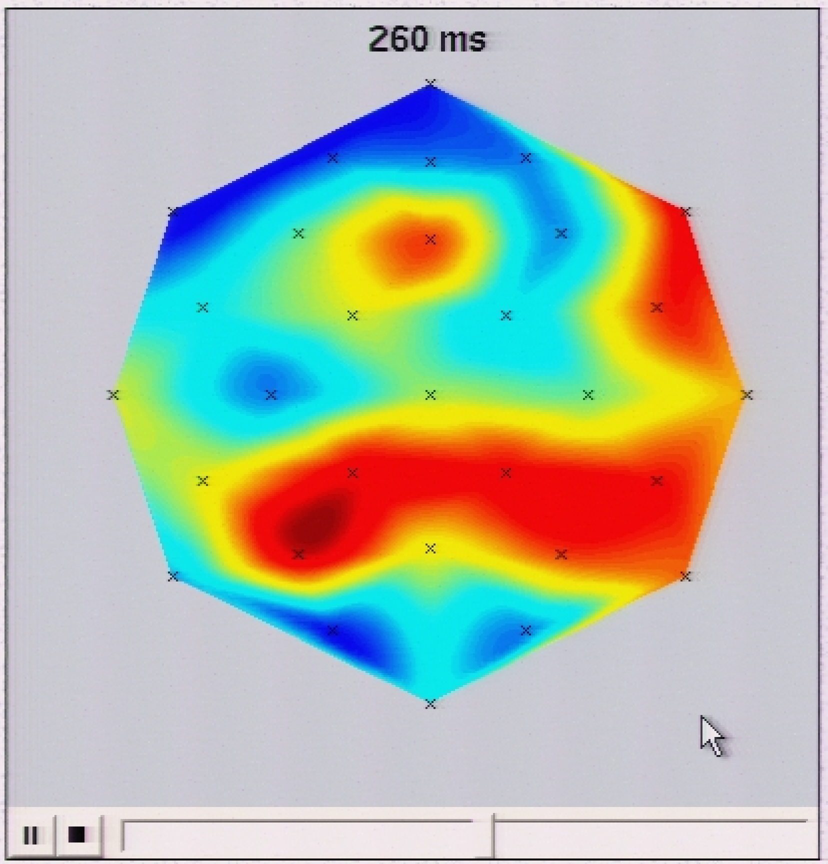

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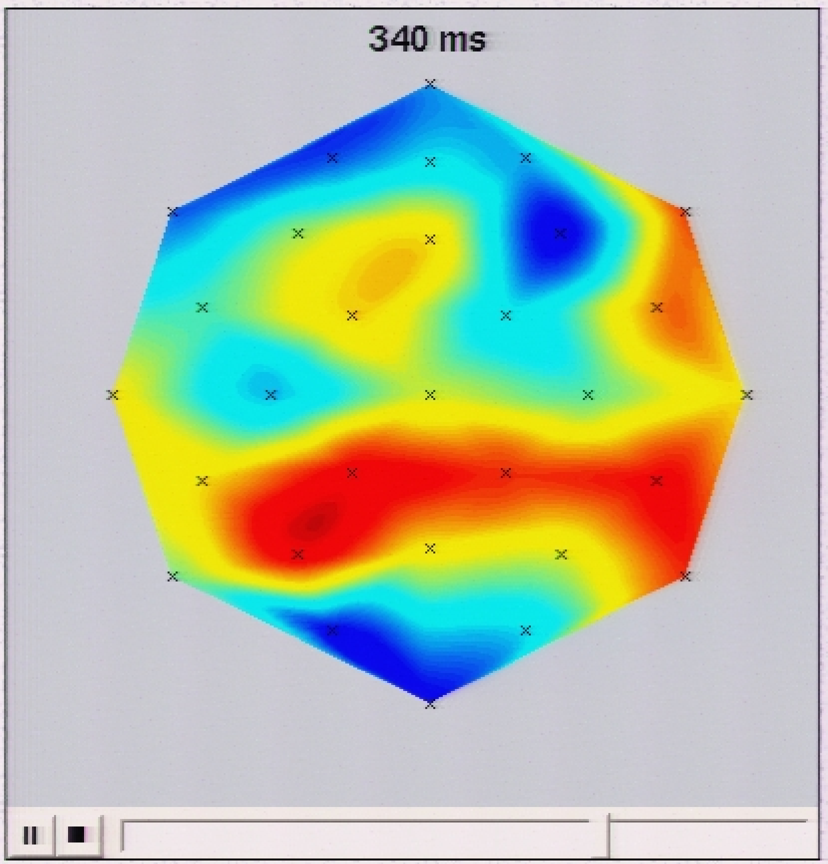

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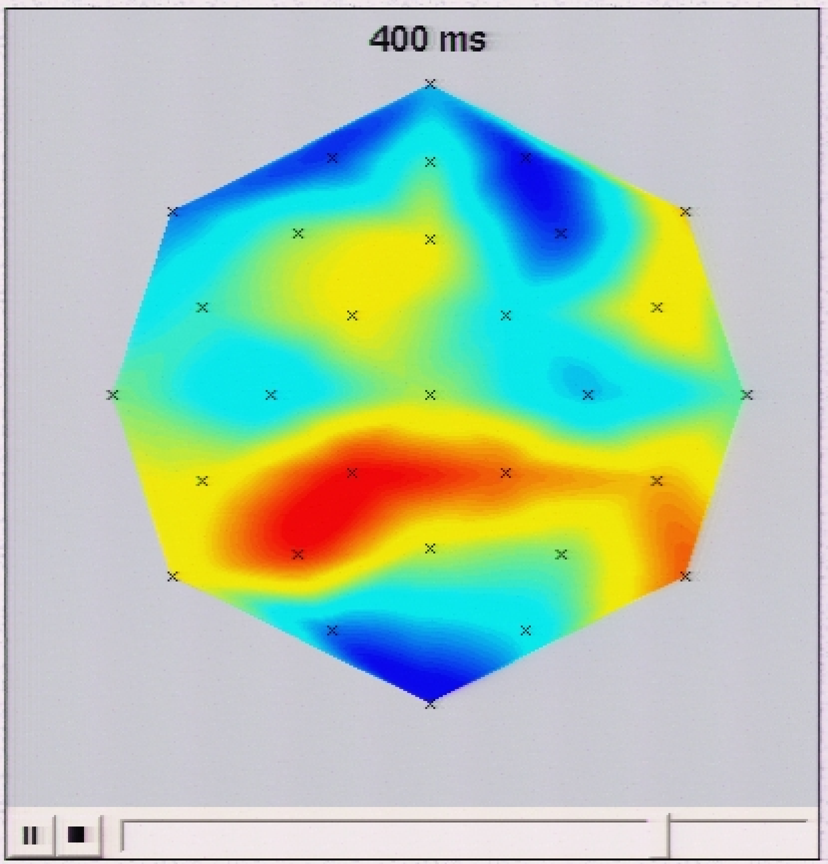

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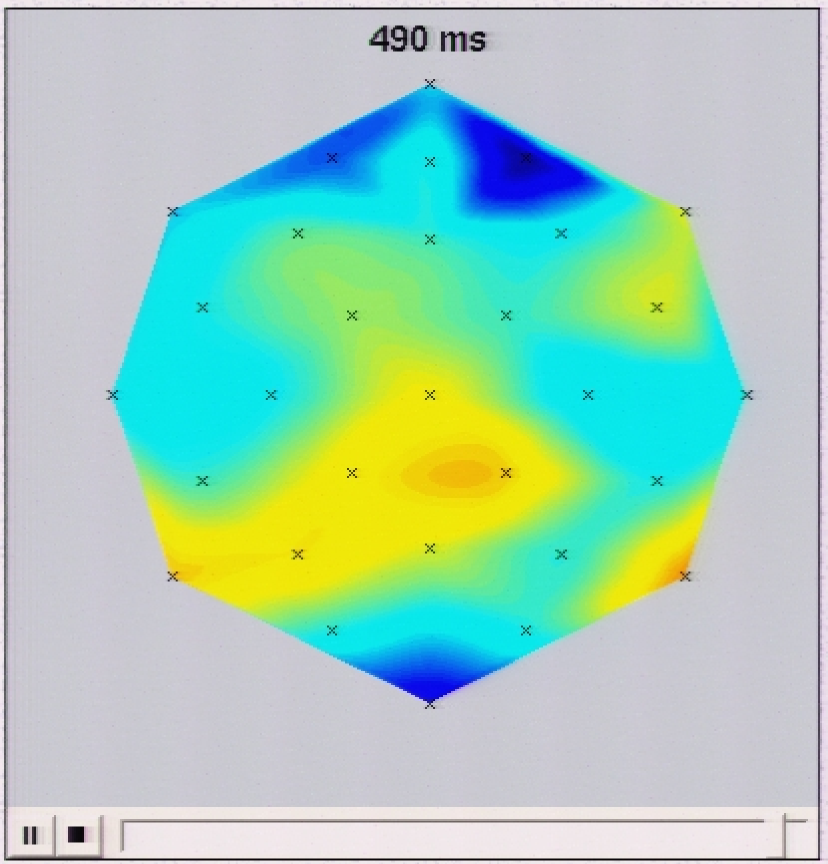

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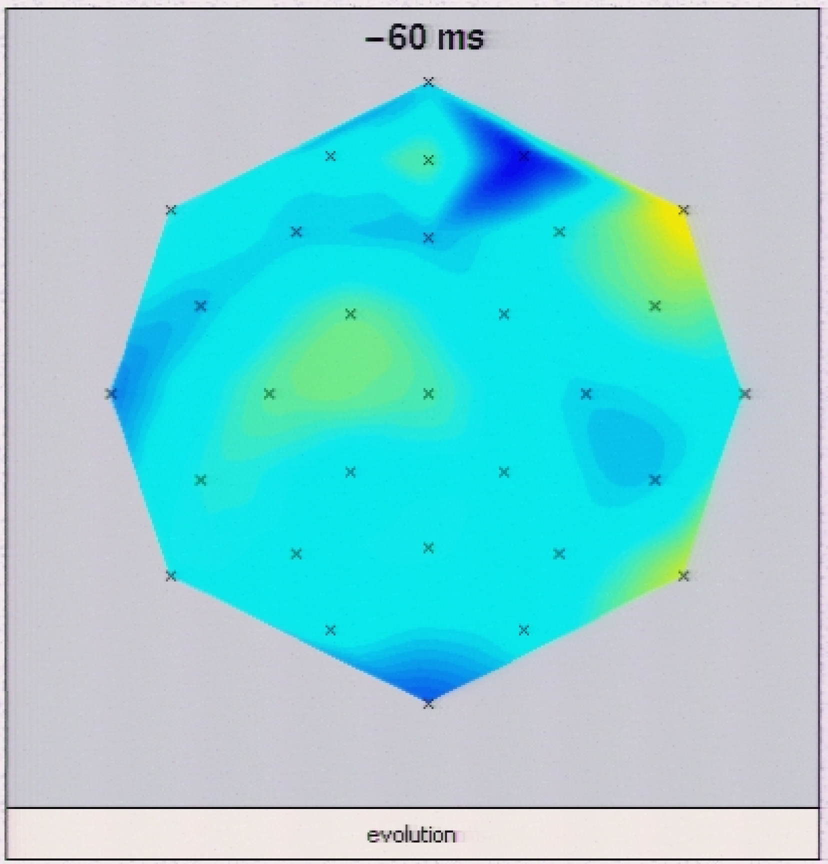
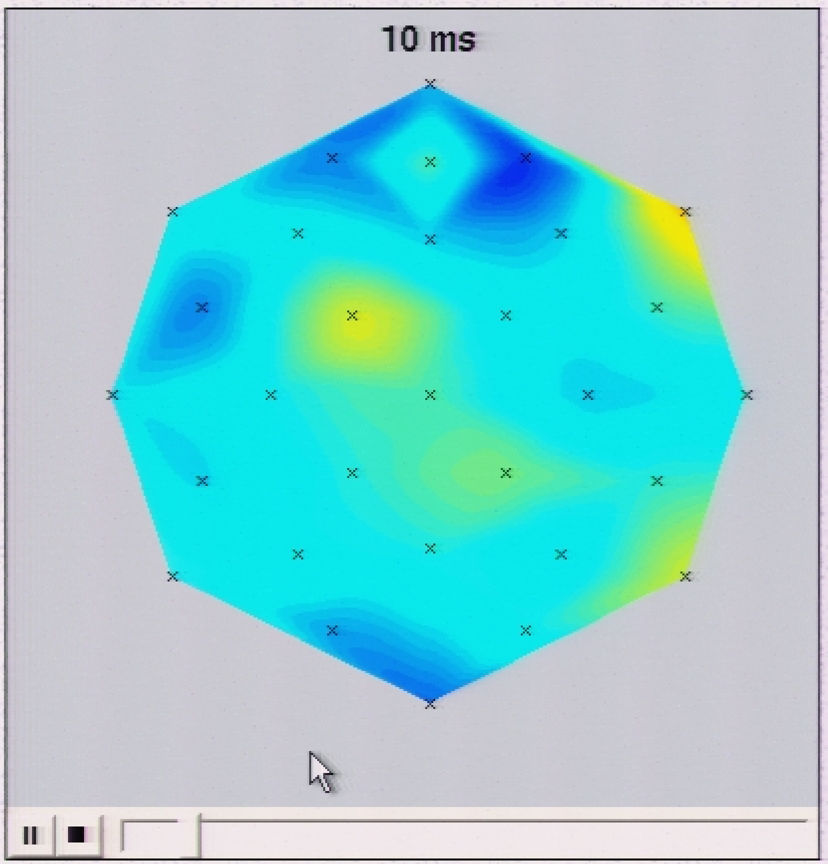
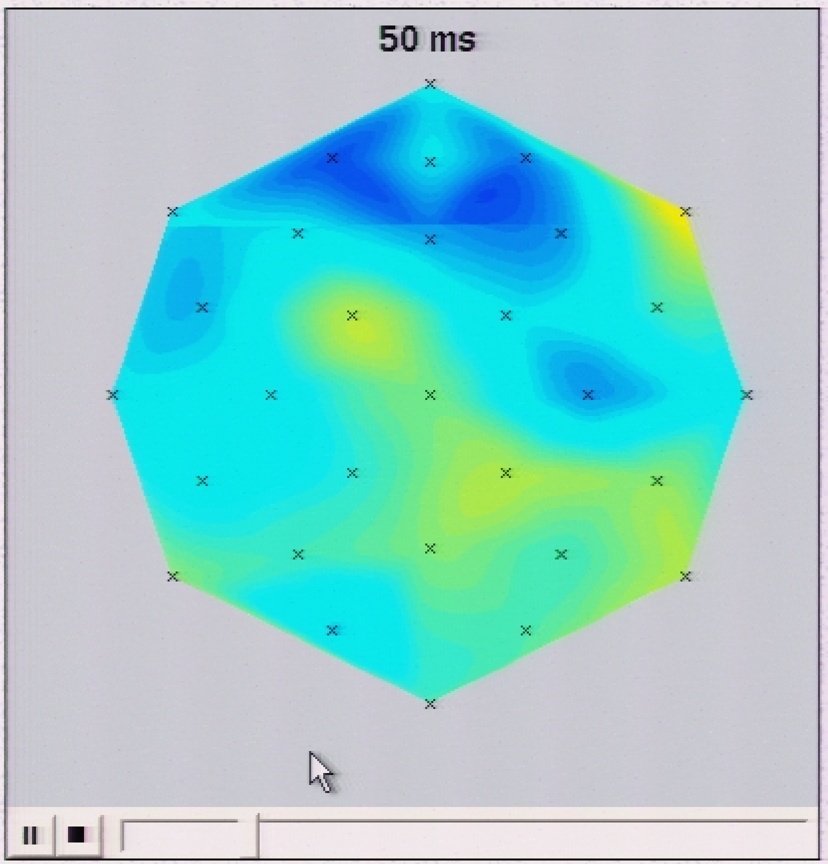


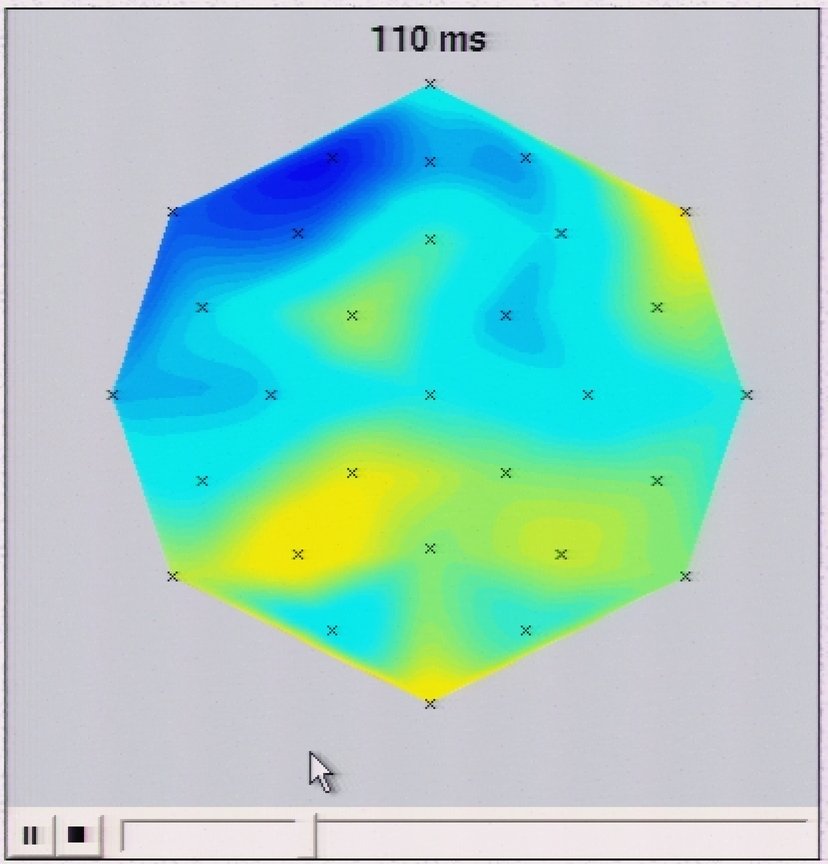
EEG (Cognition)



EEG (Cognition)



EEG (Cognition)



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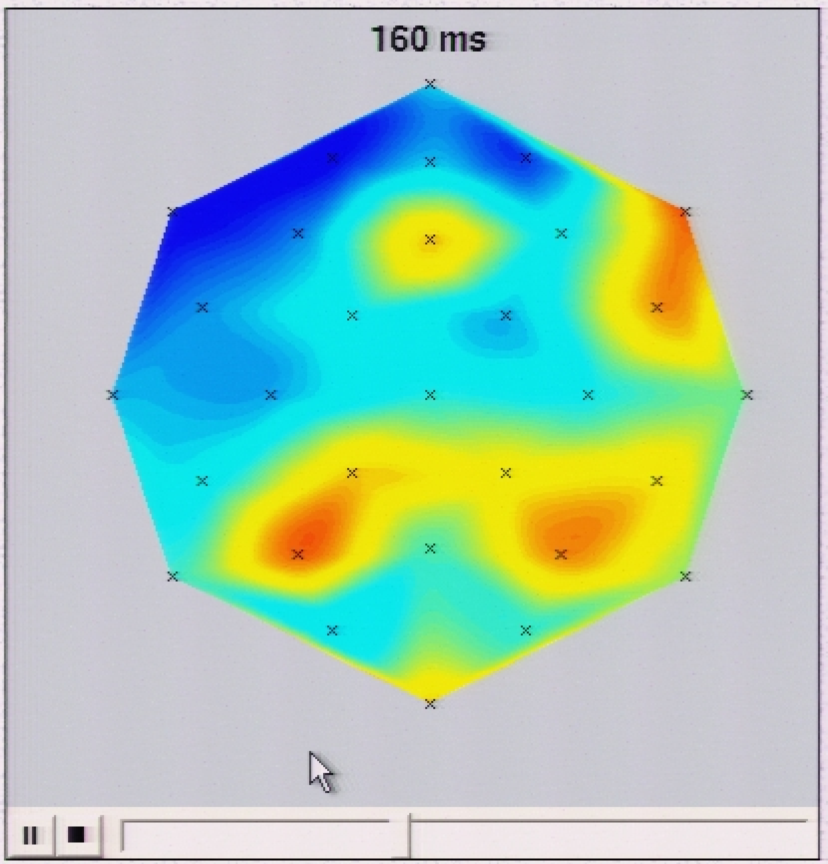

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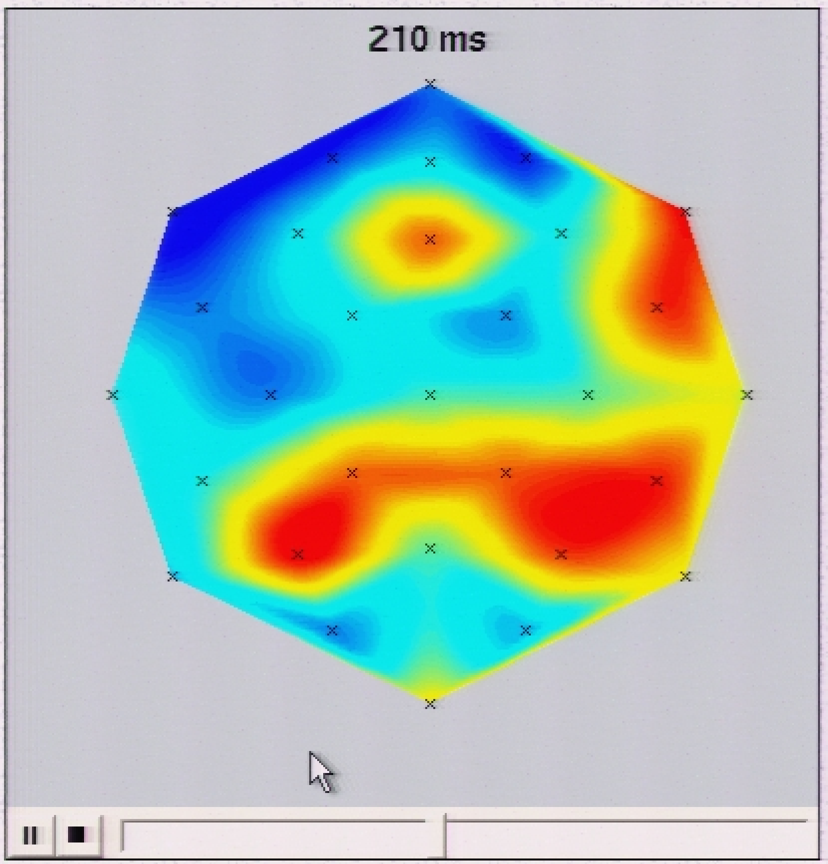

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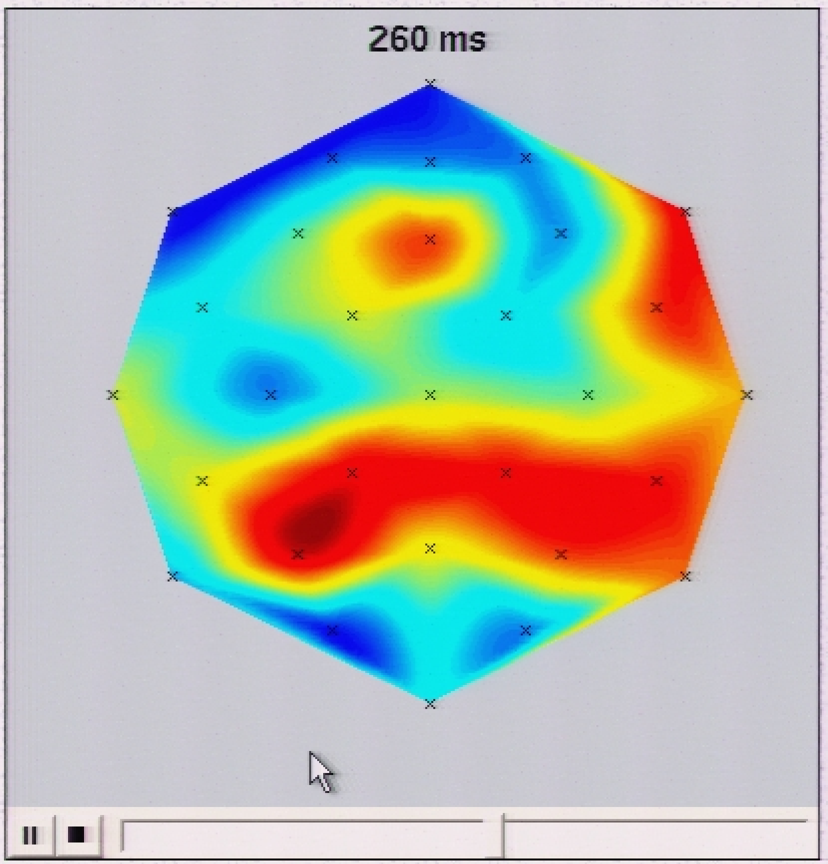

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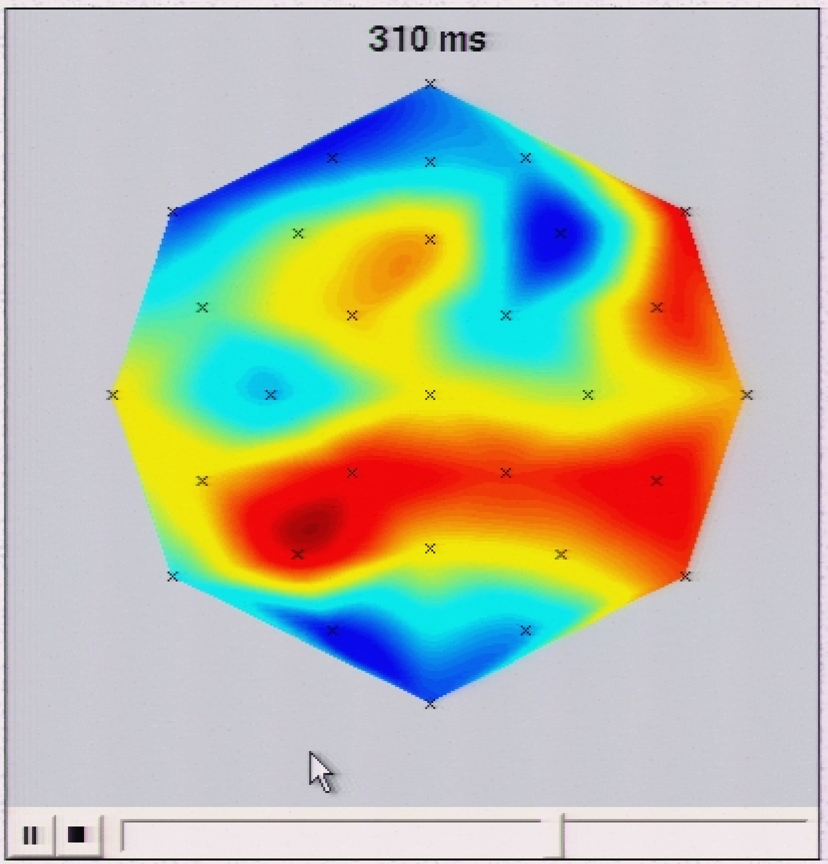

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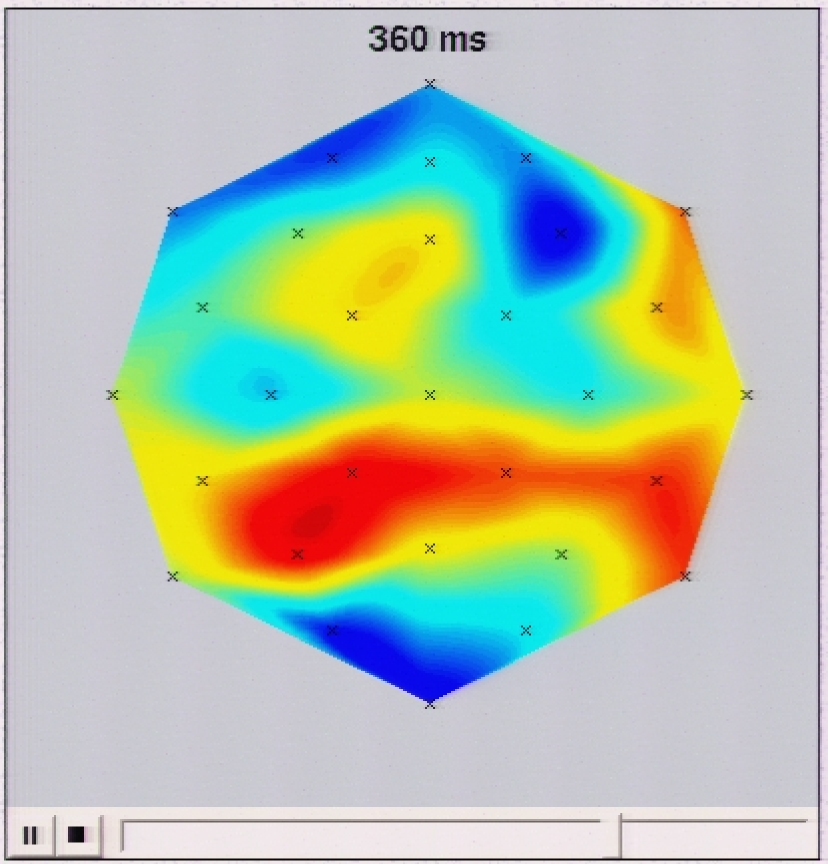

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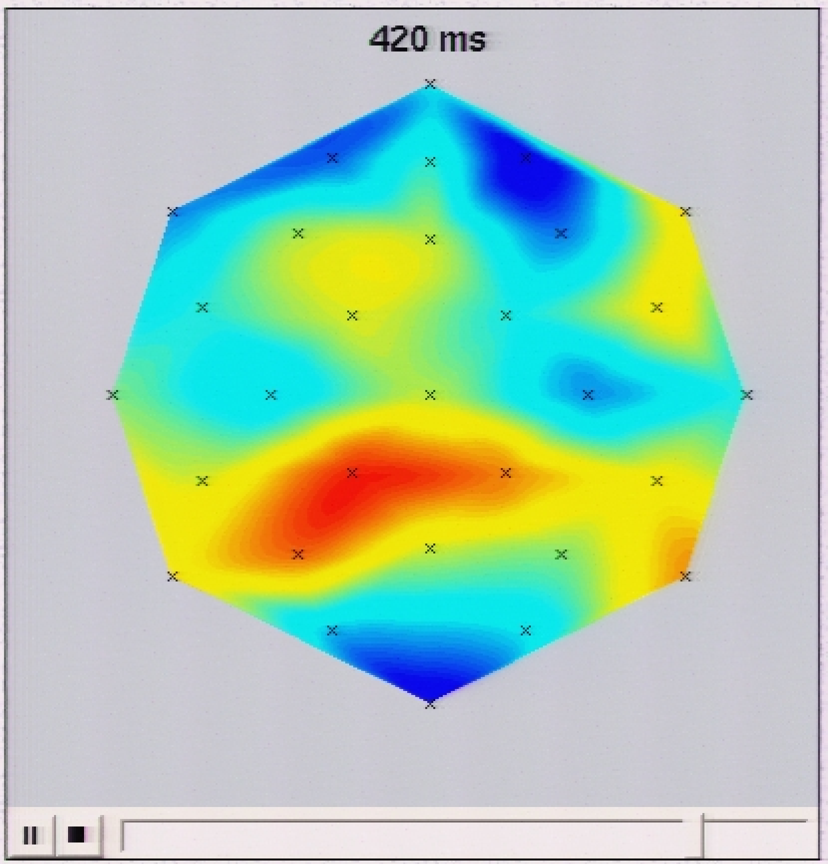

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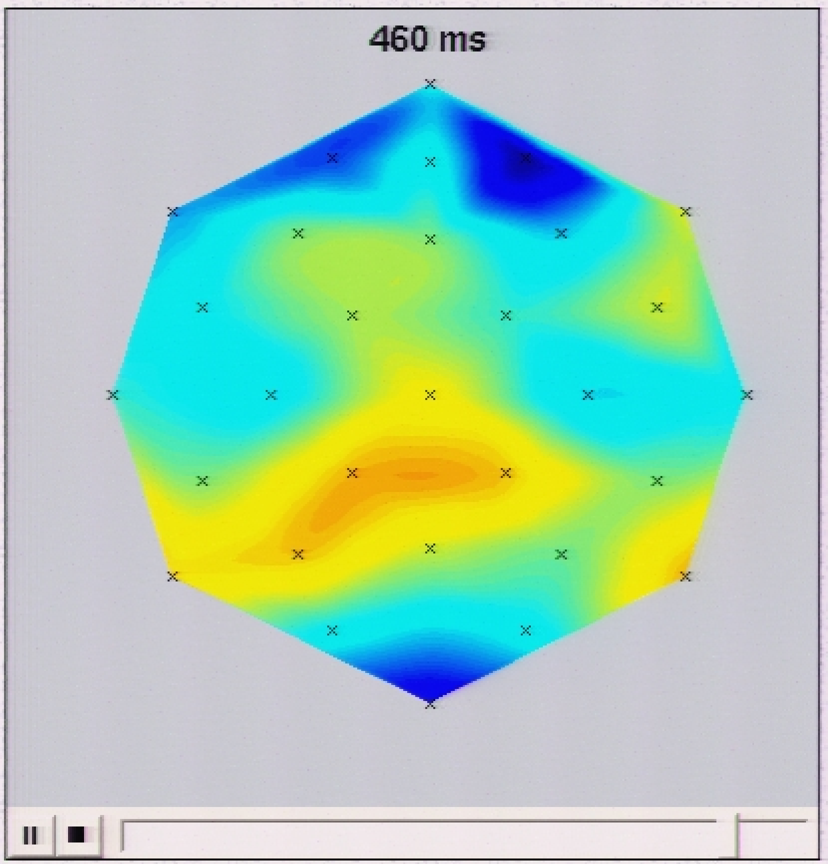

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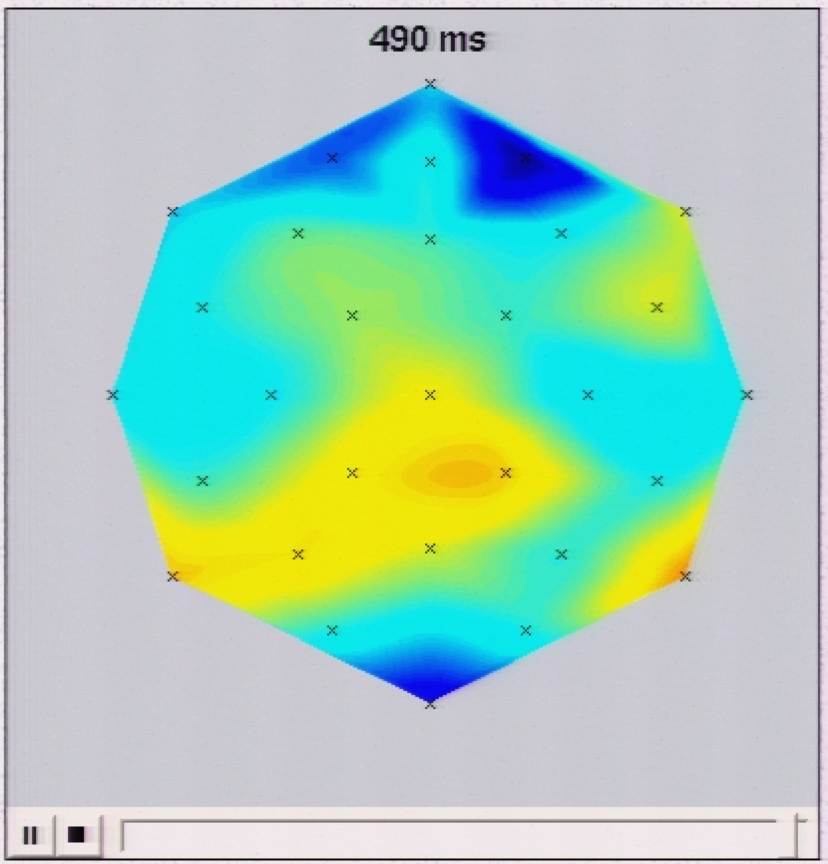

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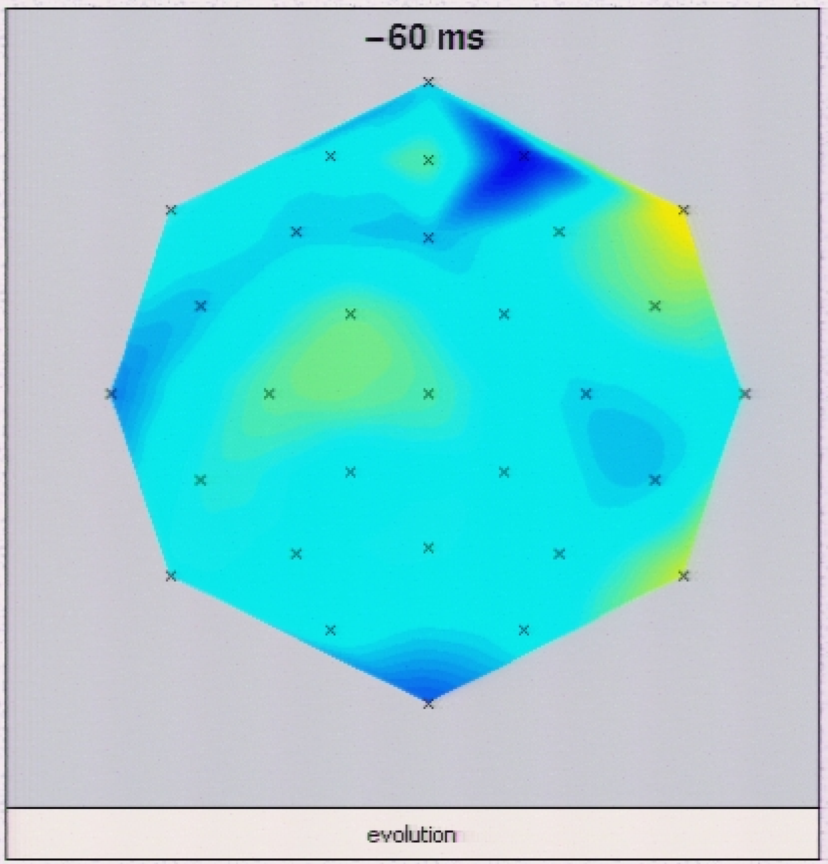

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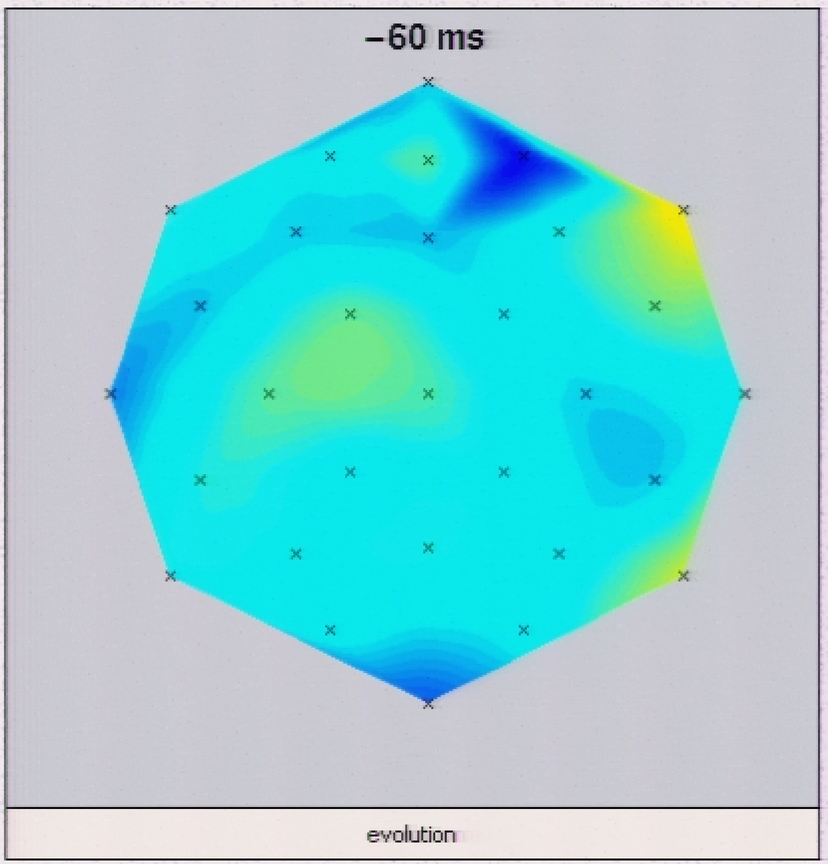


Cognition

Language processing

- Approach via sentences with conflicts

Der Priester wurde *geholt*.

The priest was *sent for*.

Der Priester wurde **asphaltiert**. (semantic conflict)

The priest was **asphalted**.

N400-activity

Press a key 2 s

Activity: -500 ... 300 ms

Synchro effect in mu-band (11 Hz)

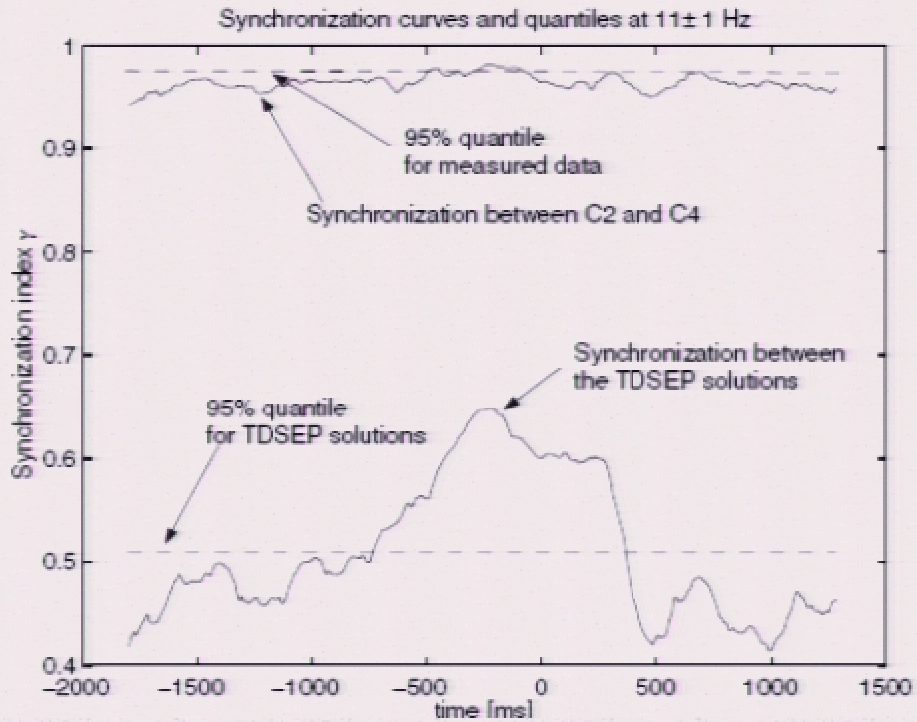
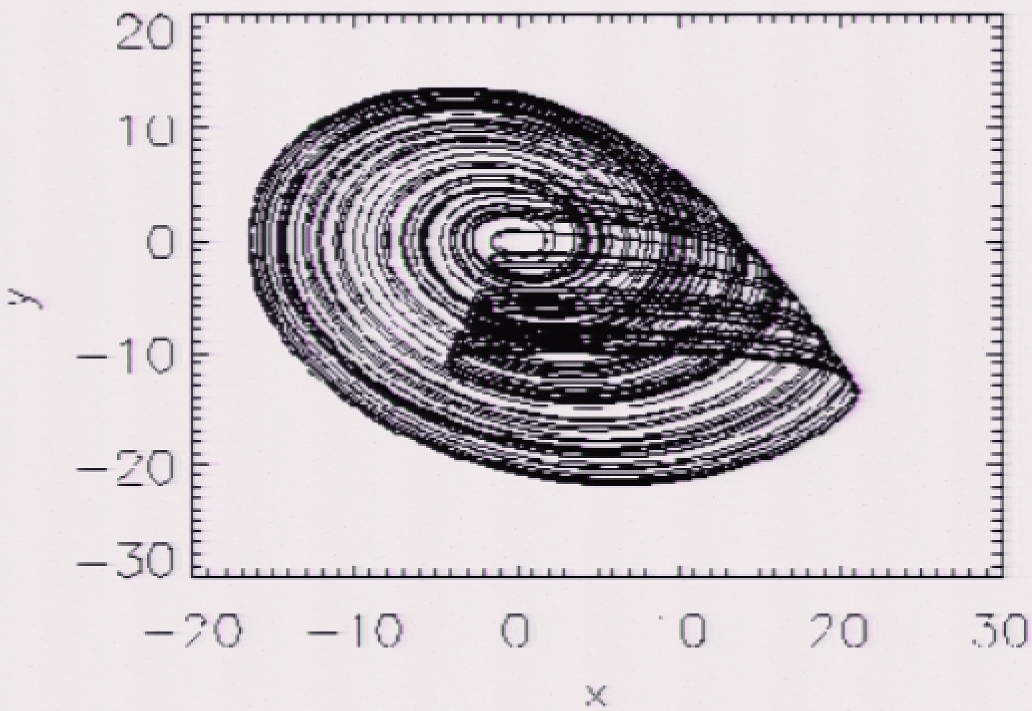


FIG. 3. Synchronization between EEG signals during finger movements measured by electrodes located over the motor cortex. The high phase locking values of the originally measured signals (upper curve) do not reveal any interesting temporal structure and can in large part be explained by superposition effects. The TDSEP-source estimates show a significant synchronization peak directly before the movement.

Meinecke, Ziehe, Kurths, Müller: Phys. Rev. Lett. 94, 084102 (2005)

Roessler Funnel – Non-Phase coherent



Synchronization in more complex topology

- Systems are often non-phase-coherent (e.g. funnel attractor)
- How to study phase dynamics there?
- 1st Concept: **Curvature**

  (Osipov, Hu, Zhou, Kurths: Phys. Rev. Lett., 2003)

$$\phi = \arctan \frac{\dot{y}}{\dot{x}}.$$

Dynamics in non-phase-coherent oscillators

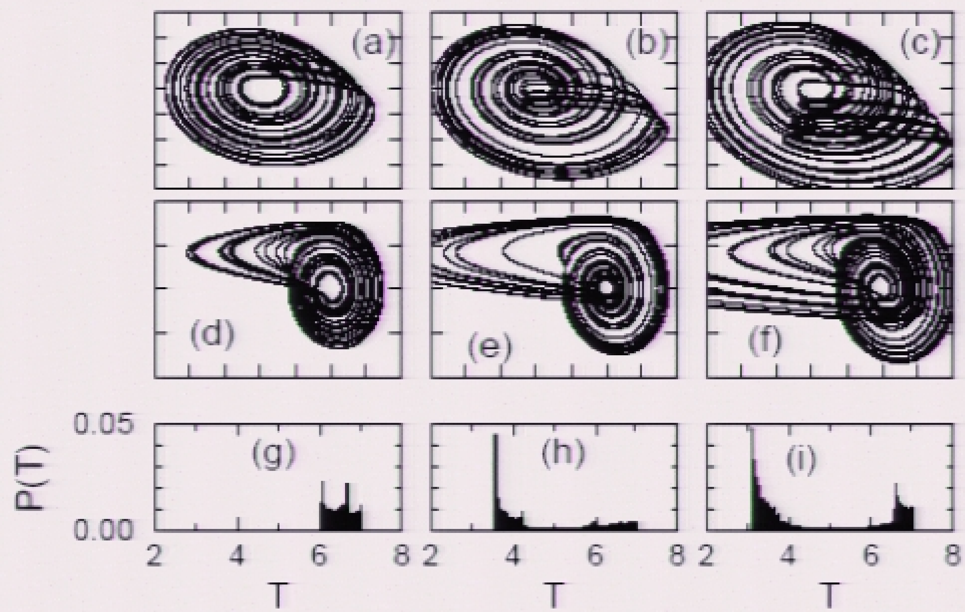


FIG. 1: Upper panel (a,b,c): projections of the attractors of the Rössler systems (1) onto the plane (x, y); middle panel: (d,e,f): projections onto $(\dot{x}, \dot{y})$; lower panel (g,h,i): distribution of the return times T. The parameters are $\omega = 0.98$ and $a = 0.16$ (a,d,g), $a = 0.22$ (b,e,h) and $a = 0.28$ (c,f,i).

Mutually coupled Rössler oscillators

$$
\begin{aligned}
\dot{x}_{1,2} &= -\omega_{1,2} y_{1,2} - z_{1,2}, \\
\dot{y}_{1,2} &= \omega_{1,2} x_{1,2} + a y_{1,2} + d(y_{2,1} - y_{1,2}), \\
\dot{z}_{1,2} &= 0.1 + z_{1,2}(x_{1,2} - 8.5),
\end{aligned}
$$

d – coupling strength

a – system parameter

Different types of synchronization transitions

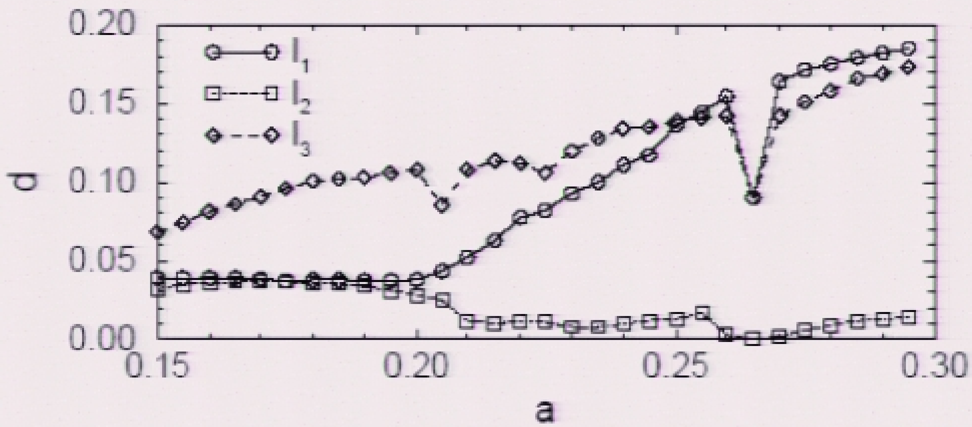


FIG. 3: Critical coupling curves. l_1 corresponds to the onset of CPS, i.e., below this line the oscillations are not synchronized, and above the phase and frequency locking conditions are fulfilled; l_2 to the transition of one of zero LEs to negative value and l_3 to zero-crossing of one of the positive LEs. Note: In this figure we do not separate the cases, where the synchronization occurs between regular and chaotic oscillations.

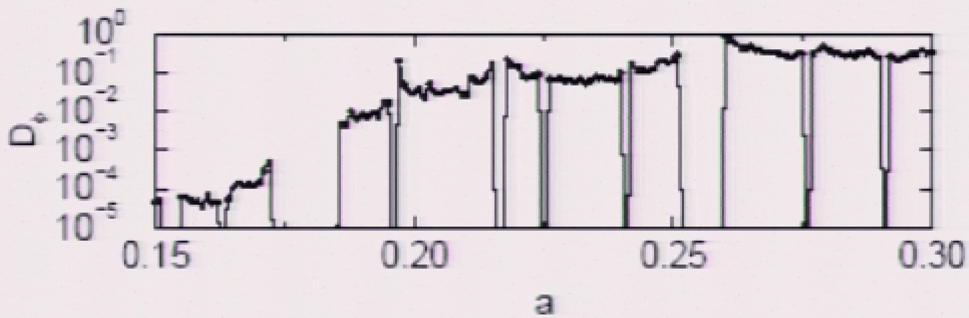


FIG. 2: Phase diffusion coefficient D_ϕ (3) vs a. $\omega = 0.98$.

Three types of transition to phase synchronization

- **Phase-coherent**: one zero Lyapunov exponent becomes negative (small phase diffusion); phase synchronization to get for rather weak coupling, whereas generalized synchronization needs stronger one
- **Weakly non-phase-coherent**: inverse interior crises-like
- **Strongly non-phase-coherent**: one positive Lyapunov exponent becomes negative (strong phase diffusion)

Application: El Niño vs. Indian monsoon

- El Niño/Southern Oscillation (ENSO) – self-sustained oscillations of the tropical Pacific coupled ocean-atmosphere system
- Monsoon - oscillations driven by the annual cycle of the land vs. Sea surface temperature gradient
- ENSO could influence the amplitude of Monsoon – Is there phase coherence?
- Monsoon failure coincides with El Niño

- (Maraun, Kurths, Geophys Res Lett 32, 023225 (2005))

El Niño vs. Indian Monsoon

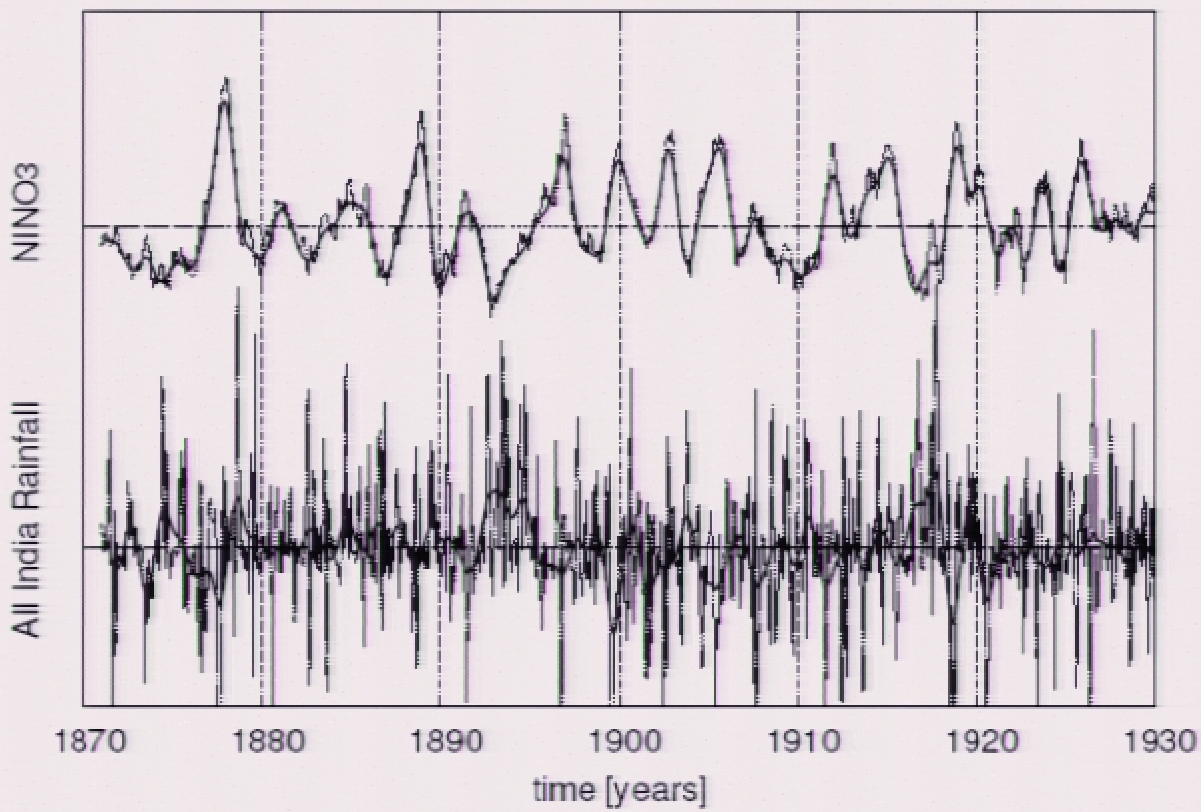


Figure 1. Section of the NINO3 (upper graph) and AIR anomalies (lower graph) time series. The dotted lines depict the raw data, **the solid lines show** the low-pass filtered data used for the further analysis.

El Niño – non phase-coherent

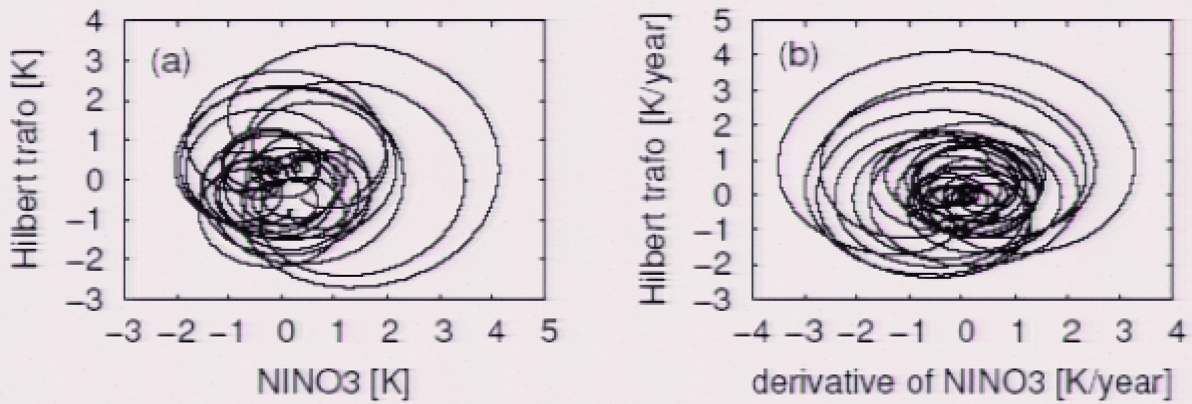


Figure 2. (a) Embedding of low-pass filtered NINO3 time series by Hilbert transformation. Many oscillations are not centered around a common center. (b) The same, but for the time derivative of the NINO3 time series. All pronounced oscillations circle around the origin.

Phase coherence between El Niño and Indian monsoon

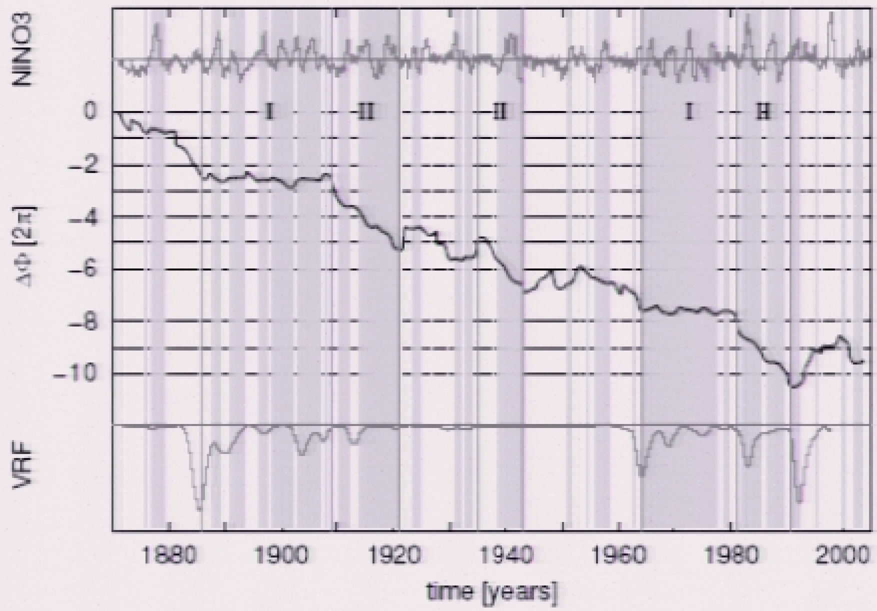
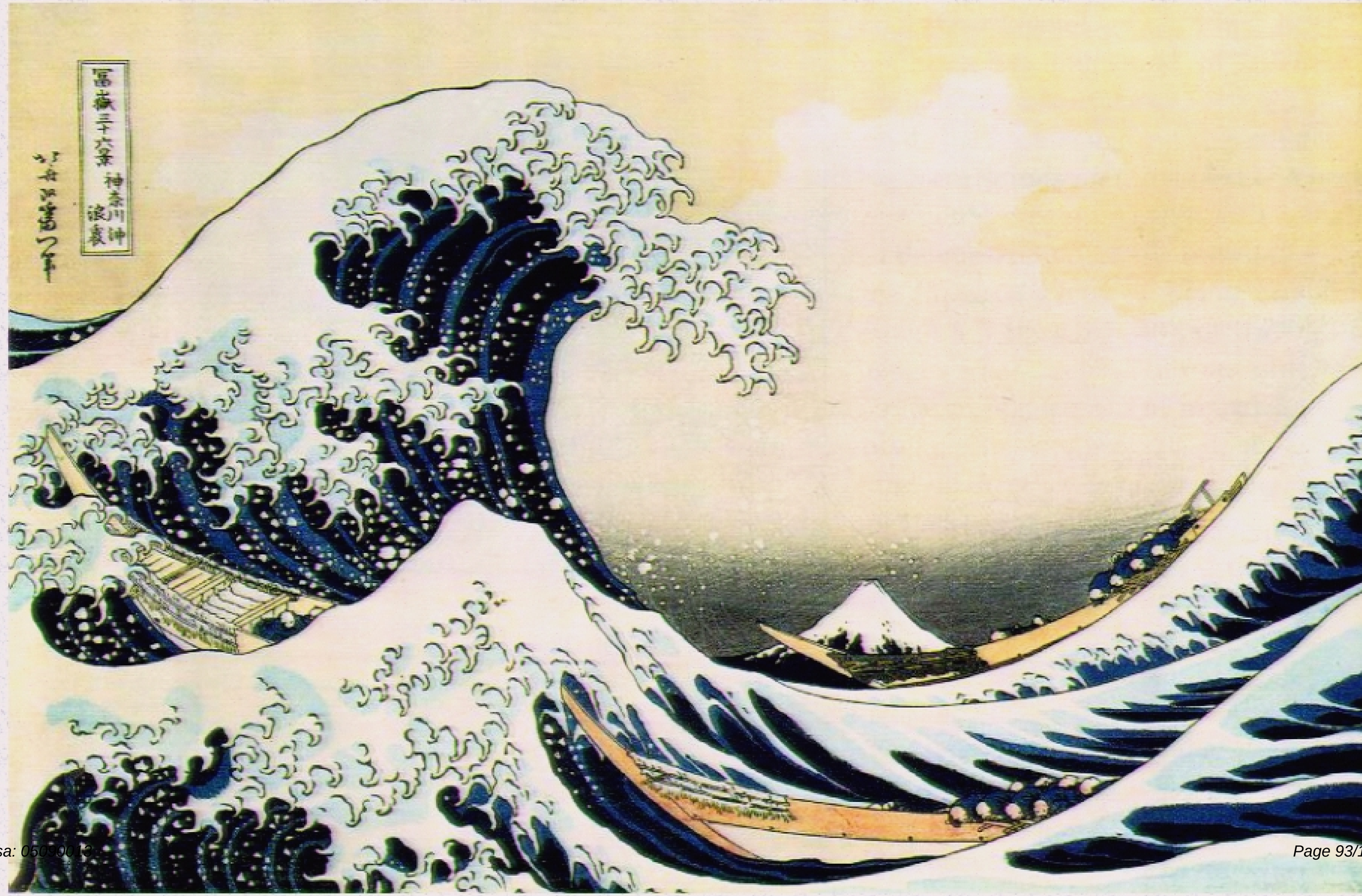


Figure 5. Phase difference of ENSO and Monsoon (black). Grey shading marks intervals of jointly well defined phases. 1886-1908 and 1964-1980 (I): plateaus indicate phase coherence. 1908-1921, 1935-1943 and 1981-1991 (II): Monsoon oscillates with twice the phase velocity of ENSO. During these intervals, both systems exhibit distinct oscillations (NINO3 time series, upper graph). 1921-1935 and 1943-1963: phases are badly defined, both processes exhibit irregular oscillations of low variance (upper graph). Lower graph shows volcanic radiative forcing index (VRF).

The Great Wave (Tsunami) by Katsushika Hokusai (1760-1850)



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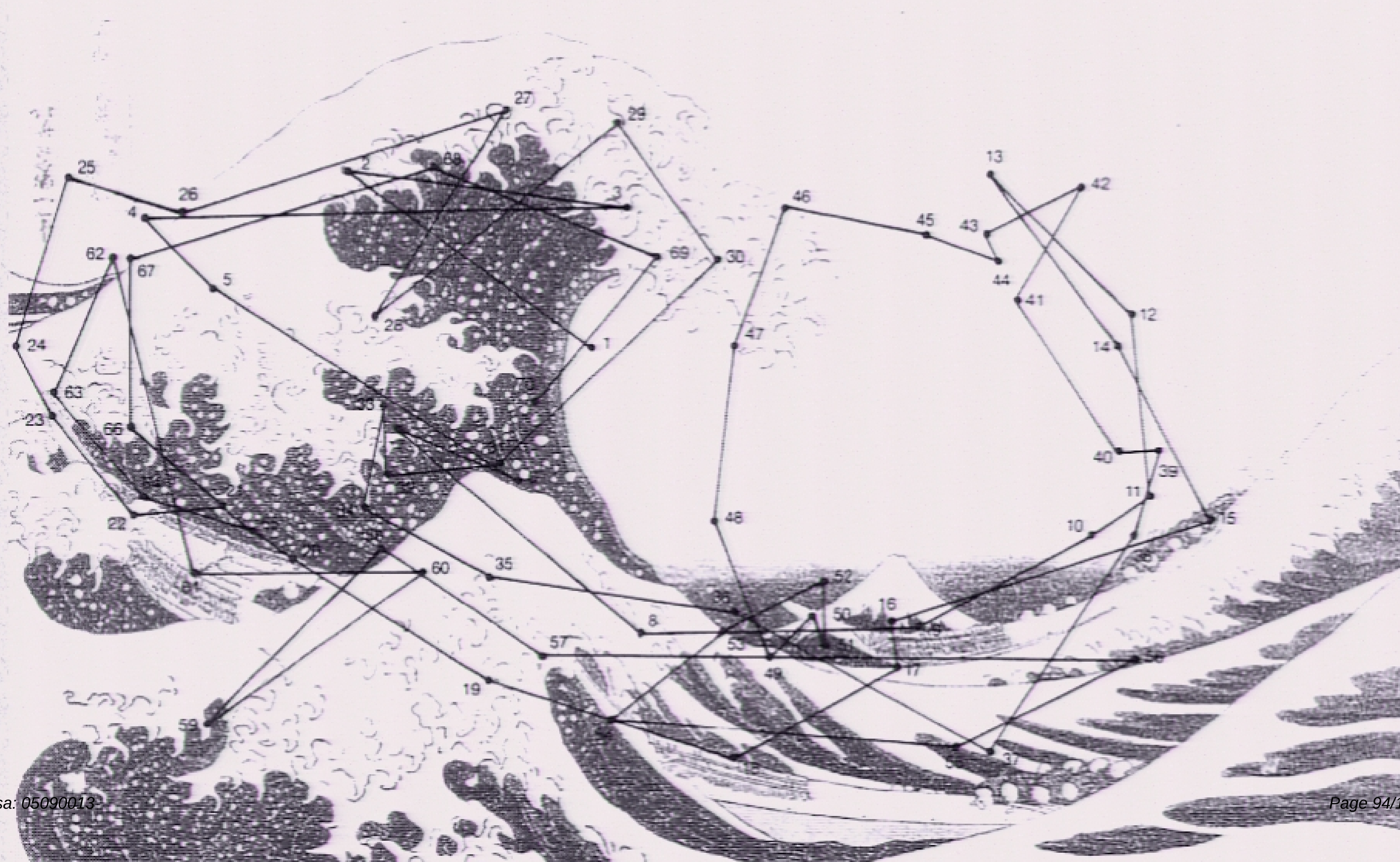

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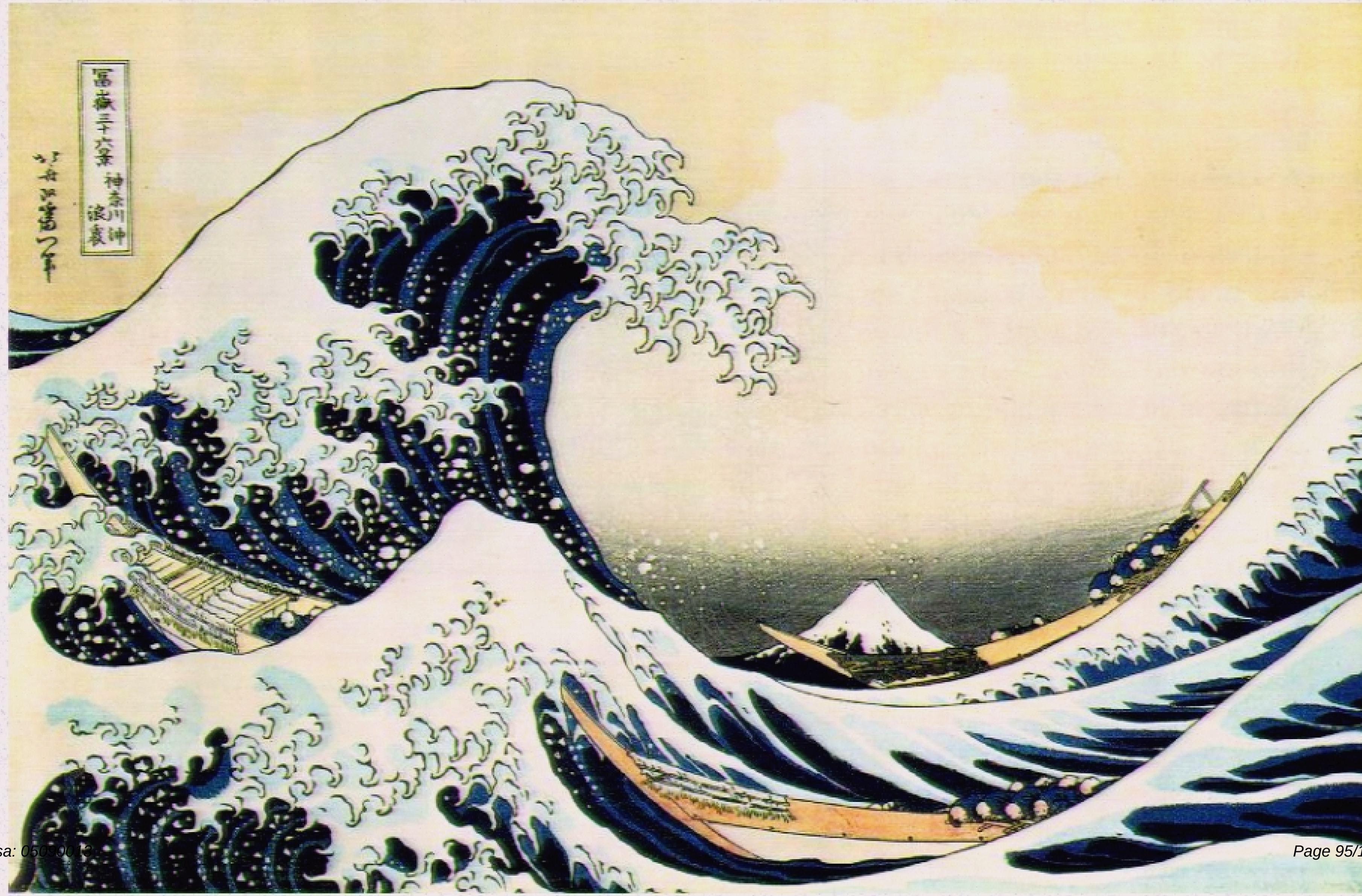

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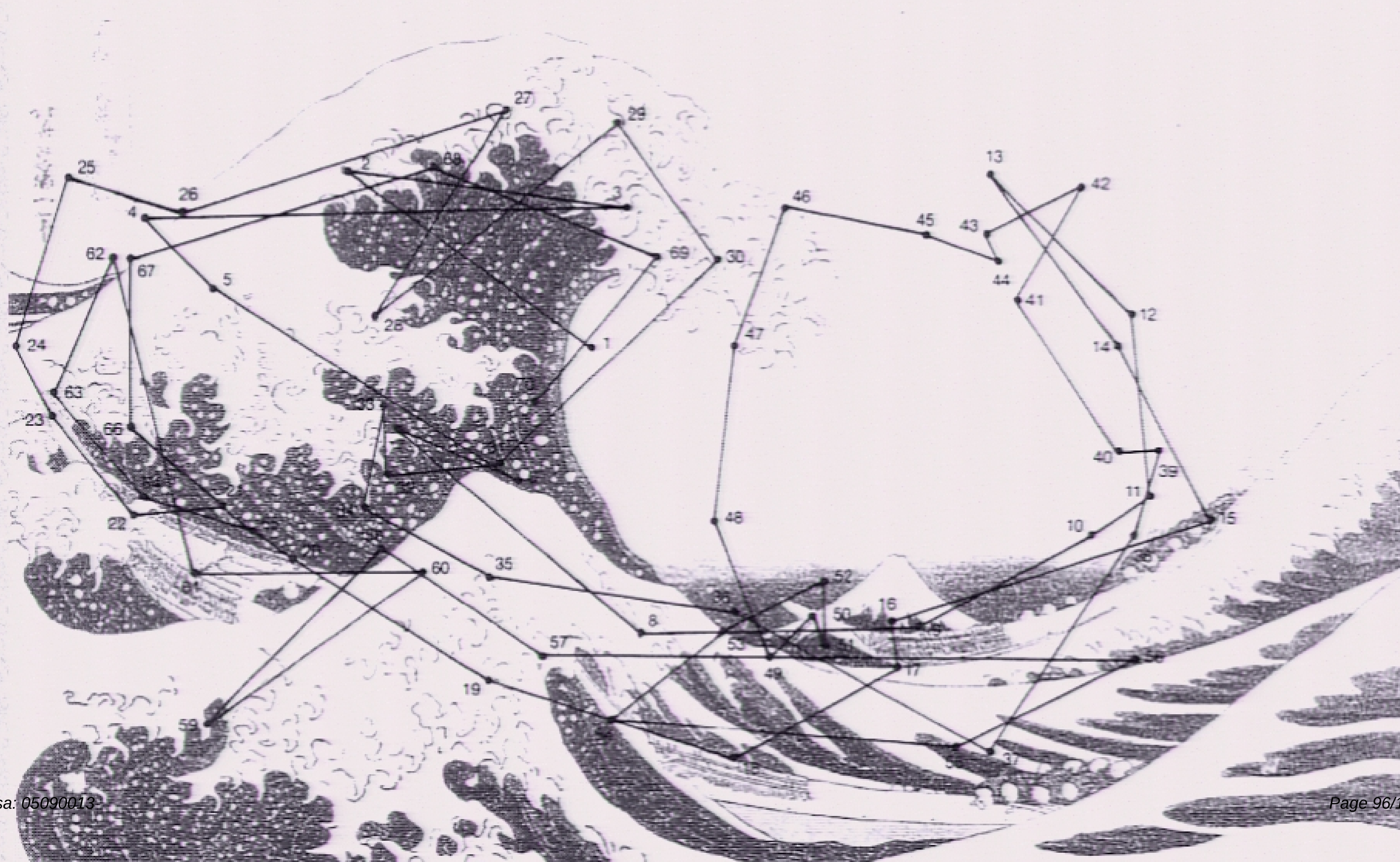
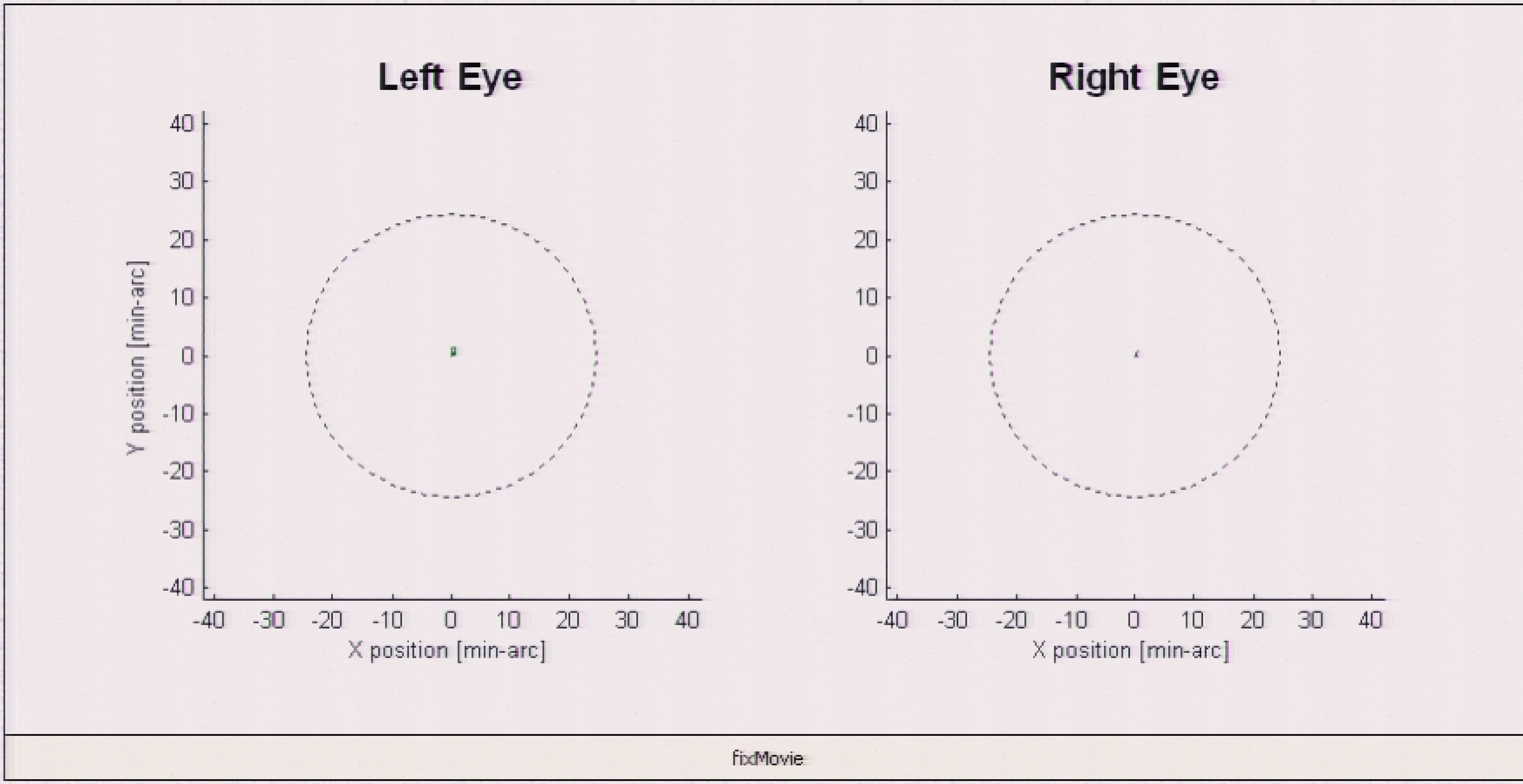


Eyes directed to one point ➔ Mikrosaccades



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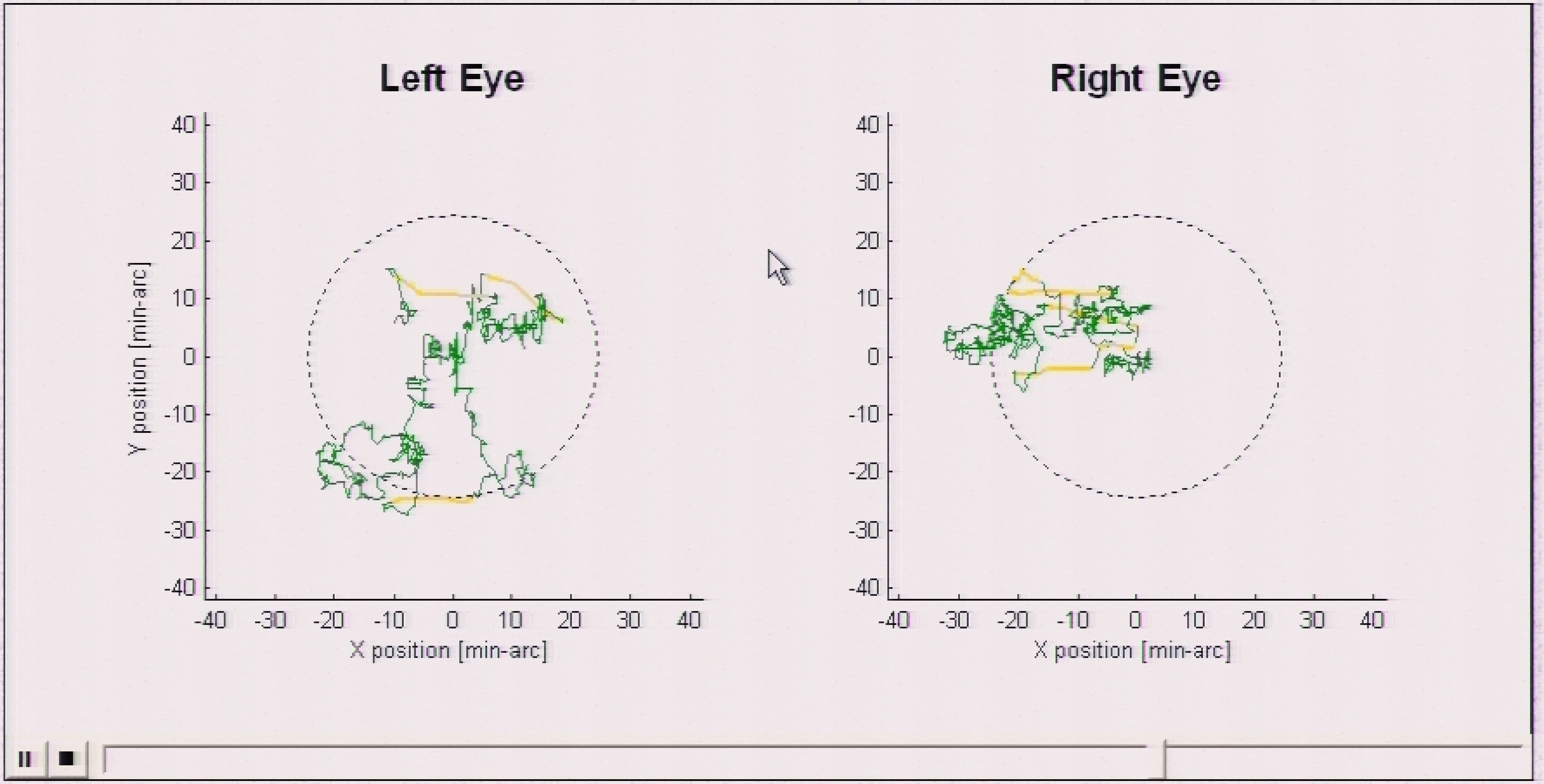

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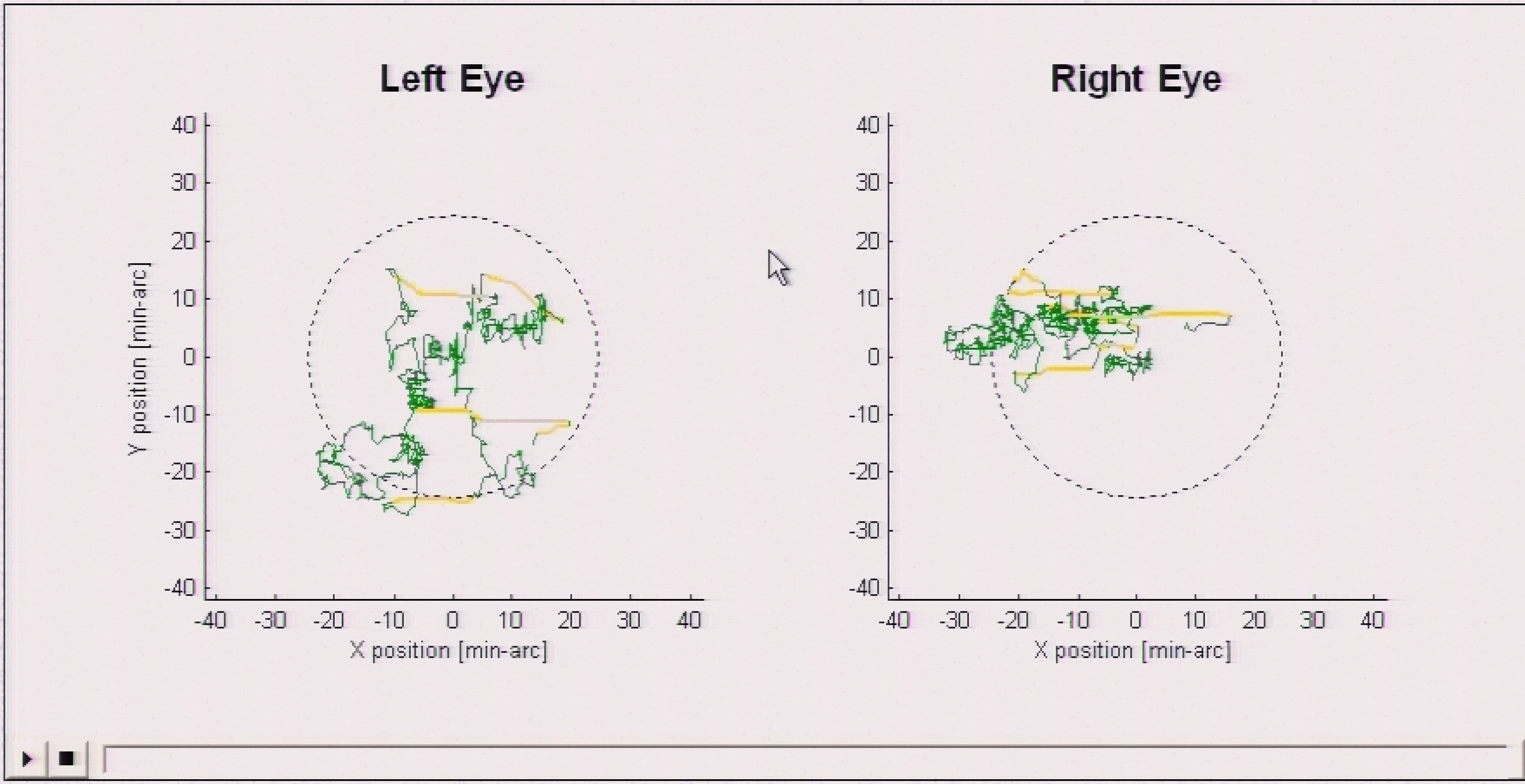

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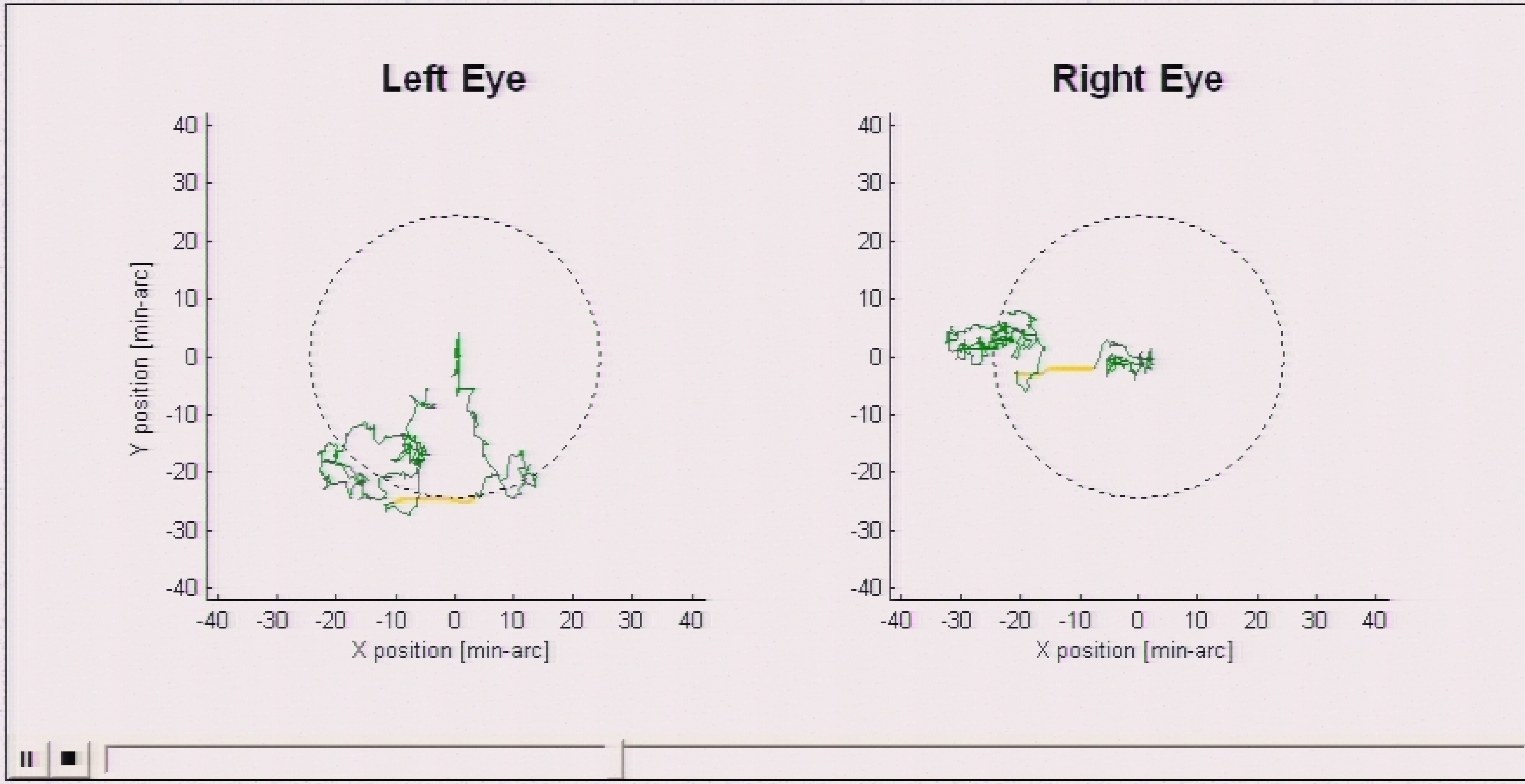

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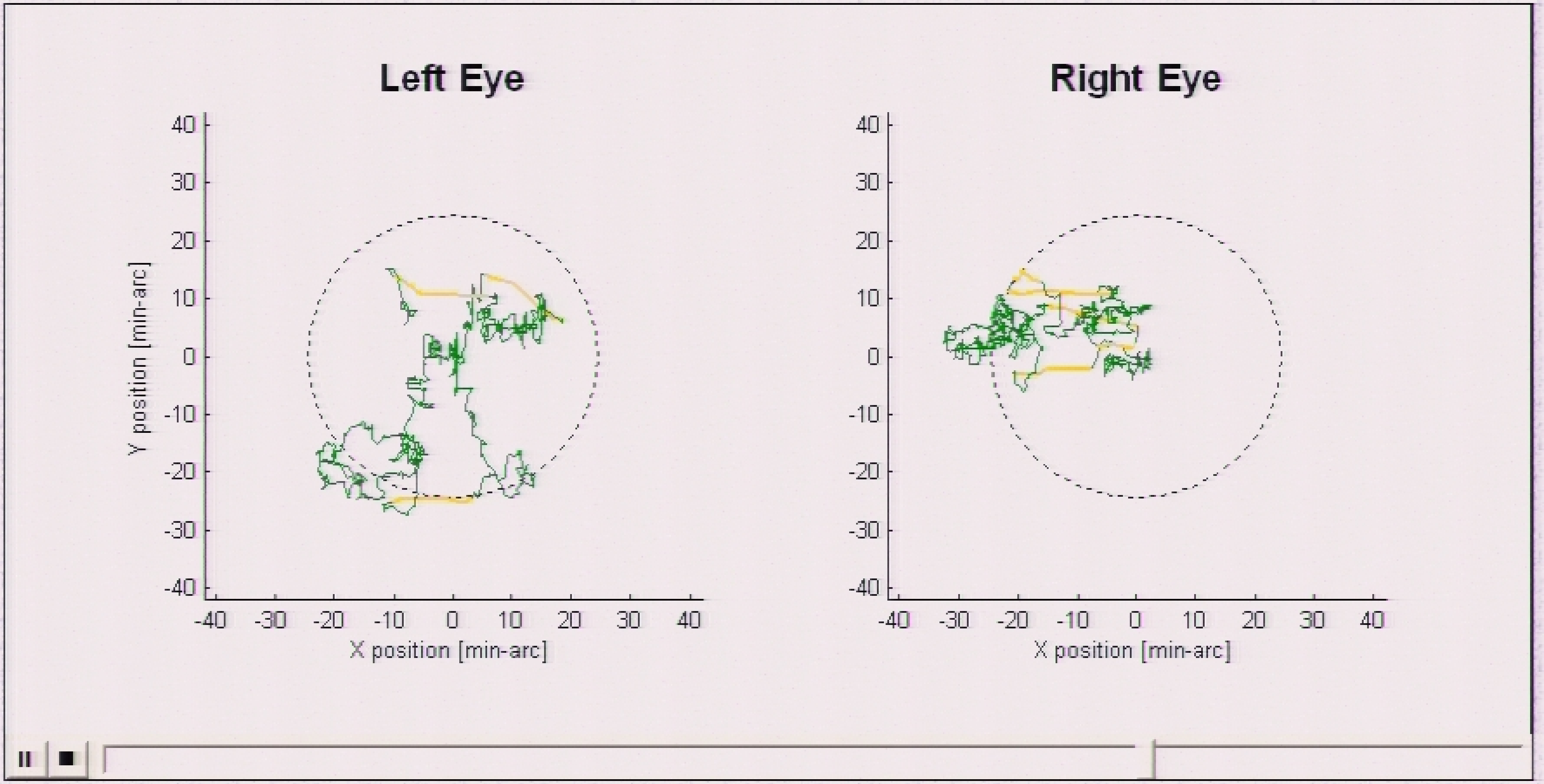


Generalized Description of such fluctuations by the Hurst-Exponent H

$$\langle \Delta x^2 \rangle = \langle (x_i - x_{i-\tau}) \rangle \sim \tau^{2H}$$

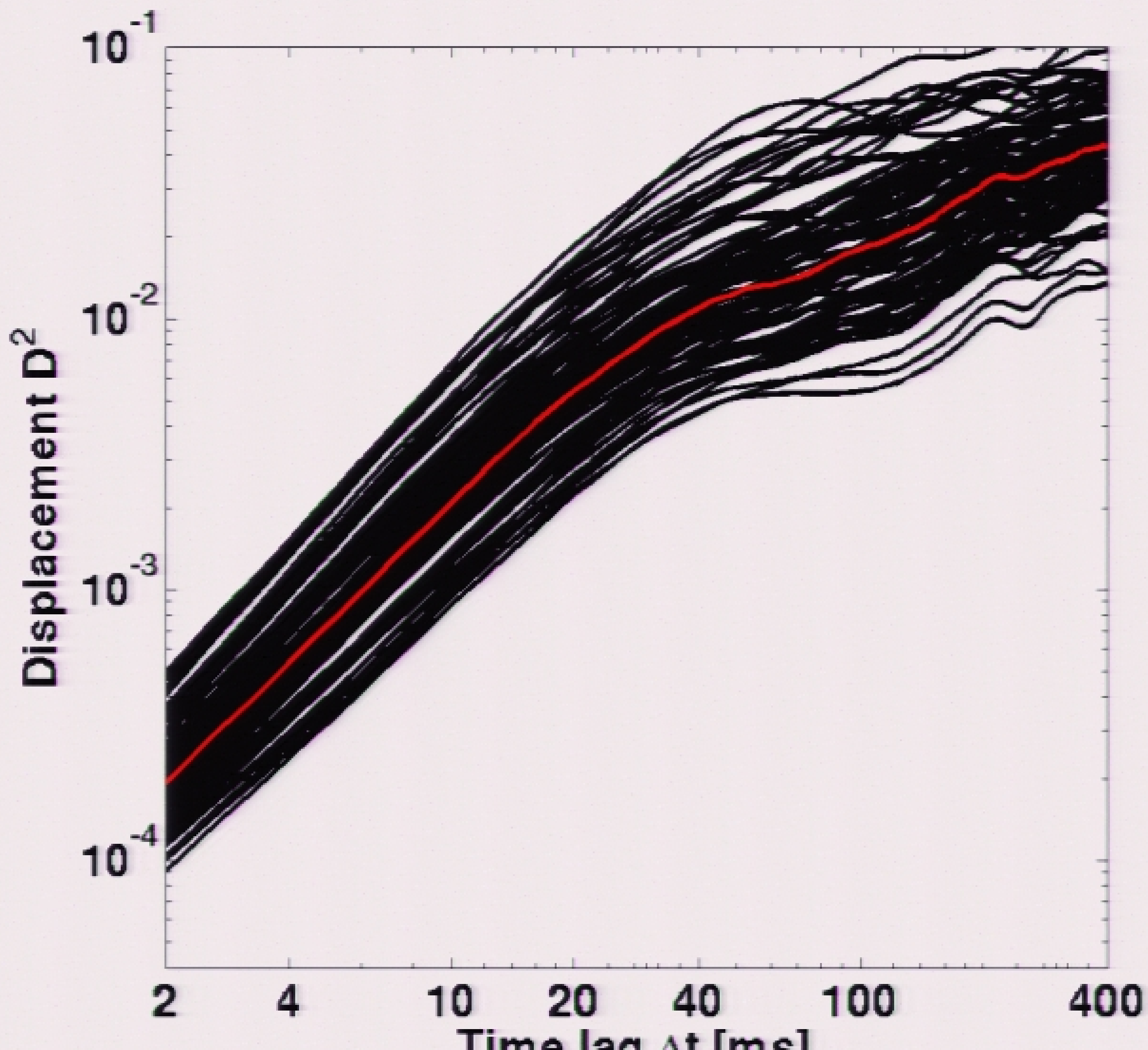
H = 0.5 – Brownian motion

H > 0.5 – pos. Persistency

(Tendencies improved)

H < 0.5 – Anti-Persistency

Mikrosaccades: Transition from Persistency to Anti-Persistency



Synchronization analysis

- Eye tracker measurements (x-y-coordinates, left and right eye)
- Simple filtering (first order difference) – to reduce long-term effects
- Are there synchronized activities?

Results:

- Fixational movements of the left and right eye are phase synchronized

- Hypothesis: there might be one center only in the brain that produces the fixational movement in both eyes

Data and twin surrogates

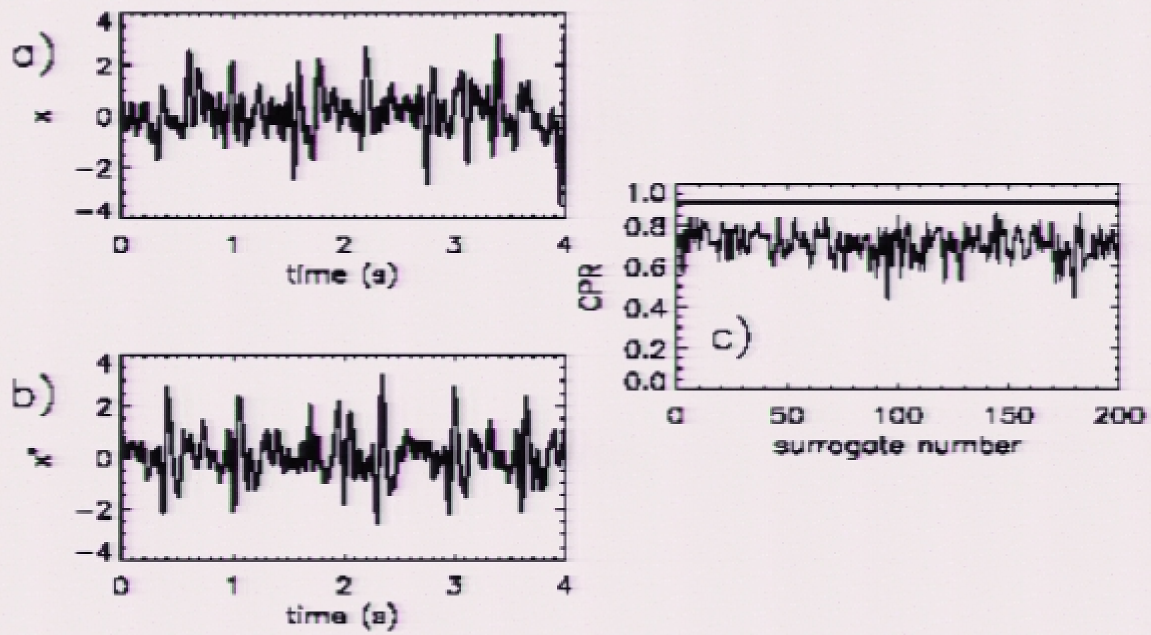


FIG. 3: a) Filtered horizontal component of the left eye of one participant. b) Horizontal component of one surrogate of the left eye. c) Results of the test performed for one trial of one participant. The PS index for the original data (bold line) is significantly different from the one of the surrogates (solid).

Concept of Recurrence

- Curvature → new theoretical insights, but problems with noisy data

- Other concepts necessary

  Filtering (Rosenblum et al., Phys. Rev. Lett., 2002)

  Recurrence

What is CHAOS?

Mathematical price to celebrate the 60th birthday of Oskar II, king of Norway and Sweden, 1889:

„Is the solar system stable?"

- Henri Poincaré (1854-1912)

SUR LE

PROBLÈME DES TROIS CORPS

ET LES

ÉQUATIONS DE LA DYNAMIQUE

PAR

H. POINCARÉ

À PARIS

MÉMOIRE COURONNÉ

DU PRIX DE S. M. LE ROI OSCAR II

[illegible]

AVEC DES NOTES

PAR L'AUTEUR.

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[illegible]

AVEC DES NOTES

PAR L'AUTEUR.

H. Poincare

If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at the succeeding moment.

but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws.

But it is not always so; it may happen that **small differences in the initial conditions produce very great ones in the final phenomena**. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

(1903 essay: Science and Method)

Weak Causality

Concept of Recurrence

Recurrence theorem:

Suppose that a point P in phase space is covered by a conservative system. Then there will be trajectories which traverse a small surrounding of P infinitely often.
That is to say, in some future time the system will return arbitrarily close to its initial situation and will do so infinitely often.
(Poincare, 1885)

Recurrence

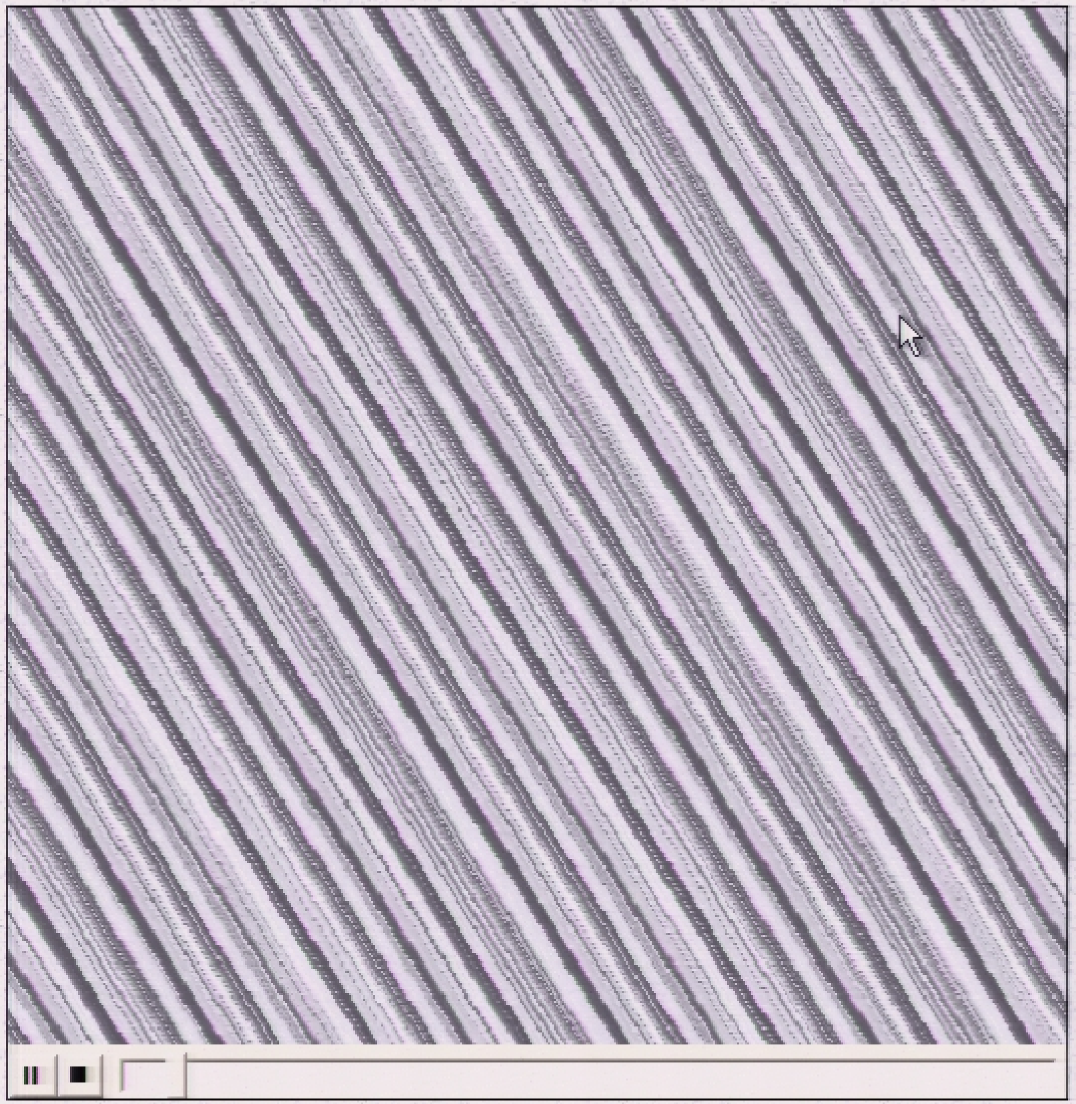
- Was ist´s, das geschehen ist? Eben das hernach geschehen wird. Was ist´s, das man getan hat? Eben das man hernach wieder tun wird; und geschieht nichts Neues unter der Sonne.
- The thing that has been, it is that which shall be; and that which is done is that which shall be done: and there is no new *thing* under the sun.

(Ecclesiastes, Der Prediger Salomo, Kohelet, Kap. 1, Vers 9)

Poincare's Recurrence - demo

poincare_res488_anim

Poincare's Recurrence - demo



Poincare's Recurrence - demo

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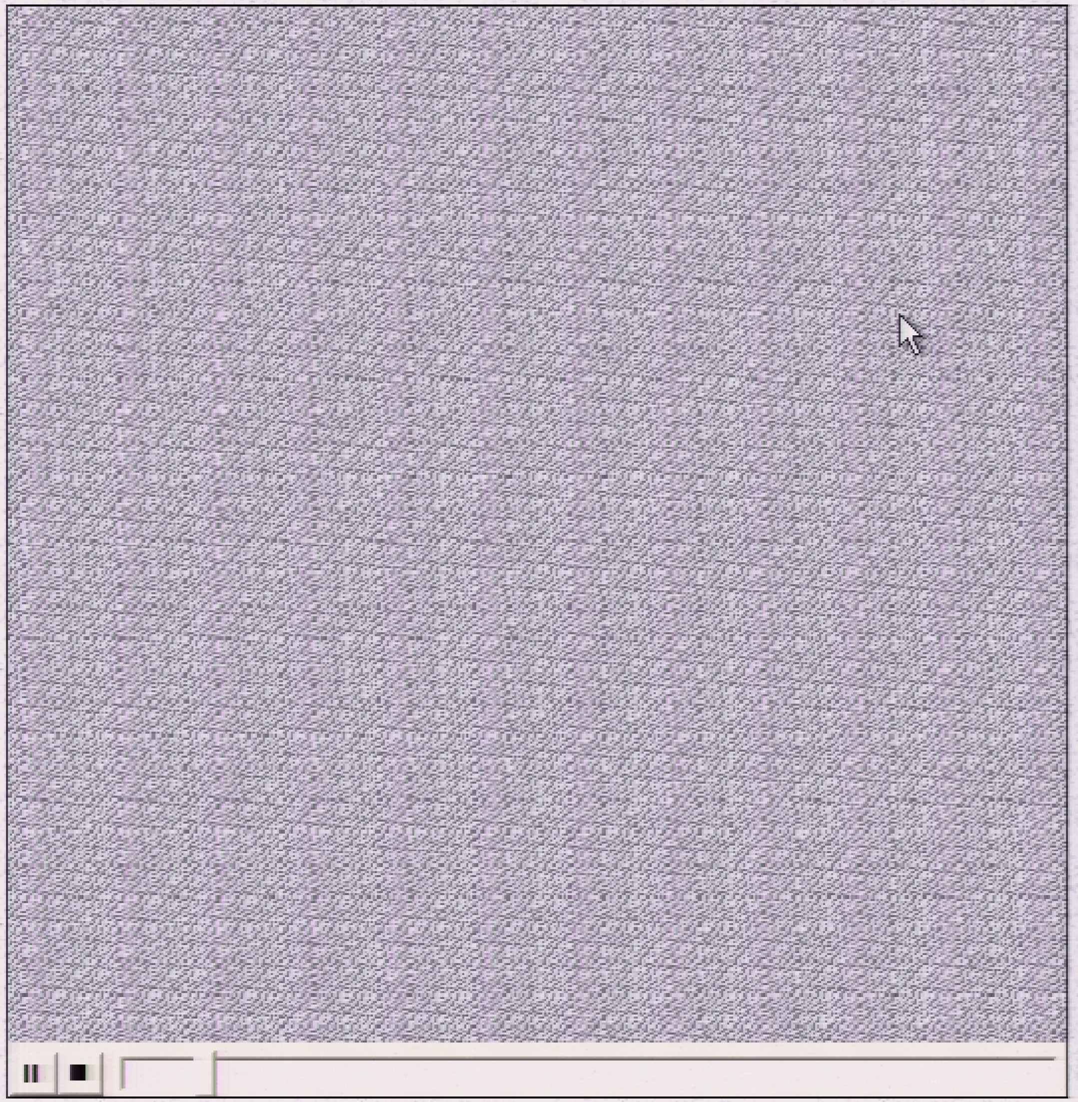

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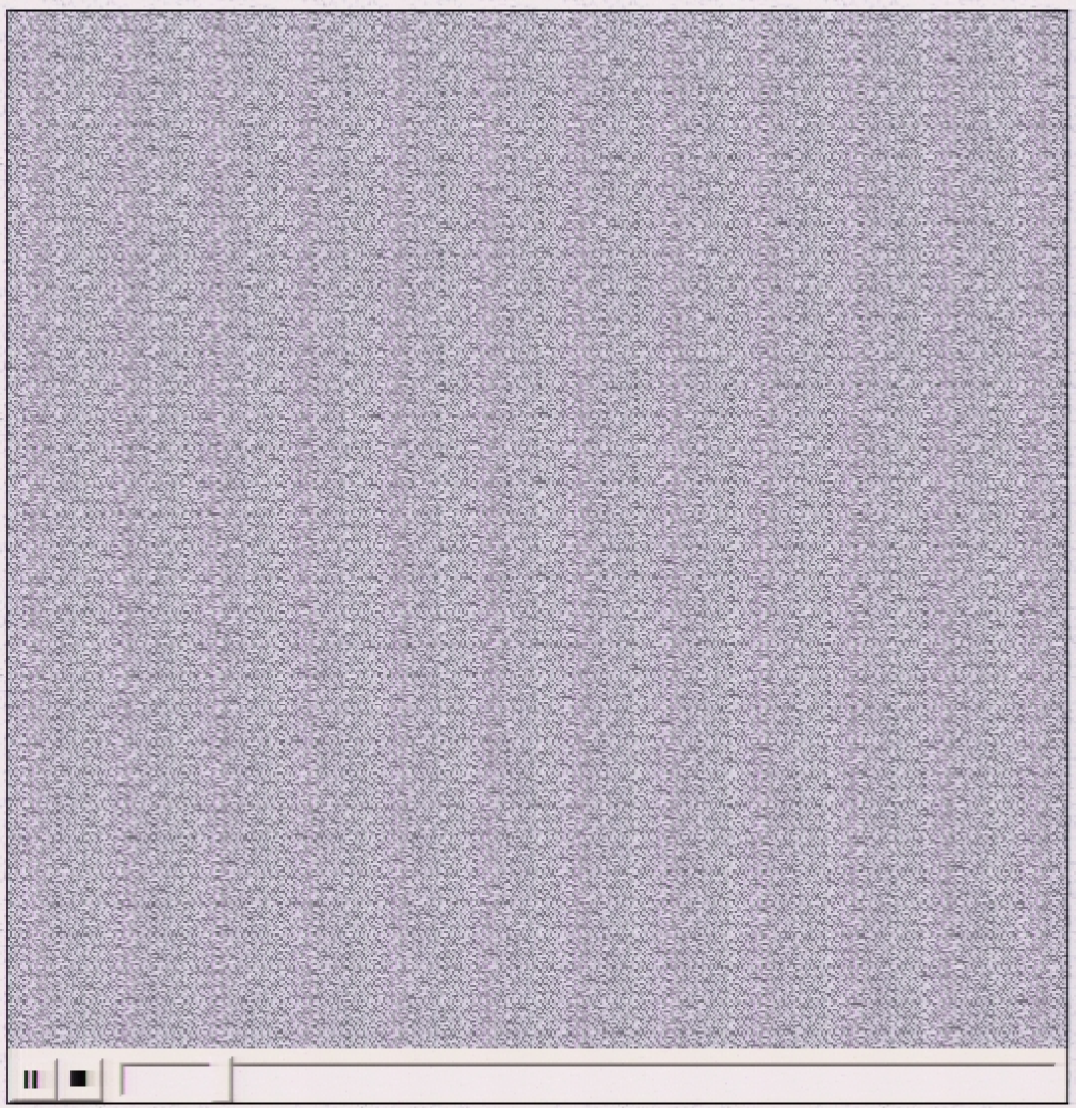

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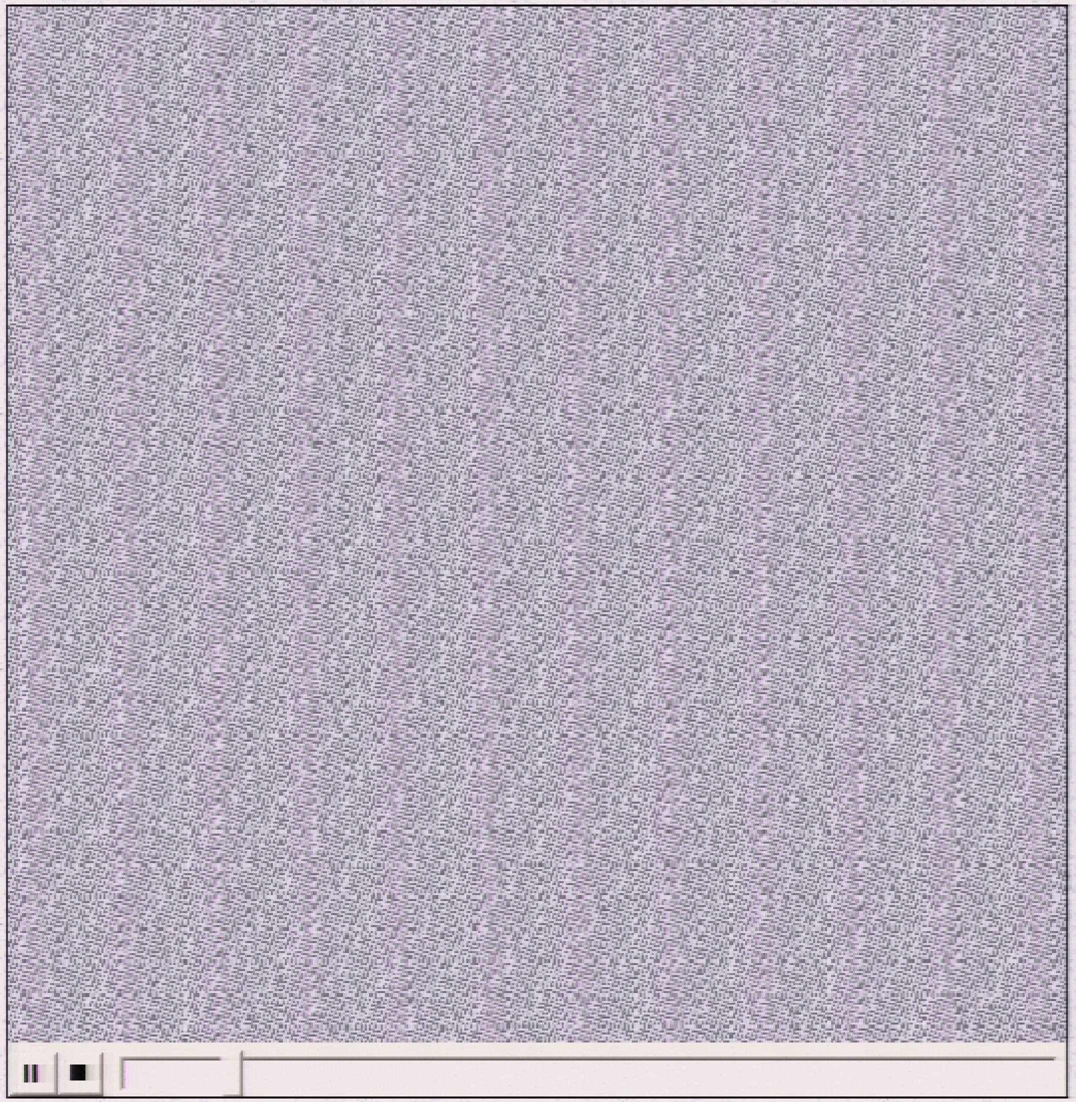

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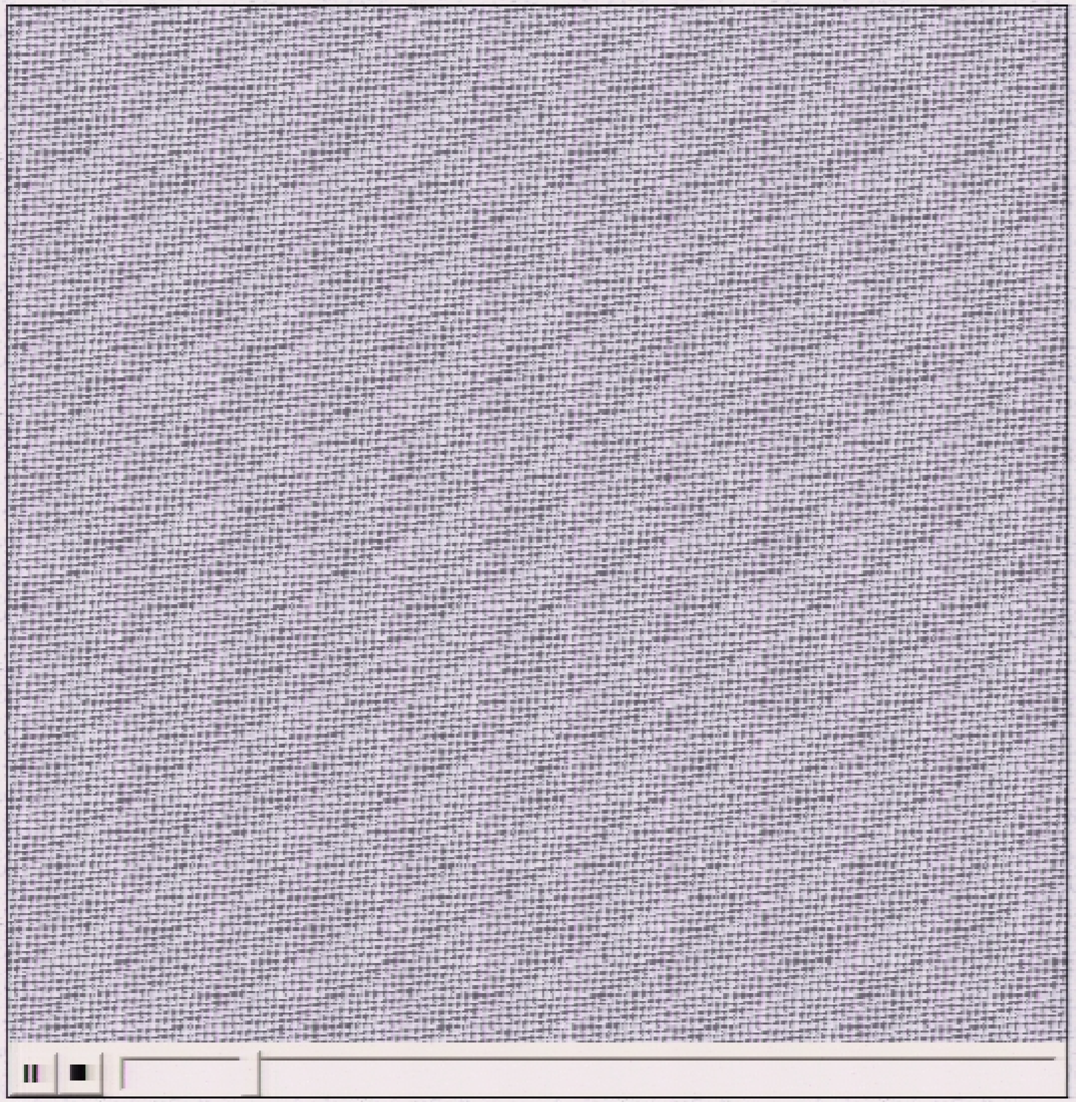
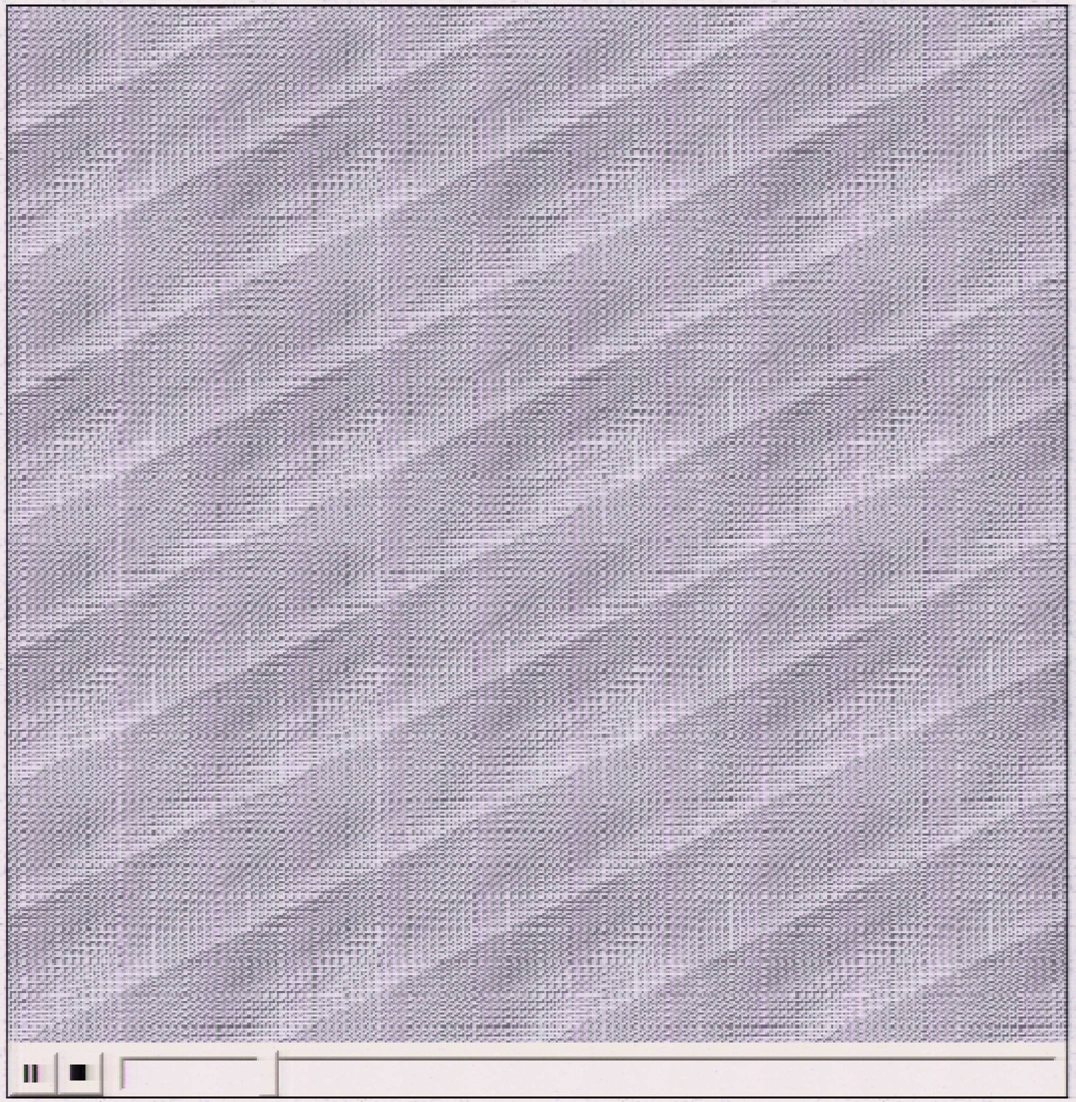


Poincare's Recurrence - demo



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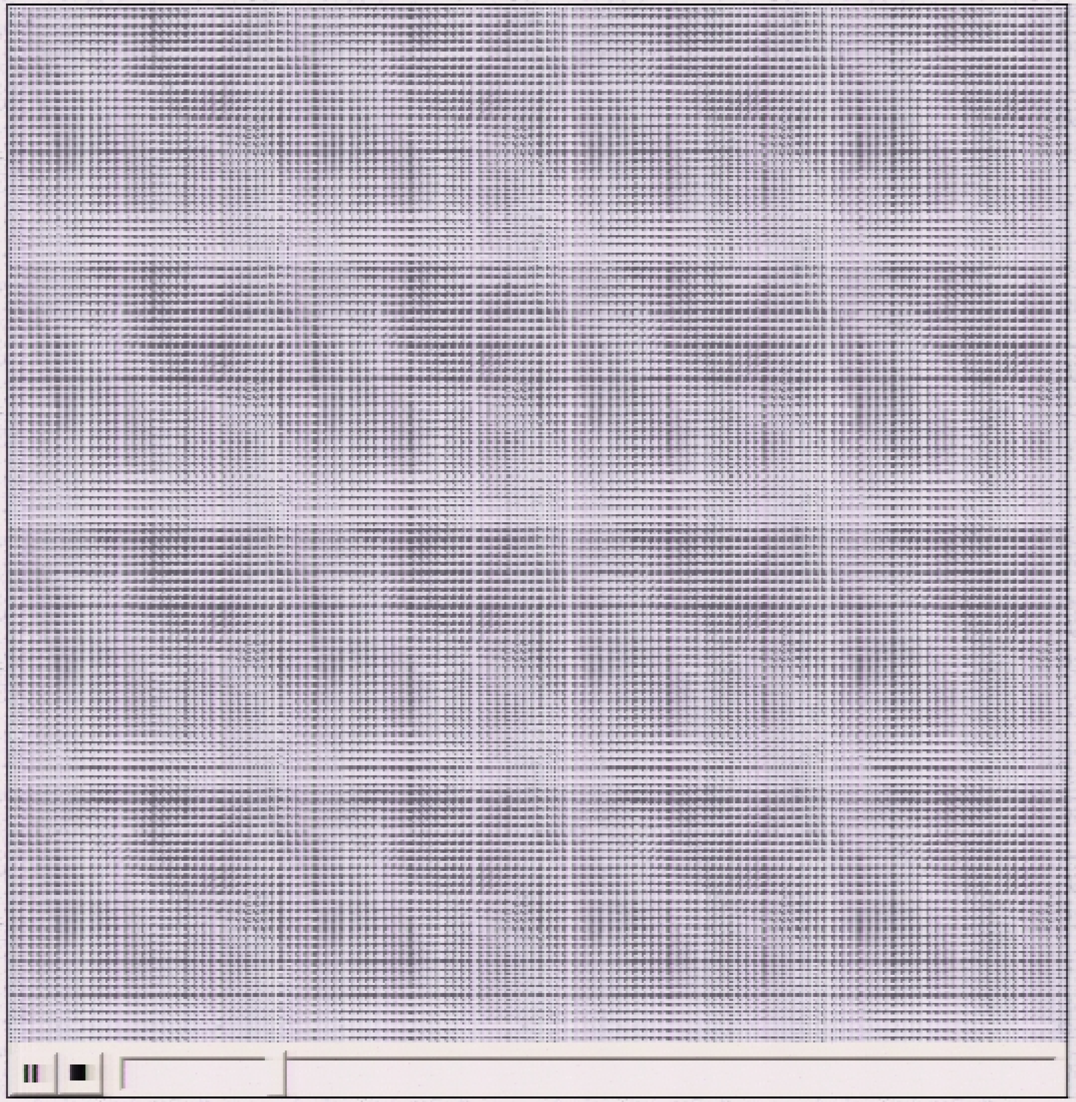

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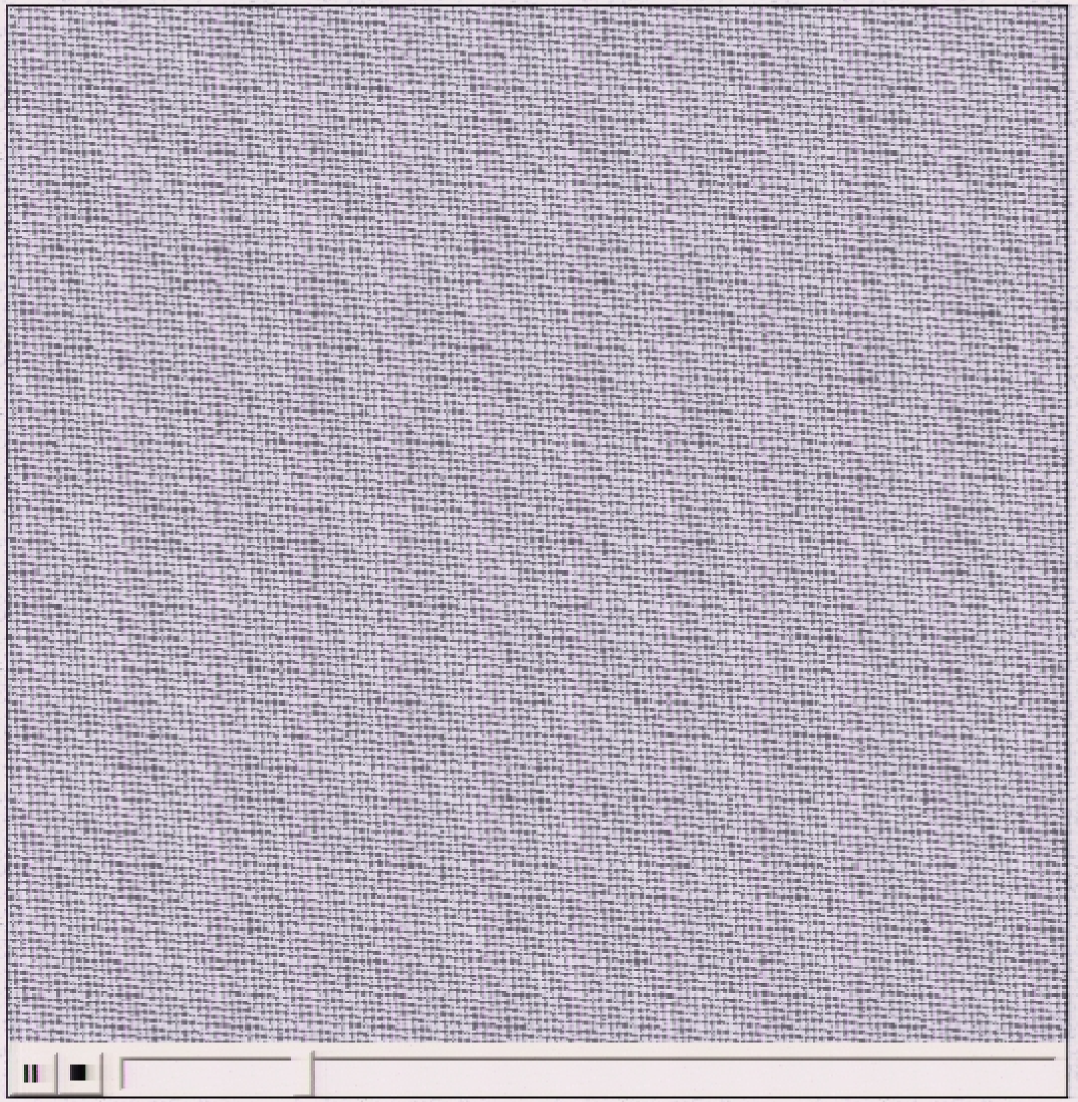

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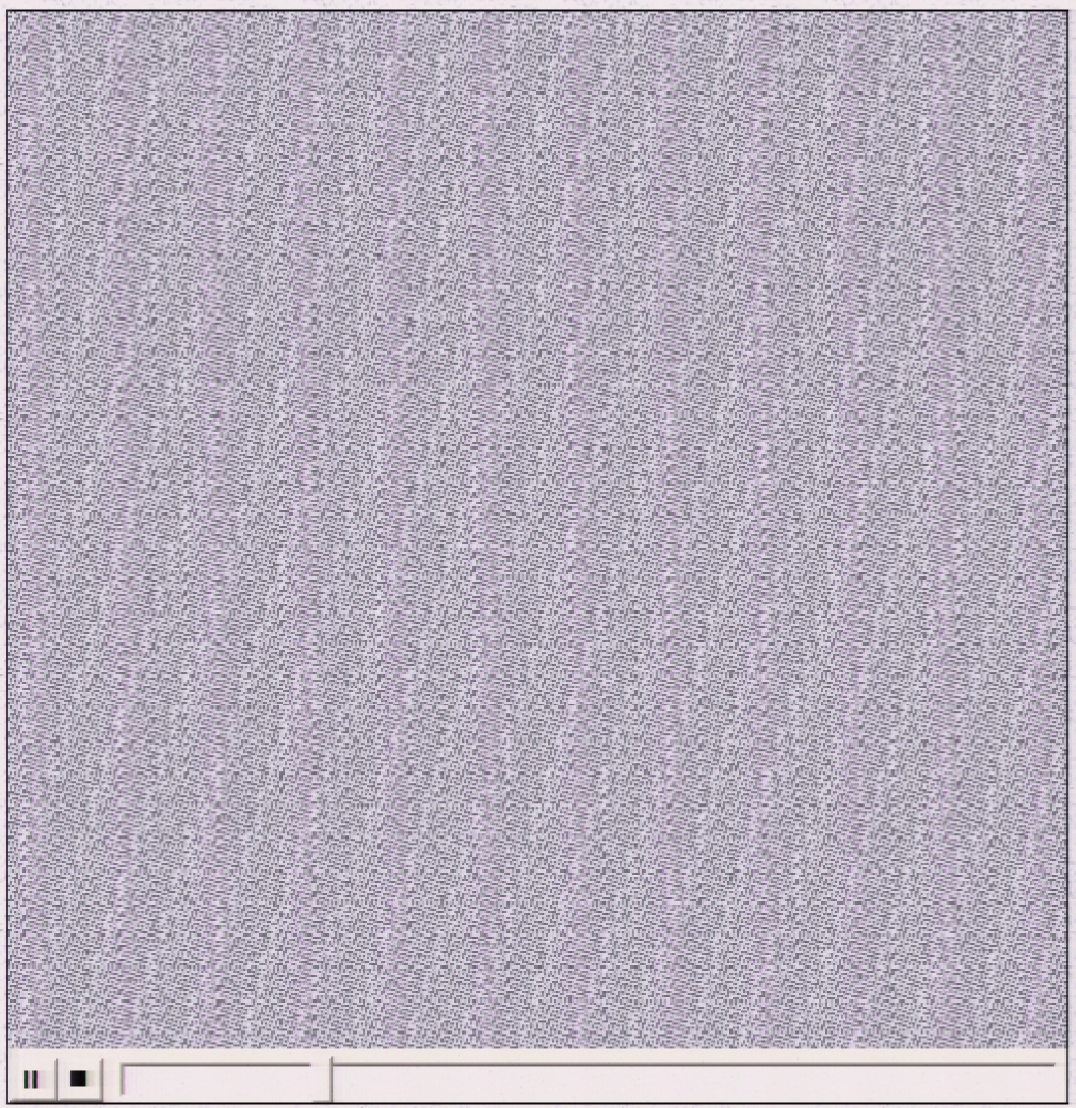

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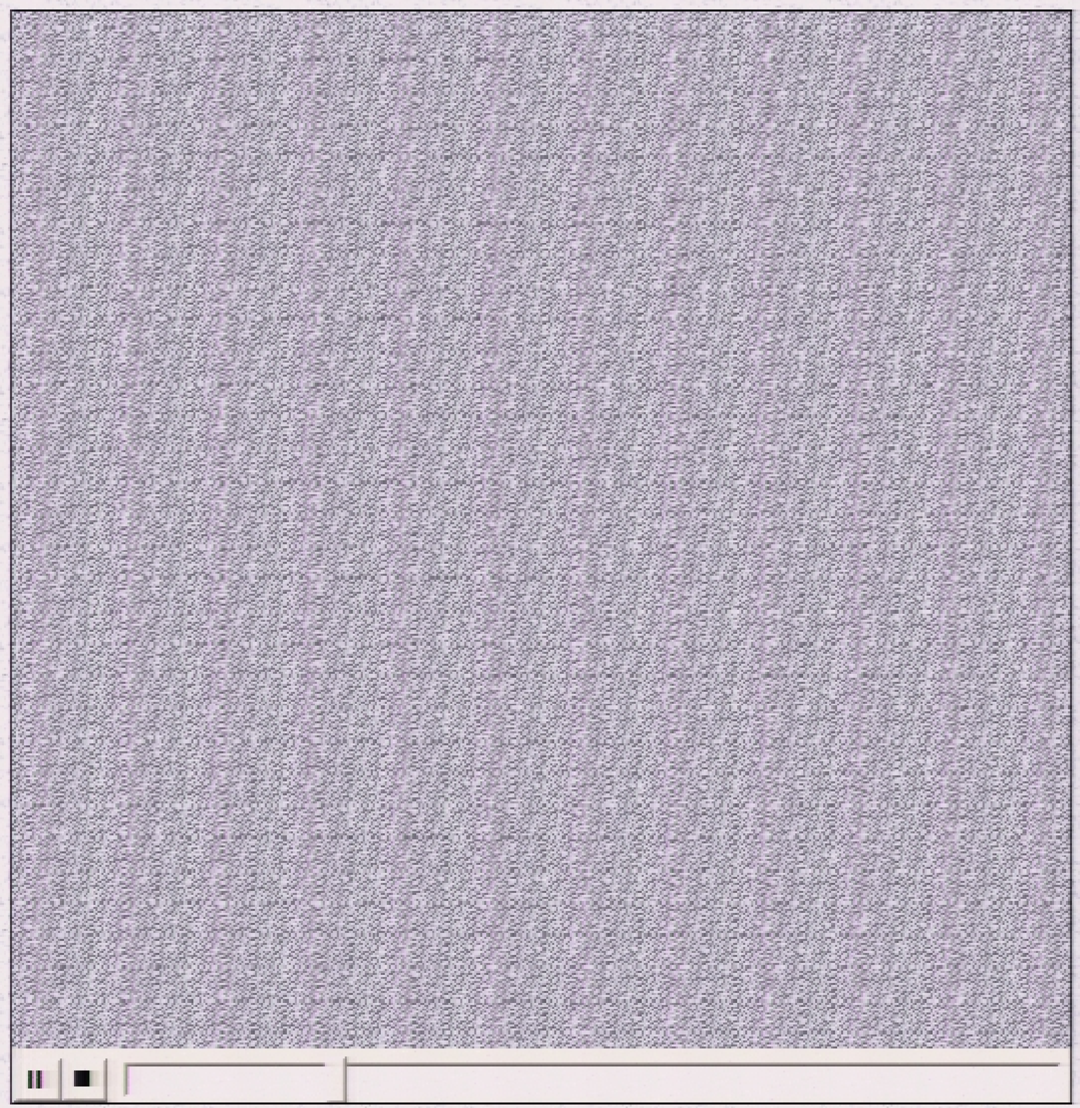

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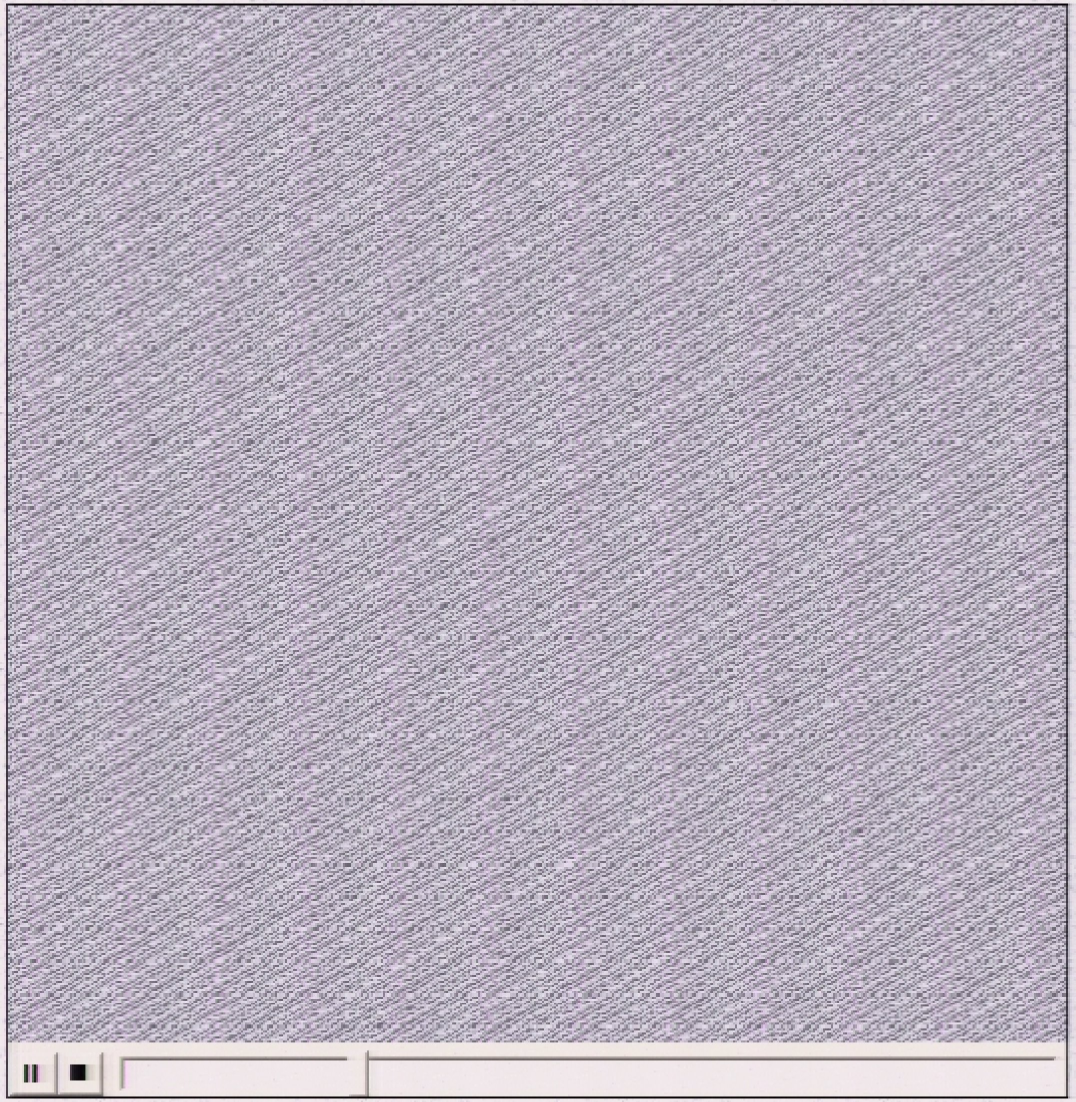

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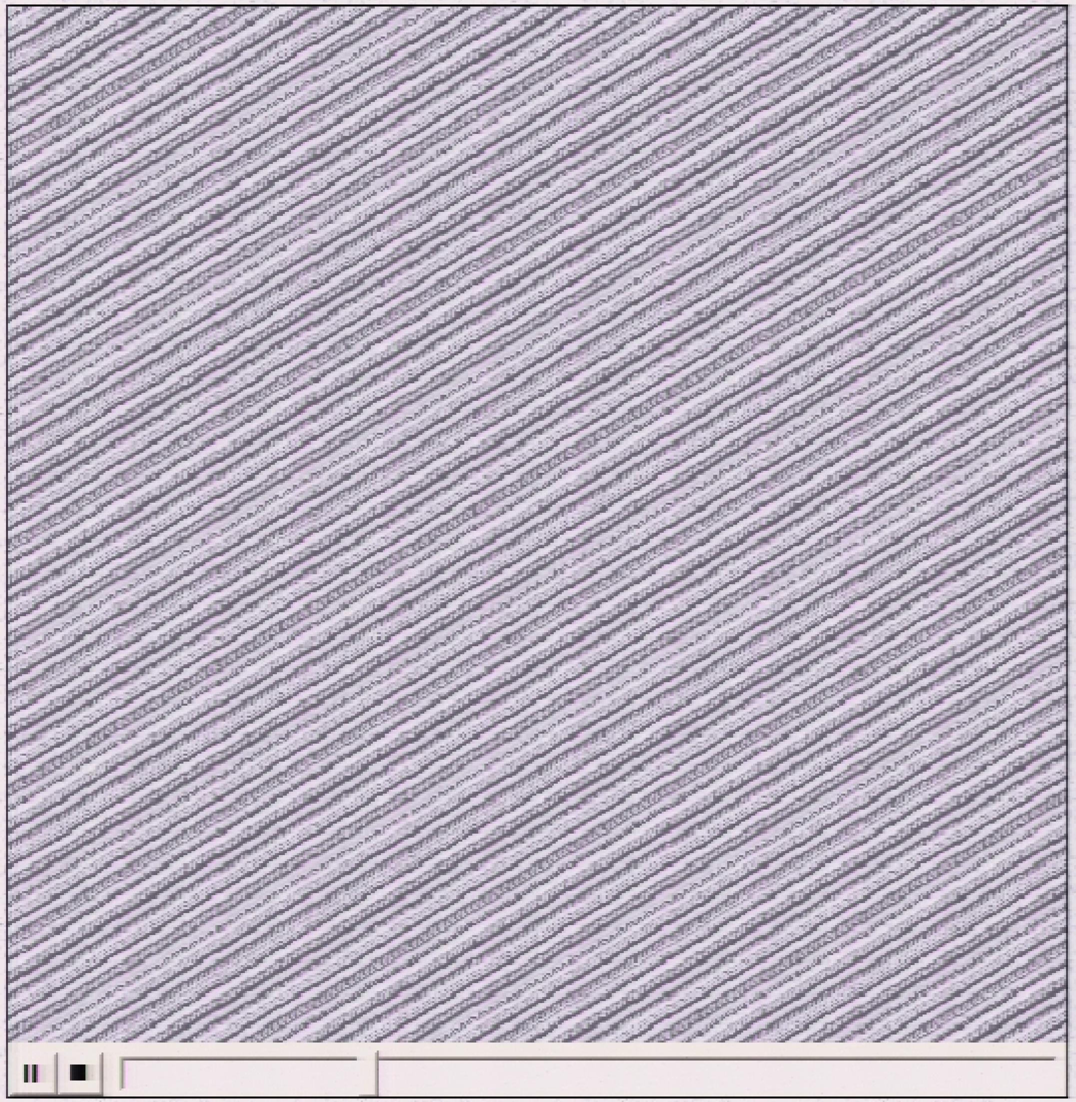

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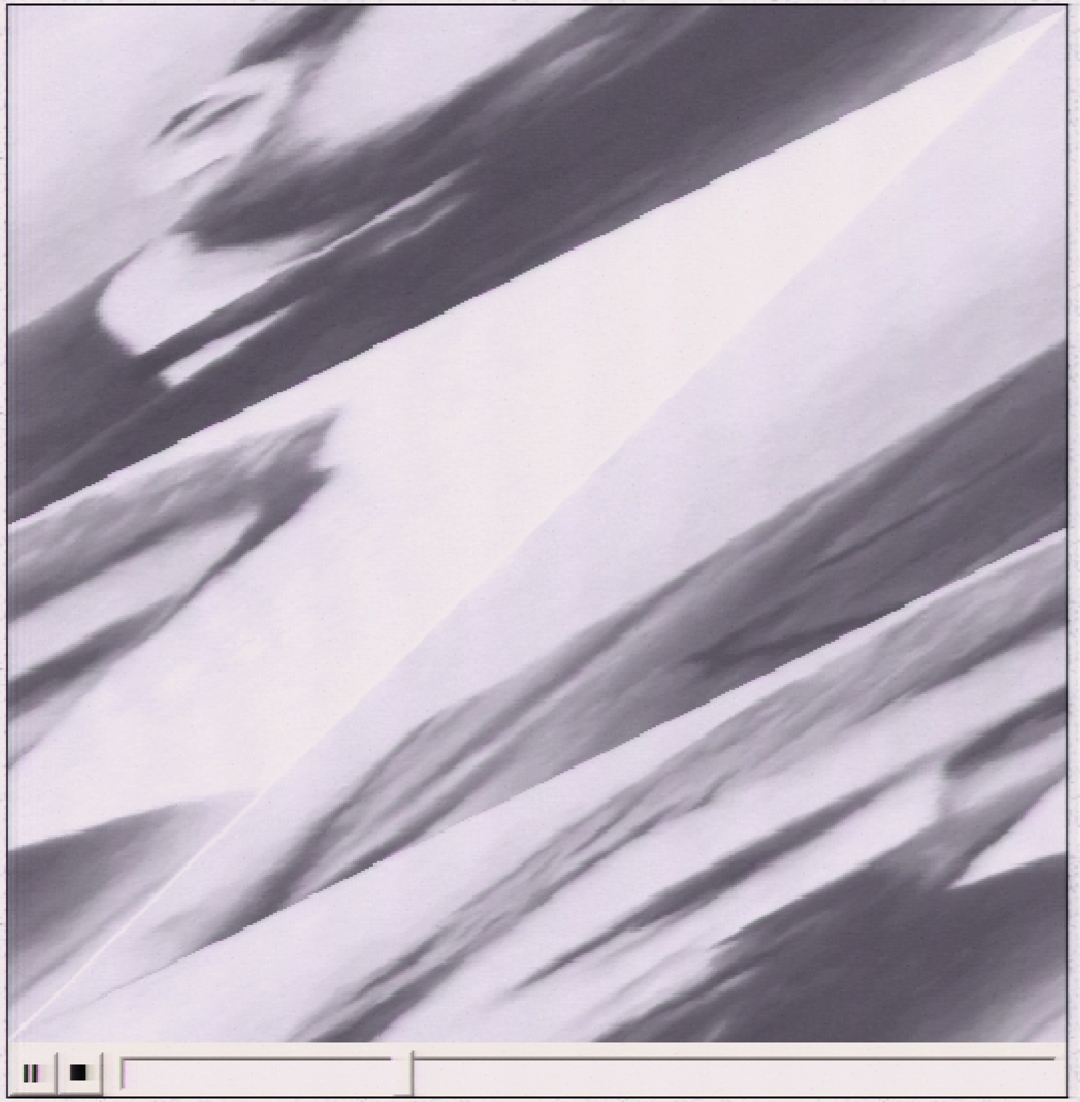

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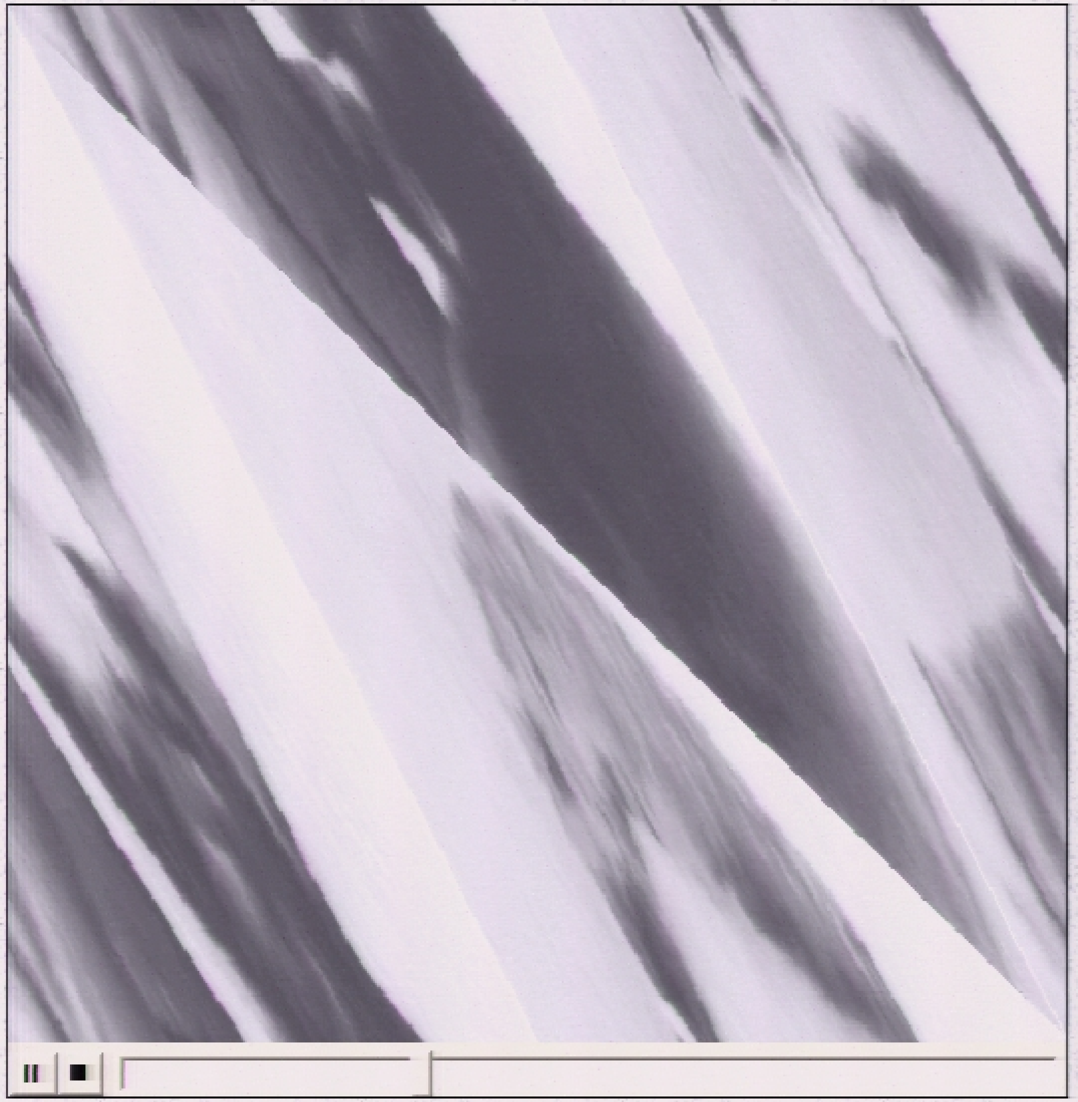

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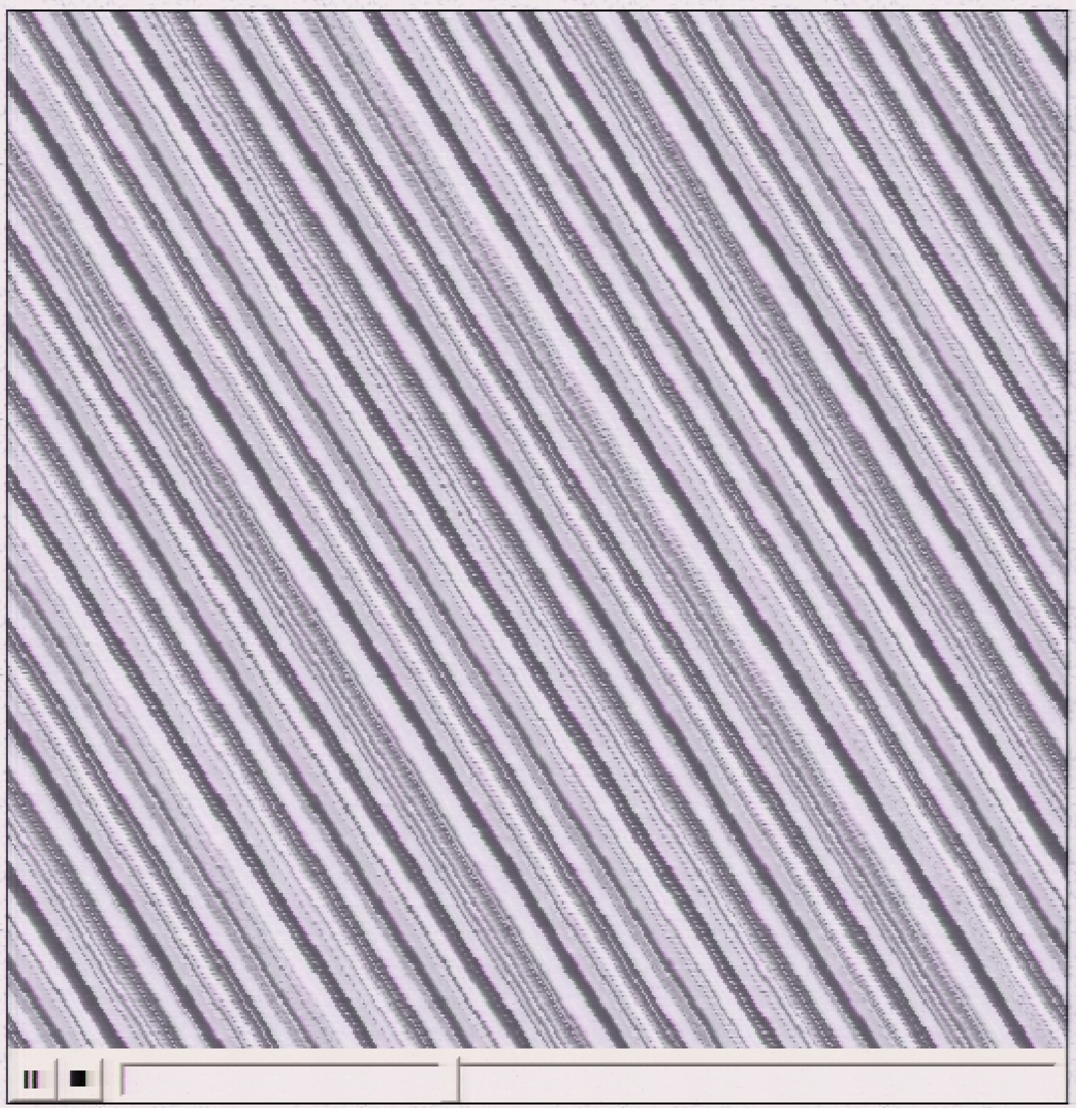

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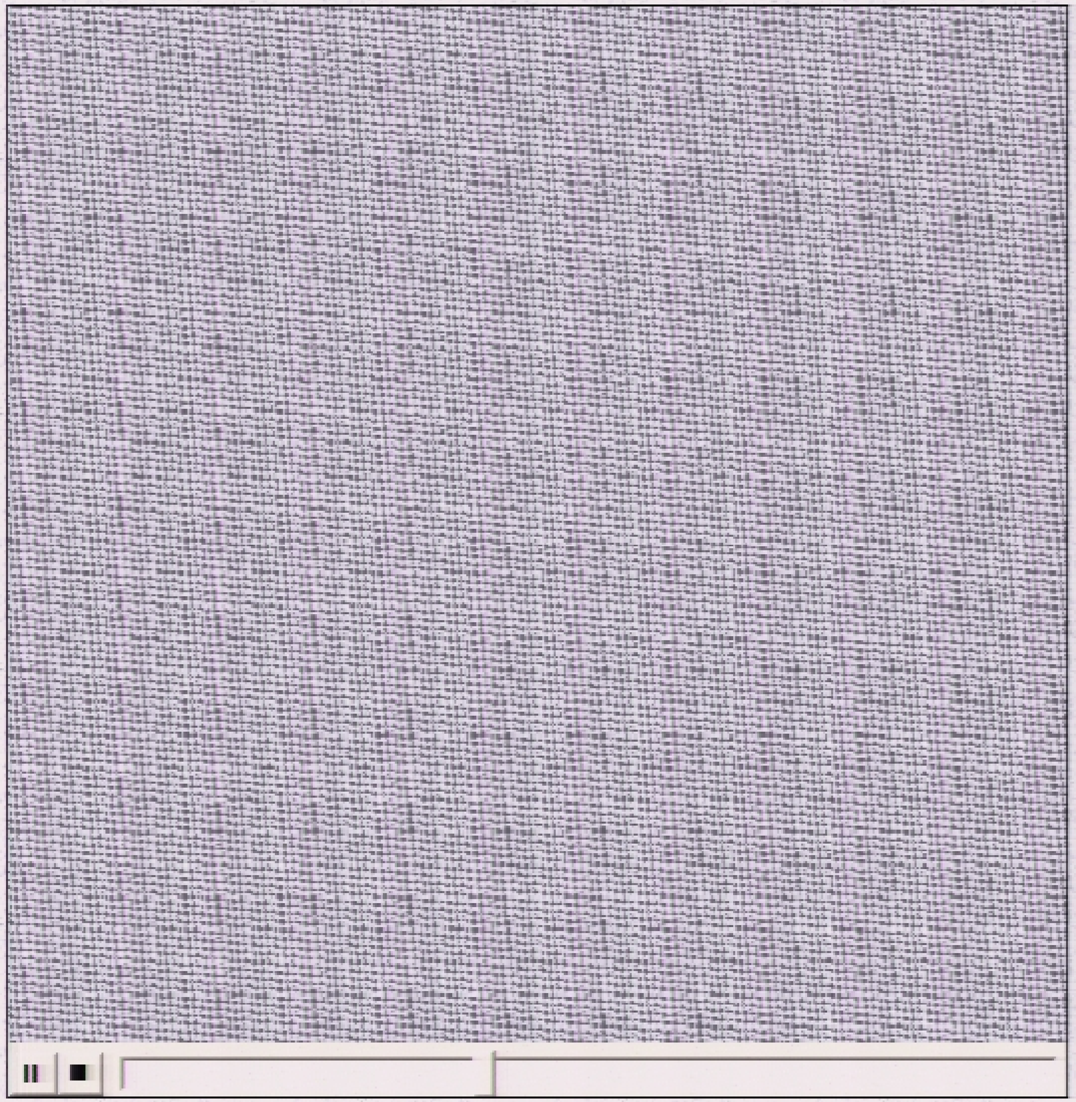


Recurrence plot analysis

- Recurrence plot

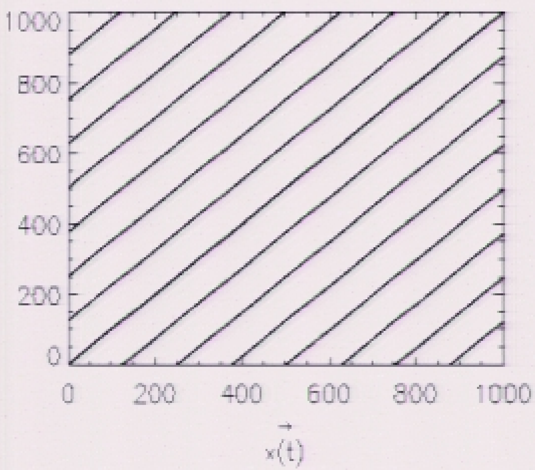
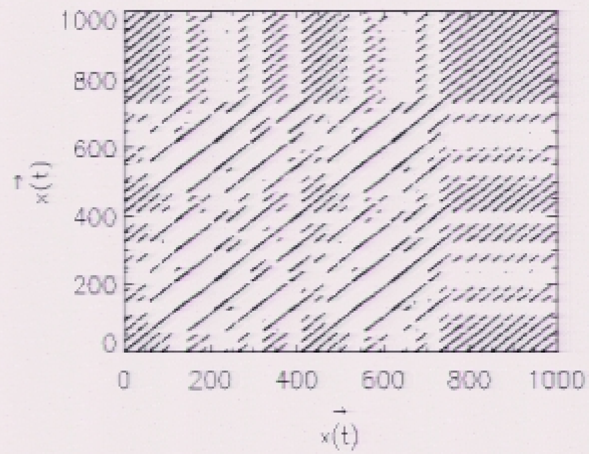
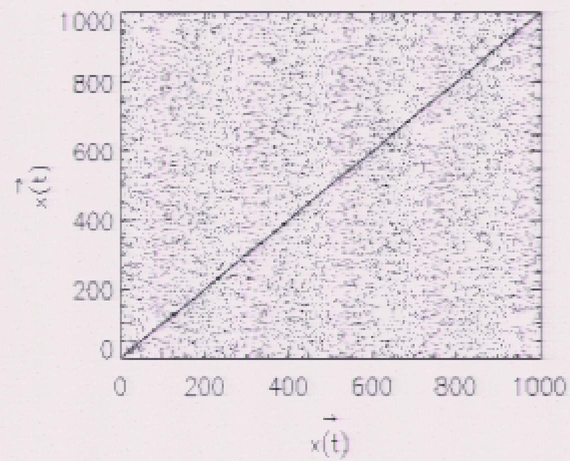
  $R(i, j) = \Theta(\varepsilon - |x(i) - x(j)|)$

Θ – Heaviside function

ε – threshold for neighborhood (recurrence to it)

Measures based on: Recurrence rate, length of diagonals

(Eckmann et al., 1987)

Sine	Rössler oscillator	white noise
		

Predictability
Long diagonals

Unpredictability
Short diagonals

Applications in Astrophysics

- Solar activity synchronized with solar inertial motion (significant during some epochs: 1727-1757, 1802-1832, 1863-1922) – weak interaction between gravity and solar activity
- Search for planets outside the solar system (recurrence analysis)
- Synchronized interaction of black holes???
- Other examples??? I would be very interested in!

Co-workers

- C. Allefeld, D. Maraun, M. Romano, M. Rosenblum, A. Pikovsky, M. Thiel, C. Zhou – Potsdam (Physics)
- R. Engbert, R. Kliegl – Potsdam (Cognitive Science)
- K.-R. Müller, F. Meinecke – Berlin
- G. Osipov, M. Ivanchenko – Nizhny Novgorod
- B. Hu – Hong Kong
- R. Roy – Maryland
- T. Arrecchi, S. Boccaletti – Florence
- A. Motter – Santa Fe/Northwestern
- I. Tokuda - Tokyo
- C. Grebogi – Aberdeen

- M. Palus - Prague

Selection of our papers on synchronization

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Phys. Rev. Lett. 79, 47 (1997)
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Reviews, special issues

S. Boccaletti, J. Kurths, G. Osipov, D. Valladares, C. Zhou, Phys. Rep. 366, 1 (2002)

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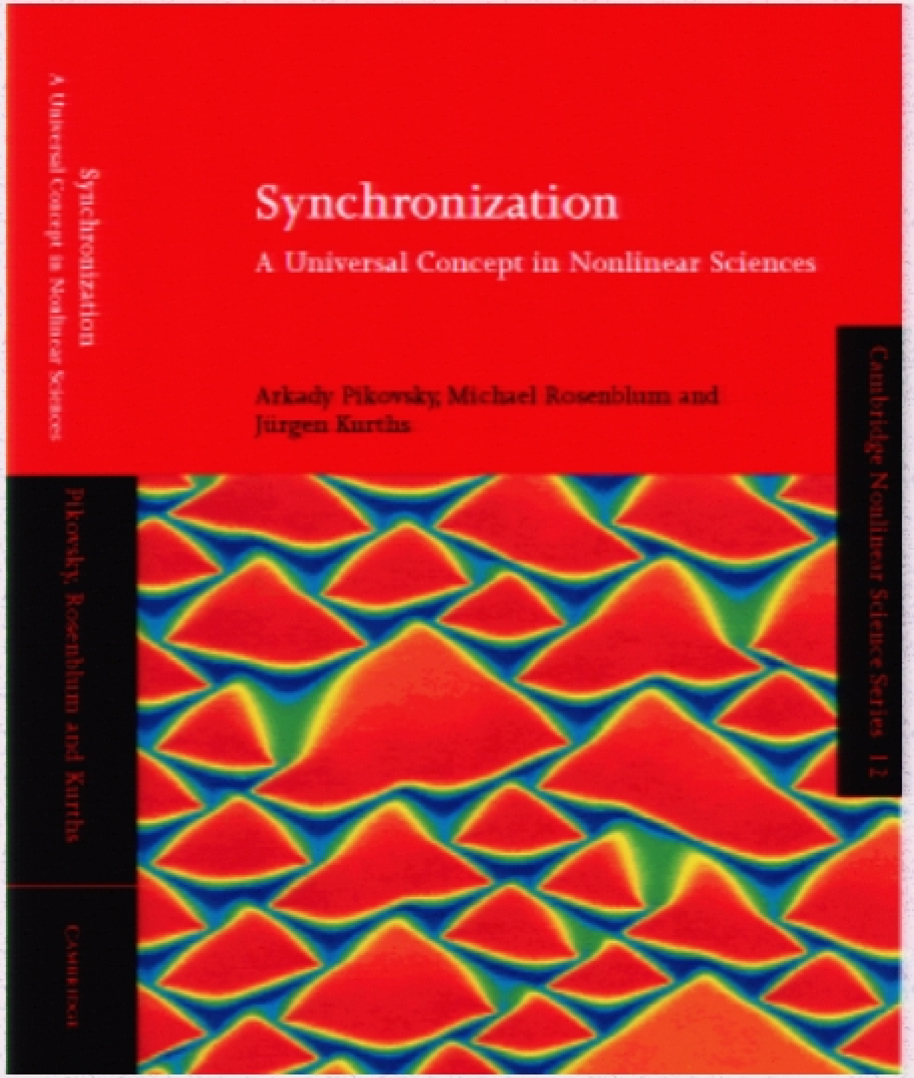
Synchronization
A Universal Concept in Nonlinear Sciences
Arkady Pikovsky, Michael Rosenblum and
Jürgen Kurths
Cambridge Nonlinear Science Series 12
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A Universal Concept in Nonlinear Sciences
Pikovsky, Rosenblum and Kurths