Title: D-barnes in noncritical strings and N=1 super Yang-Mills

Date: Sep 20, 2005 02:00 PM

URL: http://pirsa.org/05090007

Abstract: TBA

D-BRANES IN NON-CRITICAL STRINGS

&

$\mathcal{N}=1$ (MINIMAL) SUPER YANG-MILLS

hep-th/0504079

w/ SAMEER MURTHY

JAN TROOST

LOW ENERGY LIMIT OF OPEN STRING THEORY

$\mathcal{N}=1$ SUPER YANG-MILLS

LOG RUNNING OF COUPLING

CHIRAL SYMMETRY BREAKING

PERT. LEVEL

$\mathcal{N}=4$ SUPERSYMMETRY

GOAL: FIND STRING/GRAVITY DUAL OF $\mathcal{N}=1$ SUPER YANG-MILLS [POLYAKOV]

HARD...

RELATED PROBLEM: CONSTRUCT MICROSCOPIC/WORLDSHEET DESCRIPTION OF D-BRANES IN CERTAIN BACKGROUNDS

SUCH THAT LOW-ENERGY LIMIT OF OPEN STRING THEORY IS $\mathcal{N}=1$ SUPER YANG-MILLS

PROPERTIES: LOG RUNNING OF COUPLING, CHIRAL SYMMETRY BREAKING } PERT. LEVEL

$$U(1)_R \longrightarrow \mathbb{Z}_{2N}$$

SETTING: NON-CRITICAL STRINGS $\mathbb{R}^{1,d-1} \times \left(\frac{SL(2,\mathbb{R})}{U(1)}\right)_k$

$$c_{mat} = 15 = \frac{3d}{2} + \left(3 + \frac{6}{k}\right) \qquad d=4 \Rightarrow k=1$$

WHY?
- BULK HAS $\mathcal{N}=2$ SUPERSYMMETRY (4d)
  ↳ D-BRANES PRESERVE $\mathcal{N}=1$ ON WORLDVOLUME
  SO, NO NEED TO LOOK FOR MECHANISMS TO ~~SUSY~~
- NO EXTRA MATTER ↳ JUST PURE $\mathcal{N}=1$ SYM

[TESCHNER] · COSET CFT ↳ NON-RATIONAL, BUT RECENTLY BOTH OPEN/CLOSED THEORIES 'SOLVED'

BULK & BOUNDARY CORRELATORS KNOWN...

MORE ABOUT THE COSET:

$\left(\frac{SL(2,R)}{U(1)}\right)_k$ AS A GAUGED WZW MODEL [WITTEN]

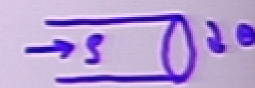
AT TREE LEVEL, INTEGRATE OUT THE GAUGE FIELD ⤳

$$ds^2 = k\left(d\rho^2 + (\tanh\rho)^2 d\theta^2\right)$$

$$\Phi = \Phi_0 - \log\cosh\rho$$

$\rho \to \infty \quad \Rightarrow \quad ds^2 = k\left(d\rho^2 + d\theta^2\right)$

$$\Phi = \Phi_0 - \rho$$

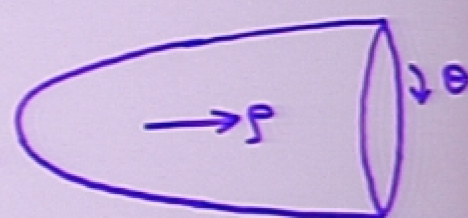


LINEAR DILATON

$\rho \to 0 \quad \Rightarrow \quad ds^2 = k\left(d\rho^2 + \rho^2 d\theta^2\right)$

$$\Phi = \Phi_0$$

BOSONIC COSET$_k$: 'CIGAR' BACKGROUND



CURVATURE $\sim \sqrt{k\alpha'}$; SO $k=1$ ⤴ PICTURE NOT VALID

YET, FOR $N=2$ VERSION, MIGHT STILL BE USEFUL...

IDEA: START WITH D-BRANES IN THIS BACKGROUND, TREATED EXACTLY IN α' AS A COSET CURRENT ALGEBRA. ⤳ FULL SOLUTION AVAILABLE

ASIDE ON AdS/CFT

FLAT SPACE — $\mathbb{R}^{1,3} \otimes$ CIGAR CFT

D3 — D3 ALONG $\mathbb{R}^{1,3}$ $\oplus$ 'LOCALIZED' BRANES IN CIGAR

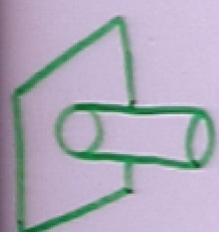
$\mathcal{N}=4$ SYM — $\mathcal{N}=1$ SYM

$AdS_5 \times S^5$ — ?!

(∃) $AdS_5 \times S^1$ DUAL TO $\mathcal{N}=1$ SCFT BUT...

IN THE ABSENCE OF A DUAL, FOCUS ON 'BOUNDARY STATE' OR WORLDSHEET DESCRIPTION OF GAUGE THEORY ⇝ STEP TOWARDS POSSIBLE STRING DUAL...

BASIC IDEA: USE BOUNDARY STATE DESCRIBING 'D-BRANE' TO DETERMINE LONG-DISTANCE BEHAVIOUR OF CLOSED STRING FIELDS SOURCED BY BRANE...



CLASSICAL SOURCE INTERACTING WITH QUANTUM STATES ⇝ AMPLITUDE FOR EMISSION OF $V_{cl} = \langle V_{cl} | D_{cl} | B \rangle$

D_{cl} → PROPAGATOR $\sim \frac{1}{L_0 - a} \sim \frac{1}{k_\perp^2}$

ROUGHLY, $|B\rangle \sim \sum_{\text{all states}} \psi_x |x\rangle$

EXAMPLE OF FLAT SPACE...

[di VECCHIA et al.]

$$J^{\mu\nu}(k) = \langle \nu^{\mu\nu}(k) | D_{cl} | B \rangle = \frac{C}{k_\perp^2} \cdot \delta^{p+1}(k_\parallel)$$

↓

FOURIER TRANSFORM

↓

$C \cdot G(x_\perp)$ ↳ GREEN'S FUNCTION FOR LAPLACE OPERATOR

WHAT IS THIS COMPUTING ?

RECALL SOLUTION IN SUPERGRAVITY :

$$ds^2 = H^{2a} dx_\parallel^2 + H^{2b} dx_\perp^2 \qquad e^{\Phi} = H^{\tau}$$

$$C_{0\cdots p} = H^{-1}$$

$$H = 1 + 2k T_p G(x_\perp)$$

↳ GREEN'S FUNCTION

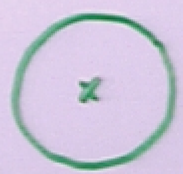
LARGE DISTANCES / FIRST ORDER BACKREACTION TO FLAT SPACE GEOMETRY DUE TO PRESENCE OF BRANE

$$e^{\Phi} \simeq 1 + \underbrace{(2k T_p \tau)\, G(x_\perp)}$$

↓

CAPTURED BY F.T. OF $J^{\mu\nu}(k)$

IN STRING THEORY,



OR DISC ONE-POINT FUNCTION ↳ Ψ_X

PLAN: REPEAT PROCEDURE FOR D-BRANES ON CIGAR

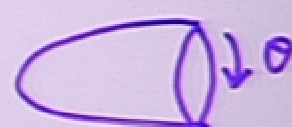
- $e^{\Phi(\rho)} \longleftrightarrow g_{YM}^{-2}(\mu)$ ↳ SCALE

RADIAL/HOLOGRAPHIC COÖRDINATE

- R-SYMMETRIES IN GAUGE THEORY GET MAPPED TO ISOMETRIES OF 'GRAVITY DUAL'

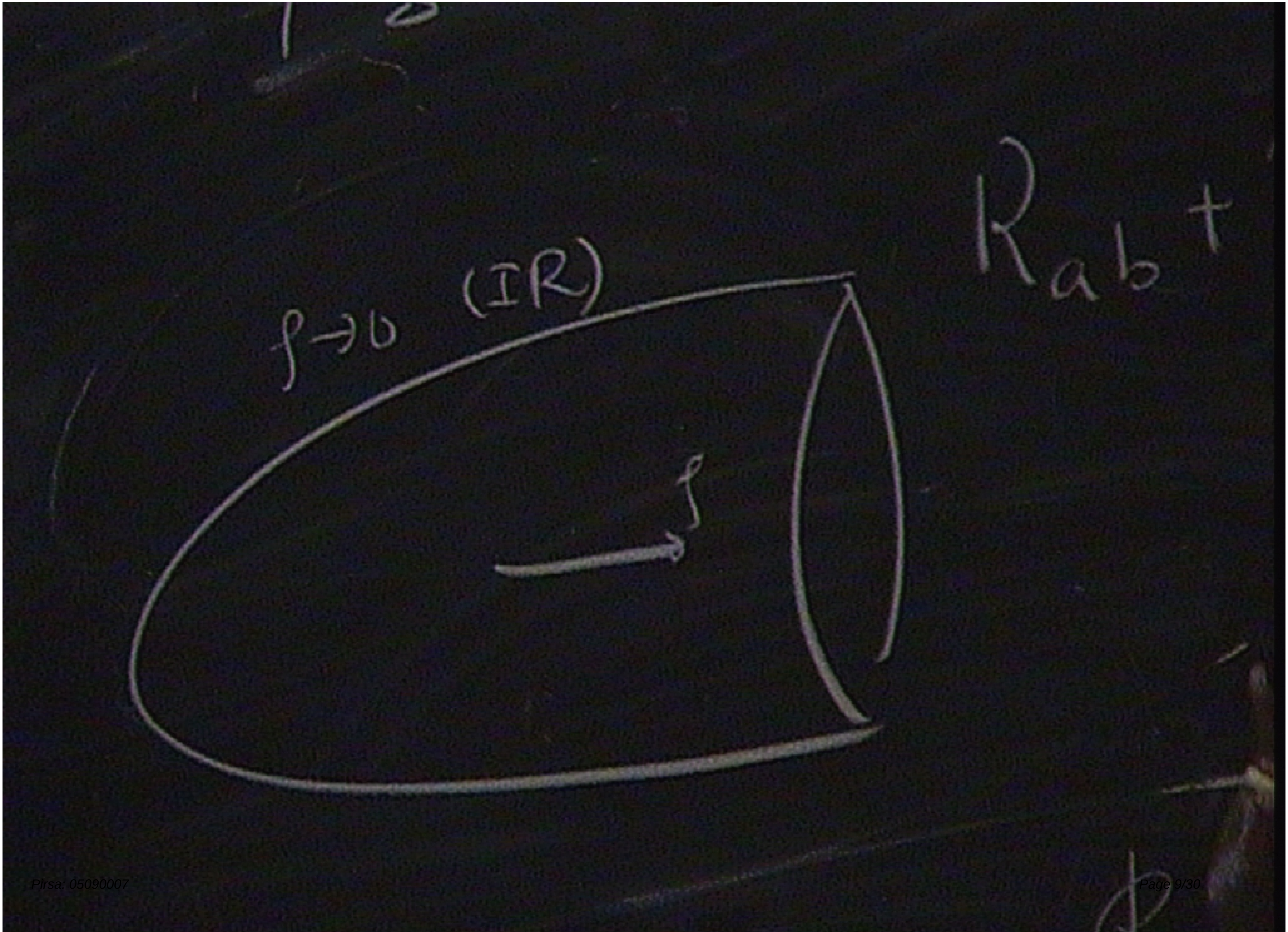
eg: $N=4 \rightsquigarrow SO(6)_R \equiv$ ISOMETRIES OF S^5

FOR $N=1 \rightsquigarrow U(1)_R \longrightarrow$ IDENTIFY THIS WITH AN ISOMETRY OF S^1



$2\theta_{YM} \sim \theta_{cig}.$

$[P^\theta, S_\alpha] = \frac{1}{2} S_\alpha$ etc...


$\rho \to 0$ (IR)
ρ
$R_{ab}\,t$

$$e^{\Phi(\rho)} \Longleftrightarrow g_{YM}^{-2}(\mu)$$

RADIAL / HOLOGRAPHIC COORDINATE

SCALE

R-SYMMETRIES IN GAUGE THEORY GET MAPPED TO ISOMETRIES OF 'GRAVITY DUAL'

eg. $N=4 \rightsquigarrow SO(6)_R \equiv$ ISOMETRIES OF S^5

FOR $N=1 \rightsquigarrow U(1)_R \Longrightarrow$ IDENTIFY THIS WITH AN ISOMETRY OF S^1

$$2\theta_{YM} \approx \theta_{e.g.}$$

$$[P^\theta, g_\alpha] = \frac{1}{2} g_\alpha \quad \text{etc.}$$

WHAT DO WE NEED?

- DESCRIPTION OF CLOSED STRING OPERATORS Φ
- DESCRIPTION OF D-BRANE BOUNDARY STATE $|B\rangle$
- PROVE THAT IT RESPECTS
  - CARDY CONSTRAINTS
  - SUPERSYMMETRY (GSO PROJECTION)
- SHOW THAT IN OPEN STRING CHANNEL
  - SPECTRUM MATCHES $\mathcal{N}=1$ SYM
- COMPUTE $\langle \Phi | D_a | B \rangle$

"FOURIER TRANSFORM" AND GET

LINEAR BACK REACTION

CHECK QUALITATIVE PROPERTIES...

CIGAR THEORY : BULK OPERATORS...

$$\mathcal{N}=2\left(\frac{SL(2,\mathbb{R})}{U(1)}\right)_k = \text{BOSONIC}\left(\frac{SL(2,\mathbb{R})}{U(1)}\right)_{k+2} \cdot \left(\text{2 FREE FERMIONS}\right)$$

$$\Phi(j,m,\bar{m},n,\bar{n}) \sim \Phi(j,m_{bos},\bar{m}_{bos}) \cdot e^{inH+\bar{n}\bar{H}}$$

- $n \sim$ 'FERMION NUMBER' (NS/R)
- $j \sim SL(2,\mathbb{R})$ QUANTUM NUMBER
- $m = m_{bos} + n$

GAUGING $\Rightarrow$ $m = \frac{p+k\omega}{2}$; $\bar{m} = -\left(\frac{p-k\omega}{2}\right)$

$p, \omega \sim$ ASYMPTOTIC MOMENTUM/WINDING AROUND CIRCLE DIRECTION

$$\Delta\left(\Phi(j,m,\bar{m},n,\bar{n})\right) = \frac{-j(j+1)}{k} + \frac{m^2}{k} + \frac{n^2}{2}$$

$$Q\left(\Phi(j,m,\bar{m},n,\bar{n})\right) = \frac{2m}{k} + n$$

FULL OPERATOR : $\left(\begin{matrix}\text{FLAT SPACE}\\ \text{PART}\end{matrix}\right) \cdot \Phi(j,m_{bos},\bar{m}_{bos},n,\bar{n})$

- IMPOSE PHYSICAL CONDITIONS...

EXAMPLES :

GRAVITON $\psi^I_{-\frac{1}{2}} \tilde{\psi}^J_{-\frac{1}{2}} |0,k\rangle_{4d} \otimes \Phi(j,0..)|0\rangle_{cig} \otimes |0\rangle_{gh}$

PHYSICAL STATES OBEY $L_0 - \frac{1}{2} = 0$ [$(-1,-1)$ PICTURE]

$$L_0 - \frac{1}{2} = \frac{1}{2} + \frac{1}{2}k_\mu^2 - \frac{j(j+1)}{k} - \frac{1}{2} = 0 \qquad [j = -\frac{1}{2} + iP]$$

$$k_\mu^2 = 0 \implies \boxed{P^2 = -\frac{1}{4}} \implies \{ j = -1 \rightarrow \text{DISCRETE STATE}$$

RR AXION

$$S^\alpha \tilde{S}_\alpha |0,k\rangle_{4d} \otimes \Phi(j,\tfrac{1}{2},\tfrac{1}{2})|0\rangle_{cig} \otimes |0\rangle_{gh}$$

$[(-\frac{1}{2},-\frac{1}{2})$ PICTURE$]$

$$L_0^{tot} - 1 = \frac{3}{8} + 2\cdot\left(\frac{1}{8}\right) + \left(P^2 + \frac{1}{4}\right) + \frac{1}{8} - 1 = P^2 \quad (k_\mu^2 = 0)$$

$$\boxed{P = 0} \qquad \left\{ \begin{matrix} m_{bos} = -n = \frac{1}{2} = \bar{m}_{bos} \implies m = 0 \\ j = -\frac{1}{2} \end{matrix} \right\}$$

- EFFECTIVE MASS IN 4D FOR GRAVITON $(\sim \frac{1}{2})$
- 'MASSLESS' RR SCALAR

↳ DIFFERENT FALL-OFF RATES IN PROFILES IN THE PRESENCE OF A D-BRANE

$= L_0$

$$R_{ab} + D_a D_b \Phi = 0$$

$$\begin{cases} \dfrac{1}{L_0 - 1} = \dfrac{1}{p^2 + 1/4} & \text{graviton} \\ \quad = \dfrac{1}{p^2} & \text{RR scalar} \end{cases}$$

EXAMPLES :

GRAVITON ~~(NS)~~ $\psi^{I}_{-\frac{1}{2}}\,\tilde{\psi}^{J}_{-\frac{1}{2}}\,|0,k\rangle_{4d}\otimes\Phi(j,0\ldots)\,|0\rangle_{cig}\otimes|0\rangle_{gh}$

PHYSICAL STATES OBEY $L_0-\frac{1}{2}=0$ [(-1,-1) PICTURE]

$$L_0-\frac{1}{2} = \frac{1}{2}+\frac{1}{2}k_\mu^2-\frac{j(j+1)}{k}-\frac{1}{2}=0 \qquad [\,j=-\tfrac{1}{2}+iP\,]$$

$$k_\mu^2=0 \Rightarrow \boxed{P^2=-\frac{1}{4}} \Rightarrow \left\{ j=-1 \rightarrow \text{DISCRETE STATE} \right.$$

RR AXION

$$S^\alpha\,\tilde{S}_\alpha\,|0,k\rangle_{4d}\otimes\Phi(j,\tfrac{1}{2},-\tfrac{1}{2})\,|0\rangle_{cig}\otimes|0\rangle_{gh}$$

[$(-\frac{1}{2},-\frac{1}{2})$ PICTURE]

$$L_0^{tot}-1 = \frac{3}{8}+2\cdot\left(\frac{1}{8}\right)+\left(P^2+\frac{1}{4}\right)+\frac{1}{8}-1 = P^2 \quad (k_\mu^2=0)$$

$$\boxed{P=0} \qquad \left\{ \begin{matrix} m_{bos}=-n=\frac{1}{2}=\bar{m}_{bos} \Rightarrow m=0 \\ j=-\frac{1}{2} \end{matrix} \right\}$$

- EFFECTIVE MASS IN 4D FOR GRAVITON $(\sim\frac{1}{2})$
- 'MASSLESS' RR SCALAR

↳ DIFFERENT FALL-OFF RATES IN PROFILES IN THE PRESENCE OF A D-BRANE

BOUNDARY STATE :

ROUGHLY. $|B\rangle = \sum_{\alpha} |NS\rangle^{\alpha}_{x} \otimes |DO\rangle^{\alpha}_{cig} \otimes |gh\rangle^{\alpha}$

BOUNDARY CONDITIONS / SPIN STRUCTURE

$$|DO\rangle^{\alpha}_{cig} = \sum_{j, m_{bos}, \bar{m}_{bos}} \Psi^{\alpha}(j, m_{bos}, \bar{m}_{bos}) \cdot | \Phi(j, m_{bos}, \bar{m}_{bos}) \rangle$$

$$\Psi^{(NS)}(j, m_{bos}, \bar{m}_{bos}) = \nu^{\frac{1}{2}+j} \frac{\Gamma(-j+m_{bos})\Gamma(-j-\bar{m}_{bos})}{\Gamma(-2j-1)\Gamma\left(1-\frac{1+2j}{k}\right)} \delta_{m_{bos}-\bar{m}_{bos}}$$

- OBTAINED FROM 'DESCENT' OF BRANES IN AdS_3 OR $SL(2,\mathbb{R})$ [RIBAULT, SCHOMERUS] CARDY CHECK ✓

- BRANES IN AdS_3 ↳ OBTAINED FOLLOWING TECHNIQUES OF ZAMOLODCHIKOVS [PONSOT SCHOMERUS TESCHNER]

FURTHER INGREDIENTS : GSO PROJECTION

$$Z(t) = \frac{1}{2}\left(Z^{NS} - Z^{\widetilde{NS}} - Z^{R} - Z^{\widetilde{R}}\right) = 0 \Rightarrow \text{SPACETIME SUSY.}$$

↳ COMPUTED USING 'EXTENDED CHARACTERS' [TROOST, ISRAËL, PAKMAN, EGUCHI, SUGAWARA...]

↳ NEEDED FOR INTEGRAL $U(1)_R$ CHARGES

SPECTRUM :

MASSLESS MODES :

$$\epsilon_\nu \psi^\nu_{-\frac{1}{2}} |k_\mu, NS\rangle \quad , \quad u^\alpha |k_\mu, \Sigma_\alpha, NS\rangle$$

GAUGE BOSON GAUGINO

EARLIER, WE SAW $Z_{open} = 0 \Rightarrow$ SUPERSYMMETRY.

- CAN EXPLICITLY CONSTRUCT OPERATORS ON THE WORLDSHEET THAT CORRESPOND TO THE $\underline{\underline{4}}$ SUPERCHARGES PRESERVED BY D-BRANE (AT LEAST ASYMPTOTICALLY)

CONCLUDE THAT WORLD-VOLUME THEORY IS $N=1$ SYM IN $d=4$. SIMILAR ANALYSIS FOR $d=2,4,6 \rightsquigarrow$ MINIMAL SUSY GAUGE THEORY

MORAL: AT LOW ENERGIES, OPEN STRING THEORY ON BRANE $\rightsquigarrow$ $N=1$ PURE SYM.

$$\bar{\Phi} = -\frac{Q}{2} S$$

$$Q = \sqrt{\frac{2}{k}} = \sqrt{2}$$

SPECTRUM :

MASSLESS MODES :

$$\epsilon_\nu \psi^\nu_{-\frac{1}{2}} |k_\mu, NS\rangle \quad , \quad u^\alpha |k_\mu, \Sigma_\alpha, NS\rangle$$

GAUGE BOSON GAUGINO

EARLIER, WE SAW $Z_{open} = 0 \Rightarrow$ SUPERSYMMETRY.

- CAN EXPLICITLY CONSTRUCT OPERATORS ON THE WORLDSHEET THAT CORRESPOND TO THE $\underline{\underline{4}}$ SUPERCHARGES PRESERVED BY D-BRANE (AT LEAST, ASYMPTOTICALLY)

CONCLUDE THAT WORLD-VOLUME THEORY IS $N=1$ SYM IN $d=4$. SIMILAR ANALYSIS FOR $d=2,4,6 \rightsquigarrow$ MINIMAL SUSY GAUGE THEORY

MORAL: AT LOW ENERGIES, OPEN STRING THEORY ON BRANE $\rightsquigarrow$ $N=1$ PURE SYM.

BACKREACTION :

$$\tilde{h}^{IJ}(k_\mu, P) = \langle \nu^{IJ}(k_\mu, P) | D_{cl} | B \rangle = \frac{\eta^{IJ}}{k_\mu^2 - j(j+1)} \cdot \delta^4(k^\mu) \cdot \psi_{00}^{j,NS}$$

$$= \eta^{IJ} \frac{\delta^4(k^\mu)\, \psi_{00}^{j,NS}}{P^2 + 1/4}$$

$h^{IJ}(x^\mu, \rho)$ = "FOURIER TRANSFORM" OF $\tilde{h}^{IJ}(k_\mu, P)$

[REPLACE $e^{ik.x}$ WITH SOLUTION TO LAPLACE EQN IN $R^{1,3} \times$ CIGAR]

$\{ j = -\frac{1}{2} + iP \}$

$$= \eta^{IJ} \int_0^\infty dP \, \frac{\phi_0(\rho, P)}{P^2 + 1/4} \psi_{00}(P)$$

CLASSICALLY, $\phi_0^{grav}(\rho, P) = -\frac{\Gamma(\frac{1}{2} + iP)^2}{\Gamma(2iP)} F\left(\tfrac{1}{2} - iP, \tfrac{1}{2} + iP; 1; -\sinh^2\rho\right)$

$\to 1$ as $\rho \to 0$

$\to e^{-\rho}\left[e^{2iP\rho} + R^{cl}(-P)\, e^{-2iP\rho} \right]$

[use CONNECTION FORMULA]

$$ds^2 = k\left(d\rho^2 + \tanh^2\rho \, d\theta^2\right)$$

Φ

$$R_{ab} + D_a D_b \Phi = 0$$

$$\left\{ \frac{1}{L_0 - 1} = \frac{1}{p^2 + 1/4} \right\} \text{graviton}$$

$$\tilde{h}^{IJ}(k_\mu, P) = \langle \nu^{IJ}(k_\mu, P) | D_{cl} | B \rangle = \frac{\eta^{IJ}}{k_\mu^2 - j(j+1)} \cdot \delta^4(k^\mu) \cdot \psi_{00}^{j,NS}$$

$$= \eta^{IJ} \frac{\delta^4(k^\mu)\, \psi_{00}^{j,NS}}{P^2 + 1/4}$$

$h^{IJ}(x^\mu, \rho)$ = "FOURIER TRANSFORM" OF $\tilde{h}^{IJ}(k_\mu, P)$

[REPLACE $e^{ik.x}$ WITH SOLUTION TO LAPLACE EQN IN $\mathbb{R}^{1,3}$ x CIGAR]

$\left\{ j = -\frac{1}{2} + iP \right\}$

$$= \eta^{IJ} \int_0^\infty dP \, \frac{\phi_0(\rho, P)}{P^2 + 1/4} \, \psi_{00}(P)$$

CLASSICALLY, $\phi_0^{\text{grav}}(\rho, P) = -\frac{\Gamma(\frac{1}{2} + iP)^2}{\Gamma(2iP)} F\left(\frac{1}{2} - iP, \frac{1}{2} + iP; 1; -\sinh^2\rho\right)$

$\to 1$ as $\rho \to 0$

$\to e^{-\rho}\left[e^{2iP\rho} + R^{cl}(-P)\, e^{-2iP\rho} \right]$

[USE CONNECTION FORMULA]

$$\int_0 \ldots \frac{\ldots}{p^2 + 1/4} \phi_{00}(p)$$

ASSIC… . $\phi_0^{grav}(\rho, p) = -\frac{\Gamma(\frac{1}{2}+ip)^2}{\Gamma(2ip)} F\left(\frac{1}{2}-ip, \frac{1}{2}+ip; 1; -\sinh^2\rho\right)$

$\rightarrow 1$ as $\rho \rightarrow 0$

$\rightarrow e^{-\rho}\left[e^{2ip\rho} + R^{cl}(-p)\, e^{-2ip\rho} \right]$

[use connection formula]

PRESCRIPTION: REPLACE $R^{cl}(p)$ BY $R^{qu}(p,\nu)$ AND USE EXACT CONNECTION FORMULA...

$$\phi^{grav}(\rho, p) = \left\{ (\sinh\rho)^{2ip-1} F\left(\tfrac{1}{2}-ip, \tfrac{1}{2}-ip; 1-2ip; \tfrac{-1}{\sinh^2\rho}\right) + R^{qu}(-p)(\sinh\rho)^{-2ip-1} F\left(\tfrac{1}{2}+ip, \tfrac{1}{2}+ip; 1+2ip; \tfrac{-1}{\sinh^2\rho}\right) \right\}$$

SUBSTITUTE IN FORMULA FOR $h^{IJ}(x,\rho)$

- TURNS ~~…~~ INTO A CONTOUR INTEGRAL WITH POLE AT $p = +\frac{i}{2}$ $(j=-1)$

$$h^{IJ}(\rho) = \eta^{IJ}\, 2\pi N \nu^{-\frac{1}{2}} \log\left(1 + \frac{1}{\sinh^2\rho}\right)$$

PRESCRIPTION: REPLACE $R^{cl}(P)$ BY $R^{qu}(P,\nu)$ AND USE EXACT CONNECTION FORMULA...

$$\phi^{grav}(\rho,P) = \Big\{ (\sinh\rho)^{2iP-1} F\left(\tfrac{1}{2}-iP, \tfrac{1}{2}-iP; 1-2iP; \tfrac{-1}{\sinh^2\rho}\right) + R^{qu}(-P)(\sinh\rho)^{-2iP-1} F\left(\tfrac{1}{2}+iP, \tfrac{1}{2}+iP; 1+2iP; \tfrac{-1}{\sinh^2\rho}\right) \Big\}$$

SUBSTITUTE IN FORMULA FOR $h^{IJ}(x,\rho)$

- TURNS ~~into~~ INTO A CONTOUR INTEGRAL WITH POLE AT $P = +\frac{i}{2}$ $(j=-1)$

$$h^{IJ}(\rho) = \eta^{IJ}\, 2\pi N \nu^{-\frac{1}{2}} \log\left(1+\frac{1}{\sinh^2\rho}\right)$$

$\eta^{IJ}[-4\pi N \nu^{-\frac{1}{2}} \log\rho]$

$(\rho\to 0)$

IR

$\delta[e^{\Phi}] \sim \delta g^{\circ}_{YM}$

LOG RUNNING.

$\eta^{IJ} \cdot 2\pi N \nu^{-\frac{1}{2}} e^{-2\rho}$

$(\rho \to 0)$

UV

· FIX CO-EFFICIENT

$$\chi \sim \chi + c$$

$$S \sim S + \ldots$$

$$S_{D_p} = \mu_p \int C_{p+1}$$

$$S_{D-1} = \mu_{-1} \left(\mu_3 \frac{\chi}{2\pi} \right) = N\theta$$

$$S \sim S + 2\pi$$

$$N\theta \sim N\theta + 2$$

$$N\theta \sim N\theta + 2\pi$$

$$\boxed{\theta \sim \theta + \frac{2\pi}{N}}$$

$$U(1)_\theta \to \mathbb{Z}_N$$

- USE CORRECT REFLECTION AMPLITUDE
- USE APPROPRIATE 'WAVE FUNCTION' $\{\Phi^{j}_{\frac{1}{2},\frac{1}{2}}\}$
- SAME METHOD AS BEFORE, BUT MORE SUBTLETIES

$$F_{0123\rho}(\rho) = N\coth\rho$$ [NO ν-FACTOR APPEARS]

$$\chi \sim N\theta$$

CAN WE FIX CO-EFFICIENT ? / CAN WE 'SEE' CHIRAL SYMMETRY BREAKING ? $U(1)_R \to \mathbb{Z}_{2N}$?

N D3 BRANES $\Rightarrow \chi = \mu_3 . N . \frac{\theta}{2\pi}$

$\Rightarrow$ D-INSTANTON WORLD-VOLUME ACTION

$$\delta S_{D(-1)} = \mu_{-1} C_0 = \mu_{-1}\chi = \mu_{-1}\mu_3 N . \frac{\theta}{2\pi}$$

$$= N\theta$$

$\Rightarrow$ U(1) ISOMETRY $\longrightarrow \mathbb{Z}_N$

CAREFUL ACCOUNTING FOR NORMALIZATIONS

$\Rightarrow U(1)_R \longrightarrow \mathbb{Z}_{2N}$ AS EXPECTED

WHAT MORE... ?

- SOLUTION OF SUGRA (NON-CRITICAL) THAT CORRESPONDS TO D-BRANE --- NEAR HORIZON LIMIT?

- ADD FLAVOUR (D5 BRANES) [FOTOPOULOS, NIARCHOS, PREZAS]
  - SEIBERG DUALITY IN $N=1$ SCFT
  - CO-EFFICIENT OF $\delta(e^{\xi}) \sim (3N_c - N_f)$?

- THESE CLOSED STRING BACKGROUNDS ARE RELATED TO CALAB-YAU SPACES (NON-COMPACT)
  - WORLD-SHEET DESCRIPTION OF D-BRANES & THEIR MODULI SPACES
  - GEOMETRIC ENGINEERING OF GAUGE THEORIES
    ↳ WORLD-SHEET DERIVATION