Title: tba

Date: Sep 13, 2005 02:00 PM

URL: http://pirsa.org/05090003

Abstract:

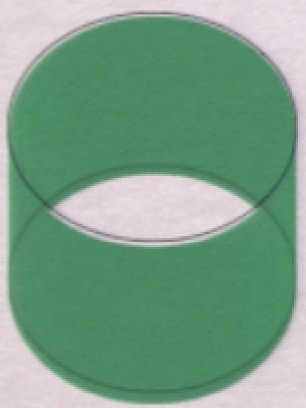
OUTLINE

- AdS/CFT (and it's dictionary)
- Quantum hall droplets in AdS/CFT
- Dual supergravity description
- Beyond half BPS
- All loop string energies

The AdS/CFT correspondence

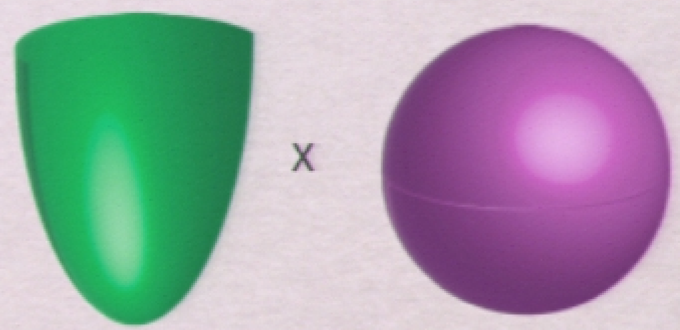
Conjectured exact quantum duality between:

N=4 SYM on a sphere



Type IIB Superstring on

$$AdS_5 \times S^5$$



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More precisely: where does geometry come from?

GENERALITIES

Map states to states

$$|s\rangle_{SYM} \equiv |s\rangle_{AdS}$$

Map operators to operators

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Superconformal algebra

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Superconformal algebra

Want to identify superconformal multiplets

In the gravity side the SYM time corresponds to global time coordinates

$$ds^2 = -\cosh^2 \rho dt^2 + \sinh^2 \rho d\Omega_3^2 + d\rho^2$$

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Notice gravitational potential:

Discrete spectrum

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Choose highest weight state of given representation
Highest weight half BPS states are build out of
only one of the scalar modes
of one complex field of the N=4 SYM theory. Let us call it
Z

Use complex basis for oscillators, so that that

$$a_Z^\dagger \sim Z \quad a_Z \sim \partial_Z$$

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All single particle (graviton) states
are descendants of

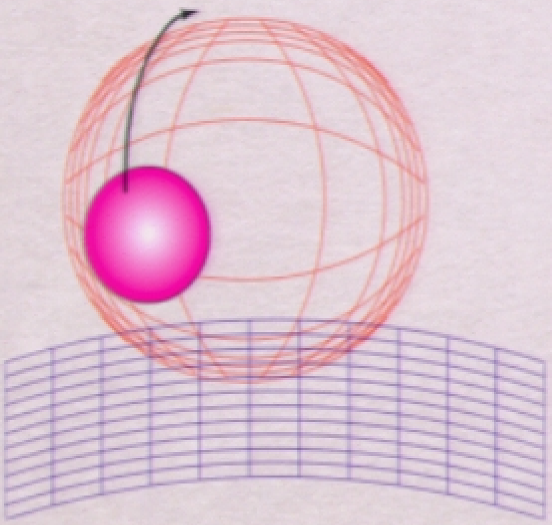
(Witten, Gubser, Klebanov, Polyakov)

$$tr(Z^n)$$

Are there other interesting protected states?

There are stable D-branes (non-perturbative states) with the same quantum numbers.

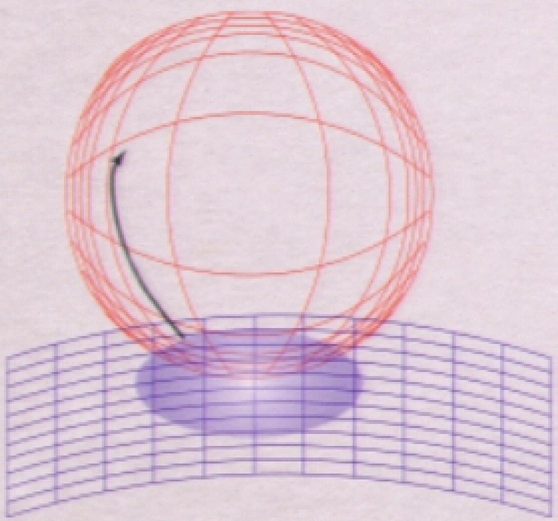
These are called giant gravitons (McGreevy, Susskind, Toumbas)



A D3 brane wrapping a 3 sphere inside the five sphere and with angular momentum on the five-sphere.

These were conjectured to resolve the technical issue that traces are not algebraically independent

There is another giant graviton



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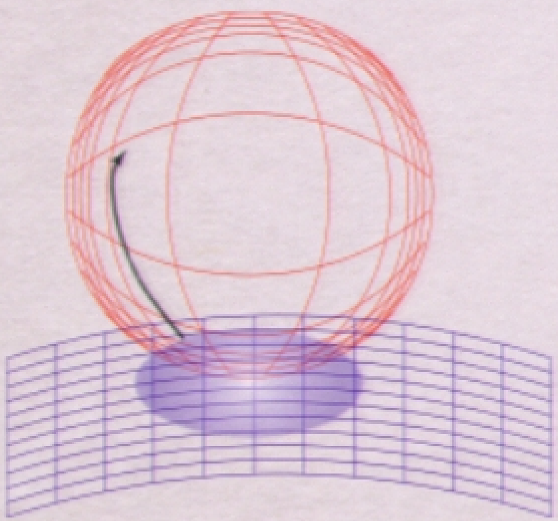

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What are the dual operators to giants?

Neither is a trace

Look at all states that are half BPS

All of these half BPS states have to be (descendants of) multi-traces of the complex scalar field Z

Because these states are supersymmetric, there are no quantum corrections to their effective dynamics

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$$\mathcal{L} = \int dt \frac{1}{2}(D_t Z)^2 - \frac{1}{2} Z^2$$

Two simple ways to solve system:

Take axial gauge, solve system (free matrix harmonic oscillators)

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Algebraically independent up to order N

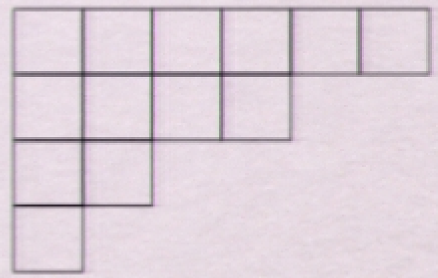
$$Tr(Z^{N+1}) = \text{Polynomial in } Tr(Z^k)$$

This basis in not orthogonal (it is approximately orthogonal in the large N limit, for fixed energy)

A more complicated way to make gauge invariants makes a complete orthogonal basis. This is via characters of U(N).

(Corely,Jevicki, Ramgoolam)

$$\chi_R(Z)$$



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Energy is the number of boxes of young tableaux

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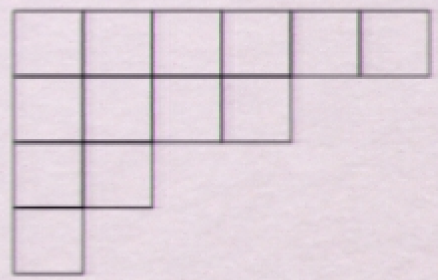

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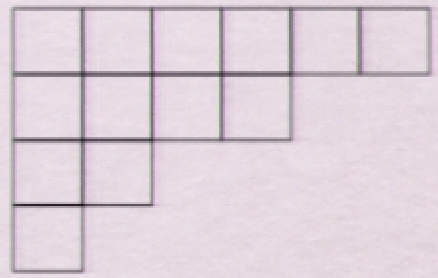

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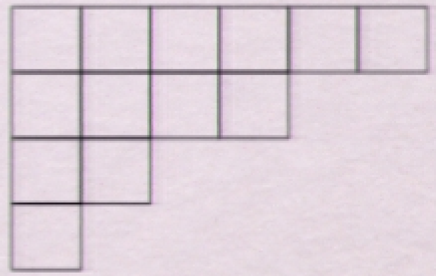

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AdS giants are rows. (CJR)

2

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Write everything in terms of eigenvalues

Classically we obtain N free particles in harmonic oscillator.

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Change of variables implies change of measure

After all of this is taken into account, we obtain N free fermions in the harmonic oscillator

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$$\frac{1}{2}\left(\sum -\mu^{-2}\partial_{z_i}\mu^2\partial_{z_i} + m z_i^2\right)$$

$$\mu^2 = \prod_{i<j}(z_i - z_j)^2 = \Delta(Z)^2$$

$$\Delta(Z) = \det \begin{pmatrix} 1 & 1 & 1 & \dots \\ z_1 & z_2 & z_3 & \dots \\ z_1^2 & z_2^2 & z_3^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \\ z_1^{N-1} & z_2^{N-2} & z_3^{N-3} & \dots \end{pmatrix}$$

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Transform

$$\psi = \Delta^{-1}\tilde{\psi}$$

Basis of Slater determinants coincides exactly with character basis (Schur functions)

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Fermi sea is a circle centered around origin.
Fermi statistics and uncertainty principle make these behave as an incompressible fluid (QHE droplet)

Basic Dictionary between QHE and BPS states.

- Edge excitations are small gravitons (follows from work of M. Stone and others)
- Giants growing into AdS are electrons away from droplet
- Giants growing into S are holes away from edge.

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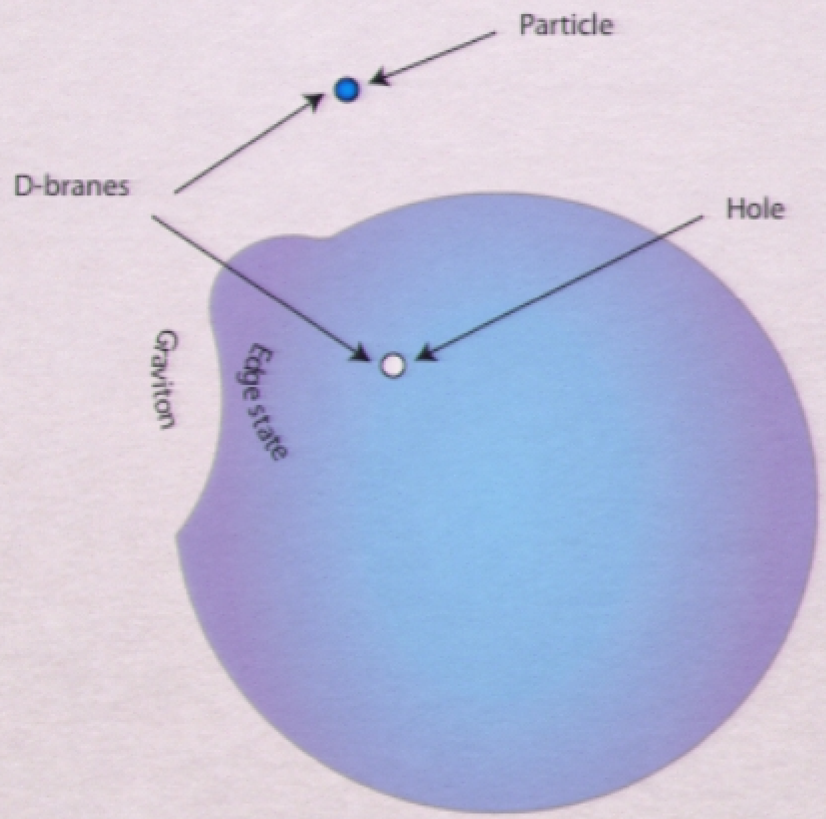

Beyond linearized supergravity

What objects are usually well
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- Coherent states of gravitons
- Large stacks of coinciding D-branes

Beyond linearized supergravity

Suggestive that one can describe arbitrary droplet shapes in supergravity.

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Indeed, there is a formalism that can reconstruct a geometry associated to any (macroscopic) droplet configuration. (Lin, Lunin, Maldecena)

LLM construction.

Supergravity solutions with isometry $SO(4) \times SO(4) \times \mathbb{R}$

$$ds^2 = g_{tt}(dt + V dx)^2 + g_3(dx dx) + \exp(G+H) d\Omega_3^2 + \exp(H-G) d\Omega_3'^2$$

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$\exp(H) = y$

Supersymmetry predicts special coodinate y

$y \geq 0$

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$$\begin{aligned} ds^2 &= -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + y\exp(G) d\Omega_3^2 + y\exp(-G) d\tilde{\Omega}_3^2 \\ h^{-2} &= 2y\cosh(G) \\ y\partial_y V_i &= \epsilon_{ij}\partial_j z, \quad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \\ z &= \frac{1}{2}\tanh G \end{aligned}$$

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3D equation $\partial_i \partial_i z + y \partial_y \left(\frac{\partial_y z}{y} \right) = 0$

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One also finds quantization of area (Dirac quantization condition)


3-sphere bundle over disc

Different topologies of the two coloring correspond to different spacetime topologies

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Suggestive: fermi-liquid density on phase space is identified as geometry

How to get fermion density?

$$\psi_F = \prod_{i<j}(z_i - z_j)\exp(-\sum z\bar{z}/2)$$

Interpret square of wave function as probability density

$$p(\vec{x}_i) = \prod_{i<j}|z_i - z_j|^2 \exp(-\sum z\bar{z}) = \exp(-\sum z\bar{z} + 2\log(|z_i - z_j|))$$

Think of p(x) as a thermodynamic ensemble for Coulomb gas in two dimensions. Introduce ρ the fermion density

$$\sum_i z_i\bar{z}_i \to \int d^2x\rho(x)\vec{x}^2$$

$$\sum_{i<j} 2\log(|z_i - z_j|) \to \iint d^2xd^2y\rho(\vec{x})\rho(\vec{y})\log(|\vec{x} - \vec{y}|)$$

$$N = \int d^2x\rho(x)$$

We can find the density ρ by taking a variational approach: maximize p(x)

One can try to push the following interpretation:

$$|\psi|^2 \sim Z_{gravity}$$

One can show that coherent states of traces in the fermion picture produce deformations of the fermion density which match the supergravity deformations of the function z at the boundary

BEYOND 1/2 BPS

States that respect 1/8 SUSY

Energy = Angular momentum

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Contributes to E but not to J

Set it to zero by hand (BPS condition)

Effective dynamics is a gauged Matrix quantum mechanics of commuting matrices

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$$H = \sum_i -\frac{1}{2\mu^2}\nabla_i \mu^2 \nabla_i + \frac{1}{2}|\vec{x}_i|^2$$

One can find ground state, and absorb square root of the measure in wave functions: copied from free fermions

$$\psi_0 \sim \exp(-\sum \vec{x}_i^2/2)$$

$$\hat{\psi} = \mu\psi$$

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

Interpret in same way as half BPS states

Interpret collection of eigenvalues as positions of particles in 6d phase space.

N Bosons in 6d with logarithmic repulsive interactions.

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

Introduce Boson density, etc, and study the thermodynamics of this ensemble in the saddle point approximation.

Density of bosons is a singular configuration. Symmetries of ensemble suggest the following density of "eigenvalues"

$$\rho = N \frac{\delta(|\vec{x}| - r_0)}{r_0^{2d-1} Vol(S^{2d-1})}$$

We get a round five sphere

Can also add coherent states by traces of the complex matrices. These keep the singular aspect of density, but deform the shape. This is very similar to what happens in gravity, we therefore identify it with gravity.

Approximations that lead to commuting matrices improve at strong coupling! Off-diagonal modes become heavy.

$$E_{osc} = \sqrt{1 + \frac{1}{2\pi^2} g_{YM}^2 |\vec{x}_j - \vec{x}_{j'}|^2}$$

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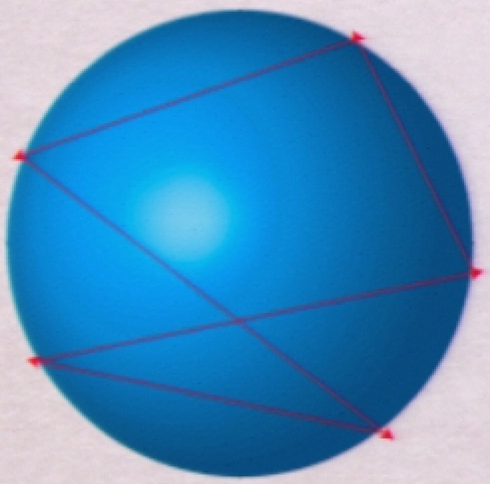
Can also suggest origin of string scale: off-diagonal modes are massive and can be represented by lines joining eigenvalues (points on the sphere): STRING BITS. Need to dress them with gravity (eigenvalues).



One can also verify string tension

One can calculate the radius of the sphere exactly

$$r_0 = \sqrt{\frac{N}{2}}$$

Size is independent of number of matrices

One can calculate string energies for BMN states:

$$O_k \sim \sum_{l=0}^{J} \exp(ikl/J)\mathrm{tr}(Z^{l-1}[Y,Z]Z^{J-l-1}[X,Z])$$

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Diagonal modes

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Diagonal modes

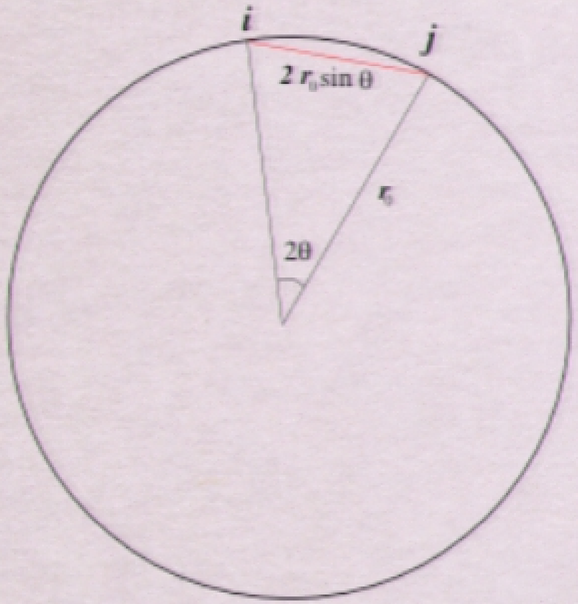
Off-diagonal modes

One treats the off-diagonal modes as free fields.

One calculates the energy of the BMN state assuming the off-diagonal modes don't affect the diagonal ones to first order.

The calculation of energies can be done in a saddle point approximation.

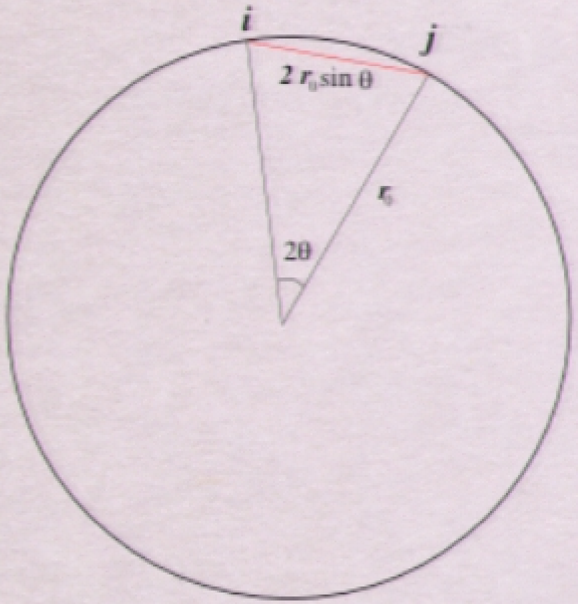
$$E \sim \frac{\langle \psi_k | H^{total} | \psi_k \rangle}{\langle \psi_k | \psi_k \rangle}$$



Result localizes to a string bit of particular fixed length

$$\theta = k/2J$$

$$\langle E^{osc} \rangle = 2\sqrt{1 + \frac{g_{YM}^2 N}{\pi^2} \sin^2(k/2J)}$$



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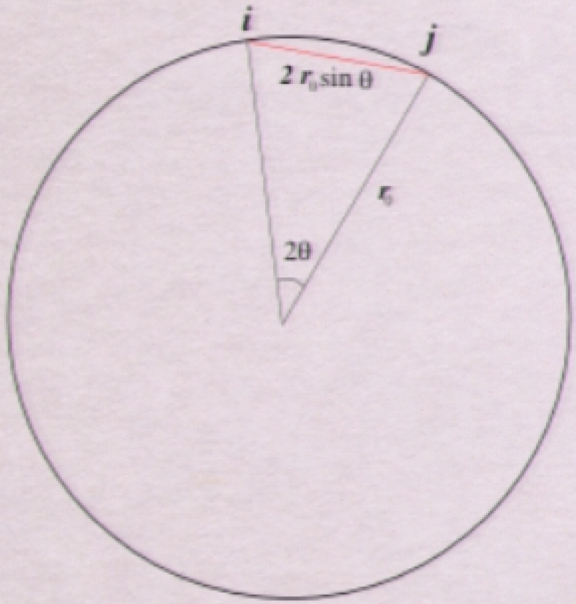
Matches BMN string calculation to all orders in the 't Hooft coupling exactly, by taking J large k fixed. (Small angles)



Result localizes to a string bit of particular fixed length

$$\theta = k/2J$$

$$\langle E^{osc} \rangle = 2\sqrt{1 + \frac{g_{YM}^2 N}{\pi^2} \sin^2(k/2J)}$$

Matches BMN string calculation to all orders in the 't Hooft coupling exactly, by taking J large k fixed. (Small angles)

Does not match string sigma model: approximation of free string bits is naive, but not in BMN limit.

Conclusion

- New understanding of AdS/CFT
- Gravity appears as hydrodynamics (collective phenomena). Locality can be addressed
- Background independence: can address many topologies in one CFT
- Picture the origin of strings and calculate some non-BPS energies exactly
- Many questions and leads to follow