

Title: Fractional branes and dynamical SUSY breaking

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Abstract: Fractional branes and dynamical SUSY breaking

Fractional branes and dynamical SUSY breaking

Sebastián Franco

Princeton University

September 2005

Based on: [hep-th/0505040](#): Franco, Hanany, Saad and Uranga

See also : [hep-th/0502113](#): Franco, Hanany and Uranga

Outline

- AdS/CFT correspondence and extensions
- Fractional branes and RG flows
- The conifold cascade
- Complex deformations and toric geometry
- Classifying fractional branes
- DSB from fractional branes
 - Complex cone over dP_1
 - Combining fractional branes
 - Cones over $Y^{p,q}$ manifolds

AdS/CFT correspondence and extensions

AdS/CFT Correspondence

$\mathcal{N} = 4$ $SU(N)$ SYM



Type IIB on $AdS_5 \times S^5$



N D3

AdS/CFT correspondence and extensions

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$$\mathcal{N} = 4 \text{ } SU(N) \text{ SYM} \longleftrightarrow \text{Type IIB on } AdS_5 \times S^5$$

Extend the duality by looking for:

Less **SUSY**

- Place D3-branes at a singularity
- SUGRA: $AdS_5 \times S^5 \rightarrow AdS_5 \times X^5$

CY 3-fold



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Break **conformal invariance**

- Add fractional D3-branes (wrapped D5-branes)
- SUGRA: turn on 3-form flux

CY 3-fold



Quiver theories

Gauge theories that typically arise in our constructions:

- **Product** gauge group

$$\prod_{i=1}^k SU(d^i)$$

- **Bifundamental** matter

| | |
|------------------|-----------|
| $SU(N_1)$ | $SU(N_2)$ |
| \overline{N}_1 | N_2 |

Quiver theories

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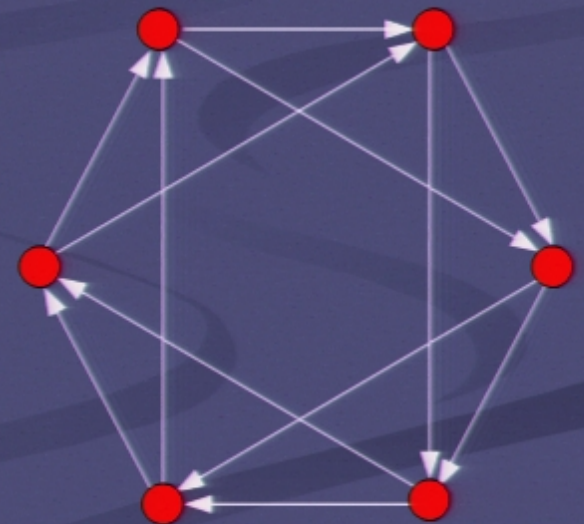
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Quiver diagram simple **diagrammatic** way of encoding the **matter content** of a gauge theory



Fractional branes and RG flows: the conifold

Regular and fractional
branes \longleftrightarrow Anomaly free rank assignments

Quiver theories

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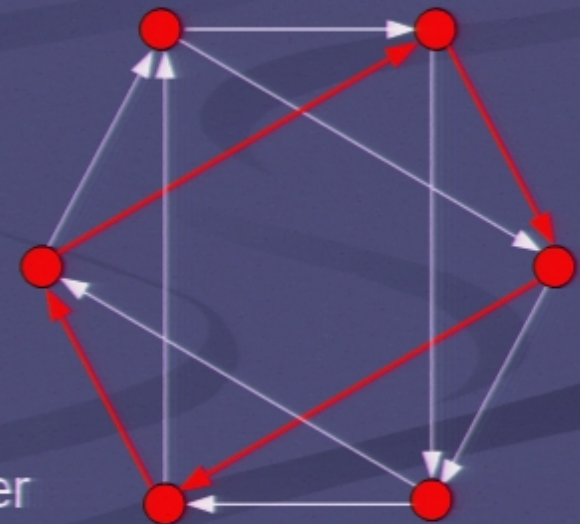
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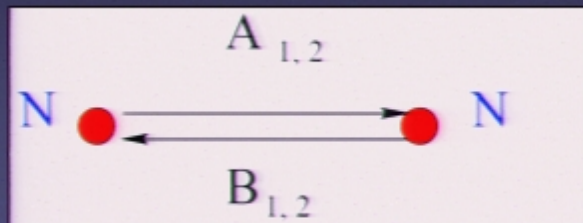


Superpotential terms: **oriented polygons** in the quiver

Fractional branes and RG flows: the conifold

Regular and fractional
branes

↔ Anomaly free rank assignments



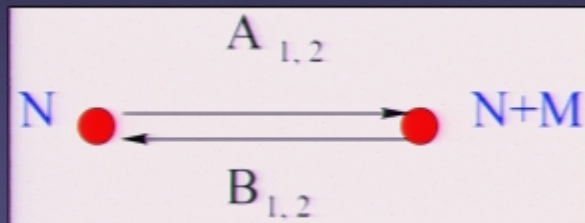
$$W = \frac{\lambda}{2} \epsilon^{ij} \epsilon^{kl} \text{Tr } A_i B_k A_j B_l$$

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| N D3-branes | $SU(N) \times SU(N)$ | conformal |

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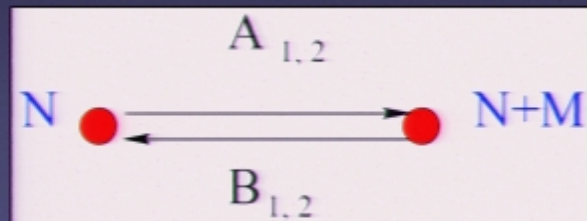
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| N D3-branes M D5-branes | SU(N) × SU(N+M) | not conformal |

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$$\beta_i = \frac{d(8\pi^2/g_i^2)}{d \ln \mu}$$

$$\beta_1 = 3 [N + (R_A - 1)(N + M) + (R_B - 1)(N + M)]$$

$$\beta_2 = 3 [(N + M) + (R_A - 1)N + (R_B - 1)N]$$

$$R(A) = 1/2 + \mathcal{O}(M/N)^2 \Rightarrow \boxed{\beta_1 = -3M}$$

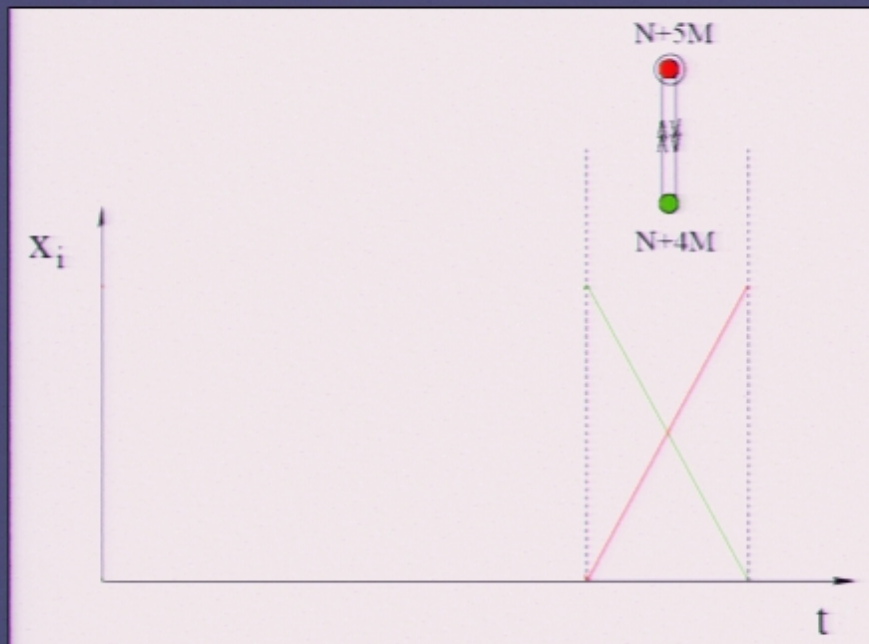
$$R(B) = 1/2 + \mathcal{O}(M/N)^2 \Rightarrow \boxed{\beta_2 = 3M}$$

Duality Cascade

Duality
Cascade

go beyond **infinite coupling** by switching
to an alternative **Seiberg dual** description
Klebanov and Strassler

The conifold example

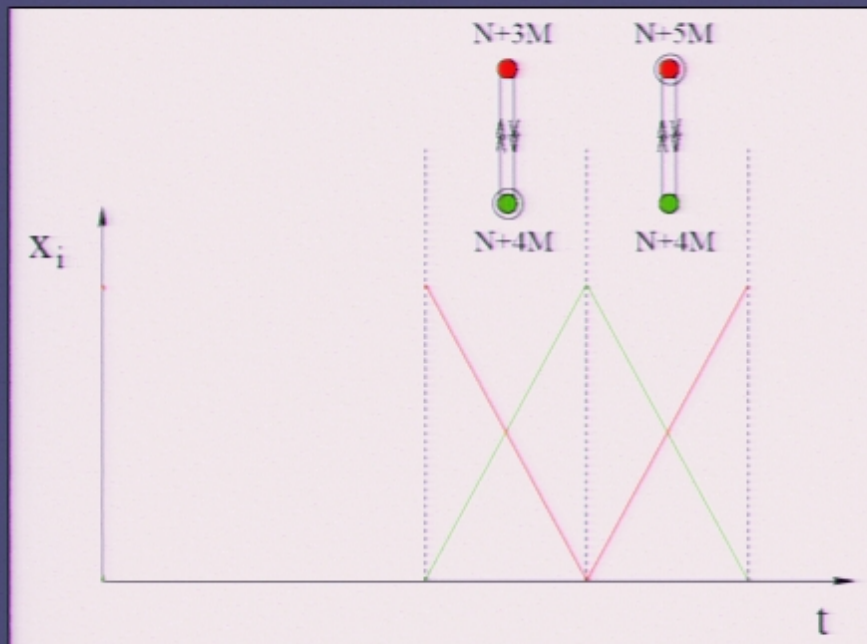


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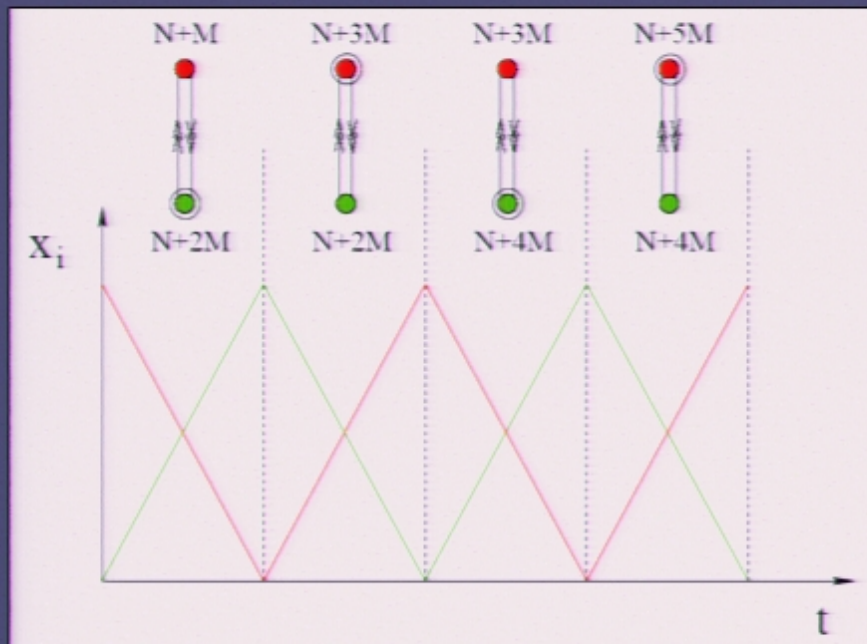


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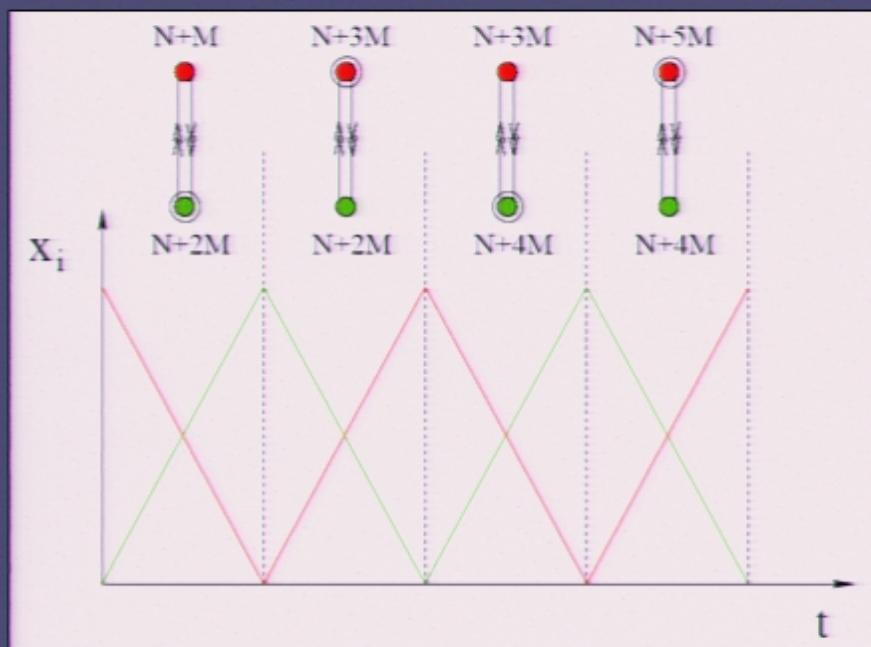


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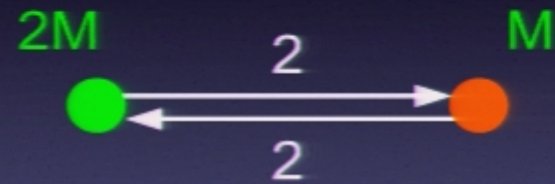
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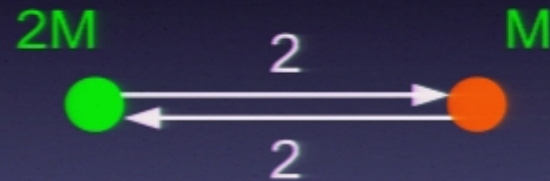
- **Periodic** flow
- # D5-branes: $M = \text{const}$
- # D3-branes: $N \sim t = \log \mu$

The end of the cascade



● : $2M$ colors with $2M$ flavors

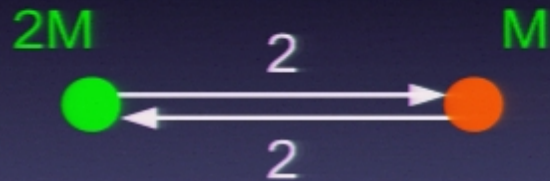
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● : 2M colors with 2M flavors \longrightarrow quantum corrected moduli space

$$\det M - B\tilde{B} = \Lambda^{4M}$$

The end of the cascade



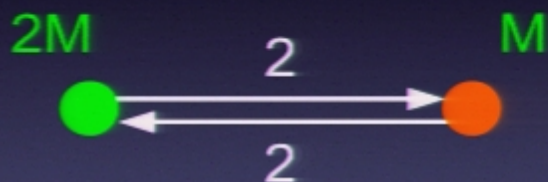
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Deformed conifold

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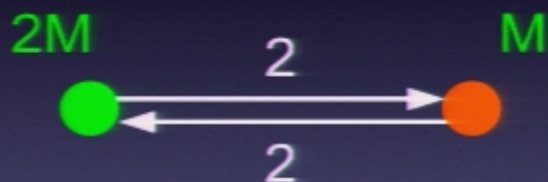
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Deformed conifold

- Logarithmic RG flow
- In the IR: confinement and chiral symmetry breaking

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Deformed conifold

- **Logarithmic** RG flow
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This behavior is nicely captured by a **SUGRA** dual
Klebanov and Strassler

Recent progress and new questions

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Related developments

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- **Brane tilings**: towards the gauge theory dual of an arbitrary toric singularity

Complex deformations and toric geometry

Toric varieties

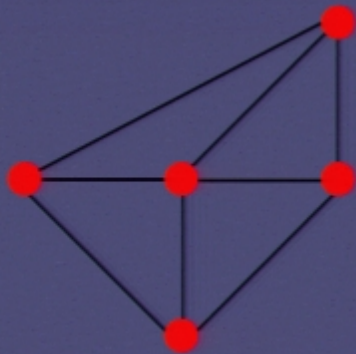
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Toric diagrams

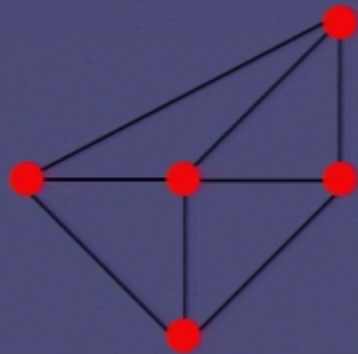


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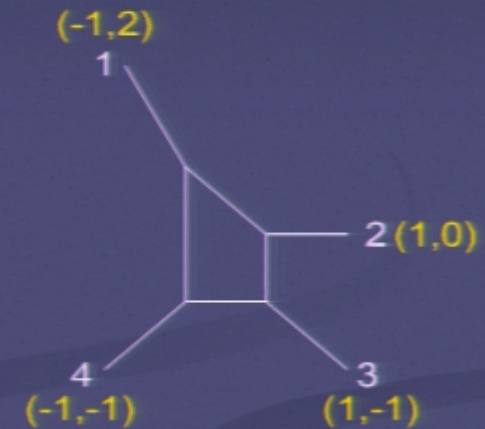
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(p,q) webs

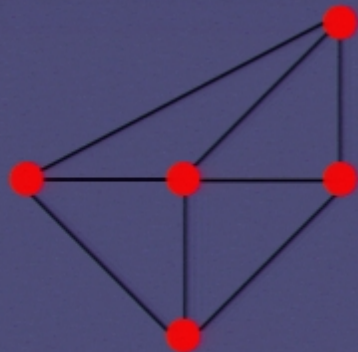


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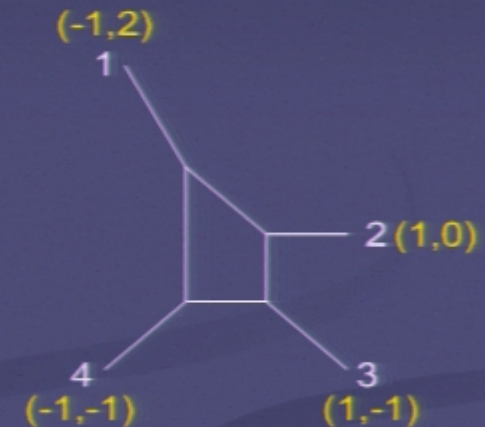
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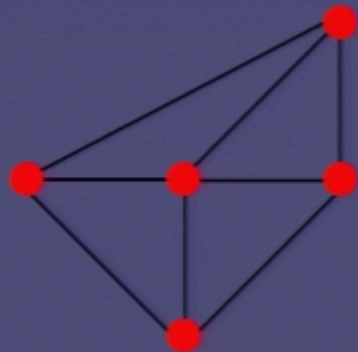


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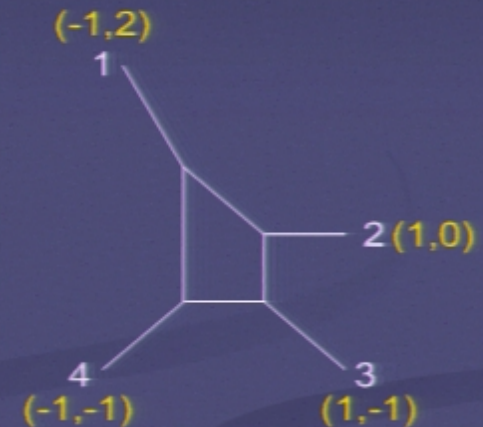
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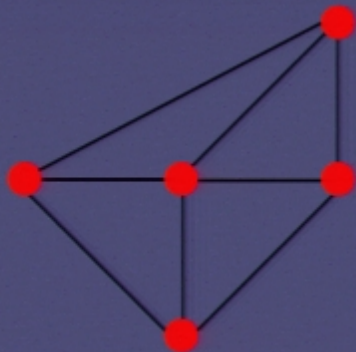
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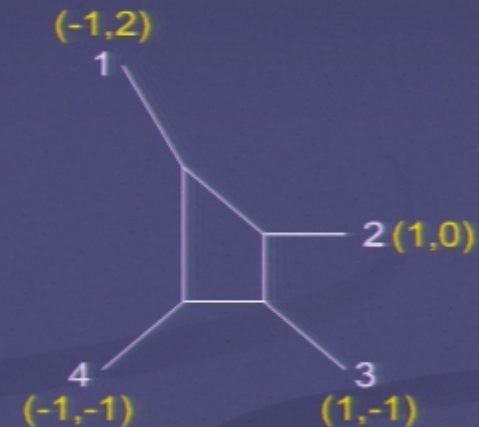
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- Decomposition of (p,q) web into **subwebs in equilibrium**
- Decomposition of toric polytope into **Minkowsky sum** **Altmann**

Obstructed deformations

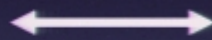
Unobstructed complex
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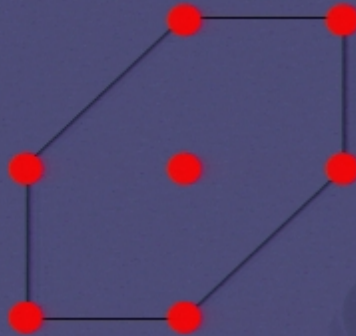
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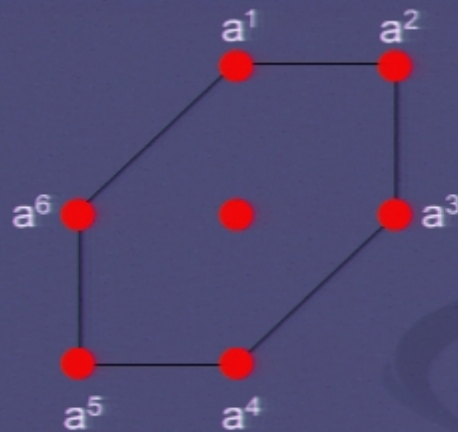
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Decomposition of toric polytope
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- a^i : vertices of toric diagram $i = 1$ to N



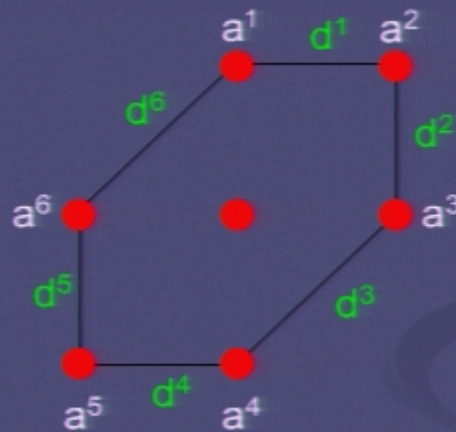
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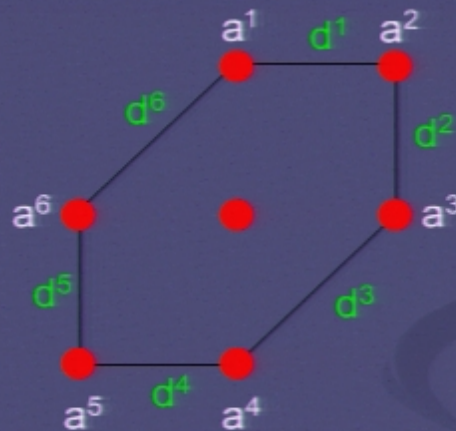
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$$0 < k \leq K$$

- Deformation unobstructed upto order K

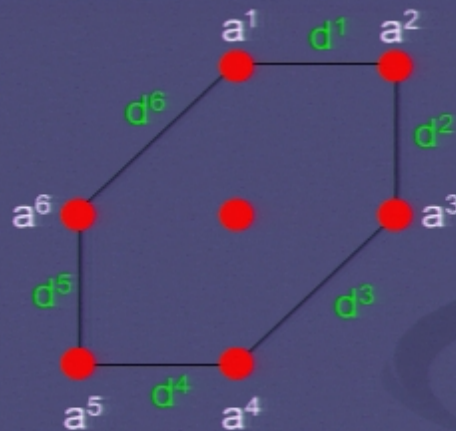
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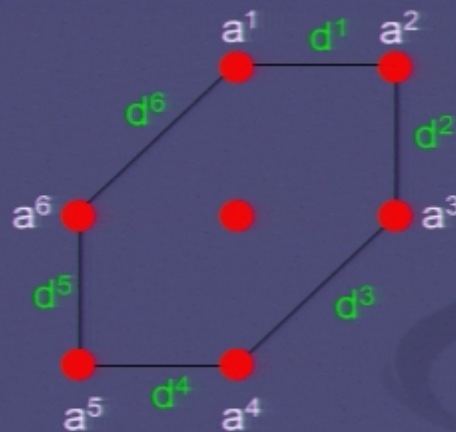
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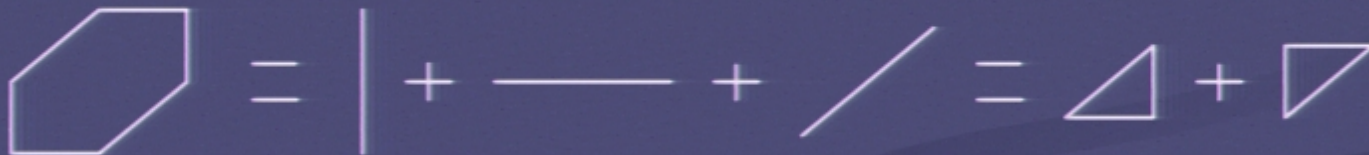
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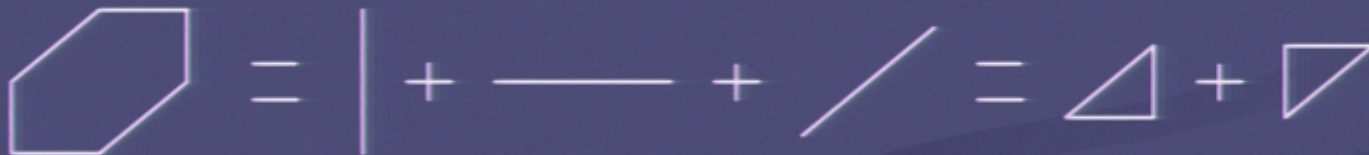
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Fractional branes can be **classified** according to the **IR dynamics** they trigger

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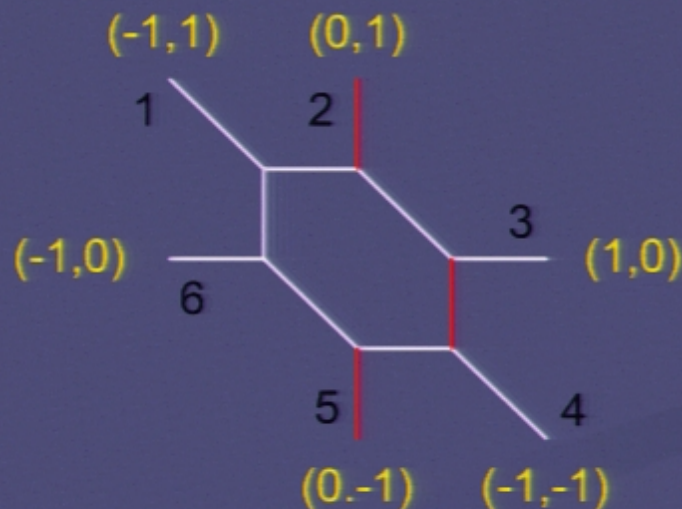
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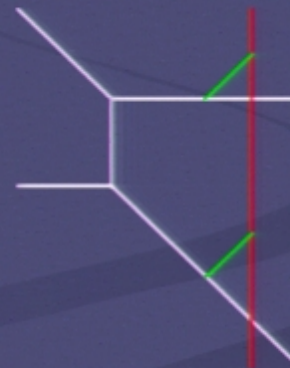
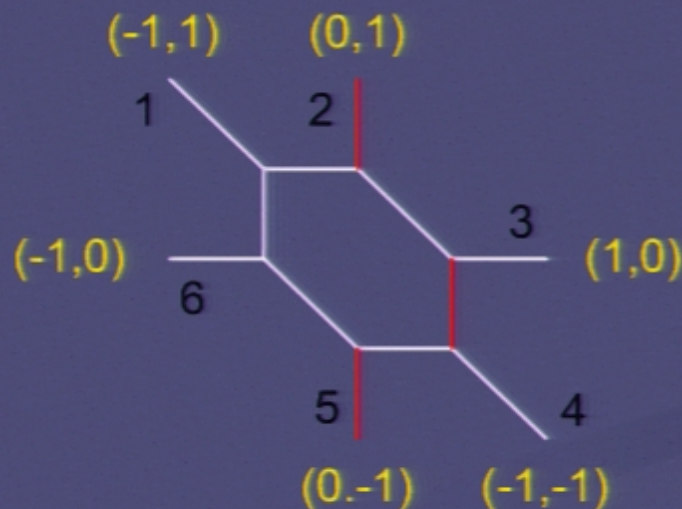


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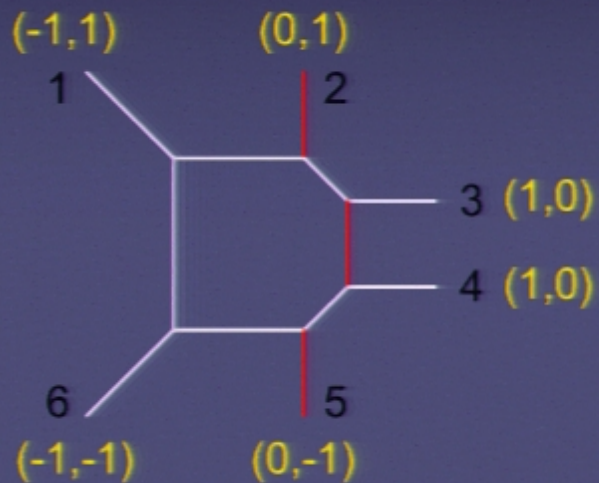


N=2 fractional branes

- flat directions along which the dynamics generically reduces to an N=2 theory
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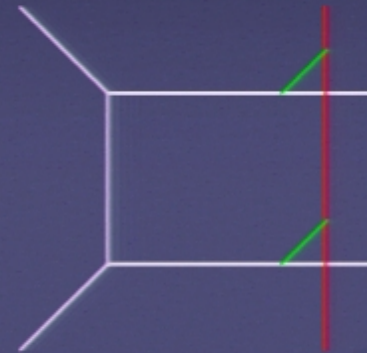
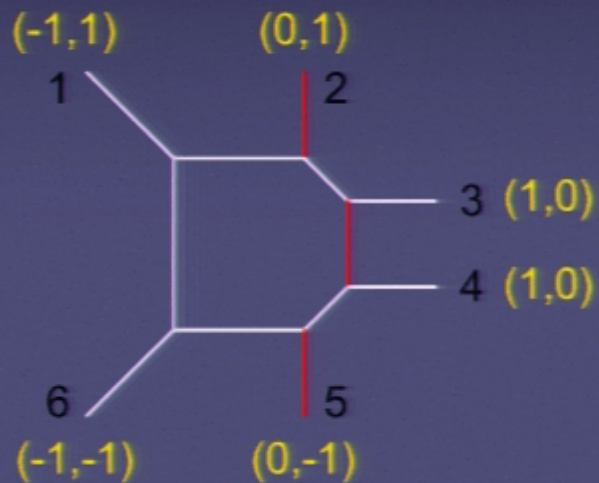
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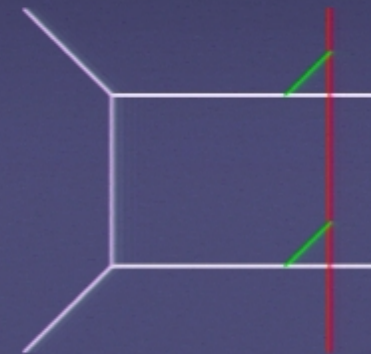
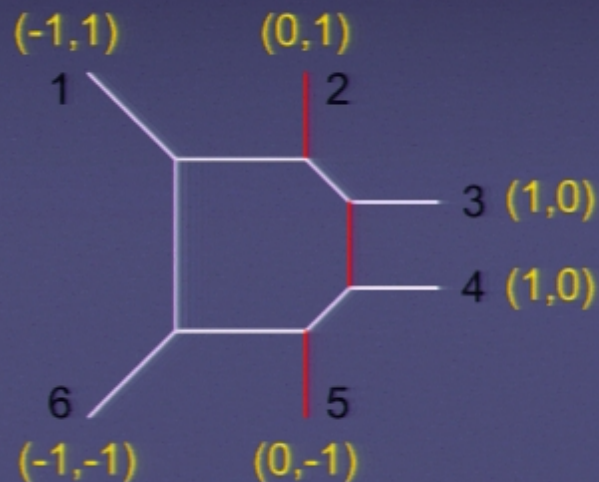
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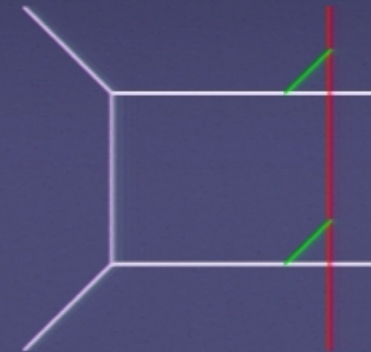
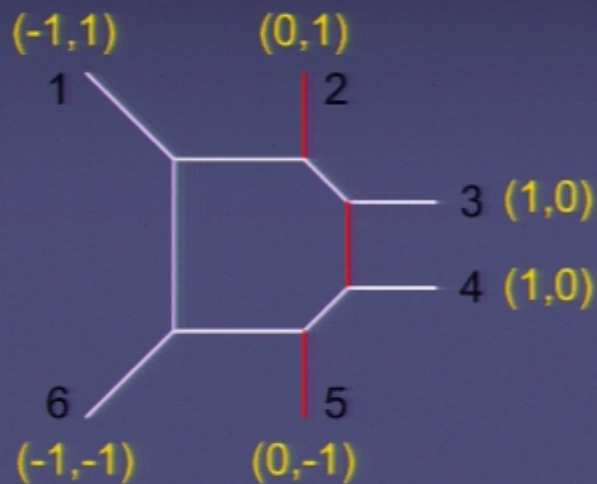
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DSB fractional branes

- All other rank assignments. Typically a gauge group generates an **ADS superpotential**

Franco, Hanany, Saad and Uranga
Berenstein, Herzog, Ouyang and Pinansky
Bertolini, Bigazzi and Cetrone

An example in detail: dP_1

Complex cone
over dP_1

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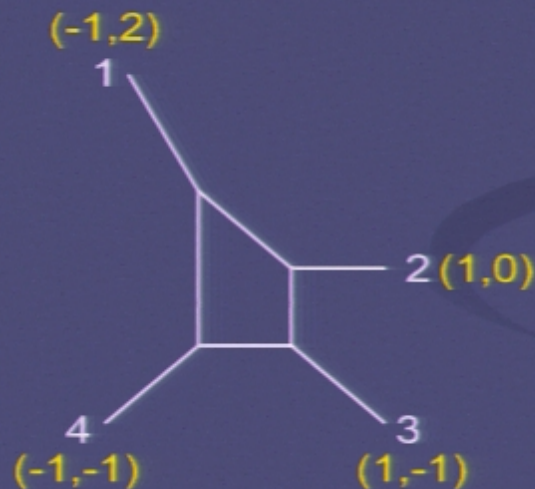
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An example in detail: dP_1

Complex cone
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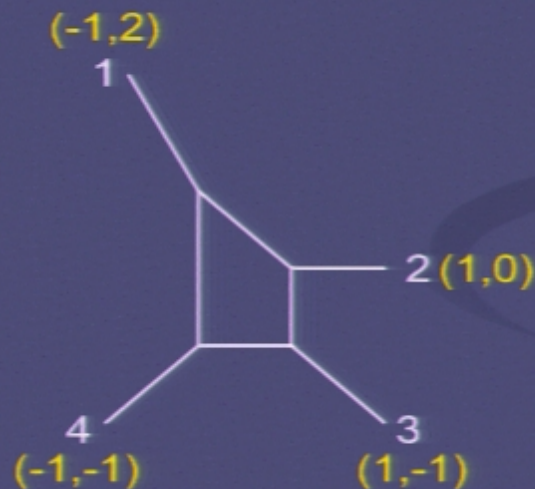
No decomposition
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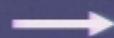
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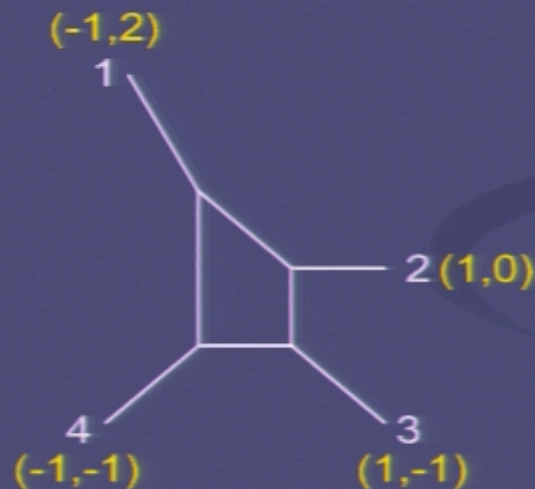
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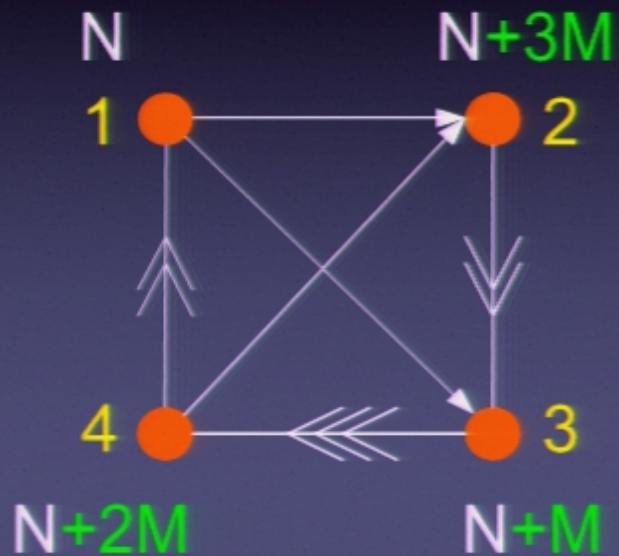


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What happens at the
IR bottom of the
cascade?

Field theory analysis

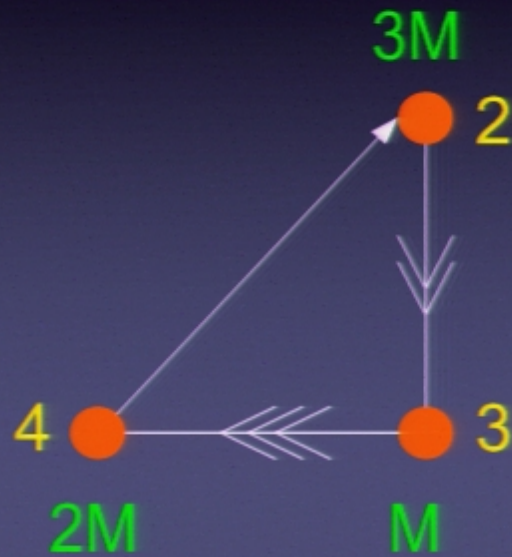


Global symmetry:

$$SU(2) \times U(1)_F \times U(1)_R \times U(1)_B$$

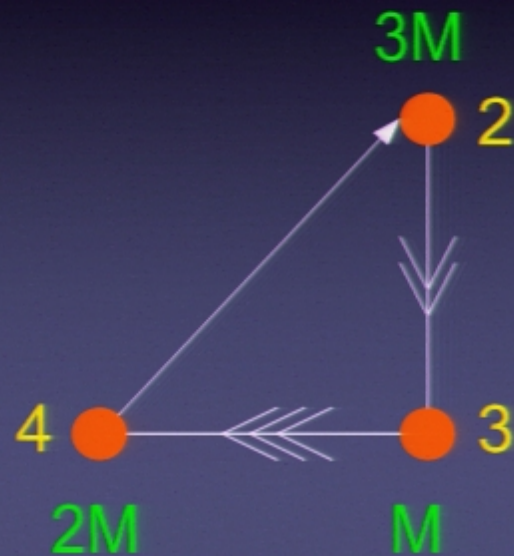
$$W = \epsilon_{\alpha\beta} X_{23}^{\alpha} X_{34}^{\beta} X_{42} + \epsilon_{\alpha\beta} X_{34}^{\alpha} X_{41}^{\beta} X_{13} - \epsilon_{\alpha\beta} X_{12} X_{23}^{\alpha} X_{34}^{\beta} X_{41}$$

Field theory analysis



$$W = X_{42}X_{23}Y_{34} - X_{42}Y_{23}X_{34}$$

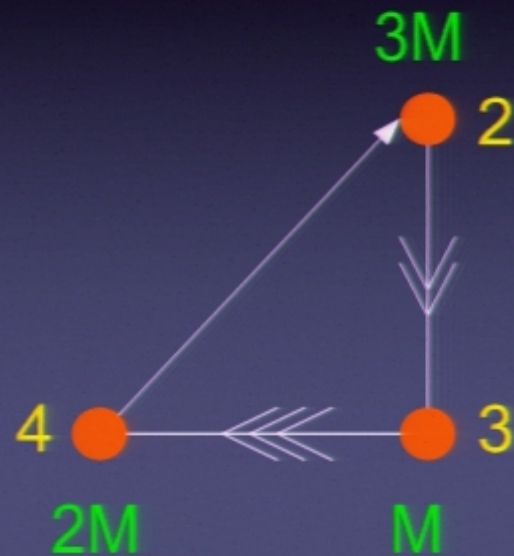
Field theory analysis



Fractional branes

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Field theory analysis



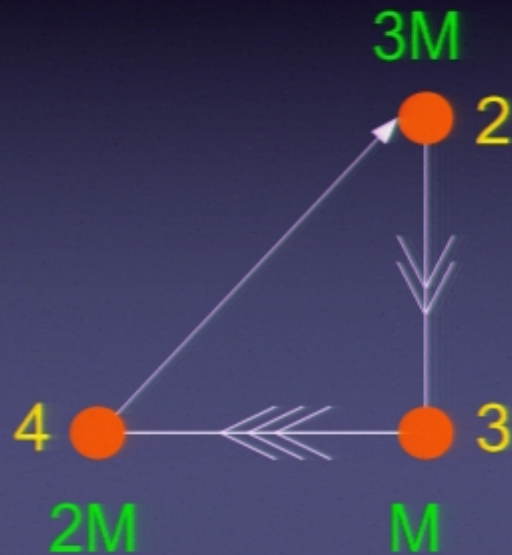
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pinched to the
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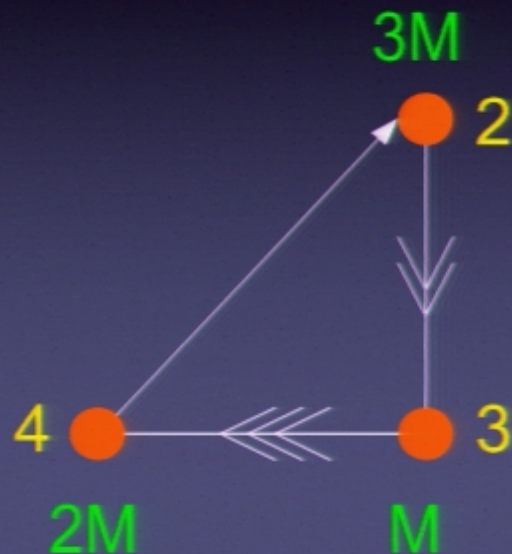
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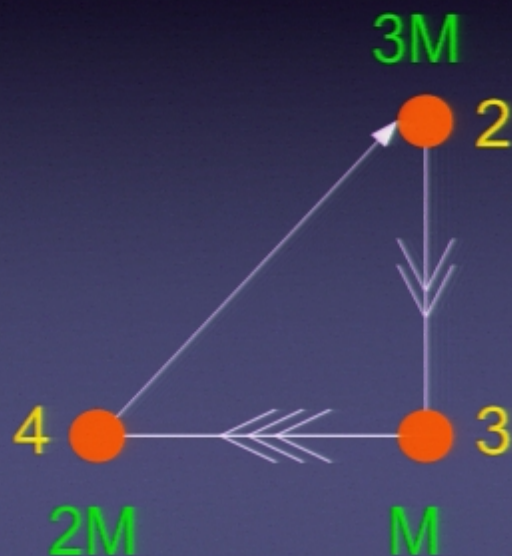
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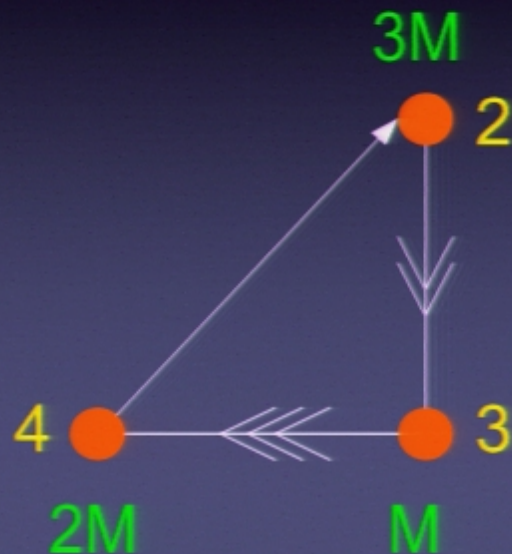
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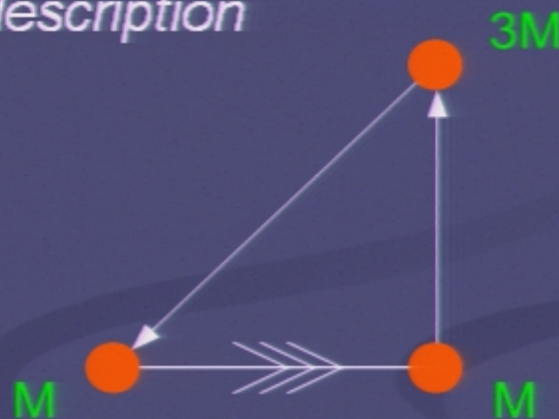
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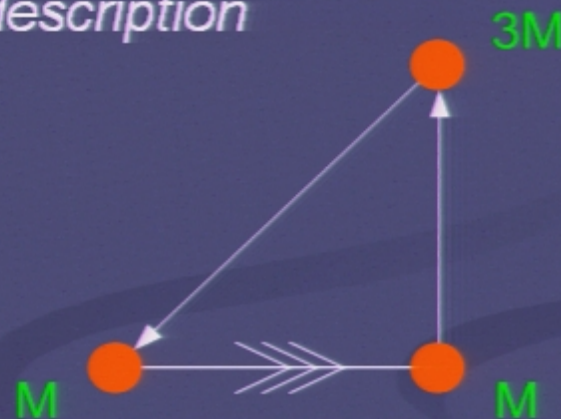
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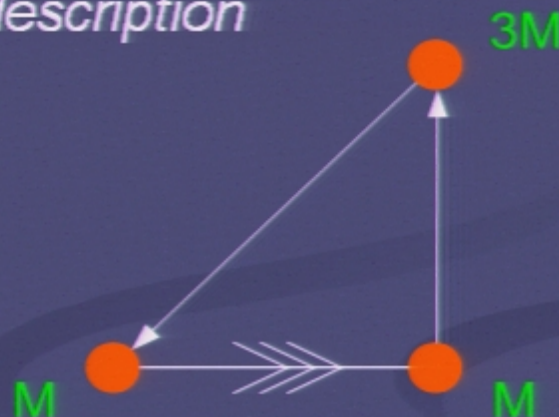
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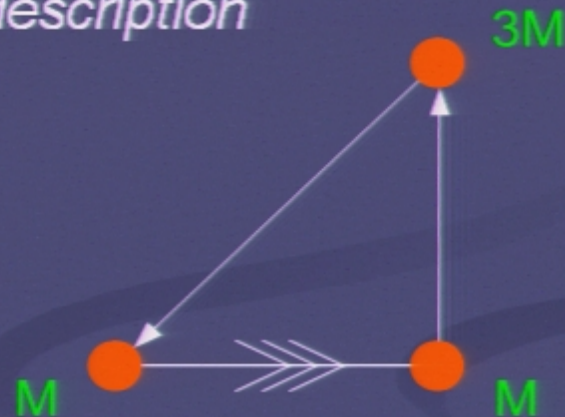
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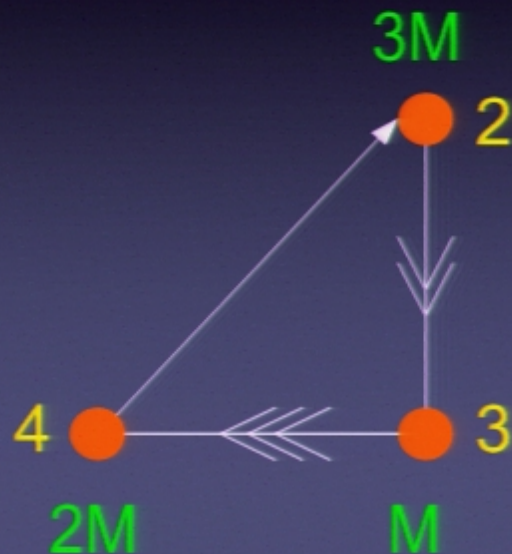
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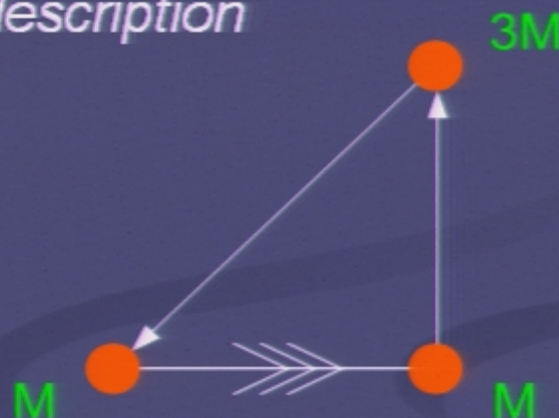
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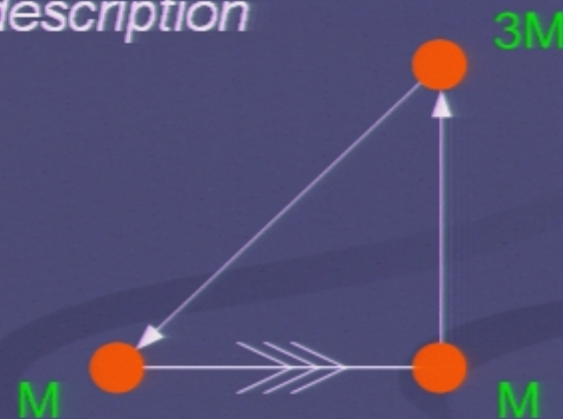
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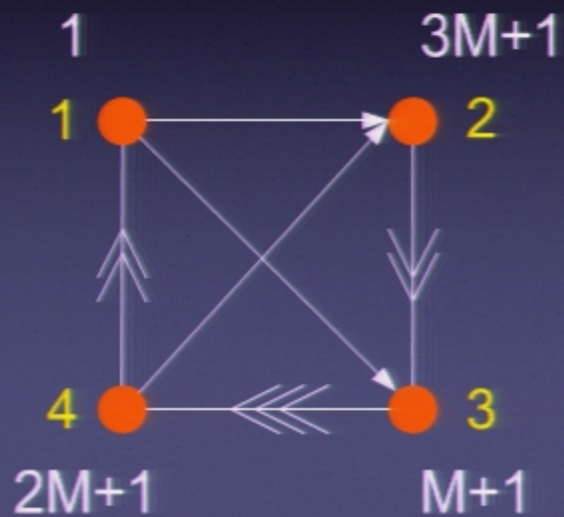


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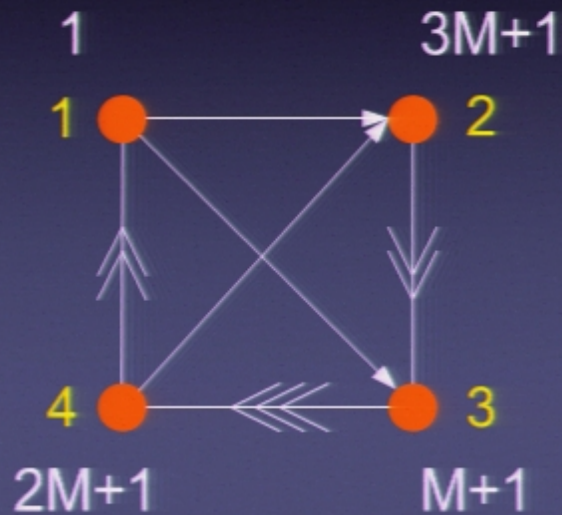
Non-susy minimum for
fixed FI parameters

D3-brane probe analysis



$$\mathrm{SU}(3M+1) \times \mathrm{SU}(M+1) \times \mathrm{SU}(2M+1)$$

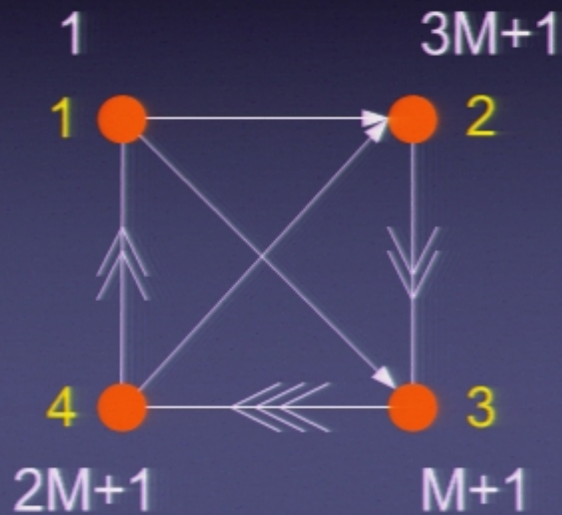
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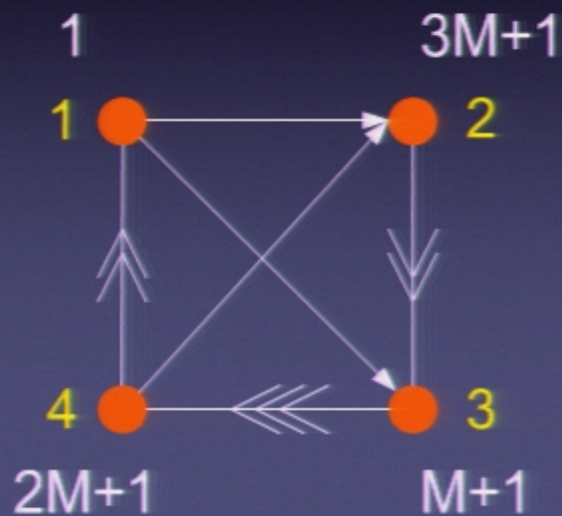
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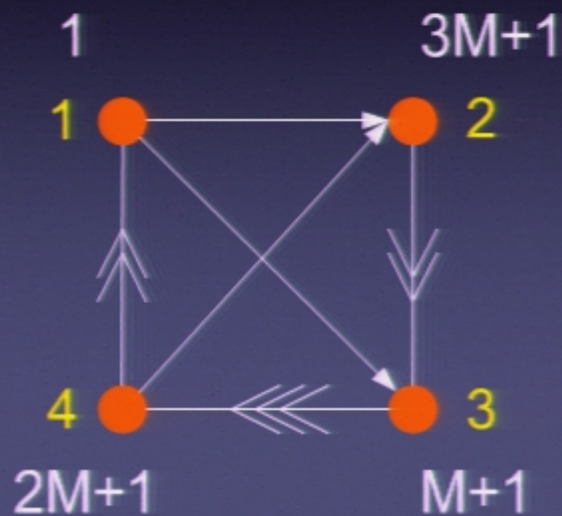
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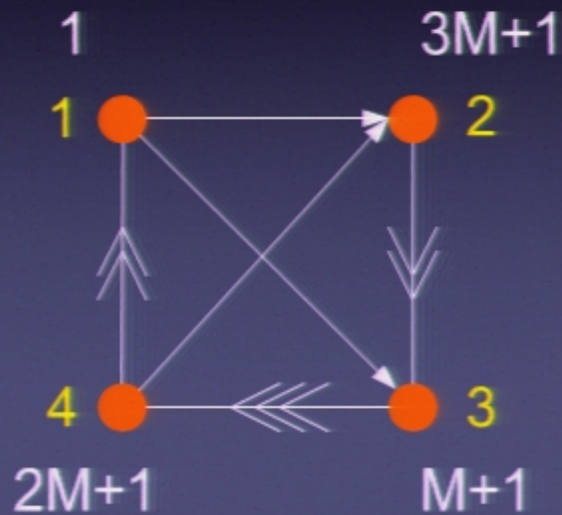
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→ Not a complex deformation
preserving **CY** with **ISD** fluxes

Combining fractional branes

different types of fractional branes
that independently lead to SUSY
RG flows

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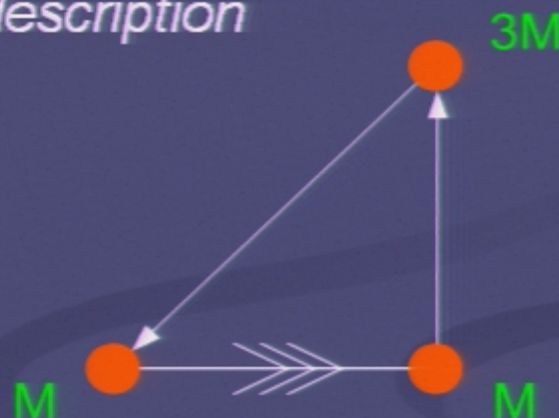
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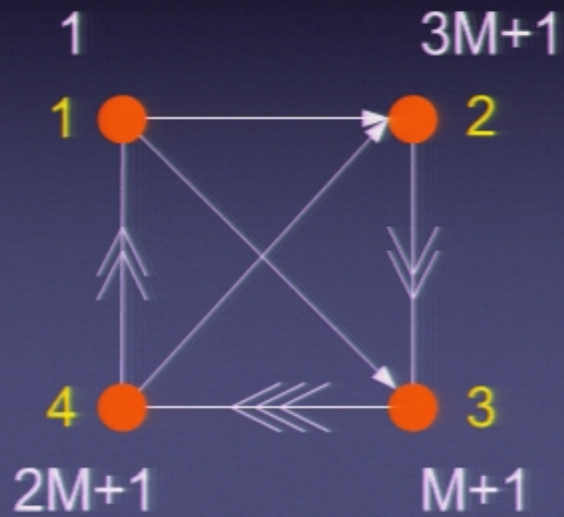


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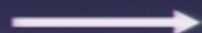
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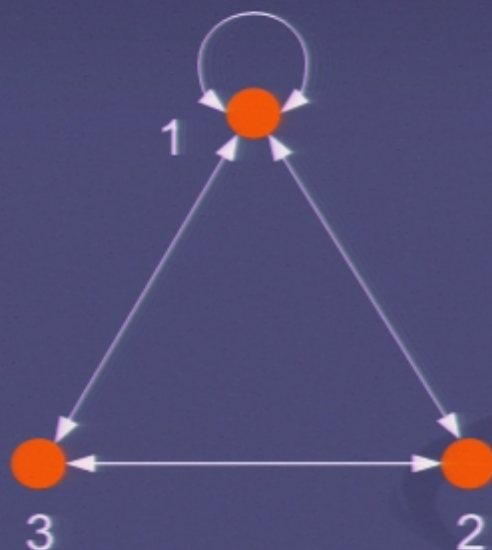
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Example: Suspended
Pinch Point (SPP)



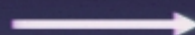
$$x y = z w^2$$

$$W = X_{21} X_{12} X_{23} X_{32} - X_{32} X_{23} X_{31} X_{13} + X_{13} X_{31} X_{11} - X_{12} X_{21} X_{11}$$

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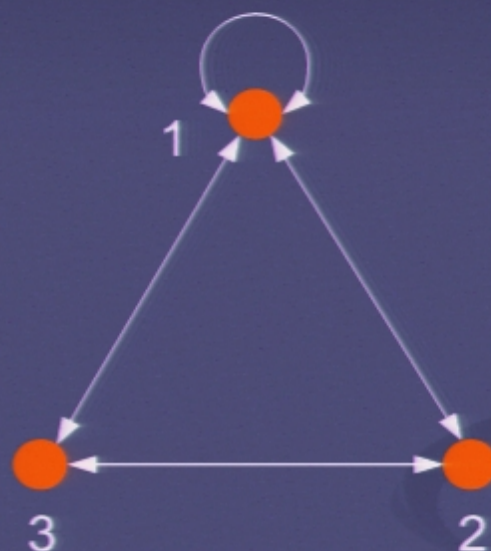
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(0,1,0)

Complex deformation

(1,0,0)

N=2 brane

moves along line of A_1 singularities
parametrized by z (X_{11})

- both types of fractional branes are **incompatible**



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We expect runaway behavior, with the $(1,0,0)$ brane escaping to infinity

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E.g. consider:

of $(0,1,0)$ branes = P

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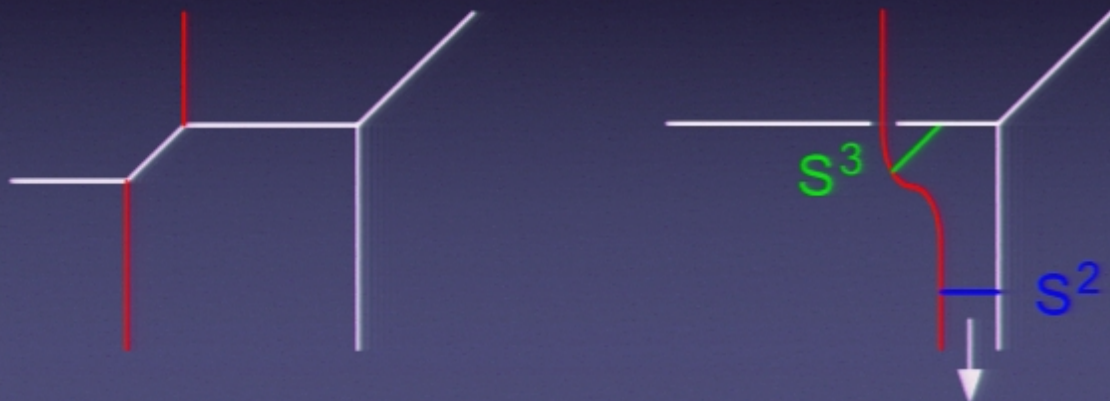
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- There is no SUSY vacuum and $X_{11} \rightarrow \infty$

$Y^{p,q}$ manifolds

- Infinite family of explicit Sasaki-Einstein metrics

Gauntlett, Martelli, Sparks and Waldram

$Y^{p,q}$ manifolds

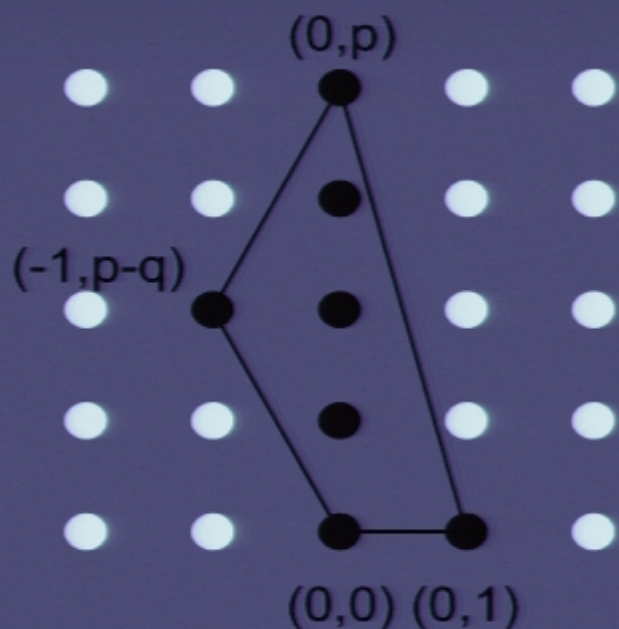
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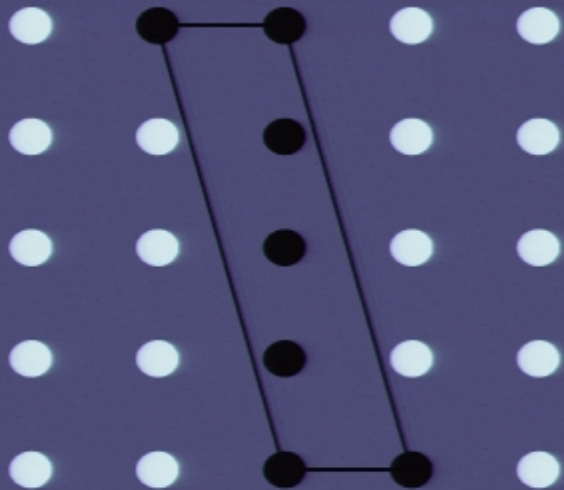
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$$0 \leq q \leq p$$

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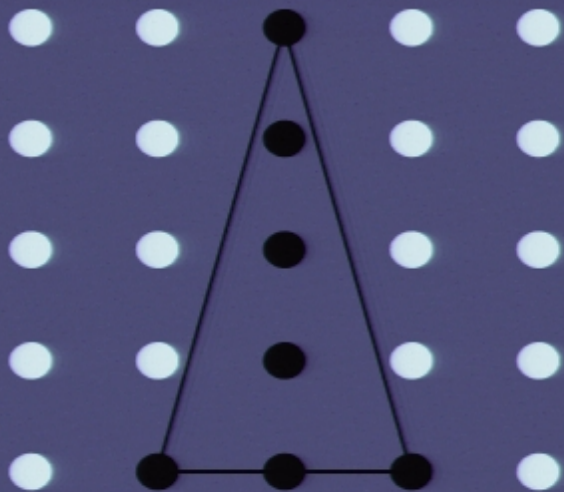
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Special cases:

- $Y^{p,0} = T^{1,1} / Z_p$

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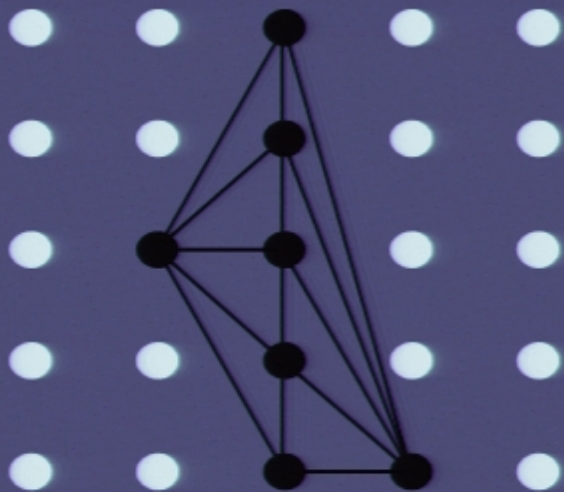
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- Supergravity solutions dual to the UV region of the cascades have been constructed

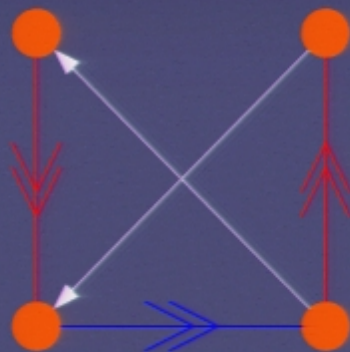
Ejaz, Herzog and Klebanov

- Complex deformation **obstructed** for all $p \neq q$

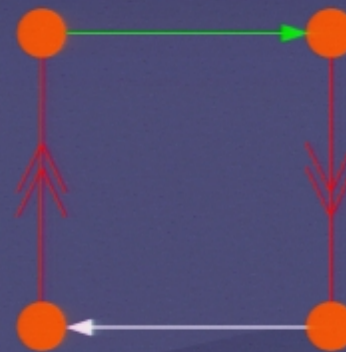
Not in contradiction with **1st order** complex deformations

Burrington, Liu, Mahato and P. Zayas

- Start from $Y^{p,p} = S^5 / Z_{2p}$ and add **(p-q)** impurities



No impurity

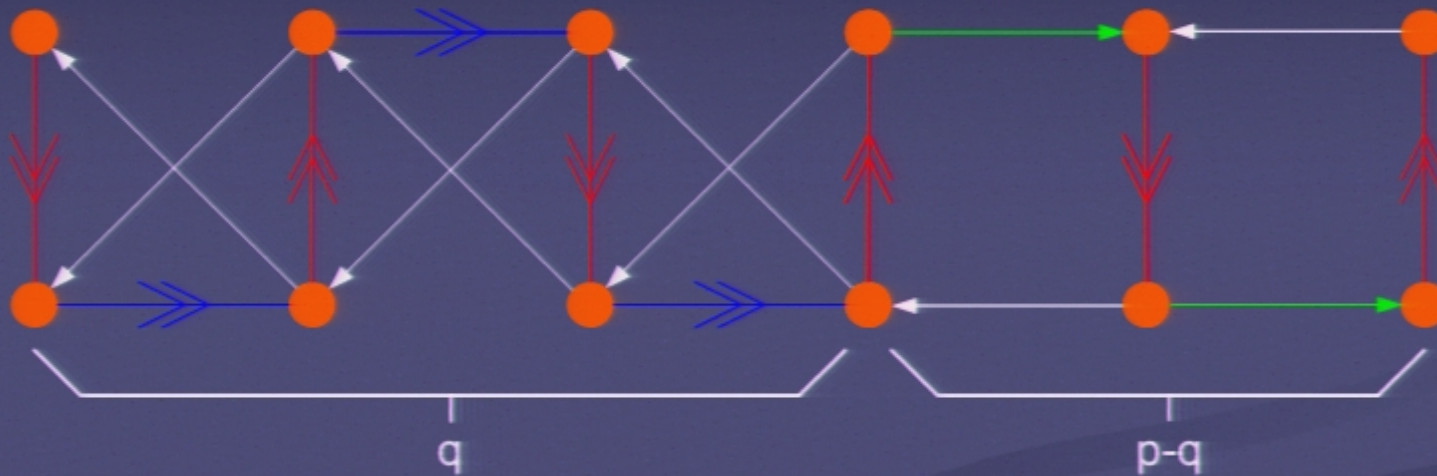


Impurity

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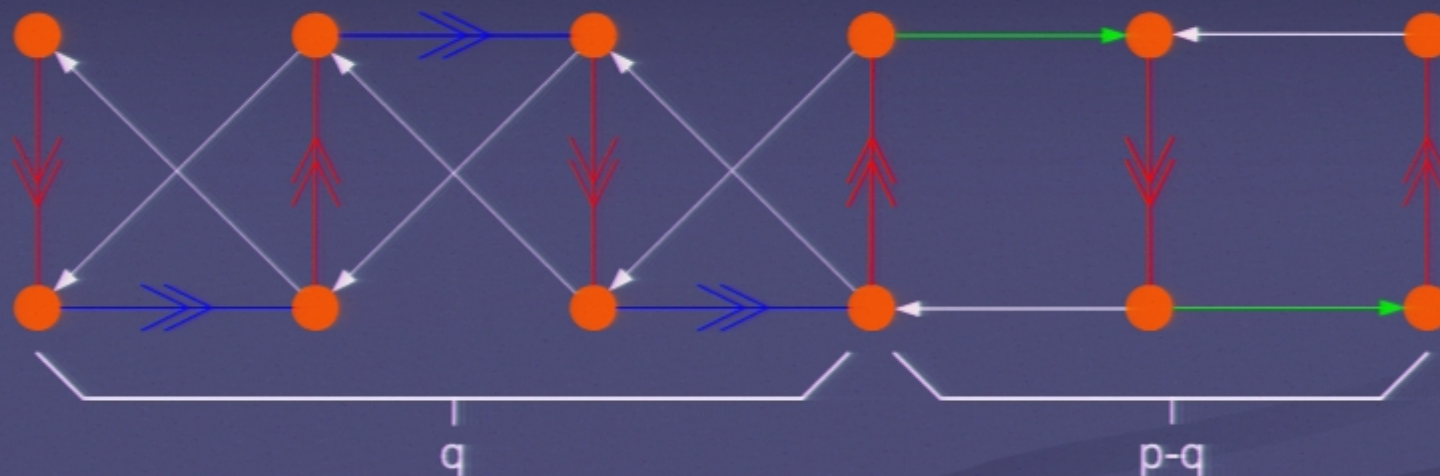
Burrington, Liu, Mahato and P. Zayas



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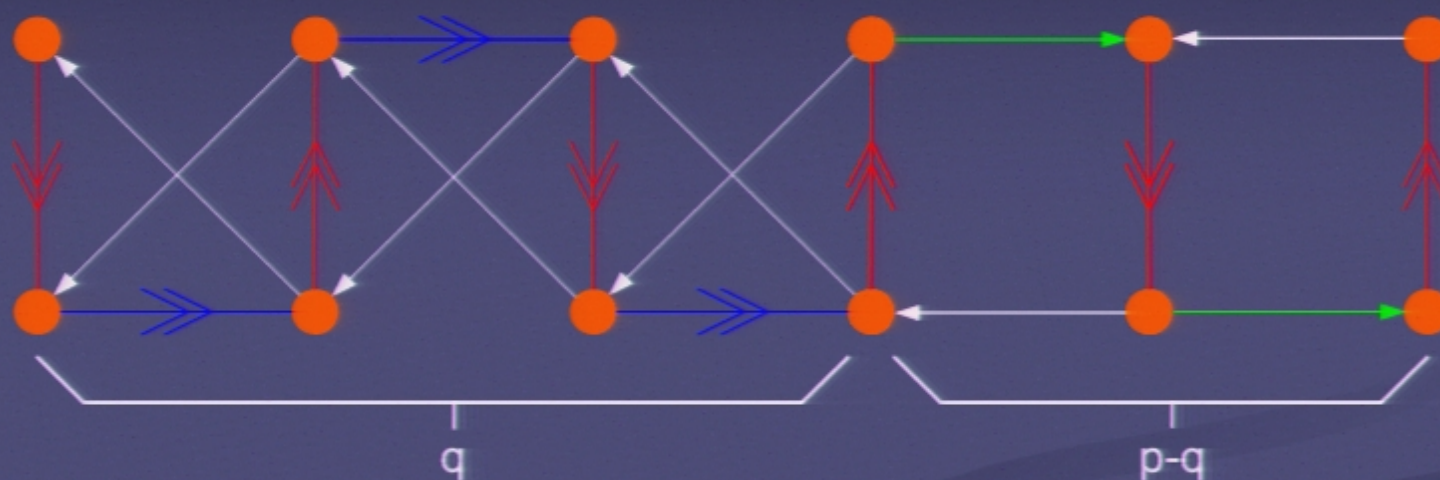


- Construct **rank vector** of fractional brane using **baryonic charges** of bifundamental fields

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Burrington, Liu, Mahato and P. Zayas



- Construct **rank vector** of fractional brane using **baryonic charges** of bifundamental fields
- For the last node:

$$N_F - N_C = -p + q < 0 \quad \text{for} \quad q < p$$

- An **ADS** superpotential is generated and **DSB** takes place

Conclusions

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Further directions

- Turn these ideas into a **simple mechanism** to generate DSB in realistic models