Title: Is a past-finite causal order the inner basis of spacetime?

Date: Sep 07, 2005 02:00 PM

URL: http://pirsa.org/05090001

Abstract: The causal set -- mathematically a finitary partial order -- is a candidate discrete substratum for spacetime. I will introduce this idea and describe some aspects of causal set kinematics, dynamics, and phenomenology, including, as time permits, a notion of fractal dimension, a (classical) dynamics of stochastic growth, and an idea for explaining some of the puzzling large numbers of cosmology. I will also mention some general insights that have emerged from the study of causal sets, the most recent one concerning the role of intermediate length-scales in discrete spacetime theories.

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Sept 28 Jungsinkel(?)

Oct

Oct 19 Reka Albert "Modeling genetic regulatory networks"

Oct 26 Charlie Bennett TBA

Nov 4 (Friday) Evelyn Fox Keller TBA

Nov 9 Sundance Bilson-Thompson Preon Models

Nov 23 Phillip Manheim TBA

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1. Order + Number $\Rightarrow$ Geometry

   Discreteness ADDS information too (as well as removing degrees of freedom)

   Volume = number

2. Discreteness and Lorentz invariance can live together

   In fact N=V *requires* Lor invar

   $\Rightarrow$ kinematic randomness

   $\Rightarrow$ UV-IR mixing

   $\Rightarrow$ locality must be given up (tho not causality)

   Recovering approximate locality $\Rightarrow$ *intermediate length scale* enters on which nonlocality survives (cf non-commutative geometry)
   [By the way, nonlocality arguably does *not* cure the perturbative infinities of qft, it worsens them! The *cutoff* cures them (but they're related)]

3. The "Problem of Time" is soluble in a discrete, past-finite cosm

A *background-free* generally covarariant dynamics exists.

Its "observables" (label-invariant predicates) are not only formally defined, but *physically accessible* (in terms of "stem predicates")

(This dynamics is classical, but covariance and background-independence aren't issues for the quantal generalization)

4. Covariance and becoming can co-exist: *dynamics as growth*

5. The Cosmological Constant might be a (nonlocal & quantal) residue of the underlying discreteness.

(i) Λ was predicted as a fluctuation effect $(1/\sqrt{N})$
(if so $d = 4$ is special, the "critical dimension")
The supernovas are illuminating the underlying discreteness!

(ii) "Why continuum?" = "why is $\Lambda \approx 0$?"
(nonlocality might be key *together with* quantal interference)

(say why!)

6. The large numbers of cosmology might be understood without inflation as reflecting the large age of cosmos: *its many cycles*

$\longrightarrow$ a new kind of renormalization induced by "bounce"

7. Causality + Covariance is restrictive $\longrightarrow$ CSG

(causality in derivation of CSG is replacing locality in derivation of GR)
we hope they will also lead us to QSG (if not then what? entropy bound? □□σ?)

8. Quantum Gravity needs Generalized Quantum Mechanics

It needs a histories formulation that can do without unitarity

QM as a generalized form of measure theory

We shouldn't expect a unitary theory if we freeze the causet (resp manifold) This is so *even if* full theory were unitary, but:

We shouldn't expect unitarity in full QG either

9. BH entropy might be understood as "number of horizon molecules"

(cf LQG)

We also see hint of "holographic entropy bound"

10. Discreteness can have phenomenological consequences even while *respecting* Lorentz-invariance

$\Rightarrow$ diffusion in velocity space

$\Rightarrow$ deviations from $\Box\phi = 0$
(maybe even too big)

$\Rightarrow$ Λ fluctuations
(incidentally, excludes "large extra dimensions")

DISCRETE GRAVITY

Why discreteness?

The infinities

- QFT ⇒ $\frac{1}{2}h\nu$ per mode ⇒ $E = \infty$ ($\Lambda = \infty$)
  (black body)
- GR ⇒ singularities where curvature → ∞
  ⇒ structure on infinitesimal scales
- std. model (prob) lacks continuum limit
  ⇒ $\nexists$ continuum?
- QFT + GR is not perturbatively renormalizable

  (gravity does not cure ∞'s of QFT
  if continuum retained)

  ⇒ something new at ℓ_{pl}, but what?

  (& $\int R dV$ must fail even earlier)

- GR + QFT ⇒ ∞ BH "entanglement entropy"
  (cutoff cures it)
- Continuum ⇒ ∞ # of horizon shapes
  which again ⇒ $S_{BH} = \infty$
  (we don't expect pure state for these vbles)

Problem is ∞'ly small distances

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DISCRETE GRAVITY

LECTURE 1 : Overview and Intro. to Causal Sets

(a) Why discreteness ?

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Problem is ∞'ly small distances

The alternative continuum-discontinuum seems to me to be a real alternative, i.e. there is no compromise... In a [discontinuum] theory space and time cannot occur... It will be especially difficult to derive something like a spatio-temporal quasi-order from such a schema... But I hold it entirely possible that the development will lead there.-

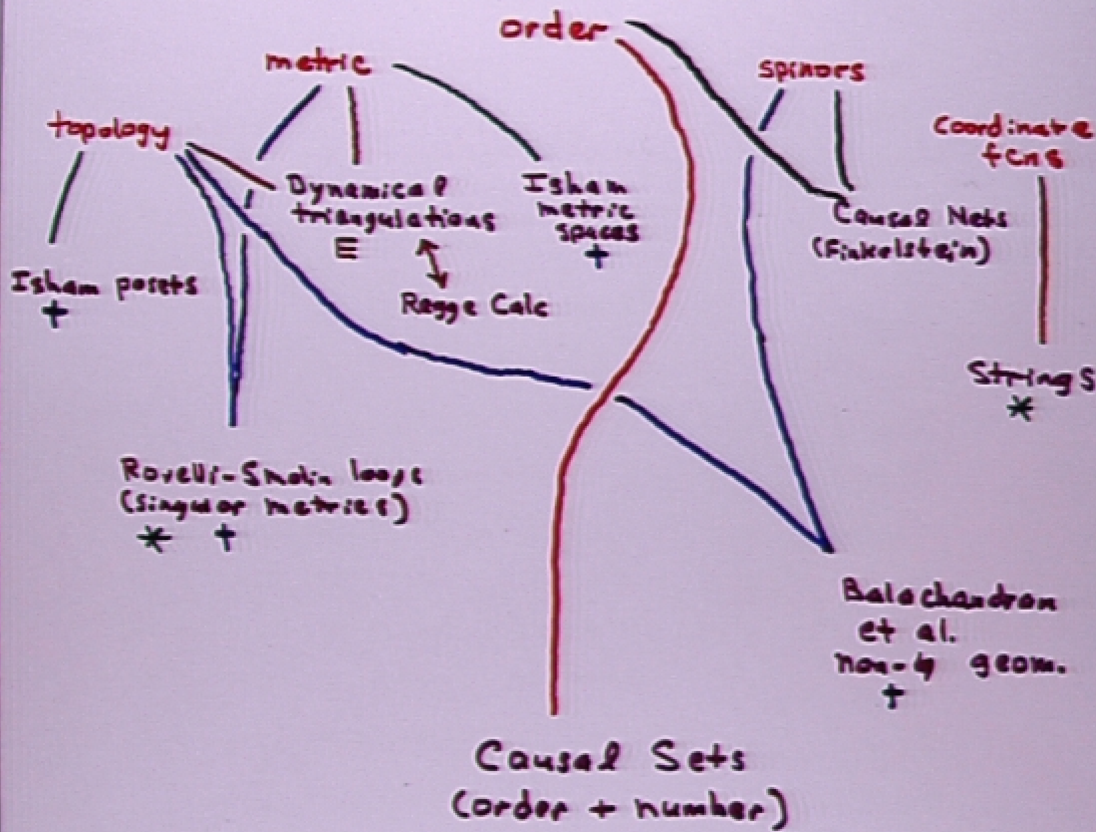
1954

Determinate parts of a manifold ... are called quanta. Their quantitative comparison occurs, for discrete [manifolds] through counting, for continuous ones through measurement.

1854

What is the underlying discrete structure?

5 bridges back to the continuum



* = manifold retained

† = only space recovered, and $t \in \mathbb{R}$

E = Euclidean Signature

The causal order is arguably
the most fundamental
spacetime structure,
and the simplest.

From it we can get all
the others

Introduction to Causets

A kind of "(modern) family tree"
(also early?)

example

Basic ideas

N $\longleftrightarrow$ Space<u>time</u> volume

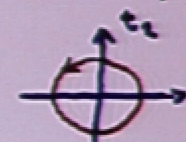
microscopic order $\longleftrightarrow$ macroscopic causal order
(light cones)

Together:
Number + Order $\Longrightarrow$ Geometry

discreteness is <u>essential</u> to get volume element

unifies: topology, diff. str., metric
in terms of order
(can it yield "matter" too?)

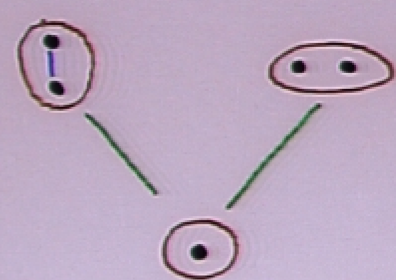
answers: why length?
explains $(-\underbrace{+++}_{D-1})$

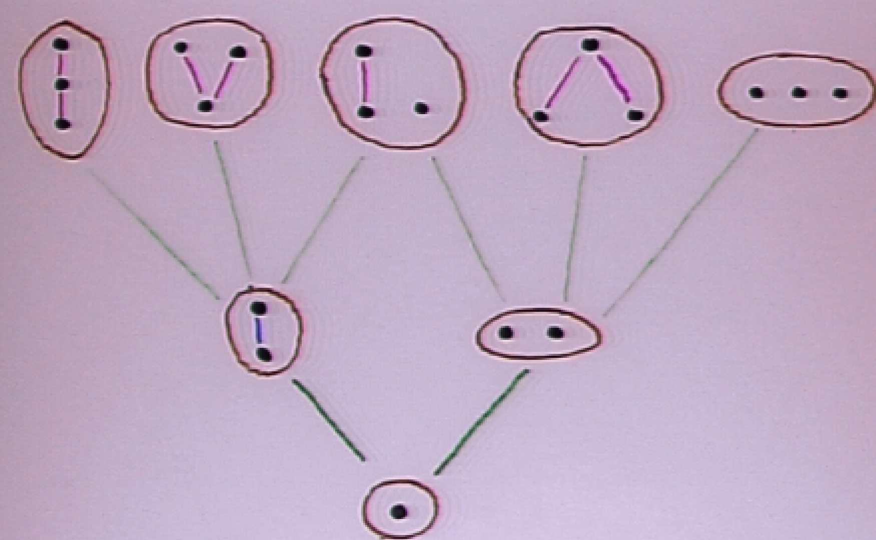


timelike loop

Also:
Coarse-graining allows description
of (eg) scale-dependent topology
(Kaluza-Klein, "foam")

$\Longrightarrow$ no closed timelike curves

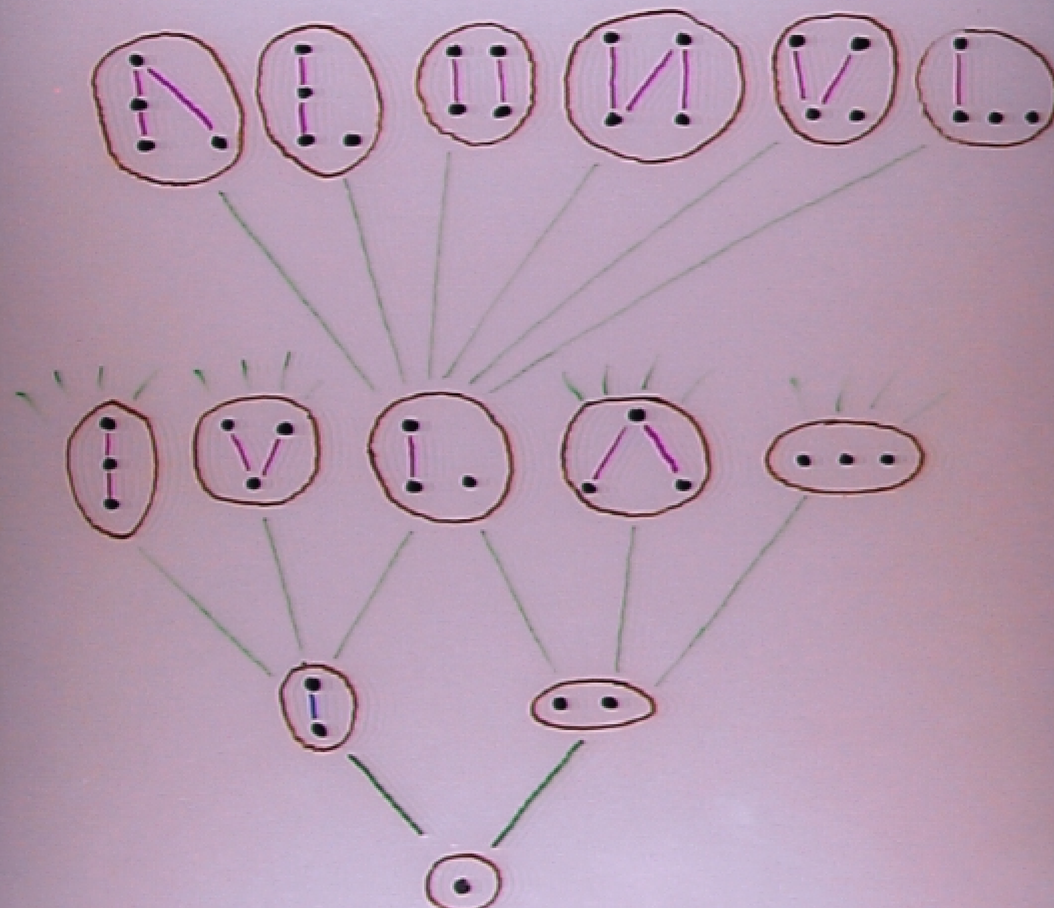




growth as an upward path thru the poset of finite causets

gen. cover. $\Rightarrow$ path independence (posets unlabeled)

$\Rightarrow$ Markovian



Causet = Causal Set

transitive $\quad x<y<z \Rightarrow x<z$

irreflexive $\quad$ not $x<x$

past-finite

$\forall x \; \{y \mid y<x\}$ a finite set

$l_{hG} \xrightarrow[h\to 0]{} 0$

$x<y$

10a

We can prove this:

(i) If C's partial stems do not characterize it then it must contain ∞ copies of some stem S.

(ii) If some S occurs infinitely often in C then some <u>level</u> of C is ∞

(iii) C almost surely has no ∞ levels if $t_2 > 0$.

(the exceptions: antichain, tree are harmless)

This gives physical meaning to our formal σ-algebra $\mathcal{R}$.

(A related issue to consider "this partial stem" vs "some partial stem")

Fluctuating Cosmological "Constant"

($\delta\lambda \sim V^{-1/2}$) (Heuristic)

In seq. growth dynamics, **N** plays formally role of time : $N \leftrightarrow T$

But we normally don't integrate over T; in Σ/histories, we hold it fixed.
$\Rightarrow$ expect to fix N in "Σ/causets"

$N \leftrightarrow V$ $\Rightarrow$ would fix V in continuum approx. (unimodular)

But this $\Rightarrow$ Λ free:

$$\delta \left(\int \left(\tfrac{1}{2\kappa} R - \Lambda_0\right) dV - \lambda V \right) = 0$$

$\Rightarrow$ only $\Lambda = \Lambda_0 + \lambda$ matters

Quantum: Λ, V conjugate $\Rightarrow$ $\delta\Lambda\, \delta V \sim \hbar$

But Poisson $\Rightarrow$ $N = V \pm \sqrt{V}$ $\Rightarrow$ $V \approx N \pm \sqrt{N}$

ie $\delta V \sim \sqrt{V}$

$\overset{(*)}{\Rightarrow}$ $\delta\Lambda \sim \dfrac{\hbar}{\delta V} \sim \dfrac{1}{\sqrt{V}}$

If $\langle \Lambda \rangle = 0$ then $\Lambda \sim \pm \dfrac{1}{\sqrt{V}}$

present epoch: $V \sim 10^{240}$ $\Rightarrow$ $\Lambda \sim 10^{-120}$

excludes large KK dim!

(*) Fourier transform argument

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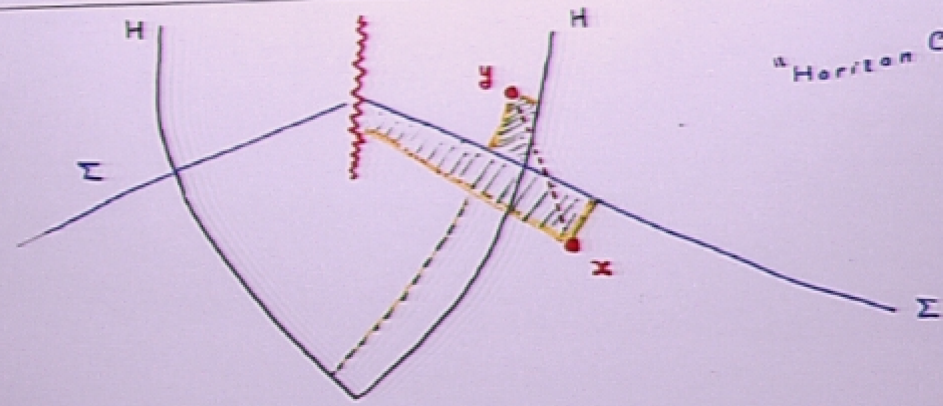
"Horizon Counting"



① $x < \Sigma, H$ $y > \Sigma, H$

② $x \prec y$ a link

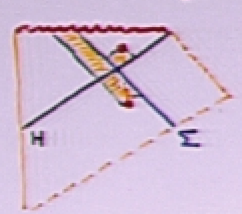
③ x maximal in (past Σ)

y minimal in (future Σ) ∩ (future H)

③' x maximal in (past Σ) ∩ (past H)

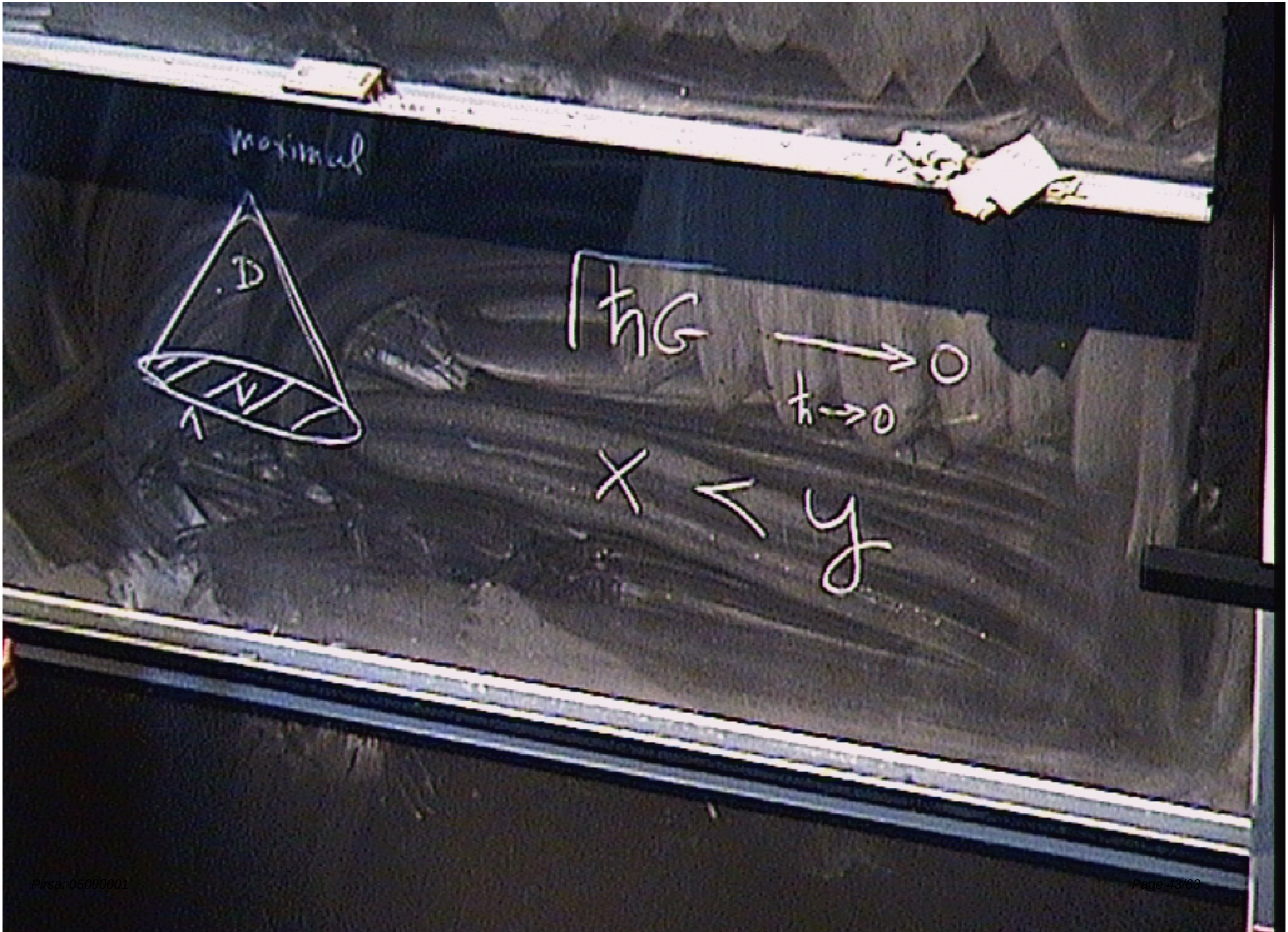
y minimal in (future H)

Count such pairs (x, y)

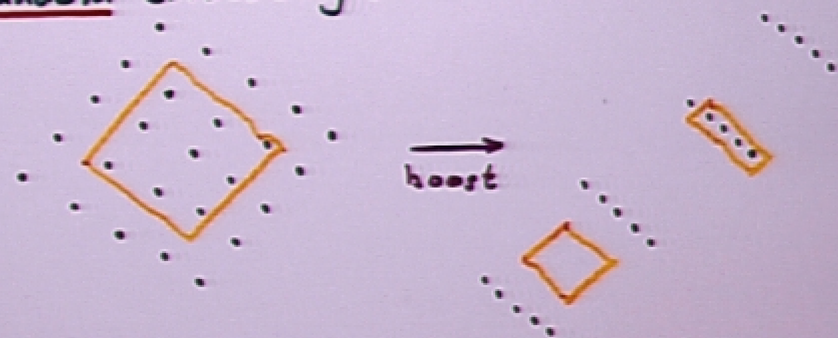


equivalent Penrose diagram

(Σ null)


maximal
D
N
$\sqrt{\hbar G} \xrightarrow[\hbar \to 0]{} 0$
$x < y$

(a) Why random embedding?

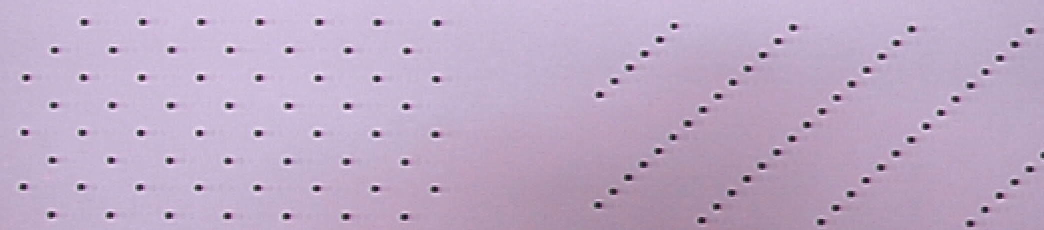


appears uniform .... but isn't!

Hence the embedding is not faithful

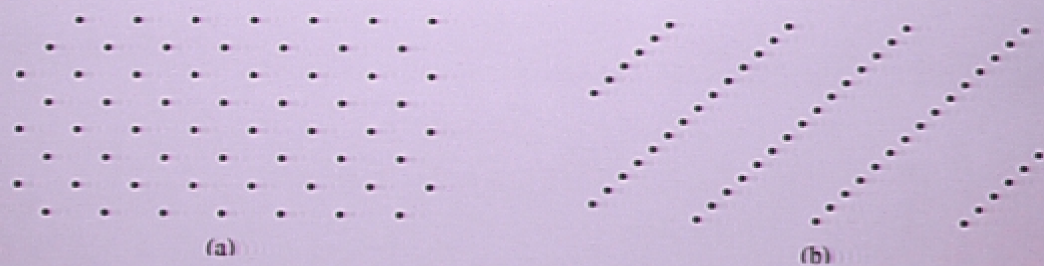
In contrast a Poisson process is Lorentz invariant and uniform.

Sprinkling produces a "random lattice" and introduces a kinematic role for randomness.



(a) (b)

Diamond lattice before and after boost



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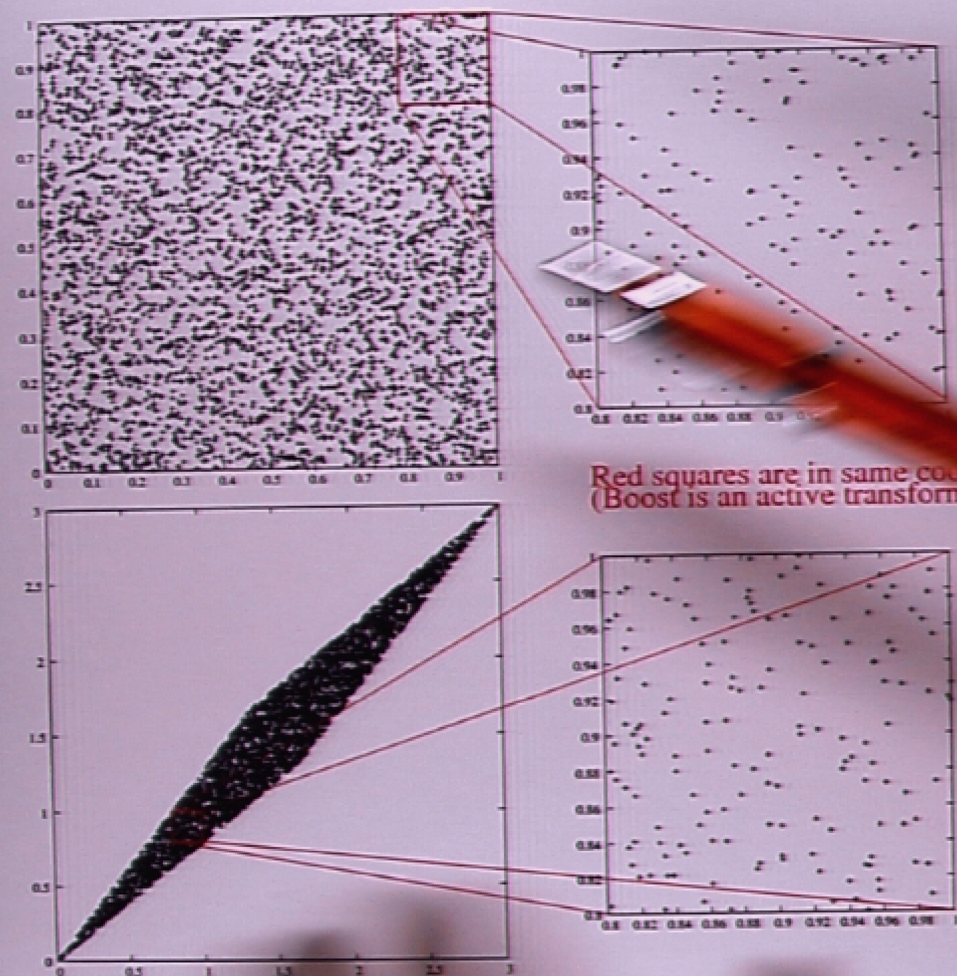
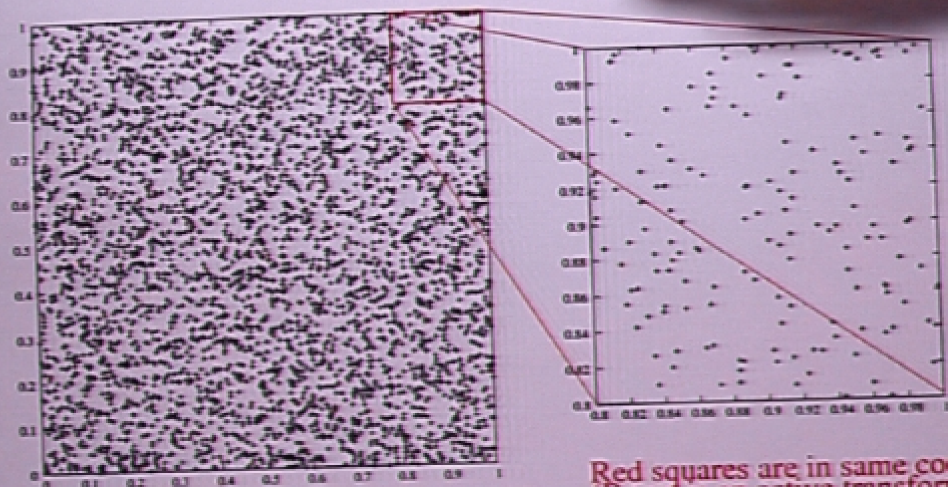
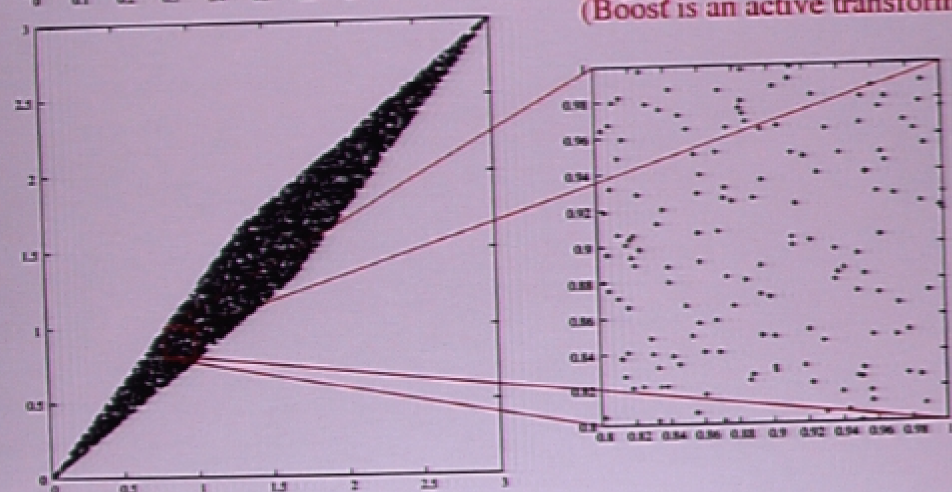
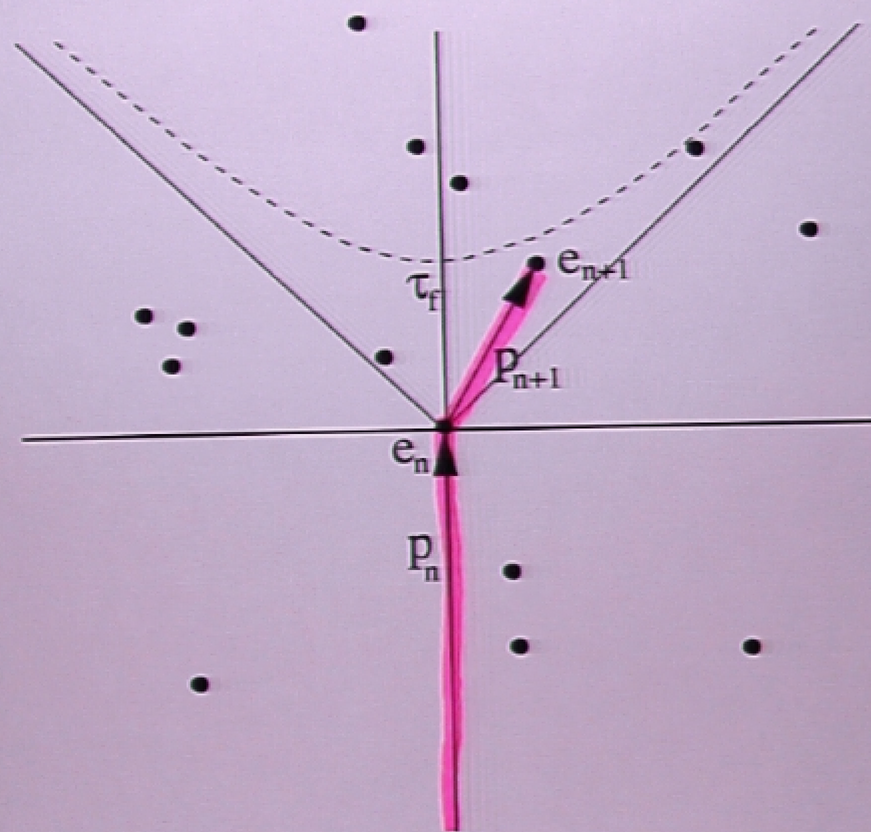


Red squares are in same coo... location.
(Boost is an active transformatio...



Red squares are in same coordinate location.
(Boost is an active transformation.)





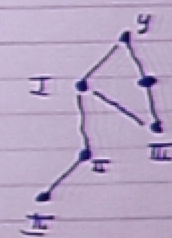
"Henson swerves" or the Lucretius effect

$$\frac{\partial \rho}{\partial \tau} = k\nabla_p^2 \rho - \frac{1}{m} p^\mu \frac{\partial}{\partial x^\mu} \rho$$

$$\frac{\partial \rho}{\partial t} = k\nabla_p^2 \left(\frac{\rho}{\sqrt{1+p^2/m^2}} \right) - \nabla_a (w^a \rho)$$

Discrete $\Box$ – operator

$$\Box\varphi(y) \propto -\tfrac{1}{2}\varphi(y) + \Big(\sum_{\mathrm{I}} - 2\sum_{\mathrm{II}} + \sum_{\mathrm{III}}\Big)\,\phi(x)$$



$$\Box\varphi(y) \leftrightarrow \frac{4\epsilon}{\ell^2}\Big[-\tfrac{1}{2}\varphi(y) + \epsilon\sum_{x\prec y} f(x)\,\varphi(x)\Big]$$

$$f(x) \equiv \left(1 - 2\,\overline{F} + \tfrac{1}{2}\,\overline{F}^2\right) e^{-\overline{F}}$$

$\overline{F} \equiv \epsilon\,|\langle x, y\rangle|$ (Cardinality)

$\ell^2 =$ area / element

(nonlocality scale $= \frac{\ell}{\sqrt{\epsilon}}$)

Causet = Causal Set

transitive $x < y < z \Rightarrow x < z$

irreflexive not $x < x$

past-finite

$\forall x \ \{y \mid y < x\}$ a finite set

$\Box \phi = 0$

$\sqrt{\hbar G} \xrightarrow[\hbar \to 0]{} 0$

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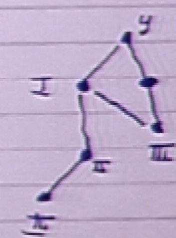

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$$f(x) = \left(1 - 2\xi + \tfrac{1}{2}\xi^2\right) e^{-\xi}$$

$$\xi = \epsilon\,|\langle x,y\rangle| \qquad \text{(cardinality)}$$

ℓ^2 = area / element

(nonlocality scale $= \frac{\ell}{\sqrt{\epsilon}}$)

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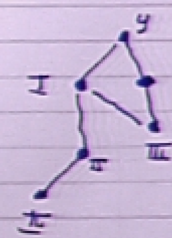

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(nonlocality sc

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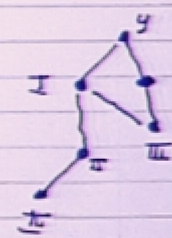


$$\Box\varphi(y) \leftrightarrow \frac{4\epsilon}{\ell^2}\left[-\frac{1}{2}\varphi(y) + \epsilon\sum_{x \prec y} f(x)\,\varphi(x)\right]$$

$$f(x) = \left(1 - 2\xi + \tfrac{1}{2}\xi^2\right) e^{-\xi}$$

$$\xi = \epsilon\ |\langle x, y\rangle| \qquad \text{(Cardinality)}$$

$$\ell^2 = \text{area / element}$$

$$\left(\text{nonlocality scale} = \frac{\ell}{\sqrt{\epsilon}}\right)$$

Many Insights (gained)

Much Work (remains)

Oct 19 Reka Albert "Modeling genetic regulatory networks"

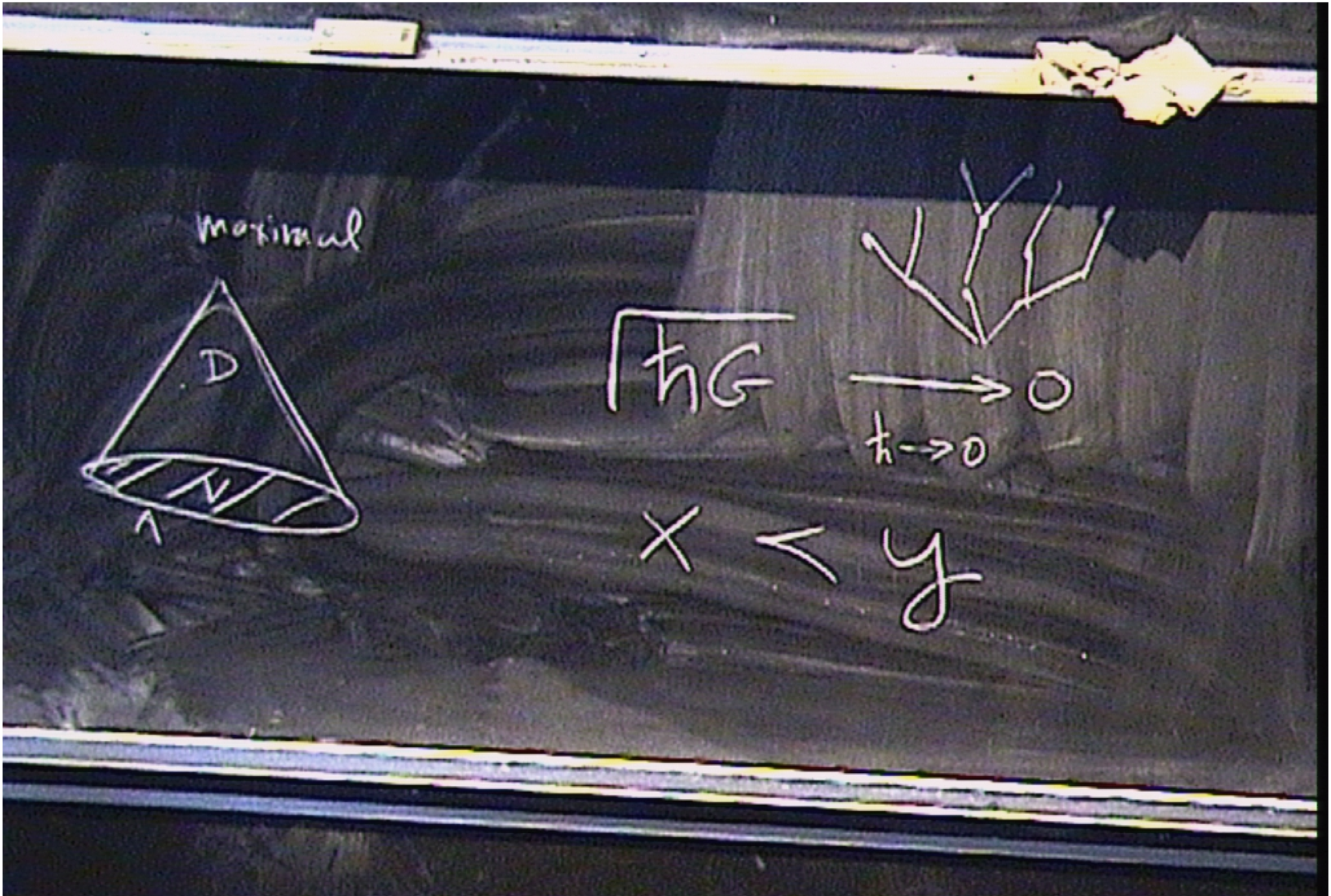
Oct 26 Charlie Bennett TBA

$$G^{ab} = \kappa T^{ab} \longrightarrow 0 = \nabla_b G^{ab} = \kappa \nabla_b T^{ab}$$

Nov 4 (Friday) Evelyn Fox Keller TBA

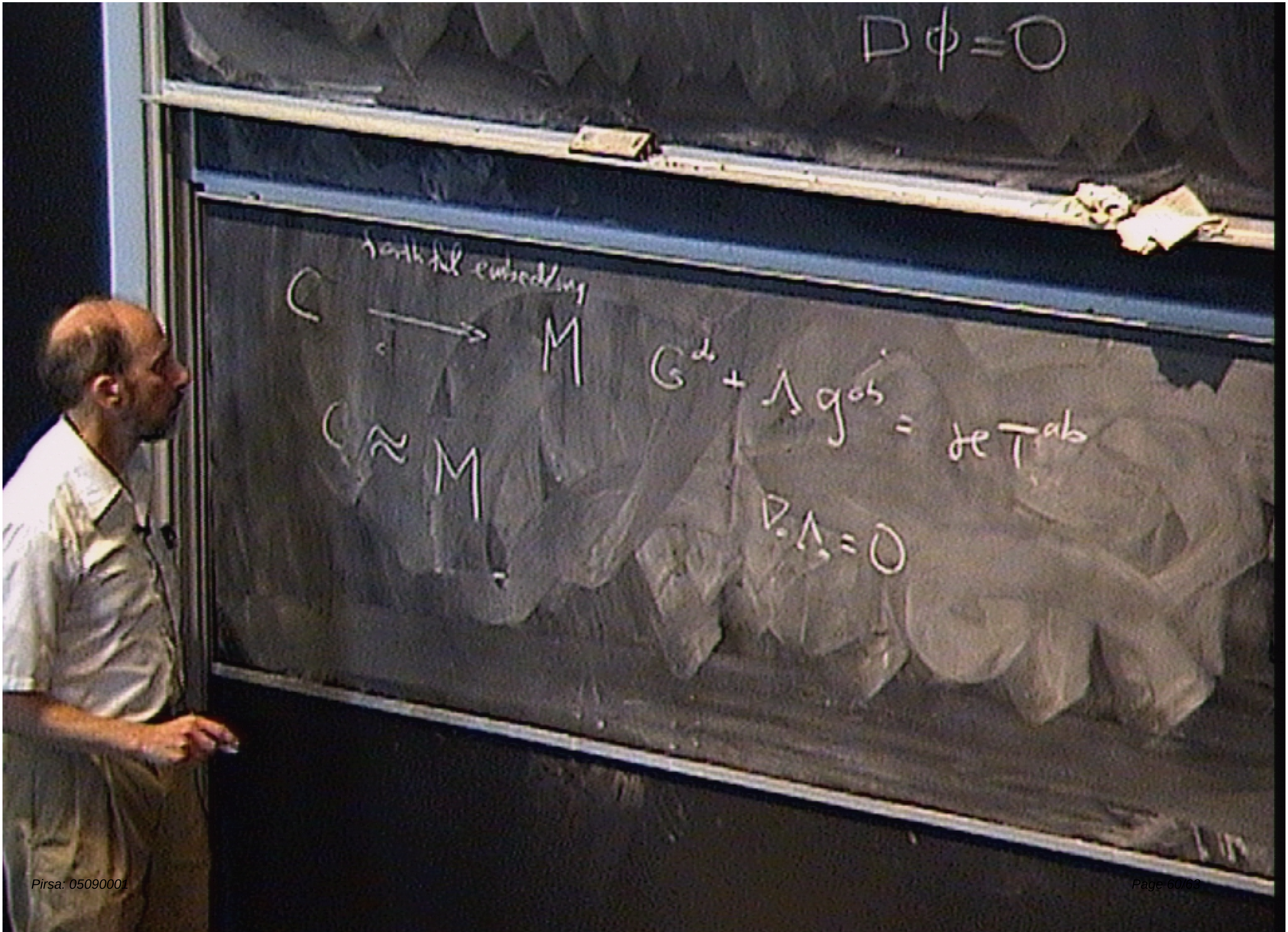
Nov 9 Sundance Bilson-Thompson Preon Models

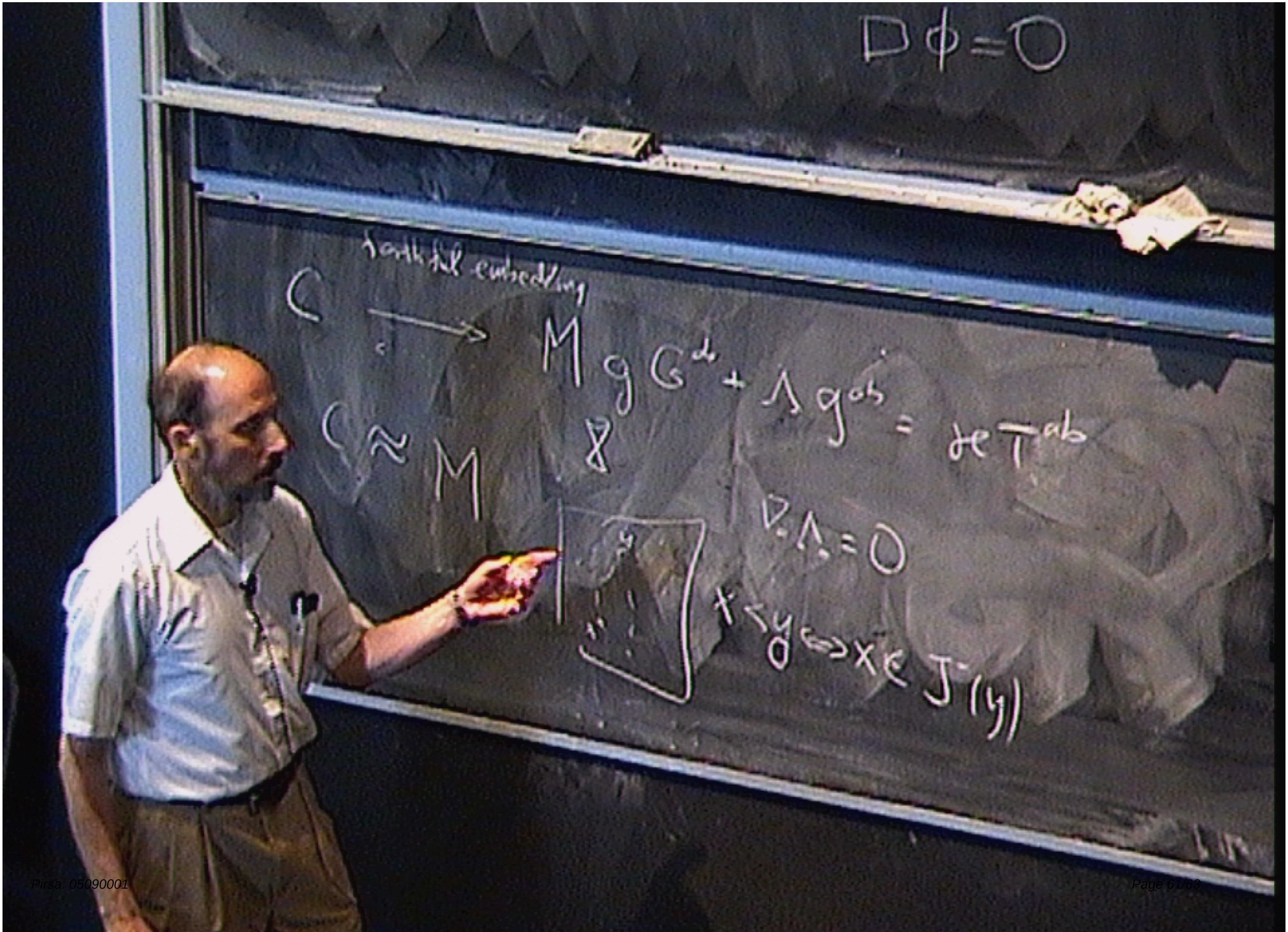
Nov 23 Phillip Manheim TBA

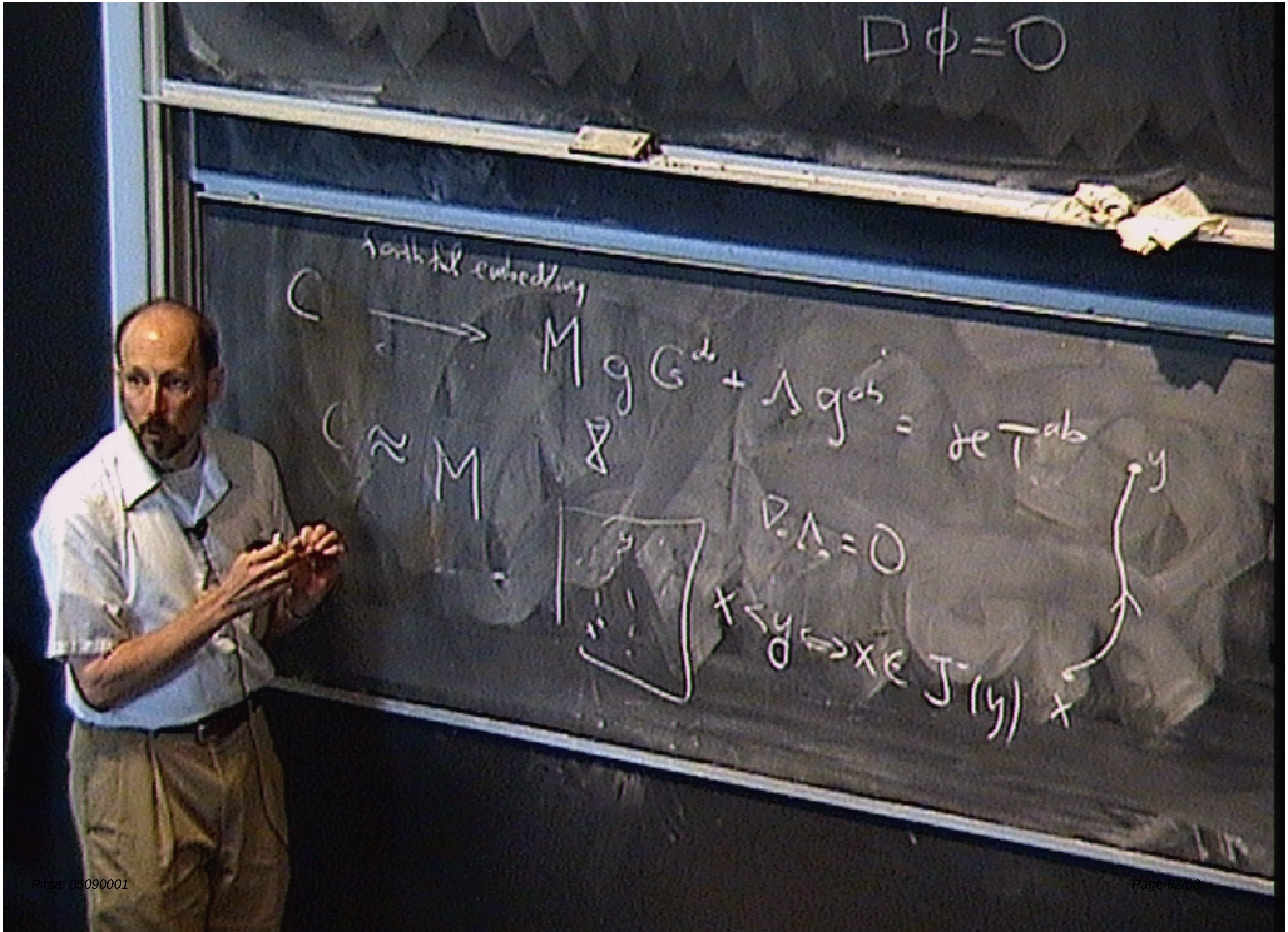

maximal
D
N
$\sqrt{\hbar G} \longrightarrow 0$
$\hbar \to 0$
$x < y$

$$G^{ab} + \Lambda g^{ab} = \kappa T^{ab}$$

$$\nabla_a \Lambda = 0$$


□φ = 0
faithful embedding
C → M
C ≅ M
∇·Λ = 0


□φ=0
faithful embedding
C ≈ M
∇·Λ=0


□ϕ=0
faithful embedding

$$C \xrightarrow{\text{conformal embedding}} M \qquad G^{ab} + \Lambda g^{ab} = 8\pi T^{ab}$$

$$C \approx M$$

$$\nabla_a \Lambda_b = 0$$

$$x < y \Leftrightarrow x \in J^-(y)$$