Title: Qubit Field Theory

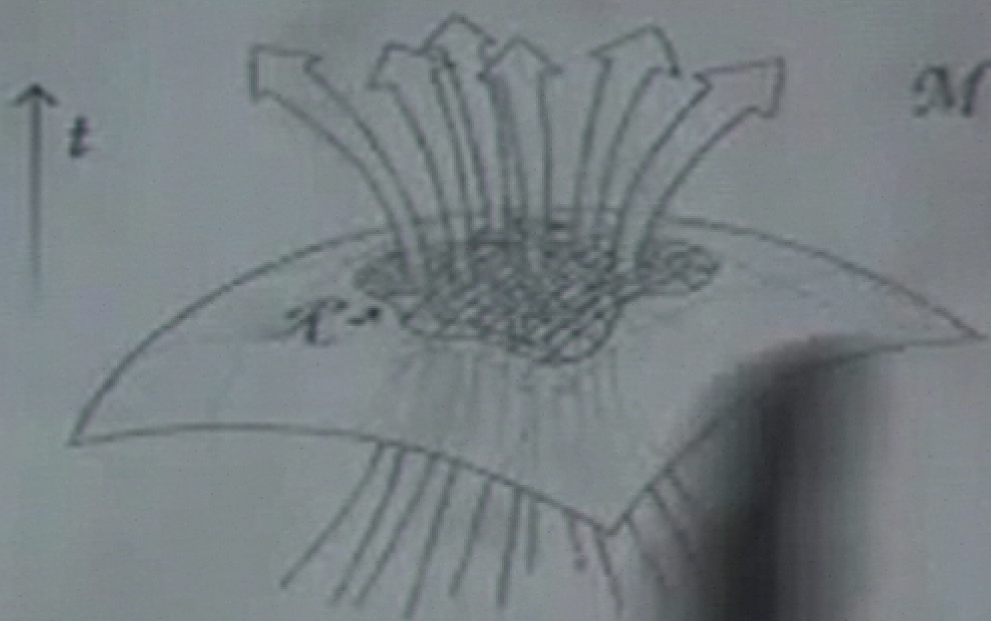
Date: Aug 24, 2005 04:00 PM

URL: http://pirsa.org/05080014

Abstract:

INFORMATION FLOW

Is there an upper bound on the amount of information that can be stored in a given finite region $\mathcal{R}$ of space?



Apparently not, if

$$\left[\hat{\varphi}(\mathbf{x},t),\hat{\varphi}(\mathbf{x}',$$

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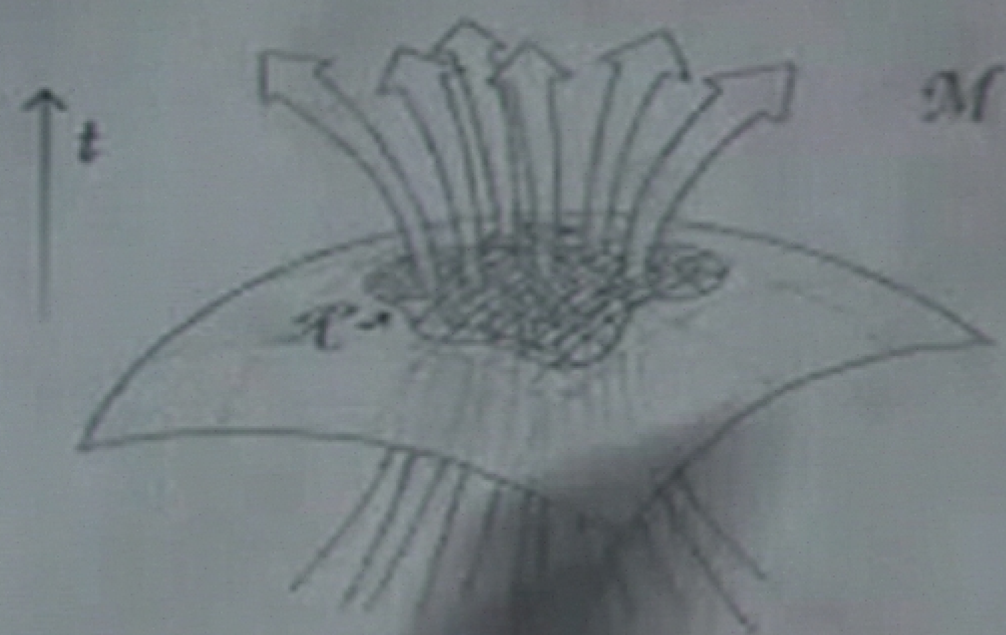

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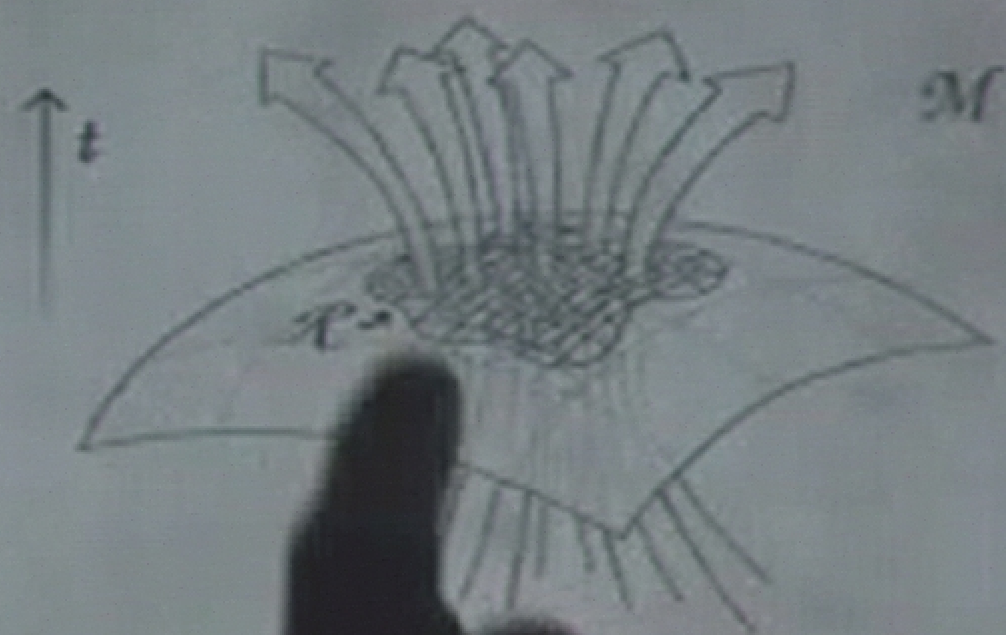

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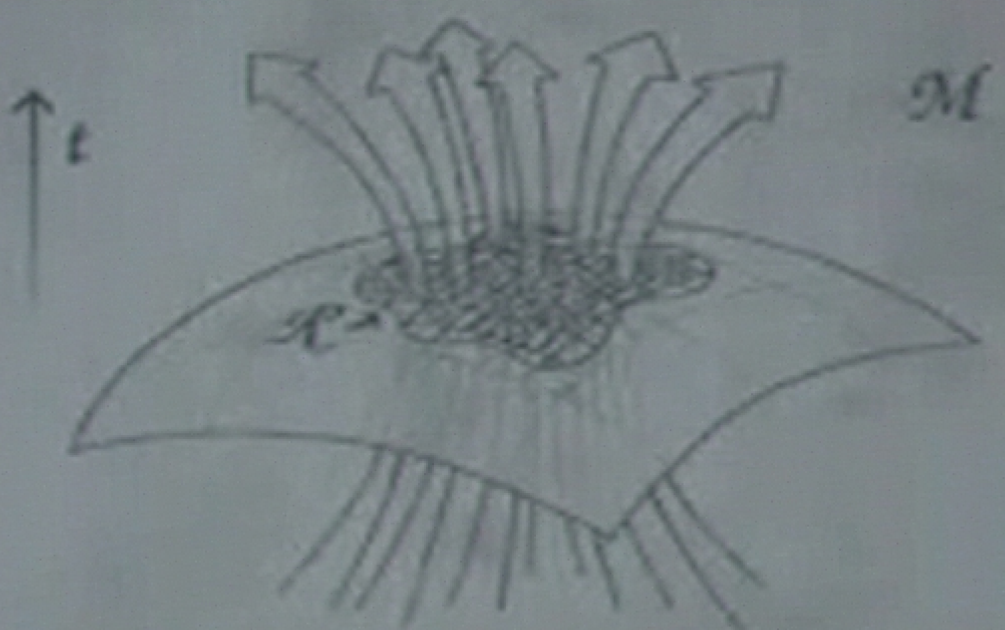

Apparently not, if

$$\left[\hat{\varphi}(\mathbf{x},t),\hat{\varphi}(\mathbf{x}',t)\right]=0$$

BEKENSTEIN BOUND

BLACK HOLE BOUND:

Energy E enclosed in surface area A will form a black hole unless

$$E < (A/\pi)^{\frac{1}{2}} c^4 / G$$

BEKENSTEIN BOUND:

Information capacity of region bounded by a surface of area A:

$$\frac{c^4 A}{4 h G \ln 2} \text{ bits}$$

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$$\left[\hat{\varphi}(\mathbf{x},t),\hat{\varphi}(\mathbf{x}',t)\right]=0$$

$$\left[\hat{\varphi}(\mathbf{x},t),\frac{d\hat{\varphi}(\mathbf{x}',t)}{dt}\right]=i\delta(\mathbf{x},\mathbf{x}')\hat{1}$$

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And it causes the divergences of quantum field theory too!

So it would be great to get rid of it. But there are some objections to that:

- Non-locality?
- What to put in its place? (Whatever you put there can't be covariant.)
- Incompatible with 'quantisation'
- Divergences ~~make us physicists feel important~~ have a rich structure which can be used to restrict, and perhaps, one day, determine, the dynamics of the fields that exist in nature.

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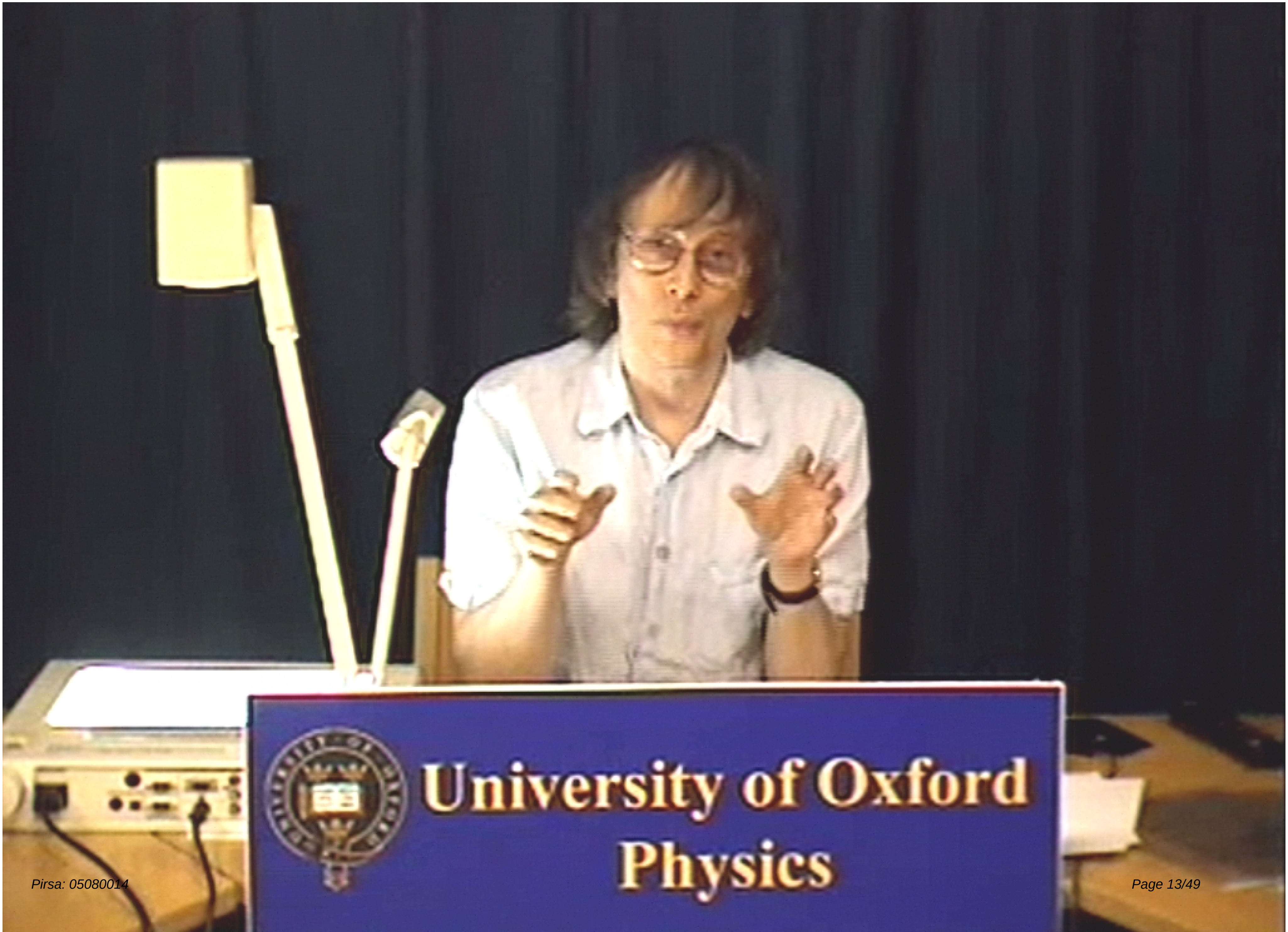
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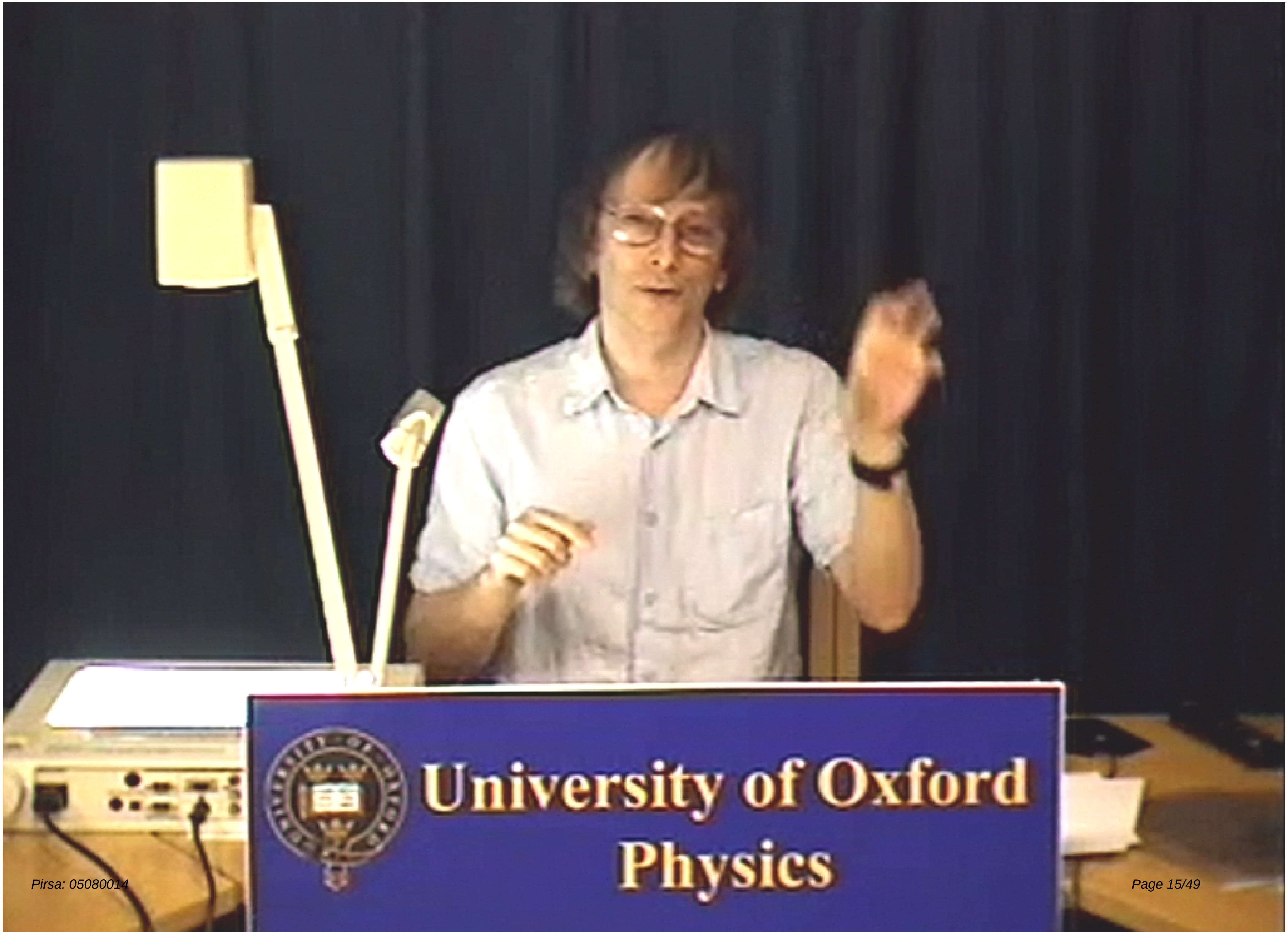
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(Heisenberg Picture)

$$\hat{q}_j(x) \quad (1 \le j \le 3)$$

$$\hat{q}_j(\ldots x) = \delta_{jk}\hat{1} + i\varepsilon_{jk}{}^{l}\hat{q}_l(x)$$

PROPERTI...

- *Motivating* ...
  Has a fin... l representation (in any finite reg...
- Need no... ke separations
  - *Let's try* ...
- Contin...
- Local ... ations above...
- With l... ow is local

All obse... at x are

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$$\hat{q}_j(x)\hat{q}_k(x) = \delta_{jk}\hat{1} + i\varepsilon_{jk}{}^{l}\hat{q}_l(x)$$

PROPERTIES:

- *Motivating property:*
  Has a finite-dimensional representation (in any finite region) – *which implies*
- Need not commute at spacelike separations
  – Let's try not changing anything else…
- Continuous, differentiable
- Local unitarity follows from the equations above
- With local dynamics, information flow is local

All observables of the qubit at x are

There exists a representation of its observables such that

$$\hat{q}_j(x) \leftrightarrow \sigma_j \otimes I$$

An arbitrary observable $\hat{A}$ of the field in this representation can be written:

$$\hat{A} \leftrightarrow I \otimes A_0 + \sigma^j \otimes A_j$$

Where

$$I \otimes A_0 \leftrightarrow \tfrac{1}{4}\left(\hat{A} + \hat{q}_j(x)\hat{A}\hat{q}^j(x)\right),$$

$$I \otimes A_j \leftrightarrow \tfrac{1}{4}\left(\left\{\hat{A}, \hat{q}_j(x)\right\} + i\varepsilon_j{}^{kl}\hat{q}_k(x)\hat{A}\hat{q}_l(x)\right).$$

The RHS are usefully regarded as super-operators acting on $\hat{A}$.

FOCUS ON THE QUBIT AT x

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The first one is the projector into the space of observables that commute with all those at x.

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$(\vec{1} - \vec{\Pi}_x)\hat{A}$

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'Hamiltonian'

Define

$$\hat{H}_\mu(x) = -\tfrac{i}{4}\hat{q}_{j,\mu}(x)\hat{q}^j(x)$$

This is the closest thing to a 'Hamiltonian' that qubit field theory has. It has the property:

$$\hat{q}_{j,\mu}(x) = i[\hat{H}_\mu(x), \hat{q}_j(x)]$$

Differences from a conventional Hamiltonian:

- It's a 4-vector field.
- It evolves the field in all directions.
- It is *kinematical*, not dynamical.
  That's not an equation of motion.

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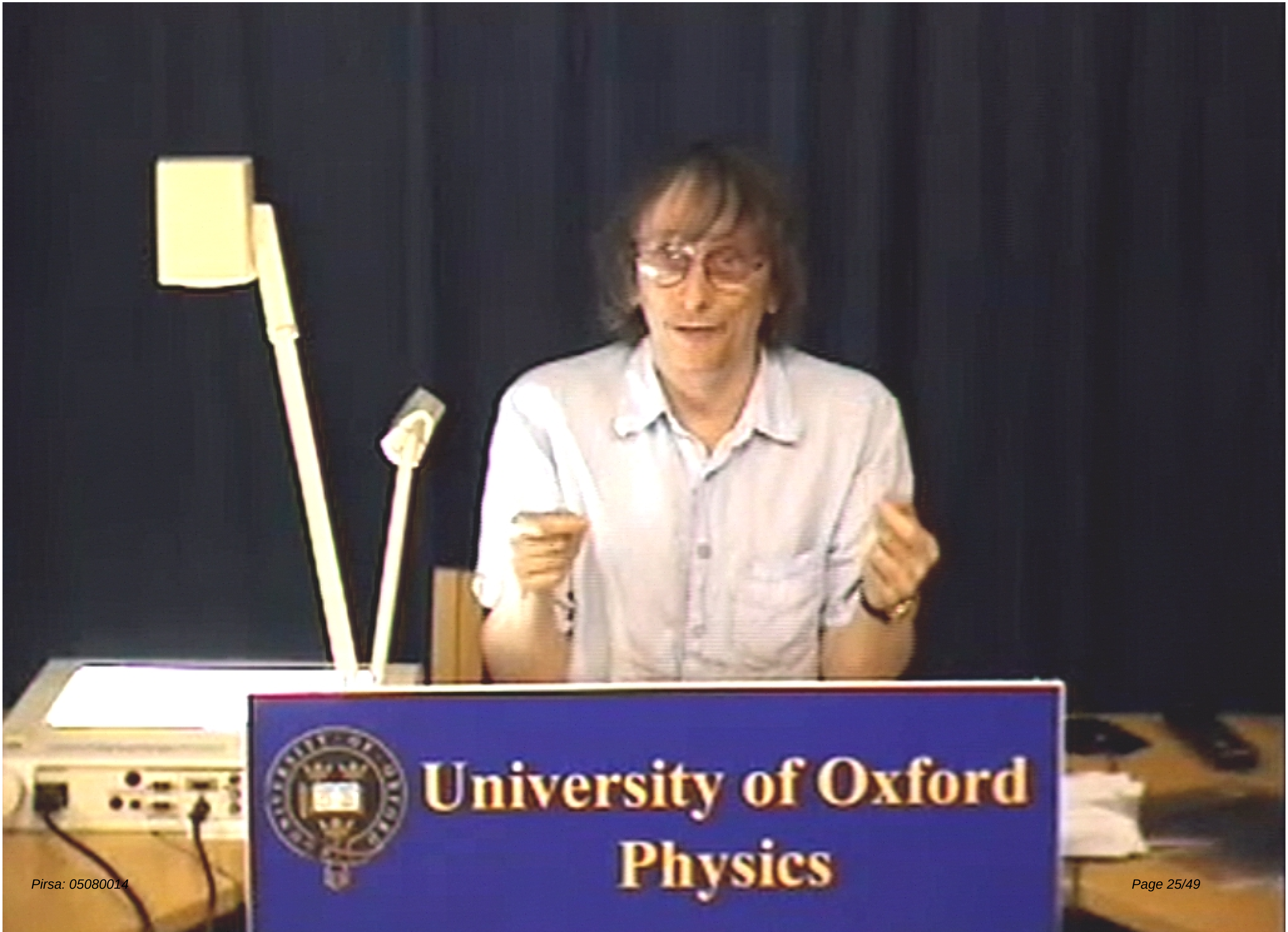
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CRITERIA FOR DYNAMICAL EQUATIONS

- ~~Derived from a classical action~~
- ~~Quantise~~
- ~~Try to avoid the generic pathologies~~
- General covariance
- Unitary evolution (~~global~~, local
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- Up to second-order equations of motion?

ms of no more than 1st degree in 2nd derivat

$$\square \hat{q}_j(x)$$

$$\hat{q}_k(x)\square\hat{q}_j(x)\hat{q}^k(x)$$

$$\hat{q}^k(x)\square\hat{q}_k(x)\hat{q}_j(x)+\hat{q}_j(x)\square\hat{q}_k(x)\hat{q}^k(x)$$

$$i\left(\hat{q}^k(x)\square\hat{q}_k(x)\hat{q}_j(x)-\hat{q}_j(x)\square\hat{q}_k(x)\hat{q}^k(x)\right)$$

$$\varepsilon_j^{\ kl}\left(\hat{q}_k(x)\square\hat{q}_l(x)+\square\hat{q}_l(x)\hat{q}_k(x)\right)$$

$$i\varepsilon_j^{\ kl}\left(\hat{q}_k(x)\square\hat{q}_l(x)-\square\hat{q}_l(x)\hat{q}_k(x)\right)$$

in terms of the following super-operators:

$$\check{O}^{(0)}\hat{A} \stackrel{\text{def}}{=} \hat{A}$$

$$\vec{\Omega}_x^{(1)}\hat{A}_j \overset{\text{def}}{=} \hat{q}_k(x)\hat{A}_j\hat{q}^k(x)$$

$$\vec{\Omega}_x^{(2)}\hat{A}_j \overset{\text{def}}{=} \varepsilon_j^{\ kl}\left(\hat{q}_k(x)\hat{A}_l + \hat{A}_l\hat{q}_k(x)\right)$$

$$\vec{\Omega}_x^{(3)}\hat{A}_j \overset{\text{def}}{=} i\varepsilon_j^{\ kl}\left(\hat{q}_k(x)\hat{A}_l - \hat{A}_l\hat{q}_k(x)\right)$$

$$\vec{\Omega}_x^{(4)}\hat{A}_j \overset{\text{def}}{=} \hat{q}^k(x)\hat{A}_k\hat{q}_j(x) + \hat{q}_j(x)\hat{A}_k\hat{q}^k(x)$$

$$\vec{\Omega}_x^{(5)}\hat{A}_j \overset{\text{def}}{=} i\left(\hat{q}^k(x)\hat{A}_k\hat{q}_j(x) - \hat{q}_j(x)\hat{A}_k\hat{q}^k(x)\right)$$

st general such equation of motion is:

$$\lambda_\alpha\vec{\Omega}_x^{(\alpha)}\Box\hat{q}_j(x) + \mu\hat{q}_j(x) = 0$$

Classifying Dynamical Equations

Composition table for the super-operators $\hat{\Omega}_i^m$

[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

In other w[illegible]

$\lambda_n \hat{\Omega}_i^m$ ha[illegible]

so if it fa[illegible]

Classifying Dynamical Equations

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	0	1	2	3	4	5
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

In other words:

$$\vec{\Omega}_x^{(\alpha)}\vec{\Omega}_y^{(\beta)} = c^{\alpha\beta}{}_{\gamma}\vec{\Omega}_r^{(\gamma)}.$$

$\lambda_\alpha\Omega_x^{(\alpha)}$ has an inverse $\bar{\lambda}_\beta\Omega_x^{(\beta)}$ if

$$\lambda_\alpha\bar{\lambda}_\beta c^{\alpha\beta}{}_{\gamma} = \delta_\gamma^0,$$

so if it fails to have an inverse,

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0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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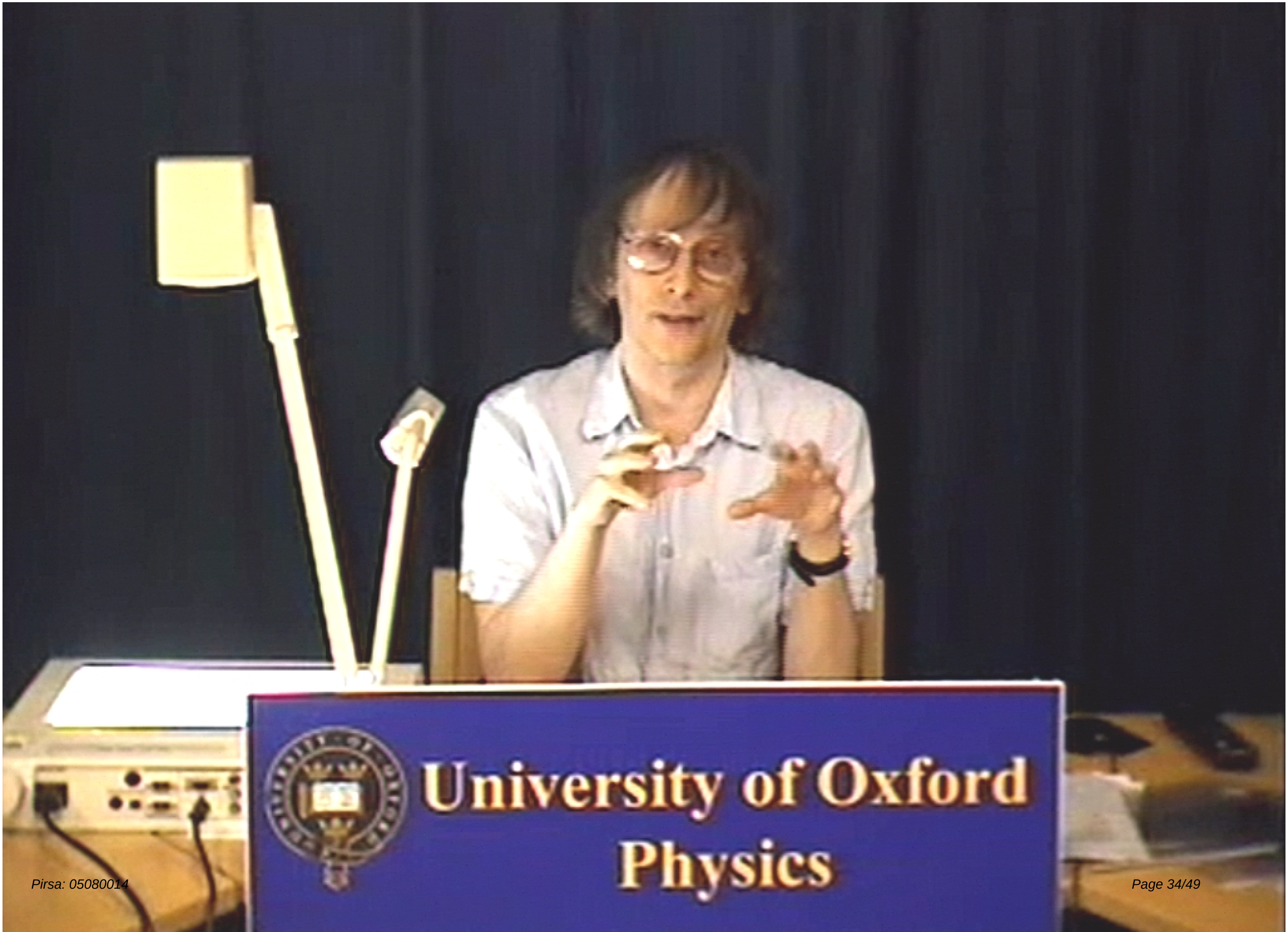
$$\det\left(\lambda_{\alpha}c^{\alpha\beta}{}_{\gamma}\right) = 0$$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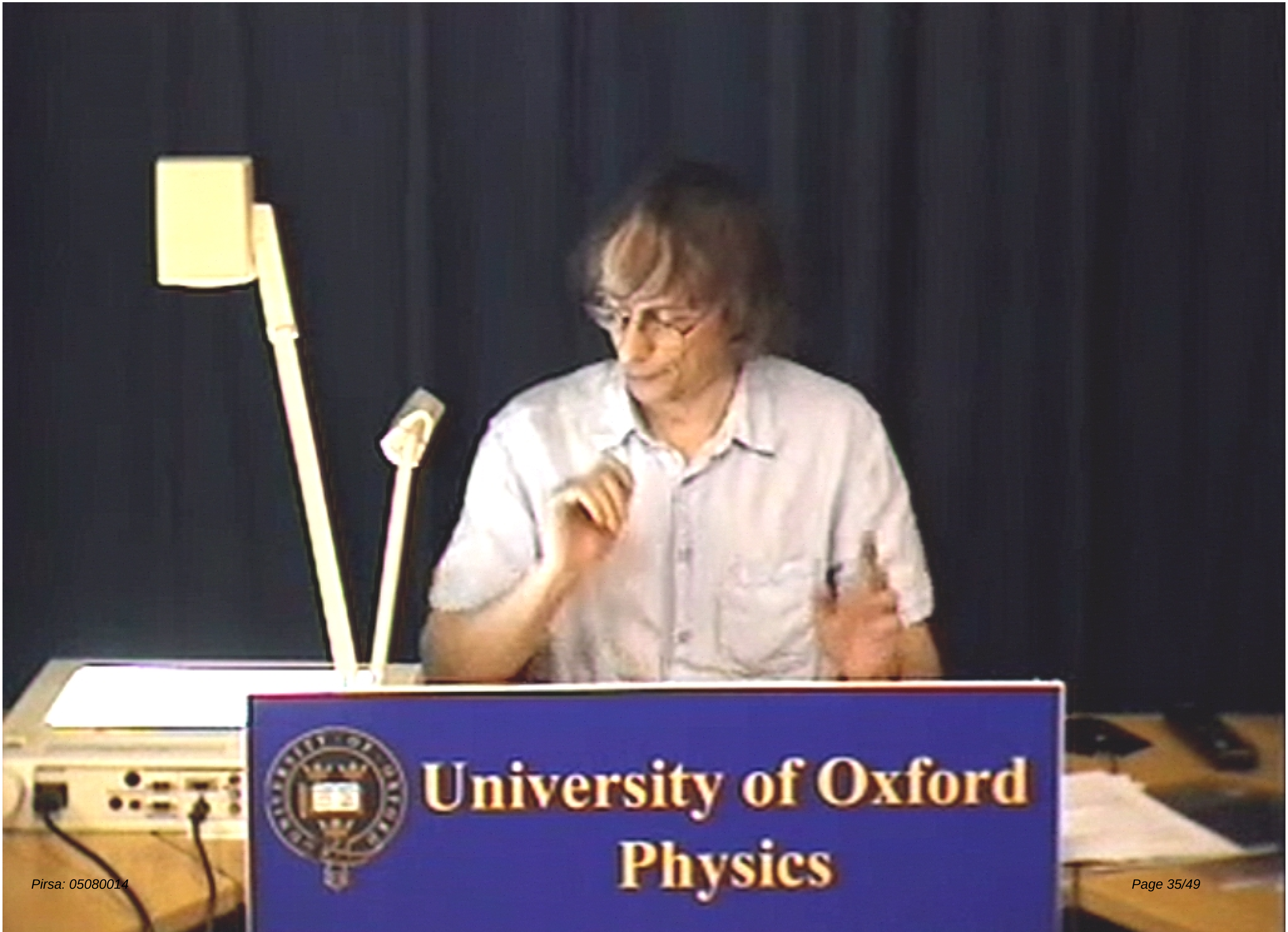
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Density Matrix

As always in quantum theory:

$$\langle \hat{A} \rangle = \mathrm{Tr}\, \hat{A}\hat{\rho}$$

The density matrix is a global q-number constant.

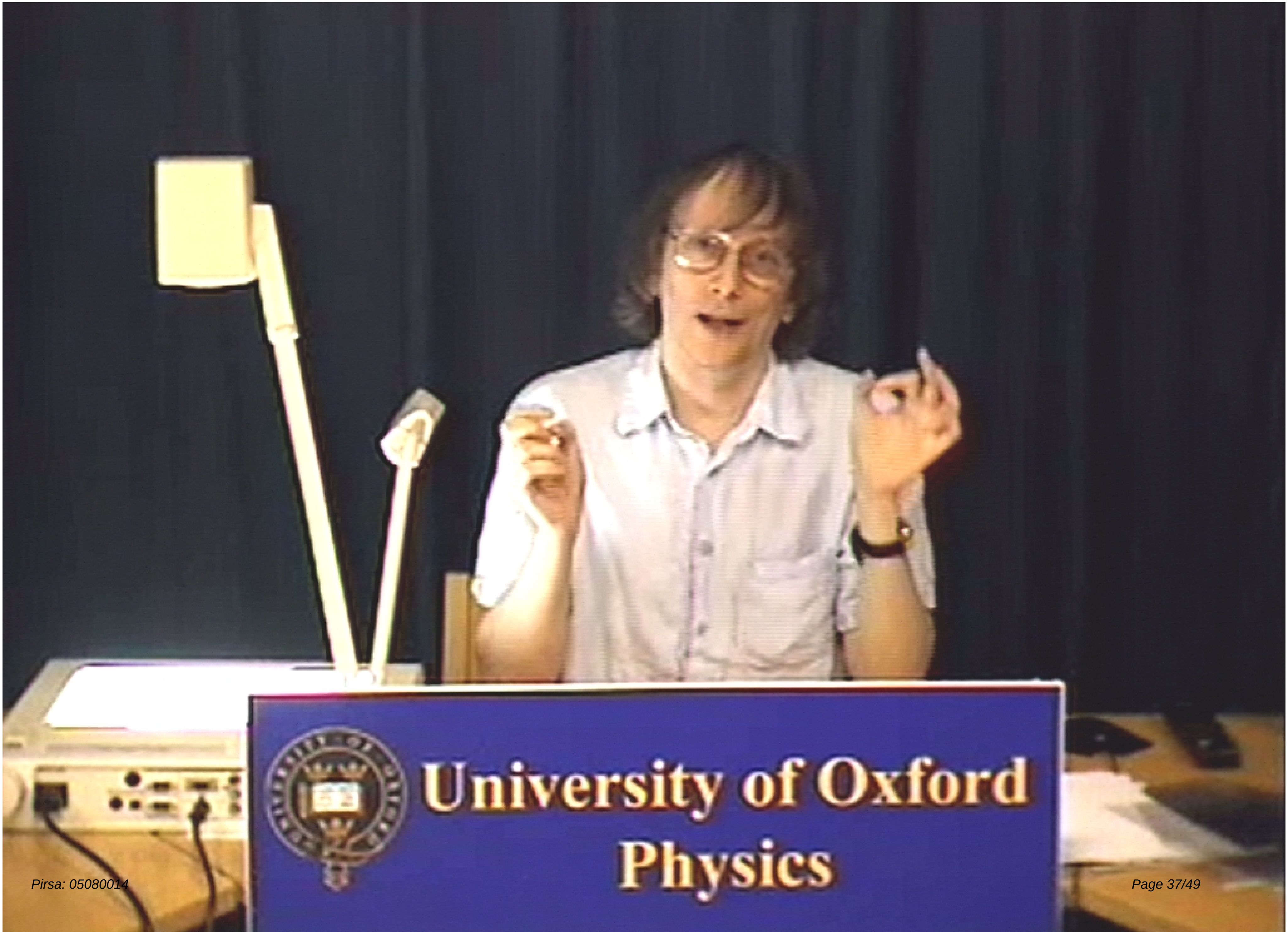
Local density operator:

$$\hat{\rho}(x) \overset{\text{def}}{\leftrightarrow} \left(\mathrm{Tr}_{H/H_x} \hat{\rho}\right) \otimes \mathrm{I}_\perp$$

$$= \tfrac{1}{2}\left(\hat{1} + \langle \hat{q}_j(x) \rangle \hat{q}^j(x)\right)$$

which has the property that for any observable of the qubit at one point x:

$$\langle \hat{A}(x) \rangle = \frac{\mathrm{Tr}\, \hat{A}(x)\hat{\rho}(x)}{\ldots}$$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WHAT HAS CHANGED?

And what hasn't?

OLD THEORY	QUBIT FIELD THEORY
	Has qubits!
Observables at spacelike separations commute	They generally do not commute
Pathological (though we try to make a virtue of this)	Finite (and there are other virtues, to compensate)
Awkward	Nice
Sinister relationship with 'classical physics'. 'Quantisation'	No such relationship; no classical analogue; no 'quantisation'
Incompatible with Bekenstein bound	Compatible?
Observables, expectation values, state	(unchanged)
Dynamics local	Dynamics local
General covariance(?)	General covariance
Heisenberg & Schrödinger pictures equivalent	No Schrödinger picture
Multiverse interpretation	

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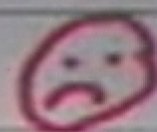
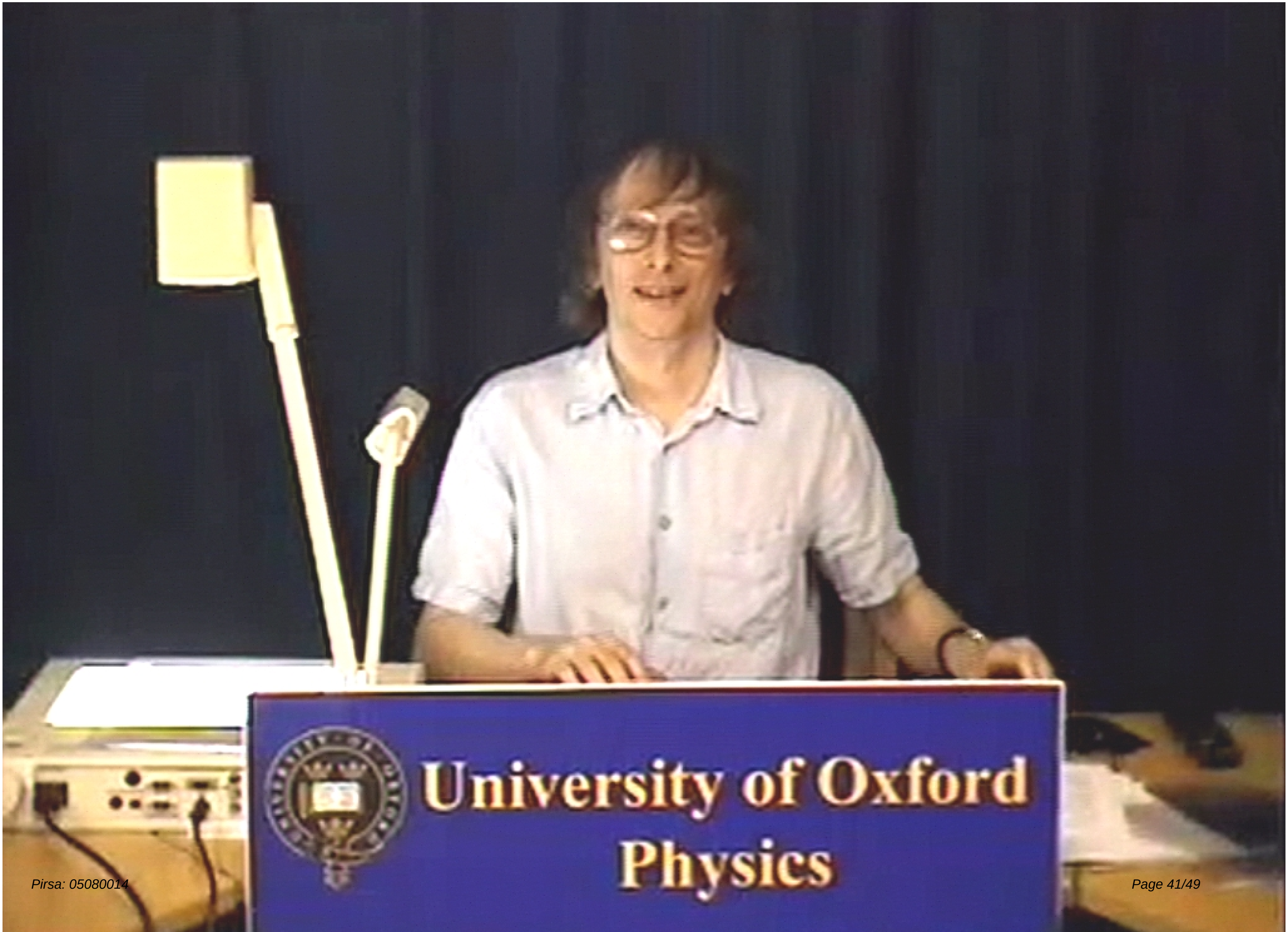
Superbly corroborated by experiment	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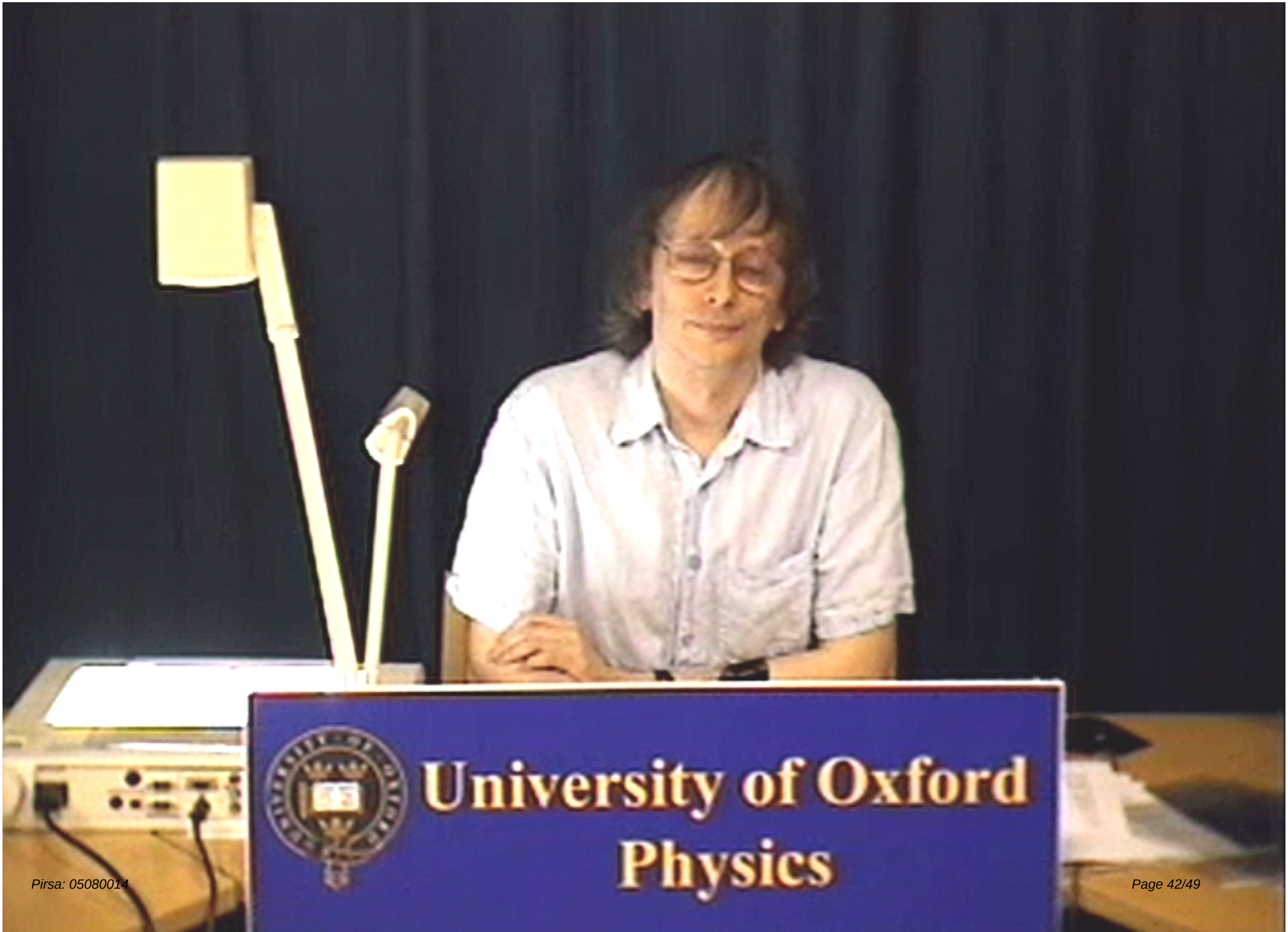
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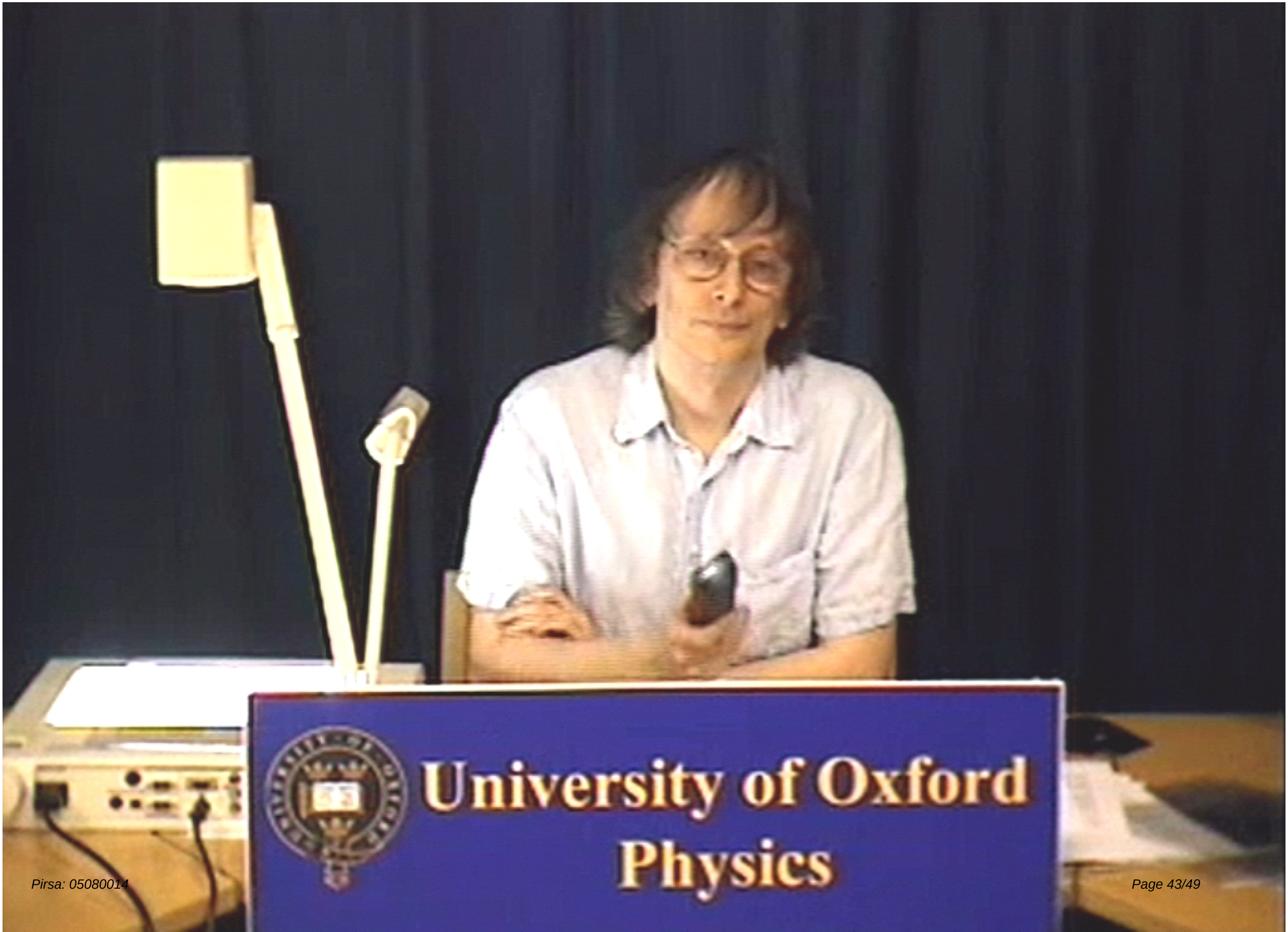
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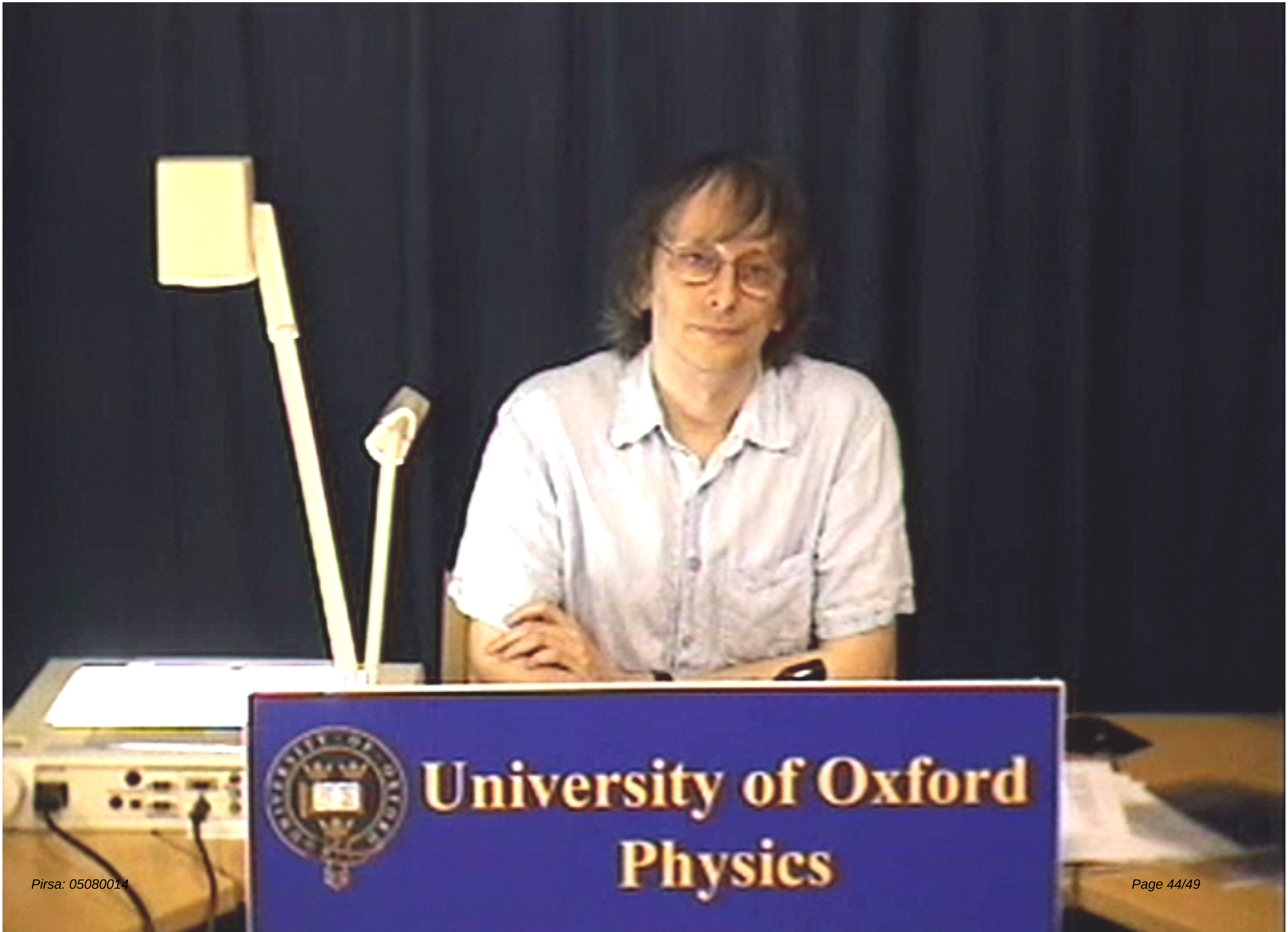
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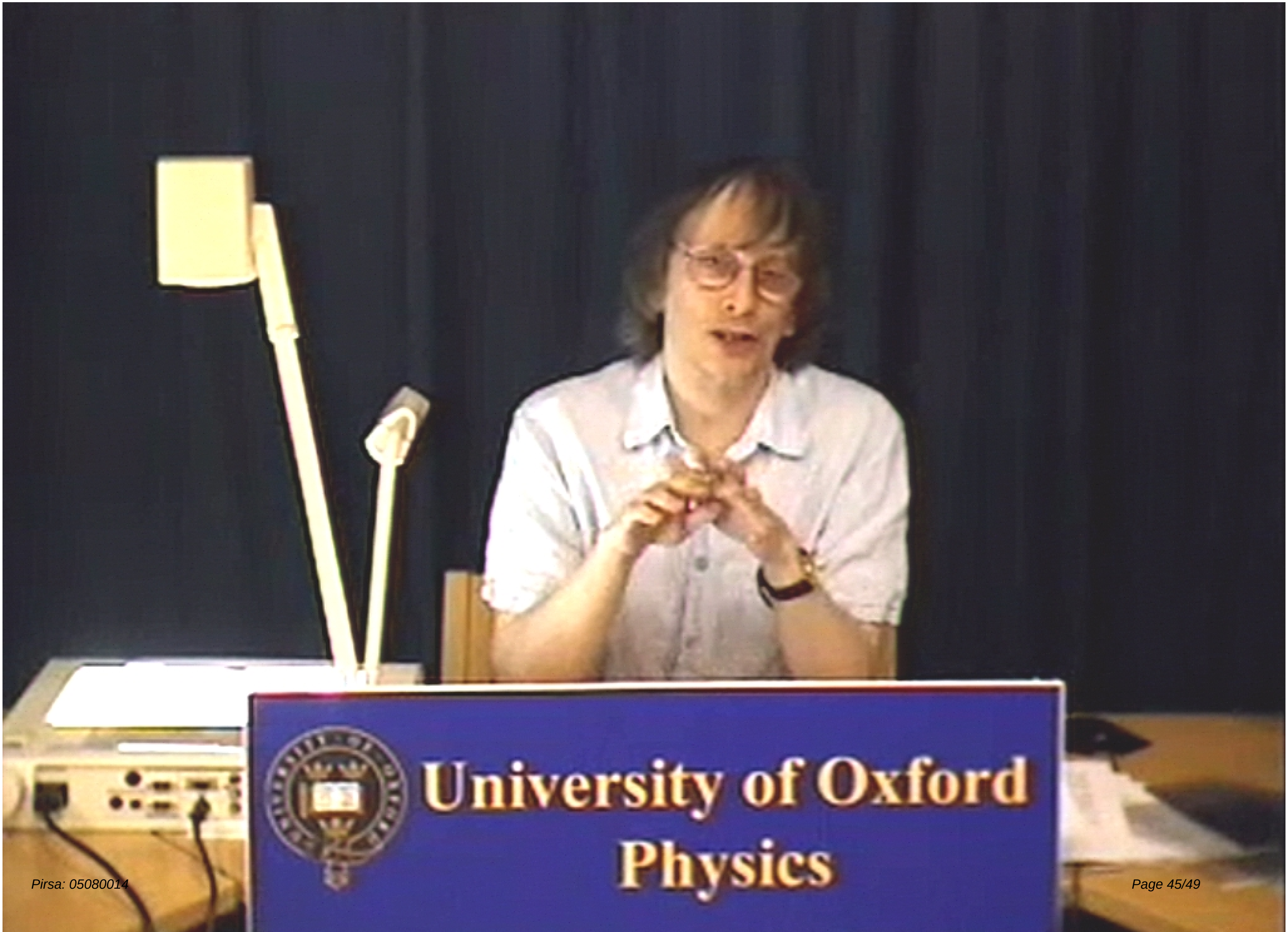
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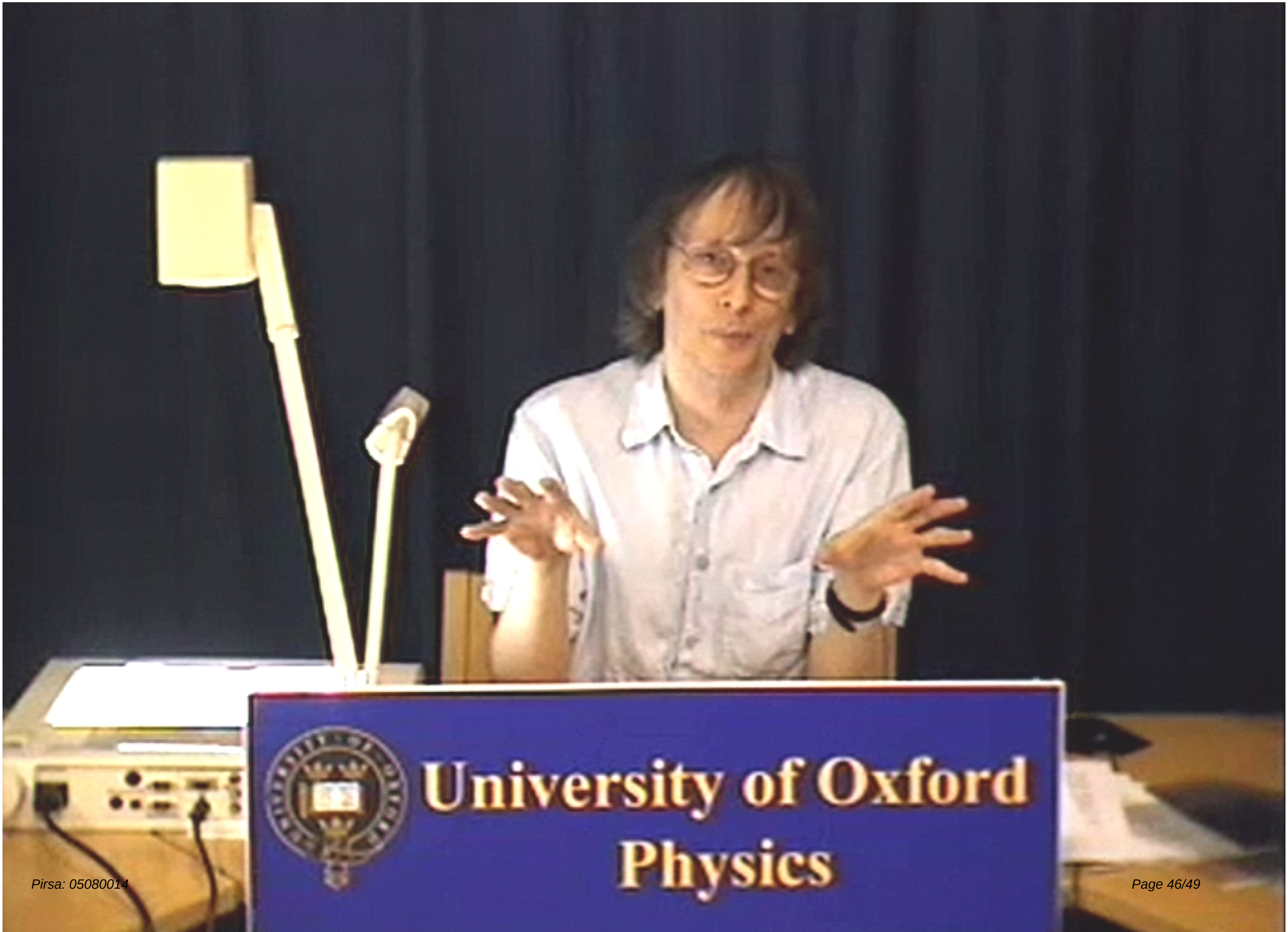
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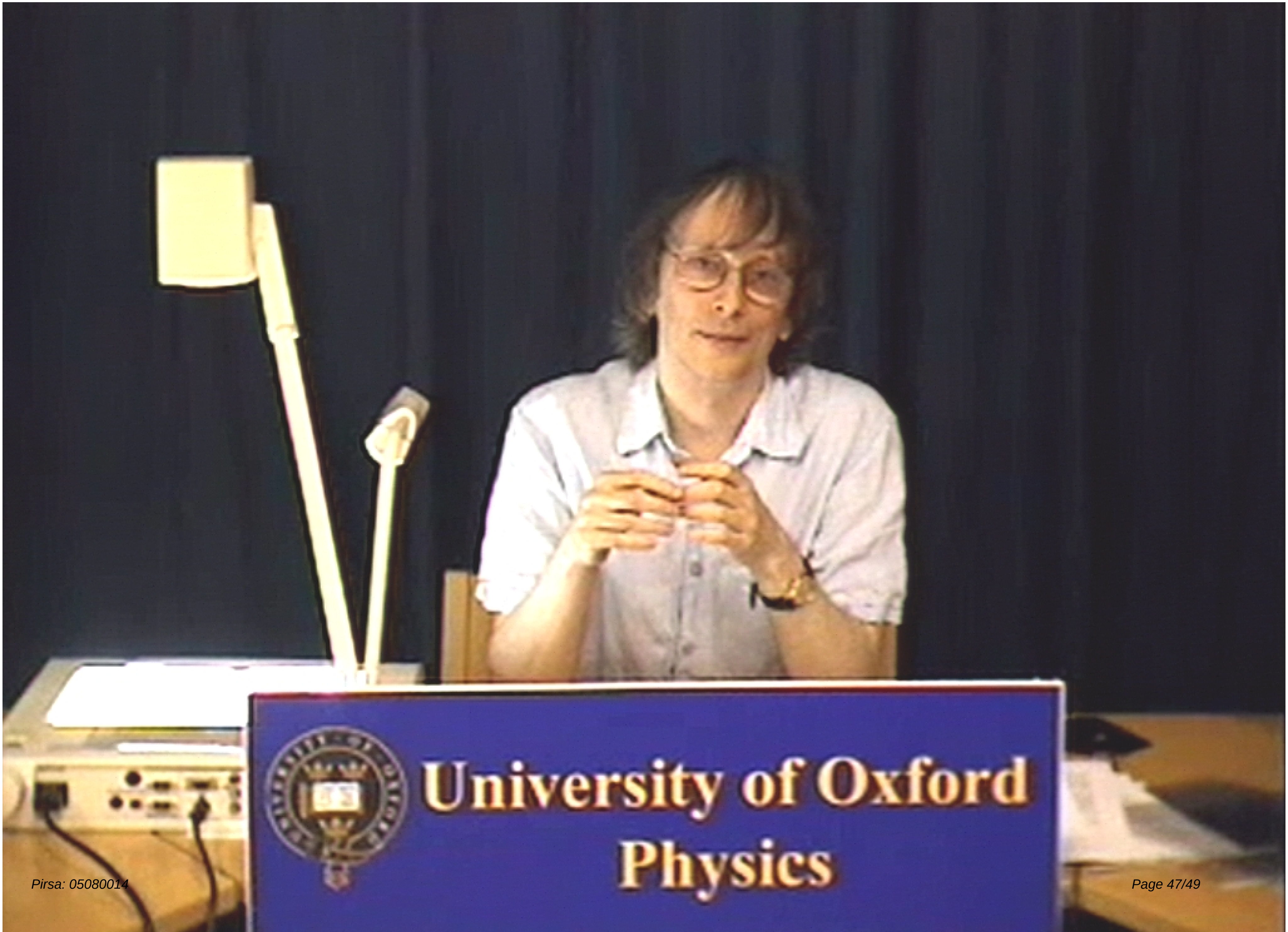
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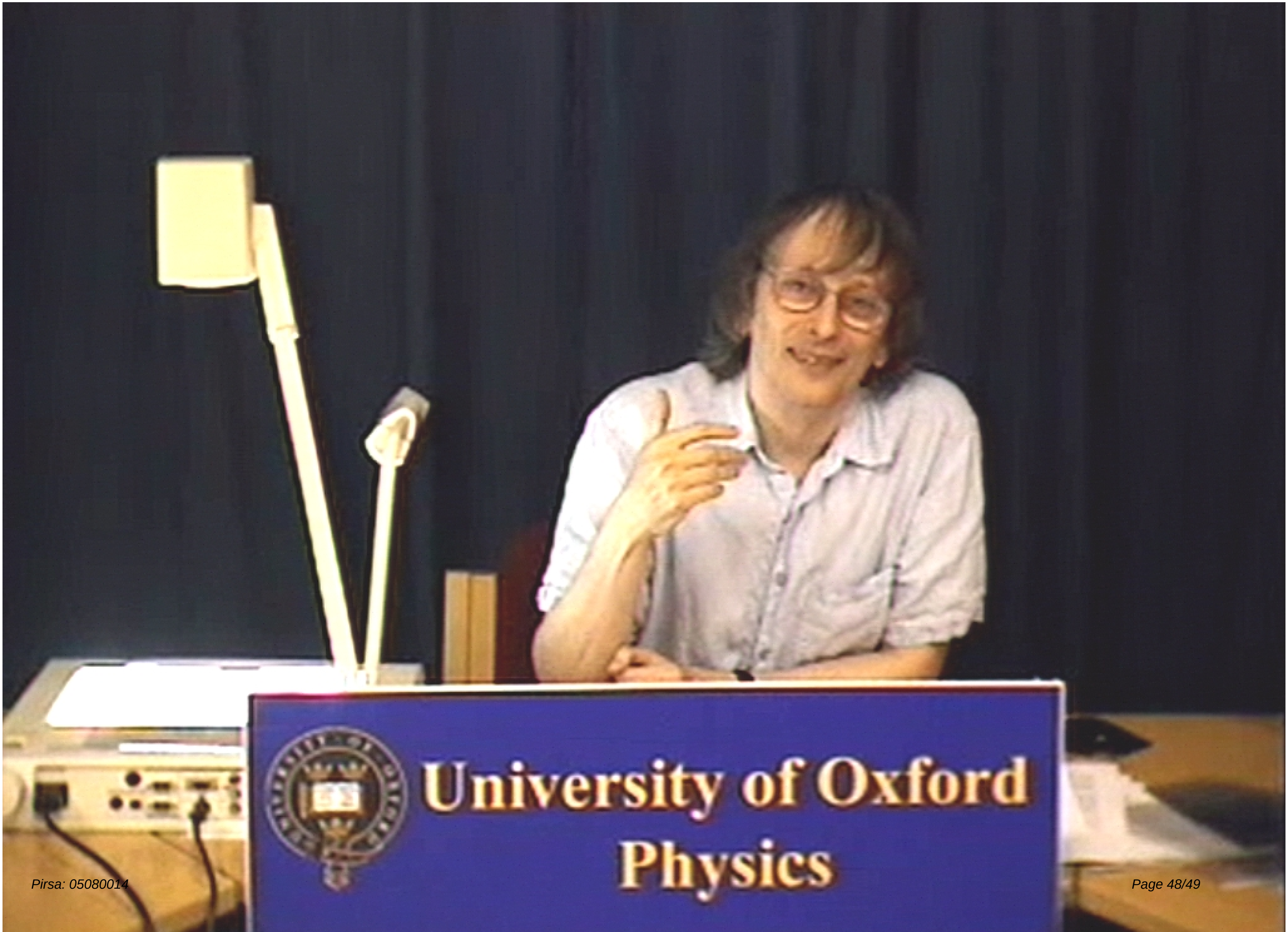
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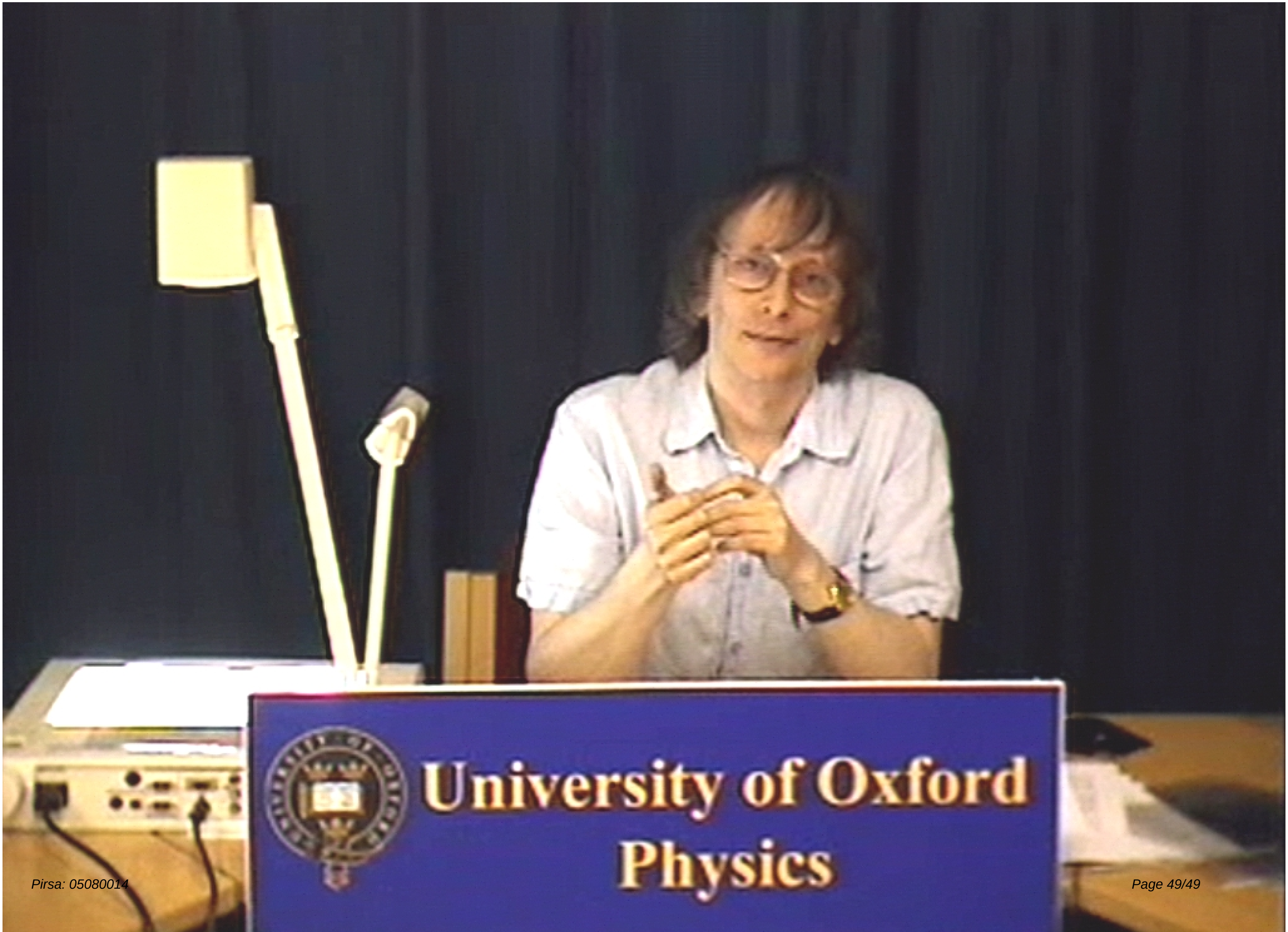
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