Title: Science Fiction Recipe for Teleportation

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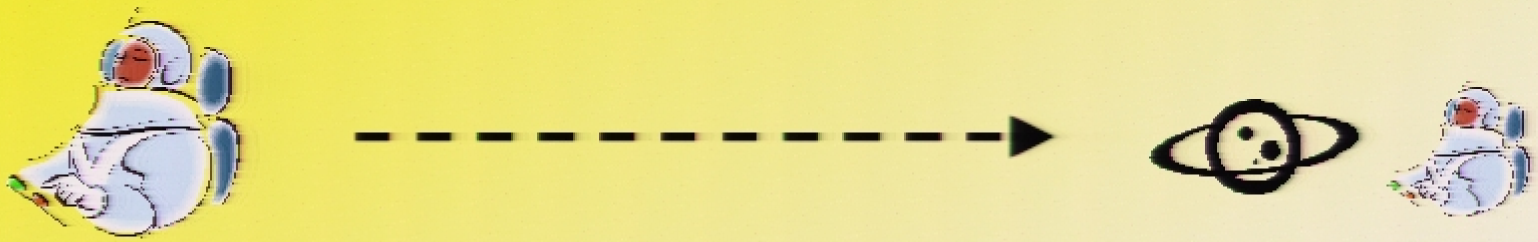
URL: http://pirsa.org/05080007

Abstract:

Science Fiction Recipe for Teleportation



- 1. A machine scans the space explorer to find out everything about her. Eg. her height, her mass, what shoes she is wearing, the colour of her eyes etc.
- 2. It then sends this information to an nearby uncharted planet.
- 3. On the planet's surface, a receiving machine takes in the information & uses it to construct a perfect copy of the astronaut.

- *Essence of teleporting: constructing a perfect copy of an object at a distant location without sending the object itself*

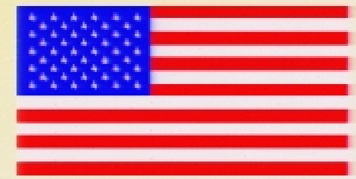
- A problem: Heisenberg's uncertainty principle prevents us from knowing everything about an object

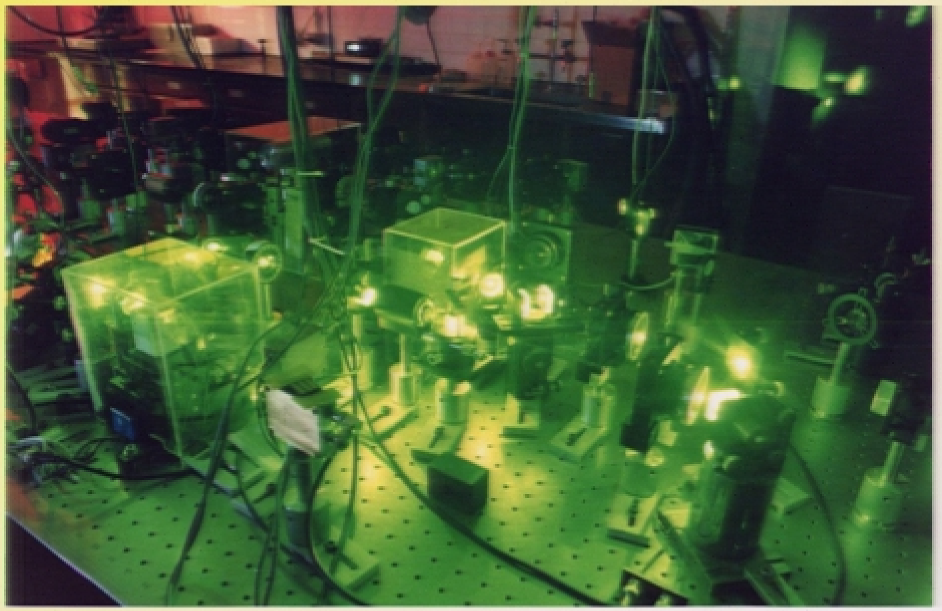
- Quantum teleportation circumvents this challenge

- KEY POINT: We do not actually teleport the object itself.

- teleport *its properties* & so get a perfect copy of it

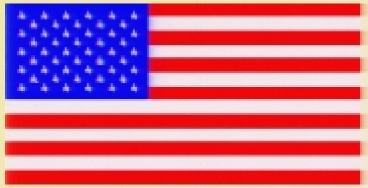
1998 & 2002: Caltech, USA & Canberra, Australia: A laser beam (About 50 cms)

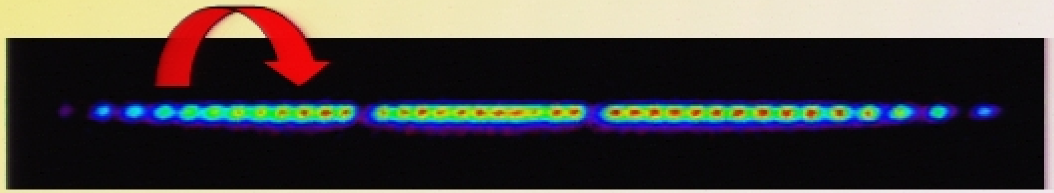




"Beam me up, mate" --- CNN

2004: Innsbruck, Austria & Boulder, Colarado, USA
teleporting atoms: c. 1 mm



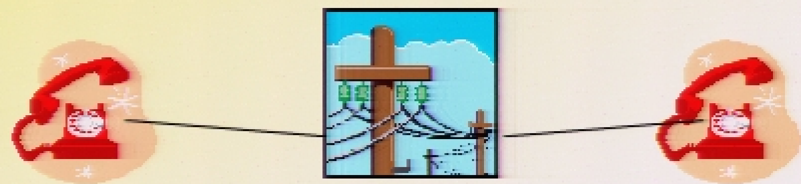


- A problem: Heisenberg's uncertainty principle prevents us from knowing everything about an object
- Quantum teleportation circumvents this challenge
- KEY POINT: We do not actually teleport the object itself.
- teleport *its properties* & so get a perfect copy of it

Quantum teleportation in three easy steps

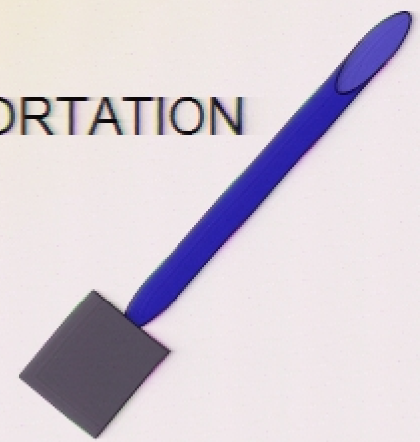
- To transmit something from A to B, we always need a means or route by which to send it.

Eg. a phone line, a wireless connection



- To teleport the properties of a laser beam from A to B, we use two routes (a.k.a. *channels*).

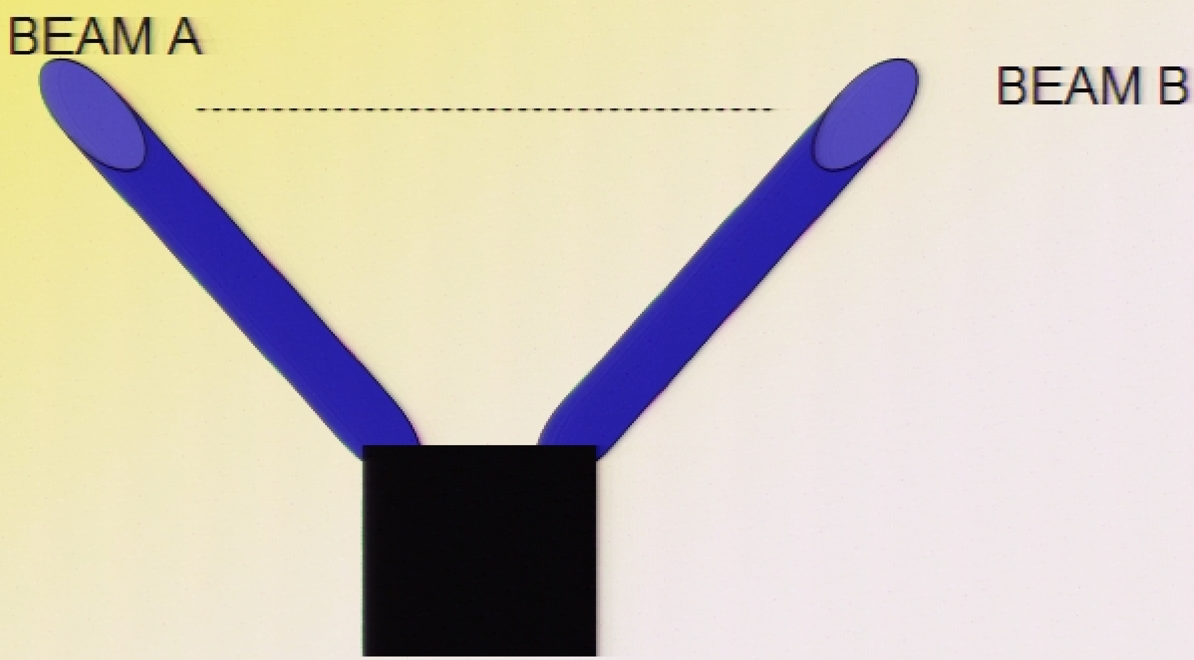
TELEPORTATION
BEAM



1st CHANNEL

- any standard means of communication, such as a telephone.

- 2^{nd} CHANNEL
- Two laser beams sharing entanglement.



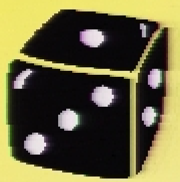
- WHAT IS ENTANGLEMENT?

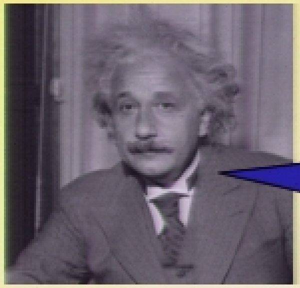
- If two quantum particles (or beams) are entangled with each other then they are instantaneously linked no matter how far apart they are

- Act as if they are a single object

- *"Entanglement means that the left hand knows what the right hand is doing even when the hands at opposite ends of the universe."*

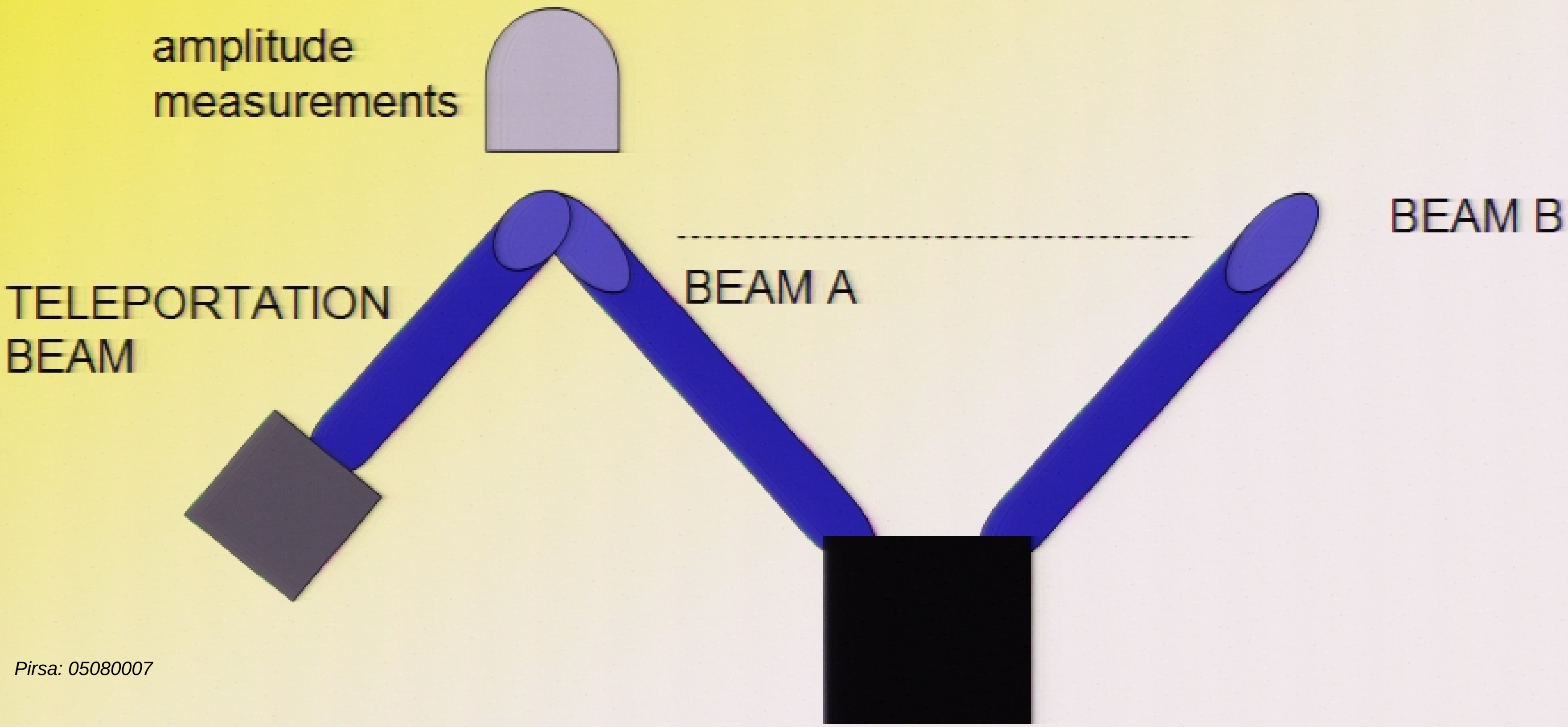
- If two dice were entangled then they would always roll the same numbers if thrown at the same time at opposite ends of the universe

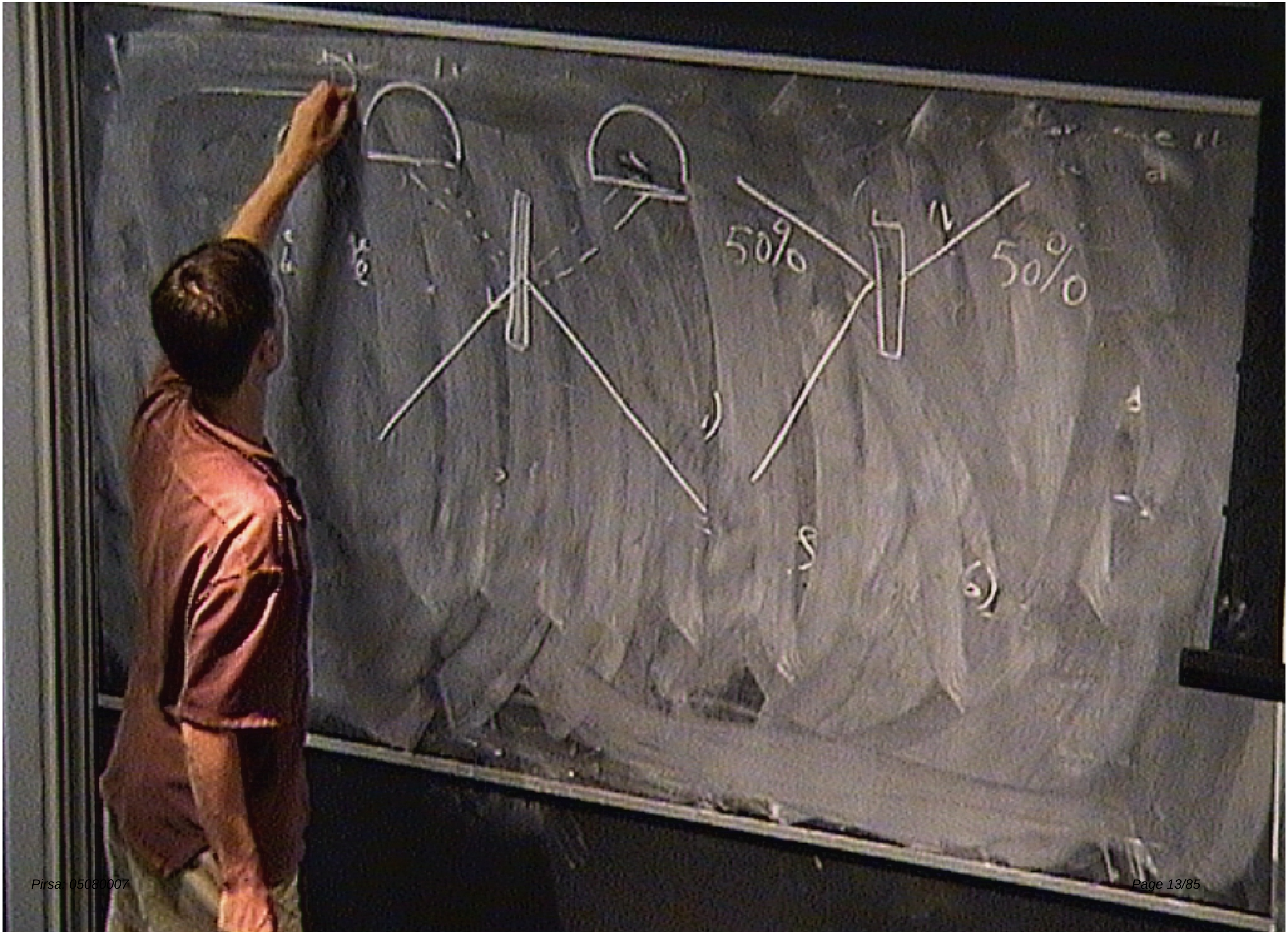




"[Entanglement is] spooky action at a distance."

- STEP 1: Combine the beam we wish to teleport with beam A & then measure two components of the overall amplitude




50%
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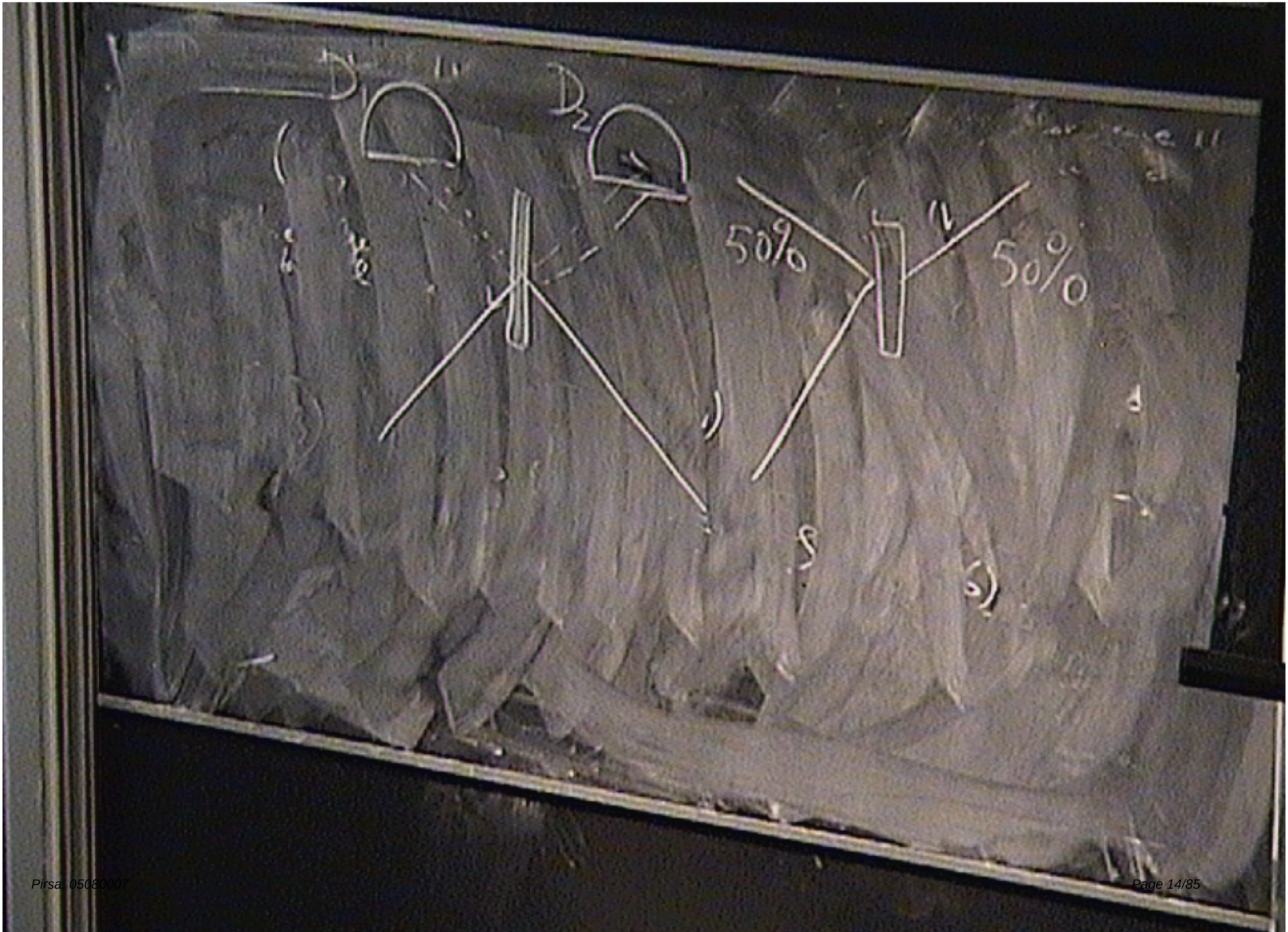

D_1
D_2
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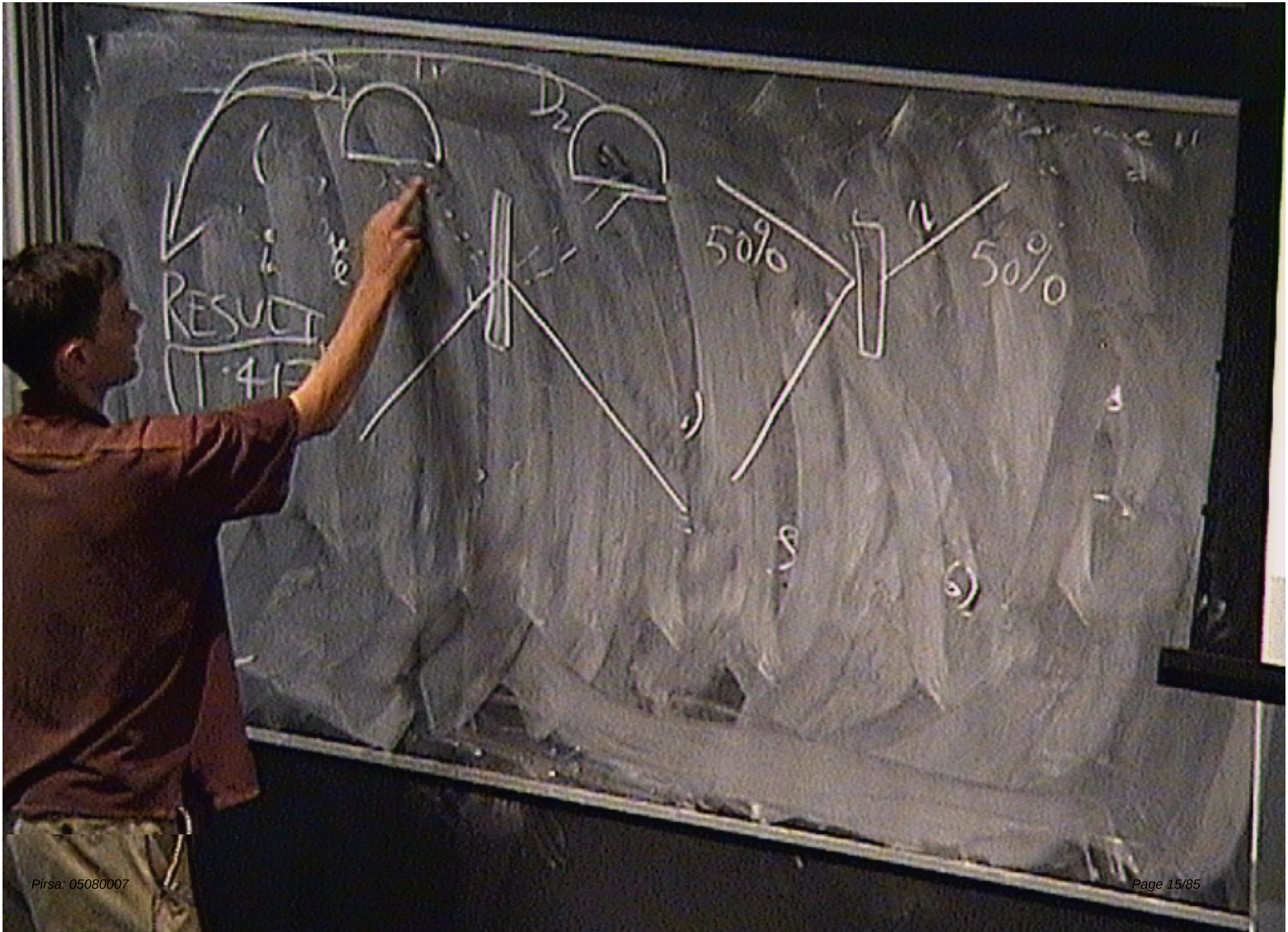
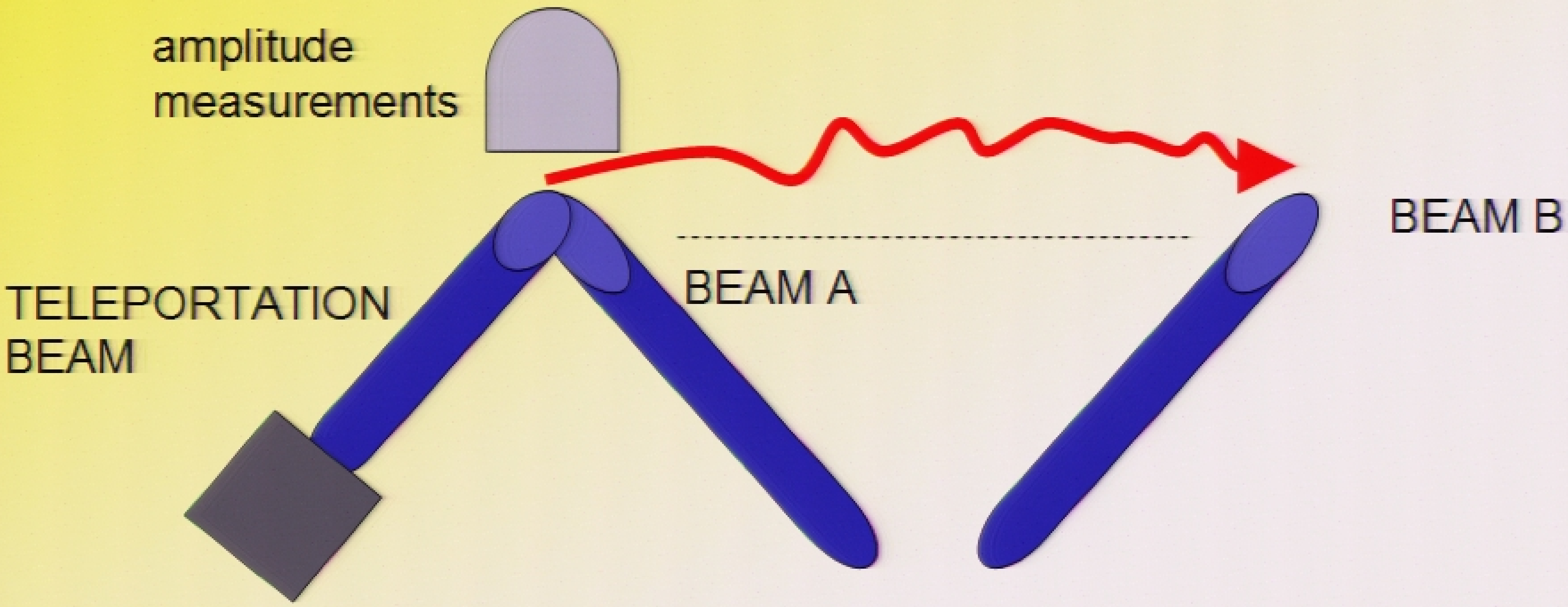
50%

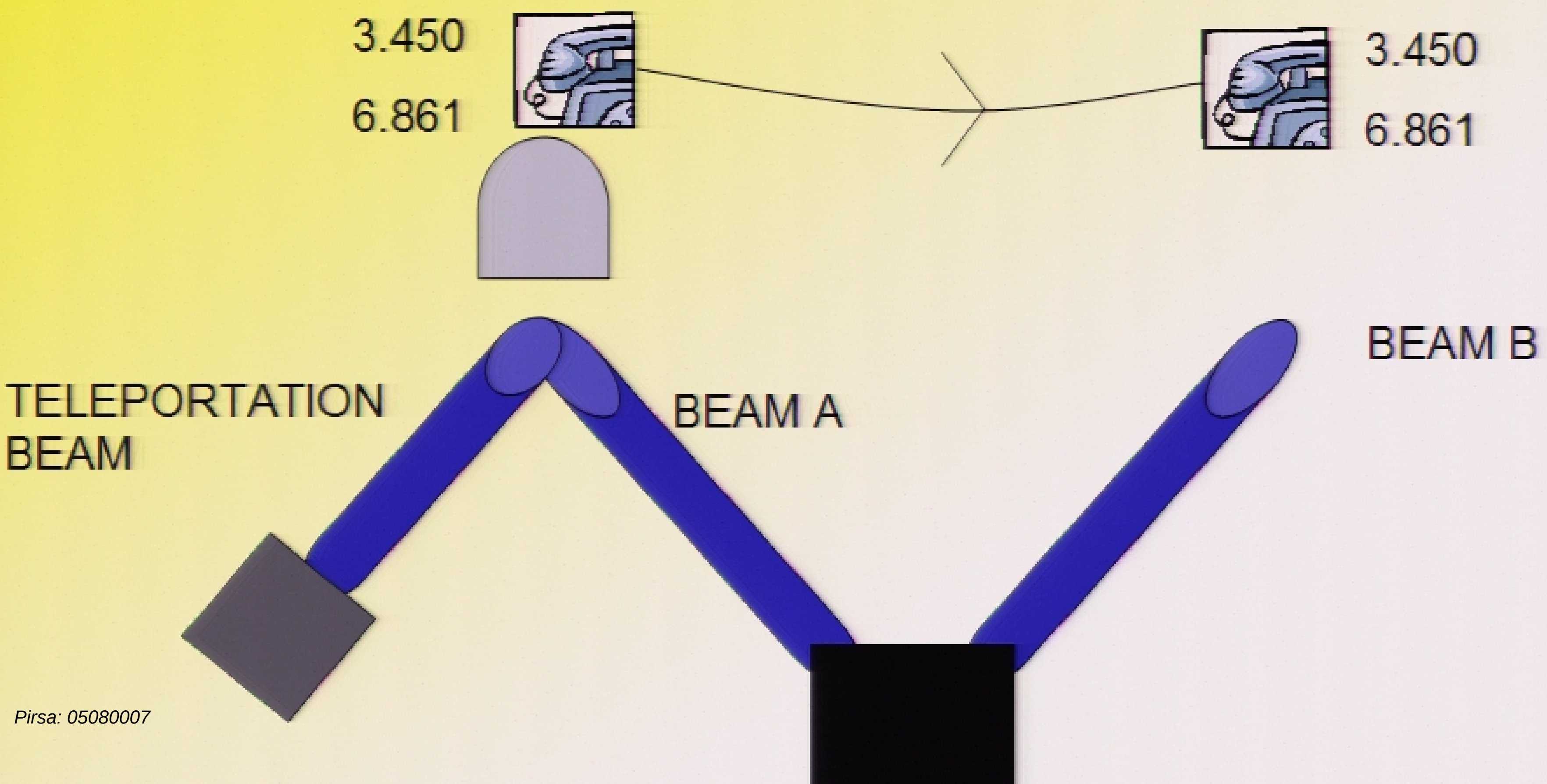


D_1
D_2
50%
50%
RESULT

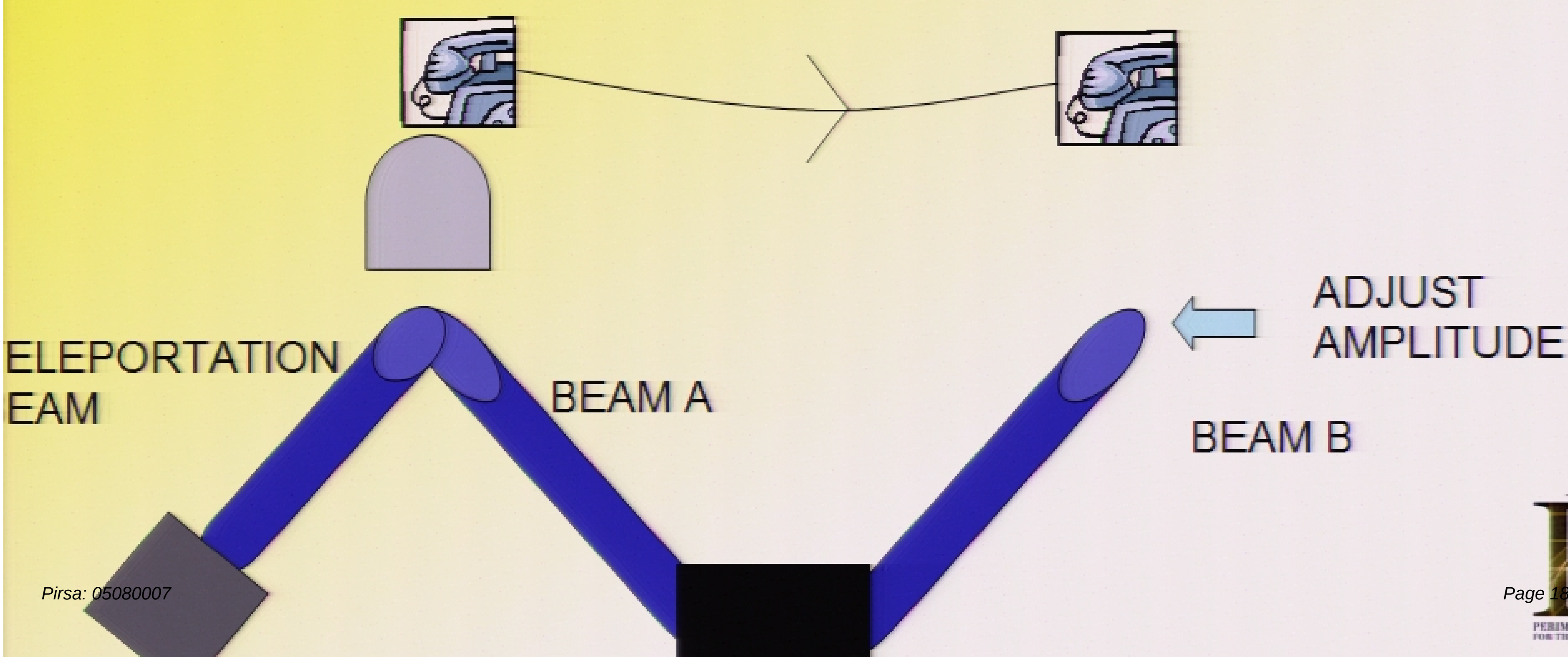
- A natural way to think of this is to say that it instantaneously (nonlocally) ‘zaps’ information about the teleportation beam to beam B.



- STEP 2: Use the 1st channel to send the result of the amplitude measurements to B.



- STEP 3: Adjust the amplitude of beam B, depending on the information received. As if by magic, beam B is now identical to the beam we initially wished to teleport. The teleportation is complete!



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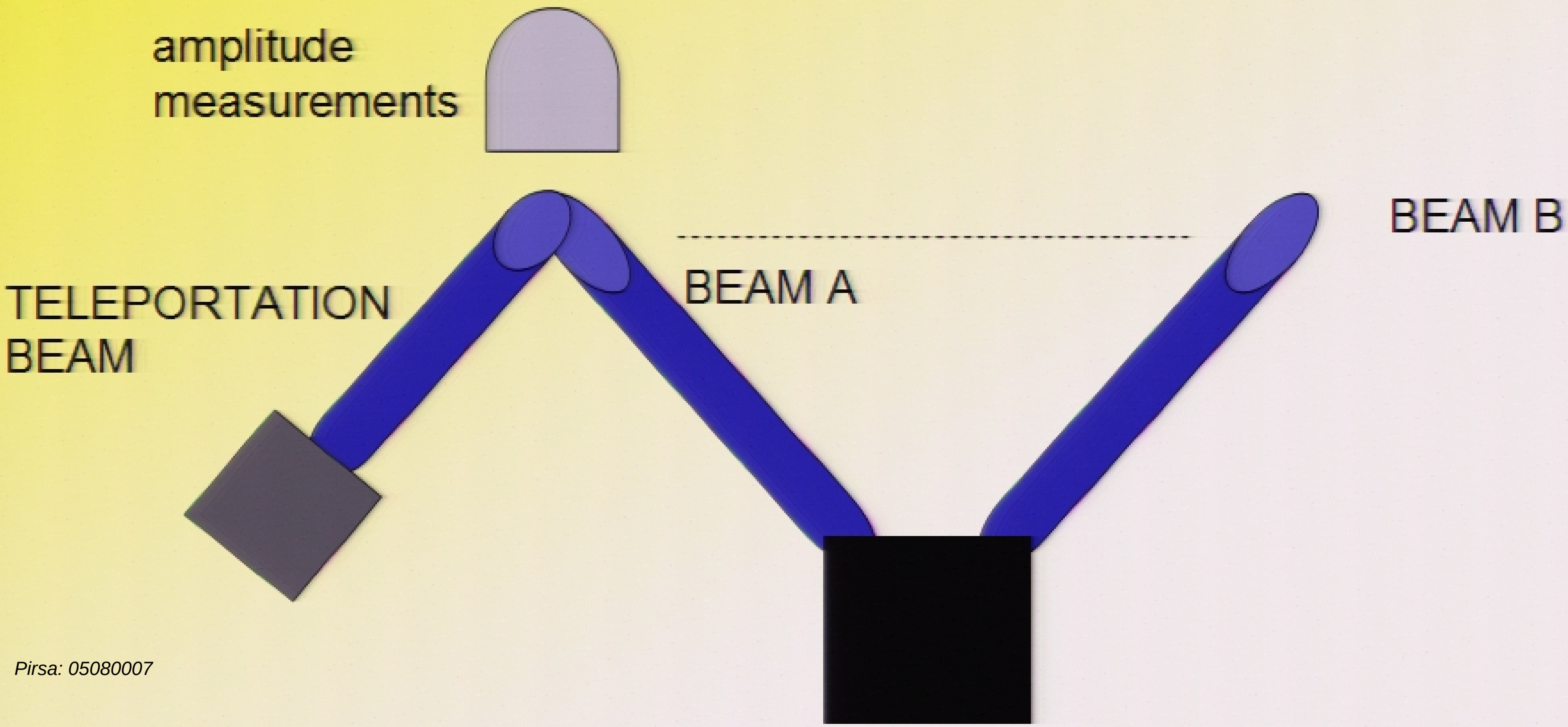

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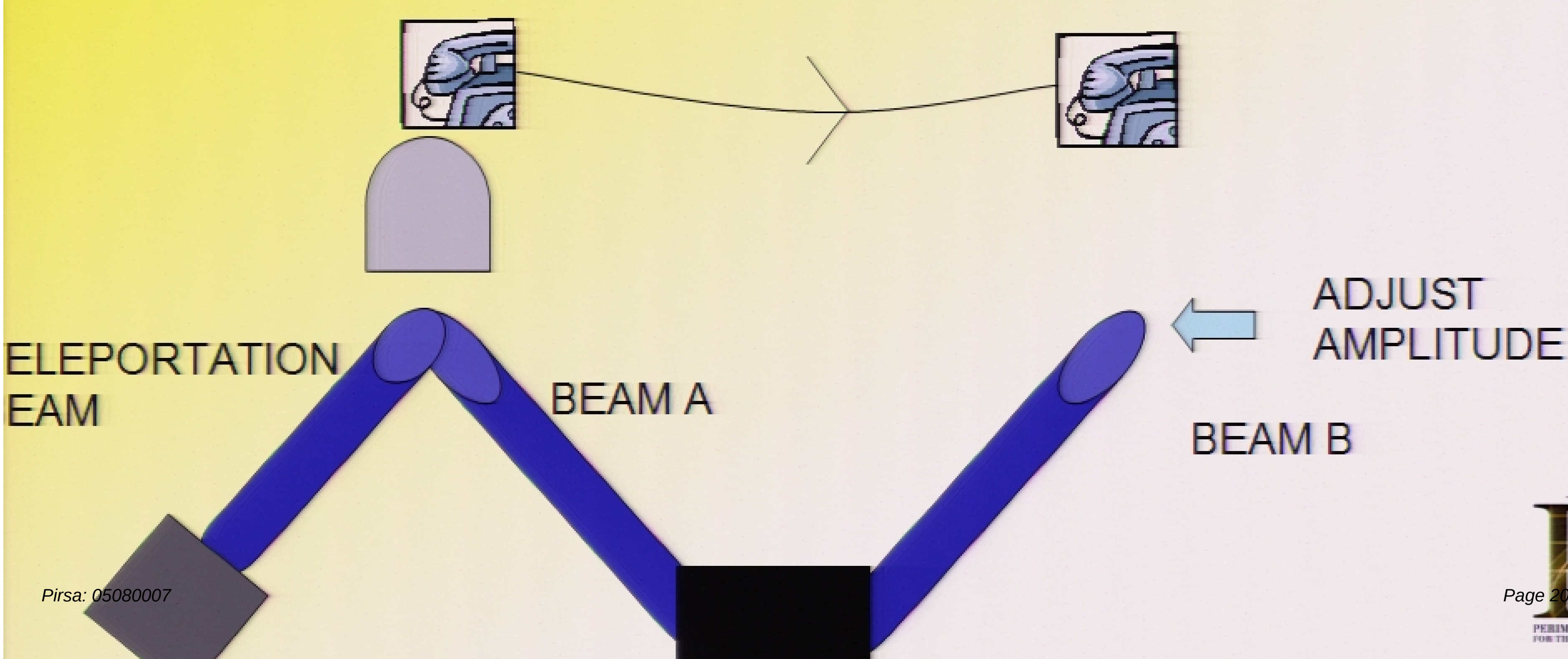

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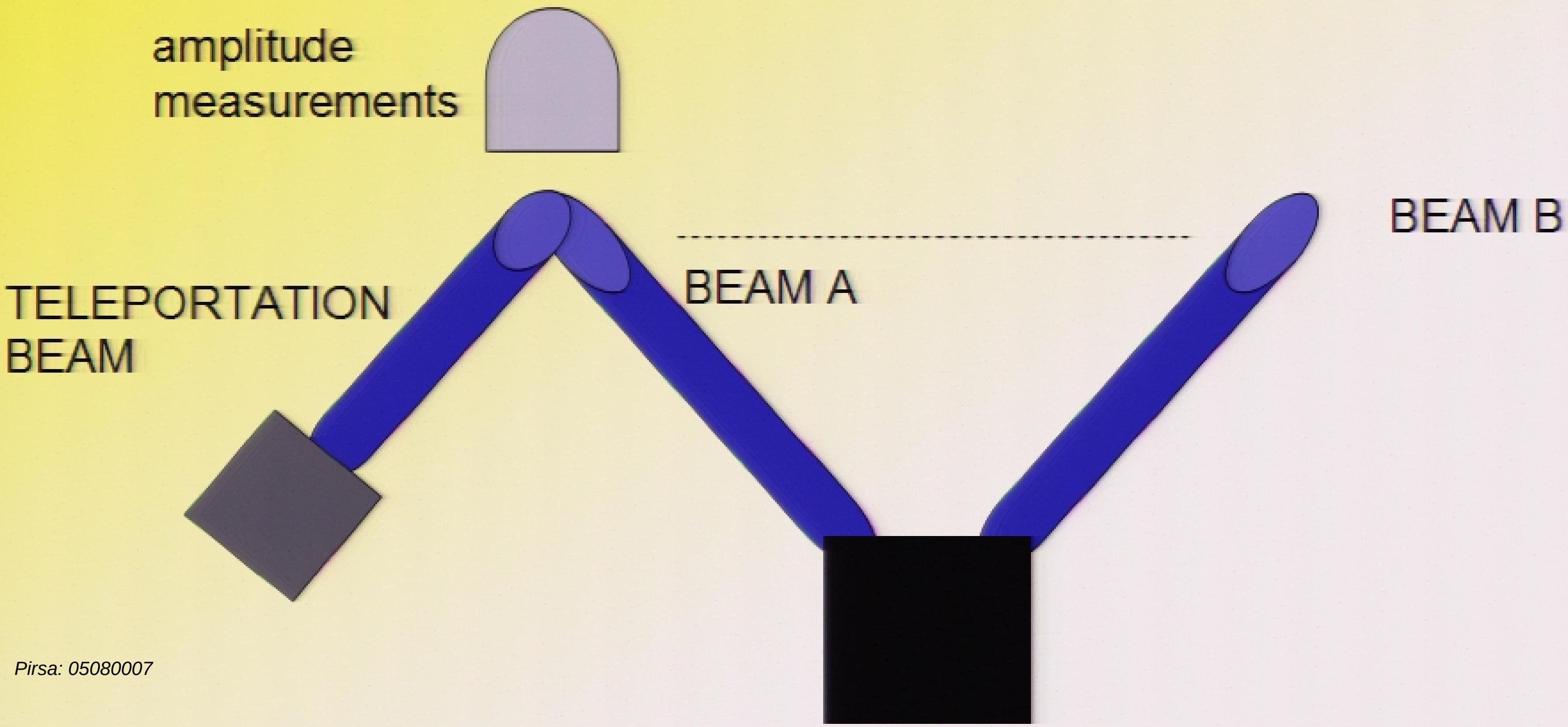

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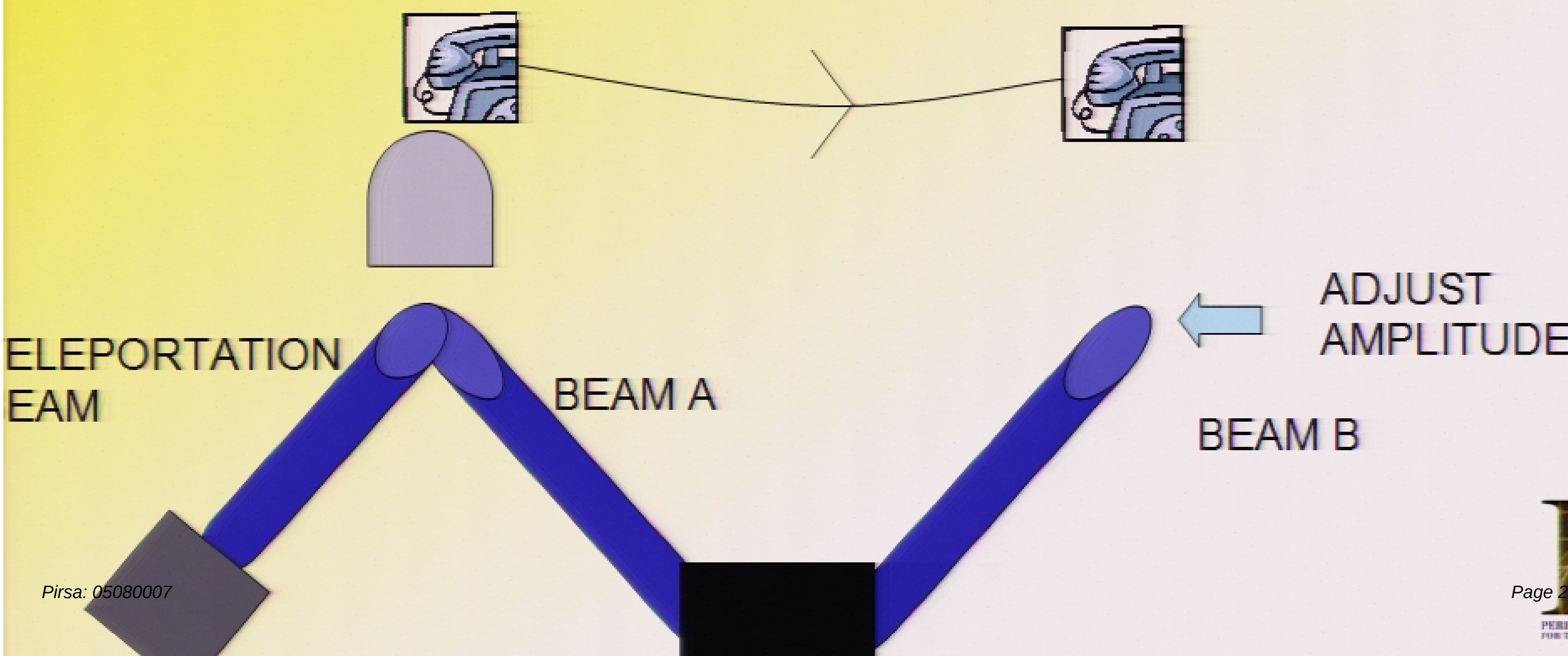
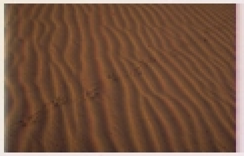


A Comparison

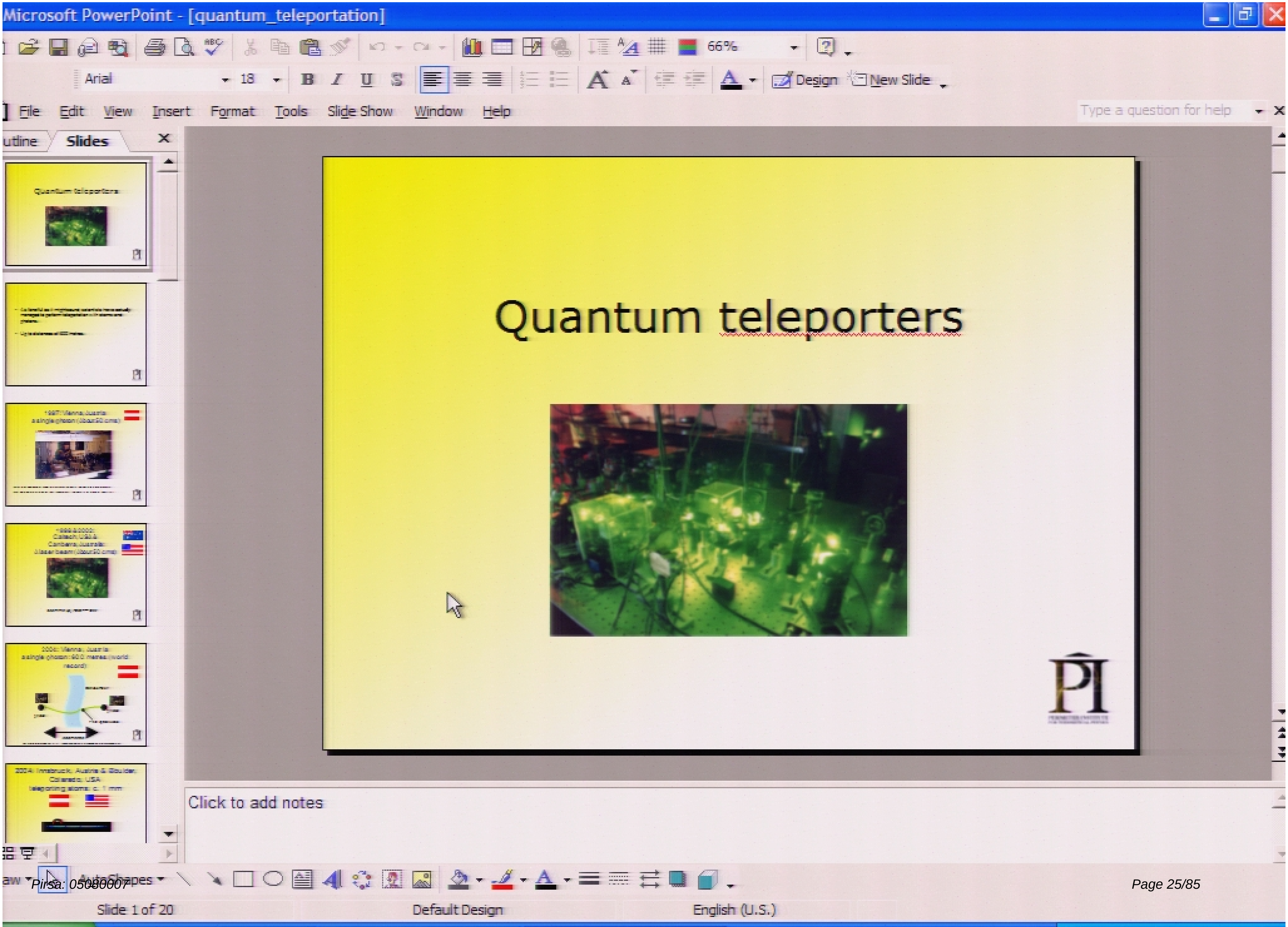
Star-Trek Teleportation	Quantum Teleportation
	
Instantaneous?	Takes a small amount of time
The object itself is teleported.	Only the structure or properties are teleported (i.e. information)
Done with people.	Done with photons & atoms
The original is destroyed in the process.	The original is destroyed in the process
Can teleport to an uncharted planet.	Need to have something set up at the teleportation destination.
Done over thousands of kilometres.	Done over 600 metres at most

What might we teleport in the future?

- Teleporting a grain of sand? One hundred years, perhaps.



- Teleporting a person? Who knows?


Microsoft PowerPoint - [quantum_teleportation]
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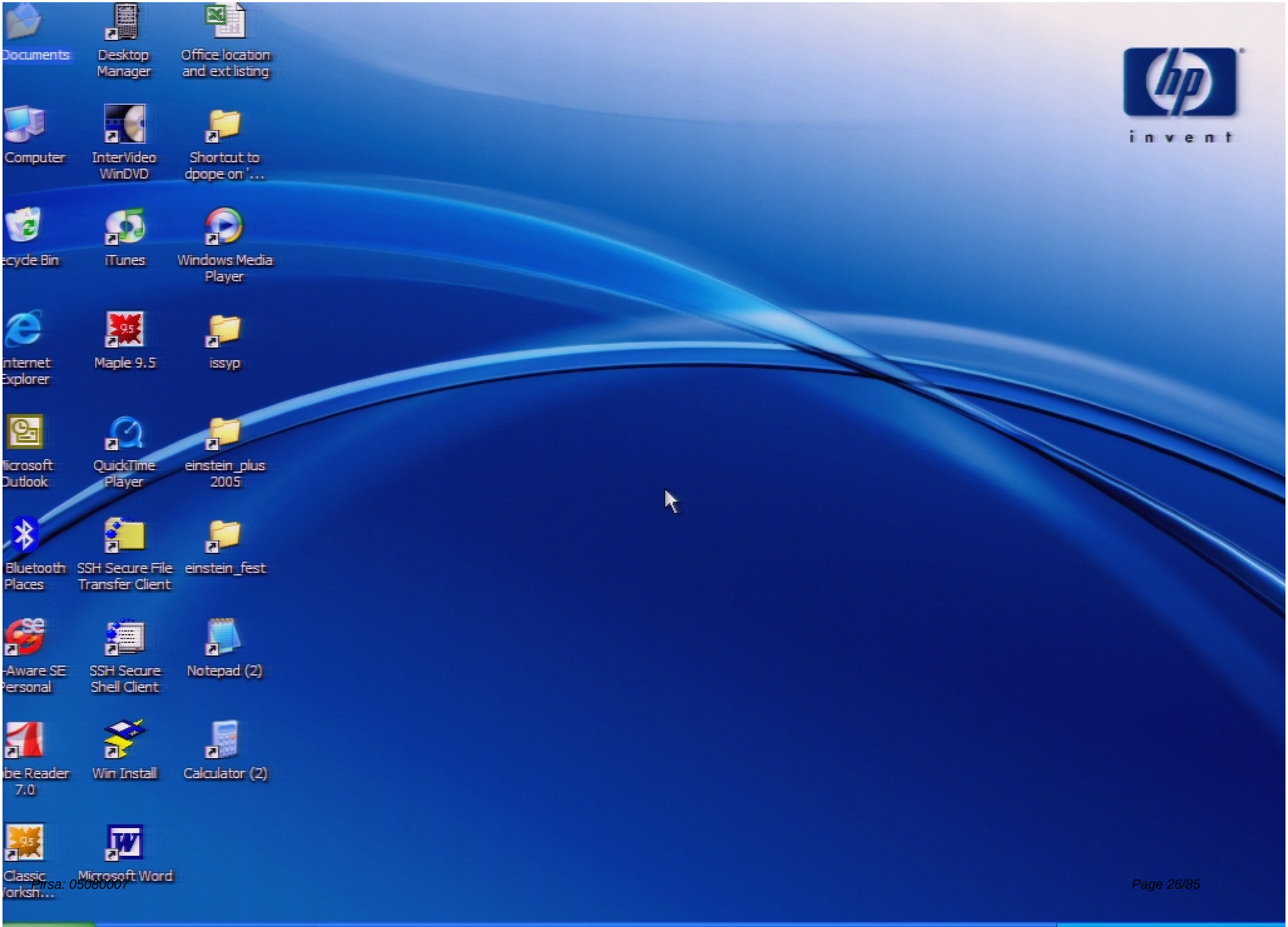
Type a question for help
Quantum teleporters
Click to add notes
Slide 1 of 20
Default Design
English (U.S.)


Documents
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Windows Media Player
nternet Explorer
Maple 9.5
issyp
licrosoft Outlook
QuickTime Player
einstein_plus 2005
Bluetooth Places
SSH Secure File Transfer Client
einstein_fest
-Aware SE Personal
SSH Secure Shell Client
Notepad (2)
be Reader 7.0
Win Install
Calculator (2)
Classic Vorksh...
Microsoft Word
hp
i n v e n t

... for spin-½

End($\mathbb{C}^2$)

$\rho(L_i) = \frac{i}{2}\sigma_i$ ..

$\begin{pmatrix} \cdot & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ "Pauli-matri...

$[\rho(L_i), \rho(L_j)] = \sum_k \epsilon_{ijk}\, \rho(L_k)$.

...ectors. $\vec{q} \longrightarrow \exp(a\, J_3)\, \vec{q}$ $a \in 2\pi\mathbb{Z}$

...rs $\xi \longrightarrow \exp(a\, \tfrac{i}{2}\sigma_3)\, \xi$ $a \in 4\pi\mathbb{Z}$

$i^2 = -1$ i ~~$\sqrt{-1}$~~

CM: Hamiltonian $H = \frac{p^2}{2m} + V(q)$

$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial q}\right)^2 \ldots$

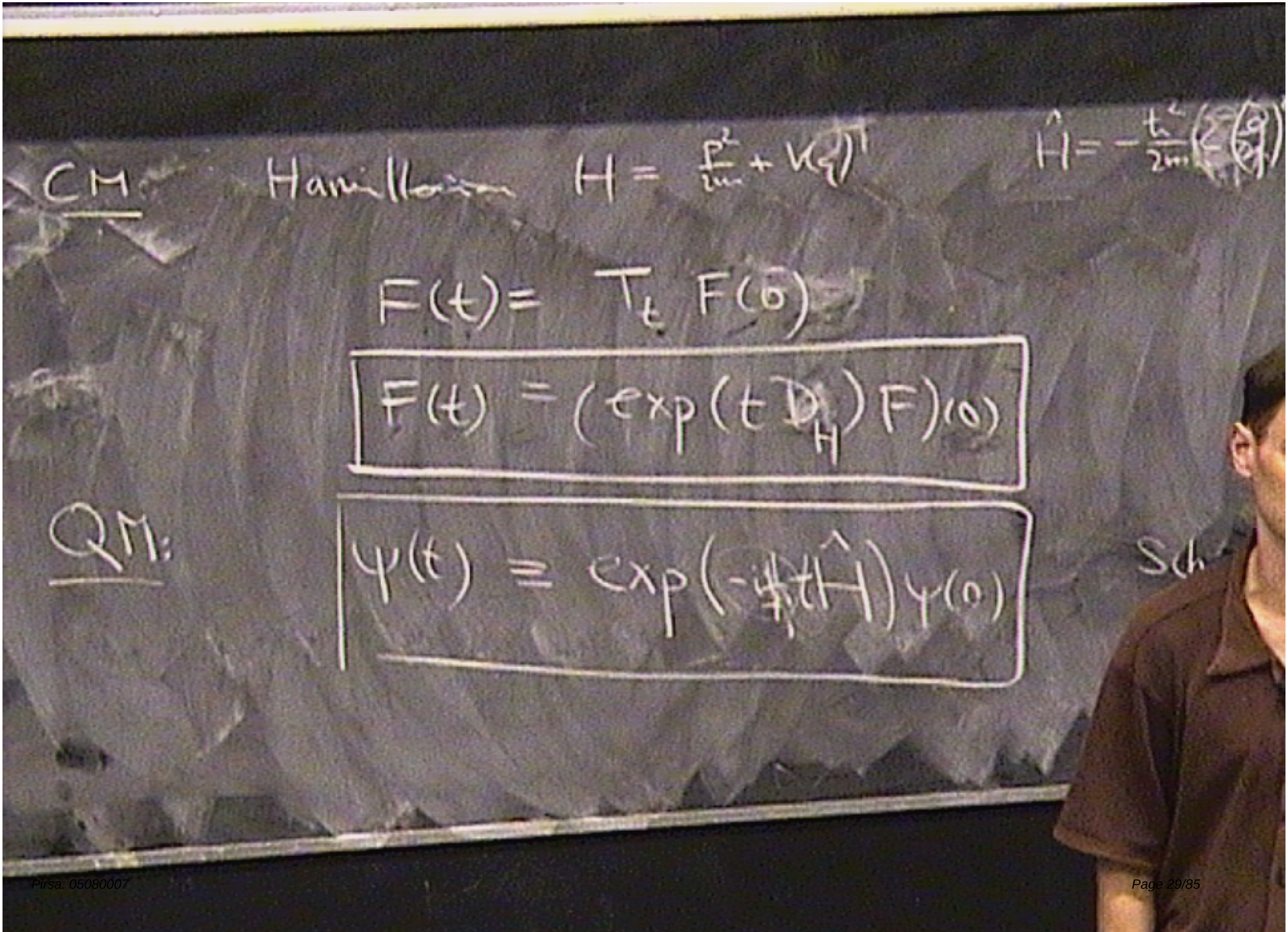
$$F(t) = T_t F(0)$$

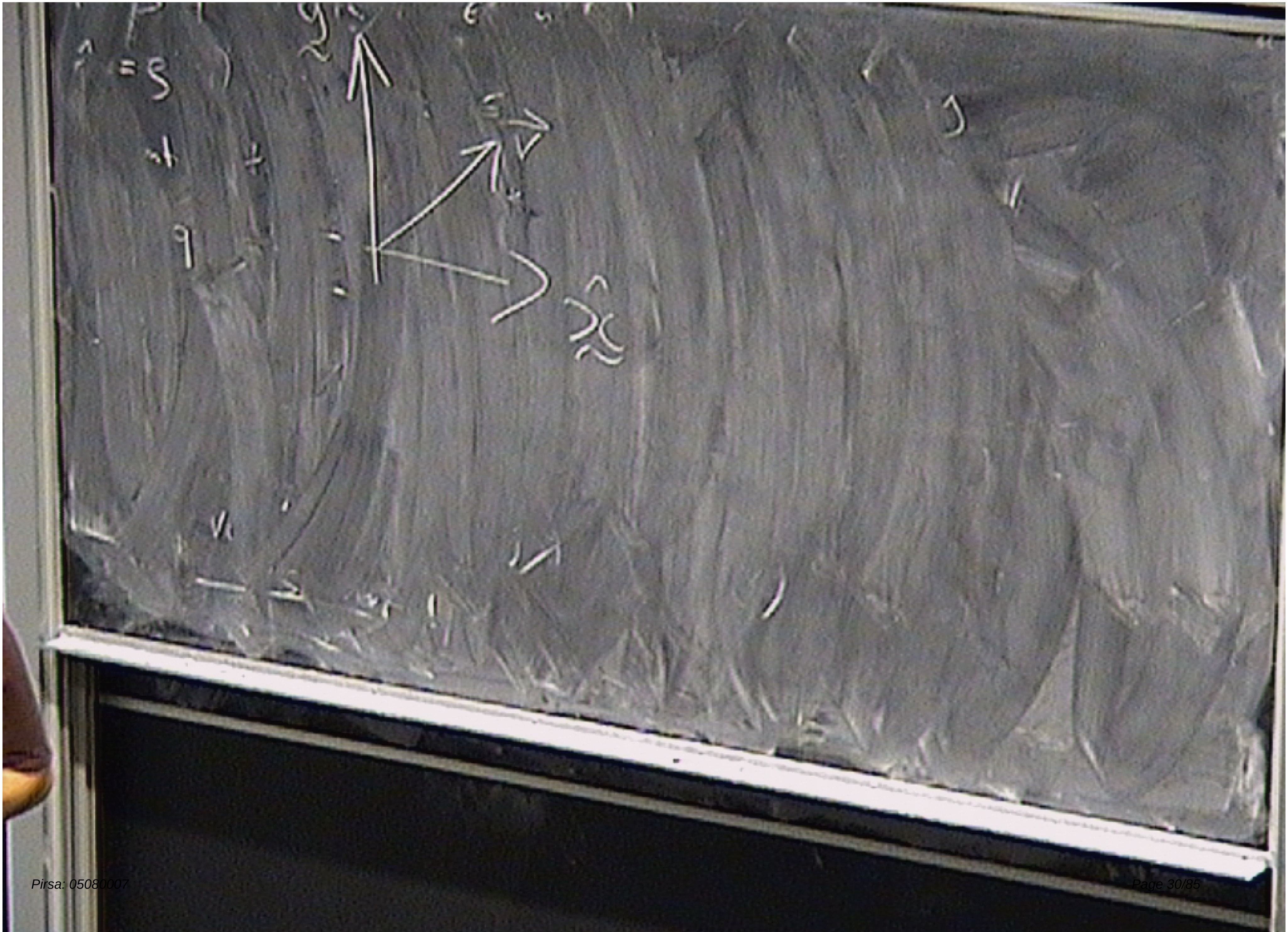
$$F(t) = (\exp(t D_H) F)(0)$$

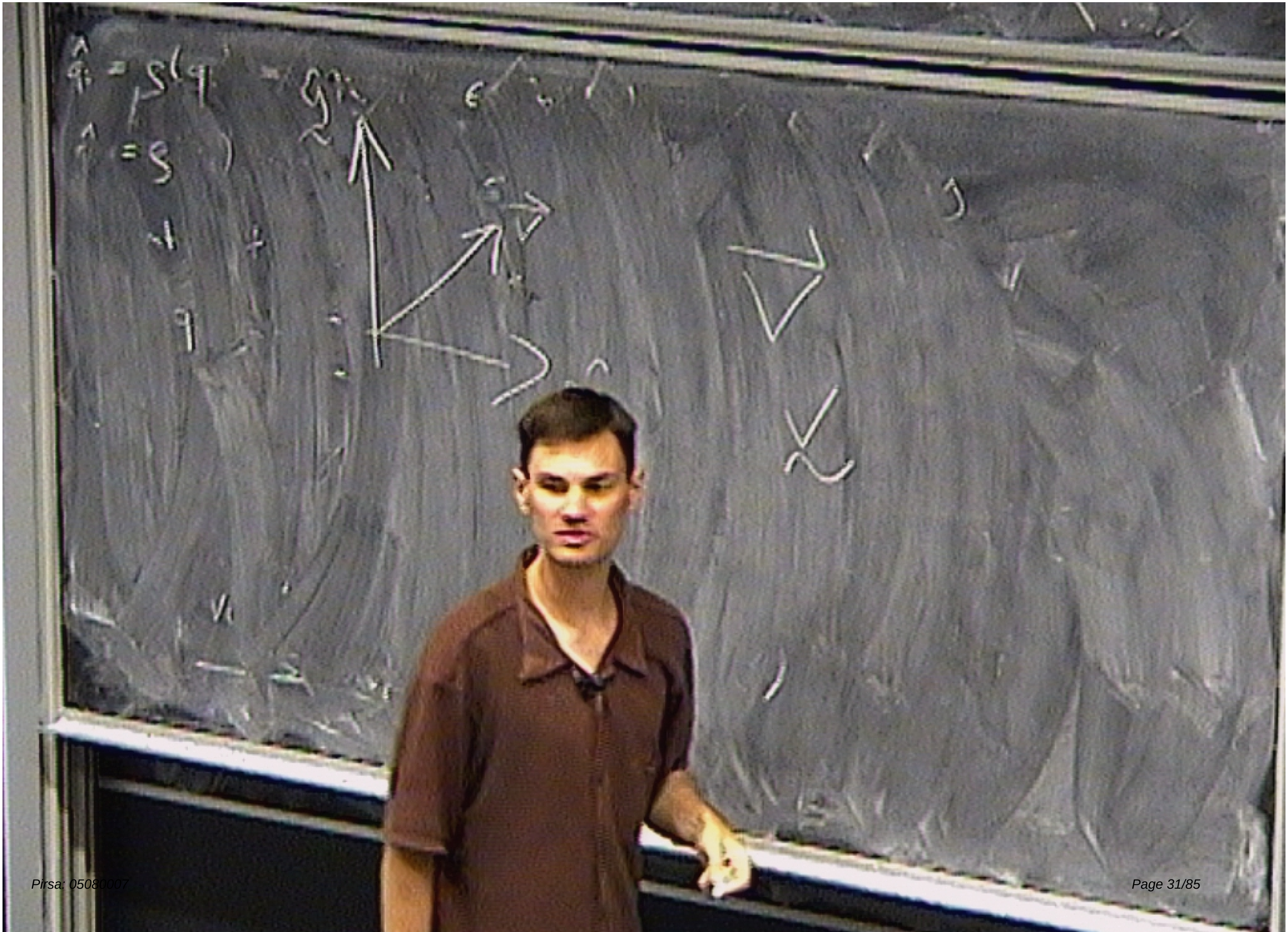
QM:

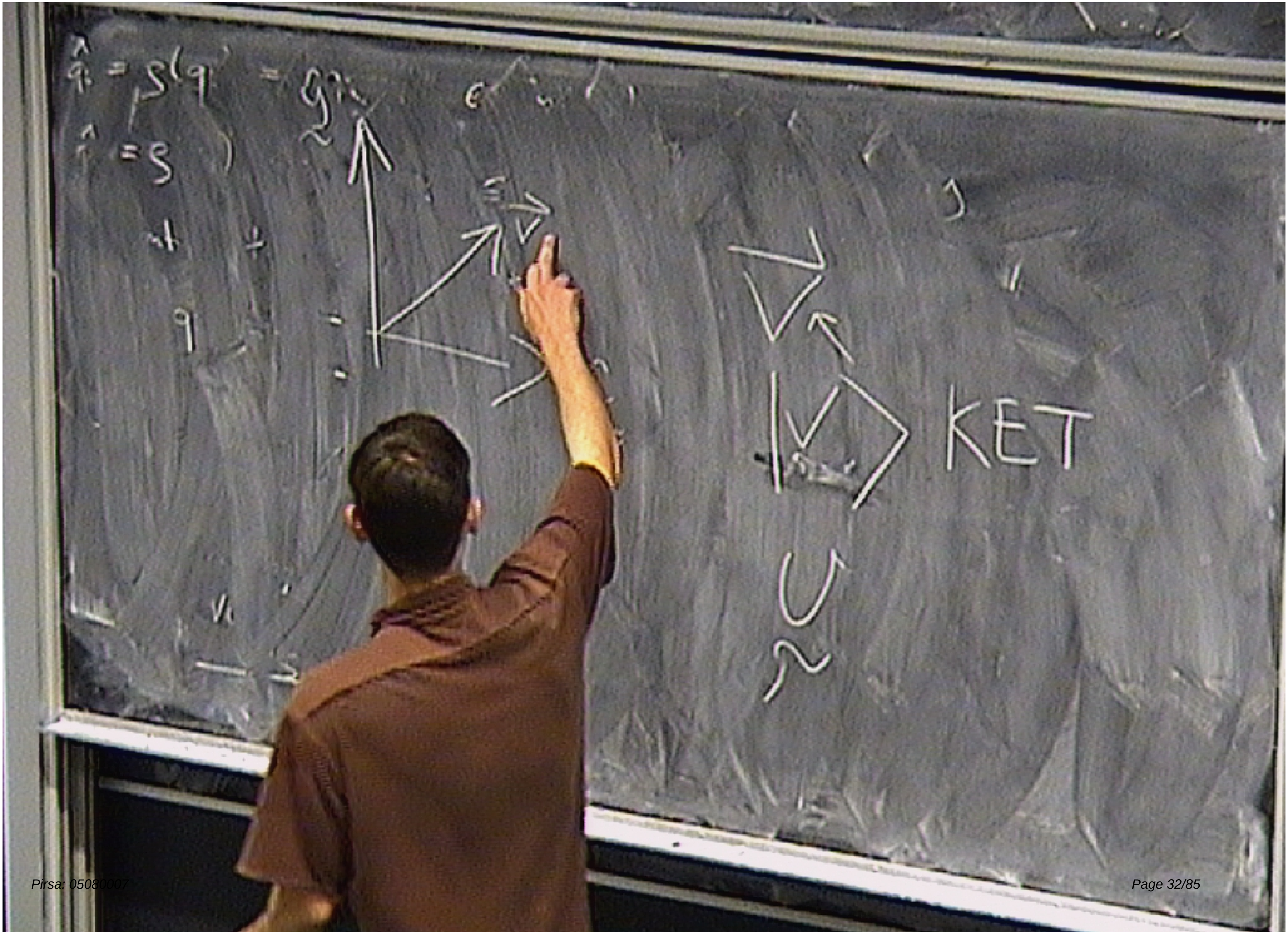
$$\psi(t) = \exp\left(-\tfrac{i}{\hbar} t \hat{H}\right) \psi(0)$$

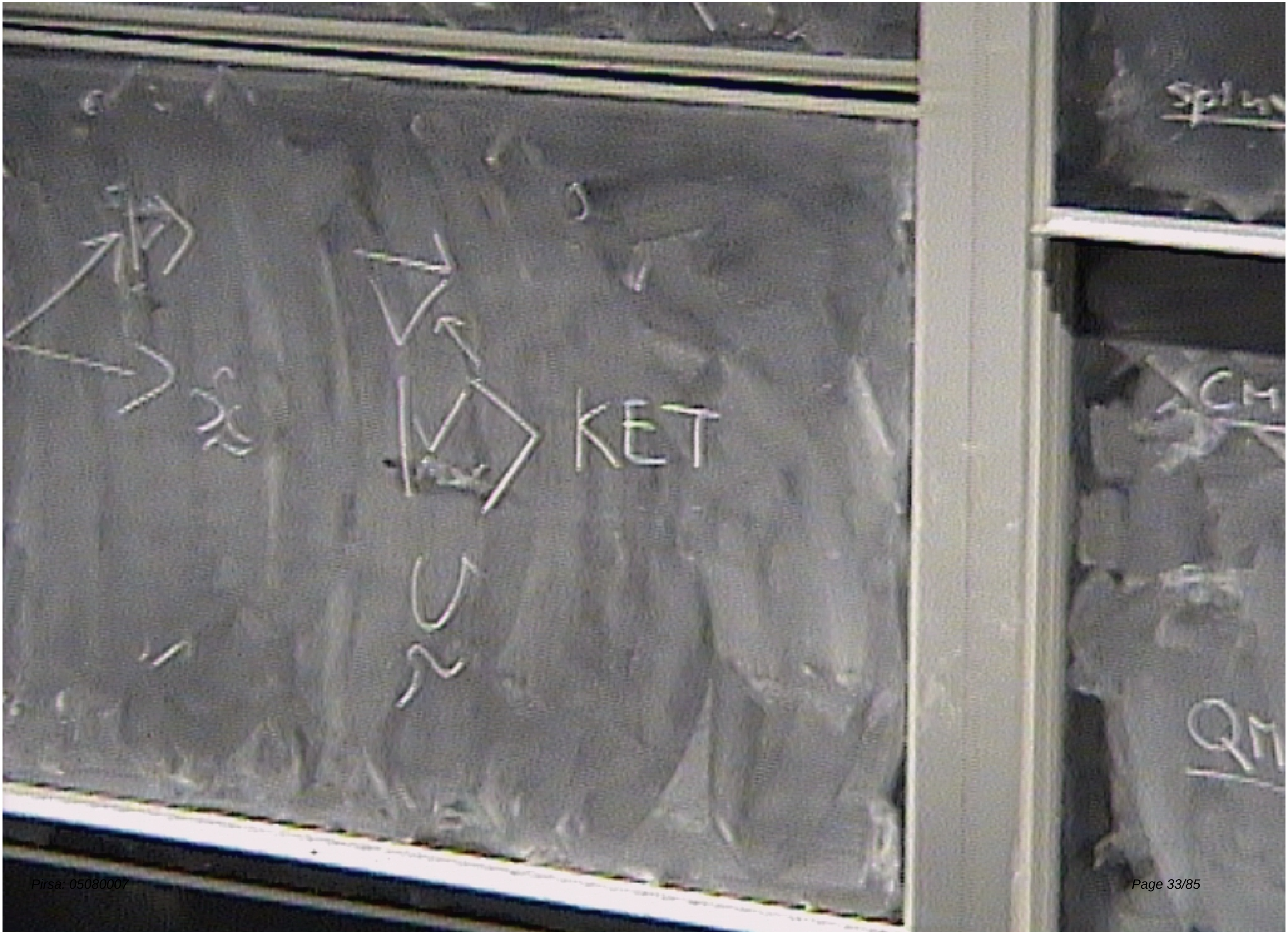
Schröd. eq.

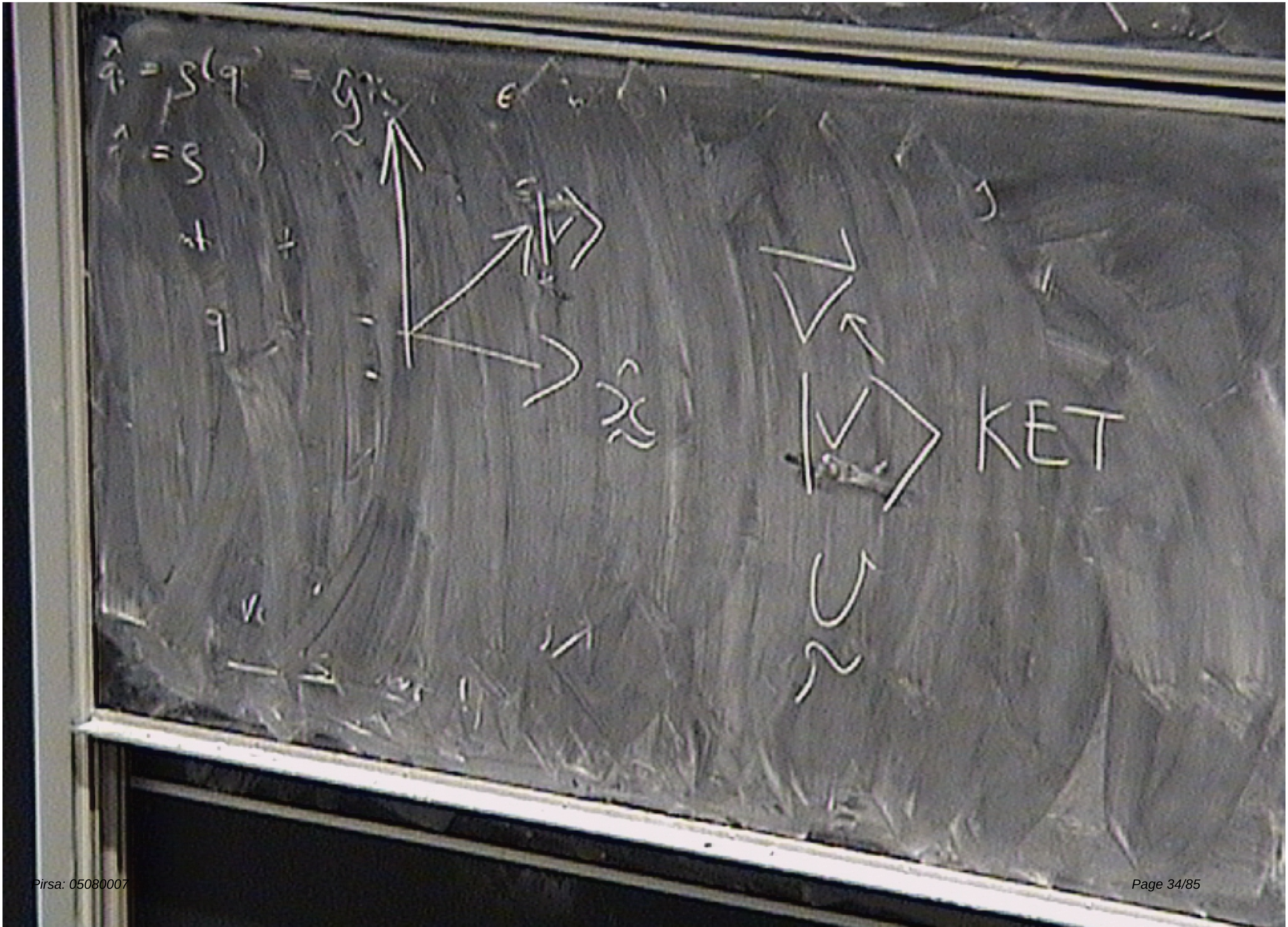

CM
Hamiltonian
H = p^2/2m + V(q)
Ĥ = −ħ^2/2m
F(t) = T_t F(0)
F(t) = (exp(t D_H) F)(0)
QM:
ψ(t) = exp(−i/ħ t Ĥ) ψ(0)

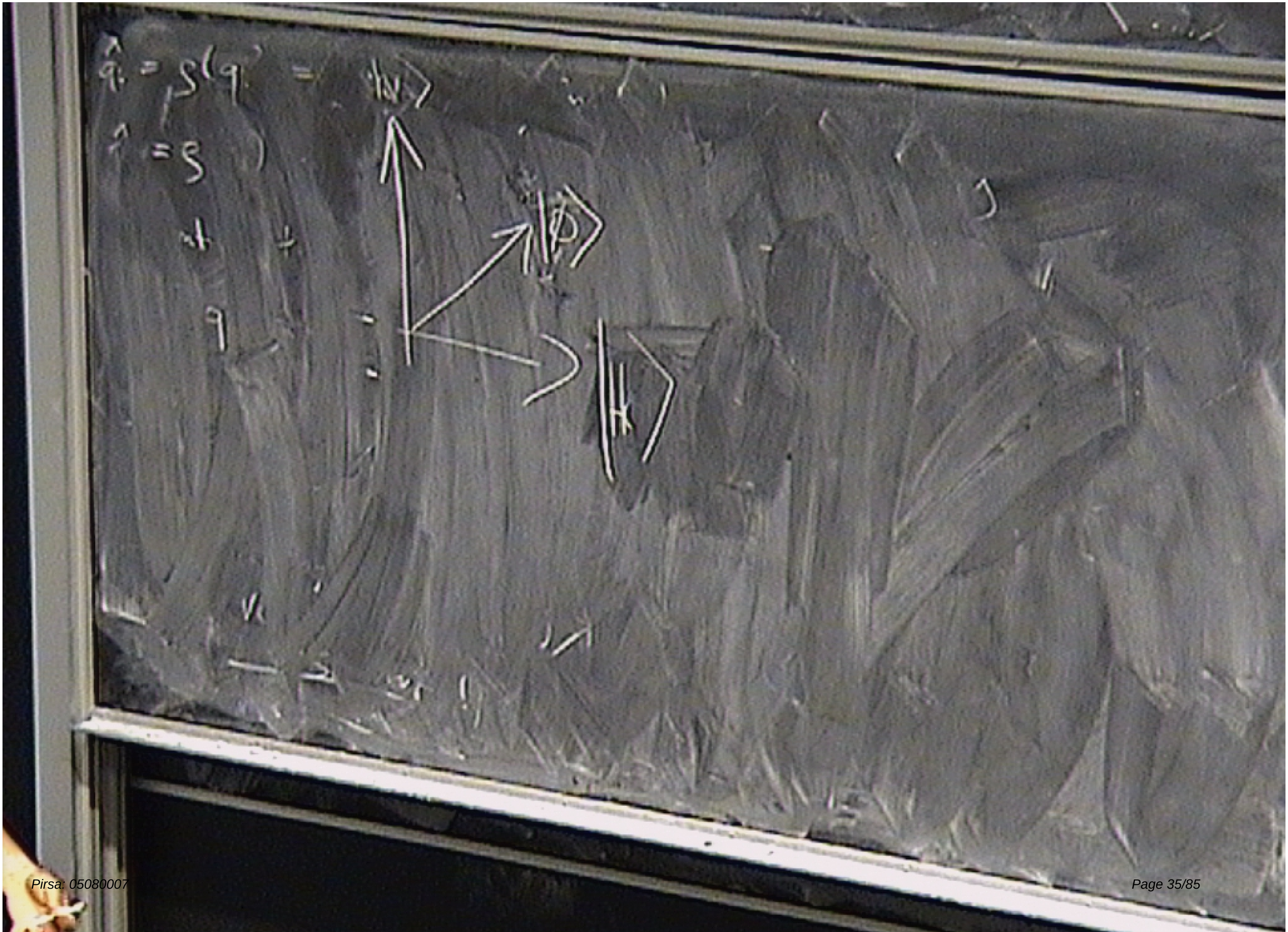


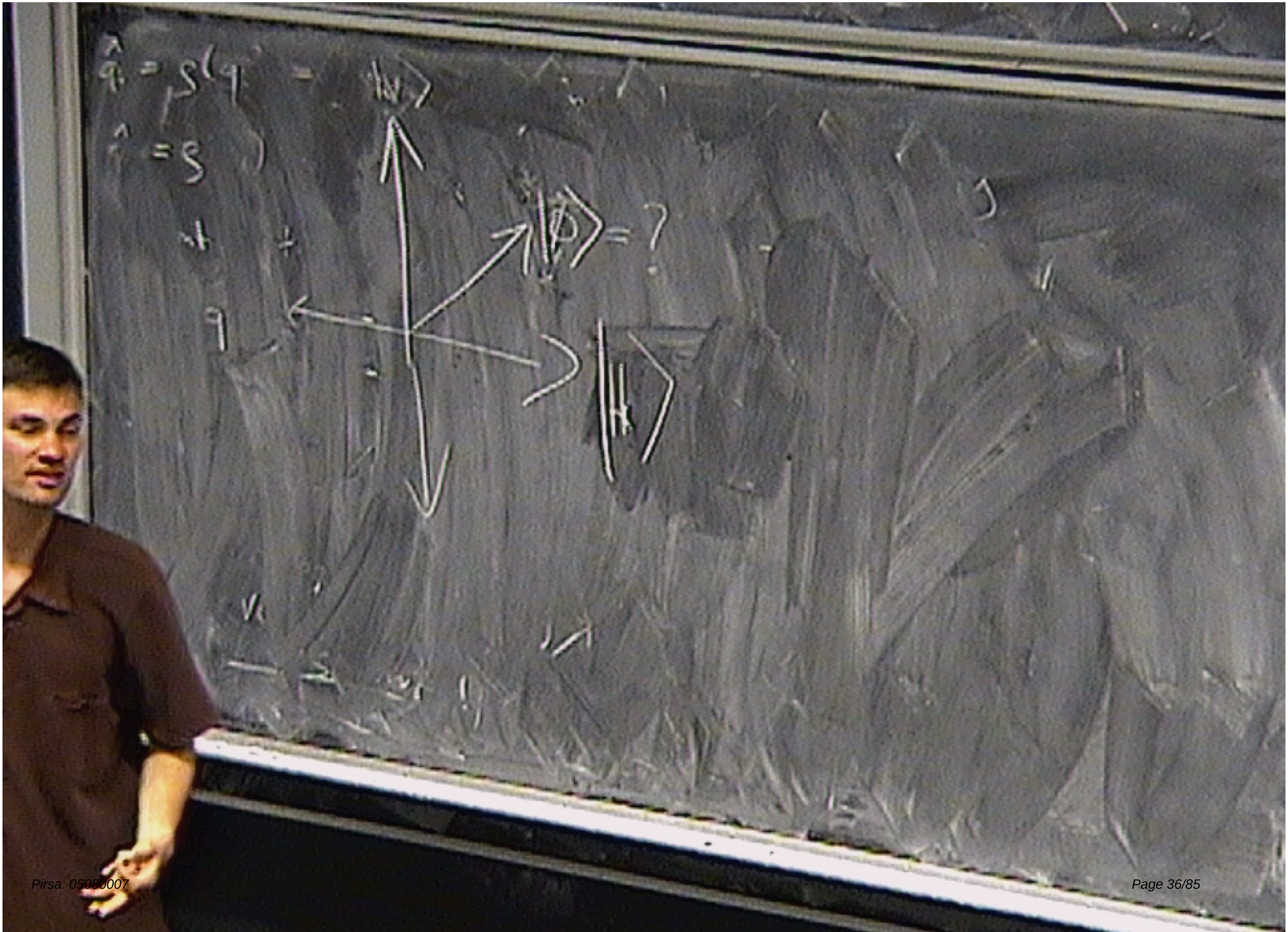


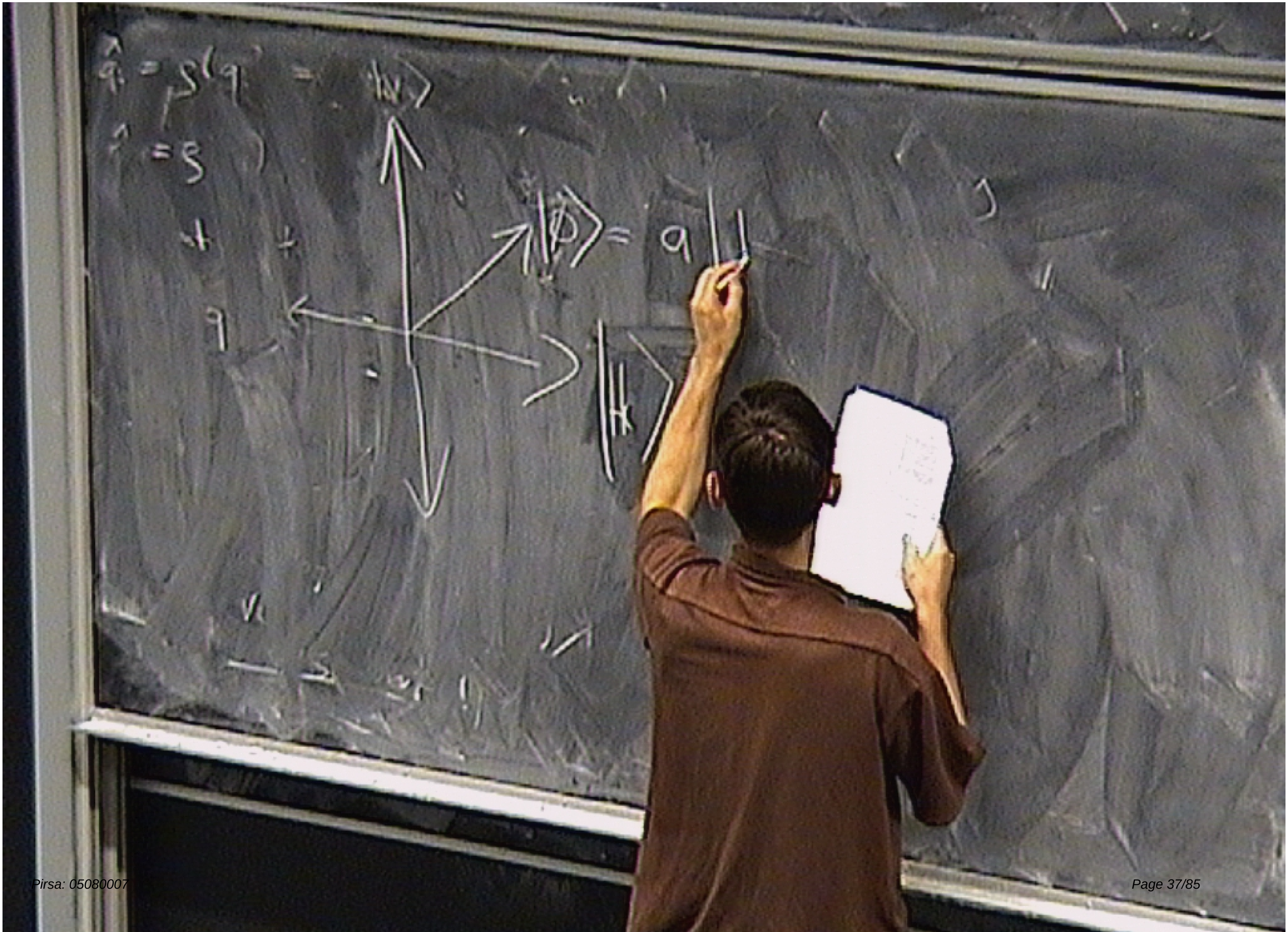

KET

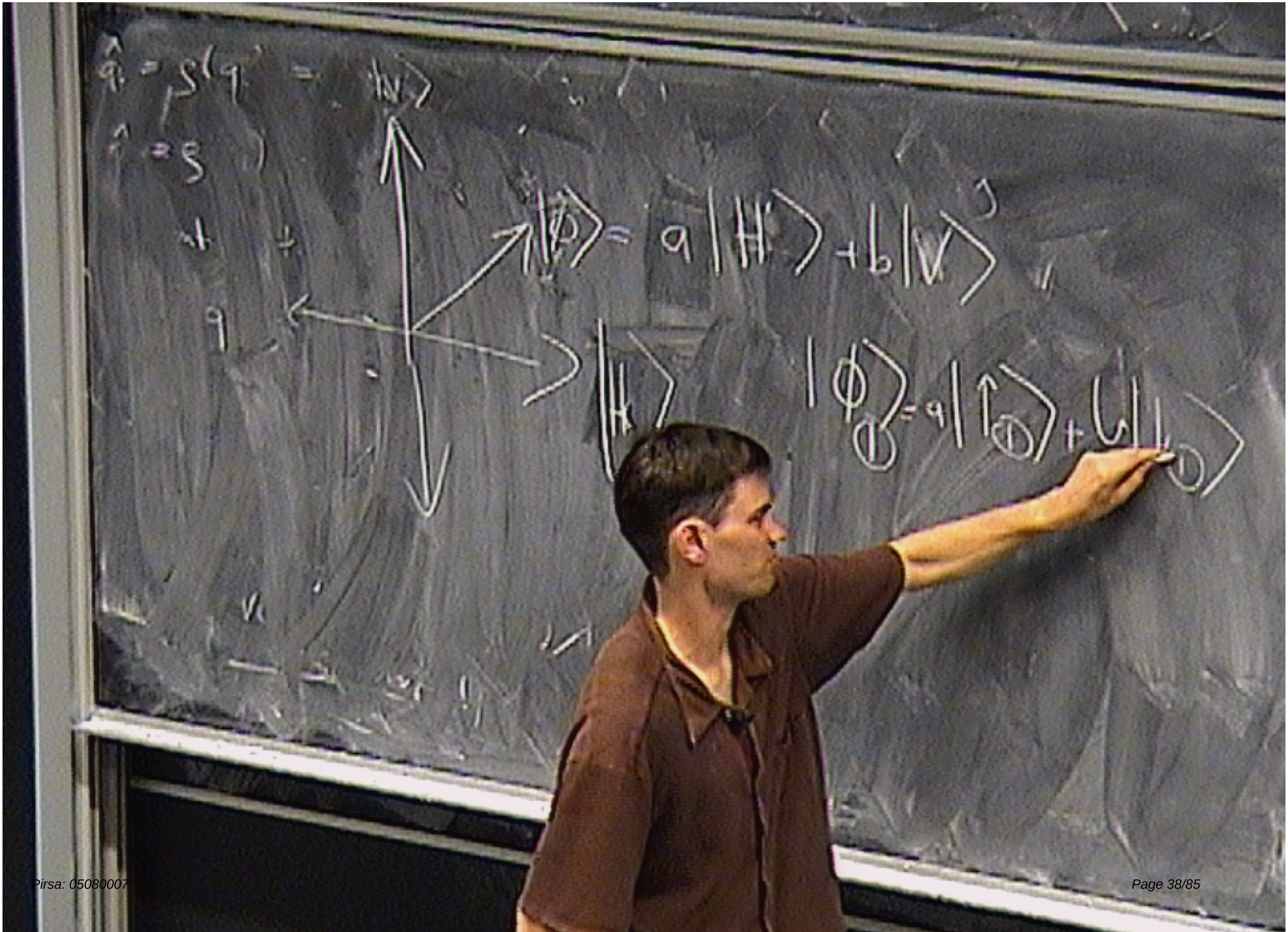

KET

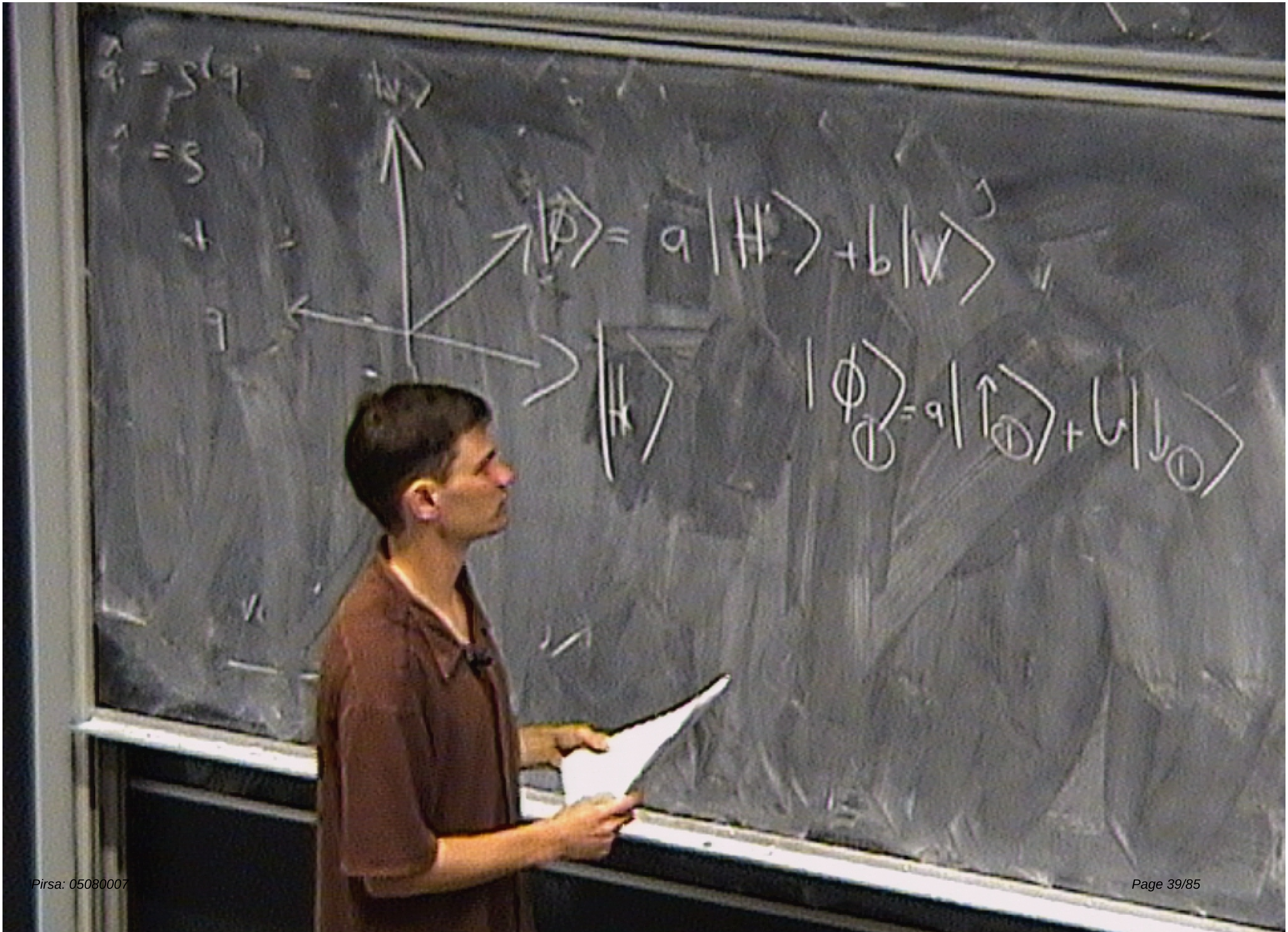

KET

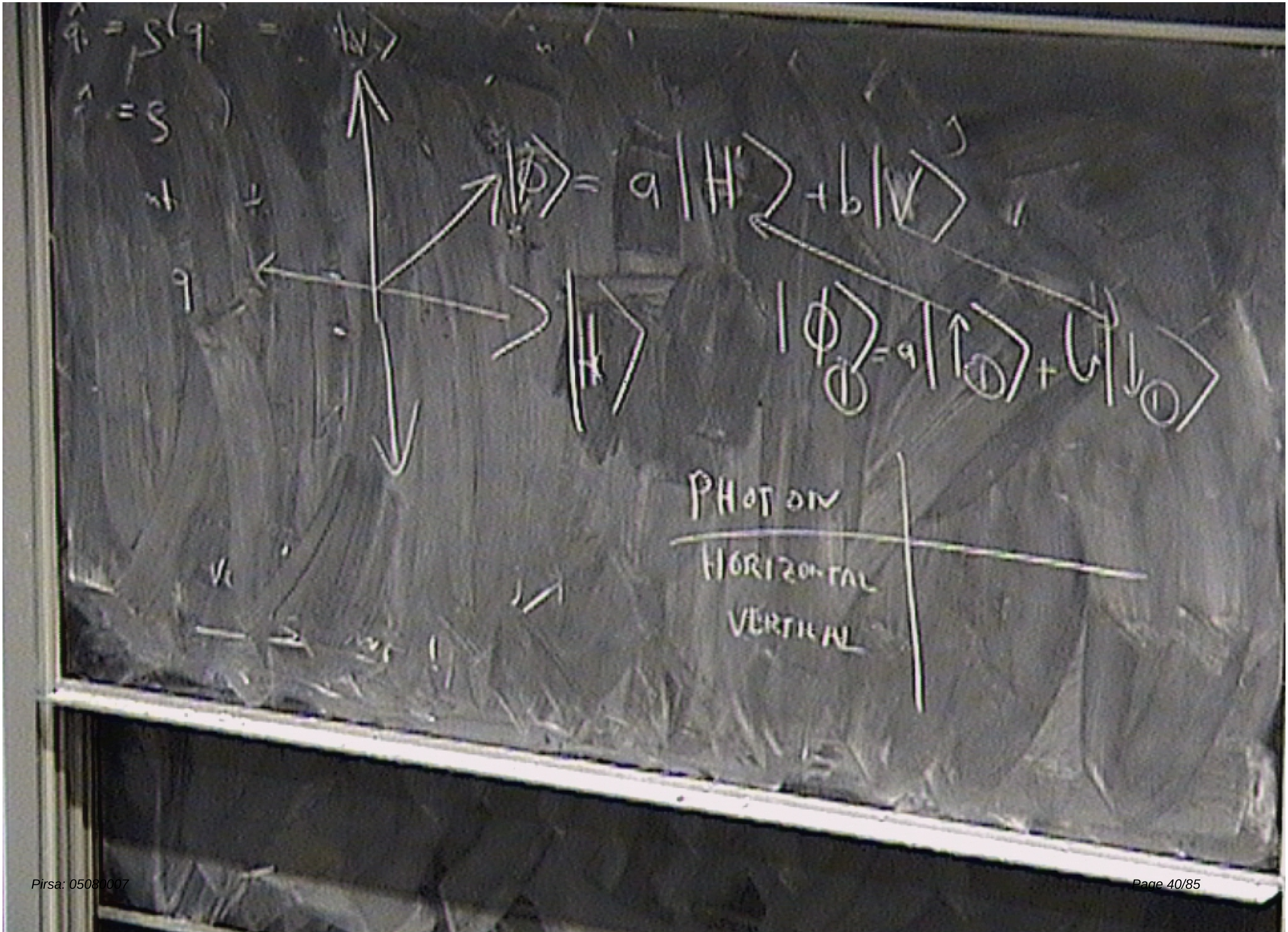


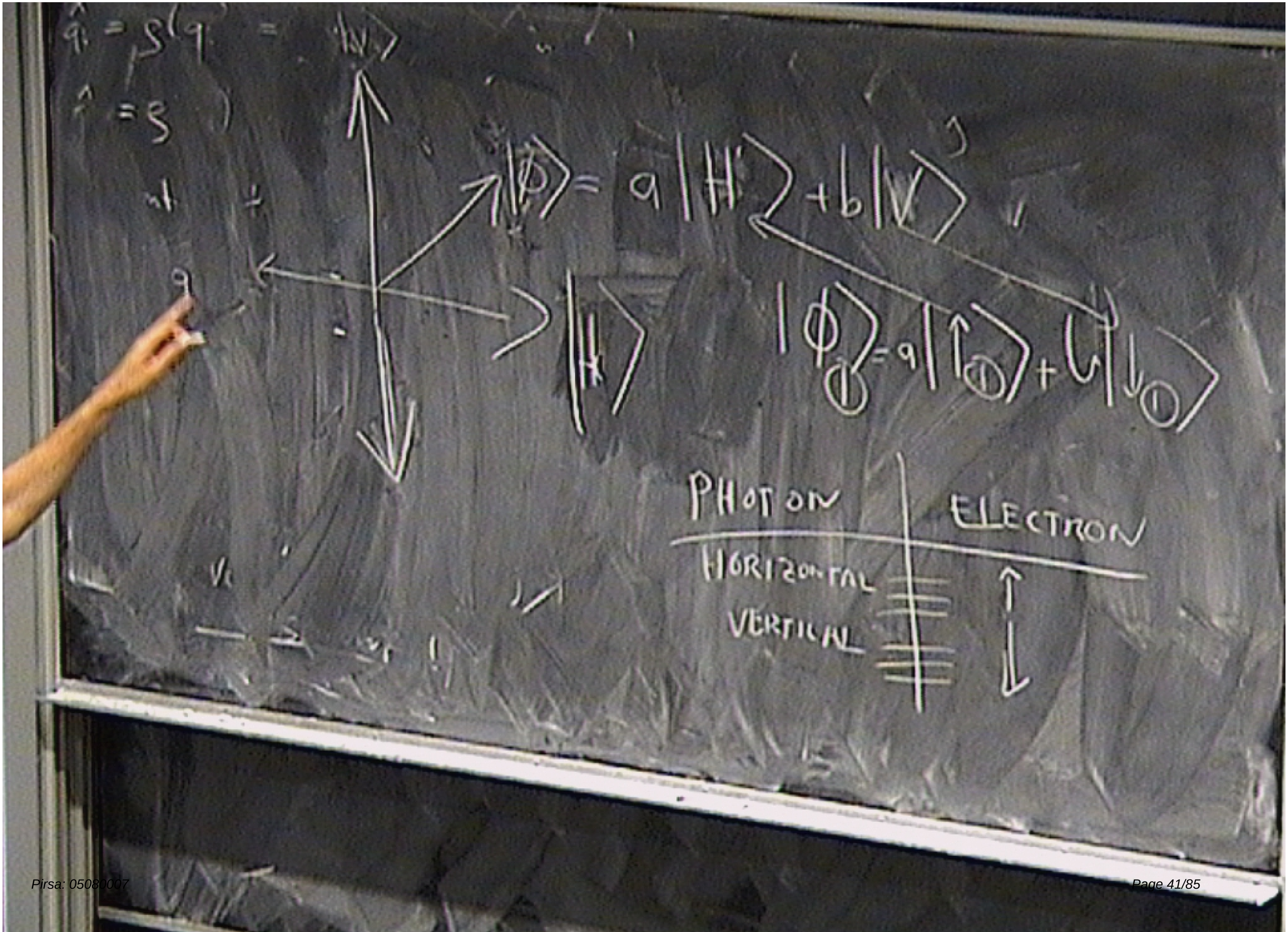


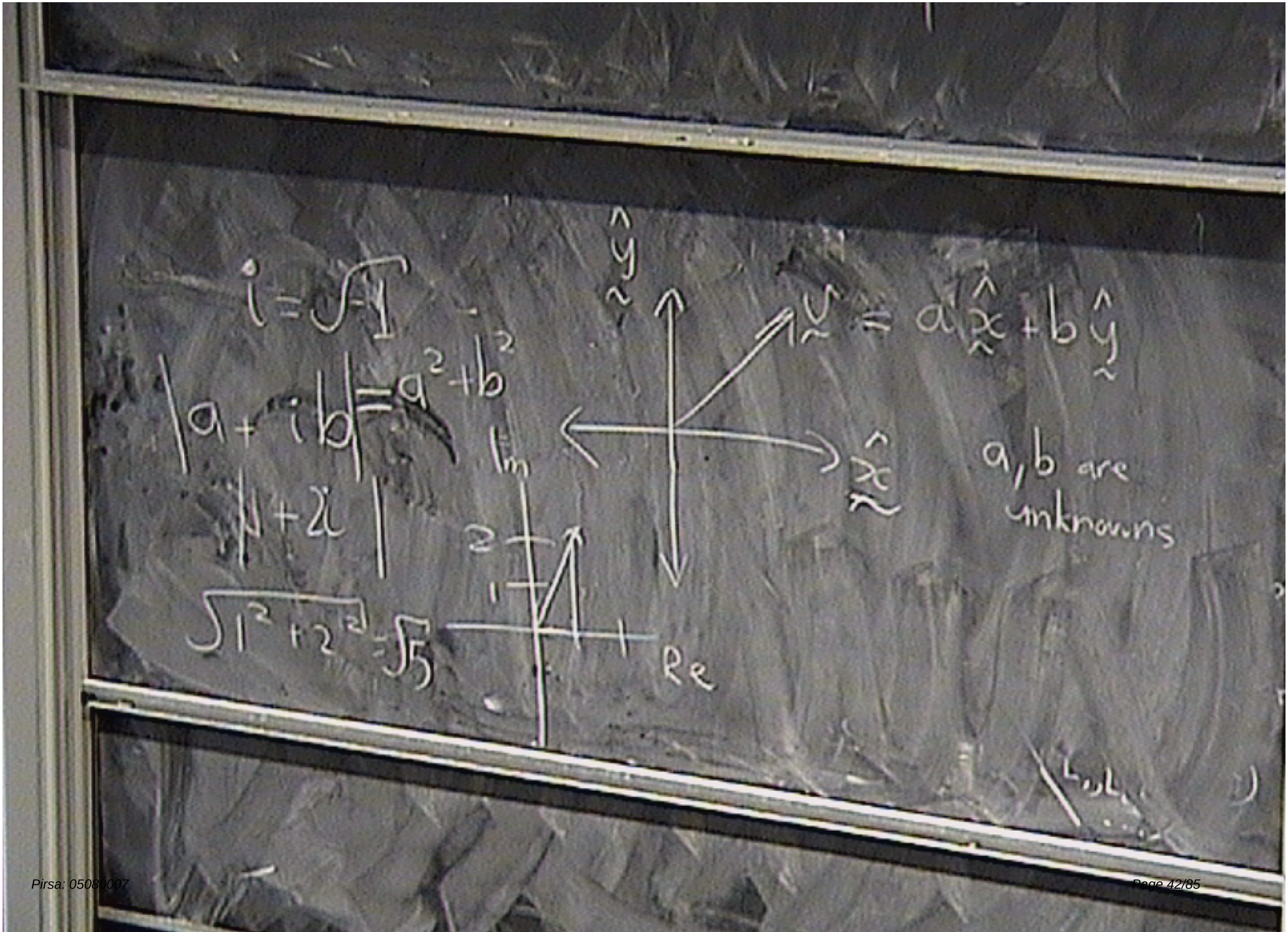



|φ⟩ = a|H⟩ + b|V⟩

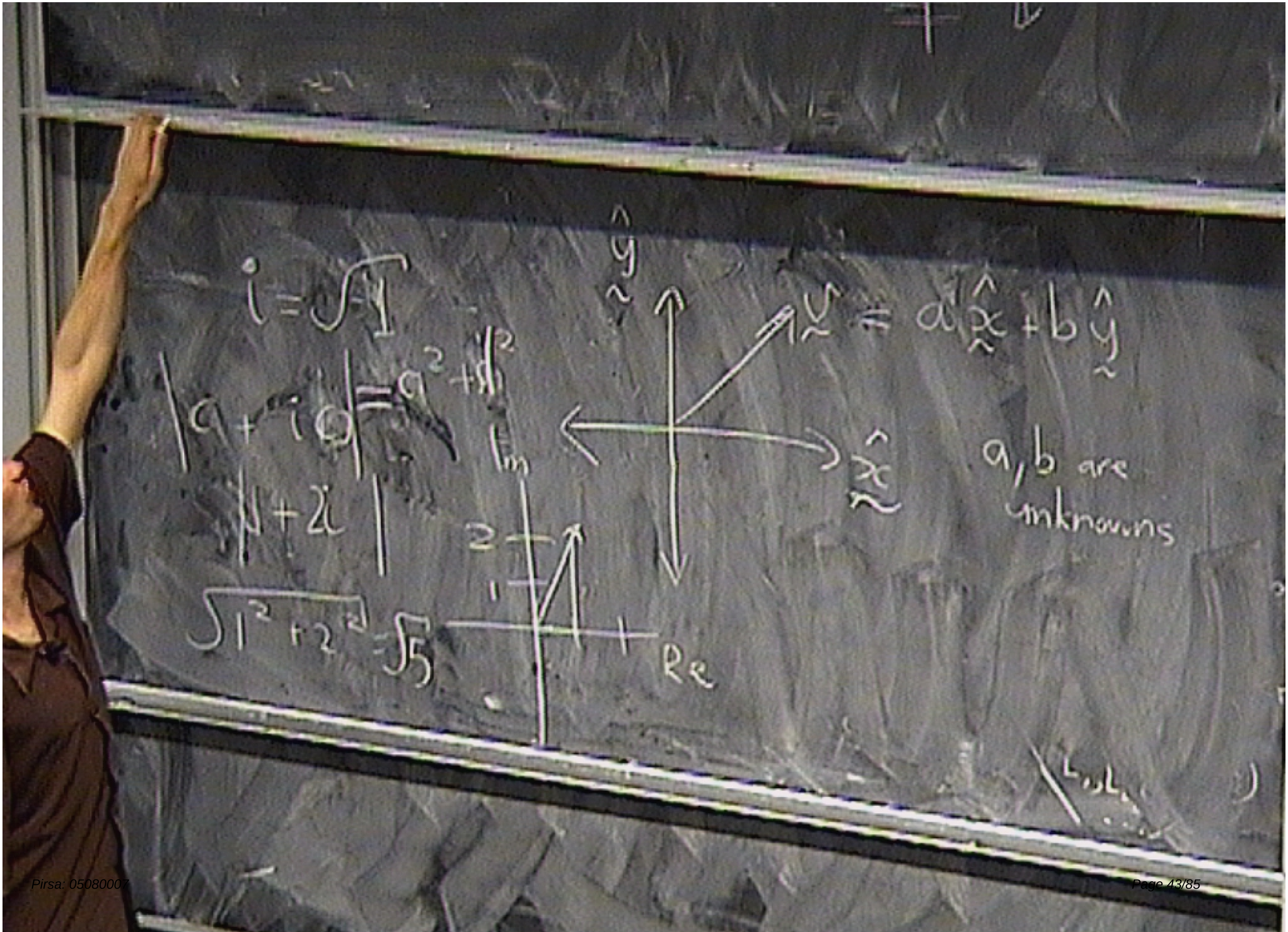

|ϕ⟩ = a|H⟩ + b|V⟩
|H⟩


|φ⟩ = a|H⟩ + b|V⟩
PHOTON
HORIZONTAL
VERTICAL


|φ⟩ = a|H⟩ + b|V⟩
|H⟩
|φ⟩ = a|↑⟩ + b|↓⟩
PHOTON
ELECTRON
HORIZONTAL
VERTICAL



i = √-1
|a + ib| = a² + b²
|1 + 2i|
√(1² + 2²) = √5
Im
Re
v = a x̂ + b ŷ
a, b are unknowns



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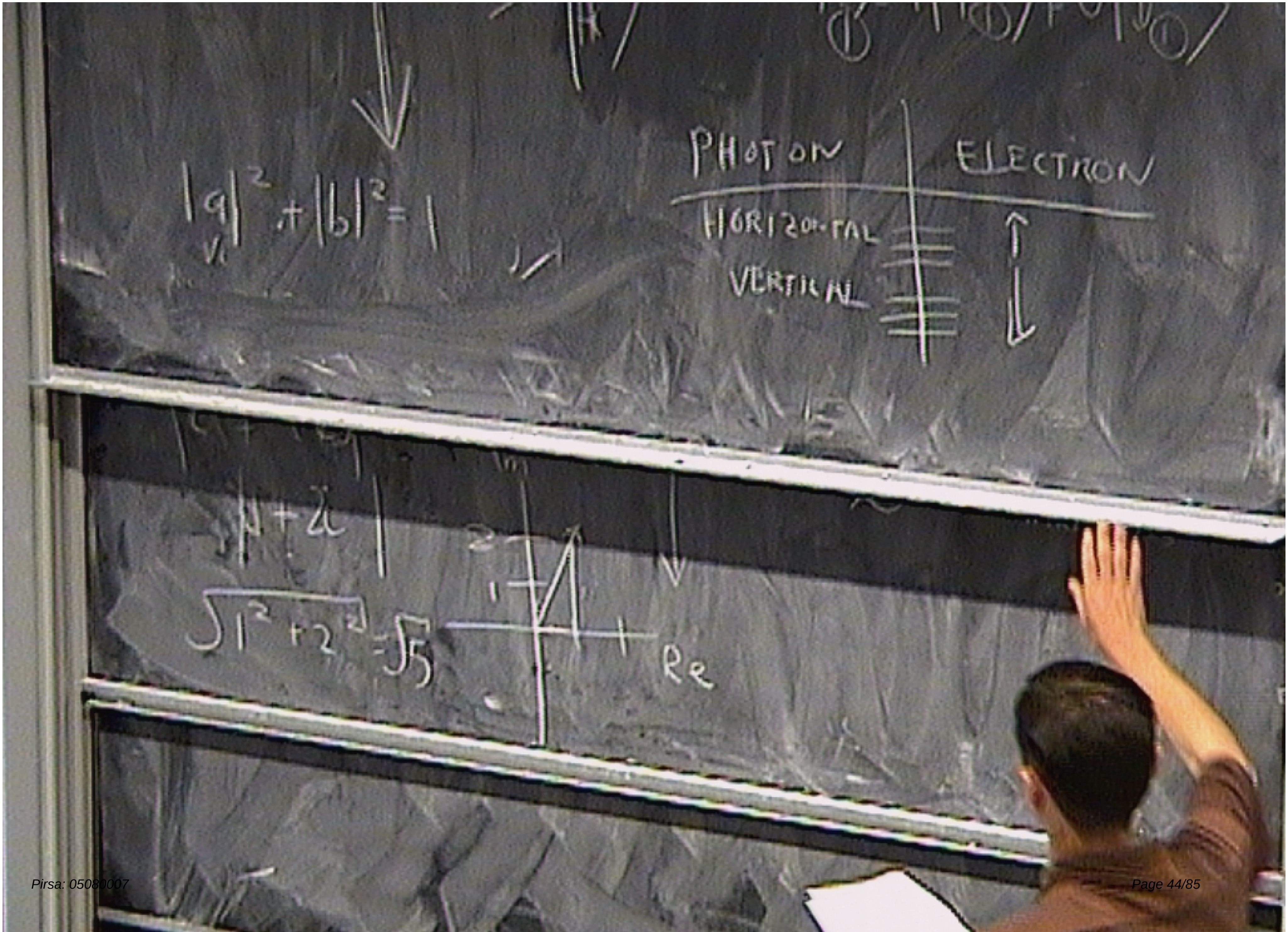
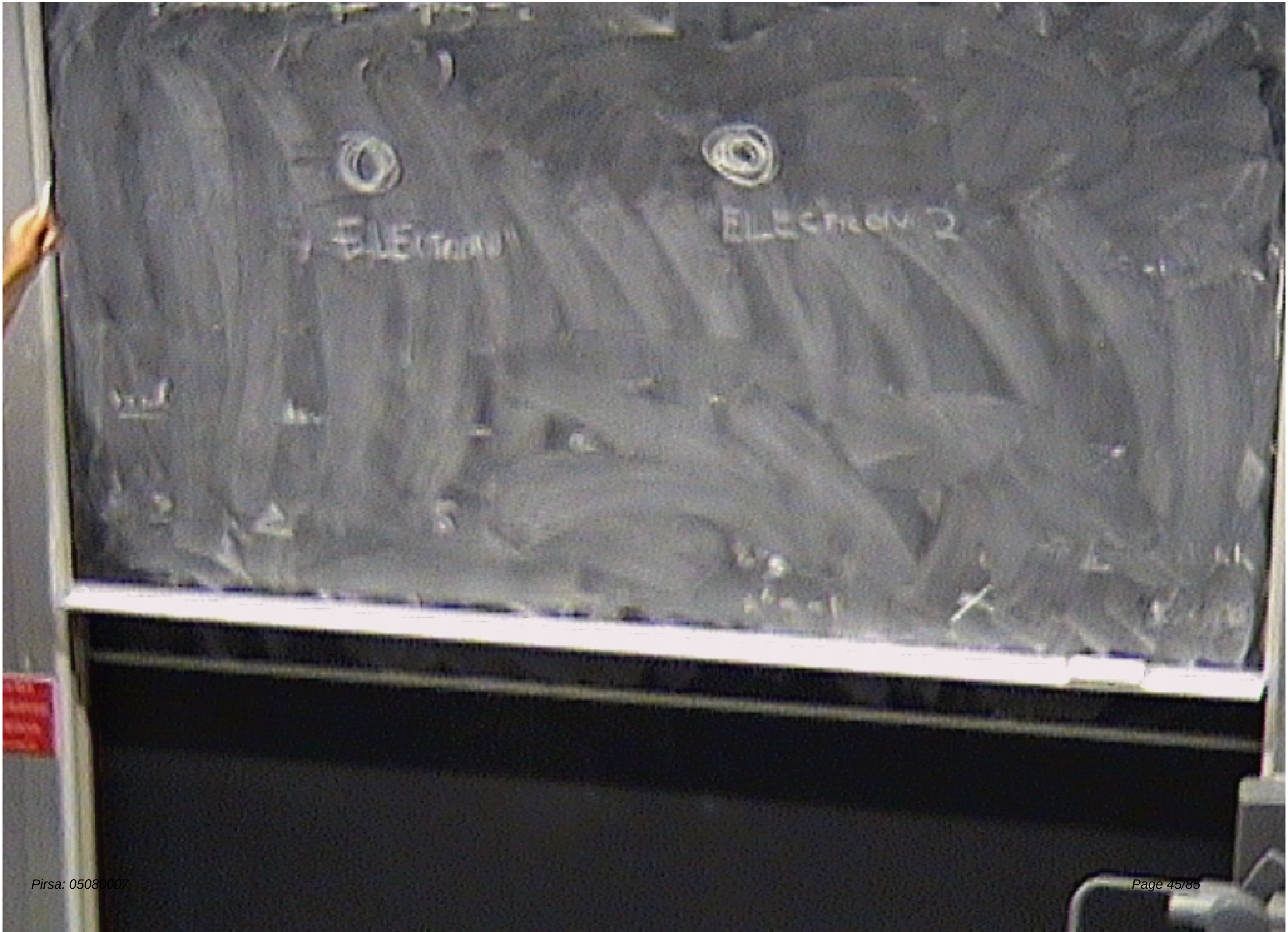
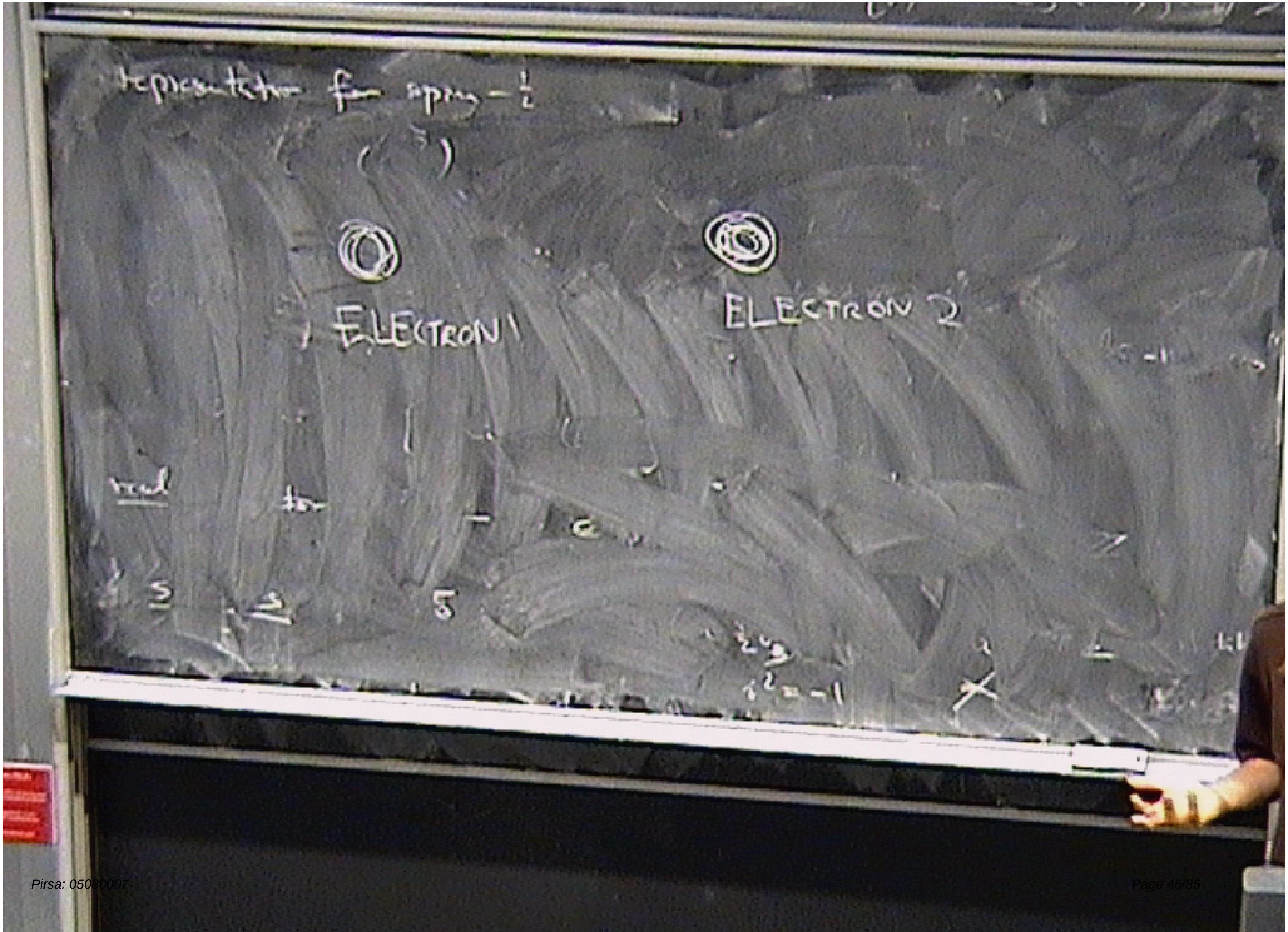
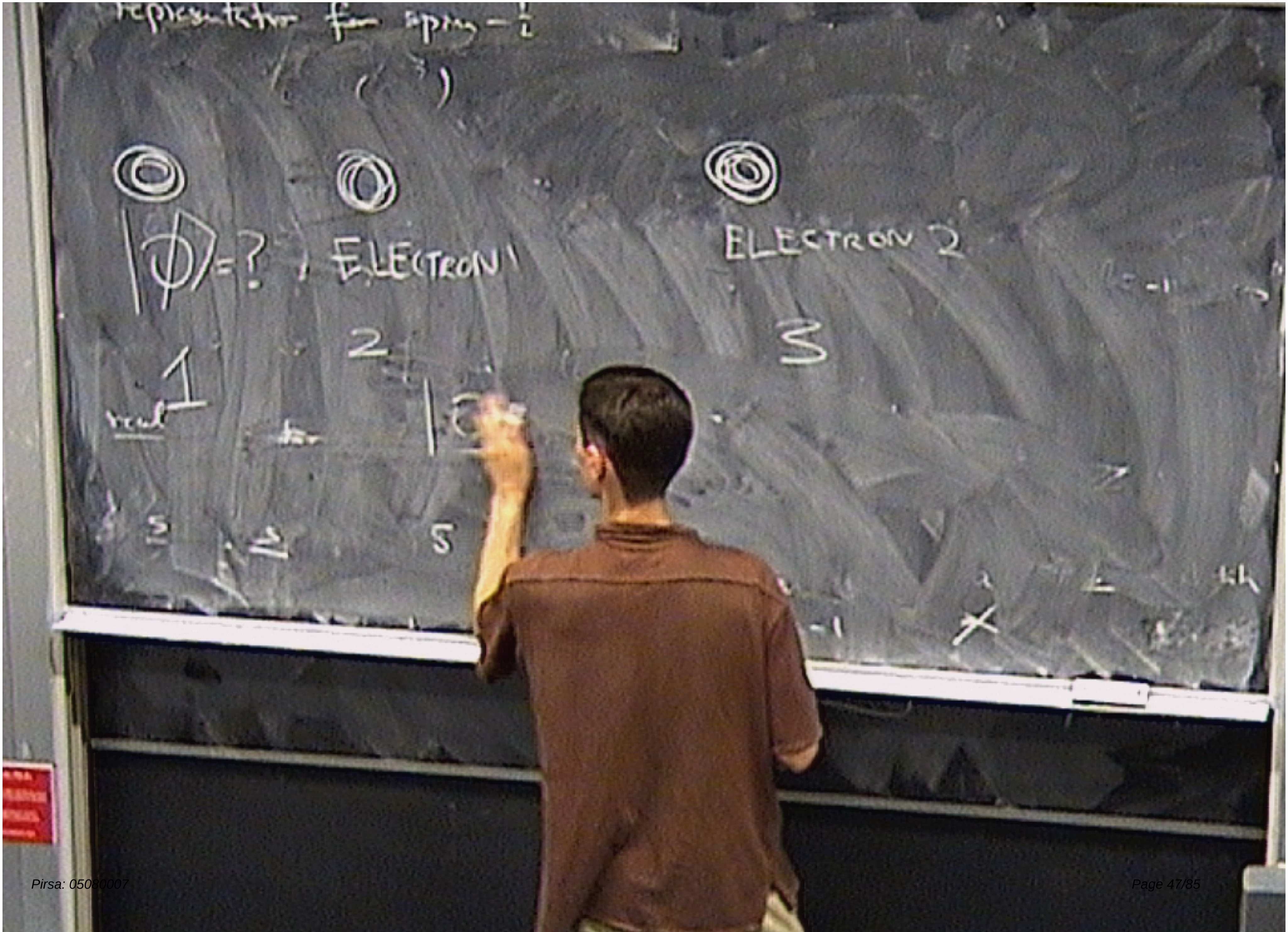
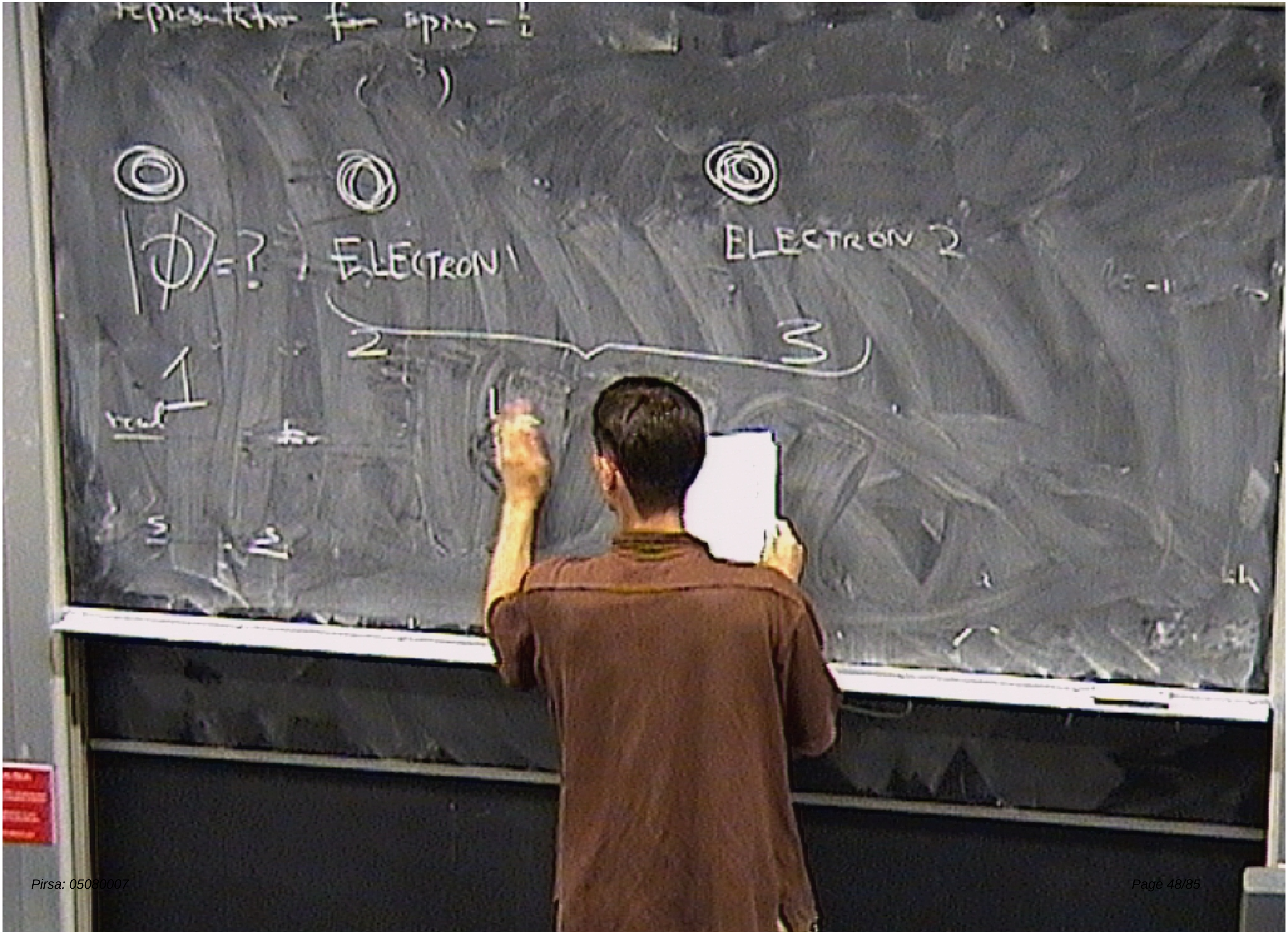

PHOTON
ELECTRON
HORIZONTAL
VERTICAL
Re

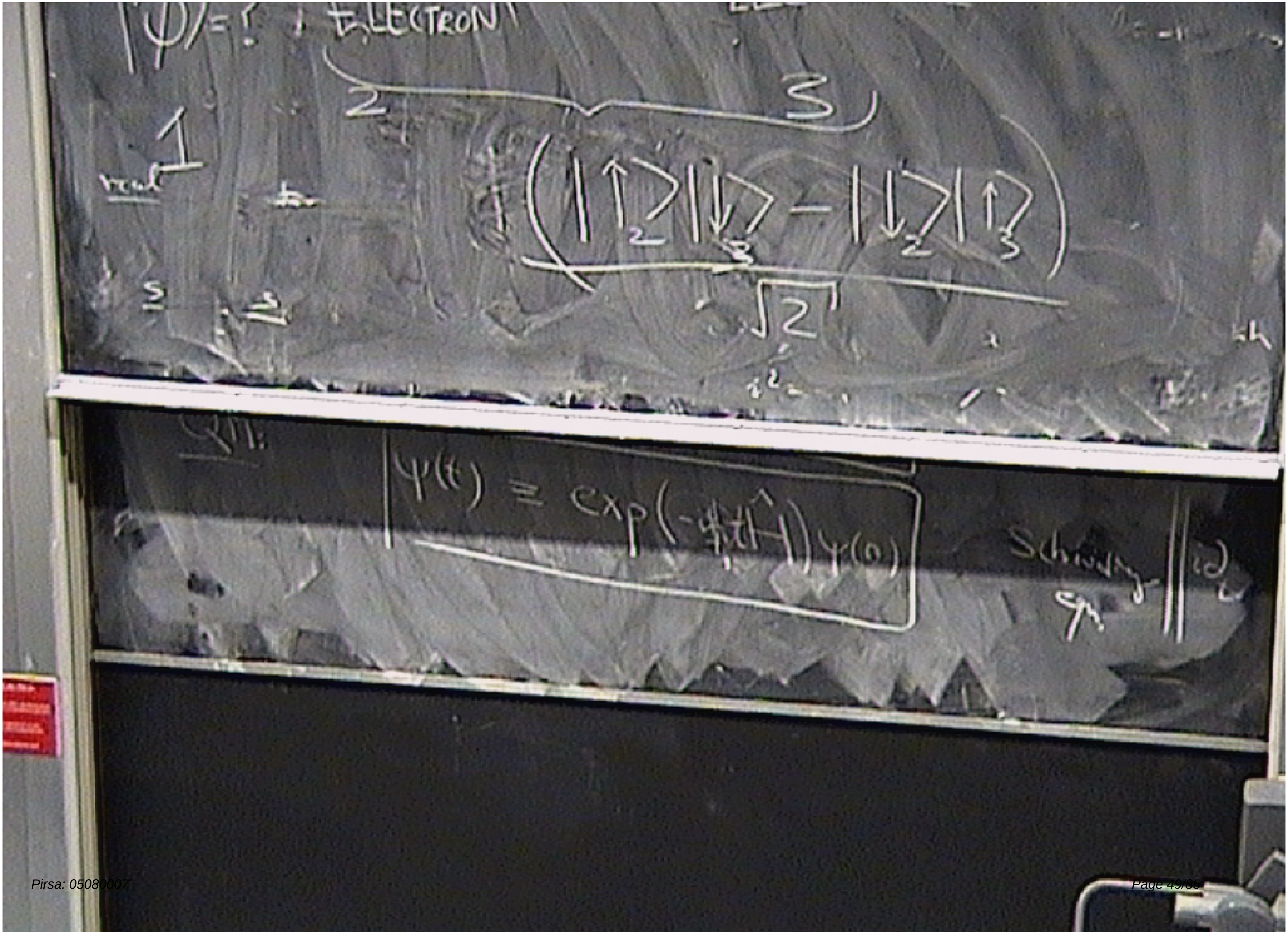

ELECTRON 2

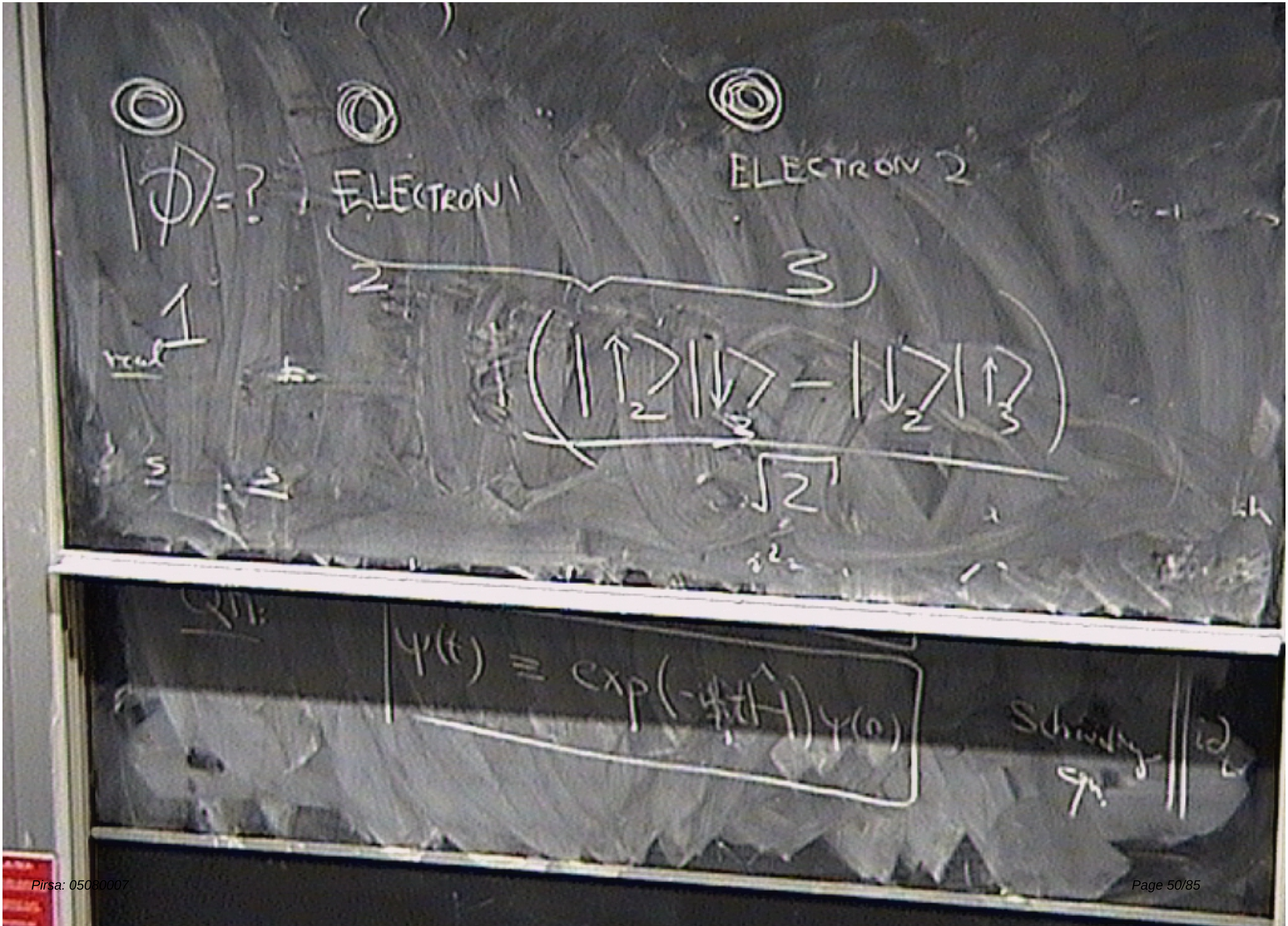

representation for spin-½
ELECTRON 1
ELECTRON 2
i² = -1

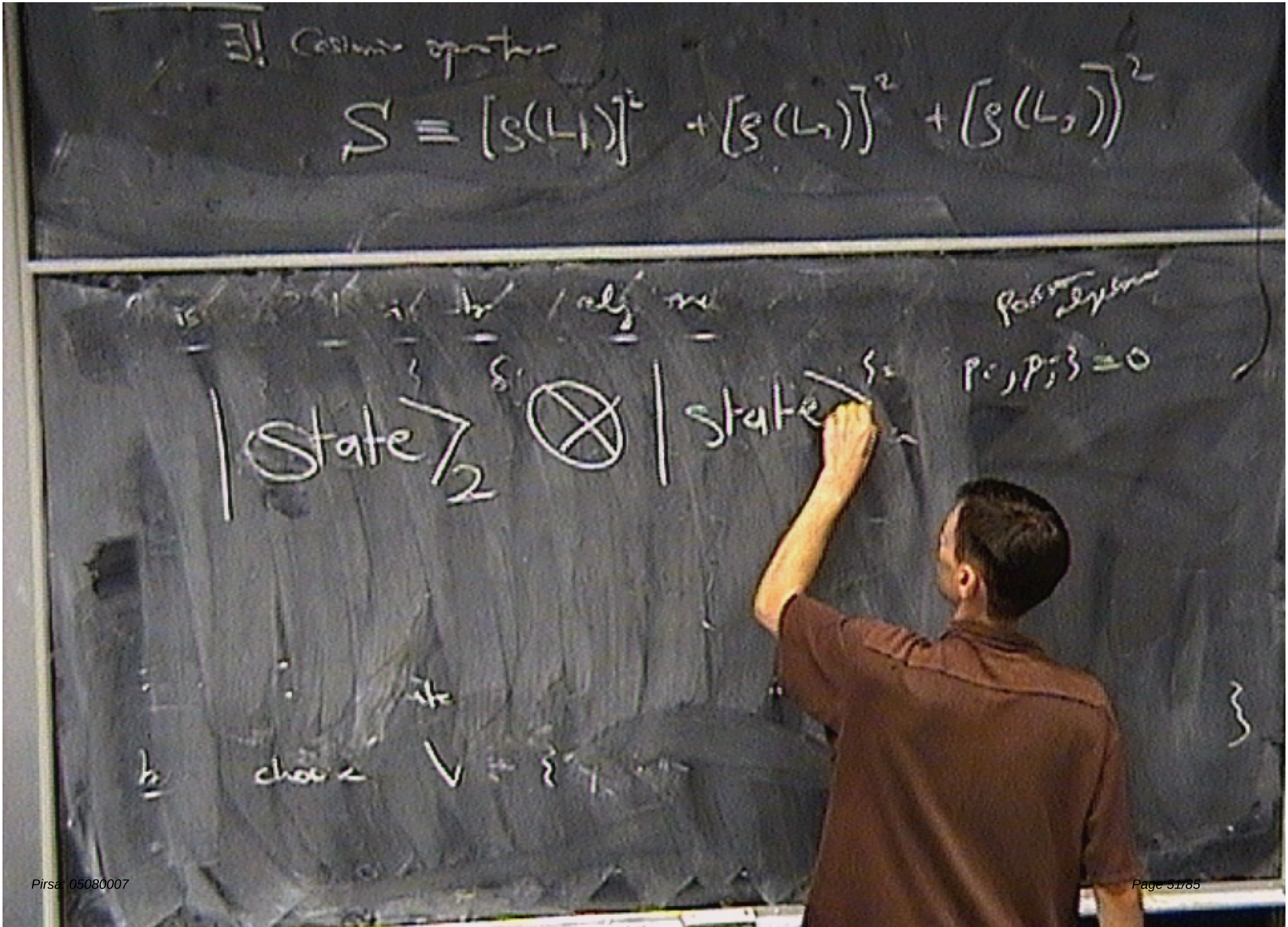


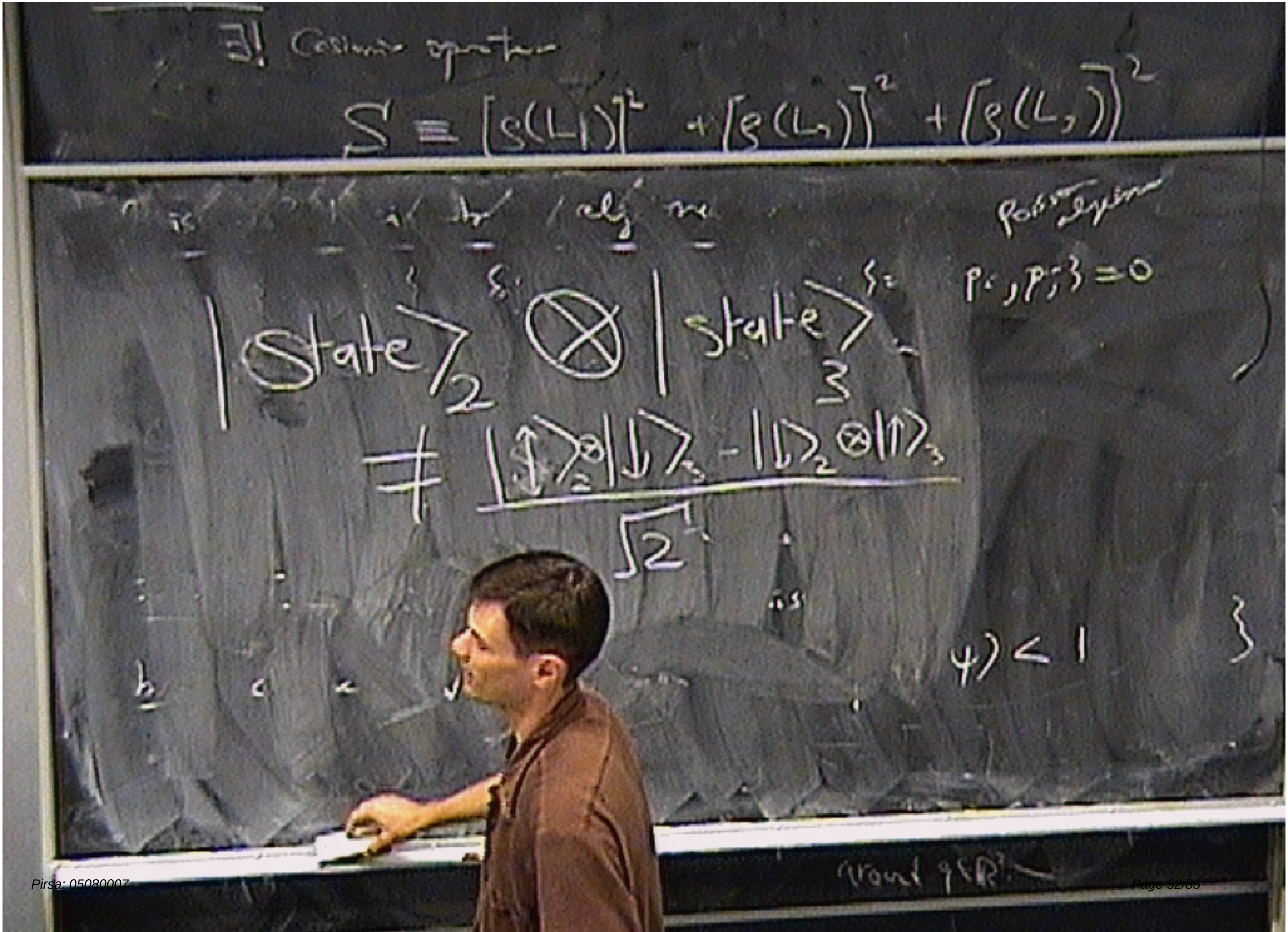
representation for spin-½
ELECTRON 1
ELECTRON 2

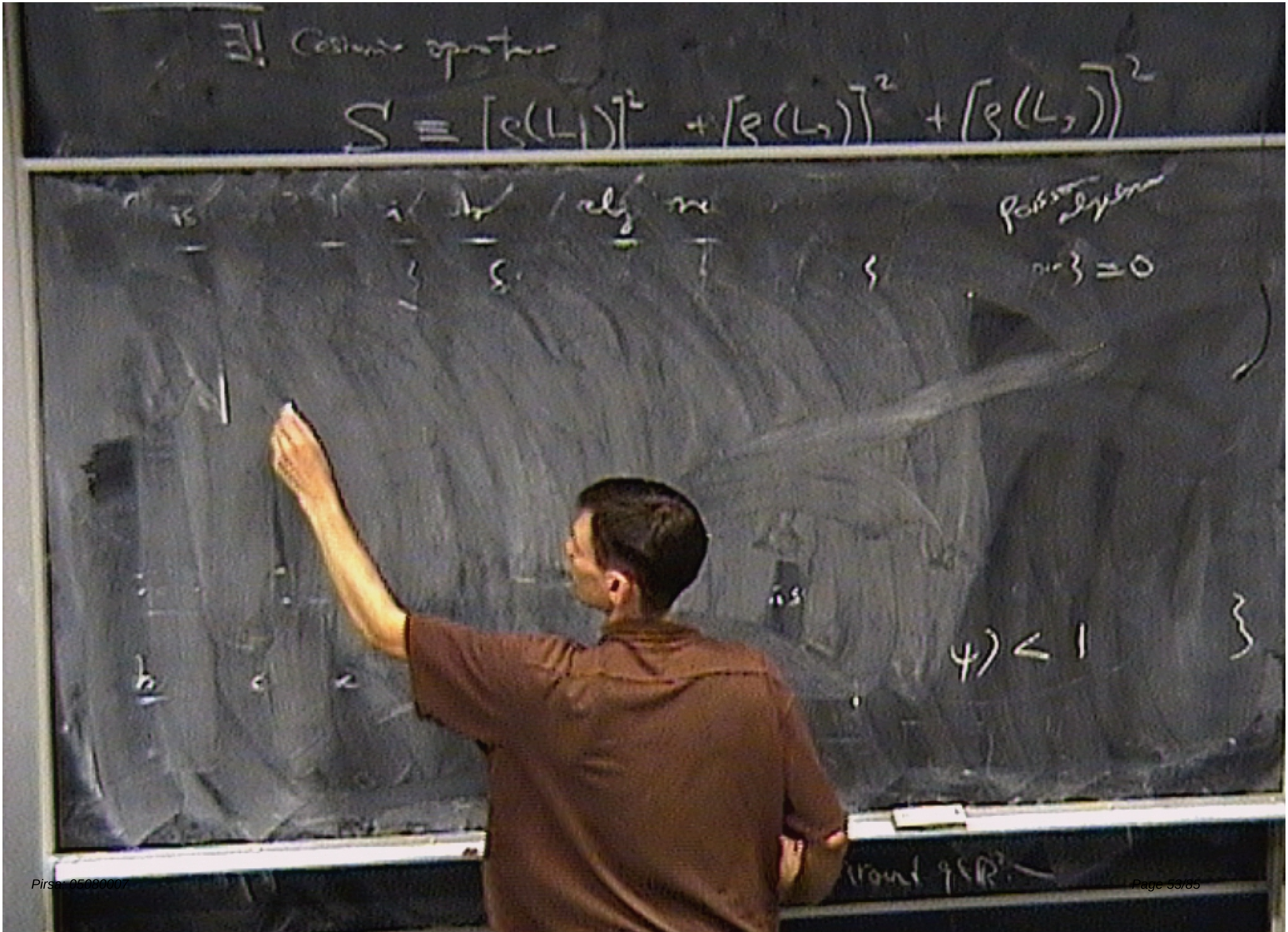

representation for spin-$\frac{1}{2}$
$|\phi\rangle = ?$
ELECTRON 1
ELECTRON 2
1
2
3

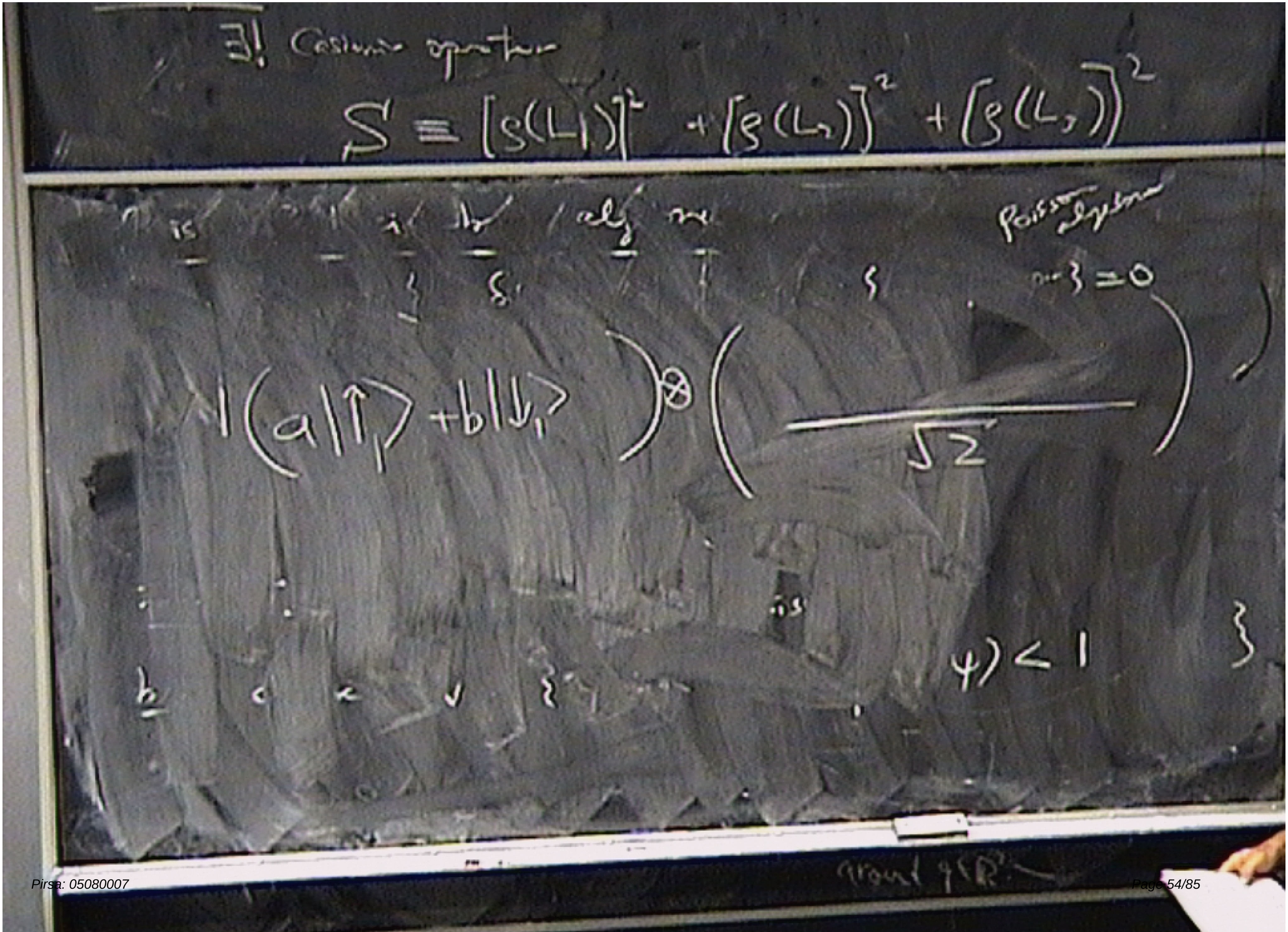

|ψ⟩ = ? ELECTRON
2
3
1
$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_2|\uparrow\rangle_3\right)$
QM:
$\psi(t) = \exp\left(-\frac{i}{\hbar}t\hat{H}\right)\psi(0)$
Schrödinger eq.


|ψ⟩ = ?
ELECTRON 1
ELECTRON 2
(|↑⟩₂|↓⟩₃ − |↓⟩₂|↑⟩₃) / √2
QM:
ψ(t) = exp(−i/ħ t Ĥ) ψ(0)
Schrödinger eqn.


S ≡ [g(L1)]2 + [g(L2)]2 + [g(L3)]2
|state⟩2 ⊗ |state
choice


∃! Casimir operator
S ≡ [ρ(L1)]^2 + [ρ(L2)]^2 + [ρ(L3)]^2
Poisson algebra
{p_i, p_j} = 0
|state⟩_2 ⊗ |state⟩_3
≠ (|↑⟩_2 ⊗ |↓⟩_3 − |↓⟩_2 ⊗ |↑⟩_3) / √2


∃! Casimir operator
S = [ρ(L1)]² + [ρ(L2)]² + [ρ(L3)]²
Poisson algebra

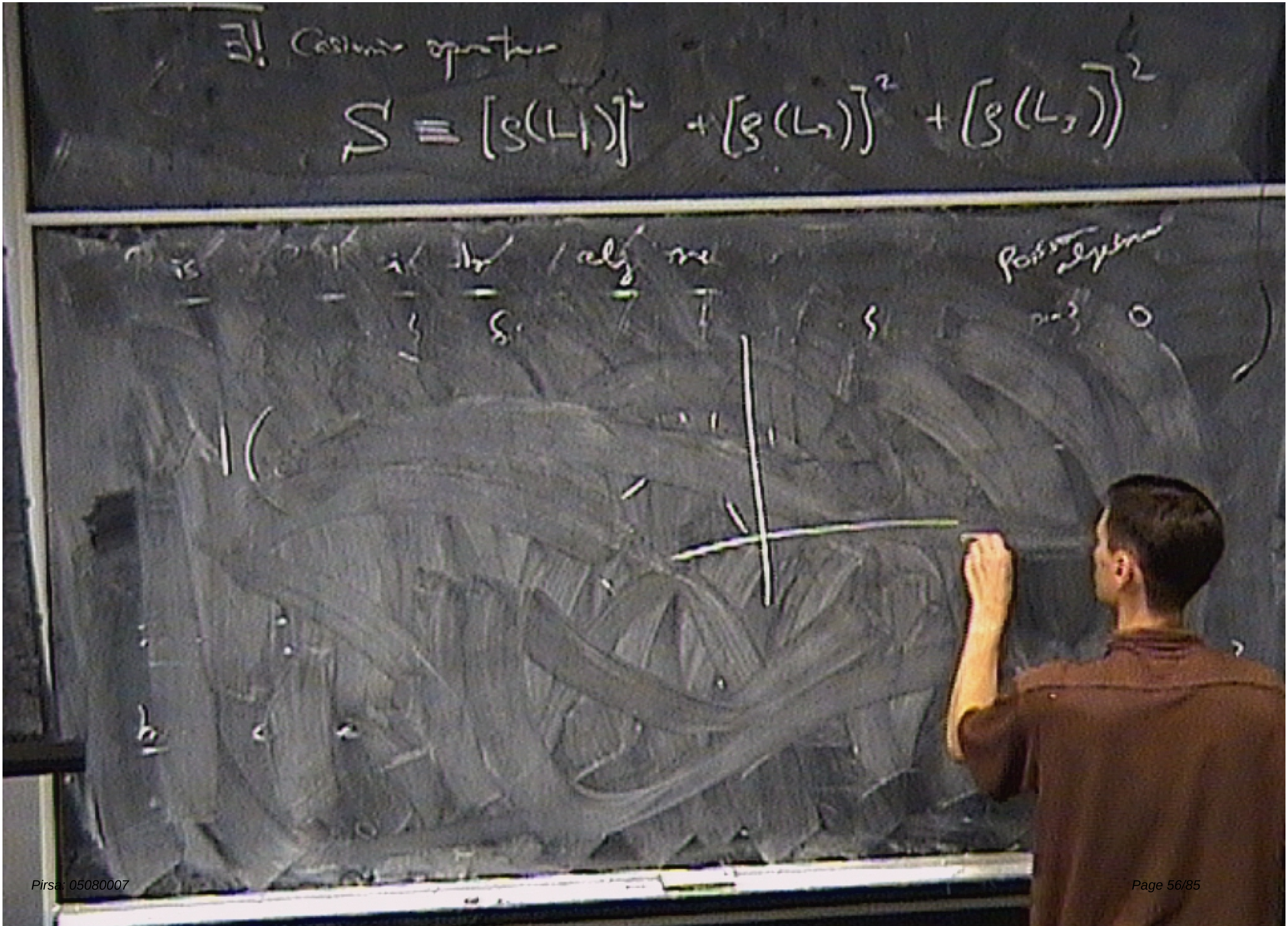

∃! Casimir operator
S ≡ [ξ(L1)]² + [ξ(L2)]² + [ξ(L3)]²
Poisson algebra
ξ = 0
(a|↑1⟩ + b|↓1⟩) ⊗ (/ √2)
ψ) < 1

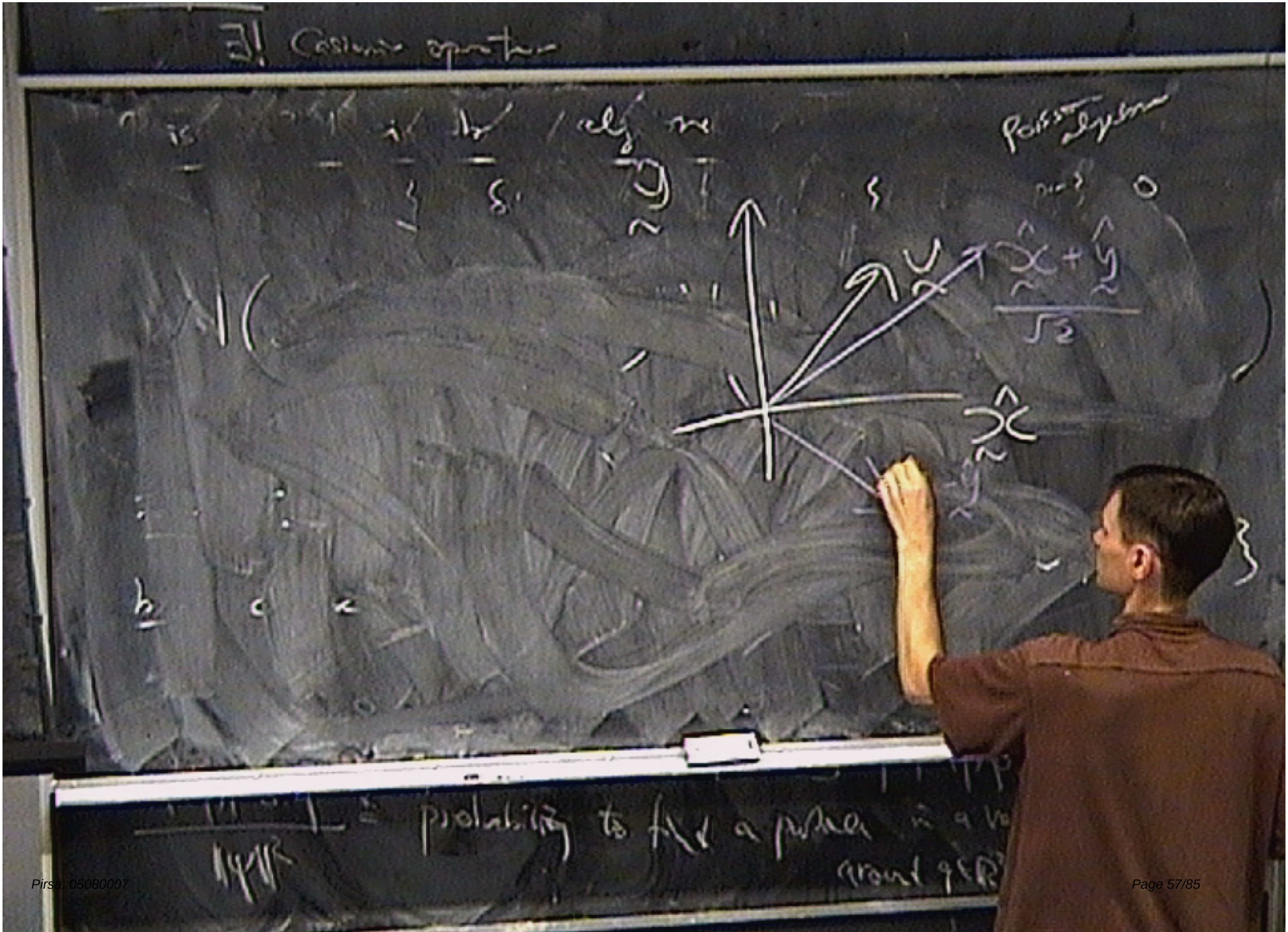
3) Canonical quantization

Poisson algebra

$$|\left(a|\uparrow\rangle_1 + b|\downarrow\rangle_1\right) \otimes \left(\frac{|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_2|\uparrow\rangle_3}{\sqrt{2}}\right)$$

$\frac{\;\;\;\;}{\|\psi\|^2}$ = probability to find a particle in a volume d^3q around $q \in \mathbb{R}^3$


∃! Casimir operator
S ≡ [ρ(L1)]² + [ρ(L2)]² + [ρ(L3)]²
Poisson algebra


Casimir operator
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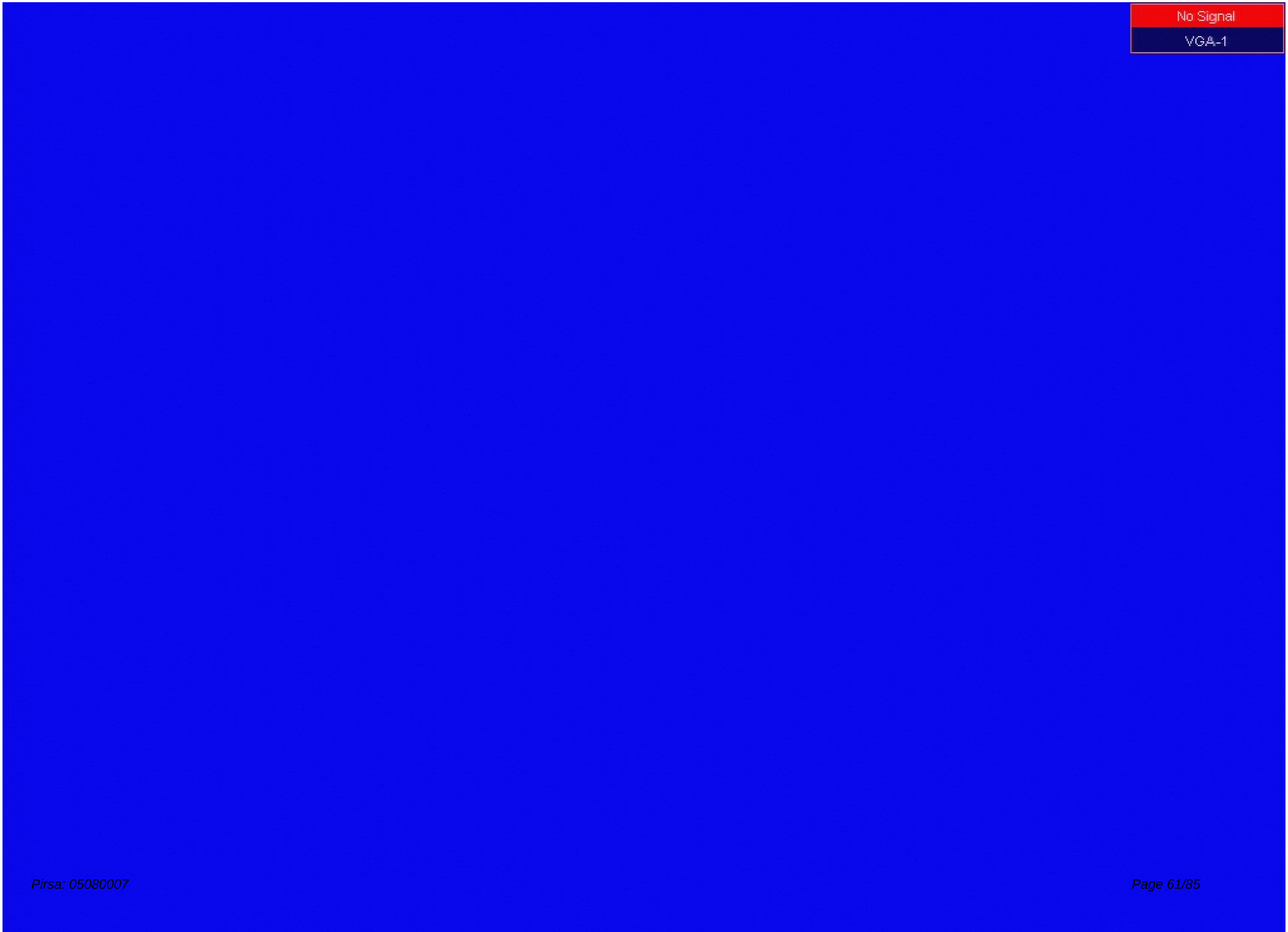
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$$|\Psi_{123}\rangle = \tfrac{1}{2}[|\Psi_{12}^{(-)}\rangle(-a|\uparrow_3\rangle - b|\downarrow_3\rangle) + |\Psi_{12}^{(+)}\rangle(-a|\uparrow_3\rangle + b|\downarrow_3\rangle) + |\Phi_{12}^{(-)}\rangle(a|\downarrow_3\rangle + b|\uparrow_3\rangle) + |\Phi_{12}^{(+)}\rangle(a|\downarrow_3\rangle - b|\uparrow_3\rangle)]. \qquad (5)$$

It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, respectively,

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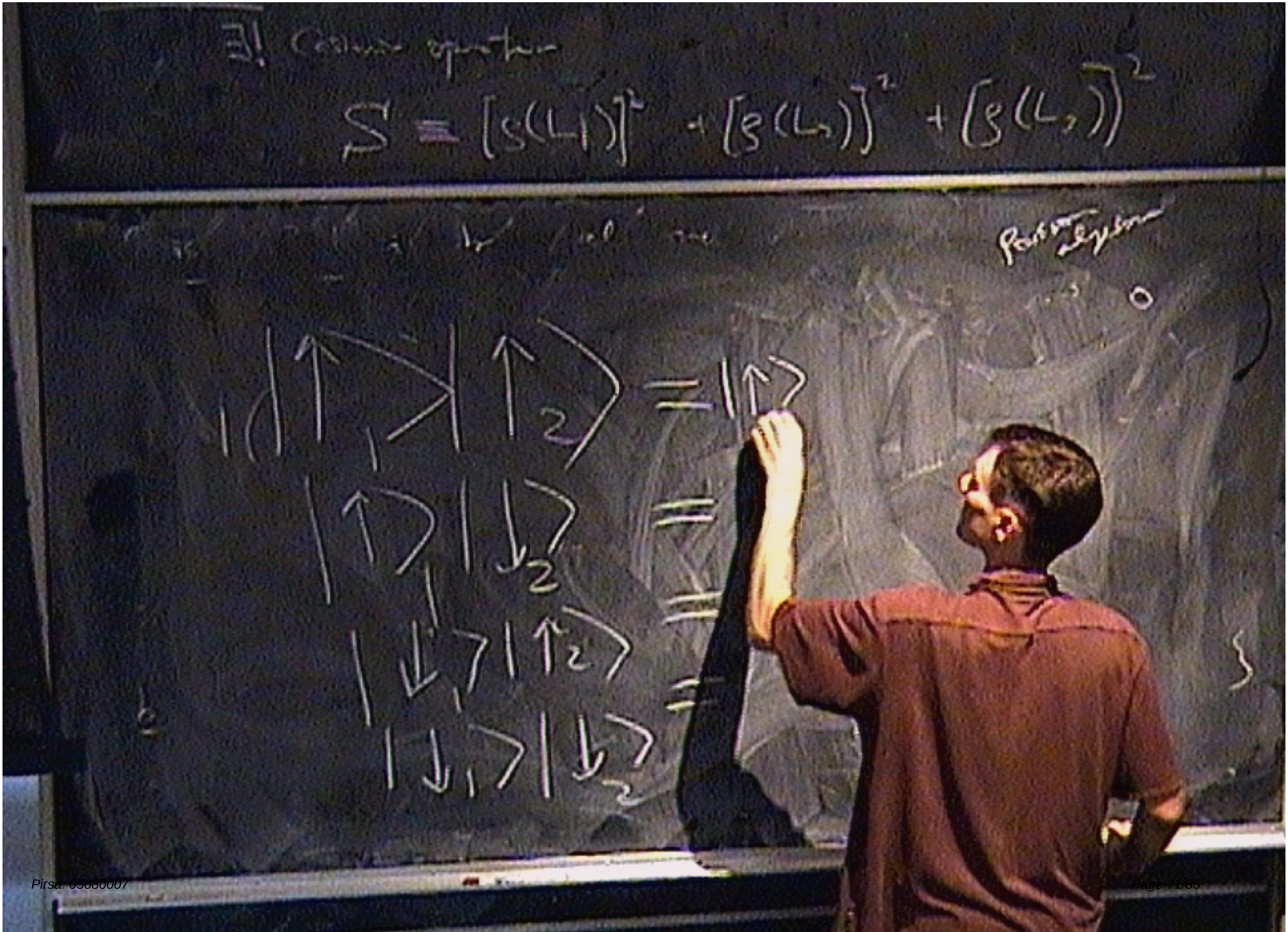
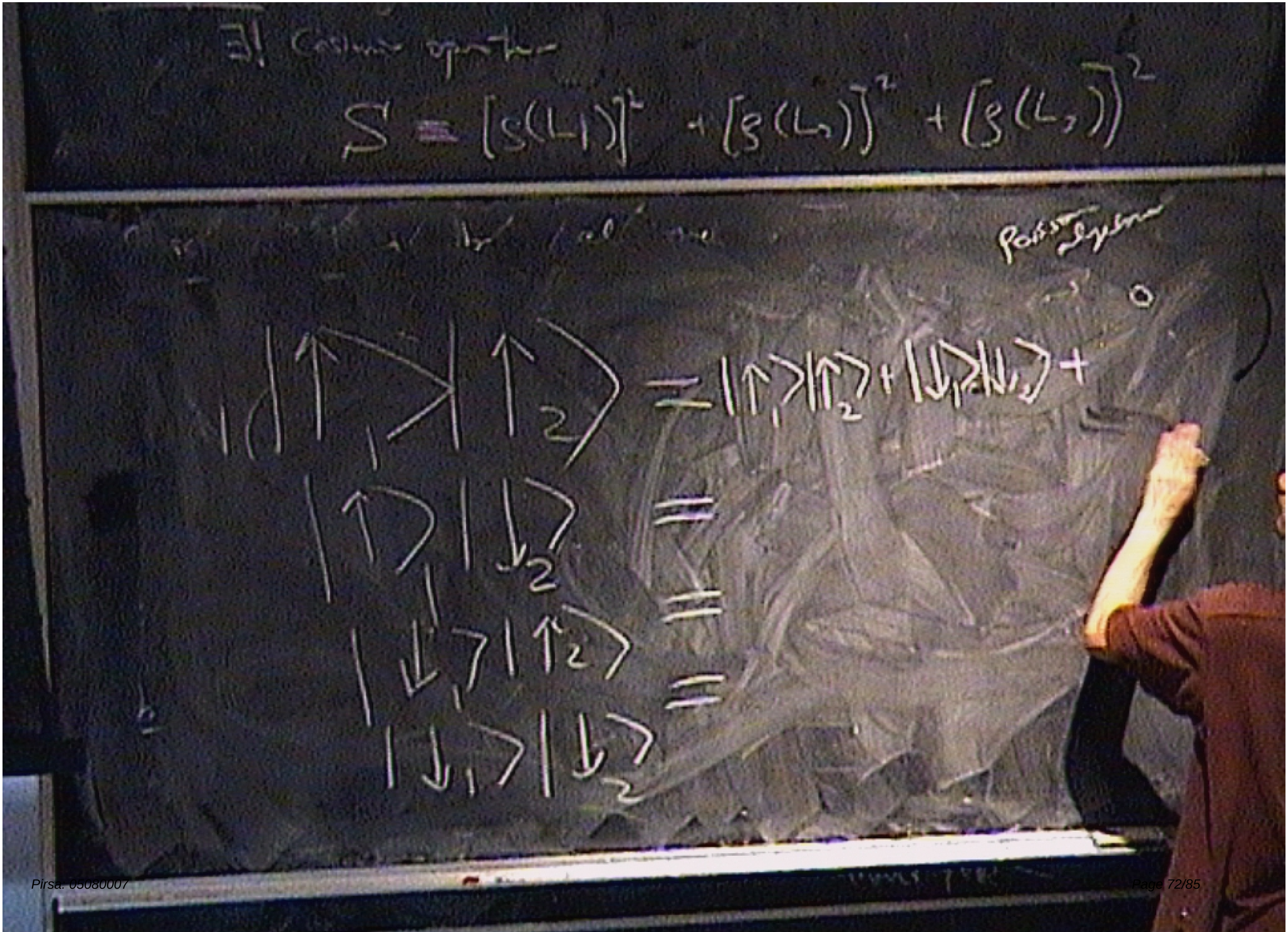
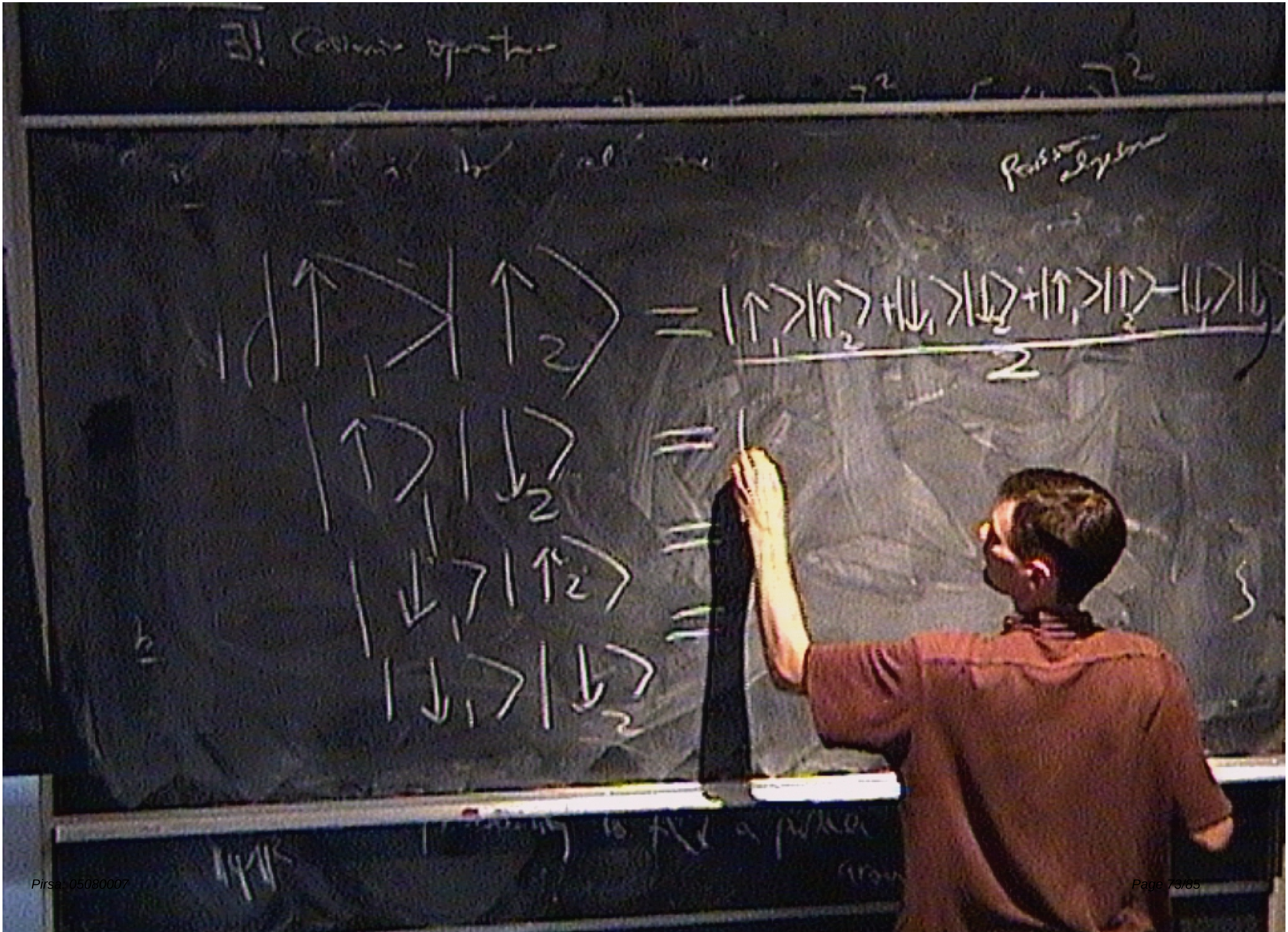
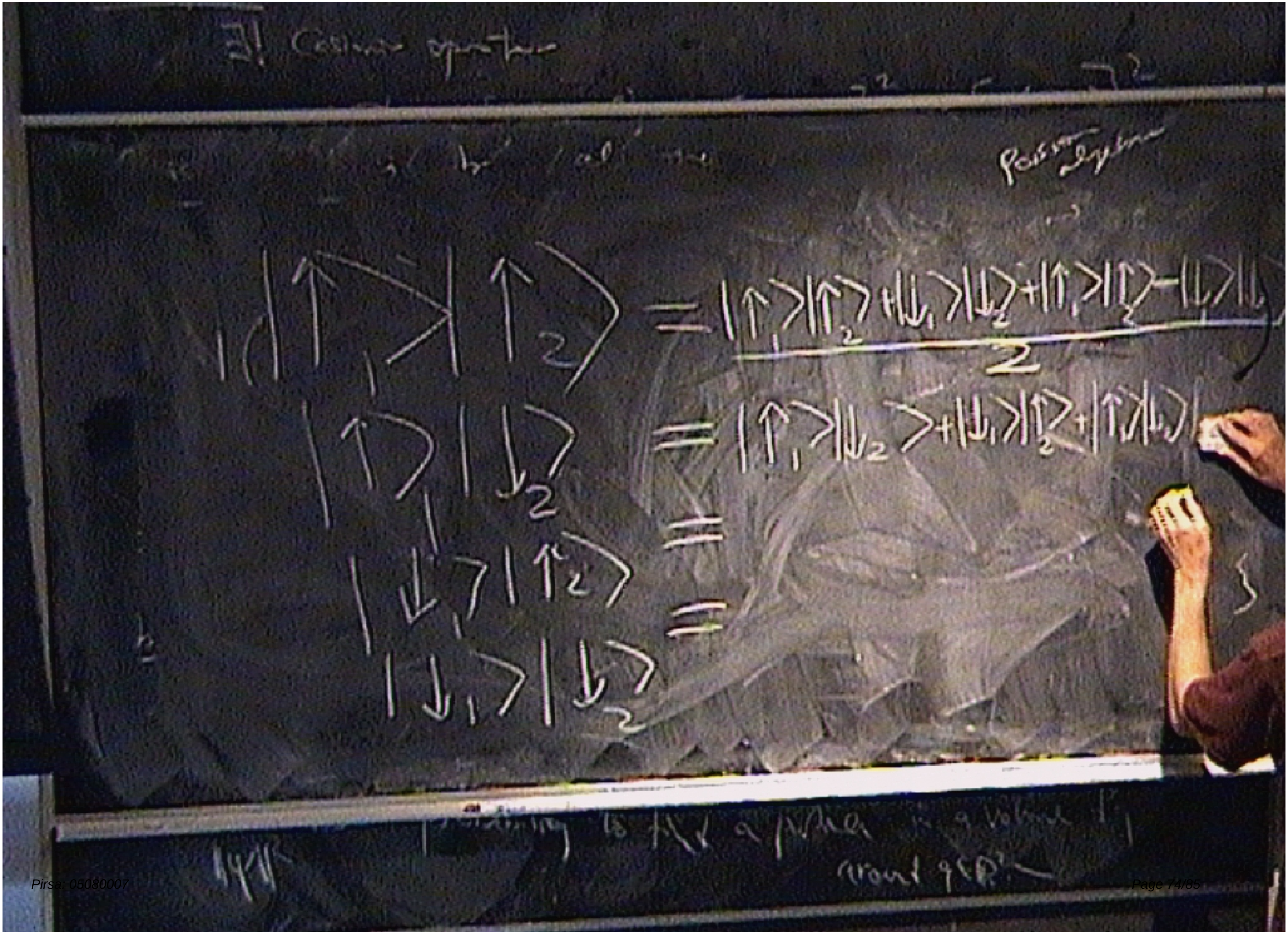

$S^2 = [S(L_1)]^2 + [S(L_2)]^2 + [S(L_3)]^2$
$|\uparrow\rangle_1 |\uparrow\rangle_2 =$
$|\uparrow\rangle_1 |\downarrow\rangle_2 =$
$|\downarrow\rangle_1 |\uparrow\rangle_2 =$
$|\downarrow\rangle_1 |\downarrow\rangle_2 =$


∃! Casimir operator
S = [ξ(L1)]² + [ξ(L2)]² + [ξ(L3)]²
Poisson algebra




Casimir operator
Poisson algebra

$$|\uparrow_1\rangle|\uparrow_2\rangle = \frac{|\uparrow_1\rangle|\uparrow_2\rangle + |\downarrow_1\rangle|\downarrow_2\rangle + |\uparrow_1\rangle|\uparrow_2\rangle - |\downarrow_1\rangle|\downarrow_2\rangle}{2}$$

$$|\uparrow_1\rangle|\downarrow_2\rangle = \frac{|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle + |\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle}{2}$$

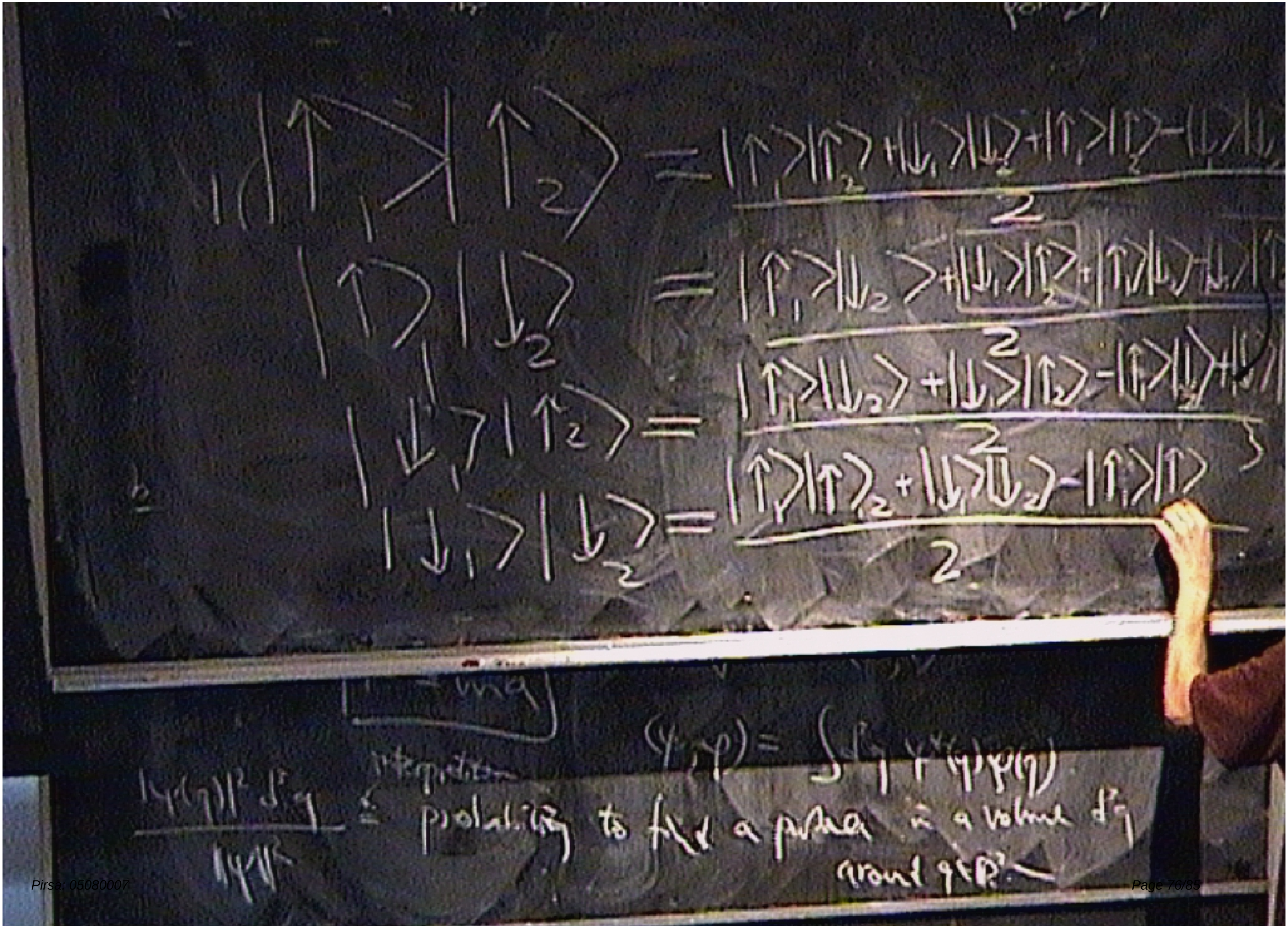
$$|\downarrow_1\rangle|\uparrow_2\rangle = \frac{|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle - |\uparrow_1\rangle|\downarrow_2\rangle + \ldots}{2}$$

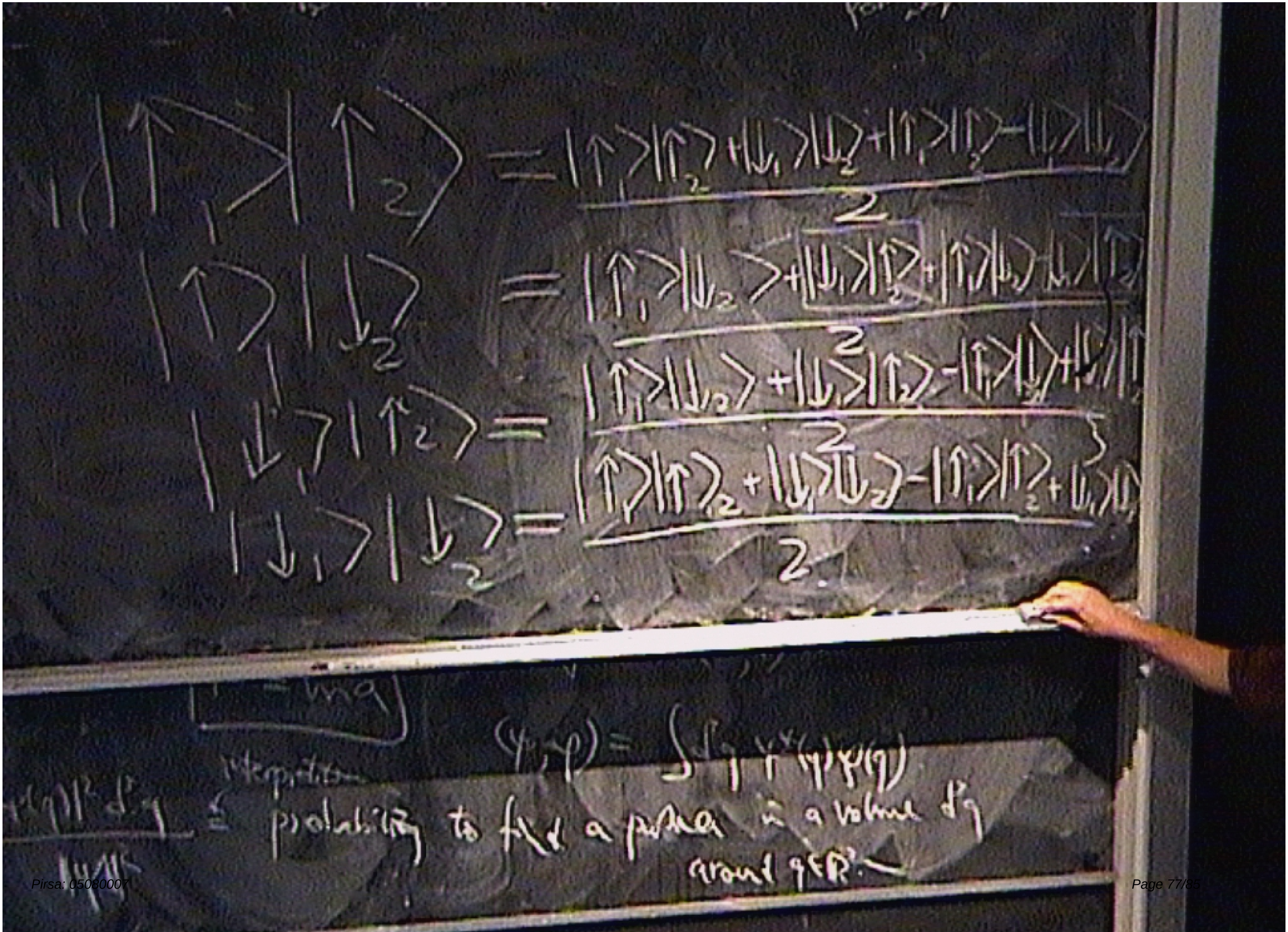
$$|\downarrow_1\rangle|\downarrow_2\rangle = \frac{\qquad}{2}$$

$$(\psi, \varphi) = \int d^3q \, \psi^*(q)\varphi(q)$$

interpretation

$$\frac{|\psi(q)|^2 d^3q}{\|\psi\|^2}$$ = probability to find a particle in a volume d^3q around $q \in \mathbb{R}^3$


|↑₁⟩|↑₂⟩ = (|↑₁⟩|↑₂⟩ + |↓₁⟩|↓₂⟩ + |↑₁⟩|↑₂⟩ − |↓₁⟩|↓₂⟩) / 2
|↑₁⟩|↓₂⟩ = (|↑₁⟩|↓₂⟩ + |↓₁⟩|↑₂⟩ + |↑₁⟩|↓₂⟩ − |↓₁⟩|↑₂⟩) / 2
|↓₁⟩|↑₂⟩ = (|↑₁⟩|↓₂⟩ + |↓₁⟩|↑₂⟩ − |↑₁⟩|↓₂⟩ + |↓₁⟩|↑₂⟩) / 2
|↓₁⟩|↓₂⟩ = (|↑₁⟩|↑₂⟩ + |↓₁⟩|↓₂⟩ − |↑₁⟩|↑₂⟩) / 2
(ψ, φ) = ∫d³q ψ*(q) φ(q)
|ψ(q)|² d³q / ||ψ||² ≙ probability to find a particle in a volume d³q around q ∈ ℝ³


(φ, ψ) = ∫ d³q φ*(q) ψ(q)
probability to find a particle in a volume d³q
around q ∈ ℝ³

tion from a pair of identically prepared particles [10].

The spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission. Below, we show how Alice can divide the full information encoded in $|\phi\rangle$ into two parts, one purely classical and the other purely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replica of $|\phi\rangle$. Of course Alice's original $|\phi\rangle$ is destroyed in the process, as it must be to obey the no-cloning theorem. We call the process we are about to describe teleportation, a term from science fiction meaning to make a person or object disappear while an exact replica appears somewhere else. It must be emphasized that our teleportation, unlike some science fiction versions, defies no physical laws. In particular, it cannot take place instantaneously or over a spacelike interval, because it requires, among other things, sending a classical message from Alice to Bob. The net result of teleportation is completely prosaic: the removal of $|\phi\rangle$ from Alice's hands and its appearance in Bob's hands a suitable time later. The only remarkable feature is that, in the interim, the information in $|\phi\rangle$ has been cleanly separated into classical and nonclassical parts. First we shall show how to teleport the quantum state $|\phi\rangle$ of a spin-$\frac{1}{2}$ particle. Later we discuss teleportation of more complicated states.

The nonclassical part is transmitted first. To do so, two spin-$\frac{1}{2}$ particles are prepared in an EPR singlet state

$$|\Psi_{23}^{(-)}\rangle = \sqrt{\tfrac{1}{2}}\,(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle). \qquad (1)$$

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state $|\phi\rangle$ she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra as a spin.

One EPR particle (particle 2) is given to Alice, while the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about $|\phi\rangle$. Indeed the entire system, comprising Alice's unknown particle 1 and the EPR pair, is in a pure product state, $|\phi_1\rangle\,|\Psi_{23}^{(-)}\rangle$, involving neither classical correlation nor quantum entanglement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about $|\phi\rangle$. An entanglement between these two subsystems is brought about in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle 1 and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of $|\Psi_{12}^{(-)}\rangle$ and

$$|\Psi_{12}^{(+)}\rangle = \sqrt{\tfrac{1}{2}}\,(|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle),$$
$$|\Phi_{12}^{(\pm)}\rangle = \sqrt{\tfrac{1}{2}}\,(|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_1\rangle|\downarrow_2\rangle). \qquad (2)$$

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

$$|\phi_1\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle, \qquad (3)$$

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle) + \frac{b}{\sqrt{2}}(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle). \qquad (4)$$

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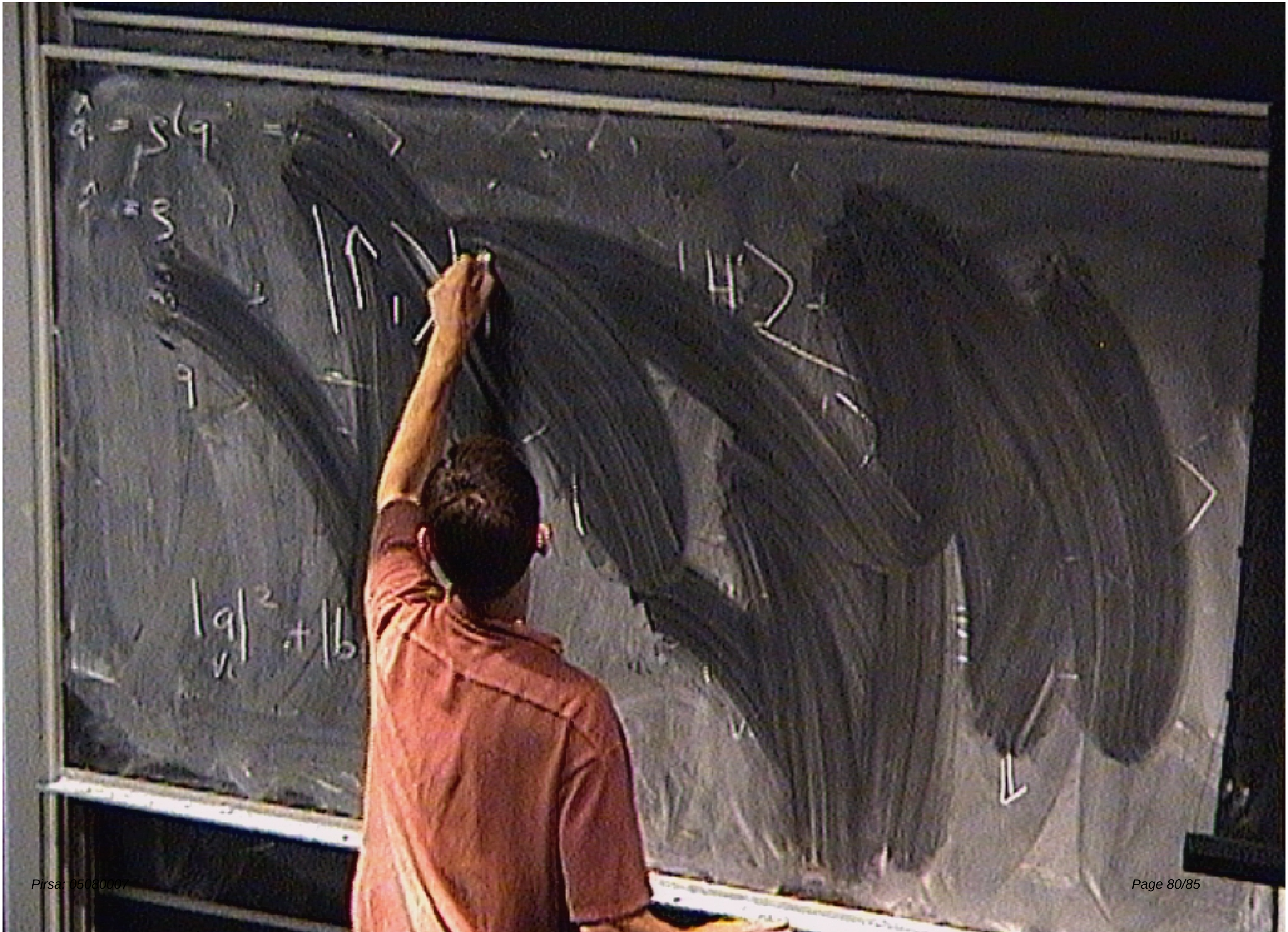
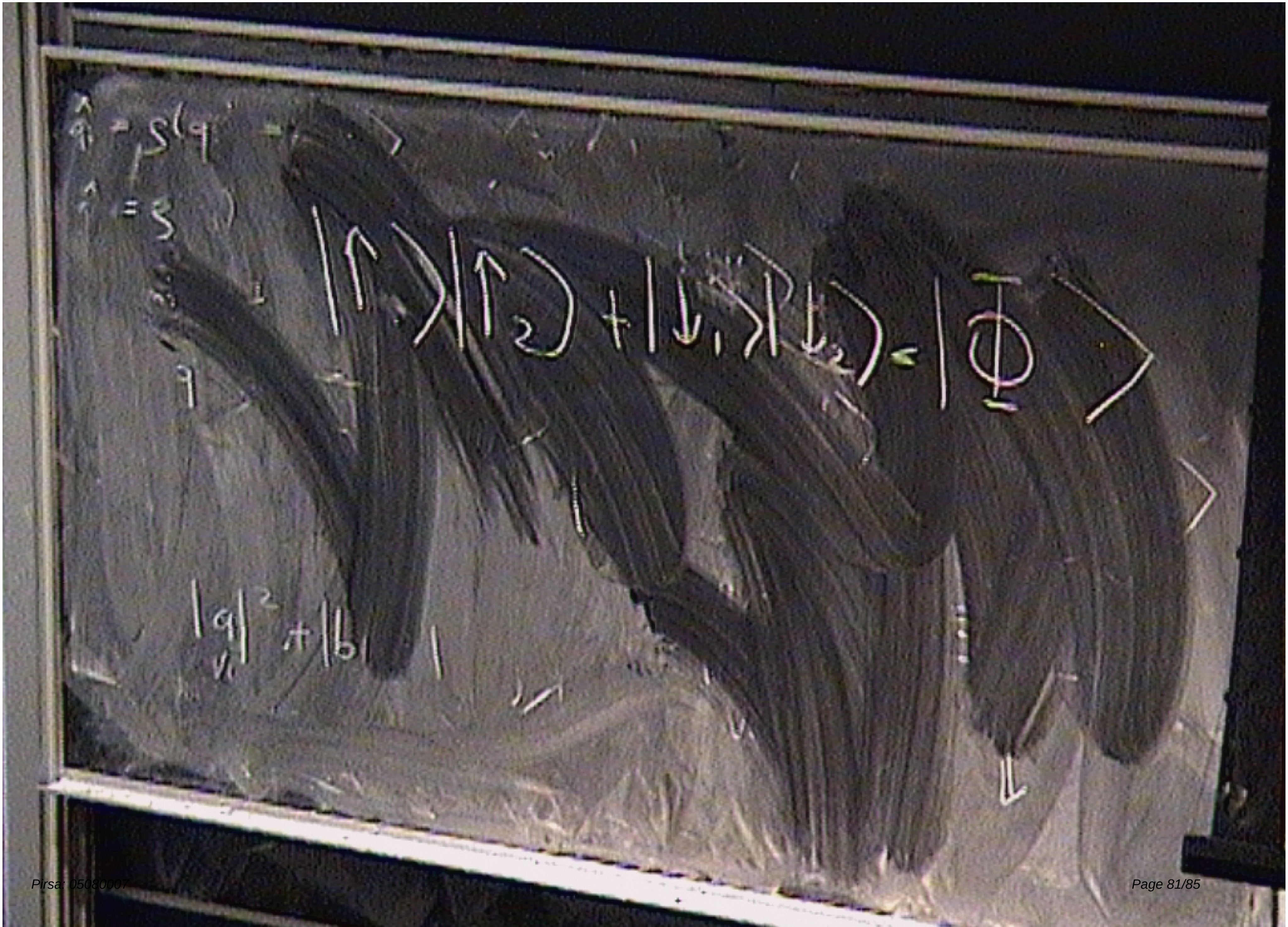
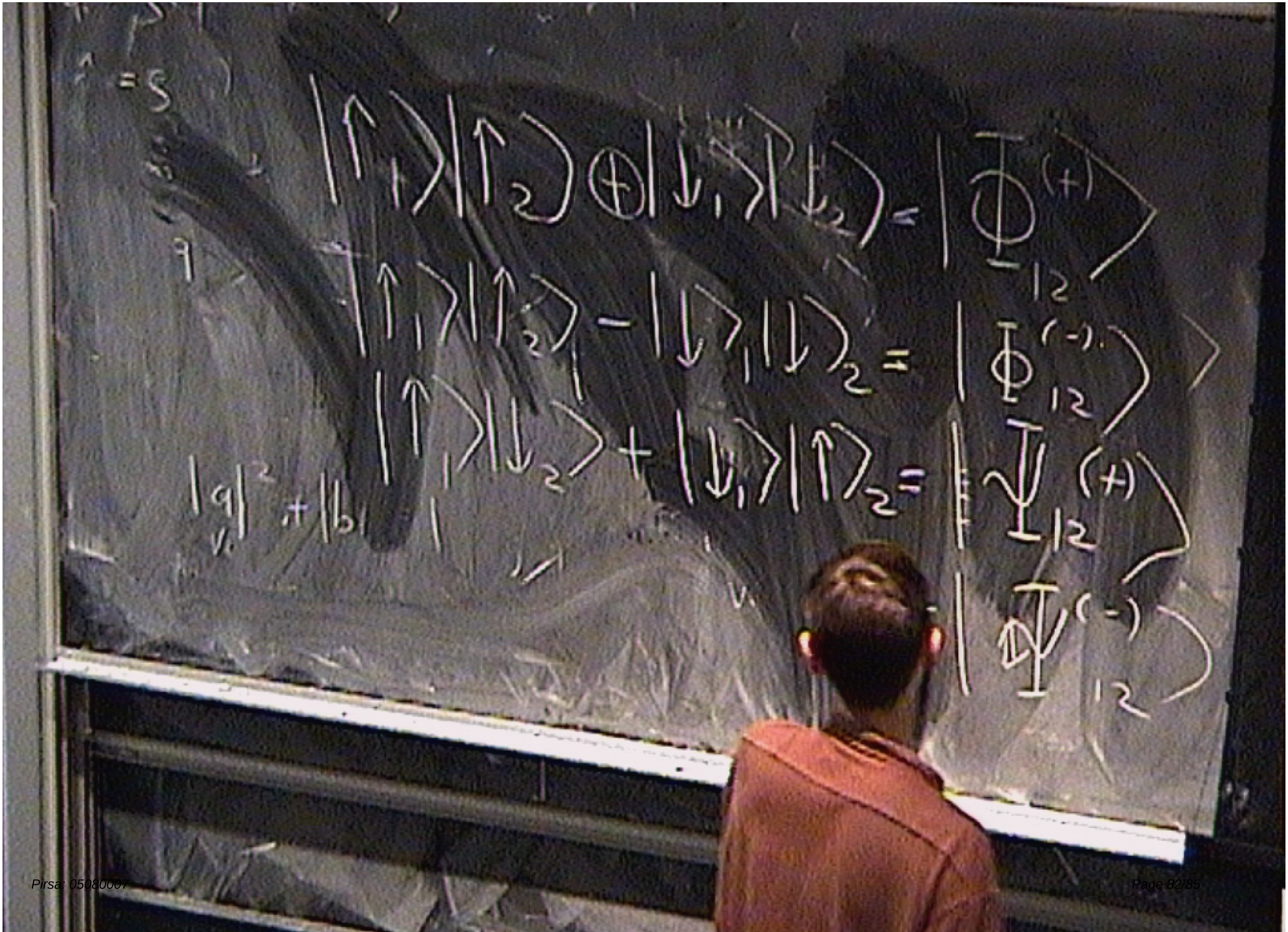
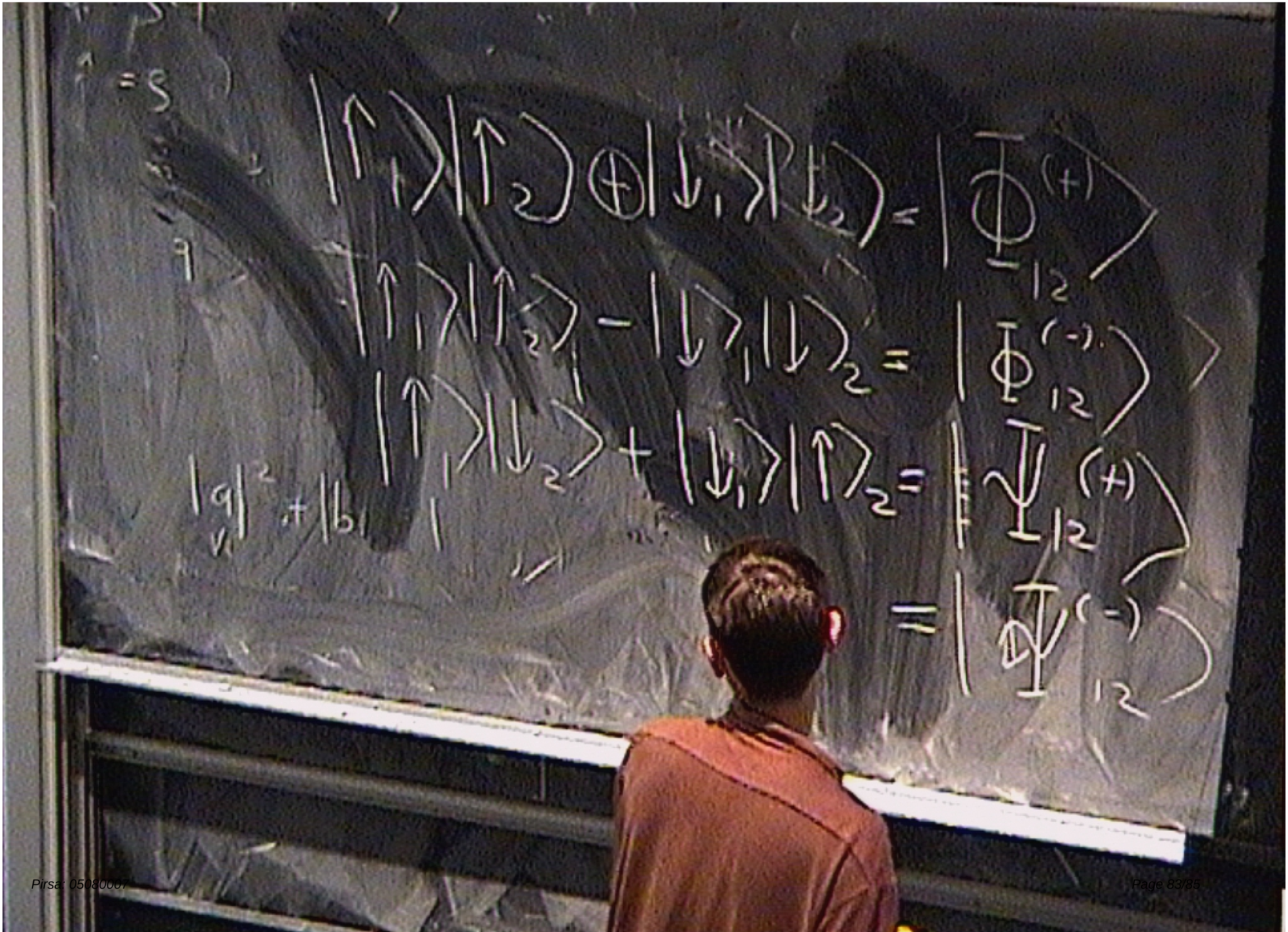
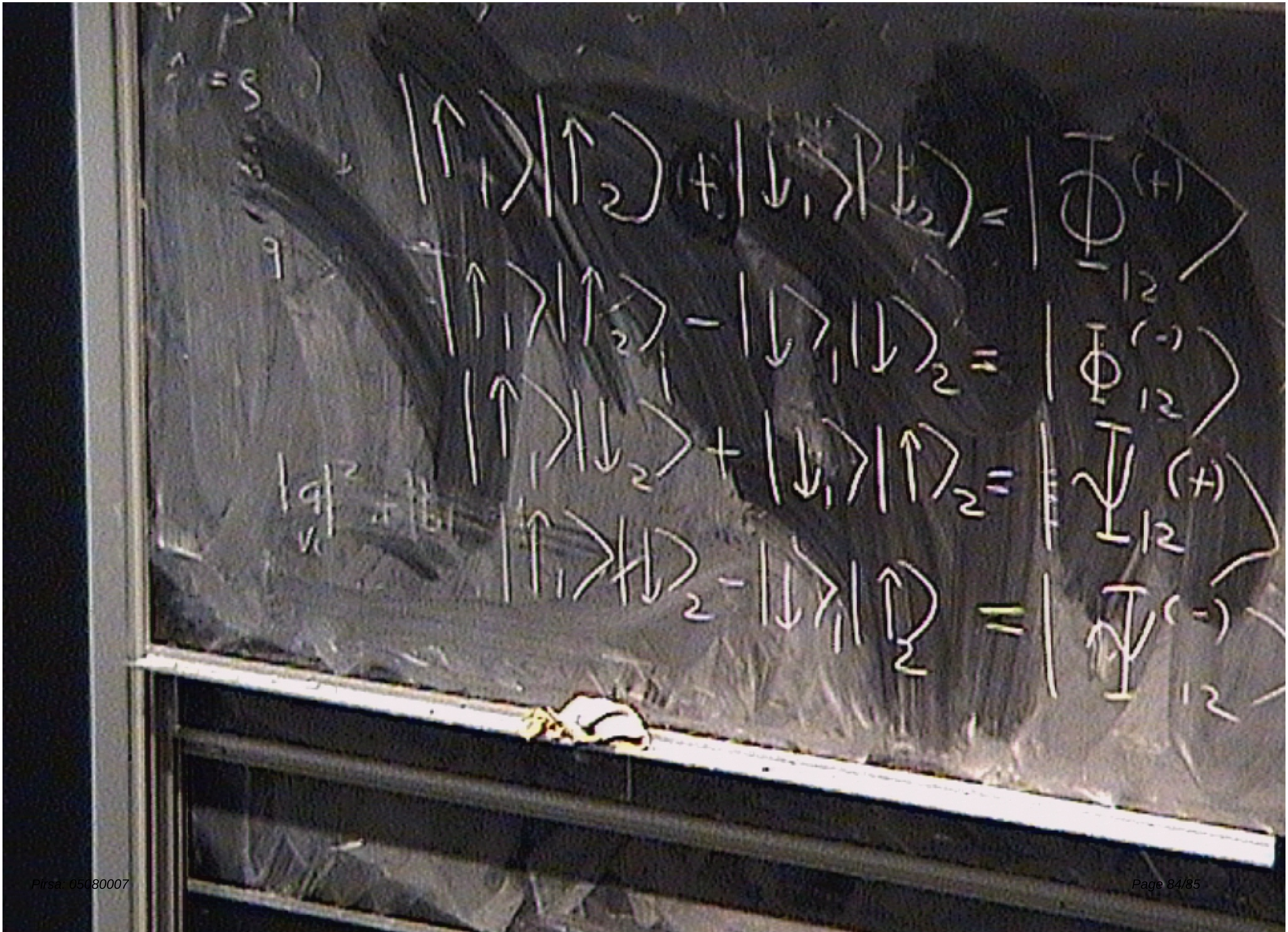
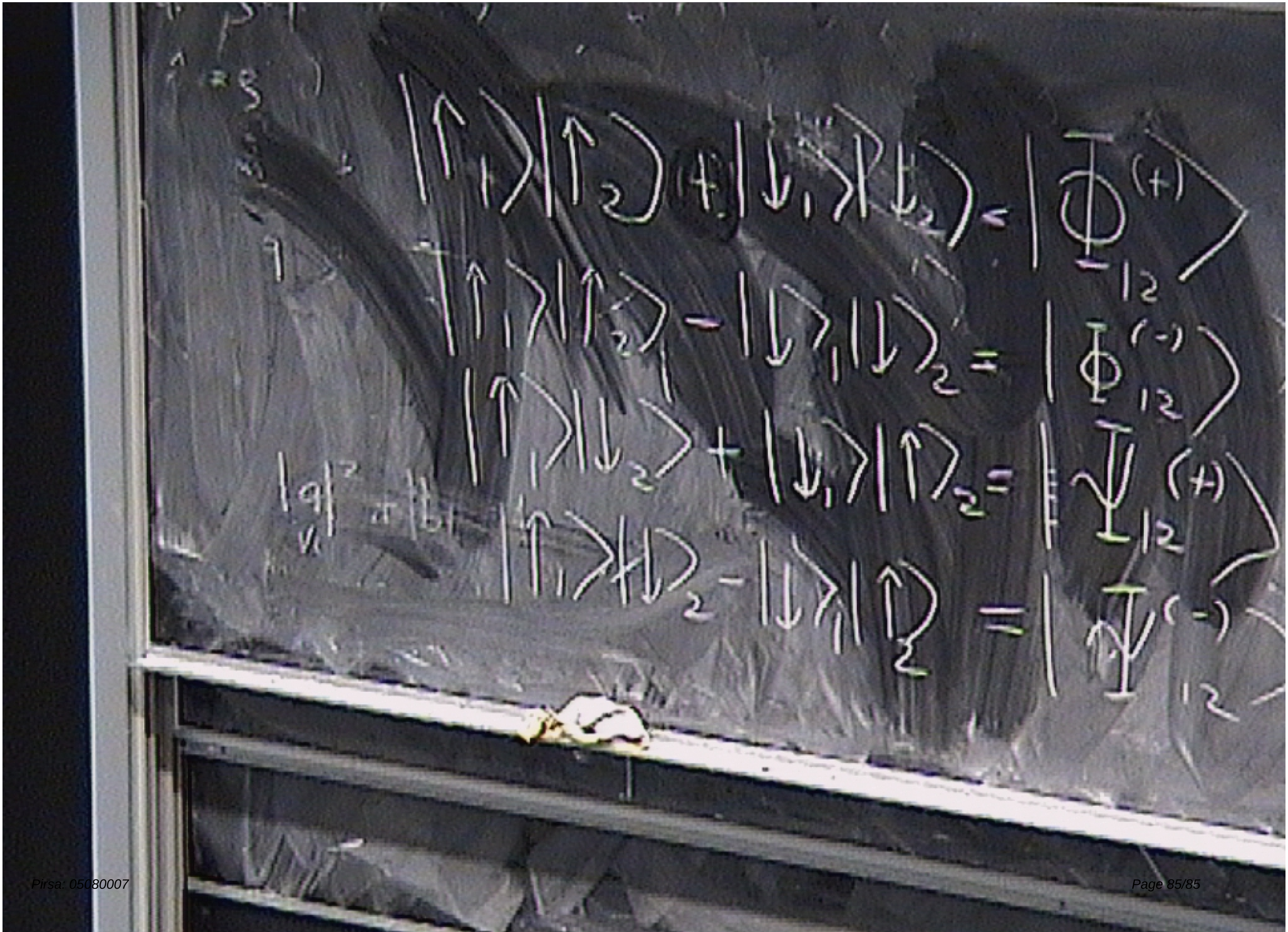





$|\uparrow_1\rangle|\uparrow_2\rangle \oplus |\downarrow_1\rangle|\downarrow_2\rangle = |\Phi^{(+)}_{12}\rangle$
$|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2 = |\Phi^{(-)}_{12}\rangle$
$|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2 = |\Psi^{(+)}_{12}\rangle$
$|\Psi^{(-)}_{12}\rangle$
$|a|^2 + |b|$


|↑⟩₁|↑⟩₂ ⊕ |↓⟩₁|↓⟩₂ = |Φ^(+)₁₂⟩
|↑⟩₁|↑⟩₂ − |↓⟩₁|↓⟩₂ = |Φ^(−)₁₂⟩
|↑⟩₁|↓⟩₂ + |↓⟩₁|↑⟩₂ = |Ψ^(+)₁₂⟩
= |Ψ^(−)₁₂⟩
|a|² + |b|


$|\uparrow_1\rangle|\uparrow_2\rangle + |\downarrow_1\rangle|\downarrow_2\rangle = |\Phi^{(+)}_{12}\rangle$
$|\uparrow_1\rangle|\uparrow_2\rangle - |\downarrow\rangle_1|\downarrow\rangle_2 = |\Phi^{(-)}_{12}\rangle$
$|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow\rangle_2 = |\Psi^{(+)}_{12}\rangle$
$|\uparrow_1\rangle|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2 = |\Psi^{(-)}_{12}\rangle$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