Title: Science Fiction Recipe for Teleportation

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Abstract:

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Science Fiction Recipe for Teleportation



- 1. A machine scans the space explorer to find out everything about her. Eg. her height, her mass, what shoes she is wearing, the colour of her eyes etc.
- 2. It then sends this information to an nearby uncharted planet.
- 3. On the planet's surface, a receiving machine takes in the information & uses it to construct a perfect copy of the astronaut.



 Essence of teleporting: constructing a perfect copy of an object at a distant location without sending the object itself



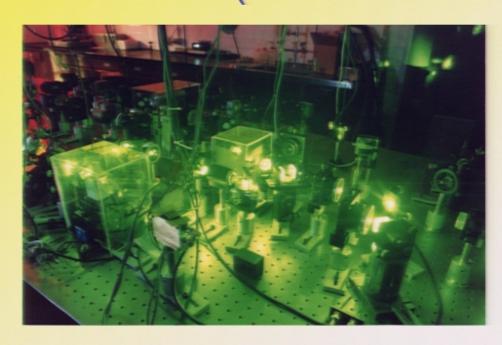
- A problem: Heisenberg's uncertainty principle prevents us from knowing everything about an object
- Quantum teleportation circumvents this challenge
- KEY POINT: We do not actually teleport the object itself.
- teleport its properties & so get a perfect copy of it



1998 & 2002: Caltech, USA & Canberra, Australia: A laser beam (About 50 cms)







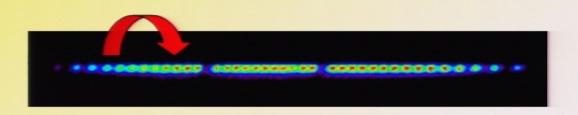
"Beam me up, mate" --- CNN



2004: Innsbruck, Austria & Boulder, Colarado, USA teleporting atoms: c. 1 mm









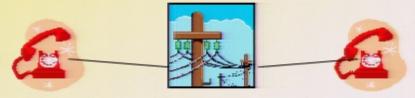
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Quantum teleportation in three easy steps

 To transmit something from A to B, we always need a means or route by which to send it.

Eg. a phone line, a wireless connection

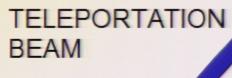


 To teleport the properties of a laser beam from A to B, we use two routes (a.k.a. channels).

1st CHANNEL

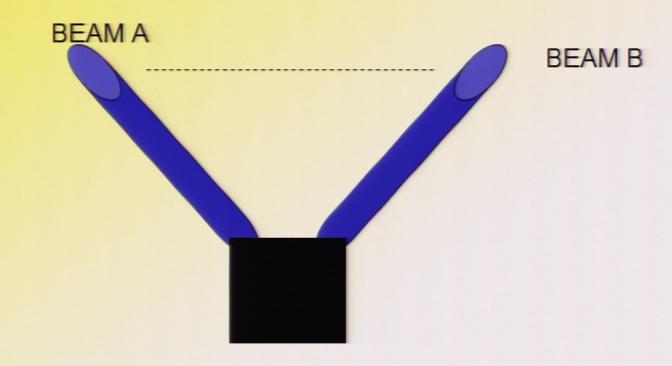


 any standard means of communication, such as a telephone.





- 2nd CHANNEL
- Two laser beams sharing entanglement.





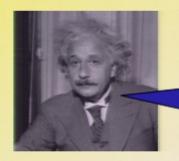
WHAT IS ENTANGLEMENT?

- If two quantum particles (or beams) are entangled with each other then they are instantaneously linked no matter how far apart they are
- Act as if they are a single object
- "Entanglement means that the left hand knows what the right hand is doing even when the hands at opposite ends of the universe."

 If two dice were entangled then they would always roll the same numbers if thrown at the same time at opposite ends of the universe



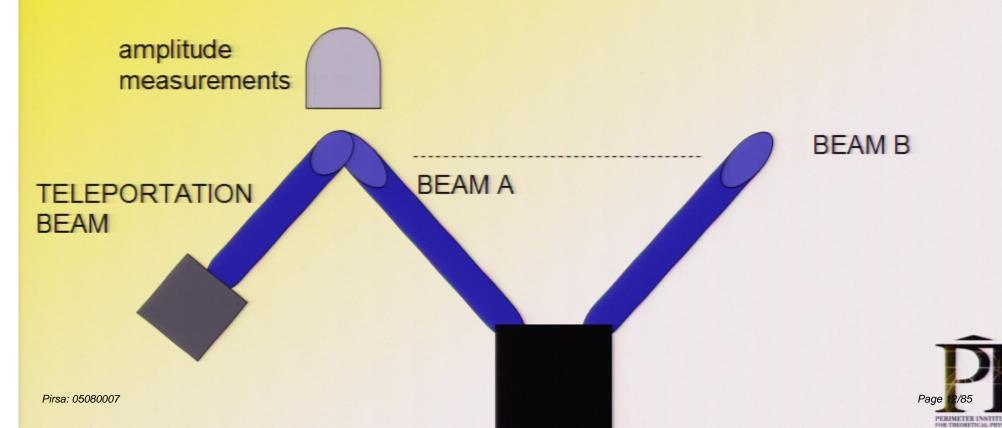


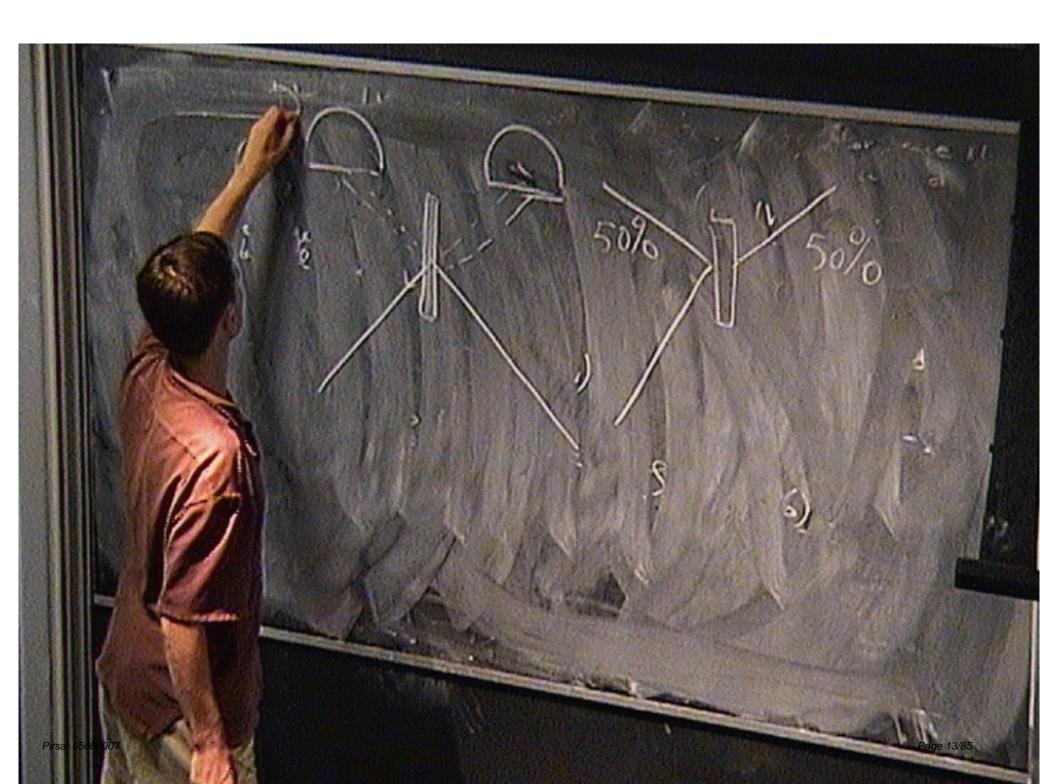


"[Entanglement is] spooky action at a distance."

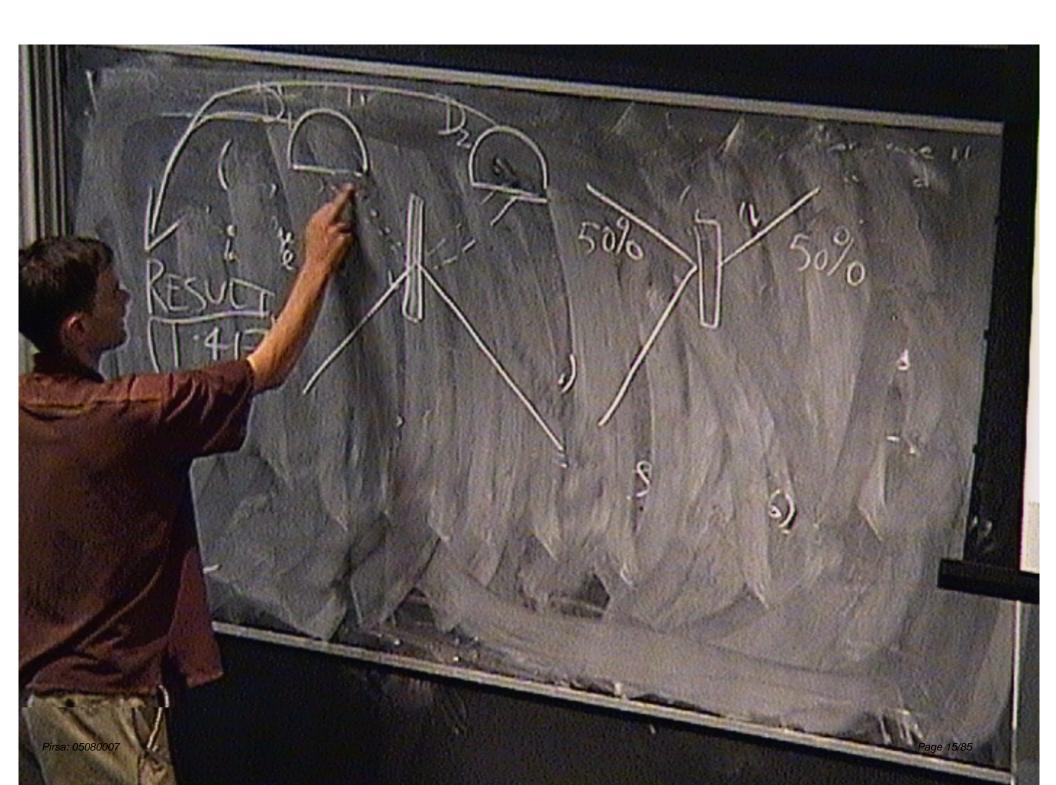


 STEP 1: Combine the beam we wish to teleport with beam A & then measure two components of the overall amplitude

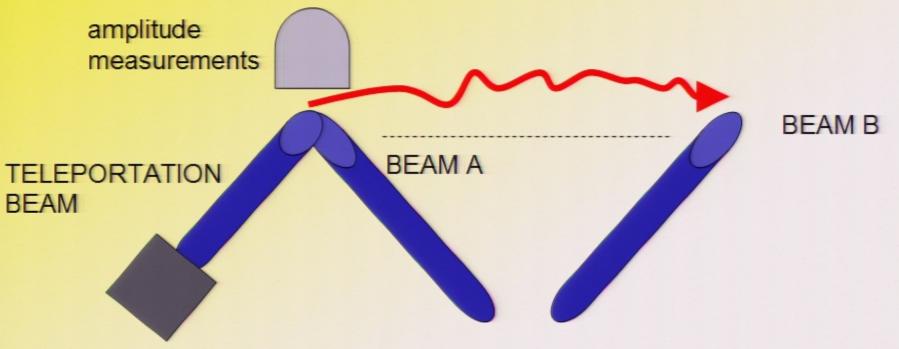








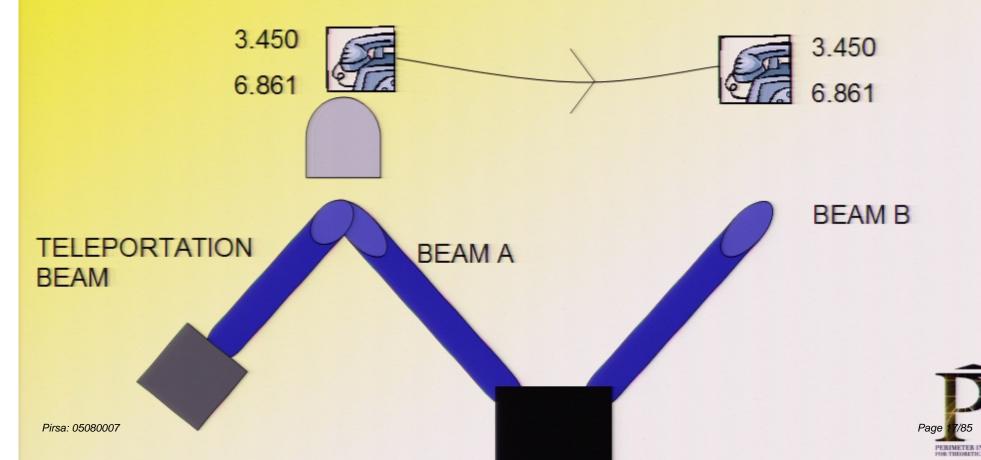
 A natural way to think of this is to say that it instantaneously (nonlocally) 'zaps' information about the teleportation beam to beam B.



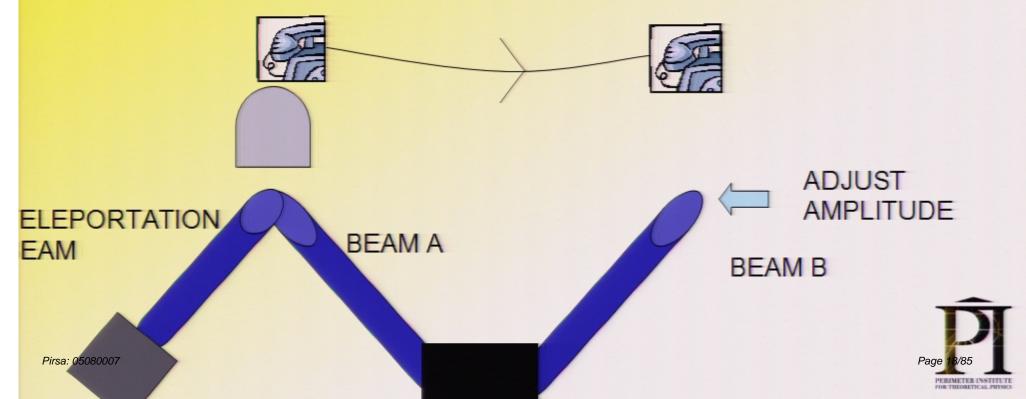


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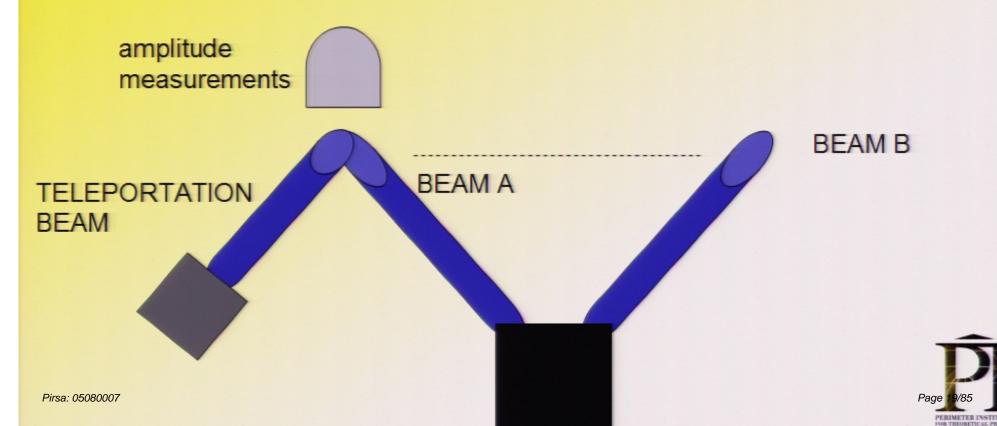
 STEP 2: Use the 1st channel to send the result of the amplitude measurements to B.



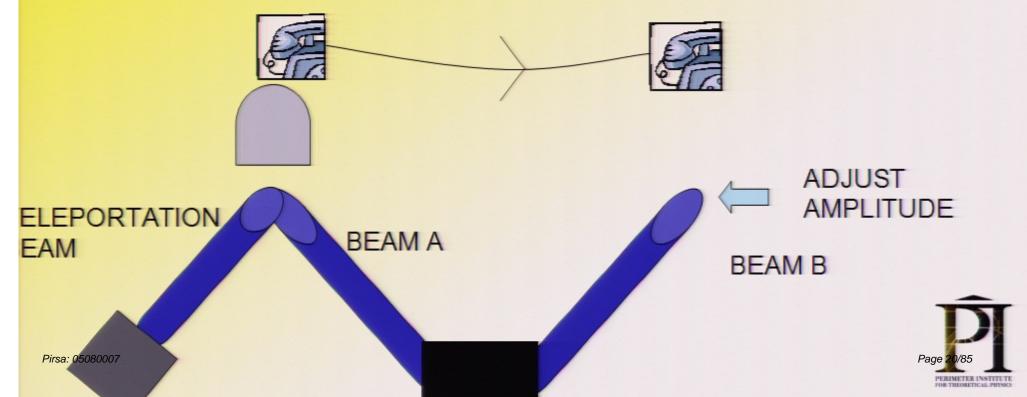
 STEP 3: Adjust the amplitude of beam B, depending on the information received. As if by magic, beam B is now identical to the beam we initially wished to teleport. The teleportation is complete!



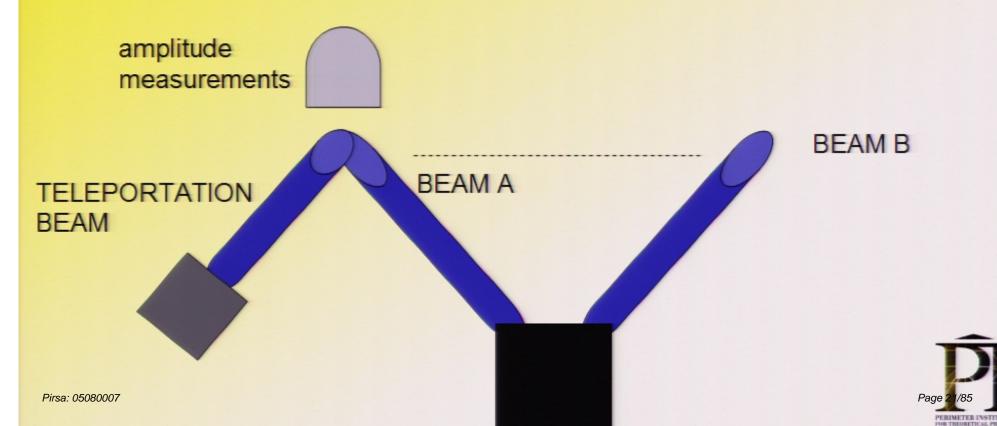
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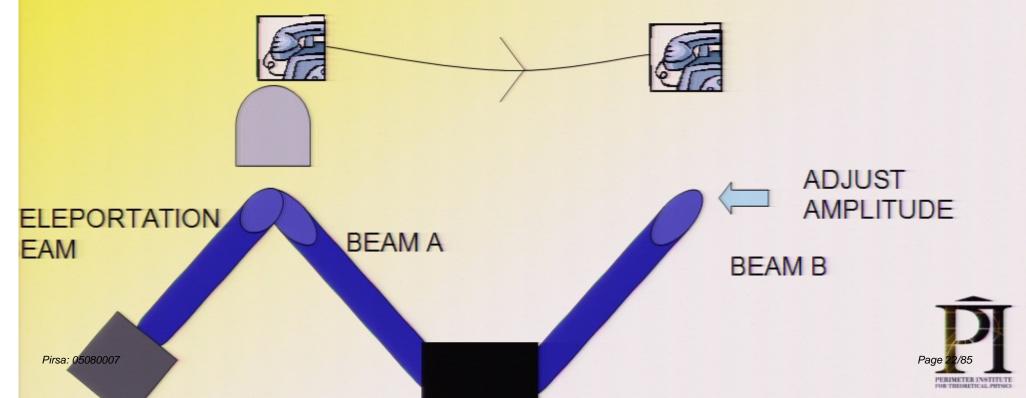
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A Comparison

Star-Trek Teleportation	Quantum Teleportation
Instantaneous?	Takes a small amount of time
The object itself is teleported.	Only the structure or properties are teleported (i.e. information)
Done with people.	Done with photons & atoms
The original is destroyed in the process.	The original is destroyed in the process
Can teleport to an uncharted planet.	Need to have something set up at the teleportation destination.
Done over thousands of wilemetres.	Done over 600 metres at most



What might we teleport in the future?

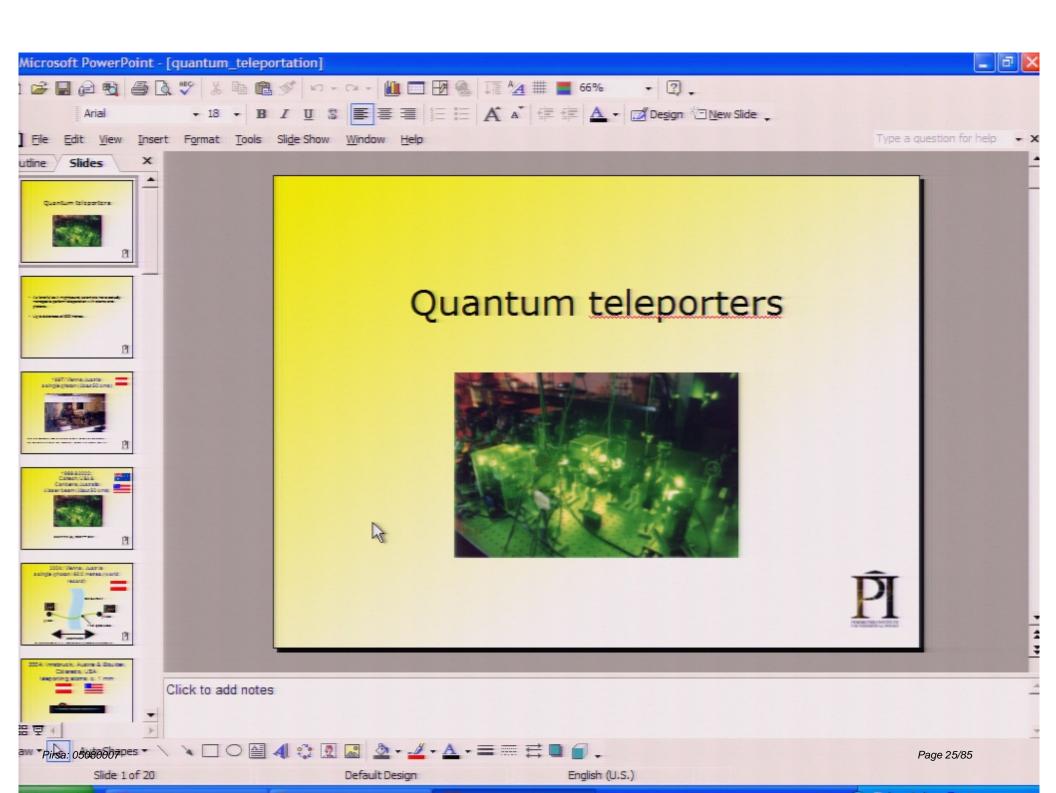
 Teleporting a grain of sand? One hundred years, perhaps.

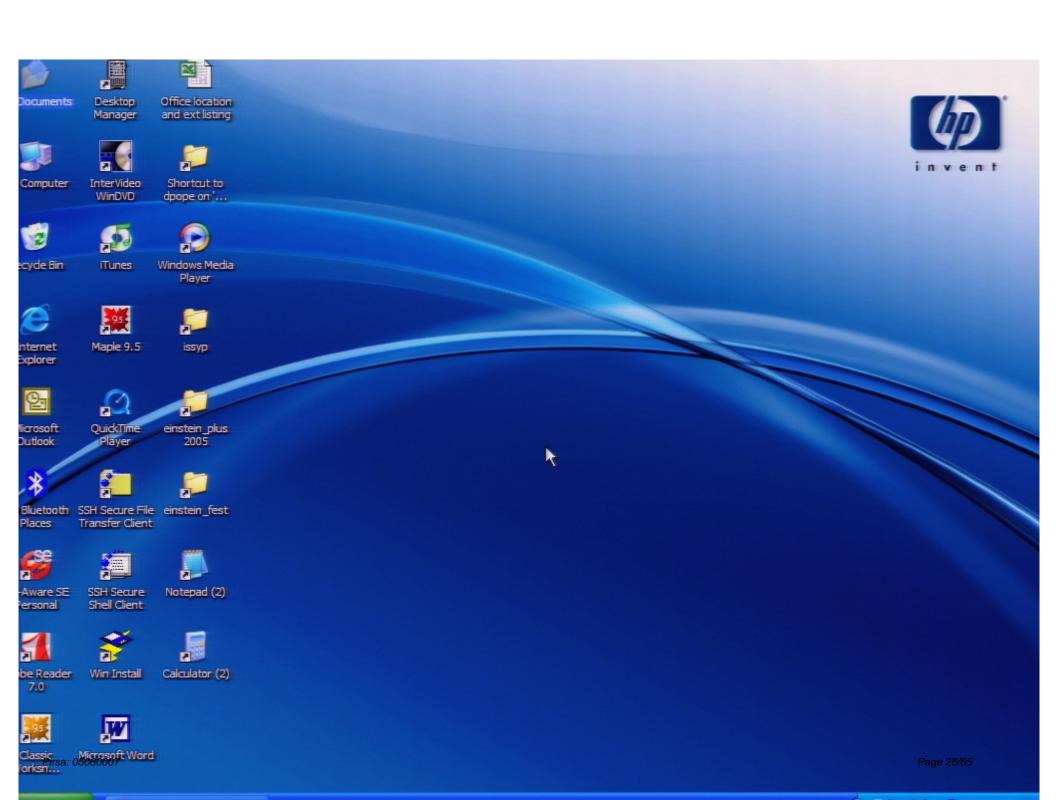


Teleporting a person? Who knows?









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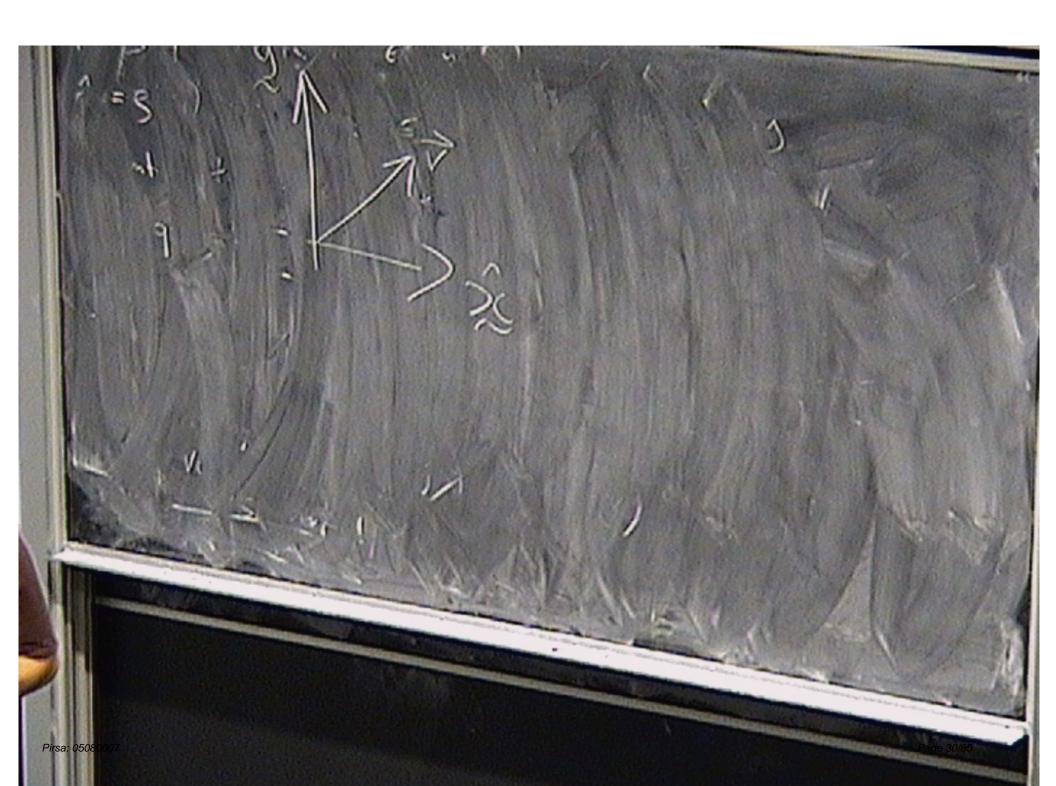
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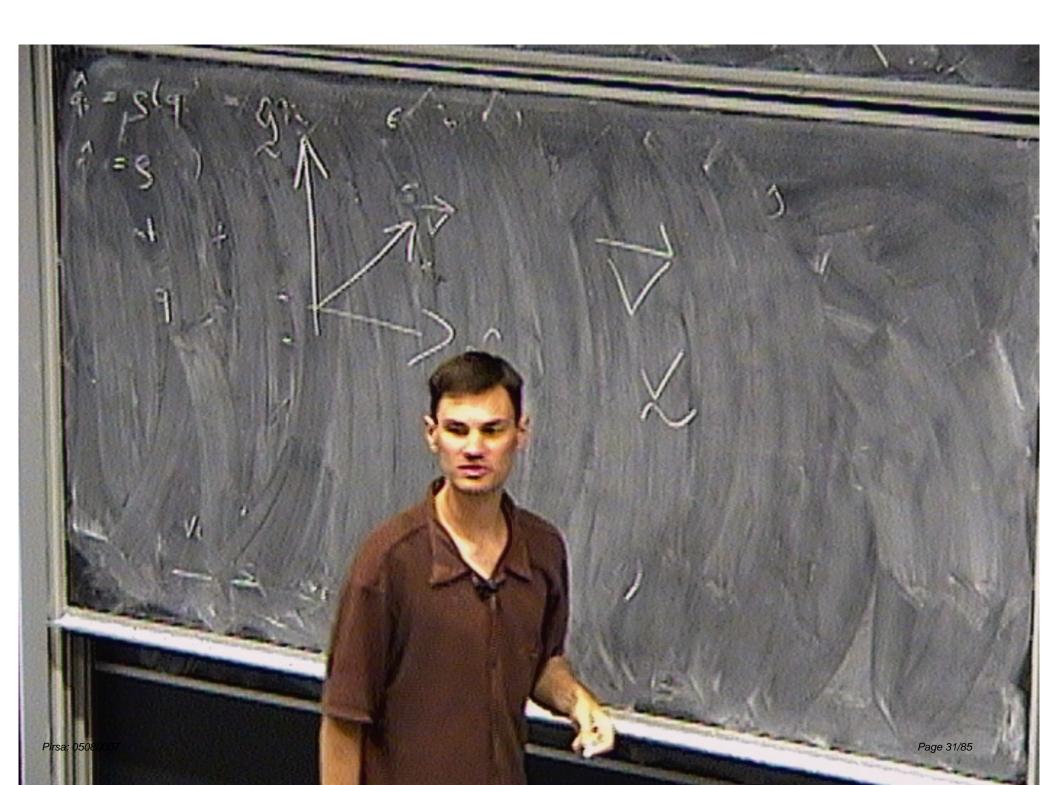
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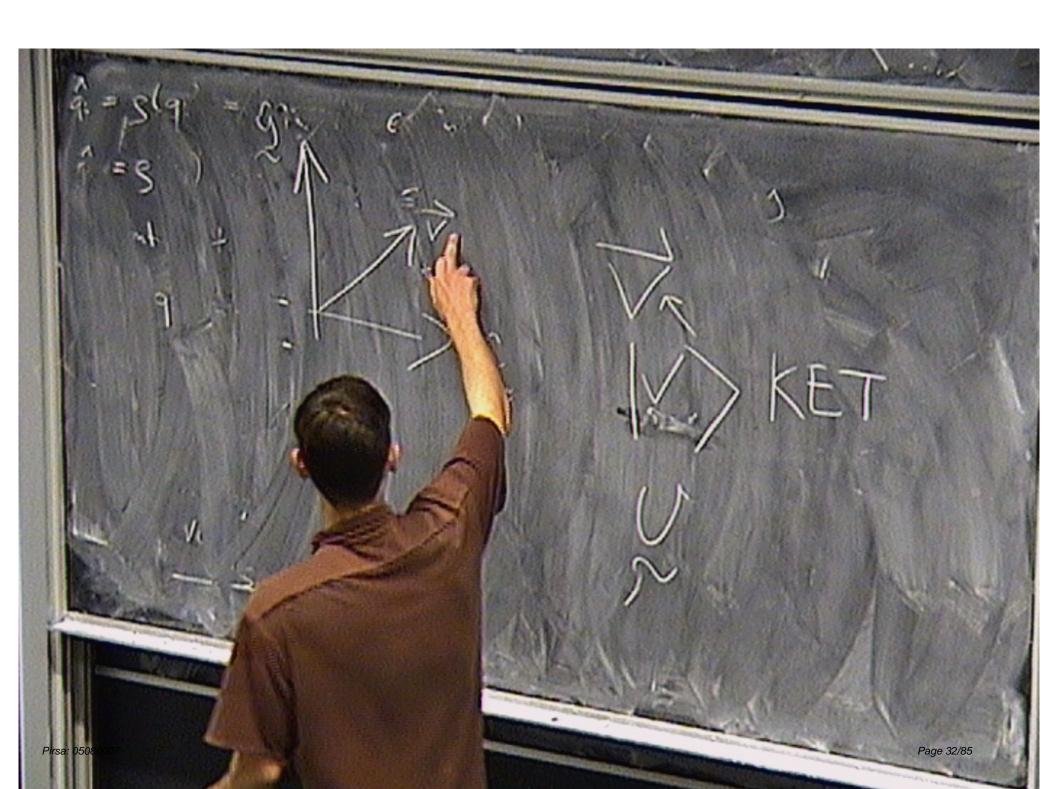
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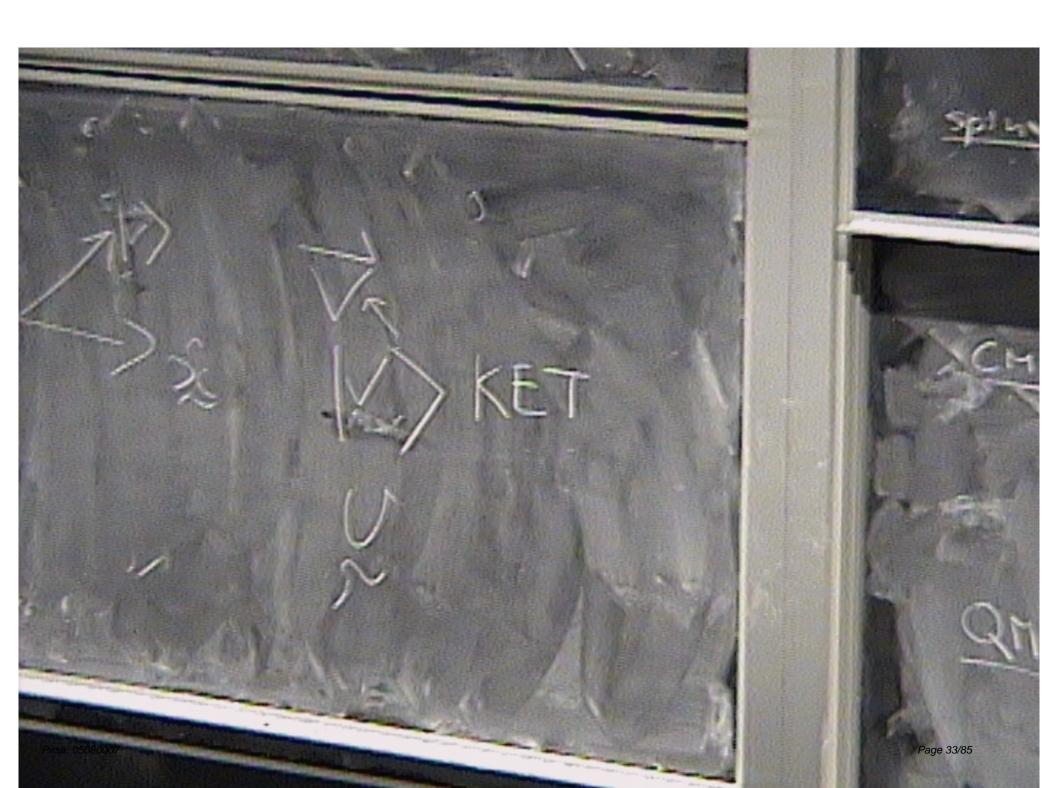
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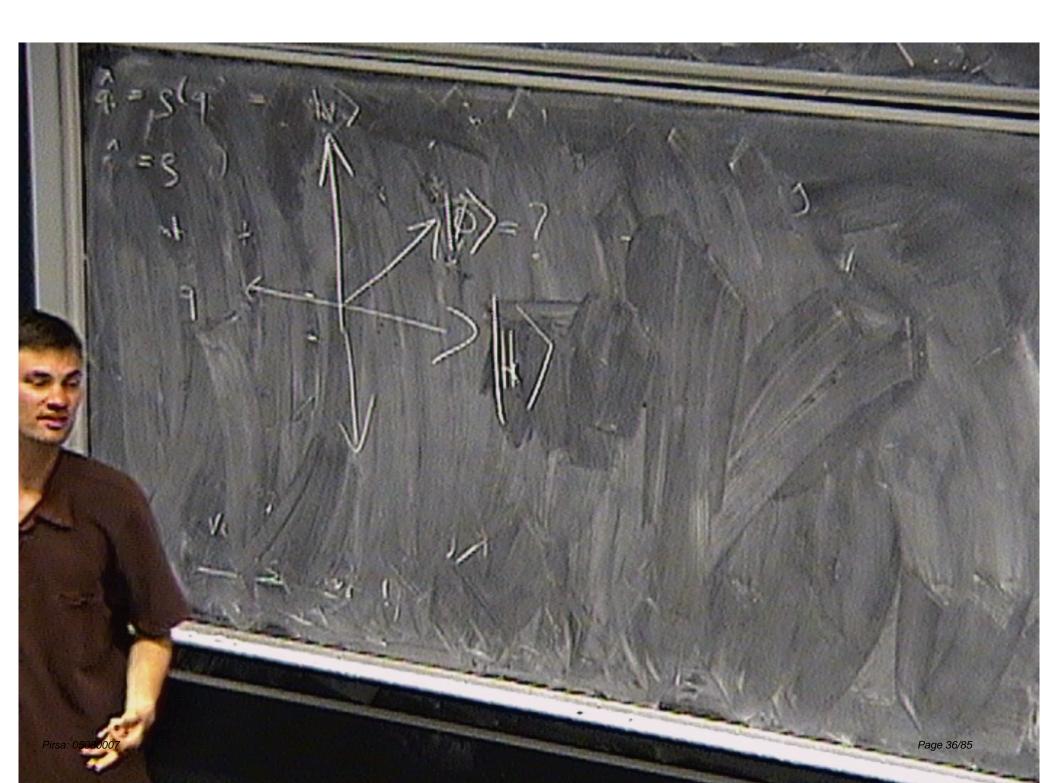


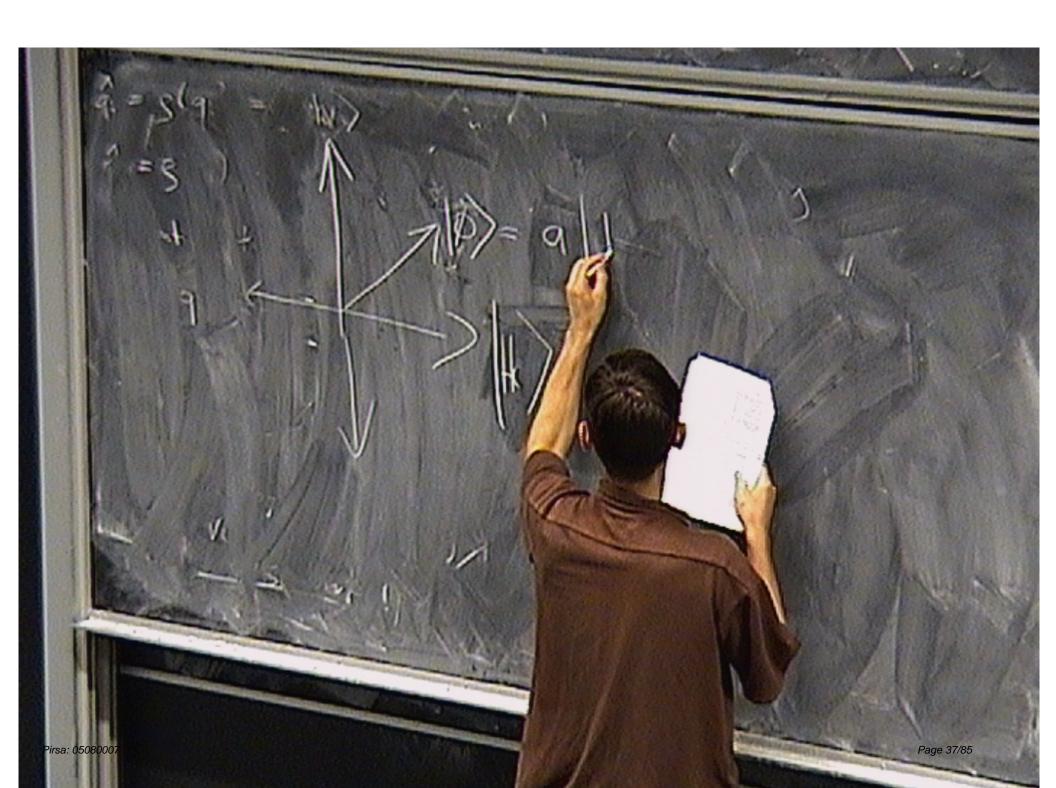


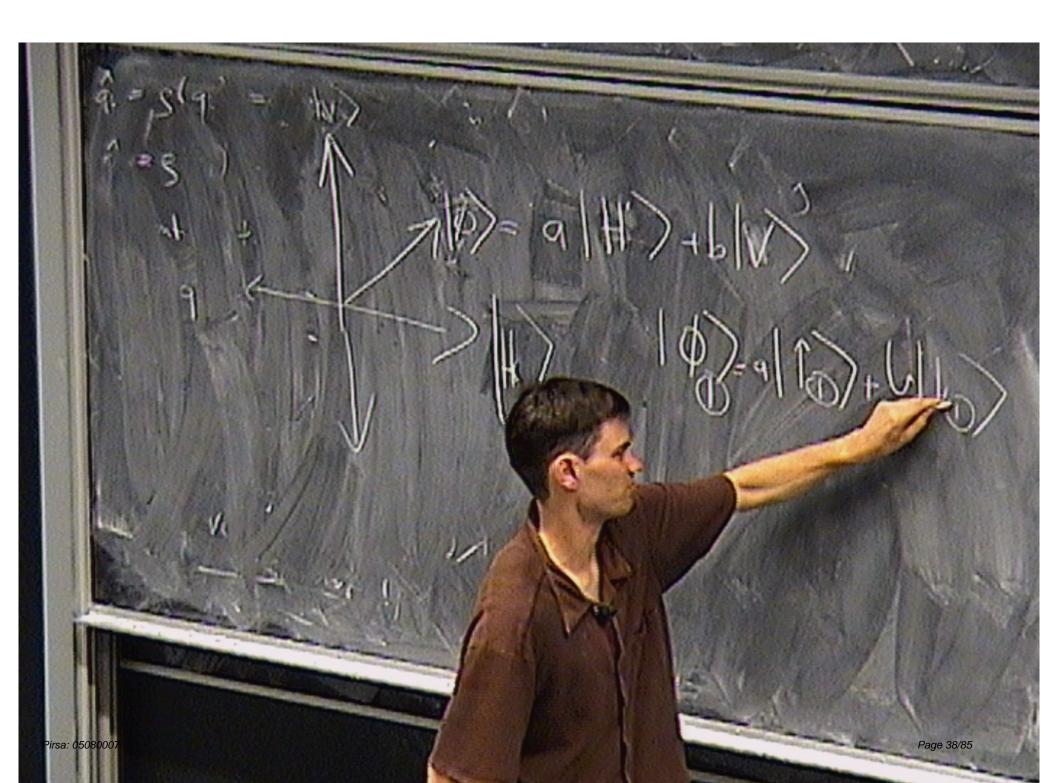


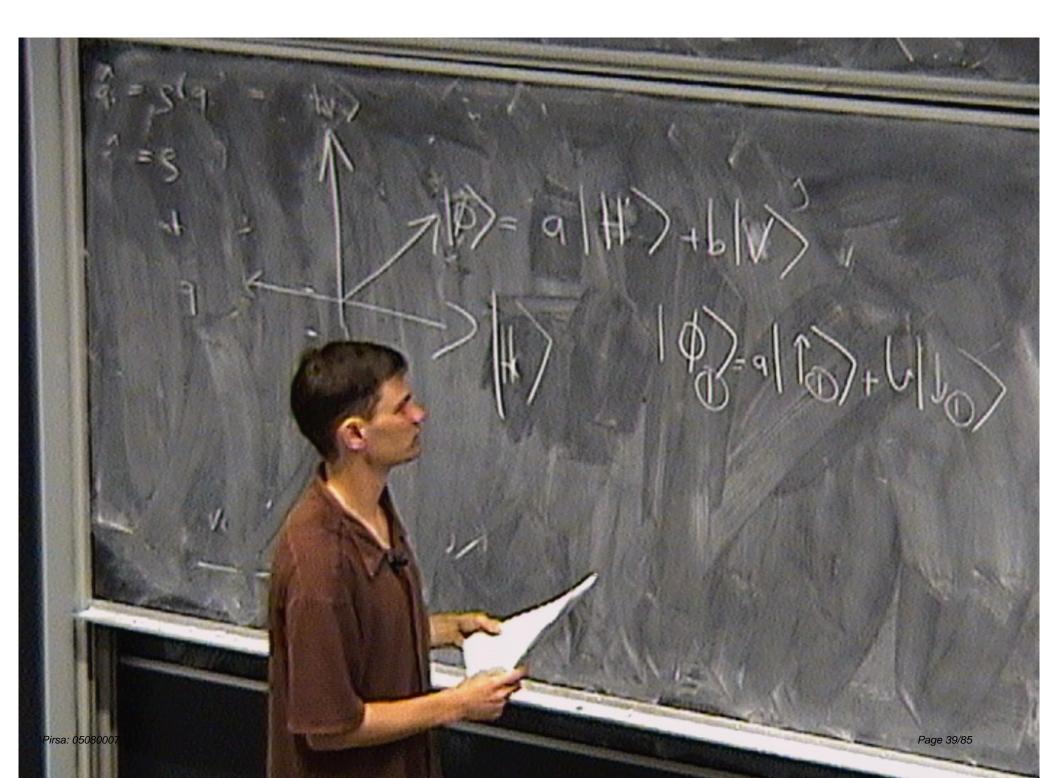


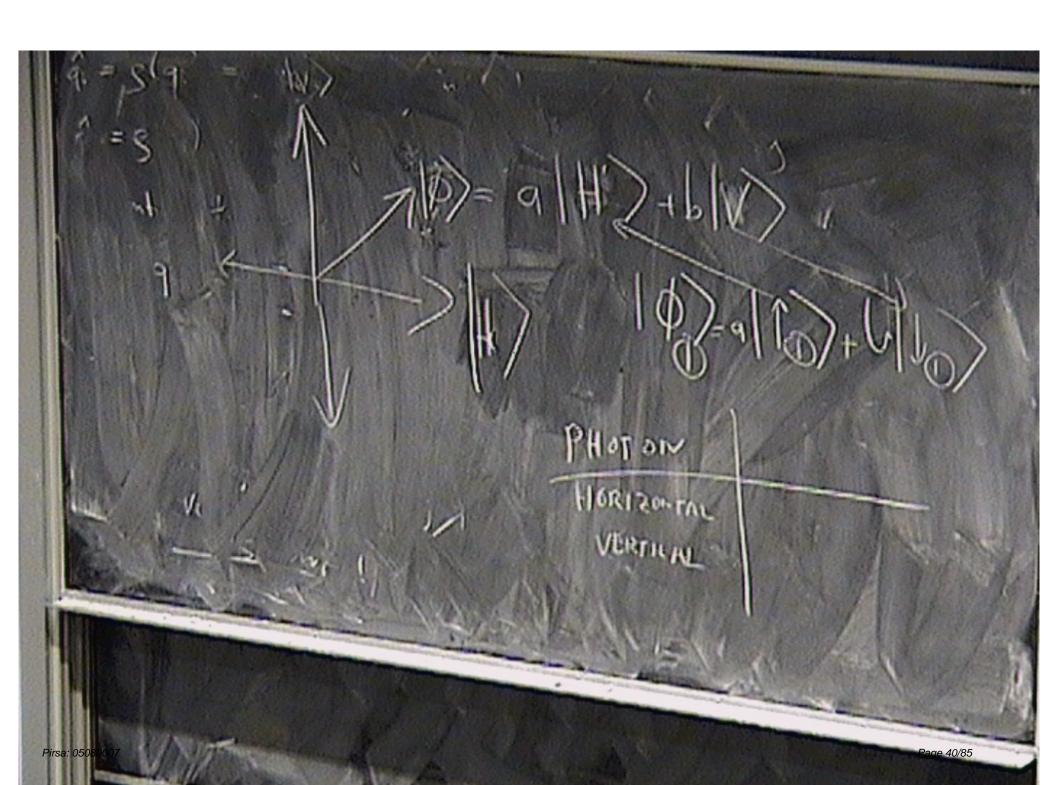


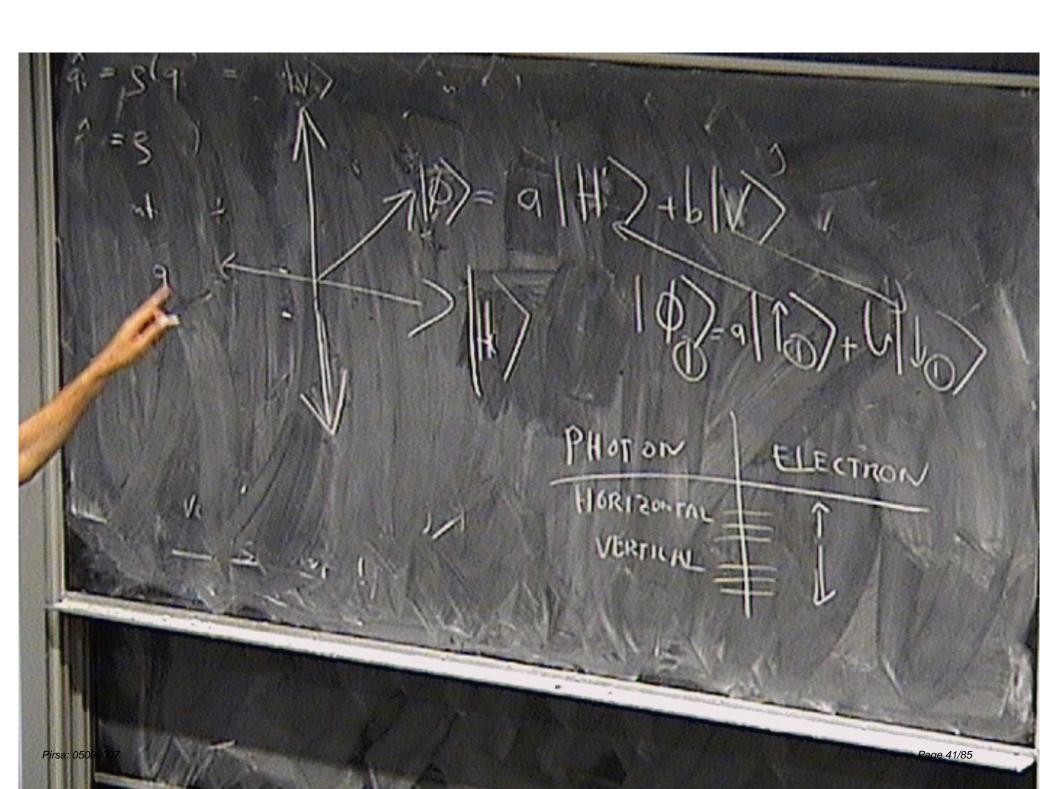


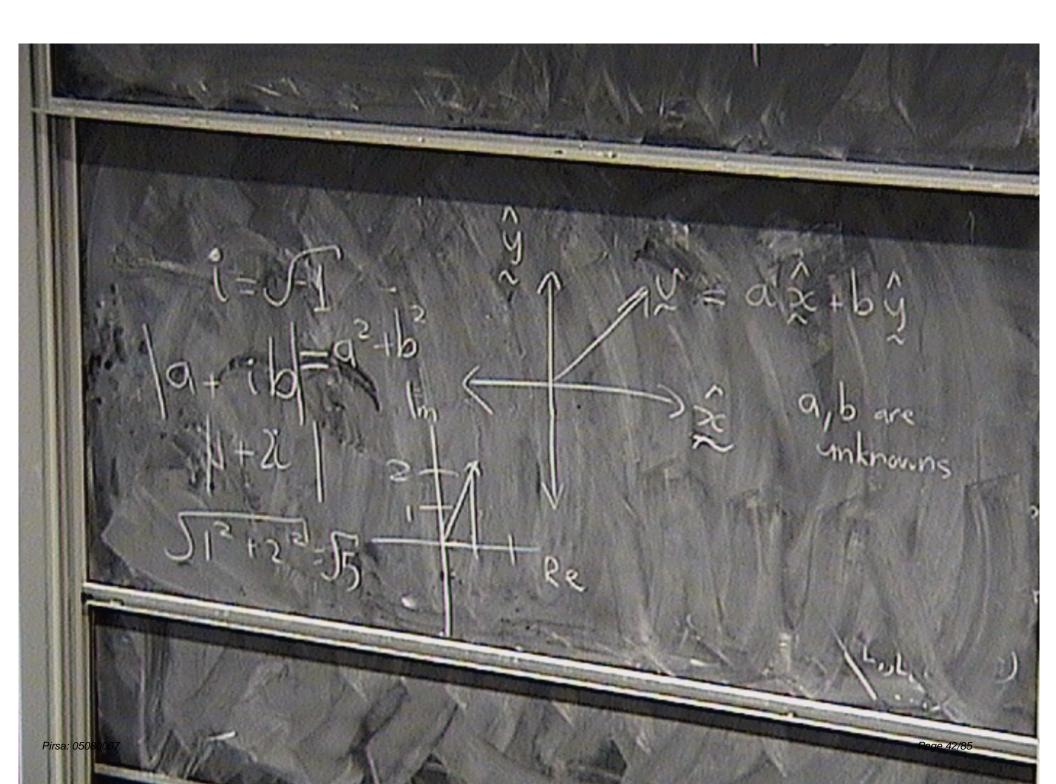


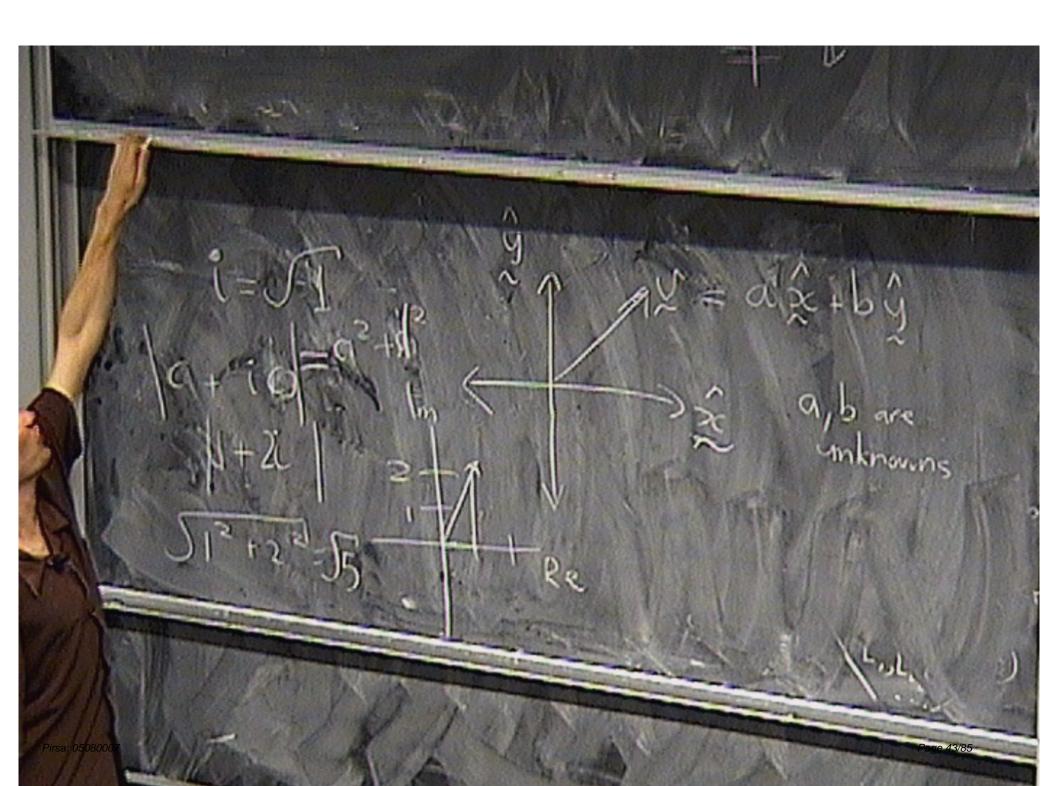


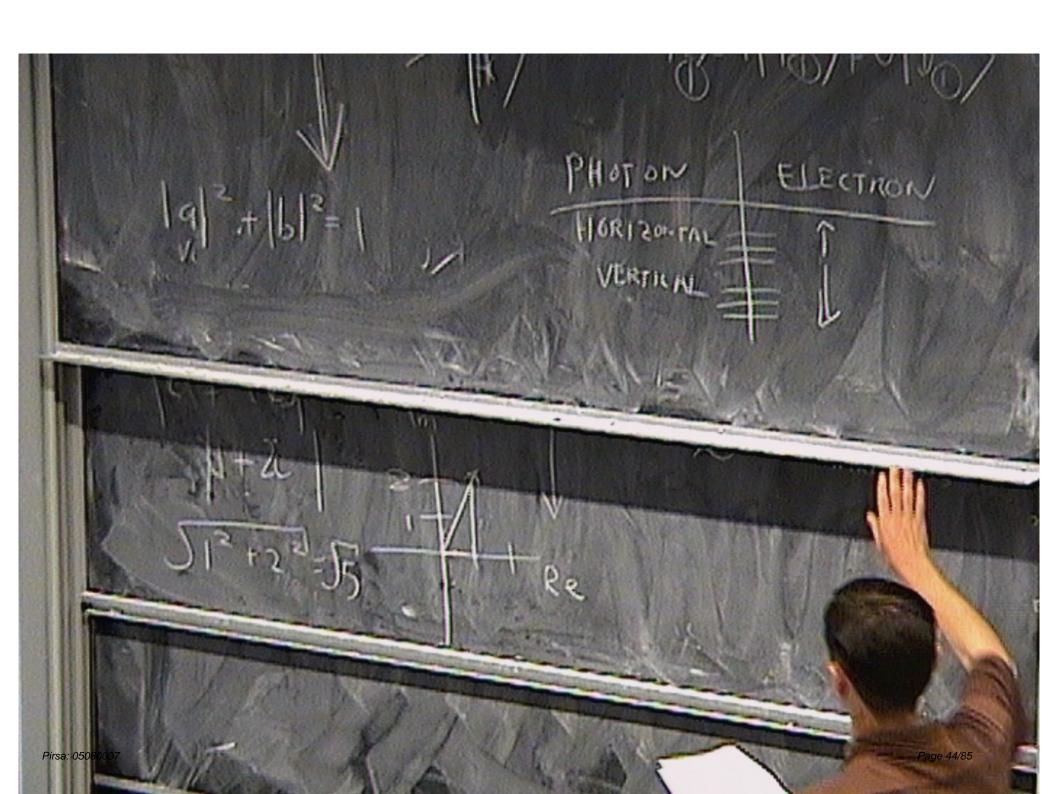










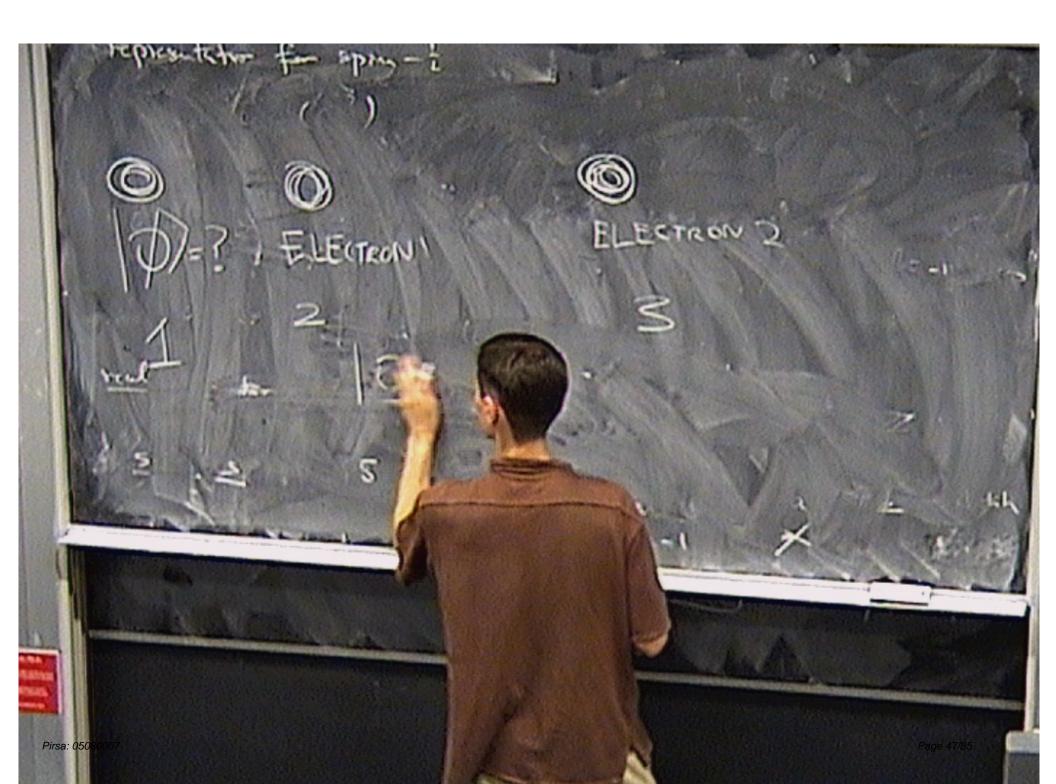


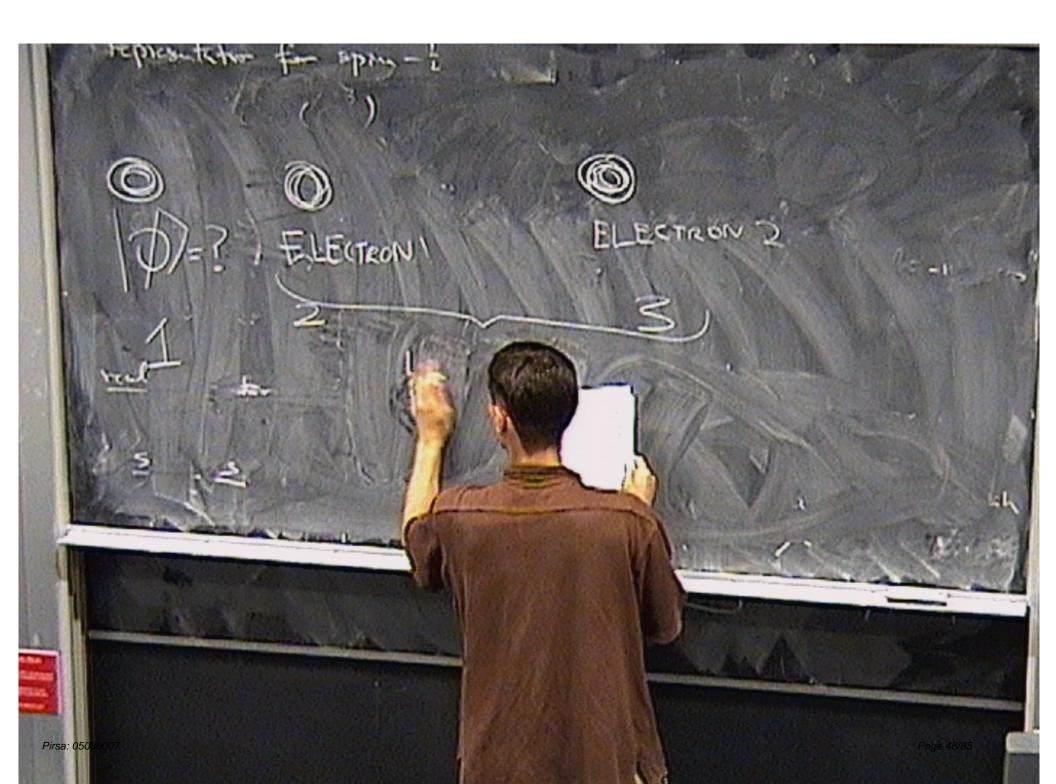


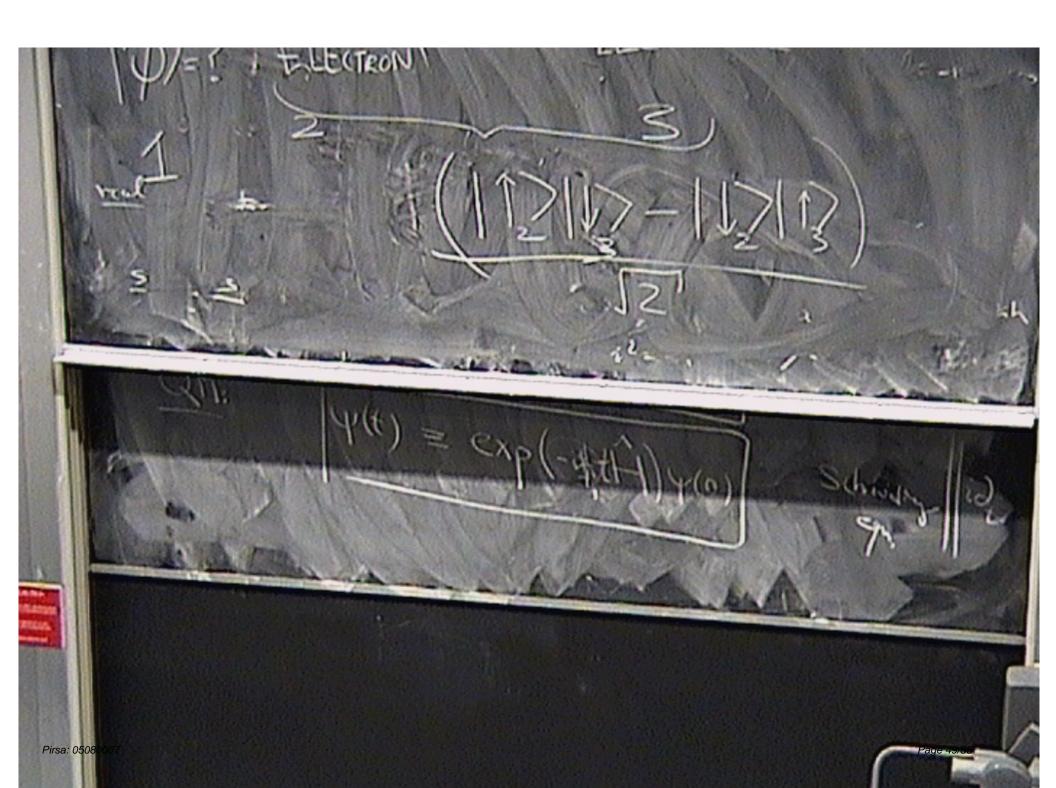
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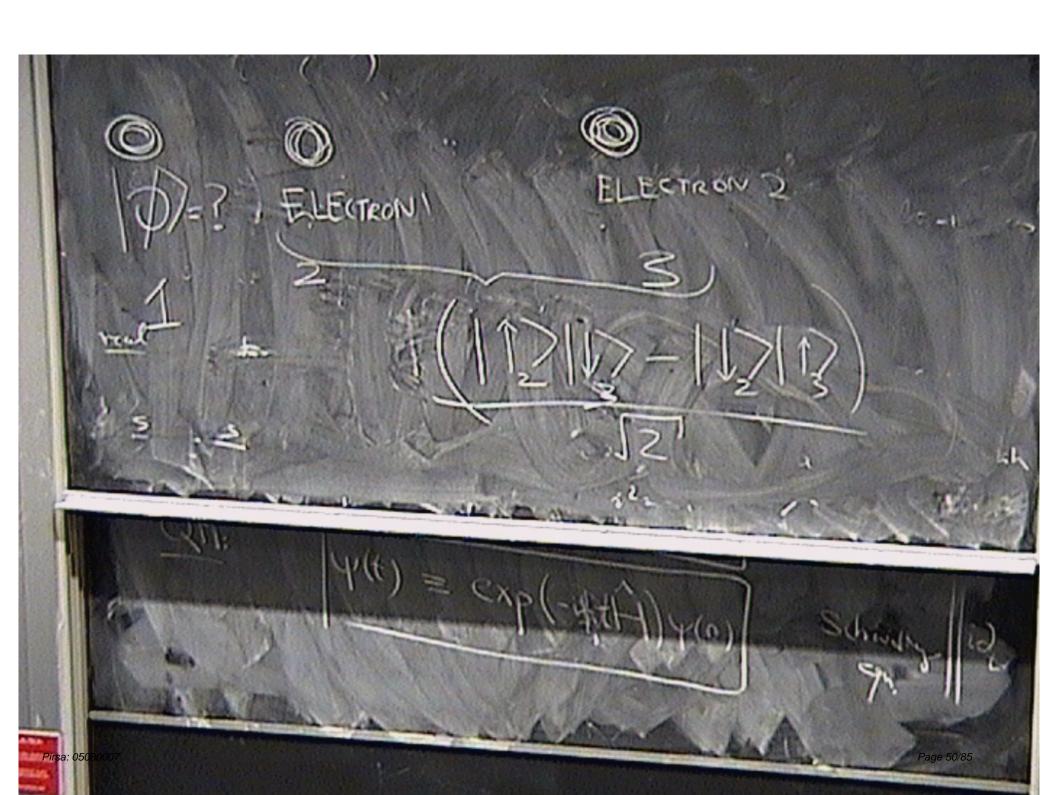
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31 Como opentur S = [s(L)] + [s(L)] + [s(L,)] P-, P; 3 =0

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3! Casining operation

S = [S(L1)] + [S(L1)] + (S(L1)) +

The spin exchange method of sending full information to Beb still lumps classical and nonclassical information ogether in a single transmission. Below, we show how Alice can divide the full information encoded in (\$\phi\$) into two parts, one purely classical and the other purely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replica of [\$\phi\$). Of course Alice's original [\$\phi\$] is destroyed in the process, as it must be to obey the mo-cloning theorem. We call the process we are about to describe teleportation, a term from science fiction meaning to make a person or object disappear while an exact replica appears consewhere also. It must be emphasized that our teleportation, unlike some science fiction versions, defice no physical laws. In particular, it cannot take place instantaneously or over a specific interval, because it requires, among other things, sanding a classical message from Alice to Bob. The net result of teleportation is completely process: the removal of [\$\phi\$) from Alice's hands and its appearance in Bob's hands a suitable time later. The only remarkable feature is that, in the interim, the information in [\$\phi\$] has been classly separated into classical and nonclassical parts. First we shall show how to teleport the quantum state [\$\phi\$] of a spin-\frac{1}{2} particle. Later we discuss teleportation of more complicated states.

The nonclassical part is transmitted first. To do so, two spin-1 particles are prepared in an EPR singlet state

$$|\Psi_{23}^{(-)}\rangle = \sqrt{\frac{1}{3}}(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle).$$
 (1)

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state [#] she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra as a spin.

One EPR particle (particle 2) is given to Alice, while

who other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair as this stage contains no information about (\$\phi\$). Indeed the entire system, comprising Alice's unknown particle 1 and the EPR pair, is in a pure product state, \$\phi_1\$ \cdots \begin{align*} \begin{align*} \lefta \text{price} \\ \text{price} \end{align*}, involving mathem classical correlation nor quantum correlations to the unknown particle and the EPR pair. Therefore no manufactured on either member of the EPR pair, or both together, can yield any information about \$\phi\$). An entarglement between these two subsystems is brought about in the part step.

To emple the first particle with the EPR pair, Alleperforms a complete measurement of the war flowers type on the joint system consisting of particle I and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of [17] and

$$\langle \Phi_{13}^{(a)} \rangle = \sqrt{\frac{1}{2}} \langle (\uparrow_3) (\downarrow_0) + (\downarrow_0) (\uparrow_0) \rangle_{a}$$

Note that these five states are a complete orthonormal basis for particles 1 and 2.

It is descendent to write the unknown state of the fire particle as

$$|\phi_1\rangle = \phi(T_1) + b(U_2)$$
. (3)

with [a] + [k] - 1. The complete state of the three particles before Aller's measurement is thus

In this equation, each direct product $|\cdot|_{2}|\cdot|_{2}$ was be as pressed in terms of the Bell operator hade vertice $|0\rangle_{2}^{(1)}$, and we obtain

$$|\Psi_{123}\rangle = \frac{1}{2} \left[|\Psi_{12}^{(-)}\rangle \left(-a|\Upsilon_3\rangle - b|\Upsilon_3\rangle \right) + |\Psi_{12}^{(+)}\rangle \left(-a|\Upsilon_3\rangle + b|\Upsilon_3\rangle \right) + |\Phi_{12}^{(-)}\rangle \left(a|\Upsilon_3\rangle + b|\Upsilon_3\rangle \right) + |\Phi_{12}^{(+)}\rangle \left(a|\Upsilon_3\rangle - b|\Upsilon_3\rangle \right)$$

It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, re-

Each of these possible resultant states for Bob's EFR.
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|\$\phi\$) which Alice sought to teleport. In the case of the first
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then from a pair of identically prepared particles [10].

The spin-exchange method of sending full information to Bob still lumps chasical and nonclassical information fagether in a single transmission. Below, we show how a fire can divide the full information encoded in [4) into two parts, one purely classical and the other parely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replice of [4]. Of course Alice's northern and accurate replice of [4]. Of course Alice's northern in the mast be to obey the non-cloning theorem. We call the process we are gabout to describe teleportation, a term from science fiction meaning to make a person or object disappear while an exact replice appears somewhere else. It must be emphasized that our teleportation, as in particular, it cannot take place instantaneously or over a spacifical of teleportation is emphasized that our teleportation and one section of teleportation as completely procasic the removal of [9] from Alice's hands and it appearance in Bob's hands a sulfable time later. The only remarkable feature is that, in the interim, the information in [9] has been cleanly separated into classical and noncleasical parts. First we spin a particle. Later we discuss teleportation of more complicated states.

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in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle I and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of [9];) and

$$|\Psi(\frac{1}{3}^2)| = \sqrt{\frac{1}{3}} (|\Gamma_1| |13) + |14| |\Gamma_2|),$$
 (2)
 $|\Psi(\frac{1}{3}^2)| = \sqrt{\frac{1}{3}} (|\Gamma_1| |\Gamma_2| + |14| |\Gamma_2|).$ (2)

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{4}} (11) [13] [13\rangle - [13] [13] [13\rangle)$$

 $+ \frac{b}{\sqrt{2}} (14) [13] [13\rangle - [1,1] [13] [13)\}.$ (4)

In this equation, each direct product $|\cdot,|\cdot|$ s) can be expressed in terms of the Bell operator basis vectors $|\Phi(\frac{1}{2})|$ and $|\Psi(\frac{1}{2})|$, and $|\Psi(\frac{1}{2})|$, and

It follows that, regardless of the unknown state [9,1], the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, respectively,

$$-|\phi_2\rangle = -\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} -10 \\ 01 \end{pmatrix}|\phi_2\rangle, \\ \begin{pmatrix} 01 \\ 10 \end{pmatrix}|\phi_2\rangle, \begin{pmatrix} 0-1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle,$$
 (6)

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state (s) which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do nothing further to produce a replace of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding, respectively, to 180° rotations around the z, z, and y axes, in order to convex has EPR particle into a replace of Alice's original state (9). (If (9) represents a photon polarization state, a suitable combination of half-

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$$|\Psi(\pm^2)\rangle = \sqrt{\frac{1}{4}} (\Gamma_1)|1_2\rangle + |1_1\rangle|\Gamma_2\rangle),$$
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$$|\Psi_{123}\rangle = \frac{9}{\sqrt{2}} (|1_1\rangle |1_2\rangle |1_3\rangle - |1_1\rangle |1_2\rangle |1_3\rangle + \frac{b}{\sqrt{2}} (|1_1\rangle |1_2\rangle |1_3\rangle - |1_1\rangle |1_3\rangle).$$
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the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about [9]. Indeed the critic system, compilating Alice's unknown particle 1 and the EPR pair, is in a pure product state, [9], [9]¹³], involving neither classical correlation nor quantum estangement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about [9]. An entantic of the cover these two subsystems is brought about

in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle 1 and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [13] consisting of [9];) and

Note that these four states are a complete orthonormal basis for particles 1 and 2. It is convenient to write the unknown state of the first particle as

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus (41) = a (11) + b (11),

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}} (|\Gamma_1\rangle |\Gamma_2\rangle |1_3\rangle - |\Gamma_1\rangle |1_2\rangle |\Gamma_3\rangle$$

$$+ \frac{b}{\sqrt{2}} (|\Gamma_2\rangle |\Gamma_3\rangle |1_3\rangle - |\Gamma_1\rangle |1_2\rangle |1_3\rangle . (4)$$

In this equation, each direct product $[\cdot, \cdot]$ $[\cdot, \cdot]$ can be expressed in terms of the Bell operator basis vectors $[\cdot 0^{(\frac{1}{2})}]$, and we obtain

$$|\Psi_{12}\rangle = \frac{1}{2} ||\Psi_{12}^{(2)}\rangle \left(-a(1_2) - b(1_2)\right) + |\Psi_{12}^{(2)}\rangle \left(-a(1_2) + b(1_2)\right) + |\Psi_{12}^{(2)}\rangle \left(a(1_2) + b(1_2)\right) + |\Psi_{12}^{(2)}\rangle \left(a(1_2) - b(1_2)\right) + |\Psi_{12}^{(2)}\rangle \left(a(1_2)$$

3

it follows that, regardless of the unknown state [91], the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. There are, respectively,

$$-|\phi_3\rangle = -\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} -10 \\ 01 \end{pmatrix} |\phi_3\rangle,$$
 (6) $\begin{pmatrix} 01 \\ 10 \end{pmatrix} |\phi_3\rangle, \begin{pmatrix} 01 \\ 10 \end{pmatrix} |\phi_3\rangle,$ (6)

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state [s] which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do soching further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding, respectively, to 150° rotations around the t. x. and y axes, in order to convert the EPR particle into a replica of Alice's original state [s). (If [s) represents a photon polarization state, a suitable combination of laif-

The spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission. Below, we show bow Alice can divide the full information encoded in |4) into two parts, one purely classical and the other purely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replica of |6). Of course Alice's original |6| is destroyed in the process, as it must be to obey the no-cloning theorem. We call the process we are about to describe teleportation, a term from science fiction meaning to make a person or object disappear while an exact replica appears comewhere else. It must be emphasized that our teleportation, unlike some science fiction versions, defies no physical laws. In particular, it cannot take place instantaneously or over a spacelike interval, because it requires, among other things, sending a classical message from Alice to Bob. The net result of teleportation is completely prosatic the removal of |6| from Alice's hands and its appearance in Bob's hands a suitable time later. The only remarkable feature is that, in the interim, the information in |6| has been cleanly separated into classical and nonclassical parts. First we shall show how to teleport the quantum state |6| of a spin-§ particle. Later we discuss teleportation of more

complicated states.

The nonclassical part is transmitted first. To do so, two spin-§ particles are prepared in an EPR singlet state

$$|\Psi_{23}^{(-)}\rangle = \sqrt{\frac{1}{2}} (|1_2\rangle |1_2\rangle - |1_2\rangle |1_3\rangle).$$
 (1)

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state |\$\phi\$| she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra

One EPR particle (particle 2) is given to Alice, while

the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about [4]. Indeed the entire system, compiling Alice's unknown particle 1 and the EPR pair, is in a pure product state, [4], [9], [1], involving neither classical correlation nor quantum entanglement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about [4]. An entanglement between these two subsystems is brought about in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle 1 and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of [9]; and

3

$$|\phi_{12}^{(\pm 1)}\rangle = \sqrt{\frac{1}{2}} (|11\rangle |12\rangle \pm |11\rangle |12\rangle).$$

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first

particle as

$$|\phi_1\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle,$$
 (3)

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}} (|1_1\rangle |1_2\rangle |1_3\rangle - |1_1\rangle |1_2\rangle |1_3\rangle)$$

$$+ \frac{b}{\sqrt{2}} (|1_1\rangle |1_2\rangle |1_3\rangle - |1_1\rangle |1_2\rangle |1_3\rangle). \quad (4)$$

In this equation, each direct product $|\cdot|$ $|\cdot|$ 2) can be expressed in terms of the Bell operator basis vectors $|\Phi|_2^{(\pm)}$, and we obtain

$$|\Phi_{122}\rangle = \frac{1}{2} \left[|\Phi_{12}^{(2)}\rangle \left(-\alpha_1 \gamma_2 \right) - \beta_1 \gamma_2 \right) + |\Phi_{12}^{(2)}\rangle \left(-\alpha_1 \gamma_2 \right) + \beta_1 \gamma_2 \right) + |\Phi_{12}^{(2)}\rangle \left(\alpha_1 \gamma_2 \right) + |\Phi_{12}^{(2)}\rangle \left(-\alpha_1 \gamma_2 \right) + |\Phi_{12}^{(2)$$

It follows that, regardless of the unknown state [\$\phi_1\$], the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (\$), according to the measurement outcome. These are, re-

$$-|\phi_3\rangle = -\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} -10 \\ 01 \end{pmatrix}|\phi_3\rangle, \begin{pmatrix} 0 \\ 10 \end{pmatrix}|\phi_3\rangle, \begin{pmatrix} 0 -1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle.$$
 (6)

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state | p which Allee sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do sothing further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (0), corresponding, respectively, to 180° rotations around the s. x. and y axes, in order to convert his EPR particle into a replica of Alice's original state | φ). (If | φ) represents a photon polarization state, a suitable combination of half-

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The nonclassical part is transmitted first. To do so, two spin-1 particles are prepared in an EPR singlet state

$$|\Psi_{22}^{(-)}\rangle = \sqrt{\frac{1}{2}}(|T_2\rangle|L_2\rangle - |L_2\rangle|T_2\rangle).$$
 (1)

The subscripts 2 and 3 label the particles in this EPR pair. Alice's criginal particle, whose unknown state $|\phi\rangle$ also seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra as a spin.

One EPR particle (particle 2) is given to Alice, while

the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about $|\phi\rangle$. Indeed the entire system, comprising Alice's unknown particle 1 and the EPR pair, is in a pure product state, $|\phi_1\rangle \langle \phi_{12}^{(-1)} \rangle$, involving neither classical correlation nor quantum entanglement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about $|\phi\rangle$. An entanglement between these two subsystems is brought about in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle I and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of $|\Psi_{1,2}^{(r)}\rangle$ and

$$|\Psi_{12}^{(\pm)}\rangle = \sqrt{\frac{1}{2}}(|T_1\rangle|1_2\rangle + |1_1\rangle|T_2\rangle),$$

$$|\Phi_{12}^{(\pm)}\rangle = \sqrt{\frac{1}{2}}(|T_1\rangle|T_2\rangle \pm |1_1\rangle|1_2\rangle).$$
(2)

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

$$|\phi_1\rangle = a|T_1\rangle + b|T_1\rangle,$$
 (3)

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{122}\rangle = \frac{a}{\sqrt{2}} (|\uparrow_1\rangle\langle\uparrow_2\rangle\langle\downarrow_2\rangle - |\uparrow_1\rangle\langle\downarrow_2\rangle\langle\uparrow_2\rangle)$$

 $+ \frac{b}{\sqrt{2}} (|\downarrow_1\rangle\langle\uparrow_2\rangle\langle\downarrow_2\rangle - |\downarrow_1\rangle\langle\downarrow_2\rangle\langle\uparrow_2\rangle).$ (4)

In this equation, each direct product $| 1 \rangle | 2 \rangle$ can be expressed in terms of the Bell operator basis vectors $|\Phi_{12}^{(\pm)}\rangle$, and we obtain

$$|\Psi_{123}\rangle = \frac{1}{2} \left[|\Psi_{12}^{(-)}\rangle \left(-a|\uparrow_3\rangle - b|\downarrow_3\rangle \right) + |\Psi_{12}^{(+)}\rangle \left(-a|\uparrow_3\rangle + b|\downarrow_3\rangle \right) + |\Phi_{12}^{(-)}\rangle \left(a|\downarrow_3\rangle + b|\uparrow_3\rangle \right) + |\Phi_{12}^{(+)}\rangle \left(a|\downarrow_3\rangle - b|\uparrow_3\rangle \right) \right]$$
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 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} |\phi_3\rangle, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} |\phi_3\rangle.$ (6)

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state $|\phi\rangle$ which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do nothing further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding, respectively, to ISO' rotations around the x, x, and y axes, in order to convert his EPR particle into a replica of Alice's original state $|\phi\rangle$. (If $|\phi\rangle$ represents a photon polarization state, a suitable combination of half-

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$$|\Psi_{12}^{(+)}\rangle = \sqrt{\frac{1}{2}}(|T_1\rangle|I_2\rangle + |I_1\rangle|T_2\rangle),$$

 $|\Phi_{12}^{(\pm)}\rangle = \sqrt{\frac{1}{2}}(|T_1\rangle|T_2\rangle \pm |I_1\rangle|I_2\rangle).$
(2)

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It is convenient to write the unknown state of the first particle as

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with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}} (|\uparrow_1\rangle\langle\uparrow_2\rangle|\downarrow_2\rangle - |\uparrow_1\rangle\langle\downarrow_2\rangle\langle\uparrow_2\rangle)$$

 $+ \frac{b}{\sqrt{2}} (|\downarrow_1\rangle|\uparrow_2\rangle\langle\downarrow_2\rangle - |\downarrow_1\rangle\langle\downarrow_2\rangle\langle\uparrow_2\rangle).$ (4)

In this equation, each direct product $|\cdot|_1\rangle|_2\rangle$ can be expressed in terms of the Bell operator basis vectors $|\Phi_{12}^{(\pm)}\rangle$ and $|\Psi_{12}^{(\pm)}\rangle$, and we obtain

$$|\Psi_{122}\rangle = \frac{1}{2} \left[|\Psi_{12}^{(-)}\rangle \left(-a|\uparrow_{2}\rangle - b|\downarrow_{2}\rangle \right) + |\Psi_{12}^{(+)}\rangle \left(-a|\uparrow_{2}\rangle + b|\downarrow_{2}\rangle \right) + |\Phi_{12}^{(-)}\rangle \left(a|\downarrow_{2}\rangle + b|\uparrow_{2}\rangle + |\Phi_{12}^{(+)}\rangle \left(a|\downarrow_{2}\rangle - b|\uparrow_{2}\rangle \right) \right]$$
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It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, respectively.

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Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state $|\phi\rangle$ which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do nothing further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding, respectively, to 180° rotations around the x, x, and y axes, in order to convert his EPR particle into a replica of Alice's original state $|\phi\rangle$. (If $|\phi\rangle$ represents a photon polarization state, a suitable combination of half-

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$$|\Psi_{23}^{(-)}\rangle = \sqrt{\frac{1}{2}}(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle).$$
 (1)

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state | 6) she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g., one or more may be photons, the polarization degree of freedom having the same algebra

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$$\begin{split} |\Psi_{12}^{(+)}\rangle &= \sqrt{\frac{1}{2}} (|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle|\uparrow_2\rangle), \\ |\Phi_{12}^{(\pm)}\rangle &= \sqrt{\frac{1}{2}} (|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_1\rangle|\downarrow_2\rangle). \end{split} \tag{2}$$

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

$$|\phi_1\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle,$$
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with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}}(|\uparrow_1\rangle||\uparrow_2\rangle||\downarrow_3\rangle - |\uparrow_1\rangle||\downarrow_2\rangle||\uparrow_3\rangle)$$

$$+ \frac{b}{\sqrt{2}}(|\downarrow_1\rangle||\uparrow_2\rangle||\downarrow_3\rangle - |\downarrow_1\rangle||\downarrow_2\rangle||\uparrow_3\rangle). \quad (4)$$

In this equation, each direct product | 1) | 2) can be expressed in terms of the Bell operator basis vectors $|\Phi_{12}^{(\pm)}\rangle$ and $|\Psi_{12}^{(\pm)}\rangle$, and we obtain

$$|\Psi_{123}\rangle = \frac{1}{2} \left[|\Psi_{12}^{(-)}\rangle \left(-a|\uparrow_3\rangle - b|\downarrow_3\rangle \right) + |\Psi_{12}^{(+)}\rangle \left(-a|\uparrow_3\rangle + b|\downarrow_3\rangle \right) + |\Phi_{12}^{(-)}\rangle \left(a|\downarrow_3\rangle + b|\uparrow_3\rangle \right) + |\Phi_{12}^{(+)}\rangle \left(a|\downarrow_3\rangle - b|\uparrow_3\rangle \right) \right]. \tag{5}$$

It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, re-007 spectively,

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$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}} (|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle)$$

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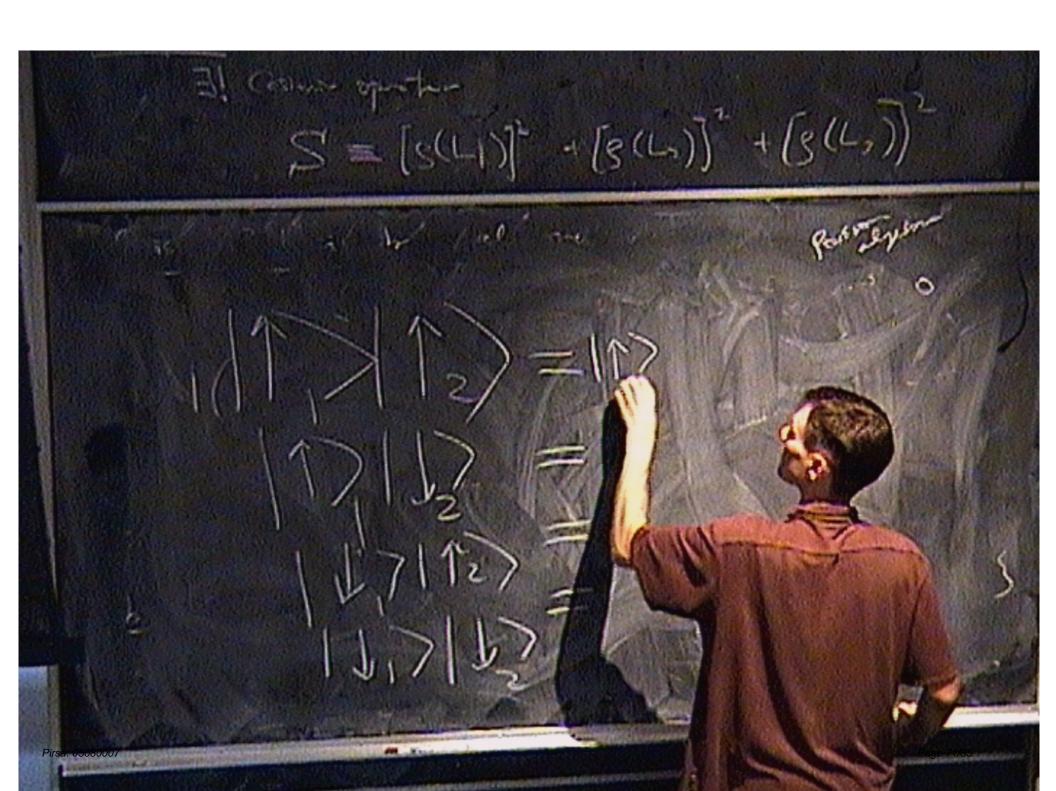
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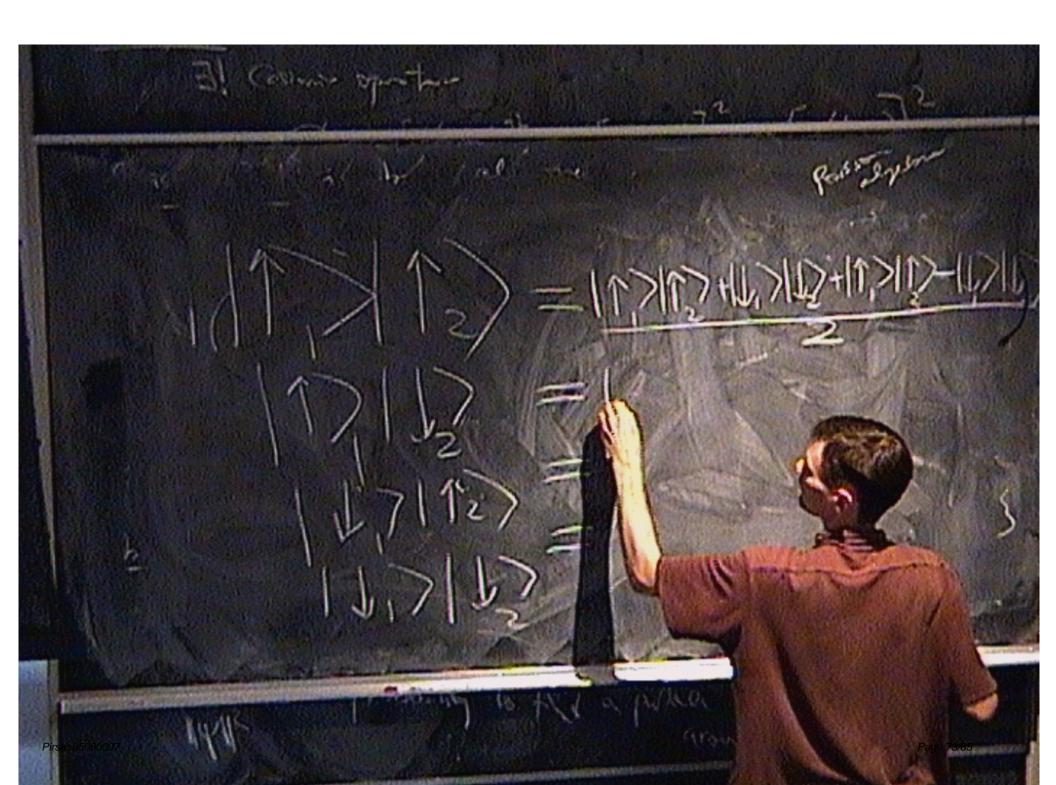
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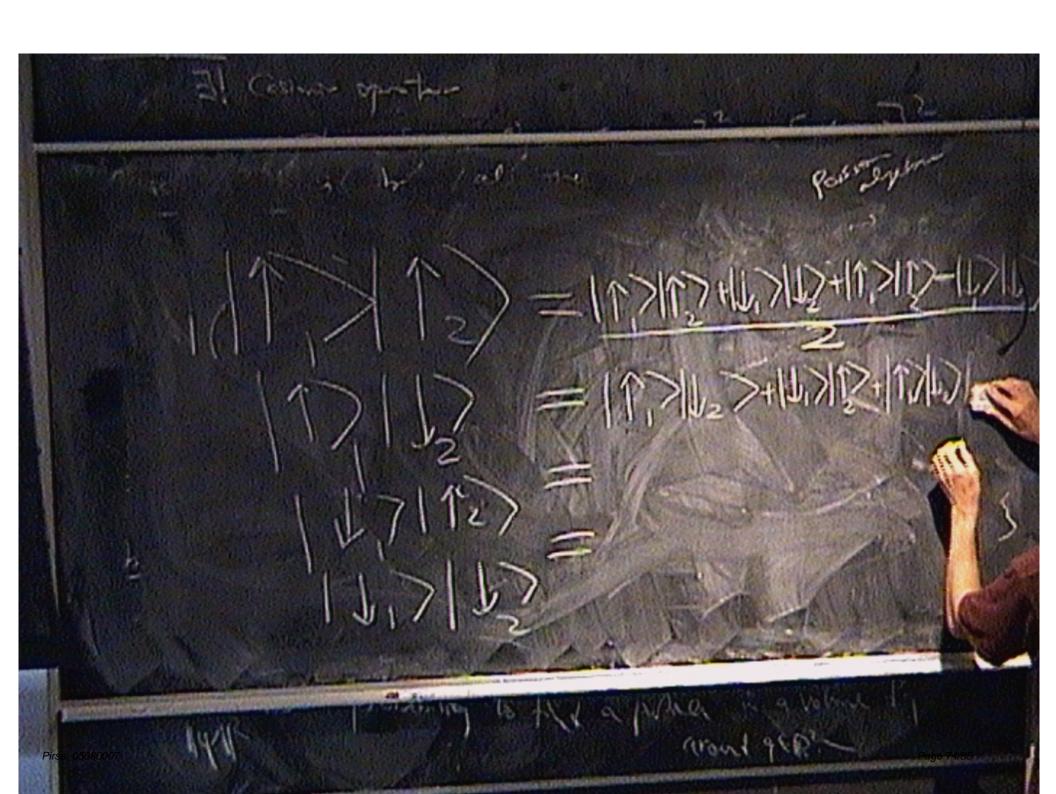
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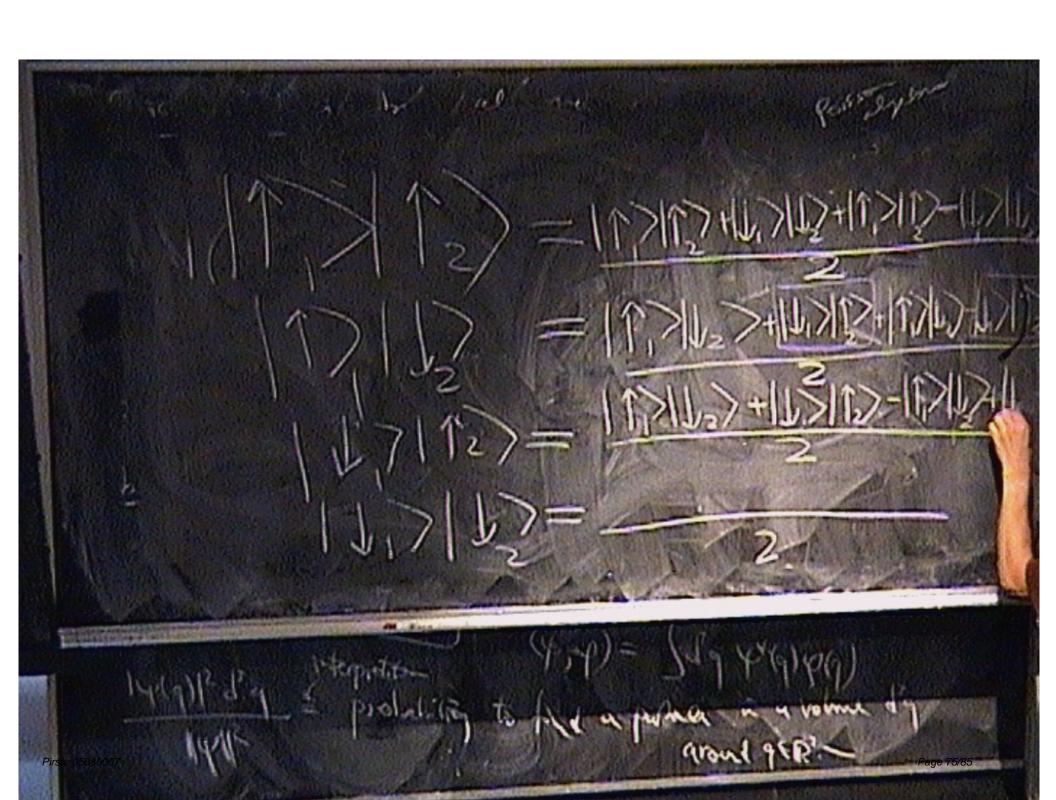
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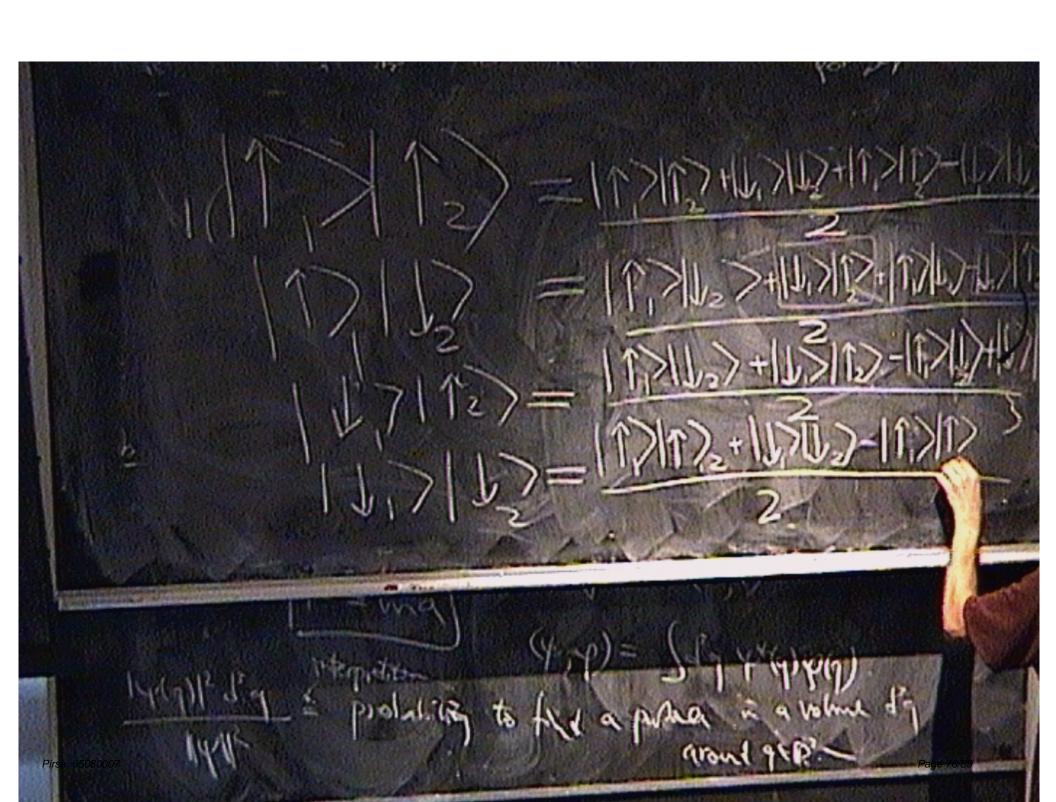


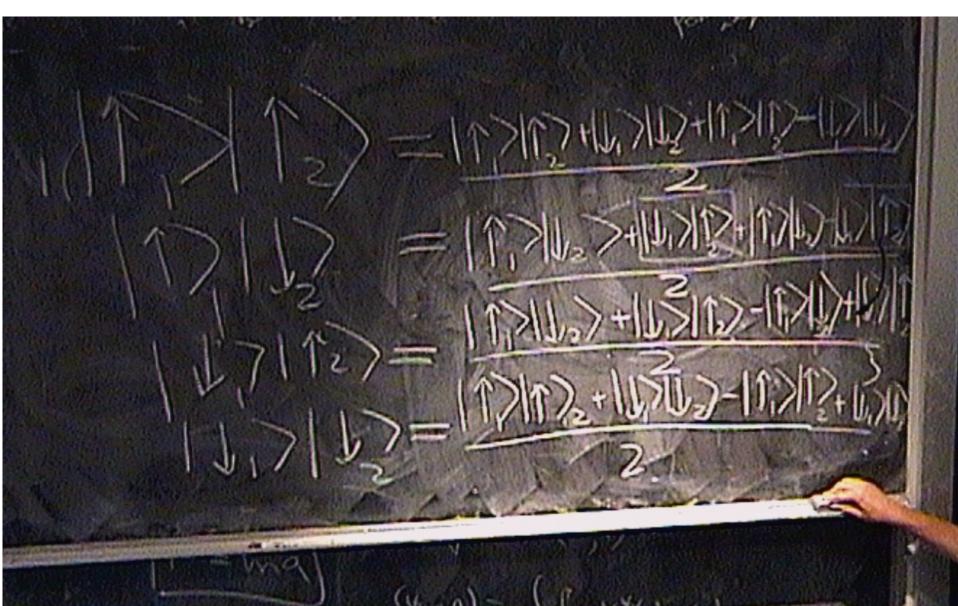
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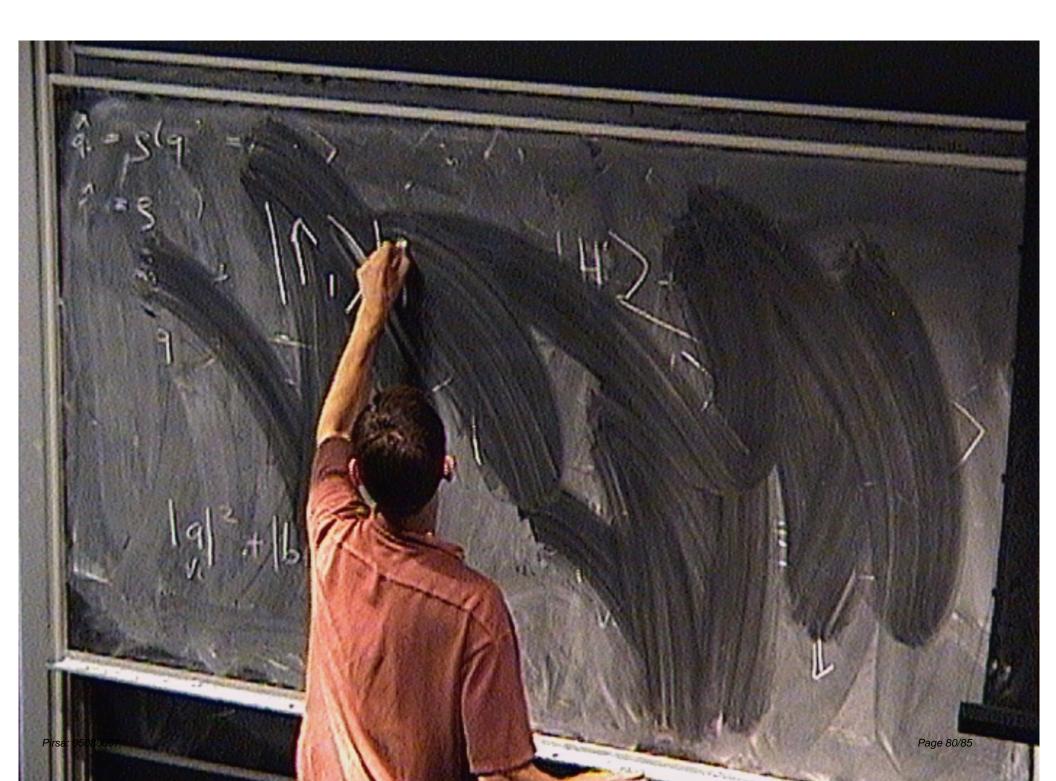
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