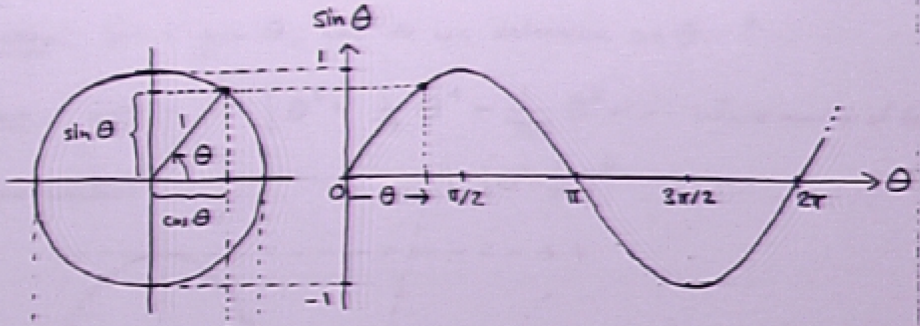
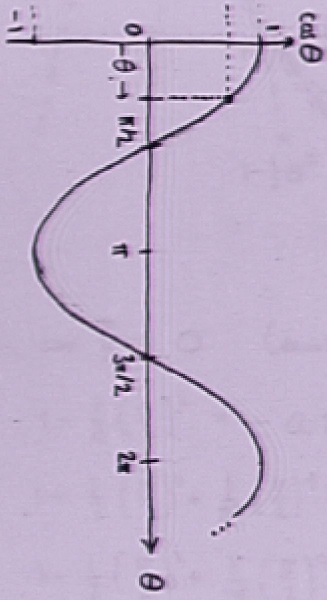
Title: ISSYP 2005 - Lecture A

Date: Aug 09, 2005 10:35 AM

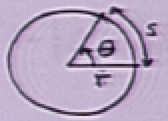
URL: http://pirsa.org/05080003

Abstract:

Series Expansions for Some Common Functions

note: the angle θ is measured in "radians"



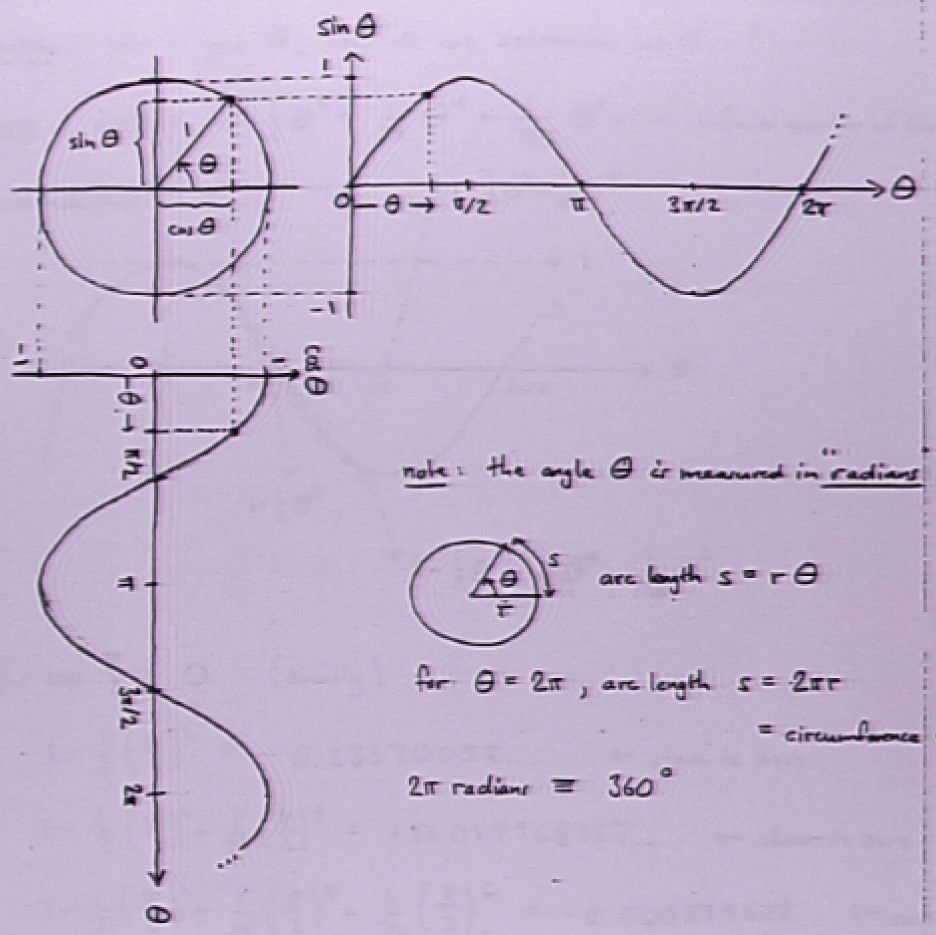
arc length $s = r\theta$

for $\theta = 2\pi$, arc length $s = 2\pi r$

= circumference ✓

2π radians $\equiv 360^\circ$

Series Expansions for Some Common Functions



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arc length $s = r\theta$

for $\theta = 2\pi$, arc length $s = 2\pi r$

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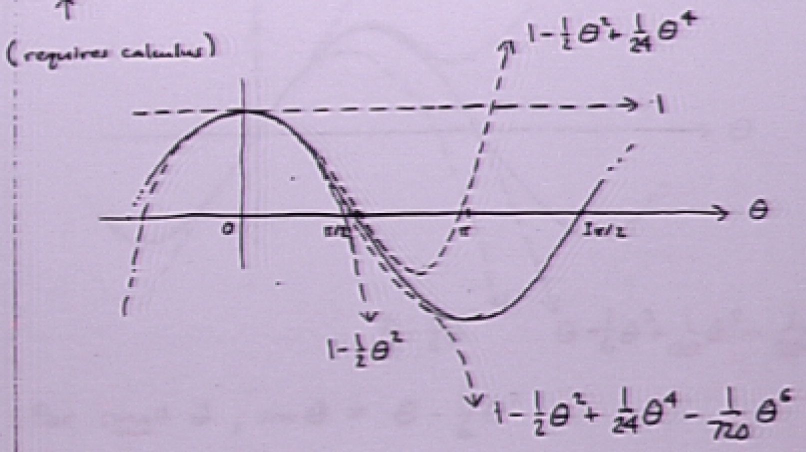
2π radians $\equiv 360^\circ$

<u>Question</u>: for a given θ, how do we determine $\cos\theta$?

<u>Answer</u>: $\cos\theta = 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 - \frac{1}{720}\theta^6 + \cdots$ infinite number of terms

↑

(requires calculus)



<u>e.g.</u> $\cos\frac{\pi}{2} = 0$ (exactly)

$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23370055\ldots$ ← close to zero

$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 = +0.019968957\ldots$ ← closer to zero

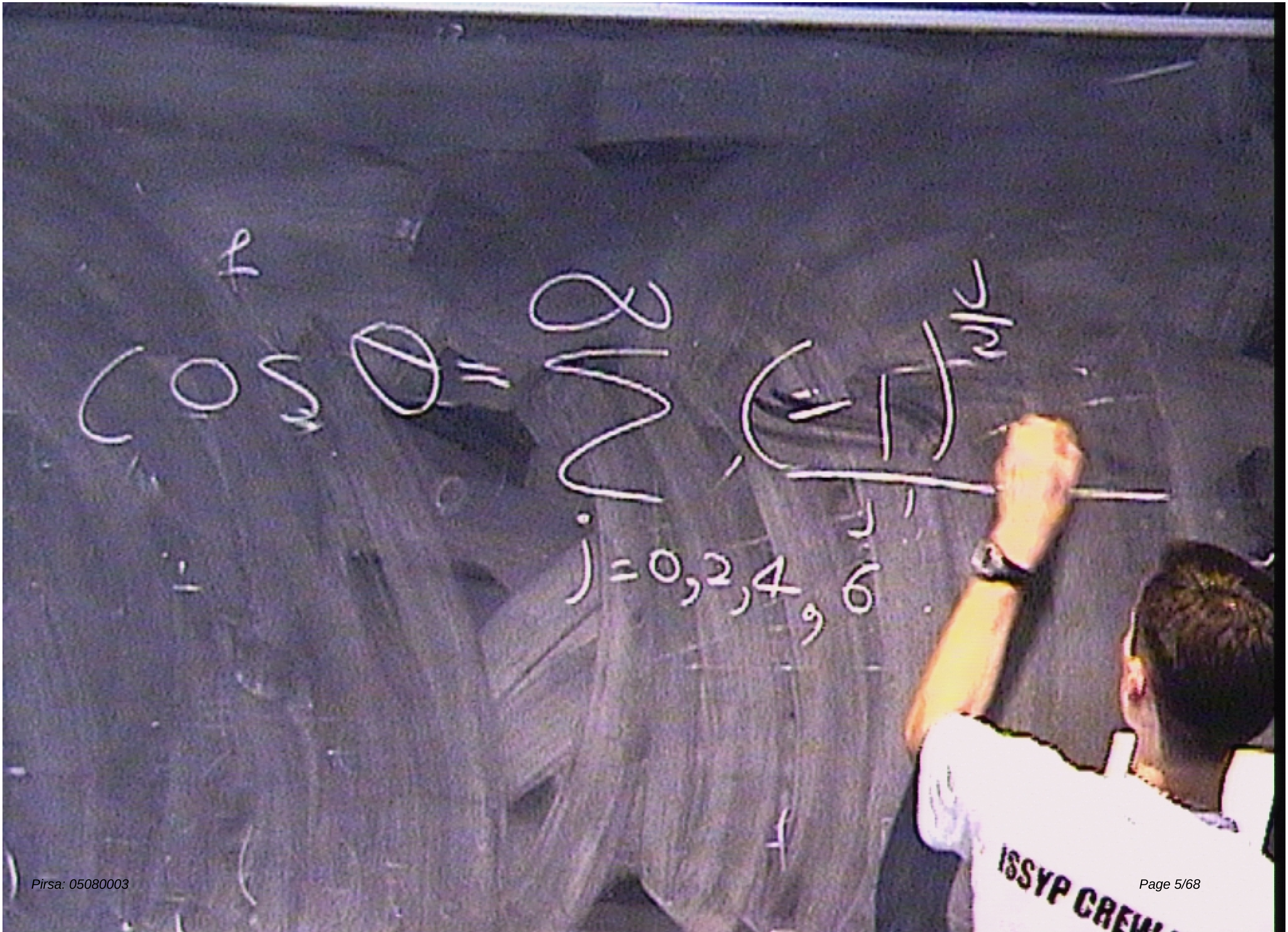
$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 - \frac{1}{720}\left(\frac{\pi}{2}\right)^6 = -0.000894522\ldots$ ← even closer to zero

$\vdots$

etc.

We get closer and closer to zero (the exact value) the more terms we include in the series expansion. Including <u>all</u> the terms will give us the <u>exact</u> value

For <u>small</u> θ, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$



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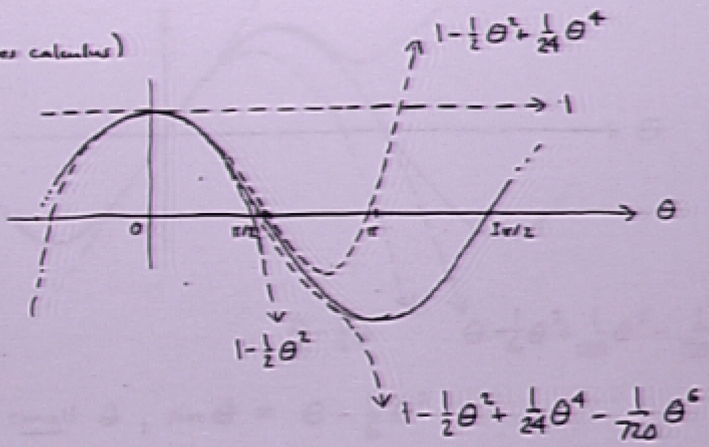

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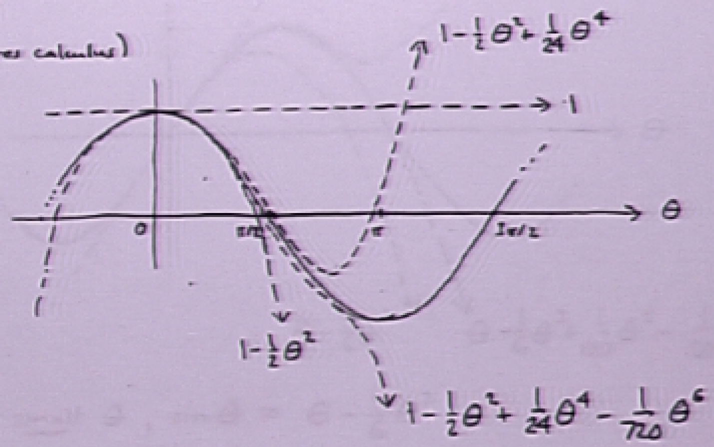

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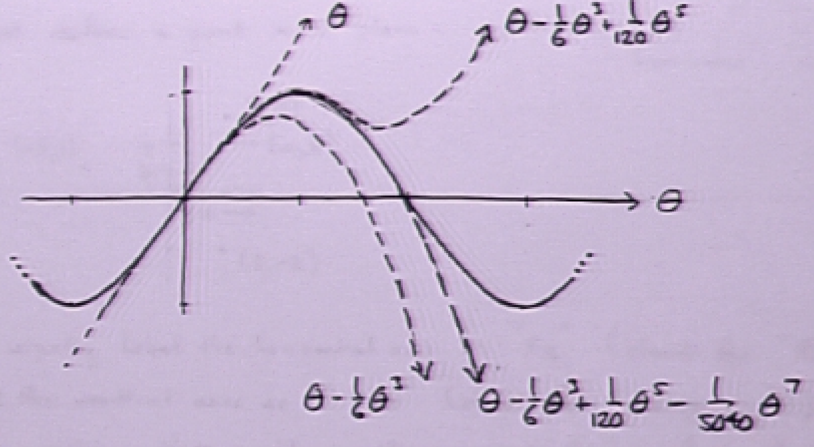
For <u>small</u> θ, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

Similarly : $\sin\theta = \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \frac{1}{5040}\theta^7 + \cdots$ infinite series



For small θ , $\sin\theta \approx \theta - \frac{1}{6}\theta^3$

Similarly : $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \cdots$ infinite series

$\uparrow$

e = 2.713... (a very special number in mathematics — like π)

note: 2 = 2·1
6 = 3·2·1
24 = 4·3·2·1
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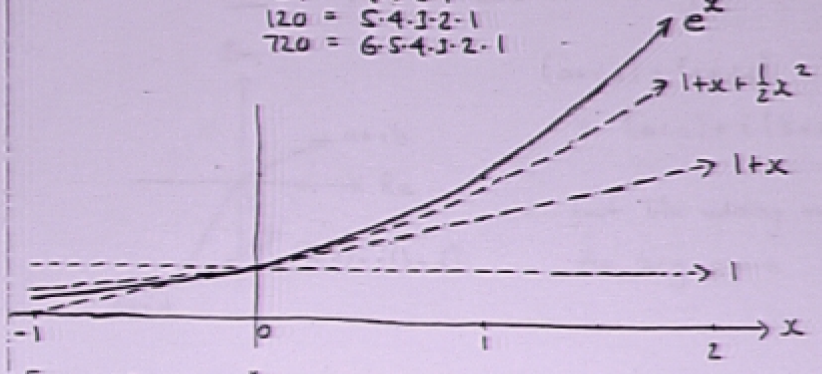
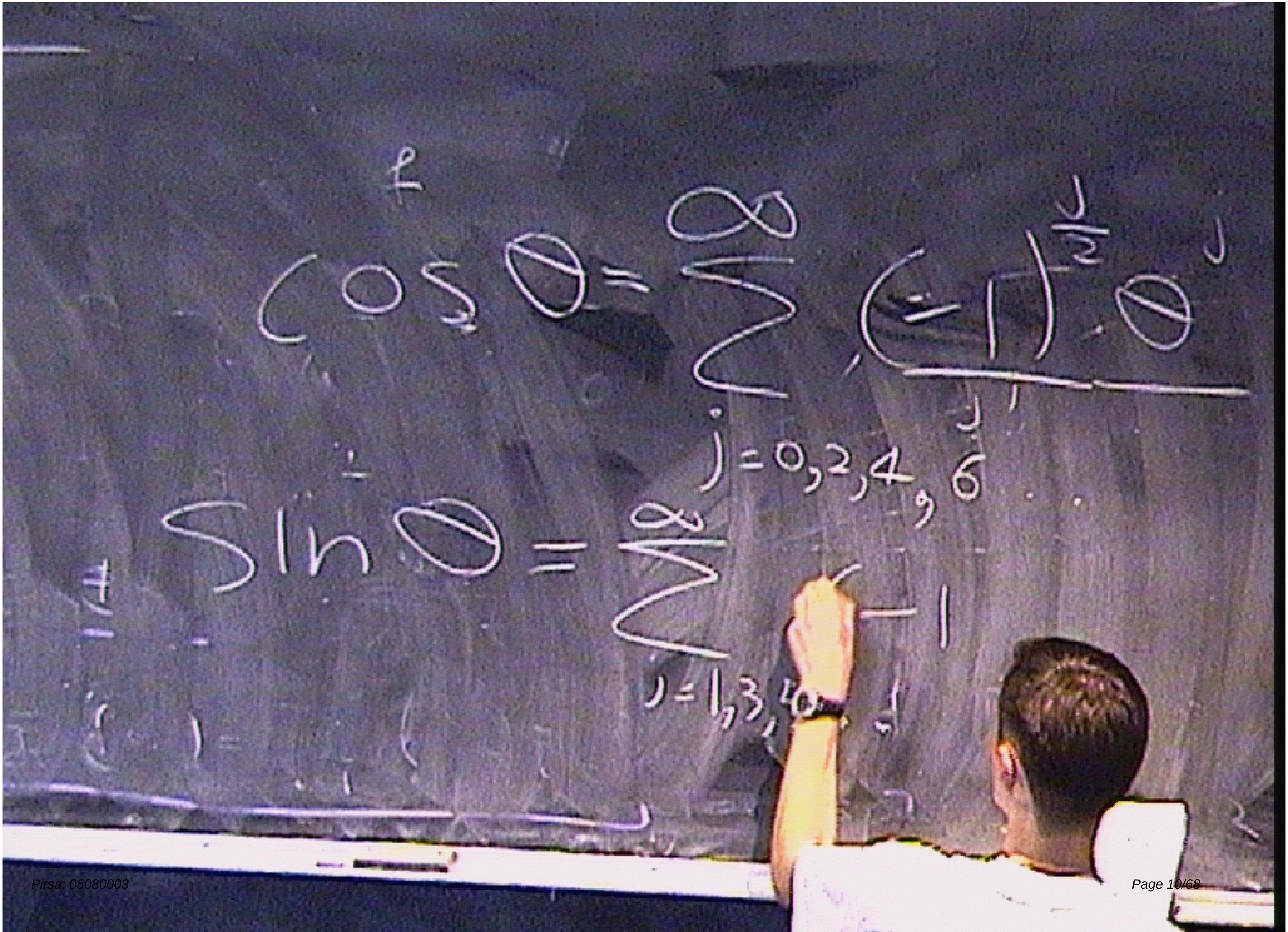


For small x , $e^x \approx 1+x$



$$\cos\theta = \sum_{j=0,2,4,6\ldots}^{\infty} \frac{(-1)^{\frac{j}{2}}\theta^{j}}{j!}$$

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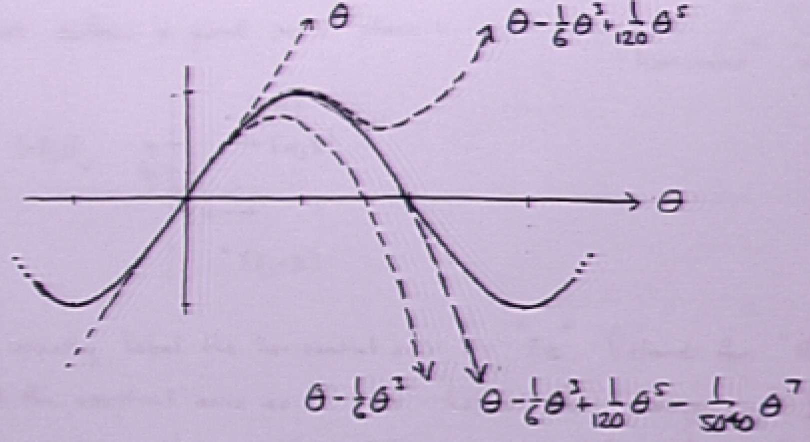

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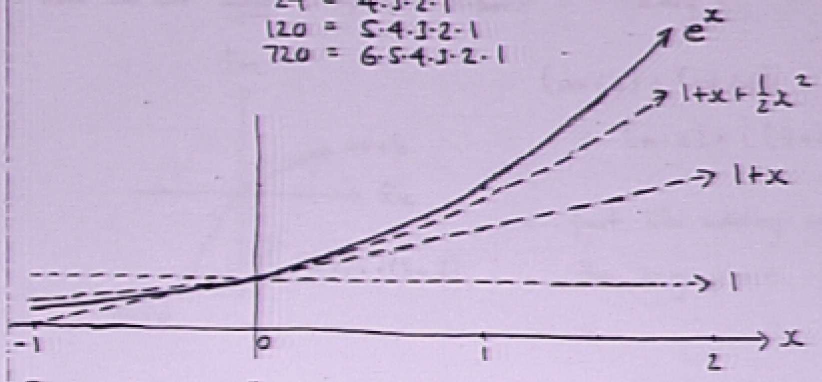

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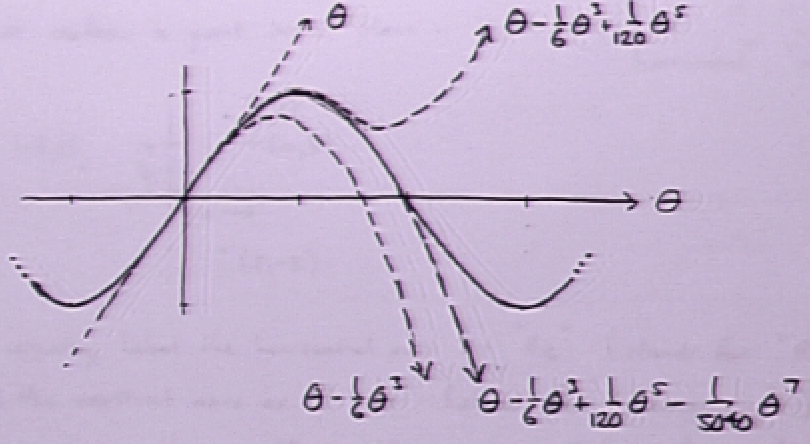

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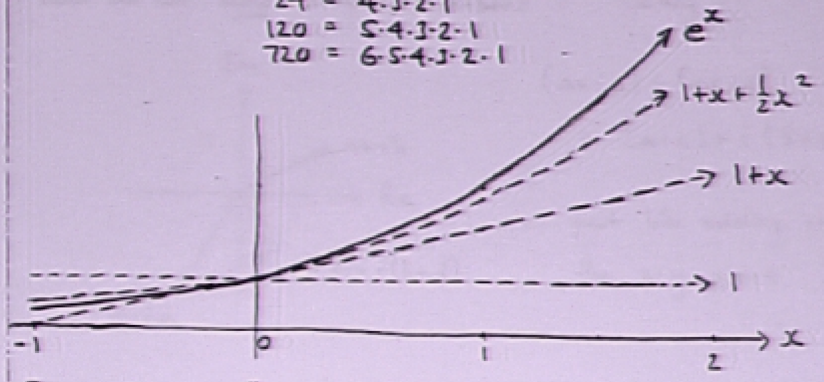


For <u>small</u> x, $e^x \approx 1+x$

$$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}$$

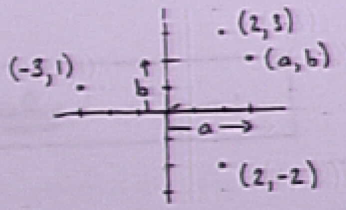
$$\frac{1)^{\frac{j}{2}}\,\theta^j}{j!}$$

$$\sin\theta = \sum_{j=1,3,5,\ldots}^{\infty} \frac{(-1)^{\frac{j-1}{2}}\,\theta^j}{j!}$$

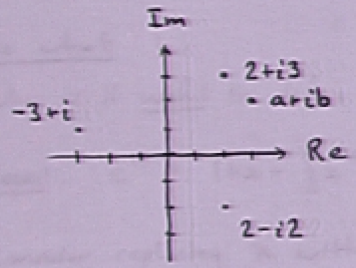
Complex Numbers

- A complex number is a pair of real numbers $\psi = (a, b)$ that defines a point in a plane:

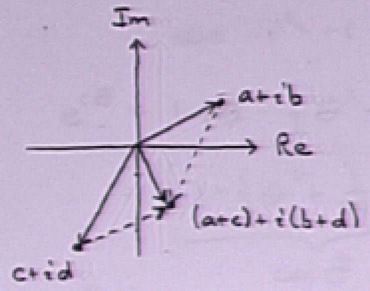
  "horizontal" "vertical"



- We usually label the horizontal axis as "Re" (stands for "Real") and the vertical axis as "Im" (stands for "Imaginary") and write $\psi = a + ib$; the symbol "i" in front of b reminds us that b is the distance along the "Imaginary" axis.



- How do we add complex numbers? Easy:



$$(a+ib) + (c+id)$$
$$= (a+c) + i(b+d)$$

--- just like adding vectors in the x-y plane!

- How do we multiply complex numbers ?

$$(a+ib)(c+id) = ac + iad + ibc + \underset{\substack{\uparrow \\ \text{what is } i^2 \text{ ?}}}{i^2} bd$$

$$\boxed{\text{define: } i^2 = -1}$$

Clearly, no number squared can be negative
— so i is not a "real" number
— it is called the (unit) "imaginary" number

With this definition of i we have

$$(a+ib)(c+id) = (ac-bd) + i(ad+bc)$$

- So what ?

Why is it useful to define a symbol i such that $i^2 = -1$?

Recall: $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \cdots$

Consider replacing x with $i\theta$ (a "pure imaginary" number):

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$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$ ← extremely important !!

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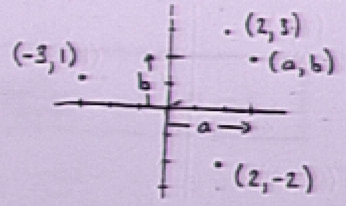

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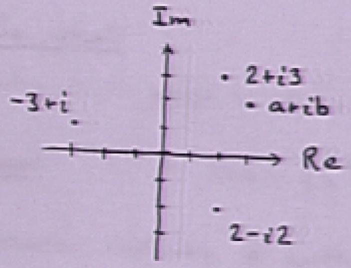

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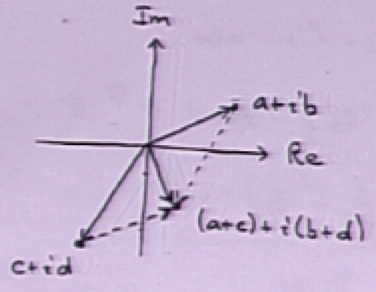

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--- just like adding vectors in the x-y plane!

- How do we <u>multiply</u> complex numbers ?

$$(a+ib)(c+id) = ac + iad + ibc + \underset{\substack{\uparrow \\ \text{what is } i^2 \text{ ?}}}{i^2} bd$$

$$\boxed{\underline{\text{define}}: \quad i^2 = -1}$$

Clearly, no number <u>squared</u> can be <u>negative</u>

— so i is not a "real" number

— it is called the (unit) "<u>imaginary</u>" number

With this <u>definition</u> of i we have

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use $i^2 = -1 \Rightarrow i^3 = -i$, $i^4 = +1$, $i^5 = i$, etc...

$$e^{i\theta} = 1 + i\theta - \frac{1}{2}\theta^2 - i\frac{1}{6}\theta^3 + \frac{1}{24}\theta^4 + i\frac{1}{120}\theta^5 + \cdots$$

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$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta} \leftarrow \text{extremely important !!}$$

- How do we multiply complex numbers ?

$$(a+ib)(c+id) = ac + iad + ibc + \underset{\uparrow}{i^2}bd$$

what is i^2 ?

$$\boxed{\text{define}: \quad i^2 = -1}$$

Clearly, no number squared can be negative

— so i is not a "real" number

— it is called the (unit) "imaginary" number

With this definition of i we have

$$(a+ib)(c+id) = (ac-bd) + i(ad+bc)$$

- So what ?

Why is it useful to define a symbol i such that $i^2 = -1$?

Recall: $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \cdots$

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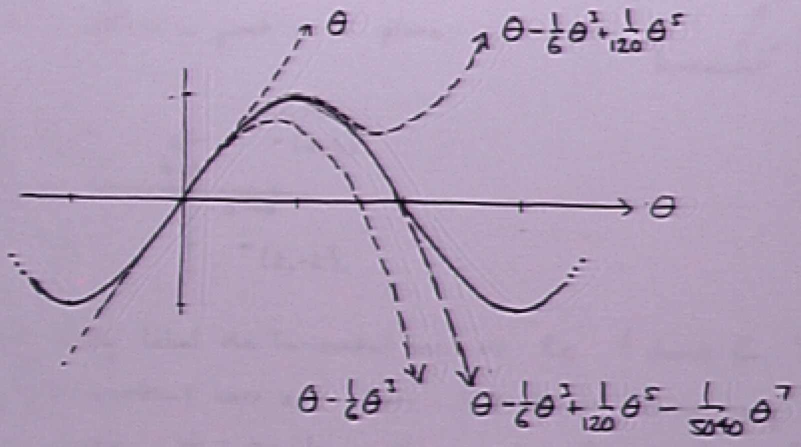
$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$ ← extremely important !!

Similarly : $\sin\theta = \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \frac{1}{5040}\theta^7 + \cdots$ infinite series

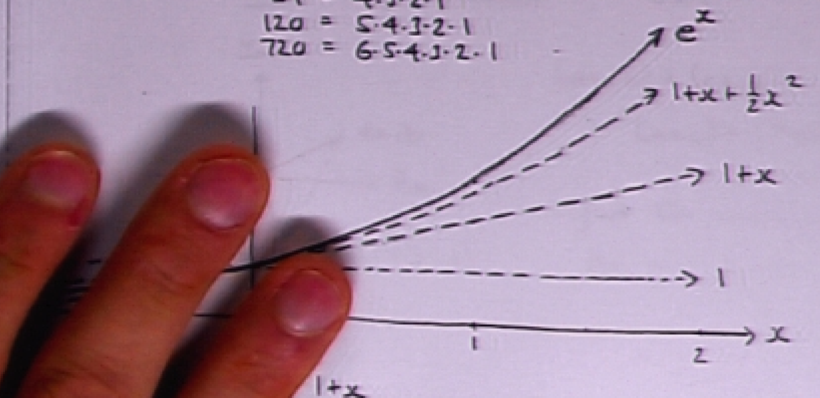


For small θ, $\sin\theta \approx \theta - \frac{1}{6}\theta^3$

Similarly : $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \cdots$
$\uparrow$
infinite series

$e = 2.713\ldots$ (a very special number in mathematics — like π)

note: $2 = 2\cdot1$
$6 = 3\cdot2\cdot1$
$24 = 4\cdot3\cdot2\cdot1$
$120 = 5\cdot4\cdot3\cdot2\cdot1$
$720 = 6\cdot5\cdot4\cdot3\cdot2\cdot1$



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$$(a+ib)(c+id) = ac + iad + ibc + \underset{\substack{\uparrow \\ \text{what is } i^2 \text{ ?}}}{i^2} bd$$

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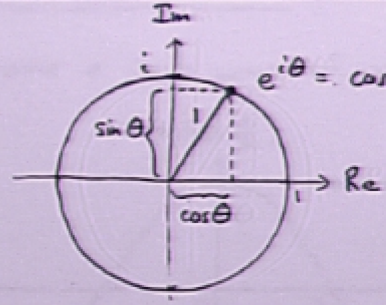
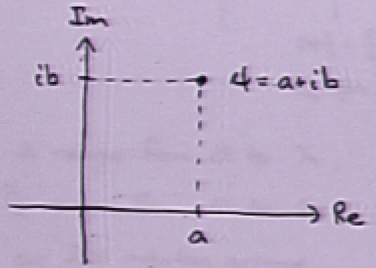
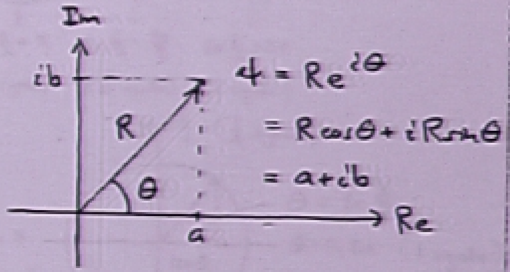


$$e^{i\theta} = \cos\theta + i\sin\theta$$ ← this is exactly the type of mathematics needed to describe quantum waves (considered on p. 17)

This provides an alternative way of expressing a complex number:





$$\psi = Re^{i\theta} = R\cos\theta + iR\sin\theta = a+ib$$

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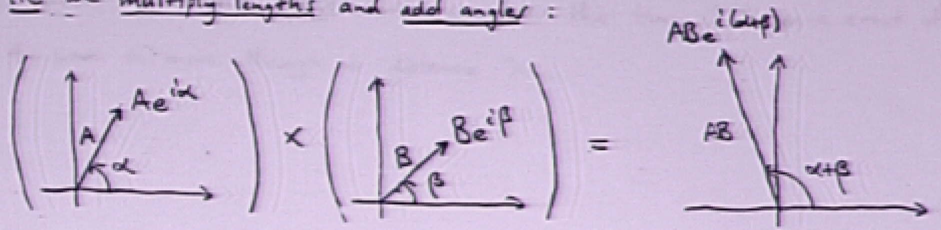

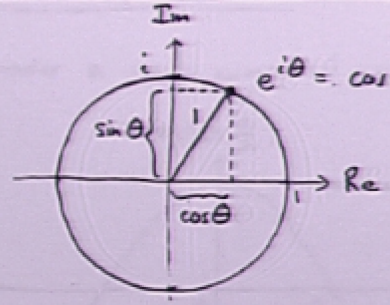

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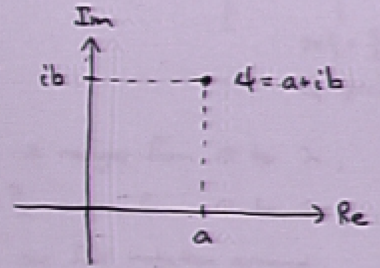

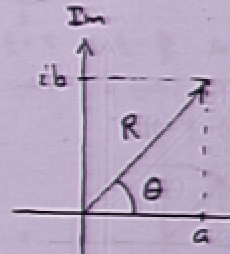

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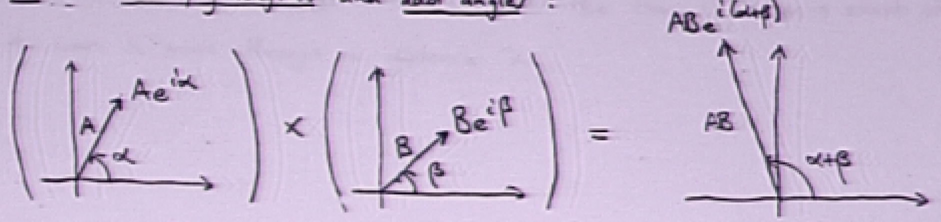

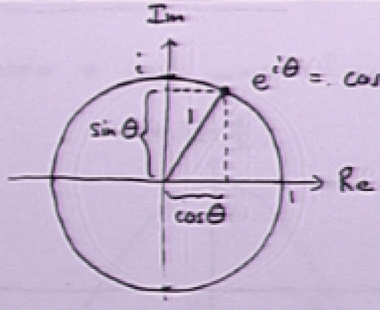

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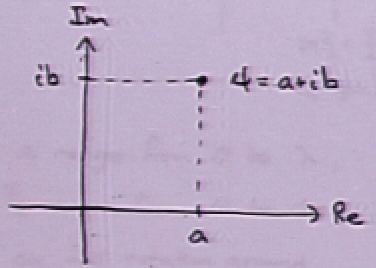

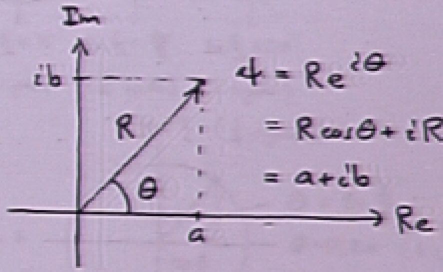

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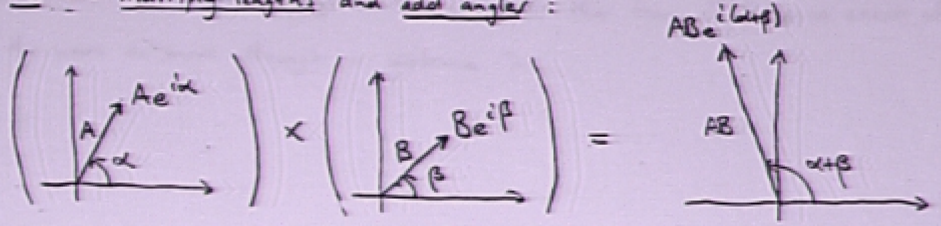

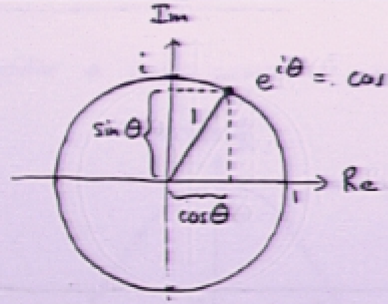

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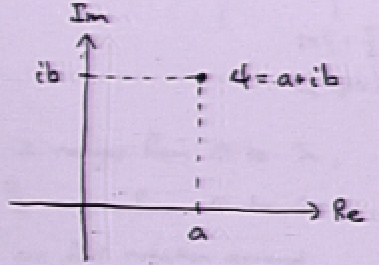

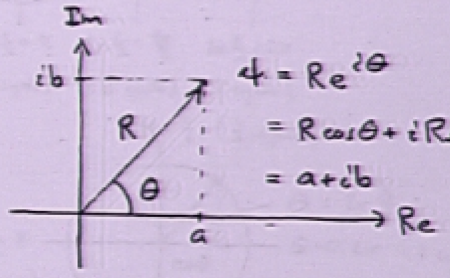

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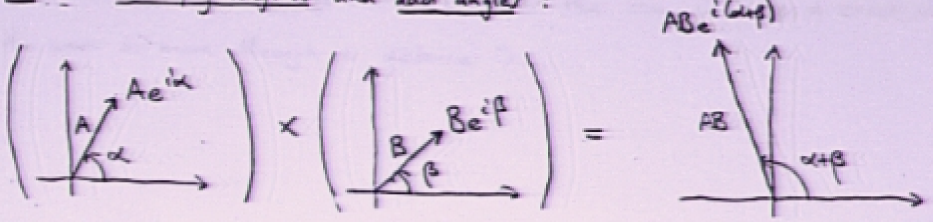

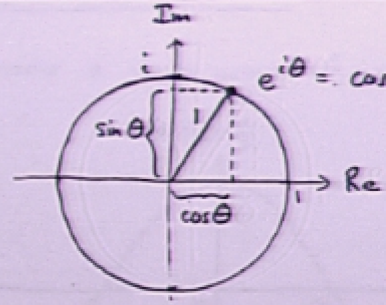

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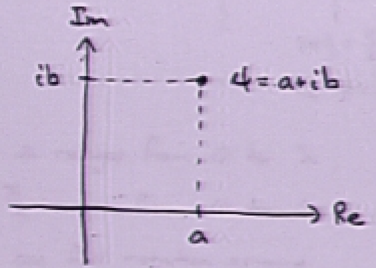

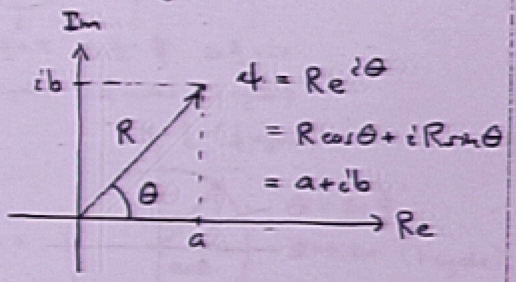

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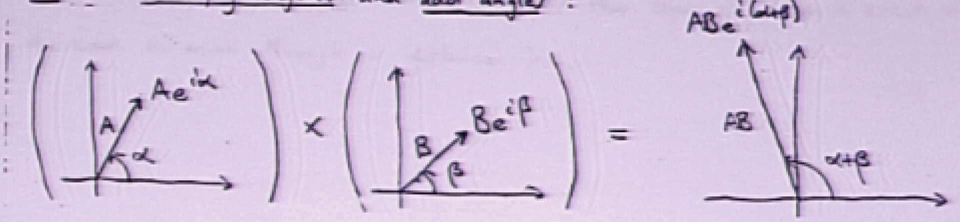

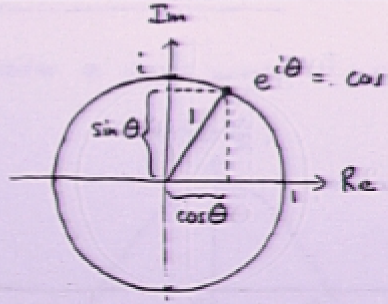

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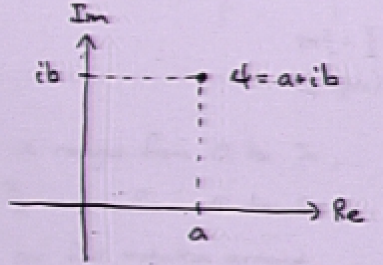

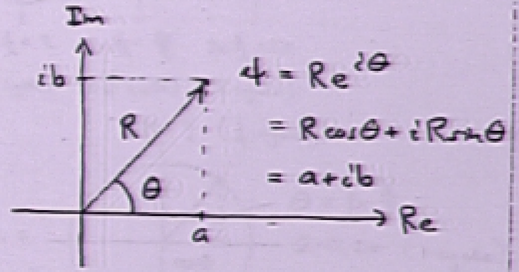

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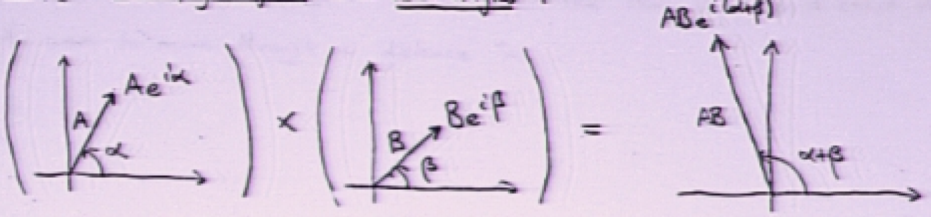

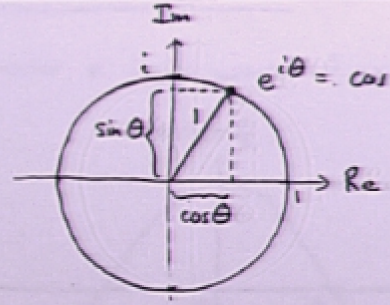

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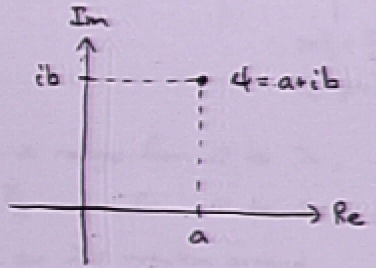

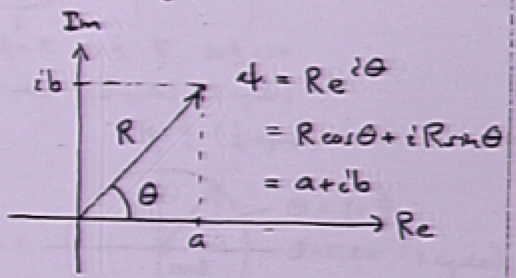

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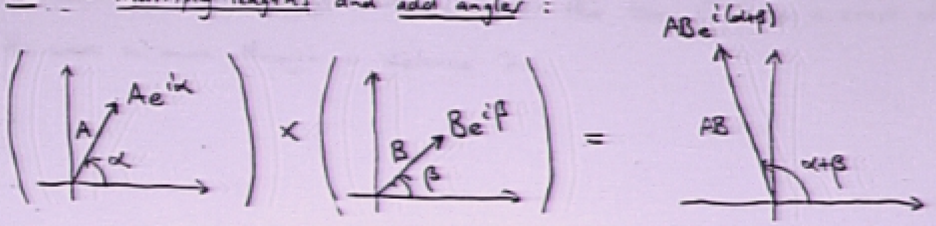

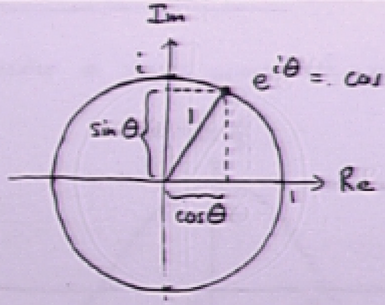

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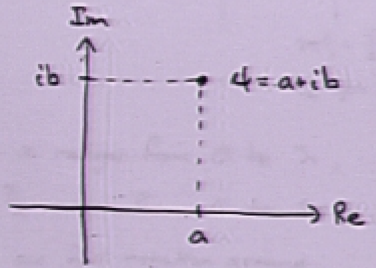

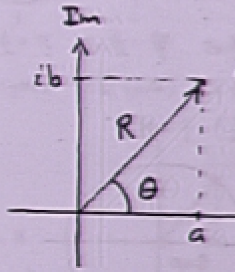

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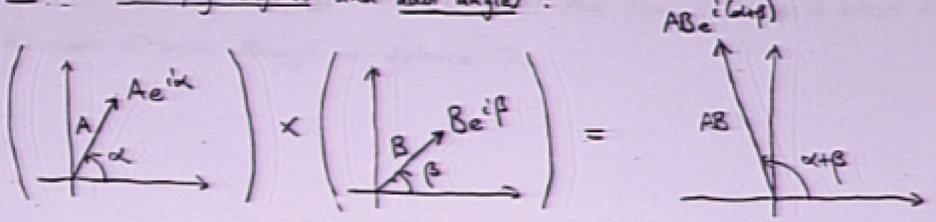
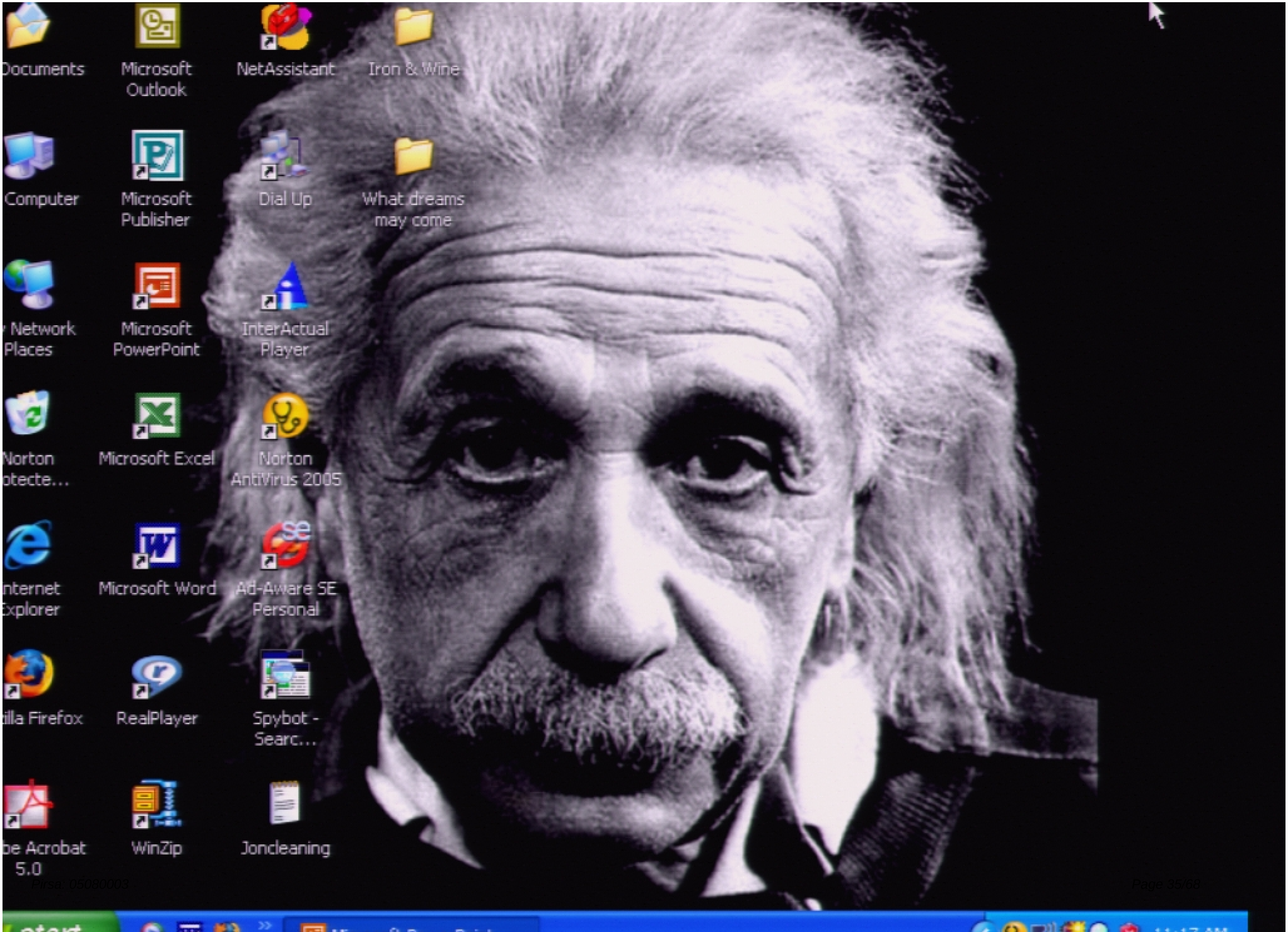



Documents
Microsoft Outlook
NetAssistant
Iron & Wine
Computer
Microsoft Publisher
Dial Up
What dreams may come
Network Places
Microsoft PowerPoint
InterActual Player
Norton otecte...
Microsoft Excel
Norton AntiVirus 2005
nternet Explorer
Microsoft Word
Ad-Aware SE Personal
illa Firefox
RealPlayer
Spybot - Searc...
be Acrobat 5.0
WinZip
Joncleaning

Actions and Reactions

Perceptions of Physics

Actions and Reactions

Perceptions of Physics

$$F = ma$$

Newton’s Laws

$$F = ma$$

$$E = mc^2$$

Special R

$$E = mc^2$$

Special Relativity

$$E = mc^2$$

$$H(t)\,|\,\psi(t)\rangle\rangle = i\hbar\frac{\partial}{\partial t}\,|\,\psi(t)\rangle\rangle$$

Quantum Mechanics

$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \Lambda g_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$$

General Relativity

$$R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R+\Lambda g_{\mu\upsilon}=8\pi T_{\mu\upsilon}$$

$$F_{ab}^{ij}\left(\vec{\tau}\right)\frac{\delta}{\delta A_a^i\left(\vec{\tau}\right)}\frac{\delta}{\delta A_b^j\left(\vec{\tau}\right)}\Psi\left[A\right]=0$$

Quantum Gravity

$$F_{ab}^{ij}\left(\vec{\tau}\right)\frac{\delta}{\delta A_{a}^{i}\left(\vec{\tau}\right)}\frac{\delta}{\delta A_{b}^{j}\left(\vec{\tau}\right)}\Psi\left[A\right]=0$$

$$\int_{C^N\times S^N} dx^N \wedge dy^N R^i{}_{jkl;\mu}\partial_\upsilon R_{ij;k}\Psi\left[\Gamma^j{}_{kl}\right]=T_{ij;k}\phi^{kl}$$

Meaningle

$$\int\limits_{C^N\times S^N} dx^N \wedge dy^N R^i{}_{jkl;\mu}\partial_\upsilon R_{ij;k}\Psi\left[\Gamma^j{}_{kl}\right]=T_{ij;k}\phi^{kl}$$

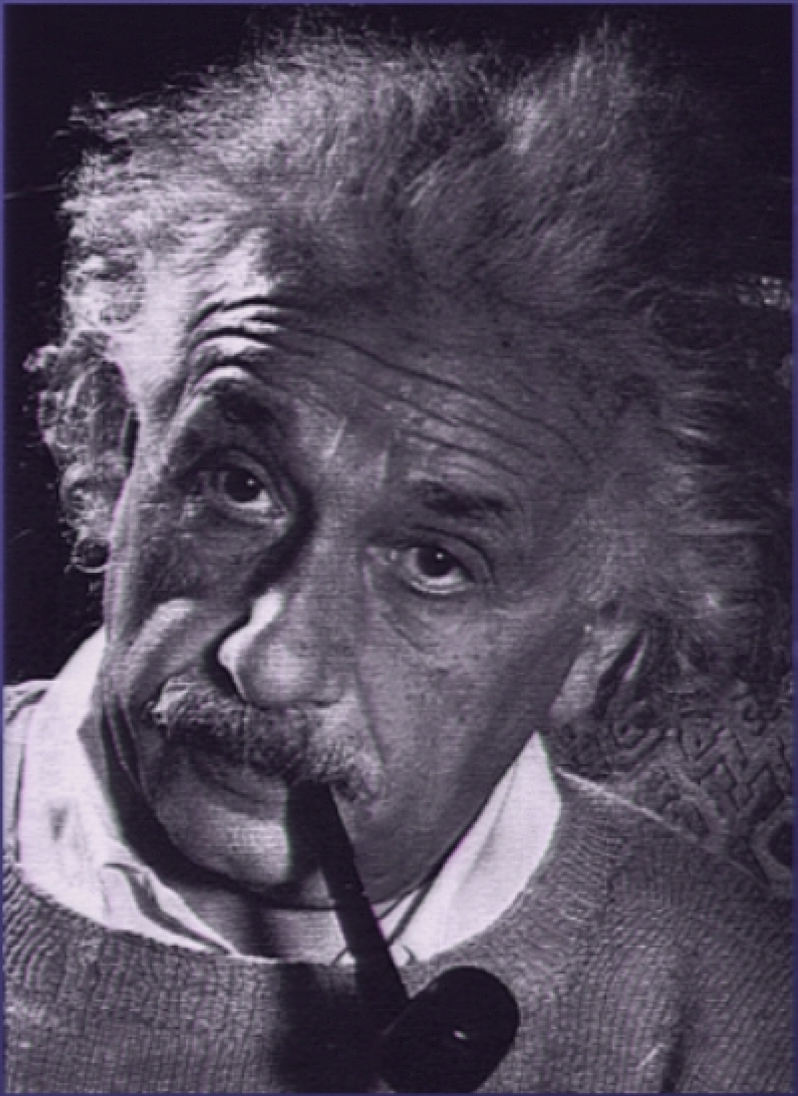
You study what?!?

“You must be a ge

“You must be a genius!”

"You must be a genius!"

“You must be a genius!”



“You’re too cute to be a ne

"You're too cute to be a nerd."

“You’re too cute to be a nerd.”

“You’re too cute to be a nerd.”

“But you’re a girl.”

Conseil de Solvay de 1927

Picard Henriot Ehernfest Herzen de Donder Schrödinger Verschaffelt Pauli Heisenberg Fowler brillouin
Debye Knudsen Bragg Kramers Dirac Compton L. de Broglie Born Bohr
Langmuir Planck Mme Curie Lorentz Einstein Langevin Guye Wilson Richardson

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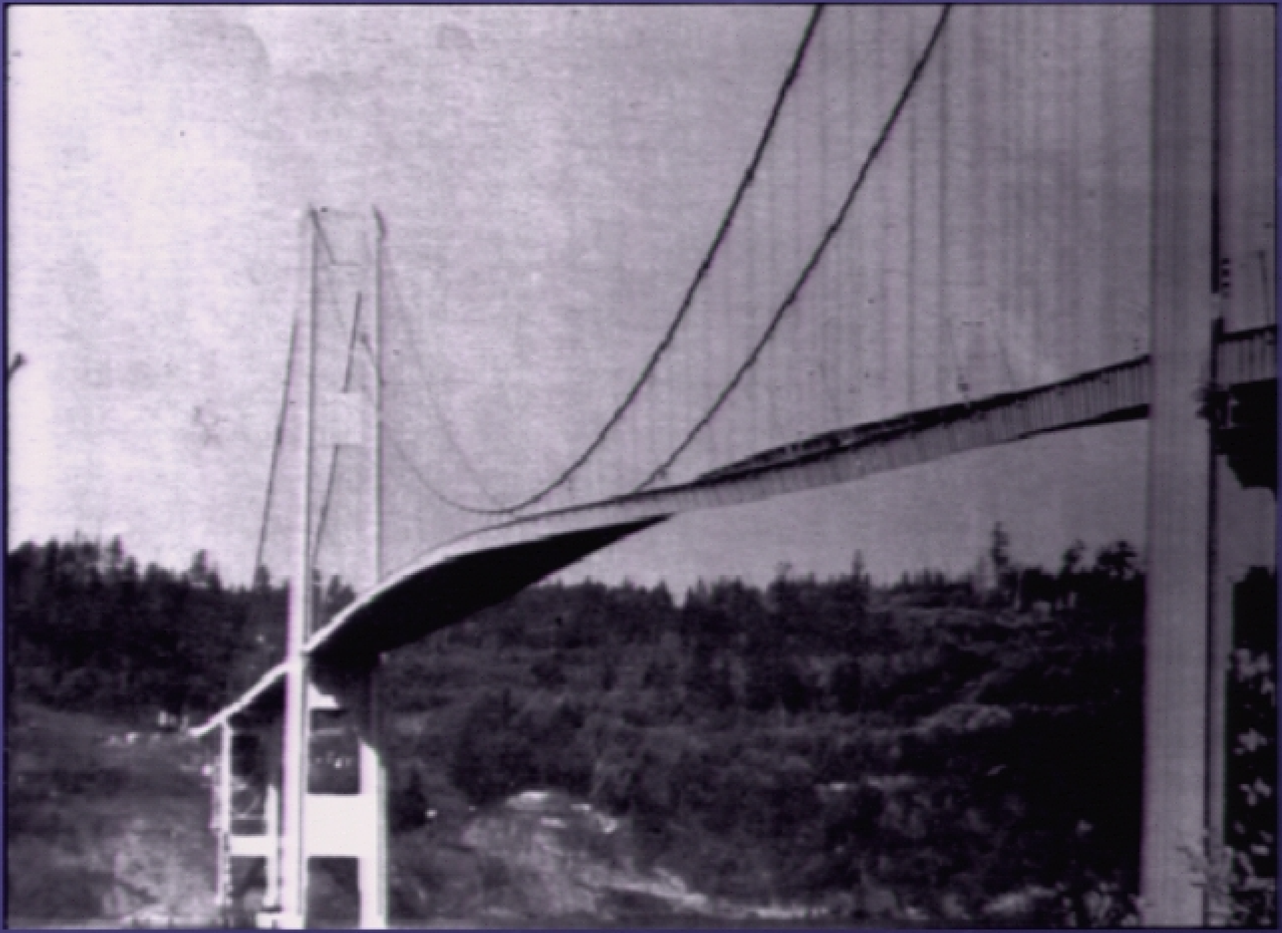
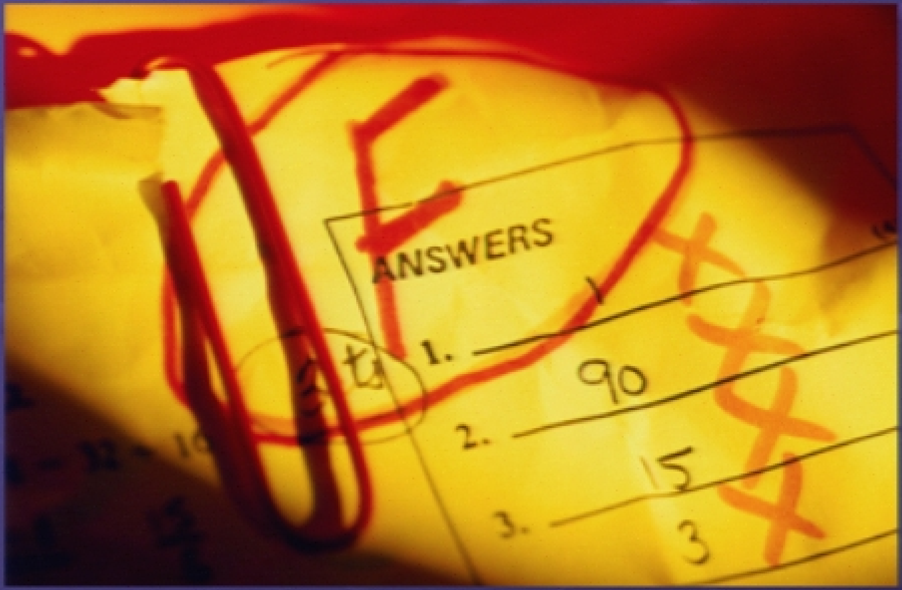
Langmuir Planck Mme Curie Lorentz Einstein Langevin Guye Wilson Richardson

“Is that like engineering or bad math?”

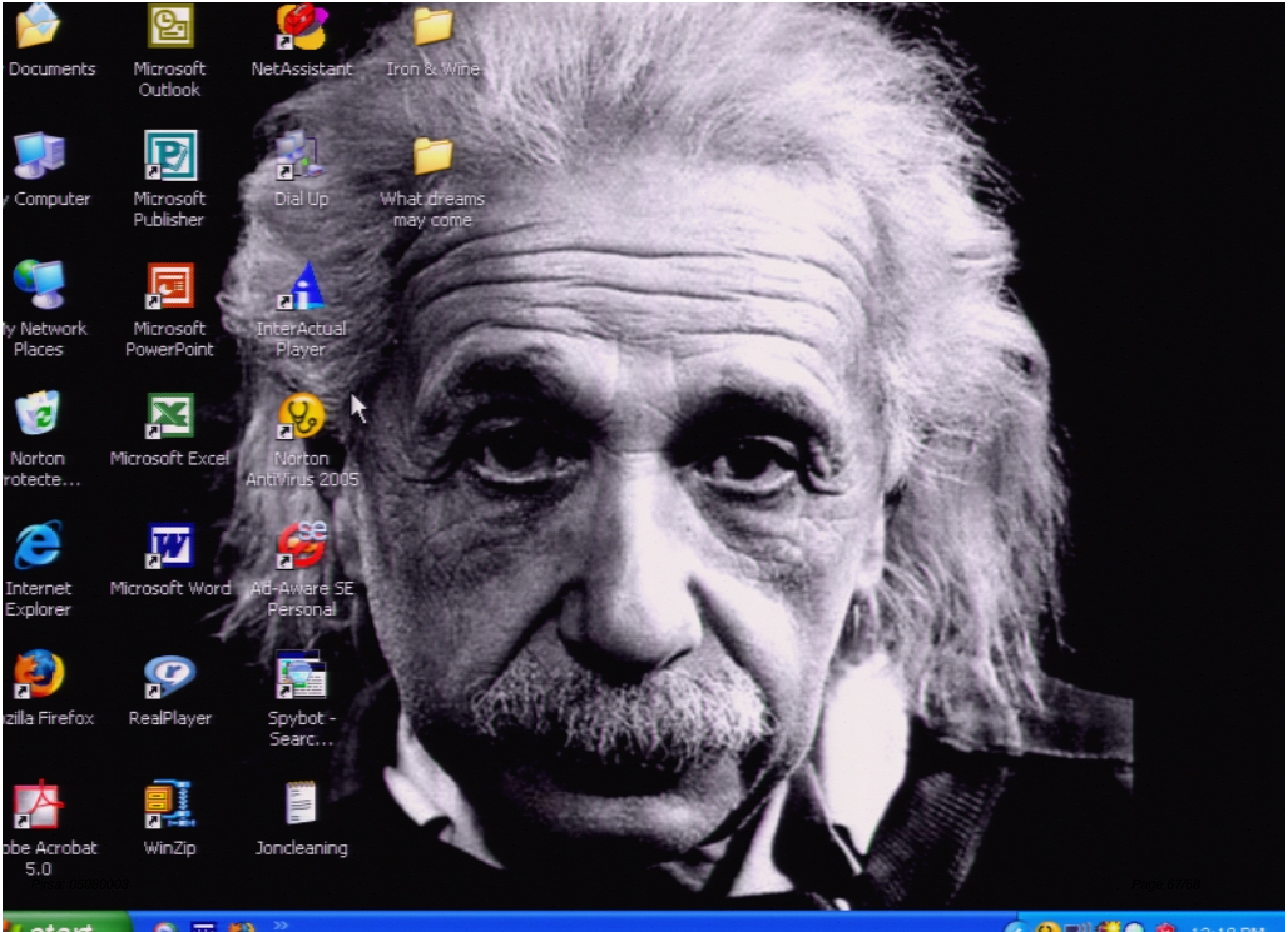


“Physics was the only course I nearly failed in high school.”



“What are you going to do with that?”

Genuine Interest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Microsoft Outlook
Microsoft Publisher
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WinZip
NetAssistant
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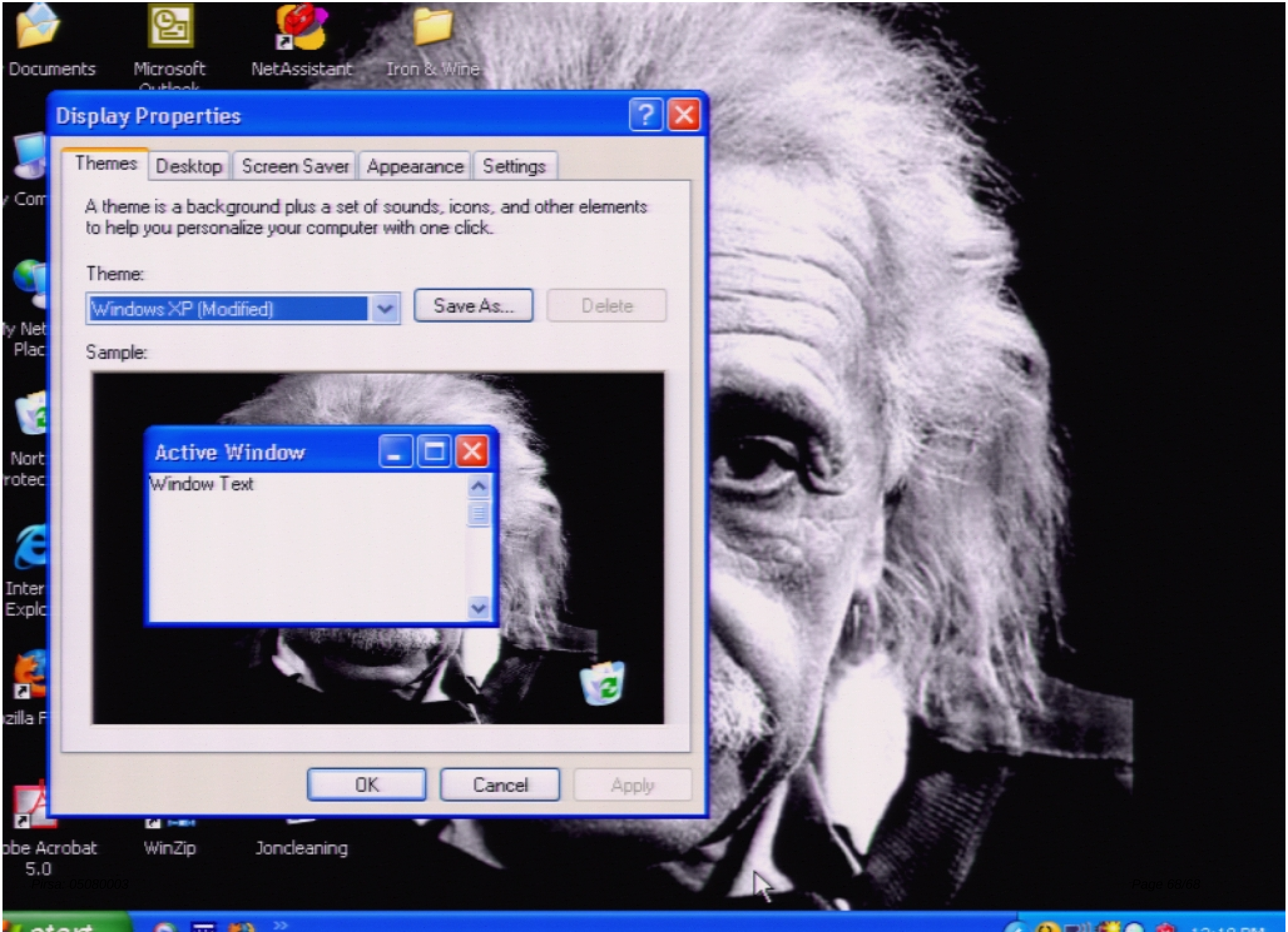
Spybot - Searc...
Joncleaning
Iron & Wine
What dreams may come



A theme is a background plus a set of sounds, icons, and other elements to help you personalize your computer with one click.

Theme: Windows XP (Modified)