Title: Introduction to Calculus

Date: Aug 08, 2005 10:35 AM

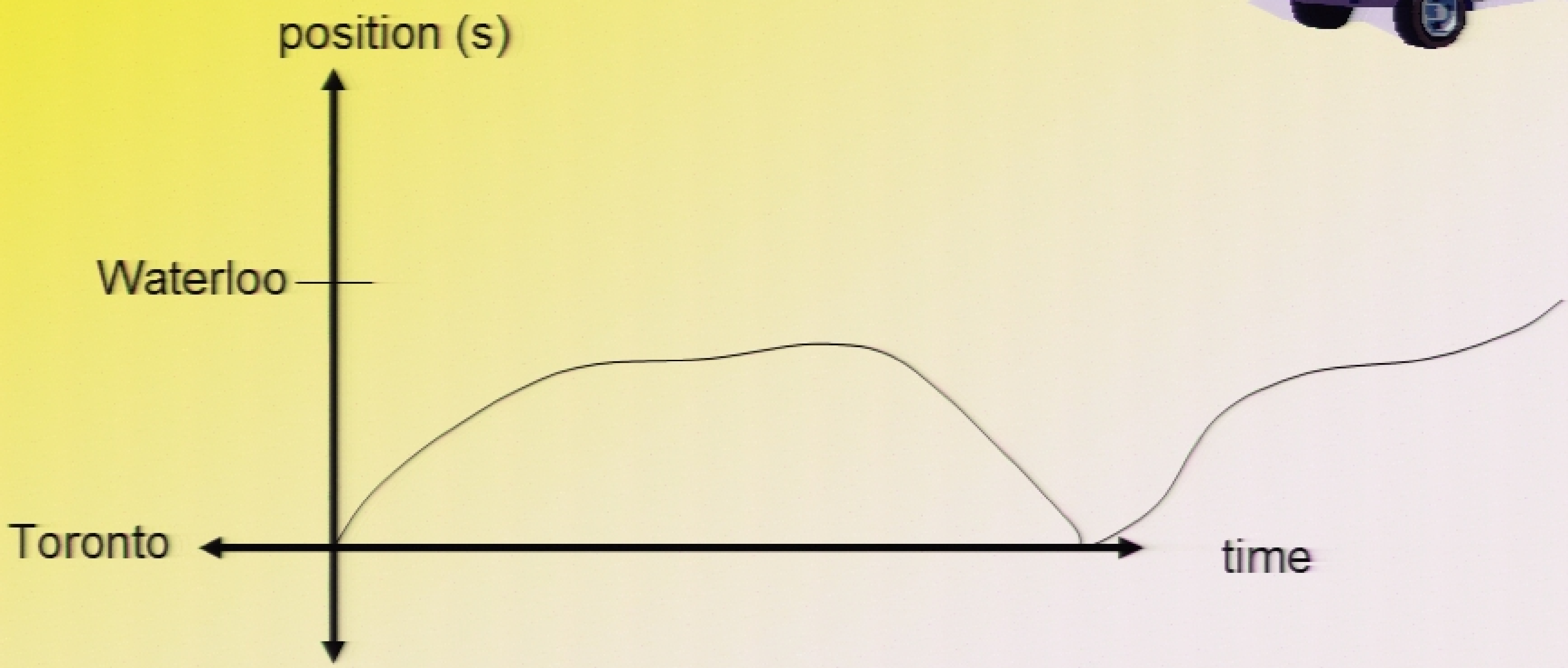
URL: http://pirsa.org/05080002

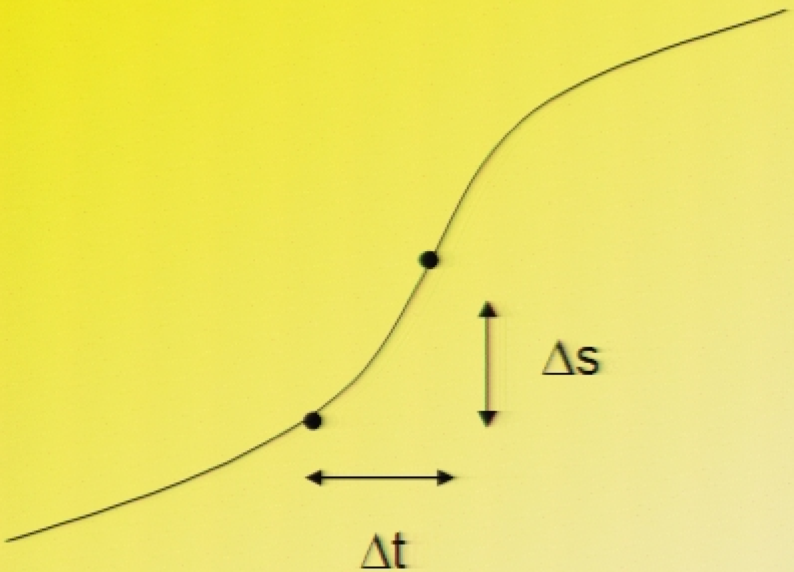
Abstract:

Calculus

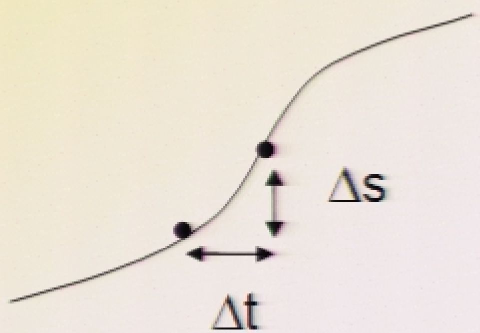
- In physics, we often come across things that are changing and evolving over time.
- eg. the location of a baseball thrown in the air
- The temperature of a pie in an oven
- position of a car over time

Position of a car over time

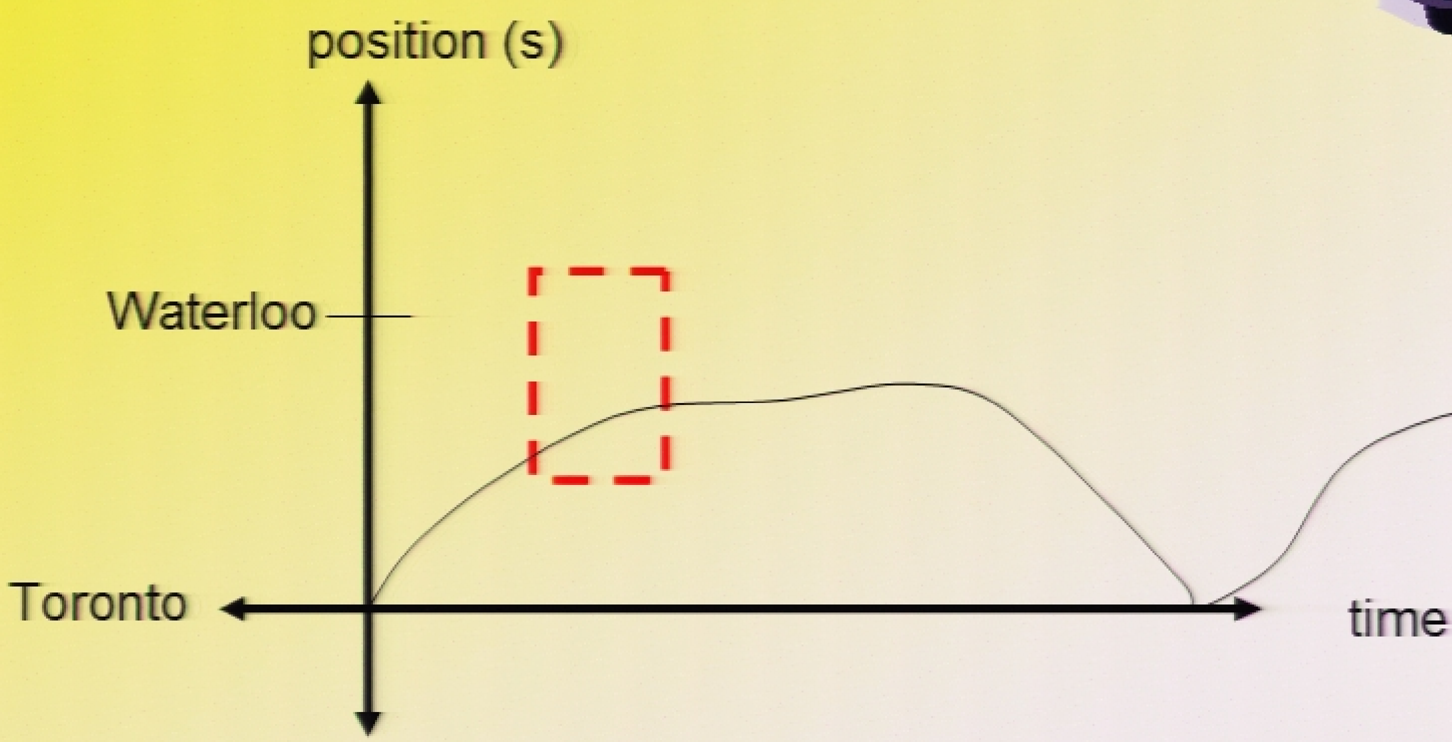


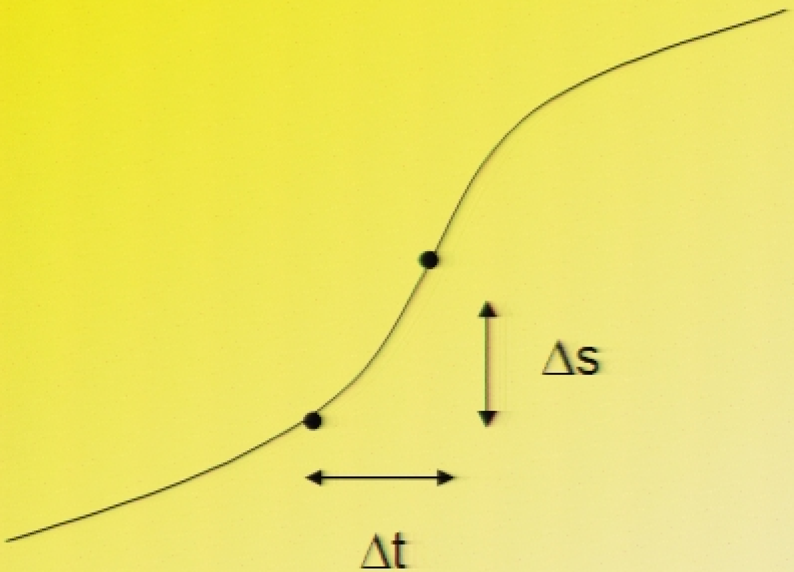


- Let us keep making Δt smaller and smaller



Position of a car over time





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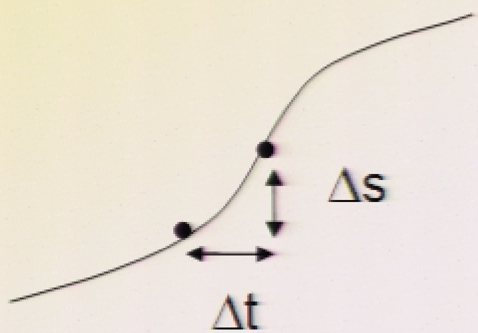
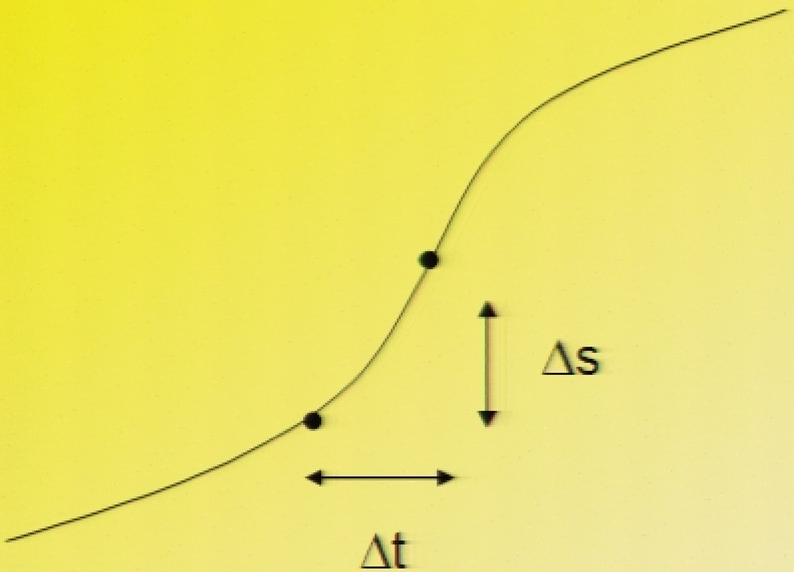


What happens when Δt is ‘really small’?

- 0/0
- What is the value of this?
- Answer: It could be anything, indeterminate.
- $\lim_{\Delta t \to 0} \Delta s/\Delta t = ds/dt$.
- ds/dt is called the *derivative* of s with respect to t
- *In general*, the derivative of f with respect to x is $df(x)/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x$
- df(x) and dx are called infinitesimals



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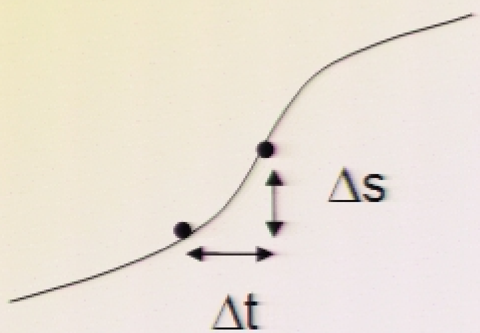

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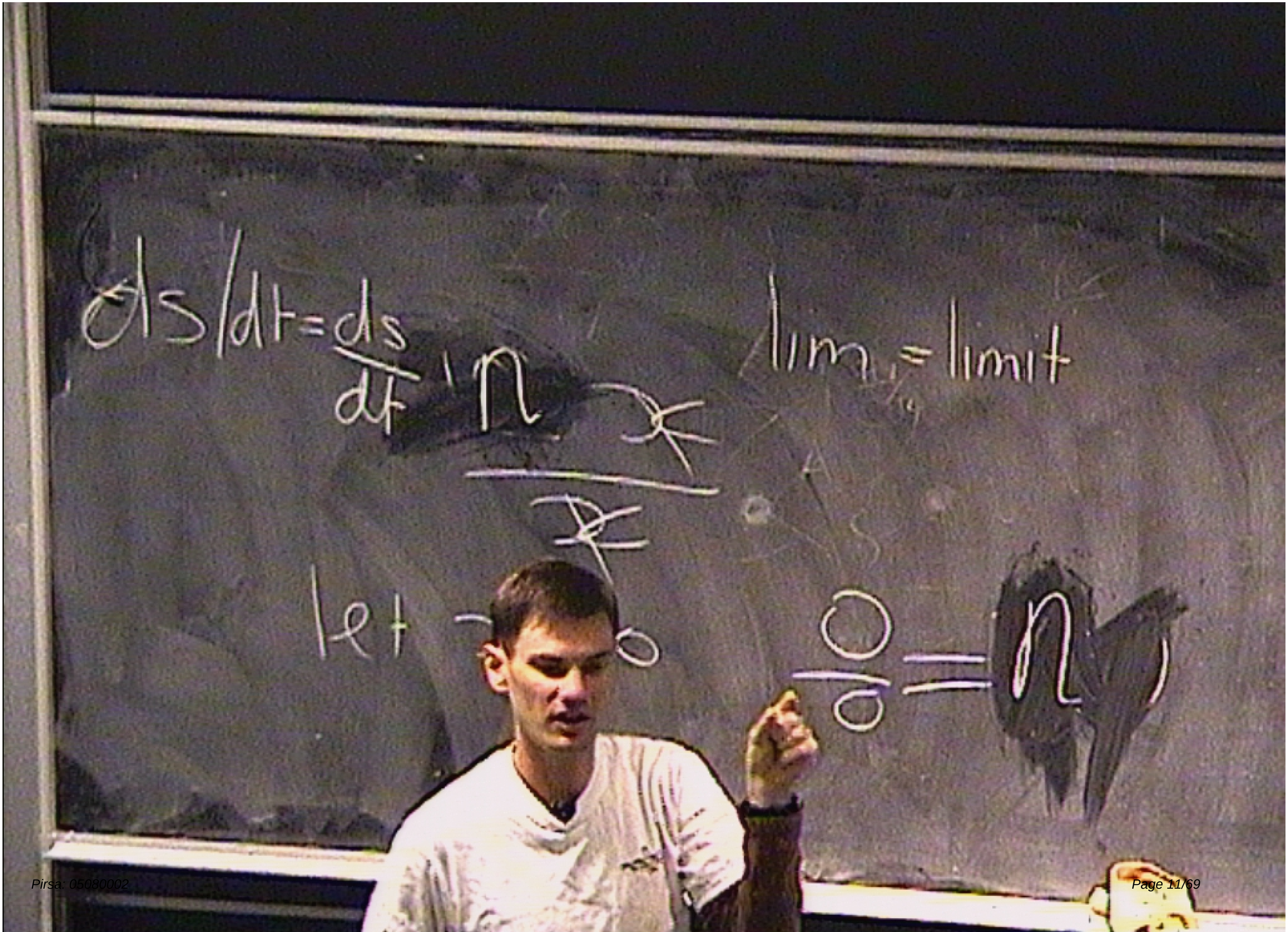
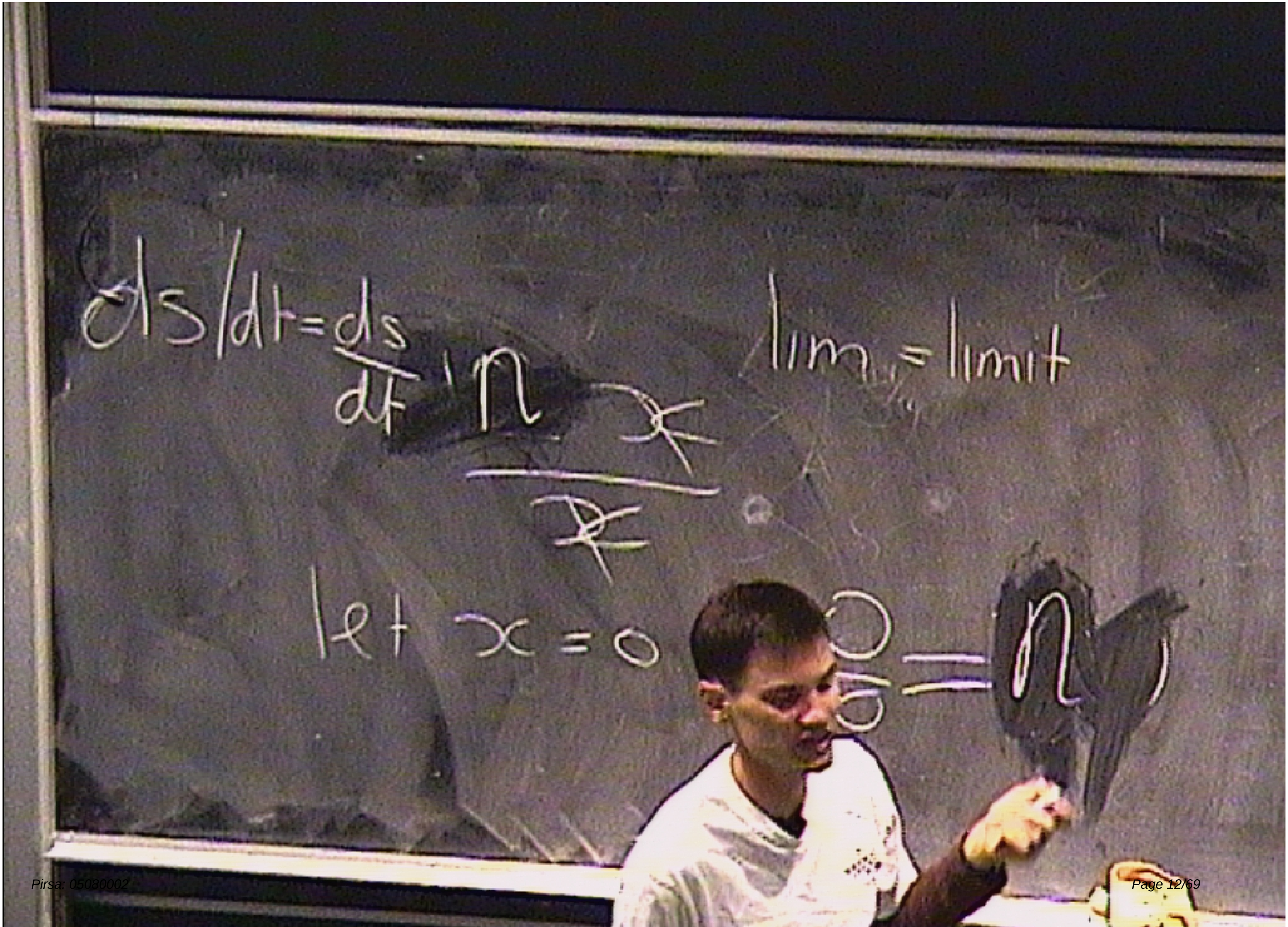

ds/dt = ds/dt
lim = limit
let


ds/dt=ds/dt
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let x = 0

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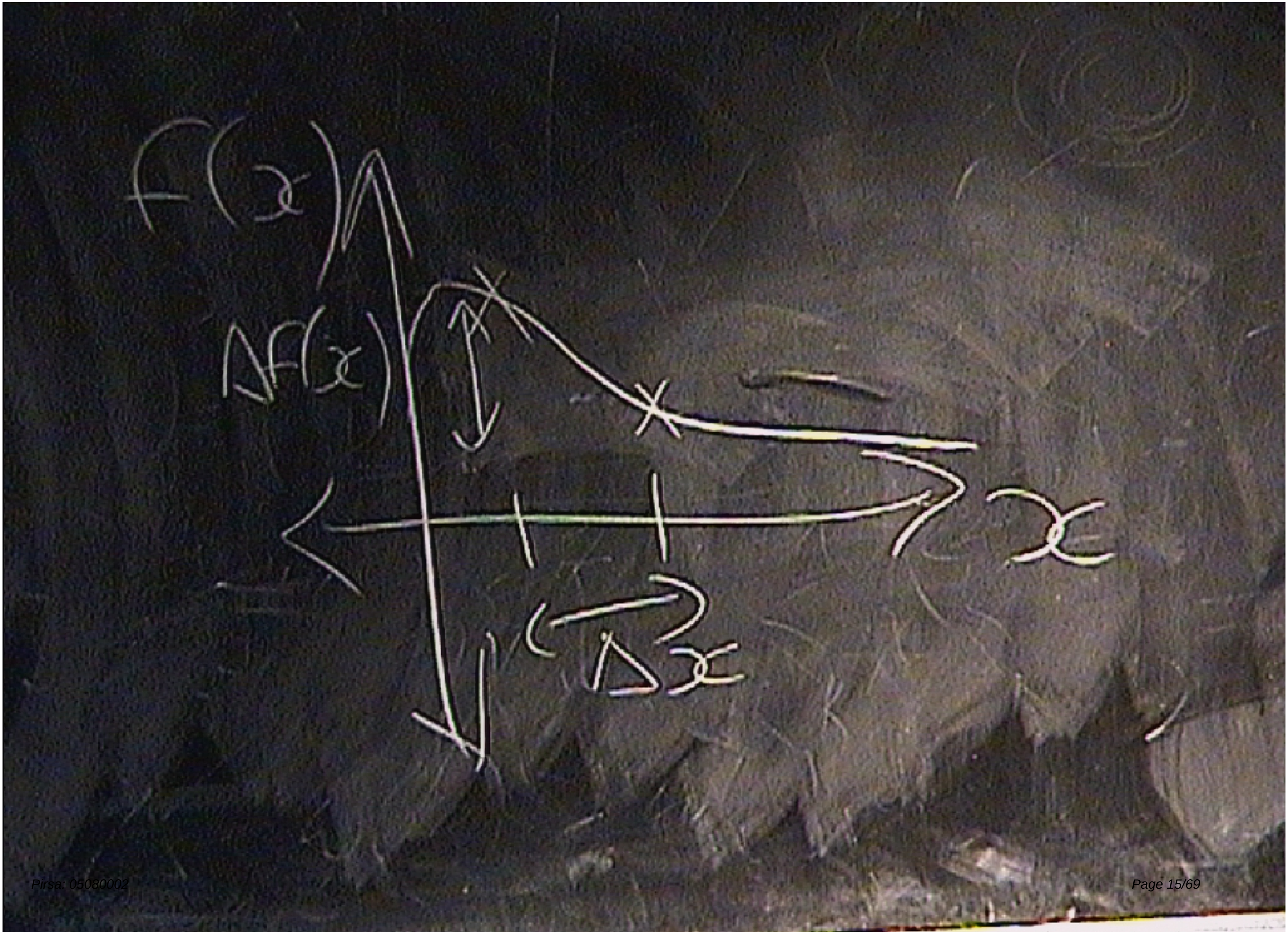
f(x)
Δf(x)
x

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- $df(x)$ and dx are called infinitesimals

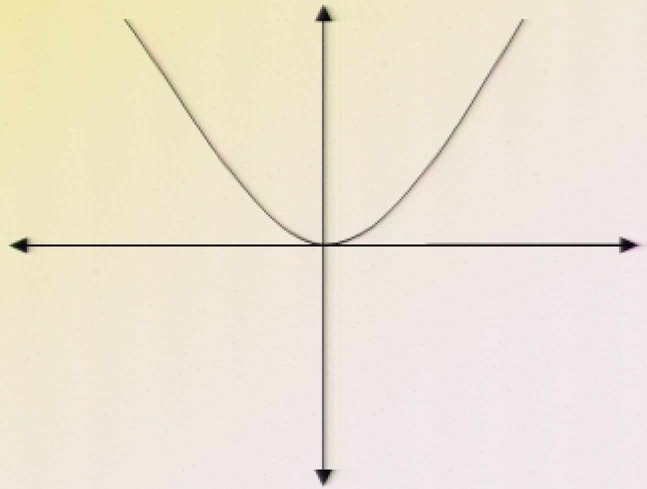
- Consider the function $f(x) = x^2$
- What is df/dx?

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

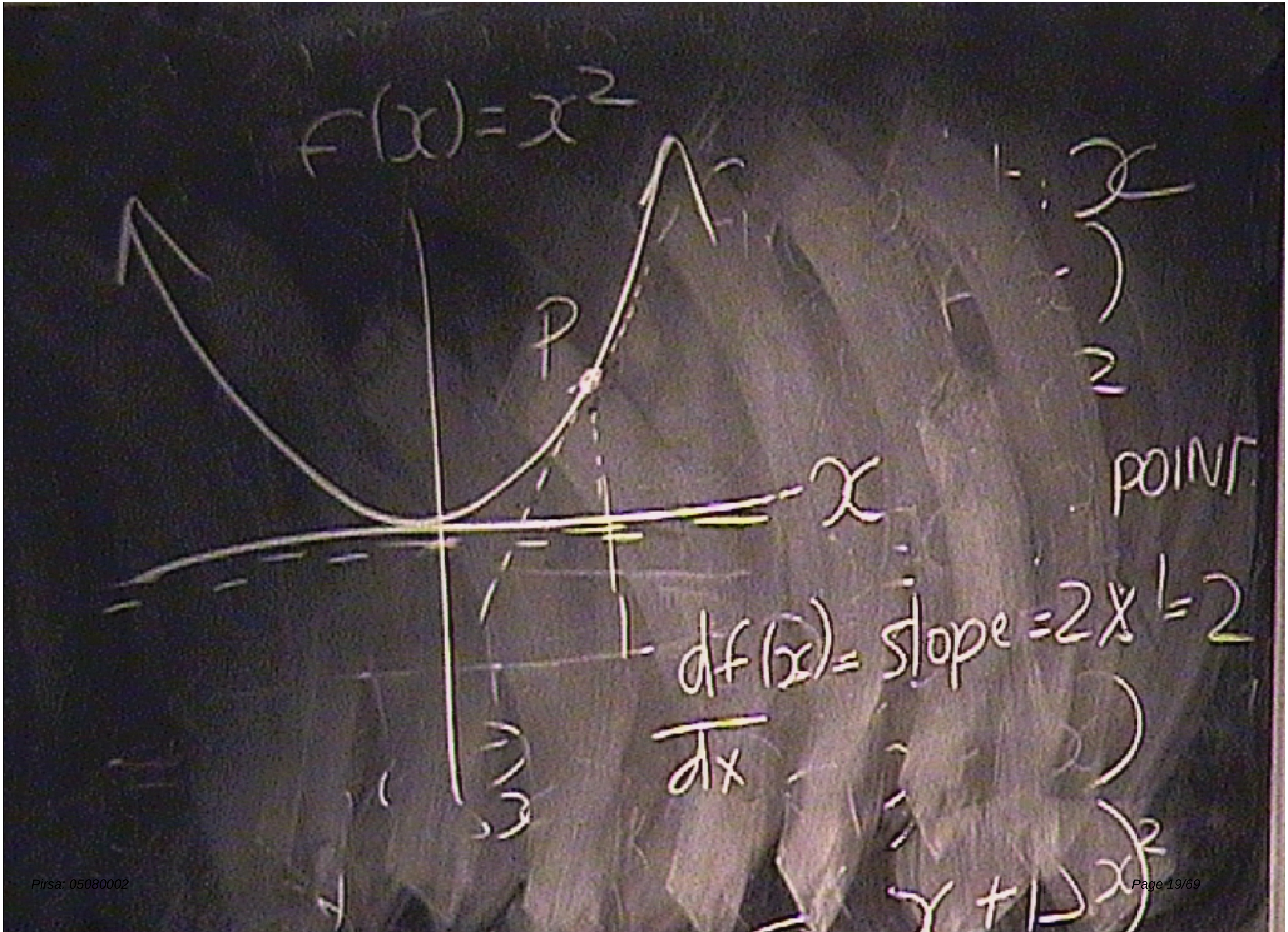
$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= 2x + \Delta x$$



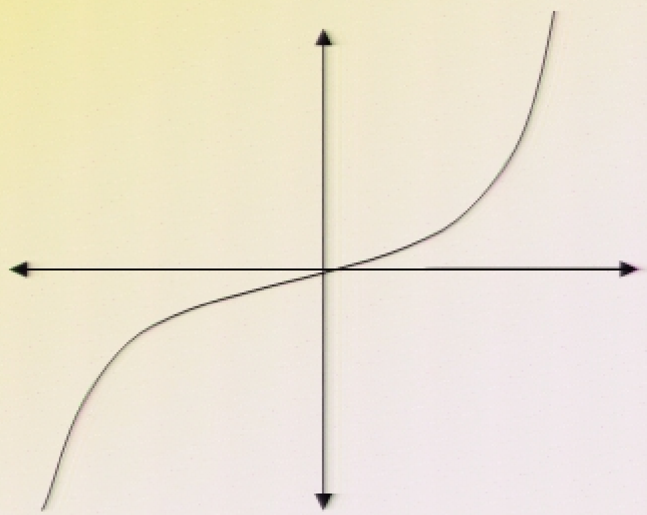
- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = 2x$


$f(x)=x^2$
P
x
POINT
$\frac{df(x)}{dx}$ = slope = 2x

- Consider the function $f(x) = x^3$
- What is df/dx?

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= ?$$



- Consider the function $f(x) = x^3$
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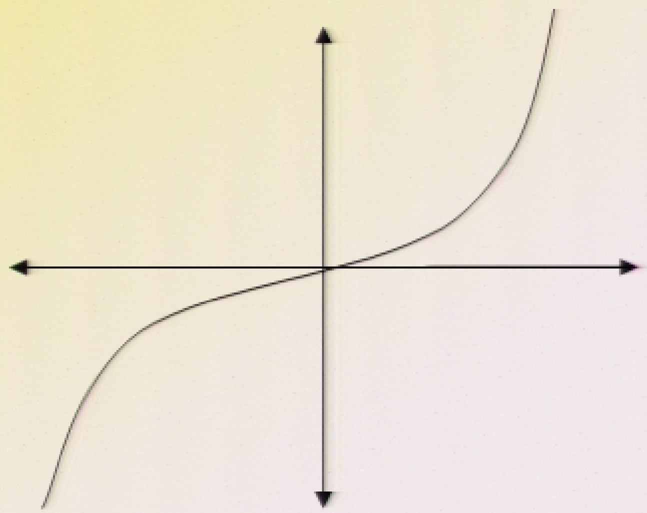
$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

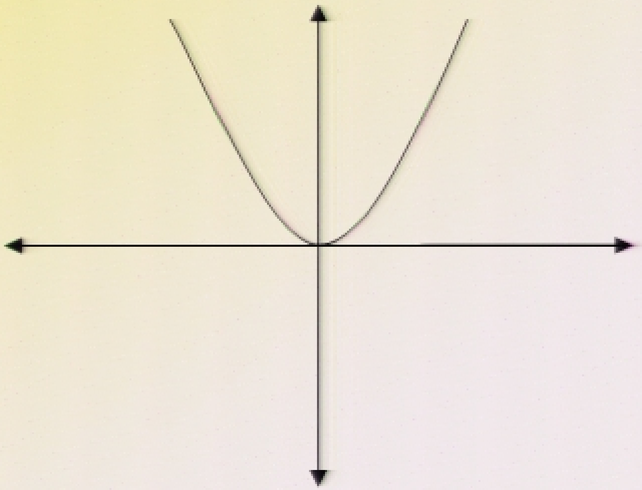
$$= \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

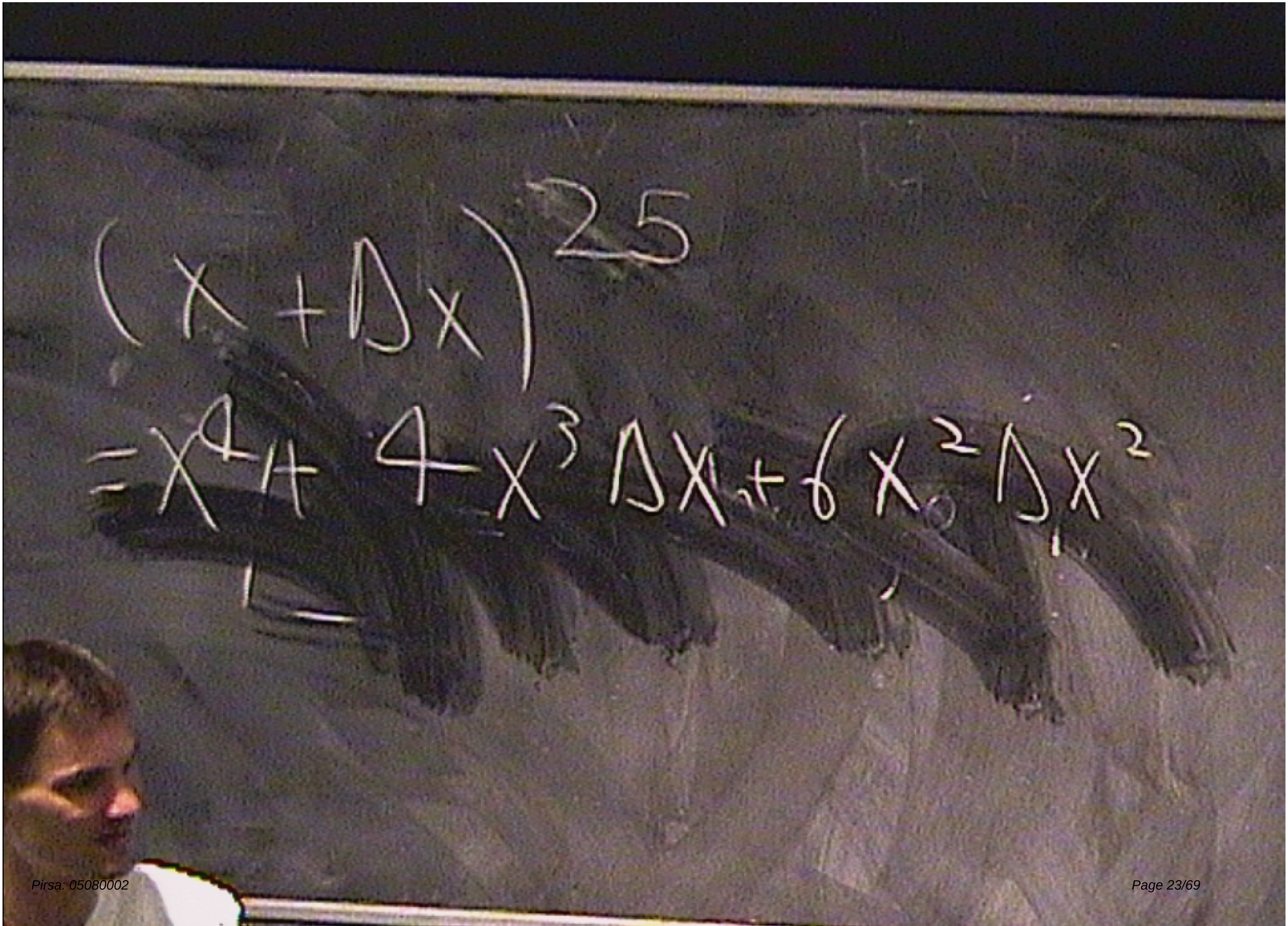
$$= 3x^2 + 3x\Delta x + (\Delta x)^2$$

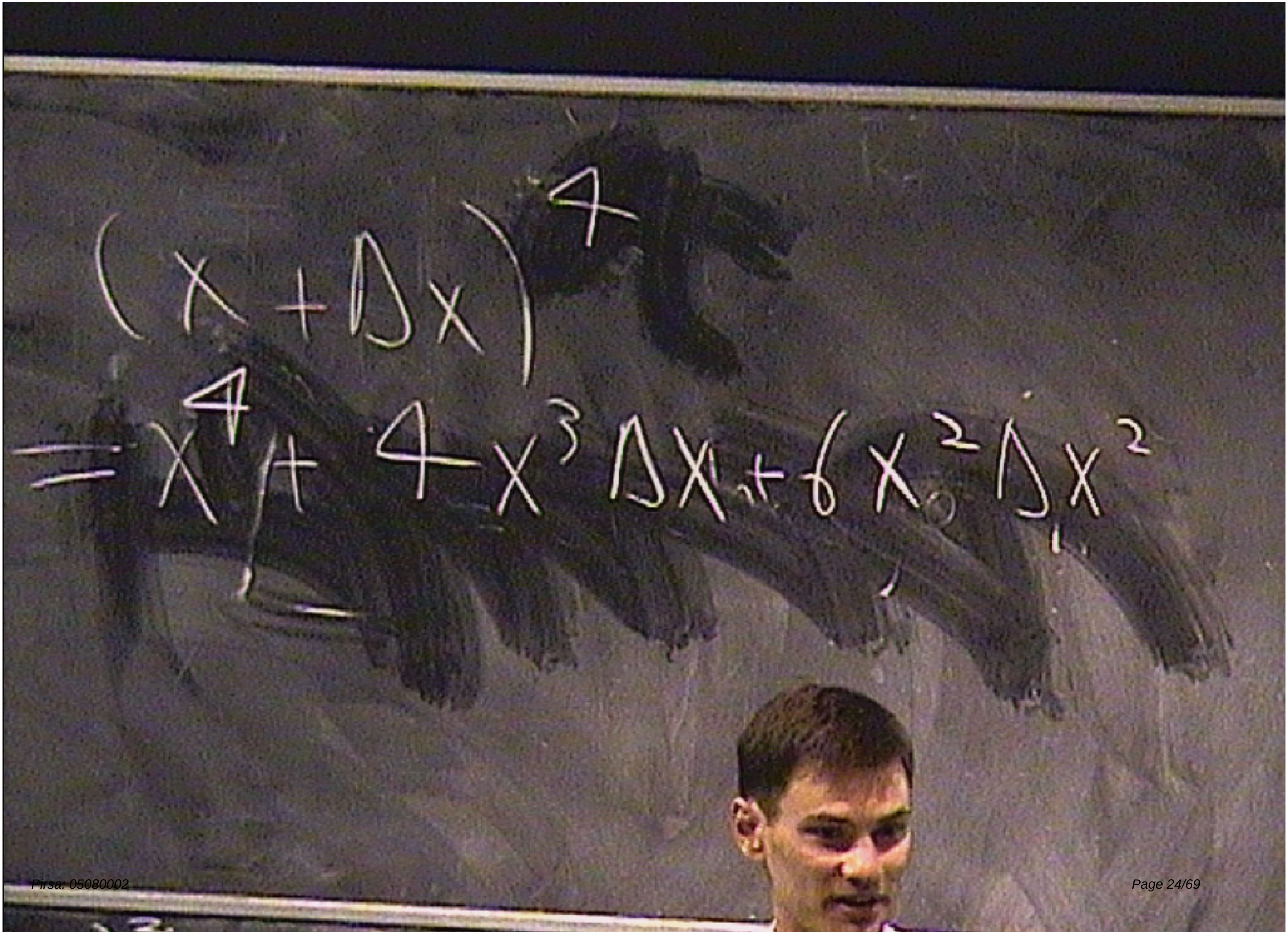


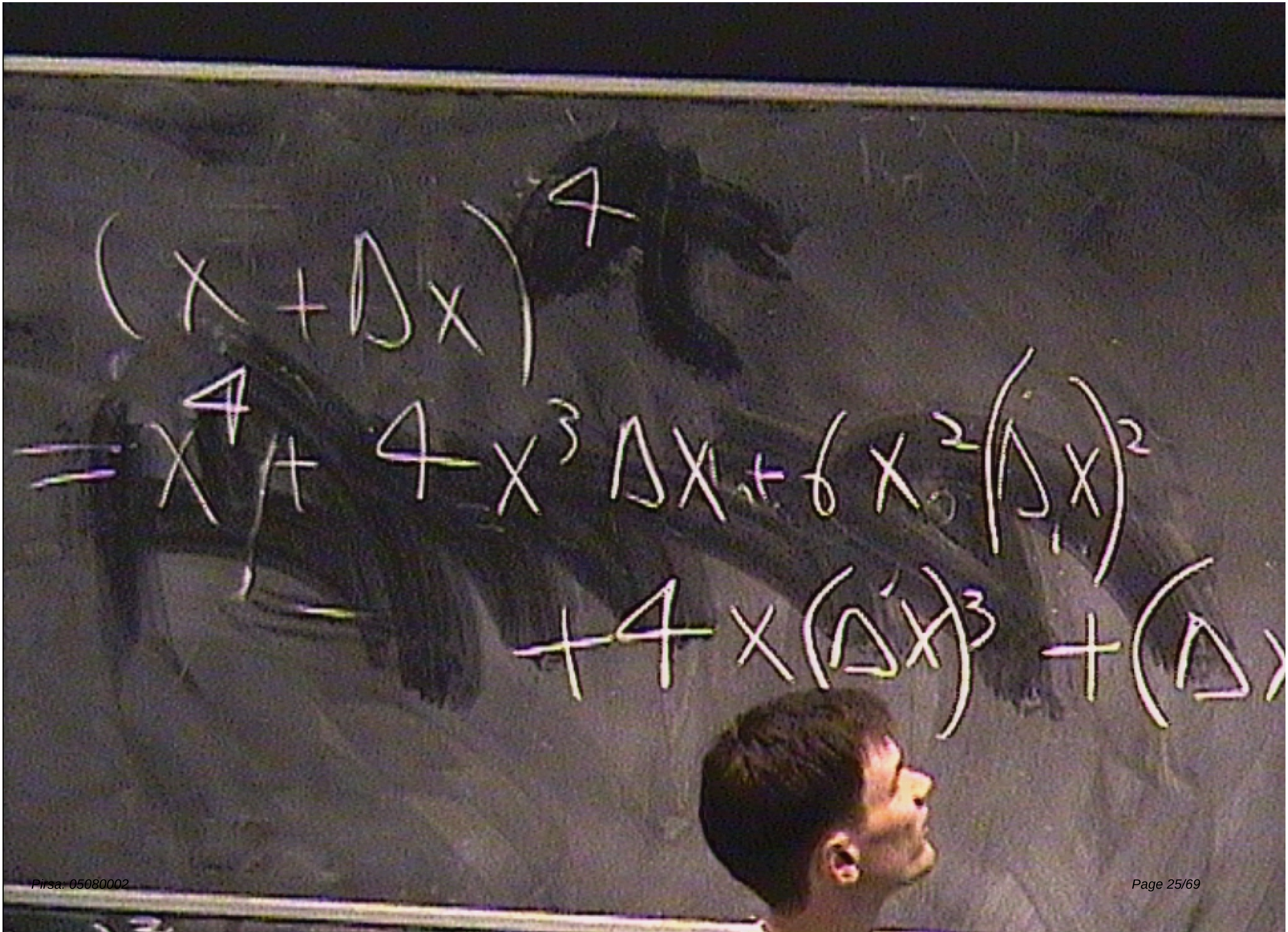
- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = 3x^2$

- Consider the function $f(x) = x^4$
- What is df/dx?
- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = 4x^3$




(x+Δx)^25
= x^4 + 4 x^3 Δx + 6 x^2 Δx^2


(x+Δx)^4
= x^4 + 4x^3Δx + 6x^2Δx^2


(x + Δx)^4
= x^4 + 4x^3 Δx + 6x^2 (Δx)^2
+ 4x (Δx)^3 + (Δx

$$\frac{d(x^2)}{dx} = 2x$$

$$\frac{d(x^3)}{dx} = 3x^2$$

$$\frac{d(x^4)}{dx} = 4x^3$$

$$\frac{d(x^5)}{dx} = 5x^4$$

In general,

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$$\frac{d(x^4)}{dx} = 4x^3$$

$$\frac{d(x^5)}{dx} = 5x^4$$

In general,

$$f(x) = A \cdot x^n$$

$$\frac{d(A \cdot x^n)}{dx} = A \cdot nx^{n-1}$$

$$\frac{d(f(x)+g(x))}{dx} = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

- Consider the function $f(x) = x^2$
- What is df/dx?

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

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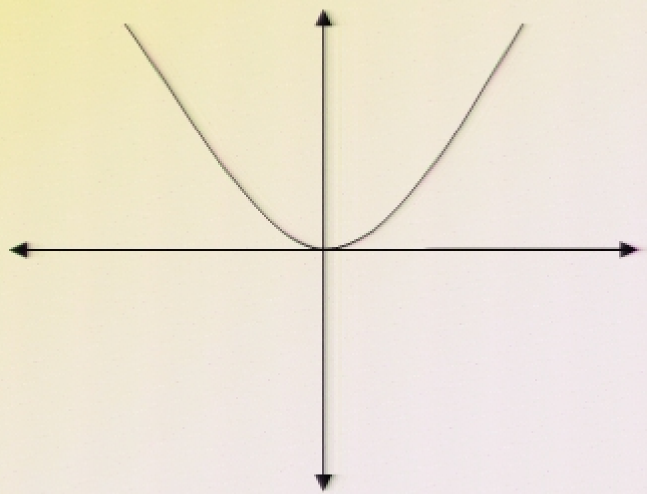

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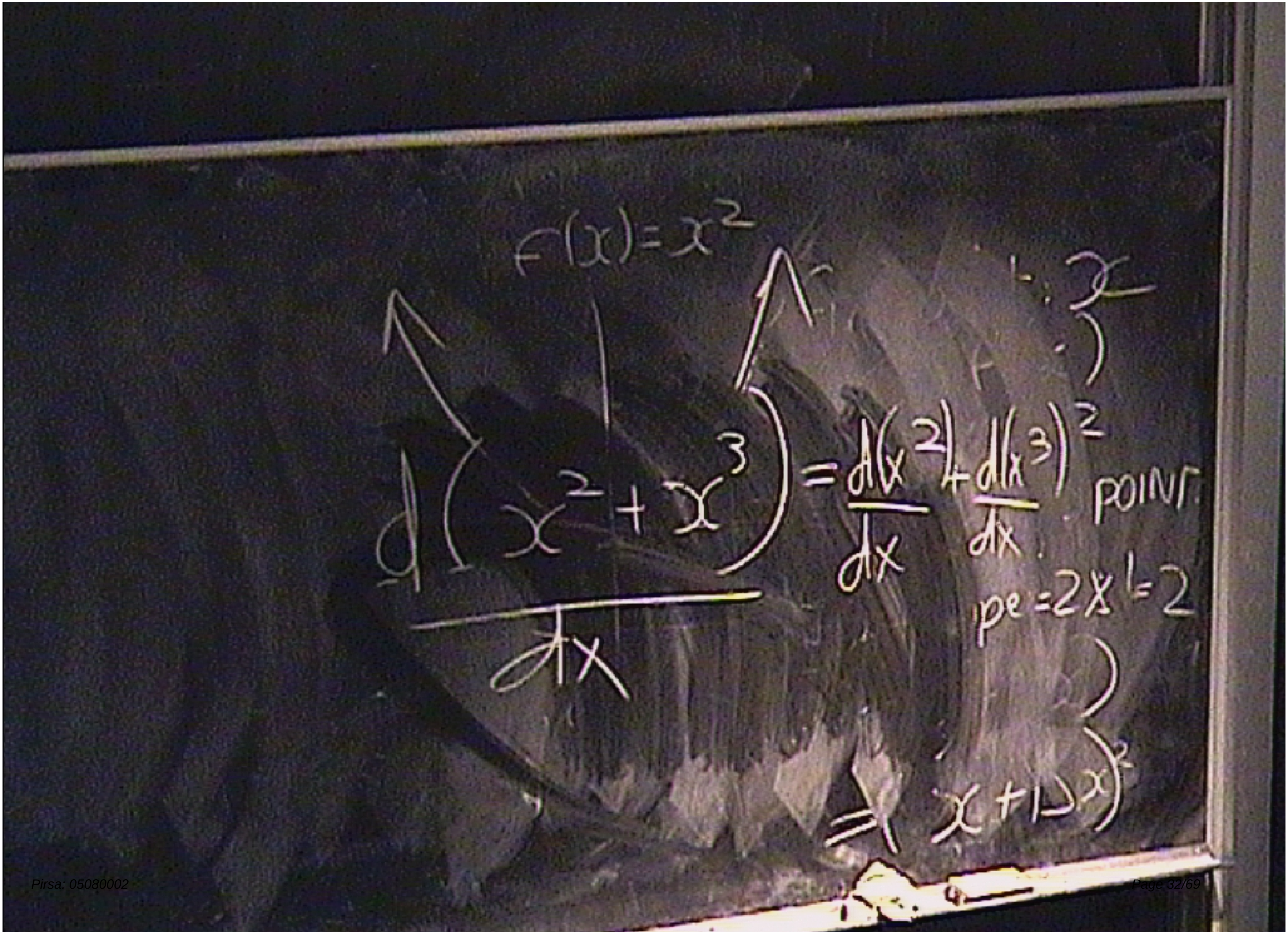
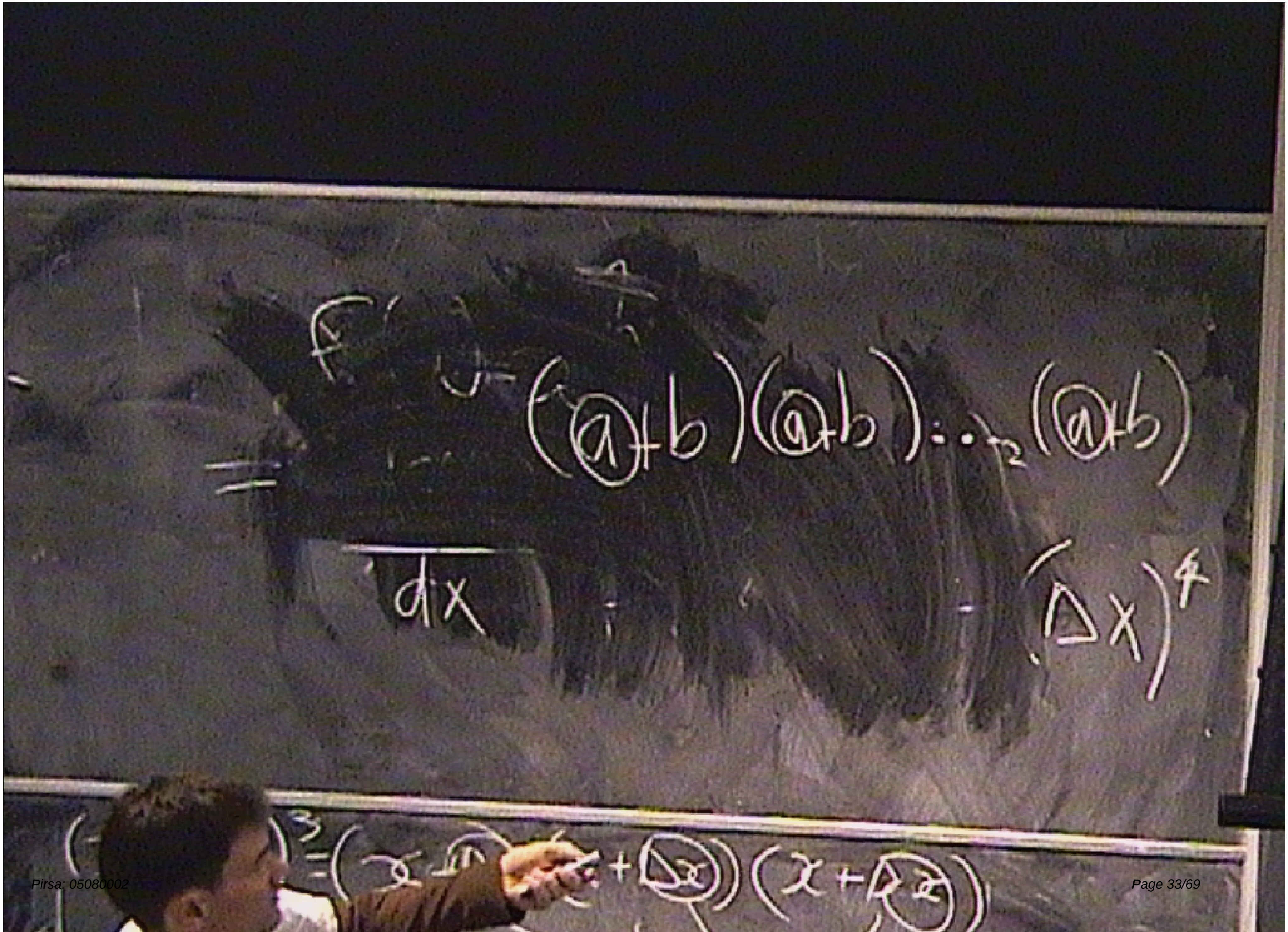

f(x)=x2
d(x2+x3)/dx = d(x2)/dx + d(x3)/dx
POINT
pe = 2x = 2


(a+b)(a+b)...(a+b)
dx
(Δx)

The binomial theorem

- very useful for calculus and many other things in physics.
- What is $(a+b)^n$?
- n=1,2,3,4…

$$(a+b)^n$$
$$= a^n + C_1 a^{n-1} b + C_2 a^{n-2} b^2 + C_3 a^{n-3} b^3 + \ldots b^n$$

- What is C_j ?

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- What is C_j ?

Pascal's triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

What are the next 4 rows?

C_j is given by the $(j+1)^{th}$ element in the n^{th} row

$$eg.\quad (a+b)^4$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Pascal's triangle

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

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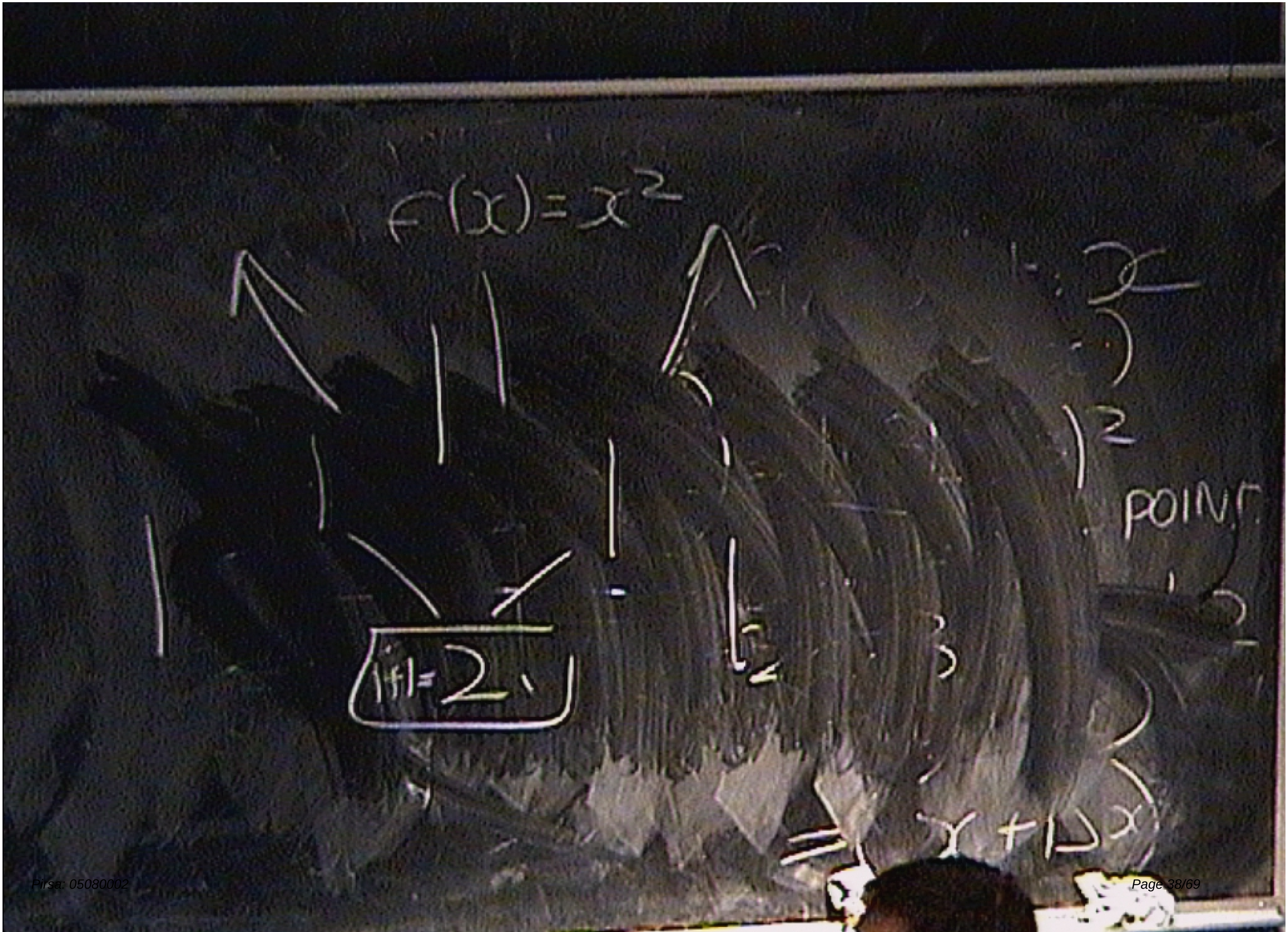
f(x)=x^2
|f|=2.
POINT

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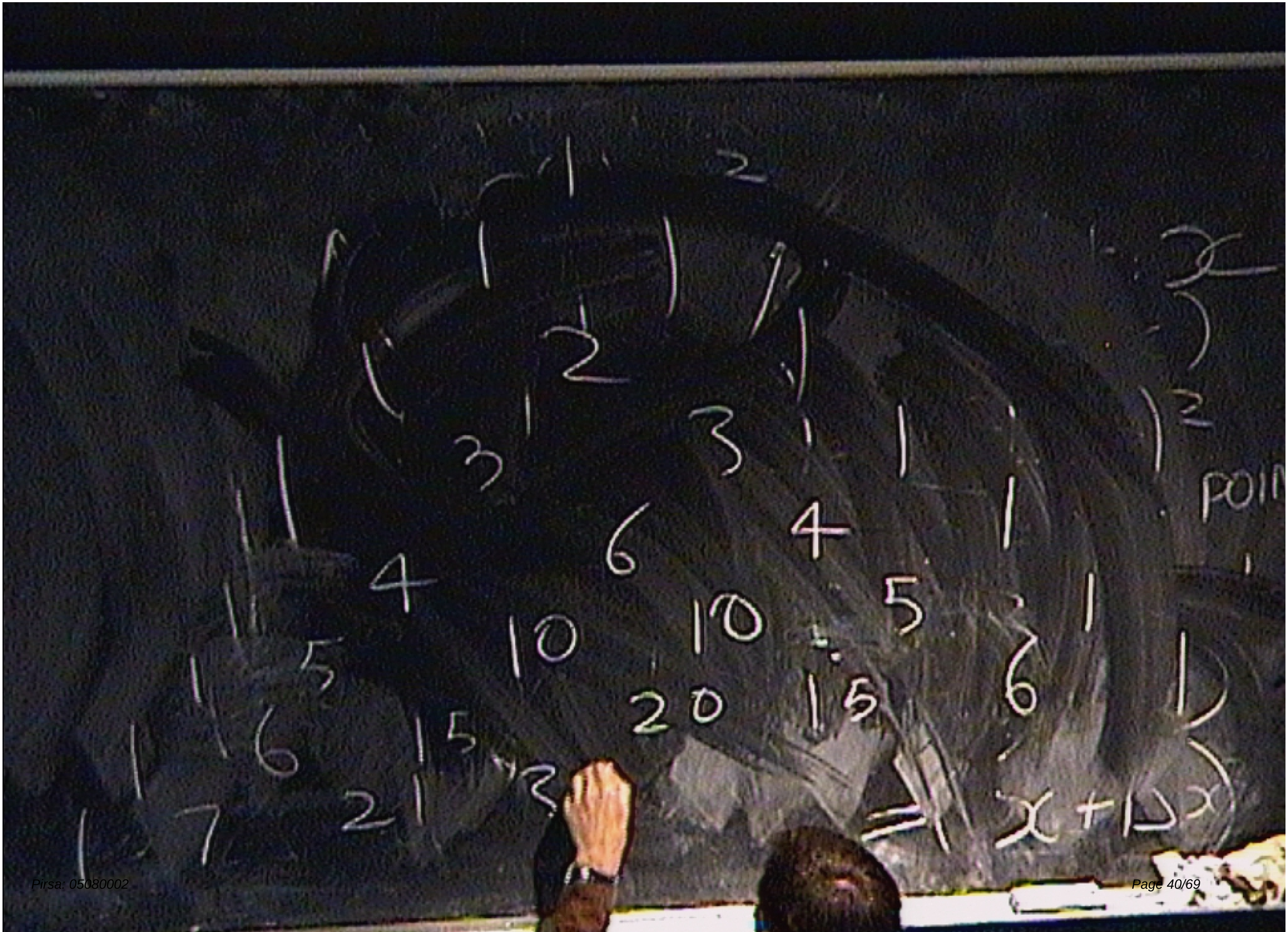
2
3 3
4 6 4
5 10 10 5
6 15 20 15 6
7 21
x + Δx

Pascal's triangle

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1 4 6 4 1

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$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$C_j = \frac{n!}{j!(n-j)!}$$

where n! = n(n-1)(n-2)(n-3)…1

What are

3!

4!

5!

and

6!

???

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- Let us check this.

- for n=3.

$$(a+b)^3$$
$$= a^3 + 3a^2b + 3a^1b^2 + b^3$$

$$C_1$$
$$= \frac{3!}{1!(3-1)!}$$
$$= \frac{6}{1 \times 2}$$
$$= 3$$
$$C_2$$
$$= \frac{3!}{2!(3-2)!}$$
$$= \frac{6}{2 \times 1}$$
$$= 3$$

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$$= 3$$

for n=4.

$$C_1 = \frac{4!}{1!(4-1)!} = \frac{24}{1\times 6} = 4$$

$$C_2 = \frac{4!}{2!(4-2)!} = \frac{24}{2\times 2} = 6$$

$$C_3 = \frac{4!}{3!(4-3)!} = \frac{24}{6\times 1} = 4$$

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$$= C_1 x^{n-1} + C_2 x^{n-2}(\Delta x) + C_3 x^{n-3}(\Delta x)^2 \ldots (\Delta x)^{n-1}$$

$$= n x^{n-1} + C_2 x^{n-2}(\Delta x) + C_3 x^{n-3}(\Delta x)^2 \ldots (\Delta x)^{n-1}$$

- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = n x^{n-1}$

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- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = n x^{n-1}$

$$e^x = \sum_j \frac{x^j}{j!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$\frac{de^x}{dx} = \sum_j \frac{1}{j!} \frac{d(x^j)}{dx} = 0 + 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots = e^x$$

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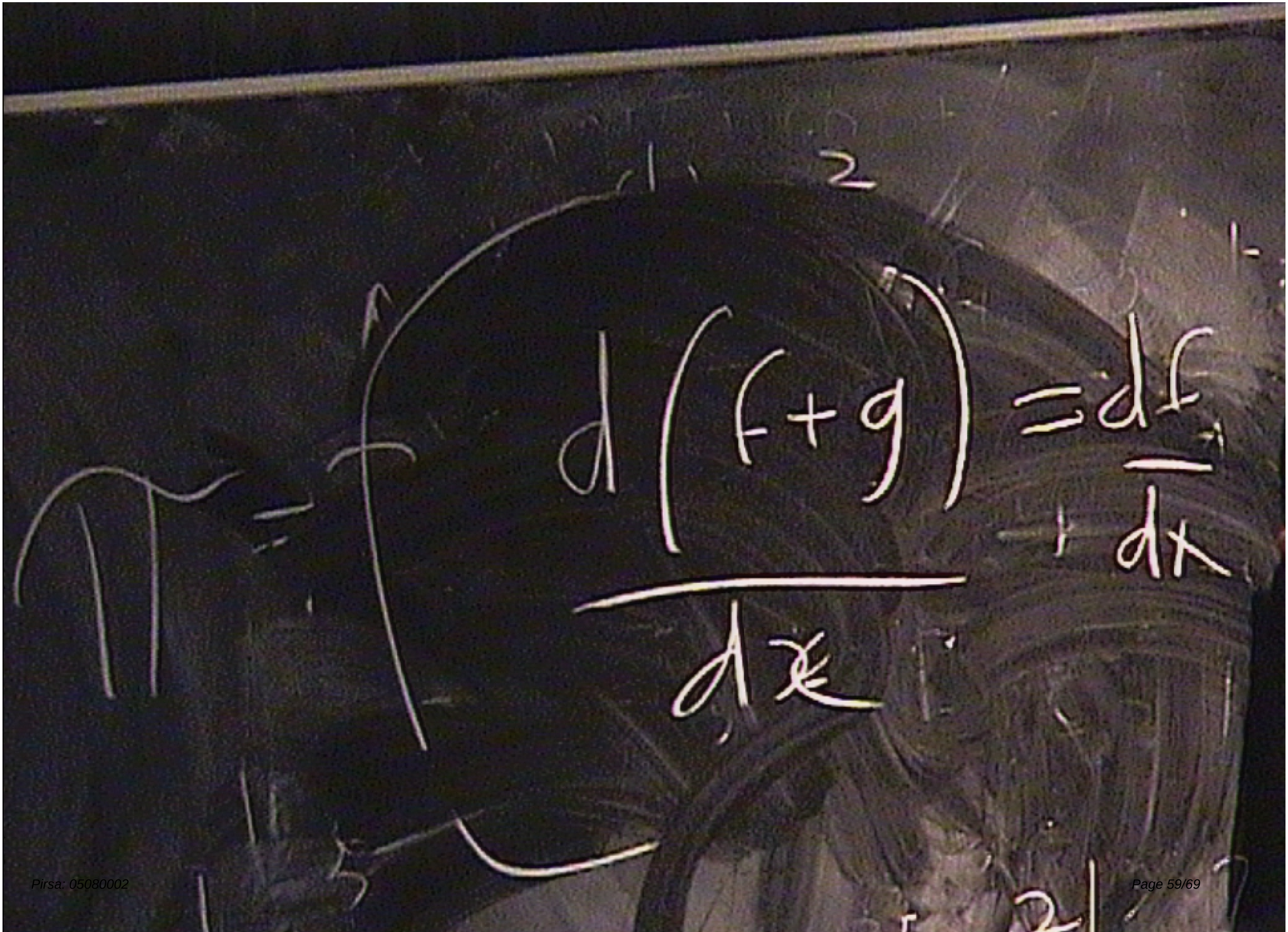
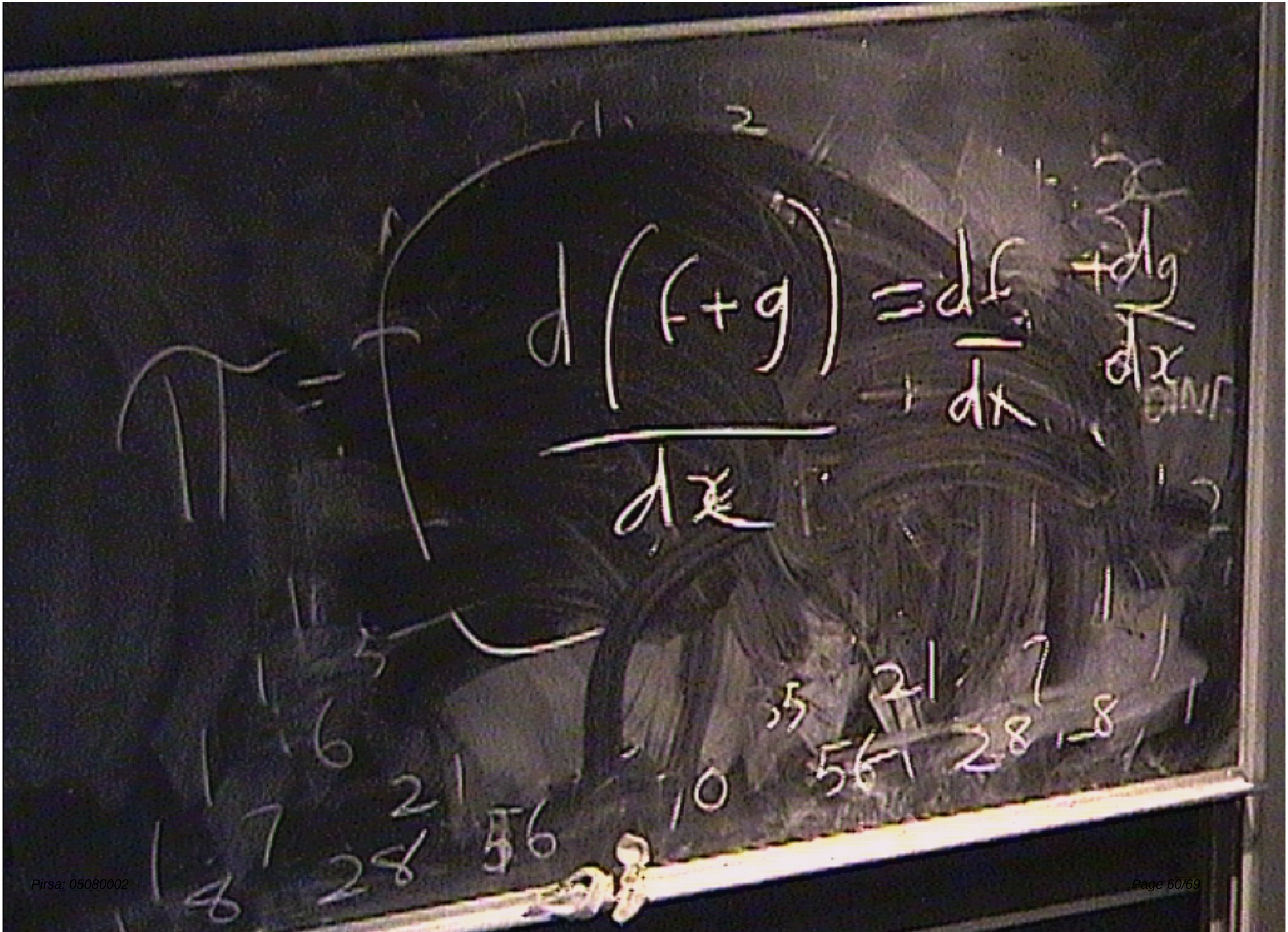
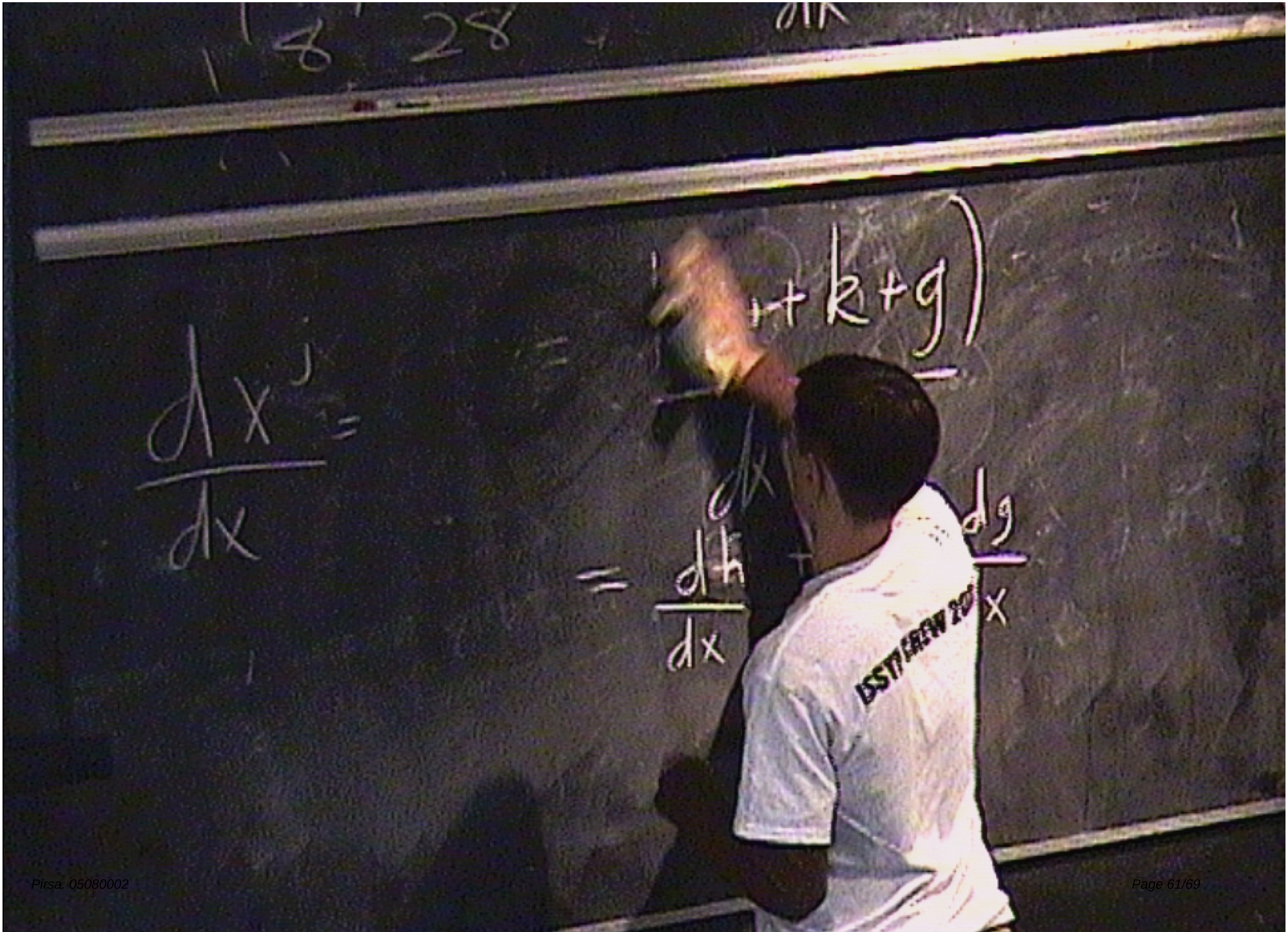
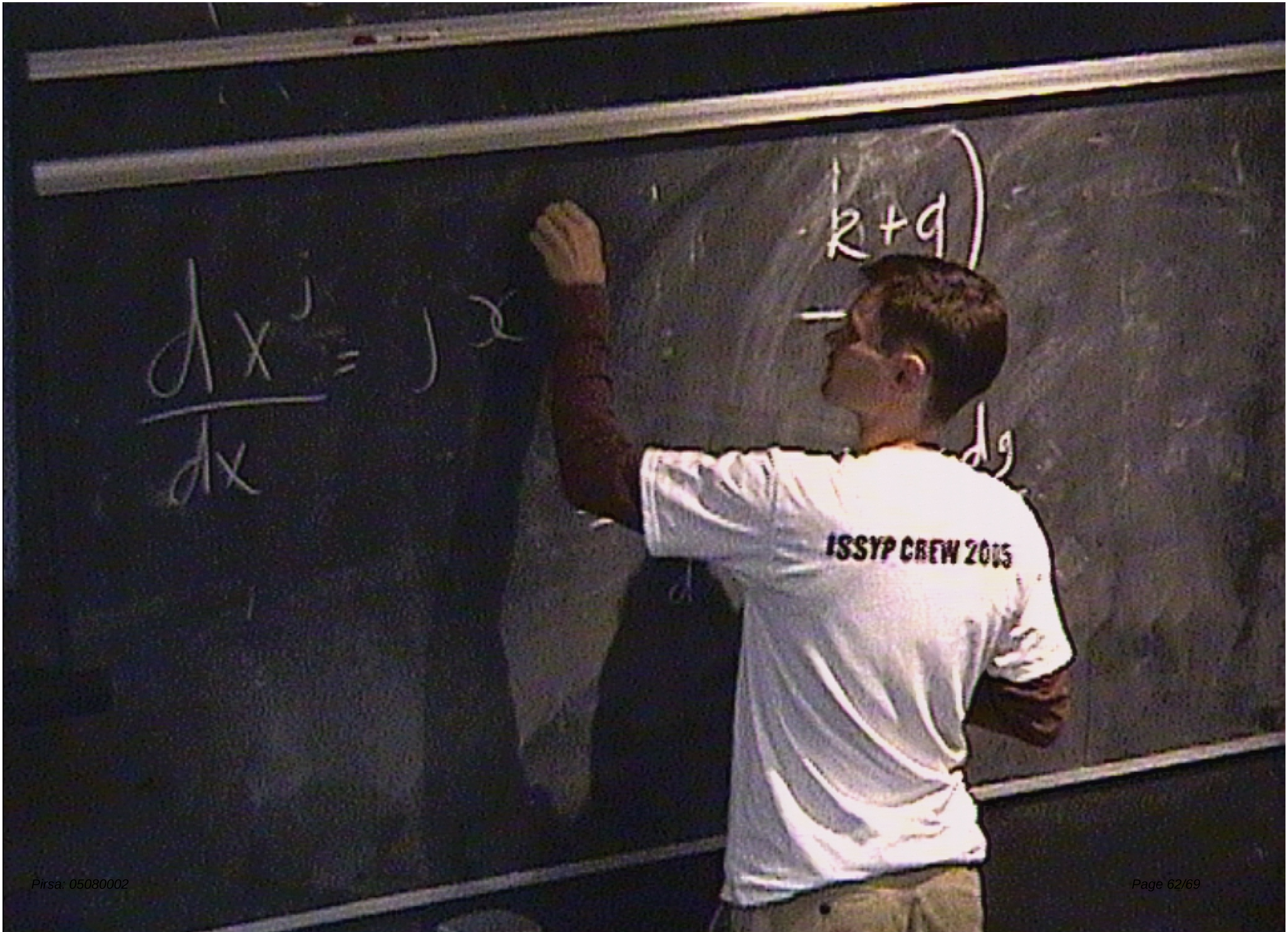
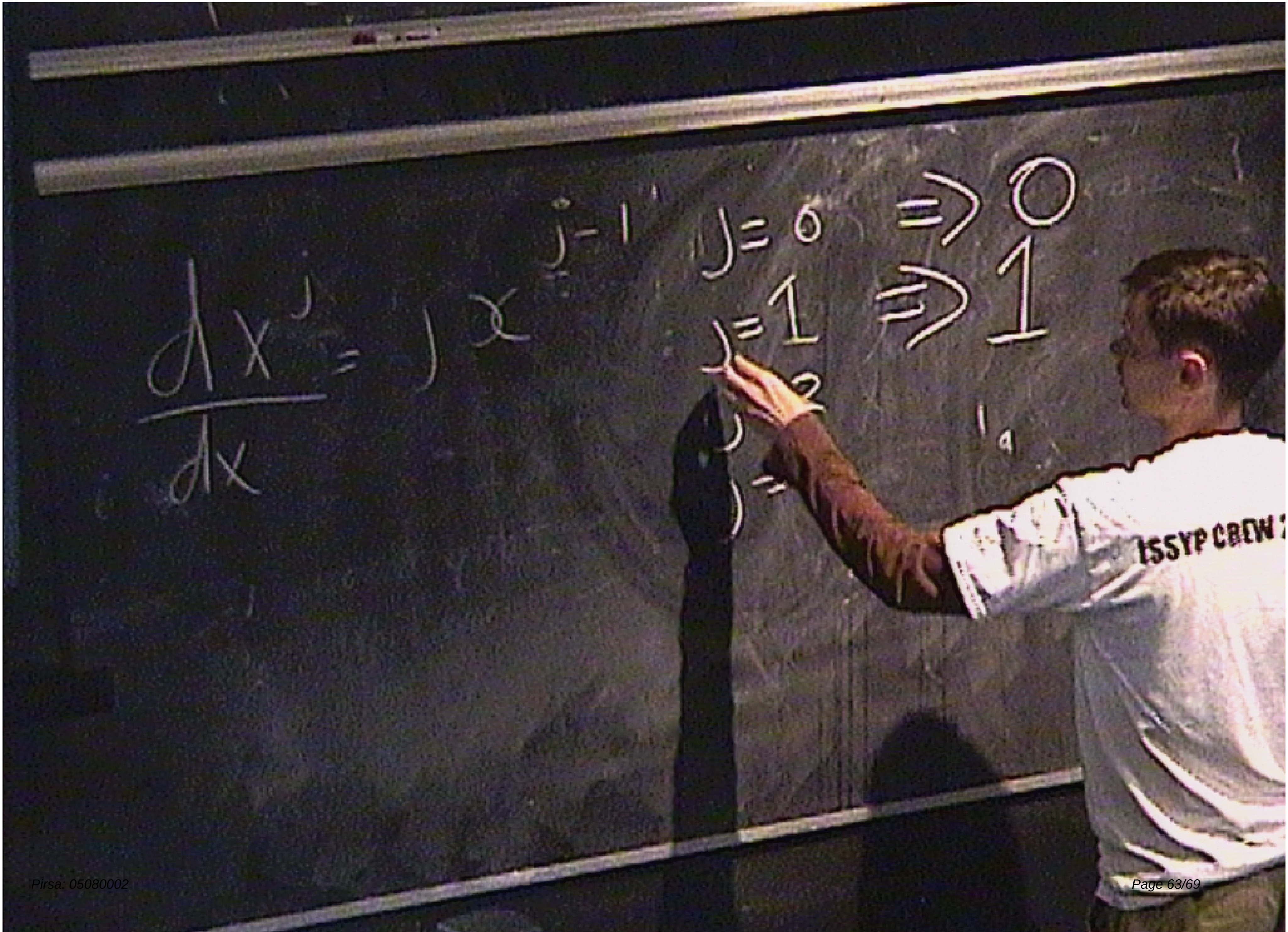

d(f+g)/dx = df/dx



$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$


dx^j / dx =
+k+g)
= dh / dx
dg


ISSYP CREW 2005
k+q


$\frac{d}{dx} x^j = j x^{j-1}$
j = 0 => 0
j = 1 => 1
ISSYP CREW

$$\frac{dx^j}{dx} = j\,x^{j-1}$$

$j=0 \Rightarrow 0$

$j=1 \Rightarrow 1$

$j=2 \Rightarrow 2x$

$j=2$

$x \cdot x$

Product Rule

$$\frac{d(f(x)\cdot g(x))}{dx} = f(x)\frac{dg(x)}{dx} + g(x)\frac{df(x)}{dx}$$

$$eg. \quad \frac{d(x^2 \cdot x^4)}{dx} = x^2\frac{d(x^4)}{dx} + x^4\frac{d(x^2)}{dx}$$

$$= x^2 \cdot 4x^3 + x^4 \cdot 2x$$

$$= 4x^5 + 2x^5$$

$$= 6x^5$$

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Chain Rule

f(g) and g(x)

$$\frac{d(f[g(x)])}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$eg. \quad f = g^2$

$g = x^3$

$$\frac{d(f[g(x)])}{dx} = \frac{d(g^2)}{dg} \cdot \frac{dx^3}{dx}$$

$= 2g \cdot 3x^2$

$= 2x^3 \cdot 3x^2$

$= 6x^5$

End of slide show, click to exit.

- Consider the function $f(x) = x^n$
- What is df/dx?

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$= \frac{x^n + C_1 x^{n-1}\Delta x + C_2 x^{n-2}(\Delta x)^2 + C_3 x^{n-3}(\Delta x)^3 \ldots (\Delta x)^n - x^n}{\Delta x}$$

$$= C_1 x^{n-1} + C_2 x^{n-2}(\Delta x) + C_3 x^{n-3}(\Delta x)^2 \ldots (\Delta x)^{n-1}$$

$$= n x^{n-1} + C_2 x^{n-2}(\Delta x) + C_3 x^{n-3}(\Delta x)^2 \ldots (\Delta x)^{n-1}$$

- $df/dx = \lim_{\Delta x \to 0} \Delta f(x)/\Delta x = n x^{n-1}$