Title: Classical and quantum indistinguishability

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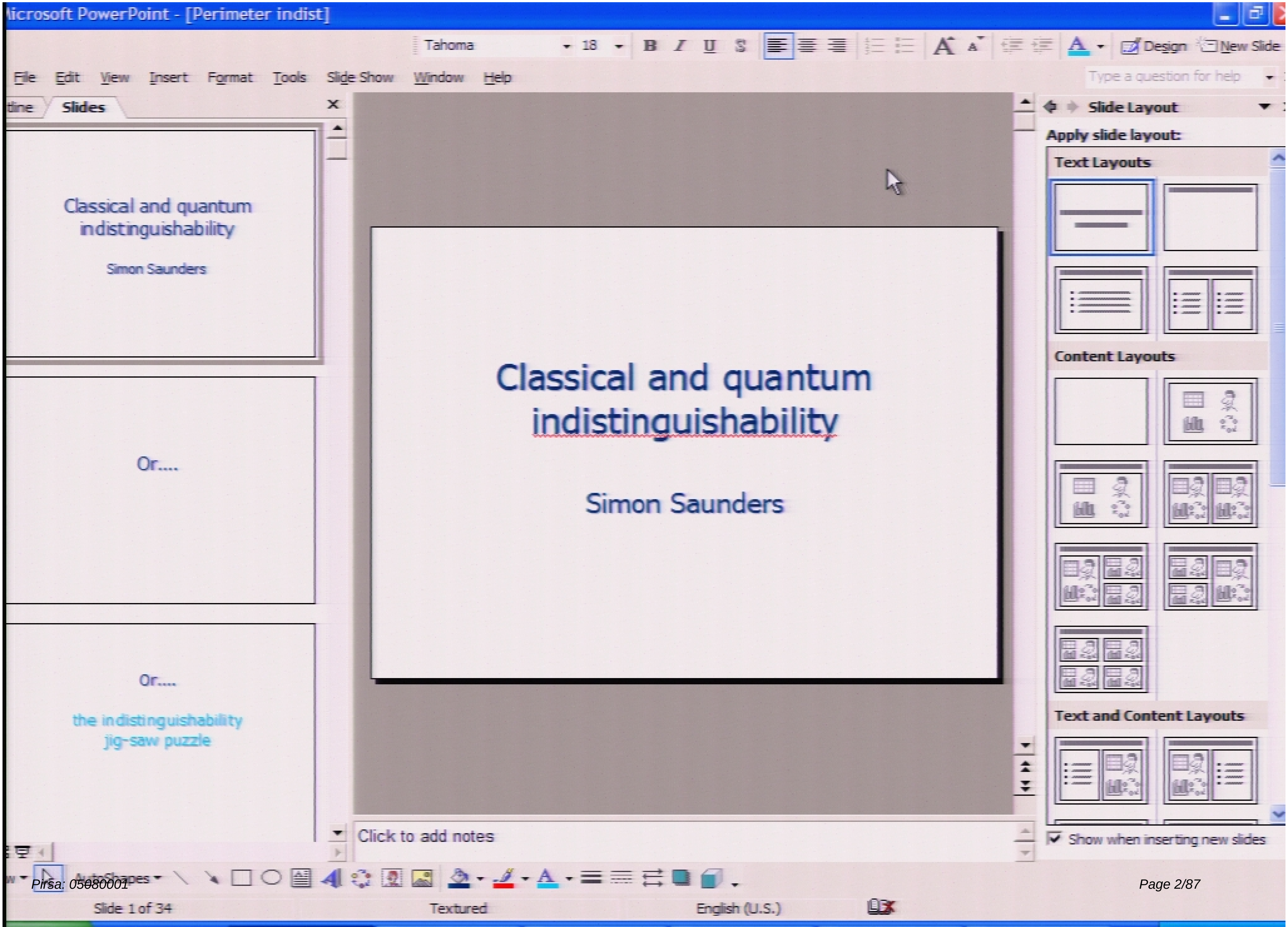
Abstract: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Simon Saunders
Or....
Or....
the indistinguishability jig-saw puzzle
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Simon Saunders
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Slide 1 of 34
Textured
English (U.S.)

# Classical and quantum indistinguishability

Simon Saunders

# Or....

# Or….

## the indistinguishability jig-saw puzzle

In conclusion, it should be emphasized that in the foregoing remarks classical statistics is considered in principle as a part of classical mechanics which deals with individuals (Boltzmann). The conception of atoms as particles losing their identity cannot be introduced into the classical theory without contradiction. (Stern, On the term N! in the entropy, Rev Mod Phys, 21, 534, 1949).

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Classical statistics thus leads to a contradiction with experience even in the range in which quantum effects in the proper sense can be completely neglected. (Munster, Statistical Thermodynamics Vol 1, Springer, p.57, 1969)

Indistinguishable Classical Particles Have No Trajectories. The unconventional role of indistinguishable classical particles is best expressed by the fact that in a deterministic setting no indistinguishable particles exist, or - equivalently - that indistinguishable classical particles have no trajectories. Before I give a formal proof I argue as follows. Suppose they have trajectories, then the particles can be identified by them and are, therefore, not indistinguishable. (Bach, Indistinguishable Classical Particles, Springer, 1997 p.7).

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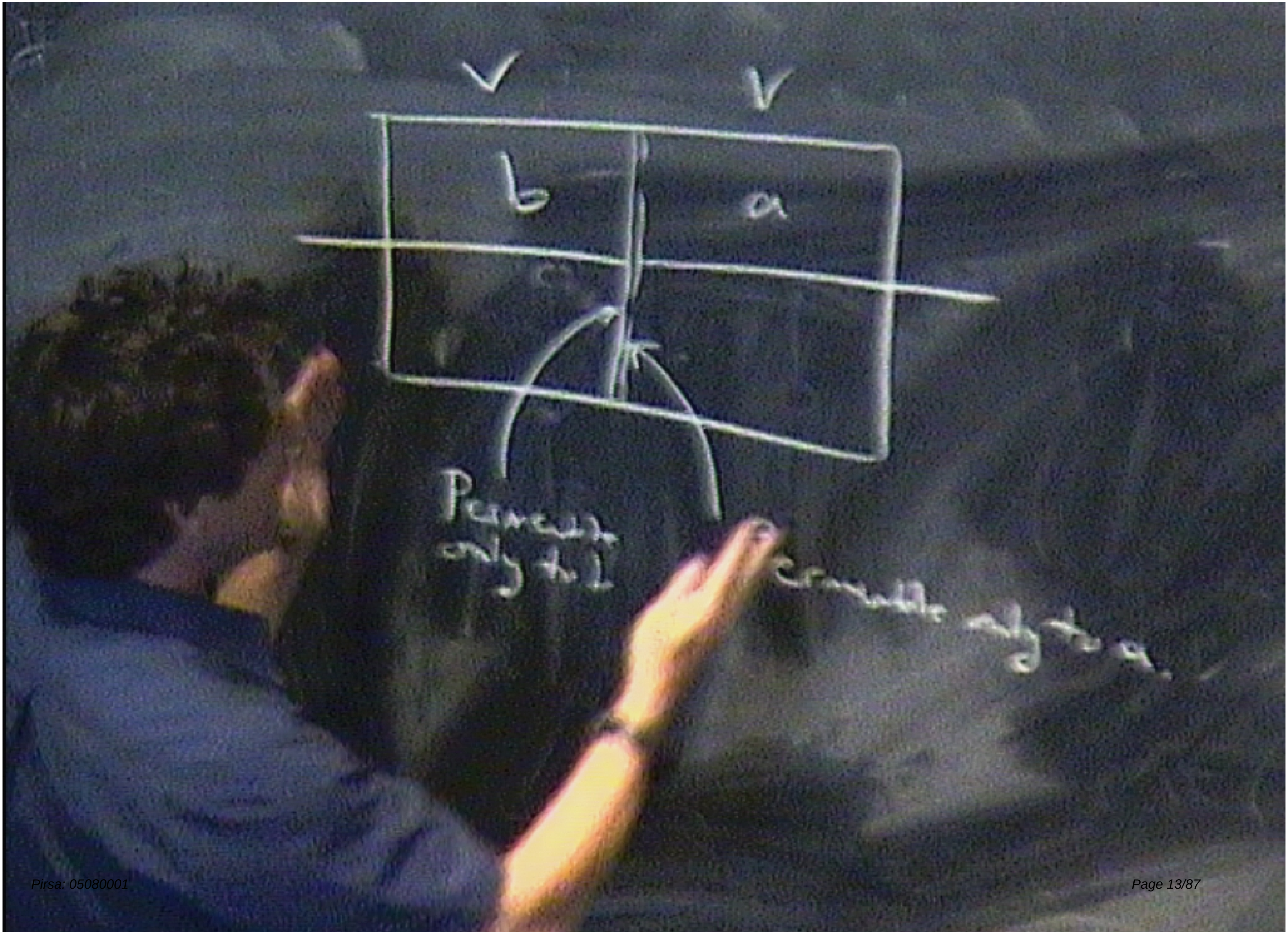
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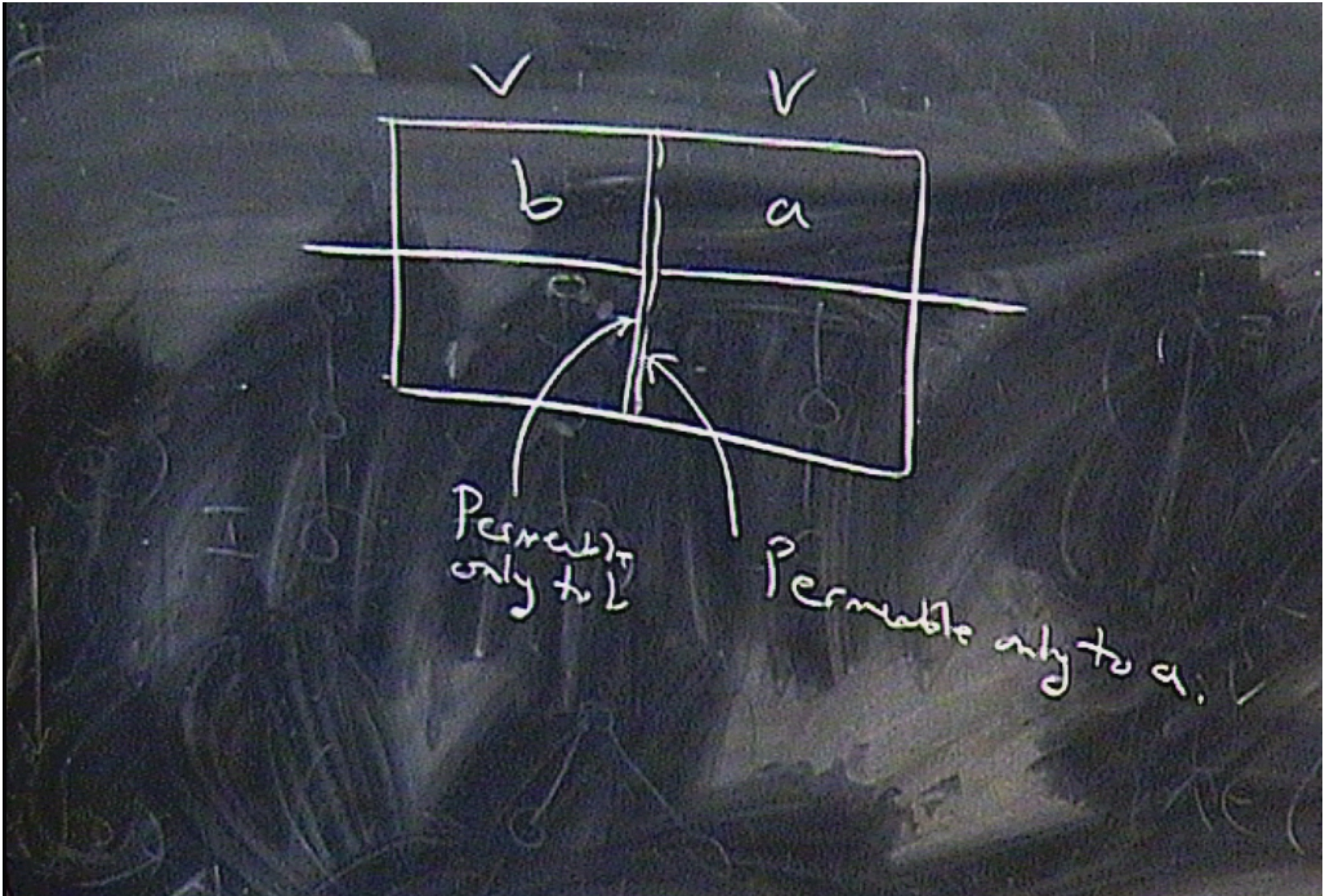
V
b
a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V
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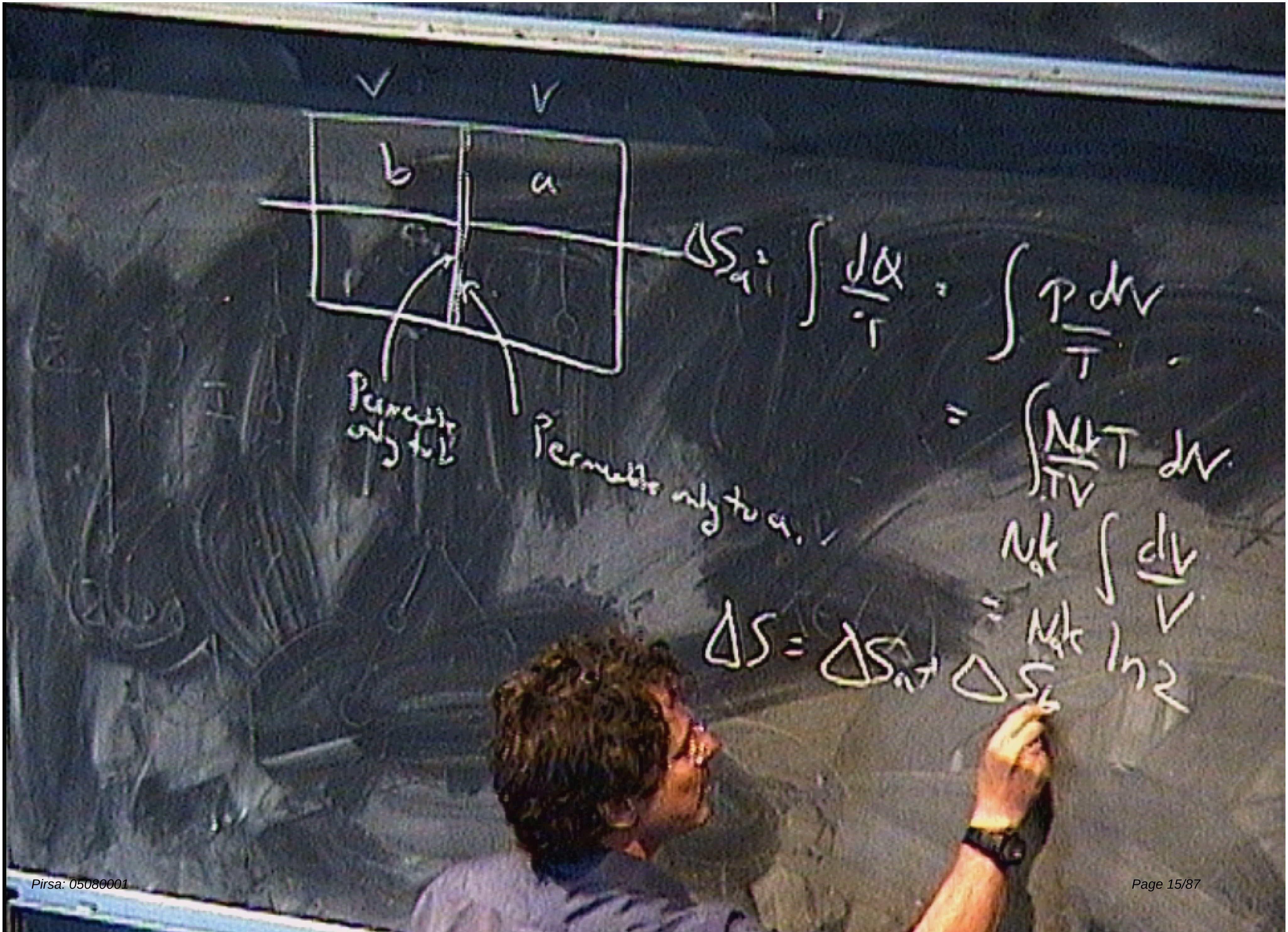
a
Permeable only to b
Permeable only to a.


V
V
b
a
Permeable only to b
Permeable only to a.
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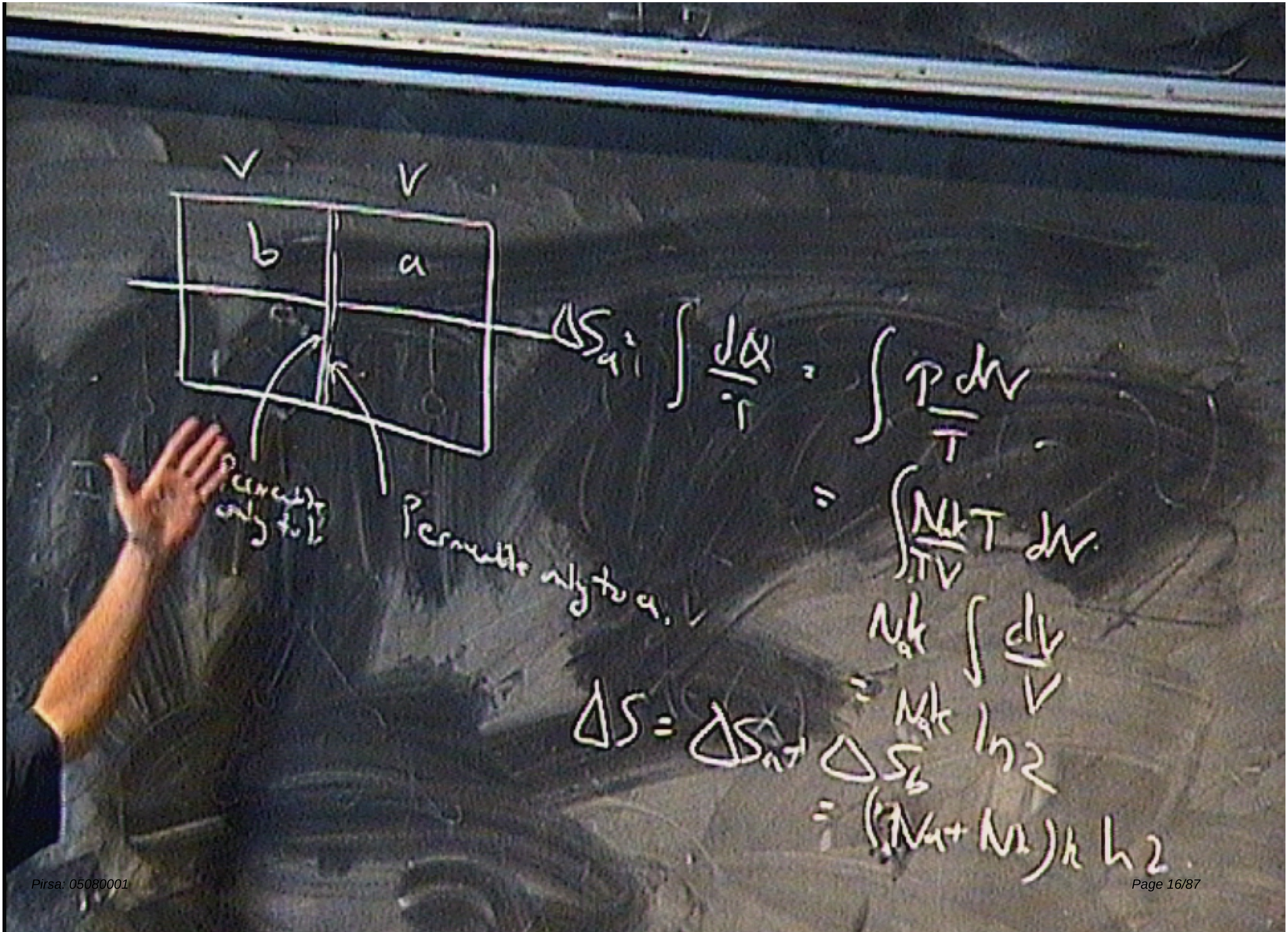
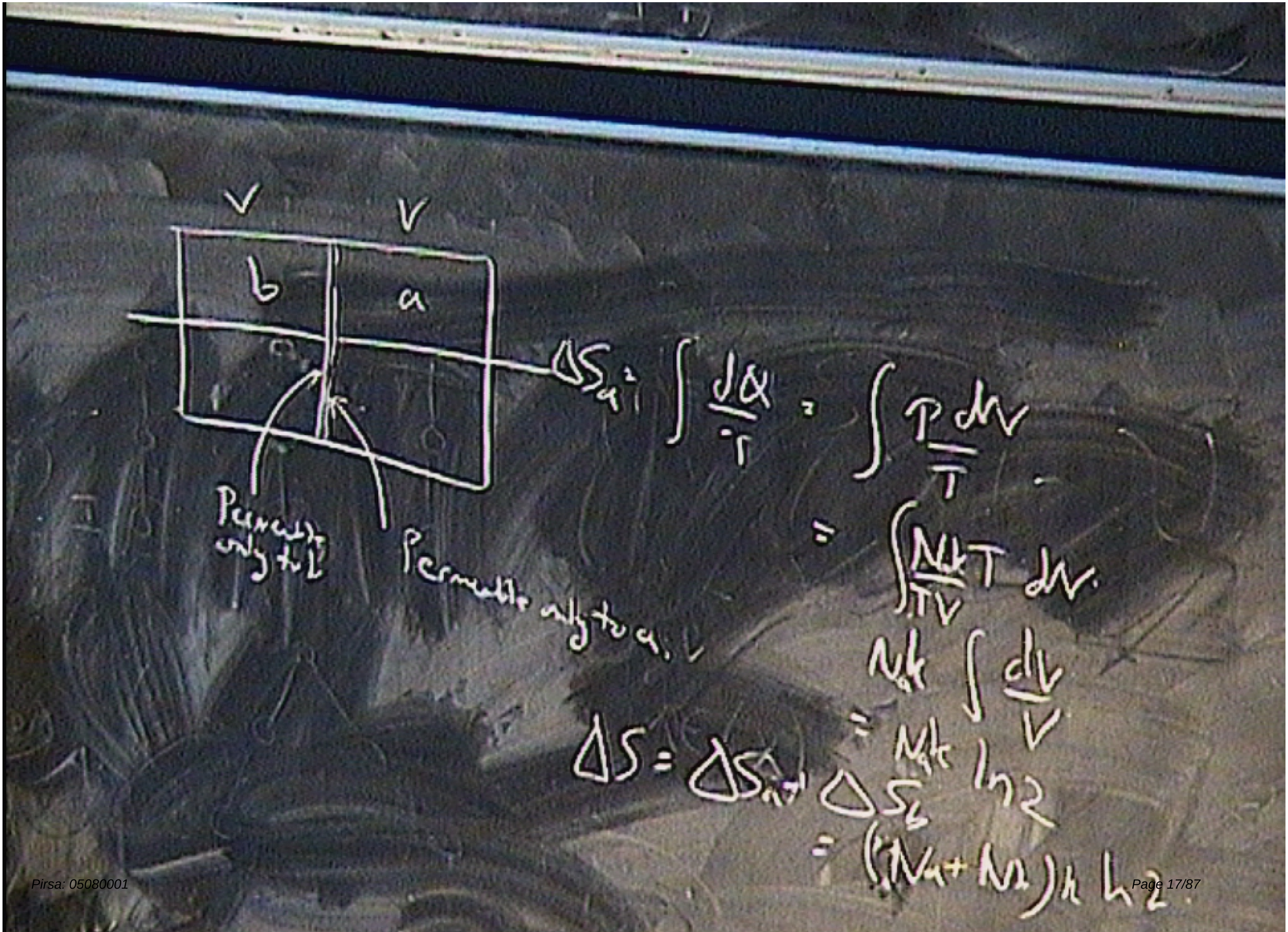
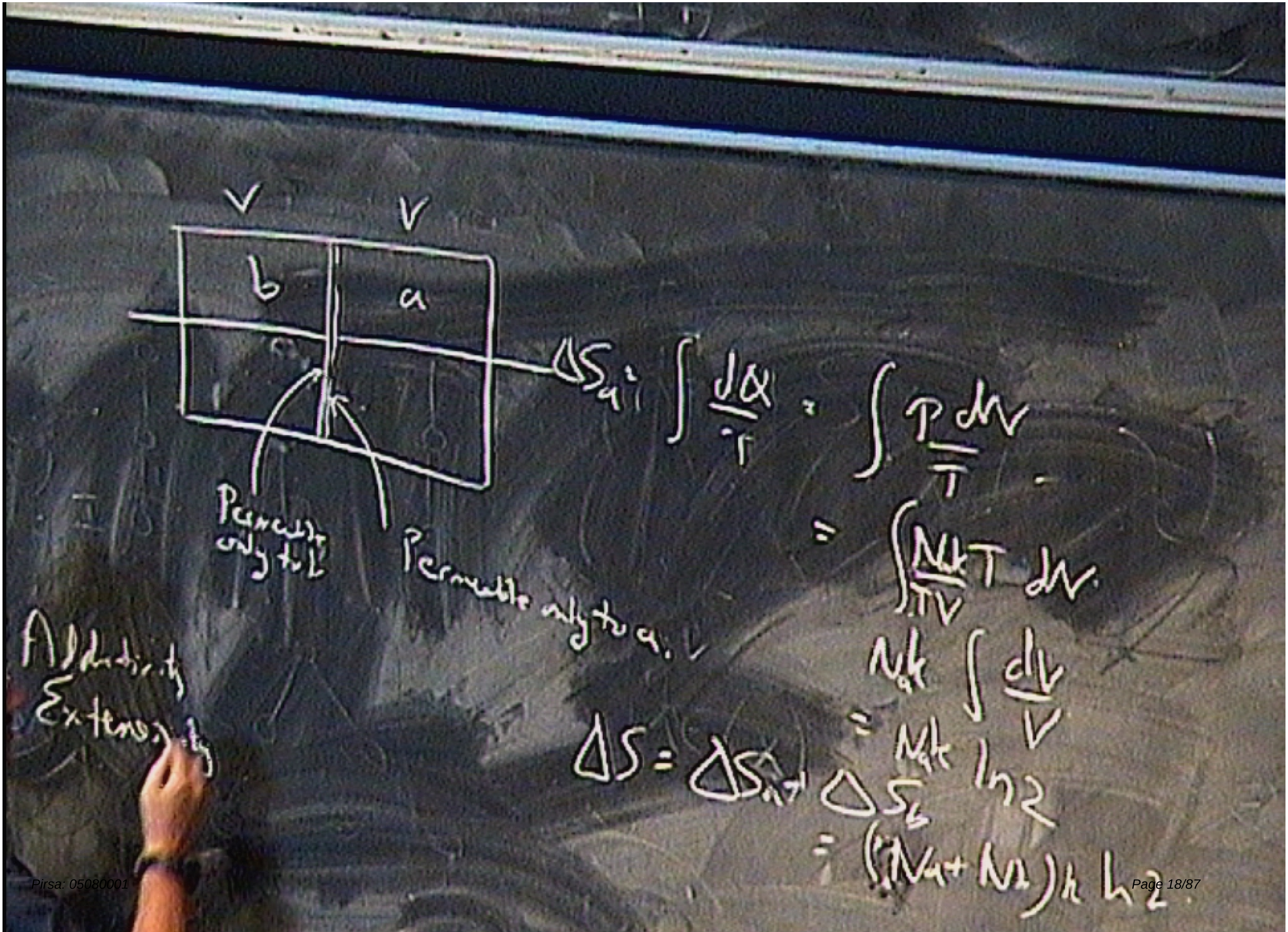
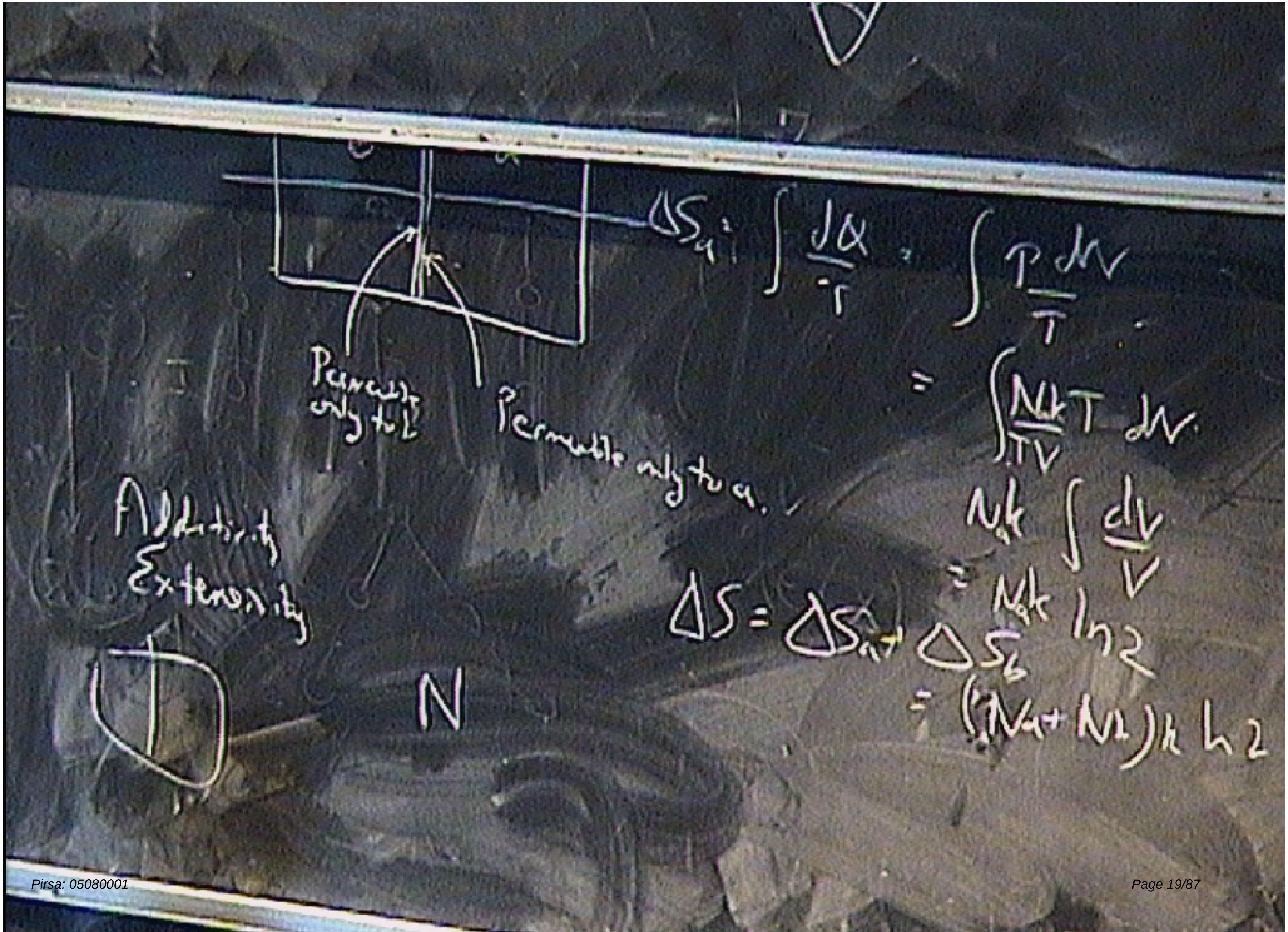
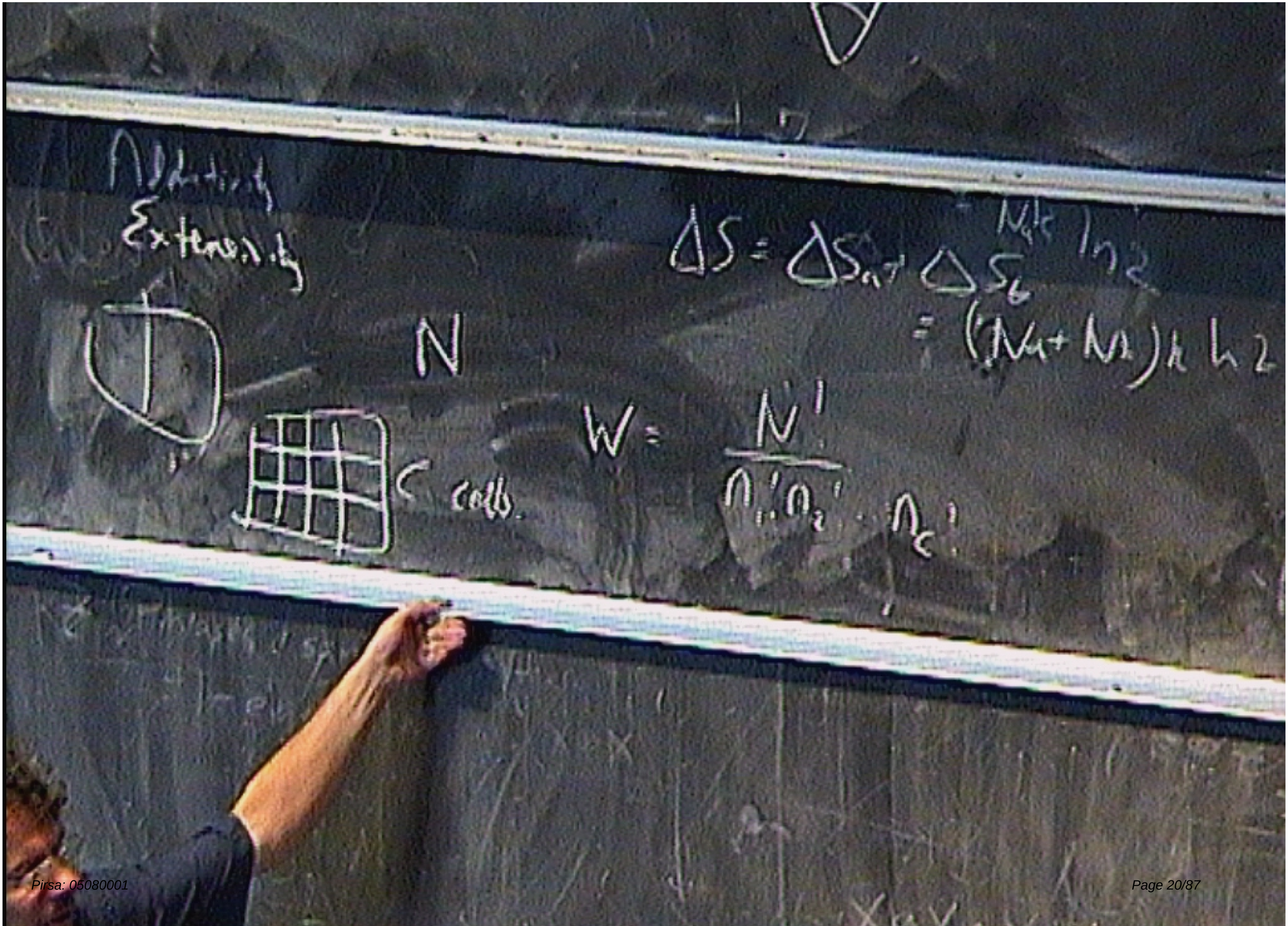
= ∫ NakT/TV dV
Nak ∫ dV/V
= Nak ln 2
ΔS = ΔS_a + ΔS_b

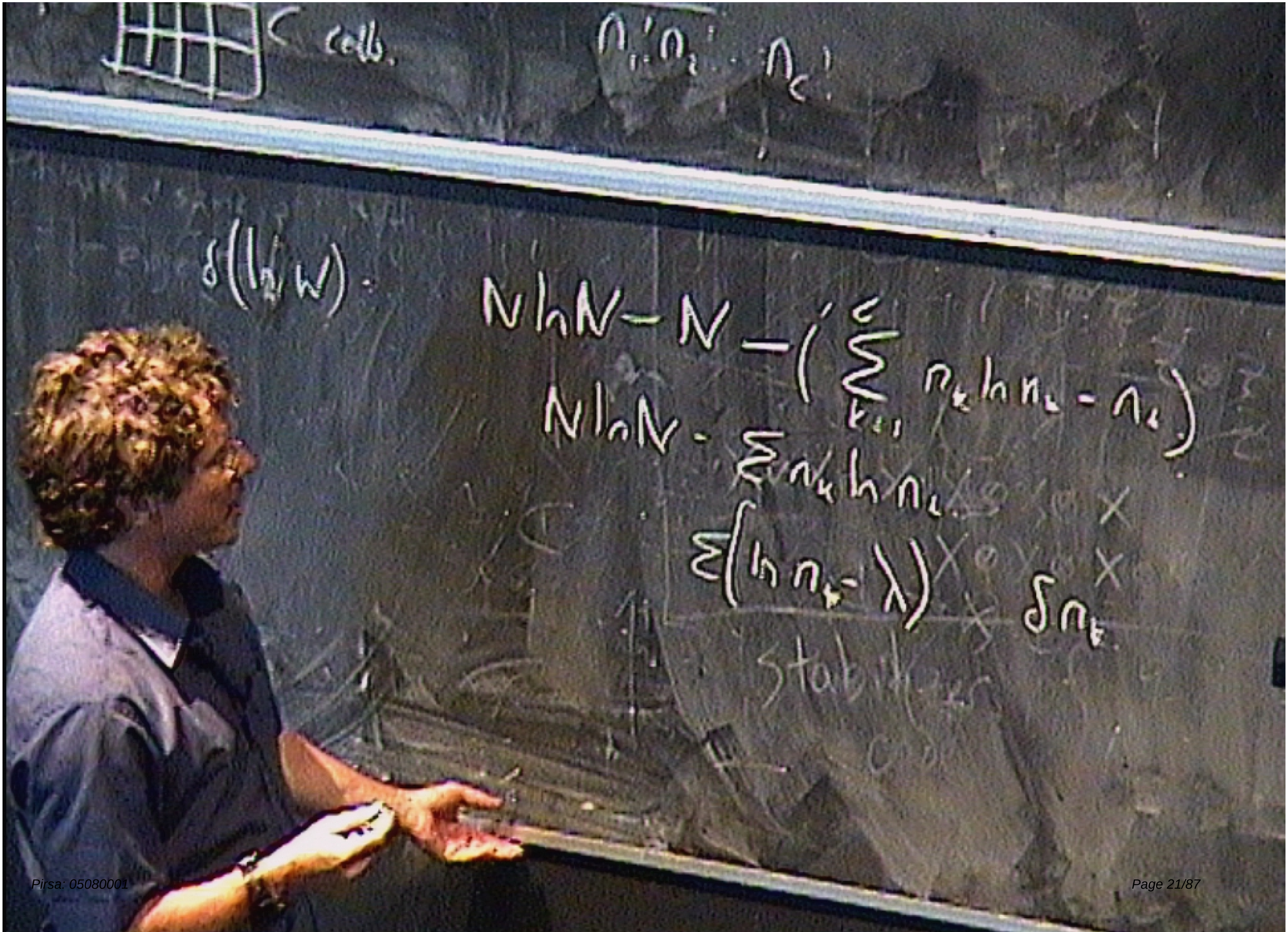

V
V
b
a
ΔS_a = ∫ dQ/T = ∫ P dV/T
= ∫ NkT/TV dV
N_a k ∫ dV/V
= N_a k ln 2
Permeable only to b
Permeable only to a
ΔS = ΔS_a + ΔS_b
= (N_a + N_b) k ln 2


V
V
b
a
Permeable only to b
Permeable only to a
$\Delta S_a = \int \frac{dQ}{T} = \int \frac{P\,dV}{T}$
$= \int \frac{N_a k T}{TV} dV$
$N_a k \int \frac{dV}{V}$
$= N_a k \ln 2$
$\Delta S = \Delta S_a + \Delta S_b$
$= (N_a + N_b) k \ln 2$


V
V
b
a
Permeable only to b
Permeable only to a
Additivity
Extensivity
ΔS_a = ∫ dQ/T = ∫ P dV/T
= ∫ N_a kT dV / TV
N_a k ∫ dV/V
= N_a k ln 2
ΔS = ΔS_a + ΔS_b
= (N_a + N_b) k ln 2


ΔSa = ∫ dQ/T = ∫ P dV/T
= ∫ Nak T dV / TV
= Nak ∫ dV/V
= Nak ln 2
Permeable only to a.
Additivity
Extensivity
N
ΔS = ΔSa + ΔSb
= (Na + Nb)k ln 2


Additivity
Extensivity
ΔS = ΔSa + ΔSb
= (Na + Nb) k ln 2
N
W = N! / n1! n2! ... nc!
c cells


cells
n₁! n₂! … n_c!
δ(ln W)
N ln N − N − ( Σ_{k=1}^{c} n_k ln n_k − n_k )
N ln N − Σ n_k ln n_k
Σ (ln n_k − λ) δn_k

$$\delta(\ln W):\quad N\ln N - N - \left(\sum_{k=1}^{c} n_k \ln n_k - n_k\right)$$

$$N\ln N - \sum n_k \ln n_k$$

$$\sum\left(\ln n_k - \lambda\right)\delta n_k$$

$$\ln n_k = \lambda$$

$n_1!\, n_2! \cdots n_c!$

$\delta(\ln W)$.

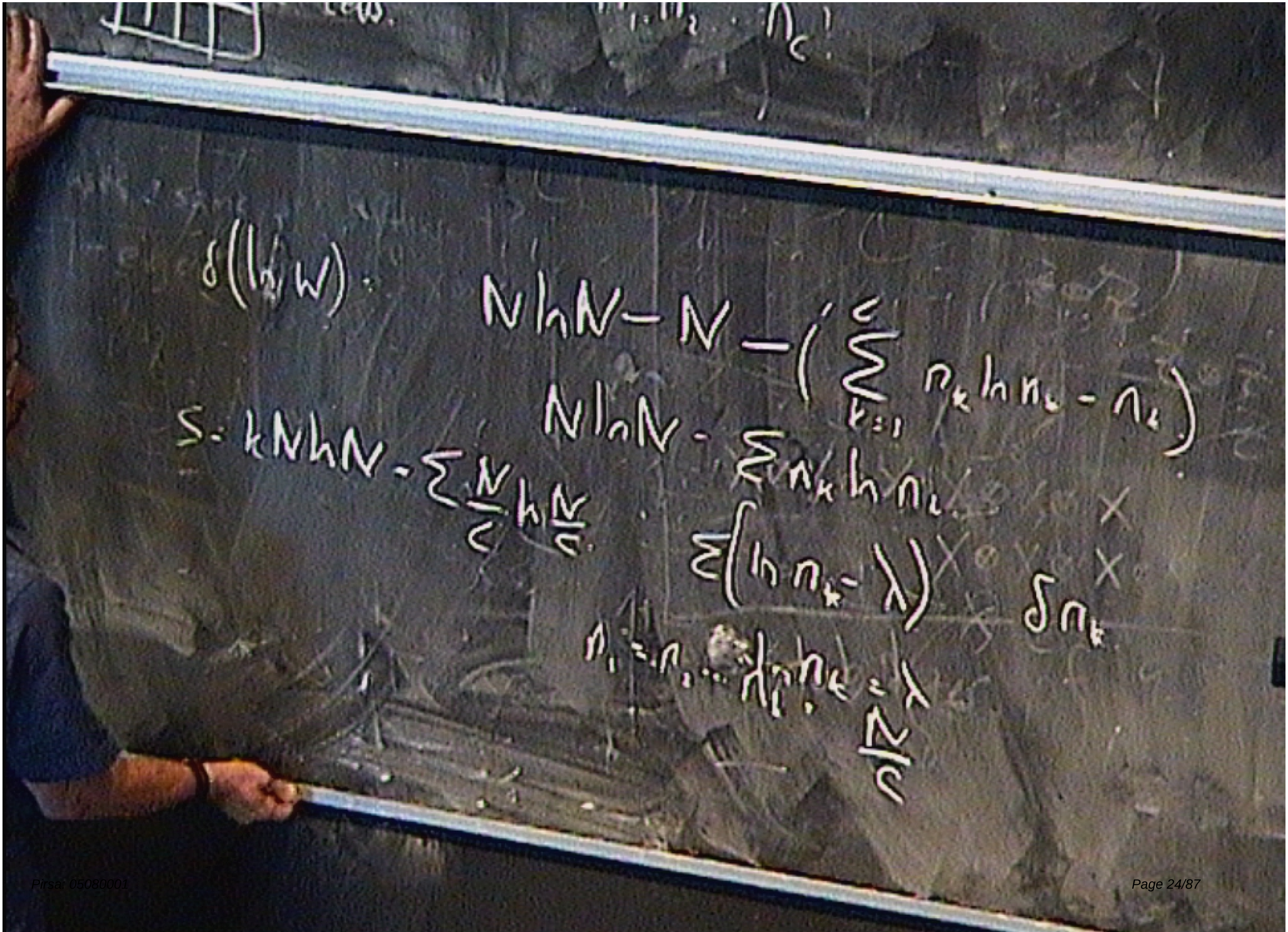
$$N \ln N - N - \left( \sum_{k=1}^{c} n_k \ln n_k - n_k \right)$$

$$N \ln N - \sum n_k \ln n_k$$

$$\sum \left( \ln n_k - \lambda \right) \delta n_k$$

$$n_1 = n_2 \ldots \quad \ln n_k = \lambda$$

$$n_k = \frac{N}{c}$$


δ(ln W) =
N ln N − N − ( Σ_{k=1}^{c} n_k ln n_k − n_k )
N ln N − Σ n_k ln n_k
S = k N ln N − Σ N/c ln N/c
Σ (ln n_k − λ) δn_k
n_1 = n_2 = … ln n_k = λ
N/c

$n_1! n_2! \cdots n_c!$

$$\delta(\ln W) : \quad N\ln N - N - \left(\sum_{k=1}^{c} n_k \ln n_k - n_k\right)$$

$$N\ln N - \sum n_k \ln n_k$$

$$S = kN\ln N - \sum \frac{N}{c}\ln\frac{N}{c}$$

$$-N\ln\frac{N}{c}$$

$$\sum\left(\ln n_k - \lambda\right)\delta n_k$$

$$n_1 = n_2 \ldots \quad \ln n_k = \lambda \quad \frac{N}{c}$$

$$\delta(\ln W) = N\ln N - N - \left(\sum_{k=1}^{c} n_k \ln n_k - n_k\right)$$

$$N\ln N - \sum n_k \ln n_k$$

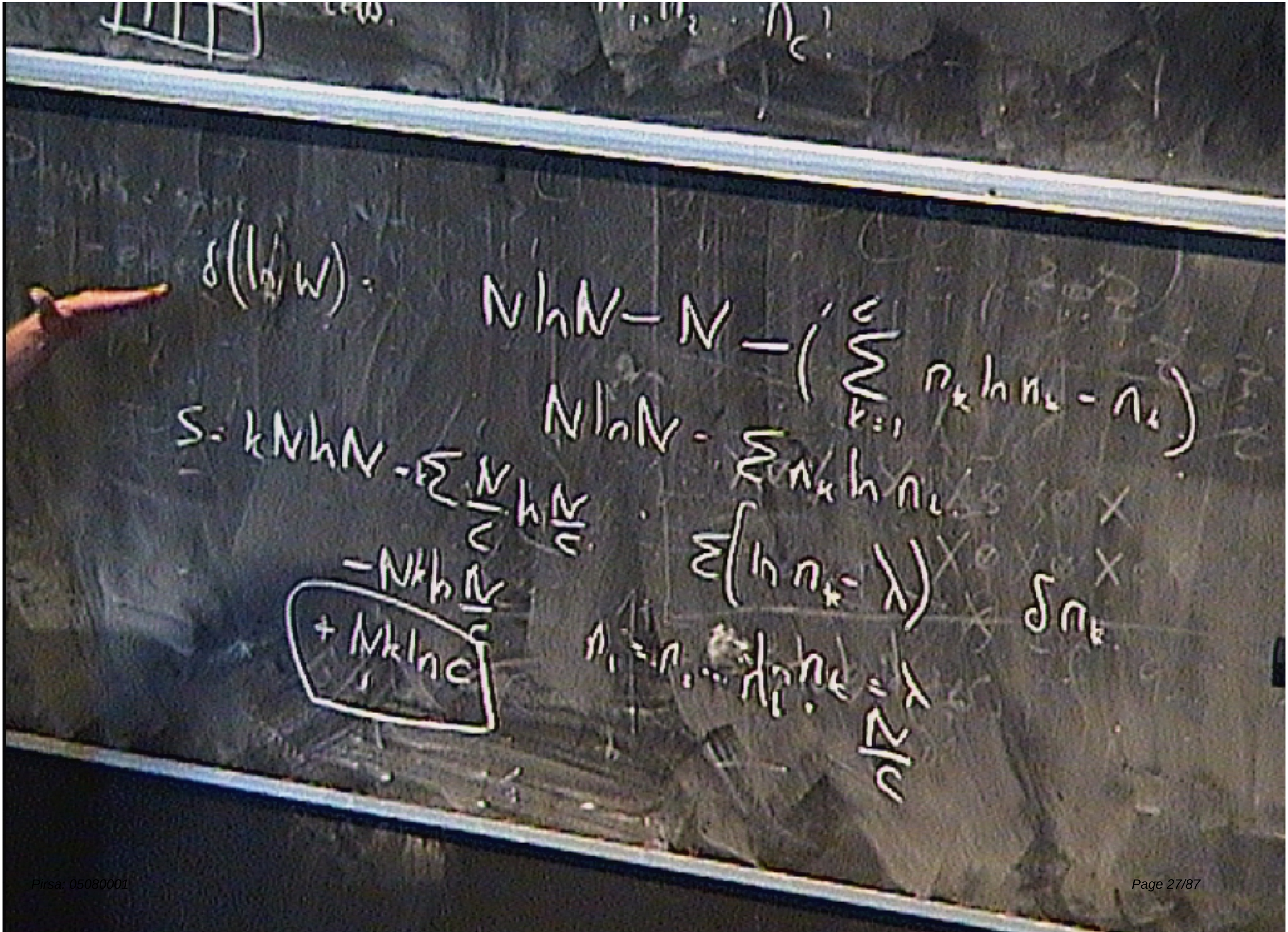
$$S = kN\ln N - k\sum \frac{N}{c}\ln\frac{N}{c}$$

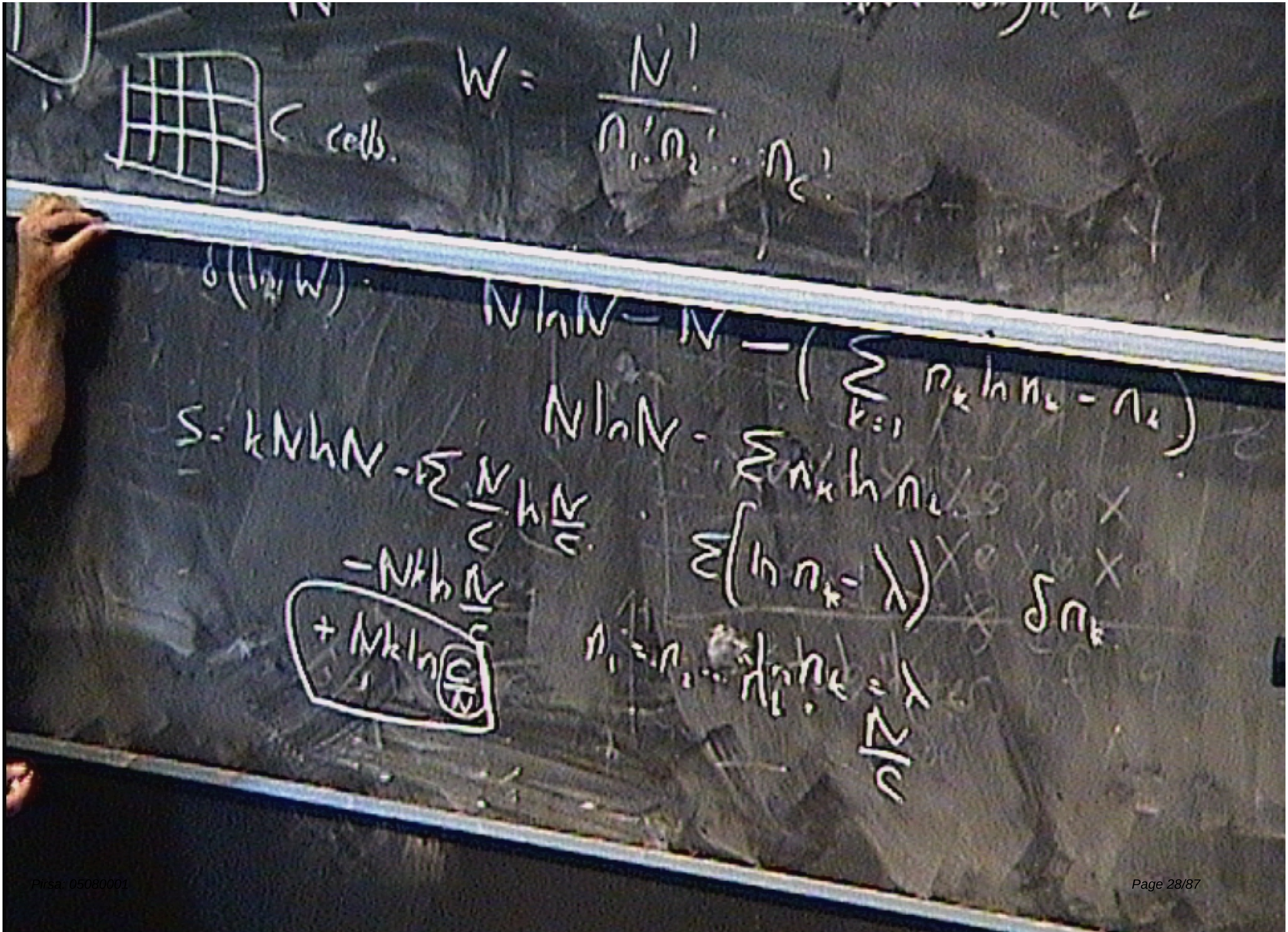
$$-Nk\ln\frac{N}{c}$$

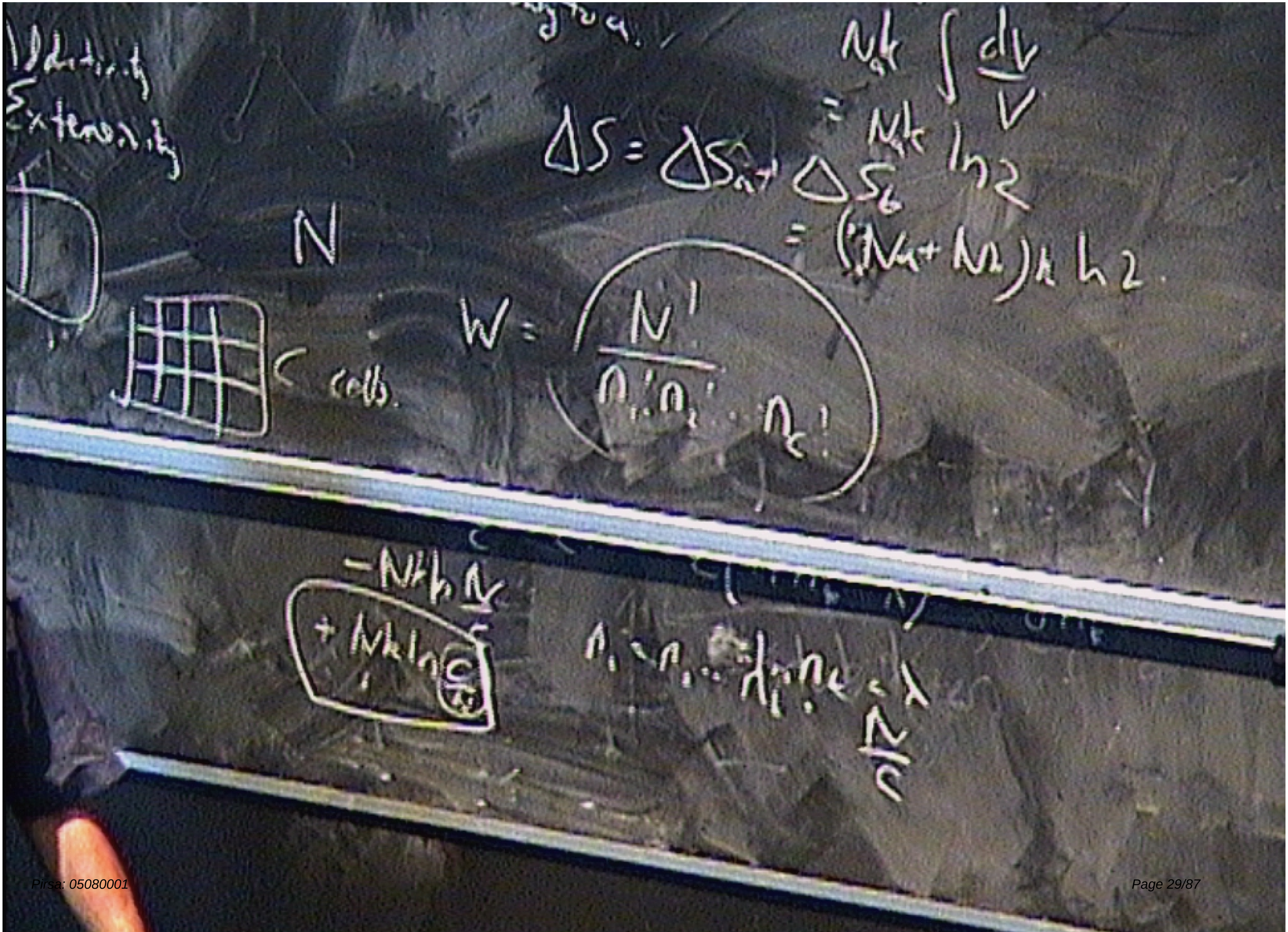
$$+Nk\ln c$$

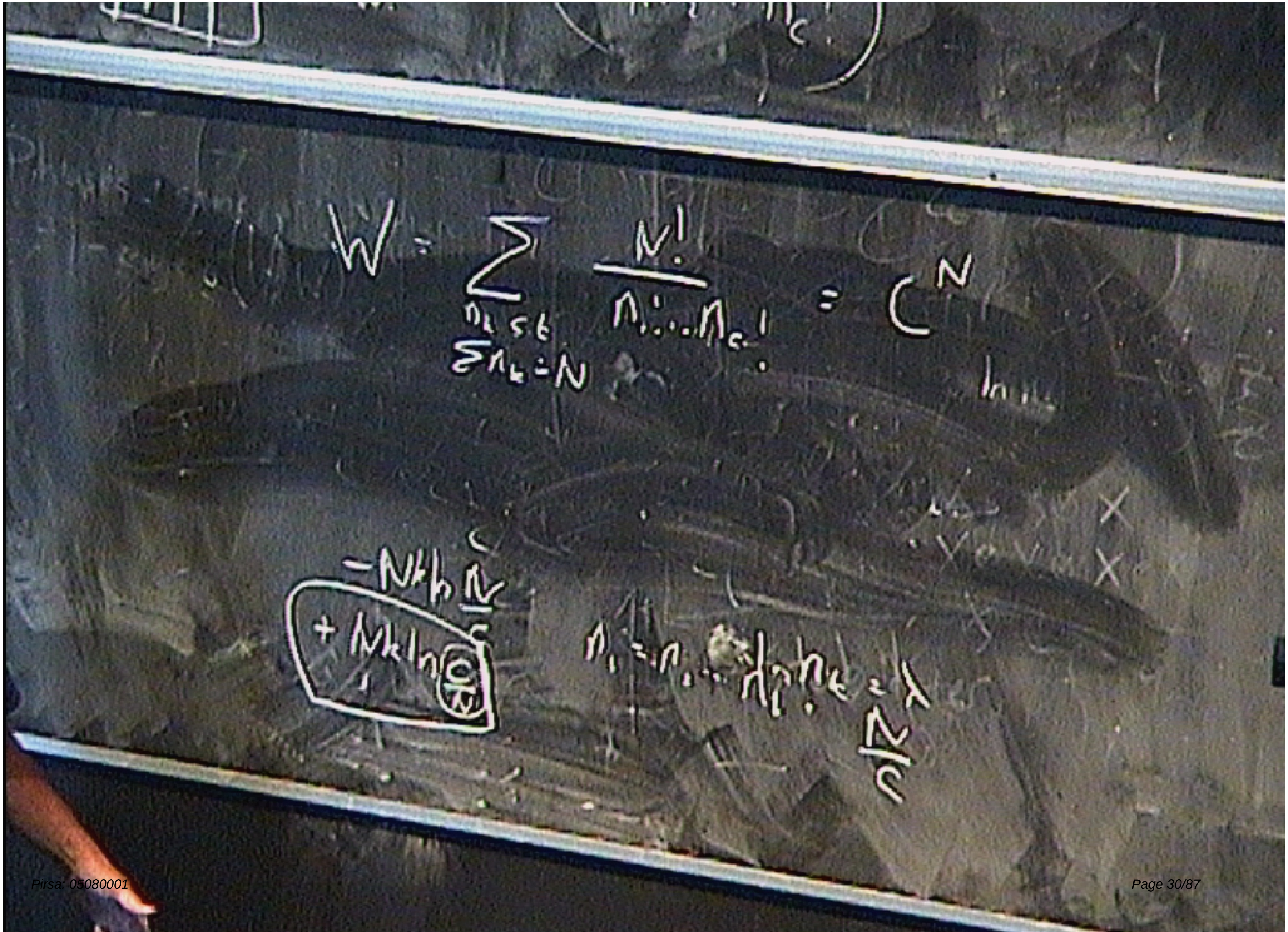
$$\sum(\ln n_k - \lambda)\,\delta n_k$$

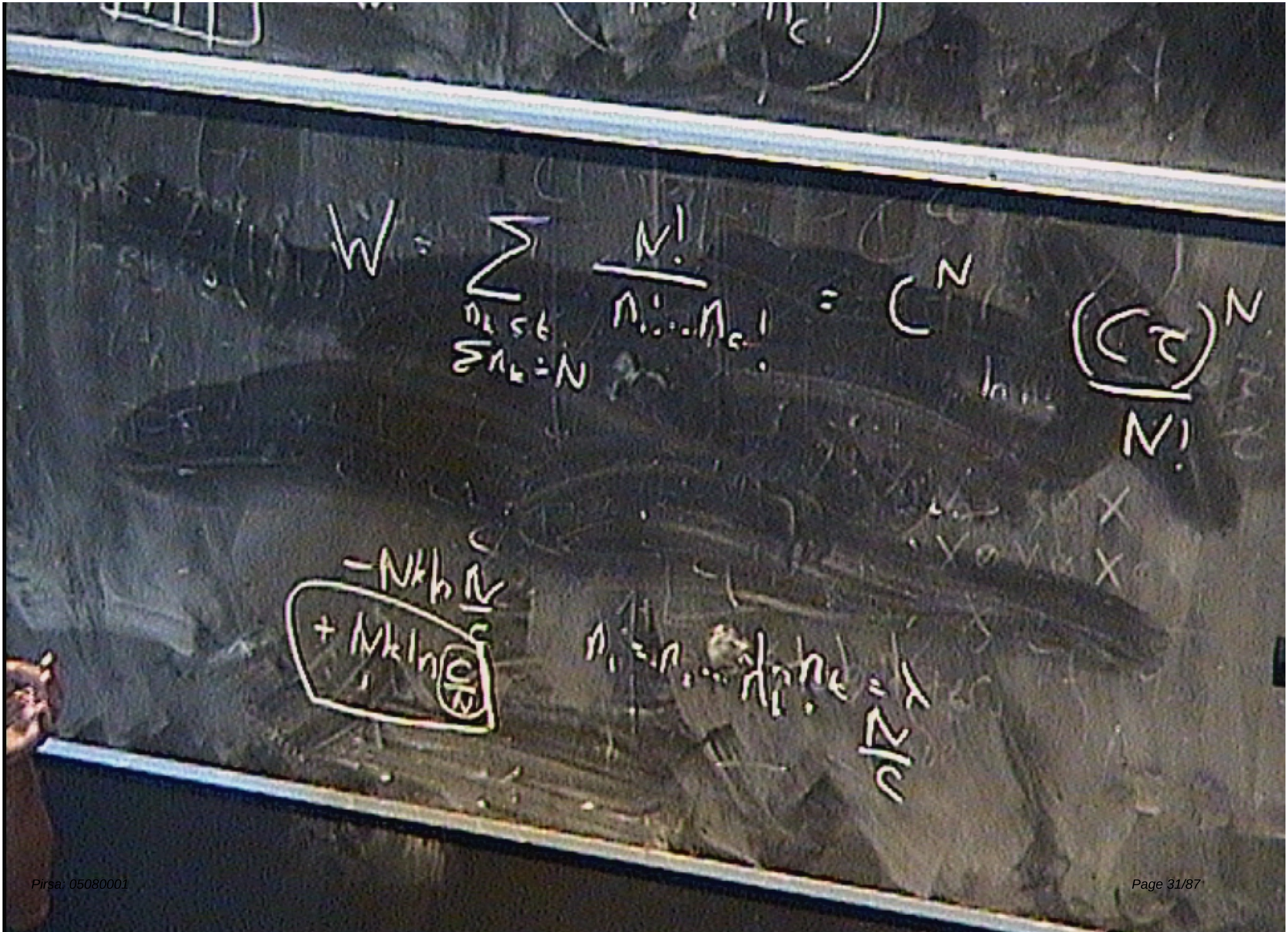
$$n_1 = n_2 = \dots \quad \ln n_k = \lambda \quad n_k = \frac{N}{c}$$


δ(ln W) =
N ln N − N − ( Σ_{k=1}^{c} n_k ln n_k − n_k )
N ln N − Σ n_k ln n_k
S = k N ln N − Σ N/c ln N/c
Σ (ln n_k − λ) δn_k
− Nk ln N/c
+ Nk ln c
n_1 = n_2 ... ln n_k = λ
N/c

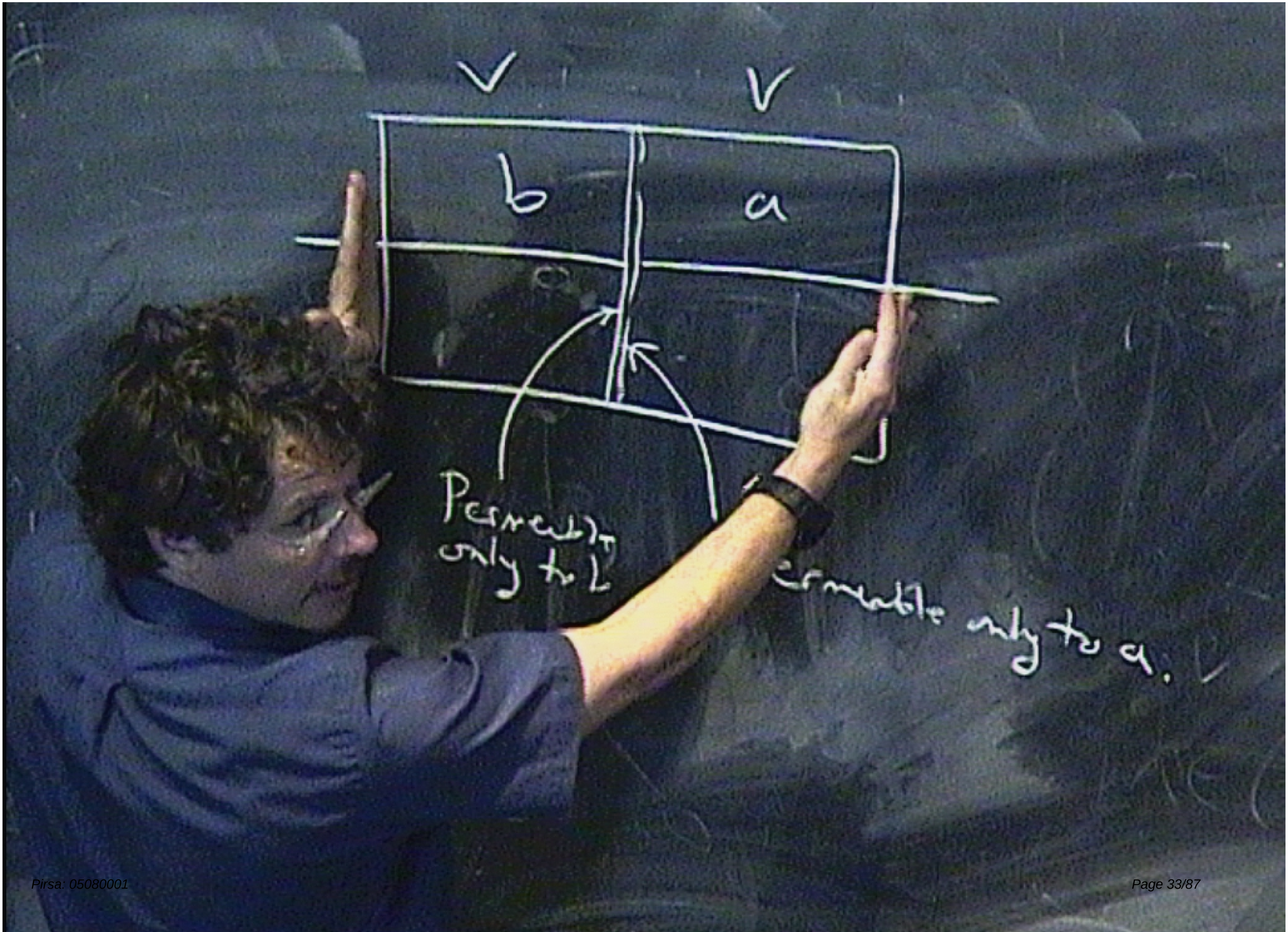

W = N! / n1! n2! ... nc!
c cells
δ(ln W)
N ln N − N − ( Σ_{k=1} n_k ln n_k − n_k )
N ln N − Σ n_k ln n_k
S = k N ln N − Σ N/c ln N/c
− N k ln N/c
+ N k ln c/N
Σ (ln n_k − λ) δn_k
n_1 = n_2 ... ln n_k = λ
N/c


Extensivity
N
cells
W = N! / n₁! n₂! ... n_c!
ΔS = ΔS_a + ΔS_b
Nk ∫ dV/V
= Nk ln 2
= (N_a + N_b) k ln 2
−Nk ln N/c
+ Nk ln c




W = Σ N! / n1!...nc! = c^N
nk s.t. Σnk = N
(cτ)^N / N!
−Nk ln N/c
+ Nk ln c/N
n1 = n2 ... = nc = λ
N/c

Indistinguishable Classical Particles Have No Trajectories. The unconventional role of indistinguishable classical particles is best expressed by the fact that in a deterministic setting no indistinguishable particles exist, or - equivalently - that indistinguishable classical particles have no trajectories. Before I give a formal proof I argue as follows. Suppose they have trajectories, then the particles can be identified by them and are, therefore, not indistinguishable. (Bach, Indistinguishable Classical Particles, Springer, 1997 p.7).


b
a
only to a.

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**Definition**. If we define
Indistinguishability = Identity of the Particles + Symmetry of the state
then all contradictions mentioned above vanish or become meaningless (Bach, ibid, p.8).

It was a famous paradox pointed out for the first time by W. Gibbs, that the same increase of entropy must not be taken into account, when the two molecules are of the same gas, although (according to naive gas-theoretical views) diffusion takes place then too, but unnoticeably to us, because all the particles are alike. The modern view [of quantum mechanics] solves this paradox by declaring that in the second case there is no real diffusion, because exchange between like particles is not a real event - if it were, we should have to take account of it statistically. It has always been believed that Gibbs's paradox embodied profound thought. That it was intimately linked up with something so important and entirely new [as quantum mechanics] could hardly be foreseen. (Schrödinger, Statistical Thermodynamics, Cambridge, 1946 p.61).

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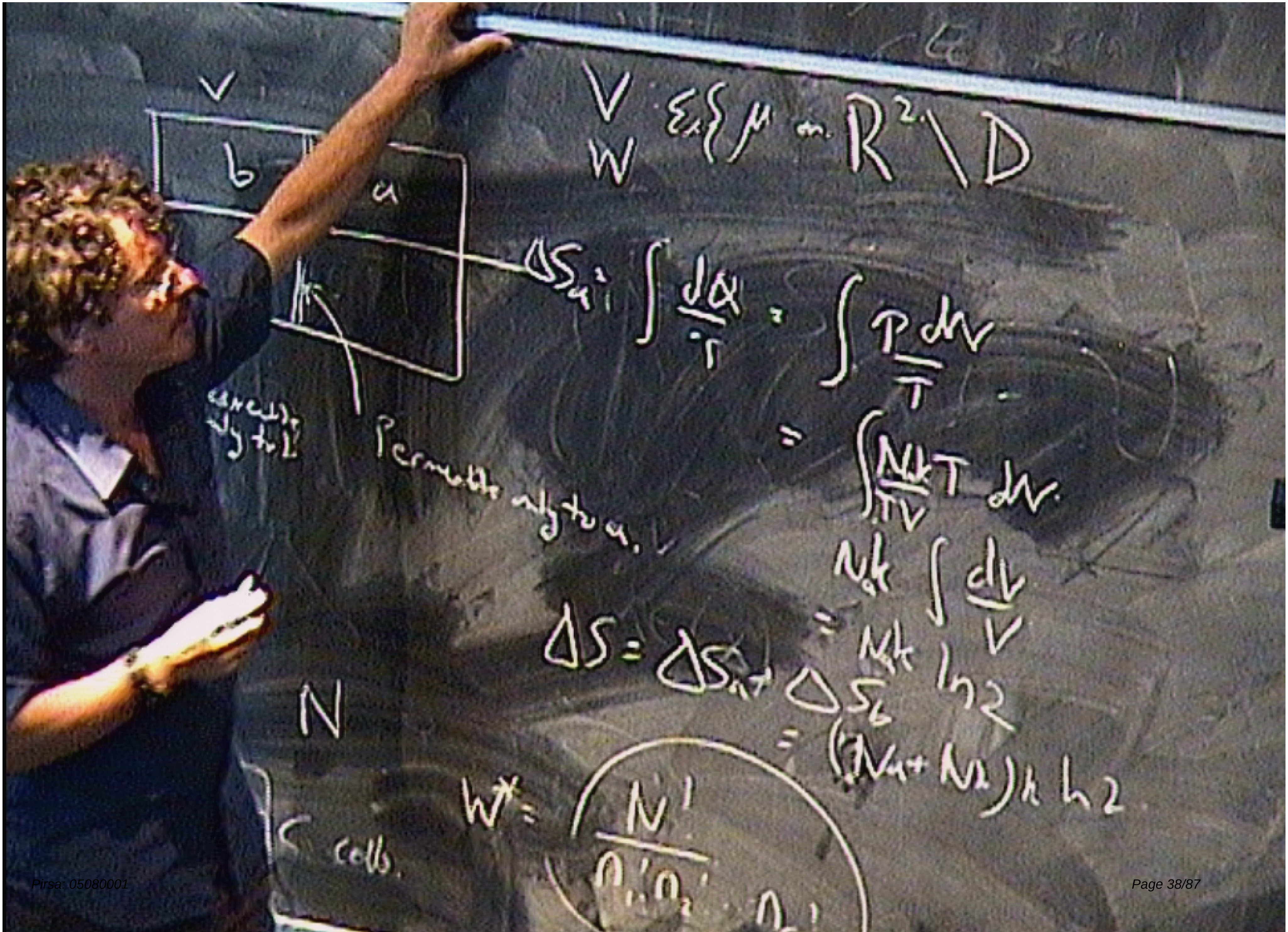
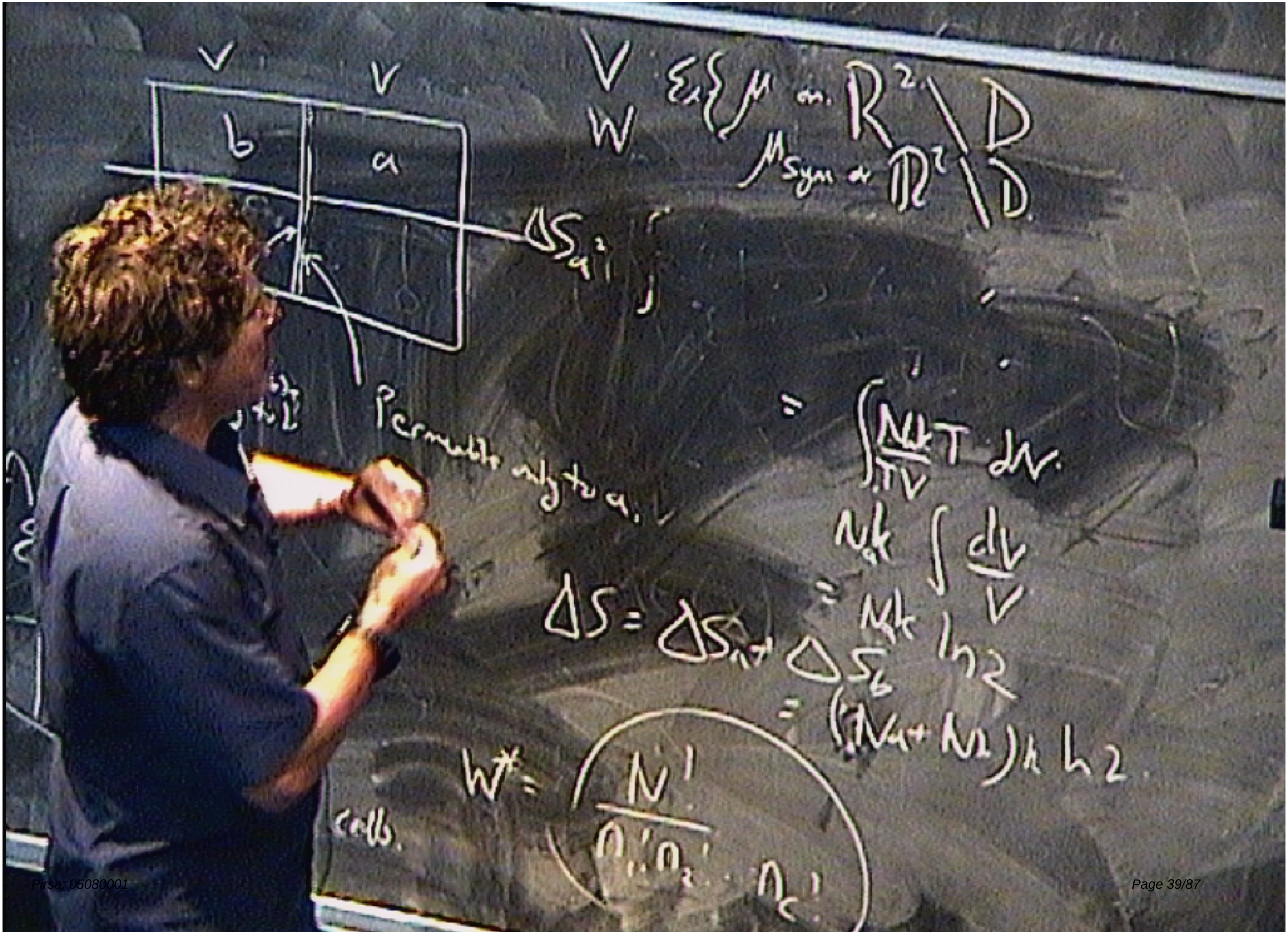
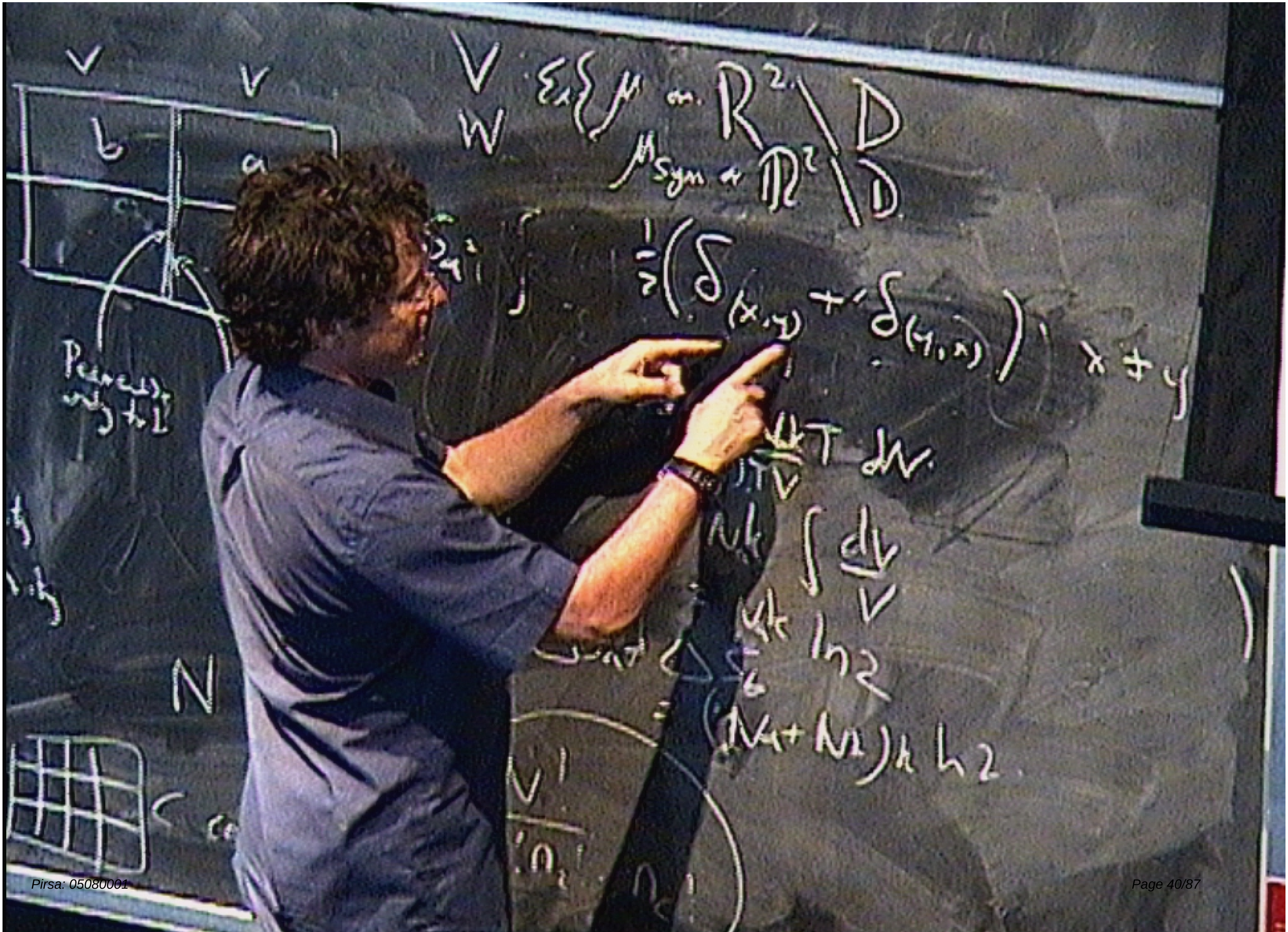
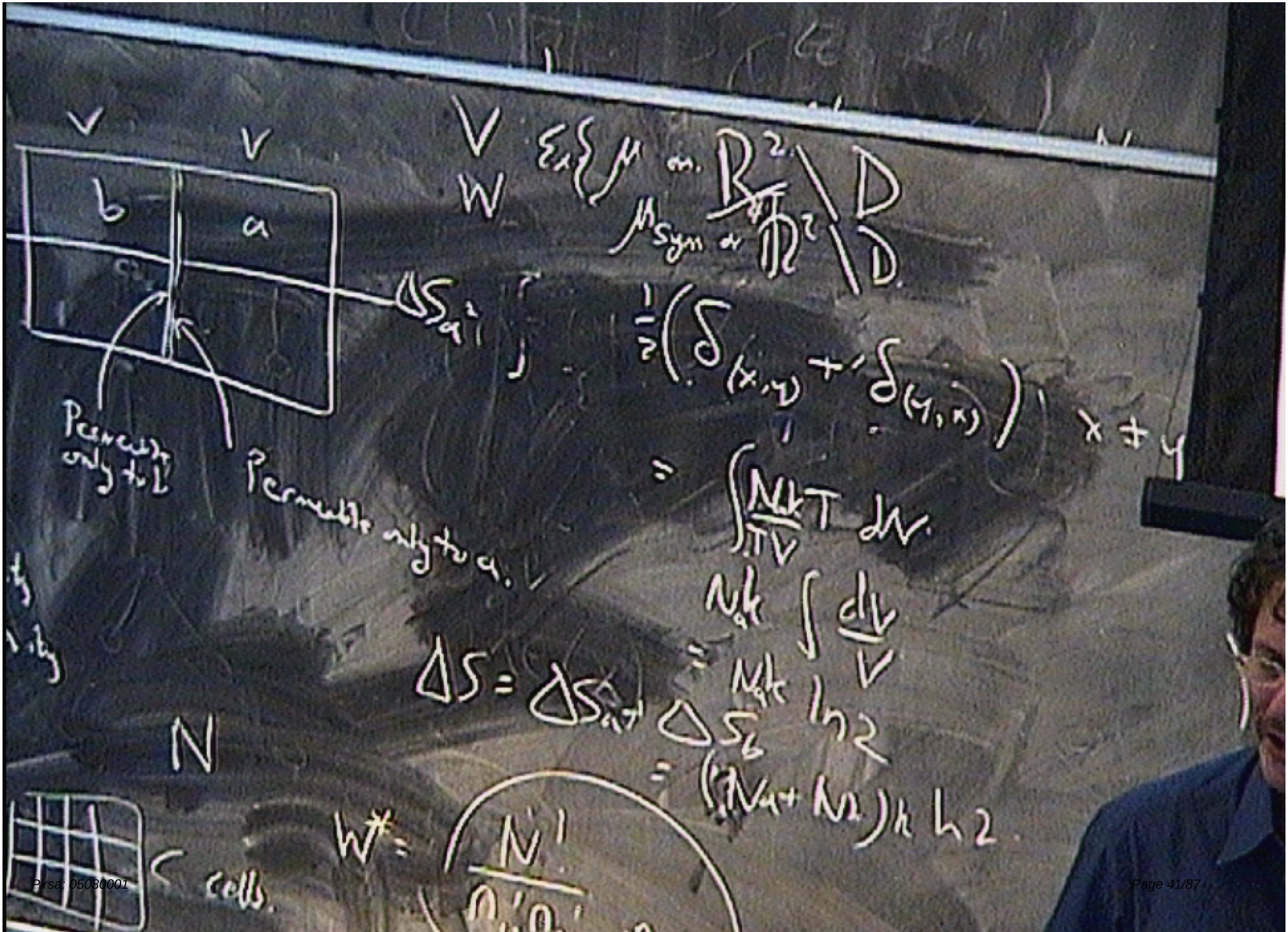


Permeable only to a
ΔS = ΔS_a + ΔS_b
= (N_a + N_b) k ln 2
N


b
a
V
W
Permeable only to a
ΔS = ΔS_a + ΔS_b
cells




b
a
Permeable only to a.
ΔS = ΔS_a + ΔS_b
= (N_a + N_b) k ln 2
W* = N!
cells

The distribution of energy over each type of resonator must now be considered, first, the distribution of the energy E over the N resonators with frequency . If E is regarded as infinitely divisible, an infinite number of different distributions is possible. We, however, consider - and this is the essential point - E to be composed of a determinate number of equal finite parts and employ in their determination the natural constant $h= 6.55\times10^{-27}$ erg sec. This constant, multiplied by the frequency, , of the resonator yields the energy element in ergs, and dividing E by h, we obtain the number P, of energy elements to be distributed over the N resonators. (Planck 1900, trans ter Haar p.239).

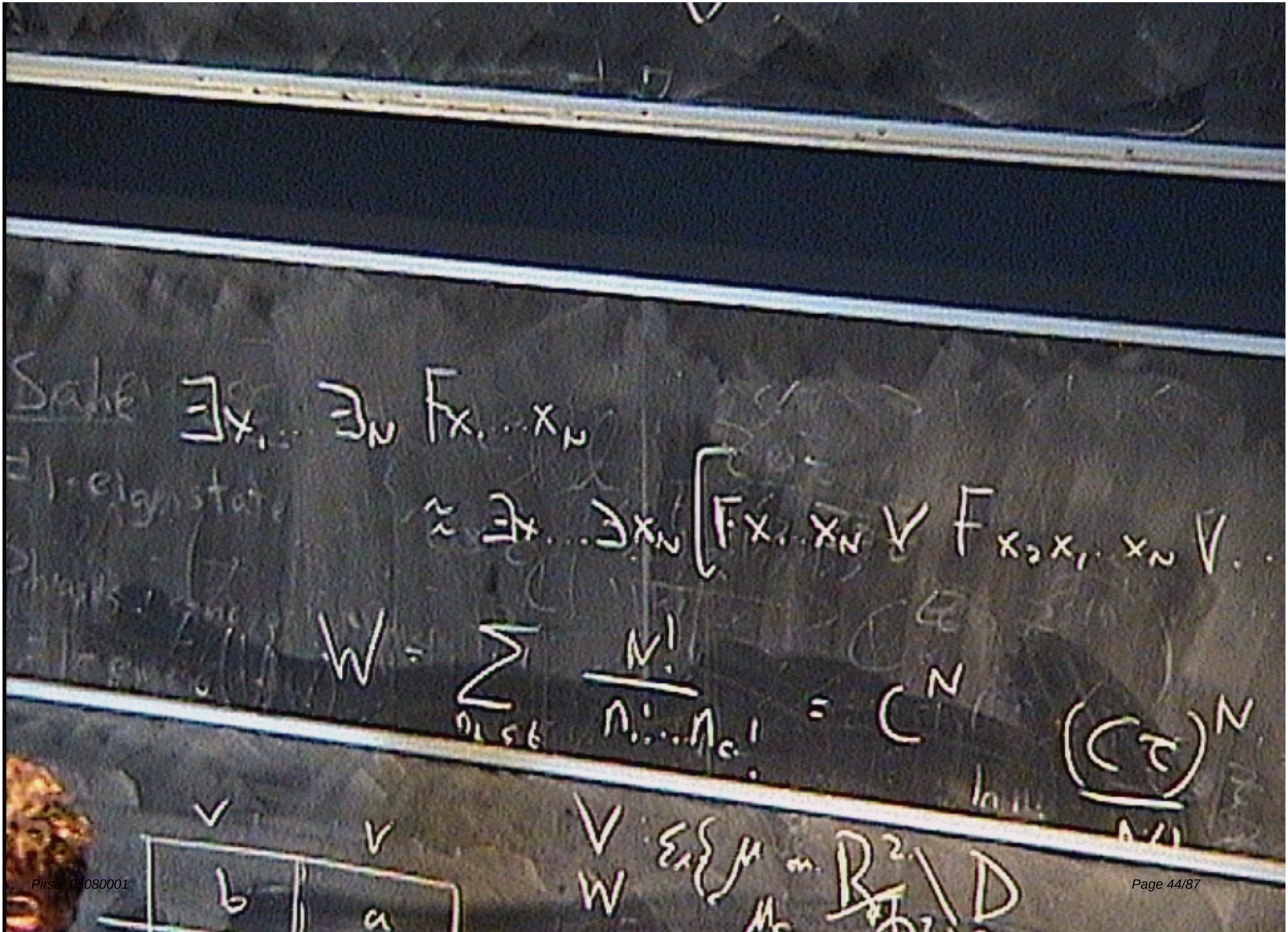
Let $\mathcal{L}$ be a first-order language without any proper names (0-ary function symbols). Let T be any $\mathcal{L}$-sentence T satisfiable only in models of cardinality N. Then there is a totally symmetric predicate

$Gx_1 \ldots x_N$_ $\in \mathcal{L}$

such that

$\exists x_1 \ldots \exists x_N \, Gx_1 \ldots x_N$

is logically equivalent to T.


∃x1 ... ∃N Fx1 ... xN
≈ ∃x1 ... ∃xN [Fx1 ... xN ∨ Fx2x1 ... xN ∨ ...
W = Σ N! / n1! ... nc! = C^N
(Cτ)^N

Bob is listening to the talk, not Alice = Fba

Bob is listening to the talk, not Alice, or Alice is listening to the talk, not Bob = Fba or Fab

Bob is listening to the talk, not Alice = Fba

Bob is listening to the talk, not Alice, or Alice is listening to the talk, not Bob = Fba or Fab

But now use a language sufficiently rich to dispense with names (Russellian descriptions):

x is B-shaped and is listening to the talk, not y who is A-shaped = F'xy

Bob is listening to the talk, not Alice = Fba

Bob is listening to the talk, not Alice, or Alice is listening to the talk, not Bob = Fba or Fab

But now use a language sufficiently rich to dispense with names (Russellian descriptions):

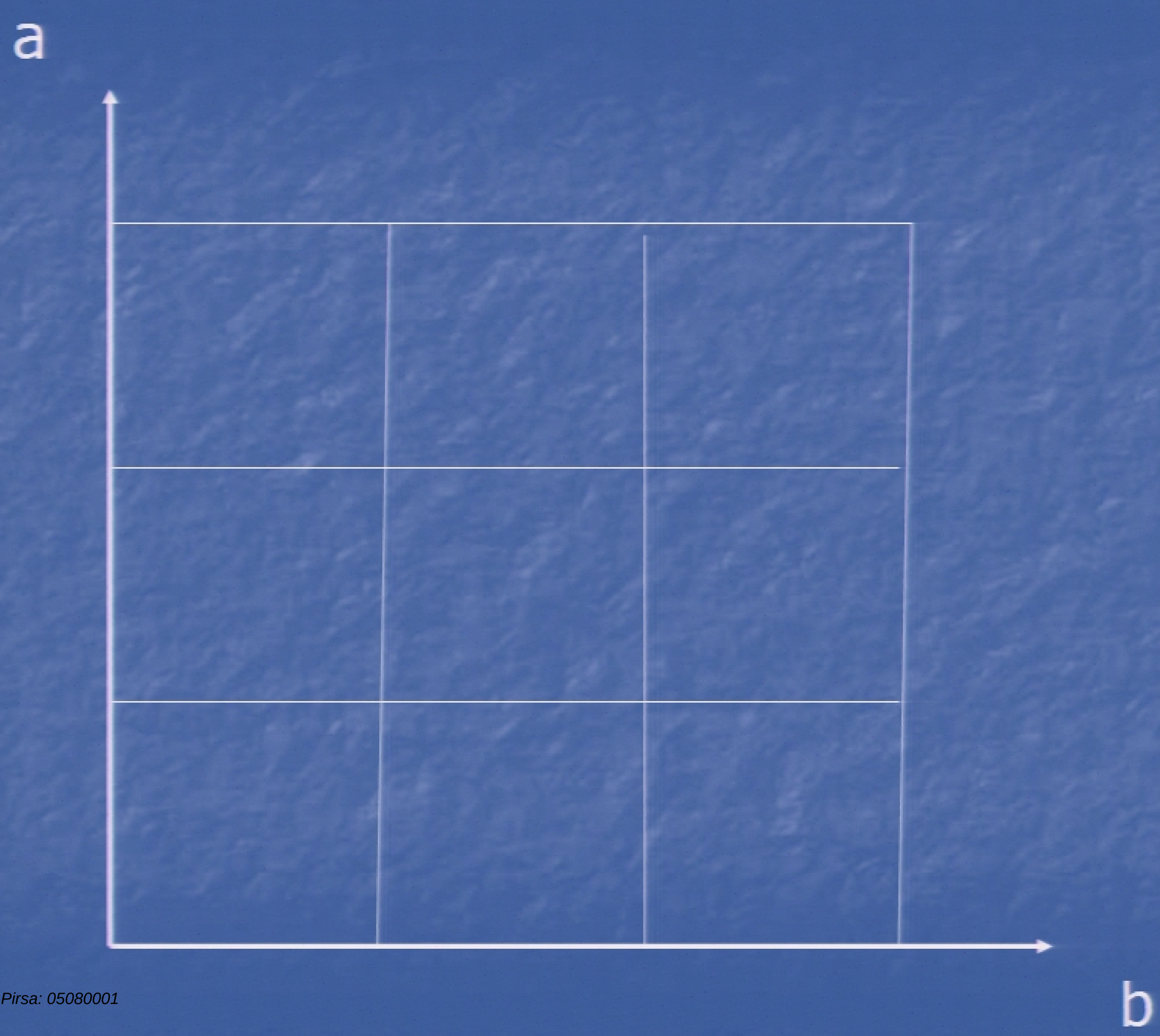
x is B-shaped and is listening to the talk, not y who is A-shaped = F'xy

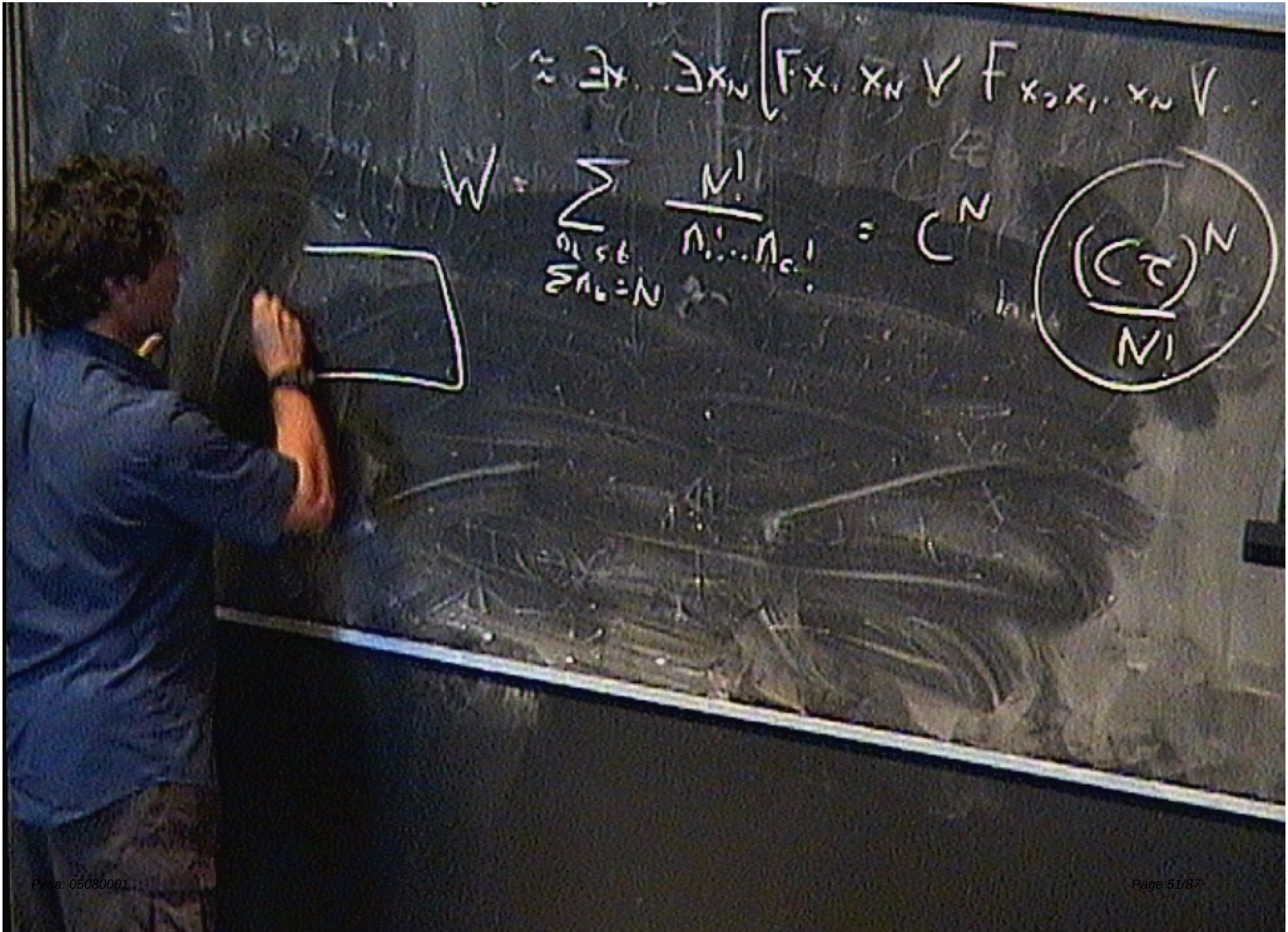
x is B-shaped and is listening to the talk, not y who is A-shaped, or y is B-shaped and is listening to the talk, not x who is A-shaped = F'xy or F'yx

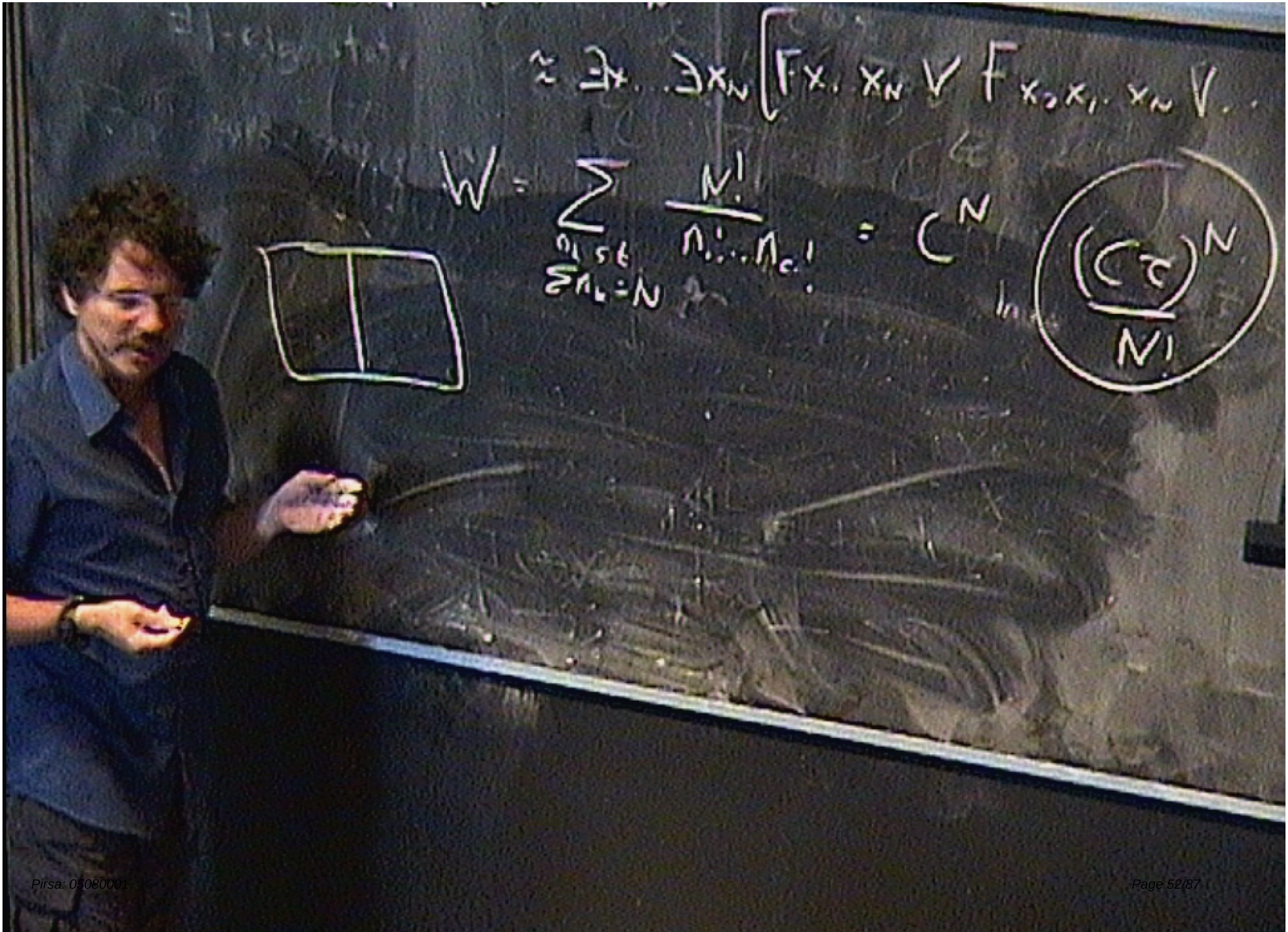
There is a certain fairly trivial sense in which it ought to have been obvious from the outset (if we had stopped to think about it) that the facts of thermodynamics cannot possibly shed any light on the truth or falsehood of the doctrine of Haecceisstism. The question of the truth or falsehood of the second law of thermodynamics is (after all) a straightforwardly empirical one; and the question of Haecceisstism, the question (that is) of whether or not certain observationally identical situations are identical simpliciter, manifestly is not. Nevertheless, it might have turned out that the statistical-mechanical account of thermodynamics is somehow radically simpler or more natural or more compelling or more of an explanatory success when expressed in a Haecceistic language than it is when expressed in a non-Haecceisstic one. And the thing we've just learned (which seems to me substantive and non-trivial and impossible to have anticipated without doing the work) is that that is not the case. (Albert p.47-48)

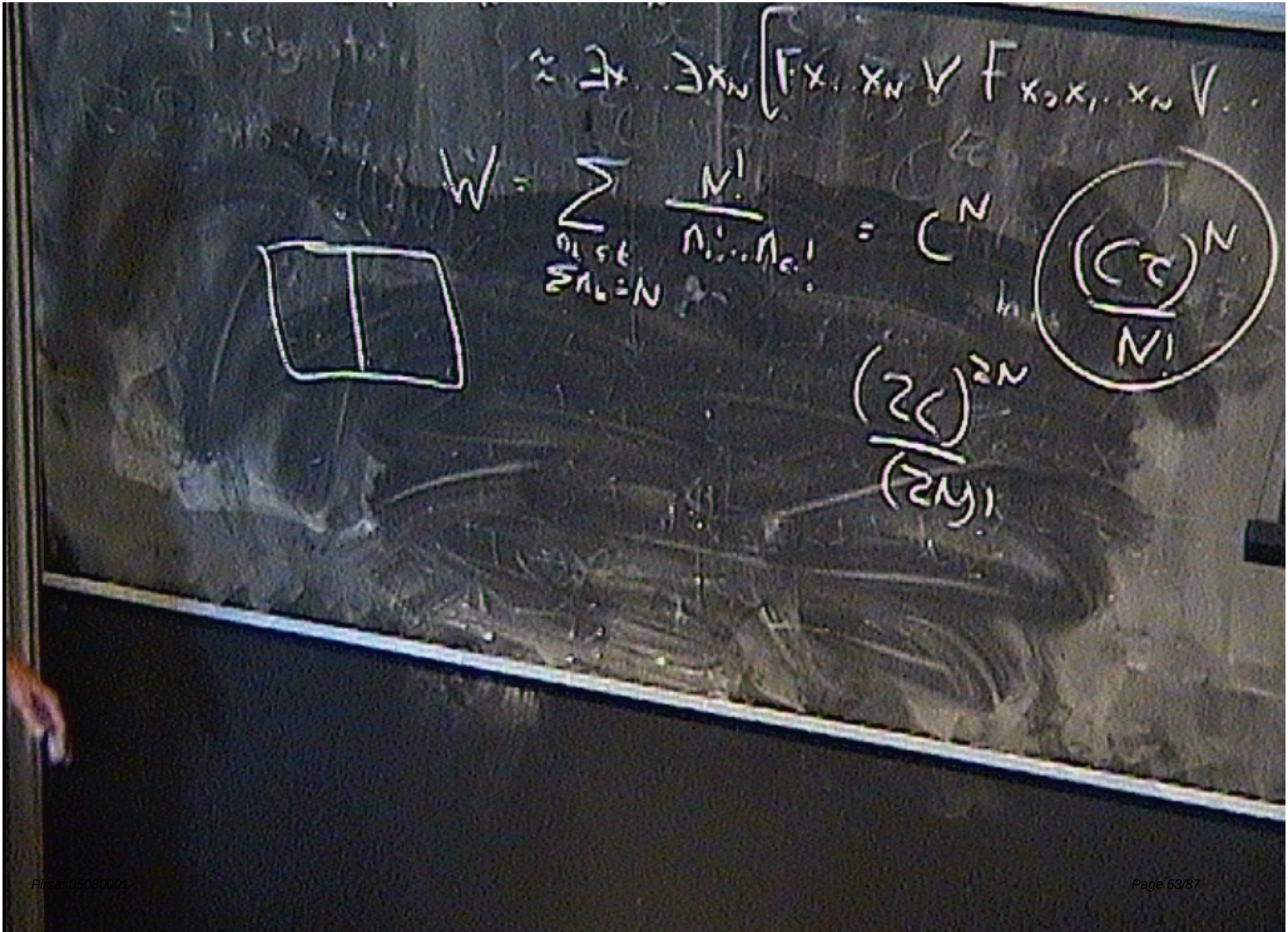
# To be expained:

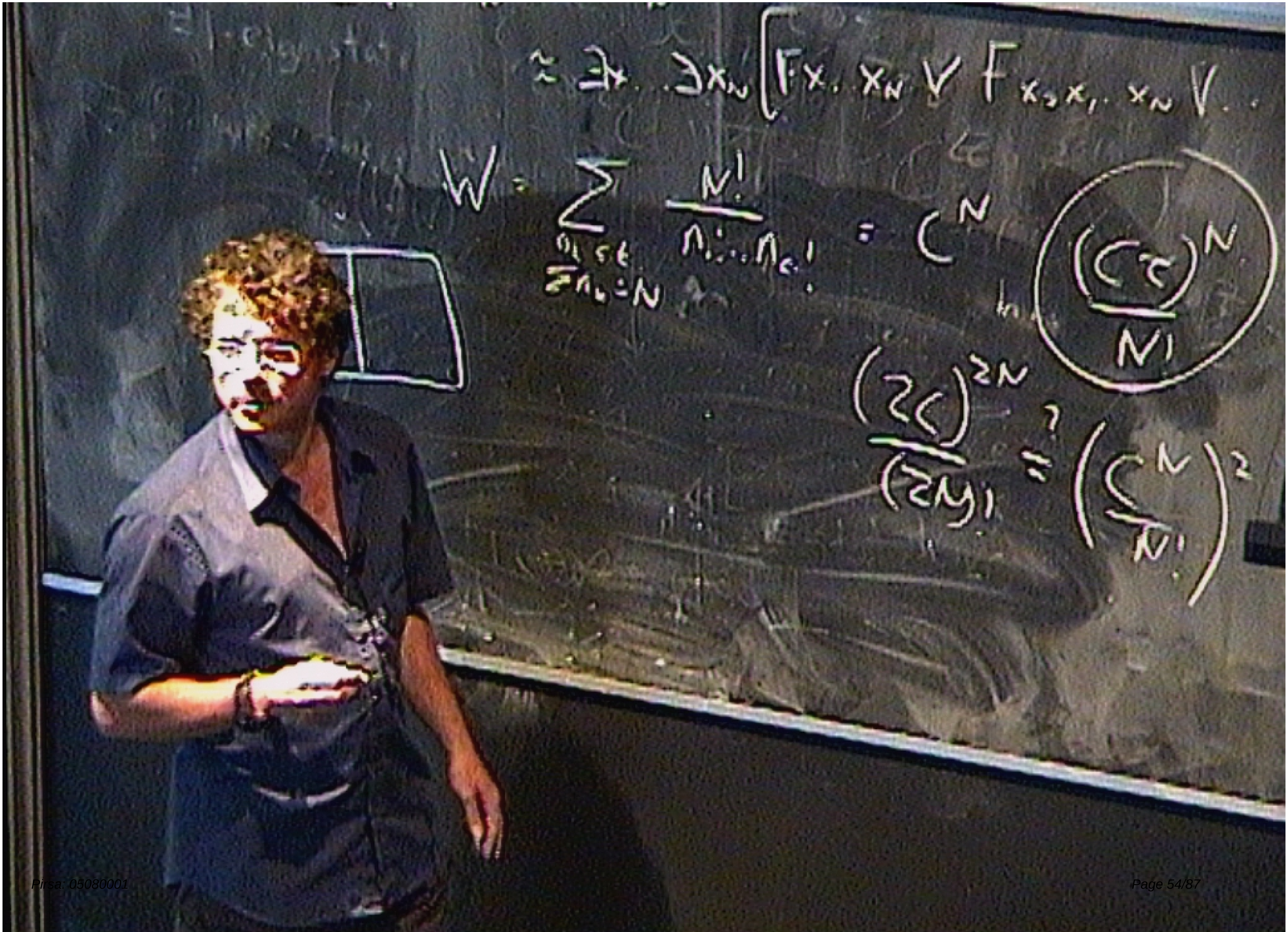
- Distinguishable classical particles obey MB statistics
- Indistinguishable classical particles obey MB statistics
- Distinguishable quantum particles obey MB statistics
- Indistinguishable quantum particles obey BE, FD, or parastatistics


a
b


≈ ∃x₁ … ∃x_N [Fx₁ … x_N V Fx₂x₁ … x_N V …
W = Σ N! / n₁! … n_c! = C^N
Σn_i = N
(Cτ)^N / N!


≈ ∃x₁ … ∃x_N [F x₁ … x_N ∨ F x₀ x₁ … x_N ∨ …
W = Σ N! / n₁! … n_c! = C^N
n_i s.t. Σn_i = N
(C τ)^N / N!


∃x₁ … ∃x_N [F x₁ … x_N ∨ F x₀ x₁ … x_N ∨ …
W = Σ N! / (n₁! … n_c!) = C^N
n_i s.t. Σn_i = N
(Cτ)^N / N!
(2C)^{2N} / (2N)!

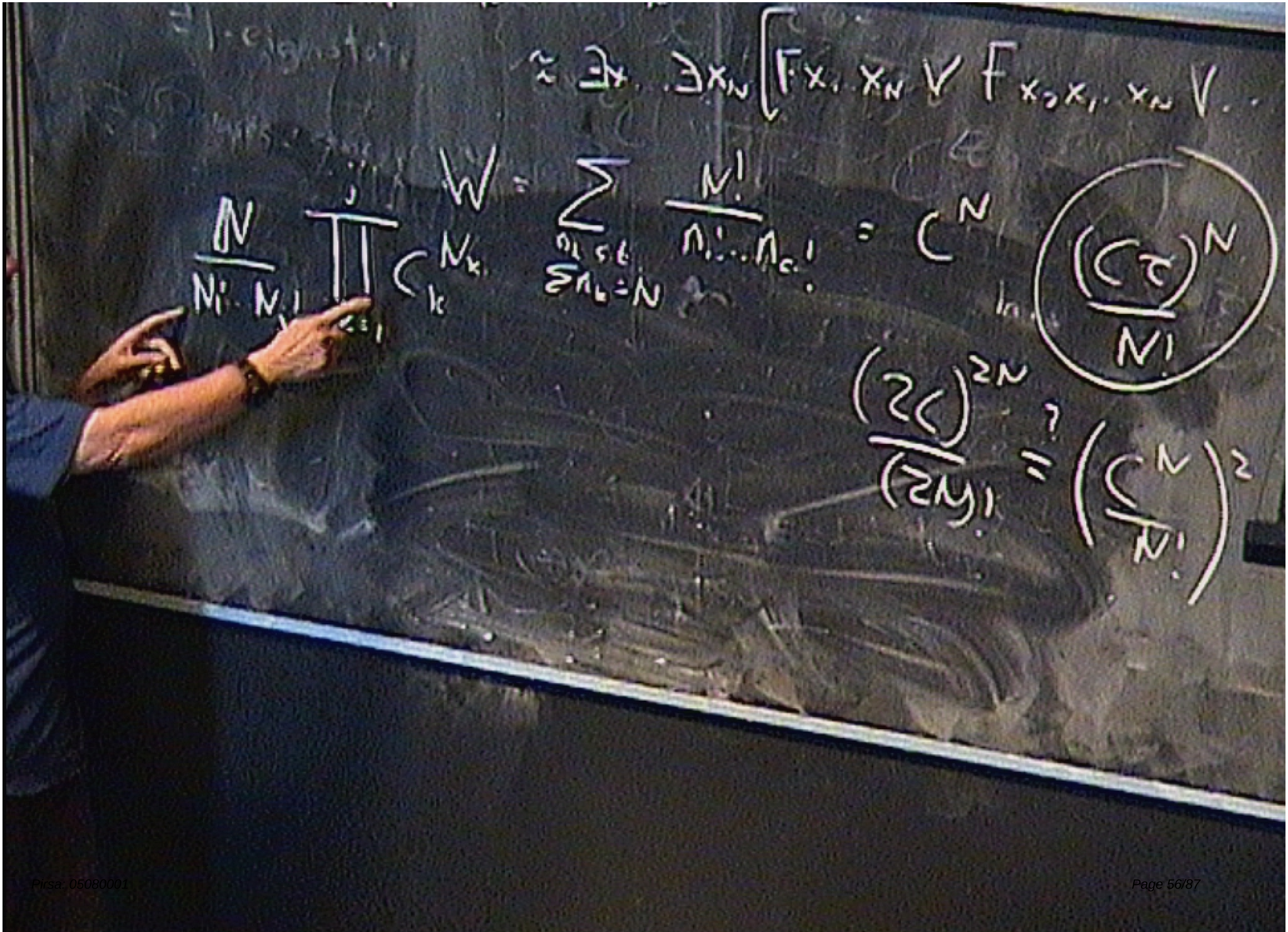


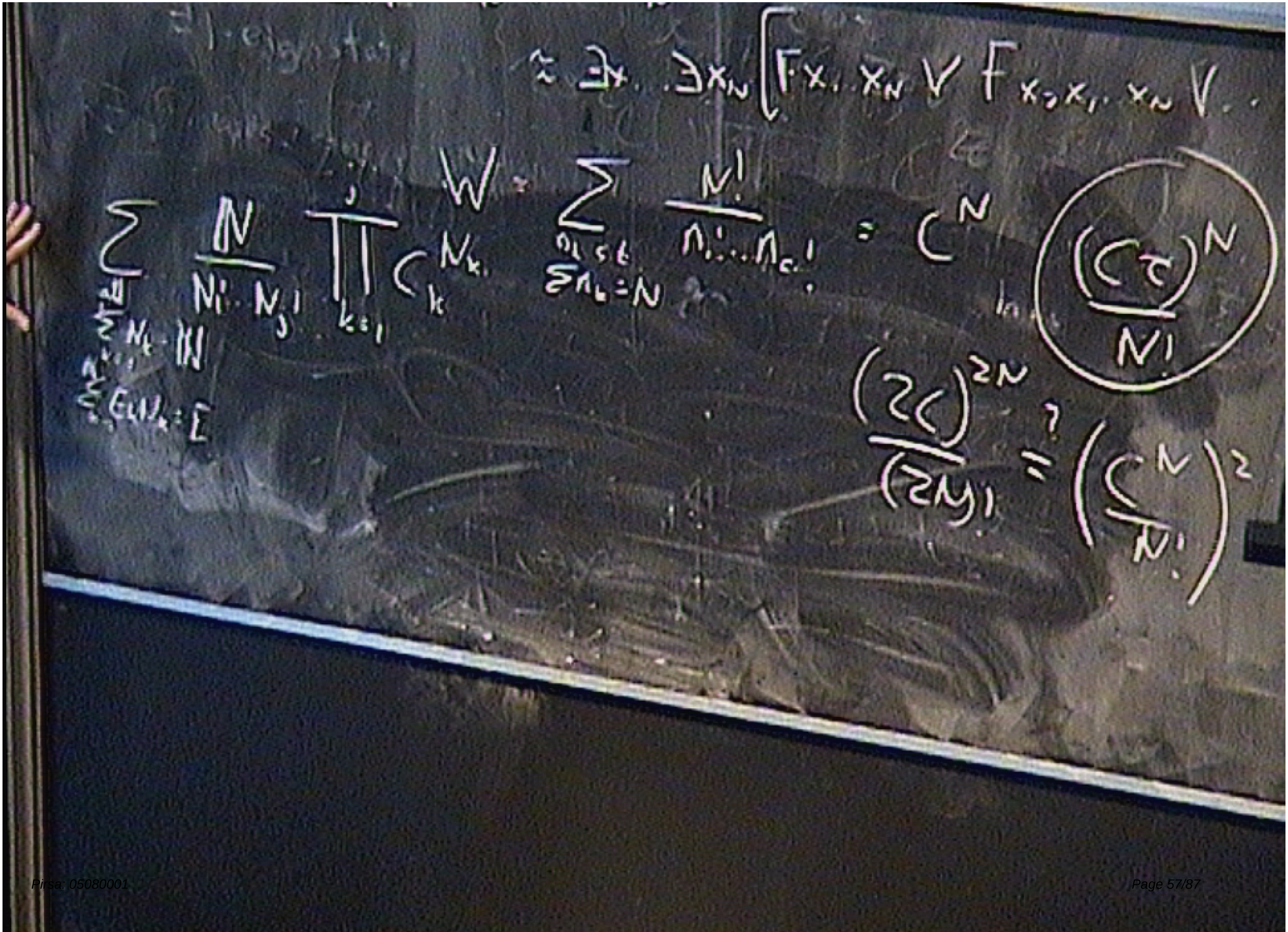
$$\approx \exists x_1 \ldots \exists x_N \left[ F x_1 \ldots x_N \vee F x_2 x_1 \ldots x_N \vee \ldots \right.$$

$$W = \sum_{\substack{n_i \text{ s.t.} \\ \sum n_i = N}} \frac{N!}{n_1! \ldots n_c!} = C^N$$

$$\frac{(C\tau)^N}{N!}$$

$$\frac{(2C)^{2N}}{(2N)!} \stackrel{?}{=} \left( \frac{C^N}{N!} \right)^2$$



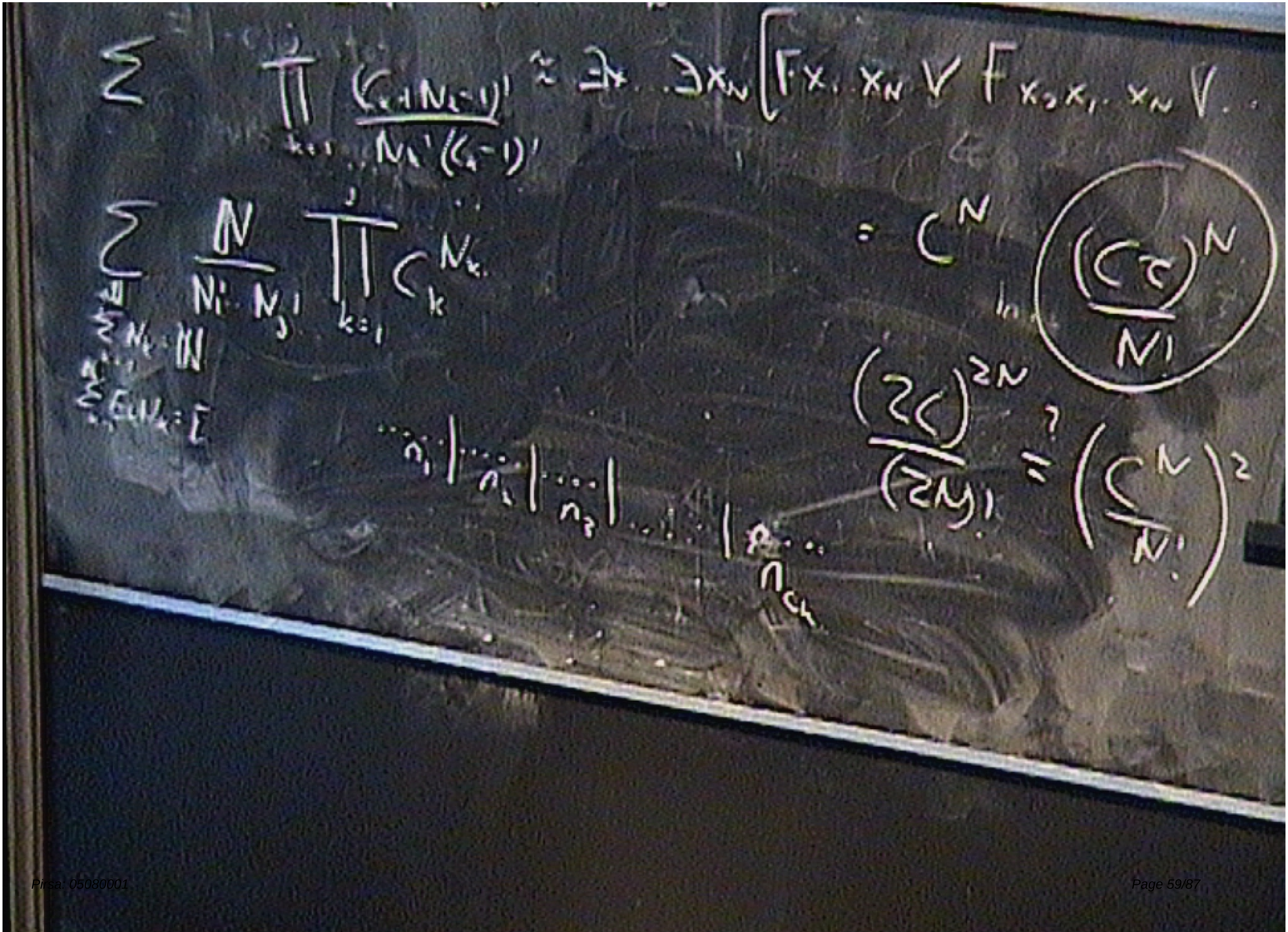

W = Σ N!/(n_1! … n_c!) = c^N
Σn_k = N
(cτ)^N / N!
(2c)^{2N} / (2N)! =? (c^N / N!)^2

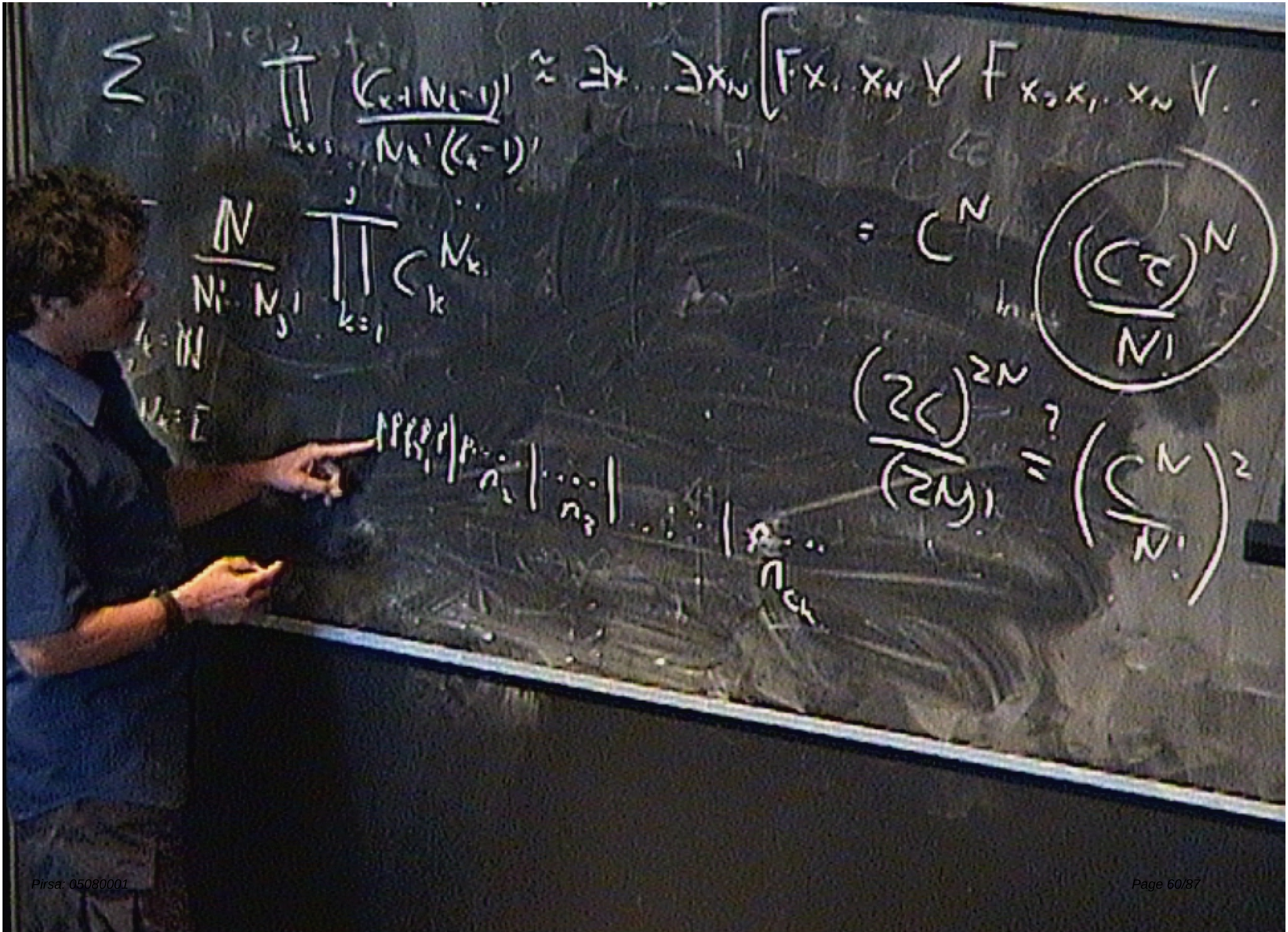
$$\sum \prod_{k=1} \frac{(c_k+N_k-1)!}{N_k!\,(c_k-1)!} \approx \exists x_1 \ldots \exists x_N \left[ F x_1 \ldots x_N \vee F x_2 x_1 \ldots x_N \vee \ldots \right.$$

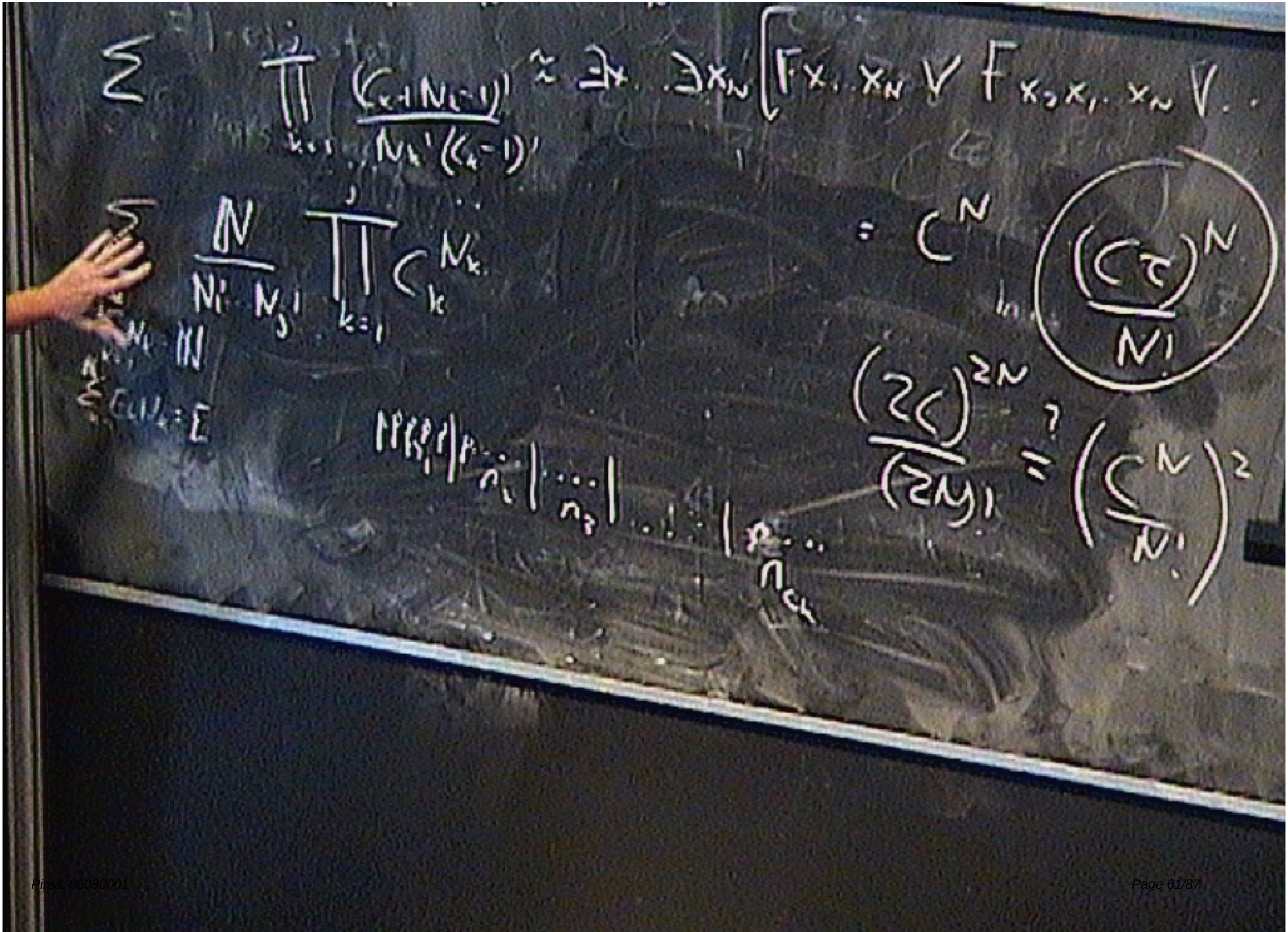
$$\sum_{\substack{\sum N_k = N \\ \sum \epsilon_k N_k = E}} \frac{N!}{N_i!\, N_j!} \prod_{k=1} c_k^{N_k} = C^N$$

$$\frac{(C\tau)^N}{N!}$$

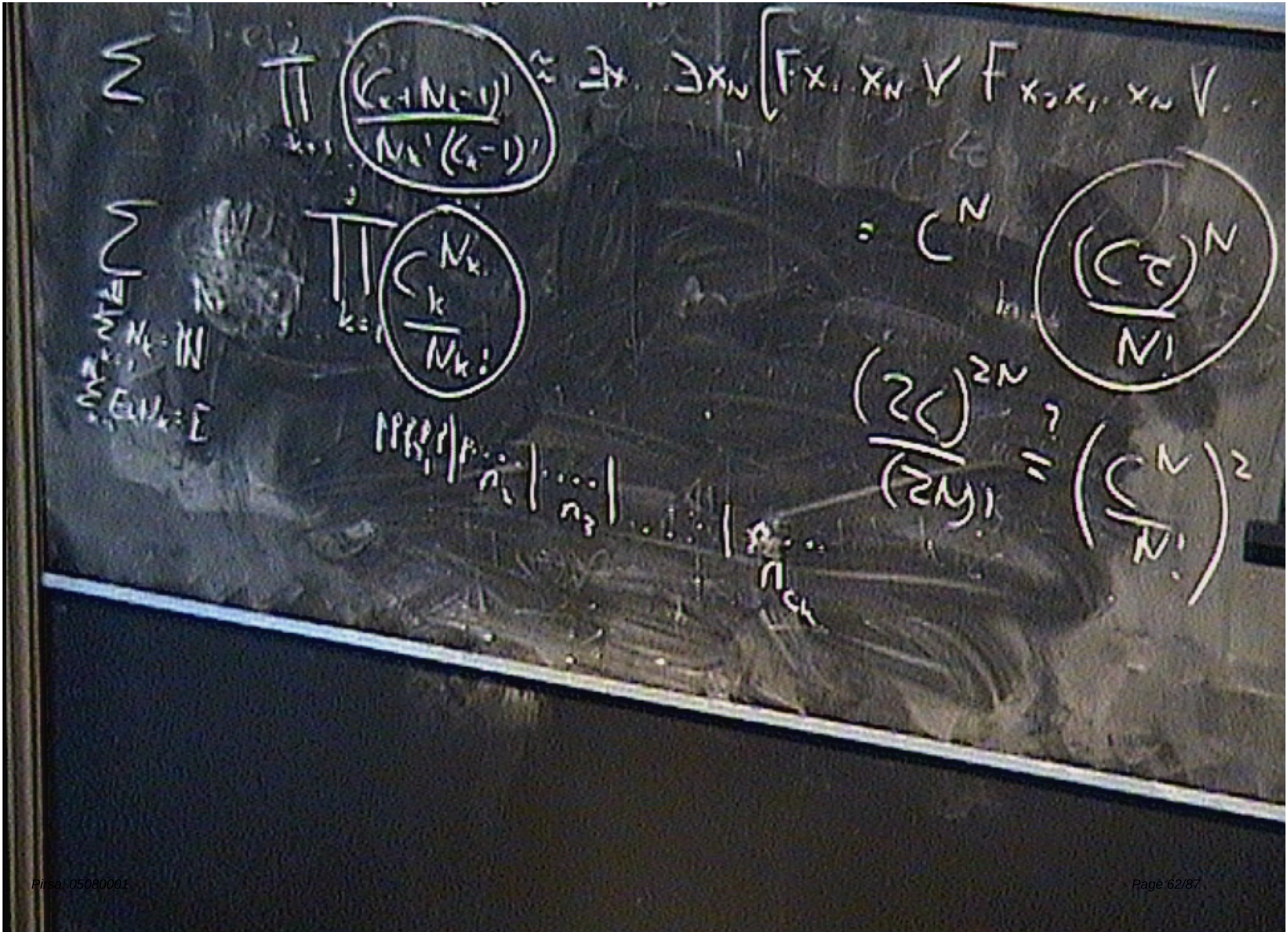
$$\frac{(2C)^{2N}}{(2N)!} \overset{?}{=} \left( \frac{C^N}{N!} \right)^2$$







= C^N
(2c)^2N / (2N)! =? (C^N / N!)^2



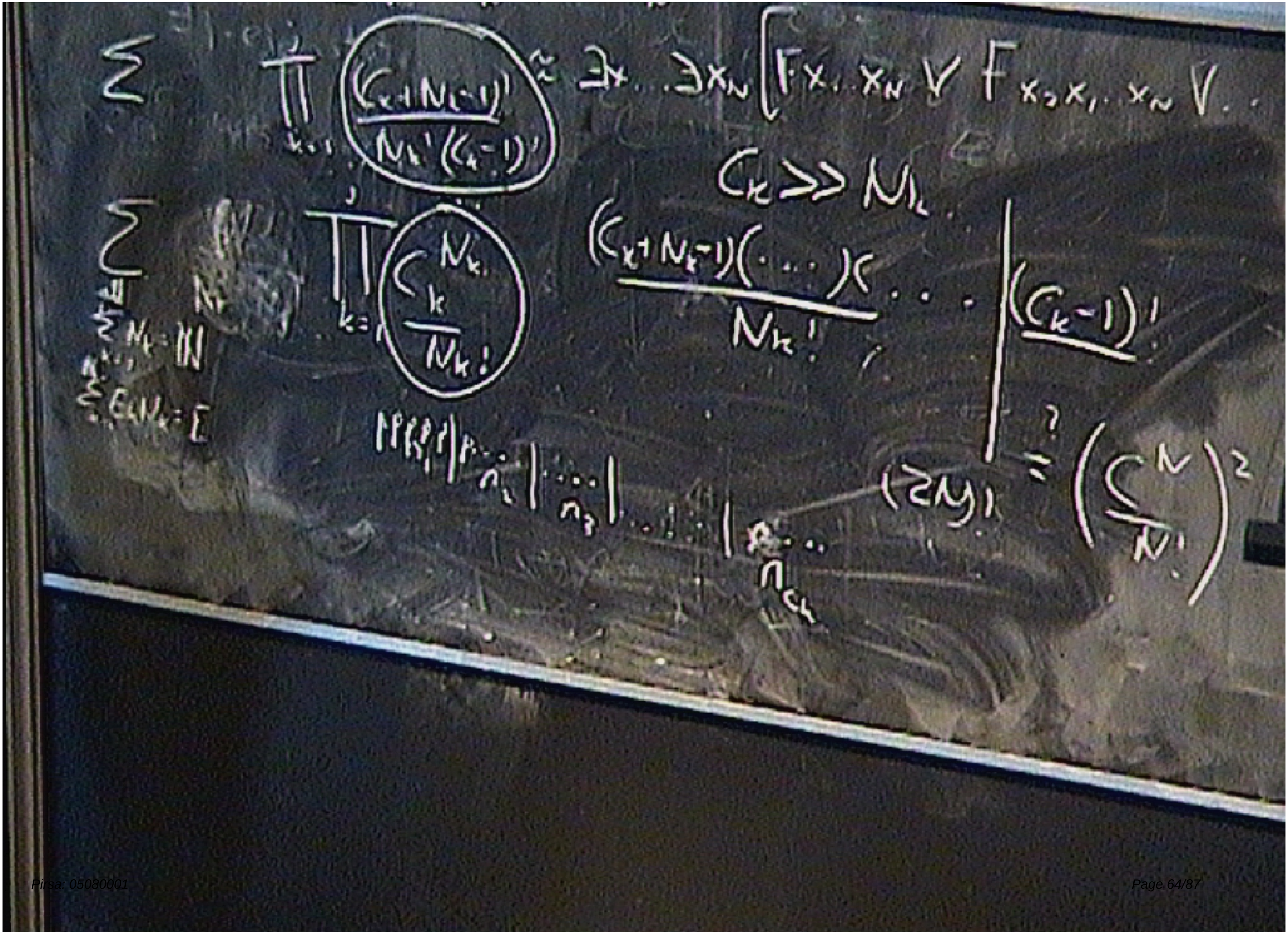
$$\sum \prod_{k=1} \frac{(C_k+N_k-1)!}{N_k!\,(C_k-1)!} \simeq \exists x_1 \ldots \exists x_N \left[F x_1 \ldots x_N \vee F x_2 x_1 \ldots x_N \vee \ldots\right.$$

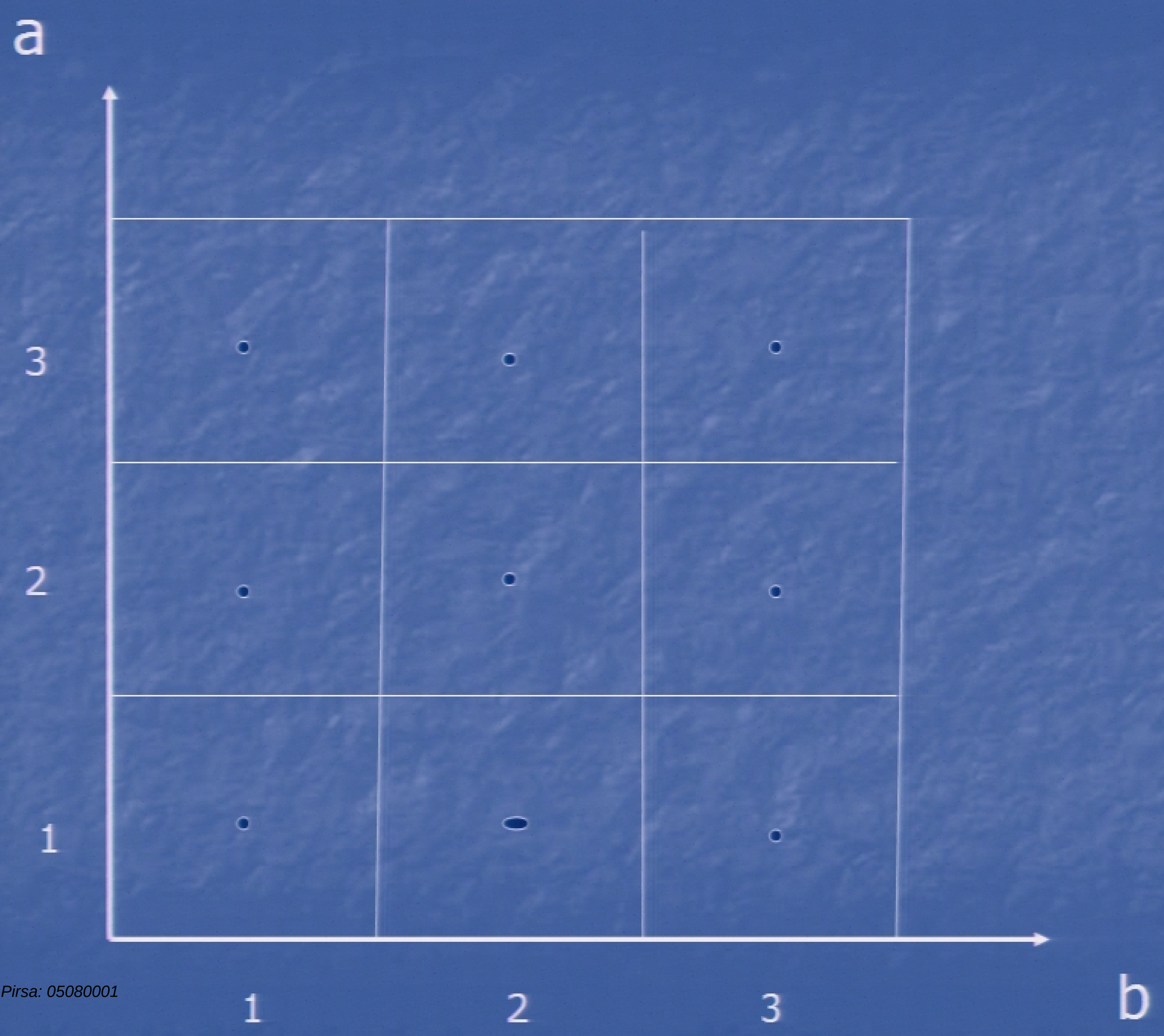
$$C_k >> N_k$$

$$\sum_{\substack{\sum N_k = N \\ \sum \epsilon_k N_k = E}} \prod_{k=1} \frac{C_k^{N_k}}{N_k!}$$

$$|||\ldots|\; n_1 \;|\; \ldots \;|\; n_2 \;|\; \ldots \;|\; n_{C_k} \ldots$$

$$(2N)! \overset{?}{=} \left(\frac{C^N}{N!}\right)^2$$

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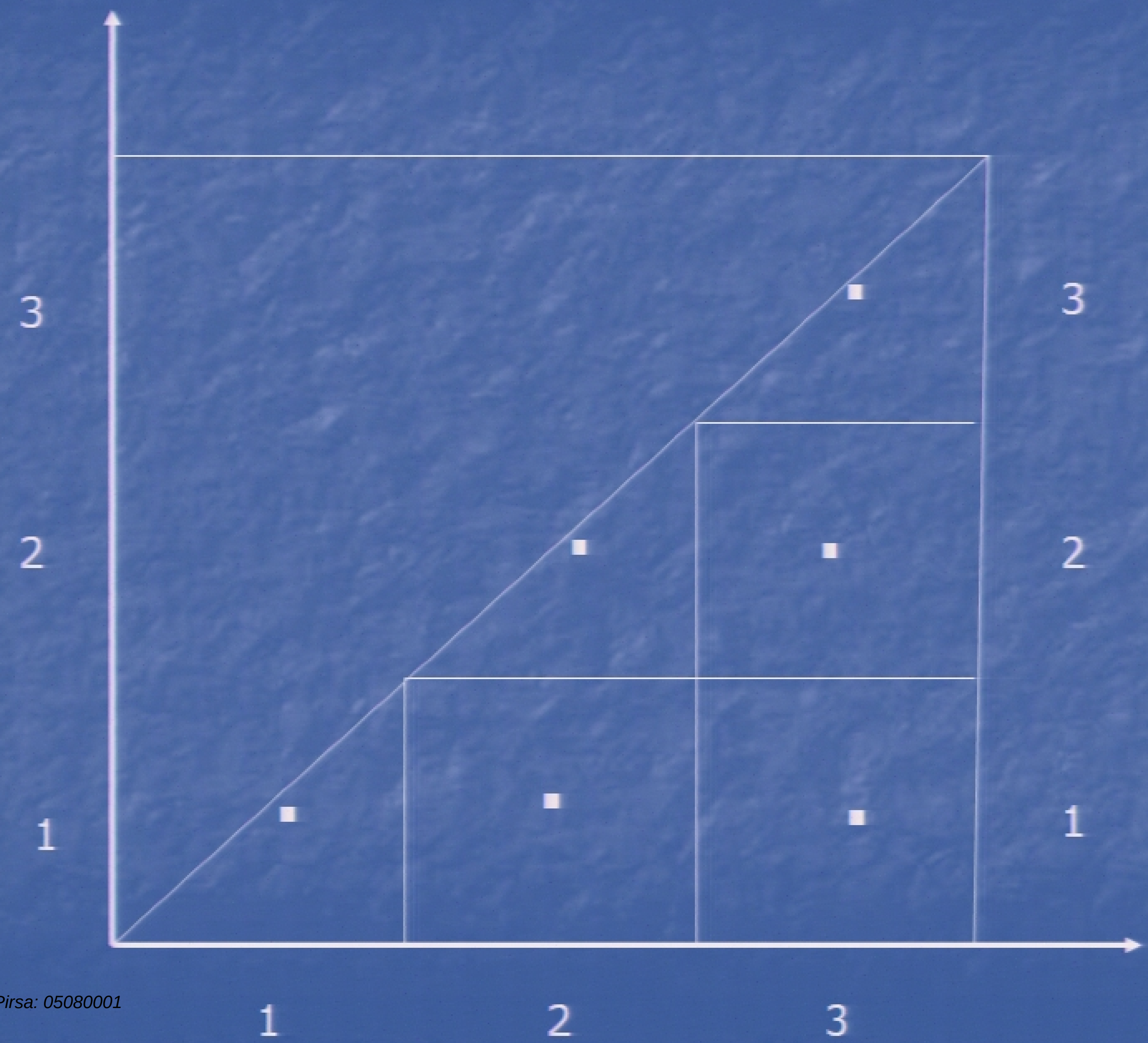

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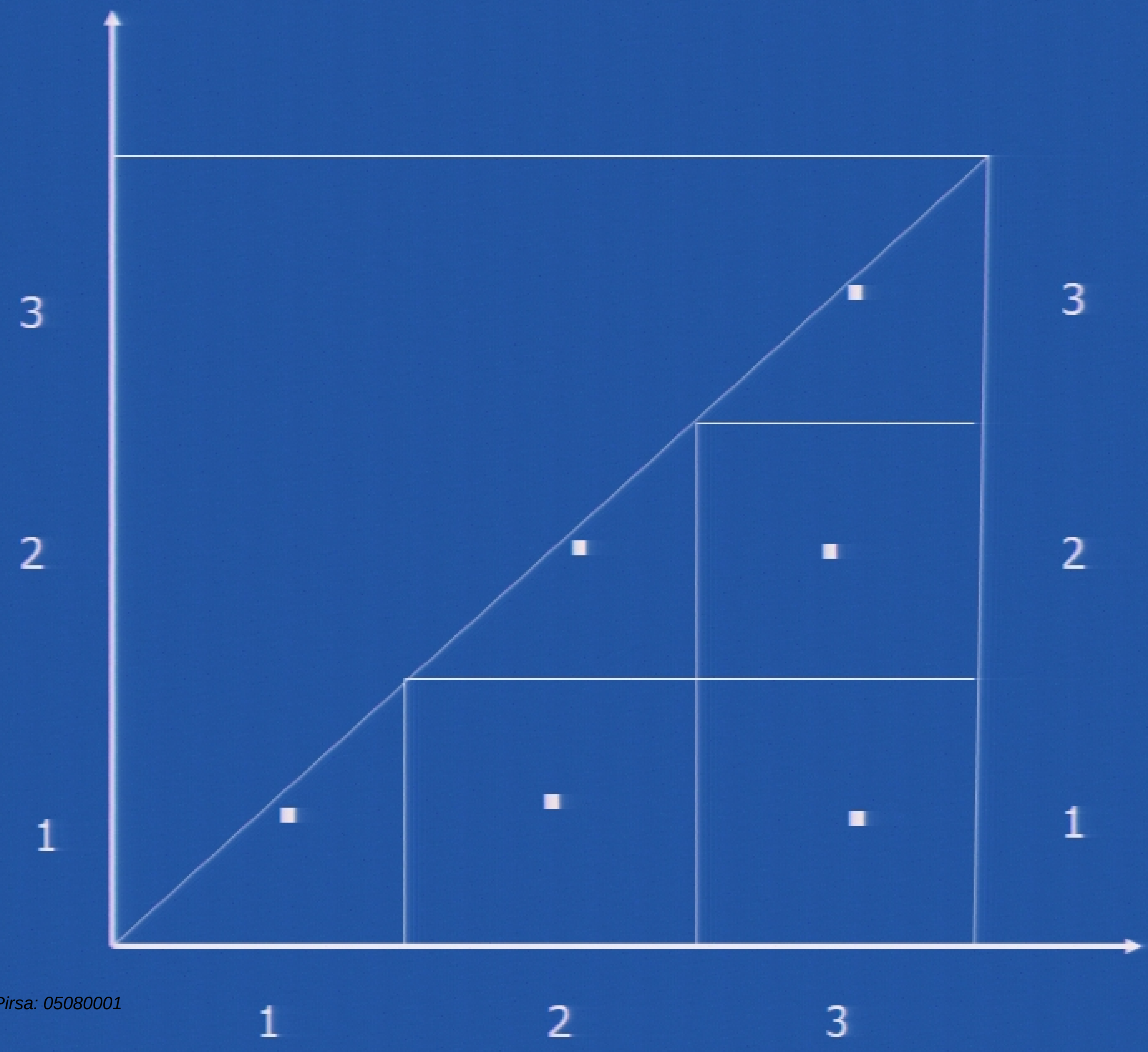
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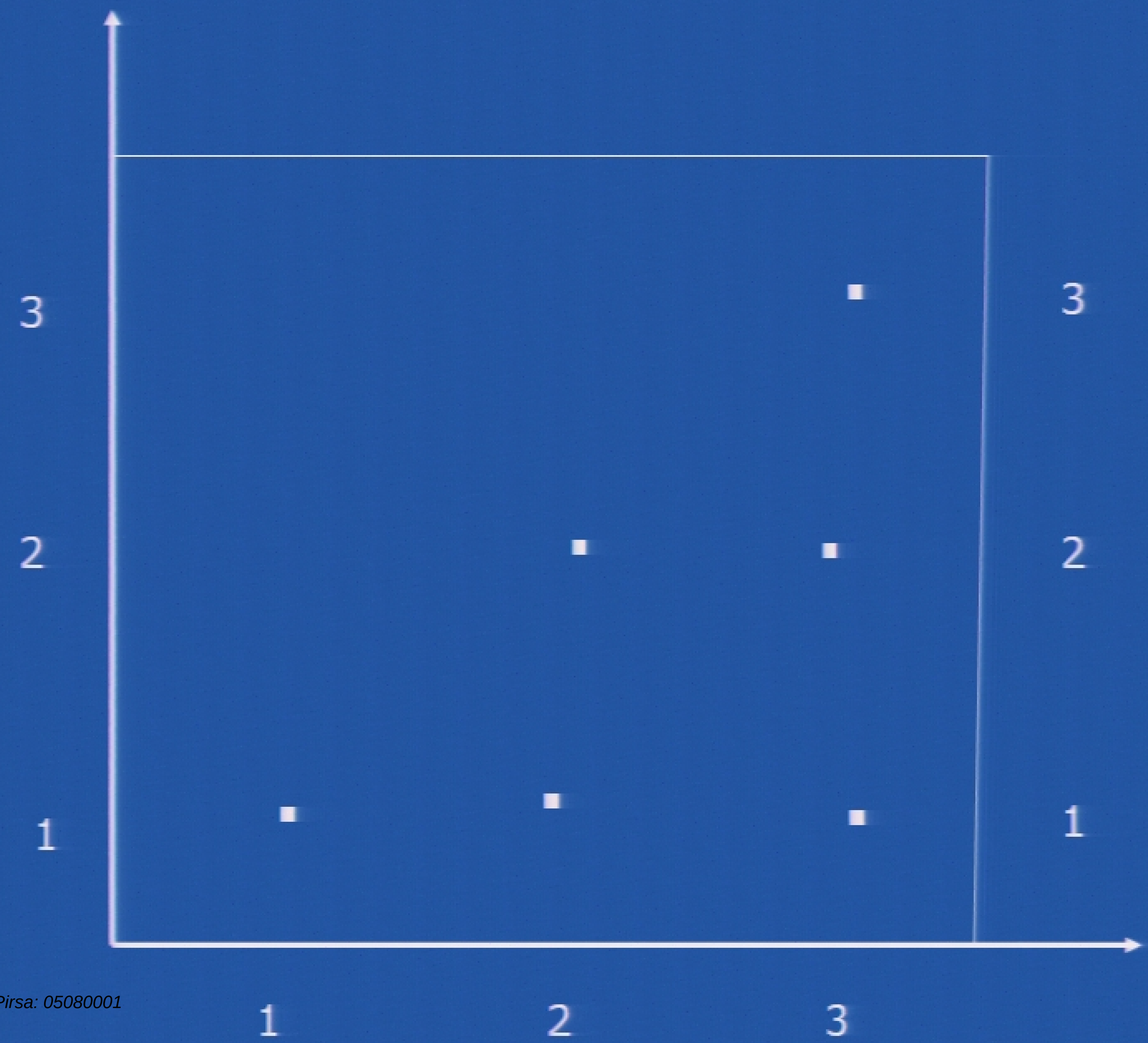
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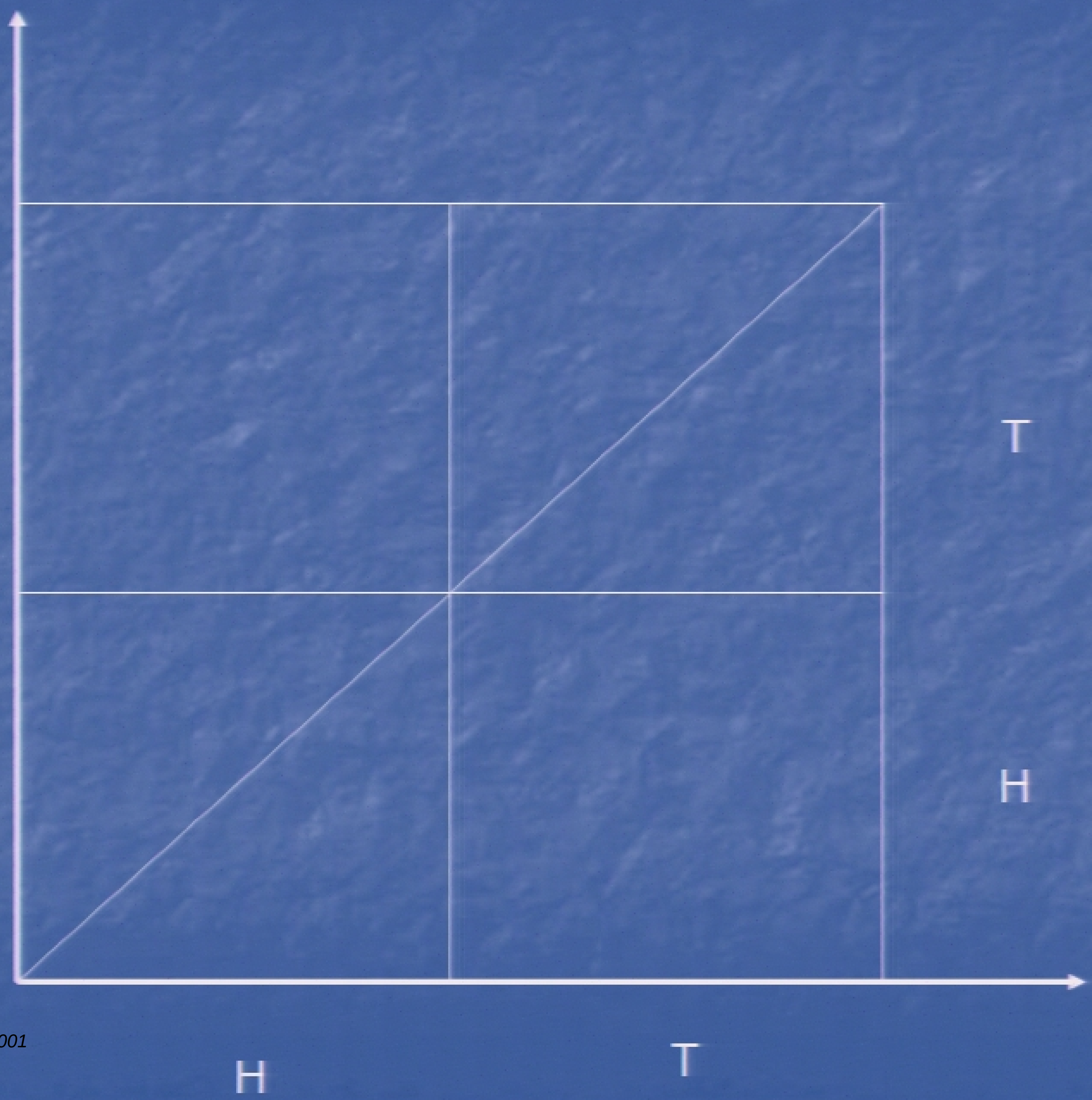
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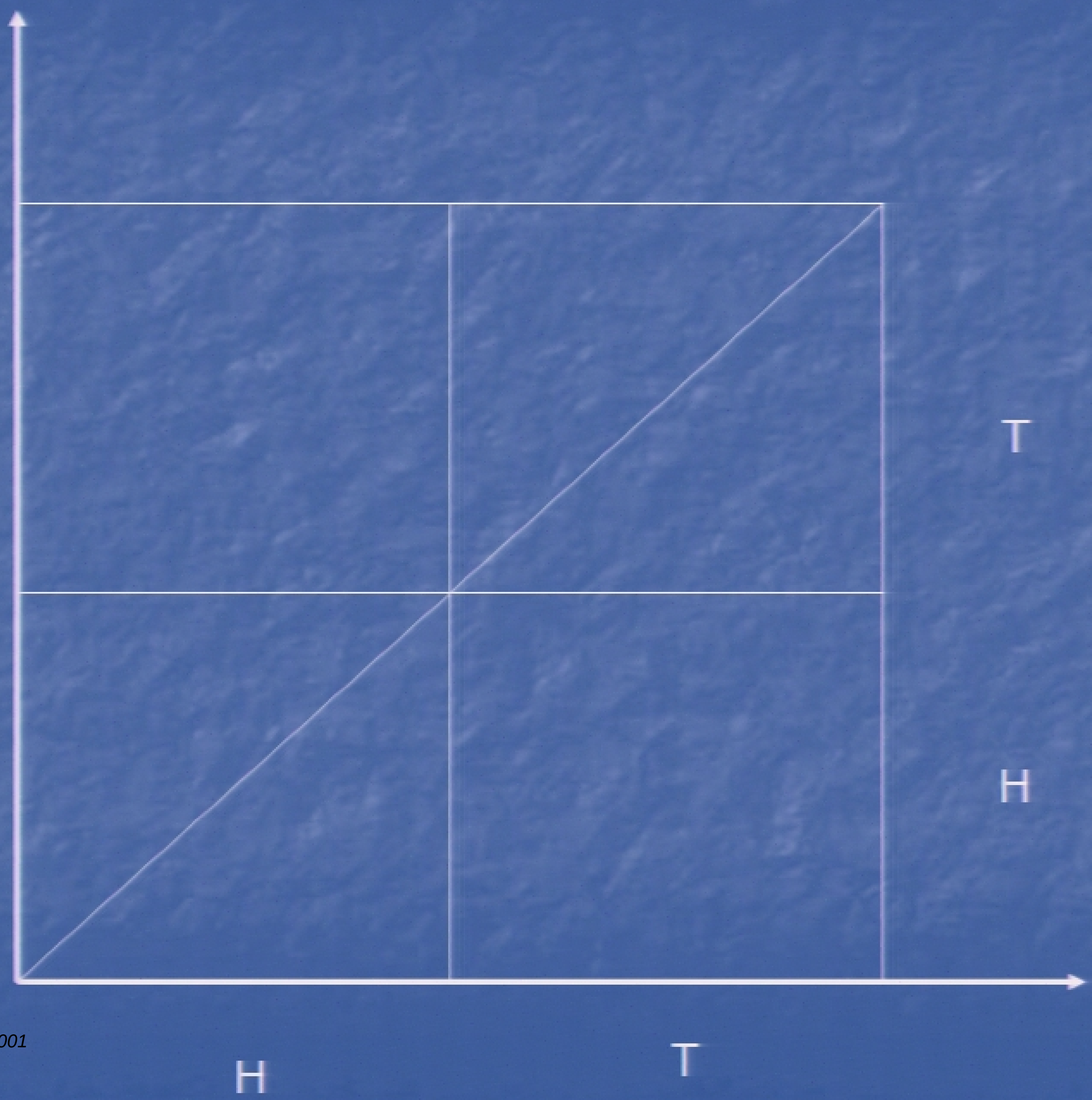
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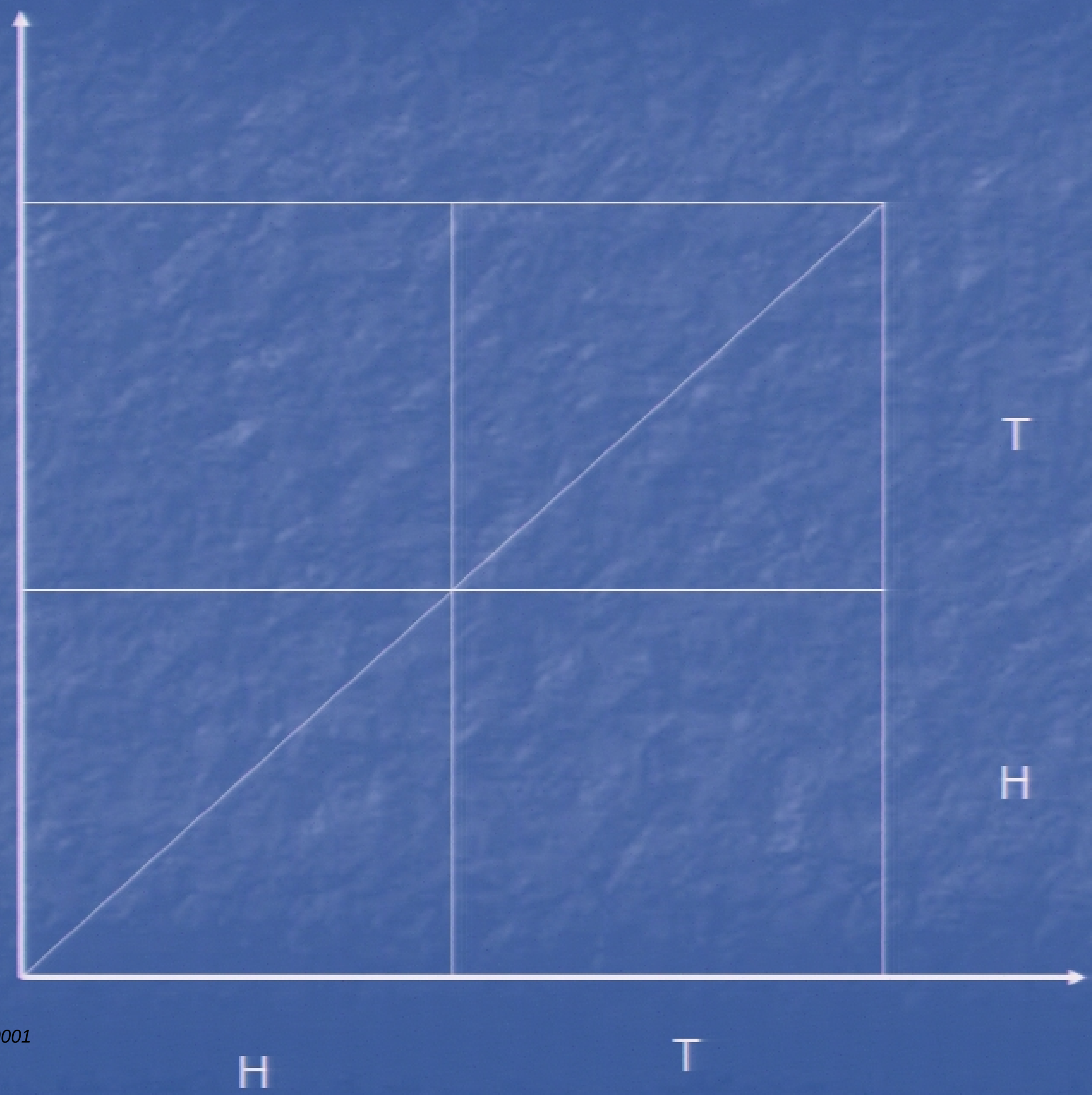
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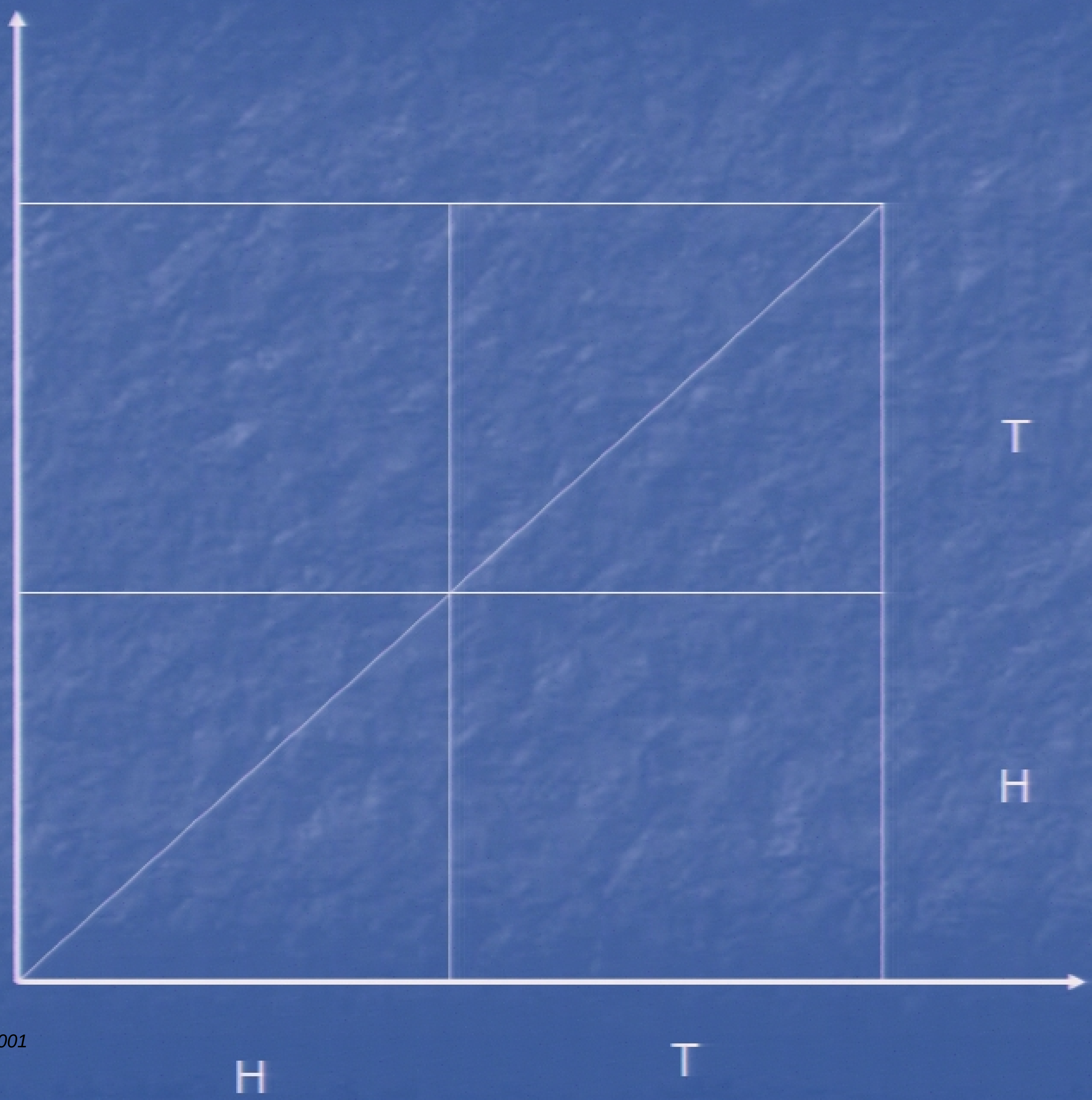
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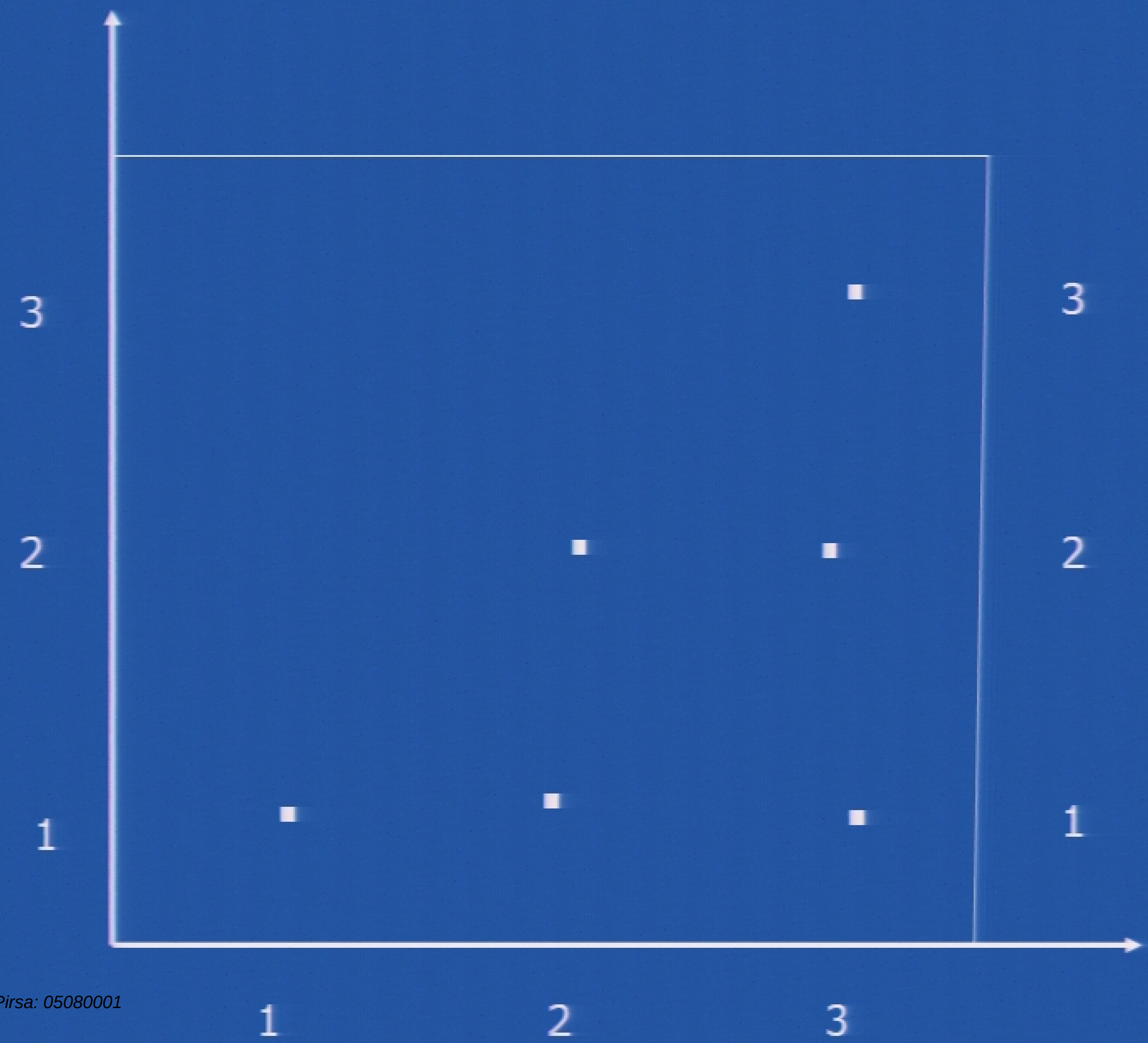
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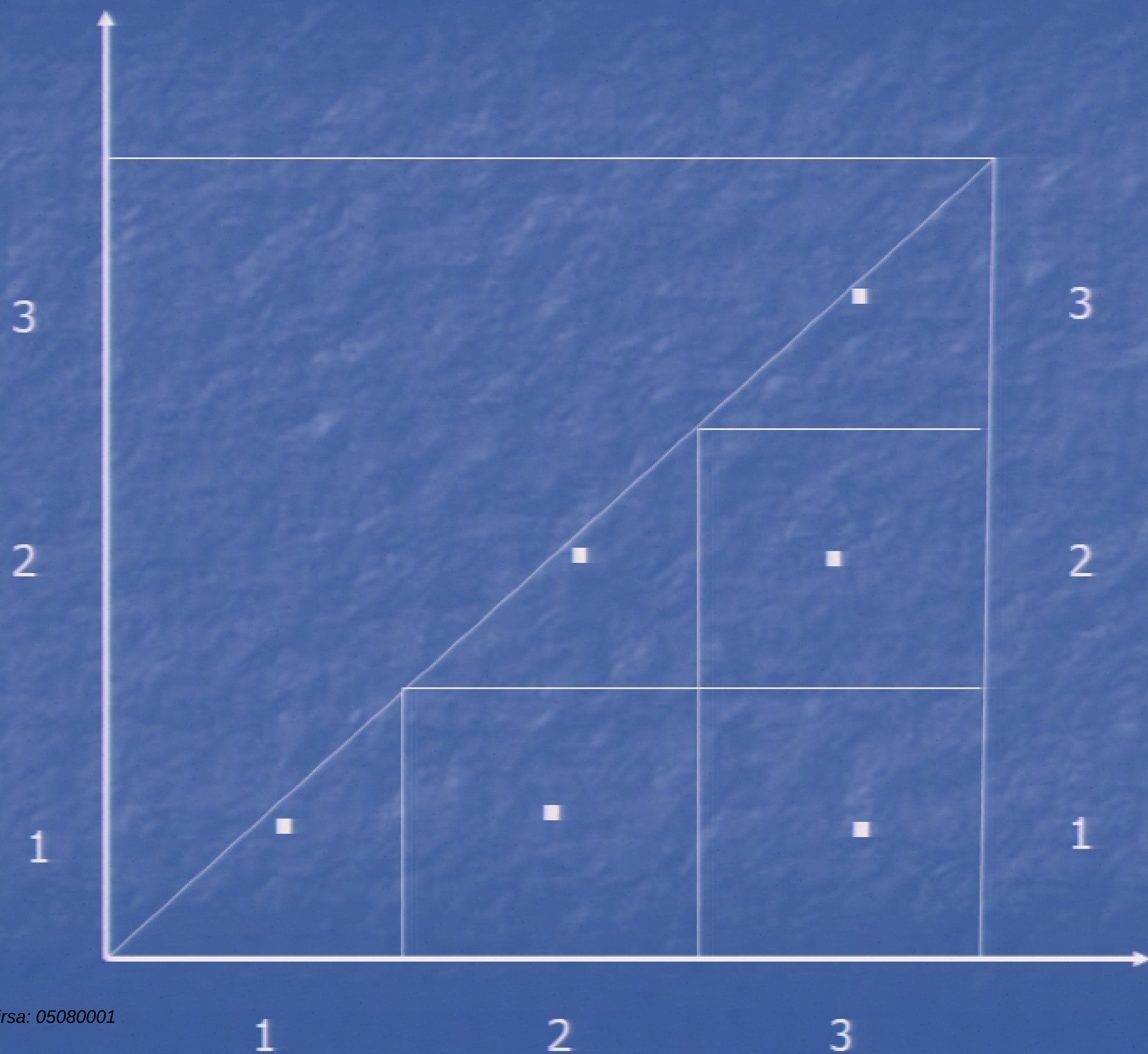
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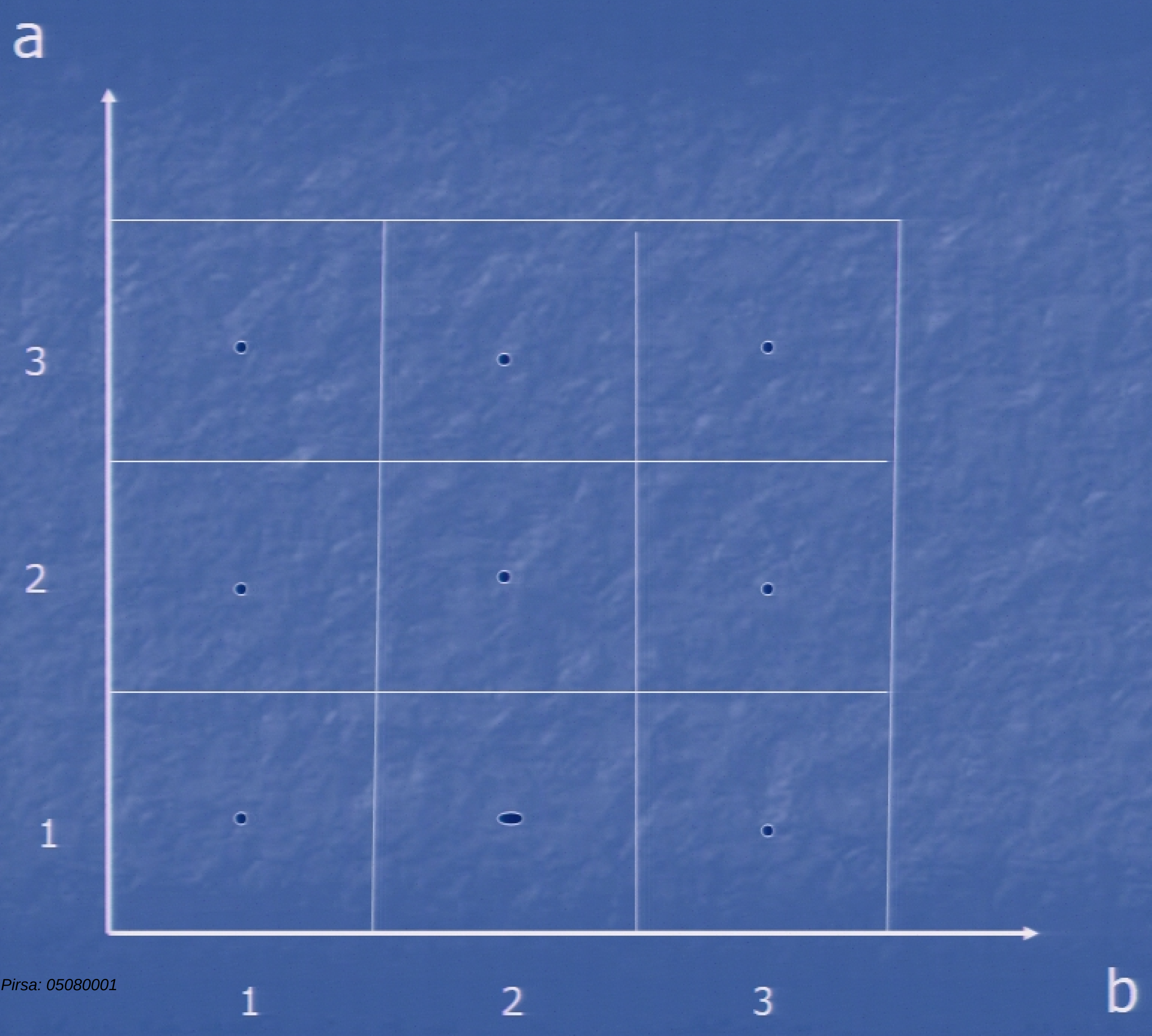
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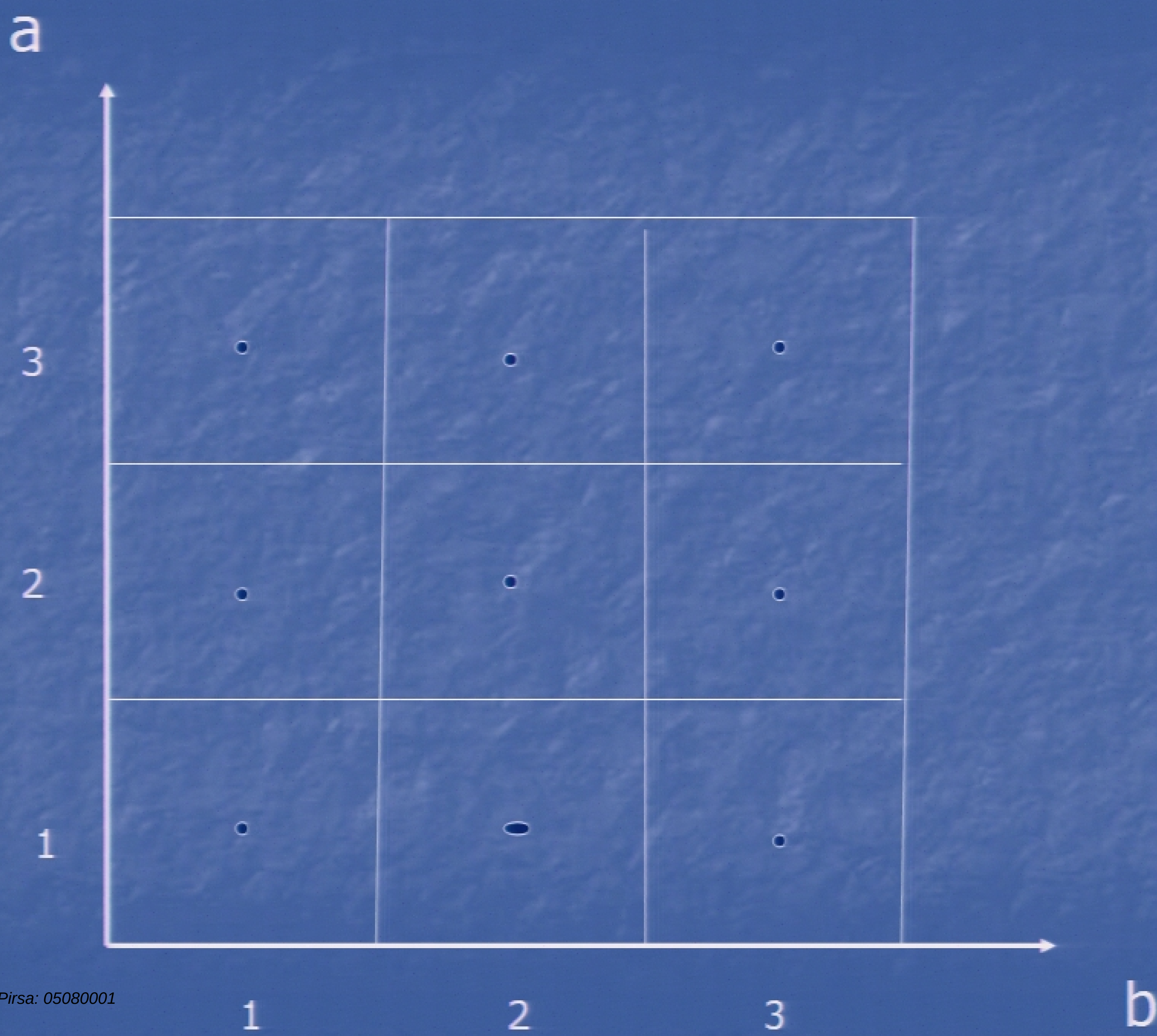
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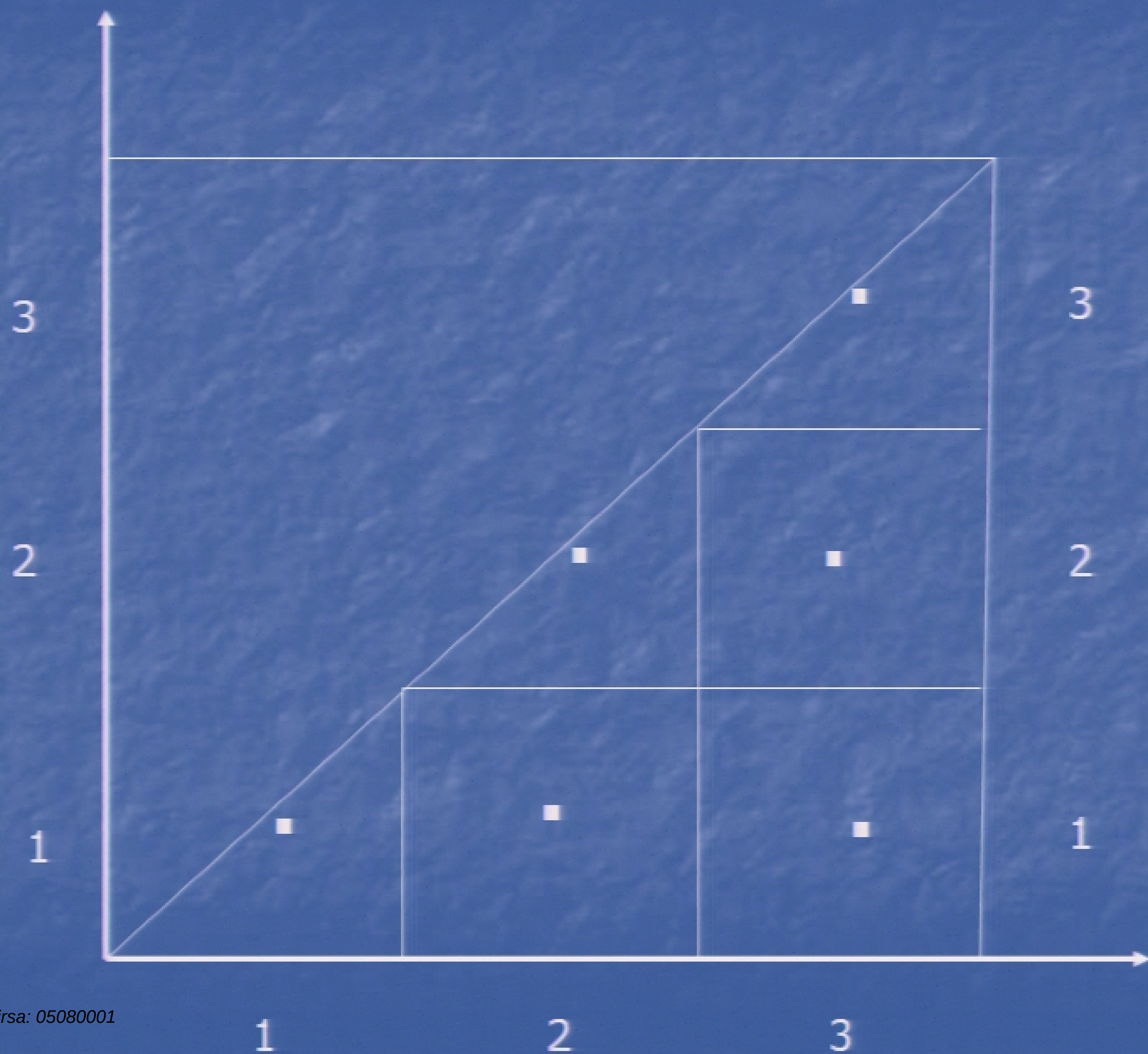
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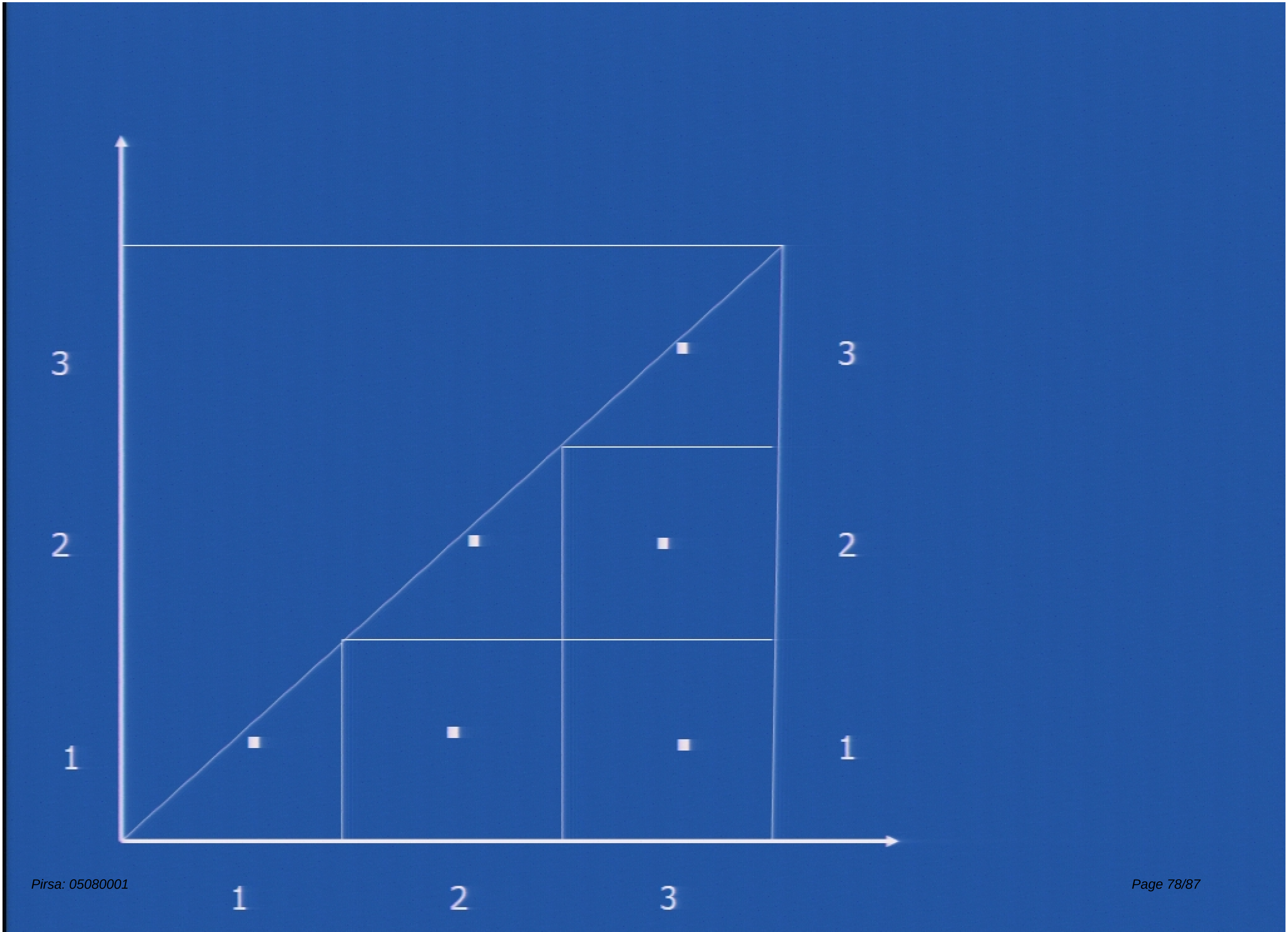

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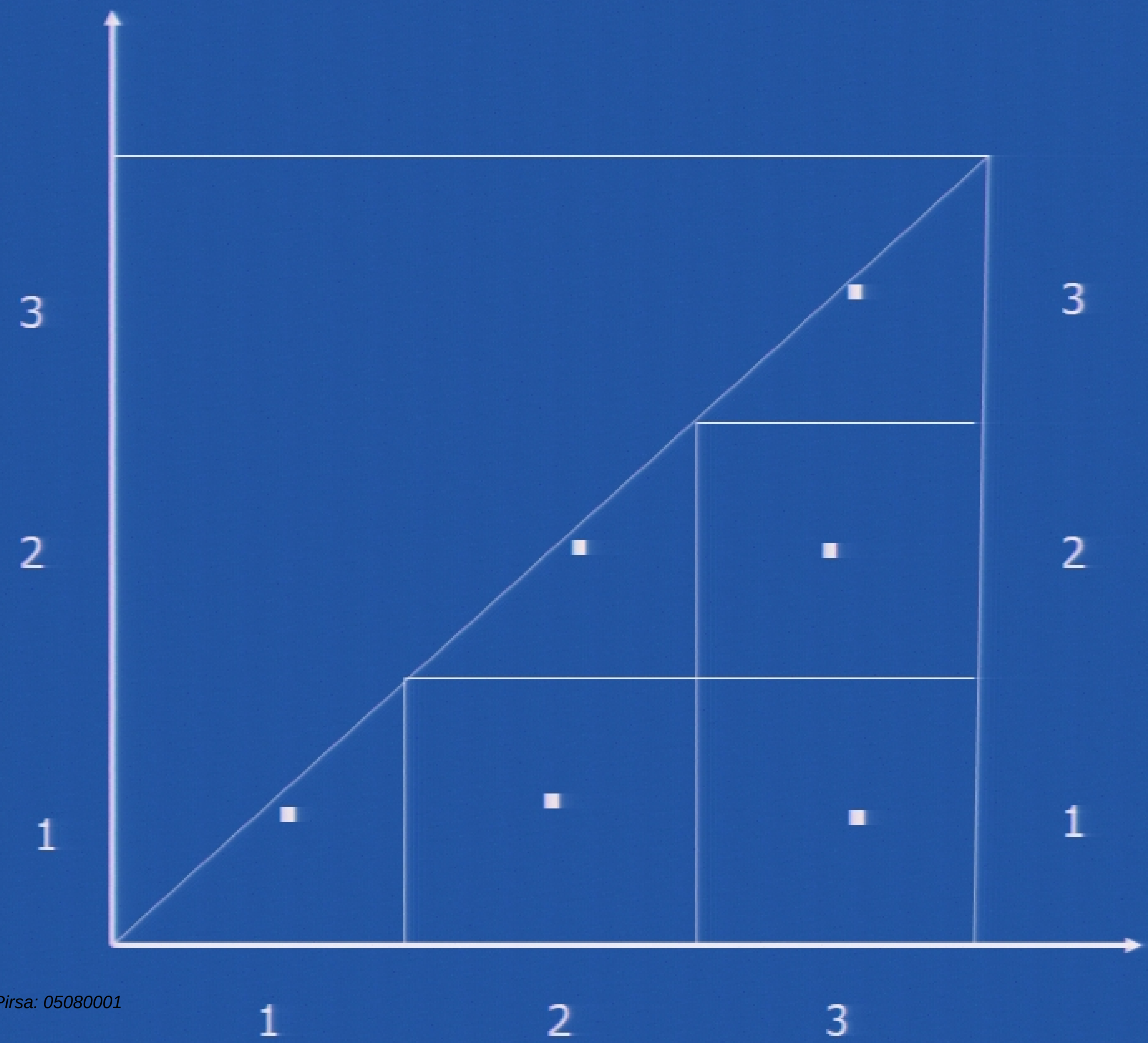
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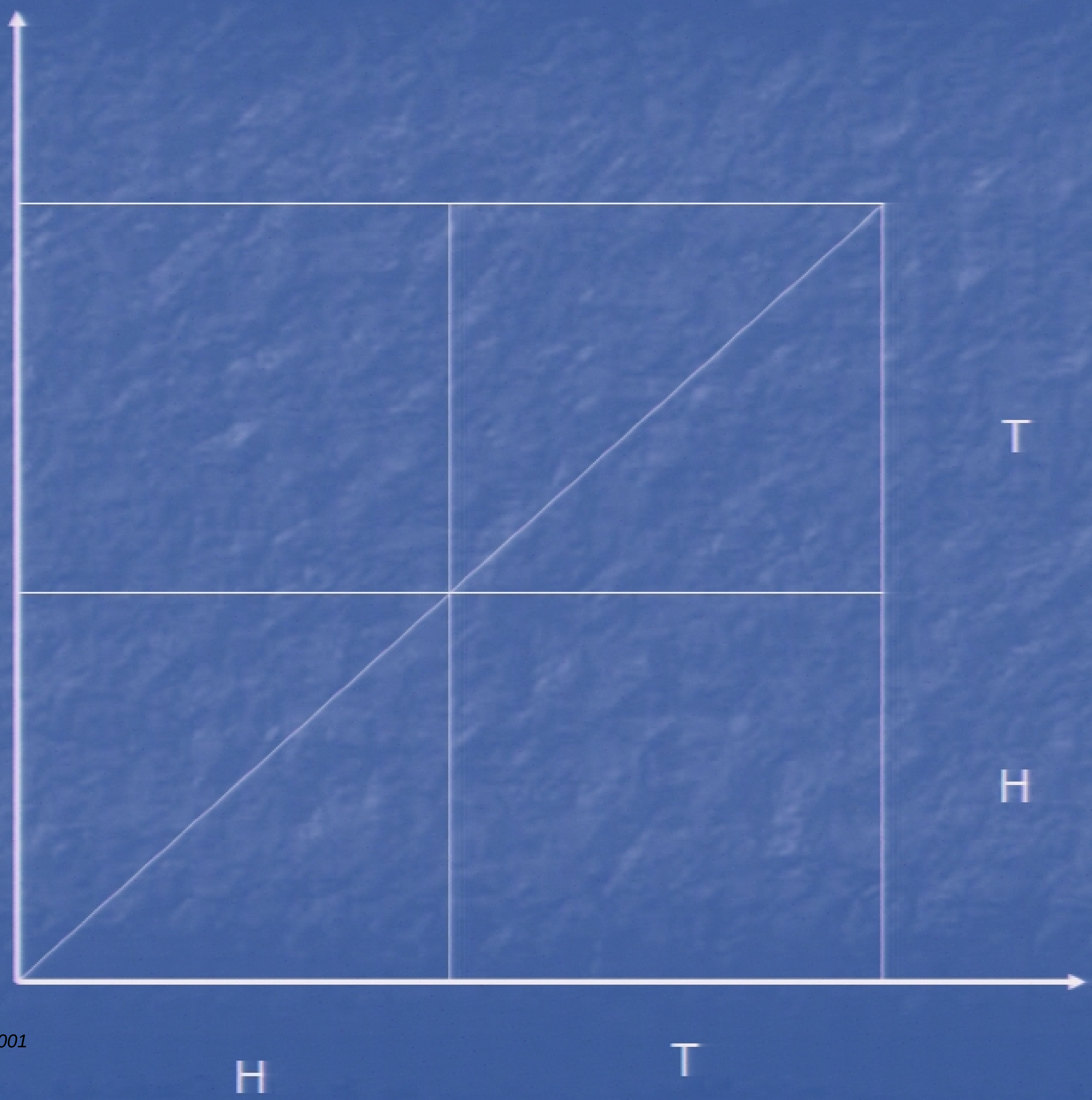
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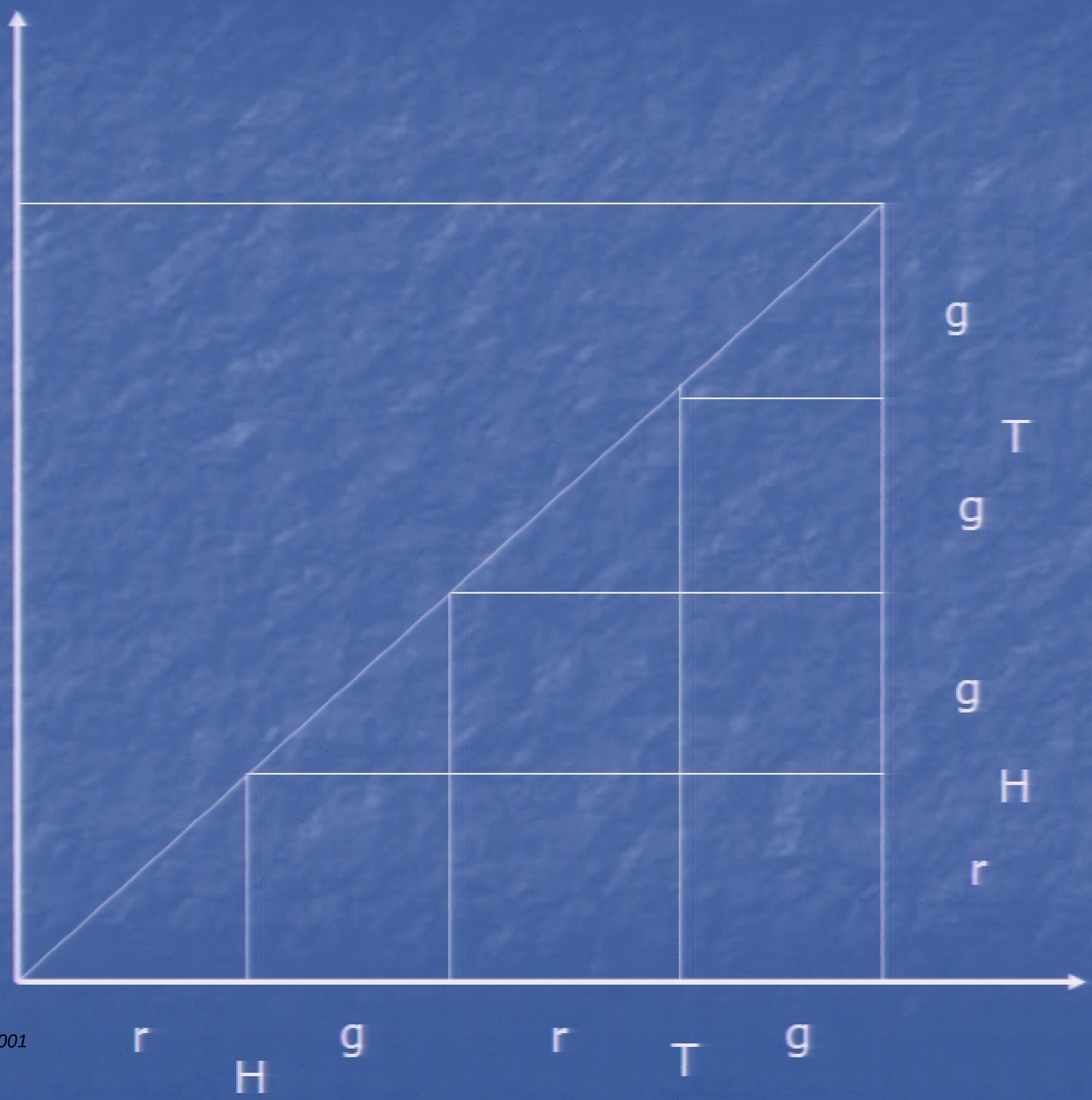
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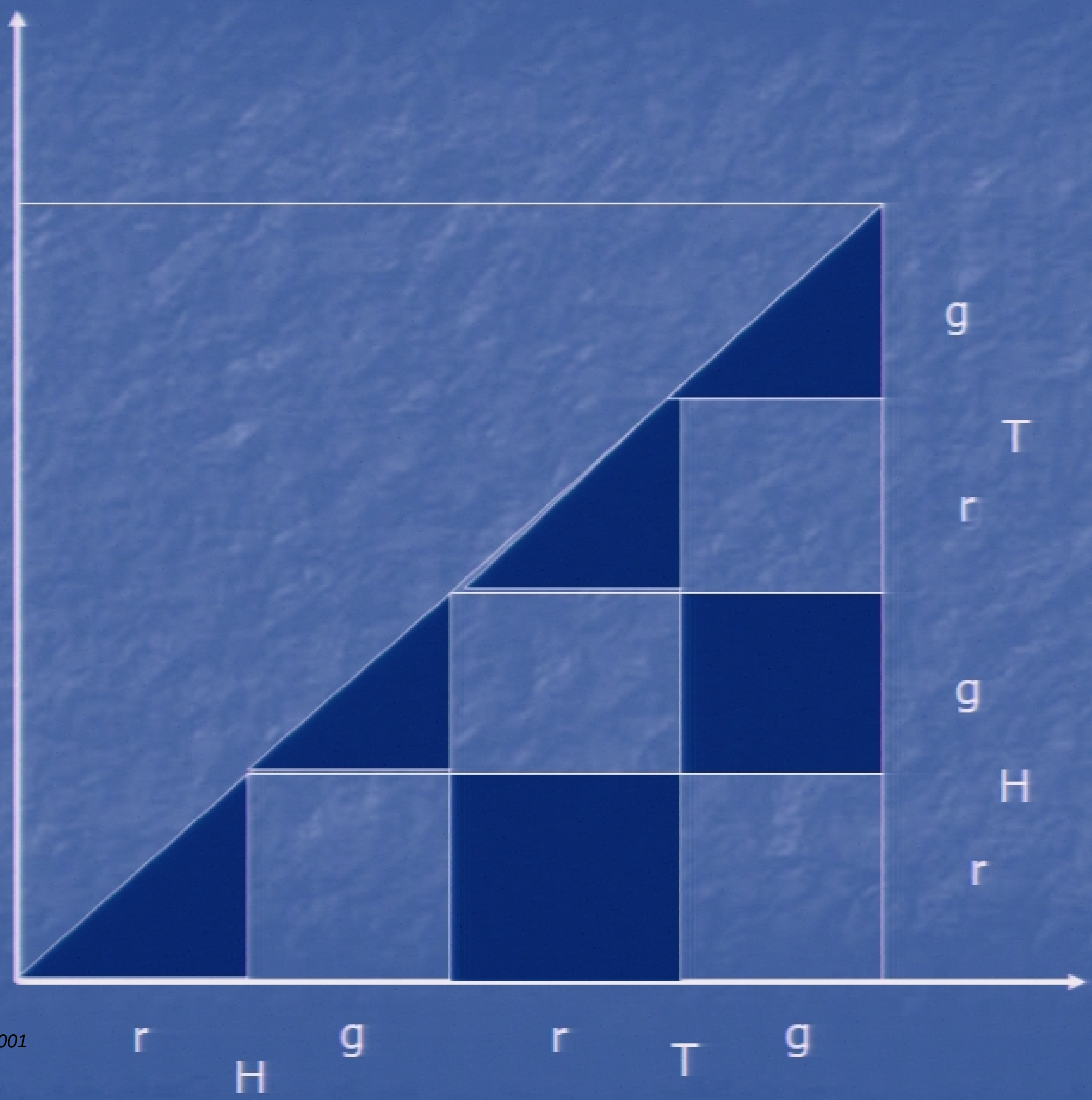
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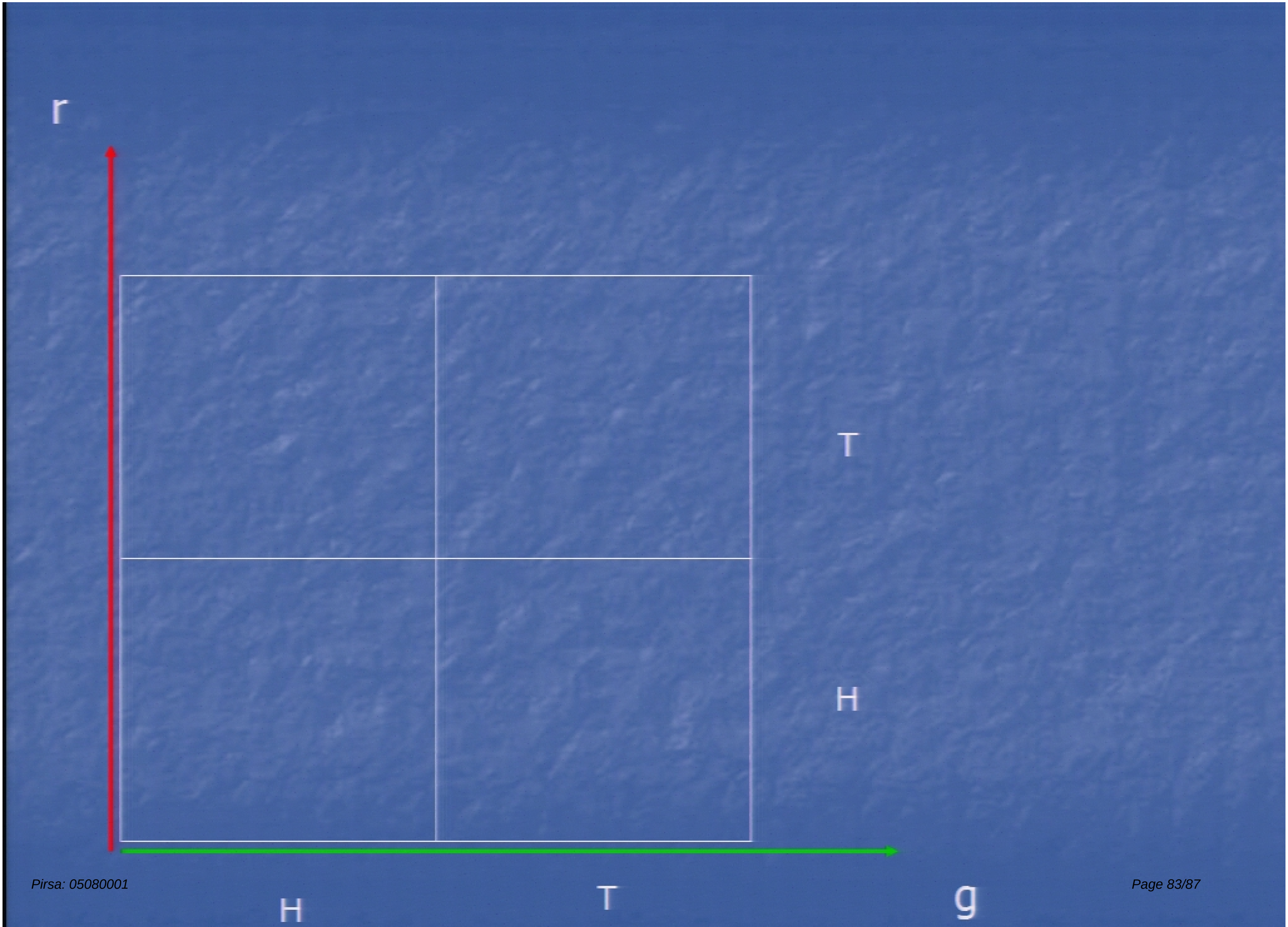
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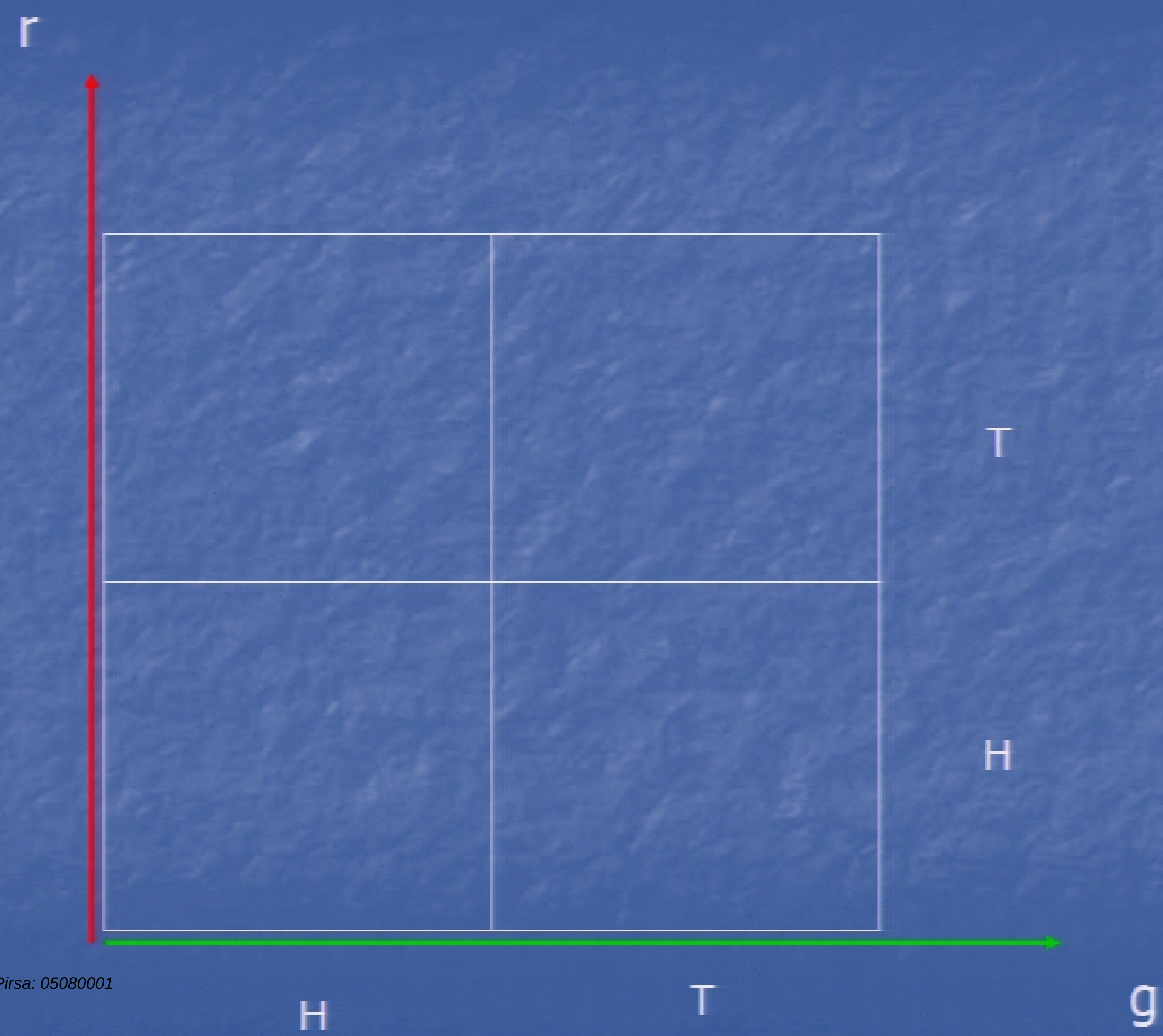
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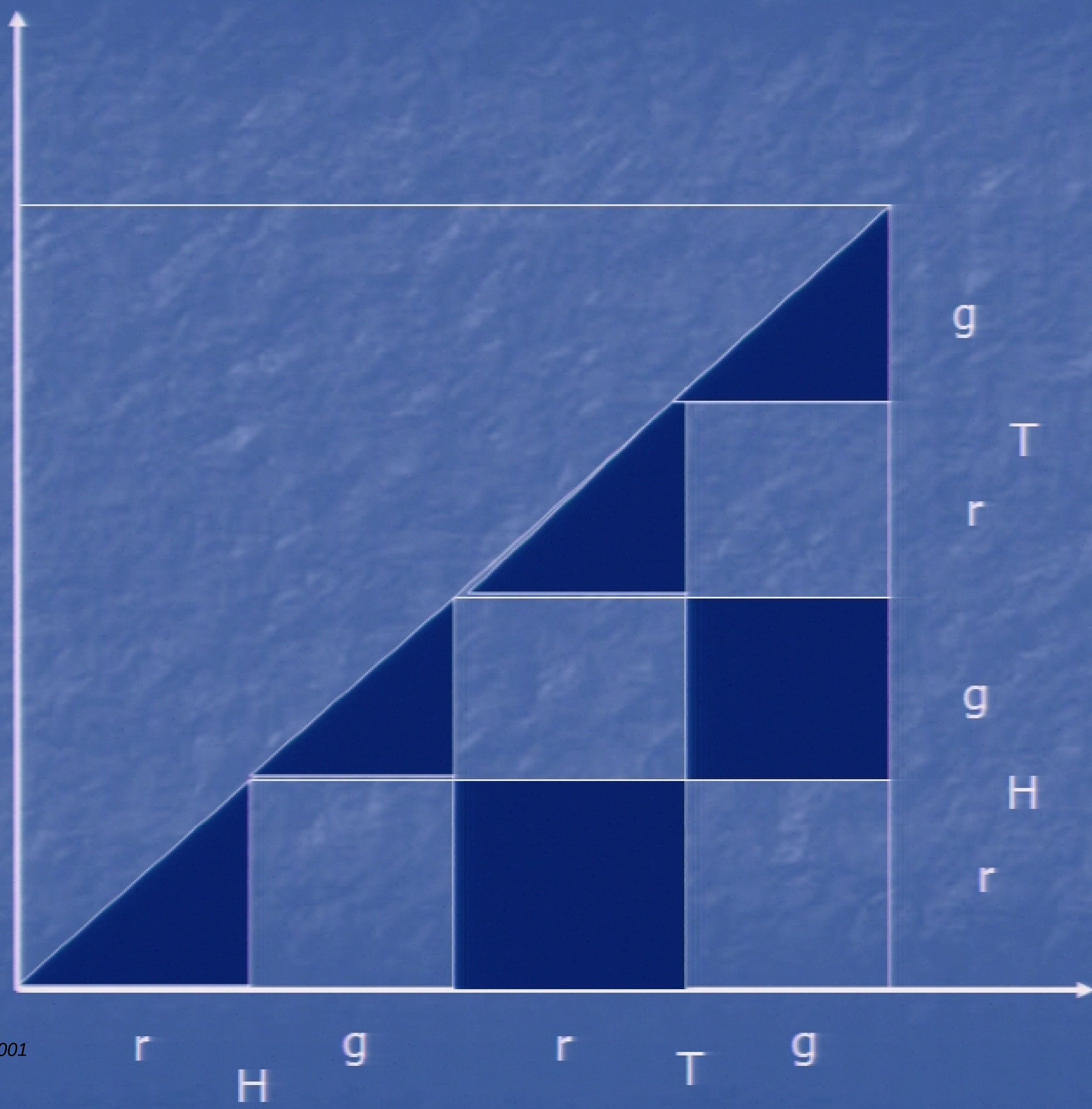
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# Conclusions:

- Indistinguishabilty (permutativity) classically innocuous
- Desirable to give extensive entropy
- Quantum statistics a consequence of (i) indistinguishability and (ii) replacement of continuous state space measure by one concentrated on points.

# Or, the finished jig-saw -:)

- Indistinguishabilty (permutativity) classically innocuous
- Desirable to give extensive entropy
- Quantum statistics a consequence of (i) indistinguishability and (ii) replacement of continuous state space measure by one concentrated on points.