Title: The Frequency Operator in Quantum Mechanics.

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Abstract:

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The Frequency Operator in Quantum Mechanics

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1. Overview
2. Frequency operator: finite N
3. Finkelstein's theorem
4. F^∞: naïve attempts
5. Laws of large numbers
6. Constructing $\mathcal{H}^{\otimes\infty}$
7. F^∞: Gutmann 1995 approach
8. F^∞: Farhi, Goldstone and Gutmann 1989
9. Status of F^∞

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Frequency

Outcomes of $N = 40$ repeated measurements of an observable B:

3910239109234723462619281911921923199911

The outcome 1 occurs $n = 10$ times.

The frequency of 1 is $f = n/N = 1/4$.

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Frequency operator

Observable B defined on a Hilbert space $\mathcal{H}$.

A particular outcome, labeled 1.

On $\mathcal{H}^{\otimes N}$, define an observable F^N with eigenvalues $f = n/N$, where n counts how often 1 occurs in N measurements of B.

> Define F^∞.
>
> Prove that $|\Psi_\infty\rangle = |\psi\rangle \otimes |\psi\rangle \otimes \cdots$ is an eigenvector of F^∞ with eigenvalue $q = |\langle 1|\psi\rangle|^2$.

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Quantum Probability Postulate

QPP: Let $B = \sum_\lambda \lambda P_\lambda$ be an observable, where λ denotes the different eigenvalues of B and P_λ are orthogonal projectors onto the eigenspaces of B. If B is measured for a system in state $|\psi\rangle$, the probability of outcome λ is $||P_\lambda|\psi\rangle||^2$.

Postulate of Definite Outcomes (QPP$'$): If an observable B is measured for a system in an eigenstate $|\psi\rangle$ of B, i.e., $B|\psi\rangle = \lambda|\psi\rangle$, the outcome is λ with certainty.

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Farhi, Goldstone and Gutmann 1989

"Without considering a limit of frequency operators F^N, we will define a frequency operator, F^∞, and we will prove

$$F^\infty|\Psi_\infty\rangle = q|\Psi_\infty\rangle\ .$$

We can then apply QPP′ to this exact eigenvector equation and the probabilistic interpretation follows."

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Frequency operator: finite N

Observable B: $B|B,j\rangle = \lambda_j|B,j\rangle$

($j = 1, \ldots, D$ if $D > 2$, and $j = 0, 1$ if $D = 2$)

$P_1 = |B,1\rangle\langle B,1|$
$P_0 = 1 - P_1$

Outcomes $j_1, \ldots, j_N$ (eigenvalues $\lambda_{j_1}, \ldots, \lambda_{j_N}$)

Frequency of outcome 1: $f = \frac{1}{N}\sum_{r=1}^{N} \delta_{1 j_r}$

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F^N: Hartle version

Projector on all sequences of N outcomes giving rise to frequency $f = n/N$:

$$\Pi_n^N = \sum_{k_1,\ldots,k_N \in \{0,1\}} P^1_{k_1} \otimes \cdots \otimes P^N_{k_N}\, \delta\left(n, \sum_{r=1}^{N} k_r\right) \qquad (1)$$

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Finkelstein's theorem

Let $|\Psi_N\rangle = |\psi\rangle^{\otimes N} \equiv |\psi\rangle \otimes \cdots \otimes |\psi\rangle$

Then there is a unique number q such that

$$\lim_{N\to\infty} ||F^N|\Psi_N\rangle - q|\Psi_N\rangle|| \equiv \lim \Delta = 0 \,,$$

namely $q = |\langle B, 1|\psi\rangle|^2$.

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namely $q = |\langle B, 1|\psi\rangle|^2$.

> Finkelstein: "[...] the ensemble $|\Psi_N\rangle$ is nearly an eigenstate of the mean F^N, as measured by the error Δ."

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F^{∞} : naïve attempt

Let $|\Psi_\infty\rangle = |\psi\rangle^{\otimes\infty} \equiv |\psi\rangle \otimes |\psi\rangle \otimes \cdots$

Then define $F^{\infty}|\Psi_\infty\rangle = \lim_{N\to\infty} \left(F^N|\Psi_\infty\rangle\right)$

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Assuming QPP . . .

$$\Pr\left(f = \frac{n}{N}\right) = ||\Pi_n^N|\Psi_N\rangle||^2 = \binom{N}{n} q^n (1-q)^{N-n}.$$

and thus (Finkelstein's theorem)

$$E\left((f-q)^2\right) \to 0 \quad \text{as} \quad N \to \infty$$

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Gleason's theorem

Assume there is a function h from the one-dimensional projectors acting on a Hilbert space of dimension greater than 2 to the unit interval, with the property that for each orthonormal basis $\{|\psi_k\rangle\}$,

$$\sum_k h\big(|\psi_k\rangle\langle\psi_k|\big) = 1 \; .$$

Then there exists a density operator ρ such that

$$h\big(|\psi\rangle\langle\psi|\big) = \langle\psi|\rho|\psi\rangle \; .$$

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Assuming QPP . . .

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Strong law of large numbers

The probability laws $\Pr\left(f = \frac{n}{N}\right)$ for $N = 1, 2, \ldots$ determine a measure on the infinite sequences of outcomes.

Define f^∞ to be the frequency of outcome 1.

Strong law of large numbers:

$$\boxed{f^\infty = q \quad \text{almost certainly.}}$$

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Assuming QPP ...

$$\Pr\left(f = \frac{n}{N}\right) = ||\Pi_n^N|\Psi_N\rangle||^2 = \binom{N}{n} q^n (1-q)^{N-n}.$$

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$\mathcal{H}^{\otimes\infty}$

The states $|B, j\rangle$ form an orthonormal basis for $\mathcal{H}$.

The states $|B, j_1\rangle \otimes \cdots \otimes |B, j_N\rangle$

form an orthonormal basis for $\mathcal{H}^{\otimes N}$.

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Constructing $\mathcal{H}^{\otimes\infty}$

Notation: $\{\psi\} = |\psi_1\rangle, |\psi_2\rangle, \ldots$

$|\{\psi\}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots$

Equivalence classes of sequences: $\{\phi\} \sim \{\psi\}$ iff there exists $N \geq 1$ such that

$$\prod_{r=N}^{\infty} |\langle\phi_r|\psi_r\rangle| > 0 \, .$$

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Components

The *component* $\mathcal{H}^{\otimes\infty}_{\{\psi\}}$ is the subspace spanned by all $\{\phi\} \sim \{\psi\}$.

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Countable basis of $\mathcal{H}_{\{\psi\}}^{\otimes\infty}$

$\{\psi\} = |\psi_1\rangle, |\psi_2\rangle, \ldots$

For each vector $|\psi_r\rangle$ in the sequence, choose an orthonormal basis $|\psi_r, 0\rangle, \ldots, |\psi_r, D-1\rangle$ such that $|\psi_r, 0\rangle = |\psi_r\rangle$.

$$\mathcal{H}\otimes\mathcal{H}\otimes\cdots$$

$$|\psi_1\rangle\otimes|\psi_2\rangle\otimes\cdots$$

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Countable basis of $\mathcal{H}_{\{\psi\}}^{\otimes\infty}$

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For each vector $|\psi_r\rangle$ in the sequence, choose an orthonormal basis $|\psi_r, 0\rangle, \ldots, |\psi_r, D-1\rangle$ such that $|\psi_r, 0\rangle = |\psi_r\rangle$.

Now define the sequences $\{i\} = i_1, i_2, \ldots$ $(0 \leq i_k \leq D-1)$ to be those with a *finite* number of nonzero terms.

The vectors

$$|\psi;\{i\}\rangle = |\psi_1, i_1\rangle \otimes |\psi_2, i_2\rangle \otimes \cdots$$

span $\mathcal{H}_{\{\psi\}}^{\otimes\infty}$.

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Countable basis of $\mathcal{H}^{\otimes\infty}_{\{v\}}$

$\{v\} = |v_1\rangle, |v_2\rangle, \ldots$

For each vector $|v_r\rangle$ in the sequence, choose an orthonormal basis $|v_r, 0\rangle, \ldots, |v_r, D-1\rangle$ such that $|v_r, 0\rangle = |v_r\rangle$.

Now define the sequences $\{i\} = i_1, i_2, \ldots$ $(0 \leq i_k \leq D-1)$ to be those with a *finite* number of nonzero terms.

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F^∞: Gutmann 1995 approach

Goal: Define F^∞ on $\mathcal{H}^{\otimes\infty}$ and derive the following "strong law of large numbers":

For any component $\mathcal{H}^{\otimes\infty}_{\{\psi\}}$ and any state $|\Psi\rangle \in \mathcal{H}^{\otimes\infty}_{\{\psi\}}$,

$$F^\infty|\Psi\rangle = f_{\{\psi\}}|\Psi\rangle \ ,$$

where

$$f_{\{\psi\}} = \frac{1}{2}\left(\limsup_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N} q_r + \liminf_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N} q_r\right) \ ,$$

and

$$q_r = |\langle\psi_r|B,1\rangle|^2 \ .$$

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The measure

The measures for all sequences beginning with $j_1, \ldots, j_N$ (for all N)

$$\nu_{|\psi;\{i\}\rangle}(j_1, \ldots, j_N) = \int d\nu_{|\psi;\{i\}\rangle}(\{j'\}) \prod_{r=1}^{N} \delta_{j_r j'_r}$$

determine the measure $d\nu_{|\psi;\{i\}\rangle}$.

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Frequency of component (1)

Frequency of 1 in $\{j\} = j_1, j_2, \ldots$

$$f(\{j\}) = \frac{1}{2}\left(\limsup_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\delta_{1 j_r} + \liminf_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\delta_{1 j_r}\right)$$

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Projector on frequency subspace

Projector Π_f^∞ onto frequency f:

$$||\Pi_f^\infty|\psi;\{i\}\rangle||^2 = \int d\nu_{|\psi;\{i\}\rangle}(\{j\})\,\Pi_f(\{j\})\ ,$$

where

$$\Pi_f(\{j\}) = \begin{cases} 1 & \text{if } f(\{j\}) = f, \\ 0 & \text{if } f(\{j\}) \neq f \end{cases}$$

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Frequency operator

We can thus define

$$\Pi_f^{\infty}|\psi;\{i\}\rangle = \begin{cases} |\psi;\{i\}\rangle & \text{if } f_{\{\psi\}} = f, \\ 0 & \text{if } f_{\{\psi\}} \neq f \end{cases}$$

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Projector on frequency subspace

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$$||\Pi_f^\infty|\psi;\{i\}\rangle||^2 = \int d\nu_{|\psi;\{i\}\rangle}(\{j\})\,\Pi_f(\{j\})\ ,$$

where

$$\Pi_f(\{j\}) = \begin{cases} 1 & \text{if } f(\{j\}) = f, \\ 0 & \text{if } f(\{j\}) \neq f \end{cases}$$

Quantum strong law of large numbers:

$$||\Pi_f^\infty|\psi;\{i\}\rangle||^2 = \begin{cases} 1 & \text{if } f_{\{\psi\}} = f, \\ 0 & \text{if } f_{\{\psi\}} \neq f \end{cases}$$

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Frequency operator

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Alternative measures

The freedom in choosing the measure $d\nu$ leads to the following freedom in the probabilities q_r that determine the eigenvalues $f_{\{\psi\}}$ of F^∞:

$$q_r = \frac{|g(\langle\psi_r|B,1\rangle)|^2}{\sum_k |g(\langle\psi_r|B,k\rangle)|^2}$$

instead of $q_r = |\langle\psi_r|B,1\rangle|^2$ (i.e., $g(x) = x$).

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Frequency of component (1)

Frequency of 1 in $\{j\} = j_1, j_2, \ldots$

$$f(\{j\}) = \frac{1}{2}\left(\limsup_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\delta_{1j_r} + \liminf_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\delta_{1j_r}\right)$$

$$\int d\nu_{|\psi;\{i\}\rangle}(\{j\})\, f(\{j\}) = f_{\{q\}}\ ,$$

where

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The measure

The measures for all sequences beginning with $j_1, \ldots, j_N$ (for all N)

$$\nu_{|\psi;\{i\}\rangle}(j_1, \ldots, j_N) = \int d\nu_{|\psi;\{i\}\rangle}(\{j'\}) \prod_{r=1}^{N} \delta_{j_r j'_r}$$

determine the measure $d\nu_{|\psi;\{i\}\rangle}$.

We choose

$$\nu_{|\psi;\{i\}\rangle}(j_1, \ldots, j_N) = \prod_{r=1}^{N} |\langle \psi_r, i_r | B, j_r \rangle|^2 \ .$$

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Frequency operator

We can thus define

$$\Pi_f^\infty |\psi;\{i\}\rangle = \begin{cases} |\psi;\{i\}\rangle & \text{if } f_{\{\psi\}} = f, \\ 0 & \text{if } f_{\{\psi\}} \neq f \end{cases}$$

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F^{∞}: Farhi, Goldstone and Gutmann 1989 approach

Restrict attention to the component $\mathcal{H}_{\{\psi\}}^{\otimes\infty}$ where $|\{\psi\}\rangle = |\psi\rangle^{\otimes\infty}$

Within $\mathcal{H}_{\{\psi\}}^{\otimes\infty}$, construct (an uncountable set of unnormalized) simultaneous eigenstates $|b;\{j\}\rangle$ of B^1,$B^2, \ldots,$ such that

$$\frac{\langle\psi;\{i\}|\,b;\{j\}\rangle}{\langle\psi;\{0\}|\,b;\{j\}\rangle} = \prod_{r=1}^{\infty} \frac{\langle\psi, i_r|B, j_r\rangle}{\langle\psi, 0|B, j_r\rangle}\,,$$

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F^∞: Farhi, Goldstone and Gutmann 1989 approach

With this inner product, define a transformation between representations for any $|\Psi\rangle \in \mathcal{H}_{\{\psi\}}^{\otimes\infty}$ by

$$\langle\psi;\{i\}|\Psi\rangle = \int d\mu(\{j\})\,\langle\psi;\{i\}|\,b;\{j\}\rangle\langle b;\{j\}|\Psi\rangle\ ,$$

$$\langle b;\{j\}|\Psi\rangle = \sum_{\{i\}}\langle b;\{j\}|\psi;\{i\}\rangle\langle\psi;\{i\}|\Psi\rangle\ .$$

Now derive the form of $d\mu$ from the requirement that this transformation is an isometry.

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F^∞: Farhi, Goldstone and Gutmann 1989 approach

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$$\frac{\langle\psi;\{i\}|\,b;\{j\}\rangle}{\langle\psi;\{0\}|\,b;\{j\}\rangle} = \prod_{r=1}^{\infty} \frac{\langle\psi, i_r|B, j_r\rangle}{\langle\psi, 0|B, j_r\rangle} ,$$

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Status of F^∞

Conclusion: Unless QPP is assumed, there is no unique F^∞ such that $F^\infty|\Psi_\infty\rangle = q|\Psi_\infty\rangle$.

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Status of F^{∞}

Conclusion: Unless QPP is assumed, there is no unique F^{∞} such that $F^{\infty}|\Psi_{\infty}\rangle = q|\Psi_{\infty}\rangle$.

Now assume, hypothetically, that there exists such a unique frequency operator. One still cannot derive QPP by applying the Postulate of Definite Outcomes QPP′.

1. Probability 1 versus certainty: QPP′ applied to F^{∞} is different from standard QM.

2. Frequency is a tail property: Any initial finite sequence of outcomes is independent of the limiting frequency

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Quantum Probability Postulate

QPP: Let $B = \sum_\lambda \lambda P_\lambda$ be an observable, where λ denotes the different eigenvalues of B and P_λ are orthogonal projectors onto the eigenspaces of B. If B is measured for a system in state $|\psi\rangle$, the probability of outcome λ is $||P_\lambda|\psi\rangle||^2$.

Postulate of Definite Outcomes (QPP$'$): If an observable B is measured for a system in an eigenstate $|\psi\rangle$ of B, i.e., $B|\psi\rangle = \lambda|\psi\rangle$, the outcome is λ with certainty.

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Status of F^∞

Conclusion: Unless QPP is assumed, there is no unique F^∞ such that $F^\infty|\Psi_\infty\rangle = q|\Psi_\infty\rangle$.

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2. Frequency is a tail property: Any initial finite sequence of outcomes is independent of the limiting frequency.

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Conclusion

1. If the quantum probability postulate QPP is assumed, we can define F^{∞} and derive a quantum strong law of large numbers.

2. The probabilistic interpretation of the quantum formalism does not follow from the Postulate of Definite Outcomes QPP$'$.

Gleason's theorem

Assume there is a function h from the one-dimensional projectors acting on a Hilbert space of dimension greater than 2 to the unit interval, with the property that for each orthonormal basis $\{|\psi_k\rangle\}$,

$$\sum_k h\big(|\psi_k\rangle\langle\psi_k|\big) = 1 \ .$$

Then there exists a density operator ρ such that

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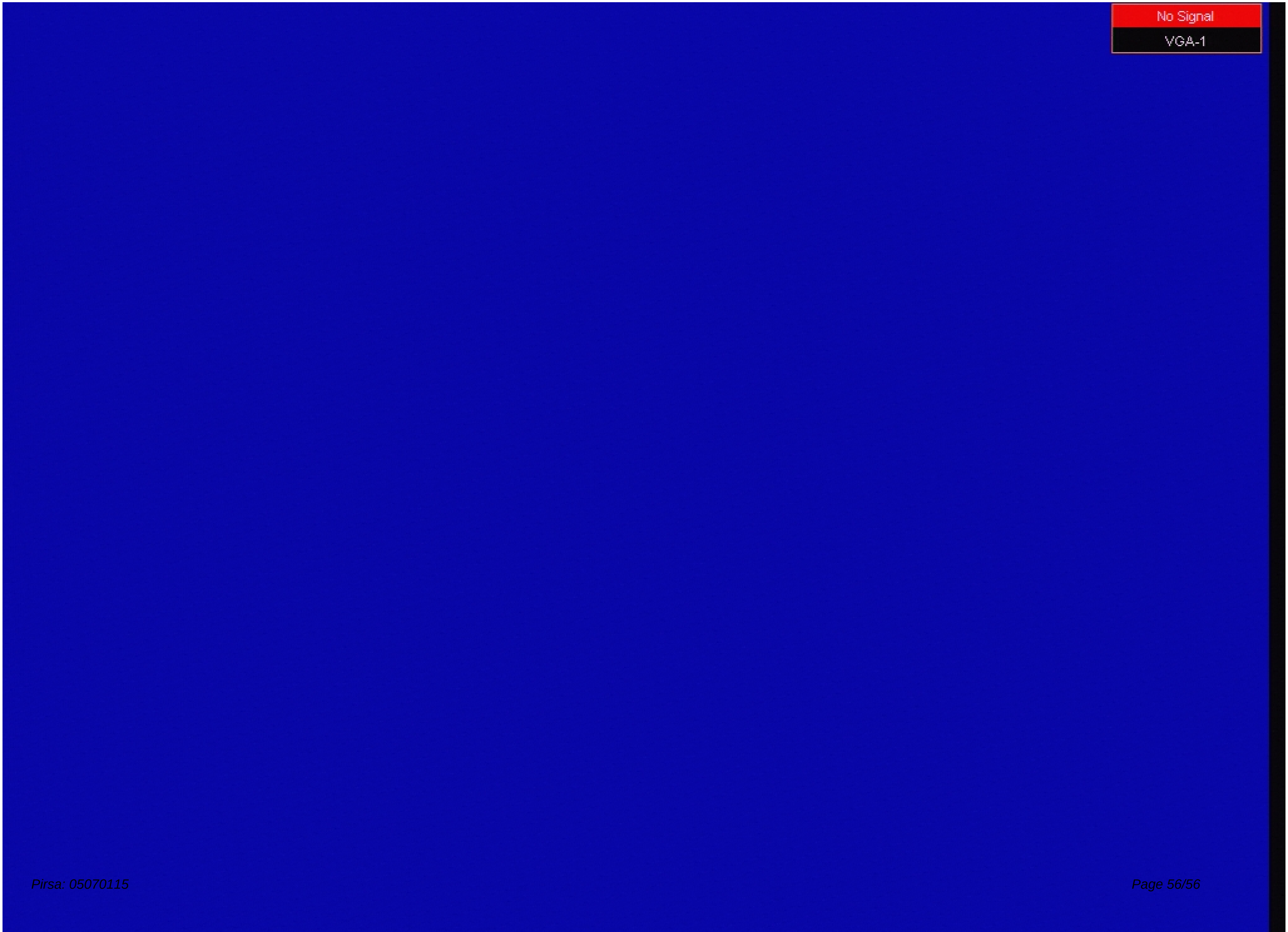
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