Title: Quantum Logics, Spaces of States,Ordered Banach Spaces, and their Connections to the Works of Foulis- Randall, Gunson, Ludwig, Birkhoff- von Neumann ,...

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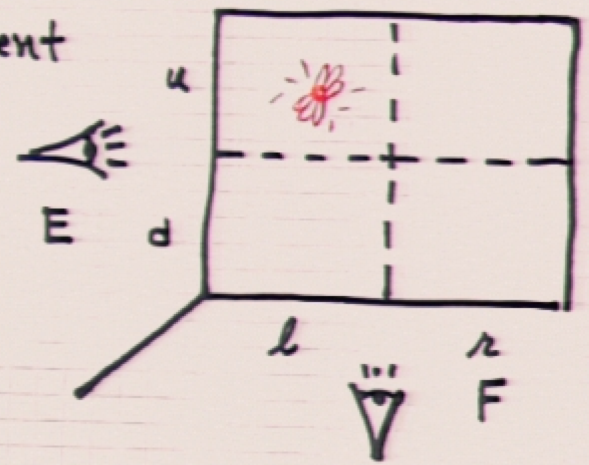
Abstract:

The Experiment



Outcomes

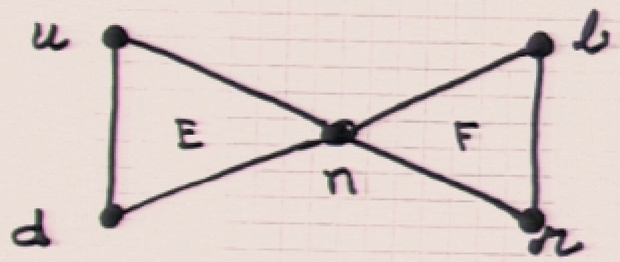
$\underline{X} = \{u, d, n, l, r\}$

Operations:

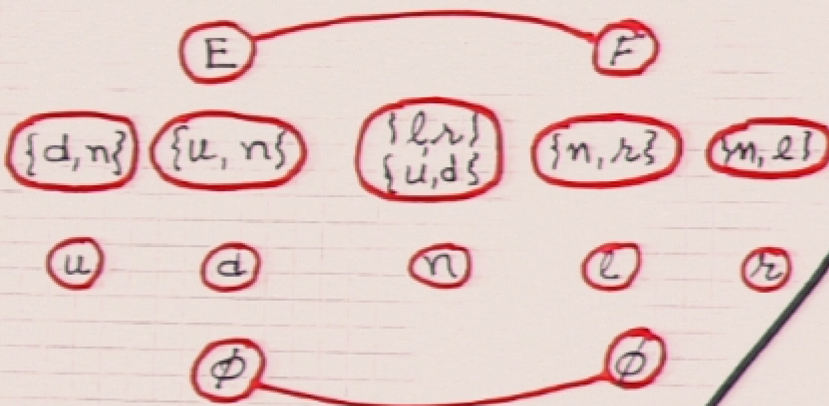
$E = \{u, d, n\}$

$F = \{l, r, n\}$

Orthogonality Space



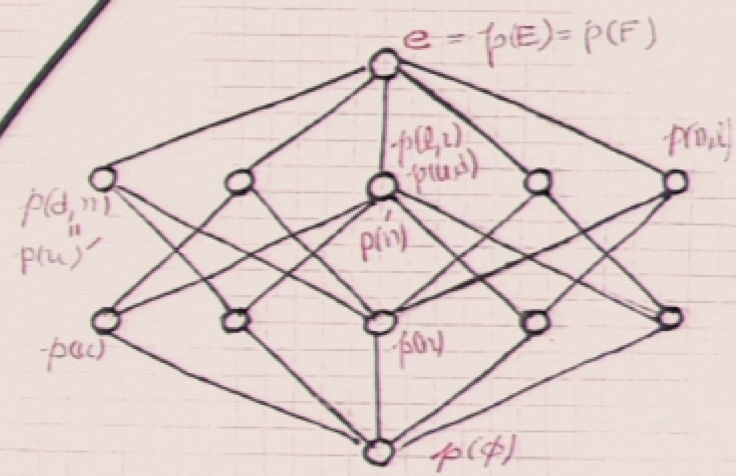
Testable events $\mathcal{E}$.



non testable sets

$\{d, l\}$, $\{u, r\}$, $\cdots$

Operational Logic Π



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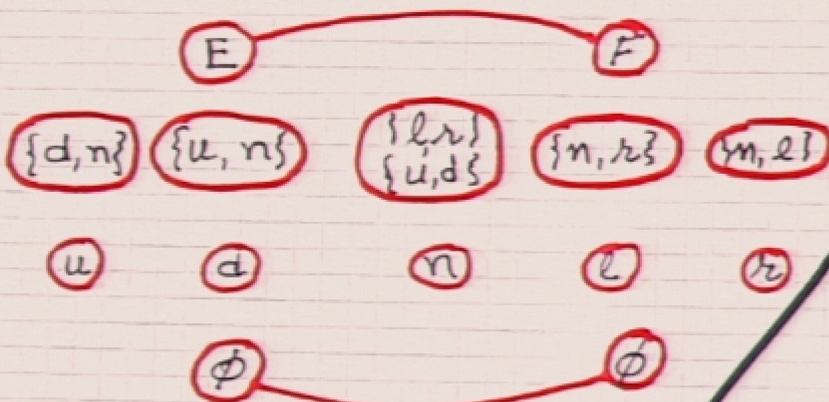
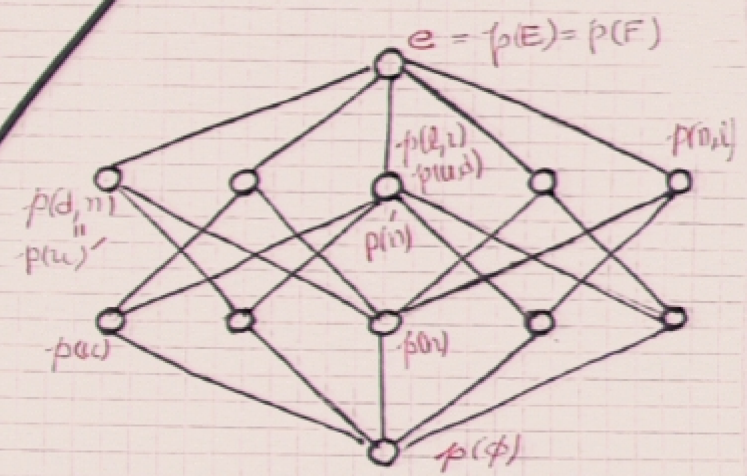
Testable events $\mathcal{E}$



non testable sets

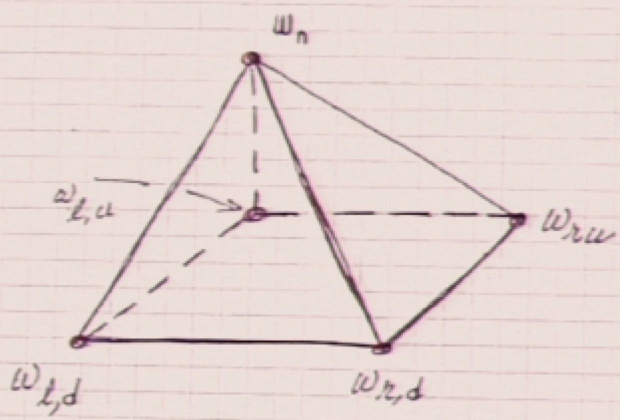
$\{d,l\}, \{u,r\}, \dots$

Operational Logic Π



A __weight__ $\omega : \underline{X} \to [0,1]$ is a f'ct such that

$\omega(E) = \omega(u) + \omega(d) + \omega(n) = 1$ & $\omega(F) = 1$



$\omega_{l,d}(d) = \omega_{l,d}(l) = 1$

zero on other values in $\underline{X}$

Set of all probability weights form a compact convex subset of $\mathbb{R}^3$ — Call this Ω_ω

Use this as a __base__ for a cone in $\mathbb{R}^4$

Each element of Ω_ω can be lifted to an orthogonally additive state on Π by

$$\omega(P(A)) \equiv \sum_A \omega(a)$$

This is where we begin! 4.

→ (Some facts about B-spaces)

Let Π be an orthomodular poset ~~and~~ and Δ – the convex set of all finitely orthogonally additive probability ~~states~~ states on Π.

Let $V(\Delta) = \mathrm{span}(\Delta) \subset \mathcal{F}(\Pi; \mathbb{R})$

$V(\Delta) = C_\Delta - C_\Delta$ — Let $U_\Delta = \mathrm{con}(\Delta \cup -\Delta)$

U_Δ is $w(V(\Delta), \Pi)$ – compact. The Minkowski fct'l of U_Δ makes $V(\Delta)$ into a p.o. Banach sp.

Each $p \in \Pi$ acts as a linear functional on $V(\Delta)$ by the formula $p(\omega) = \omega(p)$.

Span$(p \in \Pi)$ can be organized into a ~~normed~~ normed space $f \geq 0$ when $f(\omega) \geq 0 \ \forall \omega \in \Delta$.

$L_1(\mu) \qquad L_\infty$

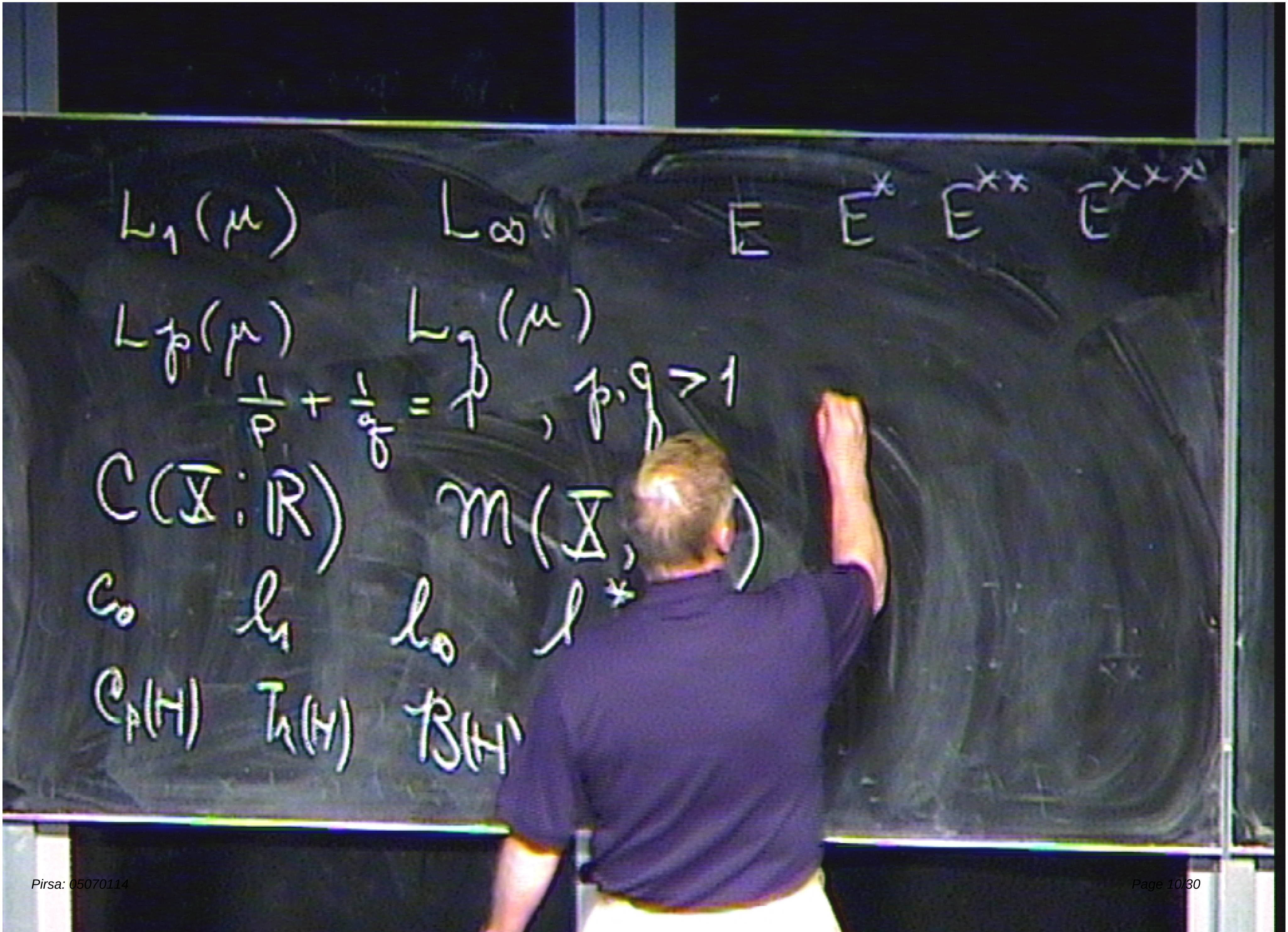
$L_p(\mu) \qquad L_q(\mu)$

$\frac{1}{p} + \frac{1}{q} = 1, \quad p, q > 1$

$C(X; \mathbb{R}) \qquad \mathcal{M}(X, \mathbb{R})$

$c_0 \qquad l_1 \qquad l_\infty \qquad l_\infty^*$

$\mathcal{C}_p(H) \qquad T_r(H) \qquad \mathcal{B}(H) \qquad \mathcal{B}(H)^*$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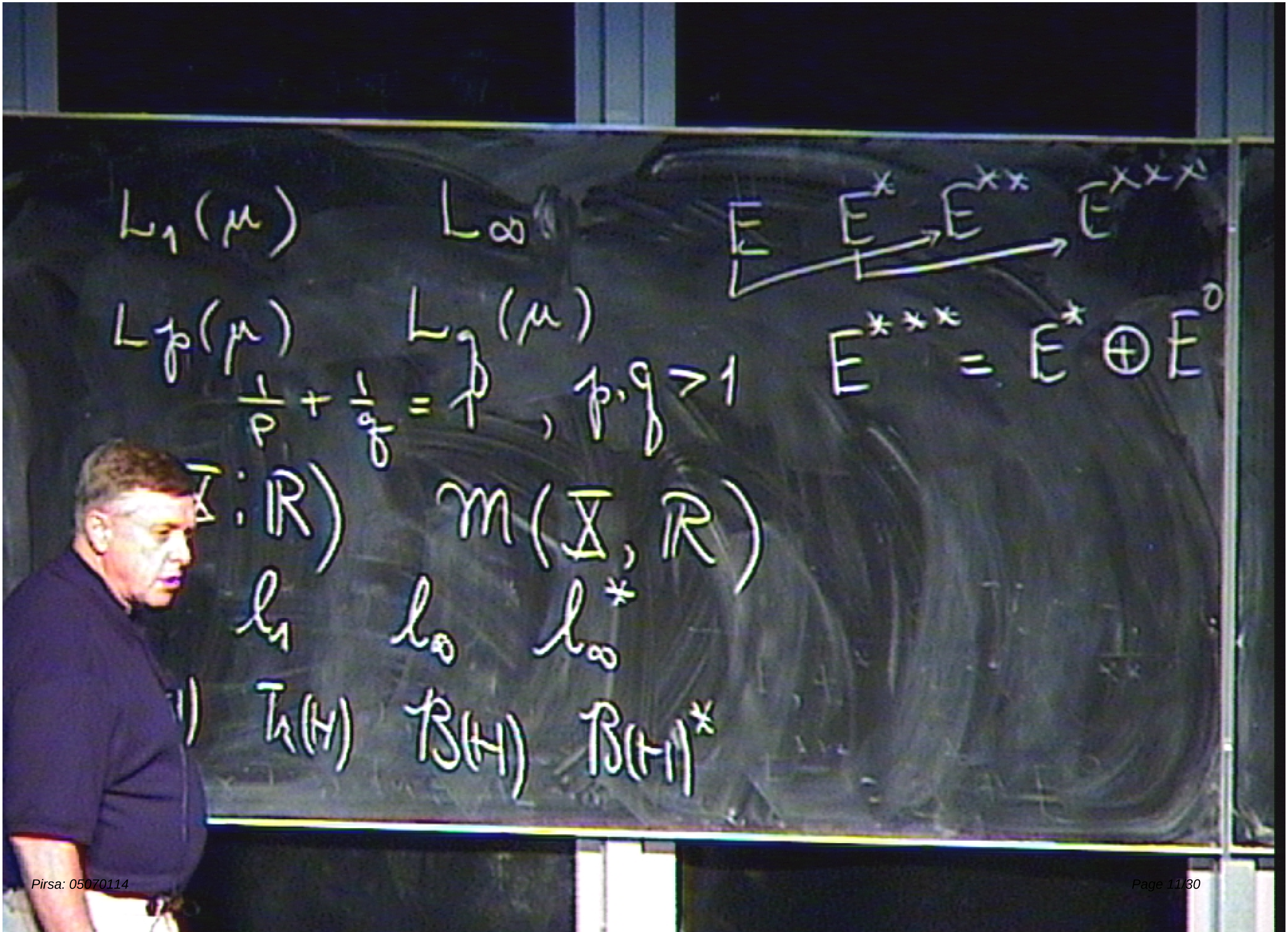
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l_1 l_0 l_∞^*
$T_1(H)$ $B(H)$ $B(H)^*$

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5.

The unit e in Π acts as an <u>order unit</u> in $\mathrm{Span}(\Pi)$ and the Minkowski functional of $[-e, e]$ is a norm on $\mathrm{Span}(\Pi)$ — Complete this space in this order unit norm and denote it with $\mathcal{L}$, $\mathfrak{A}(\mathcal{L}, e)$.

<u>Theorem</u> $\mathcal{L}^* = V(\Delta)$ — ~~Denote $\mathcal{L}^*$ with $(\mathcal{L}, e)$~~

1. Does $(\mathcal{L}$ have a predual ?

Not alwa (ex.) But if it does then the predual ace normed space and consist mpletely <u>additive</u> measures in Π

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Not always! (Ot.) But if it does then the predual is a base normed space and consists of <u>completely</u> <u>additive</u> measures on Π

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The unit e in Π acts as an order unit in Span(Π) and the Minkowski functional of $[-e, e]$ is a norm on Span(Π) — Complete this space in this order unit norm and denote it with $\mathcal{L}$, ~~$\mathfrak{A}$~~$(\mathcal{L}, e)$.

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6.

Denote this space $V(\Delta\Omega_\nu)$

2. Are all the completely additive measures in $V(\Delta\Omega_\nu)$ (as in a v. N alg.) ?

Not necessarily.

Let us suppose ~~$V(\Delta\Omega_\nu)$~~ $\Delta\Omega_\nu$ is strong over Π

$\left[\!\left[\, p \le q \text{ in } \Pi \quad \text{iff} \quad \omega(p) = 1 \Rightarrow \omega(q) = 1 \,\right]\!\right]$ and

$V(\Delta\Omega_\nu)$ is a separable Banach Space

Theorem If $V(\Delta\Omega_\nu)$ is a dual space then the unit ball $U_{\Omega\nu} = \mathrm{Con}(\Delta\Omega_\nu \cup -\Delta\Omega_\nu)$ is the closed convex hull of its exposed points

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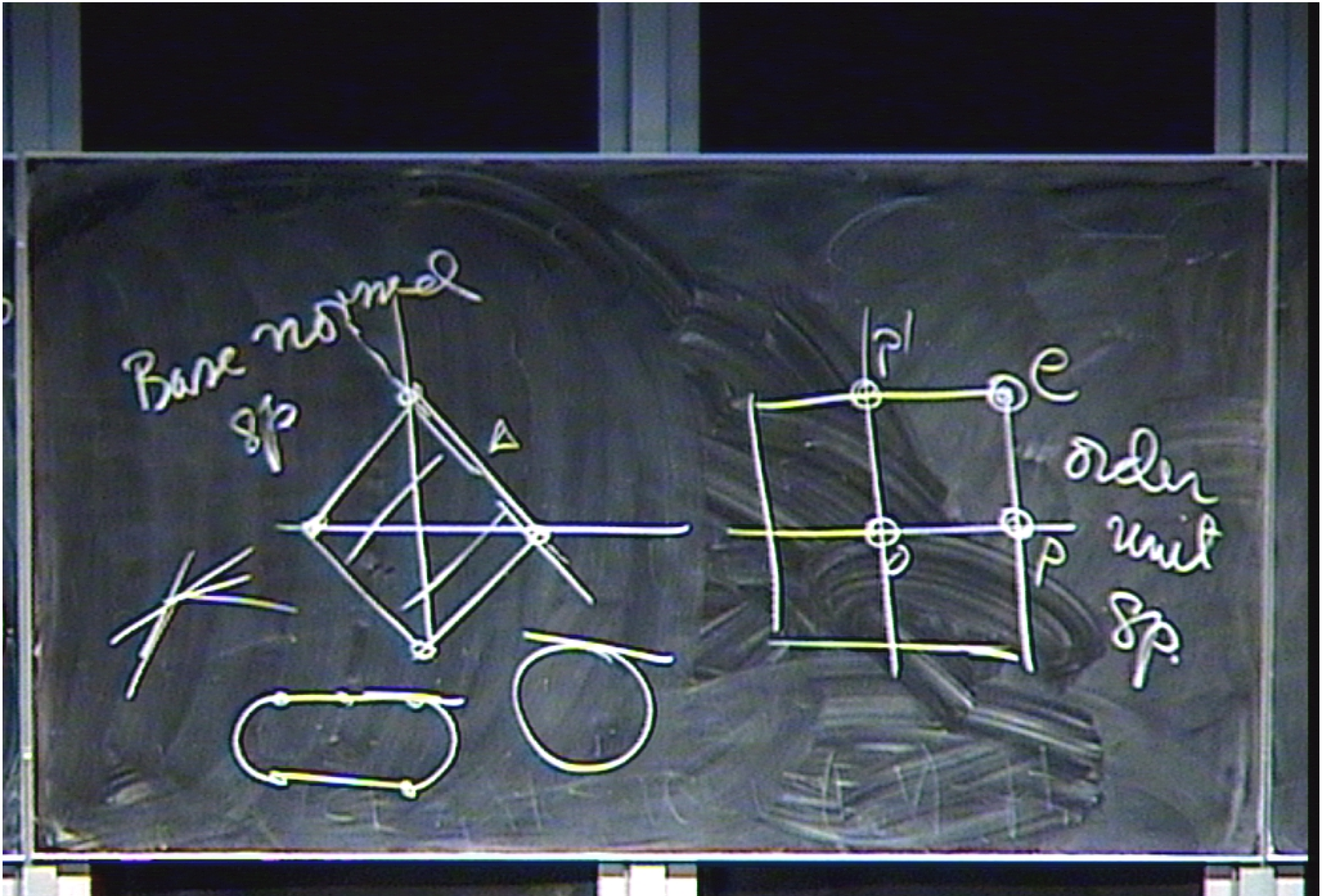
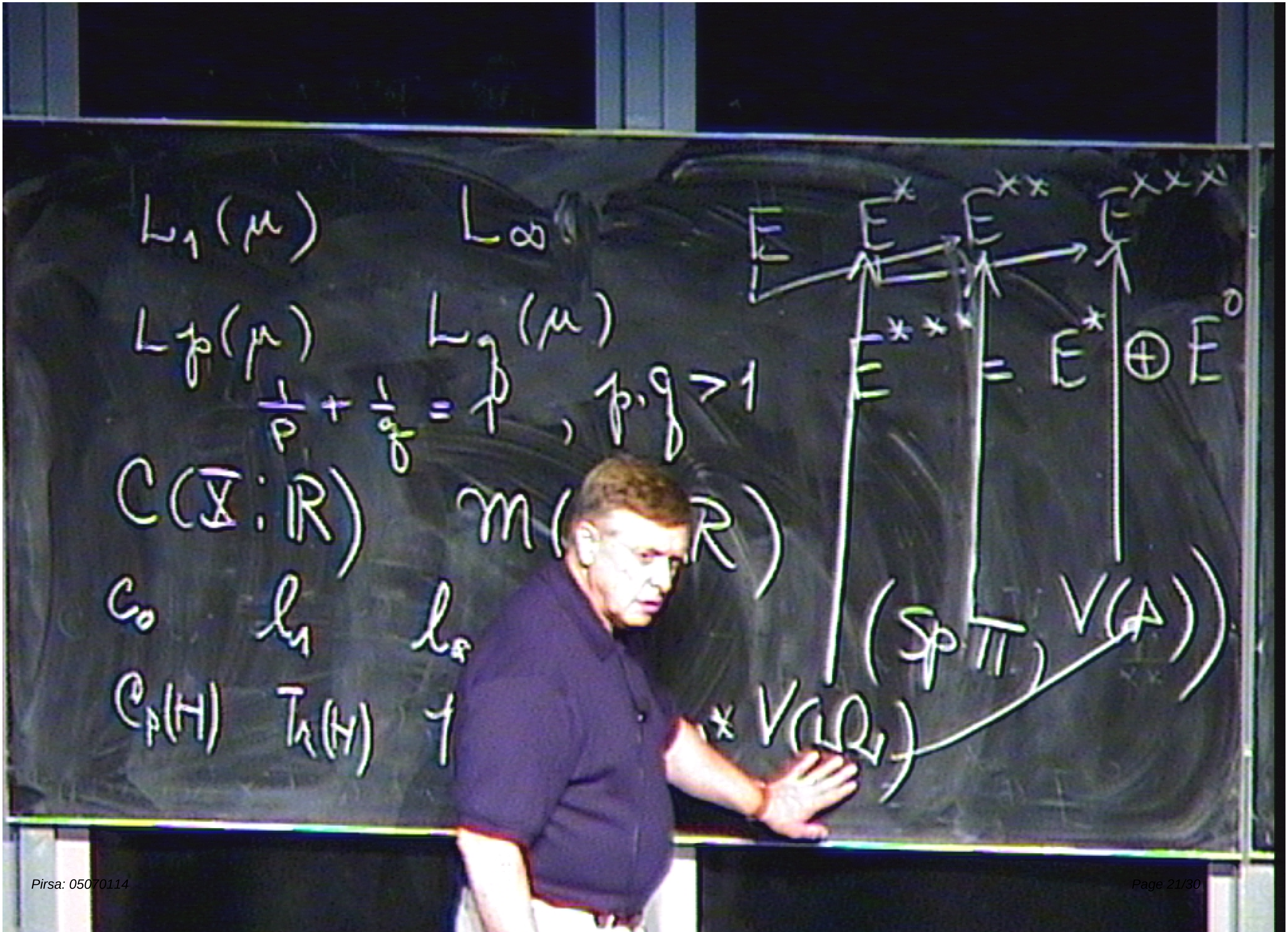


Base normal
sp
Δ
P'
e
order
unit
sp.
p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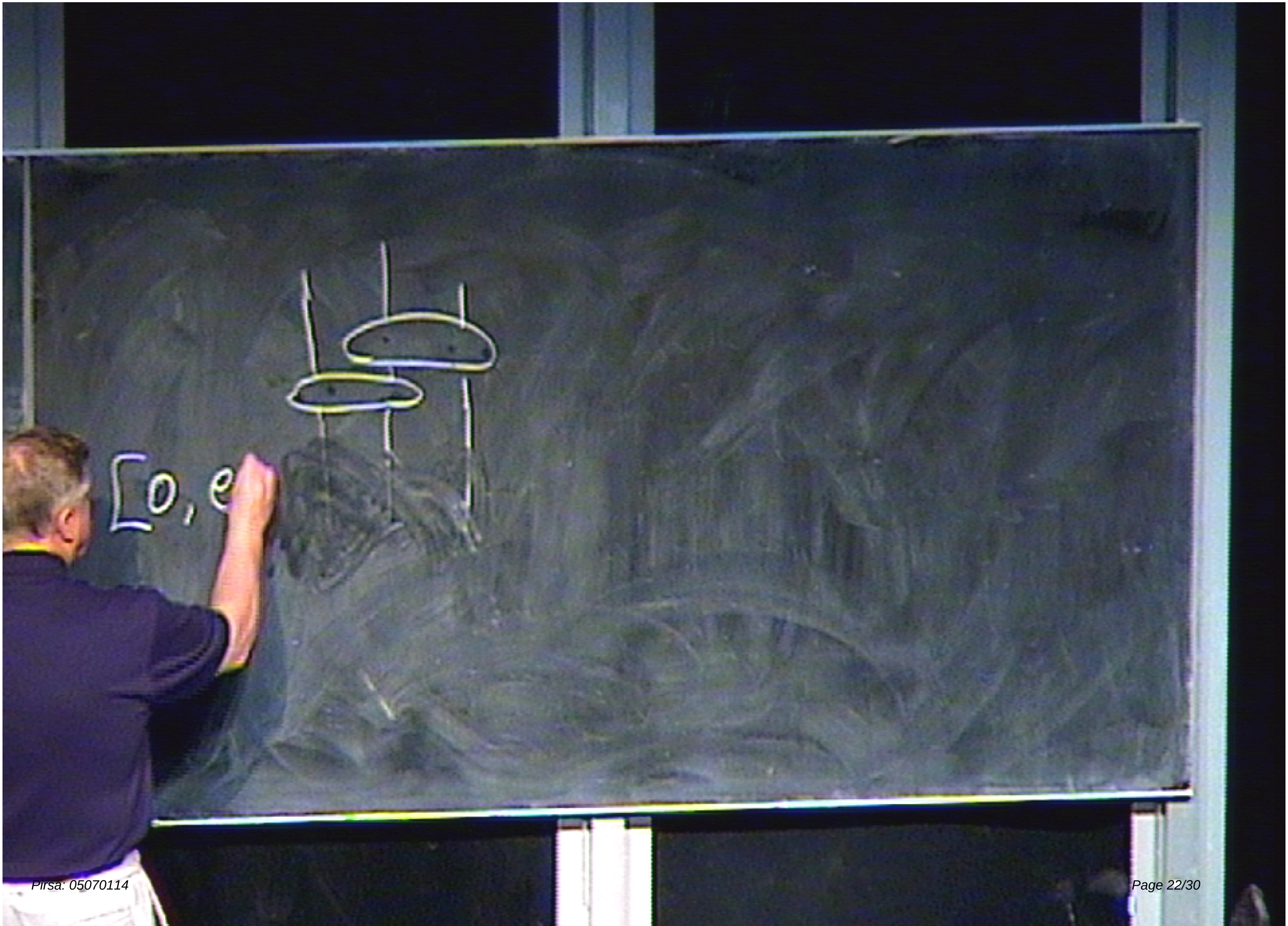
$(Sp\,\pi, V(A))$
$V(L\Omega)$


[0, e

$[0,e]$:

$\forall g_1, g_2 \in [0,e]_1 \quad \exists g \in [0,e]_1$

$g \leq g_1, g_2 \quad (g_1)^1 \cap (g_2)^1 = (g)^1$

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Denote this space $V(\omega\Omega_0)$

2. Are all the completely additive measures $V(\omega\Omega_0)$ (as in a v.N alg.)?

Theorem

the unit ball ... closed

Convex hull of its exposed