Title: Generalised Probability Theories as a setting for Quantum Theory and other theories.

Date: Jul 21, 2005 02:00 PM

URL: http://pirsa.org/05070113

Abstract:

Outline.

Motivation.
scheme
states
definitions
5 Axioms
Beyond QT?

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states

definitions

5 Axioms

Beyond QT?

Motivation.

QT $\quad \hat{\rho}, \hat{A}, \hat{U}, \$(\cdot), \mathcal{H}, \otimes$

$\Rightarrow$ Abstract.

Can we account for this structure by some reasonable axioms or principles?

Historical examples.

Kepler $\rightarrow$ Newton $\longrightarrow$ beyond ([illegible] cannot)

Lorentz $\rightarrow$ Einstein (SR) $\longrightarrow$ beyond (GR)

Abstract

reasonable axioms $\Longrightarrow$ beyond (GR)

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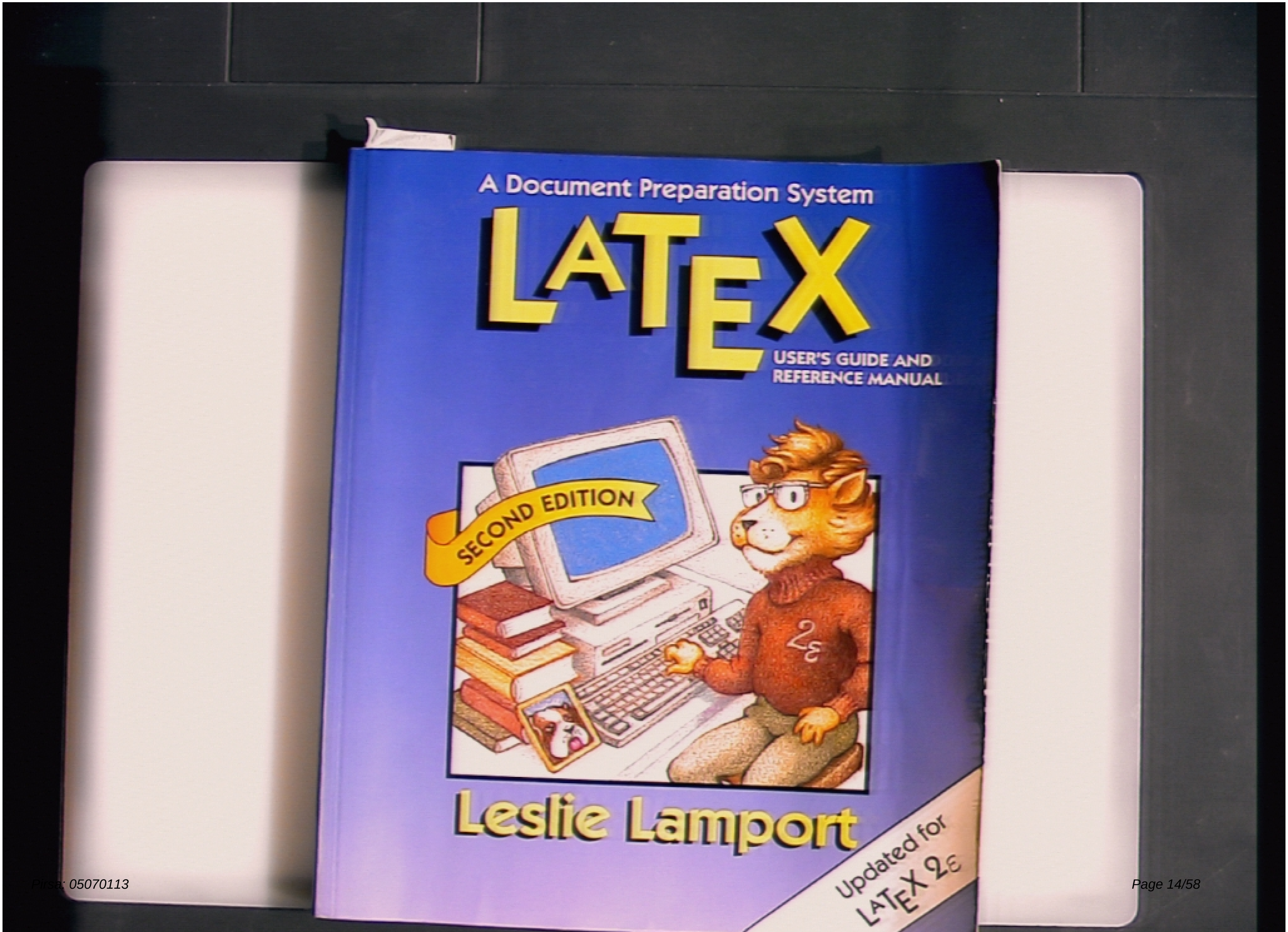
A Document Preparation System
LaTeX
USER'S GUIDE AND
REFERENCE MANUAL
SECOND EDITION
Leslie Lamport
Updated for
LaTeX 2ε

example of a user-defined environment:
environment produces items that are
[illegible]
defined in terms of LaTeX's itemize en-
[illegible] and \em command.

```
\newenvironment{emphit}{\begin{itemize}
    \em}{\end{itemize}}
... example of a user-defined environment:
\begin{emphit}
 \item This environment produces
\end{emphit}
```

An optional argument of the `\newenvironment` command allows you to [illegible] that has arguments; it works the same as described [illegible]

[illegible]—in this ex-
of a single item—can
[illegible] environments.
[illegible] of
[illegible] by the

```
\newenvironment{descit}[1]{\begin{quote}
   \textit{#1}:}{\end{quote}}
...
defined in terms of existing environment
\begin{descit}{Armadillos}
This witty description of the armadillos
\end{descit}
```

[illegible] (the #1, #2, etc.) can appear only in the *begin text*. [illegible] about the scope of declarations that appear inside argu[illegible] defined with `\newcommand` apply to the arguments of [illegible] with `\newenvironment`. Section C.8.2 explains how to define [illegible] with an optional argument.

[illegible]`vironment` command produces an error if the environment [illegible]. Use `\renewenvironment` to redefine an existing environment. [illegible] complains that an environment you've never heard of [illegible] different environment name. Use `\renewenvironment` only when [illegible] you're doing; don't try redefining an environment that you [illegible]

3.4.3 Theorems and Such

Mathematical text usually includes theorems and/or theorem-like structures, such as lemmas, propositions, axioms, conjectures, and so on. Nonmathematical text may contain similar structures: rules, laws, assumptions, principles, etc. Having a built-in environment for each possibility is out of the question, so LaTeX provides the `\newtheorem` declaration to define environments for the particular theorem-like structures in your document.

The `\newtheorem` command has two arguments: the first is the name of the environment, the second is the text used to label it.

Conjectures are numbered consecutively from the beginning of the document; this is the fourth one:

Conjecture 4 *All conjectures are interesting, but some conjectures are more interesting than others.*

```
\newtheorem{guess}{Conjecture}
...
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The `\newtheorem` declaration is best put in the preamble, but it can go anywhere in the document.

A final optional argument to `\newtheorem` causes the theorem-like environment to be numbered within the specified sectional unit.

This is the first Axiom of Chapter 3.

Axiom 3.1 *All axioms are very dull.*

```
\newtheorem{axiom}{Axiom}[chapter]
...
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\end{axiom}
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Theorem-like environments can be numbered within any sectional unit; using `section` instead of `chapter` in the example above causes axioms to be numbered within sections.

Sometimes one wants different theorem-like structures to share the same numbering sequence—so, for example, the hunch immediately following Conjecture 5 should be Hunch 6.

Conjecture 5 *Some good conjectures are numbered.*

Hunch 6 *There are no sure-fire hunches.*

```
\newtheorem{guess}{Conjecture}
\newtheorem{hunch}[guess]{Hunch}
...
\begin{guess} Some good ... \end{guess}
\begin{hunch} There are ... \end{hunch}
```

The optional argument `guess` in the second `\newtheorem` command specifies that the `hunch` environment should be numbered in the same sequence as the `guess` environment.

A theorem-like environment defined with `\newtheorem` has an optional argument that is often used for the inventor or common name of a theorem, definition, or axiom.

Conjecture 7 (Wiles, 1985) *There do exist integers $n > 2$, x, y, and z such that $x^n + y^n = z^n$.*

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QT is a probability calculus

Classical probability theory

Quantum (probability theory)

. . . .

other probability theories

- Newtonian mech.
- electromagnetism
- gambling
- stat. mech
- ⋮

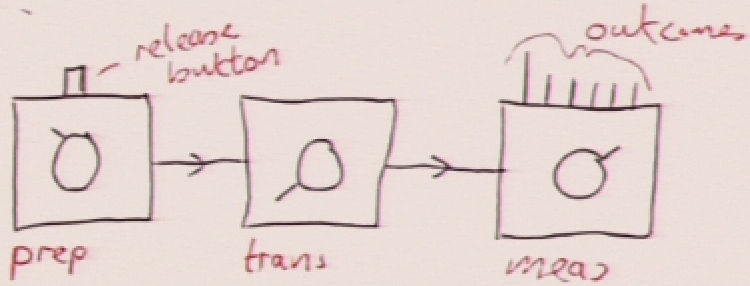
- Q. particle in well
- QED
- Q. computing
- ⋮

. . . .

new physics.

Axioms must apply to some framework.

We consider the preparation, (transformation)^n, measurement framework.



Axiom 1 The quantity

$$p = \lim_{n \to \infty} \frac{n_{\text{given outcome}}}{n}$$

takes same value whenever given measurement follows a given preparation.

States

Definition The state associated with a preparation is that thing represented by any mathematical object which can be used to calculate the probability for any outcome for any measurement

$$\text{state} \Leftrightarrow \underline{P} = \begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \end{pmatrix}$$

α runs over all outcomes of all measurements.

$$\text{prob} = \underline{R} \cdot \underline{P} \qquad \underline{R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow \text{1 in appropriate position.}$$

But expect physical theory to have some structure so don't need such a long list. Represent state by ~~P~~ $\underline{p}$ where

$$\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{pmatrix}$$

K is the 'number of fiducial measurements'

where just sufficient to calculate general prob by a linear formula

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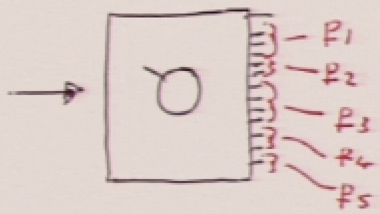
Information carrying capacity



N is the maximum number of reliably distinguishable states.

A system is constrained to have information carrying capacity $\log_2 M$ if the allowed states are constrained such that, for a measurement set to distinguish all N states, we only get outcomes associated with some subset of M of these states.

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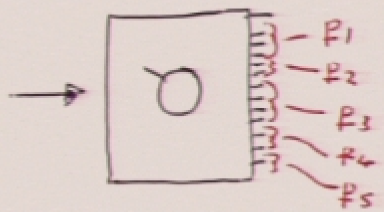

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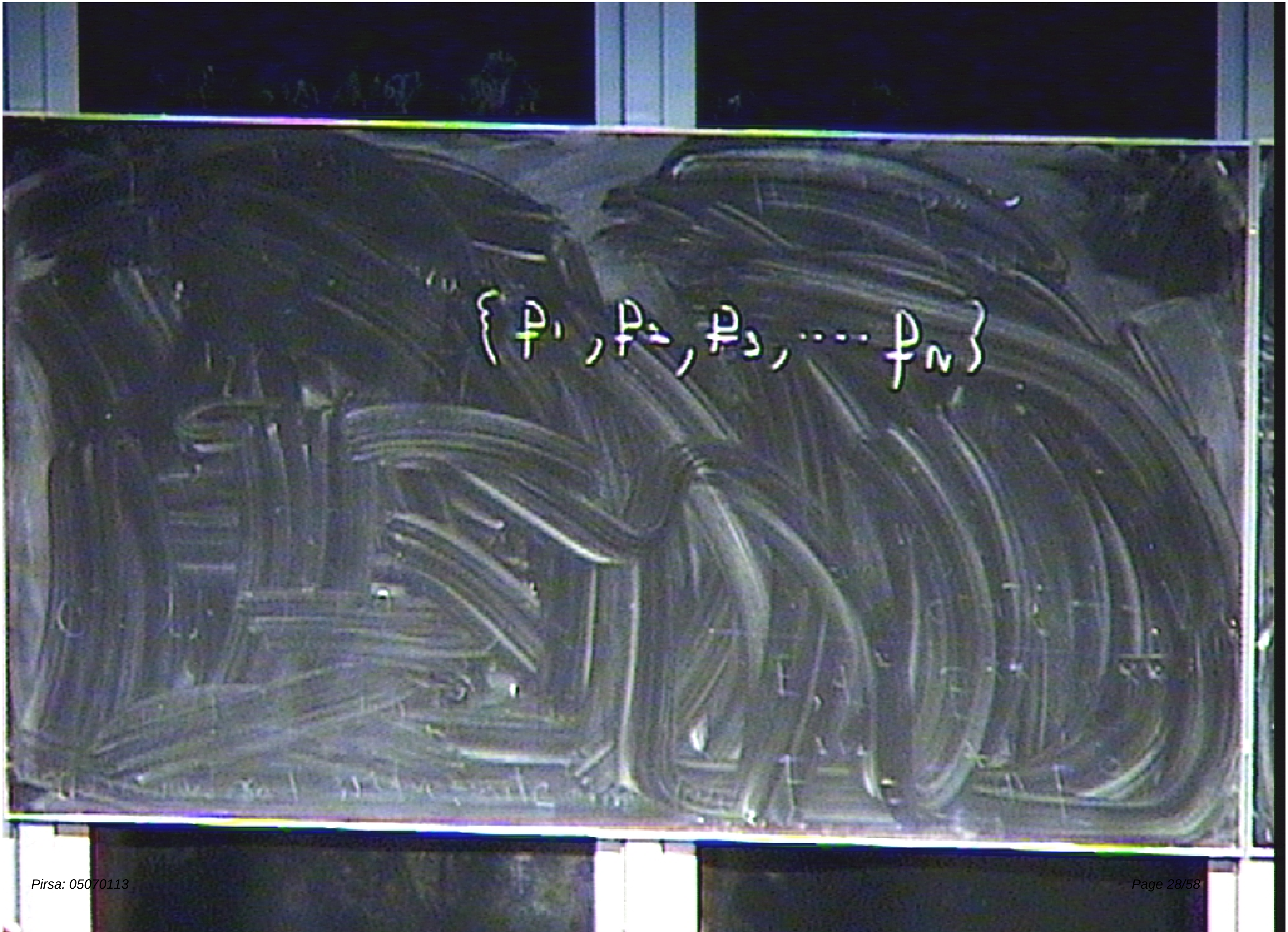
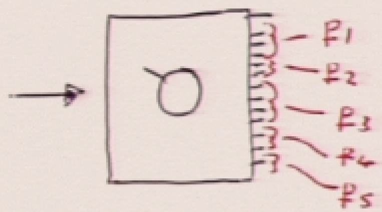
$$\{p_1, p_2, p_3, \dots p_N\}$$


$\{p_1, p_2, p_3, \cdots p_N\}$

$\{\rho_1, \rho_2, \rho_3, \cdots \rho_N\}$

Information carrying capacity



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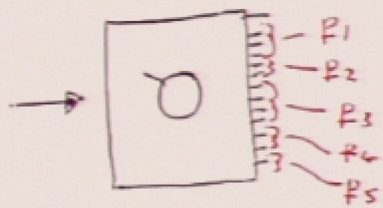
Information carrying capacity $\log_2 N$



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Transformation

$$\underline{f} \longrightarrow \boxed{O} \longrightarrow Z_{\underline{f}}$$

Have

$\underline{f} \in S$

$\underline{r} \in R$

$Z \in \Gamma$

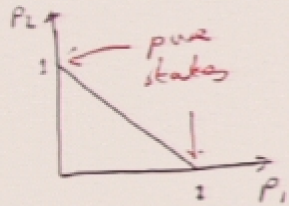
K

N

Classical Prob. Theory

$$\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{pmatrix} \qquad K = N$$

Example $N=2$



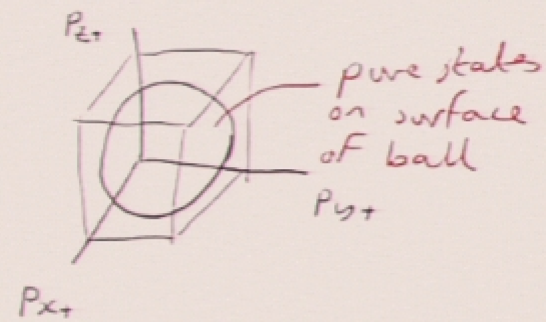
pure states form a discrete set

reversible transformations form a discrete set (permutations)

Quantum Theory

$$\hat{\rho} = \begin{pmatrix} p_1 & \square & & \\ \square & p_2 & & \\ & & \ddots & \\ & & & p_N \end{pmatrix} \qquad K = N^2$$

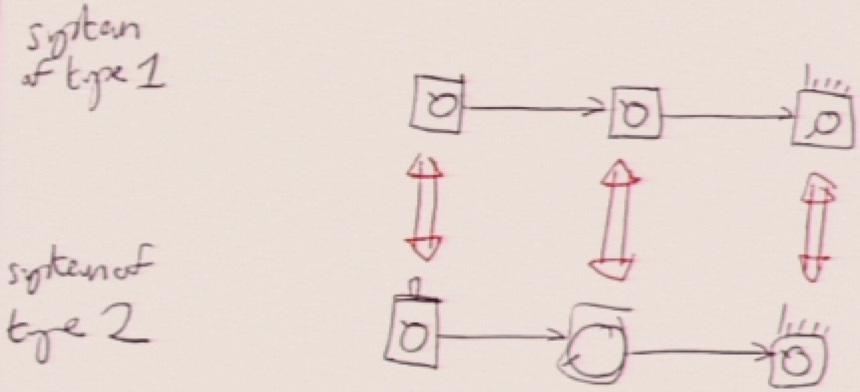
Example $N = 2$ (qubit)



pure states form a continuous set.

reversible transformations form a continuous set (unitaries)

Two systems 'have the same properties' if . . .



under mapping between equivalence classes get same prob.

Pure states are states which cannot be simulated by mixtures of distinct states.

A reversible transformation

$$\rho \rightarrow \boxed{T} \rightarrow \boxed{\exists T^{-1}} \rightarrow \rho \qquad \forall \rho$$

A continuous transformation

$$\rightarrow \boxed{T} \rightarrow \quad \equiv \quad \rightarrow \boxed{\delta} \rightarrow \boxed{\delta'} \rightarrow \cdots\cdots \boxed{\delta''} \rightarrow$$

where $\rightarrow \boxed{\delta} \rightarrow$ has an infinitesimal effect on all probabilities.

Classical Prob. Theory

$$\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{pmatrix} \qquad K = N$$

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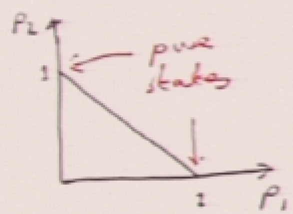

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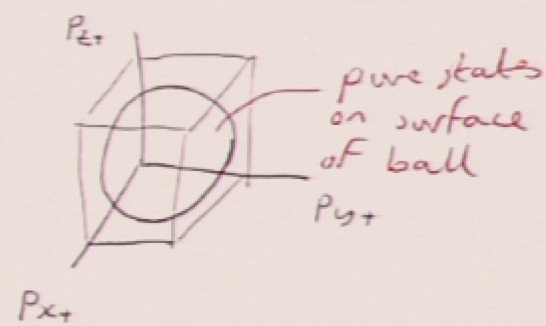

pure states form a continuous set.

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Five Axioms for QT

Axiom 1 Probabilities.

$P = \lim_{n \to \infty} \frac{n_c}{n}$ takes same value for any ensemble with same prep. and meas.

Axiom 2 Information Systems having, or constrained to have, the same information carrying capacity have the same properties

Axiom 3 Composite systems $N_{AB} = N_A N_B$, $K_{AB} = K_A K_B$

Axiom 4 Continuity There exists a continuous reversible transformation between any two pure states.

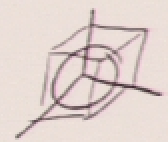
Axiom 5 Simplicity For each N, K takes minimum value consistent with other axioms.

Outline of reconstruction

1) Ax2 $\Rightarrow$ $K = K(N)$, $K(N+1) > K(N)$

Ax3 $\Rightarrow$ $K(N_A N_B) = K(N_A)K(N_B)$ $\Rightarrow$ $K = N^r$

2) $r=1$ doesn't work. $\Rightarrow$ $r=2$ by Ax5.

3) Apply Ax4 to $N=2$, $K=4$ case $\Rightarrow$  Bloch sphere

4) Use Ax2 in successive 2-dim subspaces. Get $\hat{\rho}$, $\hat{A}$ for general N along with $\text{prob} = \text{tr}(\hat{A}\hat{\rho})$.

5) Use Ax3 to construct tensor product structure.

6) General considerations to restrict Z to completely positive maps (also get general measurement theory).

Beyond?

QT

probabilistic.

fixed causal structure.

GR

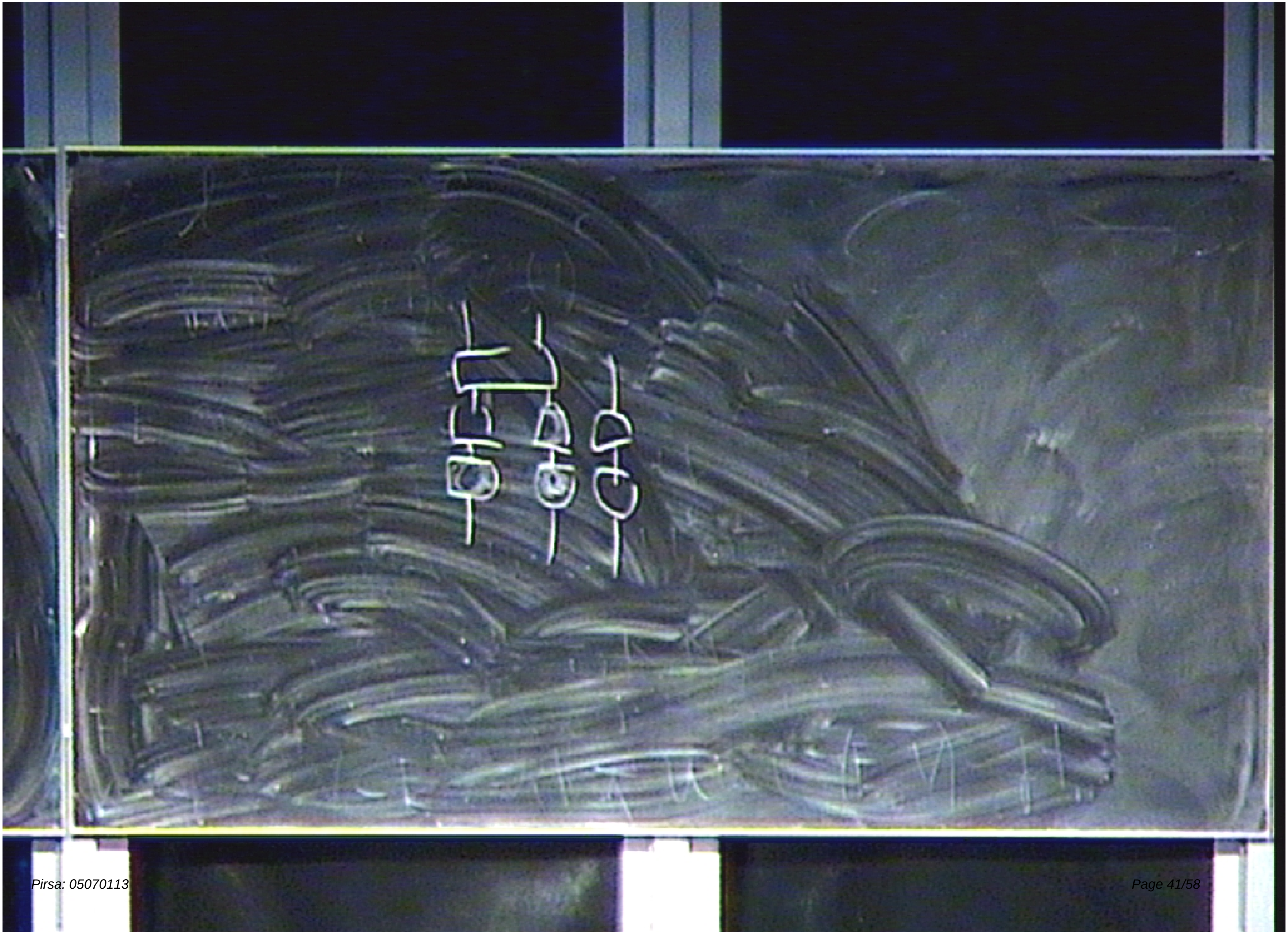
deterministic.

dynamic causal structure.

probability theories with dynamic causal structure

$\Downarrow$

Quantum Gravity.



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Quantum Gravity.

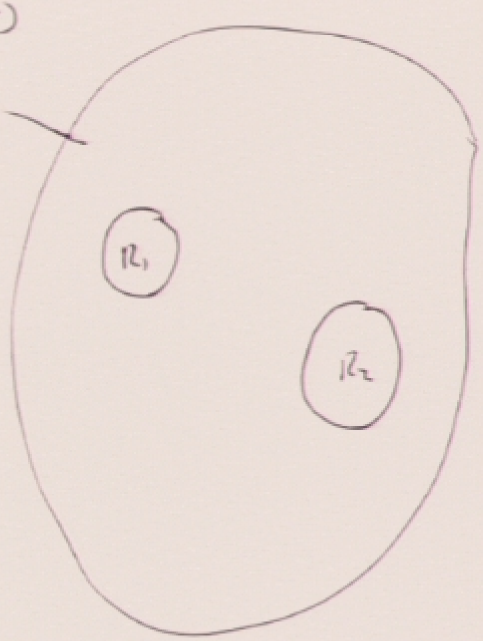
The Causaloid $\underline{\Lambda}$

coming soon to an ~~archi~~ arXive near you.

Theory specifics go into $\underline{\Lambda}$

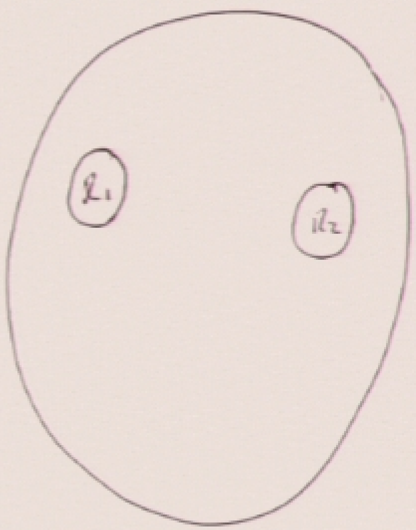
Remaining eqns are same for all theories.

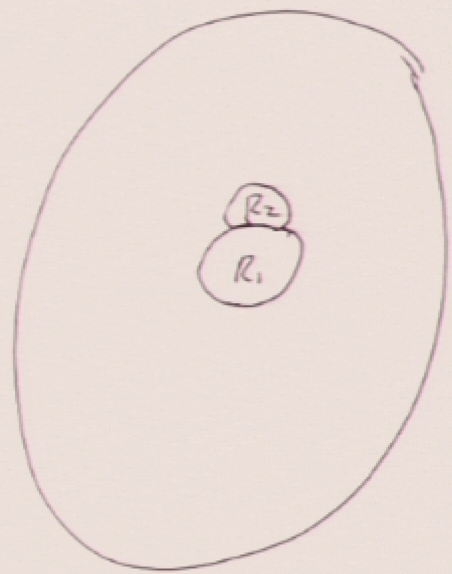
operationally defined "space-time"



$\underline{\Lambda}$ gives a quantitative measure of how number of fiducial measurements is reduced when we go to composite region $R_1 \cup R_2$ for all such regions.

Special cases





'spacelike'

$$K_{12} = K_1 K_2$$

sequential.

$$K_{12} = K_1 = K_2 .$$

Conclusions.

Probability theories can be seen as a general setting for physical theories.

Q.T. is a probability theory and follows from 5 axioms.

Why do this?

1) cozy feeling

2) Beyond

Motivation.

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Historical examples.

Kepler $\longrightarrow$ Newton $\longrightarrow$ beyond ([illegible])

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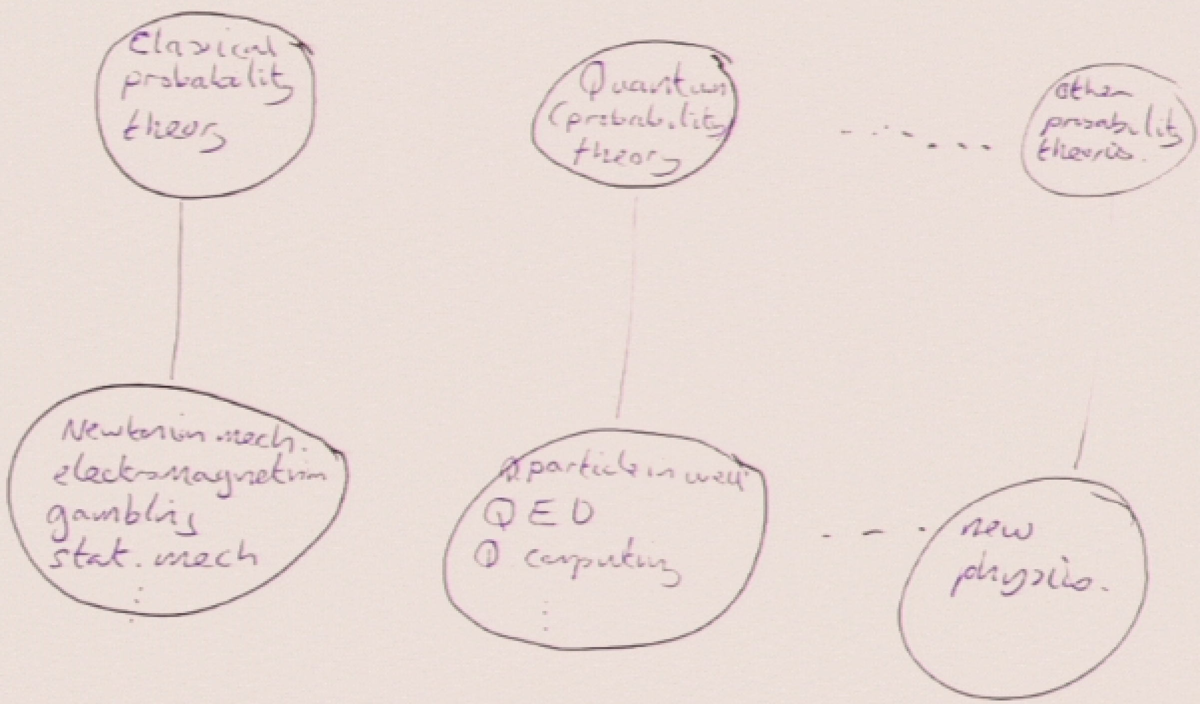
Abstract | reasonable axioms

QT is a probability calculus



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