Title: Information flow in graph states computing

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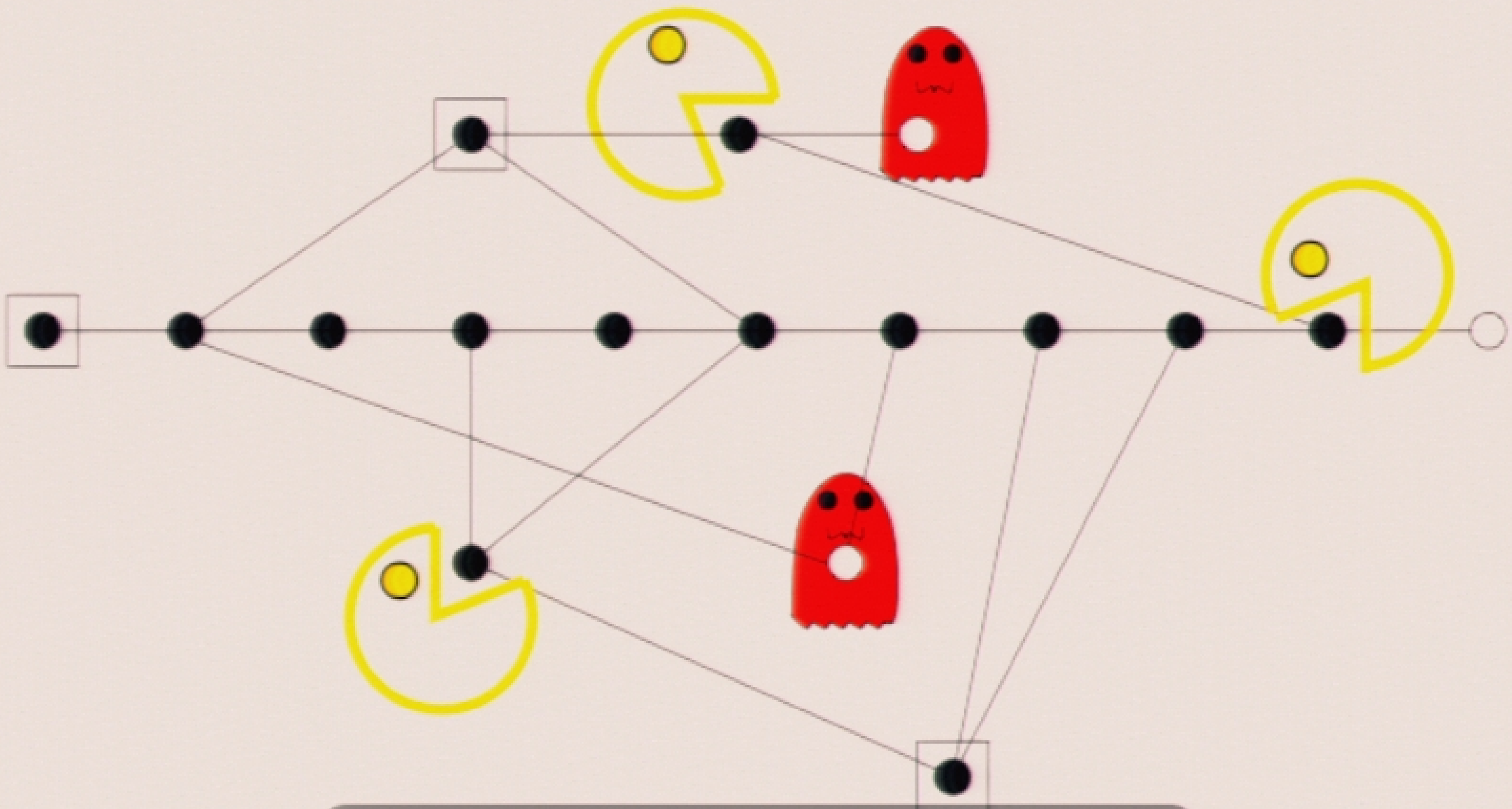
Abstract:

Measurements

Usually measurements are thought of as something one does at the end of the computation, not an integral part of the computation; with measurement based models the situation is very different, measurements play a central rule. However, measuring induces non-deterministic evolutions. This probabilistic drift can be controlled.

Quantum Pacman



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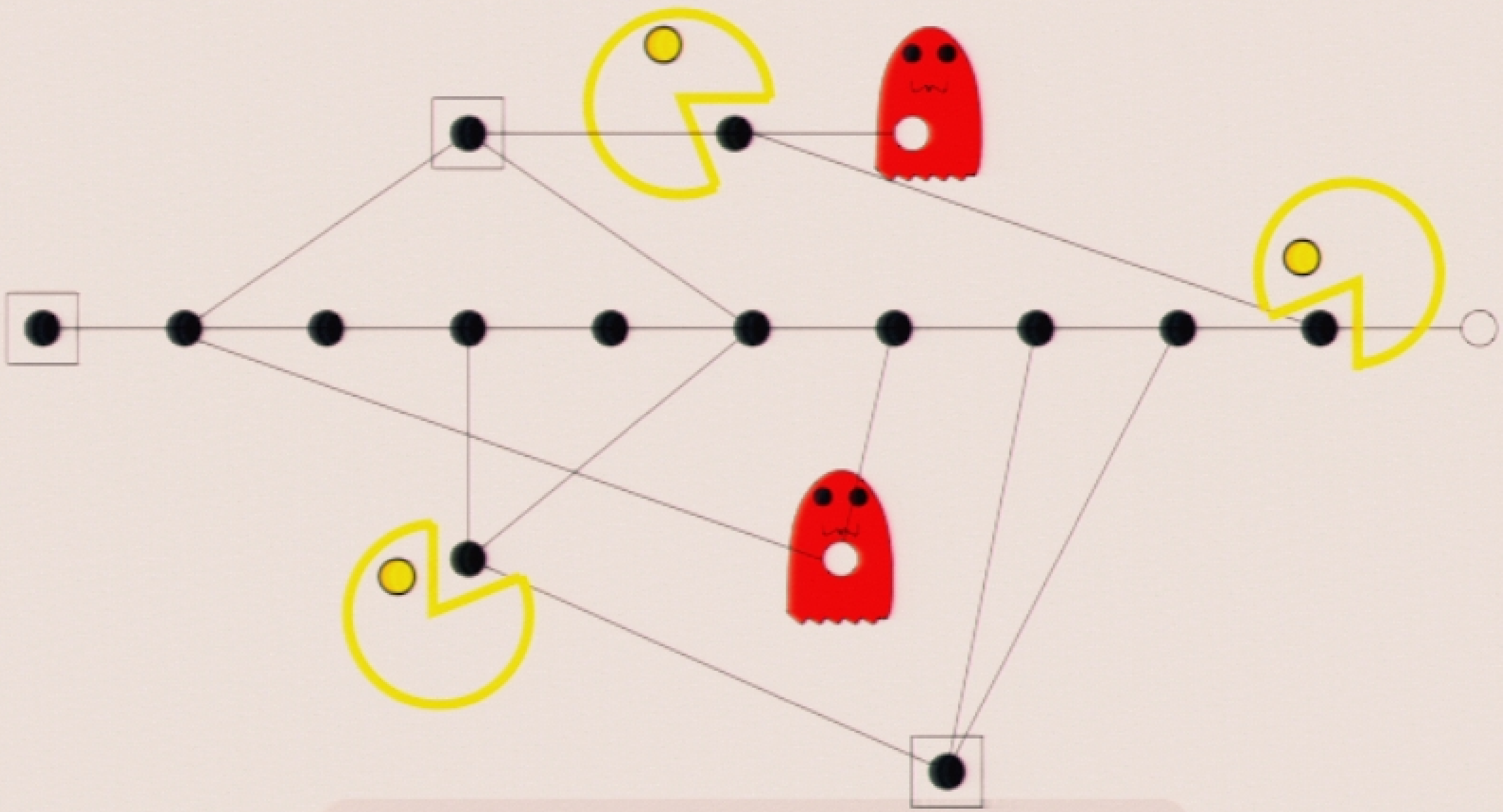

Commands

- N_i prepares qubit i in $|+\rangle$
- M_i^α projects qubit i with $\langle +_\alpha|$ or $\langle -_\alpha|$ *
  — measurement outcome is $s_i = 0$ or 1
- E_{ij} is controlled-Z applied at qubits i and j
- Local Pauli corrections: X_i, Z_i

Feed forward. measurements and corrections commands are allowed to depend on previous measurements outcomes, for example: C_i^s.

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Patterns of computation

The basic computation unit consists of finite lists:

$$(V, I, O, A_n \ldots A_1)$$

* Inputs and outputs may overlap, and this leads to optimization, in the sense of using fewer qubits.

Example: pattern $\mathfrak{H} := (\{1,2\}, \{1\}, \{2\}, X_2^{s_1} M_1^0 E_{12} N_2^0)$ implements Hadamard H.

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Properties

- Close under composition
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4. Determinism

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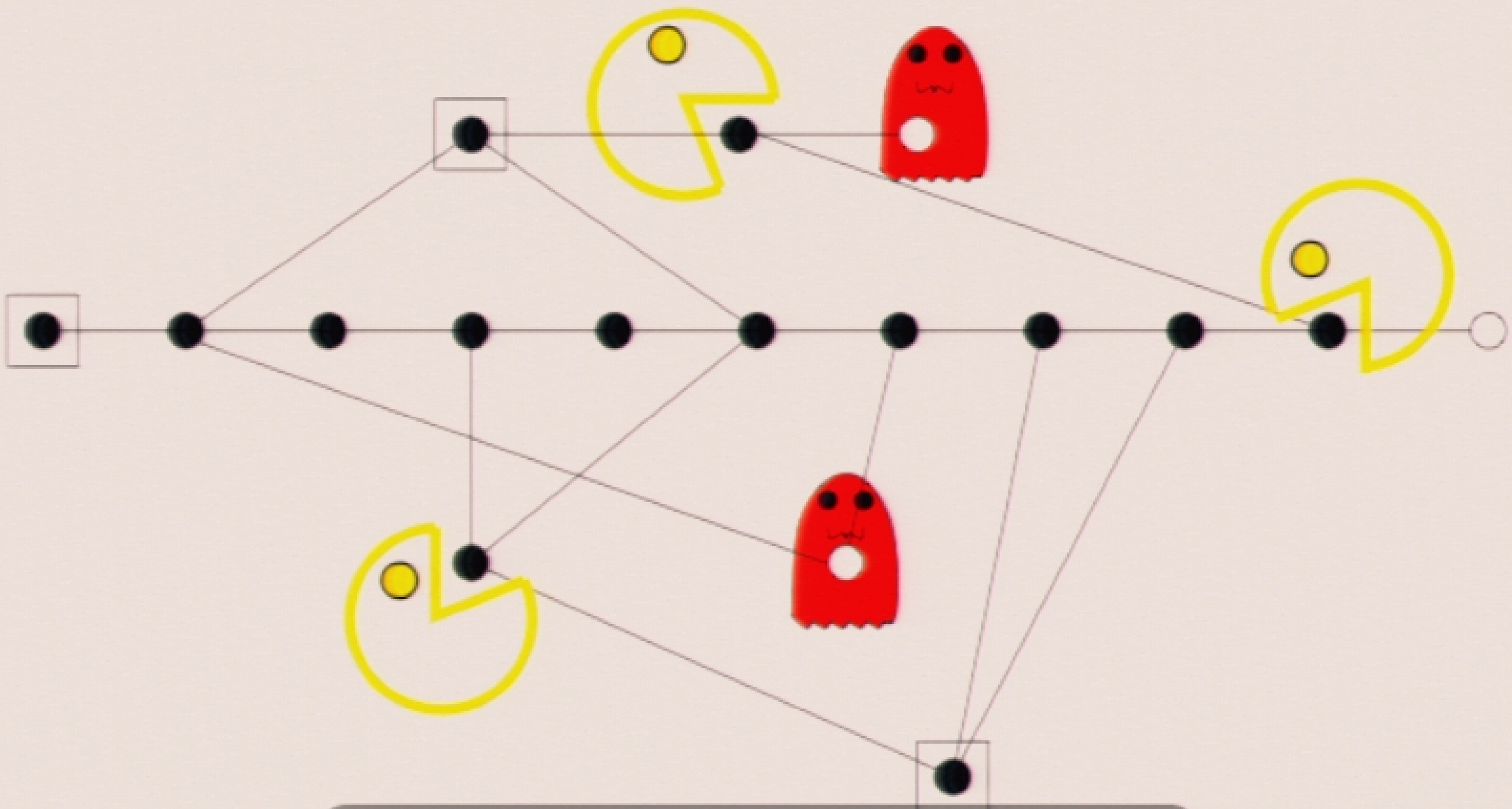

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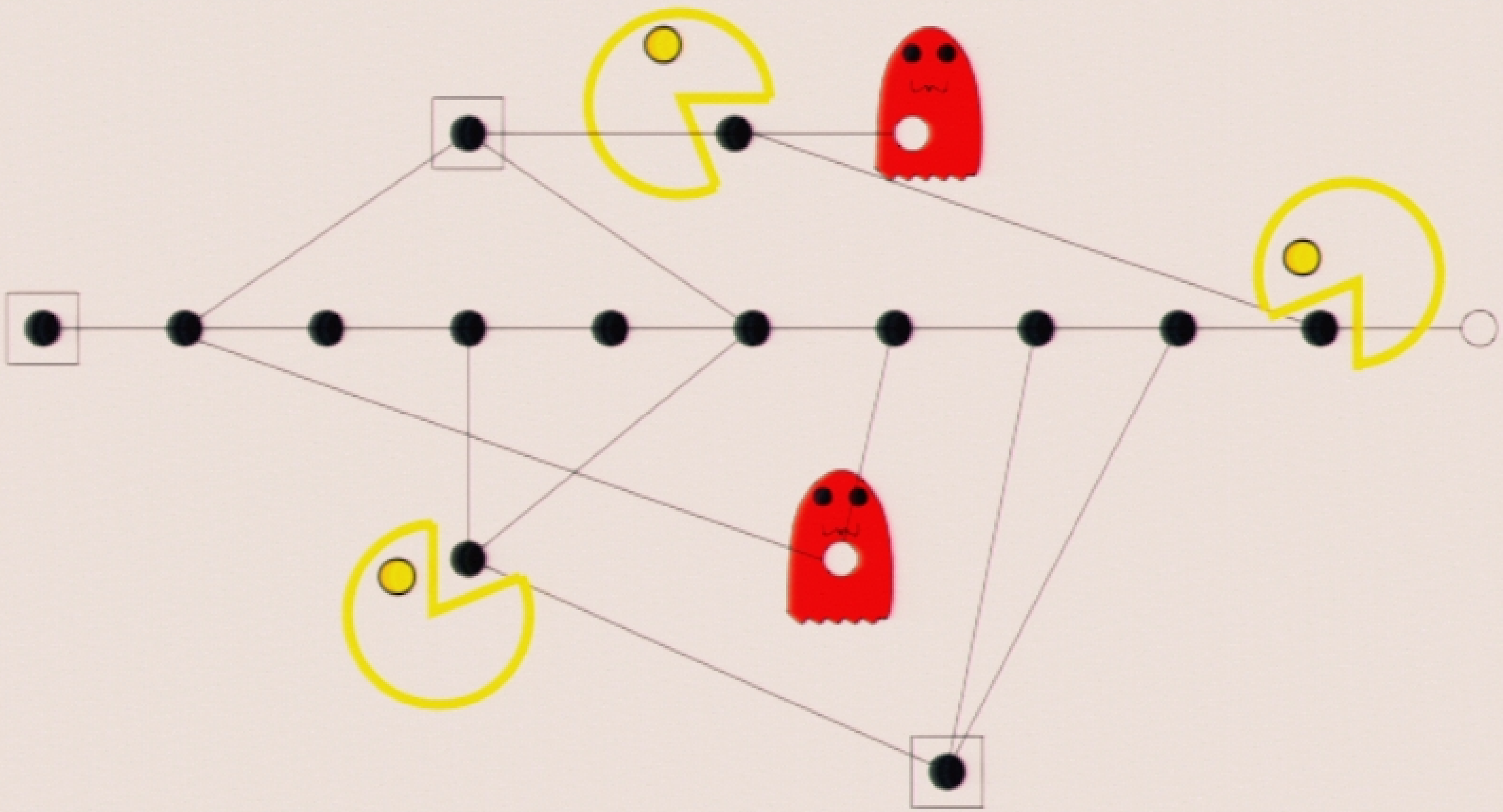

Flow

An entanglement graph with inputs and outputs, (G, I, O), has *flow*, if there exists $f : O^c \to I^c$ such that:

- $(i, f(i)) \in G$,
- there exist a partial order $>$ such that:

a) $f(i) > i$,

b) for all $k \neq i$ neighbour of $f(i)$ in G, $k > i$.

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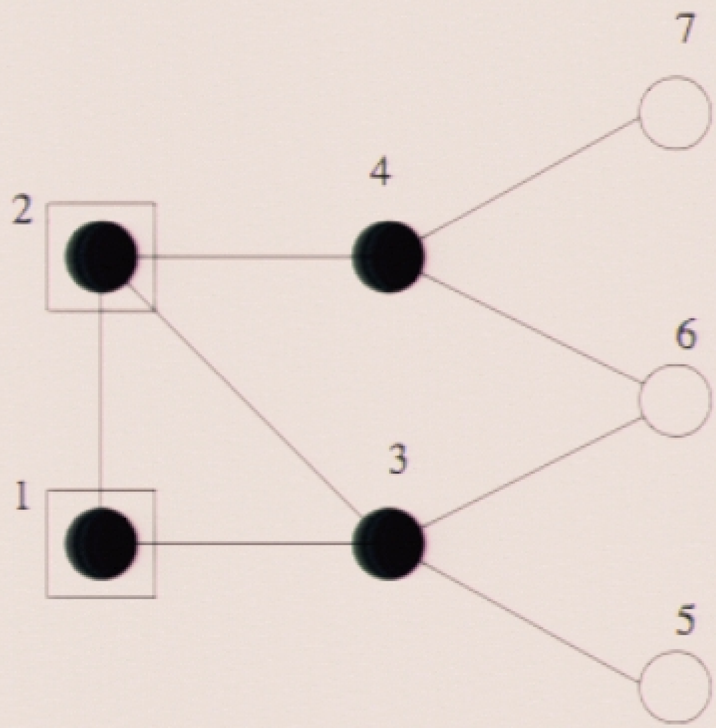

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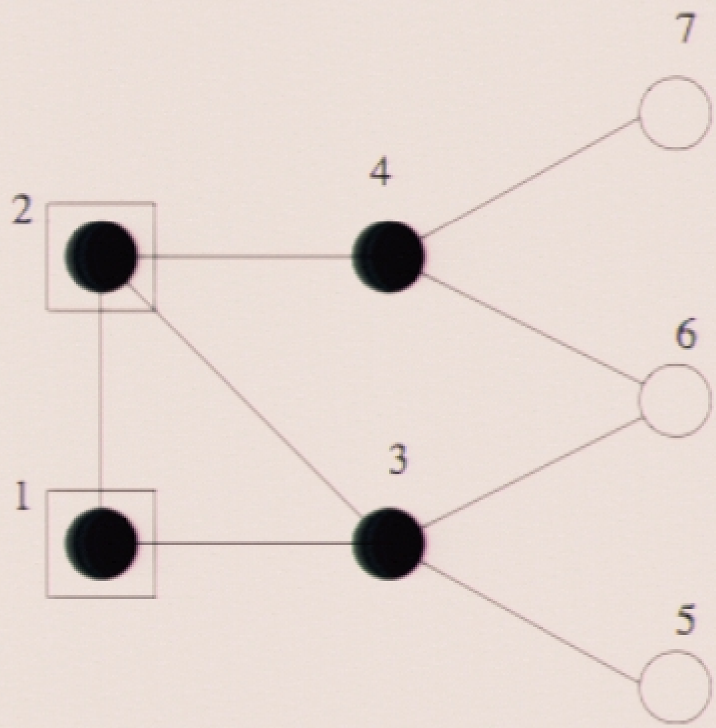

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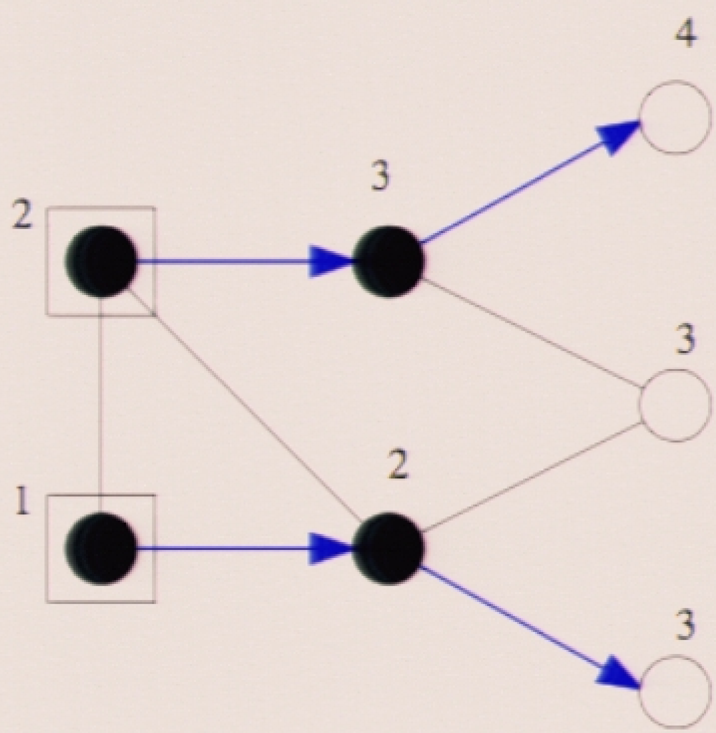

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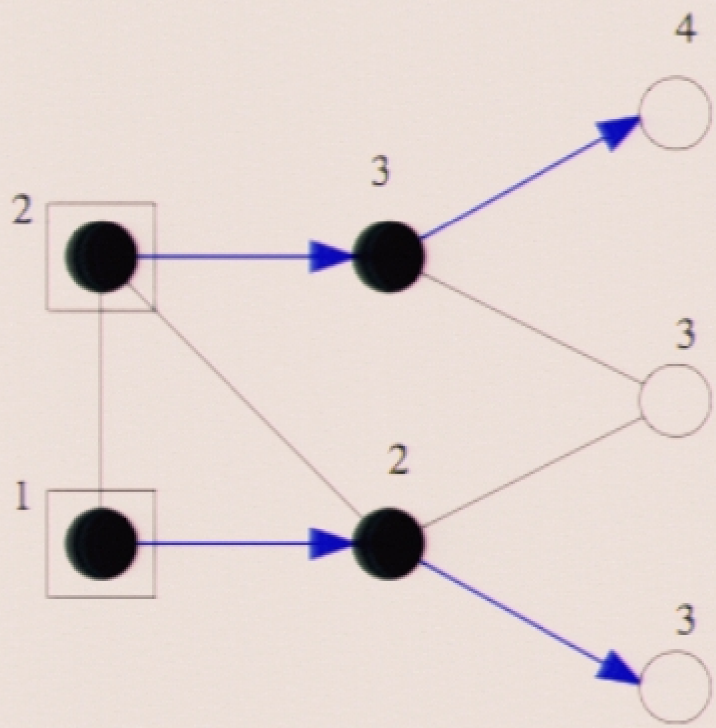
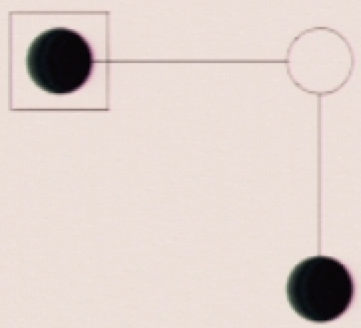


no-Flow example

Here is a geometry with no flow:



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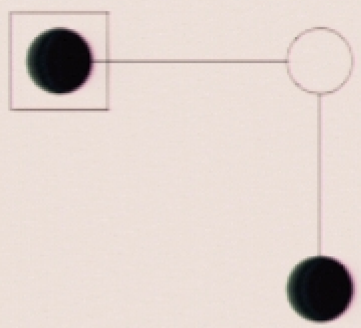

Determinism Theorem

Anachronical measurements $M_i^\alpha Z_i^{s_i} = \langle +_\alpha|_i$ are deterministic, since they are *projections*.

- [Theorem]. If (G, I, O) has flow, then the following pattern is deterministic:

$$\prod_{i\in O^c}(X_{f(i)}^{s_i} \prod_{k\in N_G(f(i))\smallsetminus\{i\}} Z_k^{s_i} M_i^{\alpha_i}) E_G$$

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and computes $\prod_{i \in O^c} \langle +_\alpha |_i \, E_G$

Proof

First we remark three things:

$$\langle +_\alpha|_i = M_i^\alpha Z_i^{s_i} \quad (1)$$

$$Z_i^{s_i} E_{ij} = X_j^{s_i} E_{ij} X_j^{s_i} \quad (2)$$

$$X_j^{s_i}(|+\rangle) = |+\rangle \quad (3)$$

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Proof (continued)

Next we consider the totally positive branch:

$$(\prod_{i\in O^c} \langle +_\alpha|_i)E_G =_1$$

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$$\cdots Z_i^{s_i} E_{if(i)} \prod_{k\neq i, k\in N_G(f(i))} E_{f(i)k} E_{G'_i} =_2$$

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Remarks

The intuition of the proof is that the transfer equation converts an anachronical Z correction at i, into a pair of a 'future' X correction, the one sent to $f(i)$ (so in the future, by condition (a)) and a 'past' X correction, sent to the past, until it reaches a preparation, where it is absorbed because of equation (3).

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Another Proof

Consider geometry $G(V, I, O)$ with flow function f. Let

$$K^{s_i}_{f(i)} = X^{s_i}_{f(i)} \prod_{k \in N_G(f(i))} Z^{s_i}_k$$

be the dependent stablizer operator at vertex $f(i)$. We have:

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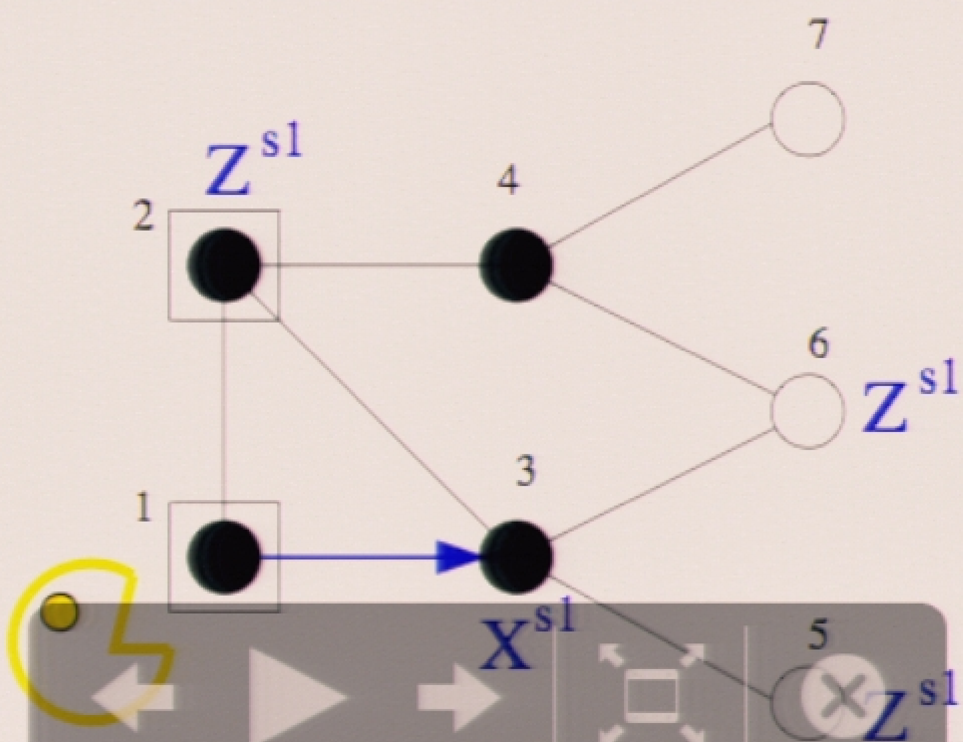

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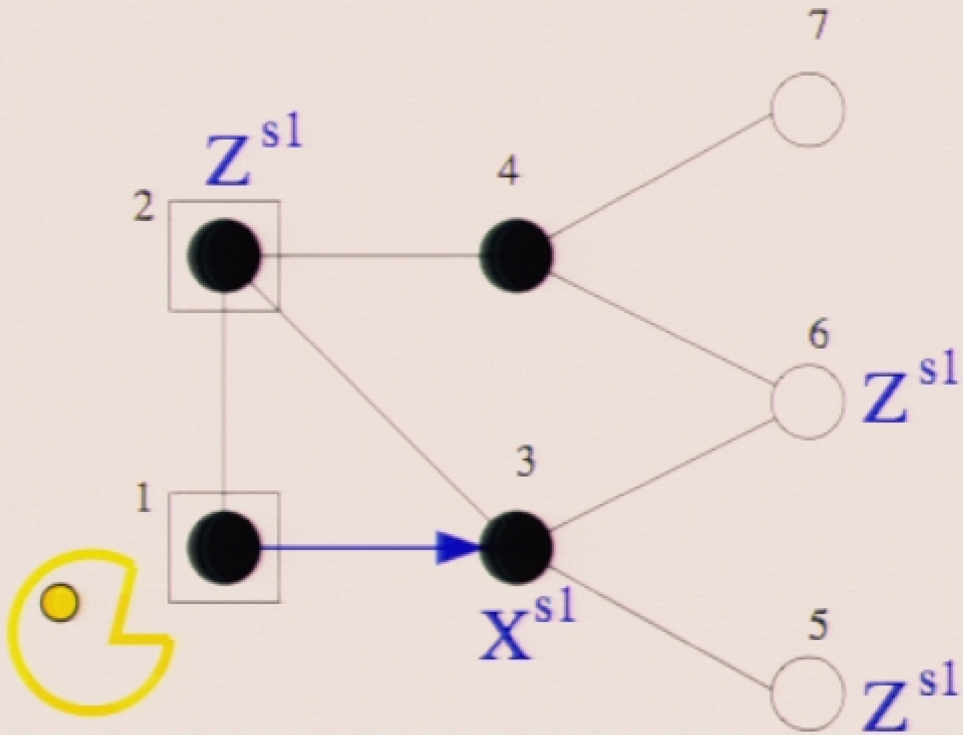

Special case, Y measurement

We required $(i, f(i)) \in G$ but if we measure the quibt i with angle $\frac{\pi}{2}$ we can let $f(i) = i$ since:

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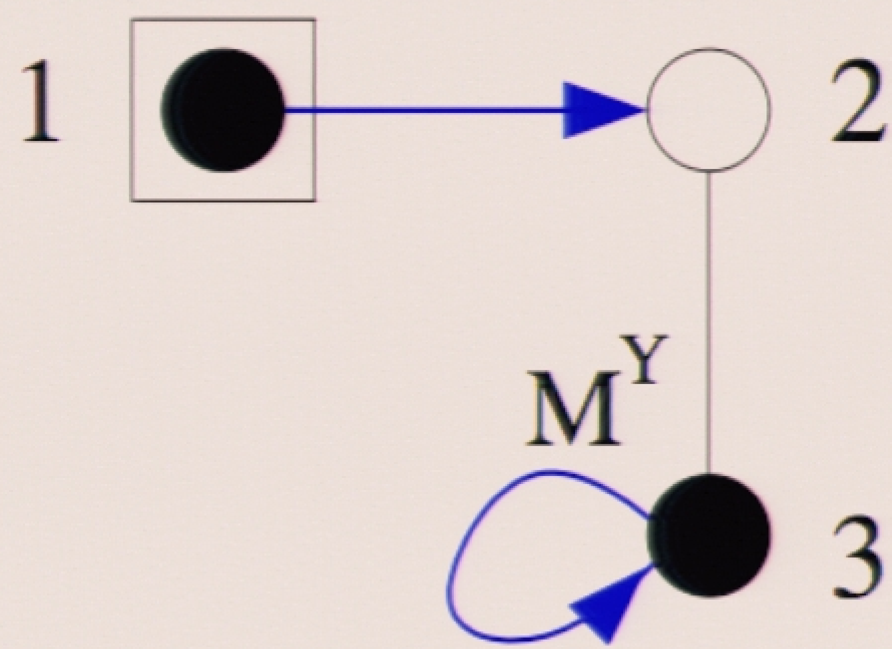

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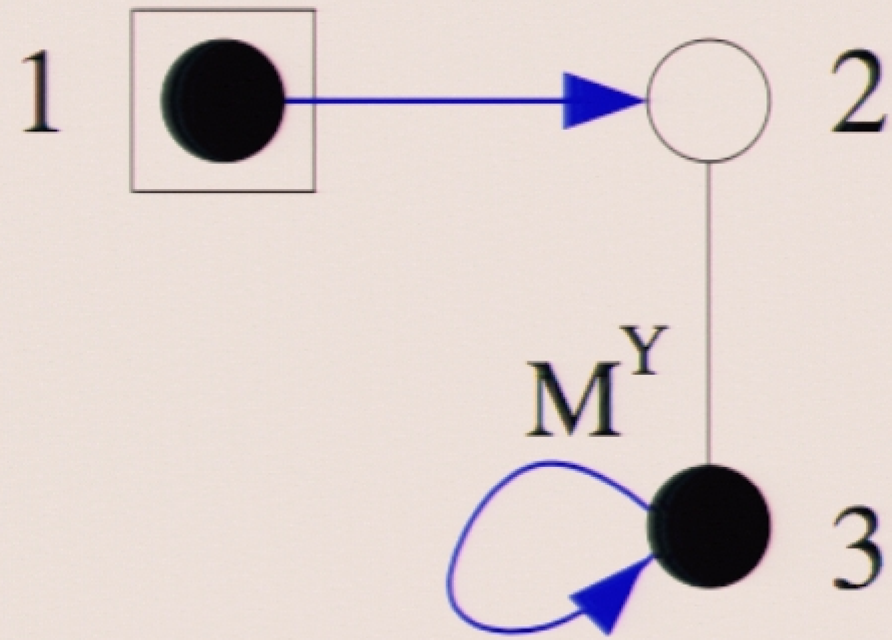

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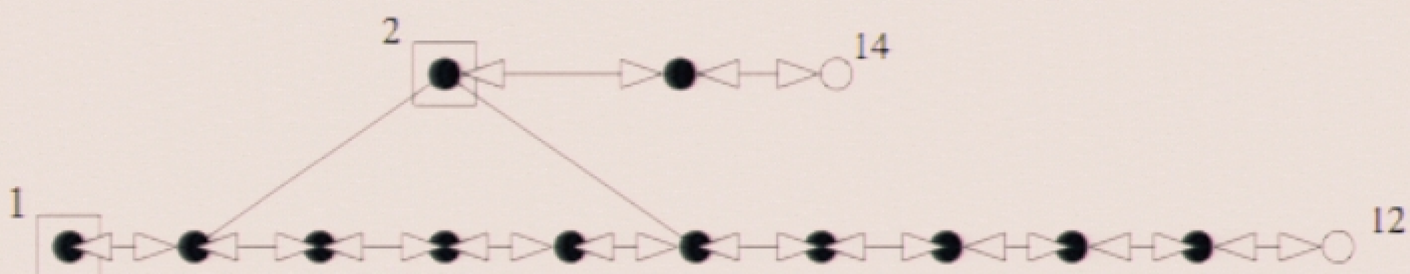
An entanglement graph with inputs and outputs has *bi-flow* if both (G, I, O) and (G, O, I) has flow.



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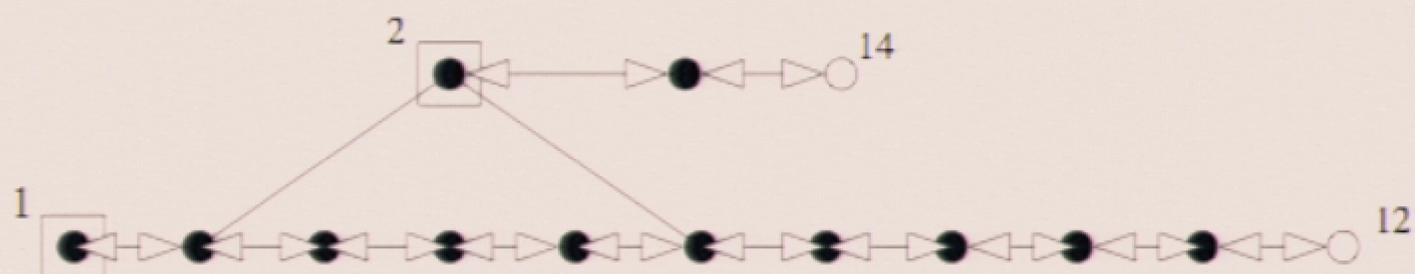

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Proof

- In positive branch:

$$\mathfrak{H}_O \xrightarrow{|+_\alpha\rangle} \mathfrak{H}_V \xrightarrow{U} \mathfrak{H}_V \xrightarrow{\langle +_\beta|} \mathfrak{H}_I$$

where:
— Preparation (adjoint of a projection)
— U is a unitary (including entanglement and corrections)
— Projection

- The *mirror branch* implementing $T^\dagger$ is given by:

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Since $\wedge Z$ and Pauli corrections are self-adjoint, and preparations and projections are symmetric under adjunction, this branch belongs to the *adjoint pattern* $\mathfrak{P}^\dagger$, with inputs and outputs exchanged.

Pattern adjunction (examples)

Hadamard:

$$\mathfrak{H} = X_2^{s_2} M_1^0 E_{12} N_2^0$$

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so $\mathfrak{H}$ is self-adjoint with inputs $\{2\}$, and outputs $\{1\}$.

Likewise for $J(\alpha)$:

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- The combinations of two such graphs has flow
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Further directions

- Abstract condition on N, E, M, C commands for having flow
- Including local complementation operation in the flow search
- The converse theorem
- Full characterization of graph with & without flow

  → Fault tolerant and stability of preparation
  → Blind quantum computing and information security
  → New paradigm for quantum algorithms

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