Title: Quantum algorithm and measurement based models of computation.

Date: Jul 21, 2005 10:15 AM

URL: http://pirsa.org/05070111

Abstract:

What do we want from quantum computation?

* poly vs exp time algorithms !

But: most hard problems we want to solve are NPC and Q. Comp most likely cannot do these in poly time.

! However:

What really is "feasible" computation?

Poly vs exp time mostly just a "mathematical convenience".

Can look for more subtle ("realistic") distinctions, especially inside poly time P.

One such feature:

Parallelisability of (poly time) algorithms.

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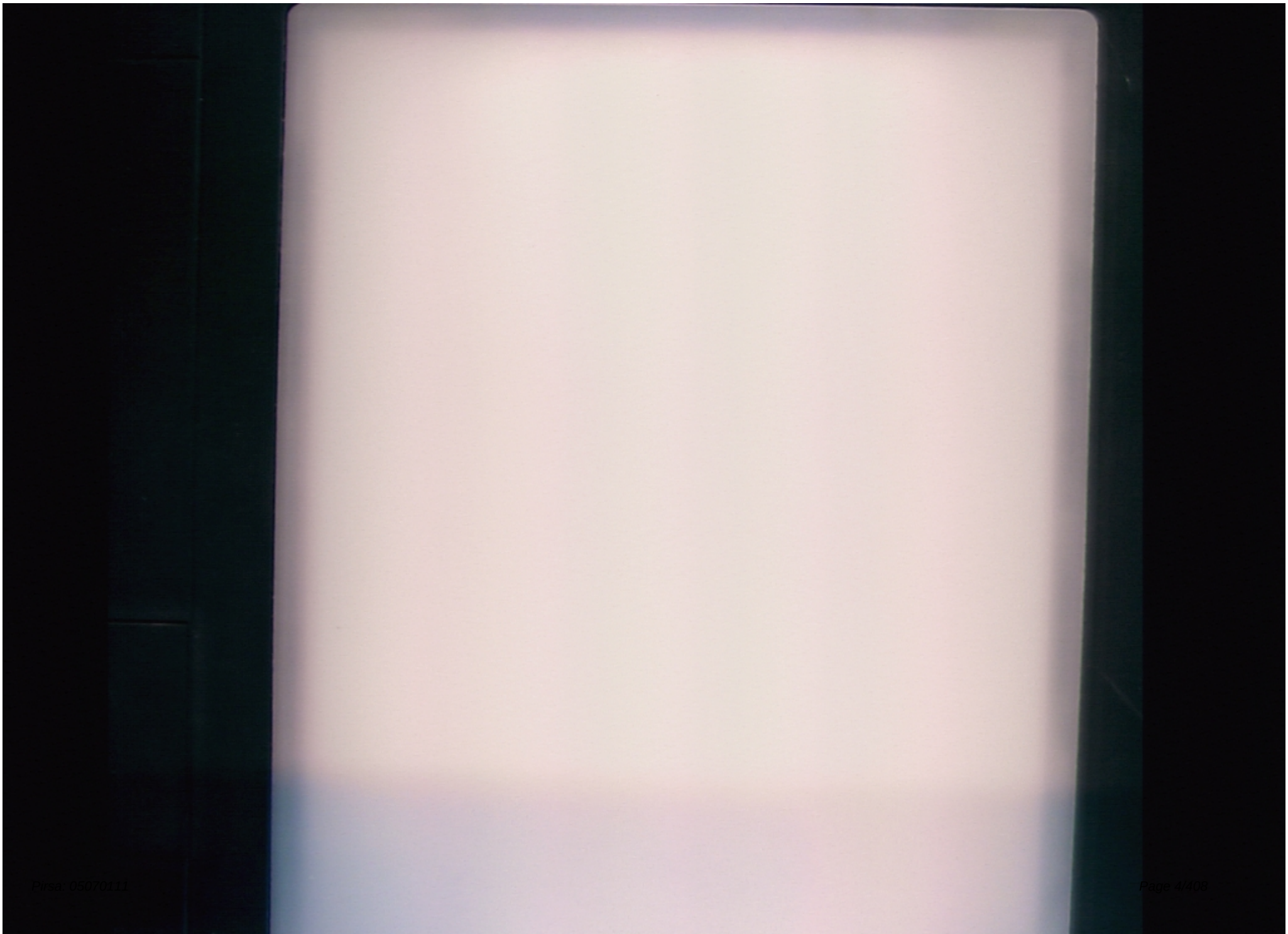


Parallelisability of Algorithms

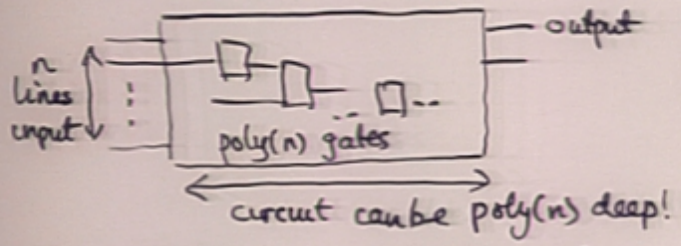
not "quantum [illegible] of doing classical or quantum operations in parallel.

Structure inside P: P vs NC

P: poly sized circuits (can have poly depth)

NC: poly sized circuits with poly log depth ← an exponential benefit of "parallel time"

Polynomial time algorithm, input size n, poly(n) gates



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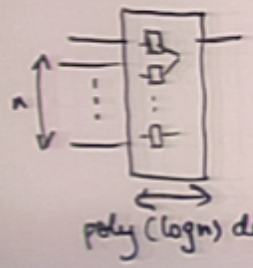

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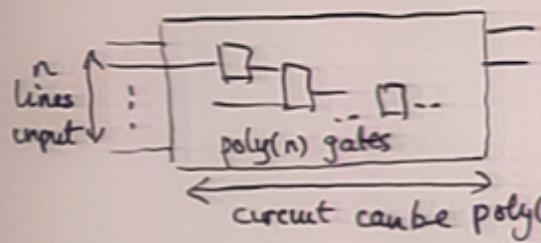

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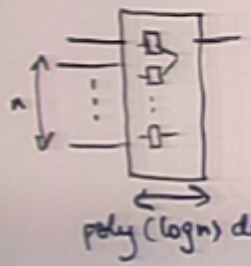

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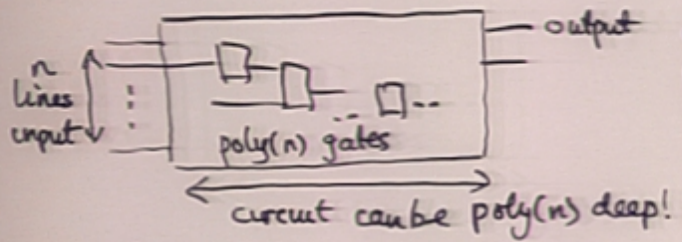

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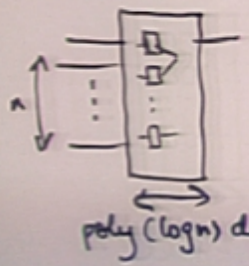

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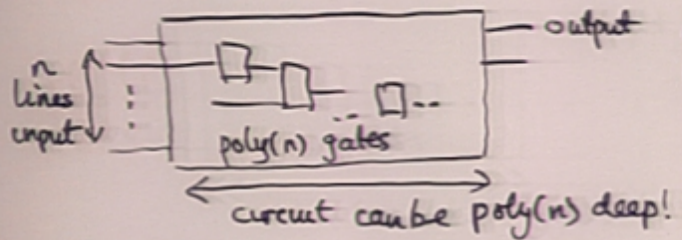

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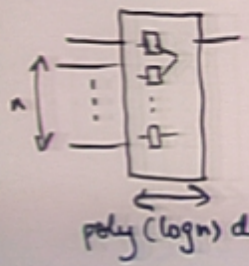

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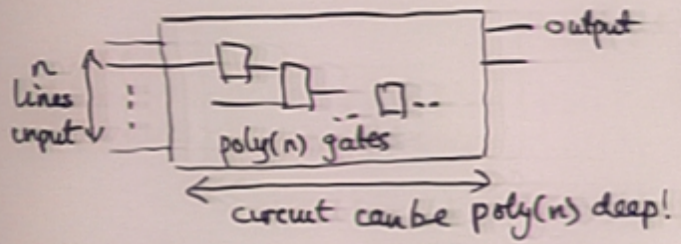

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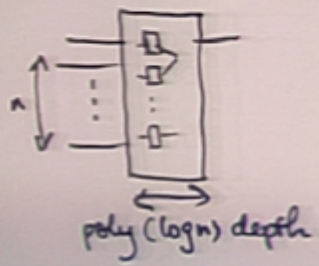

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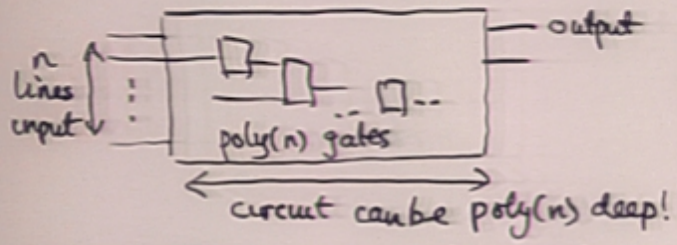

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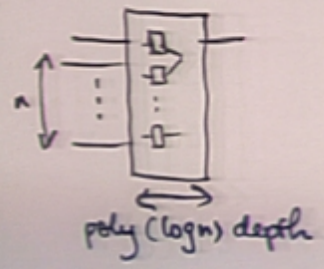

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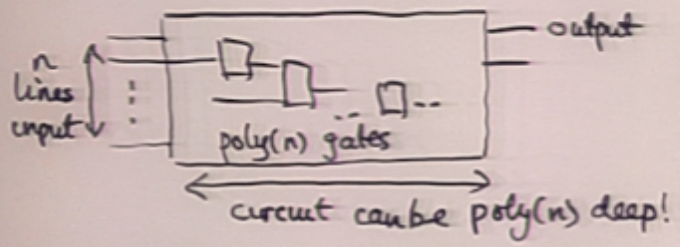

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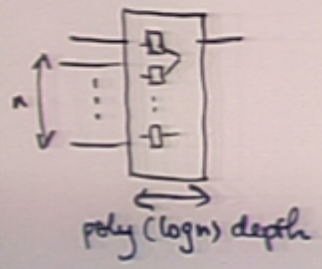

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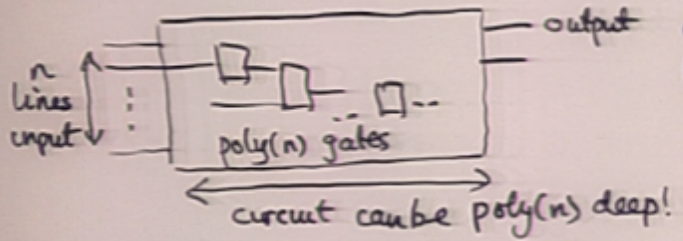

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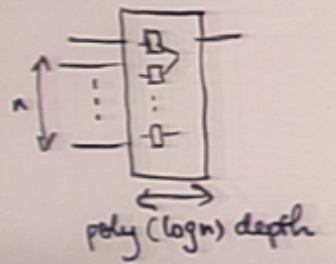

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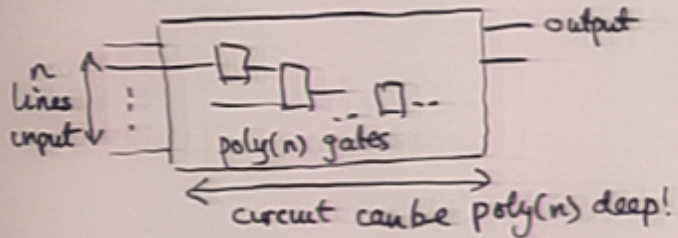

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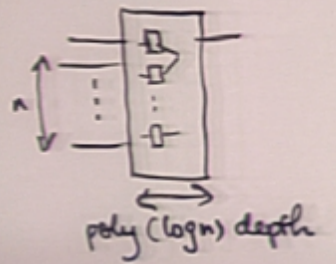

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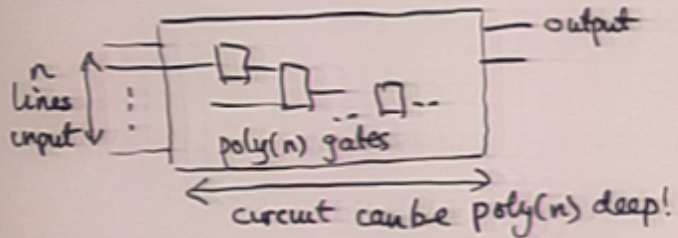

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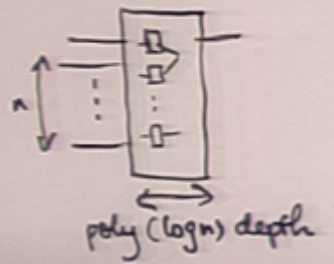

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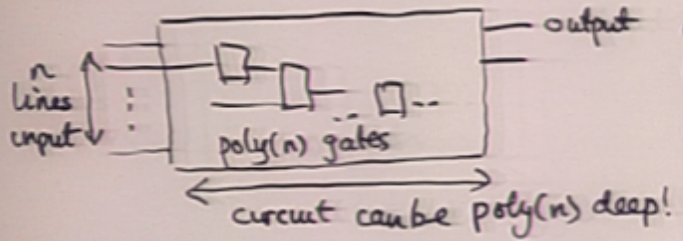

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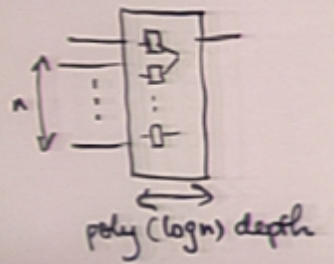

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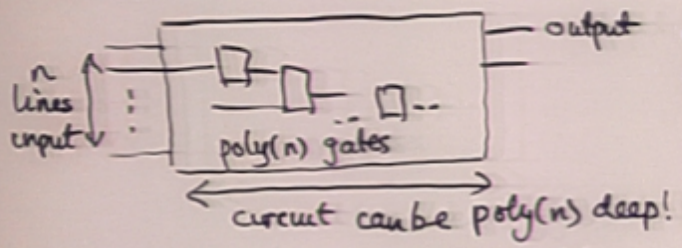

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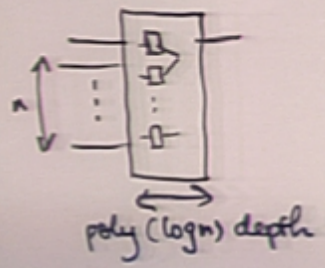

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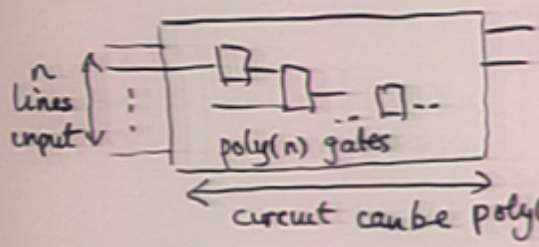

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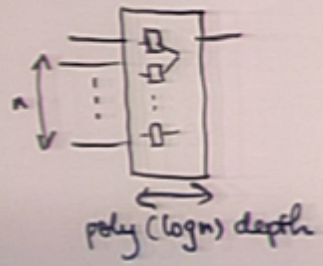

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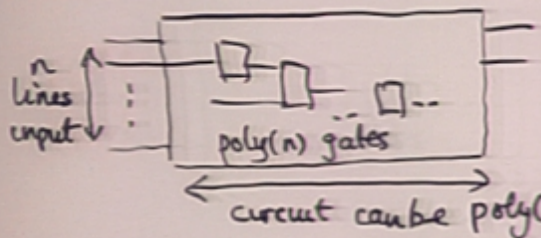

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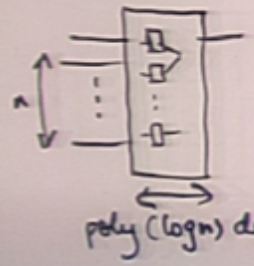

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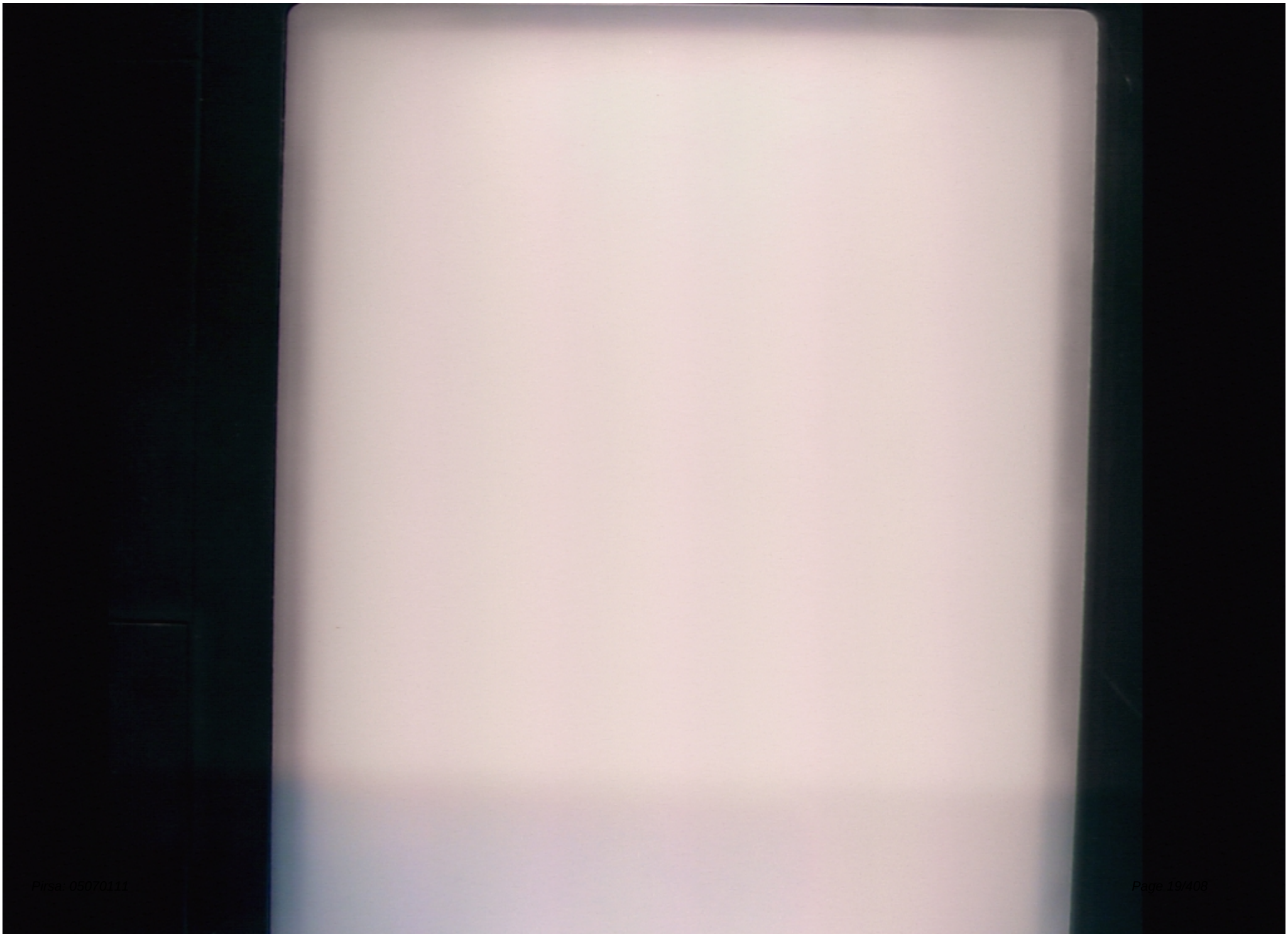

Measurement based models of computation

- Teleportation-based QC (TQC)
  Gottesman & Chuang, Nielsen, Leung & others
- "One way" QC or cluster model (1WQC)
  Raussendorf & Briegel

These models start with large (fixed) entangled state of many qubits (cluster state or grid of EPR pairs) and perform universal computation by measurements only, on the qubits ← ("dual" to gate array concept)

These models are poly-time equivalent to gate array model (each can simulate the other with only poly overhead of resources) – so why study them?

Poly-time equivalence preserves the class of poly-time computations but <u>not</u> more subtle features such as parallelisability!

e.g. in classical computation, one-tape TMs and gate arrays are poly-time equivalent but TMs <u>do not even have</u> any natural notion of parallel computation!

Parallelism in measurement based models:
Mmts on different qubits can be done in parallel
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Pirsa: 05070111

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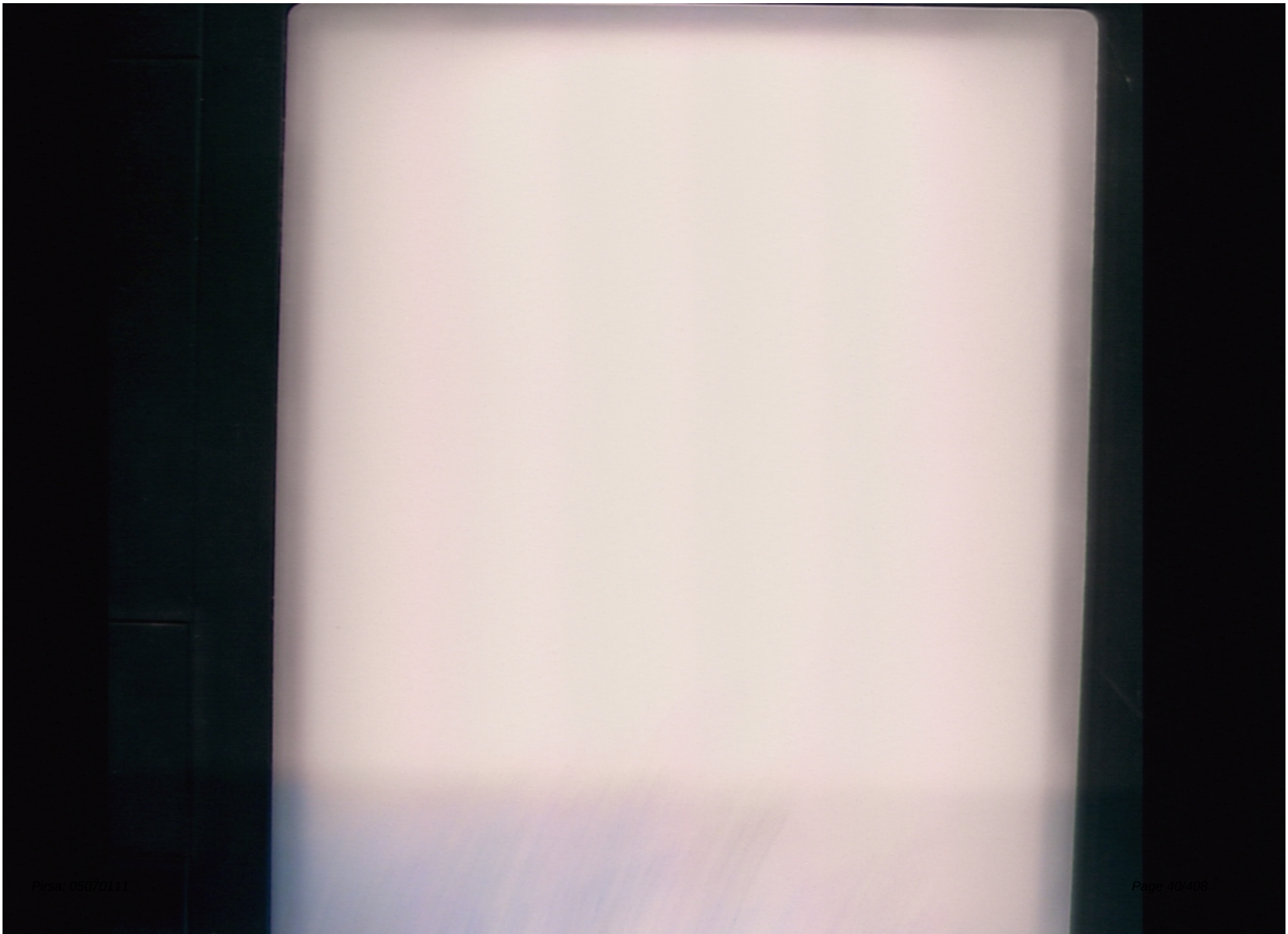

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Conjecture?: any poly time quantum algorithm can be done with $O(\log n)$ layers, interspersed with poly(n) classical computation.

(Here can generalise mmt-based models to allow unitary gates in layers too..)

Cleve & Watrous 2000: Shor's algorithm has this form.
quant-ph/0006004

Can view classical & quantum computation as different resources

Quantu... ... "quantum part" &
"quantum part" with aim of minimising the quantum part.

Gate array: views every step as quantum!

View poly time quantum algorithm as poly time classical algorithm punctuated by injection of results from quantum processes

Want to:

- minimise number of injections
- each quantum process has shallow (const) depth
- quantum state can be passed from one injection to the next (kept stationary while classical comp. being done).

Mmt based models:

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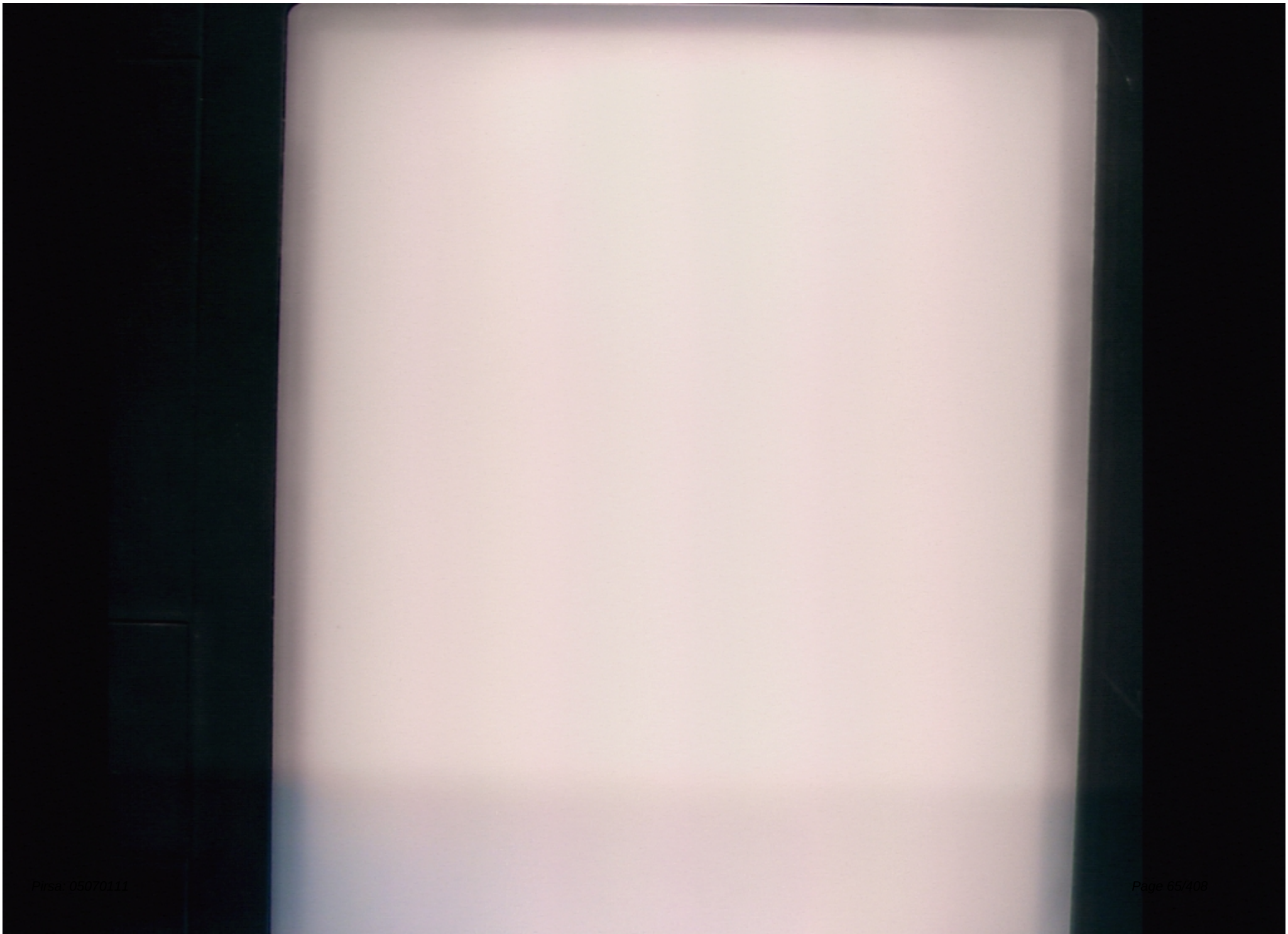

TQC – the ...

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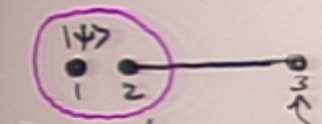

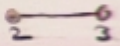

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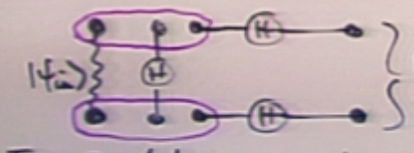

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Pirsa: 05070111

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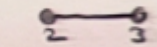

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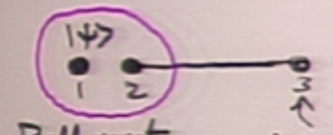

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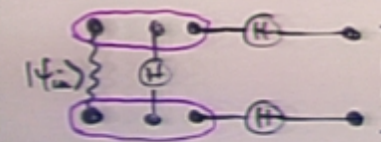

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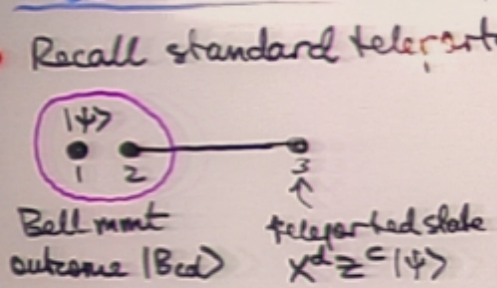

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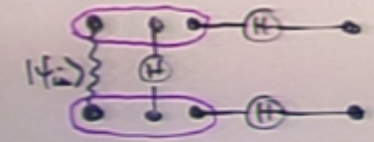

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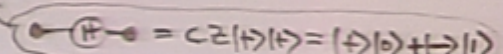

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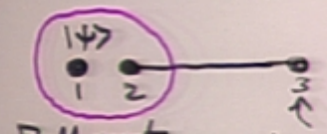

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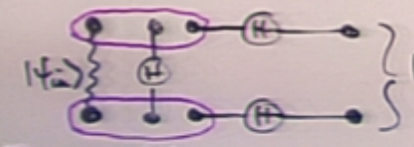

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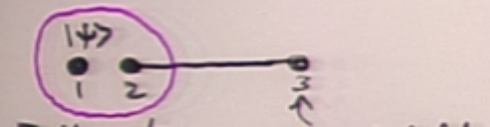

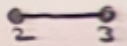

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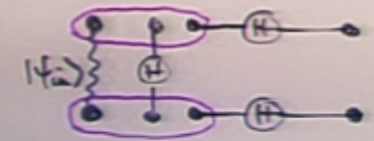

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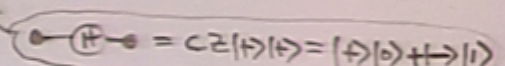

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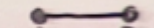

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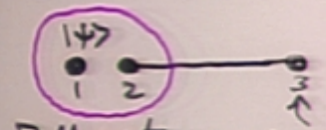

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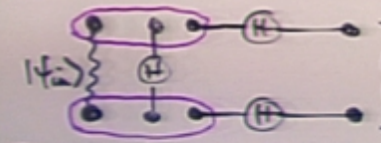

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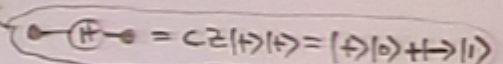

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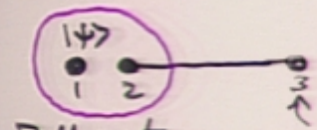

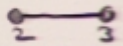

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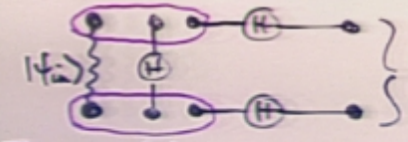

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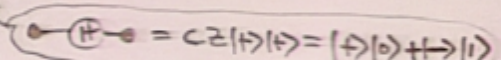

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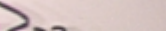

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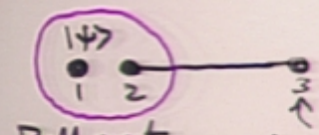

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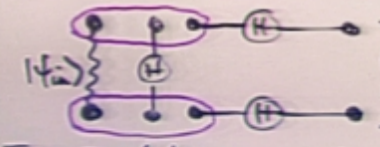

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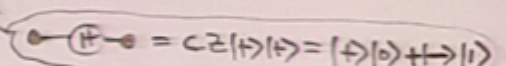

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Pirsa: 05070111

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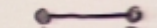

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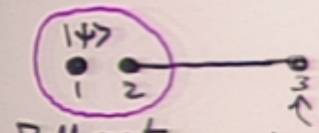

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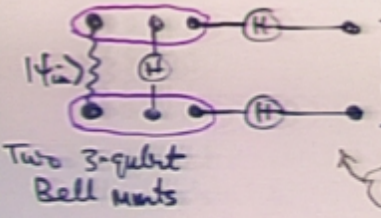

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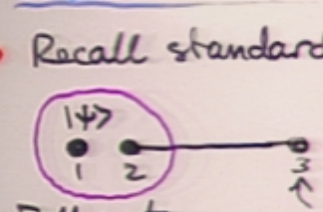

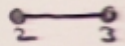

is $|B_{00}\rangle_{23}$ (or any max entg state)

Bell basis

$|B_{00}\rangle = |00\rangle + |11\rangle = I \otimes I$ (—•)

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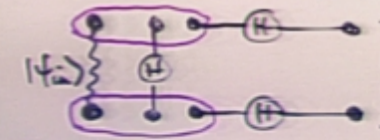

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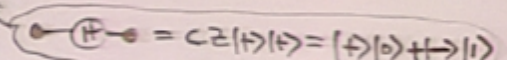

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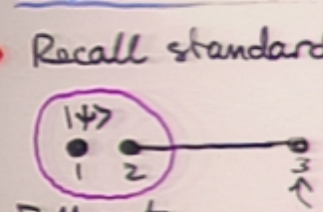

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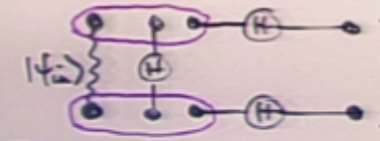

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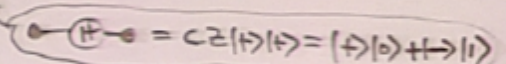
Two 3-qubit Bell mmts



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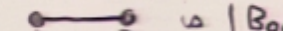

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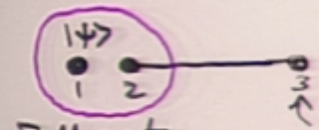

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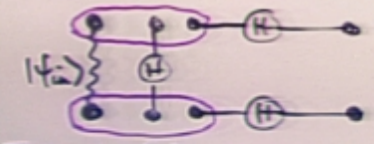

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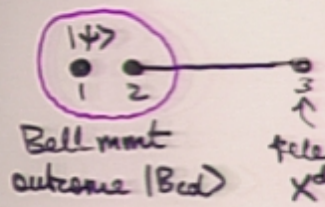

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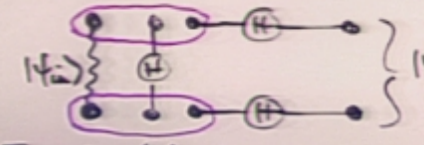

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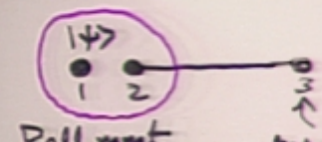

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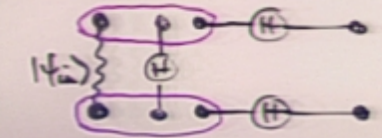

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Pirsa: 05070111

TQC – the basic idea

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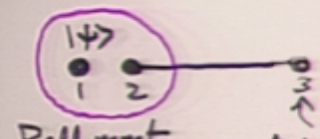

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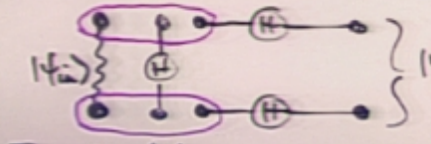

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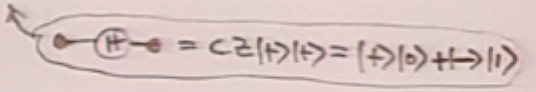

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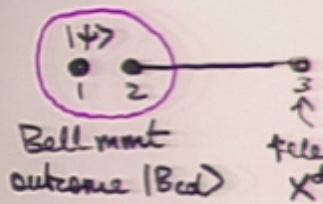

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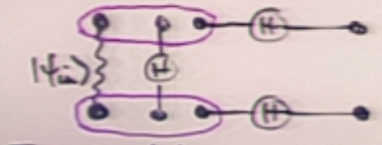

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Pirsa: 05070111

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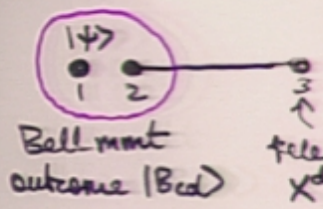

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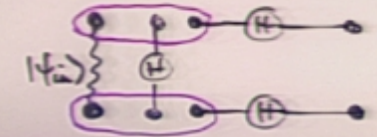

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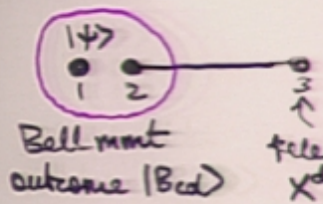

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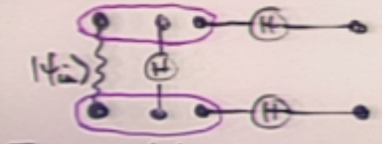

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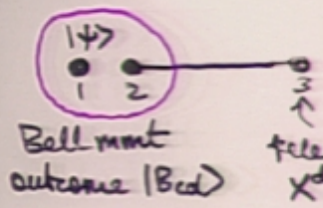

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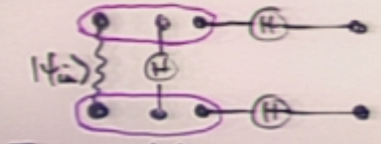

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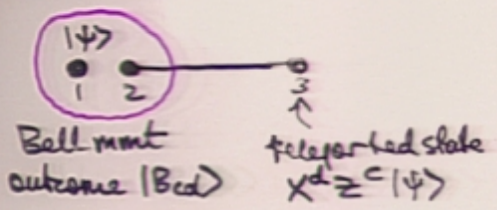

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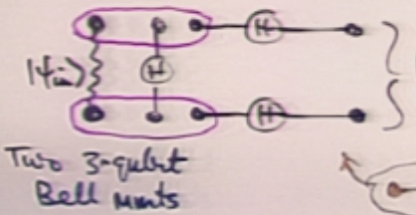

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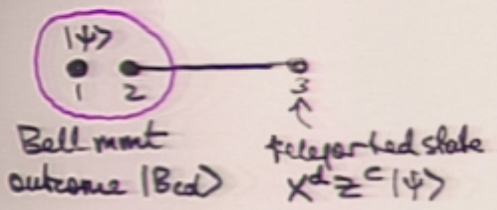

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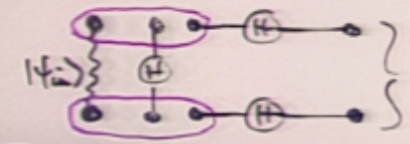

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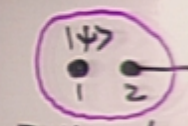
Perm: 05070111

- Recall standard teleportation 2 — 3 is $|B_{00}\rangle_{23}$ state



Bell msmt outcome $|B_{cd}\rangle$

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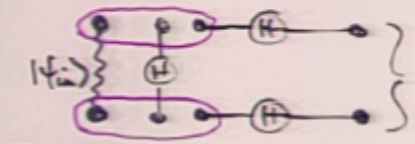

Two 3-qubit Bell msmts

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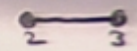
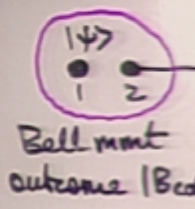
- Recall standard teleportation  is $|B_{00}\rangle_{23}$ state)



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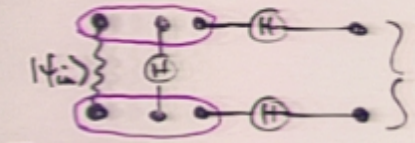

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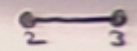
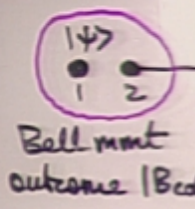
- Recall standard teleportation  is $|B_{00}\rangle_{23}$ state)



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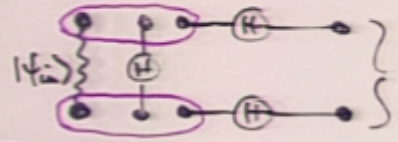

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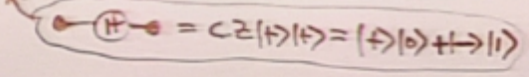

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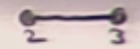
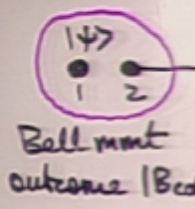
- Recall standard teleportation  is $|B_{00}\rangle_{23}$ state)



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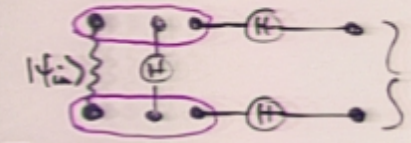

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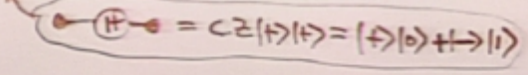

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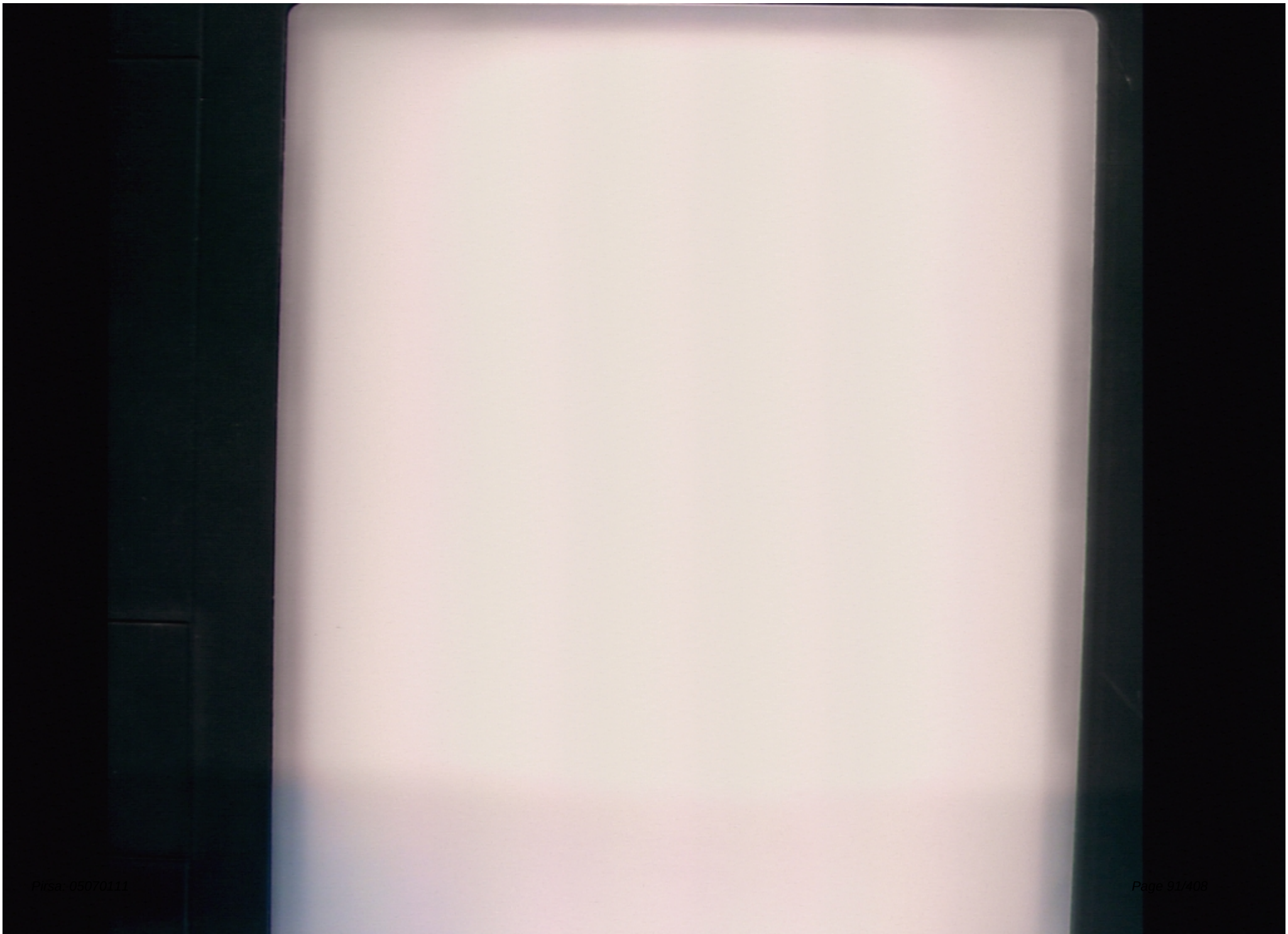

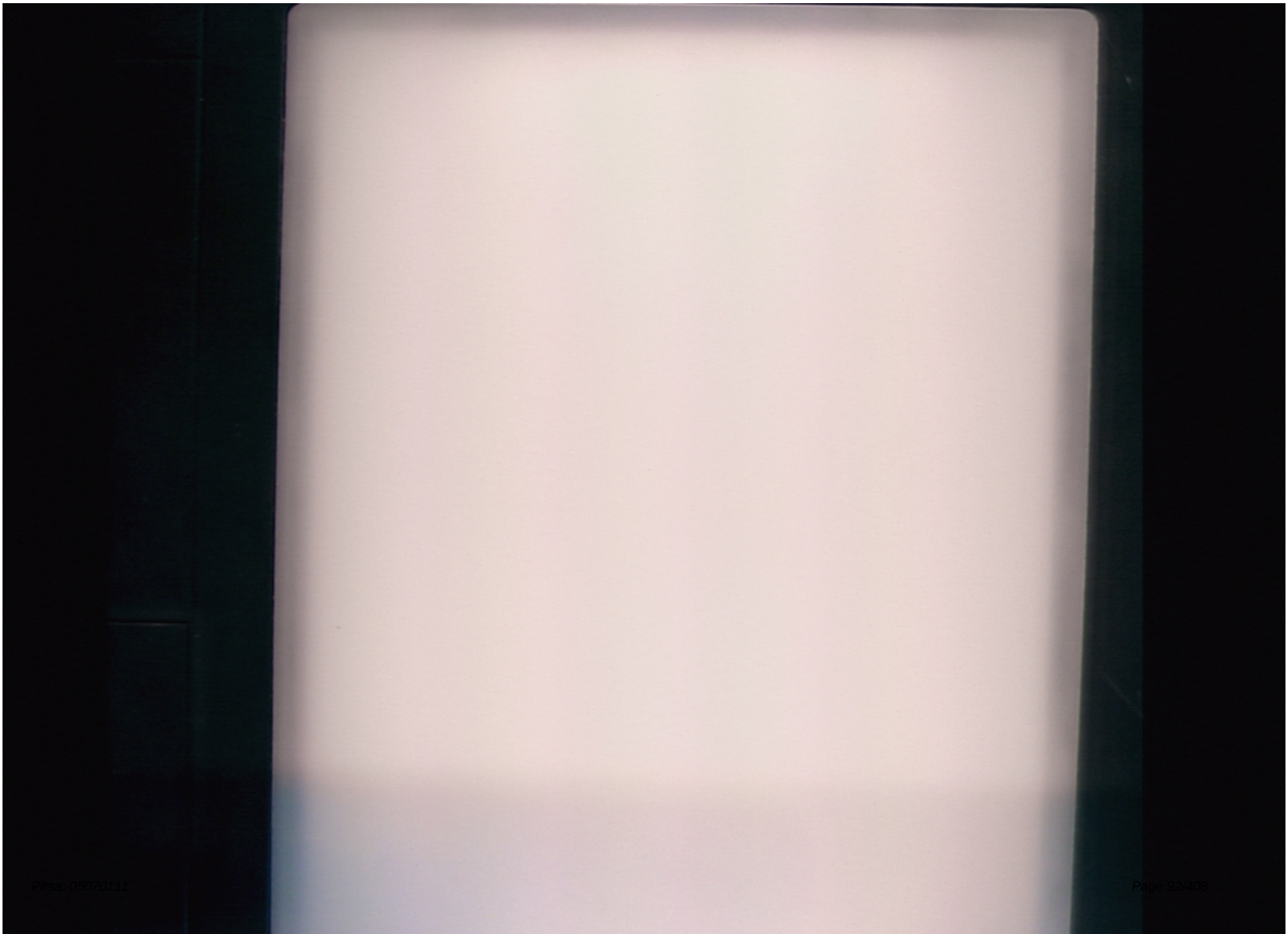

Universal simulation

Want to implement arbitrary sequences of gates from a universal set such as

$$\{ CZ,\ R_x(\theta) = e^{-i\theta X},\ R_z(\varphi) = e^{-i\varphi Z} \}$$

But: for $\ldots U_3 U_2 U_1 |\psi\rangle$

Successive teleportations give instead

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with P_i's pauli's depending on measurement outcomes.

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Pirsa: 05070111

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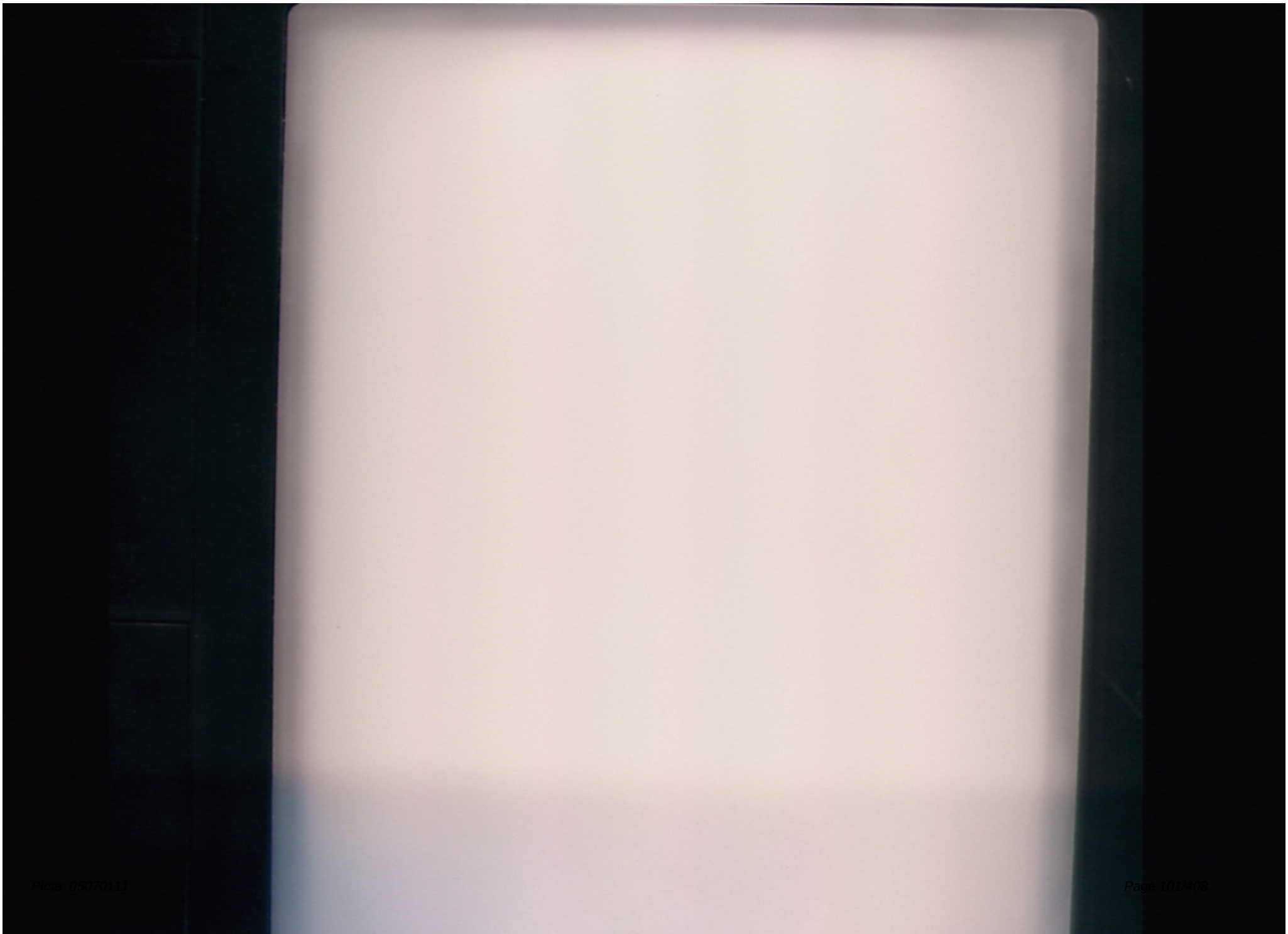

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eg. 1-qubit gates. Pauli "propagation" relations:

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Now commuting $Z^a X^b$ to the left through $R_z(\beta)$ has:

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Continuing in this way adapting measurement bases to earlier measurement results gives

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Perg: 05070111

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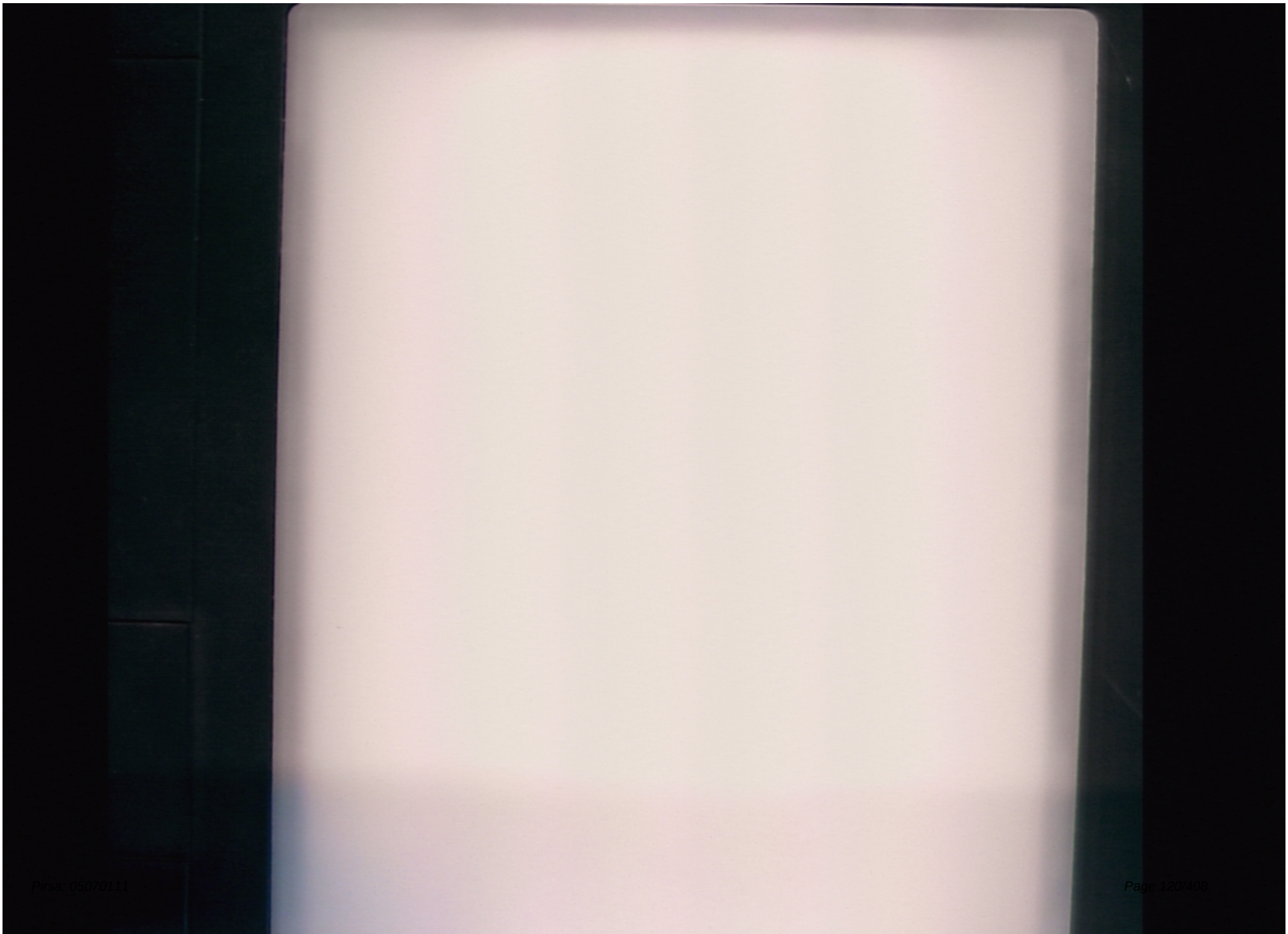
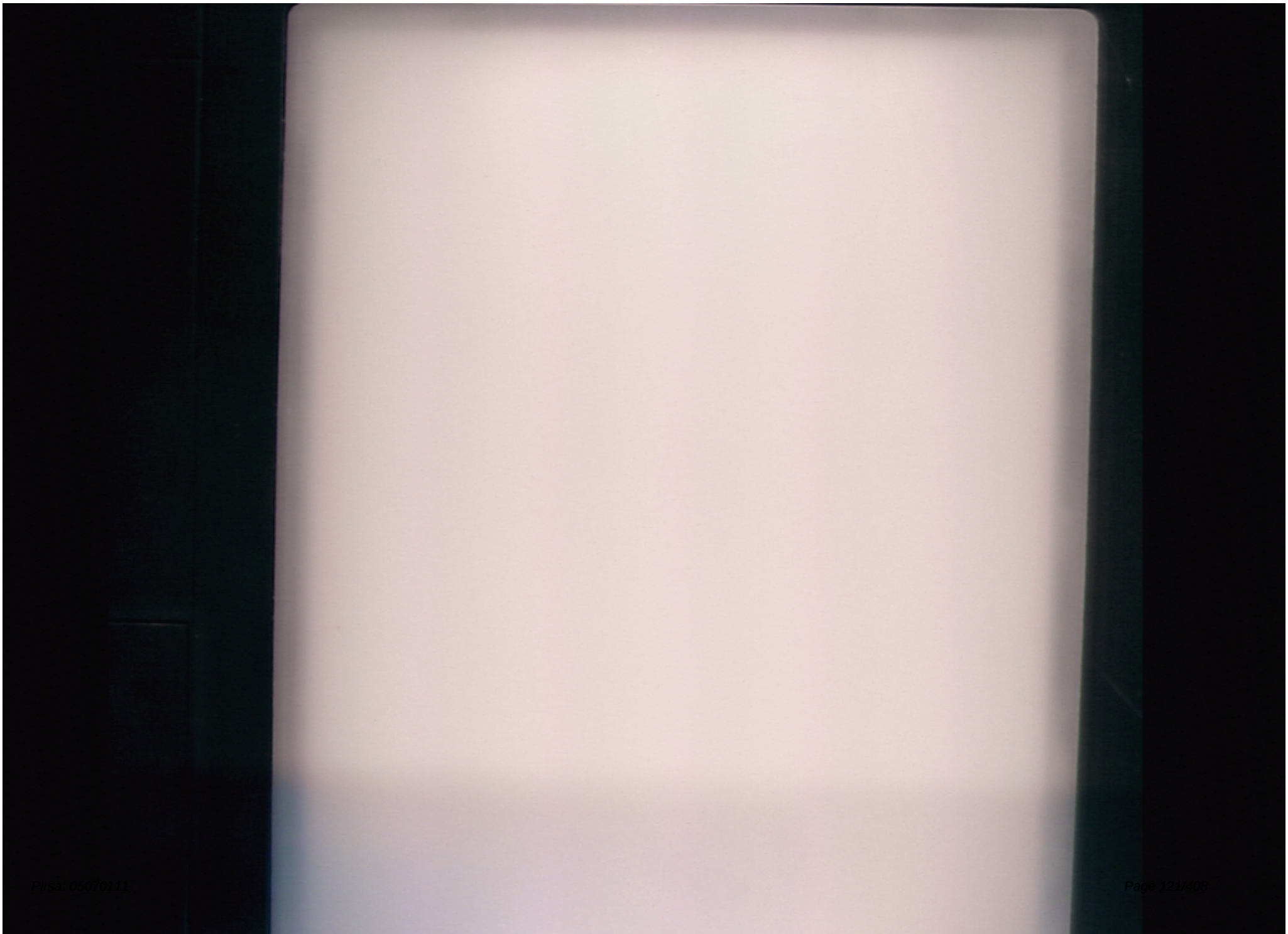
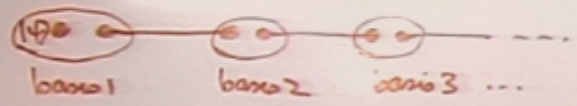




Parallelism for measurement operations

Recall basic ...



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Clifford Operations (on n qubits)

Pauli group P_n on n qubits: generated by

$\pm I, \pm i, X, Z$ on each qubit

e.g. P_3 has $X \otimes X \otimes Z$, $iZ \otimes Y \otimes I$, $I \otimes I \otimes X$ etc

C is a Clifford operation on n qubits if

$$C P_n C^{-1} = P_n \quad \text{i.e.} \quad C P_n = P_n C$$

i.e. For every Pauli $P \in P_n$ there is another Pauli $P' \in P_n$ such that $CP = P'C$

so can propagate Paulis while C stays the same!

Examples: H: $HX = ZH$ $HZ = XH$ so $HY \simeq YH$

2 qubit CX and CZ are Clifford

eg have $(CZ)(X \otimes I) = (X \otimes Z)(CZ)$ etc.

Theorem: The Clifford group on n qubits is generated by (i.e. all possible gate arrays on n qubits of:)

$Z, H, P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, CX.

Pirsa: 05070111

Parallelisation of [illegible] operations

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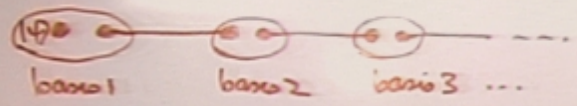

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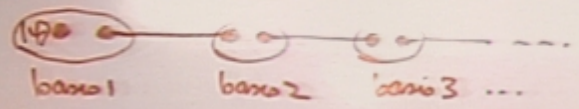

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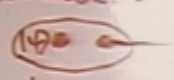

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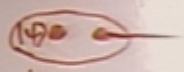

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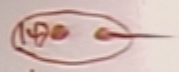

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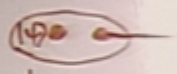

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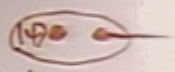

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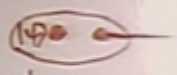
Pirsa: 05070111

Parallelisability Feature: Clifford Operations

Recall basic circuit



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Clifford Operations (on n qubits)

Pauli group P_n on n qubits: generated by

$\pm I, \pm i, X, Z$ on each qubit

eg. P_3 has $X\otimes X\otimes Z$, $iZ\otimes Y\otimes I$, $I\otimes I\otimes X$ etc

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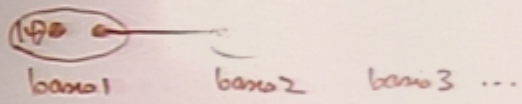

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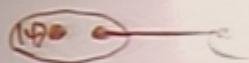

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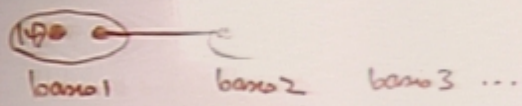

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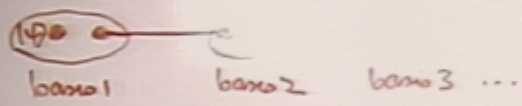

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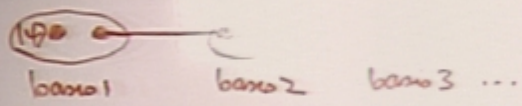

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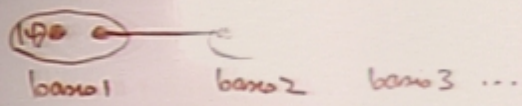

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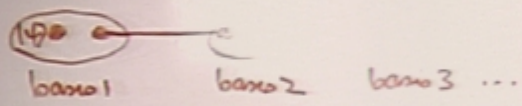

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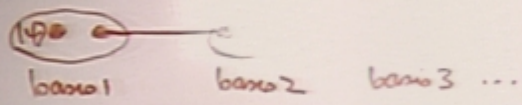

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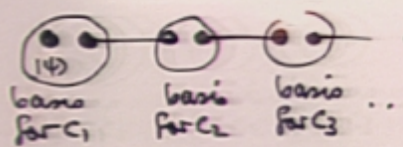
In measurement model, to apply sequence of Clifford operations

$C_k \cdots$



we can apply all measurements simultaneously in parallel (no adaptation) to get

$P_k C_k \cdots \quad P_2 C_2 P_1 C_1 |\psi\rangle \qquad P_1, \ldots P_k$ Paulis

Then commute P's through all C's to get

$X_n^{a_n} Z_n^{b_n} \cdots X_1^{a_1} Z_1^{b_1} (C_k \cdots C_2 C_1)|\psi\rangle$

i.e. Clifford operation is done in depth 1 process
(also starting state ——— can be made in const depth)

But:

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in its depth here

Pisa: 05070111

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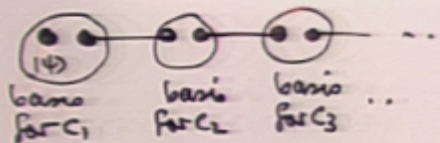

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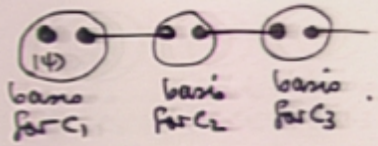
$C_k \cdots C_2 C_1 |\psi\rangle$ of n qubits



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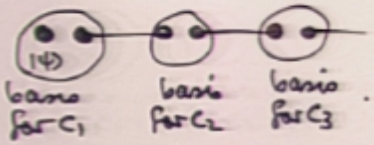

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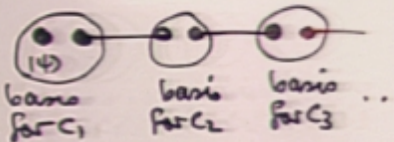

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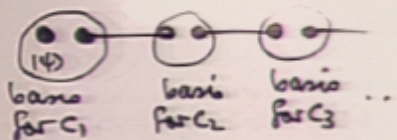

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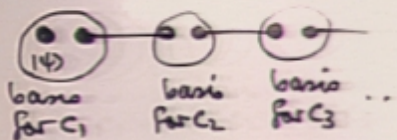

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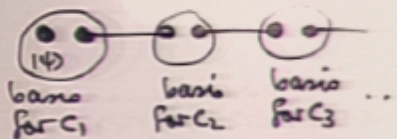

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in its depth here

In measurement model, to apply sequence of Clifford operations

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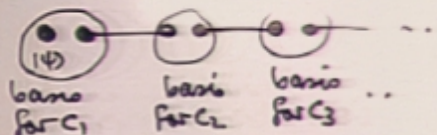

we can apply all measurements simultaneously in parallel (no adaptation) to get

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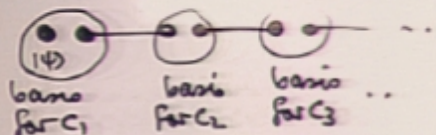

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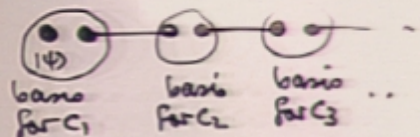

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Pirsa: 05070111 Page 150/403

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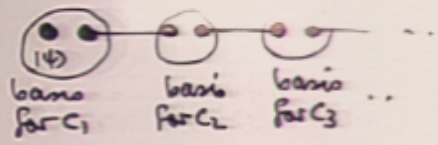

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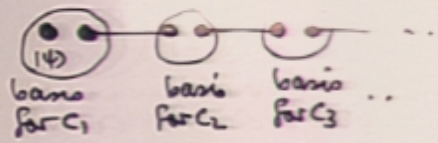

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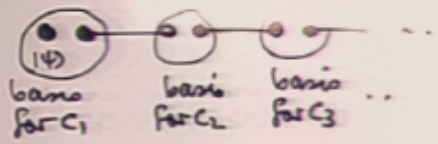

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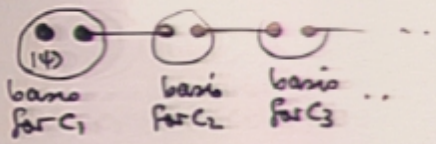

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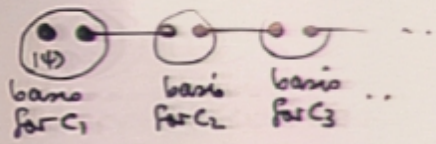

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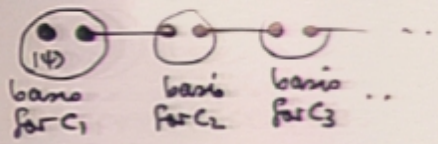

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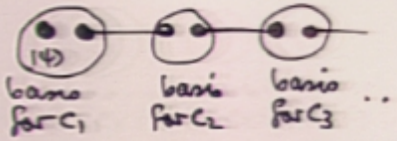
$C_k \cdots C_2 C_1 |\psi\rangle$ $|\psi\rangle$ of n qubits



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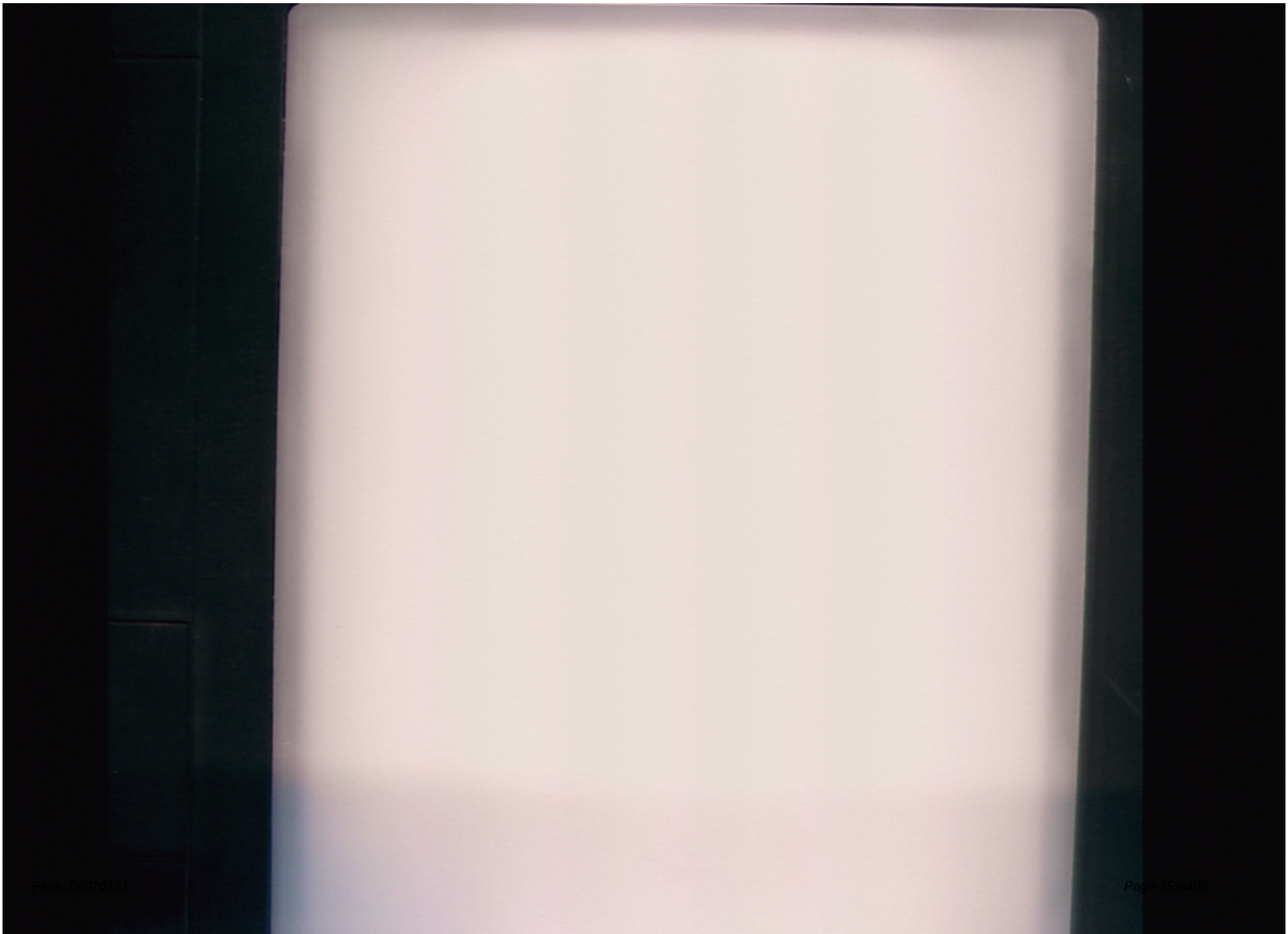
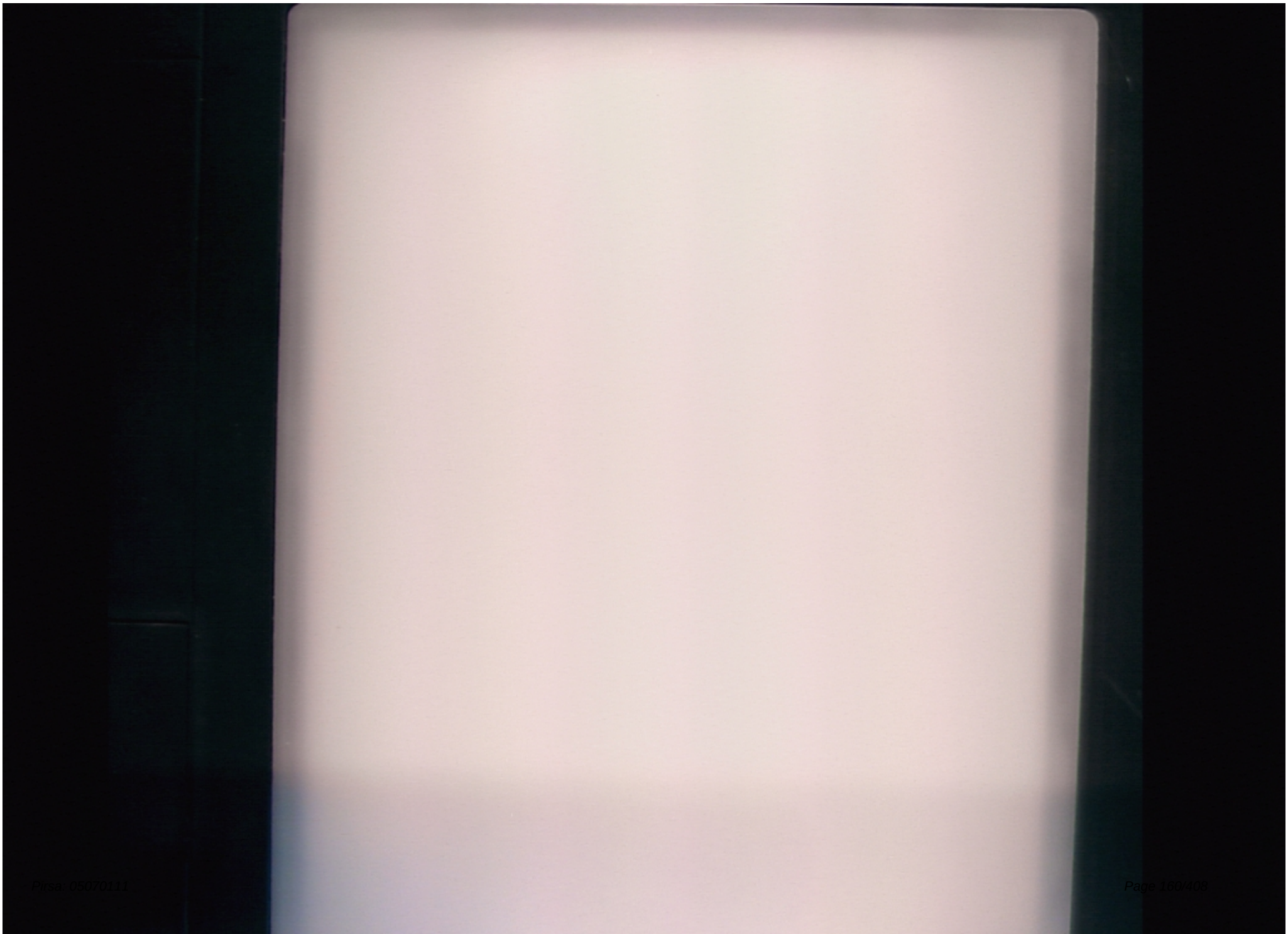


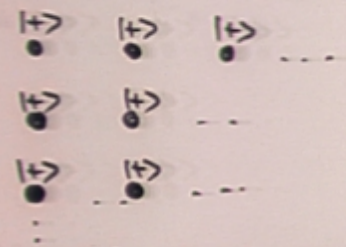


Raussendorf - BI

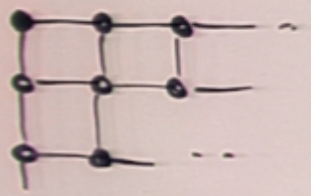
At first sight looks rather different from TQC but turns out to be very closely related!

The cluster state (2-D)

Start with rectangular grid of $|+\rangle$ states



Then apply CZ to each nearest neighbour pair



Cluster state.

All these CZ's commute so can do in constant depth

Notation:

- straight line denotes CZ
- wiggly line denotes some (arbitrary) entangled state.

We will also use 1-D cluster states - as above with 1-dim array of $|+\rangle$ states giving

Raussendorf-Briegel

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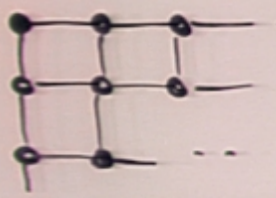

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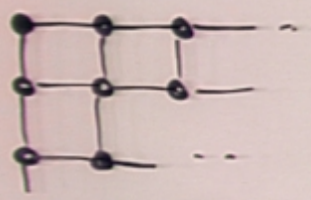

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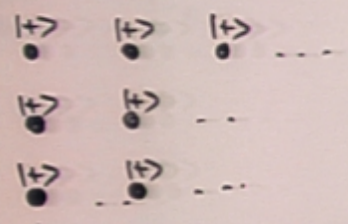

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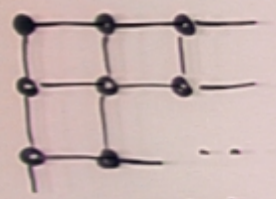

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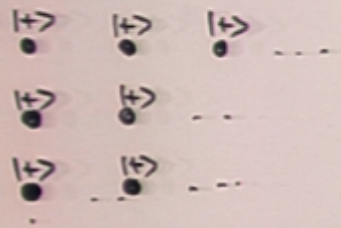

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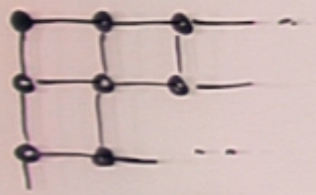

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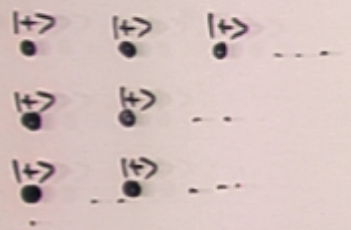

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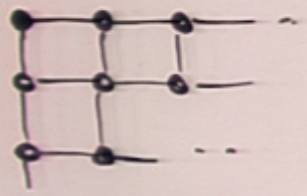

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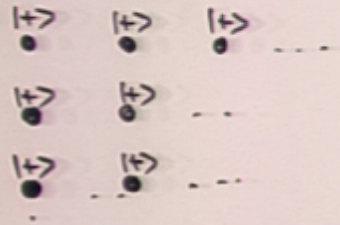

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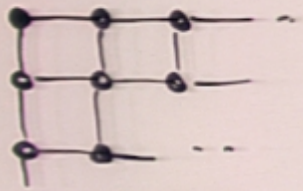


Cluster state.

All these CZ's commute so can do in constant depth

Notation:

- straight line denotes CZ
- wiggly line denotes some (arbitrary) entangled state.

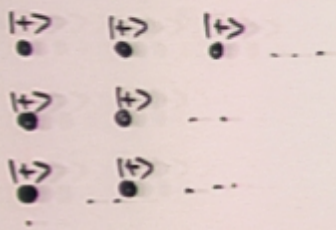
We will also use 1-D cluster states - as above with 1-dim array of $|+\rangle$ states giving

Raussendorf

At first sight looks rather different from 1Q C but turns out to be very closely related!

The cluster state (2-D)

Start with rectangular grid of $|+\rangle$ states



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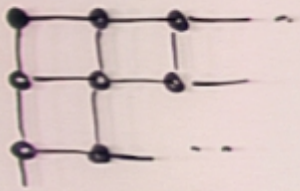

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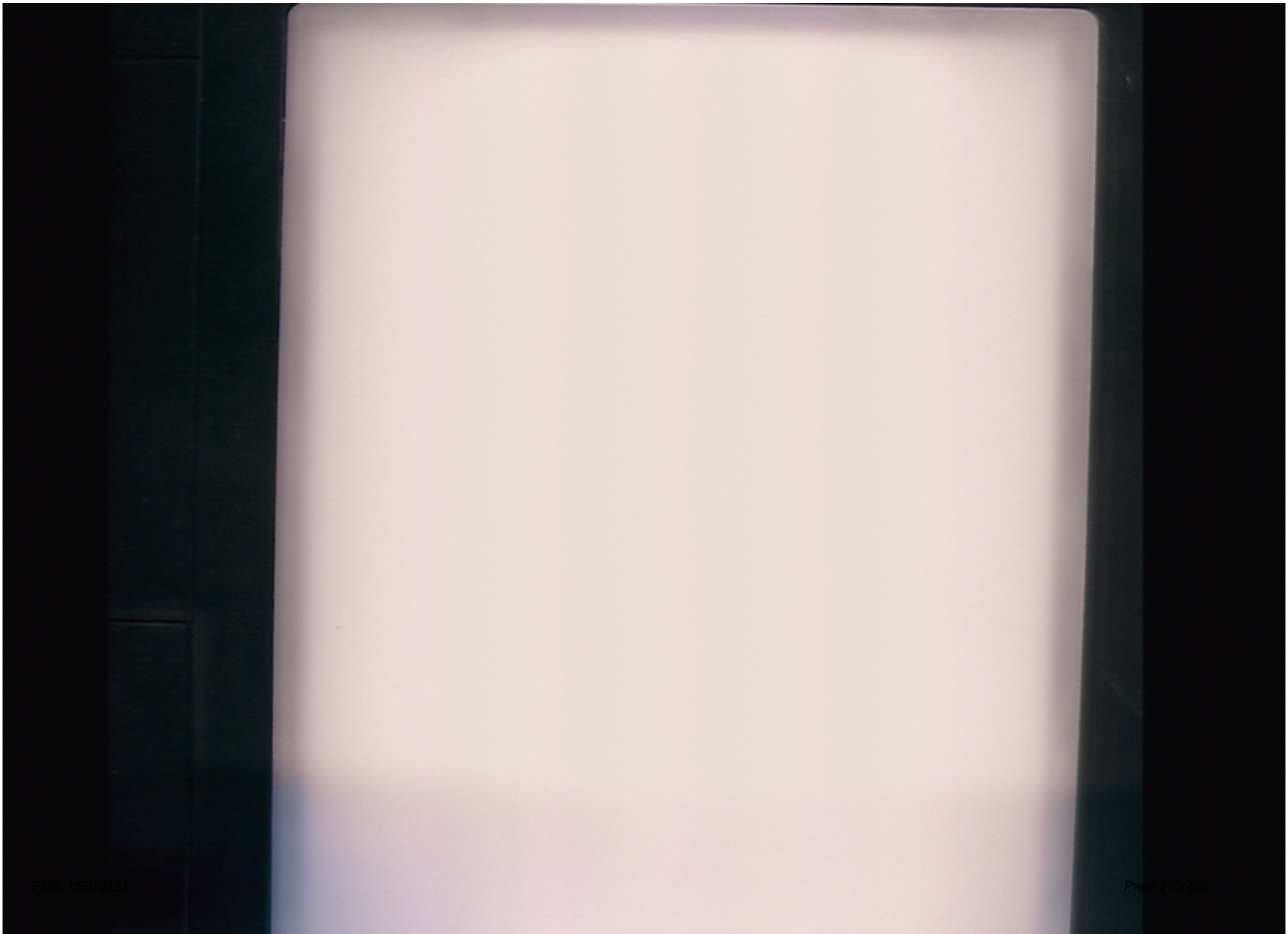

- Any quantum circuit can be implemented by a pattern of 1-qubit measurements on a cluster state
- Measurements are only in bases

  Note X-basis is $M(\theta)$

  $M_Z = \{|0\rangle, |1\rangle\}$ and $M(\theta) = \{|0\rangle \pm e^{i\theta}|1\rangle\}$
- Measurement outcomes 0 or 1 are always 50/50 random
- As in teleportation, quantum gates are implemented up to Pauli errors $X^a Z^b$ where a, b depend on measurement outcomes

  (These Paulis $X^a Z^b$ are called byproduct operators in MBQC literature)

  Hence we'll get same feature of <u>adaptive</u> measurements as before

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Print: 05070111

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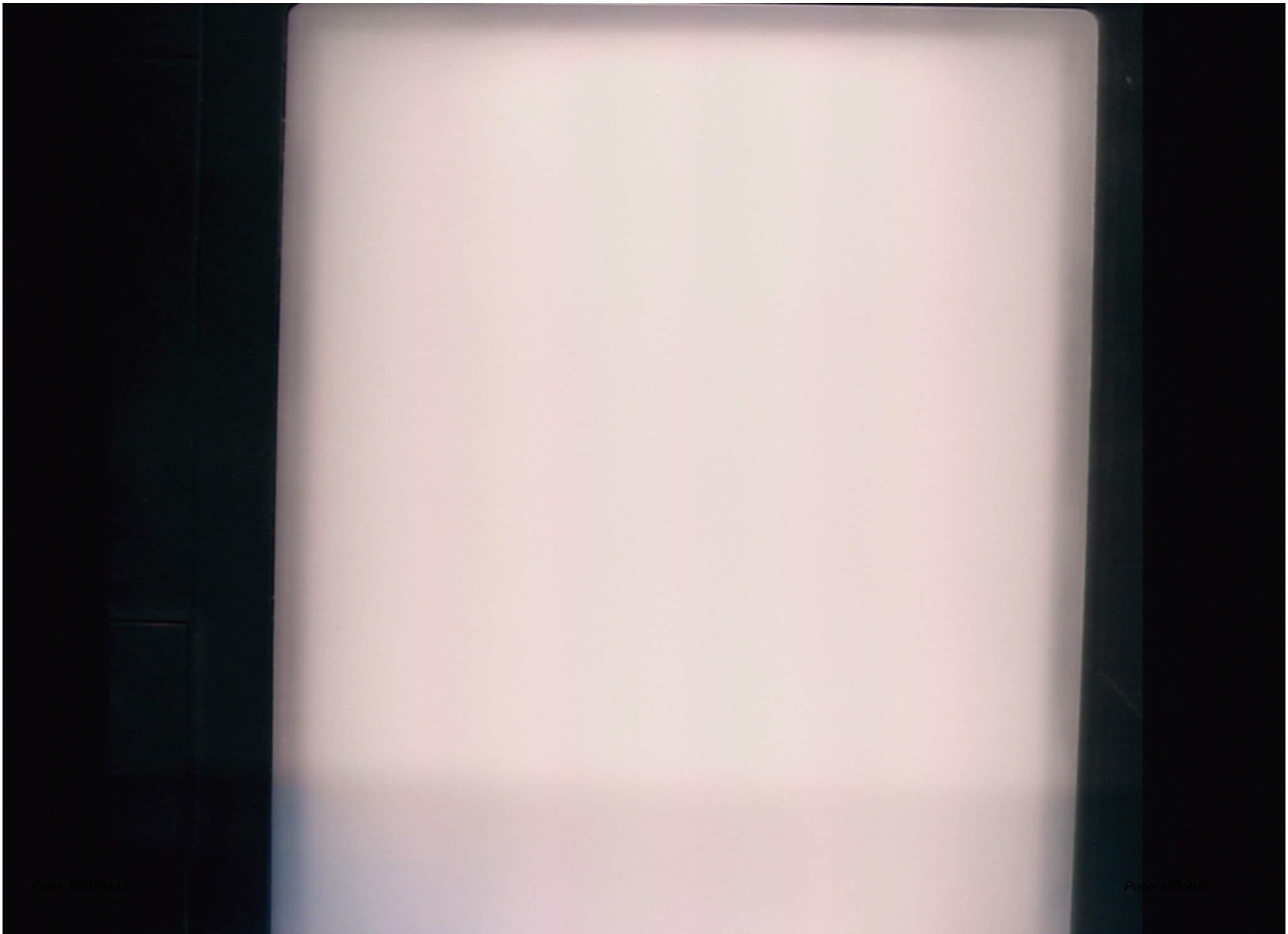


"Valence bond" ... 311130

Interpret each 1-qubit measurement in 1WQC in terms of a full teleportation.

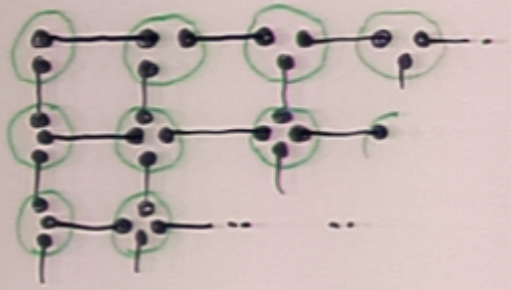
The basic idea:

Use $|H\rangle = CZ|+\rangle|+\rangle = |0\rangle|+\rangle + |1\rangle|-\rangle$ = ●—●
$= |+\rangle|0\rangle + |-\rangle|1\rangle$

(max entg so can use for teleportation)

Consider 2D grid of ●—● states:

$|\text{grid}\rangle$ =



only nearest neighbour links "valence bonds"
Each site (circled) has 4 (or 3 or 2) qubits

At each site consider 2D subspace spanned by

$\{|00..0\rangle, |11..1\rangle\}$ i.e. all $a|0..0\rangle + b|1..1\rangle$'s

Projector $\Pi = |\tilde{0}\rangle\langle 00..0| + |\tilde{1}\rangle\langle 11..1|$

"Valence" ... Field Circa 311130

Interpret each [illegible] in LHQC in terms of a full teleportation.

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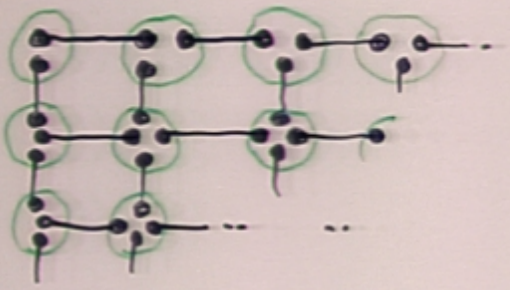

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"Valence bond ..." ... Simple 1 Qubit ...

Interpret each 1-qubit unitary in UQC in terms of a full teleportation.

The basic idea:

Use $|\Phi\rangle = CZ|+\rangle|+\rangle = |0\rangle|+\rangle + |1\rangle|-\rangle$ =

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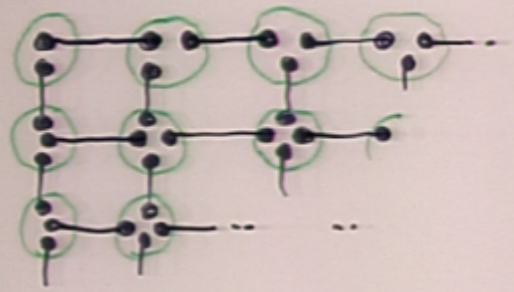

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"Valence bond solid" ... Simulated Quac
5.1130

Interpret each ground state in MBQC in terms of a full teleportation.

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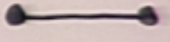
Use $|H\rangle = CZ|+\rangle|+\rangle = |0\rangle|+\rangle + |1\rangle|-\rangle$ = 
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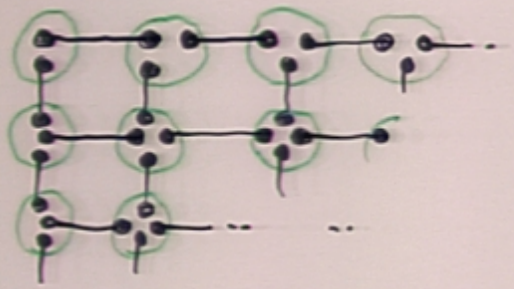

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"Valence bond solids" interpretation Verstraete & Cirac

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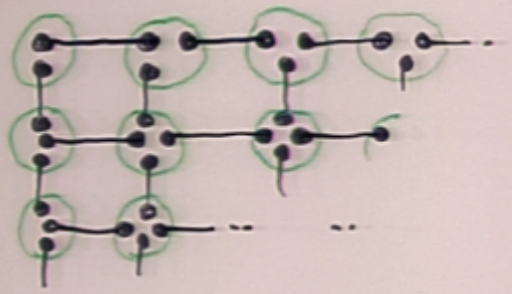

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Pirsa: 05070111

"Valence bond solids" interpretation

Verstraete & Cirac

Interpret
a full teleportation

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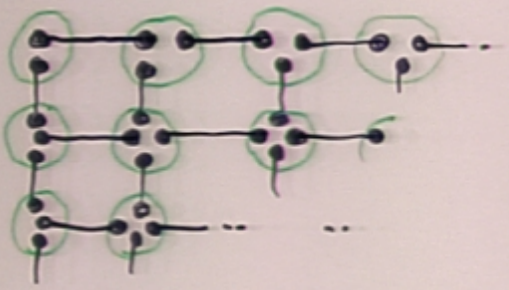

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0311130

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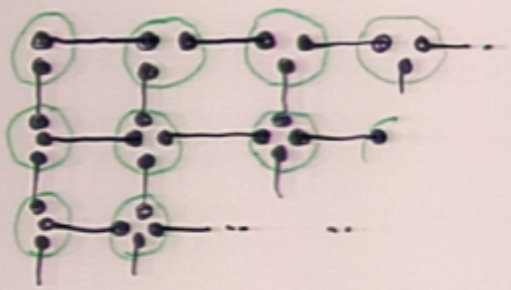

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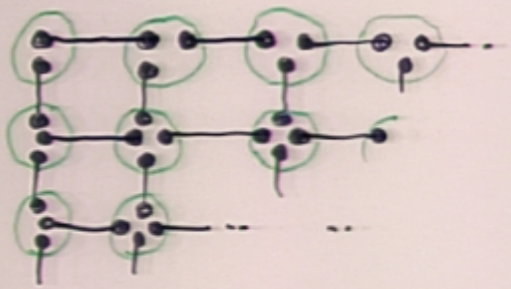

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Verstraete & Cirac
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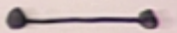
Use $|\Psi\rangle = CZ|+\rangle|+\rangle = |0\rangle|+\rangle + |1\rangle|-\rangle$ = 
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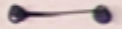
Consider 2D grid of  states:

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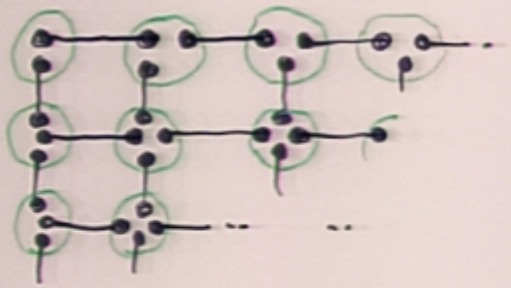

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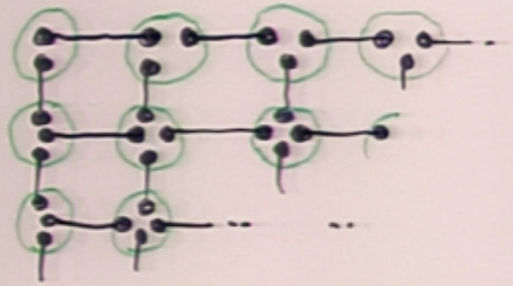

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"Valence bond solids" interpretation

Verstraete & Cirac
0311130

Interpret
a full ...

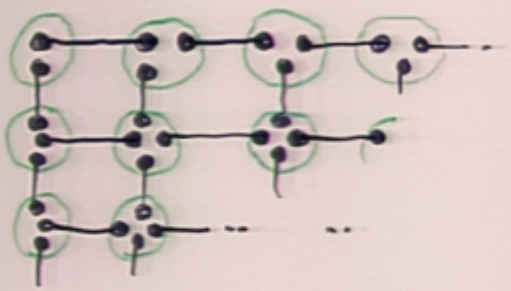
The basic idea:

Use $|\Psi\rangle = CZ(|+\rangle|+\rangle) = |0\rangle|+\rangle + |1\rangle|-\rangle$ =
$= |+\rangle|0\rangle + |-\rangle|1\rangle$

(max entg so can use for teleportation)

Consider 2D grid of states:

$|\text{grid}\rangle =$



only nearest neighbour links "valence bonds"
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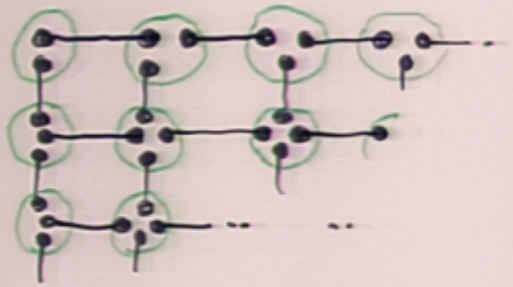

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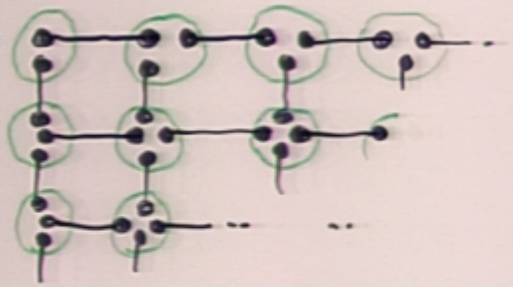

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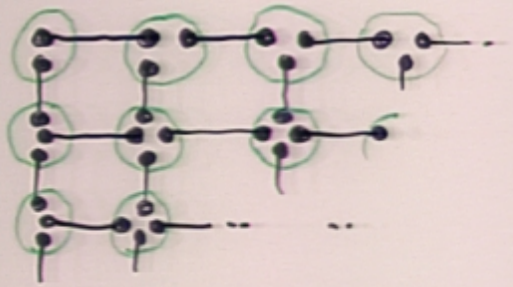

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Verstraete & Cirac ~ ?11130

Interpret
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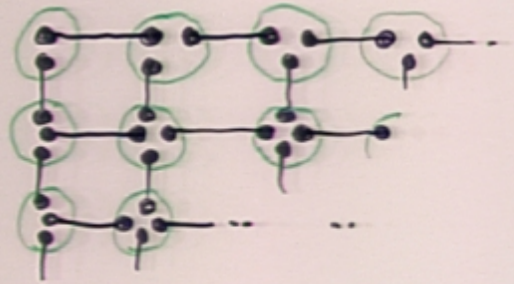

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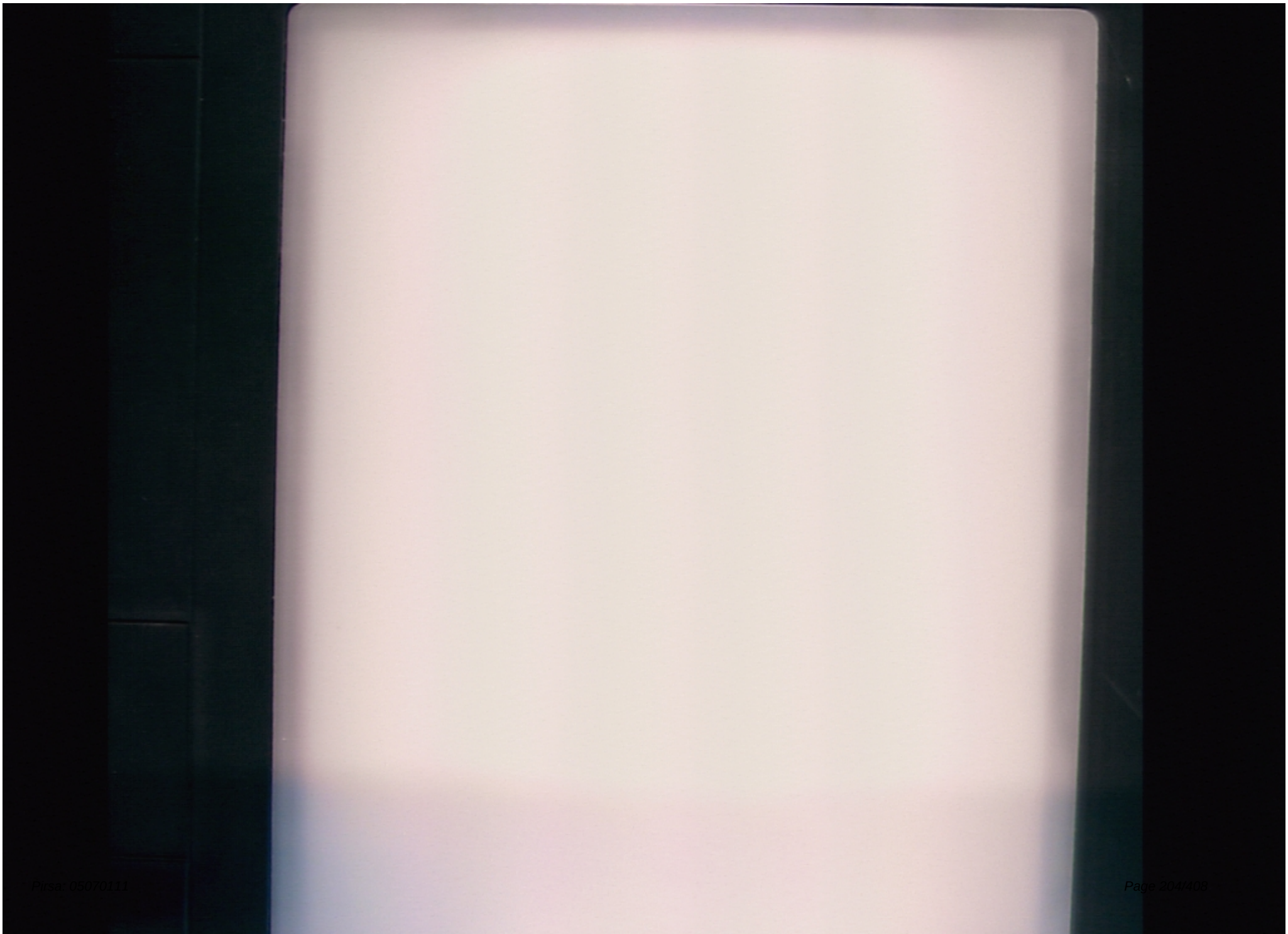

Consider projecting $|grid\rangle$ down to two 2-dim subspaces at each site!

Theorem Resulting state is precisely IWQC cluster state (of qubits span$\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$ at each site.

i.e. "cluster state sits naturally inside the bigger $|grid\rangle$ state"

Proof idea for 1D grid $\Rightarrow$ 1D cluster (2D case similar)

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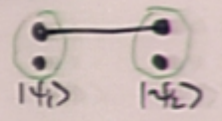
If we start with  and then

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we get $CZ|+_1\rangle|+_2\rangle$ as resulting 2-qubit state.

Now consider

Each ●—● is already $CZ|+\rangle|+\rangle$ by definition.

Apply projector Π at each node.

By above fact 23 & 45 projections simply apply CZ to qubits 2 & 5 etc.

Hence whole projected state is just CZ applied to all consecutive pairs in $|+\rangle|+\rangle|+\rangle\ldots$

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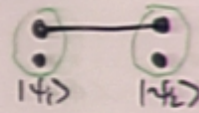
If we start with  and then

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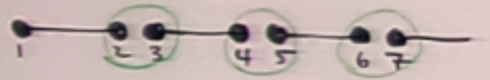

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Pirsa: 05070111

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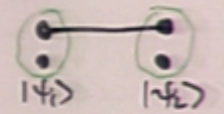
If we start with  and then

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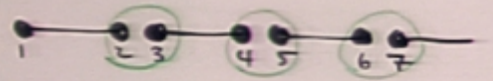

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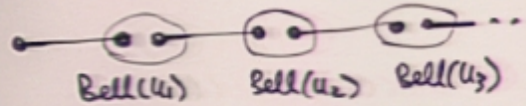
Phys. 05070111

Look at 1D case
(2D case similar)

TQC: to apply U_1 then U_2 then U_3... (up to Pauli errors):



$|H\rangle$ here as we're using $|H\rangle$ to teleport

Bell(U) basis: $|a\rangle = (U^\dagger \sigma_a \otimes I)|H\rangle$

where $a=0,1,2,3$ $\sigma_0 = I$, $\sigma_1 = X$, $\sigma_2 = Y$ $\sigma_3 = Z$

Measurement outcome "a" does $\sigma_a U$ on input state.

Now consider $U = U(\theta) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & e^{-i\theta} \\ 1 & -e^{-i\theta} \end{bmatrix}$

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i.e. at each circled node these 2 Bell states lie in the 1-qubit π-subspace (spanned by $|\tilde{0}\rangle, |\tilde{1}\rangle$) which realises cluster state as "sub-state" of full grid state and the "restricted" Bell mmt $\{|a=0\rangle, |a=1\rangle\}$ is $M(\theta)$ mmt on cluster state.

i.e. $M(\theta)$ mmts on cluster states can be thought of as "teleportations where only 2 of the 4 Bell outcomes occur."

* Similarly all other ingredients of 1WQC can be seen as descendants of teleportation constructions under the π projection.

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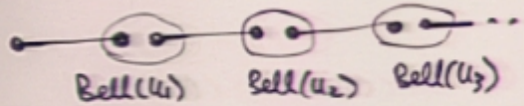

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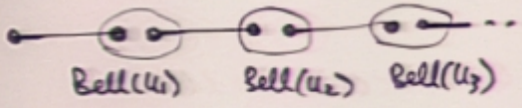

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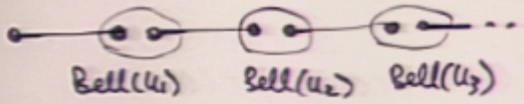

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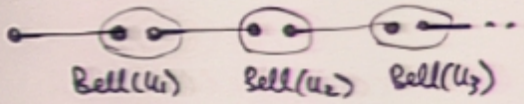

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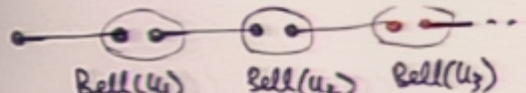

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Pirsa: 05070111

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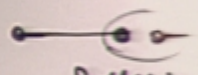

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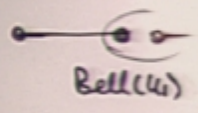

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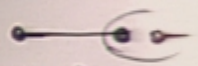

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where $a=0,1,2,3$ $\sigma_0 = I$, $\sigma_1 = X$, $\sigma_2 = Y$ $\sigma_3 = Z$

Measurement outcome "a" does $\sigma_a U$ on input state.

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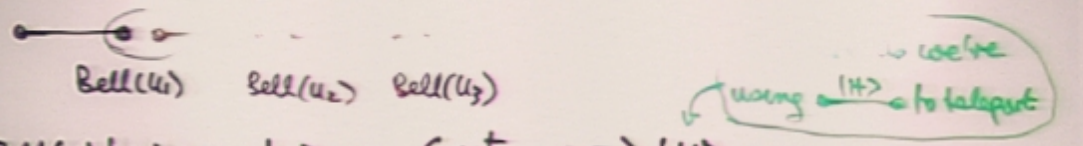

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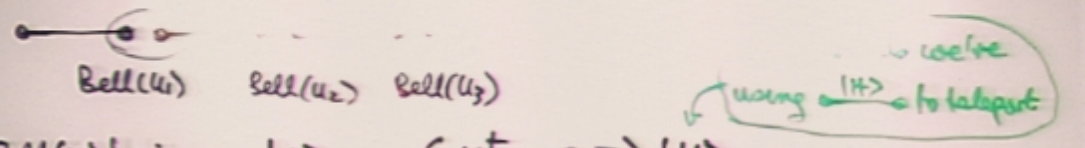

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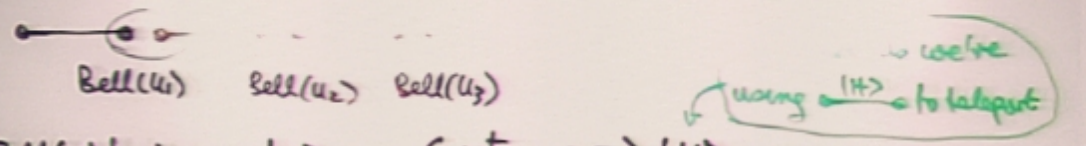

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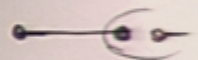

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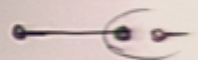

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Pirsa: 05070111

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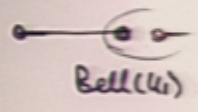

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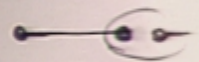

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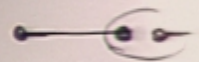

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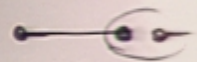

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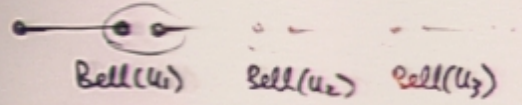

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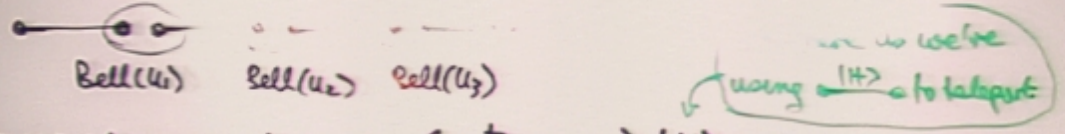

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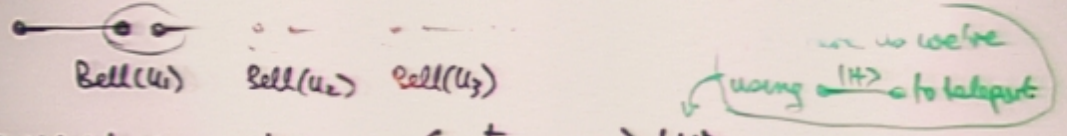

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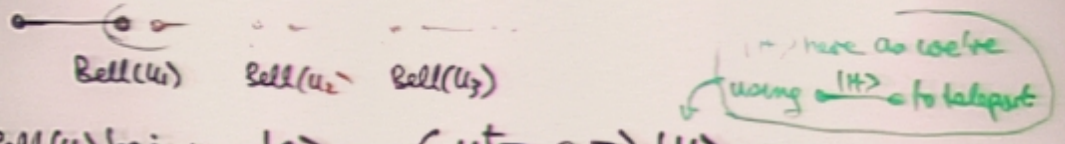

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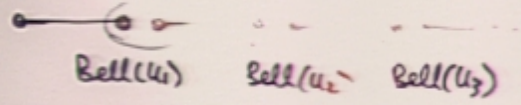
i.e. $M(\theta)$ mmts on cluster states can be thought of as "teleportations where only 2 of the 4 Bell outcomes occur."

* Similarly all other ingredients of 1WQC can be seen as descendants of teleportation constructions under the Π projection.

(2D case similar)

TQC: to apply U_1, then U_2 then U_3 (up to Pauli errors):



more as resolve using $|H\rangle$ to teleport

Bell(U) basis: $|a\rangle = (U^\dagger \sigma_a \otimes I)|H\rangle$

where $a=0,1,2,3$ $\sigma_0 = I$, $\sigma_1 = X$, $\sigma_2 = Y$ $\sigma_3 = Z$

Measurement outcome "a" does $\sigma_a U$ on input state.

Now consider $U = U(\theta) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & e^{-i\theta} \\ 1 & -e^{-i\theta} \end{bmatrix}$

Then get $a=0$ & $a=1$ Bell states:

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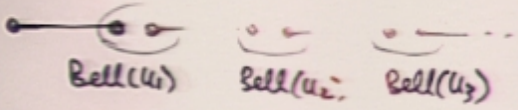

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Generalizing

Teleport in higher dimensions $d > 2$

More exotic groups for Bob's correction operators $P \subseteq U(d)$ finite subgroup

Associated Clifford groups:

Normaliser: $N(P) = \{C : CPC^\dagger = P\}$

want $\supsetneq P$ ← always a normal subgroup

$P_2 = P \otimes P$

$N(P_2) = \{C : CP_2C^\dagger = P_2\}$

always contains SWAP and $N(P) \otimes N(P)$

Want more: imprimitive $C \in N(P_2)$

= not SWAP or product

want to generate qudit entanglements by Clifford operations

any imprimitive gate plus all 1-qudit gates are universal

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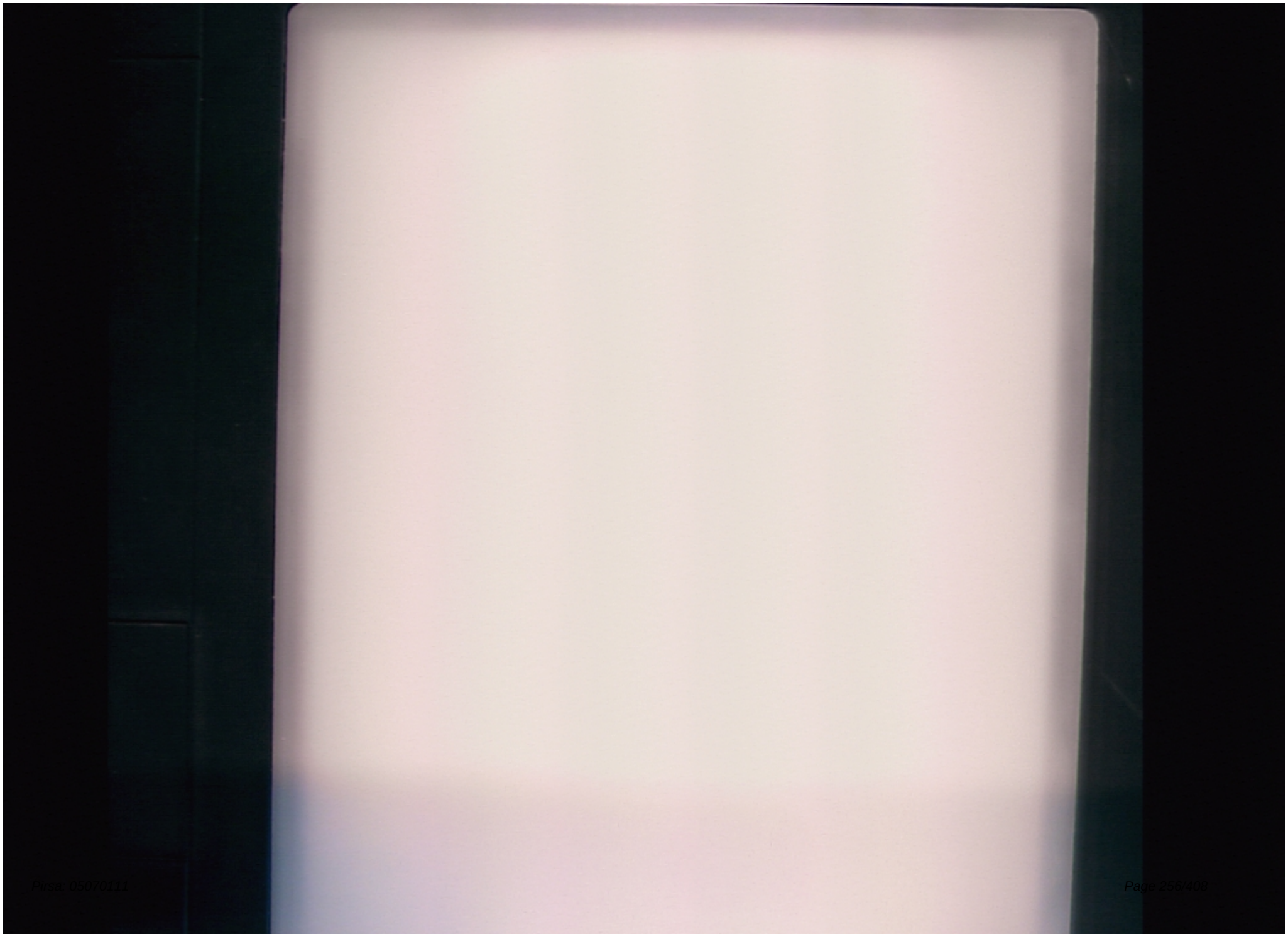


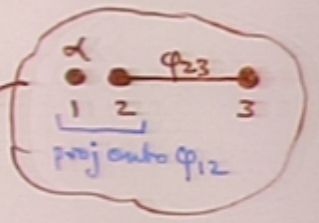
Teleportation

Let $|\varphi\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d} |i\rangle|i\rangle$

be any max. entg. state in $d \times d$ dimensions

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{j,k} a_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

$$= \frac{1}{d} \sum_{ijk} a_j \delta_{ij} \delta_{ik} |k\rangle = \frac{1}{d} \sum_k a_k |k\rangle. \;//$$

Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | U b\rangle$

So introduce

$$\boxed{|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 \, |\varphi\rangle_{12}}$$

Then Fact 1 immediately gives

Fact 2 $\langle \varphi_U |_{12} |\alpha\rangle_1 |\varphi\rangle_{23} = \langle \varphi_{12} | U\alpha\rangle_1 |\varphi_{23}\rangle = \frac{1}{d} U|\alpha\rangle_3 \;//$

Pirsa: 05070111

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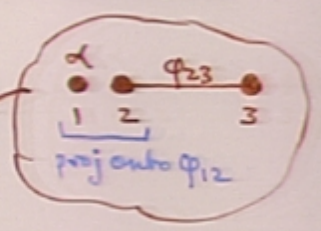

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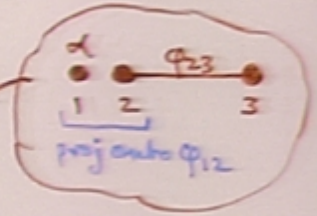

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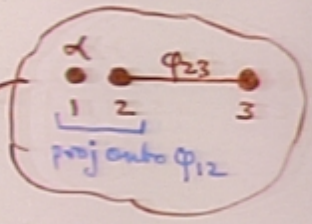

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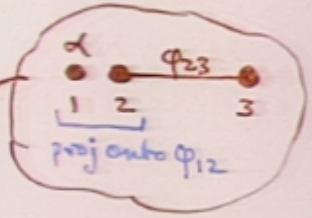

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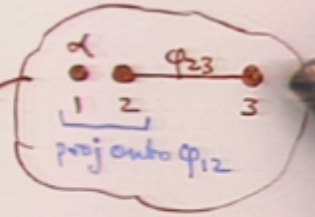
Fact 2 $\langle \varphi_U |_{12} |\alpha\rangle_1 |\varphi\rangle_{23} = \langle \varphi_{12} | U\alpha\rangle_1 |\varphi_{23}\rangle = \frac{1}{d} U|\alpha\rangle_3 \quad //$

Teleg

Let $|\varphi\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle|i\rangle$

be any max. entg. state in $d \times d$ dimensions

Fact 1 $\langle \varphi_{12} | x_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |x\rangle_3$



Proof: Let $|x\rangle = \sum q_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{jk} q_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

$$= \frac{1}{d} \sum_{ijk} q_j \delta_{ij} \delta_{ik} |k\rangle = \frac{1}{d} \sum_k q_k |k\rangle . \quad //$$

Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | U b\rangle$

So introduce

$$|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 |\varphi\rangle_{12}$$

Then Fact 1 immediately gives

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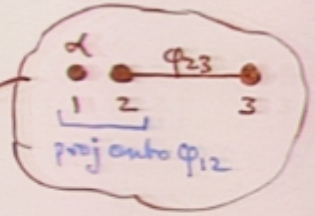
Fact 1 $\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$



Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{j,k} a_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

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Pirsa: 05070111

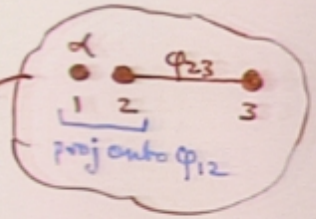
Teleportation Calculation

Let $|\Omega\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle|i\rangle$ ...

be any max. entg. state in $d \times d$ dimensions

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



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Now for $U^+|a\rangle \equiv |U^+ a\rangle$ we have $\langle U^+ a | b\rangle = \langle a | U b\rangle$

So introduce

$$|\varphi_U\rangle_{12} = U_1^+ \otimes I_2 |\varphi\rangle_{12}$$

Then Fact 1 immediately gives

Fact 2 $\langle \varphi_U |_{12} |\alpha\rangle_1 |\varphi\rangle_{23} = \langle \varphi_{12} | U\alpha\rangle_1 |\varphi_{23}\rangle = \frac{1}{d} U|\alpha\rangle_3$. //

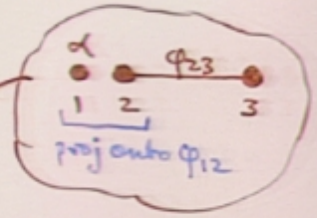
Teleportation Calculation

Let $|\varphi\rangle = \frac{1}{d} \sum_{i=1}^{d-1} |i\rangle|i\rangle$

be any max. entg. state on $\mathbb{C}^d \otimes \mathbb{C}^d$

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



Proof: Let $|\alpha\rangle = \sum q_j |j\rangle$

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Pirsa: 05070111

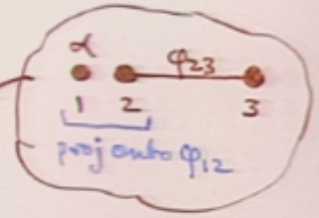
Teleportation Calculation

Let $|\varphi\rangle = \frac{1}{d} \overleftrightarrow{\sum_{i=1}^{d}} |i\rangle|i\rangle$

be any max. entg. state in $d \times d$ dimensions

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



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Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | U b\rangle$

So introduce

$$|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 |\varphi\rangle_{12}$$

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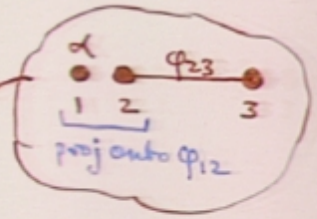
Pirsa: 05070111

Teleportation Calculation

Let $\sum_{j=1}^{d} |j\rangle|j\rangle$ be any max. entg. state in $d \times d$ dimensions

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



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$$= \frac{1}{d} \sum_{ijk} q_j \delta_{ij} \delta_{ik} |k\rangle = \frac{1}{d} \sum_k q_k |k\rangle . \quad //$$

Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | Ub\rangle$

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Pirsa: 05070111

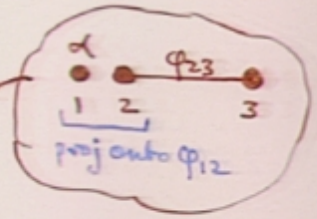
Teleportation Calculation

Let $|\varphi\rangle = \sum_{j=1}^{d} |j\rangle|j\rangle$

be any max. entg. state in $d \times d$ dimensions

Fact 1

$$\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$$



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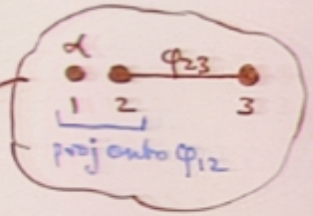
Pirsa: 05070111

Teleportation Calculation

Let $|\varphi\rangle$ ($d-1$)

be any max. entg. state in $d \times d$ dimensions

Fact 1 $\langle \varphi_{12} | \alpha_1 \rangle | \varphi_{23} \rangle = \frac{1}{d} |\alpha\rangle_3$



Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{j,k} a_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

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Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b \rangle = \langle a | U b \rangle$

So introduce

$$\boxed{|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 \, |\varphi\rangle_{12}}$$

Then Fact 1 immediately gives

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Teleportation Calculation

Let $|\varphi\rangle$ ($d-1$)

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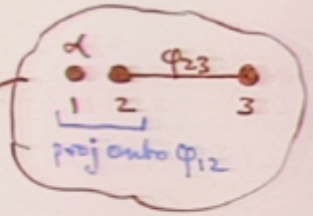

Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

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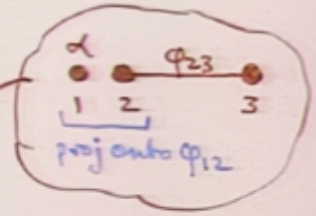
Pirsa: 05070111

Teleportation Calculation

Let $|\varphi\rangle$ be any max. entg. state in $d \times d$ dimensions ($d=4$)

Fact 1 $\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$



Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{j,k} a_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

$$= \frac{1}{d} \sum_{i,j,k} a_j \delta_{ij} \delta_{ik} |k\rangle = \frac{1}{d} \sum_k a_k |k\rangle . \;//$$

Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | U b\rangle$

So introduce

$$|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 \, |\varphi\rangle_{12}$$

Then Fact 1 immediately gives

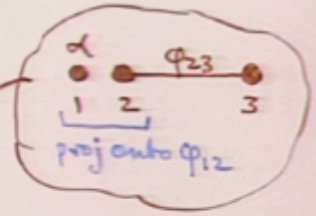
Fact 2 $\langle \varphi_U |_{12} |\alpha\rangle_1 |\varphi\rangle_{23} = \langle \varphi_{12} | U\alpha\rangle_1 |\varphi_{23}\rangle = \frac{1}{d} U|\alpha\rangle_3 \;//$

Teleportation Calculation

Let $|\varphi\rangle$ ($d-1$)

be any max. entg. state in $d \times d$ dimensions

Fact 1 $\langle \varphi_{12} | \alpha_1 \rangle |\varphi_{23}\rangle = \frac{1}{d} |\alpha\rangle_3$



Proof: Let $|\alpha\rangle = \sum a_j |j\rangle$

$$\text{LHS} = \frac{1}{d} \left(\sum_i \langle i|_1 \langle i|_2 \right) \left(\sum_{j,k} a_j |j\rangle_1 |k\rangle_2 |k\rangle_3 \right)$$

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Now for $U^\dagger|a\rangle \equiv |U^\dagger a\rangle$ we have $\langle U^\dagger a | b\rangle = \langle a | U b\rangle$

So introduce

$$|\varphi_U\rangle_{12} = U_1^\dagger \otimes I_2 |\varphi\rangle_{12}$$

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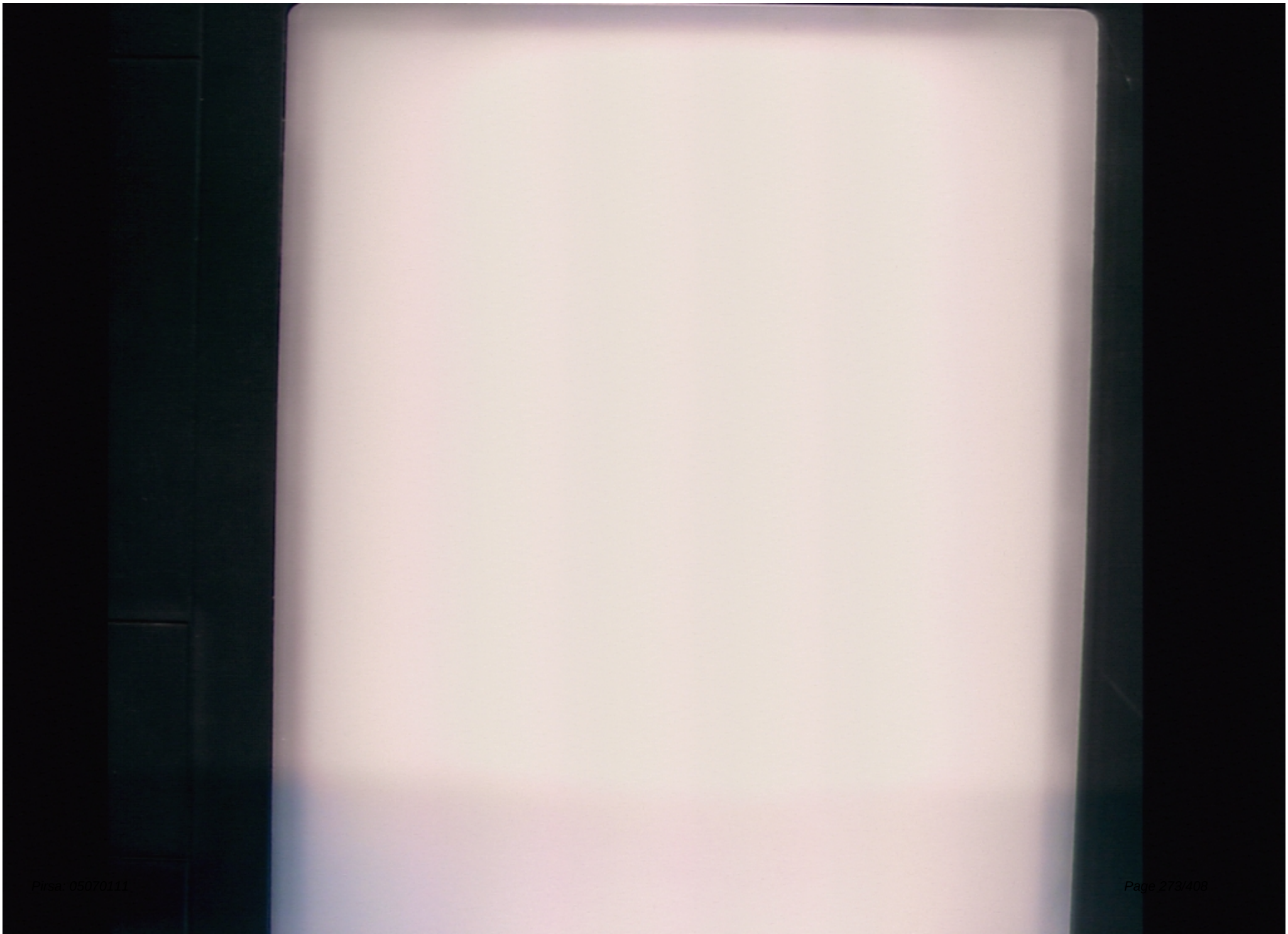

Generalized Pauli matrices

Quantum error correction

Choose d^2 unitaries U_i so that $|\psi_{U_i}\rangle$'s are orthonormal basis in 12-space

i.e. need $\mathrm{tr}(U_i U_j^\dagger) = \delta_{ij}$

Bob's correction operators are then the U_i's.

eg standard quant teleportation: U_i's are I, X, Z, XZ Pauli group

Clifford group generated by $Z, H, P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, $CX = CNOT$

This generalizes to:
"standard" Pauli group in dimension d:

$$X = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & & 1 & & \\ 0 & & & 1 & 0 \end{bmatrix} \qquad X|j\rangle = |j+1\rangle$$

$\omega = e^{2\pi i/d}$

$$Z = \begin{bmatrix} 1 & & & \\ & \omega & & 0 \\ & & \omega^2 & \\ & 0 & & \ddots \\ & & & & \omega^{d-1} \end{bmatrix} \qquad Z|j\rangle = \omega^j |j\rangle$$

$ZX = \omega XZ$

Pauli group $= \{ X^a Z^b : a, b = 0, \ldots d-1 \}$ a unitary operator basis.

Generalised Bell measurements I

Unitaries

Choose d^2 unitaries U_i so that $|\varphi_{U_i}\rangle$'s are orthonormal basis in 12-space

i.e. need $\mathrm{tr}(U_i U_j^\dagger) = \delta_{ij}$

Bob's correction operators are then the U_i's.

e.g. standard qubit teleportation: U_i's are I, X, Z, XZ Pauli group

Clifford group generated by $Z, H, P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, CX = CNOT

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$ZX = \omega XZ$

Pauli group $= \{ X^a Z^b : a, b = 0, \ldots d-1 \}$ a unitary operator basis!

Generalized Bell measurement w.r.t. Unitary operator basis

Choose d^2 unitaries U_i so that $|\varphi_{U_i}\rangle$'s are orthonormal basis in 12-space

i.e. need $\mathrm{tr}(U_i U_j^\dagger) = \delta_{ij}$

Bob's correction operators are then the U_i's.

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e.g. standard qubit teleportation: U_i's are I, X, Z, XZ Pauli group

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Generalized Bell measurement & T

Unitary operator basis

Choose d^2 unitaries U_i so that $|\varphi_{U_i}\rangle$'s are orthonormal basis in 12-space

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Bob's correction operators are then the U_i's.

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e.g. standard qubit teleportation: U_i's are I, X, Z, XZ Pauli group

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Generalized Bell measurement & Unitary operator basis

Choose d^2 unitaries U_i so that $|\varphi_{U_i}\rangle$'s are orthonormal basis in 12-space

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Generalized Bell measurement & T

Unitary operator basis:

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$\omega = e^{2\pi i/d}$

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$ZX = \omega XZ$

Pauli group $= \{ X^a Z^b : a, b = 0, \ldots d-1 \}$ a unitary operator basis!

Generalized Bell measurement w.r.t. T

Unitary operator version:

Choose d^2 unitaries U_i so that $|\varphi_{U_i}\rangle$'s are orthonormal basis in 12-space

i.e. need $\mathrm{tr}(U_i U_j^\dagger) = \delta_{ij}$

Bob's correction operators are then the U_i's.

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e.g. standard qubit teleportation: U_i's are I, X, Z, XZ Pauli group

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Generalized Bell measurement and Teleportation

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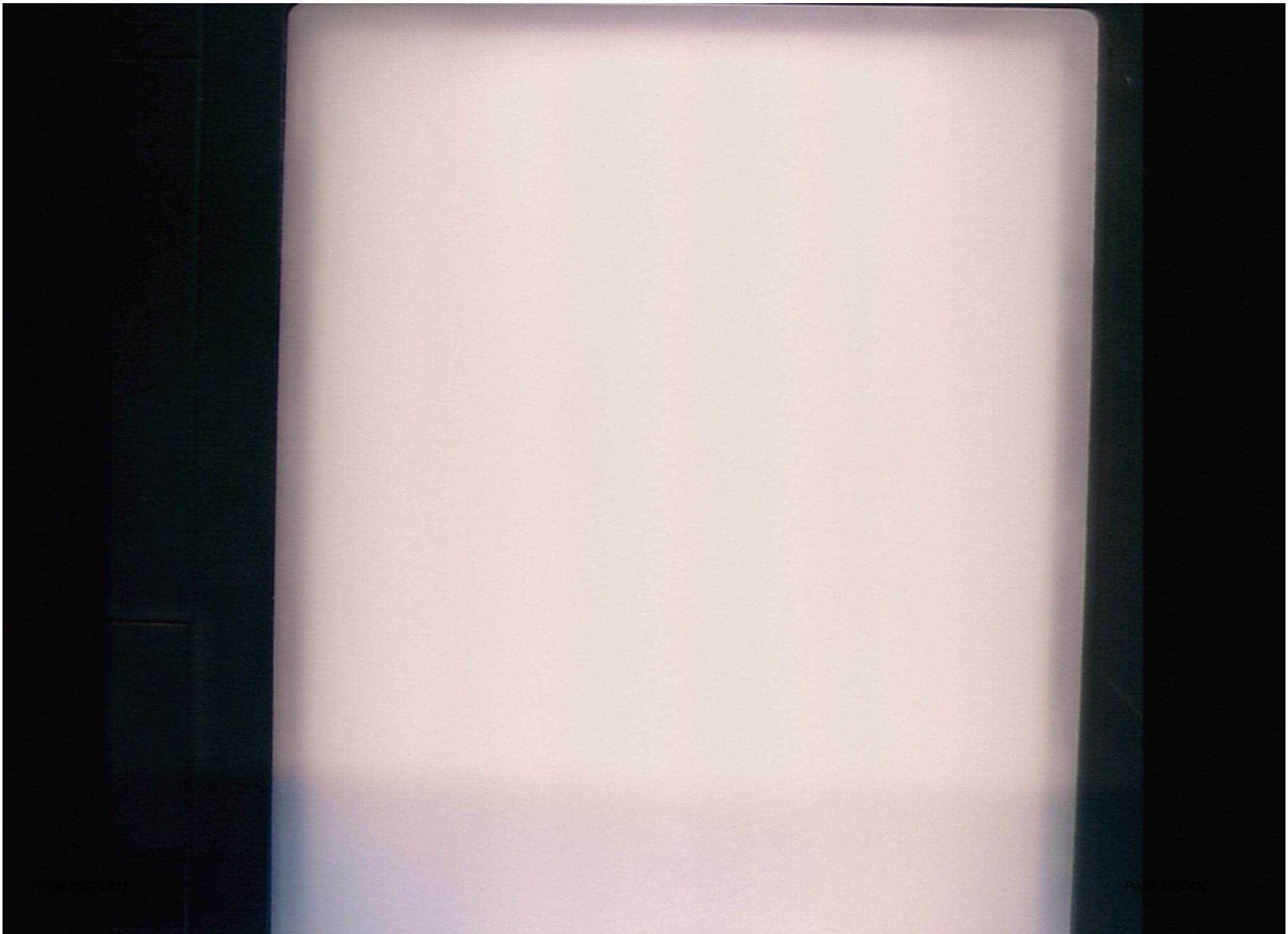

Clifford group for d dimensions (d prime)
and standard basis notation:

Introduce

$$CX\,|j\rangle|k\rangle = |j\rangle X^j|k\rangle = |j\rangle|j+k\rangle$$

$$S = \mathrm{diag}\left(1, \omega, \omega^3, \omega^6, \omega^{10}, \ldots \omega^{\frac{k(k+1)}{2}} \ldots\right)$$

constructed so that $SXS^\dagger = ZX$

Theorem: If d is prime then Clifford group on n qudits is generated by

$$Z,\ F = QFT_d,\ S,\ CX.$$

Hence all circuits of these gates can be parallelised to 1 measurement layer, in T&C/1WQC formalism.

Remark: Valence bond formalism based on maximally entangled $\sum_{i=0}^{d-1} |i\rangle QFT|i\rangle$ bonds and projection of sites to span $\{|0\ldots0\rangle, |1\ldots1\rangle, \ldots |d-1\ldots d-1\rangle\}$ gives generalisation of 1WQC to dimension d.
(cf Zhou, Zeng, Xu, Sun c304e54 for a different approach)

Clifford [illegible] and 'standard' Pauli construction:

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Pirsa: 05070111

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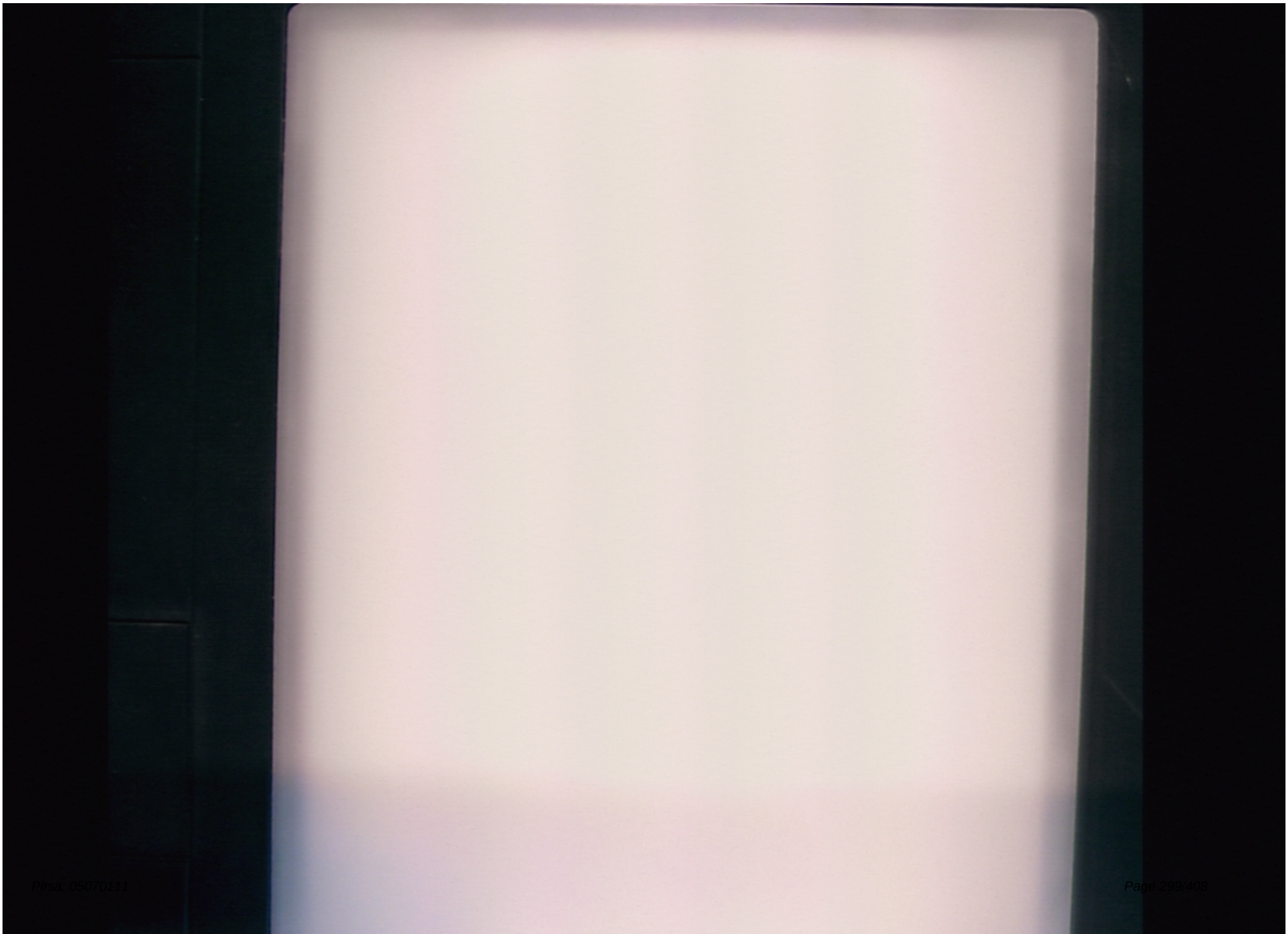

More general constructions of unitary error bases

- "Shift & Multiply bases" (Werner 000307o)

$$U_{ij}|k\rangle = H^{j}_{ik}\,|\lambda(j,k)\rangle \qquad i,j,k = 0,\ldots,d-1.$$

  - $\lambda(j,k)$ Latin square
  - H^{j}_{ik} for each j is $d\times d$ "complex Hadamard matrix" i.e. $|H^{j}_{ik}| = 1$ & $HH^{\dagger} = dI$.

- Example
  Latin square is addition table for group $\mathbb{Z}_d$
  $H^{j}_{ik} = [\mathrm{QFT}]_{ik}$ (indep of j)
  ⇒ get previous "standard" Pauli group for d dims.

- Many further choices:
  e.g. Latin squares $d=5$: 56 (Euler)
  $d=10$: $>10^{24}$ known
  Full classification: unsolved math. problem
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Pirsa: 05070111

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Pirsa: 05070111

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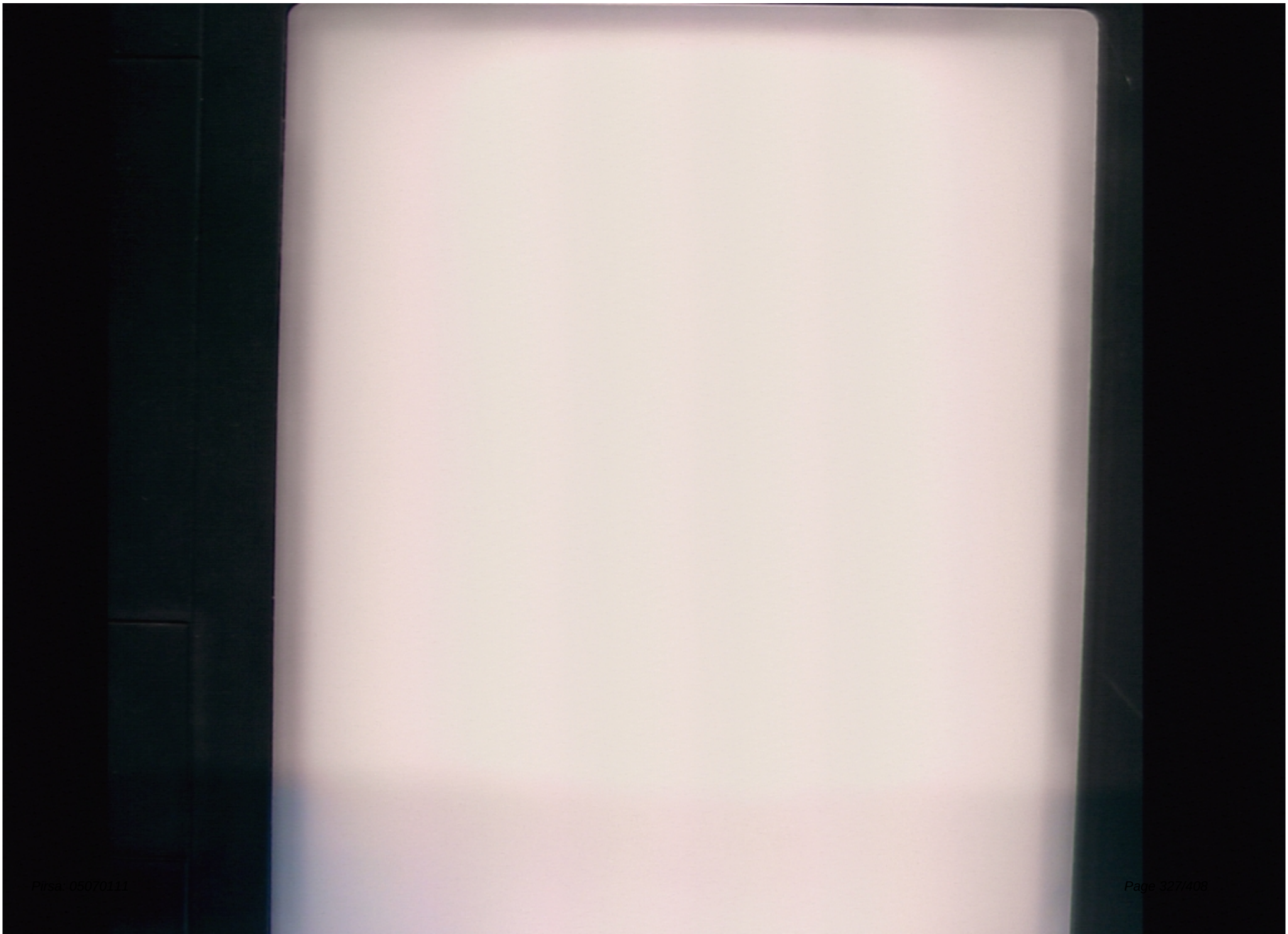

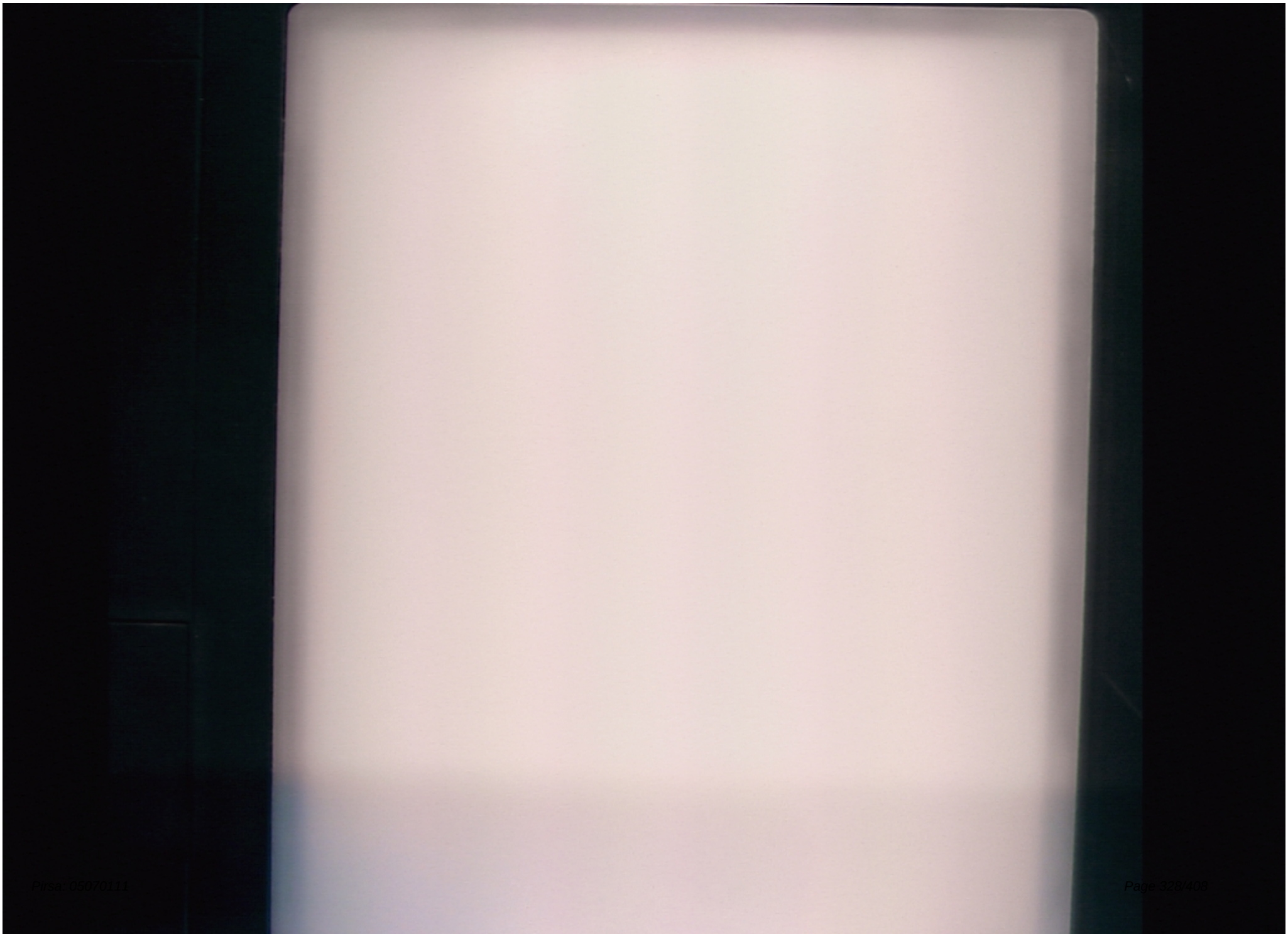

Difficult to identify Clifford operations at 2-qudit level for such unitary operator bases.

small d: get (subgroups of) known Cliffords

larger d: groups too big to handle to find Cliffords, even with computer algebra

So far: no interesting new Clifford operations found in this formalism (using computer algebra)

Question:

what structure should a group $G \subset SU(d)$ have, to lead to interesting normalizers of $G \subset SU(d)$ and $G \otimes G \subset SU(d) \otimes SU(d)$??

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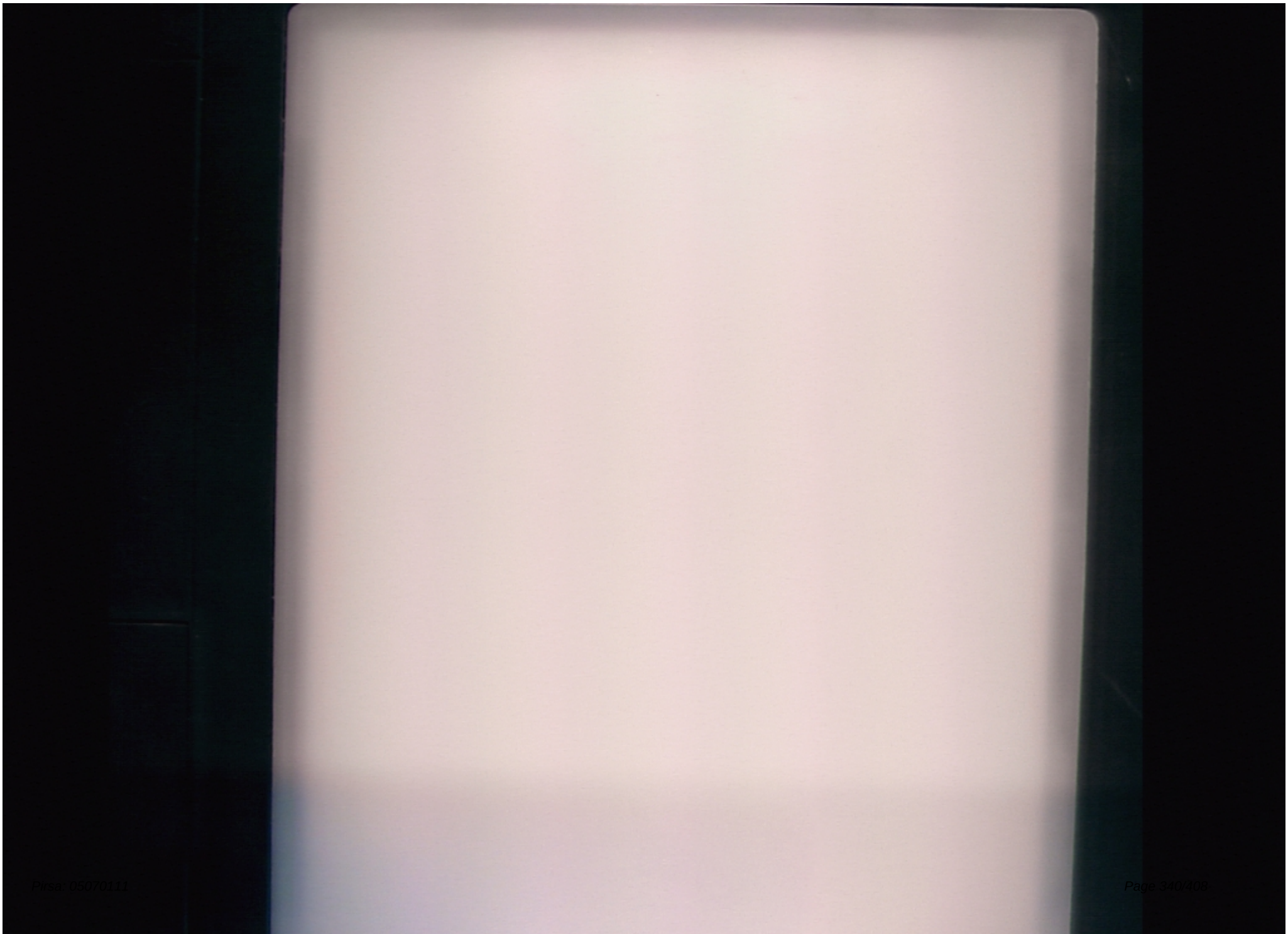

Further [illegible]

Can use POVM measurements!

We seek families $\{U_i\}$ (now not operator bases!) such that

$\{ k_i U_i^\dagger \otimes I \, |\varphi\rangle\langle\varphi| \, U_i \otimes I \}$ is POVM ($k_i > 0$ constants)

Recall $|\varphi\rangle = \sum_i |i\rangle|i\rangle$

i.e. $\boxed{\sum_i k_i \, |\varphi_{U_i}\rangle\langle\varphi_{U_i}| = I_{d\times d}}$ (for some $k_i > 0$)

Many examples exist, even in $d = 2$ dimensions.

Theorem: Suppose $\{U_i\}_{i=1}^n \subseteq SU(d)$ is an irreducible representation of a group.
Then the POVM condition holds with $k_i = d^2/n$.

[Proof ingredients:
Need Schur lemma (if A commutes with all U_i's then $A \propto I$)
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Further generalization of teleportation

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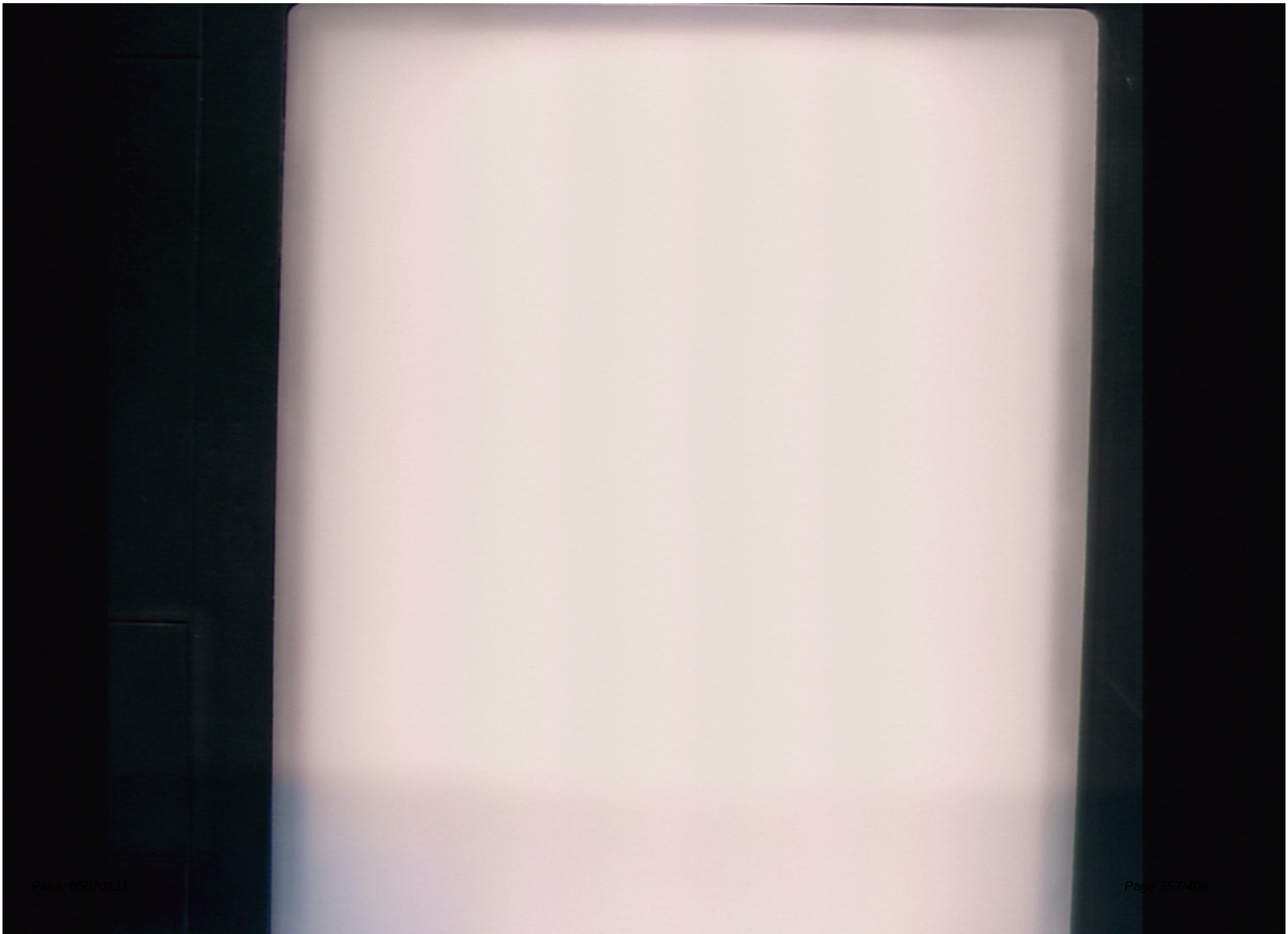

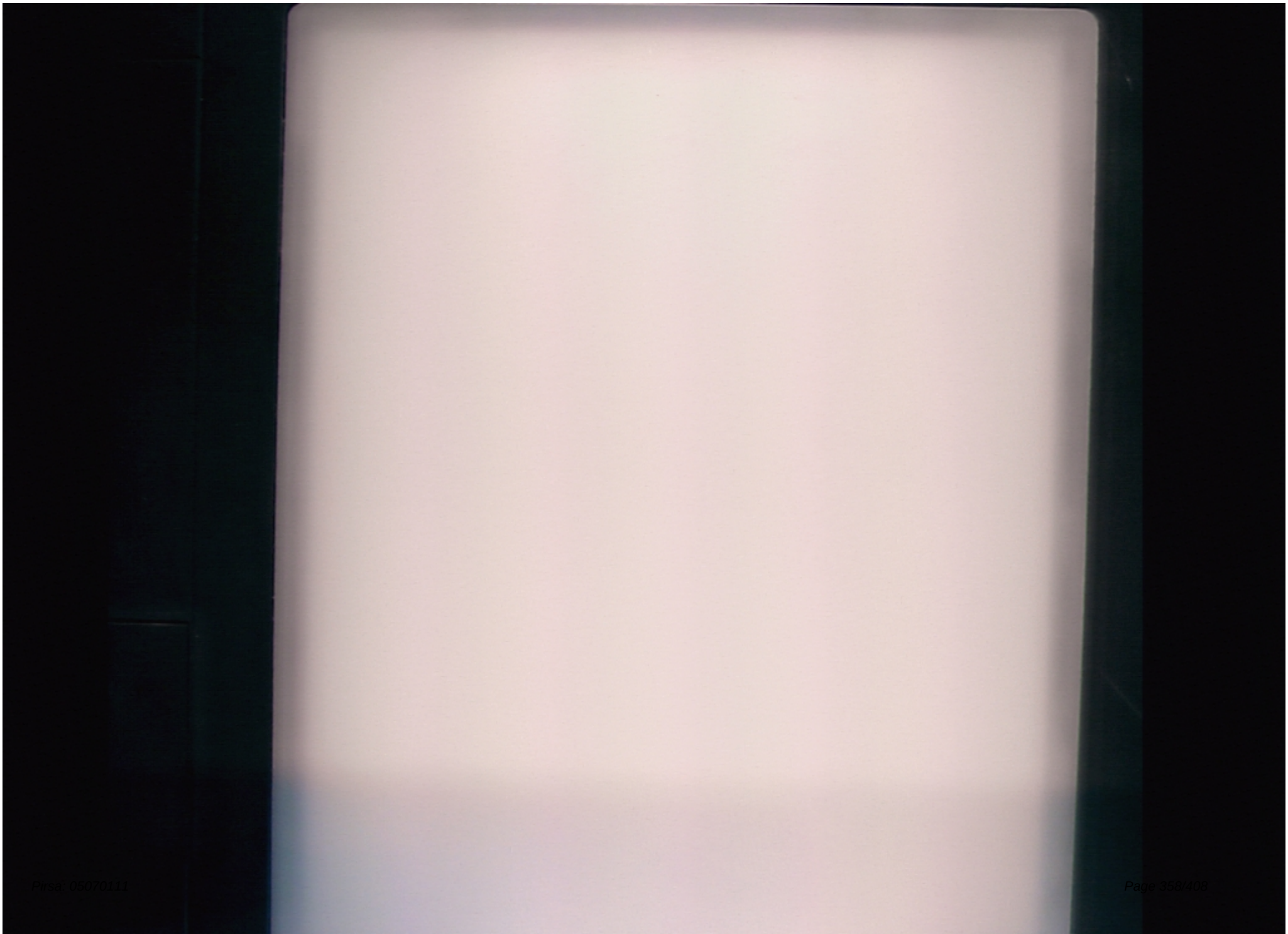

Examples of POVM-teleportation

- qubit states ~ Bloch sphere
- Unitaries ~ rotations of sphere
- Candidate finite groups: look at rotational symmetry groups of Platonic solids.

Get POVM-teleportation schemes associated to Platonic solids.

Cube group S_4 (24 elements)

Fact: standard 1-qubit Clifford group isomorphic to cube group.

Pauli group $\{\pm I, \pm X, \pm Z, \pm XZ\}$ is normal subgroup of π-rotations about x, y, z axes.

Tetrahedral group A_4 (12 elements)

Only one proper normal subgroup.

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? So: for interesting normaliser group structures need $d > 2$?

Further constructions for $d > 2$ under investigation....

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No new normalisers found at 1 or 2 qubit level (beyond standard 1 & 2 qubit Cliffords)

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Examples of POVM-teleportation for $d=2$

- qubit states ~ Bloch sphere
- Unitaries ~ rotations of sphere
- Candidate finite groups [illegible] groups of Platonic solids.

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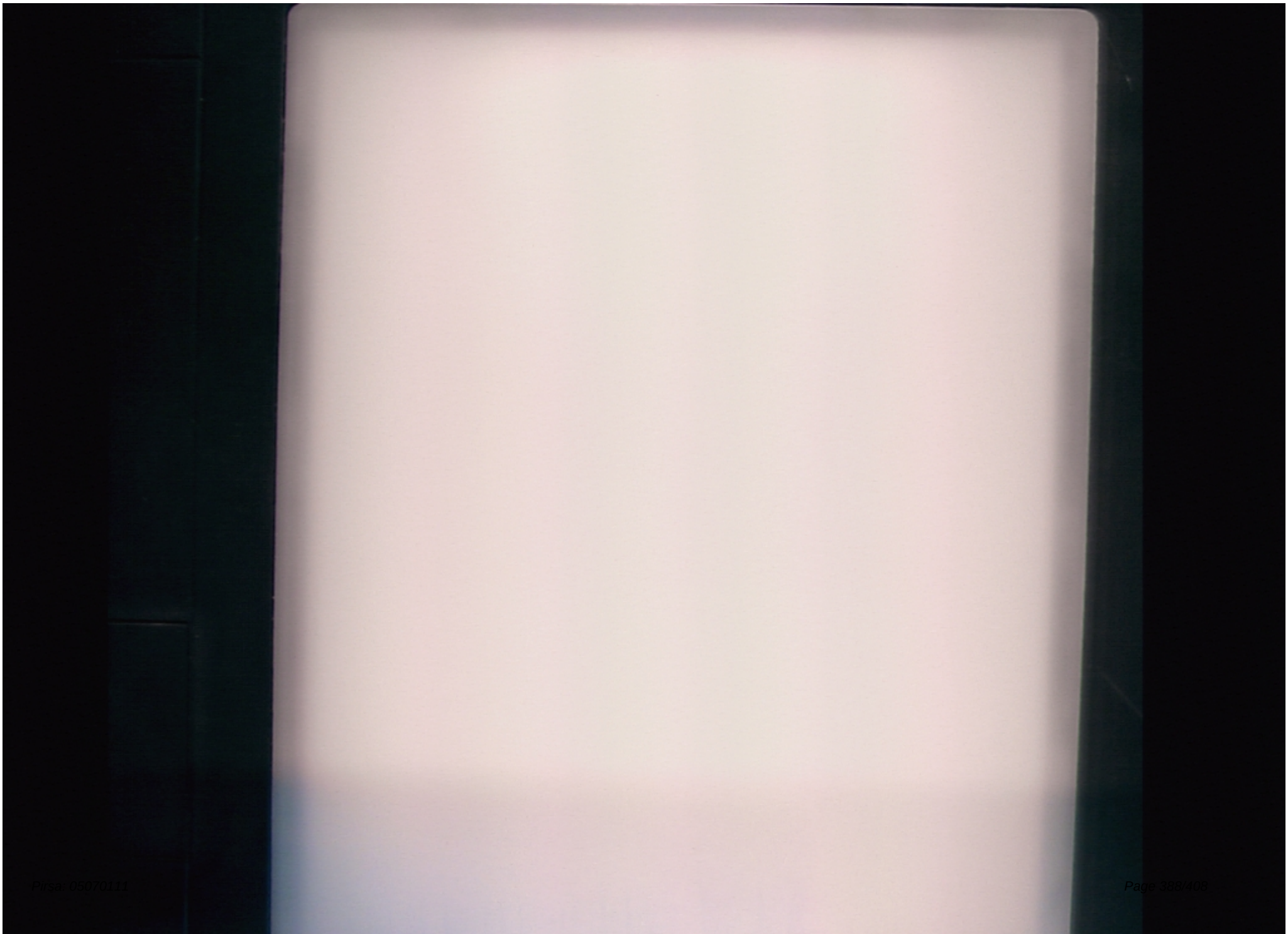

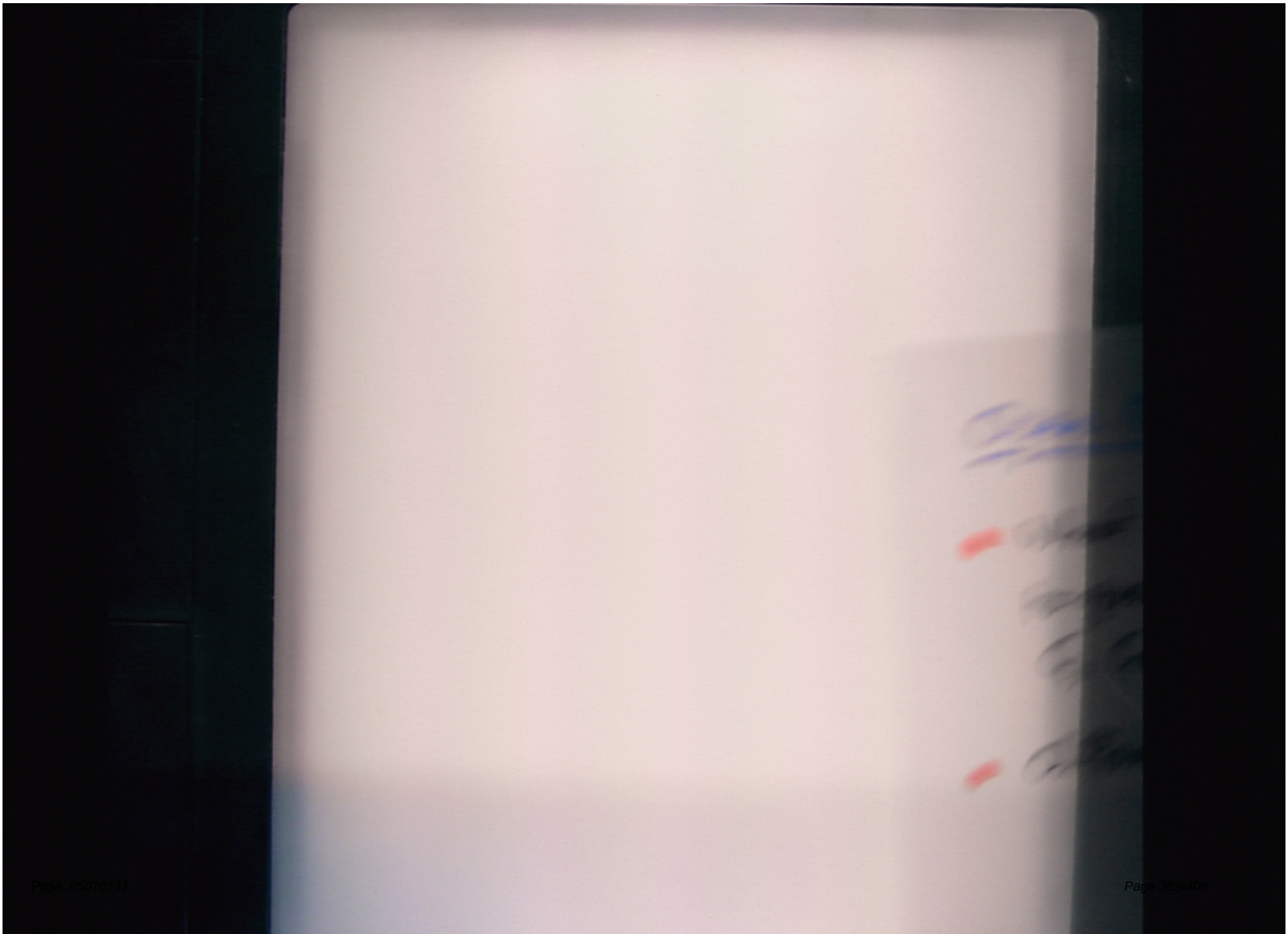

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