

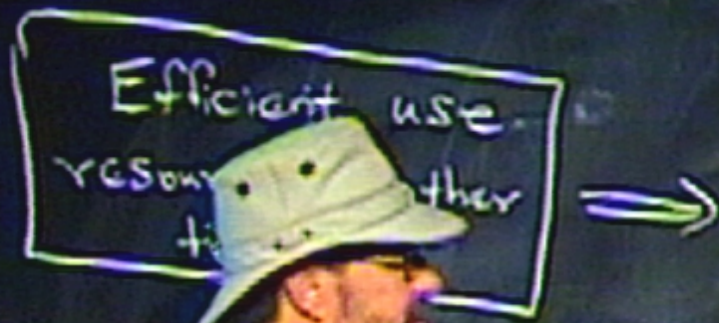
Title: Introduction to logics as type theories for quantum processes

Date: Jul 19, 2005 11:00 AM

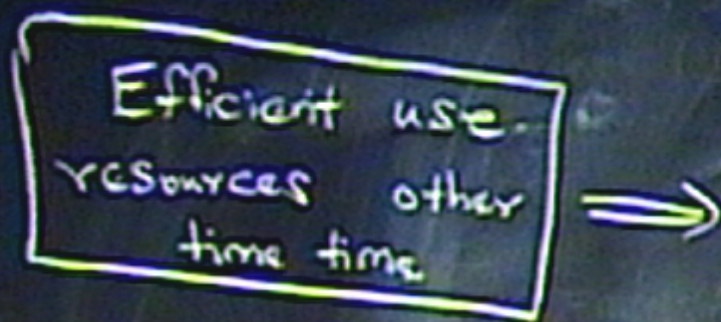
URL: <http://pirsa.org/05070099>

Abstract:

Efficient use
resources
together



Efficient use
resources other
time time



Efficient use
resources other
time time



Tensor-product
Structure

No efficient realistic
simulation of computation



Entanglement cannot be
restricted to blocks of
parts of fixed size

Efficient use
resources other
time time



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Entanglement cannot be
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Global
entanglement

No
efficient
realistic
description

Global
entanglement

No
efficient
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description

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entanglement

No
efficient
realistic
description

Global
entanglement

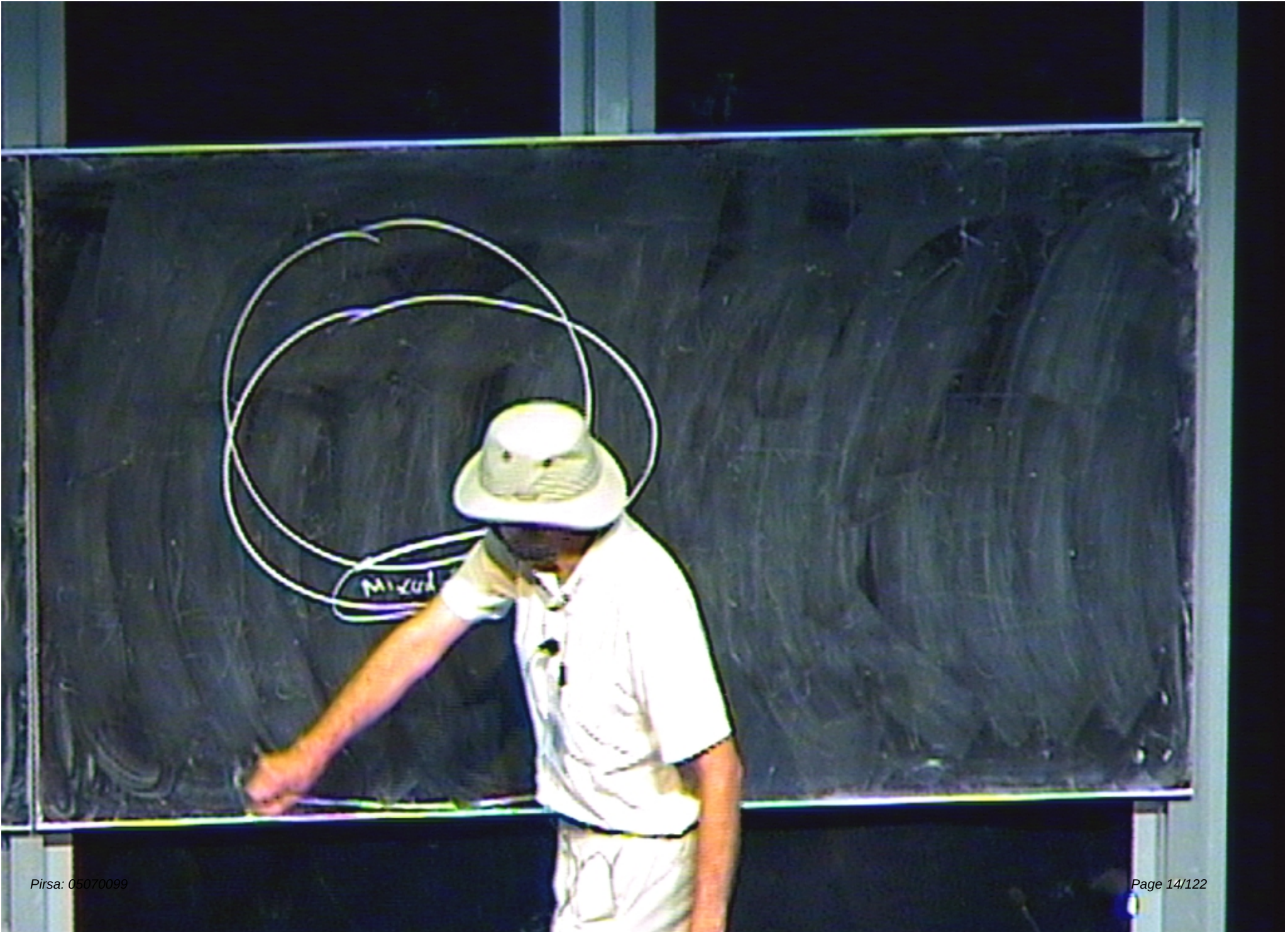
No
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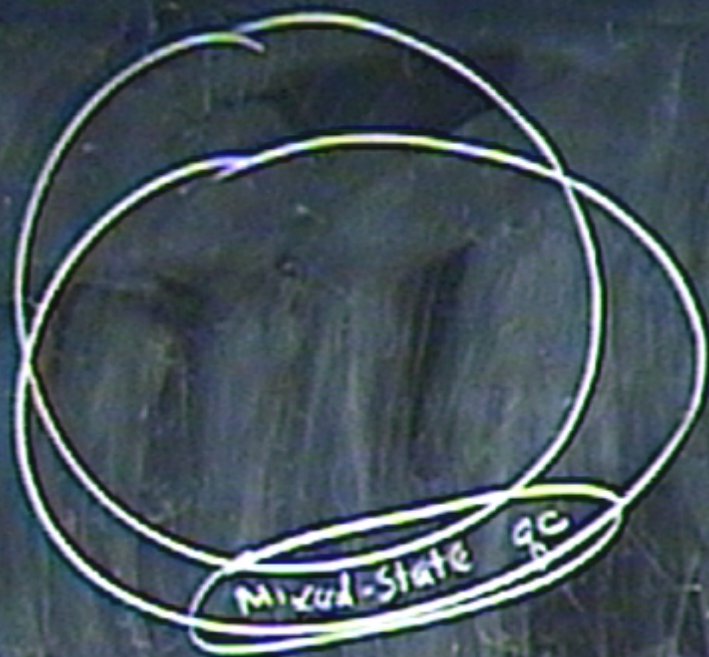
Global
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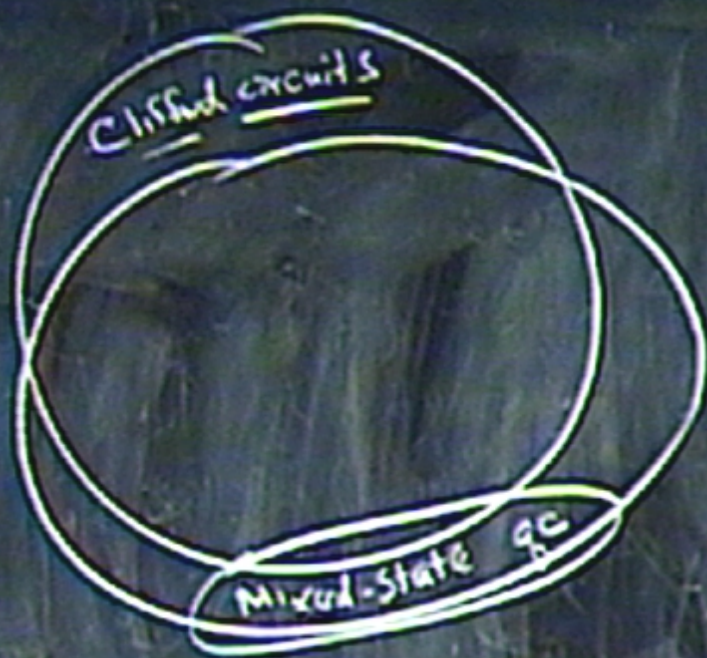
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G-K theorem

- Start in $|0 \dots 0\rangle$

G-K theorem

- Start in $|0 \dots 0\rangle$
- Allowed gates

$H, S, \cdot$

G-K theorem

- Start in $|0 \dots 0\rangle$
- Allowed gates $H, S, C\text{-NOT}$

G-K theorem

- Start in $|0 \dots 0\rangle$

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H, S, C-NOT

X, Y, Z

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H, S, C-NOT
X, Y, Z

G-K theorem

- Start in $|0 \dots 0\rangle$
- Allowed gates $H, S, C\text{-NOT}$
 X, Y, Z
- Measurements. Z, X, Y

G-K theorem

- Start in $|0 \dots 0\rangle$
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G-K theorem

Efficient simulation

- Start in $|0 \dots 0\rangle$

$O(N^2)$

- Allowed gates

H, S, C-NOT

X, Y, Z

- Measurements. Z, X, Y

G-K theorem

Efficient simulation

- Start in $|0 \dots 0\rangle$

$O(N^2)$

- Allowed gates

H, S, C-NOT

- Measurements. Z, X, Y

X, Y, Z

Stabilizer formalism

- Pauli group

Stabilizer formalism

• Pauli group $\left(\begin{smallmatrix} \pm 1 \\ \pm i \end{smallmatrix} \right) \sigma_1 \otimes \dots \otimes \sigma_N$

Stabilizer formalism

- Pauli group $\left(\begin{smallmatrix} \pm 1 \\ \pm i \end{smallmatrix} \right) \sigma_1 \otimes \dots \otimes \sigma_N$ $g^2 = I$

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$$g^2 = I$$

$$gh = \pm hg$$

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Stabilizer formalism

- Pauli group $(\begin{smallmatrix} \pm 1 \\ \pm i \end{smallmatrix}) \otimes_{\alpha=1}^N \sigma_{\alpha}$ $g^2 = I$
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Stabilizer formalism

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Stabilizer formalism

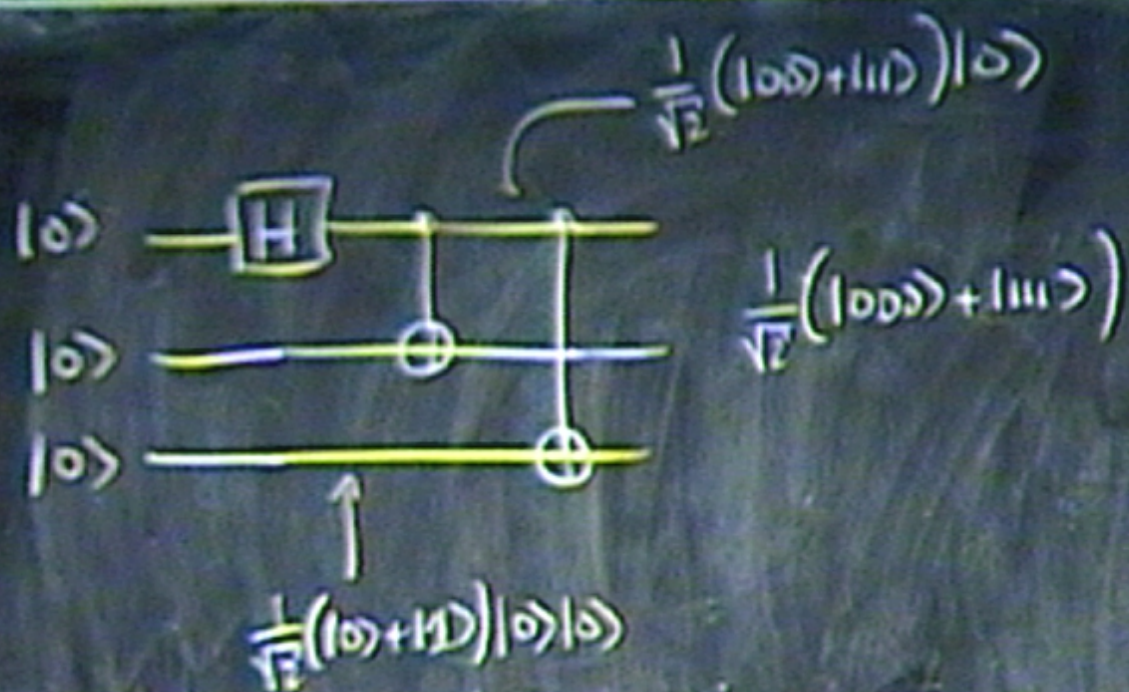
- Pauli group $(\pm 1) \mathbb{I} \otimes \dots \otimes \sigma_{\alpha_N}$ $g^2 = \mathbb{I}$
 $gh = \pm hg$
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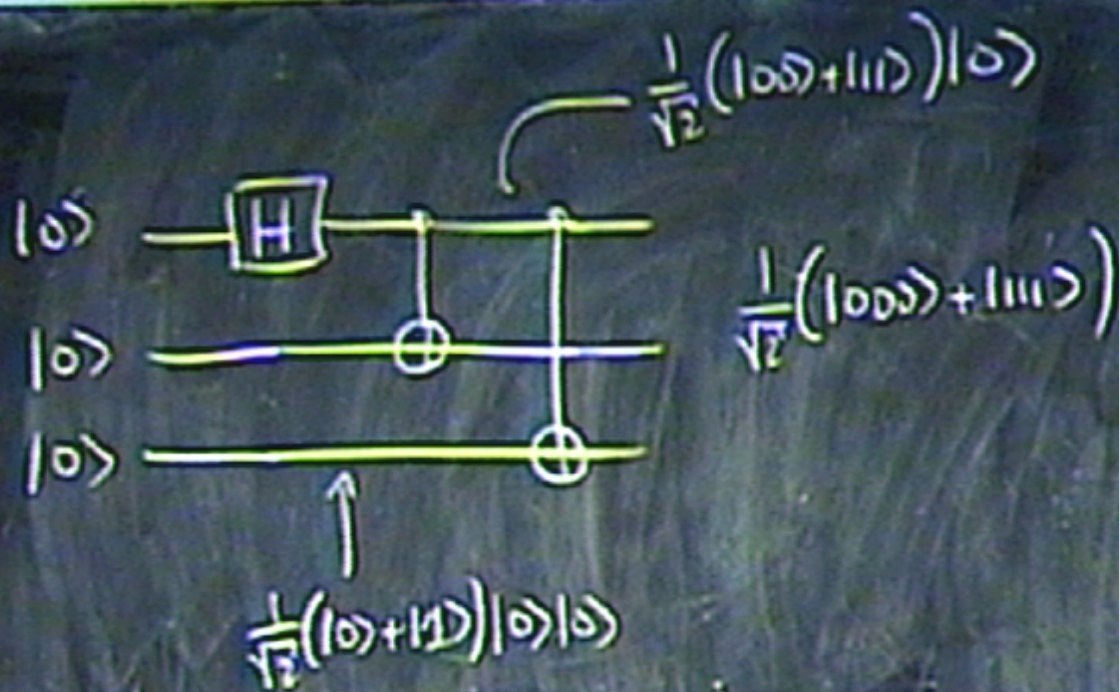
$$\sim 2^{N^2}$$

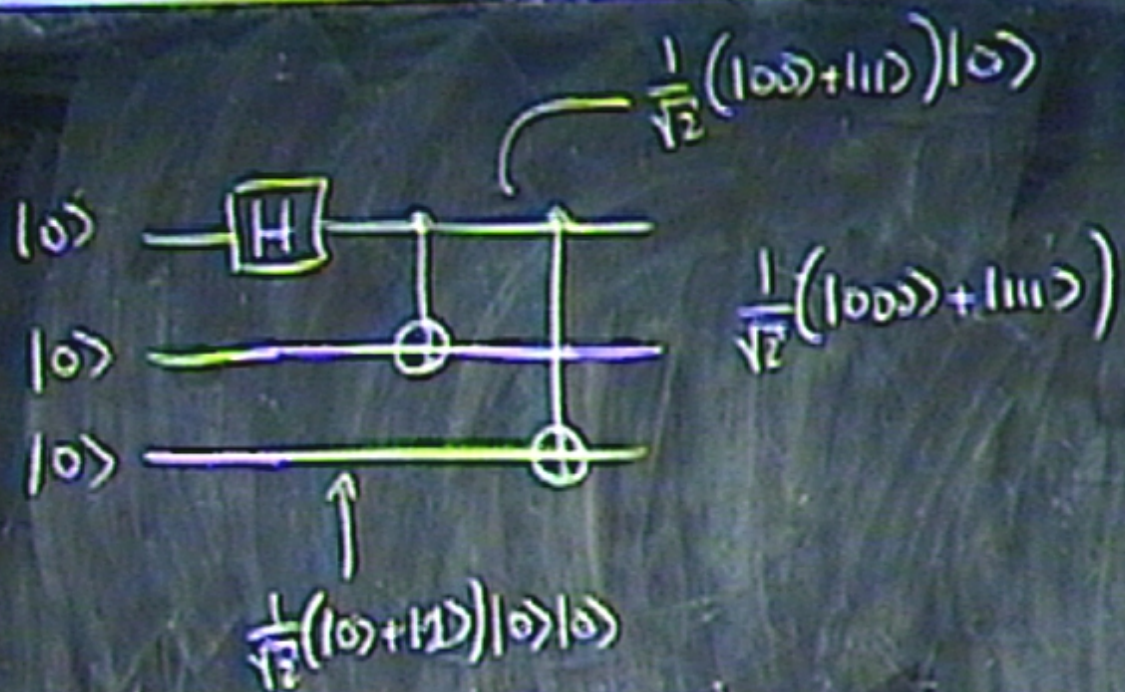
$$N=1: 6$$

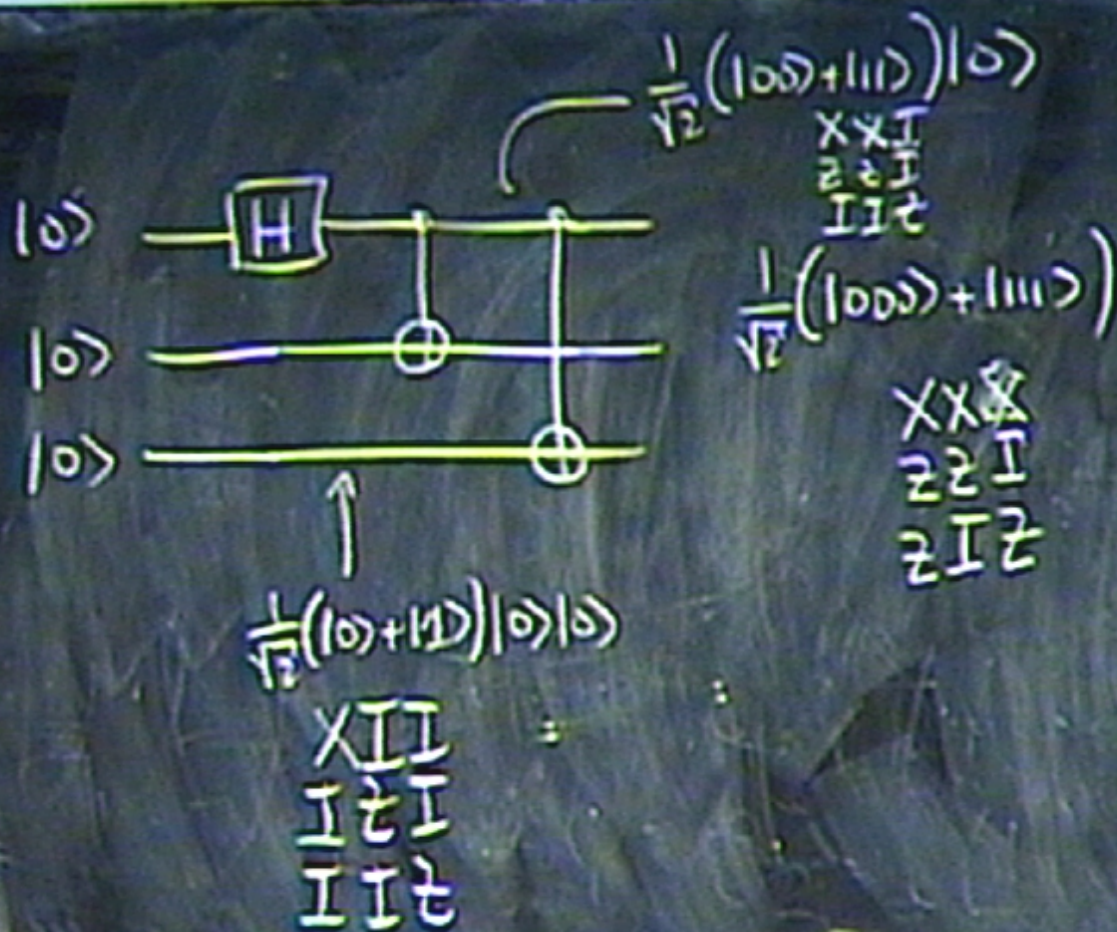
$$N=2: 60$$

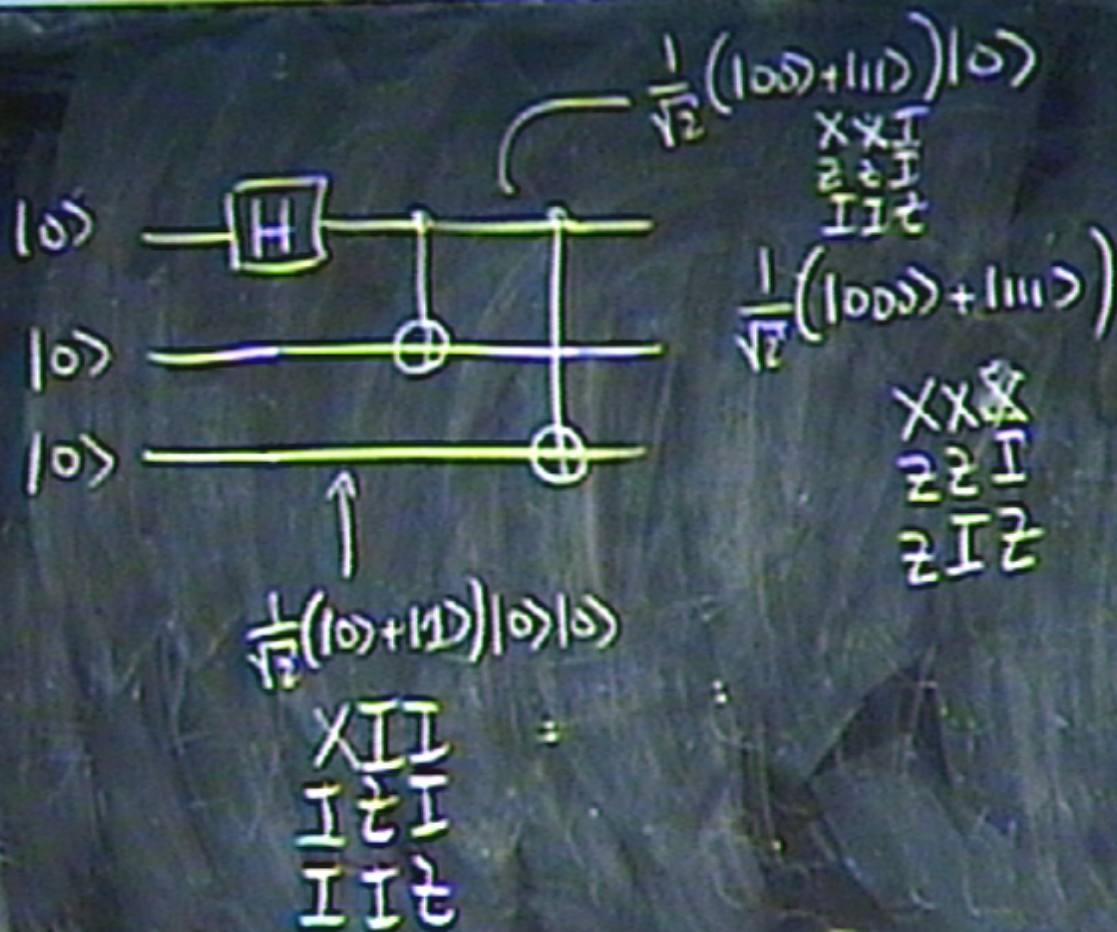
$$N=3: 1080$$

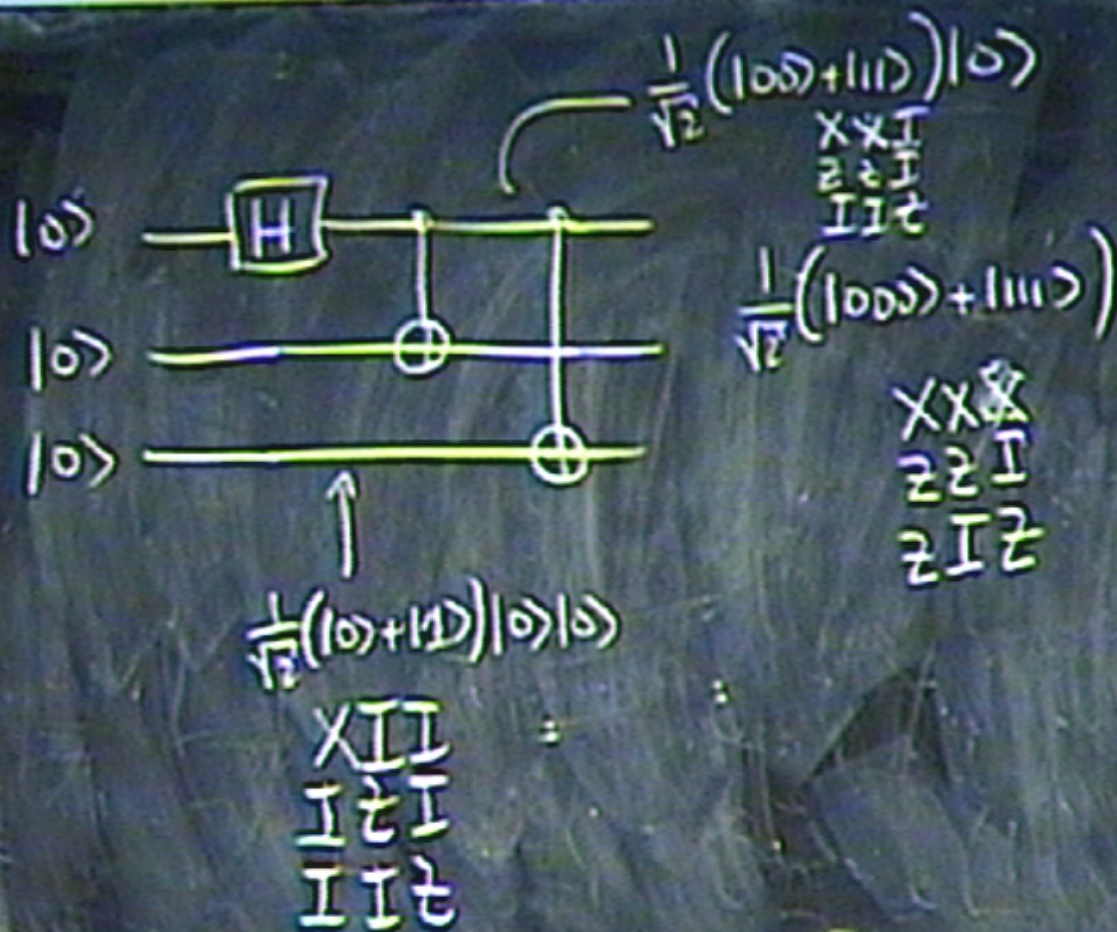


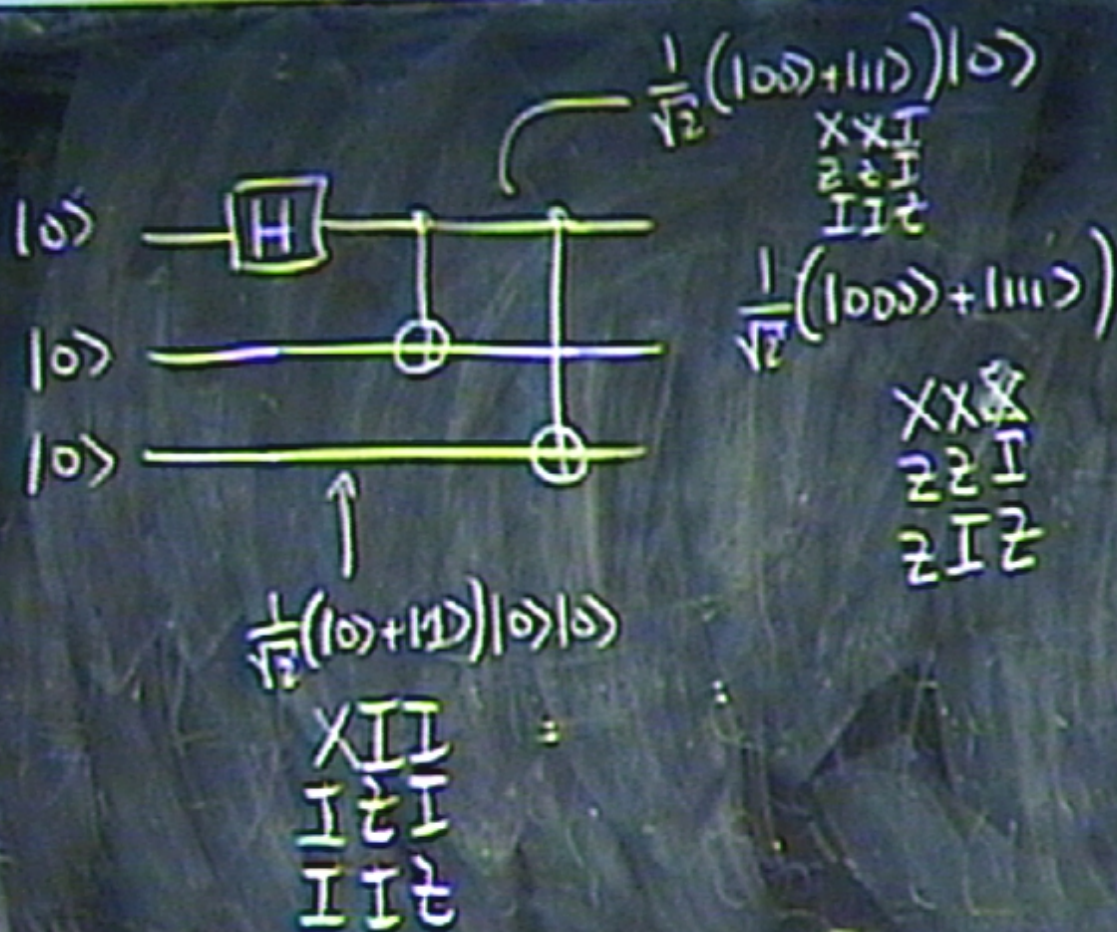


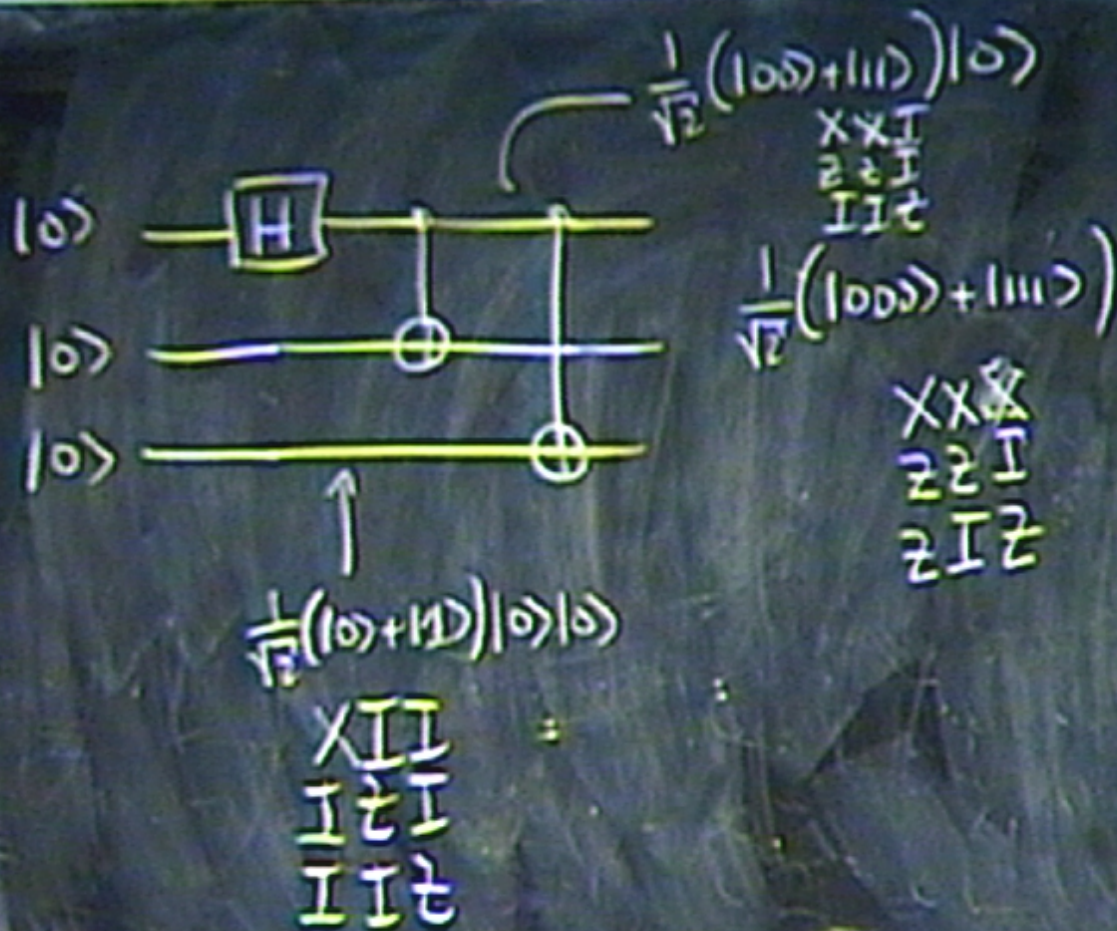












Stabilizer formalism

- Pauli group $(\pm 1) \sigma_1 \otimes \dots \otimes \sigma_N$

$$g^2 = I$$

$$gh = \pm hg$$

$$\begin{matrix} \sigma_1 \sigma_2 \\ \sigma_2 \sigma_1 \\ \sigma_1 \sigma_3 \\ \sigma_3 \sigma_1 \\ \sigma_2 \sigma_3 \\ \sigma_3 \sigma_2 \end{matrix}$$

- Stabilizer group S : commuting g 's

$$S = \langle g_1, \dots, g_r \rangle$$

- $|\psi\rangle$ simultaneous ± 1 eigenstate

- Measurements: $g \in S$

state stabilized
by S

Stabilizer formalism

- Pauli group $(\pm 1) \mathbb{I}, \sigma_1, \dots, \sigma_N$ $g^2 = \mathbb{I}$
 $gh = \pm hg$

$\mathbb{I}\mathbb{I}$
$\mathbb{I}\mathbb{I}$
$\mathbb{I}\mathbb{I}$
- Stabilizer group S : 2^N commuting g 's
- $S = \langle g_1, \dots, g_r \rangle$
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- Measurements: $\pm g \in S \quad \pm 1$

Stabilizer formalism

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$\mathbb{I}\mathbb{I}$
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- Measurements: $\pm g \in S$ ± 1

Stabilizer formalism

- Pauli group $(\pm 1) \sigma_1 \otimes \dots \otimes \sigma_N$ $g^2 = I$ $gh = \pm hg$

II
II
II
II
- Stabilizer group S : 2^N commuting g 's
- $S = \langle g_1, \dots, g_N \rangle$
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Stabilizer formalism

- Pauli group $(\pm 1) \mathbb{I}, \sigma_1, \dots, \sigma_N$ $g^2 = \mathbb{I}$ $gh = \pm hg$ $\begin{matrix} \mathbb{Z}\mathbb{I}\mathbb{I} \\ \mathbb{I}\mathbb{Z}\mathbb{I} \\ \mathbb{I}\mathbb{I}\mathbb{Z} \end{matrix}$
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Stabilizer formalism

- Pauli group $(\pm 1) \sigma_1 \otimes \dots \otimes \sigma_N$ $g^2 = I$
 $gh = \pm hg$ $\begin{matrix} ZII \\ IZI \\ IIZ \end{matrix}$
- Stabilizer group S : 2^N commuting g 's
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- Measurements: $\pm g \in S \quad \pm 1$

Stabilizer formalism

- Pauli group $(\pm 1) \mathbb{I}, \sigma_1, \dots, \sigma_N$

$$g^2 = \mathbb{I}$$

$$gh = \pm hg$$

$$\begin{matrix} \mathbb{Z} & \mathbb{I} & \mathbb{I} \\ \mathbb{I} & \mathbb{Z} & \mathbb{I} \\ \mathbb{I} & \mathbb{I} & \mathbb{Z} \end{matrix}$$

- Stabilizer group S : 2^N commuting g 's

$$S = \langle g_1, \dots, g_N \rangle$$

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- Measurements: $\pm g \in S \quad \pm 1$

LHV table

	x	y	z
qubit 1	x_1	y_1	z_1
$\vdots$			
qubit N	x_N	y_N	z_N

LHV table

	x	y	z
qubit 1	x_1	y_1	z_1
$\vdots$			
qubit N	x_N	y_N	z_N

LHV table

	x	y	z
qubit 1	x_1	y_1	z_1

$\vdots$

$\vdots$

qubit N

x_N	y_N	z_N
-------	-------	-------

$$xyz = \pm 1$$

LHV table

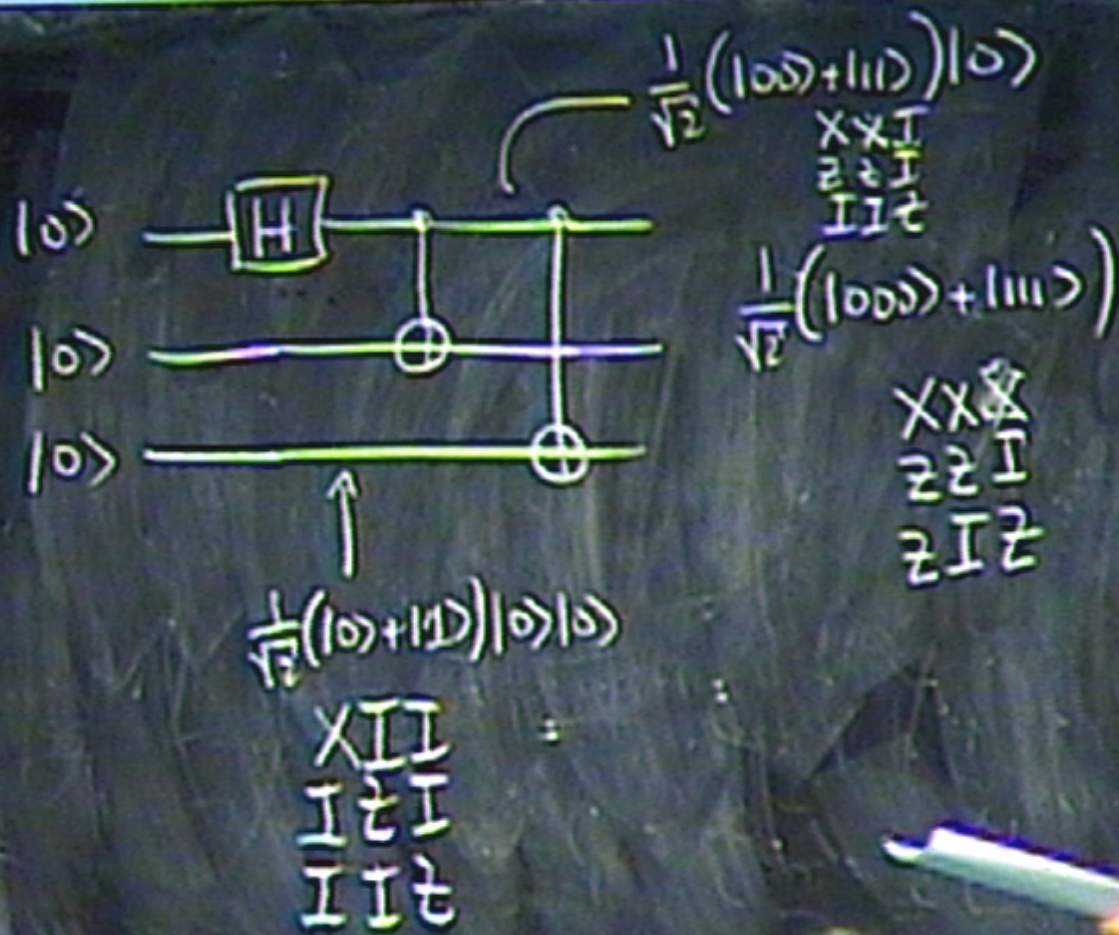
	x	y	z
qubit 1	x_1	y_1	z_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
qubit N	x_N	y_N	z_N

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LHV table

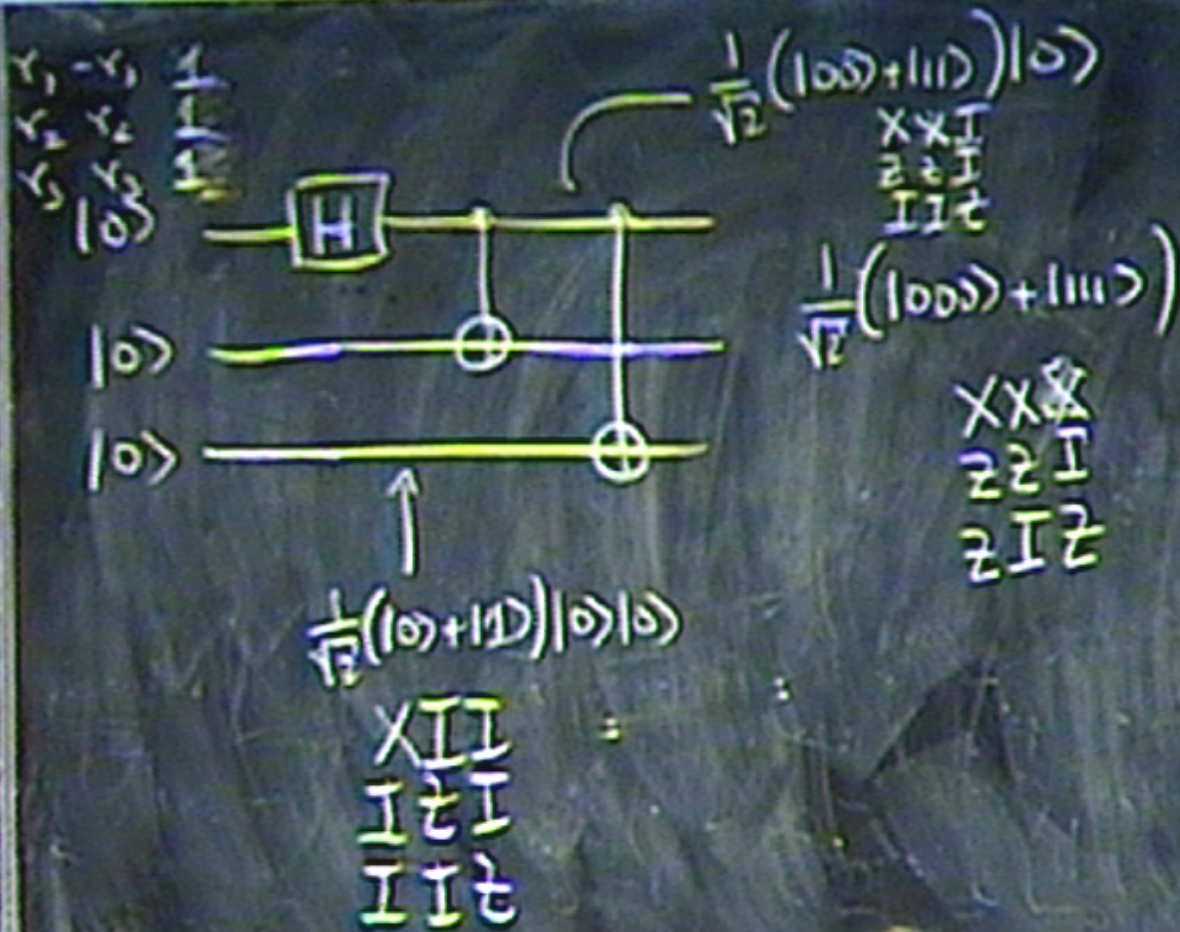
	x	y	z
qubit 1	x_1	y_1	z_1
$\vdots$			
qubit N	x_N	y_N	z_N

$$xyz = \pm 1$$

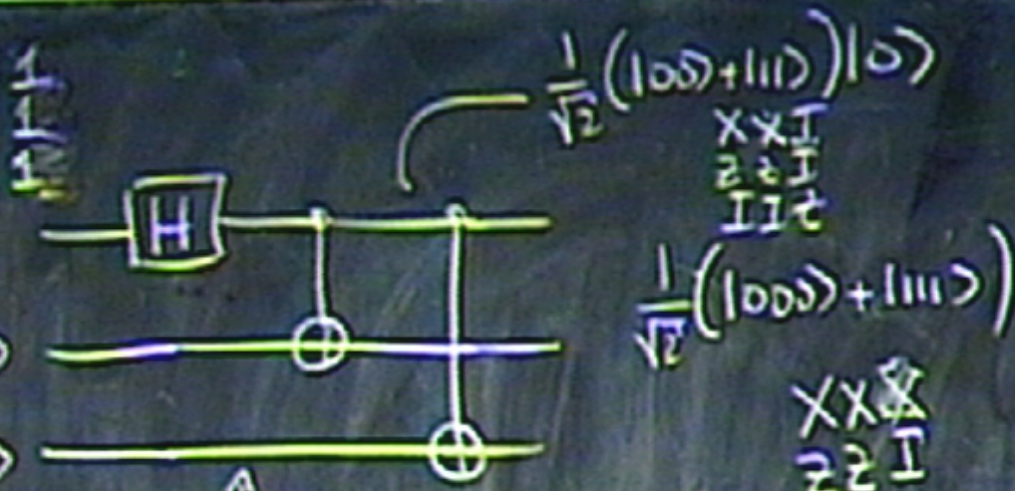


$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$\begin{matrix} X & X & I \\ Z & Z & I \\ Z & I & Z \end{matrix}$$



$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $|0\rangle$

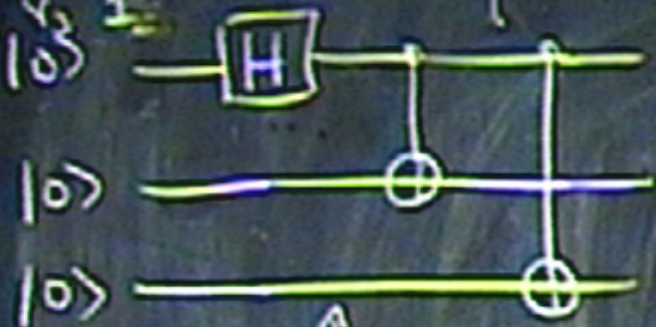


$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) |0\rangle |0\rangle$

$\begin{matrix} X & I & I \\ I & Z & I \\ I & I & Z \end{matrix}$

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$\begin{array}{l}
 \gamma_1, -\gamma_1, \frac{1}{\sqrt{2}} \\
 \gamma_2, \gamma_2, \frac{1}{\sqrt{2}} \\
 \gamma_3, \gamma_3, \frac{1}{\sqrt{2}}
 \end{array}$$



$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) |0\rangle$$

$$\begin{array}{c}
 XXII \\
 ZZII \\
 IIZI
 \end{array}$$

$$\frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

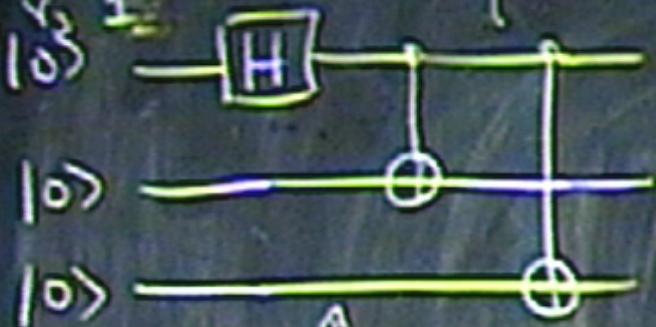
$$\begin{array}{c}
 XXXX \\
 ZZII \\
 ZZII
 \end{array}$$

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle |0\rangle$$

$$\begin{array}{c}
 XIII \\
 IIZI \\
 IIZI
 \end{array}$$

$$\begin{array}{ccc}
 \gamma_2 \gamma_3 & \gamma_1 \gamma_2 \gamma_3 & \gamma_1 \\
 \gamma_2 & \gamma_1 \gamma_2 & \gamma_1 \\
 \gamma_3 & \gamma_1 \gamma_3 & \gamma_1
 \end{array}$$

$$\begin{matrix}
 Y_1 & -Y_1 & \frac{1}{\sqrt{2}} \\
 Y_2 & Y_2 & \frac{1}{\sqrt{2}} \\
 Y_3 & Y_3 & \frac{1}{\sqrt{2}}
 \end{matrix}$$



$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) |0\rangle$$

$$\begin{matrix}
 X & X & I \\
 Z & Z & I \\
 I & I & Z
 \end{matrix}$$

$$\frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

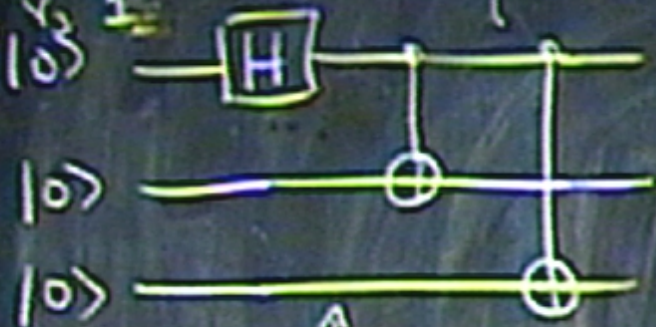
$$\begin{matrix}
 X & X & X \\
 Z & Z & I \\
 Z & I & Z
 \end{matrix}$$

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle |0\rangle$$

$$\begin{matrix}
 X & I & I \\
 I & Z & I \\
 I & I & Z
 \end{matrix}$$

$$\begin{matrix}
 Y_2 Y_3 & Y_1 Y_2 Y_3 & Y_1 \\
 Y_2 & Y_1 Y_2 & Y_1 \\
 Y_3 & Y_1 Y_3 & Y_1
 \end{matrix}$$

$$\begin{array}{l}
 \gamma_1, -\gamma_1, \frac{1}{\sqrt{2}} \\
 \gamma_2, \gamma_2, \frac{1}{\sqrt{2}} \\
 \gamma_3, \gamma_3, \frac{1}{\sqrt{2}}
 \end{array}$$



$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) |0\rangle$$

$\begin{matrix} X & X & I \\ Z & Z & I \\ I & I & Z \end{matrix}$

$$\frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

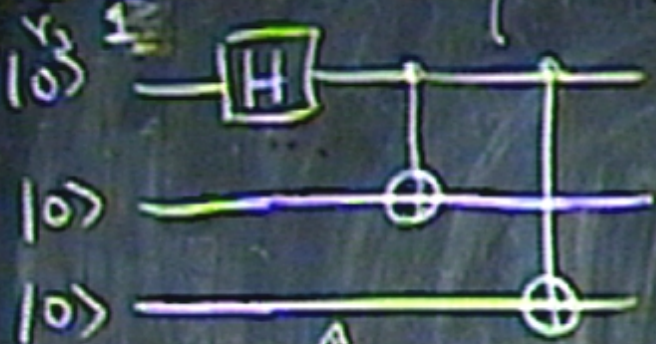
$\begin{matrix} X & X & X \\ Z & Z & I \\ Z & I & Z \end{matrix}$

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle |0\rangle$$

$\begin{matrix} X & I & I \\ I & Z & I \\ I & I & Z \end{matrix}$

$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$\begin{matrix}
 Y_1 & -Y_1 & 1 \\
 Y_2 & Y_2 & 1 \\
 Y_3 & Y_3 & 1
 \end{matrix}
 \begin{matrix}
 1 \\
 1 \\
 1
 \end{matrix}$$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$$\begin{matrix}
 X & I & I \\
 I & Z & I \\
 I & I & Z
 \end{matrix}$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

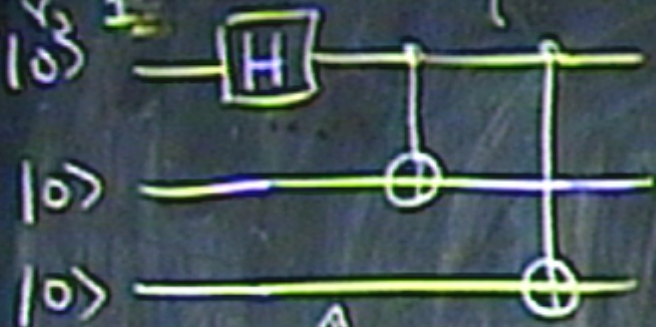
$$\begin{matrix}
 X & X & I \\
 Z & Z & I \\
 I & I & Z
 \end{matrix}$$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$\begin{matrix}
 X & X & X \\
 Z & Z & I \\
 Z & I & Z
 \end{matrix}$$

$Y_2 Y_3$	$Y_1 Y_2 Y_3$	Y_1
Y_2	$Y_1 Y_2$	Y_1
Y_3	$Y_1 Y_3$	Y_1

$$\begin{matrix}
 \gamma_1 & -\gamma_1 & 1 \\
 \gamma_2 & \gamma_2 & 1 \\
 \gamma_3 & \gamma_3 & 1
 \end{matrix}$$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$$\begin{matrix}
 XXII \\
 ZZII \\
 IIZI
 \end{matrix}$$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$\begin{matrix}
 XXX \\
 ZZI \\
 ZIZ
 \end{matrix}$$

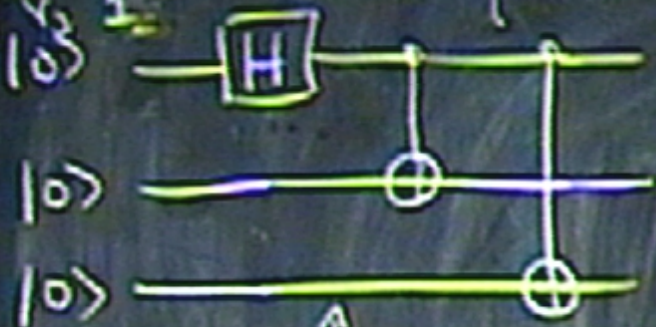
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$$\begin{matrix}
 XII \\
 IZI \\
 IIZ
 \end{matrix}$$

$$\begin{matrix}
 \gamma_2\gamma_3 & \gamma_1\gamma_2\gamma_3 & \gamma_1 \\
 \gamma_2 & \gamma_1\gamma_2 & \gamma_1 \\
 \gamma_3 & \gamma_1\gamma_3 & \gamma_1
 \end{matrix}$$

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \times I$
 $Z \times I$
 $I \times I$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \times X \times$
 $Z \times Z \times I$
 $Z \times I \times Z$

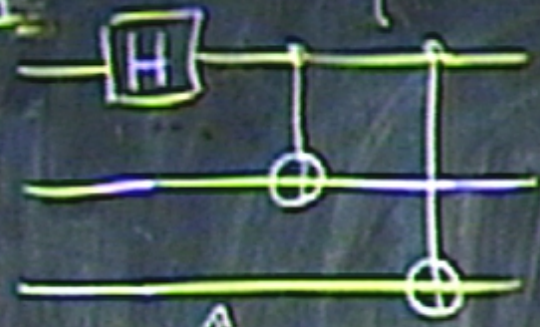
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$X I I$
 $I Z I$
 $I I Z$

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $10\rangle$



$$\frac{1}{\sqrt{2}}(100\rangle + 111\rangle)|0\rangle$$

$X \otimes I$
 $Z \otimes I$
 $I \otimes Z$

$$\frac{1}{\sqrt{2}}(1000\rangle + 1111\rangle)$$

$X \otimes X$
 $Z \otimes Z$
 $Z \otimes I$

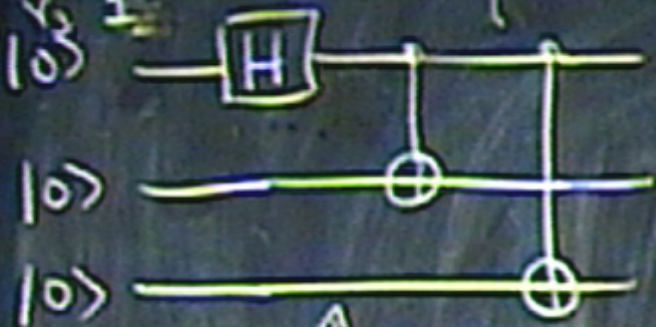
$$\frac{1}{\sqrt{2}}(10\rangle + 11\rangle)|0\rangle|0\rangle$$

$X \otimes I$
 $I \otimes Z$
 $I \otimes Z$

A
 $\gamma_2 \gamma_3$
 γ_2
 γ_3
 $\gamma_1 \gamma_2 \gamma_3$
 $\gamma_1 \gamma_2$
 $\gamma_1 \gamma_3$
 γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \otimes X \otimes I$
 $Z \otimes Z \otimes I$
 $I \otimes I \otimes Z$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \otimes X \otimes X$
 $Z \otimes Z \otimes I$
 $Z \otimes I \otimes Z$

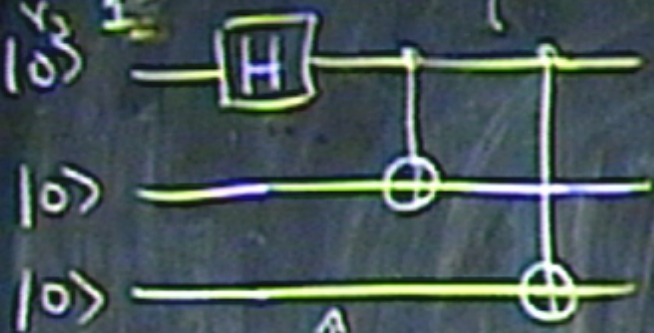
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$X \otimes I \otimes I$
 $I \otimes Z \otimes I$
 $I \otimes I \otimes Z$

A
 $\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$

XII
 $I\bar{I}I$
 $I\bar{I}\bar{I}$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$

XXI
 $Z\bar{I}I$
 $I\bar{I}\bar{I}$

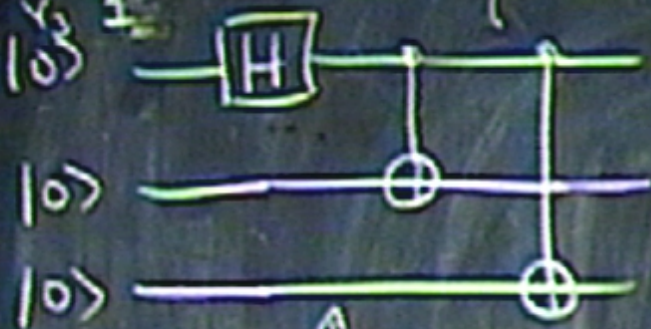
$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

XXX
 $Z\bar{Z}I$
 $ZI\bar{Z}$

A
 $\gamma_2\gamma_3$ $\gamma_1\gamma_2\gamma_3$ γ_1
 γ_2 $\gamma_1\gamma_2$ γ_1
 γ_3 $\gamma_1\gamma_3$ γ_1

$-XYY - YXY - YYX$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 III

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

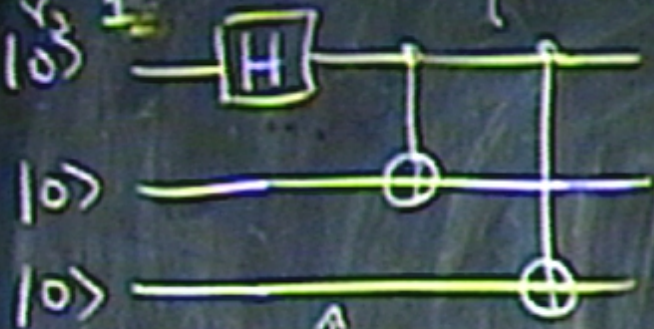
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 IIZ
 IIZ

A		
$\gamma_2\gamma_3$	$\gamma_1\gamma_2\gamma_3$	γ_1
γ_2	$\gamma_1\gamma_2$	γ_1
γ_3	$\gamma_1\gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \otimes I$
 $Z \otimes I$
 $I \otimes Z$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \otimes X \otimes X$
 $Z \otimes Z \otimes I$
 $Z \otimes I \otimes Z$

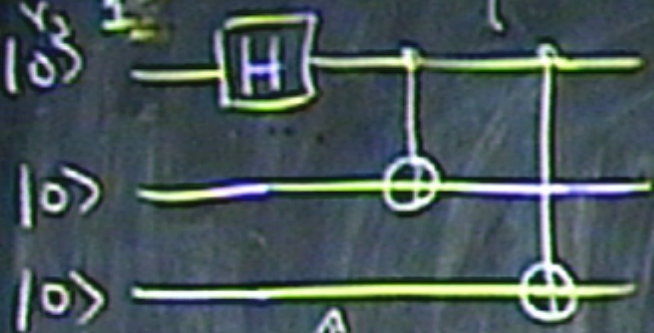
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$X \otimes I \otimes I$
 $I \otimes Z \otimes I$
 $I \otimes I \otimes Z$

A
 $\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

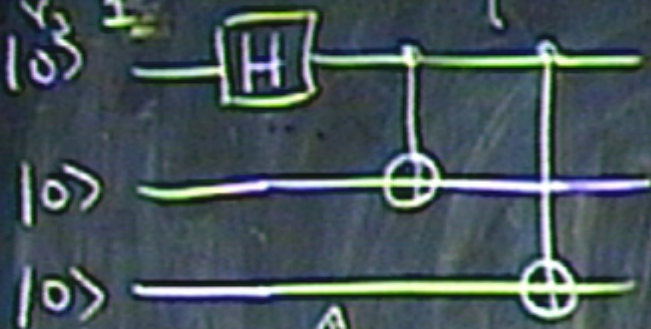
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 IIZ
 IIZ

A
 $\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$

XII
 $I\bar{I}I$
 $III\bar{I}$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$

XXI
 $Z\bar{I}I$
 $II\bar{I}$

$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

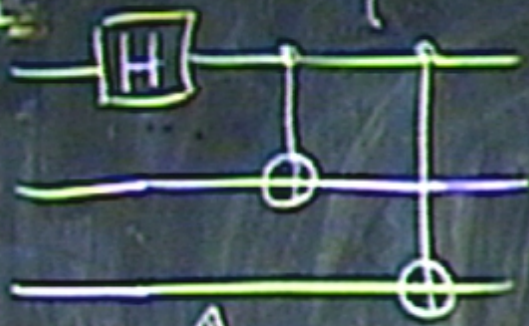
XXX
 ZZI
 ZIZ

N-2 GHZ states

A		
$\gamma_2\gamma_3$	$\gamma_1\gamma_3$	γ_1
γ_2	$\gamma_1\gamma_2$	γ_1
γ_3	$\gamma_1\gamma_3$	γ_1

$-XYX - YXY - YYX$

$\gamma_1, \gamma_2, \gamma_3$
 $\frac{1}{\sqrt{2}}$
 $\frac{1}{\sqrt{2}}$
 $\frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

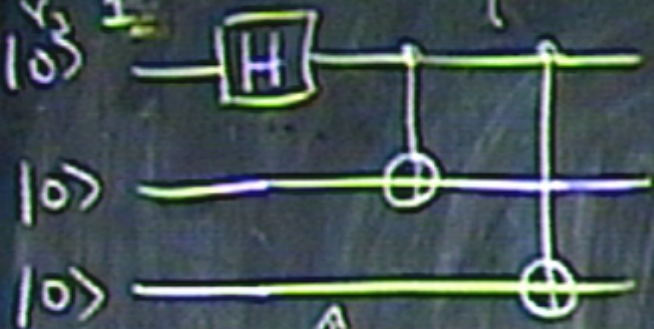
XII
 IIZ
 IIZ

N-2 GHZ states

A	B	C
$\gamma_2\gamma_3$	$\gamma_1\gamma_3$	γ_1
γ_2	$\gamma_1\gamma_2$	γ_1
γ_3	$\gamma_1\gamma_3$	γ_1

$-XYY$ $-YXY$ $-YYX$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

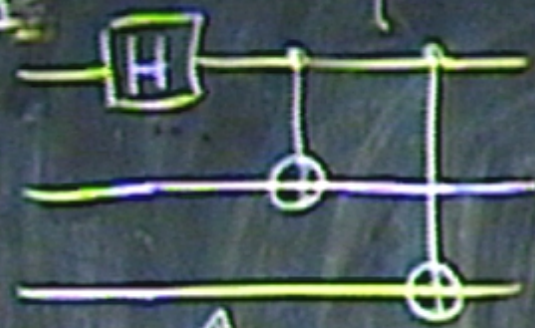
XII
 IIZ
 IIZ

N-2 GHZ states

$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \otimes I$
 $Z \otimes I$
 $I \otimes Z$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \otimes X$
 $Z \otimes Z$
 $Z \otimes I$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

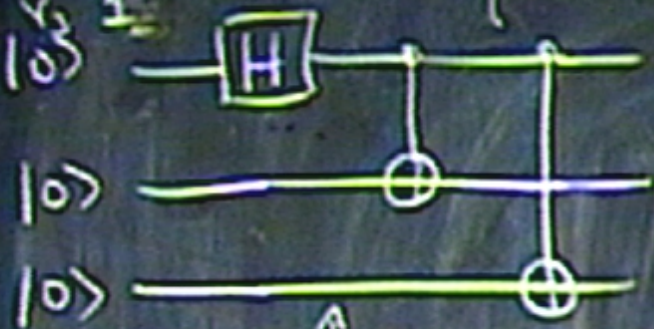
$X \otimes I$
 $I \otimes Z$
 $I \otimes Z$

N-2 GHZ states

A	B	C
γ_2, γ_3	γ_1, γ_3	γ_1
γ_2	γ_1, γ_2	γ_1
γ_3	γ_1, γ_3	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

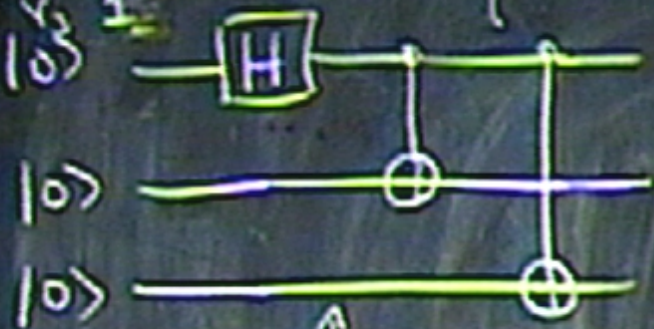
XII
 IIZ
 IIZ

N-2 GHZ states

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 $I\bar{I}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 $Z\bar{Z}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

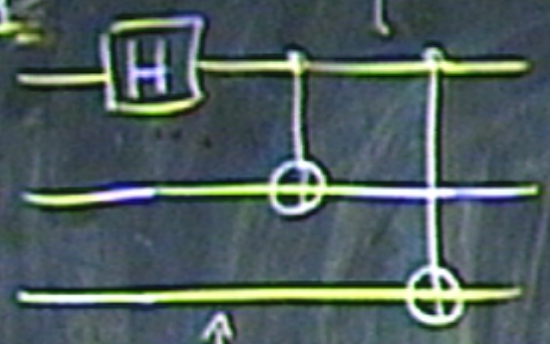
XXX
 $Z\bar{Z}I$
 $Z\bar{I}\bar{Z}$

N-2 GHZ states

$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$-XYY - YXY - YYX$

$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 $I\bar{I}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 $Z\bar{I}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 $Z\bar{Z}I$
 $ZI\bar{Z}$

N-2 GHZ states

A		
$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$$X$$

$$\{I, X\}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$Y$$

$$Z$$

$$|0\rangle$$

$$-X$$

$$-Y$$

$$-Z$$

$$|1\rangle$$

$$\begin{array}{c} X \\ \{I, X\} \\ |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \end{array}$$

$$\begin{array}{c} Y \\ |i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{array}$$

$$\begin{array}{c} Z \\ |0\rangle \end{array}$$

$$\begin{array}{c} -X \\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{array} \quad \begin{array}{c} -Y \\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{array} \quad \begin{array}{c} -Z \\ |1\rangle \end{array}$$

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

Z

$$|0\rangle$$

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-Y

$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

-Z

$$|1\rangle$$

$$\begin{array}{c}
 X \\
 \{I, X\} \\
 |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
 \pm
 \end{array}$$

$$\begin{array}{c}
 Y \\
 |i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)
 \end{array}$$

$$\begin{array}{c}
 Z \\
 |0\rangle \\
 \gamma \quad -\gamma \quad \frac{1}{\sqrt{2}}
 \end{array}$$

$$\begin{array}{c}
 -X \\
 |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)
 \end{array}$$

$$\begin{array}{c}
 -Y \\
 |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)
 \end{array}$$

$$\begin{array}{c}
 -Z \\
 |1\rangle
 \end{array}$$

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Z

$$|0\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

-Z

$$|1\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Z

$$|0\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-Z

$$|1\rangle$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

1	0	0
---	---	---

--	--	--	--

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

1	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1, 1	-1, 1	1, -1	-1, -1
------	-------	-------	--------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

0	1	0
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-1	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

1	-1	0
---	----	---

-Z

1	1	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-1	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1, 1	-1, 1	1, -1	-1, -1
------	-------	-------	--------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-0	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-1	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-1	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	0	0
----	---	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-0	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-0	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

0	0	-1
---	---	----

1,1	-1,1	1,-1	-1,-1
-----	------	------	-------

X
 $\{I, X\}$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

1	0	0
---	---	---

Y

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	1	0
----	---	---

Z

$$|0\rangle$$

0	0	1
---	---	---

--	--	--	--

X, Z

-X

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

-1	-1	0
----	----	---

-Y

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

0	-1	0
---	----	---

-Z

$$|1\rangle$$

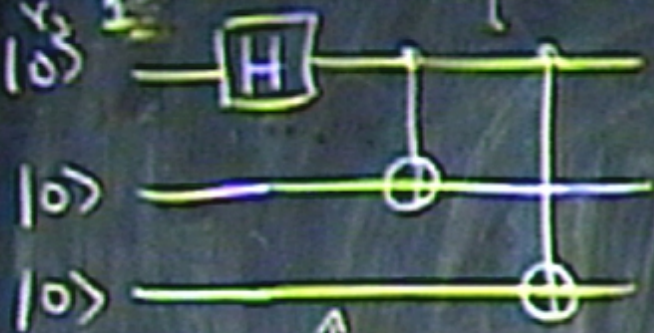
0	0	-1
---	---	----

1, 1	-1, 1	1, -1	-1, -1
------	-------	-------	--------

Stabilizer formalism

- Pauli group $\left(\begin{smallmatrix} \pm 1 \\ \pm i \end{smallmatrix}\right) \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$ $g^2 = I$
 $gh = \pm hg$ $\begin{smallmatrix} \pm II \\ \pm II \\ \pm II \end{smallmatrix}$
- Stabilizer group S : 2^N commuting g 's
- $S = \langle g_1, \dots, g_N \rangle$
- $|\psi\rangle$ simultaneous ± 1 eigenstate of $S \rightarrow$ state stabilized by S
- Measurements: $g \in S$ ± 1

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \otimes I$
 $Z \otimes I$
 $I \otimes Z$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \otimes X \otimes X$
 $Z \otimes Z \otimes I$
 $Z \otimes I \otimes Z$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

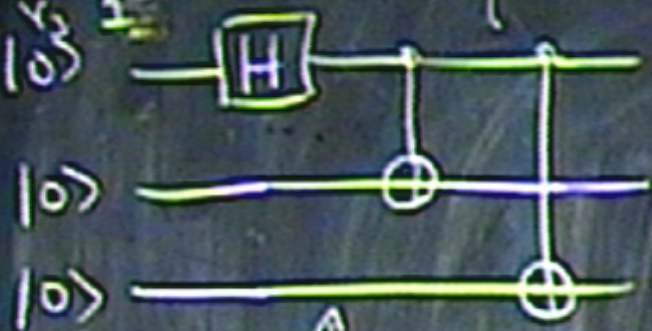
$X \otimes I \otimes I$
 $I \otimes Z \otimes I$
 $I \otimes I \otimes Z$

N-2 GHZ States

A		
$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

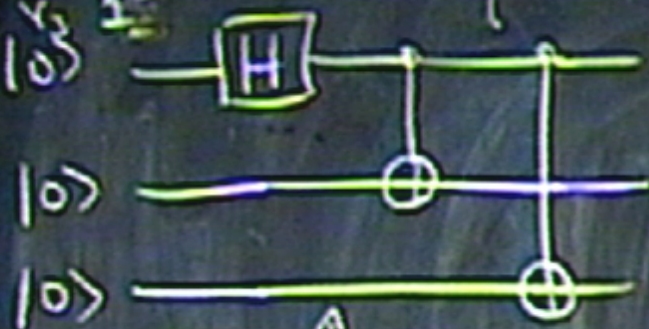
XII
 IIZ
 IIZ

N-2 GHZ states

$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 $I\bar{I}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 $Z\bar{Z}I$
 $I\bar{I}\bar{I}$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

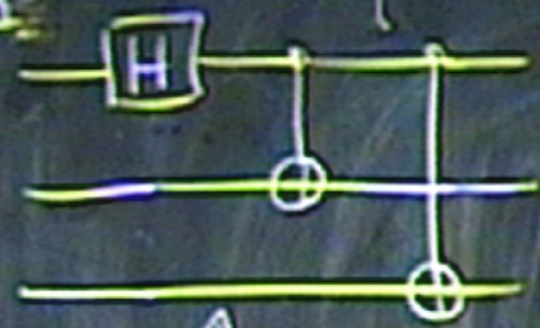
XXX
 $Z\bar{Z}I$
 $Z\bar{I}\bar{Z}$

N-2 GHZ states

$\gamma_2\gamma_3$ $\gamma_1\gamma_2\gamma_3$ γ_1
 γ_2 $\gamma_1\gamma_2$ γ_1
 γ_3 $\gamma_1\gamma_3$ γ_1

$-XYY$ $-YXY$ $-YYX$

$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

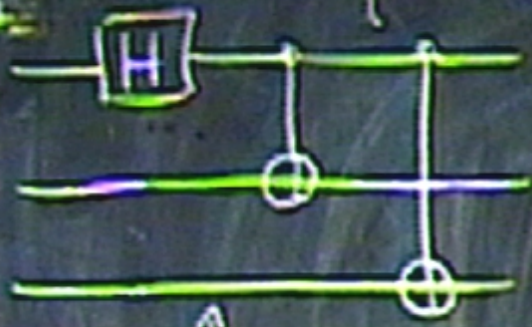
XII
 IIZ
 IIZ

N-2 GHZ states

A	B	C
$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

$$-XYY - YXY - YYX$$

$\gamma_1, -\gamma_1$
 γ_2, γ_2
 γ_3, γ_3
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 III

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

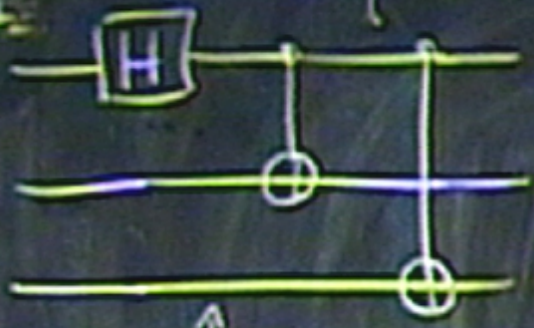
XII
 IIZ
 III

N-2 GHZ states

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

$-XYY$ $-YXY$ $-YYX$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$X \times I$
 $Z \times I$
 $I \times I$

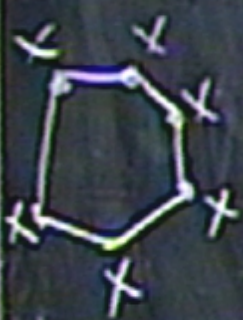
$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$X \times$
 Z

N-2 GHZ states

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

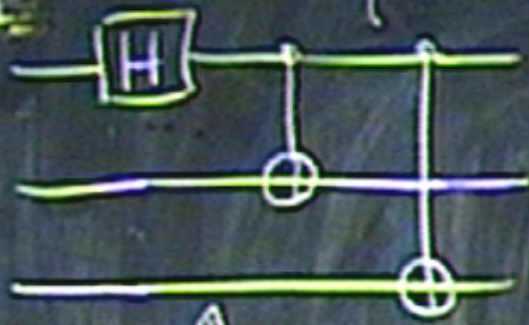
$$- \gamma_1 \gamma_2 \gamma_3 - \gamma_1 \gamma_2 \gamma_3$$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$X I I$
 $I Z I$
 $I I Z$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 IIZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

N-2 GHZ states

A		
$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

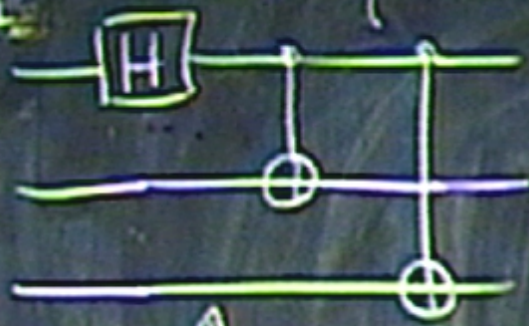


$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 IIZ
 IIZ

$$-XYX - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$\begin{matrix} XXII \\ ZZII \\ IIZI \end{matrix}$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$\begin{matrix} XXXX \\ ZZII \\ ZIZI \end{matrix}$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

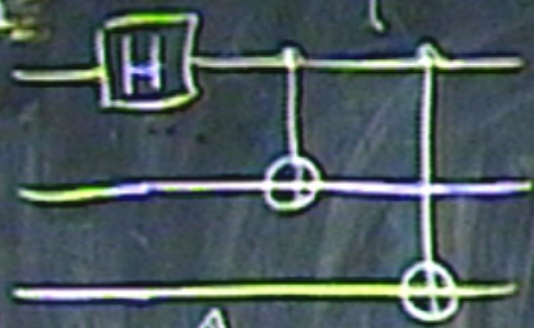
$\begin{matrix} XIII \\ IZII \\ IIZI \end{matrix}$

N-2 GHZ states

A	B	C
$\gamma_2\gamma_3$	$\gamma_1\gamma_3$	γ_1
γ_2	$\gamma_1\gamma_2$	γ_1
γ_3	$\gamma_1\gamma_3$	γ_1

$$-XYX - YXY - YYX$$

$\gamma_1, -\gamma_1$
 $\gamma_2, -\gamma_2$
 $\gamma_3, -\gamma_3$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 III

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

N-2 GHZ states

A		
$\gamma_2 \gamma_3$	$\gamma_1 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1

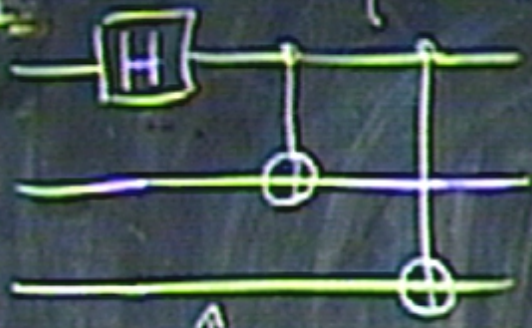


$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 IIZ
 IIZ

$$-XYX - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI
 ZZI
 III

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

XXX
 ZZI
 ZIZ

N-2 GHZ states

$\gamma_2 \gamma_3$ $\gamma_1 \gamma_2 \gamma_3$ γ_1
 γ_2 $\gamma_1 \gamma_2$ γ_1
 γ_3 $\gamma_1 \gamma_3$ γ_1

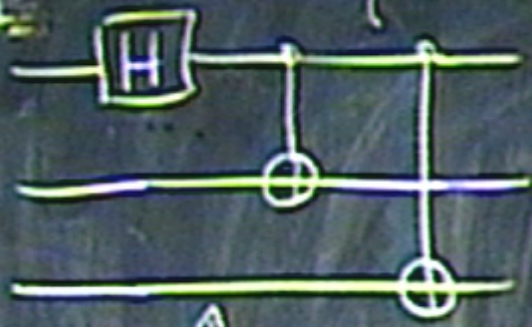


$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

XII
 IIZ
 IIZ

$$-XYX - YXY - YYX$$

$\gamma_1, -\gamma_1, \frac{1}{\sqrt{2}}$
 $\gamma_2, \gamma_2, \frac{1}{\sqrt{2}}$
 $\gamma_3, \gamma_3, \frac{1}{\sqrt{2}}$
 $|0\rangle$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

$\begin{matrix} XXII \\ ZZII \\ IIZI \end{matrix}$

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$\begin{matrix} XXX \\ ZZI \\ ZIZ \end{matrix}$

N-2 GHZ states

$\gamma_2 \gamma_3$	$\gamma_1 \gamma_2 \gamma_3$	γ_1
γ_2	$\gamma_1 \gamma_2$	γ_1
γ_3	$\gamma_1 \gamma_3$	γ_1



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle|0\rangle$$

$\begin{matrix} XII \\ IZI \\ IIZ \end{matrix}$

$$-XYY - YXY - YYX$$