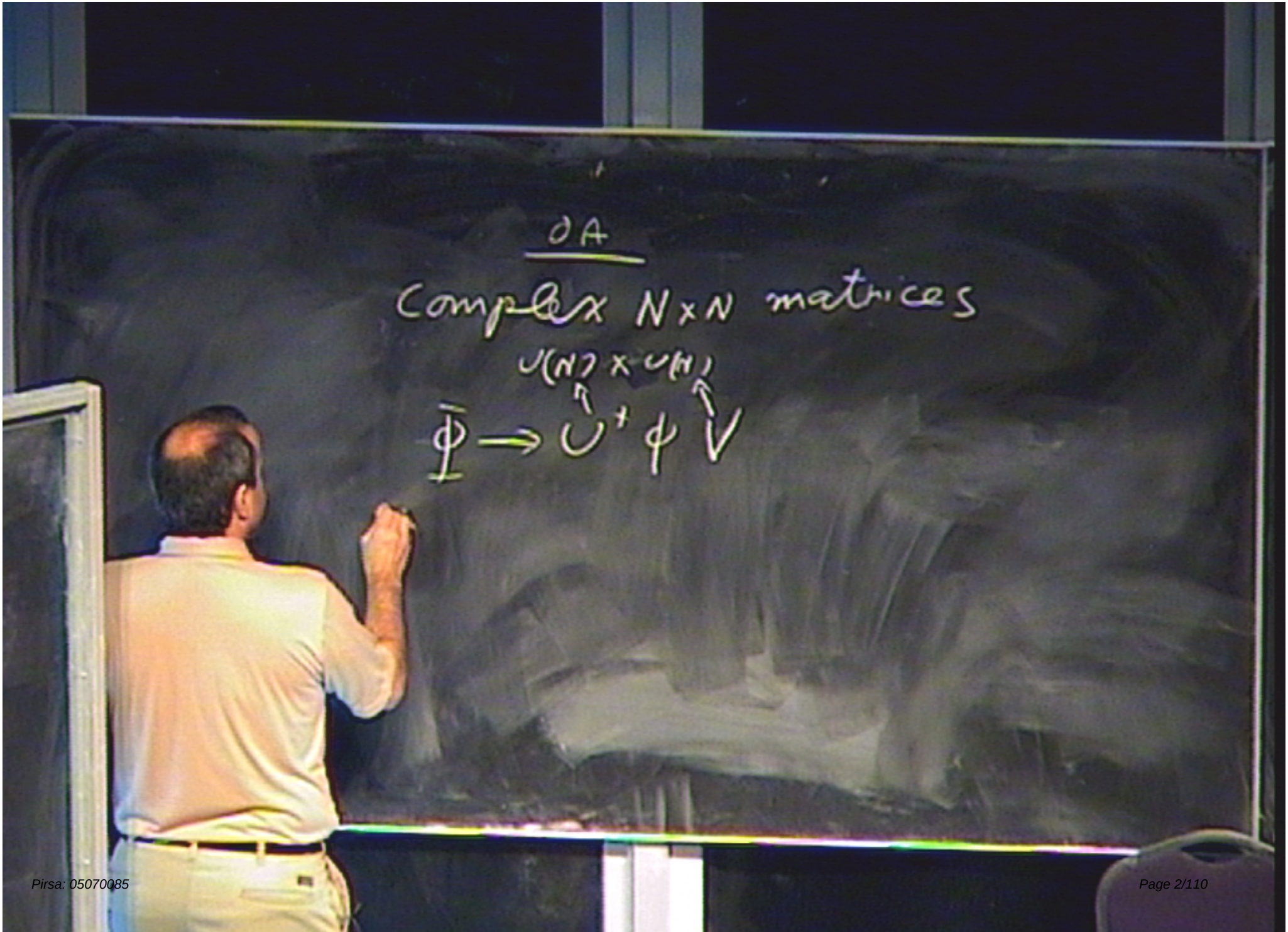
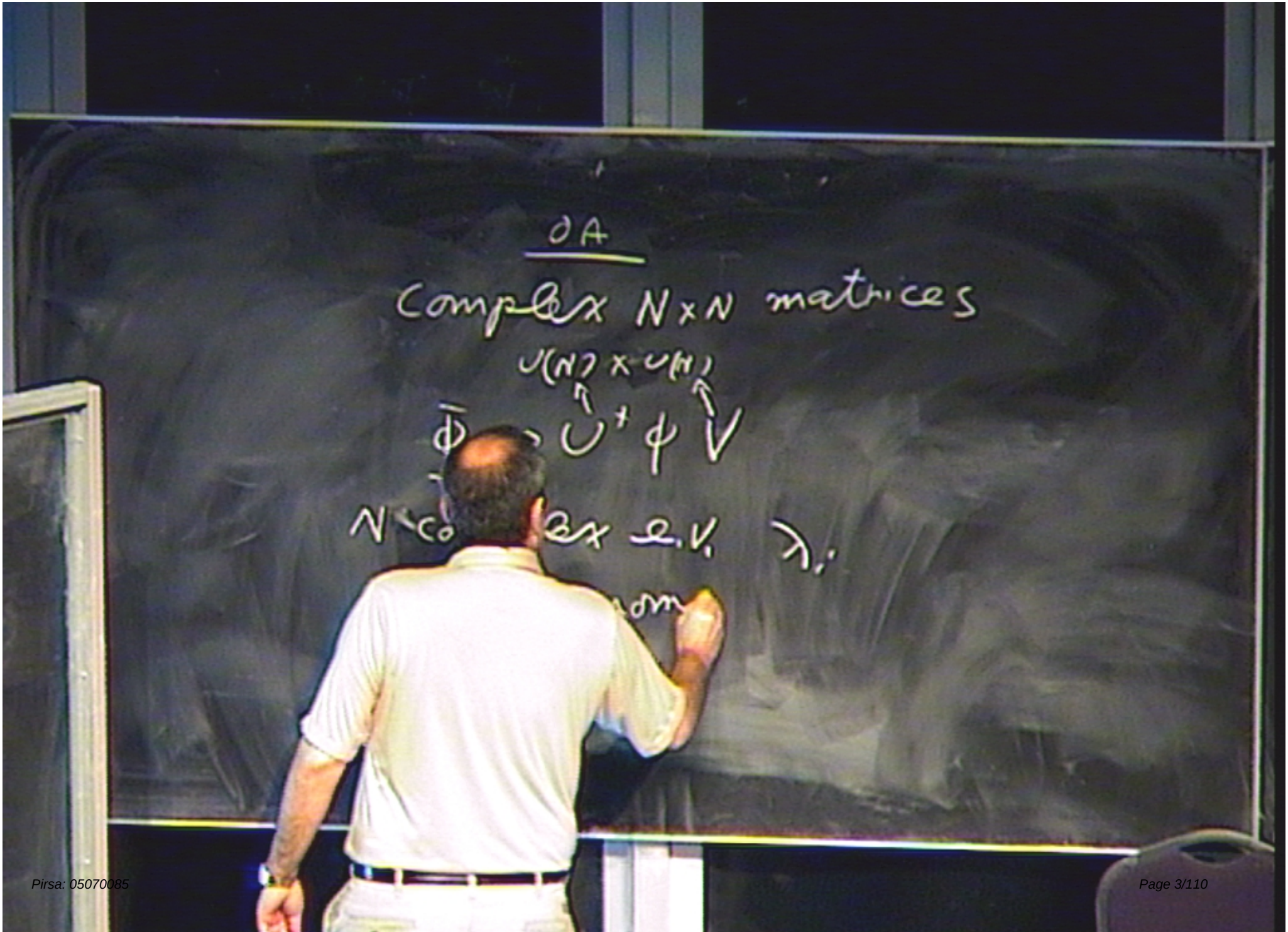
Title: Matrix Models and Noncritical Strings

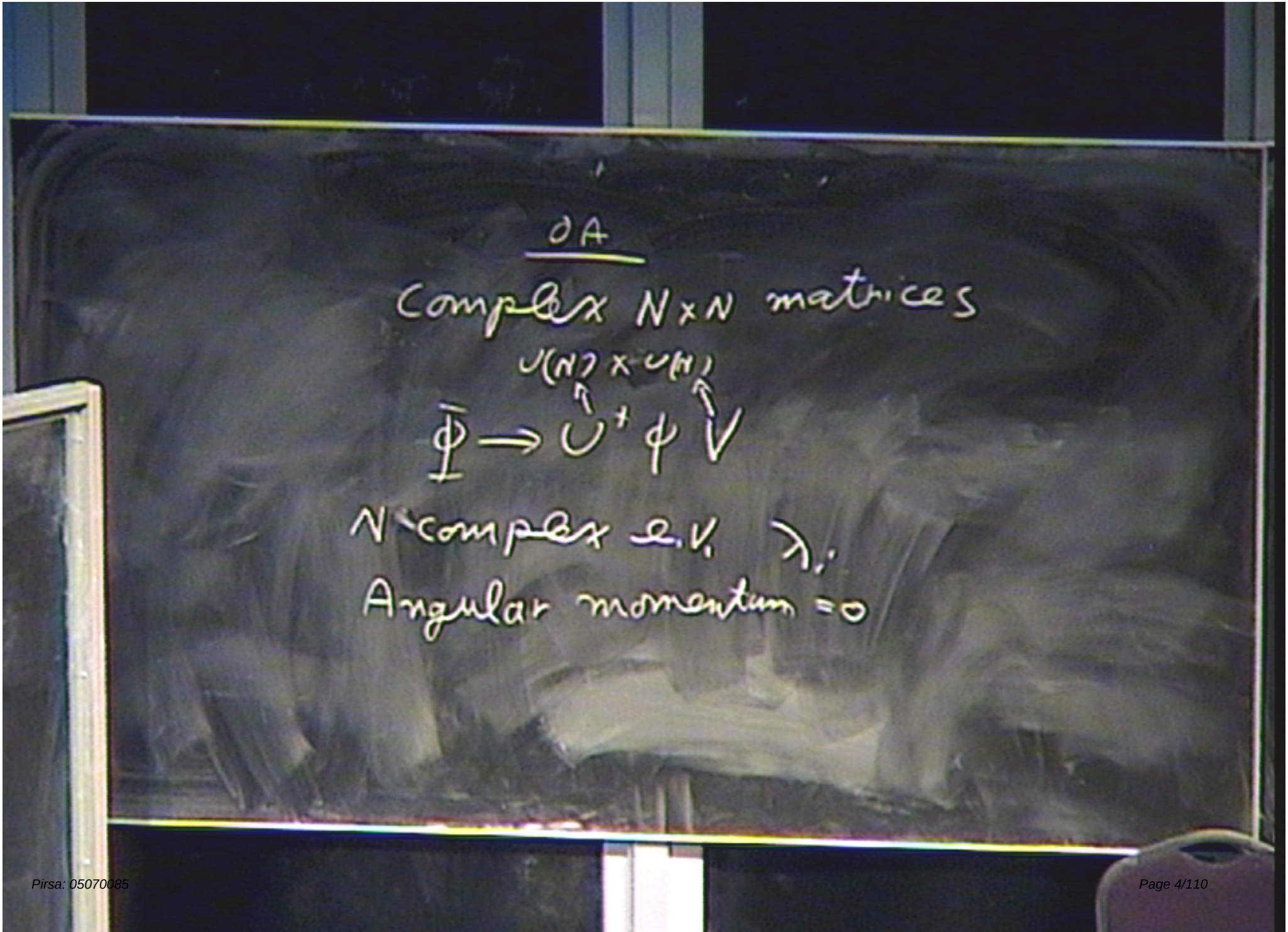
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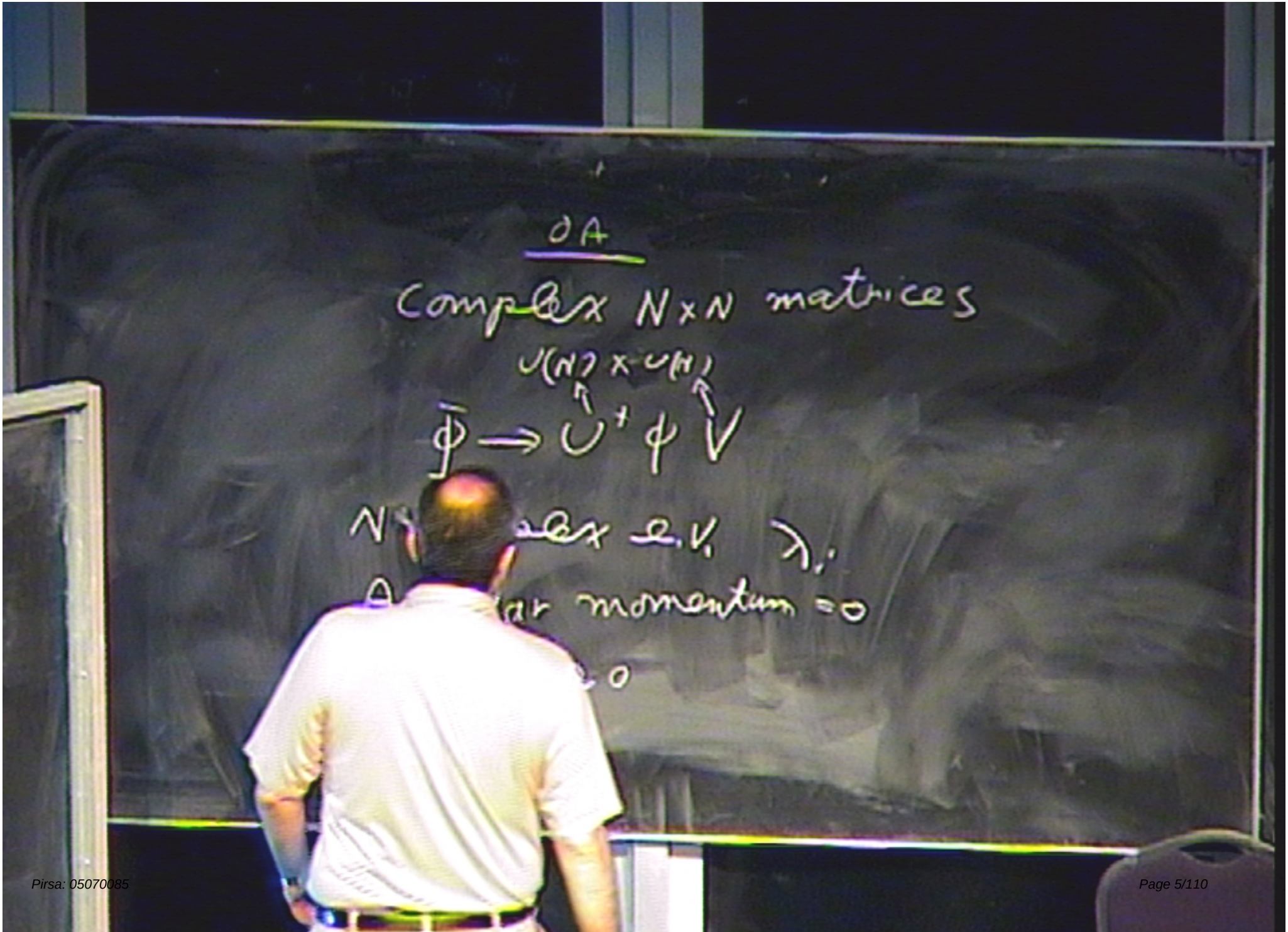
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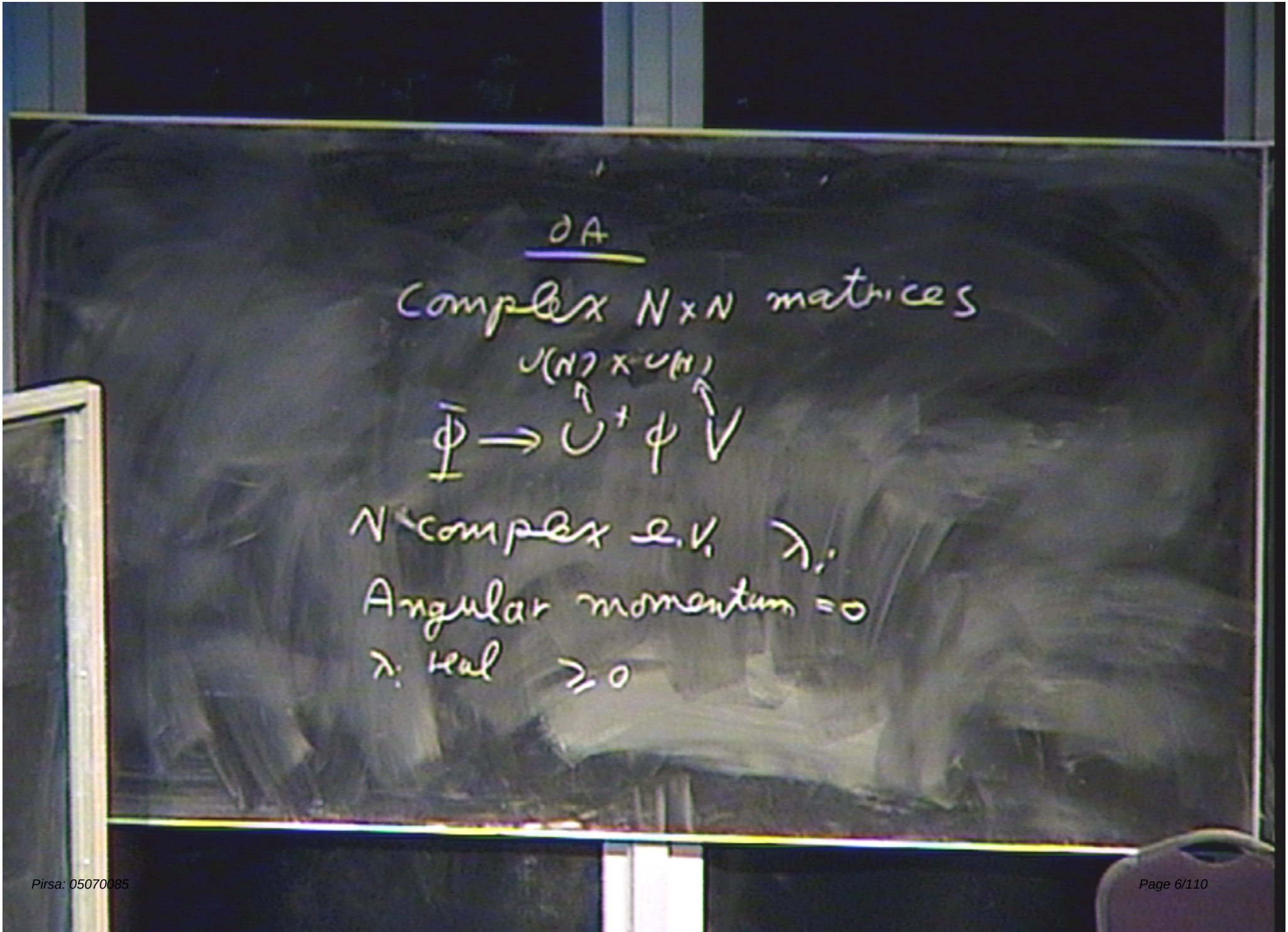
Abstract:

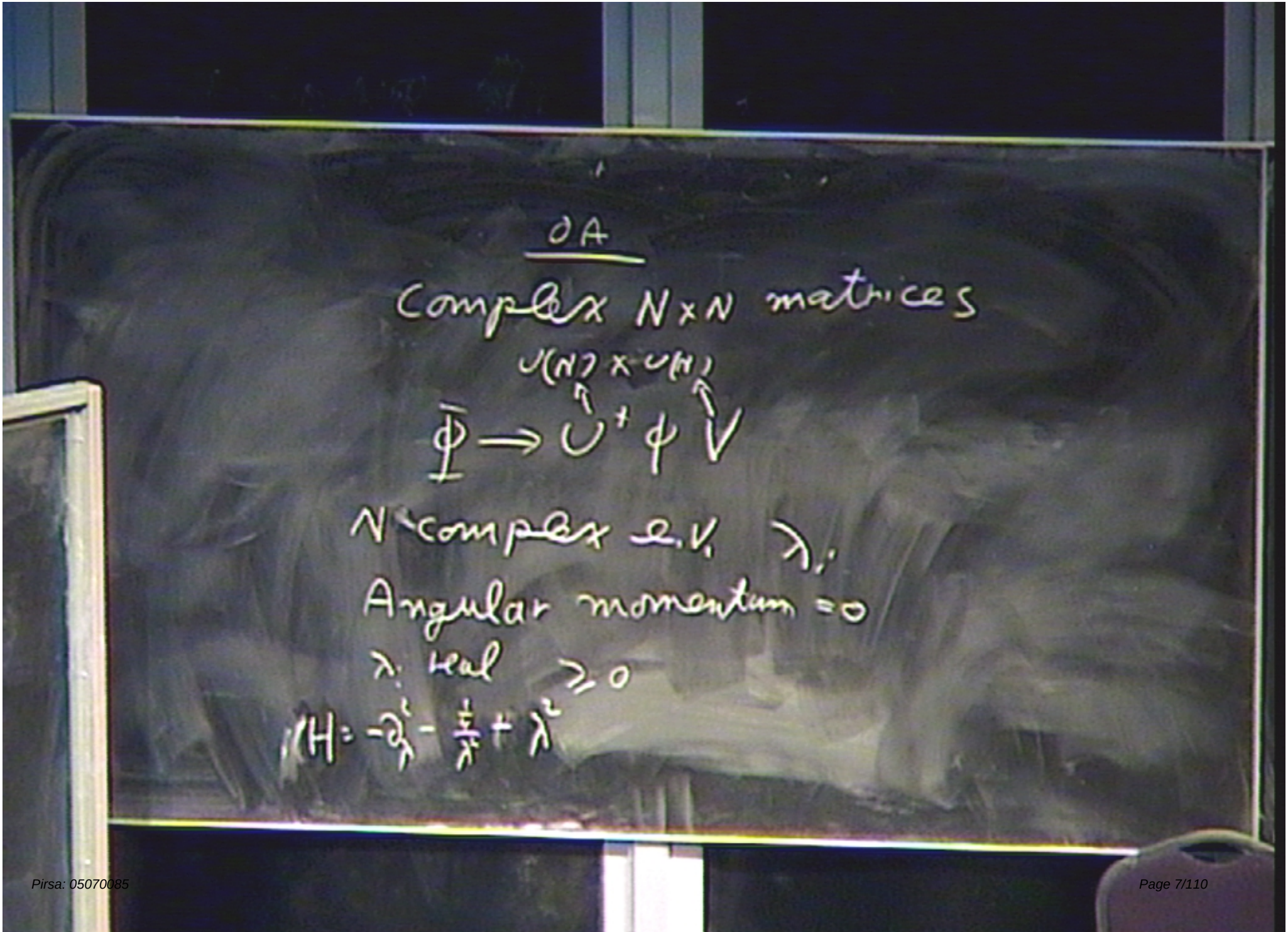

0A
Complex N×N matrices
U(N) × U(N)
Φ → U† φ V


0A
Complex N×N matrices
U(N) × U(N)
Φ → U† φ V


0A
Complex N×N matrices
U(N) × U(N)
Φ → U† φ V
N complex e.v. λ_i
Angular momentum = 0


0A
Complex N×N matrices
U(N) × U(N)
$\Phi \to U^\dagger \Phi V$
momentum = 0


0A
Complex N×N matrices
U(N) × U(N)
Φ → U† φ V
N complex e.v. λ_i
Angular momentum = 0
λ_i real ≥ 0


0A
Complex N×N matrices
U(N) × U(N)
Φ → U† Φ V
N complex e.v. λ_i
Angular momentum = 0
λ_i real ≥ 0
H = -∂_λ² - 1/λ² + λ²

0A

Complex $N \times N$ matrices

$U(N) \times U(N)$

$\Phi \to U^{\dagger} \Phi V$

$H = -\partial^2 - \frac{1}{4} - \lambda^2$

N complex e.v. λ_i

Angular momentum $= 0$

λ real ≥ 0



0A

Complex $N \times N$ matrices

$U(N) \times U(N)$

$\Phi \to U^{\dagger} \phi V$

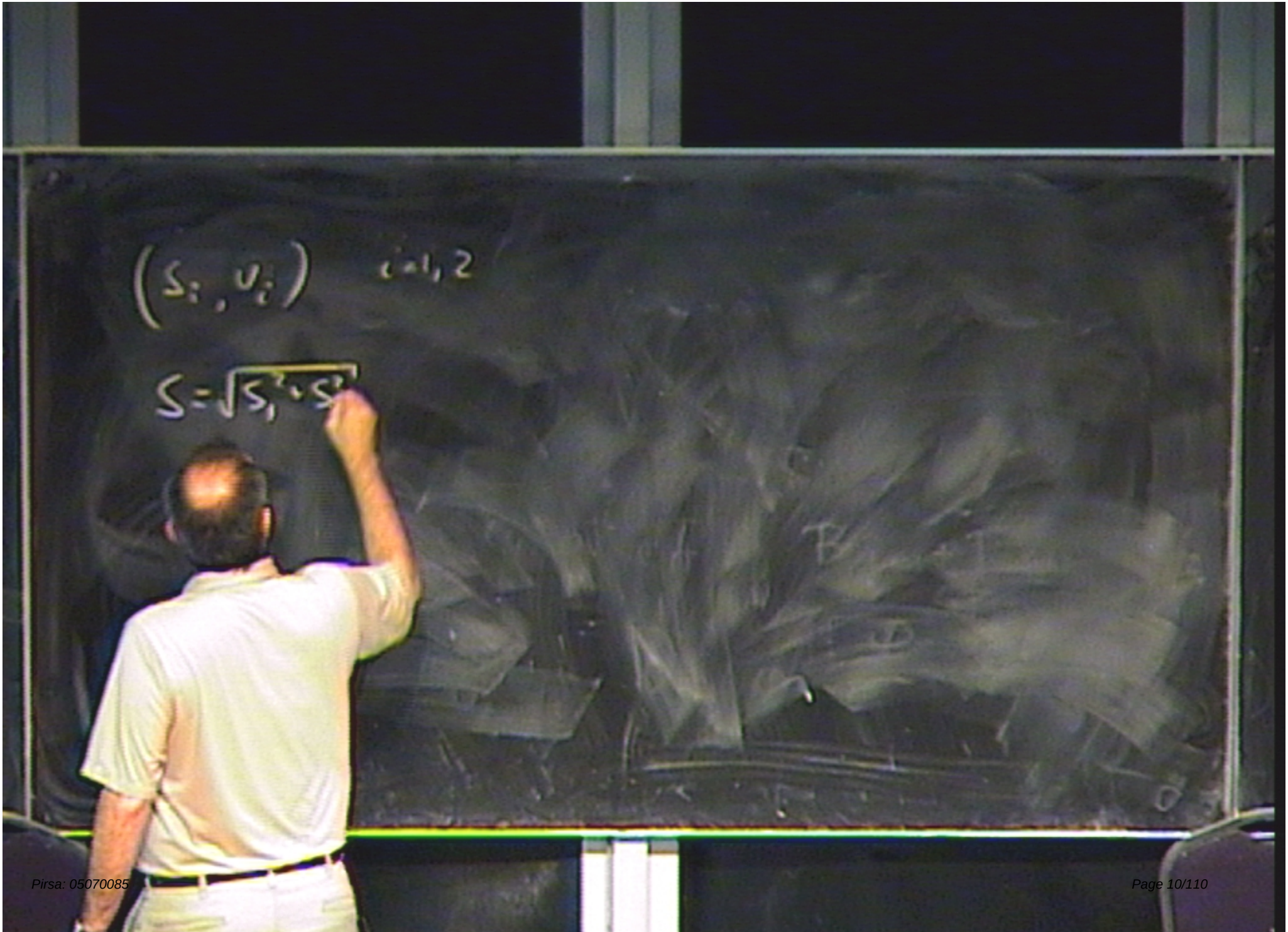
$H = -\partial^2 - \frac{1}{4} - \lambda^2$

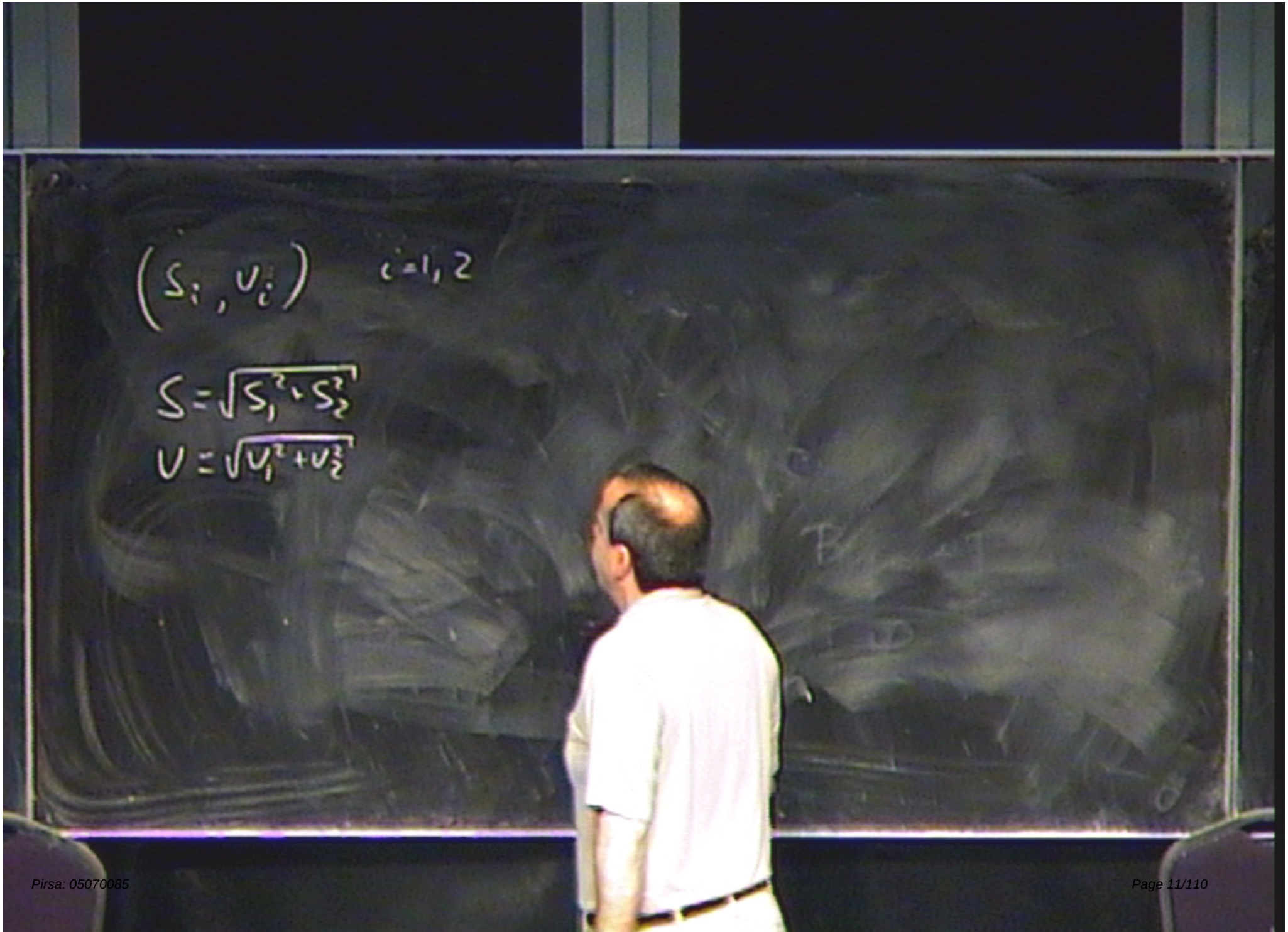
N complex e.v. λ_i

Angular momentum $= 0$

λ real ≥ 0




(S_i, U_i) $i=1,2$
$S=\sqrt{S_1 \cdot S_2}$


(S_i, V_i) $i=1,2$
$S=\sqrt{S_1^2+S_2^2}$
$V=\sqrt{V_1^2+V_2^2}$

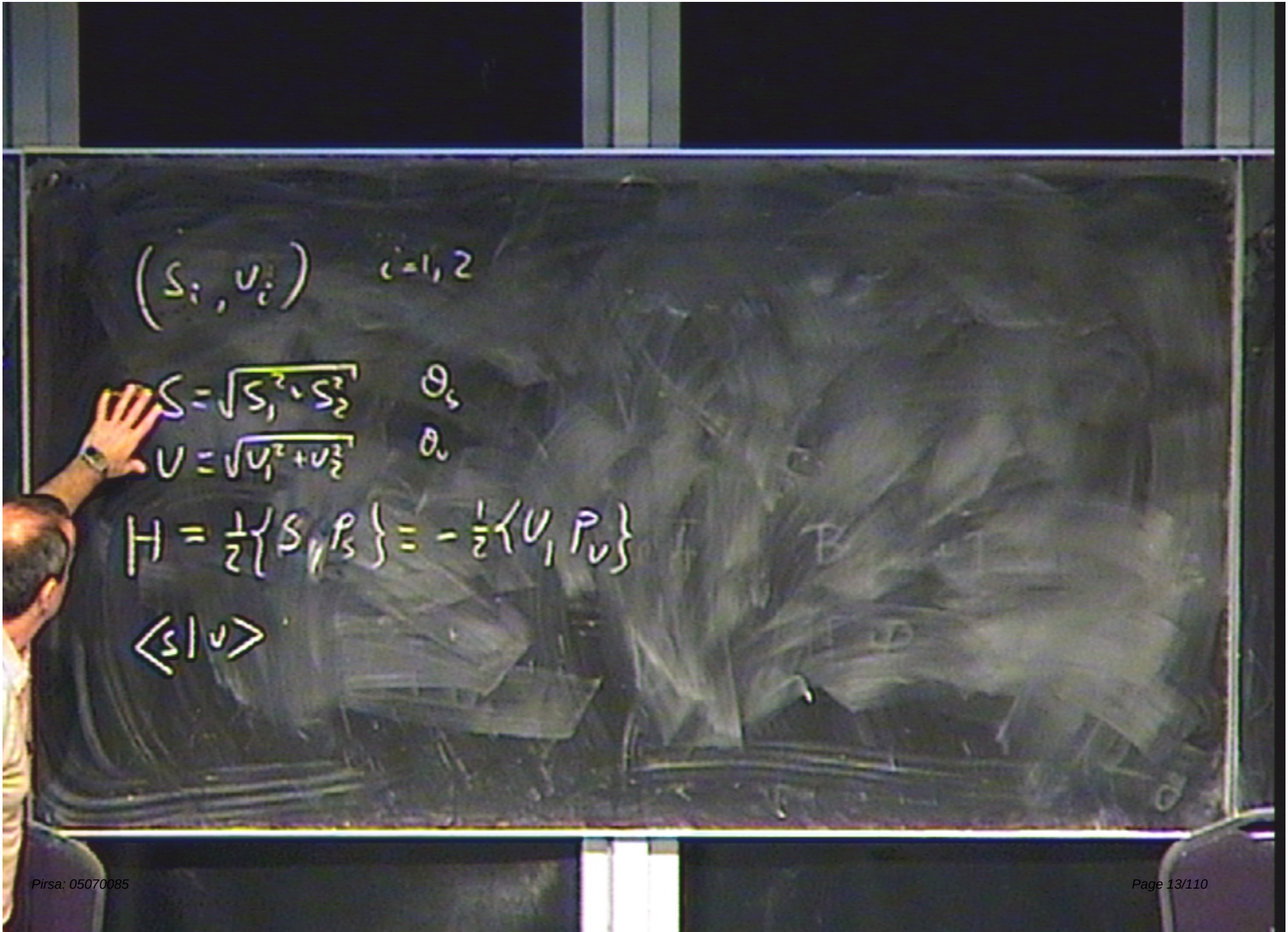
(S_i, U_i) $i = 1, 2$

$\rightarrow S = \sqrt{S_1^2 + S_2^2}$ θ_s

$\rightarrow U = \sqrt{U_1^2 + U_2^2}$ θ_u

$H = \frac{1}{2}\{S_1, P_S\} = -\frac{1}{2}\{U_1, P_U\}$

$\langle s | u \rangle$


(S_i, U_i) $i=1,2$
$S = \sqrt{S_1^2 + S_2^2}$ θ_s
$U = \sqrt{U_1^2 + U_2^2}$ θ_u
$H = \frac{1}{2}\{S_1, P_S\} = -\frac{1}{2}\{U_1, P_U\}$
$\langle S | U \rangle$

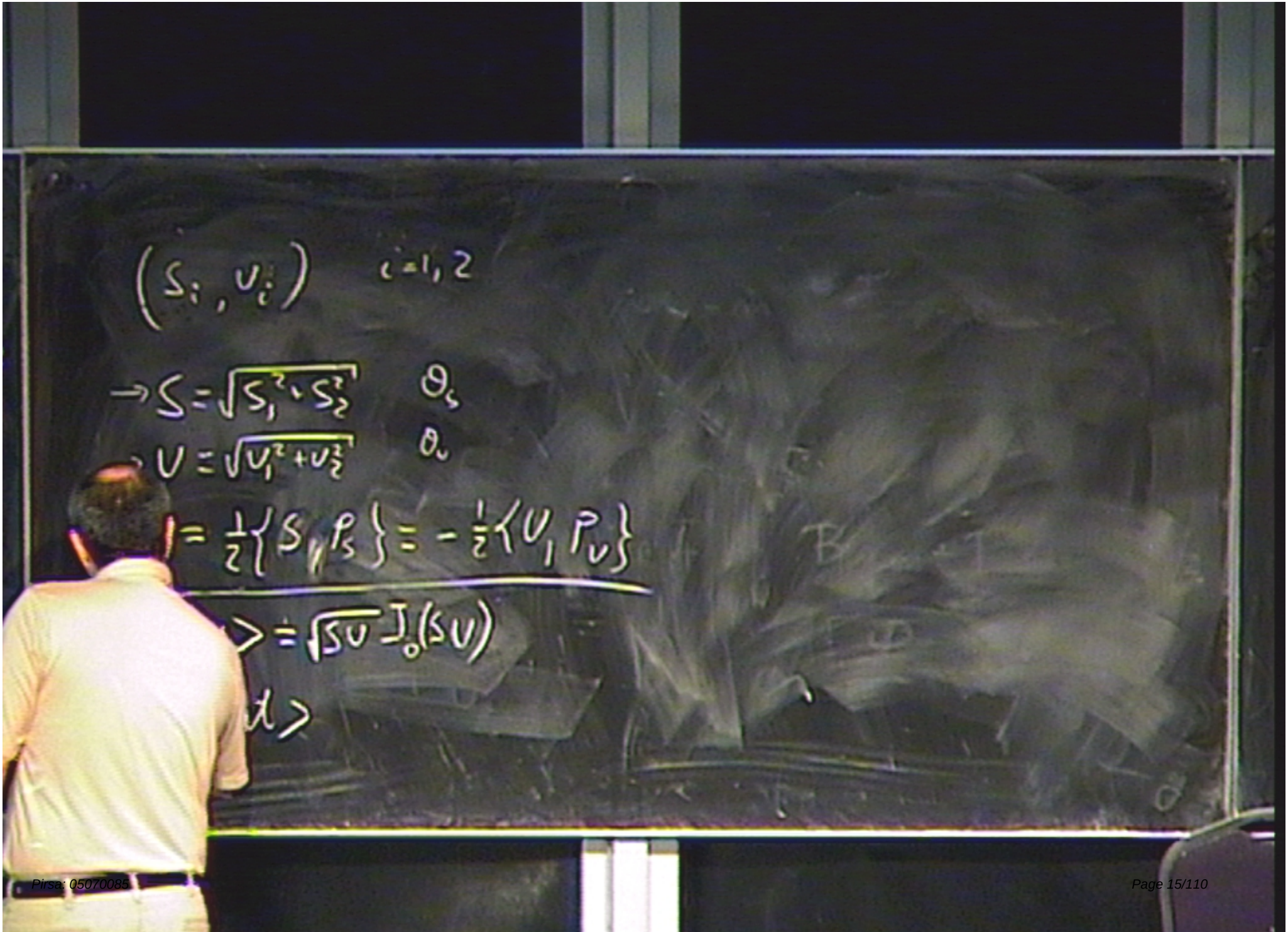
(S_i, V_i) $\quad i = 1, 2$

$\rightarrow S = \sqrt{S_1^2 + S_2^2}$ $\quad \theta_s$

$\rightarrow V = \sqrt{V_1^2 + V_2^2}$ $\quad \theta_v$

$$H = \frac{1}{2}\{S, P_S\} = -\frac{1}{2}\{V, P_V\}$$

$$\langle S | V \rangle = \sqrt{SV}\, J_0(SV)$$


(S_i, U_i) $i = 1, 2$
$\rightarrow S = \sqrt{S_1^2 + S_2^2}$ θ_s
$U = \sqrt{U_1^2 + U_2^2}$ θ_u
$= \frac{1}{2}\{S_1, P_S\} = -\frac{1}{2}\{U_1, P_U\}$
$> = \sqrt{SU}\, J_0(SU)$
$u>$

(S_i, U_i) $i = 1, 2$

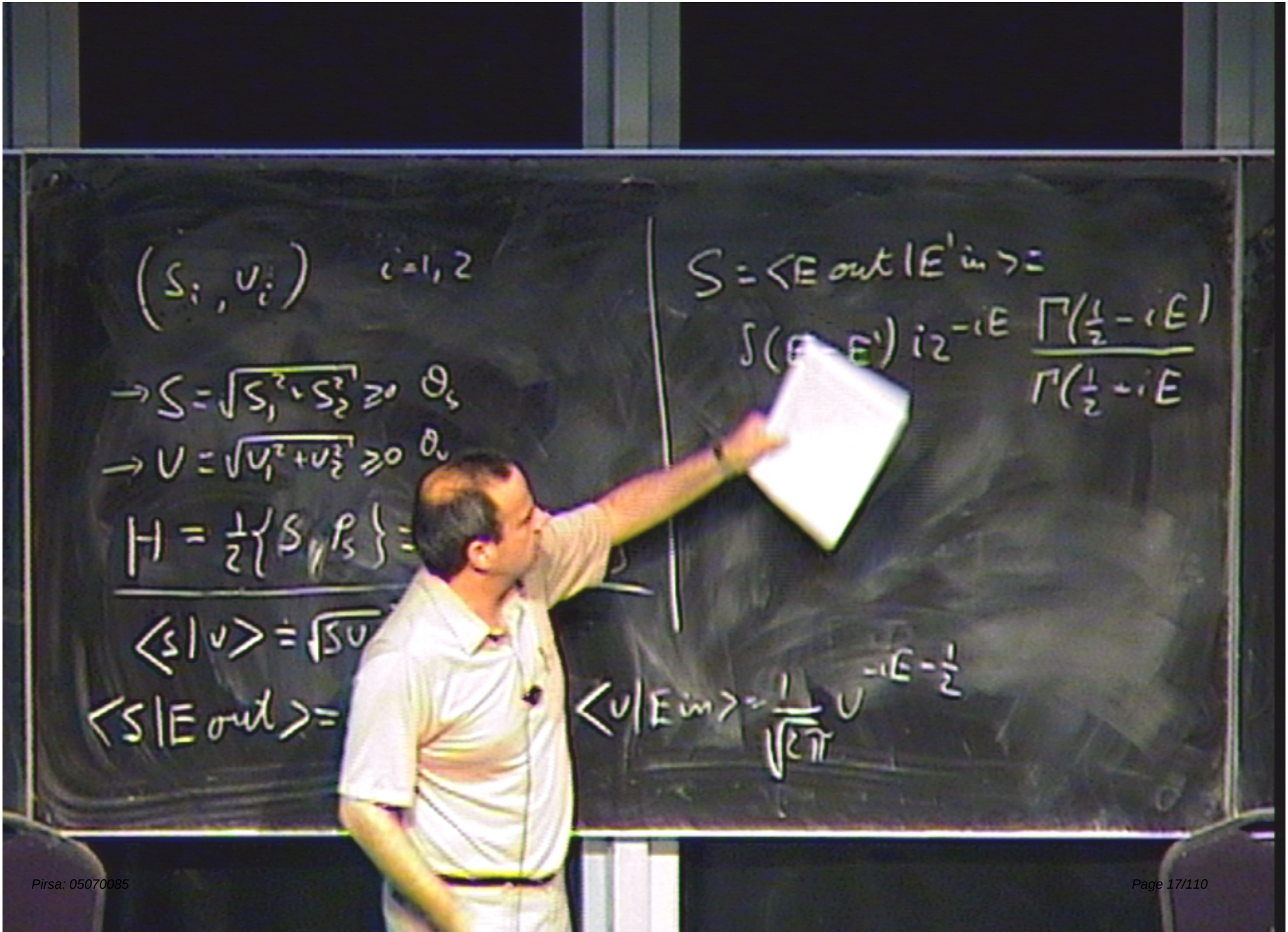
$\rightarrow S = \sqrt{S_1^2 + S_2^2}$ θ_s

$\rightarrow U = \sqrt{U_1^2 + U_2^2}$ θ_u

$$H = \frac{1}{2}\{S, P_S\} = -\frac{1}{2}\{U, P_U\}$$

$$\langle S | U \rangle = \sqrt{SU}\, J_0(SU)$$

$$\langle S | E_{out} \rangle = \frac{1}{\sqrt{2\pi}} S^{iE - \frac{1}{2}} \qquad \langle U | E_{in} \rangle = \frac{1}{\sqrt{2\pi}} U^{-iE - \frac{1}{2}}$$


(S_i, U_i) $i=1,2$
$\rightarrow S = \sqrt{S_1^2 + S_2^2} \geq 0$ θ_s
$\rightarrow U = \sqrt{U_1^2 + U_2^2} \geq 0$ θ_u
$H = \frac{1}{2}\{S, P_S\} =$
$\langle s | v \rangle = \sqrt{su}$
$\langle S | E\,out \rangle =$
$S = \langle E\,out | E'\,in \rangle =$
$\delta(E - E')\, i2^{-iE} \frac{\Gamma(\frac{1}{2} - iE)}{\Gamma(\frac{1}{2} + iE}$
$\langle U | E\,in \rangle = \frac{1}{\sqrt{2\pi}} U^{-iE - \frac{1}{2}}$

(S_i, U_i) $i=1,2$

$\rightarrow S = \sqrt{S_1^2 + S_2^2} \geq 0$ θ_s

$\rightarrow U = \sqrt{U_1^2 + U_2^2} \geq 0$ θ_u

$$H = \frac{1}{2}\{S, P_S\} = -\frac{1}{2}\{U, P_U\}$$

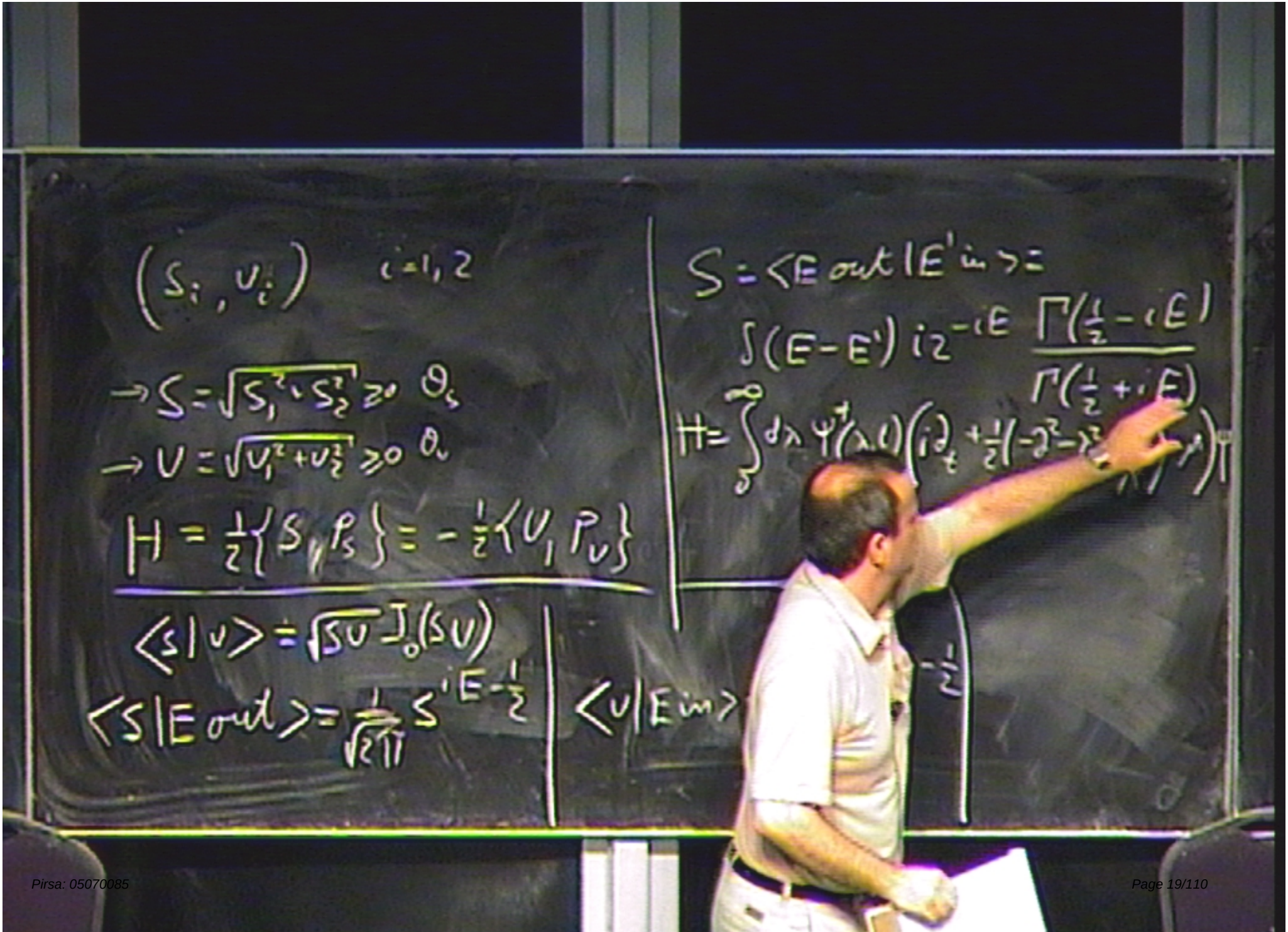
$$\langle s | u \rangle = \sqrt{su}\, J_0(su)$$

$$\langle s | E\,out \rangle = \frac{1}{\sqrt{2\pi}} s^{iE-\frac{1}{2}}$$

$$\langle u | E\,in \rangle = \frac{1}{\sqrt{2\pi}} u^{-iE-\frac{1}{2}}$$

$$S = \langle E\,out | E'\,in \rangle =$$

$$\delta(E-E')\, i2^{-iE}\, \frac{\Gamma(\frac{1}{2} - iE)}{\Gamma(\frac{1}{2} + iE)}$$


(s_i, v_i) $i=1,2$
$\rightarrow s = \sqrt{s_1^2 + s_2^2} \geq 0$ θ_s
$\rightarrow v = \sqrt{v_1^2 + v_2^2} \geq 0$ θ_v
$H = \frac{1}{2}\{s, p_s\} = -\frac{1}{2}\{v, p_v\}$
$\langle s|v\rangle = \sqrt{sv}\, J_0(sv)$
$\langle s|E\,out\rangle = \frac{1}{\sqrt{2\pi}} s^{iE-\frac{1}{2}}$
$S = \langle E\,out|E'\,in\rangle =$
$\int (E-E')\, i2^{-iE} \frac{\Gamma(\frac{1}{2} - iE)}{\Gamma(\frac{1}{2} + iE)}$
$H = \int_0^\infty d\lambda\, \Psi^\dagger(\lambda, v)$
$\langle v|E\,in\rangle$

(s_i, v_i) $i=1,2$

$\rightarrow s = \sqrt{s_1^2 + s_2^2} \geq 0$ θ_s

$\rightarrow v = \sqrt{v_1^2 + v_2^2} \geq 0$ θ_v

$$H = \frac{1}{2}\{s, p_s\} = -\frac{1}{2}\{v, p_v\}$$

$$\langle s | v \rangle = \sqrt{sv}\, J_0(sv)$$

$$\langle s | E\,\text{out} \rangle = \frac{1}{\sqrt{2\pi}} s^{iE-\frac{1}{2}}$$

$$\langle v | E\,\text{in} \rangle = \frac{1}{\sqrt{2\pi}} v^{-iE-\frac{1}{2}}$$

$$S = \langle E\,\text{out} | E'\,\text{in} \rangle =$$

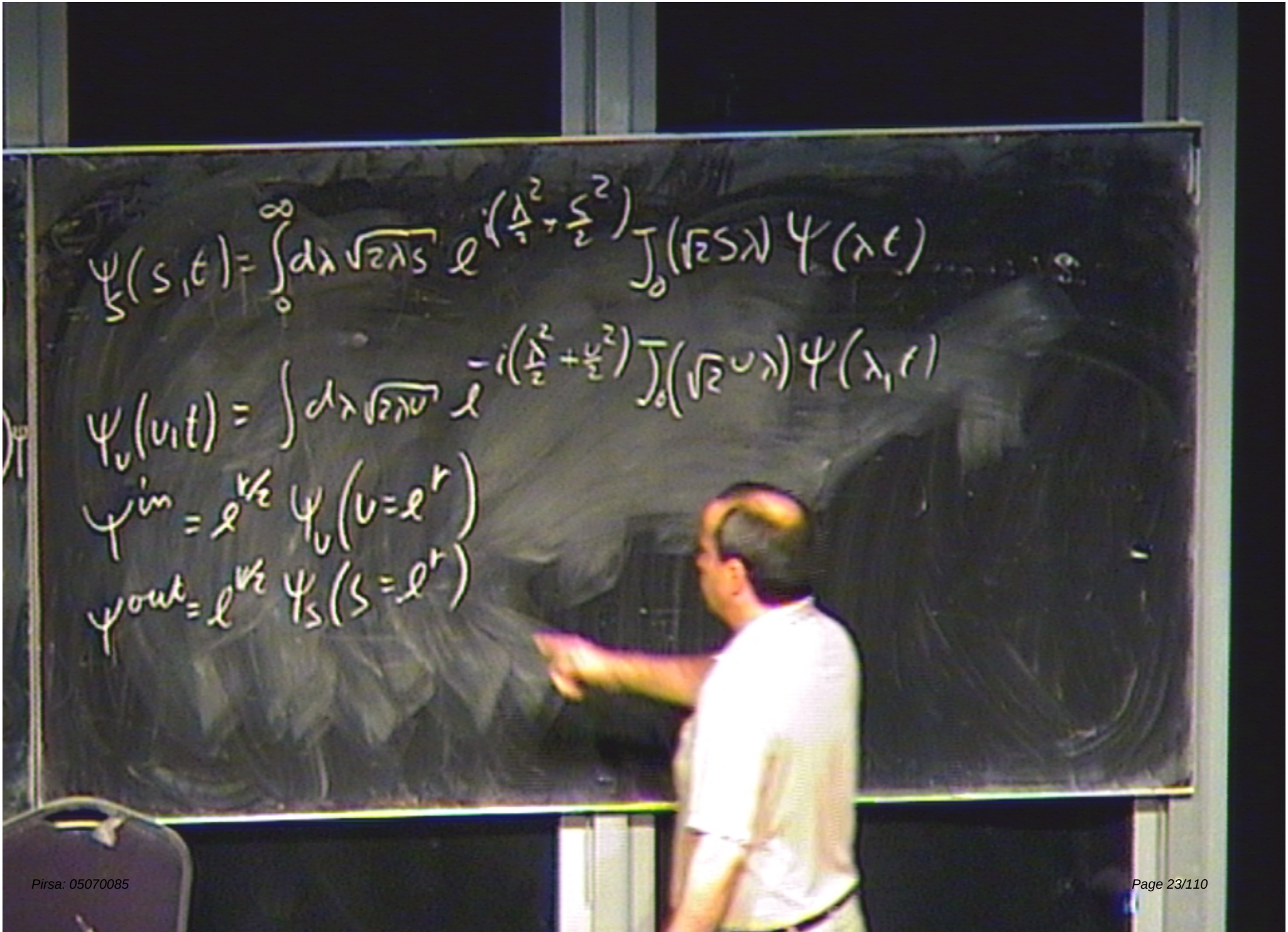
$$\delta(E-E')\, i 2^{-iE}\, \frac{\Gamma(\frac{1}{2} - iE)}{\Gamma(\frac{1}{2} + iE)}$$

$$H = \int_0^\infty d\lambda\, \Psi^\dagger(\lambda, t)\left(i\partial_t + \frac{1}{2}\left(-\partial^2 - \lambda^2 - \frac{\frac{1}{4}}{\lambda^2}\right) - \mu\right)\Psi$$


$\Psi_s(s,t)$
$\Psi_v(v,t) =$

$$\Psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2\lambda s})\, \Psi(\lambda,\zeta) \quad \ldots\ldots (3)$$

$$\Psi_v(v,t) = \int d\lambda \sqrt{2\lambda v}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2\lambda v})\, \Psi(\lambda,\zeta)$$

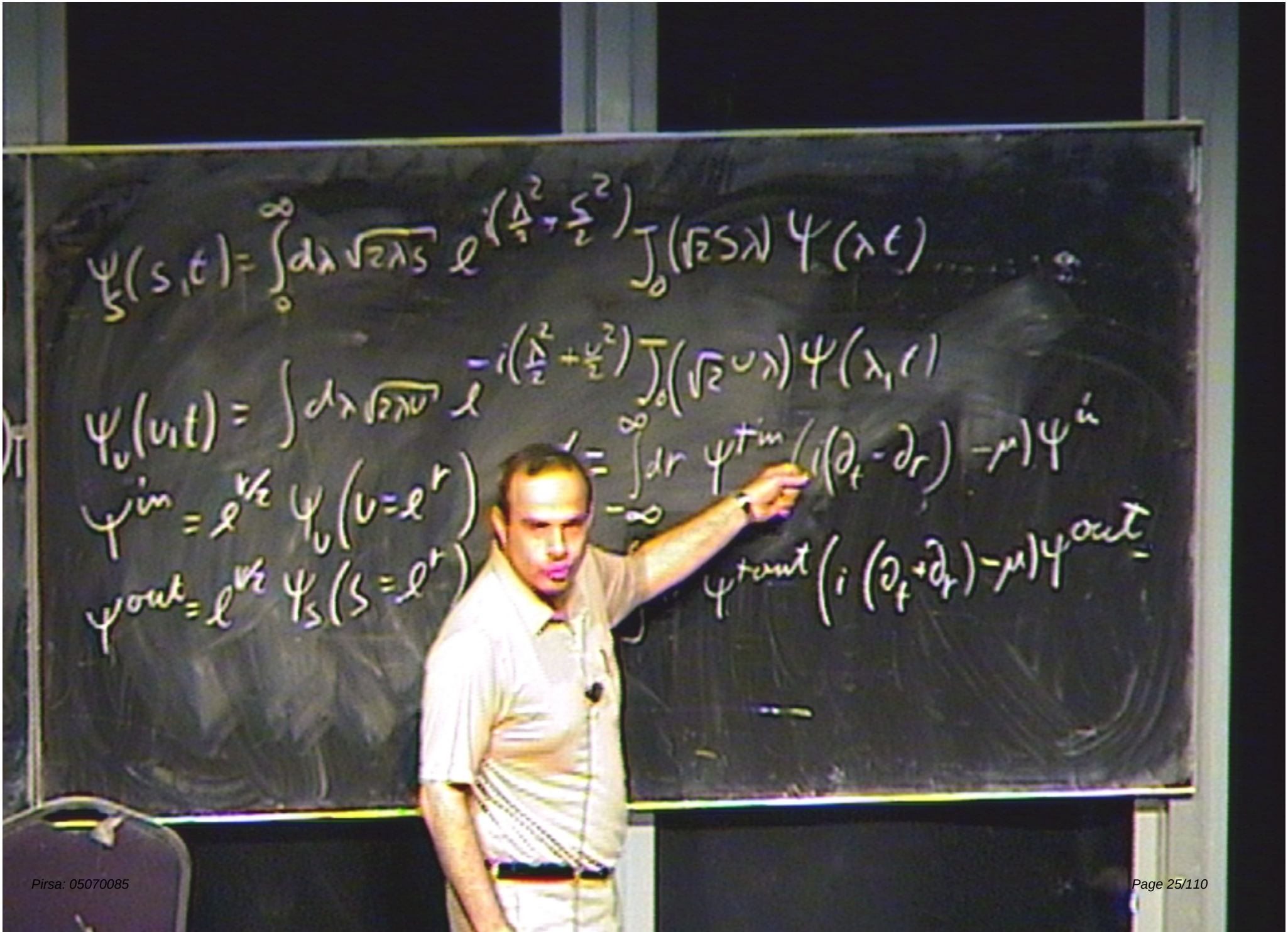

$\psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2\zeta s})\, \psi(\lambda,\zeta)$
$\psi_v(v,t) = \int d\lambda \sqrt{2\lambda v}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2v})\, \psi(\lambda,\zeta)$
$\psi^{in} = e^{r/2}\, \psi_v(v = e^r)$
$\psi^{out} = e^{r/2}\, \psi_s(s = e^r)$

$$\psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{t^2}{2}\right)} J_0(\sqrt{2}s\lambda)\, \psi(\lambda t) \quad \ldots (2)$$

$$\psi_v(v,t) = \int d\lambda \sqrt{2\lambda v}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{t^2}{2}\right)} J_0(\sqrt{2}v\lambda)\, \psi(\lambda, t)$$

$$\psi^{in} = e^{r/2}\, \psi_v(v = e^r) \qquad \mathcal{L} = \int_{-\infty}^{\infty} dr\, \psi^{+in}\left(i(\partial_t - \partial_r) - \mu\right)\psi^{in}$$

$$\psi^{out} = e^{r/2}\, \psi_s(s = e^r) \qquad = \int dr\, \psi^{+out}\left(i(\partial_t + \partial_r) - \mu\right)\psi^{out}$$


$\psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2}s\lambda)\, \psi(\lambda,\zeta)$
$\psi_v(v,t) = \int d\lambda \sqrt{2\lambda v}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{\zeta^2}{2}\right)t} J_0(\sqrt{2}v\lambda)\, \psi(\lambda,\zeta)$
$\psi^{in} = e^{r/2}\, \psi_v(v=e^r)$
$\psi^{out} = e^{r/2}\, \psi_s(s=e^r)$
$= \int_{-\infty}^{\infty} dr\, \psi^{tout}\left(i(\partial_t - \partial_r) - \mu\right)\psi^{in}$
$\psi^{tout}\left(i(\partial_t + \partial_r) - \mu\right)\psi^{out} =$

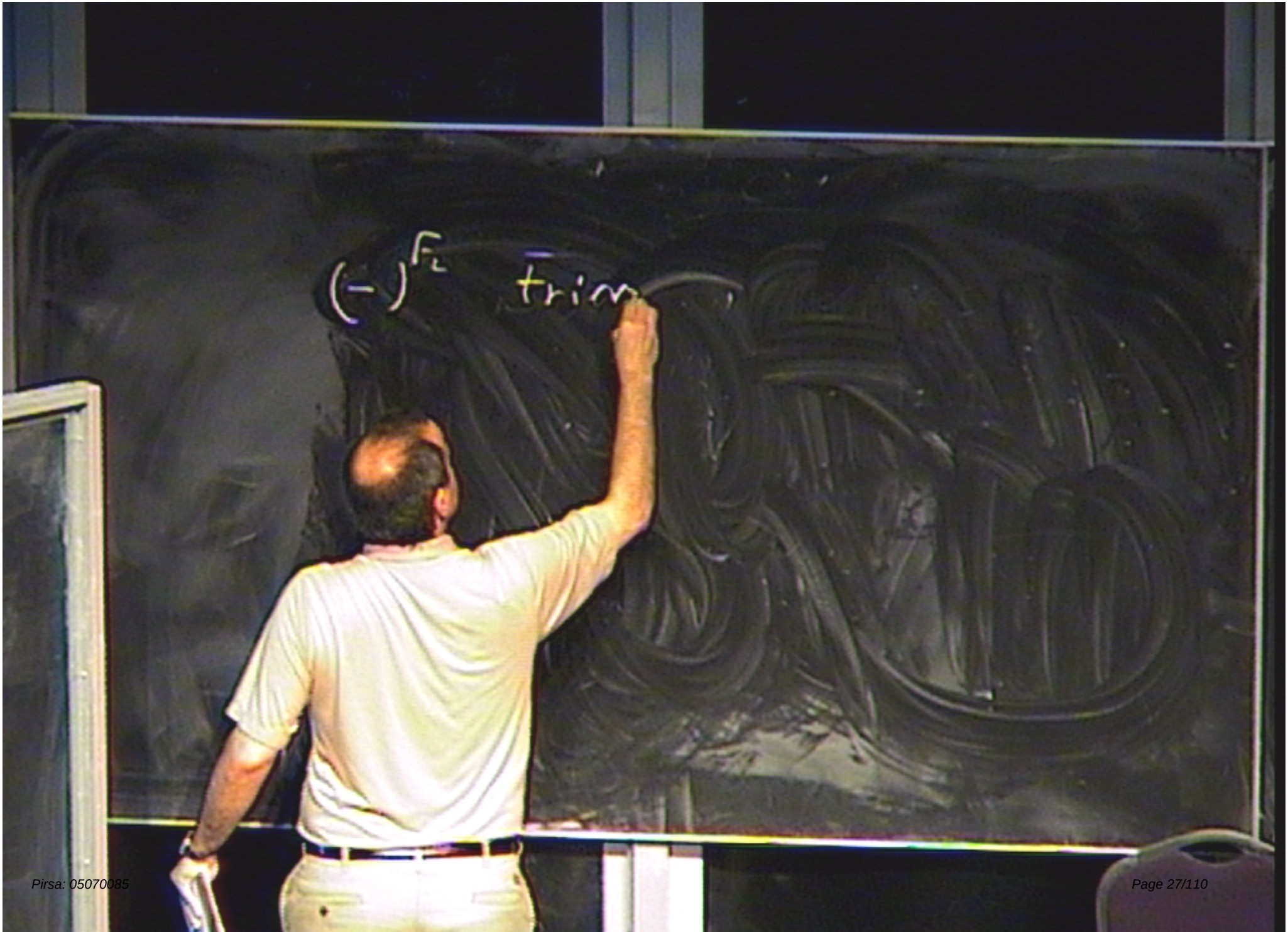
$$\psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{t^2}{2}+\frac{\lambda^2}{2}\right)} J_0(\sqrt{2}s\lambda)\, \psi(\lambda t) \quad \ldots\ldots (3)$$

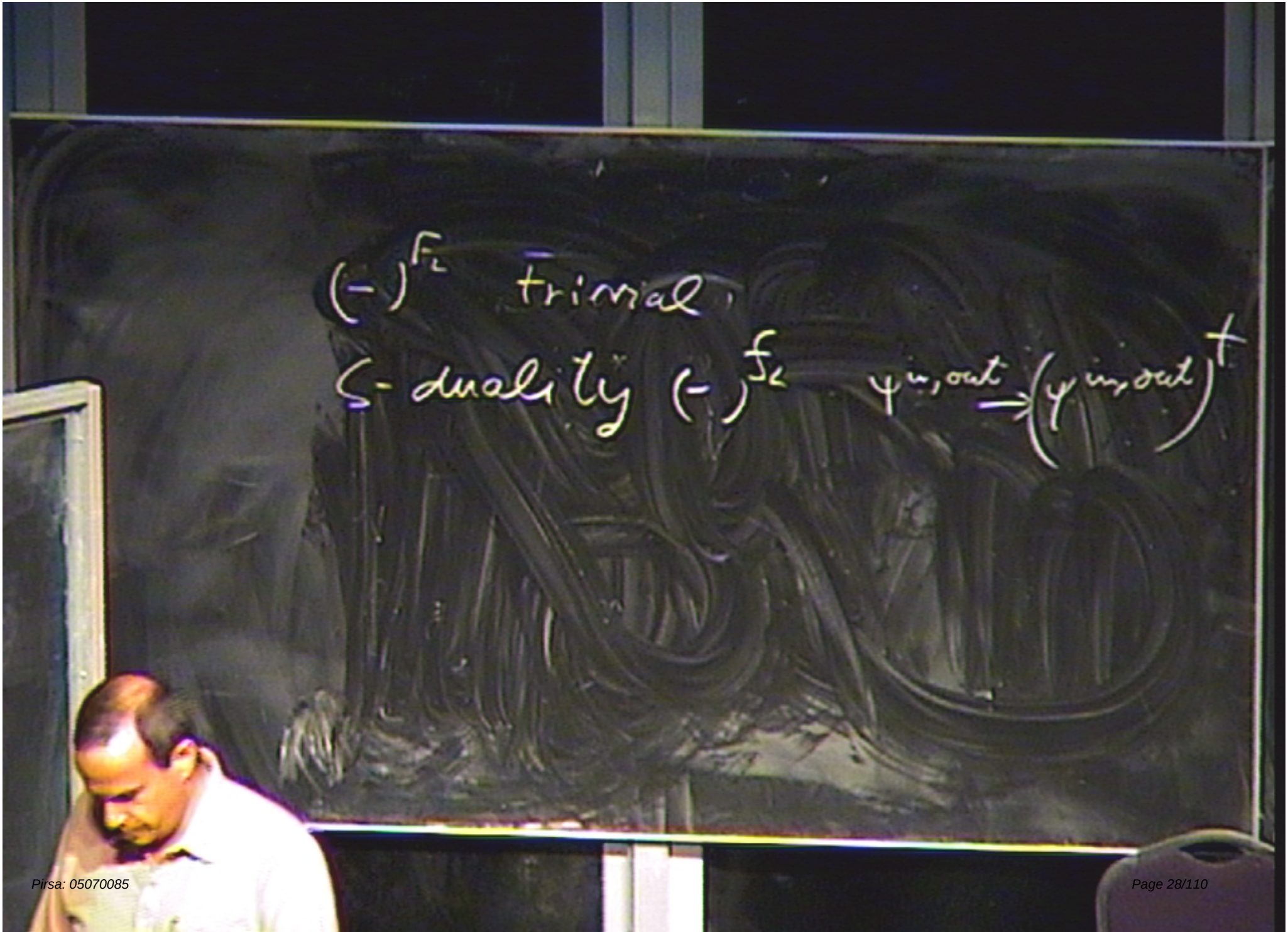
$$\psi_u(u,t) = \int d\lambda \sqrt{2\lambda u}\, e^{-i\left(\frac{t^2}{2}+\frac{\lambda^2}{2}\right)} J_0(\sqrt{2}u\lambda)\, \psi(\lambda, t)$$

$$\psi^{in} = e^{r/2}\, \psi_u(u = e^r) \qquad \mathcal{L} = \int_{-\infty}^{\infty} dr\, \psi^{tin}\left(i(\partial_t - \partial_r) - \mu\right)\psi^{in}$$

$$\psi^{out} = e^{r/2}\, \psi_s(s = e^r) \qquad = \int_{-\infty}^{\infty} dr\, \psi^{tout}\left(i(\partial_t + \partial_r) - \mu\right)\psi^{out}$$

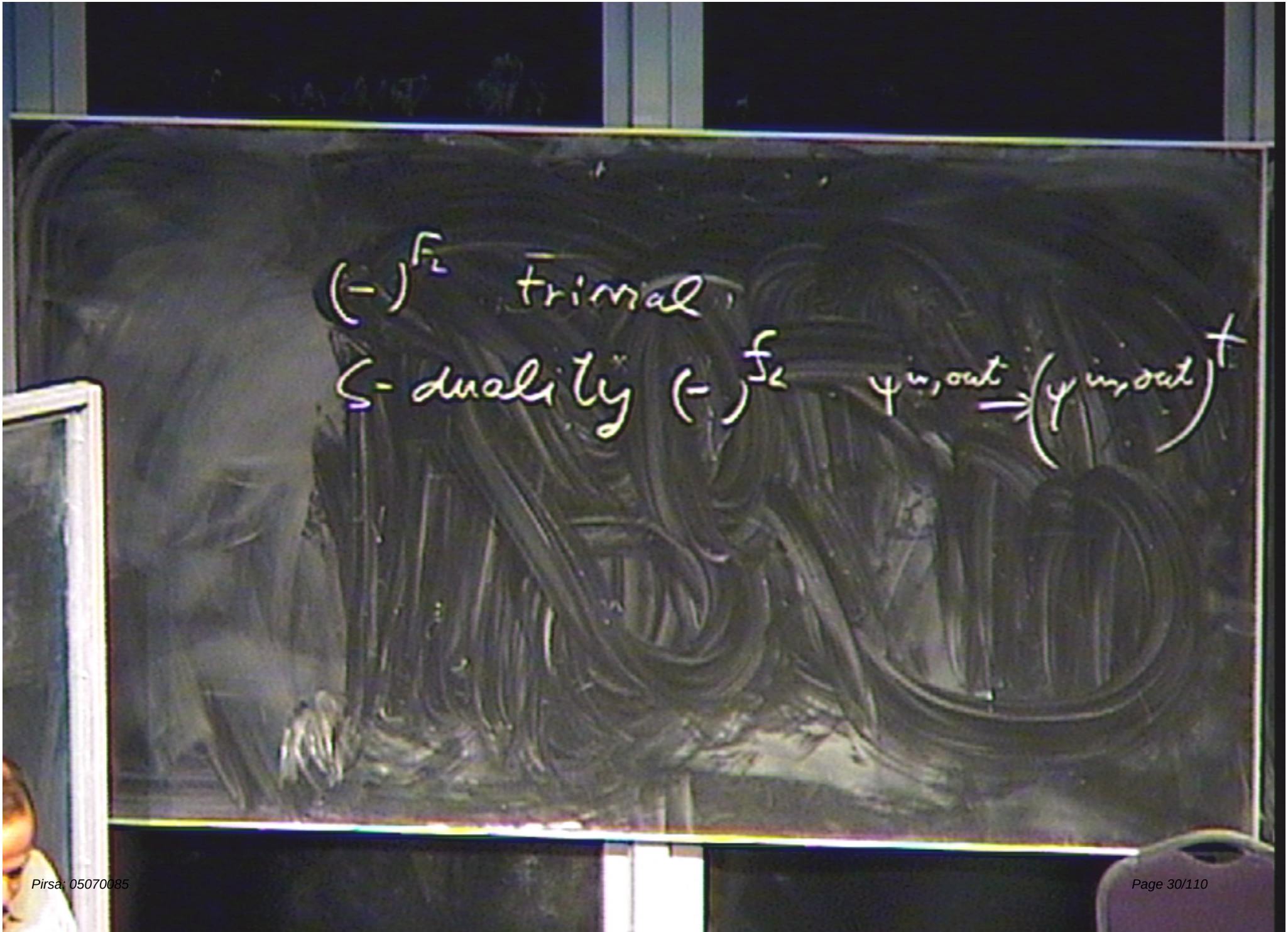
$$\boxed{\psi^{out}(r,t) = \int dr'\, e^{r-r'} J_1(e^{r-r'})\, \psi^{in}(r',t)}$$

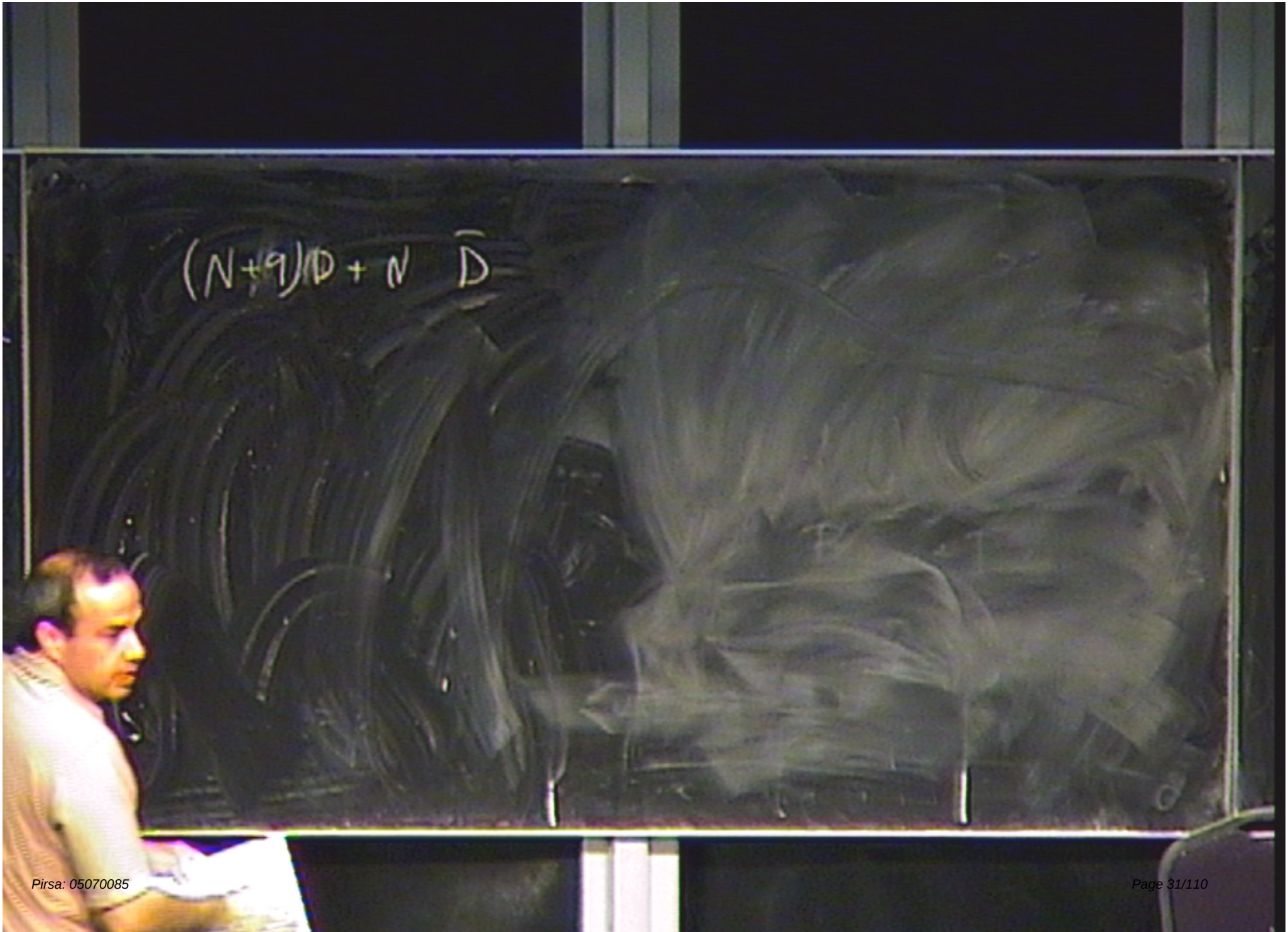

$(-)^{F_L}$ trim

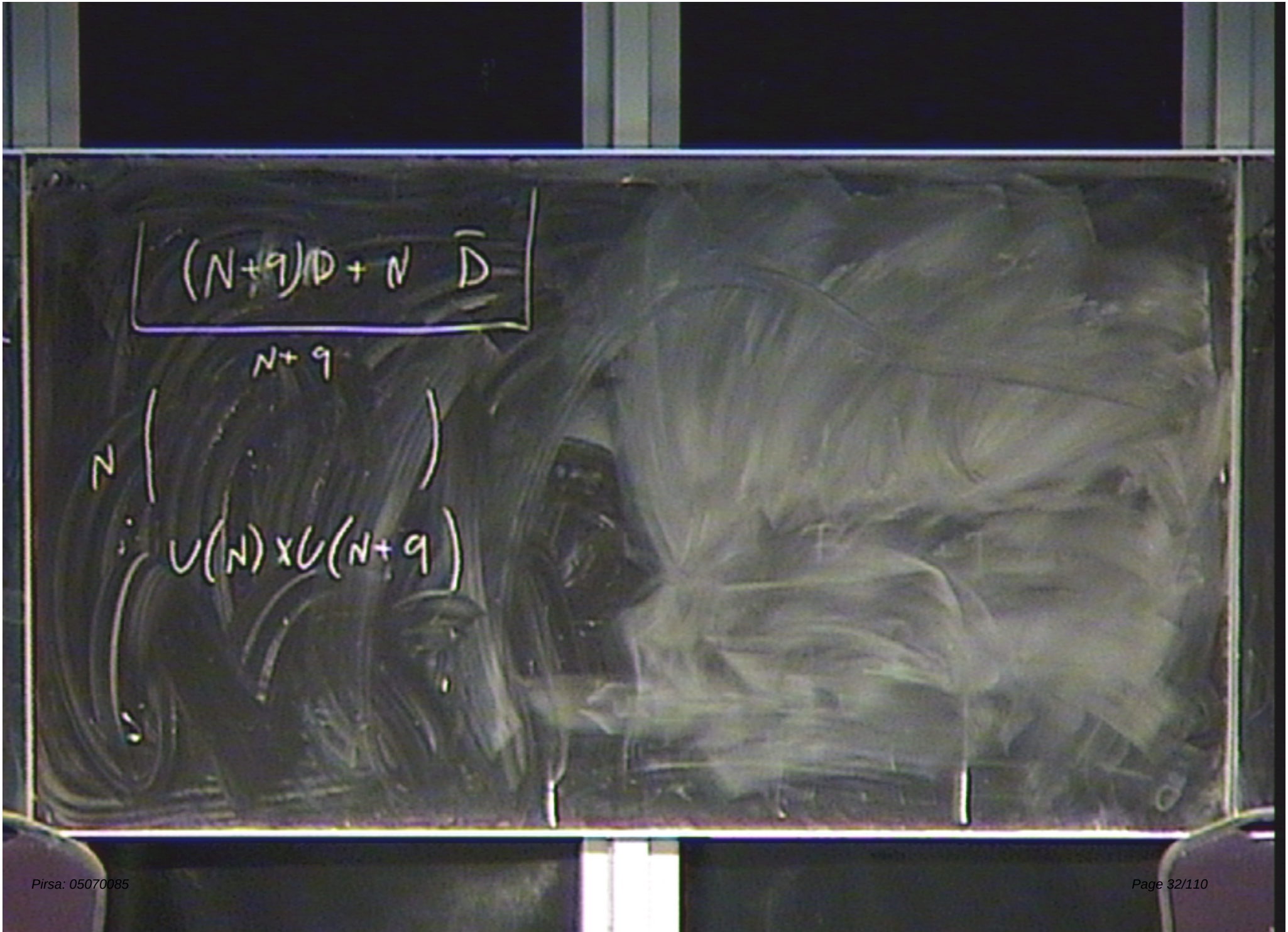

$(-)^{F_L}$ trivial
ζ-duality $(-)^{F_L}$ $\psi^{in,out} \Rightarrow (\psi^{in,out})^\dagger$

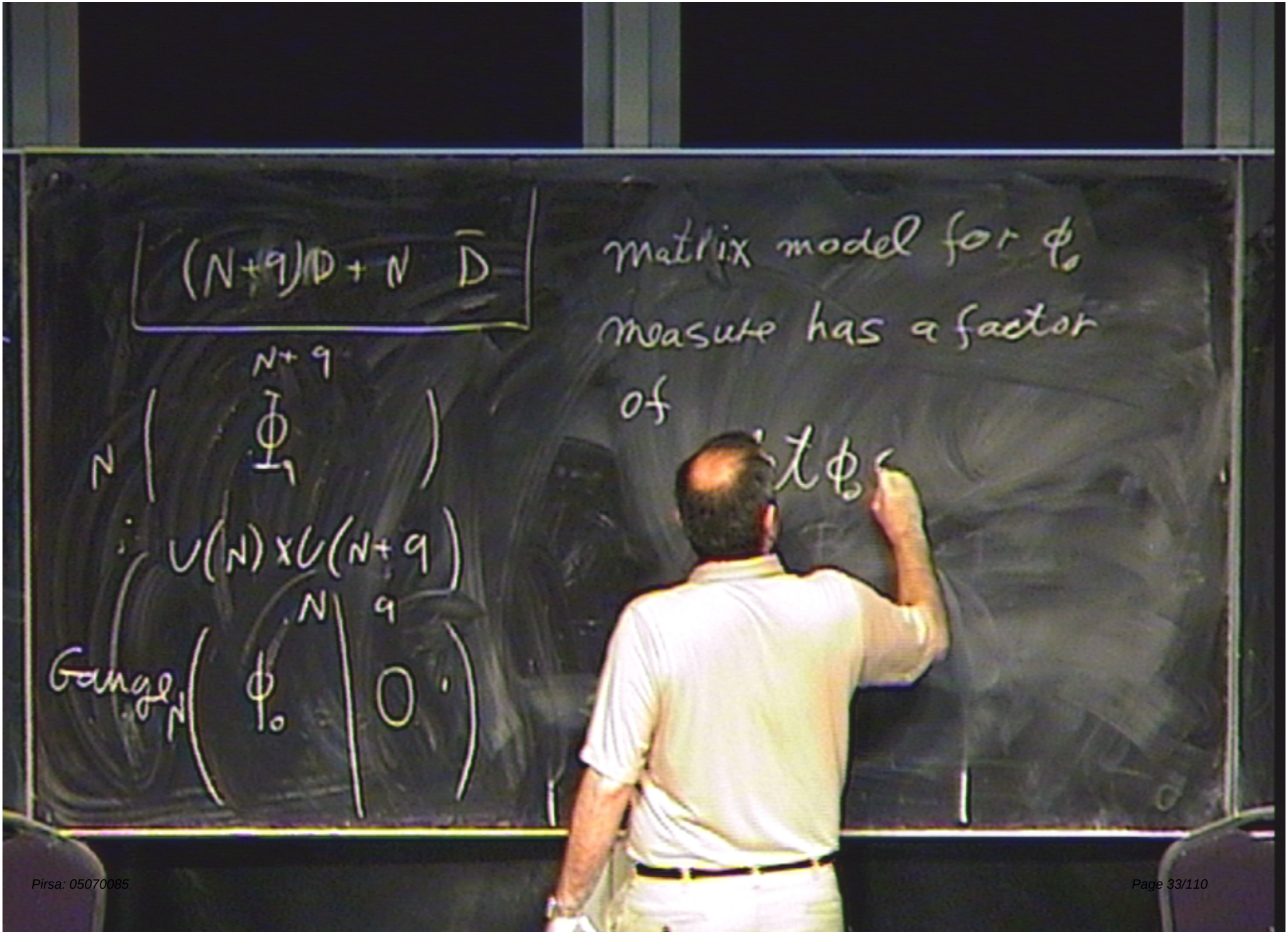
$(-)^{F_L}$ trivial

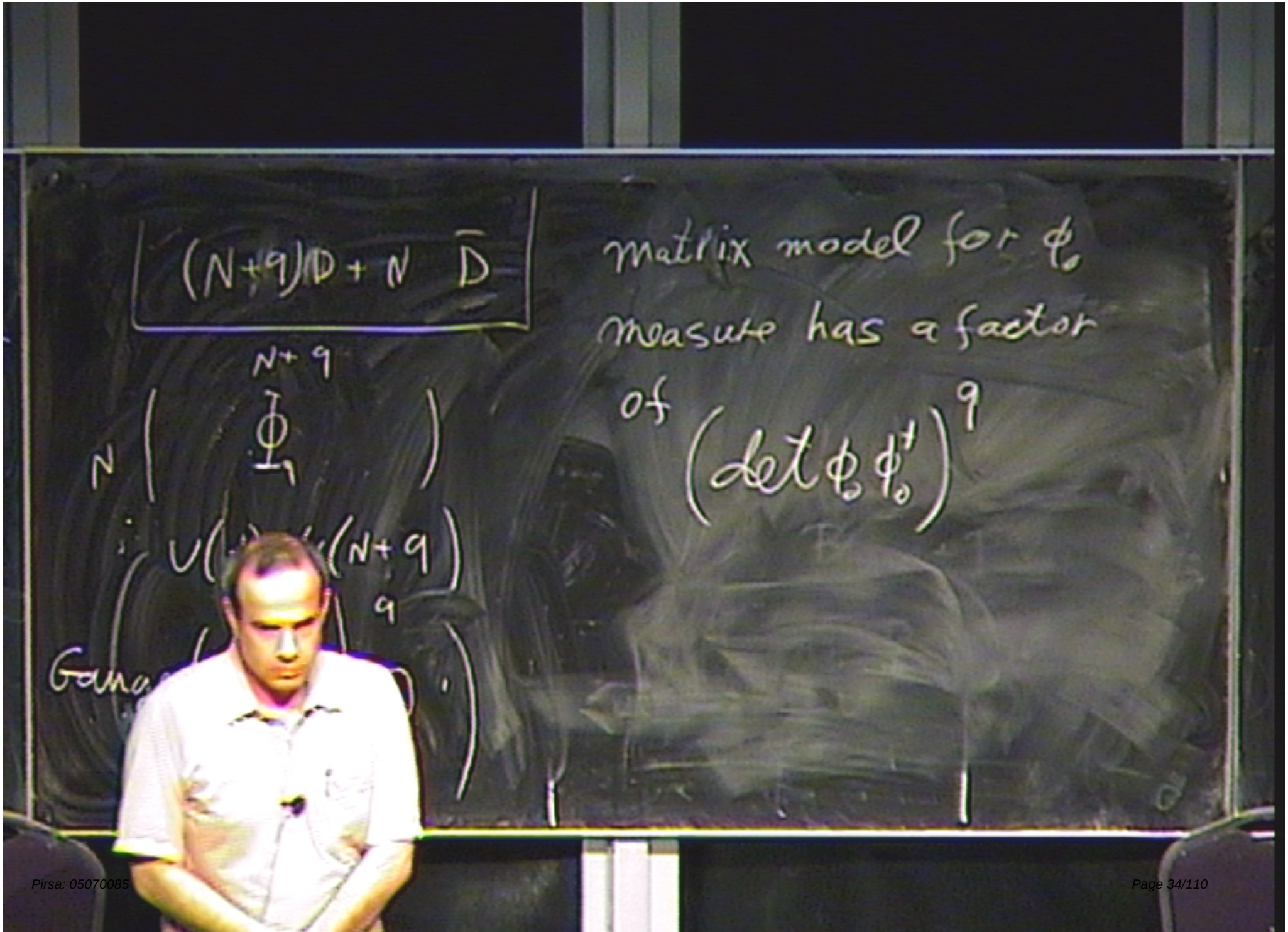
ζ-duality $(-)^{F_L}$ $\psi^{in,out} \Rightarrow (\psi^{in,out})^{\dagger}$

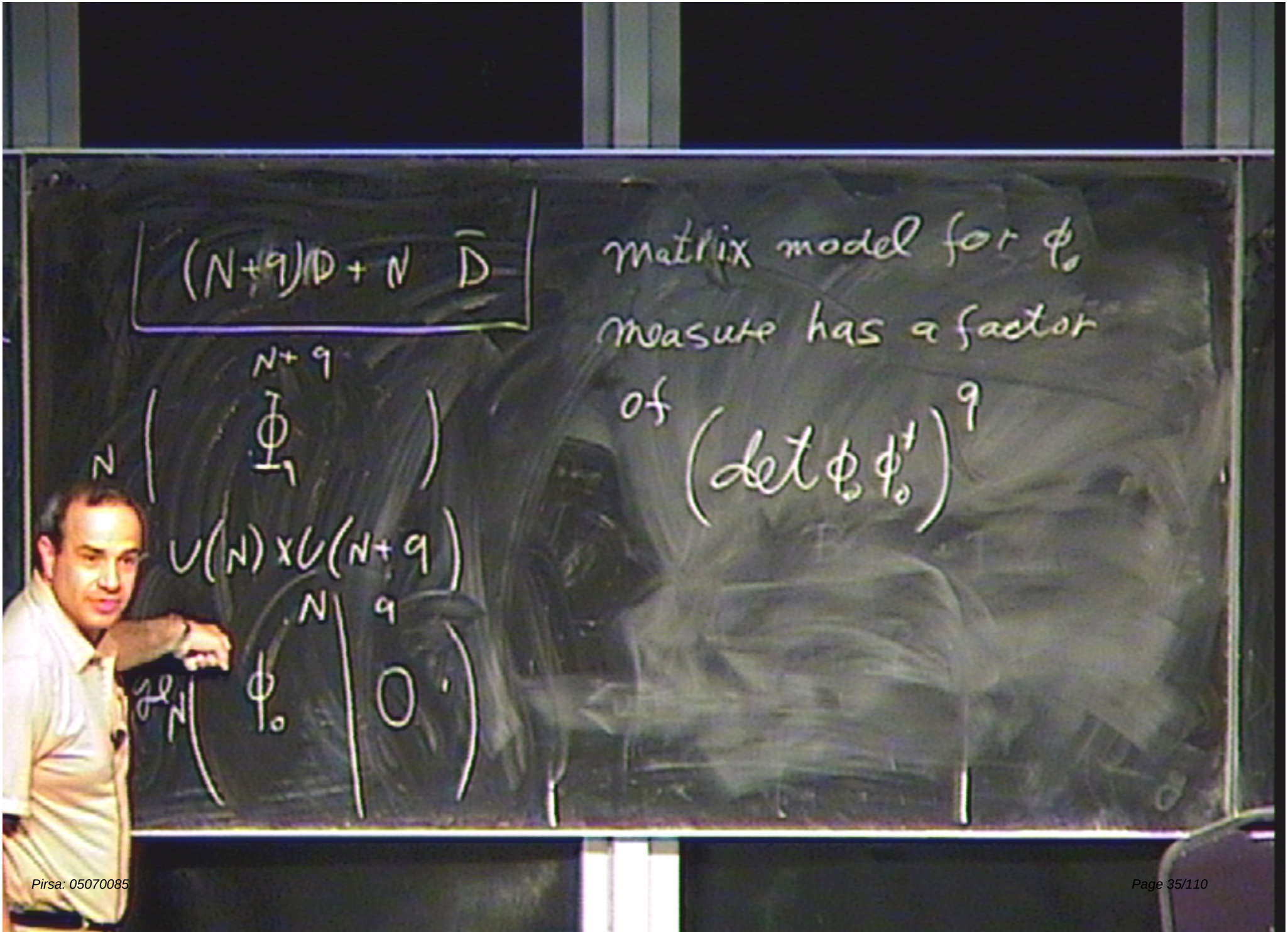

$(-)^{F_L}$ trivial
ζ-duality $(-)^{F_L}$ $\psi^{in,out} \to (\psi^{in,out})^\dagger$

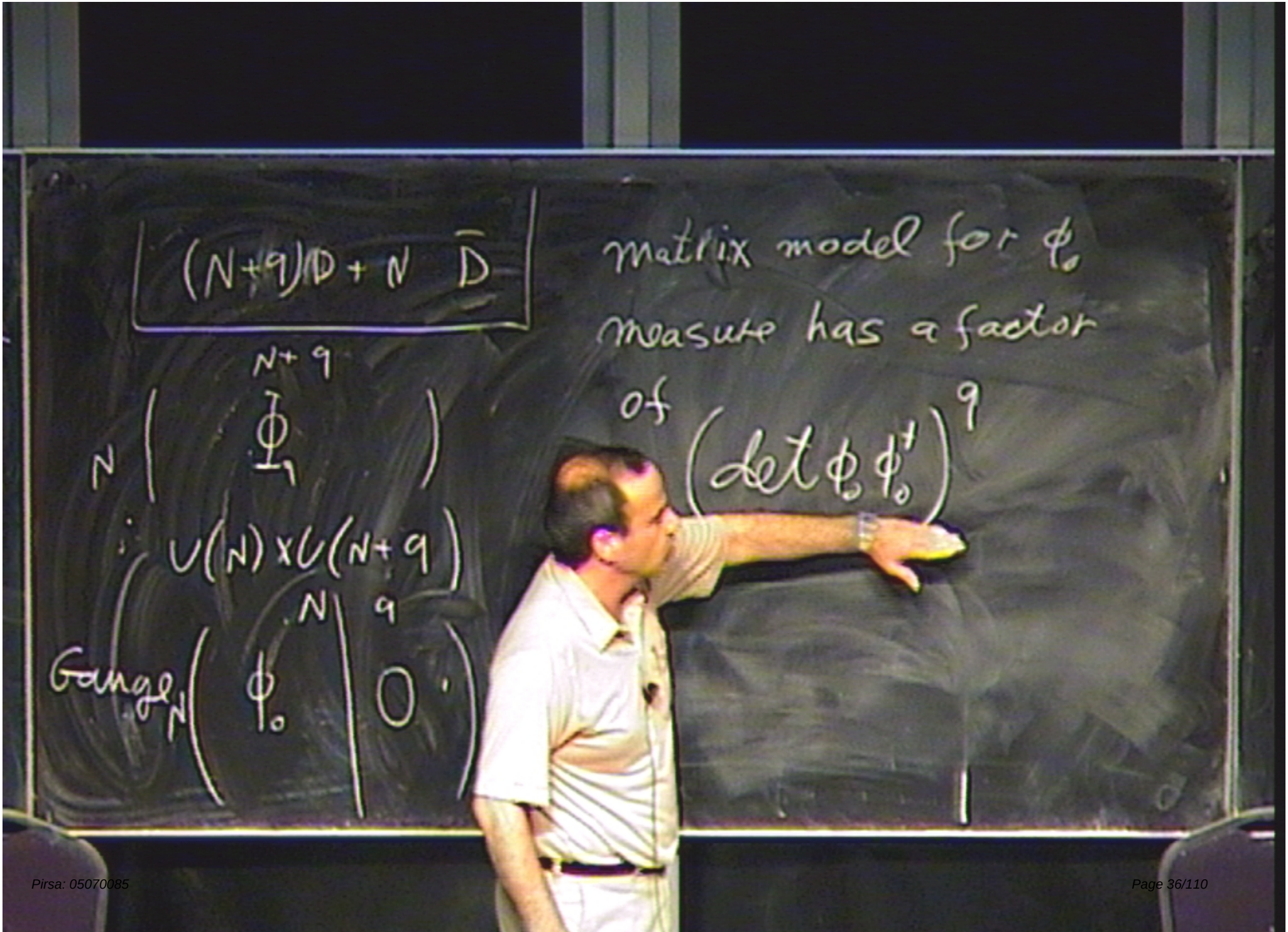

(N+9)D + N D̄


(N+9)D + N D̄
N+9
N
U(N) x U(N+9)


$(N+9)D + N\bar{D}$
matrix model for ϕ_0
measure has a factor
of
$U(N) \times U(N+9)$
Gauge
ϕ_0
0


(N+9)D + N D̄
matrix model for φ₀
measure has a factor
of
(det φ₀ φ₀†)^9


(N+9)D + N D̄
N+9
N
Φ
U(N) x U(N+9)
N
9
φ0
0
matrix model for φ0
measure has a factor
of
(det φ0 φ0†)^9


(N+9)D + N D̄
matrix model for φ0
measure has a factor
of (det φ0 φ0†)^9
N+9
N
Φ1
U(N) x U(N+9)
N
9
Gauge N
φ0
0

$(N+9)D + N\bar{D}$

matrix model for ϕ_0

measure has a factor

of $(\det \phi_0 \phi_0^\dagger)^9$

$N+9$

$N \left(\begin{array}{c} \\ \Phi \\ \\ \end{array} \right)$

$U(N) \times U(N+9)$

$N \quad 9$

Gauge $_N \left(\begin{array}{c|c} \phi_0 & 0 \end{array} \right)$

$(N+9)D + N\bar{D}$

$N+9$

$N \begin{pmatrix} \Phi_1 \end{pmatrix}$

$U(N) \times U(N+9)$

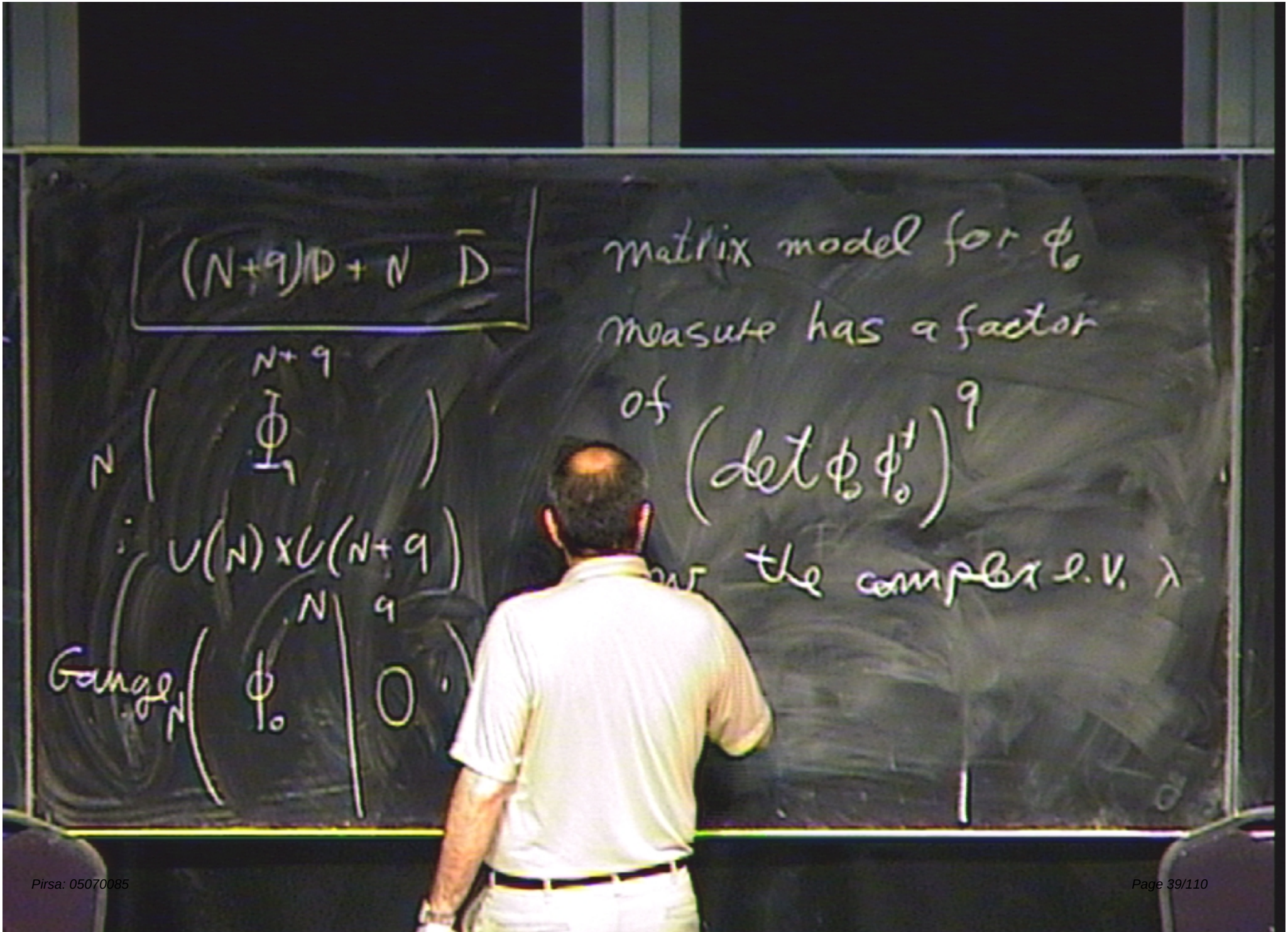
$N \quad 9$

Gauge $_N \begin{pmatrix} \phi_0 & 0 \end{pmatrix}$

matrix model for ϕ_0

measure has a factor

of $(\det \phi_0 \phi_0^\dagger)^9$


(N+9)D + N D̄
matrix model for Φ₀
measure has a factor
of (det Φ₀ Φ₀†)^9
U(N) × U(N+9)
Gauge
N
N+9
9
Φ₀
0
Φ
the complex e.v. λ

$(N+9)D + N\bar{D}$

$N+9$

$N \left(\Phi \right)$

$U(N) \times U(N+9)$

$N \quad 9$

Gauge $_N \left(\Phi_0 \;\middle|\; 0 \right)$

matrix model for Φ_0

measure has a factor

of $\left(\det \Phi_0 \Phi_0^\dagger \right)^9$

Now the complex e.V. λ

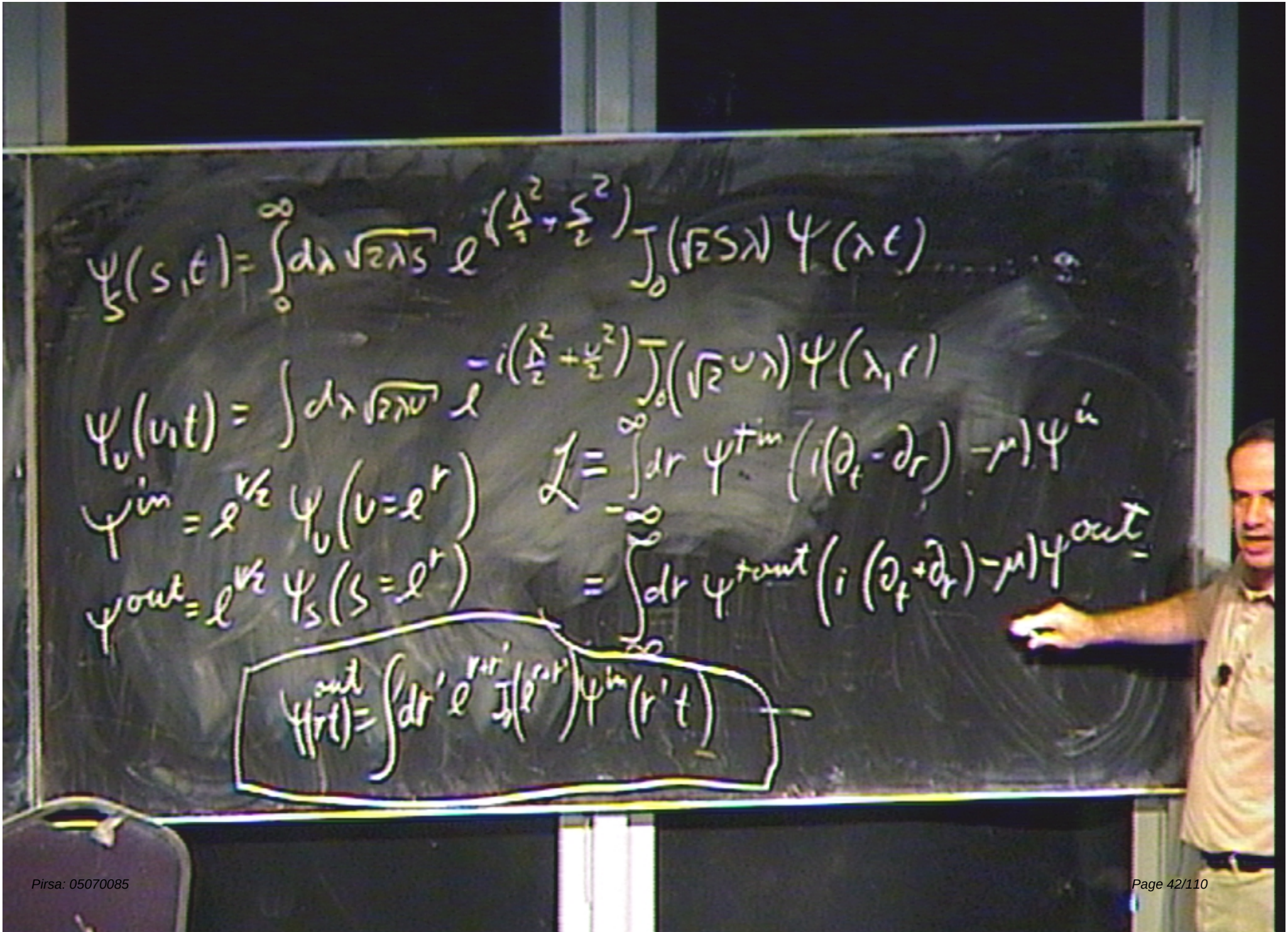
$\rightarrow$ fermions moving in F

w/ angular momentum 9

$(-)^{F_L}$ trivial

ζ-duality $(-)^{F_L}$ $\psi^{in,out} \Rightarrow (\psi^{in,out})^\dagger$

$$H = -\partial^2 - \lambda^2 + \frac{q^2 - \frac{1}{4}}{\lambda^2}$$


$\psi_s(s,t) = \int_0^\infty d\lambda \sqrt{2\lambda s}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{t^2}{2}\right)} J_0(\sqrt{2}s\lambda)\, \psi(\lambda t)$ ..... (3)
$\psi_v(v,t) = \int d\lambda \sqrt{2\lambda v}\, e^{-i\left(\frac{\lambda^2}{2}+\frac{t^2}{2}\right)} J_0(\sqrt{2}v\lambda)\, \psi(\lambda,t)$
$\psi^{in} = e^{r/2}\, \psi_v(v=e^r)$
$\psi^{out} = e^{r/2}\, \psi_s(s=e^r)$
$\mathcal{L} = \int_{-\infty}^{\infty} dr\, \psi^{t in}\left(i(\partial_t - \partial_r) - \mu\right)\psi^{in}$
$= \int_{-\infty}^{\infty} dr\, \psi^{t out}\left(i(\partial_t + \partial_r) - \mu\right)\psi^{out}$
$\psi^{out}(r,t) = \int dr'\, e^{r-r'} J_1(e^{r-r'})\, \psi^{in}(r',t)$

$\tilde{q}$ $U(N) \times U(N)$ Φ

A $\tilde{A}$

$$\tilde{q} \int dt \left(\mathrm{Tr} A - \mathrm{Tr} \tilde{A} \right)$$

$\tilde{q}$ $U(N) \times U(N)$ Φ

A $\tilde{A}$

$\tilde{q} \int dt \left(\mathrm{Tr} A - \mathrm{Tr} \tilde{A} \right)$

$\hat{q} \int dt \, \xi_i (u_i - \hat{a}_i)$

Angular momentum - $\tilde{q}$

$\tilde{q}$ $U(N) \times U(N)$ Φ

A $\tilde{A}$

$\tilde{q} \int dt \left(\mathrm{Tr} A - \mathrm{Tr} \tilde{A} \right)$

$\hat{q} \int dt \, \xi_i (u_i - \hat{a}_i)$

Angular momentum - $\tilde{q}$

$\tilde{q}$ $U(N) \times U(N)$ Φ

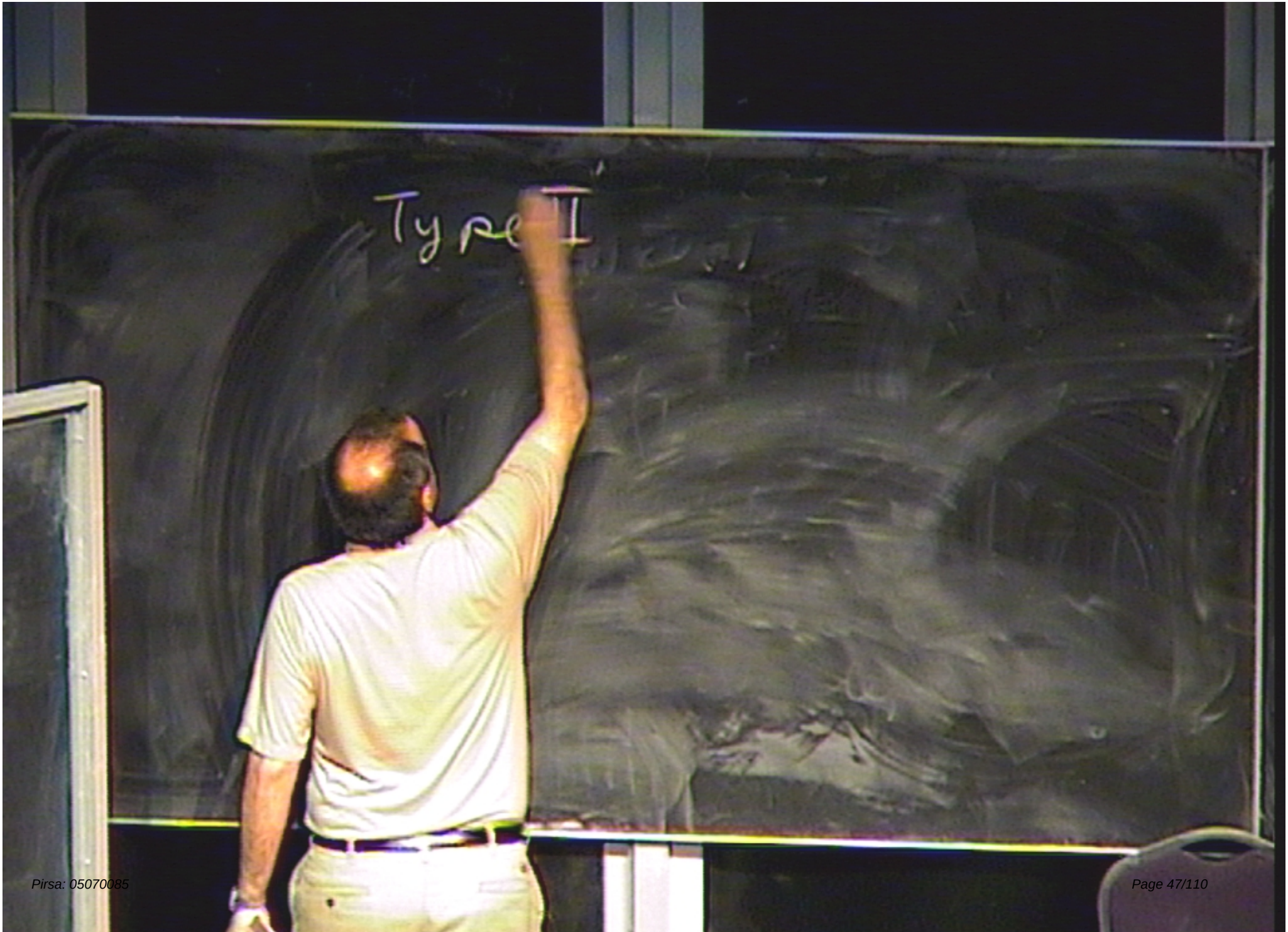
A $\tilde{A}$

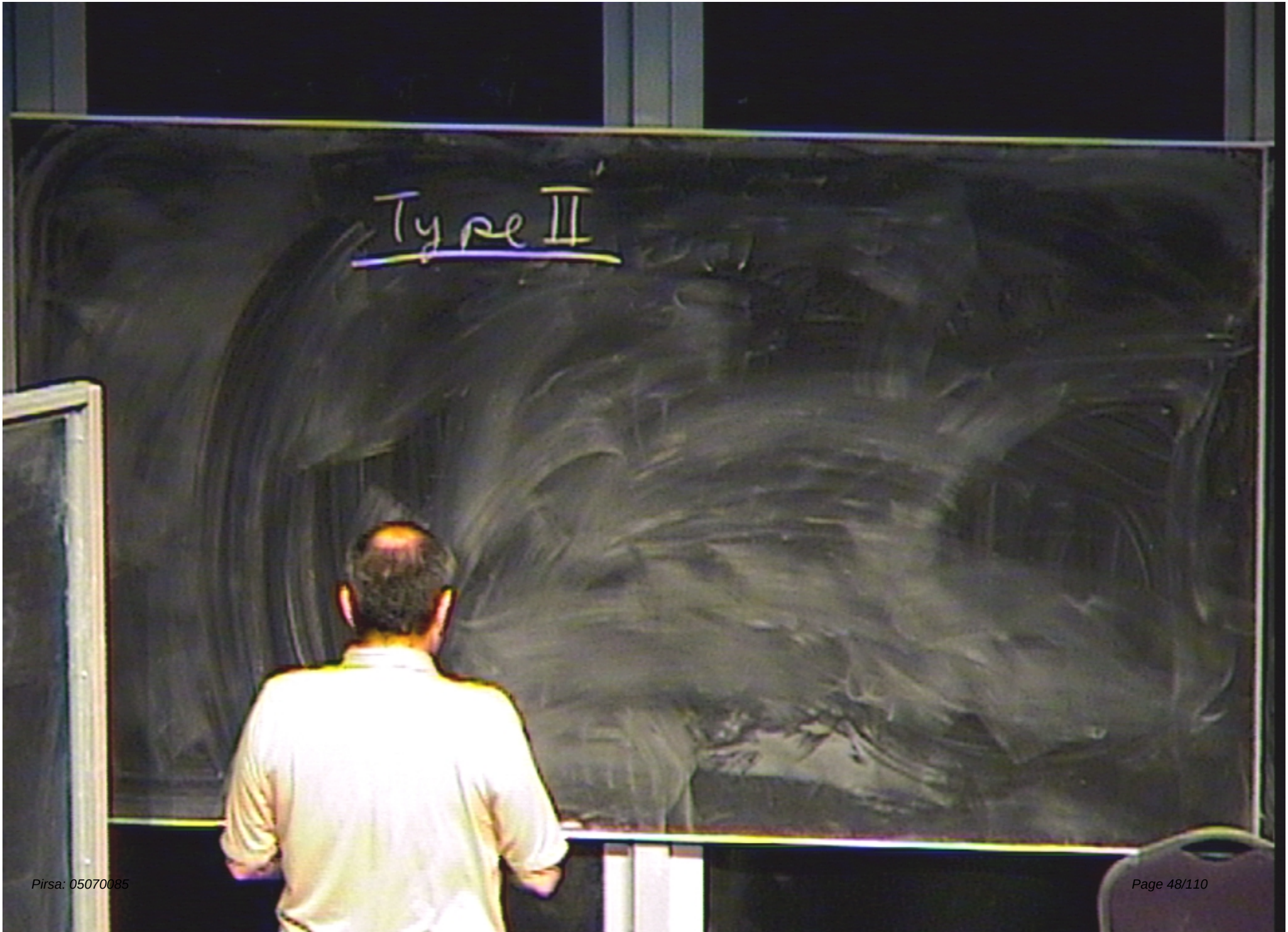
$\tilde{q} \int dt \left(\mathrm{Tr} A - \mathrm{Tr} \tilde{A} \right)$

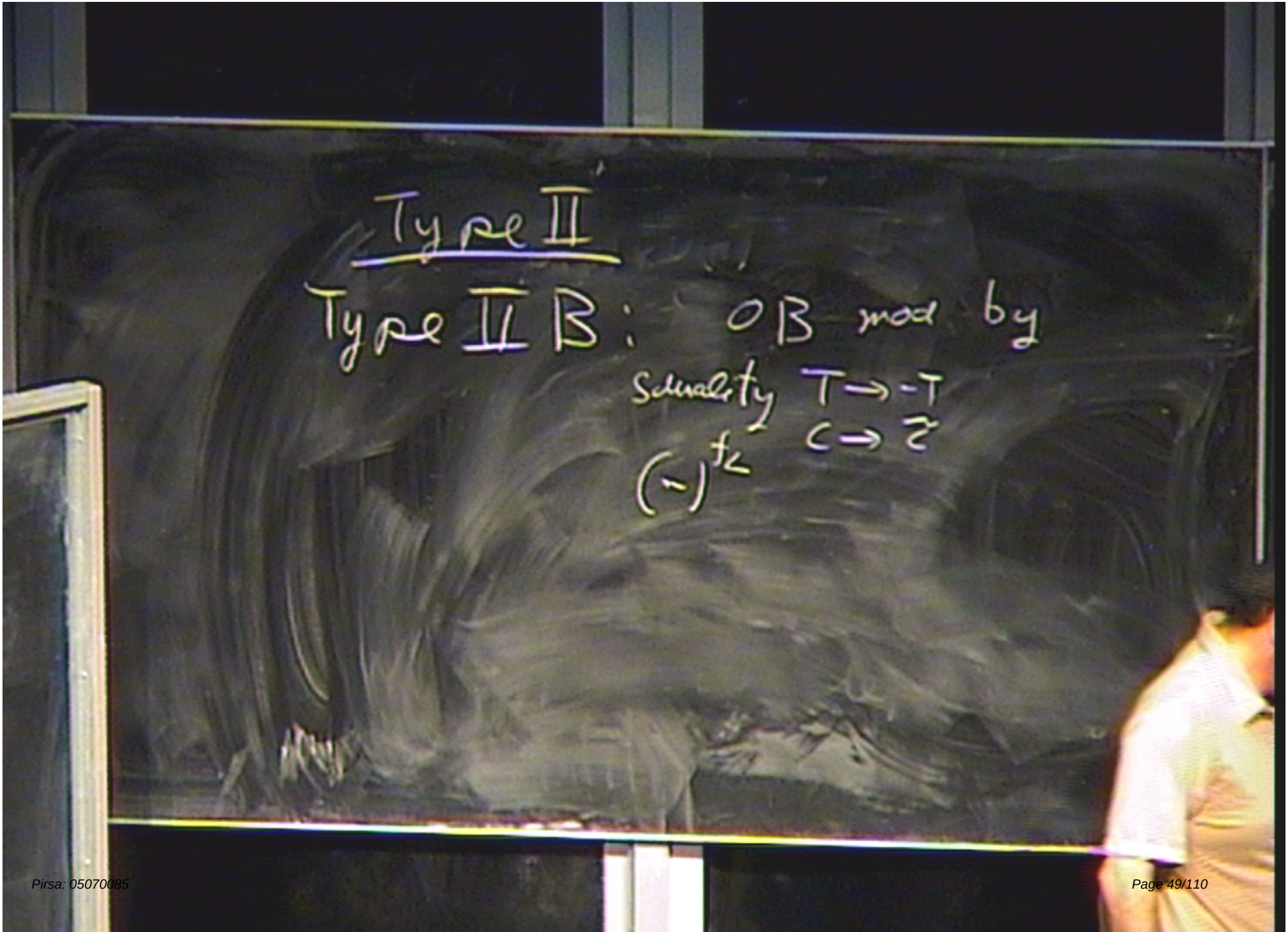
$\hat{q} \int dt \sum_i (u_i - \tilde{a}_i)$

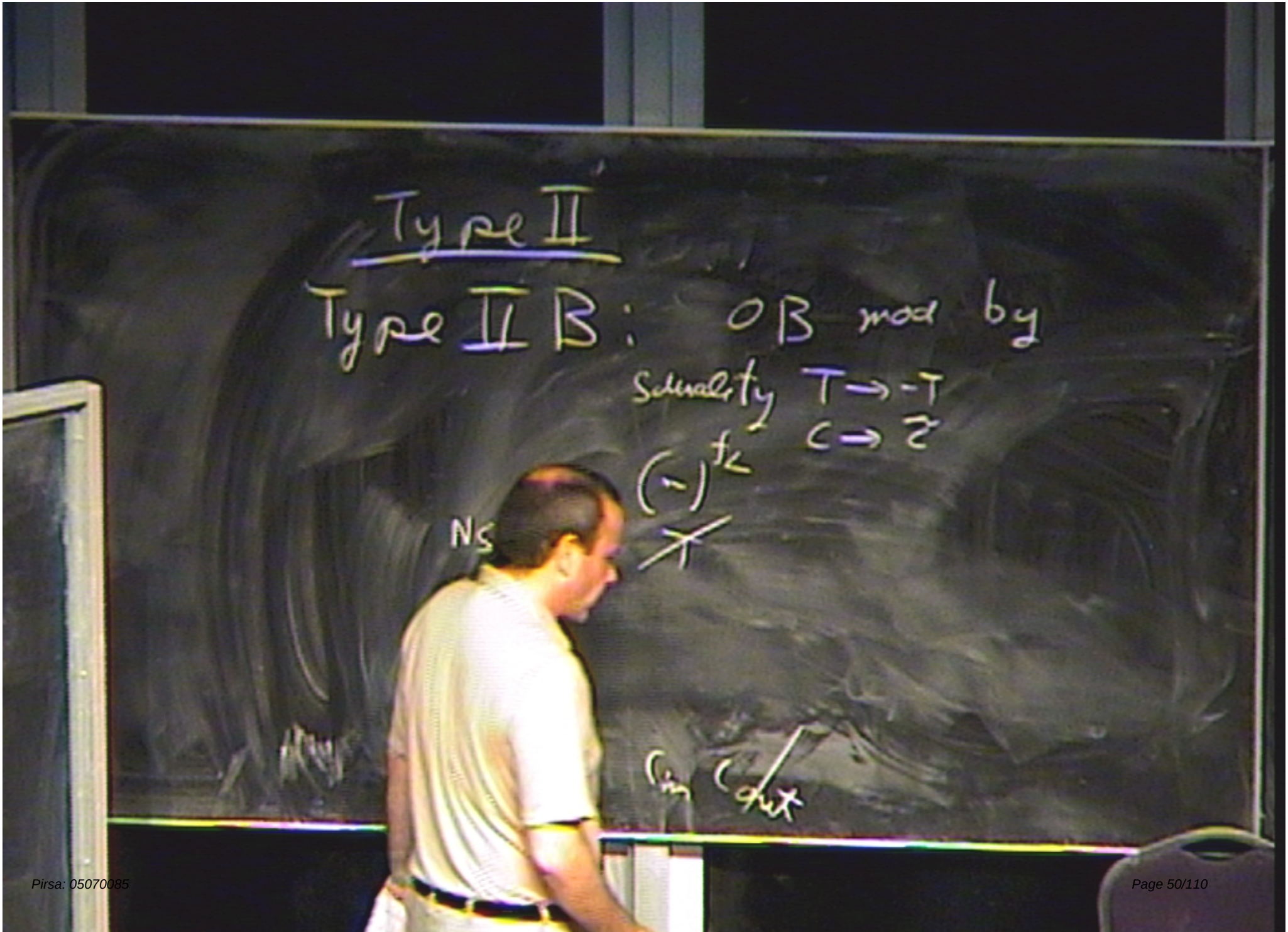
Angular momentum - $\tilde{q}$

W/ $q, \tilde{q}$ answers depend only on $|q| + |\tilde{q}|$


Type II'


Type II'


Type II'
Type II B: OB mod by
duality T → -T
C → Z
(-)^{F_L}


Type II
Type II B: OB mod by
Schwalty T → -T
C → 2
(-)^{f_L}
Ns

Type II

Type II B: OB mod by

Sduality $T \to -T$
$C \to \tilde{C}$

$(-)^{f_L}$

NS-NS ✗

RR C_{even} C_{odd} $\mathcal{O}$ $e^{-\varphi_L - \bar{\varphi}_L + \frac{i}{2}(H+\bar{H}) + ipX + ((-}$

Type II

Type II B: OB mod by

Duality $T \to -T$

$C \to \tilde{C}$

$(-)^{f_L}$

NS-NS ✗

RR C_{in} C_{out} $\mathcal{O}$ $e^{-\varphi_L - \bar{\varphi}_L + \frac{i}{2}(H+\bar{H}) + ipX + ((\cdot}$

Type II

Type II B: OB mod by

Sduality $T \to -T$
$C \to \tilde{C}$

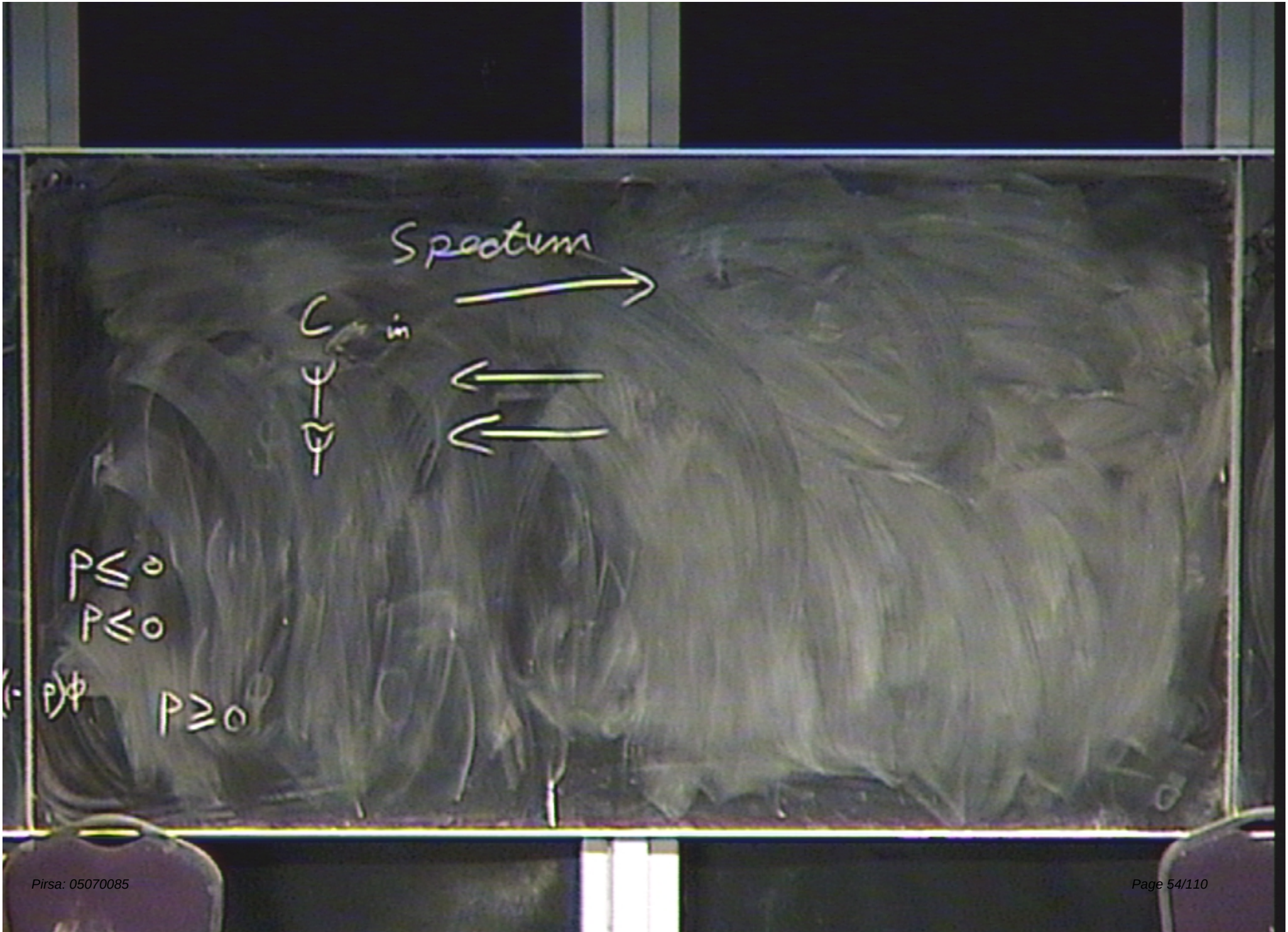
$(-)^{f_L}$

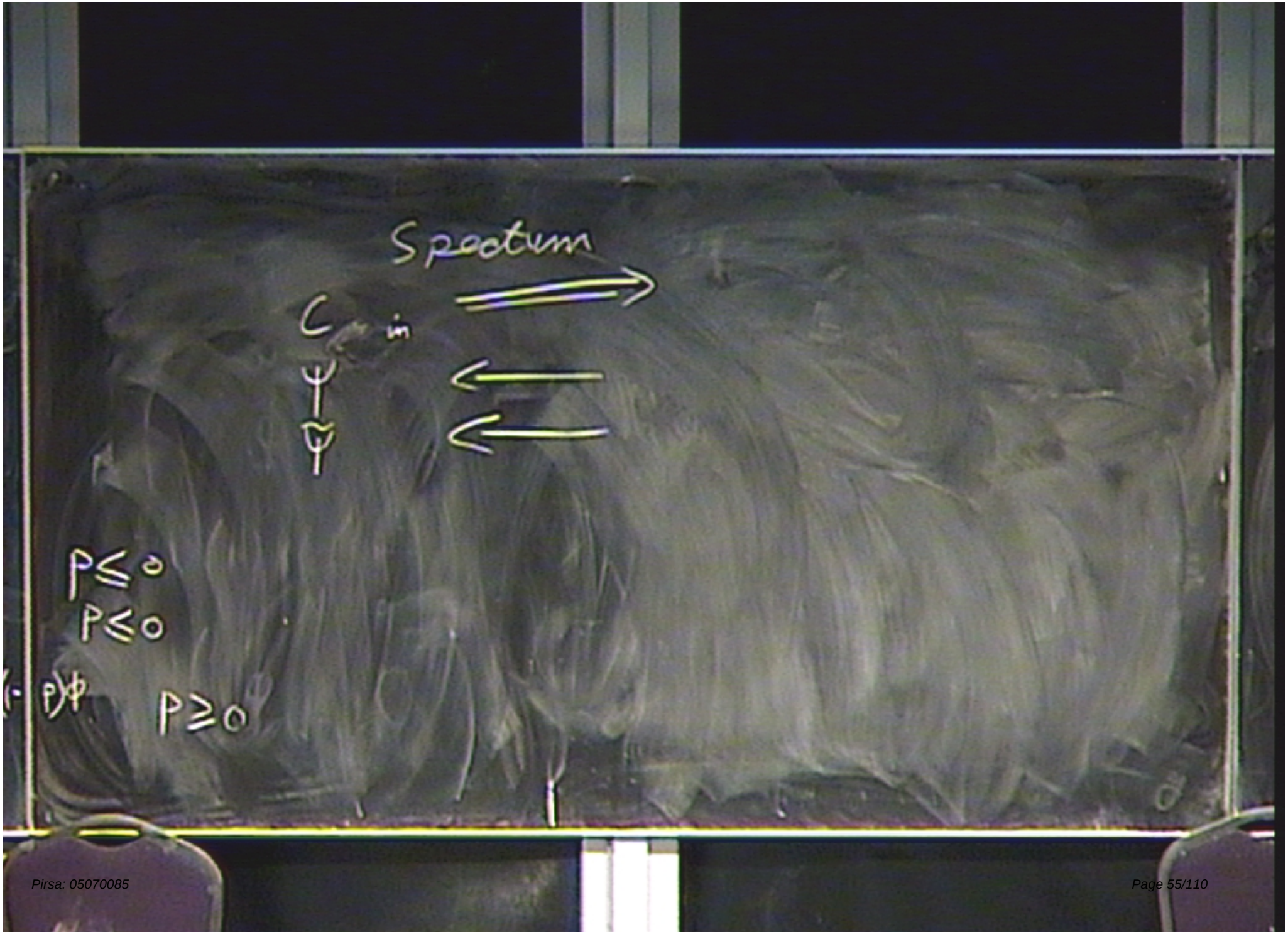
NS-NS ✗

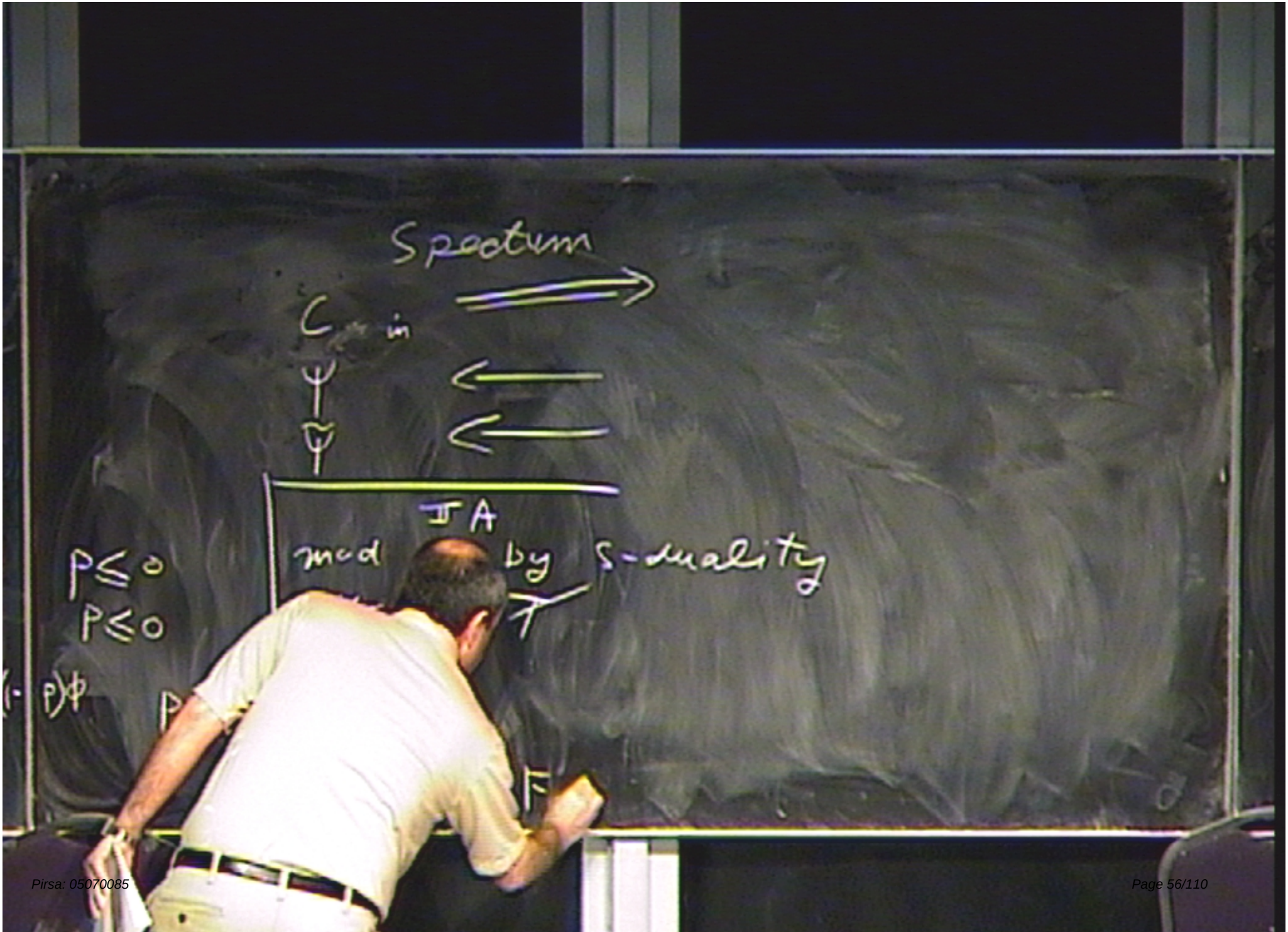
NS-R $e^{-\varphi/2 - \bar{\varphi} - i\bar{H}/2 + ipx + (1-|p|)\phi}$

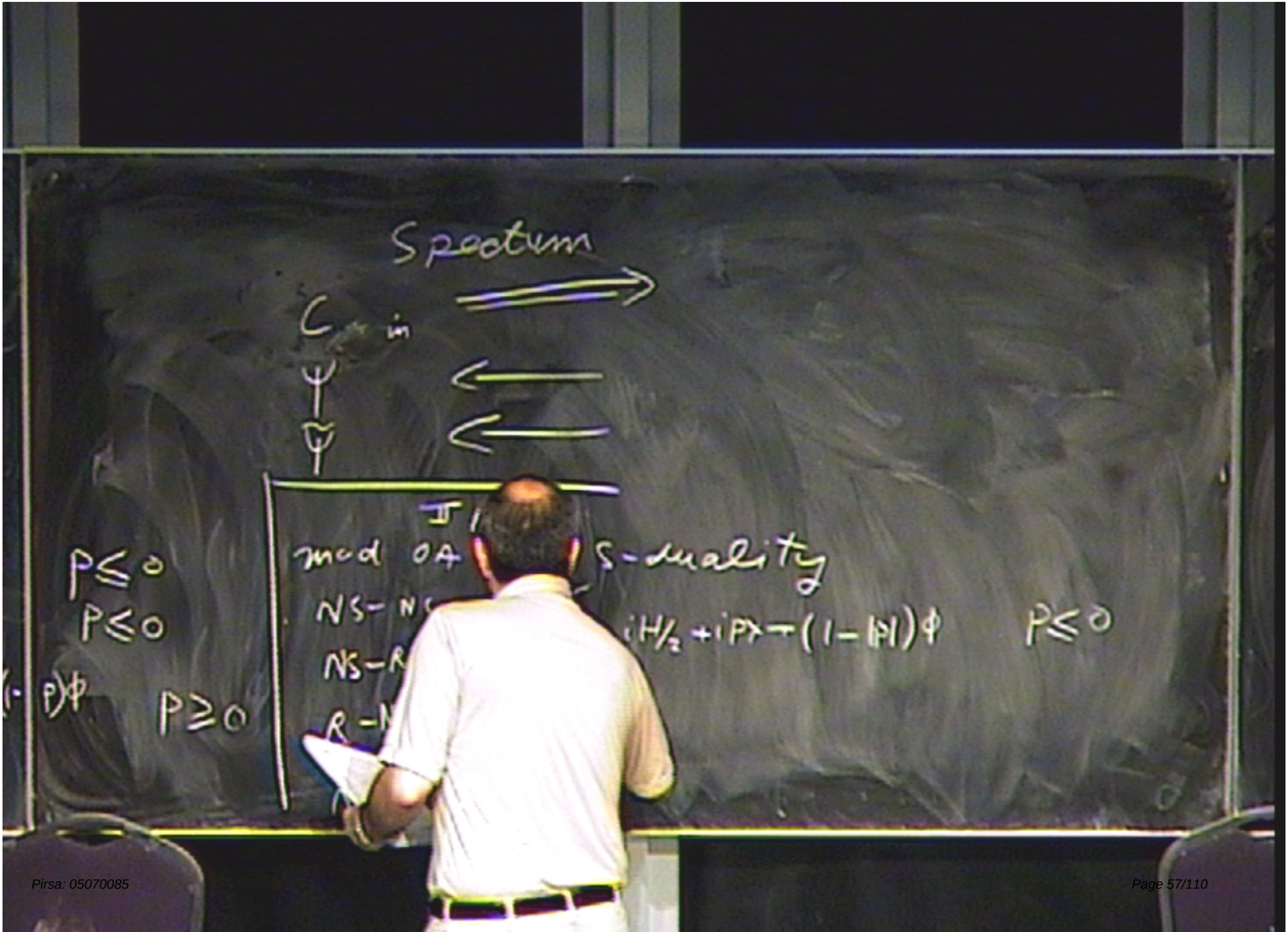
R-NS $e^{-\varphi - \bar{\varphi}/2 - i\bar{H}/2 + ipx + (1-|p|)\phi}$

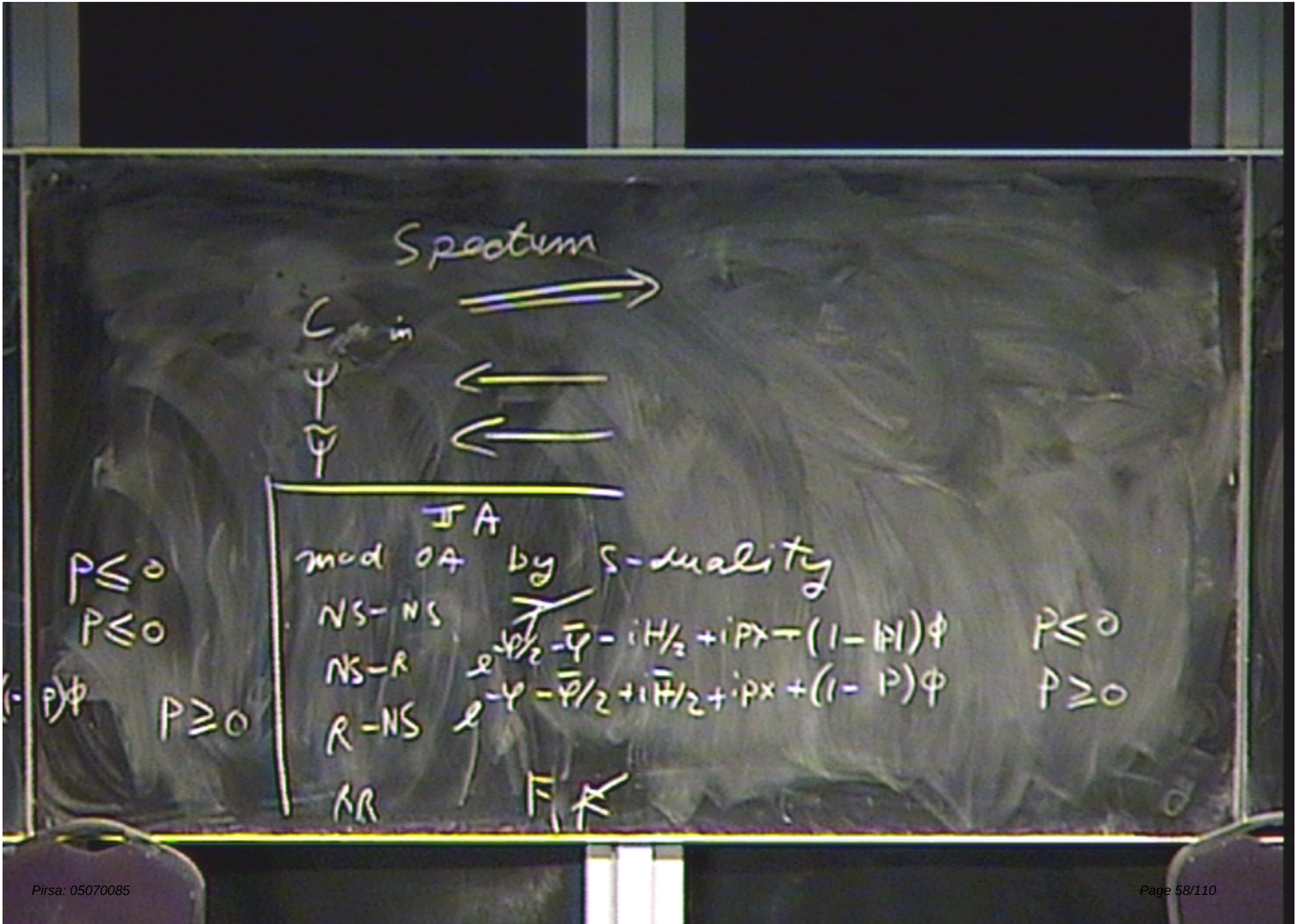
RR C_{mn} C_{mnx} $e^{-\varphi/2 - \bar{\varphi}/2 + \frac{i}{2}(H+\bar{H}) + ipx + (}$

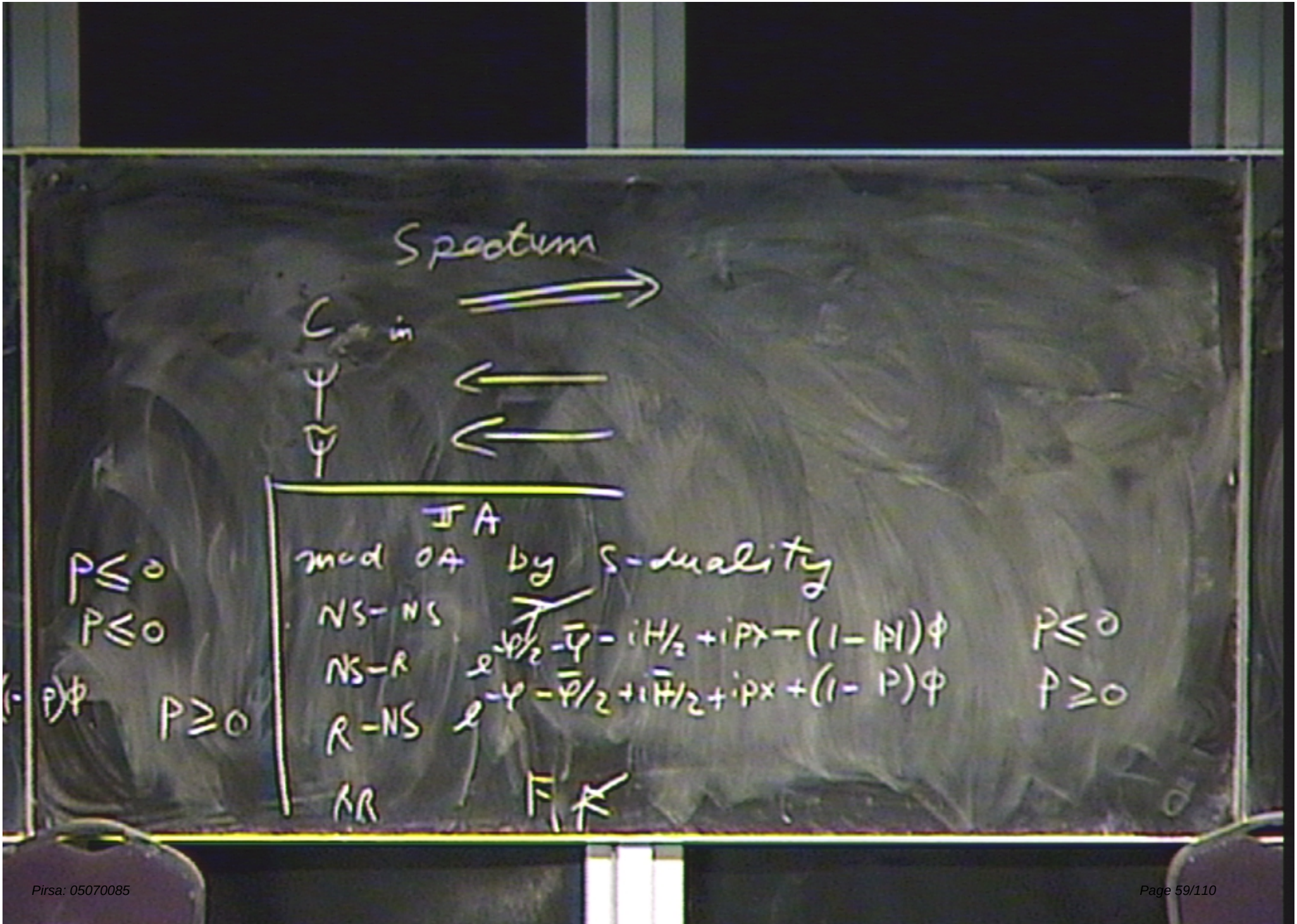

Spectrum
in
P≤0
P≤0
P≥0

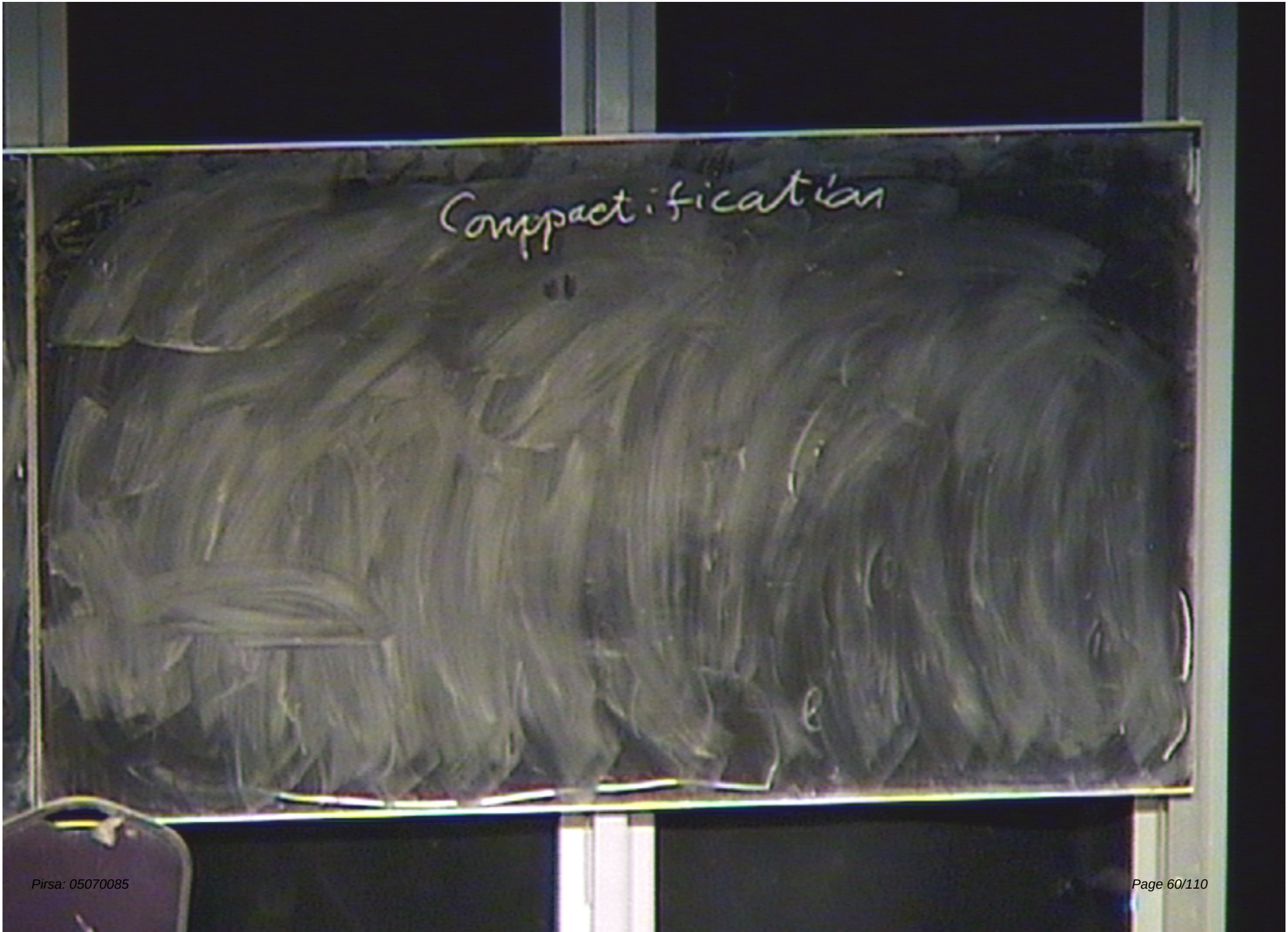

Spectrum
C in
P≤0
P≤0
P≥0

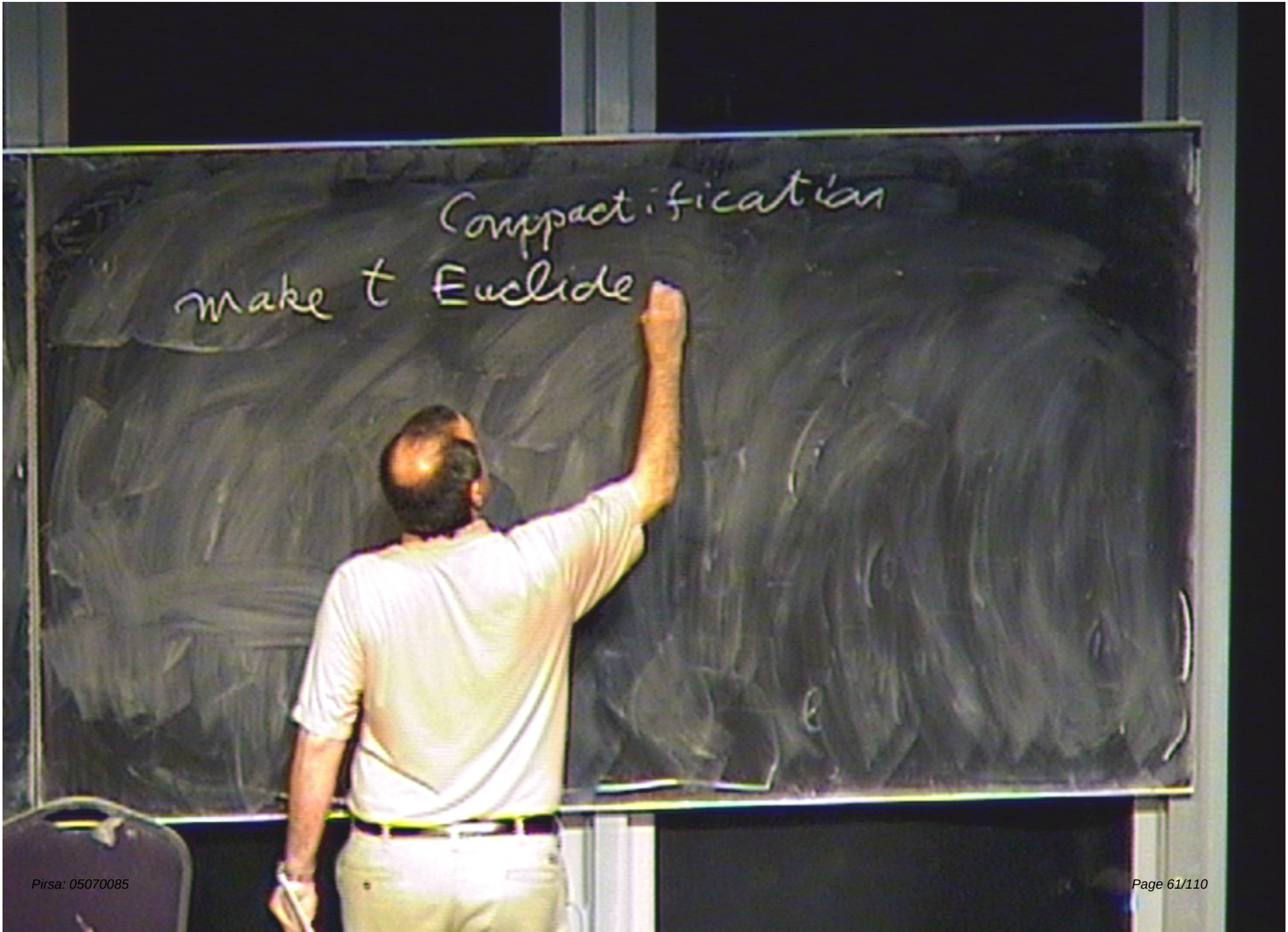

Spectrum
C in
A
mod by S-duality
P≤0
P≤0
F

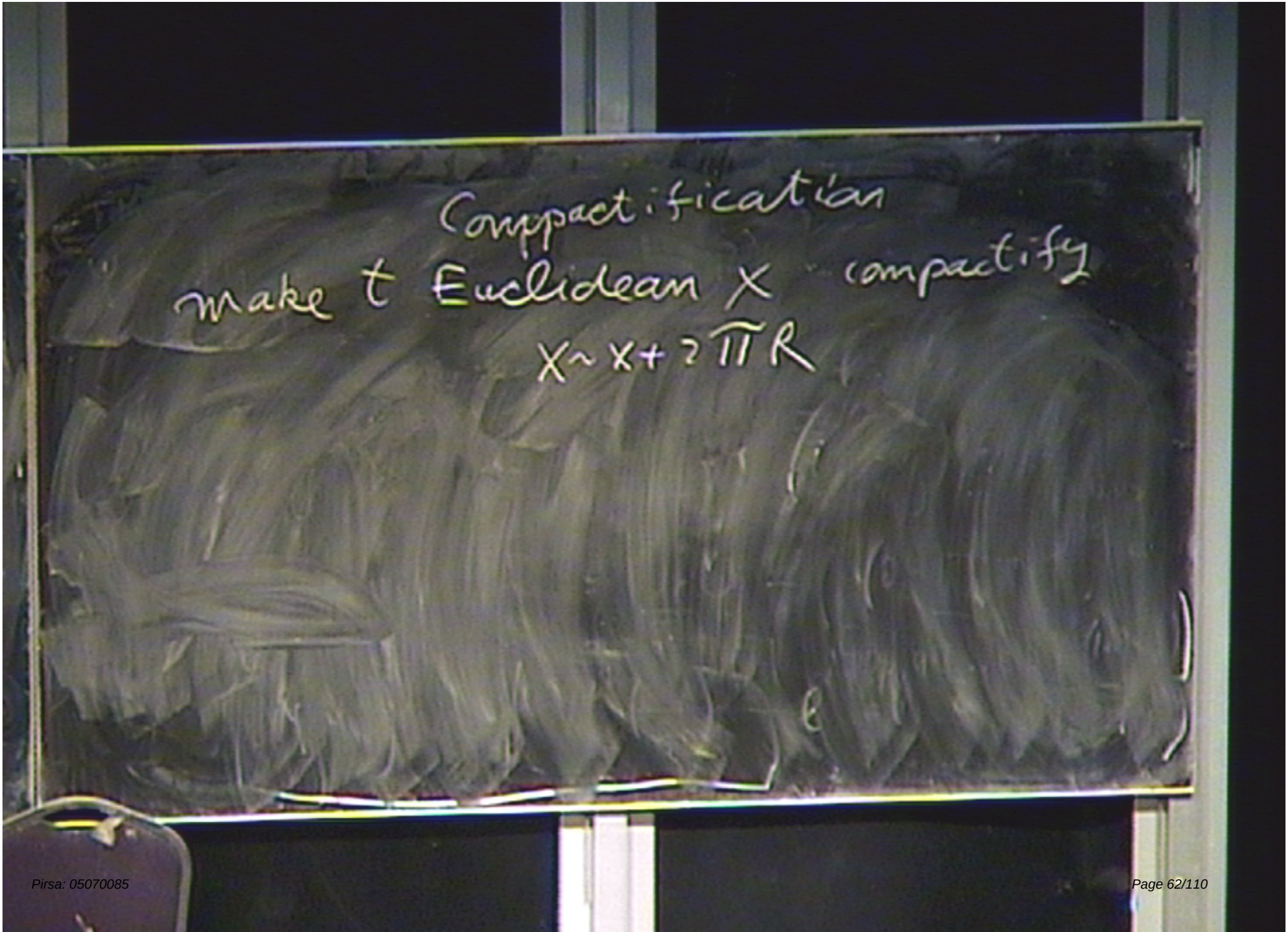

Spectrum
S-duality
P≤0
P≤0
P≥0
P≤0

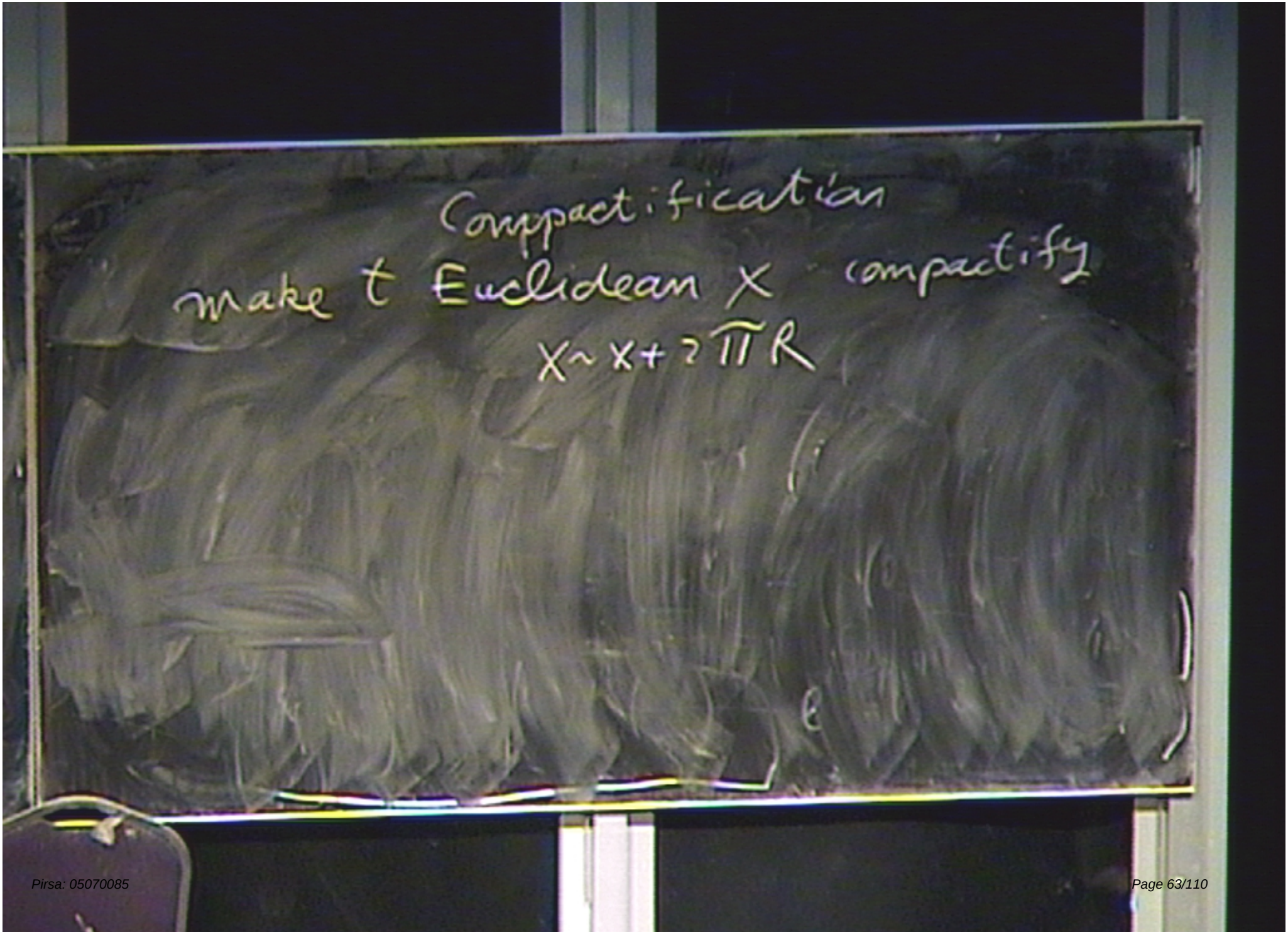

Spectrum
C in
Ψ
Ψ̄
∫ A
mod OA by S-duality
NS-NS
NS-R
R-NS
RR
P≤0
P≥0
P≤0
P≥0
F, F̃

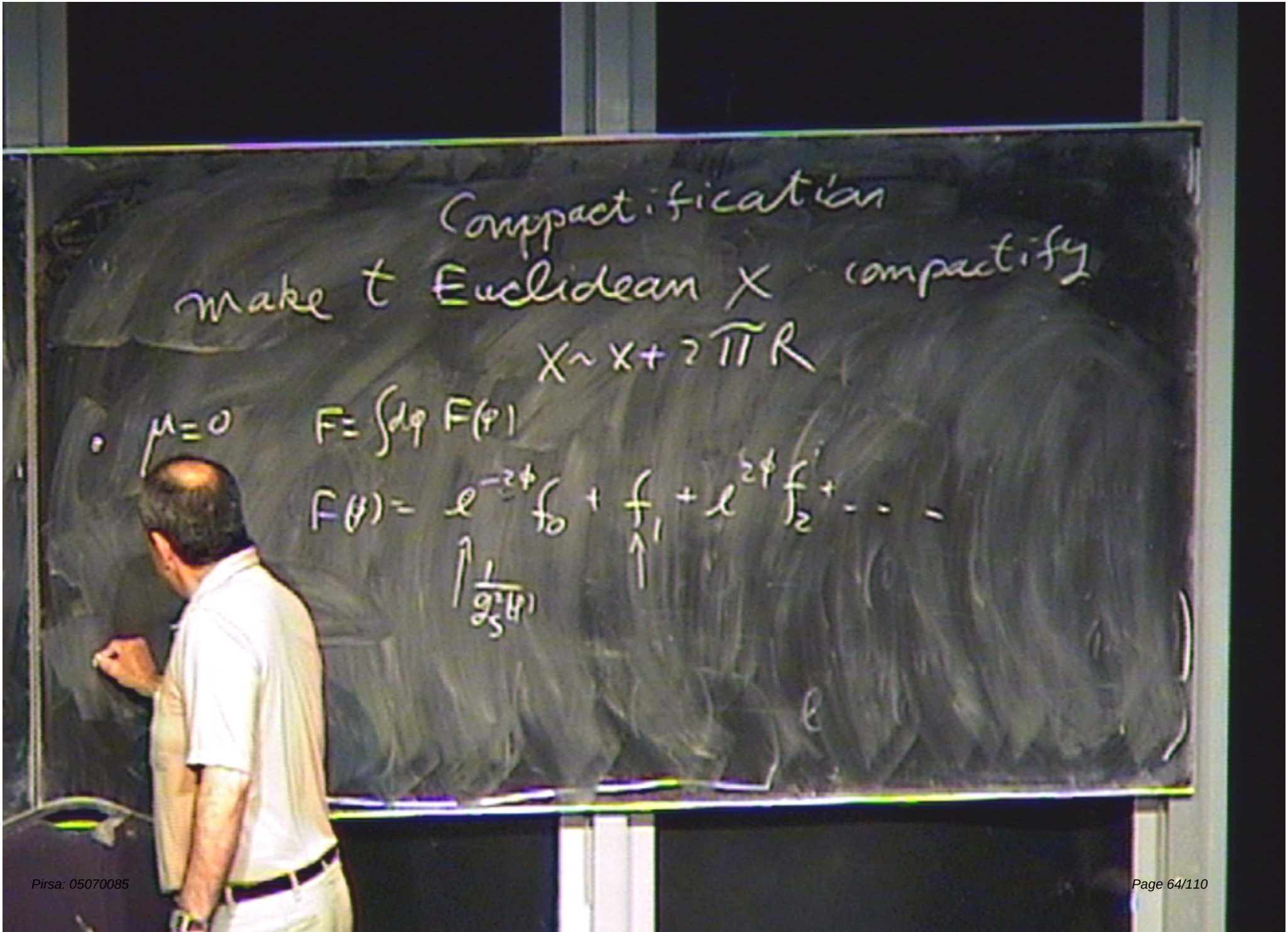

Spectrum
C in
ψ
ψ̃
∫ A
mod OA by S-duality
P≤0
P≤0
P≥0
NS-NS
NS-R
R-NS
RR
e^{-φ/2 - φ̄} - iH/2 + ipx - (1-|H|)φ
e^{-φ - φ̄/2 + iH̄/2 + ipx + (1-P)φ
P≤0
P≥0
F, F̃

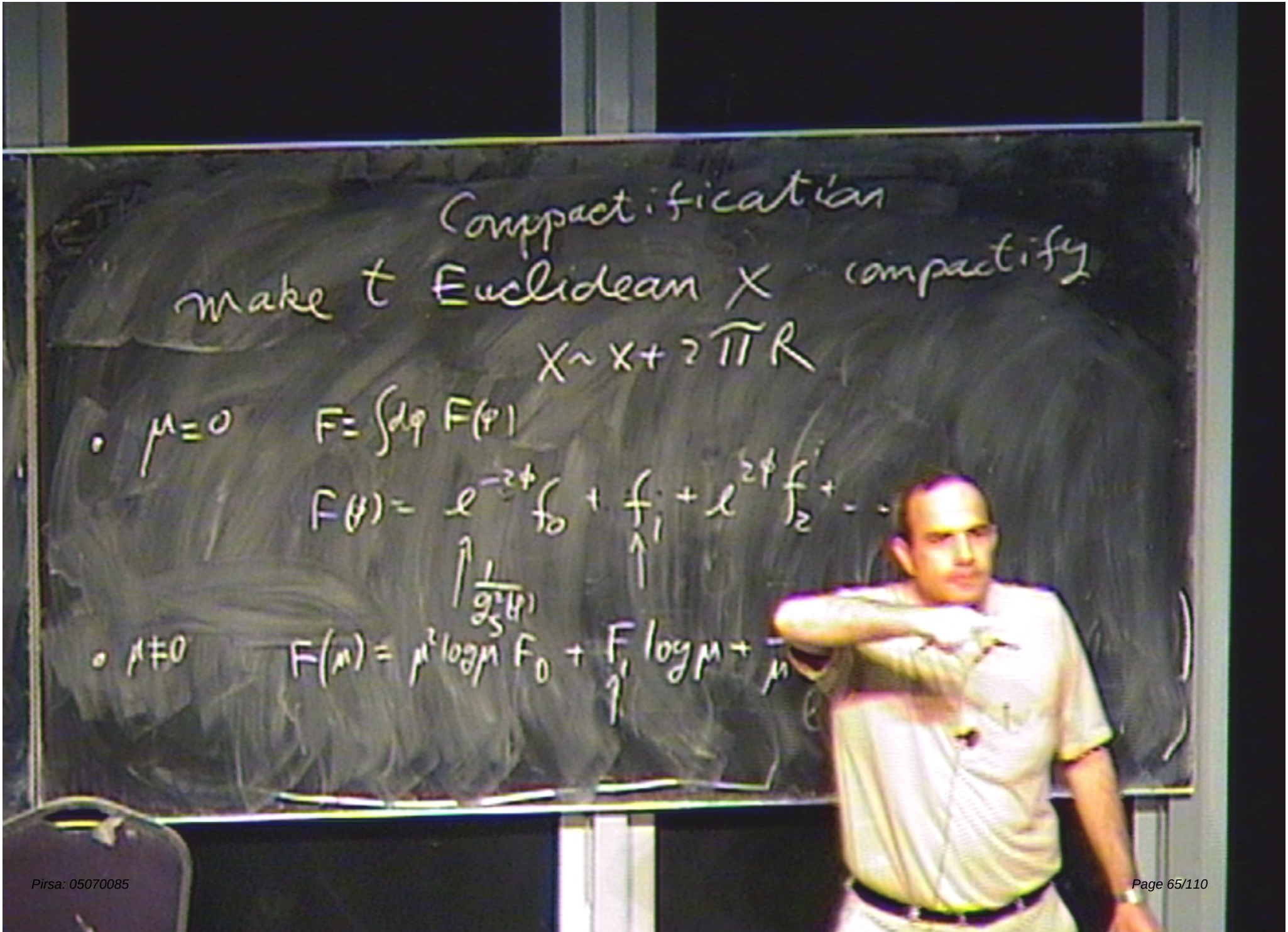

Compactification


Compactification
make t Euclide


Compactification
make t Euclidean X compactify
X ~ X + 2πR


Compactification
make t Euclidean X compactify
X ~ X + 2πR


Compactification
make t Euclidean X compactify
$X \sim X + 2\pi R$
$\mu = 0$ $F = \int dy\, F(y)$
$F(y) = e^{-2y} f_0 + f_1 + e^{2y} f_2 + \cdots$
$\frac{1}{g_5^2(y)}$


Compactification
make t Euclidean X compactify
$X \sim X + 2\pi R$
• $\mu = 0$ $F = \int dq\, F(\psi)$
$F(\psi) = e^{-2\psi} f_0 + f_1 + e^{2\psi} f_2 + \cdots$
$\frac{1}{g_s^2(\mu)}$
• $\mu \neq 0$ $F(\mu) = \mu^2 \log\mu\, F_0 + F_1 \log\mu + \frac{}{\mu}$

Compactification

make t Euclidean X compactify

$$X \sim X + 2\pi R$$

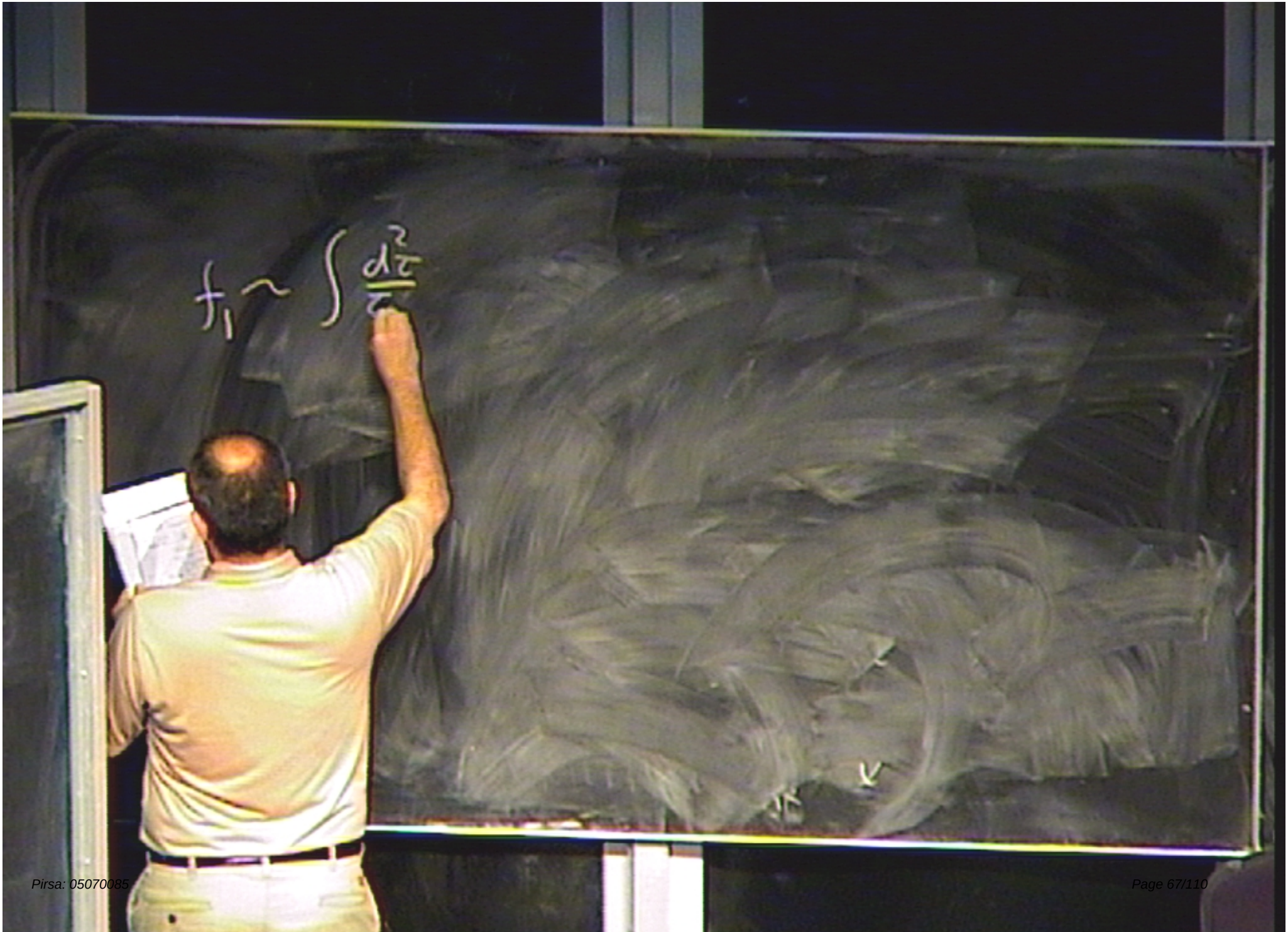
- $\mu = 0$ $\quad F = \int d\varphi\, F(\varphi)$

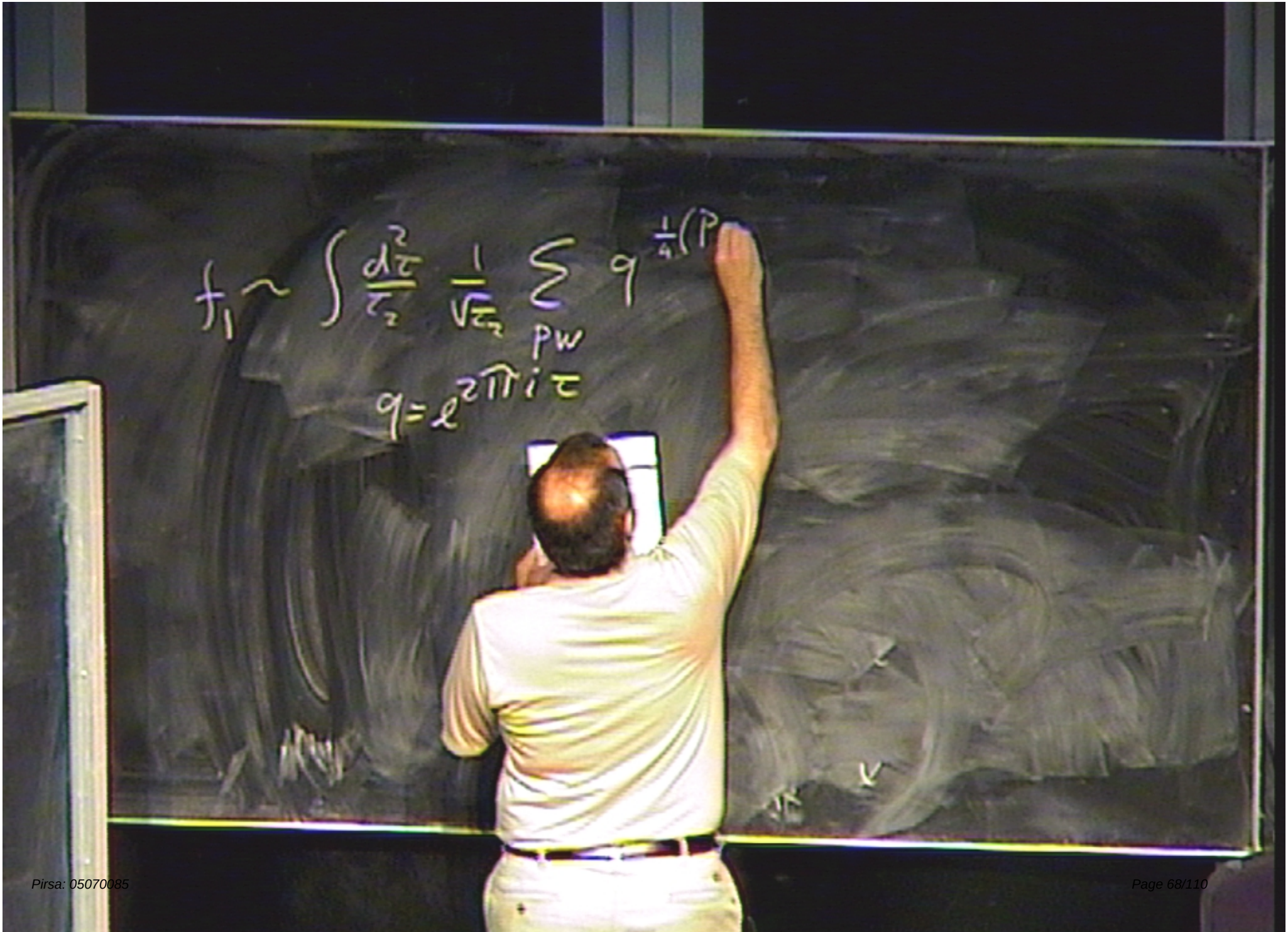
$$F(\theta) = e^{-2\theta} f_0 + f_1 + e^{2\theta} f_2 + \cdots$$

$\uparrow \frac{1}{g_s^2(\theta)}$ $\quad \uparrow$

- $\mu \neq 0$ $\quad F(\mu) = \mu^2 \log\mu\, F_0 + F_1 \log\mu + \frac{1}{\mu^2} F_2 + \cdots$

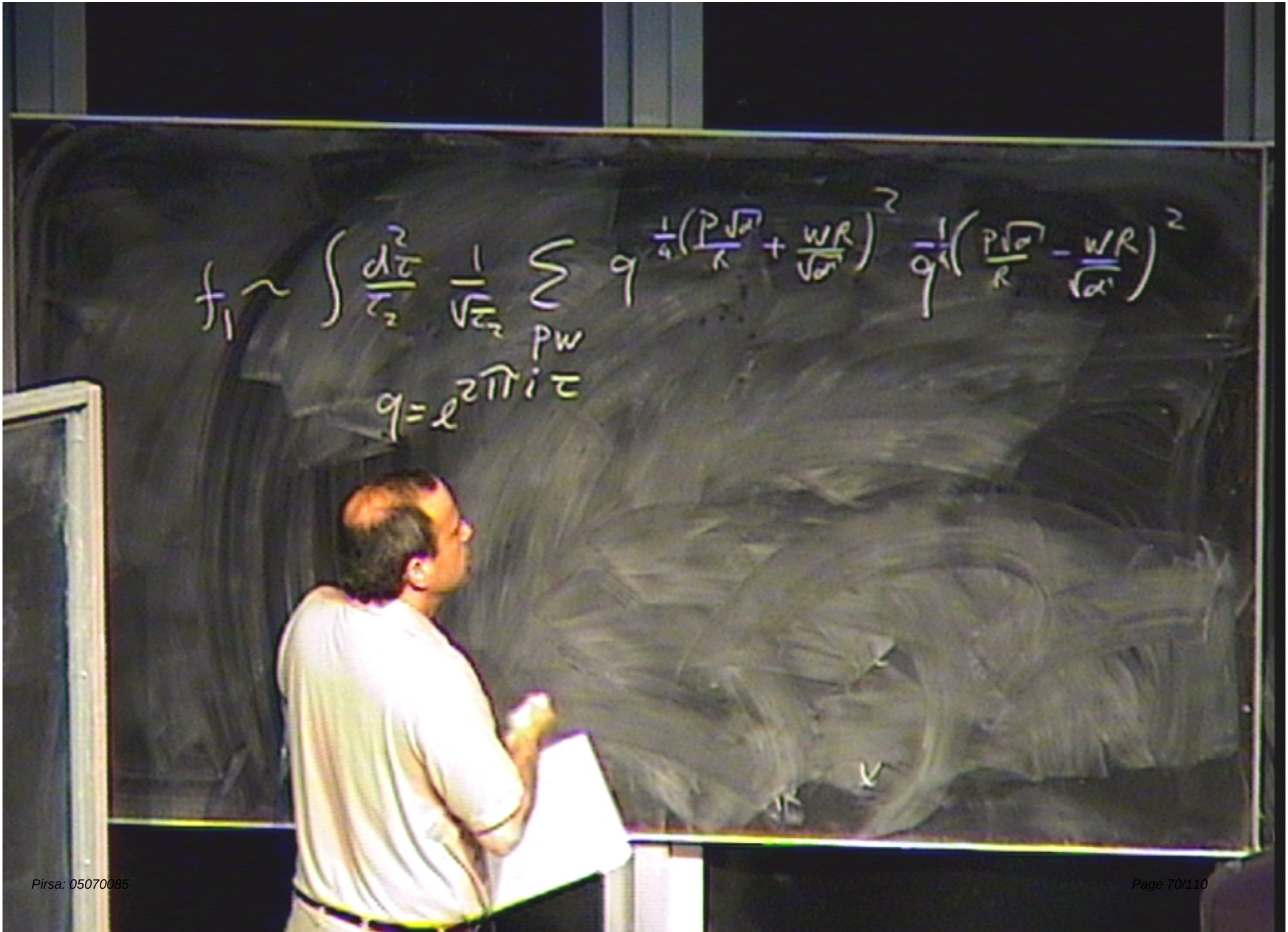
1 $\quad \ell$

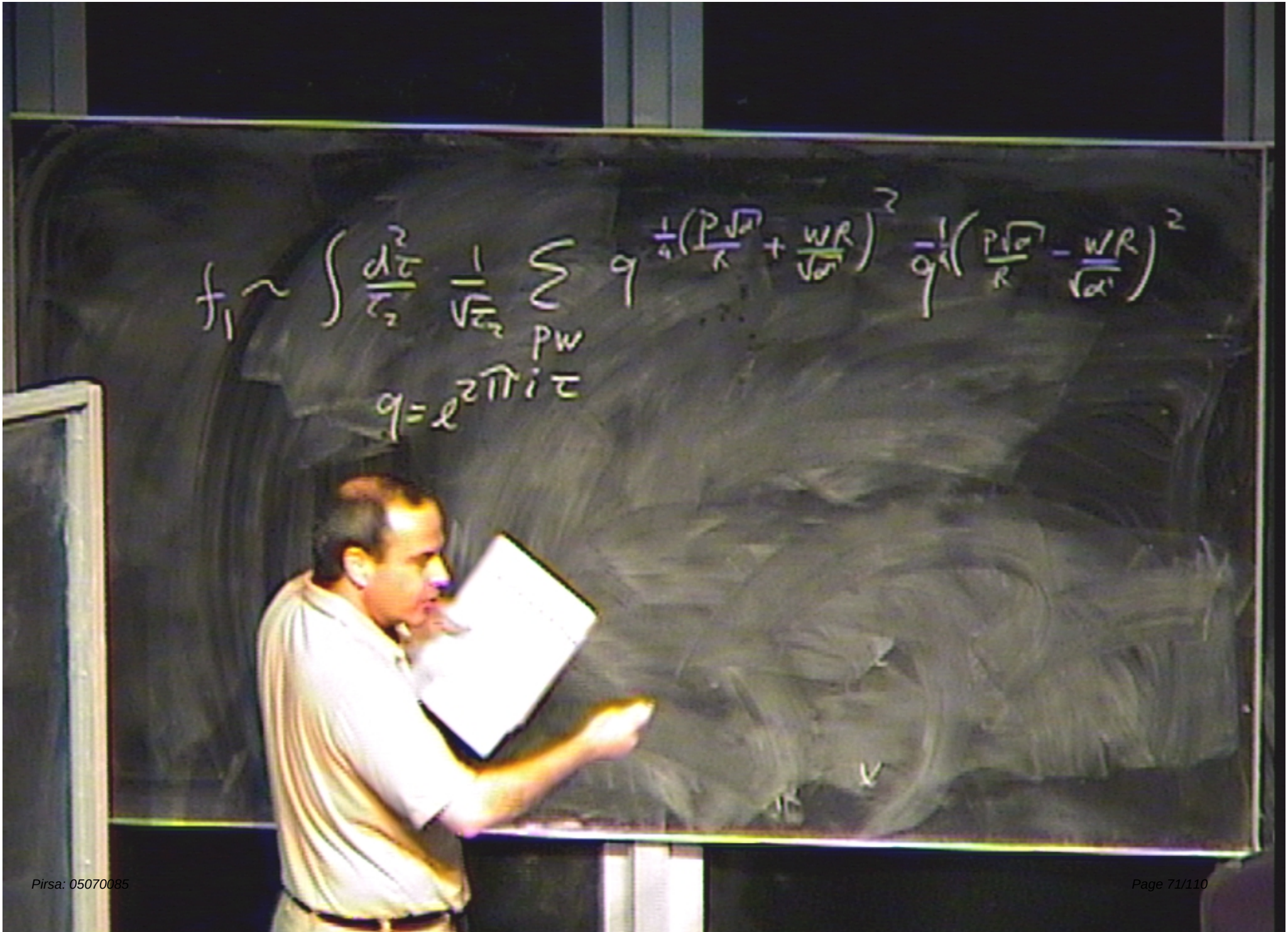

$f_1 \sim \int \frac{d^2\tau}{\tau^2}$


$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}(p$
$q = e^{2\pi i \tau}$

$$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$$

$$q = e^{2\pi i \tau}$$

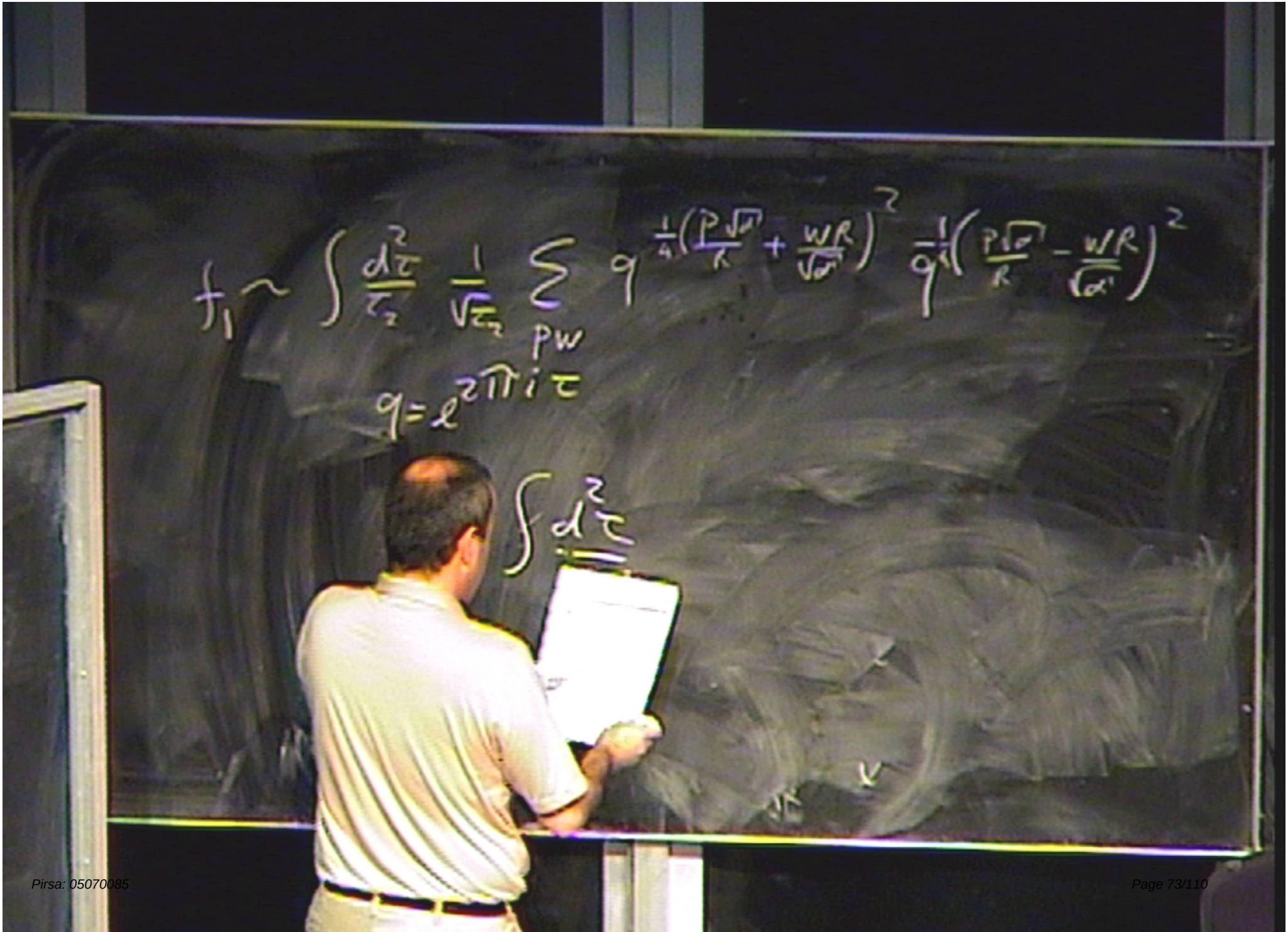

$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$


$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$

$$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$$

$$q = e^{2\pi i \tau}$$

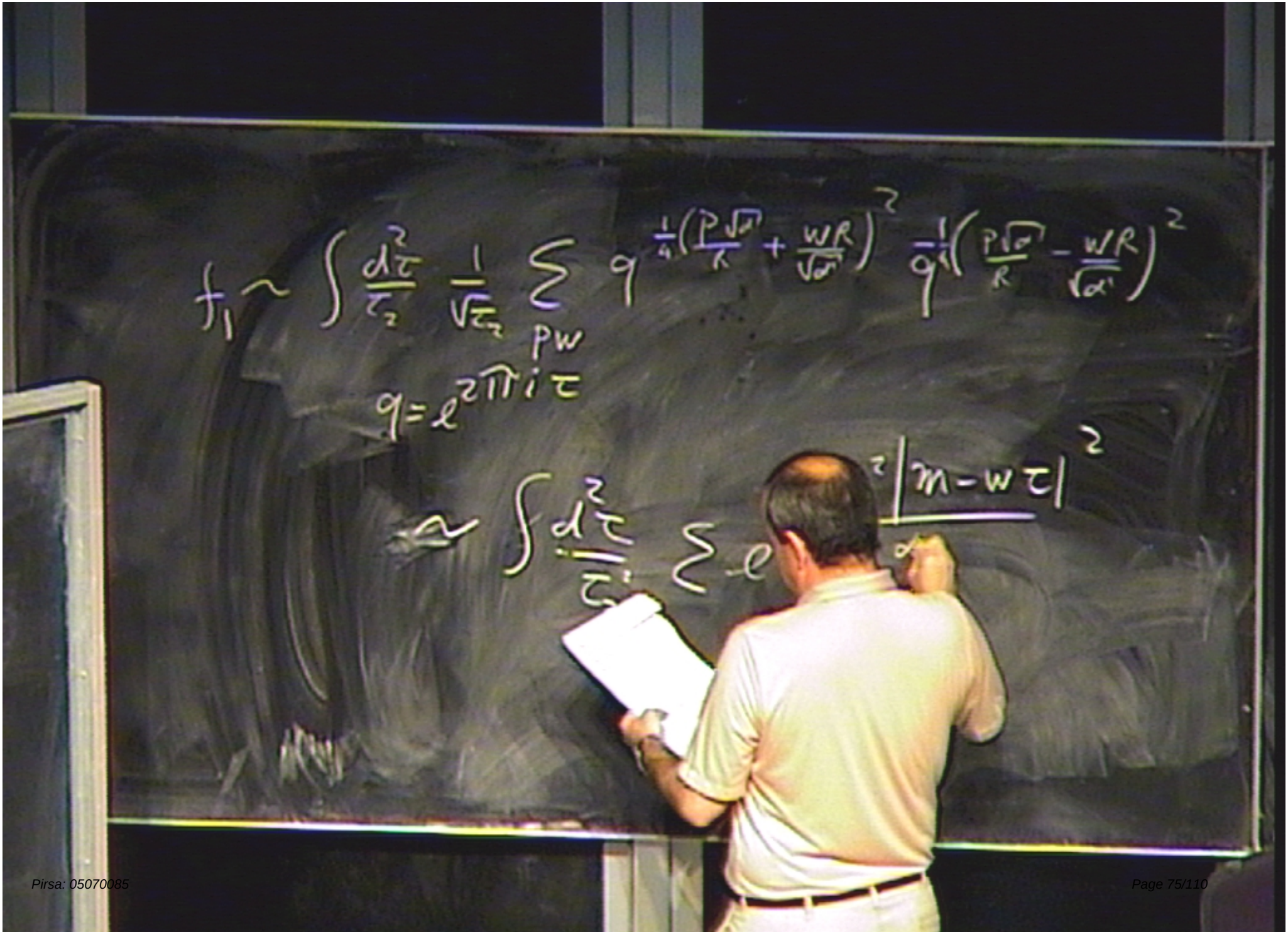
$$=$$

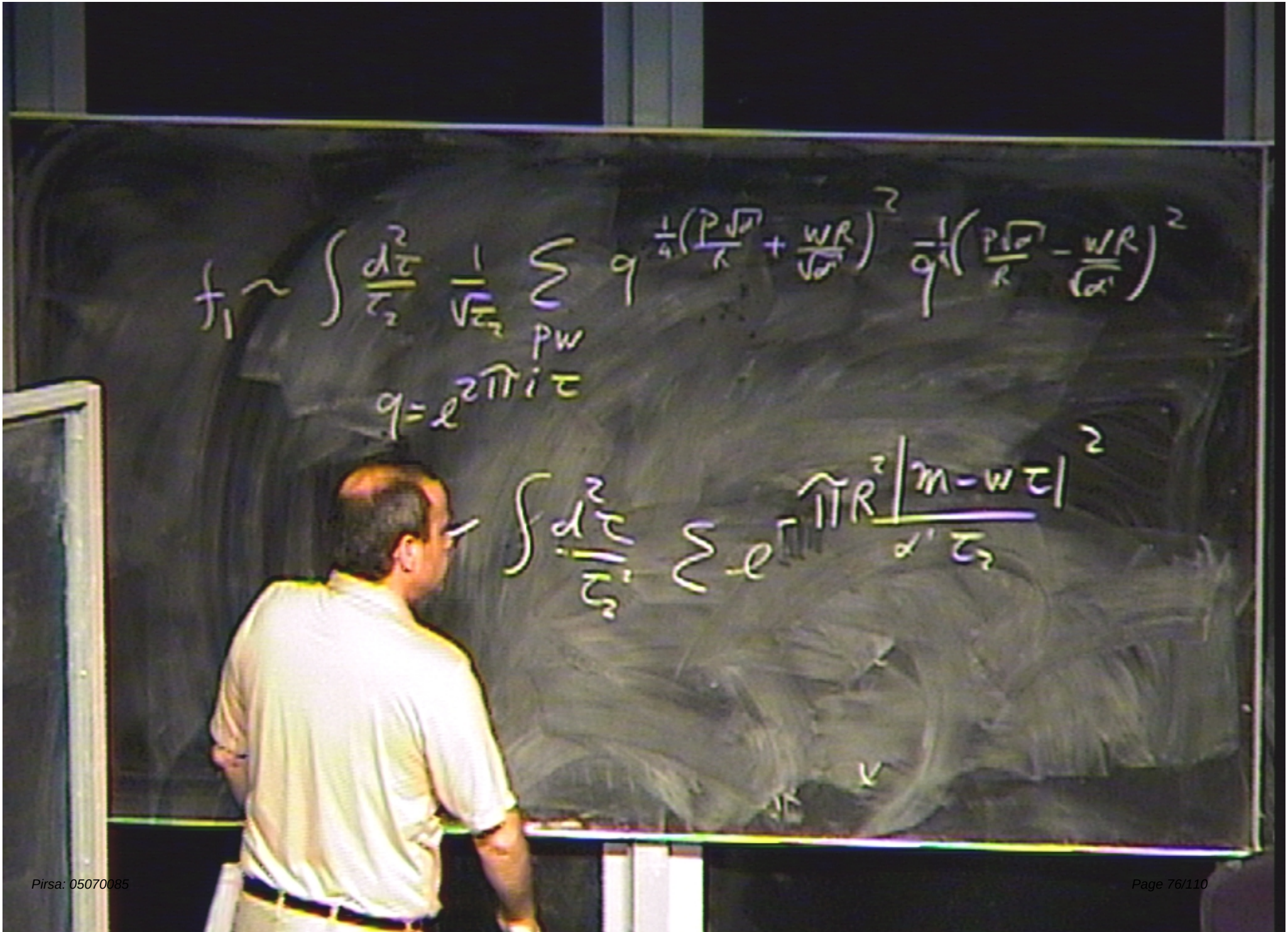

$t_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$
$\int d^2\tau$

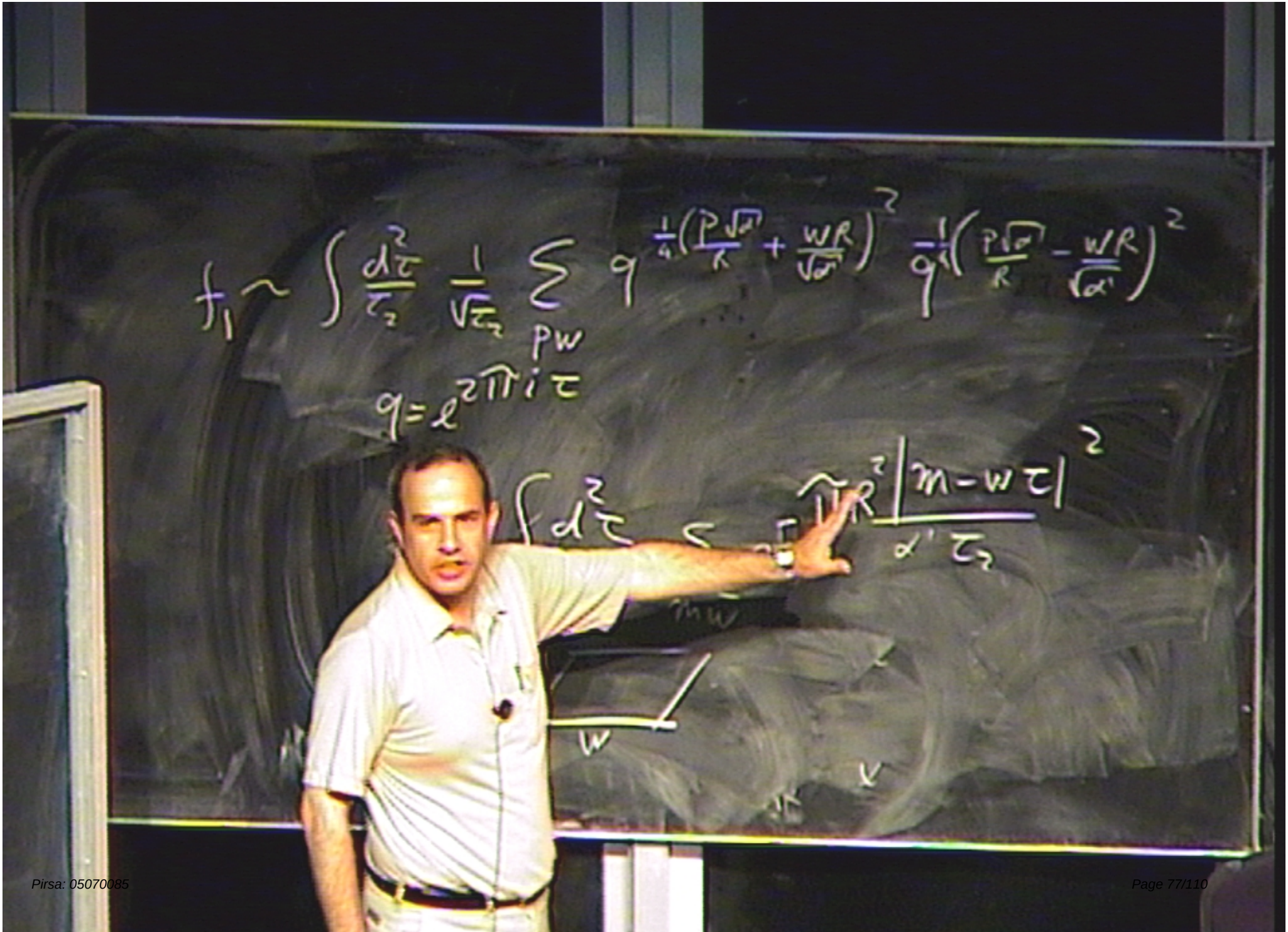
$$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$$

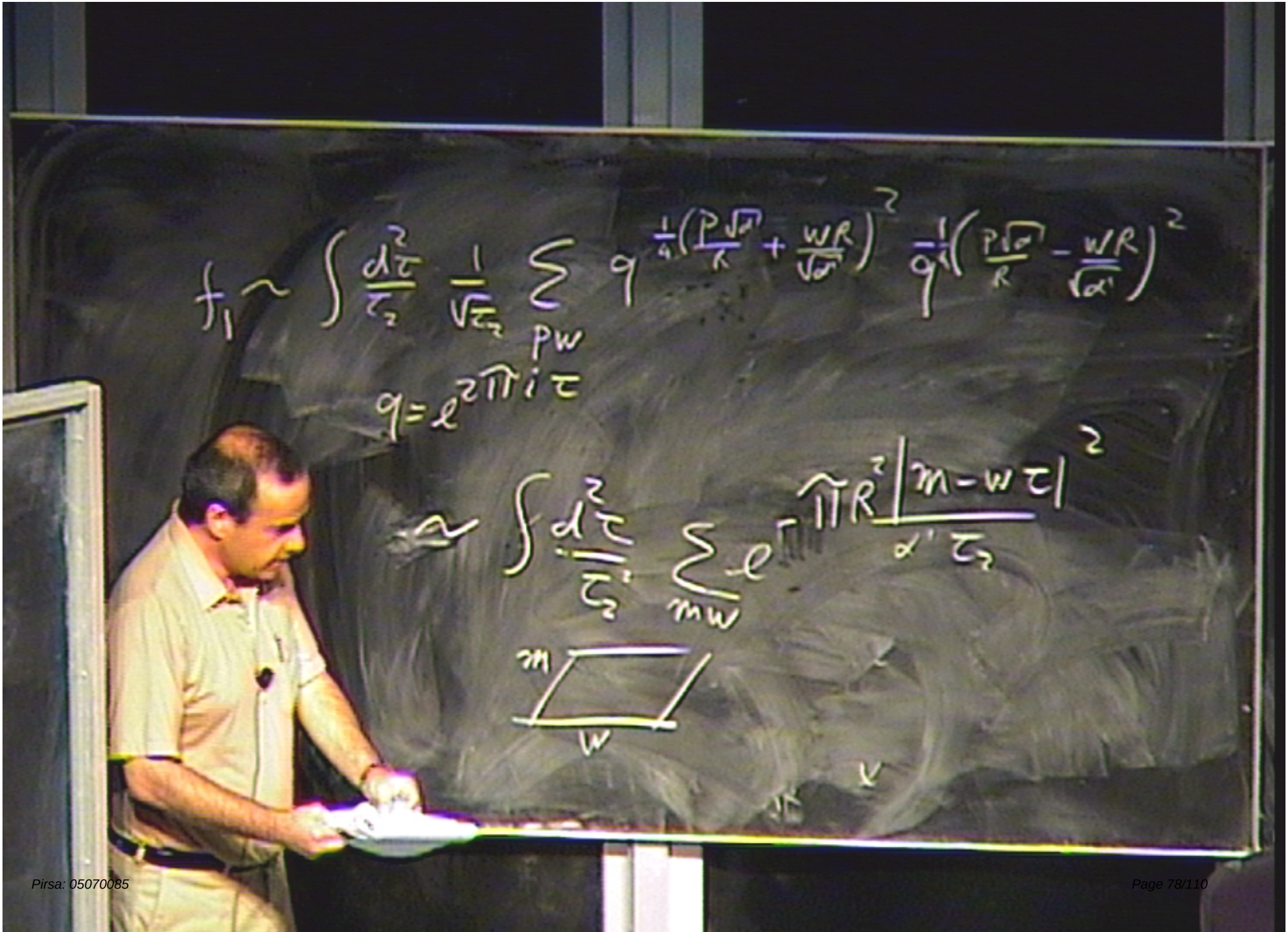
$$q = e^{2\pi i \tau}$$

$$\sim \int \frac{d^2\tau}{\tau_2^{13}} \sum e^{i}$$


$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$
$\sim \int \frac{d^2\tau}{\tau_2^2} \sum e^{-\frac{\pi}{\alpha'} \frac{|m - w\tau|^2}{\tau_2}}$


$t_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$
$\sim \int \frac{d^2\tau}{\tau_2^2} \sum e^{-\pi R^2 \frac{|m - w\tau|^2}{\alpha' \tau_2}}$


$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$
$\int d^2\tau \leq \frac{\pi R^2 |m - w\tau|^2}{\alpha' \tau_2}$
$m w$
w
v

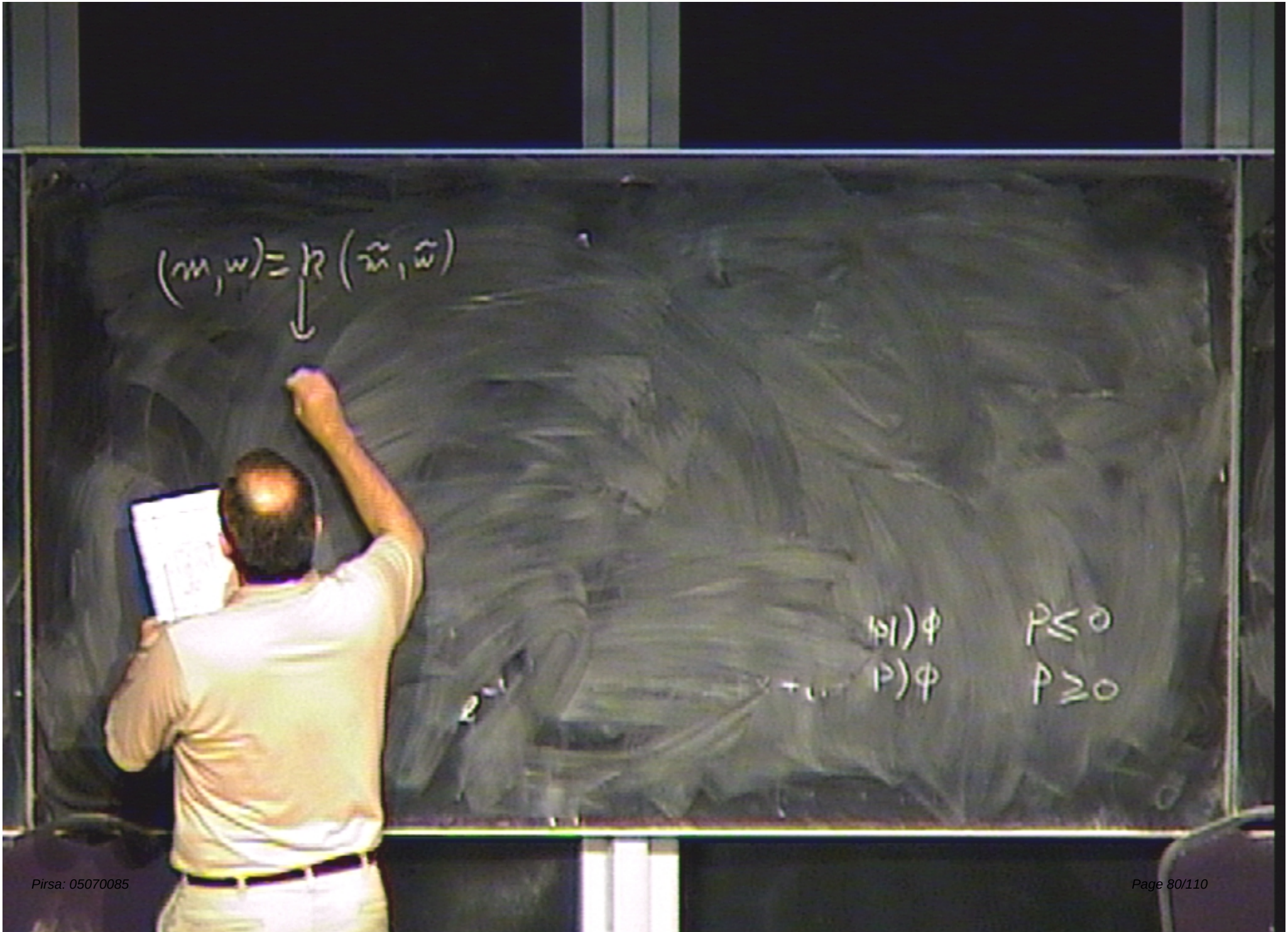

$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$
$q = e^{2\pi i \tau}$
$\sim \int \frac{d^2\tau}{\tau_2^2} \sum_{mw} e^{-\frac{\pi R^2 |m - w\tau|^2}{\alpha' \tau_2}}$
m
w

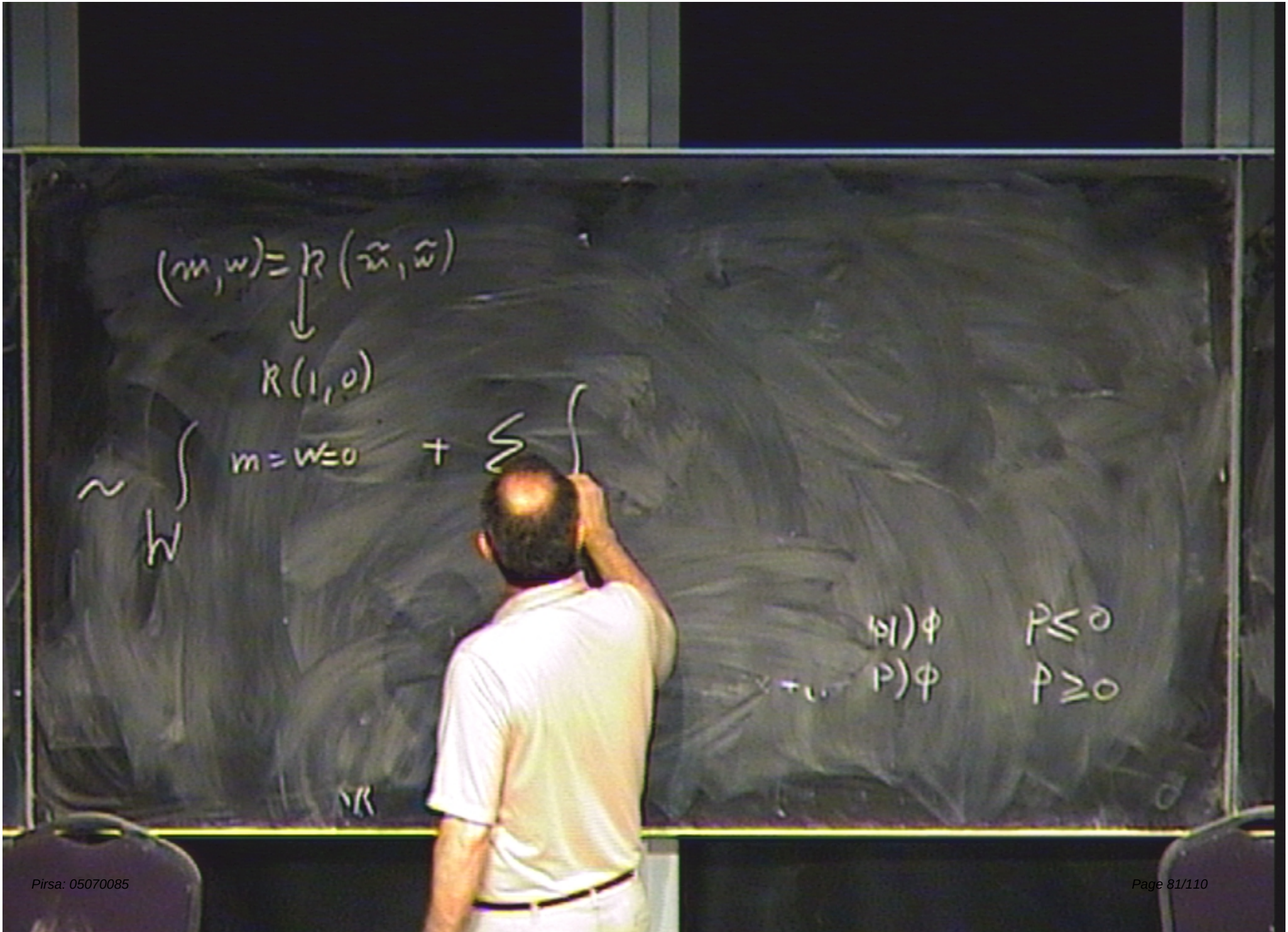
$$f_1 \sim \int \frac{d^2\tau}{\tau_2} \frac{1}{\sqrt{\tau_2}} \sum_{pw} q^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} + \frac{wR}{\sqrt{\alpha'}}\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{p\sqrt{\alpha'}}{R} - \frac{wR}{\sqrt{\alpha'}}\right)^2}$$

$$q = e^{2\pi i \tau}$$

$$\sim \int \frac{d^2\tau}{\tau_2^2} \sum_{mw} e^{-\frac{\pi R^2 |m - w\tau|^2}{\alpha' \tau_2}} =$$




$(m,w) = \Phi(\tilde{m},\tilde{w})$
$\Phi(\phi \quad P \leq 0$
$P)\phi \quad P \geq 0$


$(m,w) = R(\tilde{m},\tilde{w})$
$R(1,0)$
$m=w=0$
$+ \xi$
$\sim$
W
$\varphi \quad P \leq 0$
$\varphi \quad P \geq 0$

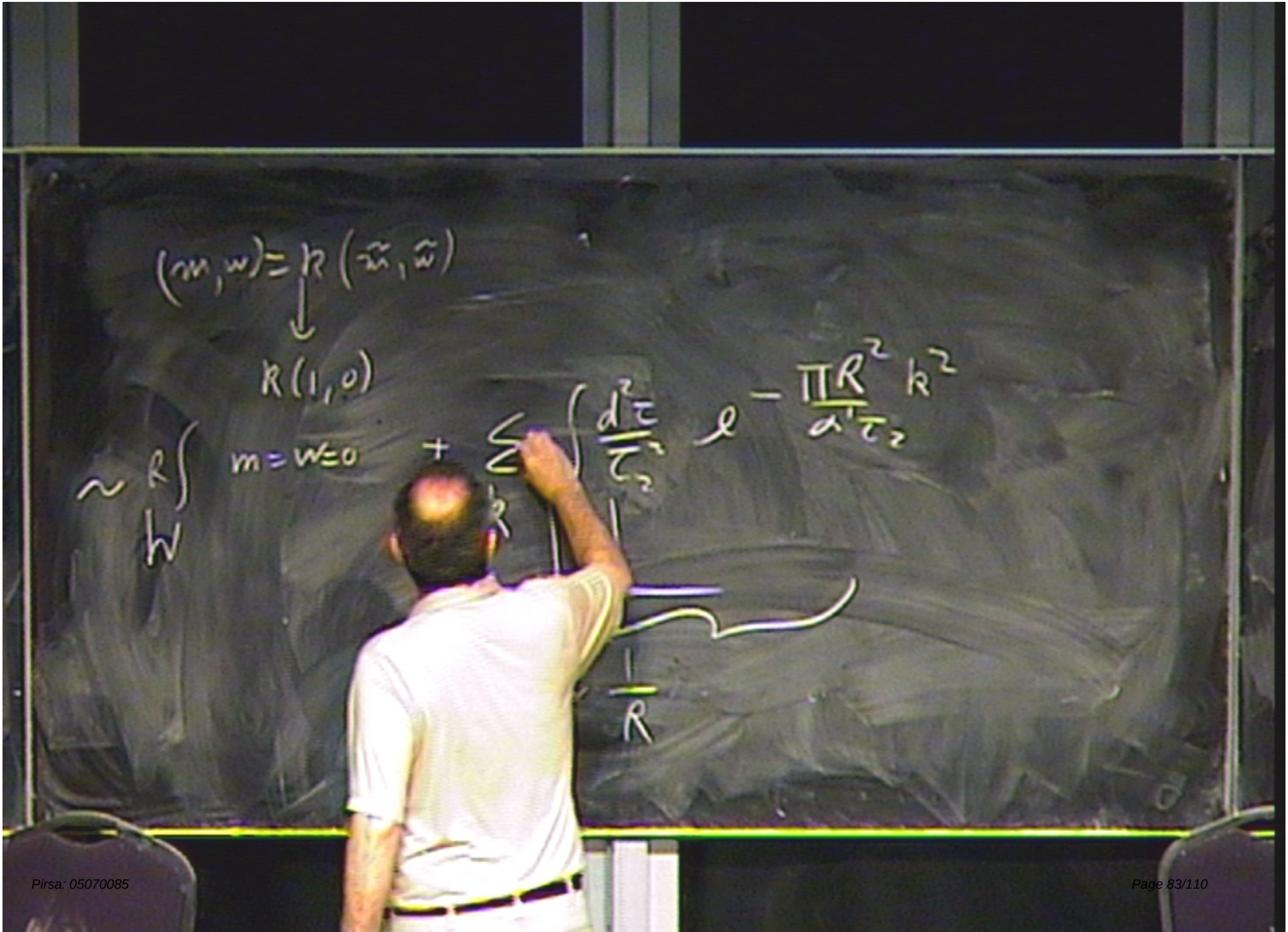
$$(m,w) = k(\tilde{m},\tilde{w})$$

$$\downarrow$$

$$K(1,0)$$

$$\sim \int_W m=w=0 \; + \sum_k \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2}{a^2}\frac{k^2}{\tau_2}}$$




$(m,w) = k(\tilde{m},\tilde{w})$
$R(1,0)$
$m=w=0$
$+ \sum_k \int \frac{d^2\tau}{\tau_2^2} e^{-\frac{\pi R^2}{a'\tau_2} k^2}$
$\frac{1}{R}$

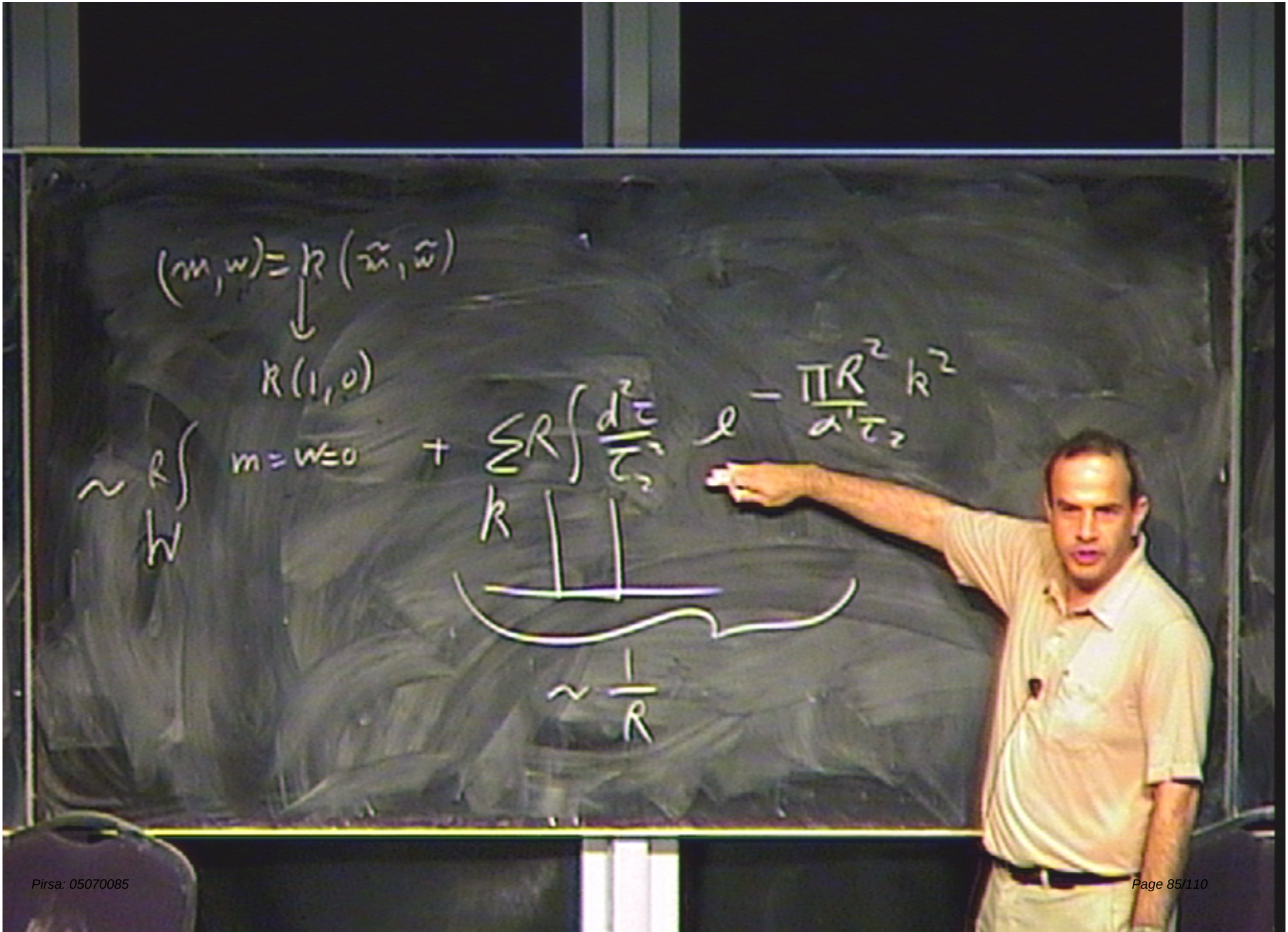
$(m,w) = k(\tilde{m},\tilde{w})$

$\downarrow$

$R(1,0)$

$\sim \int_W^R \; m=w=0 \; + \sum_k R \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2}{A\tau_2} k^2}$

$\sim \frac{1}{R}$


$(m,w) = k(\tilde{m},\tilde{w})$
$k(1,0)$
$\sim \int_W$
$m=w=0$
$+ \sum_k R \int \frac{d^2\tau}{\tau_2}$
$e^{-\frac{\pi R^2}{\alpha'\tau_2}k^2}$
$\sim \frac{1}{R}$

$(m, w) = k(\tilde{m}, \tilde{w})$

$\downarrow$

$R(1,0)$

$m = w = 0$

$\sim \int_W R$

$+ \sum_k R \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$

$\sim \frac{1}{R}$

$(m, w) = k(\tilde{m}, \tilde{w})$

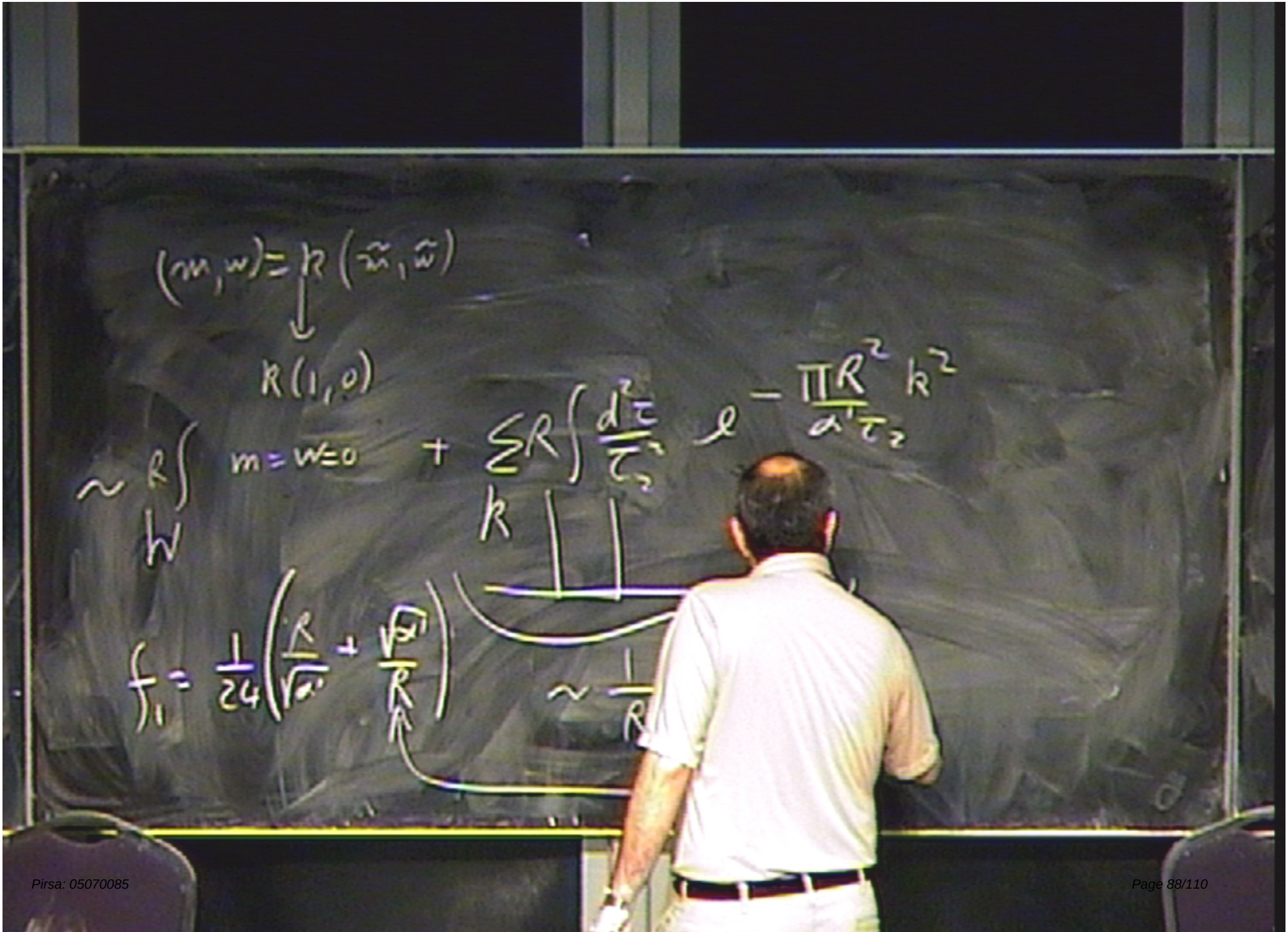
$\downarrow$

$R(1,0)$

$m = w = 0$

$$\sim \int_W \; + \sum_k R \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2}{\alpha' \tau_2} k^2}$$

$$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right) \sim \frac{1}{R}$$


$(m,w) = k(\tilde{m},\tilde{w})$
$R(1,0)$
$\sim \int_W$
$m=w=0$
$+ \sum_k R \int \frac{d^2\tau}{\tau_2}$
$e^{-\frac{\pi R^2}{\alpha'\tau_2}k^2}$
$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$
$\sim \frac{1}{R}$

$(m,w) = k(\tilde{m},\tilde{w})$

$\downarrow$

$R(1,0)$

$\sim \int_W \; m=w=0 \; + \sum_k R \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$

$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$ $\sim \frac{1}{R}$ finite in field theory

$(m,w) = k(\tilde{m},\tilde{w})$

$\downarrow$

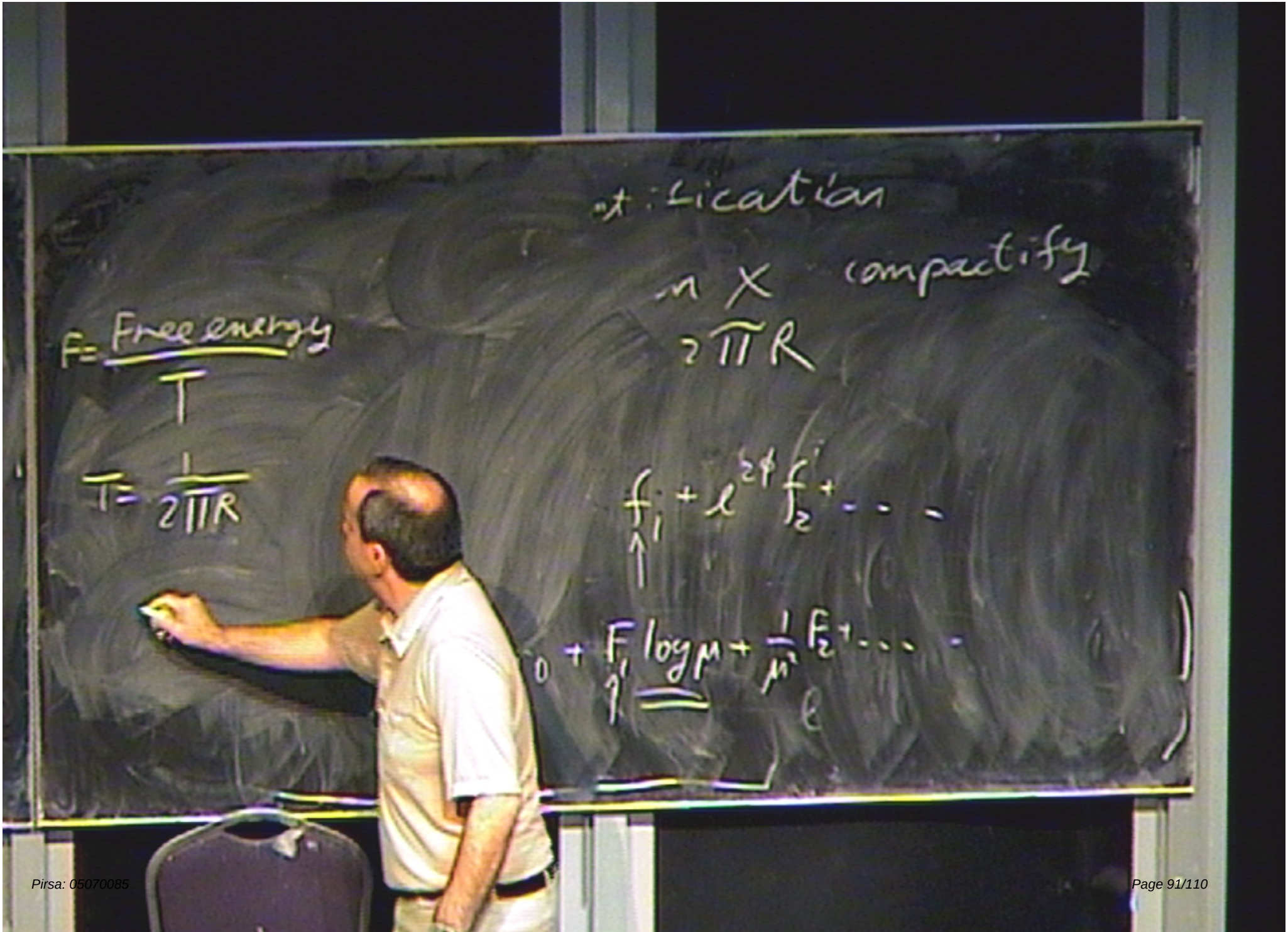
$R(1,0)$

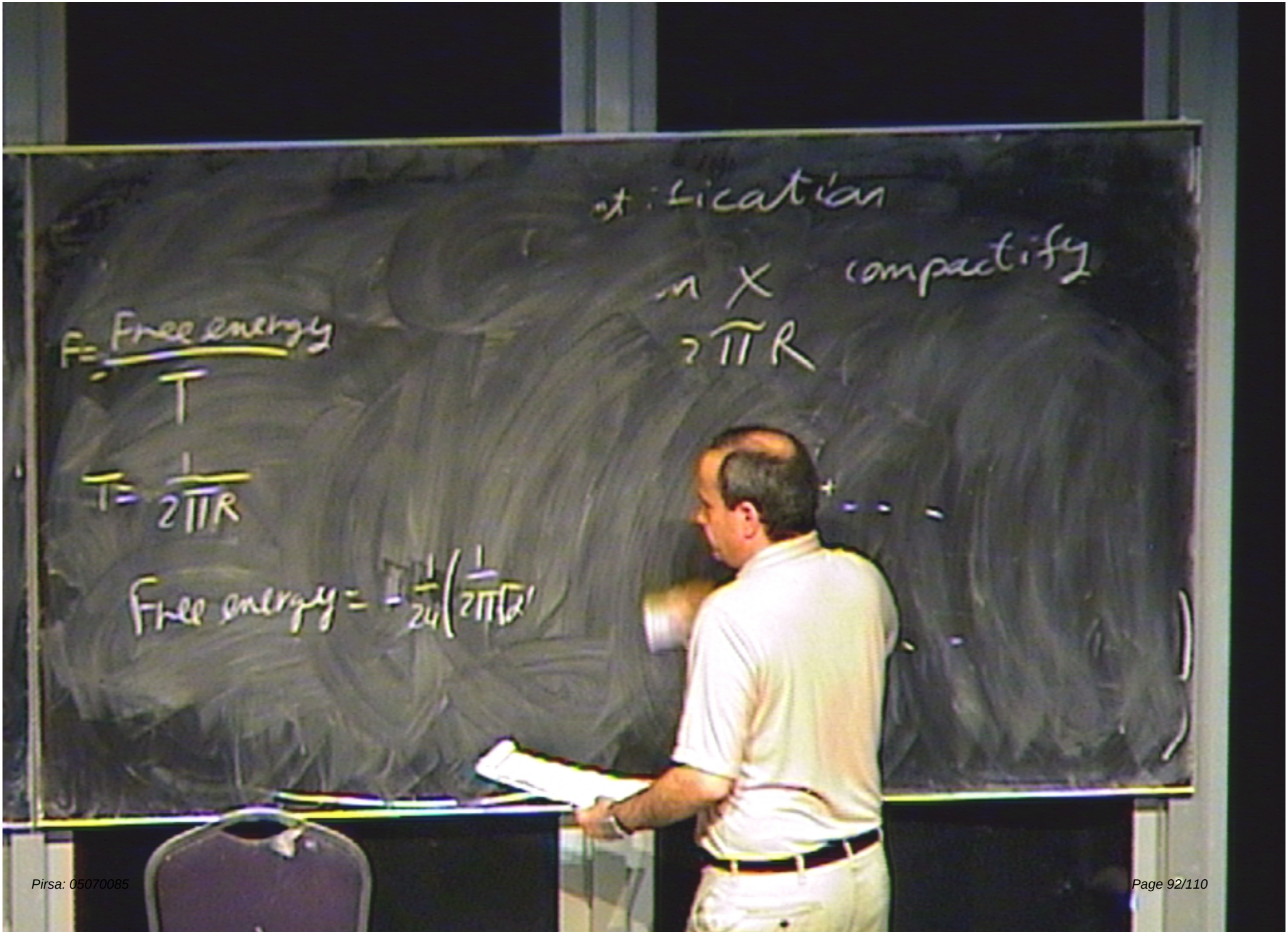
$$\sim R \int_W \; m=w=0 \; + \sum_k R \int \frac{d^2\tau}{\tau_2} \, e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$$

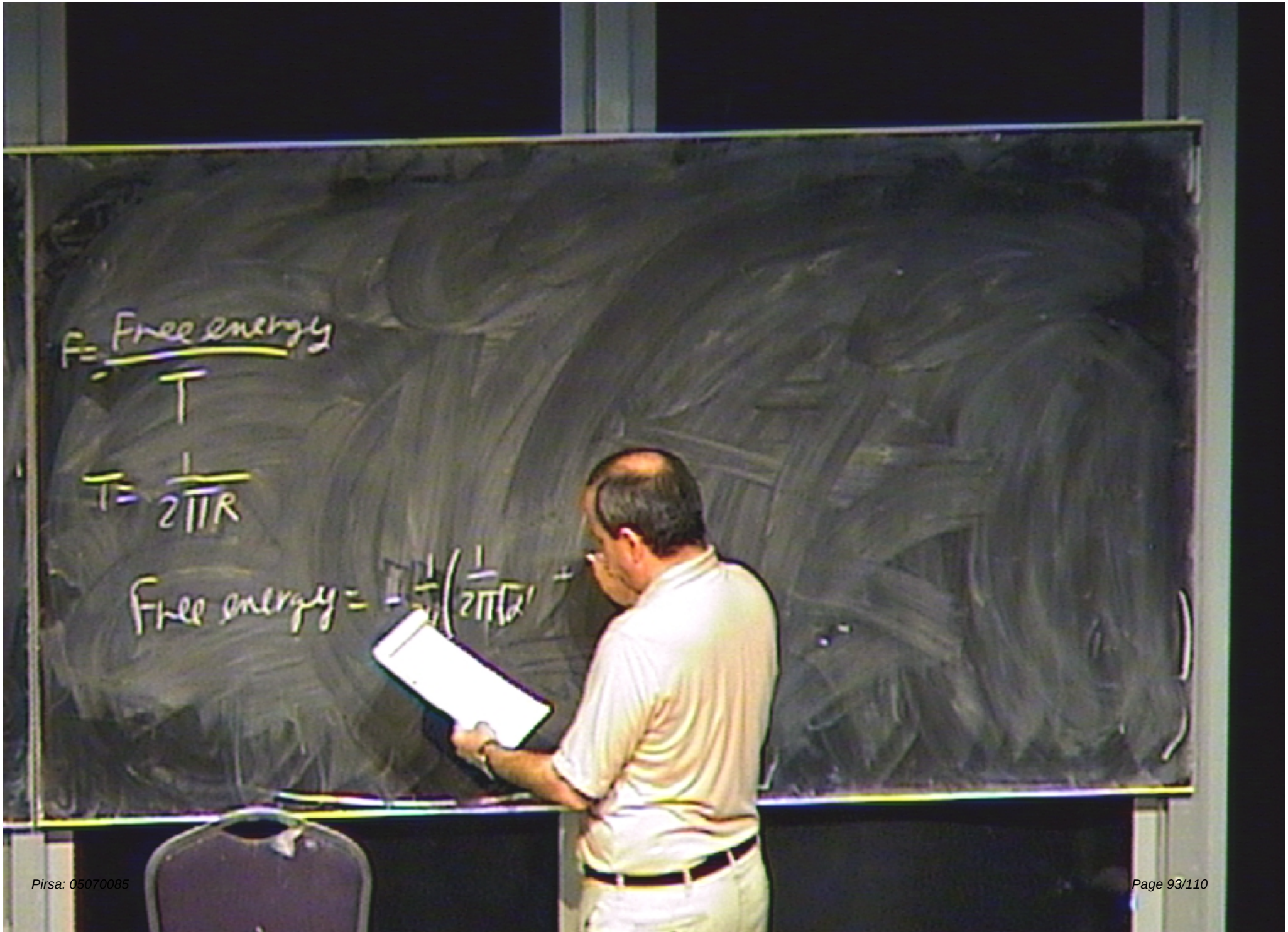
$$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right) \sim \frac{1}{R}$$

finite in field theory

∞ in field theory
finite in string


F= Free energy
T
T= 1/2πR
compactify
2πR
f1 + e^2 f2 + . . .
F1 log μ + 1/μ F2 + . . .

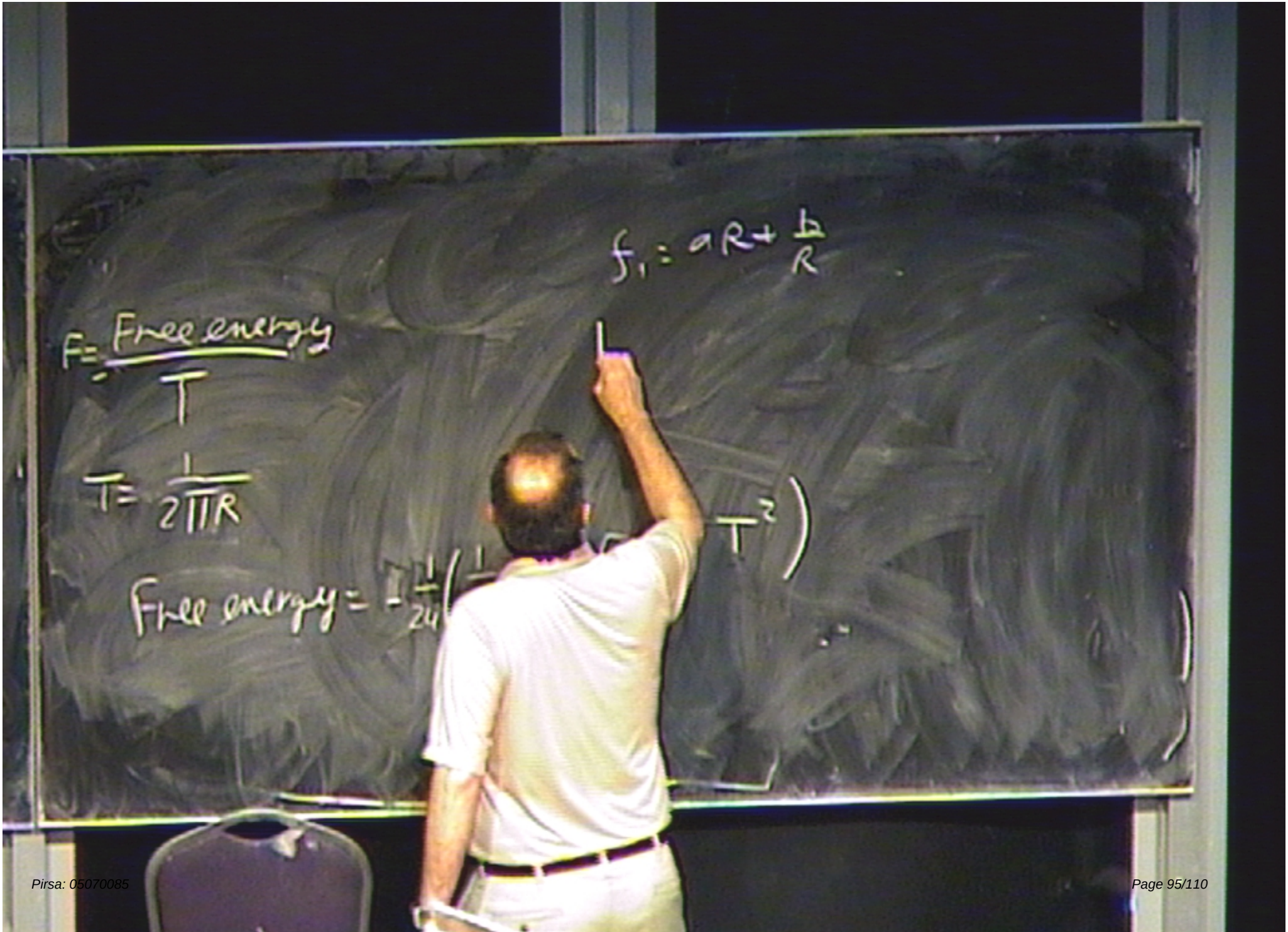

ot : Lication
n X compactify
2πR
F= Free energy
T
T= 1/2πR
Free energy = - 1/24 (1/2πR

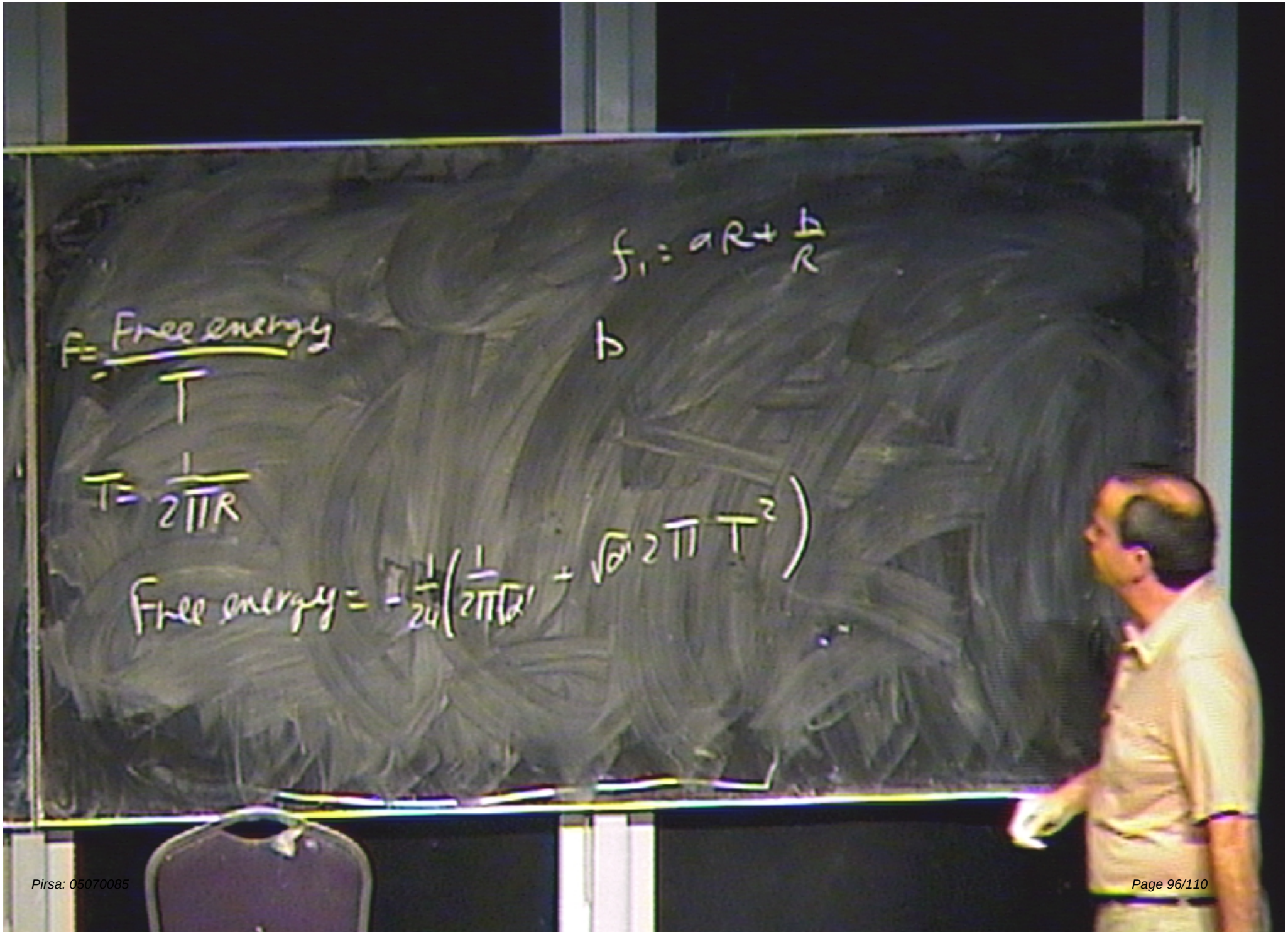

F= Free energy
T
T= 1/2πR
Free energy = -

$F = \frac{\text{Free energy}}{T}$

$T = \frac{1}{2\pi R}$

$$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi\alpha'} + \sqrt{\alpha'}\,2\pi\,T^2\right)$$


$f_1 = aR + \frac{b}{R}$
$F = \frac{\text{Free energy}}{T}$
$T = \frac{1}{2\pi R}$
Free energy = $-\frac{1}{2}\left(\frac{1}{}\right.$
T^2
$\left.\right)$


$f_1 = aR + \frac{b}{R}$
$F = \frac{\text{Free energy}}{T}$
b
$T = \frac{1}{2\pi R}$
$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi\sqrt{a}} + \sqrt{a}\,2\pi T^2\right)$

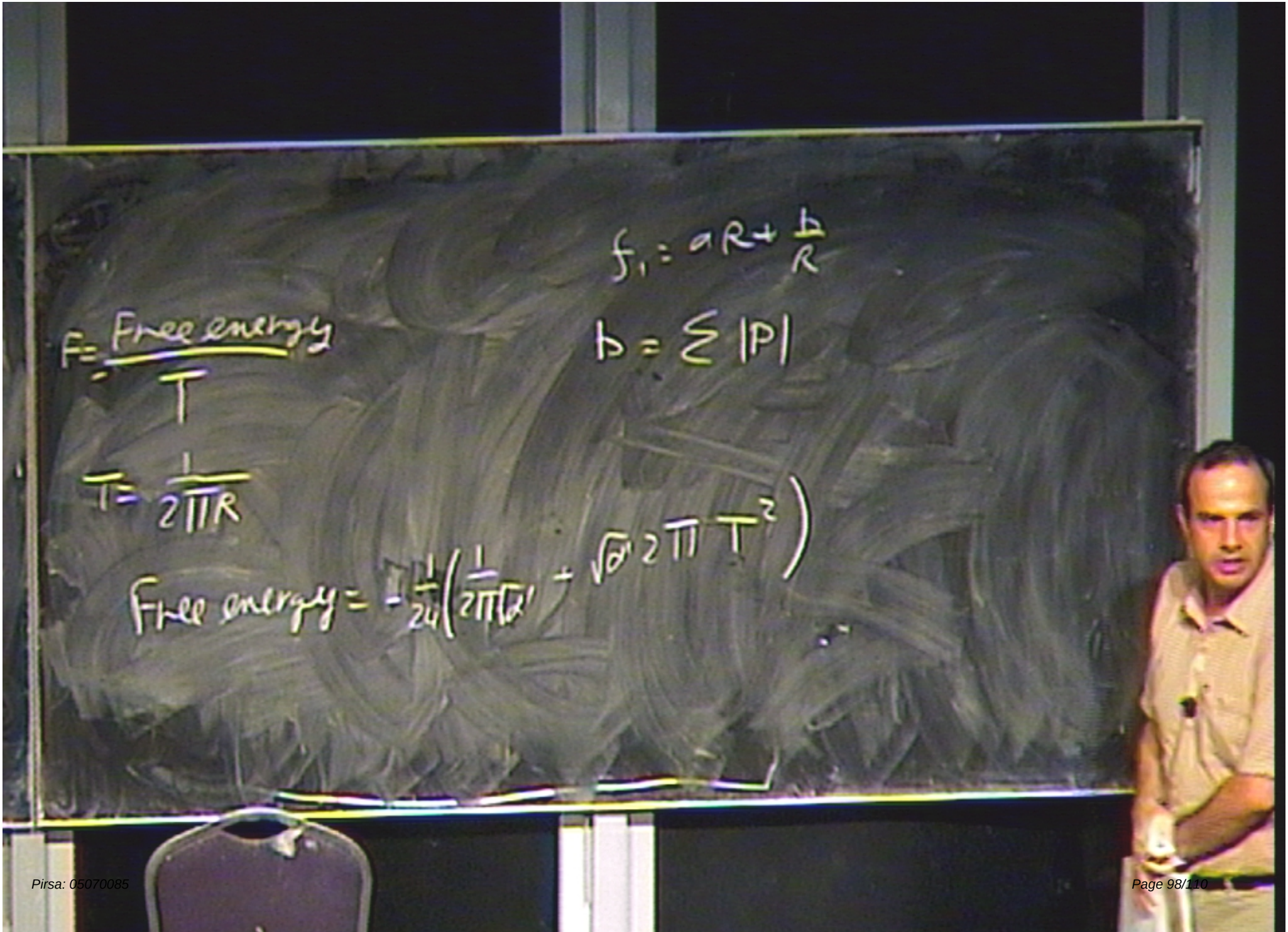
$f_1 = aR + \frac{b}{R}$

$b = \sum |P|$

$F = \frac{\text{Free energy}}{T}$

$T = \frac{1}{2\pi R}$

$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi\sqrt{\alpha'}} + \sqrt{\alpha'} 2\pi T^2\right)$


$f_1 = aR + \frac{b}{R}$
$b = \sum |p|$
$F = \frac{\text{Free energy}}{T}$
$T = \frac{1}{2\pi R}$
$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi T a'} + \sqrt{a'} 2\pi T^2\right)$

$f_1 = aR + \frac{b}{R}$

$F = \frac{\text{Free energy}}{T}$

$b = \sum |P|$

$T = \frac{1}{2\pi R}$

$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi R'} + \sqrt{a'}\, 2\pi T^2\right)$

$f_1 = aR + \frac{b}{R}$

$F = \frac{\text{Free energy}}{T}$

$b = \sum |p|$

$a = \sum |w|$

$T = \frac{1}{2\pi R}$

$\text{Free energy} = -\frac{1}{24}\left(\frac{1}{2\pi T} + \sqrt{a}\, 2\pi T^2\right)$

$(m,w) = k\,(\tilde{m},\tilde{w})$

$\downarrow$

$R(1,0)$

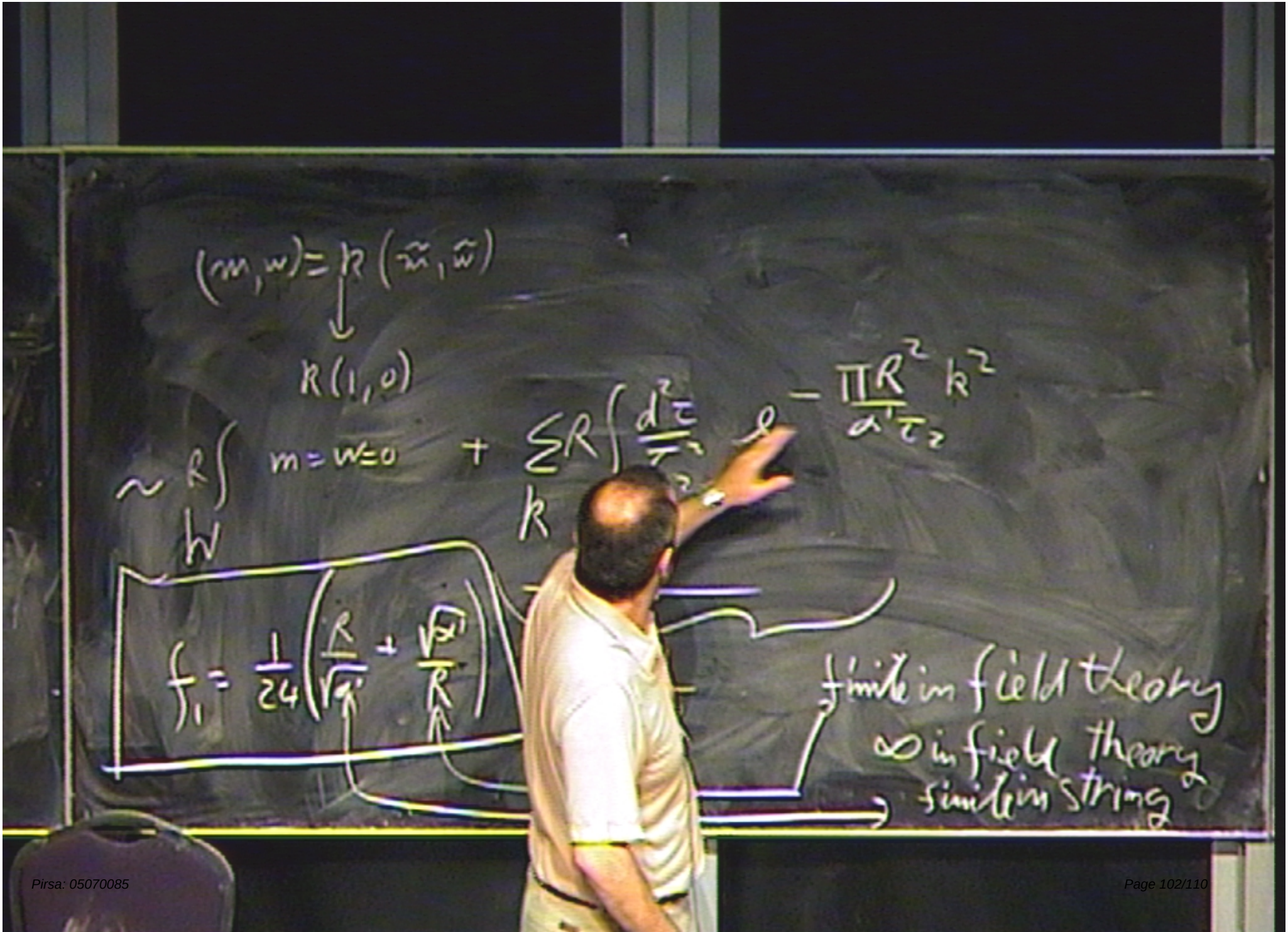
$\sim R \int_{m=w=0} + \sum_k R \int \frac{d^2\tau}{\tau_2}\, e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$

$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$

$\sim \frac{1}{R}$

finite in field theory

∞ in field theory & finite in string


$(m,w) = k(\tilde{m},\tilde{w})$
$R(1,0)$
$m=w=0$
$+ \sum_k R \int \frac{d^2\tau}{\tau_2} e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$
$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$
finite in field theory
∞ in field theory
finite in string

$(m,w) = k\,(\tilde{m},\tilde{w})$

$\downarrow$

$R(1,0)$

$m=w=0$

$\sim R \int_0^W$ $\quad + \sum_k R \int \frac{d^2\tau}{\tau_2^2}\, e^{-\frac{\pi R^2 k^2}{\alpha' \tau_2}}$

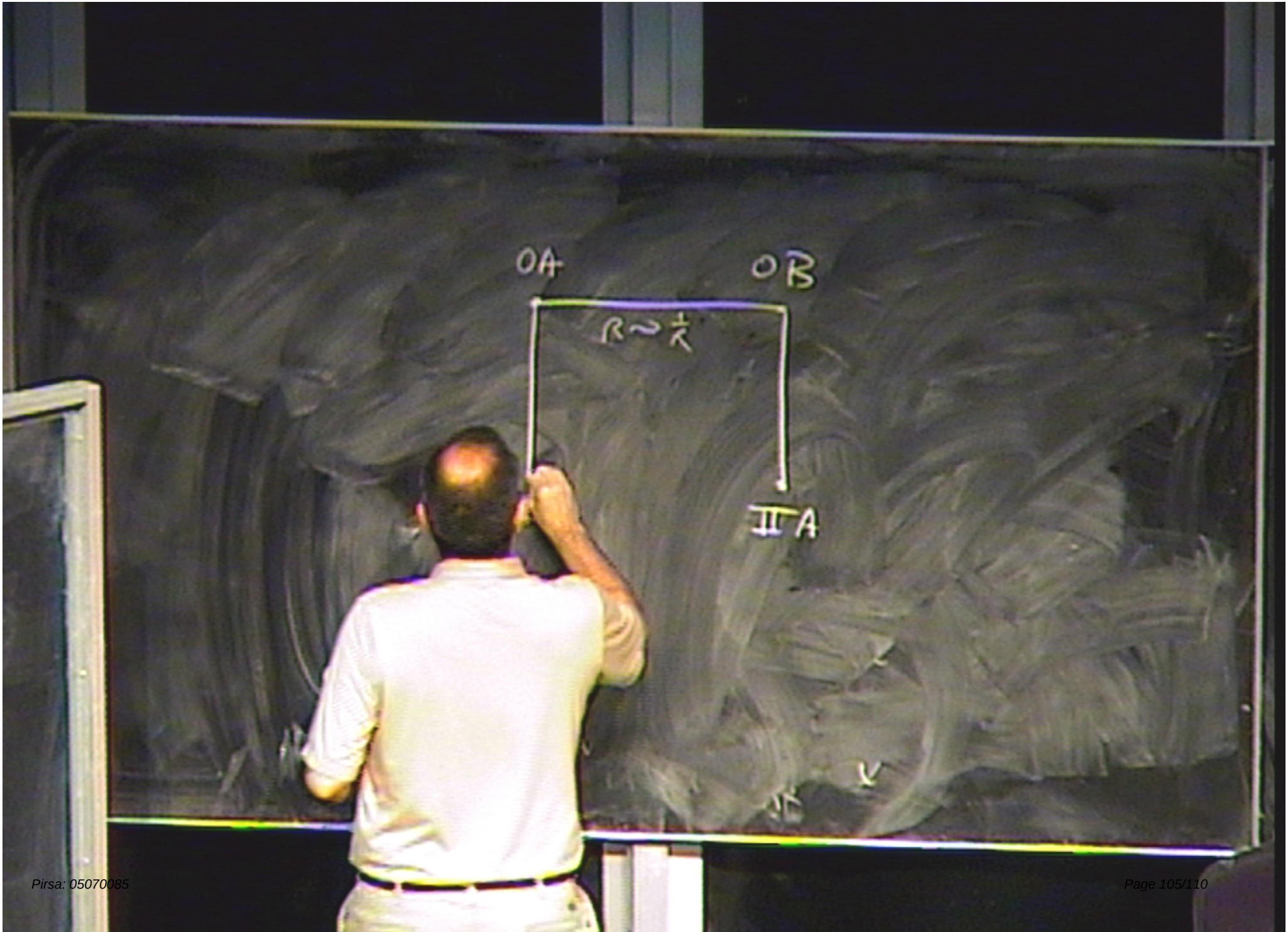
$f_1 = \frac{1}{24}\left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$

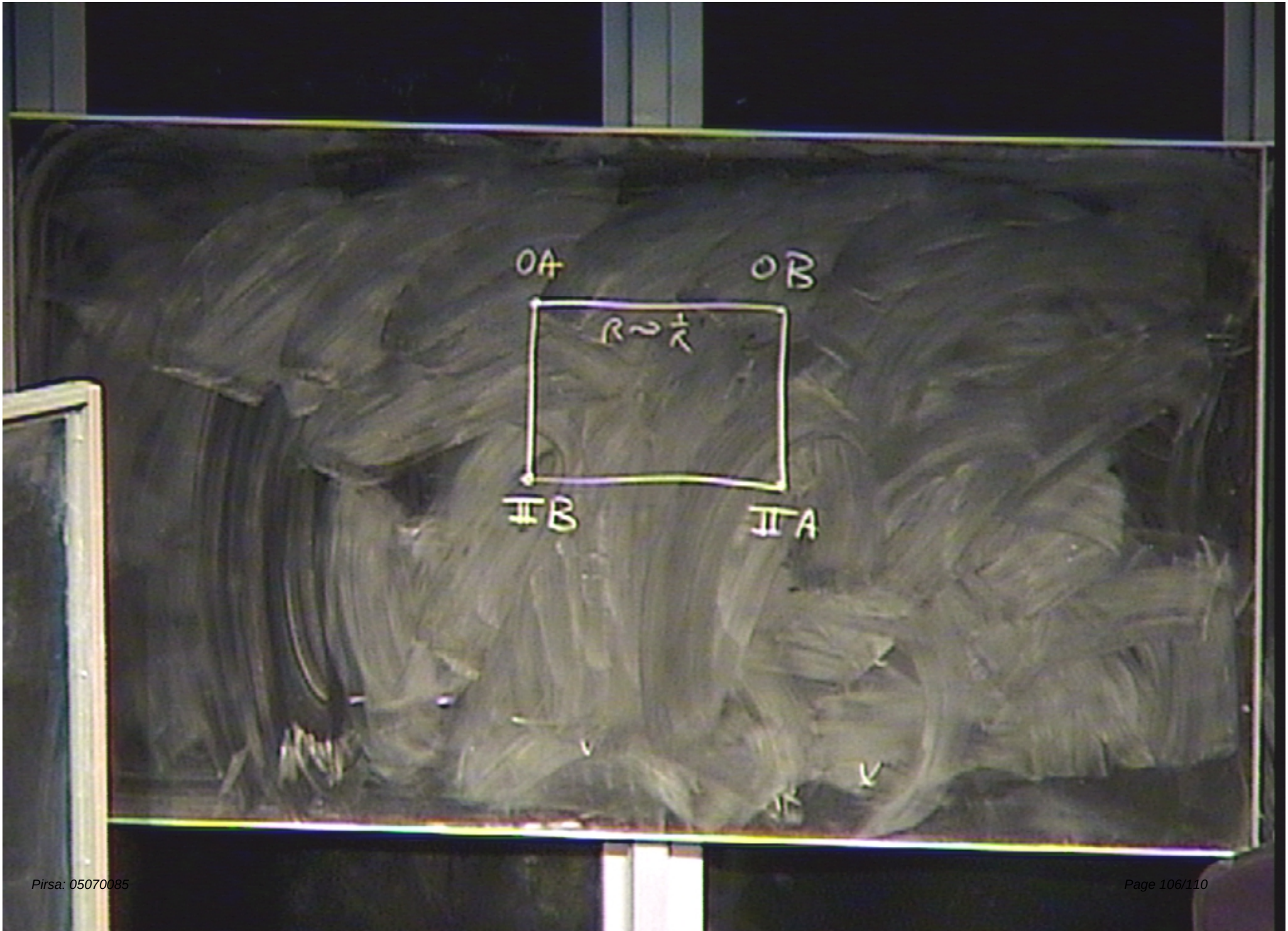
$\sim \frac{1}{R}$

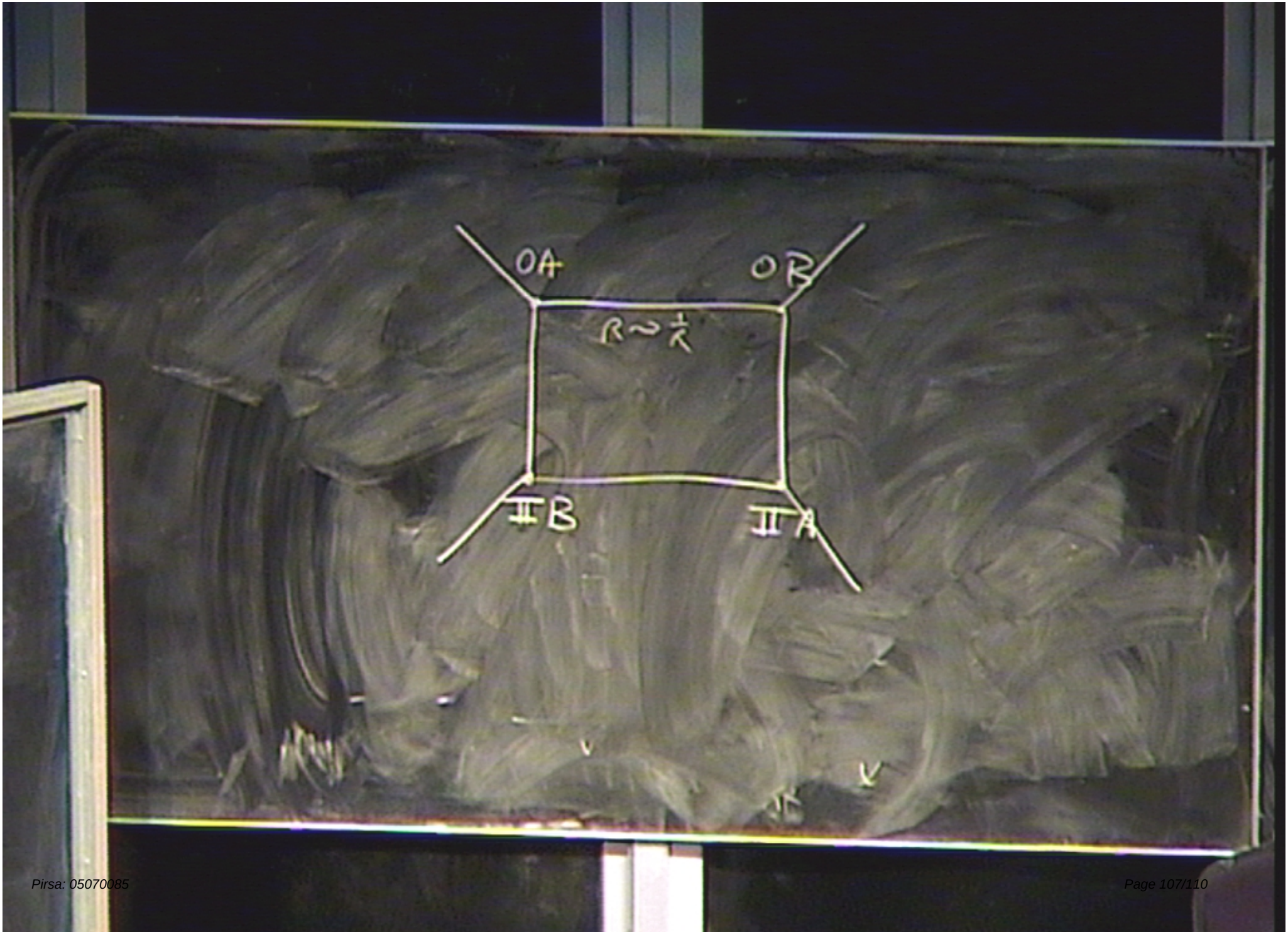
finite in field theory

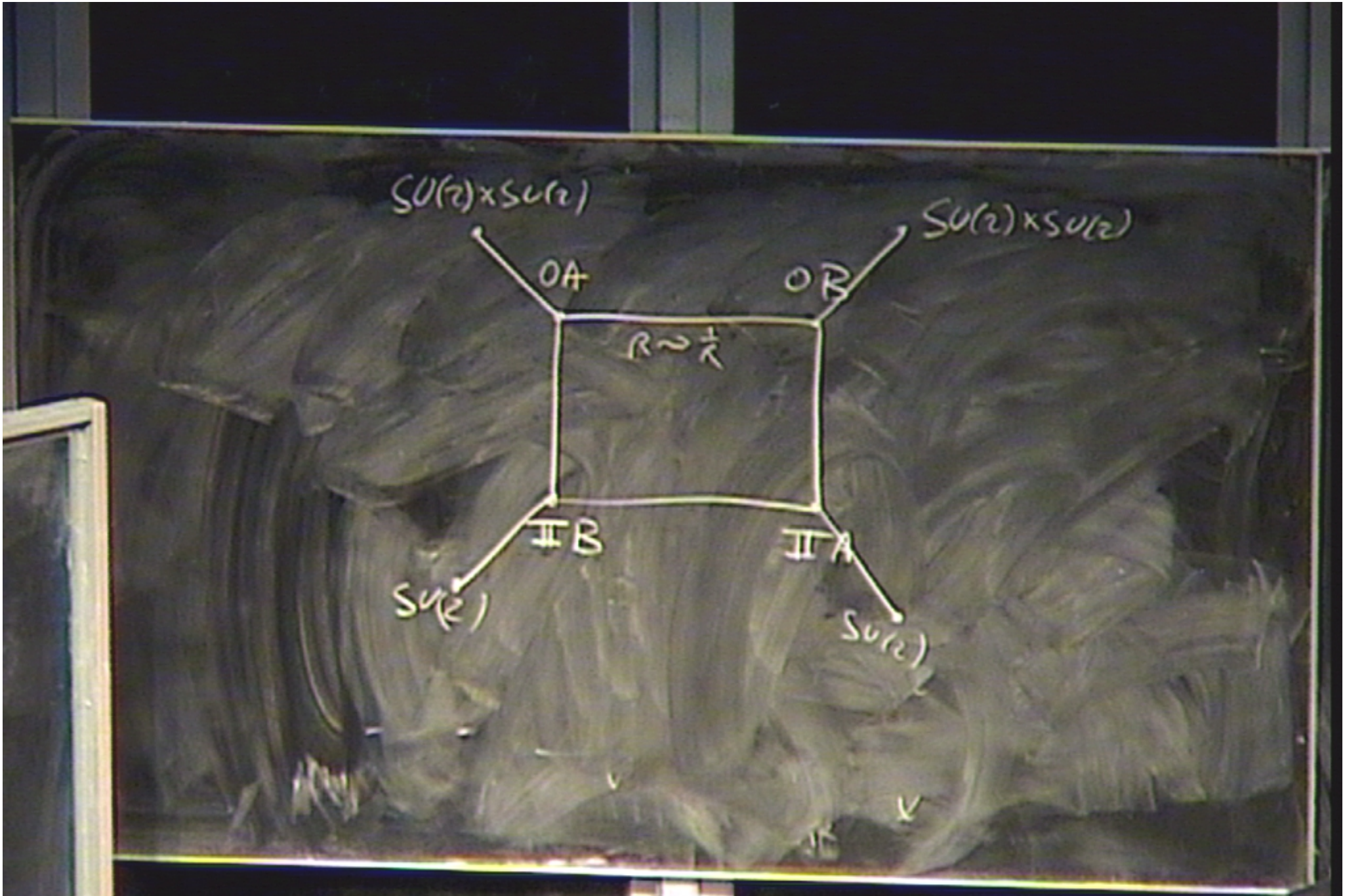
∞ in field theory
finite in string

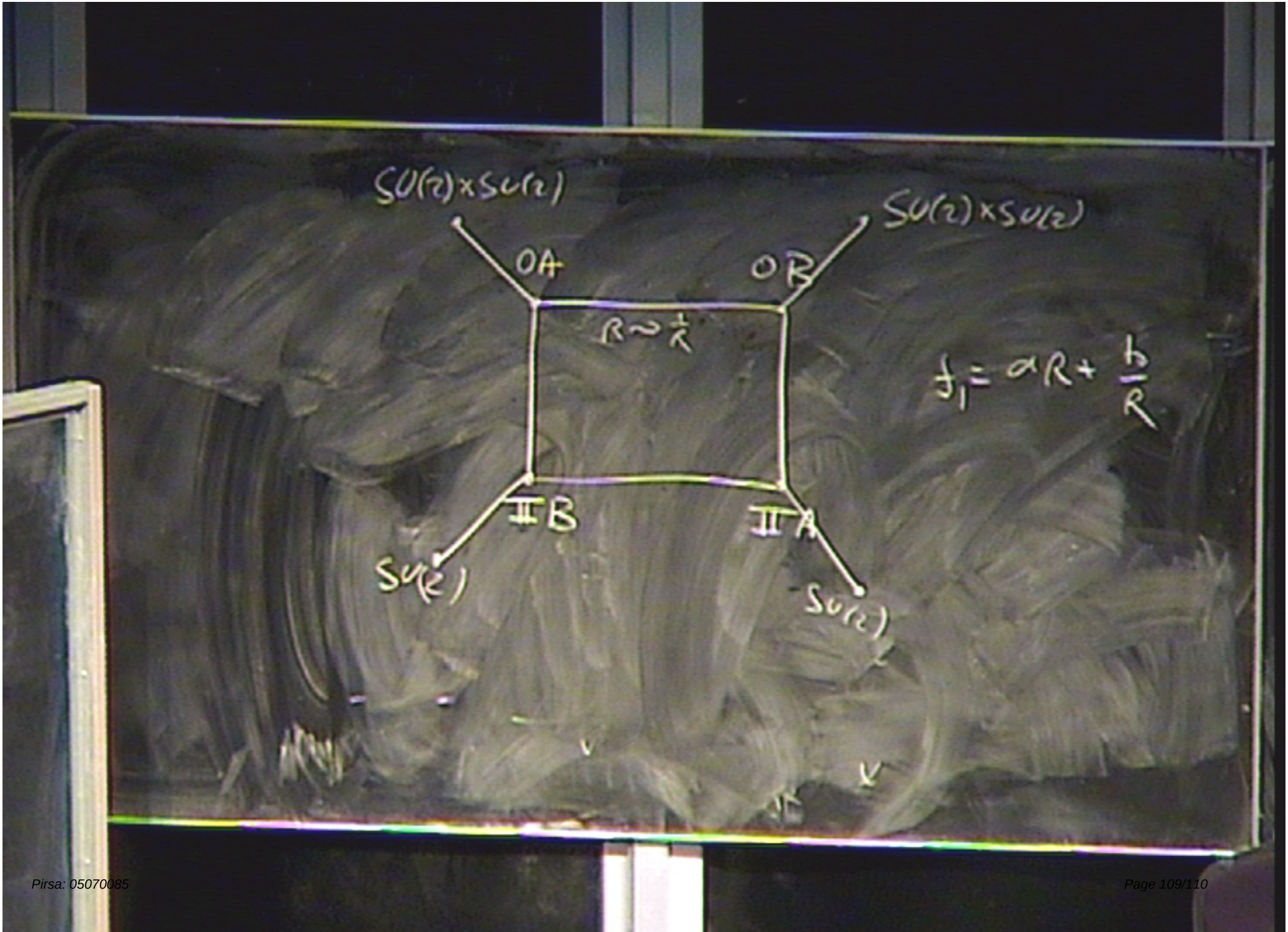

OA
OB
R~1/κ

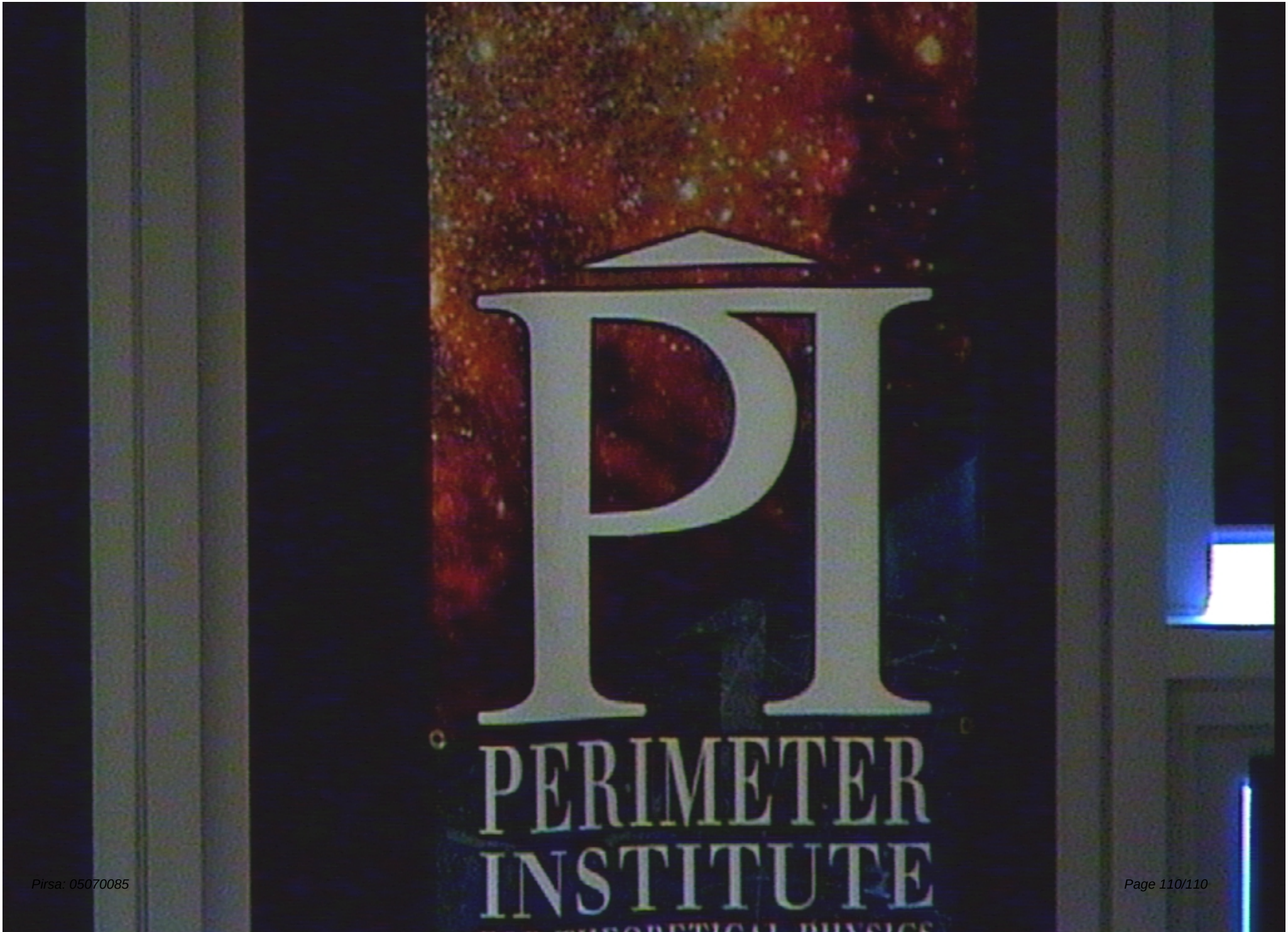

OA
OB
R~1/k
II A


OA
OB
$R \sim \frac{1}{\kappa}$
IIB
IIA


OA
OB
R~1/r
IIB
IIA


SU(2)×SU(2)
SU(2)×SU(2)
OA
OB
R~1/λ
IIB
IIA
SU(2)
SU(2)


SU(2)×SU(2)
SU(2)×SU(2)
OA
OB
R~1/r
IIB
IIA
SU(2)
SU(2)
f1 = aR + b/R


PI
PERIMETER
INSTITUTE